Does Size Matter? Bailouts with Large and Small Banks[*]

Eduardo Dávila[†] Ansgar Walther[‡]

May 2018

Abstract

We explore how large and small banks make funding decisions when the government provides system-wide bailouts to the financial sector. We show that bank size, purely on strategic grounds, is a key determinant of banks' leverage choices, even when bailout policies treat large and small banks symmetrically. Large banks always take on more leverage than small banks because they internalize that their decisions directly affect the government's optimal bailout policy. In equilibrium, small banks also choose strictly higher borrowing when large banks are present, since banks' leverage choices are strategic complements. Overall, the presence of large banks increases aggregate leverage and the magnitude of bailouts. The optimal ex-ante regulation features size-dependent policies that disproportionally restrict the leverage choices of large banks. A quantitative assessment of our model implies that an increase in the share of assets held by the five largest banks from 50% to 70% is associated with a 3.5 percentage point increase in aggregate debt-to-asset ratios (from 90.1% to 93.6%). Under the optimal policy, large banks face a "size tax" of 40 basis points (0.4%) per dollar of debt issued.

JEL numbers: G21, G28, E61

Keywords: bailouts, bank regulation, too-big-to-fail, too-many-to-fail, size tax

[*]We would like to thank Marios Angeletos, Robert Barro, Javier Bianchi, John Campbell, Emmanuel Farhi, Xavier Freixas, Xavier Gabaix, Itay Goldstein, Kinda Hachem, Ken Rogoff, Alexi Savov, Philipp Schnabl, Ricardo Serrano-Padial, Andrei Shleifer, Jeremy Stein, Alp Simsek, and Tanju Yorulmazer, as well as seminar and conference participants for helpful comments. Luke Min provided outstanding research assistance.

[†]New York University and NBER. Email: edavila@nyu.edu

[‡]University of Warwick. Email: ansgar.walther@wbs.ac.uk

A surprising number of pundits seem to think that if one could only break up the big banks, governments would be far more resilient to bailouts, and the whole "moral hazard" problem would be muted. That logic is dubious, given how many similar crises have hit widely differing systems over the centuries. A systemic crisis that simultaneously hits a large number of medium-sized banks would put just as much pressure on governments to bail out the system as would a crisis that hits a couple of large banks.

Kenneth Rogoff. All for One Tax and One Tax for All? Project Syndicate, 04/29/2010

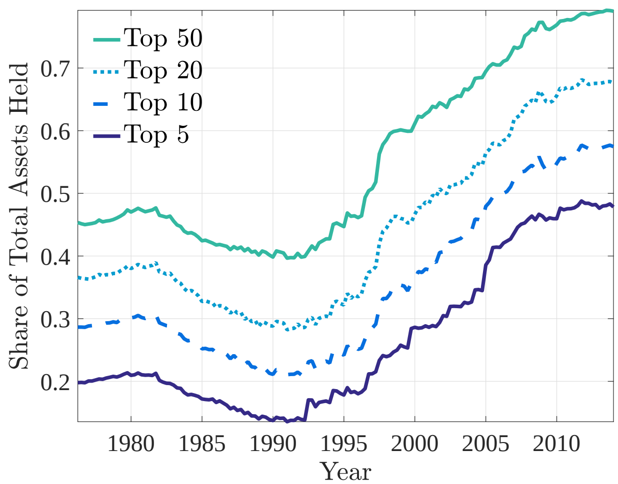
1 Introduction

The differential treatment of large financial institutions has drawn substantial interest in recent financial regulatory discussions. In particular, several regulatory measures put in place after the 2008 financial crisis have singled out large banks as subjects of increased scrutiny. At the same time, the U.S. banking industry has become increasingly concentrated, as illustrated in Figure 1. The total number of U.S. banks has dropped from 25,000 in the 1920's, to 14,000 in the 1970's, to less than 6,000 as of today, while the top 10 bank holding companies now control more than 50% of total bank assets. In parallel, an active literature has highlighted that large firms have a disproportionate impact on aggregate outcomes if they are subject to granular firm-specific shocks (Gabaix, 2011). Taken together, these developments suggest that concerns about too-big-to-fail banks are now more important than ever before.

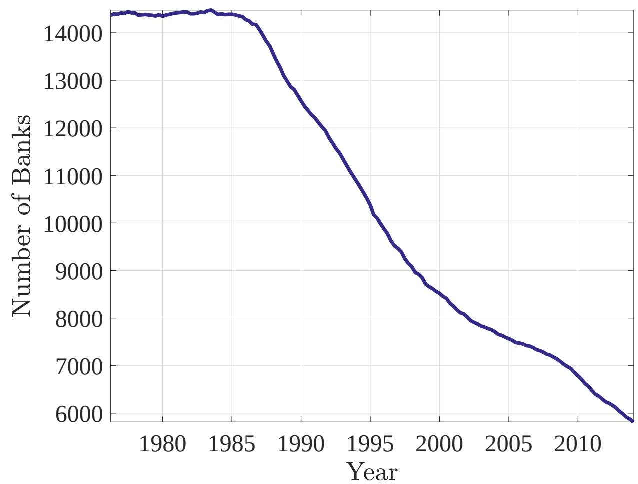
Even though the too-big-to-fail question has been a perennial subject of heated public debates – see, for instance, the forceful exposition of Stern and Feldman (2004) – the number of formal contributions to this topic remains small. In this paper, we formally study the effects of bank size on banks' funding decisions in an environment with systemic bailouts. From a positive perspective, we seek to understand whether the current levels of bank concentration have consequences for aggregate banking stability. From a normative perspective, we seek to understand whether regulators directly need to address bank concentration *per se*, or whether size-independent regulations that apply to all banks are sufficient. We address both sets of questions theoretically and provide a quantitative illustration of the mechanisms at play.

A simple example illustrates the underlying mechanism behind our results. Is the government's decision problem different when it contemplates a bailout of 10 banks of size one versus a bailout of one bank of size 10? If we assume that the losses associated with bank failure are proportional to bank size, the naive answer to this question, from an ex-post perspective, is no. This can be called the too-many-to-fail critique to the too-big-to-fail problem, or the "clones" property of bailouts.[1] The problem with this argument is that large banks are aware that their individual choices directly affect the likelihood and magnitude of a bailout, while small banks are individually unable to modify bailout policy responses. Therefore, anticipating the government's policy response and internalizing the effect

[1]See our opening quotation and the quotations reproduced in the Appendix for different formulations of this logic.



(a) Share of total assets held by largest banks



(b) Total number of U.S. banks

Figure 1: Measures of bank concentration

Note: Figure 1a shows the share of total assets held by the 5, 10, 20, and 50 largest U.S. banks in terms of assets from 1976Q1 until 2013Q4. Figure 1b shows the total number of U.S. banks each year over the same period. Note that the secular increase in concentration reaches back further. The U.S. economy had more than 25,000 banks in the 1920's; see, e.g., Davison and Ramirez (2014). Both figures are based on U.S. Call Reports data, as distributed by Drechsler, Savov and Schnabl (2016, 2017). See also Janicki and Prescott (2006) and Fernholz and Koch (2016), who study the dynamics of the distribution of bank's assets and document the sustained increase in concentration in the U.S. banking sector over the last decades. Bank concentration in other countries is even higher than in the U.S., as documented by Laeven, Ratnovski and Tong (2014).

of their size, large banks decide to be more aggressive at an ex-ante stage, increasing their leverage in equilibrium and, consequently, the likelihood of a bailout. Moreover, this effect is amplified by strategic spillovers to small banks. Aggressive leverage choices by large banks increase the implicit bailout subsidy for the banking sector as a whole. Small banks, encouraged by this shield, respond by increasing their leverage beyond what they would optimally choose in the absence of large banks. This mechanism is most salient when large banks experience granular shocks, that is, when idiosyncratic risk is not perfectly diversified within large banks. In this case, idiosyncratic shocks to a large bank become shocks to aggregate bank capital, strengthening strategic leverage incentives for large banks, spillovers among small banks and, consequently, increasing the likelihood of bailouts.

In our model, banks optimally choose their leverage trading off bankruptcy costs associated with default with costs of equity issuance, as in the canonical trade-off theory of capital structure.[2] Ex-post, to avoid bankruptcy costs, the government may find it optimal to bail out banks, and we focus our

[2] Our results remain valid under alternative theories of bank capital structure, as long as a meaningful trade-off between bank equity and debt finance remains. Our results arise from market participants anticipating system-wide bailouts, not from the specifics of the private costs and benefits of leverage.

attention on system-wide bailout policies.[3] To highlight the strategic role of bank size, we make two conservative assumptions. First, banks have constant returns to scale, so that we can rule out size effects driven by technological differences. Second, the government decides its bailout policy considering all banks equally, that is, the government's objective is simply to minimize aggregate welfare losses, regardless of whether large or small institutions generate these losses. These assumptions guarantee that a too-many-to-fail scenario, in which multiple small banks fail, can provide as strong a motivation for a bailout as the failure of a single large institution of equal size. If large banks enjoy cost advantages, or if distress in large banks are disproportionately costly, the strategic incentives that we highlight in this paper will be stronger. Our assumptions are therefore geared towards obtaining a lower bound on the relative importance of large banks.

Our first result shows that a large bank takes on more leverage than a small bank, purely on strategic grounds. Relative to a small bank, a large bank internalizes that its actions directly affect the magnitude of the government's optimal bailout, which generates an additional incentive to take on debt. Our second result establishes that small banks take on more leverage when large banks are present. This behavior arises because bank's borrowing choices are strategic complements. Intuitively, when choosing their capital structure ex-ante, banks anticipate that, ex-post, the magnitude of any bailout increases with aggregate leverage. When other banks increase their leverage, each individual bank rationally anticipates larger bailouts, which provide an effective shield against bankruptcy costs and increase the marginal value of increasing its own leverage. Thus, leverage choices become strategic complements, introducing a coordination motive among banks.[4] Because large banks have an additional incentive to take on more leverage, their presence makes small banks more willing to borrow. This strategic interaction generates amplification: Large banks take on yet more leverage in response to small banks' choices, small banks respond with a further round of increased borrowing, and so forth, until convergence. These effects arise even though bank managers do not directly exploit government bailouts. Instead, the incentive to take leverage is driven by the combination of managers' value maximizing goal with competitive capital markets, which value bank debt more generously when expected bailouts are larger.

When combined, our results about the behavior of large and small banks imply that system-wide leverage is higher in an economy in which large banks are present. Our results predict that all banks,

[3] A common example of a system-wide bailout is an asset purchase program that is accessible to all banks, such as TARP. Kelly, Lustig and Van Nieuwerburgh (2016) find direct empirical evidence consistent with market participants who expect system-wide bailouts to occur. In fact, current US legislation prohibits targeted assistance to individual banks: The Dodd-Frank act limits emergency lending programs to those with "broad-based eligibility", meaning that bailouts cannot be designed to save specific institutions.

[4] This coordination motive opens the door to multiple equilibria. Our main results do not rely on equilibrium uniqueness. Using tools from monotone comparative statics (e.g. Milgrom and Shannon, 1994), we can characterize the differential roles of large and small banks regardless of which equilibrium is selected. In our quantitative assessment, however, we find a unique equilibrium for a wide range of reasonable parameter values.

large and small, take more leverage and default more frequently, also implying that the government relies on bailouts more frequently than when banks are small. We show that an increase in bank size is associated with a leverage multiplier that increases the quantitative significance of this mechanism.[5]

After establishing the positive predictions of bank size for the behavior of banks, we characterize the optimal ex-ante bank regulation. First, we show that when the government implements a constrained efficient outcome, all banks, large and small, choose the same level of leverage. This result implies that the optimal ex-ante regulation regarding quantities, which can be implemented through binding capital requirements, is identical for large and small banks. Subsequently, we show that the optimal regulation can be equivalently implemented with size-dependent Pigouvian taxes. Large banks are charged a supplement tax on borrowing, which counteracts their incentive to increase leverage so as to maximize government subsidies. Our normative results provide a formal rationale to regulate large banks differently from small banks, simply because of their size. Our results further imply that there is a natural interaction between financial regulation and policies that directly control industry structure (i.e., antitrust policy, merger regulation, etc.).

Even though in the baseline model banks' returns are only subject to aggregate risk, we show that our results extend easily to the more realistic scenario in which banks face idiosyncratic and aggregate risk. We can parametrize in our model whether idiosyncratic shocks cancel out when small banks merge into a large bank, by appealing to a law of large numbers, or not, under a granularity hypothesis (Gabaix, 2011). Although our theoretical results remain valid in both scenarios, we show in our quantification that the granular formulation – which is the empirically plausible one – substantially amplifies the strategic forces that we study in this paper.

Although the model is stylized, it is worthwhile to provide a sense of the magnitudes that it generates when calibrated. To quantitatively assess the mechanisms studied in the paper, we illustrate the predictions of our model when selecting parameters consistent with U.S. data over the period 1990Q1 to 2013Q4. We find that our model is able to rationalize roughly half of the observed differences in leverage between large and small banks.

We use the quantitative results of the model as a laboratory to study the effects of industry concentration. We find an increasing and convex relation between the leverage choices of large and small banks and the degree of industry concentration. We show that moderate increases in industry concentration starting from the status quo, in which the top 5 largest banks hold around half of total bank's assets, are associated with substantial increases in leverage by large and small banks. In particular, an increase from 50% to 70% in the share of assets held by the 5 largest banks is associated with a 3.5 percentage point increase in aggregate debt-to-asset ratios (from 90.1% to 93.6%),

[5] The ideal experiment to test our results exploits a plausibly exogenous change in bank concentration and traces its impact on the leverage choices of all banks. It is not possible to assess our predictions using only time series correlations of concentration and leverage. For instance, while Figures 1a and 1b shows increases in concentration in the past 30 years, leverage regulation was tightened considerably over the same period as a result of the Basel Accords.

or equivalently with a 5.5 point increase in the aggregate leverage ratio (from 10.1 to 15.6), in the absence of regulation that counteracts large banks' leverage incentives.

Finally, with the goal of guiding policymakers on the magnitude of optimal corrective policies, we also compute the associated "size tax" implied by the optimal policy. Under the optimal ex-ante policy, calculated for current levels of industry concentration, large banks pay a size tax of 40 basis points (0.4%) per dollar of debt issued, over and above the Pigouvian levy that is charged to small banks. This optimal size tax increases up to roughly 60 basis points per dollar of debt issued if the share of assets held by the 5 largest banks reaches 70%.

Related Literature This paper is most closely related to the growing literature that studies the implications of bank bailouts, and other system-wide government interventions in financial crises. The core idea underlying both earlier and most recent contributions, including those of Holmstrom and Tirole (1998), Freixas (1999), Schneider and Tornell (2004), Acharya and Yorulmazer (2007), Diamond and Rajan (2012), Bianchi (2016), Keister (2016), Nosal and Ordoñez (2016), Chari and Kehoe (2016), Bianchi and Mendoza (2017), and Gourinchas and Martin (2017) is that the lack of government's commitment regarding ex-post optimal policies modifies the ex-ante behavior of banks, a phenomenon that is often described as moral hazard.

The strategic problem faced by banks in our model is most closely related to that in Farhi and Tirole (2012), who identify the strategic complementary caused by systemic, non-targeted bailouts. They refer to this behavior as collective moral hazard. While our results build on theirs, we focus on size asymmetries, while they study an environment with symmetric (small) agents. Our results extend far beyond their informal discussion of size effects.[6] Indeed, we show that bank size is relevant for bank decisions under the conservative assumption that the social consequences of bank failure (and the welfare weight on large banks) are independent of size. We therefore provide a distinct theory of how bank size influences bank decisions, which applies to a wider class of models.

Bianchi (2016) shows that non-targeted and systemic bailouts are preferred to targeted ones, since the latter exacerbate banks' ex-ante responses (moral hazard), and provides a full-fledged macroeconomic calibration. Our results highlight the importance of size asymmetries in a similar environment. Our results are also related to the growing literature on granular shocks, following Gabaix (2011).

Only a few papers refer to size asymmetries in the context of bank bailouts. Freixas (1999) rationalizes the presence of too-big-to-fail policies, through which a regulator responds more strongly to the actions of large banks, if the costs associated with bank failure are increasing in bank size. Acharya

[6] Farhi and Tirole (2012) tangentially discuss size asymmetries as follows: "In our model, breaking down banks into smaller banks would be ineffective. The set of equilibria would be unaffected. (...) This irrelevance result would break down if big banks carried a higher welfare weight than small banks per unit of investment, say because big banks' failures have bigger systemic consequences, or because the bankruptcy of a large bank is disproportionately reported in the media, creating pressure for a bailout".

and Yorulmazer (2007) formalize the too-many-to-fail argument, showing that multiple failures by small banks can generate an identical bailout response as the failure of a single large bank. By design, our environment captures the too-many-to-fail effect. In their extension with asymmetric banks, the behavior of large banks does not affect the optimal bailout policy at the margin, which is crucial for our results. Nosal and Ordoñez (2016) show that uncertainty about the ex-post bailout policy can mitigate the ex-ante effects of lack of commitment, reducing the strength of strategic complementarities and consequently reducing the incentives of banks to take on excessive risk. They show that this effect is weakest for large banks. The thorough reviews of Freixas and Rochet (2008) and Gorton and Winton (2003) do not discuss the effect of bank size on banks' funding decisions.[7]

In general, this paper belongs to the large literature that studies strategic complementarities and coordination games. In these environments, when complementarities are sufficiently strong, there exists the possibility of multiple equilibria, as emphasized by Cooper and John (1988) and Angeletos and Lian (2016). Our main results remain valid regardless of whether there is a unique equilibrium, since we exploit monotone comparative statics tools to derive results that hold in *any* equilibrium. Within that line of work, purely from a strategic viewpoint, our results are most closely related to the work by Corsetti et al. (2004), who study the effect of a large speculator in a canonical currency attack global game framework. They provide conditions under which the presence of a large agent makes coordination easier. Despite the many modeling differences between their paper and ours — their model features a binary action game with dispersed information — our paper, like theirs, solves a model with strategic complementarities with agents of different sizes. Also related is the work by Sakovics and Steiner (2012), who show that policy interventions in coordination games among small agents ought to be targeted towards players whose choices have a large impact on others' incentives. Similarly, it is optimal in our model to target macroprudential taxes at large banks, whose leverage choices generate large spillovers to other banks.[8]

Within the sizable quantitative literature, the work of Cuciniello and Signoretti (2015) – see also Corbae (2015) – studies the effects of banks size in the context the model of Corbae and D'Erasmo (2010), who provide a detailed quantitative analysis of banking dynamics in a strategic environment. Our quantitative results abstract from several of the features emphasized in this line of work to highlight the effects of system-wide guarantees.

Finally, within the small but growing empirical literature that seeks to directly quantify the effects of bank size and government guarantees, the work of Kelly, Lustig and Van Nieuwerburgh (2016) is

[7]Leaving aside the possibility of bank bailouts, there is scope to explore further how asymmetries in bank size affect bank's decisions. For instance, Hachem and Song (2017) show how large banks choose more liquid positions in an equilibrium model with endogenous interbank pricing and Egan, Hortaçsu and Matvos (2017) structurally estimate a strategic model of competition for bank deposits.

[8]The work by Drozd and Serrano-Padial (2016), which studies credit enforcement, also features heterogeneous agents in an environment with strategic complementarities. Recent work by Kacperczyk, Nosal and Sundaresan (2017) studies the effect of large players in a model of financial market trading and information acquisition.

most relevant. By comparing the price of out-of-the-money put options on individual banks with the price of identical options on a bank index, they present evidence consistent with market participants who expect collective government guarantees, which is the starting point of our theory.

Outline Section 2 describes the environment and Section 3 characterizes the equilibrium, introducing our main positive results. Section 4 conducts the normative analysis and Section 4 describes the implication of our results for policy-making. Section 5 discusses idiosyncratic risk and granular shocks. Section 6 provides a quantitative illustration of our results, and Section 7 concludes. The Appendix contains all proofs and technical derivations.

2 Environment

Our model provides a parsimonious framework to study banks' borrowing decisions in the presence of government bailouts and heterogeneity in banks' size. We first describe the economic environment and then discuss our modeling assumptions in a series of remarks.

Agents and timing There are two dates $t = \{0, 1\}$ and a single consumption good (dollar), which serves as numeraire. There is a unit mass of risk-neutral financiers with a unit discount factor and a benevolent government. Financiers hold claims in a continuum of banks, which are indexed by $i \in [0, 1]$. We denote by $\mu(A)$ the share of capital stock managed by any subset of banks $A \subset [0, 1]$. If bank i has positive point mass $\mu(i) > 0$, then we refer to it as a large bank.[9] We refer to infinitesimal banks as small banks. Without loss of generality, we normalize total assets/capital held by banks to one unit. Figure 2 illustrates the timeline of events, which we proceed to describe.

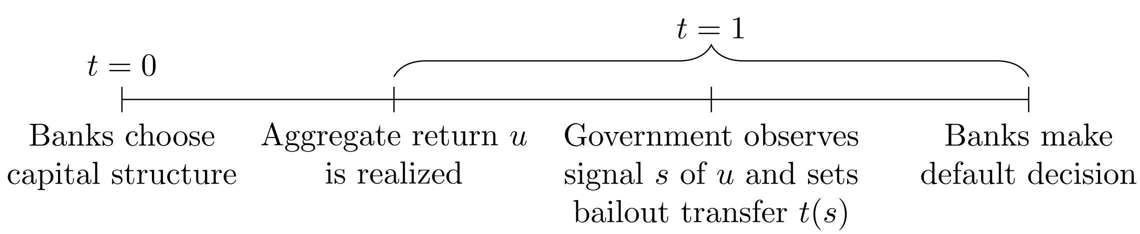


Figure 2: Timeline of events

Banks' technology and capital structure At date 0, bank i sells claims to financiers by issuing debt with face value b^i and by selling equity. Banks choose a capital structure that maximizes market value $q^i b^i + e^i$, where q^i is the market price of debt and e^i is the market value of the bank's shares.[10]

[9] More rigorously, we define the probability measure $\mu : \mathcal{B}([0, 1]) \to \mathbb{R}_+$ where $\mathcal{B}([0, 1])$ is the σ-field of Borel subsets of $[0, 1]$. For the point mass of individual banks i, we write $\mu(i)$ as shorthand for $\mu(\{i\})$ throughout the paper.

[10] Market value maximization is a standard assumption in the literature on optimal capital structure. In Appendix B.1, we provide an explicit microfoundation for this objective function in the context of our model.

Bond prices q^i and stock values e^i are endogenously determined in equilibrium. The funds raised by issuing debt and equity are used by the bank to make a fixed investment of one dollar per unit of capital at date 0.

At date 1, each bank's assets yield a random return $u \geq 0$ of date 1 dollars per unit of initial capital. The return u is common across banks and publicly observable. To clarify the exposition, we begin by analyzing the case where the aggregate shock u is the only source of uncertainty. This restriction does not affect our qualitative results. In Section 5, and in our quantitative assessment in Section 6, we additionally allow for idiosyncratic shocks to individual banks' returns.

At date 1, after u is realized and possibly after receiving a proportional government transfer $t^i \geq 0$, as described below, bank i's shareholders decide whether to default. If the bank defaults, shareholders receive nothing and creditors seize all bank's resources including government transfers and receive $\phi u + t^i$ per unit of capital, where the remainder $(1 - \phi)\, u$ measures the deadweight losses associated with default. If the bank does not default, creditors are paid b^i and shareholders receive the residual claim $(1 - \psi)\left(u + t^i - b^i\right)$ per unit of capital, where ψ captures the costs of equity issuance or tax advantages of debt. The costs of default, $1 - \phi > 0$, and of equity issuance, $\psi > 0$, imply that the Modigliani and Miller (1958) theorem does not hold. Our setup therefore mirrors the classical "trade-off theory" of capital structure (Kraus and Litzenberger, 1973; Myers, 1984). Although, to simplify our derivations, we assume that the costs of equity issuance materialize ex-post, our results easily extend to when they materialize ex-ante.

A contentious issue is whether the costs of bank equity are private or social costs. We do not take a stand in this debate and allow for either case. We therefore assume that $\bar{\psi}\left(u + t^i - b^i\right)$ dollars are reimbursed to financiers as a lump sum if the bank does not default, where $0 \leq \bar{\psi} \leq \psi$. The boundary case $\psi = \bar{\psi}$ corresponds to purely private costs, for example arising from tax considerations, while $\psi < \bar{\psi}$ implies that the private costs of bank equity issuance are also social costs.

Notice that asset values, as well as the costs of distress and equity issuance, exhibit constant returns and are independent of bank size. This allows us to define all bank-level (lower-case) variables *per unit of capital/assets*. For example, a large bank that manages a point mass $\mu(i) > 0$ share of capital issues debt with face value $\mu(i)\, b^i$, earns a return $\mu(i)\, u$ and receives $\mu(i)\, t^i$ from the government. A subset S of small banks collectively issue debt with face value $\int_S b^i d\mu$, earn returns $u\mu(S)$ and receive transfers $\int_S t^i d\mu$. We can express aggregates of bank level variables a^i as the Lebesgue integral $\int_0^1 a^i d\mu$.[11]

Government policy The existence of deadweight losses associated with bank failure provides a rationale for a benevolent government to bail out banks at date 1. Formally, we allow the government to transfer funds to banks under the following three assumptions.

[11]The Lebesgue notation is convenient since we can include large and small banks in one integral. For an illustration, suppose there is one large bank $i = 0$ with point mass $\mu(i) = \lambda$, and all $i > 0$ form a continuum of identical small banks with mass $1 - \lambda$. Then the aggregate of a variable a^i is $\int_0^1 a^i d\mu = \lambda a^0 + (1 - \lambda) \int_{i>0} a^i di$, where the second term is the standard (Riemann) integral over small banks.

First, we assume that government transfers cannot be conditioned on bank characteristics. This assumption, which we discuss in more detail below, captures the fact that some bank support policies are provided to the banking system as a whole, consistently with empirical evidence in Kelly, Lustig and Van Nieuwerburgh (2016). Second, we assume that the government must decide on the level of the bailout transfer with imperfect information about the realization of the aggregate state. Formally, the government chooses a transfer $t(s)$ contingent on a signal s about the realization of aggregate returns u. Hence, each bank i receives a transfer $t^i = t(s)$ per unit of capital. This assumption captures that bailout policies are often determined under uncertainty about fundamentals. From a formal standpoint, it guarantees that the problem solved by banks is smooth. Finally, we assume that government transfers are associated with a net deadweight loss of $\kappa(t)$ dollars, where $\kappa(t)$ is a weakly increasing and convex function that satisfies $\lim_{t\to\infty} \kappa(t) = \infty$. This assumption limits the magnitude of the optimal transfer chosen by the government. The deadweight loss of intervention can be literally interpreted as a fiscal distortion.[12]

Asset returns and government signals are drawn according to a joint distribution $F(u, s)$. We write $F_u(u|s)$ for the conditional distribution of asset values, with density $f_u(u|s)$, and $G(s)$ for the marginal distribution of signals.

We impose a mild regularity condition that prevents the conditional c.d.f. $F_u(u|s)$ from changing too quickly. Formally, we assume that the conditional density of asset returns satisfies the following condition for all marginal default states $u = b^i - t$:

$$\frac{d \log f_u(u|s)}{d \log u} = \frac{u f_u'(u|s)}{f_u(u|s)} > -1, \quad \forall s. \tag{1}$$

Intuitively, enough noise is necessary to preserve the smoothness of the capital structure problem faced by banks. As we show in the Appendix, when signals are normally distributed, a high enough variance for the signal s is a sufficient condition for Equation (1) to hold over the relevant region.

Equilibrium definition An equilibrium is defined as a set of bank capital structure decisions b^i and default decisions, prices for bank debt q^i and equity e^i, and a government bailout policy $t(s)$, such that (i) banks maximize their market value net of issuance costs, given the behavior of other banks and the government, (ii) financiers break even when purchasing debt or equity, and (iii) the government maximizes ex-post welfare at date 1 given their signal.

Remarks on the environment

Our assumptions are geared towards studying an environment in which banks have endogenous funding and default choices and government policies arise endogenously from a well-defined government's objective. Some of the assumptions deserve further discussion.

[12] A broader interpretation of $\kappa(t)$ not only captures the slack of the fiscal capacity of the government, but it can also capture the "type" of the government: hawkish vs. dovish, as in Diamond and Rajan (2012), among others.

Remark 1: Differences between large and small banks. By design, in this paper, the differences between large and small banks are strategic rather than technological. Banks' technology features constant returns to scale and government bailouts are not targeted at large banks. This allows us to speak to the "too many to fail" argument. In this paper, whether a single large bank or many small banks are in distress is irrelevant *ex-post.*[13]

We do not intend to take constant returns to scale literally. In the context of our paper, identical technologies are a conservative assumption and allow us to derive a lower bound on the difference between large and small banks.[14] Indeed, we argue that the presence of large banks renders the financial system riskier. If policy-makers provided implicit guarantees to large banks but not to small ones – as would occur, for example, if we allowed the marginal cost of default $(1 - \phi)$ to be bank specific and increasing in bank size – this effect would be strengthened. Similarly, all banks in our setup choose their leverage simultaneously, so that large banks have no timing advantage. This assumption removes herding and signaling effects in the presence of large banks, which would also strengthen the strategic importance of large banks (Corsetti et al., 2004). Our key point is, therefore, that policy bias or timing advantages are not necessary for large banks to be important: size matters purely for strategic reasons.

We focus on the role of size as the single dimension of heterogeneity across banks. Banks may differ on their degree of interconnectedness in the form of network centrality, or in richer dimensions regarding liquidity risk or maturity mismatch. While these are important factors, they are unlikely to change our main results, since ex-post government responses continue to induce strategic complementarities even when there is additional heterogeneity. Ceteris paribus, the presence of large banks then continues to lead to increased risk taking.

In this paper, the distribution μ of bank size and the scale of each bank's investment are taken as primitives of the model. Similar results would obtain in an extension of our model in which banks' technologies feature decreasing returns to scale. Under decreasing returns to scale, technological differences would induce an endogenous size distribution of banks. Banks which are large for technological reasons in that case would behave as large banks in our setup, and moreover, strategic differences between large and small banks would affect scale as well as leverage decisions. We focus on the case with constant returns and an exogenous size distribution because it allows us to directly address the "too many to fail" argument, which has dominated the public debate.

[13] This assumption is in line with recently developed empirical metrics of systemic risk, such as CoVaR (Adrian and Brunnermeier, 2016). CoVaR is defined so that many small banks generate the same measure of ex-post distress as one large bank, holding constant their asset exposures and leverage. Adrian and Brunnermeier (2016) further show that bank size predicts high contributions to CoVaR in the U.S. data. This is consistent with the equilibrium prediction of our model, since larger banks choose higher leverage and contribute more to systemic risk.

[14] The empirical evidence on technological size effects is mixed, but overall suggests that large banks have weakly lower funding or operating costs than small ones (e.g. Gandhi and Lustig, 2015; Minton, Stulz and Taboada, 2017).

Remark 2: Systemic bailouts and noisy signals. Bailouts in our model are systemic and not targeted. This assumption is relevant in the sense that it generates strategic complementarities among banks. Of course, not all bailout policies in reality are system-wide. Targeted transfers can be introduced to the model, and our qualitative results remain valid as long as government policy is not *perfectly* targeted. It is well-established that imperfect targeting can be an optimal arrangement when governments lack commitment. Indeed, perfectly targeted bailouts exacerbate moral hazard ex-ante (Bianchi, 2016), and force the government to provide distortive information rents to banks if it does not perfectly observe bank-level performance (Farhi and Tirole, 2012).

Empirically, the evidence in Kelly, Lustig and Van Nieuwerburgh (2016) strongly rejects perfect targeting of bailout assistance. The major bailout policies in the crisis of 2008, which consisted of asset purchase and lending programs such as TARP and TALF, were accessible to all banks and therefore systemic. The interpretation of ex-post government intervention can also be broader, and strategic complementarities arise naturally if the government could intervene by lowering interest rates below their otherwise optimal level (a policy often referred to as the "Greenspan put").

Finally, even if we allowed transfers to be targeted *ex-post*, it is not obvious that policy is anticipated perfectly *ex-ante*. If a large bank A fails and the government decides to conduct a targeted bailout, it can either provide a subsidy directly to bank A or a subsidy to bank B in order to purchase A's assets. The various proposals tabled before the failures of Lehman Brothers and Bear Stearns in 2008 are a case in point. If it is unclear which exact policy option the government will pursue *ex-post*, strategic complementarities will remain *ex-ante*. Indeed, ex-ante ambiguity may again be an optimal arrangement: Nosal and Ordoñez (2016), for example, argue that constructive ambiguity is valuable when governments conduct bailouts but lack commitment.[15]

Remark 3: Banks' capital structure and institutional setting. Although we have adopted a classic trade-off theory formulation, in which bankers have a meaningful source of outside equity, to determine bank funding choices, alternative assumptions that generate a well-defined capital structure decision do not modify the insights of the paper. For example, the trade-off between bank debt and equity could be driven by concerns about moral hazard among shareholders, a desire to create "money-like" and information-sensitive claims (Gorton and Pennacchi, 1990; Stein, 2012; DeAngelo and Stulz, 2015), or concerns about bank runs and market discipline as in Diamond and Rajan (2001). Our results merely rely on the presence of strategic complementarities, which arise from the non-targeted bailout, with heterogeneity in bank sizes.

Furthermore, the financial institutions in our model need not be interpreted as traditional banks. Indeed, they could represent any segment of the financial sector and the "shadow banking" system,

[15] The fact that mainly small banks failed in 2008 is not inconsistent with our findings: When most banks are about to fail, a bailout materializes but, despite the bailout, some fraction of small banks with bad idiosyncratic shocks is still likely to fail.

as long as widespread defaults cause social costs and induce the government to provide support. This observation also motivates our choice to abstract from (insured or uninsured) demand deposit contracts and focus on leverage choices more generally. More generally, the results of this paper can be adapted to other sectors of the economy in which government bailouts occur.

Finally, note that bank managers in our model act as a veil and are solely motivated by the maximization of bank value, or equivalently, by the desire to reduce banks' cost of funds. Therefore, our results do not assume that bank managers are ill-intentioned or that they directly have to form expectations of bailout probabilities and government policy, it is sufficient if market participants do so.

3 Equilibrium characterization

We characterize the equilibrium of the model in multiple steps. First, we characterize the ex-post optimal government policy. Next, we determine banks' default decisions at date 1 and their date 0 market value for a given government policy. Finally, we study banks' ex-ante funding decisions at date 0.

Ex-post optimal bailout policy

At date 1, the government observes the signal s of asset values and chooses transfers t to maximize expected social welfare. Given banks' borrowing choices $\boldsymbol{b} = \left(b^i\right)_{i=0}^{1}$, a choice of transfer t, and a signal s, we let $W_1(\boldsymbol{b}, t|s)$ denote expected social welfare from the perspective of date 1.

In our model, date 1 welfare corresponds to the sum of aggregate resources:

$$\begin{aligned} W_1\left(\boldsymbol{b}, t|s\right) = {} & \mathbb{E}\left[u|s\right] - \kappa\left(t\right) \\ & - \left(1-\phi\right) \int \Pr\left[\mathcal{D}^i|s\right] \mathbb{E}\left[u|\mathcal{D}^i, s\right] d\mu \\ & - \left(\psi - \bar{\psi}\right) \int \Pr\left[\mathcal{N}^i|s\right] \mathbb{E}\left[t + u - b^i|\mathcal{N}^i, s\right] d\mu. \end{aligned} \tag{2}$$

where $\mathcal{D}^i$ denotes the event of default by bank i, and $\mathcal{N}^i$ is the complementary non-default region. The interpretation of Equation (2) is intuitive. The first line captures the present value of banks' assets and deadweight costs of taxation. The second and third lines respectively measure the deadweight costs of bank failure and equity issuance. Note that the latter accounts for the possibility that the costs of equity are partly or exclusively private costs when $\bar{\psi} > 0$.

Hence, the government's optimal bailout policy after observing the signal s is to maximize expected welfare. It corresponds to

$$t\left(\boldsymbol{b}|s\right) = \arg\max_{t \geq 0} W_1\left(\boldsymbol{b}, t|s\right). \tag{3}$$

The optimal bailout policy $t(\boldsymbol{b}|s)$ is characterized by the following first-order condition:

$$(1-\phi)\int f_u\left(b^i - t|s\right)\left(b^i - t\right) d\mu \leq \kappa'(t) + \left(\psi - \bar{\psi}\right)\int \Pr\left[\mathcal{N}^i|s\right] d\mu, \tag{4}$$

which is satisfied with equality when the optimal t is strictly positive. Intuitively, the left-hand side in Equation (4) measures the marginal benefit of transfers, which equals the marginal reduction in default costs. The right-hand side measures the marginal cost of the bailout which consists of two terms. First, the government incurs the direct marginal cost of taxation $\kappa'(t)$. Second, an indirect cost arises when a bailout is conducted but the bank remains solvent. In this case, the bailout constitutes a transfer to shareholders, who derive a lower marginal utility from this transfer due to the social costs $\left(\psi - \bar{\psi}\right)$ of equity.[16] This second term only matters whenever bank i is solvent, which occurs with conditional probability $\Pr\left[\mathcal{N}^i|s\right]$. In the empirically relevant region where public news s are bad, this probability is low, and therefore bailouts are driven largely by the trade-off between the benefit of preventing bank failure and the cost of taxation.

The government's optimal response illustrates the key difference between small and large banks. A small bank's choice b^i has no impact on the integral in (4), and therefore no impact on ex-post optimal transfers. But if bank j is large – with point mass $\mu(j)$ – and the government chooses a strictly positive transfer, $t(\boldsymbol{b}|s) > 0$, after receiving a signal s, a marginal change in the large bank borrowing position b^j increases the optimal transfer, since

$$\frac{\partial t(\boldsymbol{b}|s)}{\partial b^j} \overset{\text{sign}}{=} \left.\left[f_u'\left(b^j - t|s\right)\left(b^j - t\right) + f_u\left(b^j - t|s\right) + \frac{\psi - \bar{\psi}}{1-\phi} f_u\left(b^j - t|s\right)\right]\right|_{u = b^j - t(\boldsymbol{b}|s)} > 0, \tag{5}$$

where the inequality is implied by our regularity condition. The formal derivation of (5) is in the Appendix, but the economics are clear. If a large bank takes more leverage, all else equal, the failure of this bank becomes more likely after adverse realizations of the public signal s. Therefore, to reduce the deadweight losses associated with bank failure, the government decides in favor of a larger bailout.

Banks' default decision and date 0 market value

At date 1, given a realization of banks' aggregate return u and after the government chooses a bailout policy $t(\boldsymbol{b}|s)$, a bank i defaults on its debt whenever $u + t(\boldsymbol{b}|s) < b^i$, and repays otherwise. As expected, holding u and $t(\boldsymbol{b}|s)$ constant, the likelihood of default increases with the level of bank borrowing b^i.

Formally, the payoff to debtholders corresponds to

$$\begin{cases} b^i, & \text{if } u + t(\boldsymbol{b}|s) \geq b^i \\ \phi u + t(\boldsymbol{b}|s), & \text{if } u + t(\boldsymbol{b}|s) < b^i, \end{cases} \tag{6}$$

[16] This indirect cost is absent if we assume that the costs of equity issuance are sunk at date 0. Assuming that these costs are paid ex-post clarifies the exposition, but does not materially affect our qualitative results.

while the payoff to shareholders corresponds to

$$\begin{cases} (1-\psi)\left(u+t\left(\boldsymbol{b}|s\right)-b^i\right), & \text{if } u+t\left(\boldsymbol{b}|s\right)\geq b^i \\ 0, & \text{if } u+t\left(\boldsymbol{b}|s\right)<b^i. \end{cases} \tag{7}$$

In equilibrium, because financiers are risk-neutral, the market value of bonds $q^i b^i$ is determined by the expected payoff to debtholders, while the market value of bank shares e^i is determined by the expected payoff to shareholders.

Therefore, taking expectations at date 0, the market value of a bank given a level of borrowing b^i and a set of possible transfers $t\left(\boldsymbol{b}|s\right)$ for each signal realization s, corresponds to

$$\begin{aligned} V\left(b^i, t\left(\boldsymbol{b}|s\right)\right) &\equiv q^i b^i + e^i \\ &= \mathbb{E}\left[u+t\left(\boldsymbol{b}|s\right)\right] - (1-\phi)\Pr\left[\mathcal{D}^i\right]\mathbb{E}\left[u|\mathcal{D}^i\right] - \psi\Pr\left[\mathcal{N}^i\right]\mathbb{E}\left[u+t\left(\boldsymbol{b}|s\right)-b^i|\mathcal{N}^i\right], \end{aligned} \tag{8}$$

where the optimal default and non-default regions are defined as

$$\begin{aligned} \mathcal{D}^i &= \left\{(u,s)|\, t\left(\boldsymbol{b}|s\right)+u<b^i\right\}, \\ \mathcal{N}^i &= \left\{(u,s)|\, t\left(\boldsymbol{b}|s\right)+u\geq b^i\right\}. \end{aligned}$$

Note that expectations and probabilities in Equation (8) account for the realizations of the return u and signal s received by the government. The value function $V\left(b^i, t\left(\boldsymbol{b}|s\right)\right)$ is therefore defined in terms of *all* possible realizations of these variables and of the corresponding government policy $t\left(\boldsymbol{b}|s\right)$. A detailed derivation of this function is in the Appendix. Equation (8) highlights that banks' leverage decisions are driven by competitive market forces. Bank managers do not directly benefit from government bailouts, and their objective function simply corresponds to their firm's market value at date 0. Nevertheless, markets generate implicit incentives to capture government bailouts, because the implicit subsidy is accounted for in security prices.

Equation (8), which effectively corresponds to the banks' objective function, further clarifies our previous remark on the difference between large and small banks. From a technological perspective, both large and small banks face an identical optimization problem. Large banks are different only in a strategic sense; unlike small banks, they expect their leverage choices to directly impact bailout policies $t\left(\boldsymbol{b}|s\right)$ in future states of the world. Equation (8) also highlights that banks' decisions are determined through classic trade-off theory: Only its second element, which corresponds to costs of distress, and its third element, which corresponds to the costs of equity issuance, are affected directly by the choice of b^i for a given value of $t\left(\boldsymbol{b}|s\right)$.

Banks' leverage choices

We now turn to banks' incentives when choosing their borrowing level b^i at date 0. Bank i's problem is

$$\max_{b^i\geq 0} V\left(b^i, t\left(\boldsymbol{b}|s\right)\right), \text{ subject to (3).} \tag{9}$$

The bank takes as given others' choices $(b^j)_{j\neq i}$, and realizes that the government will respond to the collective choice $\boldsymbol{b}$ according to Equation (3). Then, it chooses its leverage b^i to maximize the joint value of debt and equity.

The first-order condition for a small bank i, which takes the government policy $t(\boldsymbol{b}|s)$ as given, is

$$\frac{\partial V(b^i, t(\boldsymbol{b}|s))}{\partial b^i} = \psi \Pr\left[\mathcal{N}^i\right] - (1-\phi) \int\limits_{\partial \mathcal{D}^i} u dF = 0. \tag{10}$$

where $\partial D^i = \{u, s | b^i = t(\boldsymbol{b}|s) + u\}$ is the boundary of the default region.[17] This expression is familiar from the canonical trade-off theory of capital structure. The first term captures the deadweight costs of equity issuance, which encourage higher leverage and arise whenever the bank is in the solvent region $\mathcal{N}^i$. The second term measures the deadweight costs of default, which discourage higher leverage.

In addition to these terms, large banks internalize the indirect effect of its leverage on the optimal bailout $t(\boldsymbol{b}|s)$, as characterized in Equation (5). Therefore, the first-order condition for a large bank j is to set the *total* derivative of its value function equal to zero:

$$\begin{aligned}\frac{dV\left(b^j, t(\boldsymbol{b}|s)\right)}{db^j} = & \frac{\partial V\left(b^j, t(\boldsymbol{b}|s)\right)}{\partial b^j} \\ & + \mathbb{E}\left[\frac{\partial t(\boldsymbol{b}|s)}{\partial b^j}\left(1 - \psi \mathbf{1}\left(\mathcal{N}^j\right)\right)\right] + (1-\phi) \int\limits_{\partial \mathcal{D}^j} \frac{\partial t(\boldsymbol{b}|s)}{\partial b^j} u dF = 0,\end{aligned} \tag{11}$$

where $\mathbf{1}\left(\mathcal{N}^j\right)$ denotes an indicator function for the non-default states of bank j. Comparing (10) and (11), it follows that *ceteris paribus,* a large bank has strictly greater incentives to take leverage than a small bank, measured by the two terms on the second line of (11). First, increasing leverage strictly increases the expected government transfer $\mathbb{E}[t(\boldsymbol{b}|s)]$. To the extent that transfers are eventually distributed to shareholders, these benefits are adjusted for the deadweight costs of equity. Second, by increasing transfers, an increase in leverage b^j shifts the default boundary, reducing the bankruptcy region and consequently the deadweight losses associated with bankruptcy.

We next consider the impact of a large bank's leverage on small banks' payoffs and incentives. We show that, all else constant, small banks have greater market value, and a higher marginal incentive to borrow, when large banks borrow more.

Lemma 1. (Effect of large bank leverage on small banks' incentives) *Suppose that in some states a positive bailout is optimal, that is,* $t(\boldsymbol{b}|s) > 0$ *for some* s*. Then for a small bank* i *with*

[17] Technically, the second term is defined as a "line integral", along a one-dimensional curve (i.e., along ∂D^i) in a two-dimensional space (i.e., in the space of signals s and asset returns u). In terms of Riemann integrals, this line integral is defined as

$$\int\limits_{\partial \mathcal{D}^i} J(u,s)\, dF(u,s) = \int\limits_{\mathcal{S}} J\left(b^i - t(\boldsymbol{b}|s), s\right) f\left(b^i - t(\boldsymbol{b}|s), s\right) ds$$

for any function $J(u,s)$, where $\mathcal{S}$ denotes the support of signals s.

borrowing level b^i, denoting by b^j the leverage choice of a large bank, we have

$$\frac{dV\left(b^i, t\left(\boldsymbol{b}|s\right)\right)}{db^j} > 0 \tag{12}$$

$$\frac{d^2V\left(b^i, t\left(\boldsymbol{b}|s\right)\right)}{db^i db^j} > 0. \tag{13}$$

Lemma 1 shows that both the *value* of a small bank and its *marginal benefit* from leveraging up are increasing in the leverage choices of the large bank. The first result is intuitive. If a large bank takes more leverage, the government's optimal bailout policy is more generous. Financiers are now willing to pay more for claims on a small bank at date 0, because they expect to benefit from a higher bailout at date 1. Thus, other things equal, the market value of a small bank increases, which explains Equation (12)

The second result in Equation (13) shows that banks' payoffs are supermodular in each others' borrowing choices. The economic implication of supermodularity is that banks' leverage decisions become *strategic complements* in the sense of Bulow, Geanakoplos and Klemperer (1985) and Cooper and John (1988). Higher leverage for large banks strengthens the strategic incentives for small banks to increase leverage. Intuitively, small banks perceive that they are shielded by the large banks.

To understand this point, recall that a small bank's appetite for borrowing is driven by the trade-off between the costs of equity issuance and the costs of default. When a large bank levers up, bailouts are larger and, consequently, small banks remain solvent more frequently. This has two effects. First, the costs of equity issuance, which hit shareholders when the bank is solvent, become more salient, and the marginal benefit of issuing debt rises. Second, since default becomes less likely, the marginal costs of default fall, and borrowing becomes yet more attractive. Both effects increase small banks' appetite for borrowing. Moreover, the same logic applies if b^j in Equation (13) is interpreted as the borrowing among the whole sector of small banks, or at least among a subset of small banks with positive mass.

In summary, our model simultaneously allows for moral hazard in the classical sense, since large banks have incentives to take leverage in order to exploit government subsidies, and for collective moral hazard, since small banks have a bigger appetite for leverage when their peers are also levering up.

These observations lead to the two main positive results of this paper, which are stated in Propositions 1 and 2. Two technical challenges arise in characterizing equilibria. First, large banks' objective functions are not necessarily concave. Second, the strategic complementarity in leverage choices opens the door to multiple equilibria. Since we wish to keep the model general, we do not restrict the primitives to guarantee concavity or uniqueness.[18] Without such restrictions, it is not possible to derive comparative statics by totally differentiating the relevant first-order conditions. Instead, our proofs rely on a modified version of the results on monotone comparative statics in Milgrom and Shannon (1994), which require only supermodularity of payoffs.

[18]In Section 6, we show numerically that both concavity and equilibrium uniqueness are satisfied for a range of empirically plausible parameters.

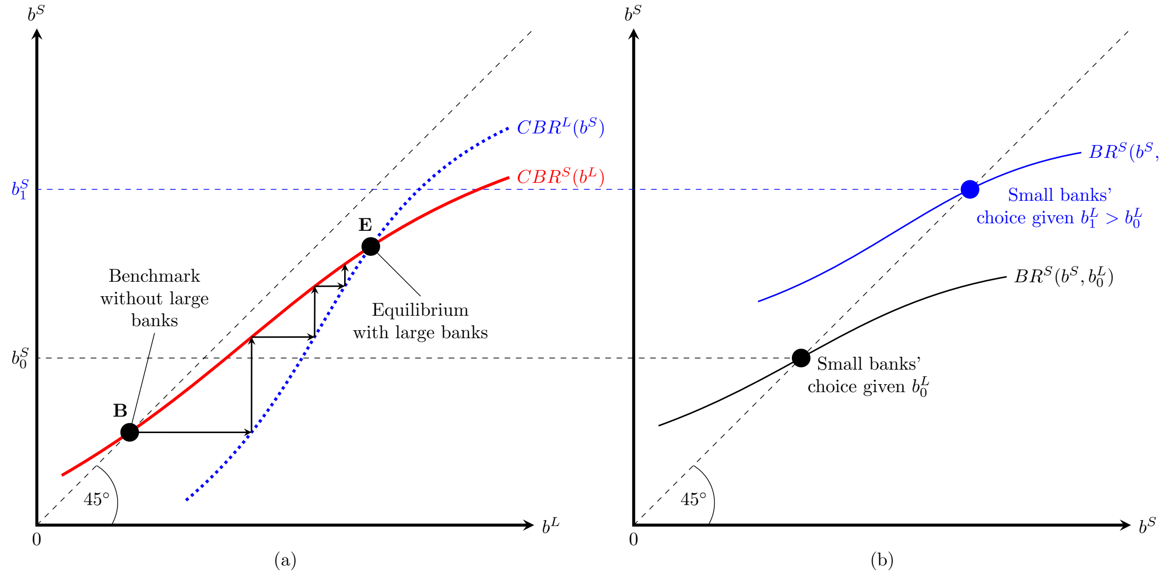


Figure 3: Equilibrium determination

Note: Figure 3a shows the best response of large banks (blue dashed line), given the borrowing choices of small banks, and the aggregate best response of small banks (red solid line), given the best responses of large banks. Figure 3b shows the best responses of a small bank given the borrowing choices of other small banks for two given values of large bank borrowing (b_0^L and b_1^L). By varying the level of b_0^L in Figure 3b, we can trace the aggregate best response of small banks (solid red line) in Figure 3a.

Proposition 1. (Large banks borrow more) *Large banks borrow strictly more than small banks in any equilibrium.*

This result follows from observing that large banks are subject to moral hazard in the classical sense. Large banks internalize that their leverage decisions directly affect the magnitude of bailouts t and, therefore, that they can increase the market prices of their debt and equity at date 0 by borrowing more and boosting their implicit government subsidy. Small banks, by contrast, are not subject to classical moral hazard because bailouts depend exclusively on aggregate conditions, and small banks rationally consider their impact on aggregates to be infinitesimal. Therefore, large banks choose to borrow strictly more than small ones.

Strategic complementarities also imply that large banks' appetite for leverage spills over to small banks. Indeed, the mere presence of large banks induces small banks to leverage up more aggressively.

Proposition 2. (When large banks are present, small banks take more leverage) *Let b_0 be the smallest borrowing level which occurs in a symmetric equilibrium with only small banks. In any equilibrium with large banks, each small bank chooses strictly higher borrowing $b^i > b_0$.*

Proposition 2 is an instance of collective moral hazard, and follows naturally from Lemma 1. Consider a hypothetical market where all banks are small and choose to borrow b_0 in equilibrium, and suppose that a subset of these banks merge to form a large bank. As shown in Proposition 1, the large bank has an independent incentive to increase their leverage to $b > b_0$. Because banks leverage decisions are strategic complements, the marginal benefit of borrowing increases for the remaining small banks, and they follow suit by choosing $b^i > b_0$.

We can combine our positive results in the following Corollary, which follows directly from both Propositions.

Corollary. (Banking systems with larger banks are associated with higher system-wide leverage) *All else constant, when larger banks are present, aggregate leverage is higher and government bailouts are larger and more frequent.*

In summary, the presence of large banks leads to an unambiguous increase in system-wide leverage and financial sector risk, implying that the government is forced to rely more frequently on larger bank bailouts than when banks are small. However, the effect of large banks on the likelihood of bank failures is ambiguous, since banks create more system-wide risk, but the government simultaneously responds with increased support. Our quantitative exercise below illustrates these competing forces.

Figure 3 provides a simple graphical illustration of equilibrium when there are large and small banks. Consider an economy where there are N large banks with total point mass λ, so that each large bank controls a share $\frac{\lambda}{N}$ of aggregate capital, and a complementary mass $1-\lambda$ of identical small banks. In this case, each small bank takes as given the borrowing choices b^L of the large bank and the choices b^S of its small peers, and its best response is to set

$$b^i = \arg\max_b V\left(b, t\left(b^S, b^L|s\right)\right) \equiv BR^S\left(b^S, b^L\right).$$

Given a borrowing level b^L for large banks, the partial equilibrium choice of the sector of small banks is found by solving the fixed point problem $b^S = BR^S\left(b^S, b^L\right)$. Figure 3b shows this for two different levels b_0^L and b_1^L of large banks' borrowing choices. This induces the "collective best response" of the small bank sector as a whole, denoted $CBR^S\left(b^L\right)$ and shown as the red (solid) curve in Figure 3a.

Similarly, we can define a large bank's individual best response, if small banks are choosing b^S and other large banks are choosing b^L, as

$$b^L = \arg\max_b V\left(b, t\left(b^S, \left(b; b_{-1}^L\right)|s\right)\right) \equiv BR^L\left(b^S, b^L\right),$$

where b_{-1}^L denotes the $(N-1)$-vector of other large banks' symmetric choices. We can again define large banks' collective best response $CBR^L\left(b^S\right)$ as the solution to the fixed point problem $b^L = BR^L\left(b^S, b^L\right)$, which is shown as the blue (dotted) curve in Figure 3a.

The economy is in equilibrium if small banks and large banks are responding optimally to each others' choices, that is, where the best response curves intersect at point E. If there were no large banks, by contrast, the economy would be in equilibrium at point B, where small banks are responding optimally to each other.

Proposition 1 establishes that, regardless of parameters, any equilibrium will be below the 45-degree line where large banks choose more leverage $b^L > b^S$ than small ones. Proposition 2 shows that small banks will always choose more leverage in an equilibrium when large banks are present (e.g. point E) than in the benchmark case with only small banks (e.g. point B).

The arrows connecting points B and E illustrate an amplification mechanism that arises in our model. Starting from the benchmark B, when large banks enter the market, they increase their leverage for strategic reasons, and the economy moves to the right in the Figure onto the collective best response CBR^L of large banks. As a result of strategic complementarities, small banks now increase their leverage, and we move upwards to the collective best response CBR^S of small banks. This move gives large banks an additional incentive to increase leverage, and so forth, until equilibrium is reached at point E. In the remainder of this section, we study the importance of this amplification mechanism when industry concentration increases.

Industry concentration and multiplier effects

We have established that the presence of large banks, or more generally increases in industry concentration, lead to higher system-wide leverage. By exploiting monotone methods, we have so far emphasized directional and qualitative results. We now show analytically that the quantitative implications of the mechanisms that we study in this paper are potentially significant because strategic complementarities amplify the initial impulse of an increase in concentration. In Section 6, we further explore the quantitative importance of our results by simulating the model for empirically plausible parameters.

For concreteness, suppose that there are N large banks with collective mass λ and that the economy is in a stable equilibrium of the form illustrated in Figure 1. Now consider an increase in industry concentration, that is, an increase in the size λ of the large bank sector. One interpretation of this change is a merger between each large bank and a subset of small banks.

First, hold constant the leverage choice b^L of the large bank, and consider the response of the remaining small banks. As before, this is determined by the fixed-point equation $b^S = BR^S\left(b^S, b^L; \lambda\right)$, where we have made explicit the dependence of best responses on industry structure. It is easy to see that, holding constant borrowing choices in the neighborhood of an equilibrium, the best response of a small bank is increasing in λ with ${\partial BR^S}/{\partial \lambda} > 0$. Intuitively, because $b^L > b^S$ in equilibrium, the increase in λ shifts weight from high-leverage to low-leverage banks, which increases aggregate leverage and leads to more generous government bailouts. By Lemma 1, this makes leverage more attractive for each small bank. However, once each small bank increases its leverage, aggregate leverage increases again, and we move into a second round of adjustments. Overall, we can characterize the response of the small bank sector as

$$\frac{\partial b^S}{\partial \lambda} = M^S \frac{\partial BR^S}{\partial \lambda}$$

where the small bank multiplier M^S is defined as

$$M^S \equiv \left(1 - \frac{\partial BR^S}{\partial b^S}\right)^{-1} > 1. \tag{14}$$

Even before large banks respond, small banks respond by increasing leverage, and this effect is amplified by strategic complementarities, as captured by the multiplier $M^S > 1$.

Second, large banks respond to the change by adjusting their own leverage, for two reasons. The first reason is that small banks have raised their borrowing, making borrowing more attractive. The second reason is that the large bank is now larger than before, and internalizes an even stronger effect of its choices on government bailouts. Computing total changes yields

$$\frac{db^L}{d\lambda} = \bar{M} \cdot \left(M^L M^S \frac{\partial BR^S}{\partial \lambda} \frac{\partial BR^L}{\partial b^S} + M^L \frac{\partial BR^L}{\partial \lambda}\right),$$

$$\frac{db^S}{d\lambda} = \bar{M} \cdot \left(M^L M^S \frac{\partial BR^L}{\partial \lambda} \frac{\partial BR^S}{\partial b^L} + M^S \frac{\partial BR^S}{\partial \lambda}\right),$$

where the large bank multiplier M^L is defined as

$$M^L \equiv \left(1 - \frac{\partial BR^L}{\partial b^L}\right)^{-1} > 1,$$

and the aggregate multiplier $\bar{M} > 1$, which is itself a function of the small and large bank multipliers, is defined as

$$\bar{M} \equiv \left(1 - M^L M^S \frac{\partial BR^L}{\partial b^S} \frac{\partial BR^S}{\partial b^L}\right)^{-1} > 1. \tag{15}$$

Equation (15) reveals that industry concentration has an effect on leverage that is amplified on two levels. First, within each subsector (of large or small banks), banks encourage each other to take more leverage, as reflected by the size-dependent multipliers M^S and M^L. Second, across bank sizes, strategic complementarities induce further amplification via the aggregate multiplier $\bar{M} > 1$ in Equation (15). The two effects reinforce each other since the aggregate multiplier $\bar{M}$ is itself increasing in M^S and M^L.

4 Optimal ex-ante policies

In our model, there is a clear case for prudential regulation. In particular, banks do not internalize the impact of their leverage choices on the social cost of ex-post bailouts. On the contrary, they endeavor to attract bailouts because funding markets at date 0 reward them for large implicit subsidies.

Constrained efficient choices

We study a constrained efficient benchmark for policy, in which a benevolent social planner can dictate banks' borrowing choices $\boldsymbol{b}$ at date 0, but cannot overcome financing frictions, that is, the costs of default and the social cost of equity issuance.

We first compute a general expression for expected social welfare at date 0. Given banks' choices $\boldsymbol{b}$ and a state-contingent bailout policy $t(s)$ at date 1 (not necessarily equal to the ex-post optimal policy $t(\boldsymbol{b}|s)$), expected welfare at date 0 is

$$\begin{aligned} W_0(\boldsymbol{b}, t(s)) = {} & \mathbb{E}[u] - \mathbb{E}[\kappa(t(s))], \\ & -(1-\phi)\int \Pr\left[\mathcal{D}^i\right]\mathbb{E}\left[u|\mathcal{D}^i\right]d\mu \\ & -\left(\psi-\bar{\psi}\right)\int \Pr\left[\mathcal{N}^i\right]\mathbb{E}\left[t(s)+u-b^i|\mathcal{N}^i\right]d\mu. \end{aligned} \tag{16}$$

This is intuitive: The constrained social planner measures welfare as the expected value of asset returns, less the deadweight social costs of bailout transfers, bank default, and equity issuance. Comparing (16) with banks' private objective function in (8) highlights the rationale for regulation. In particular, the government transfer $t(s)$ enters negatively in social welfare due to deadweight costs $\kappa(t(s))$. This cost is not accounted for in banks' market values. Moreover, $t(s)$ enters positively in banks' market values because it represents a subsidy from taxpayers to the owners of bank debt and equity.

Two maximization problems are of potential interest. First, we can assume that the planner can commit to any state-contingent bailout policy $t(s)$ at date 0. Then he solves

$$W^c = \max_{\{\boldsymbol{b}, t(s)\}} W_0(\boldsymbol{b}, t). \tag{17}$$

Second, we can assume that the planner cannot commit to transfers, and chooses them optimally ex-post as in Section 3. In this case he solves

$$W^{nc} = \max_{\{\boldsymbol{b}, t(s)\}} \{W_0(\boldsymbol{b}, t(s)) \text{ subject to } t(s) = t(\boldsymbol{b}|s)\}, \tag{18}$$

where $t(\boldsymbol{b}|s)$ is the government's ex-post best response, defined in (3). Clearly, the loss of commitment in (18) cannot increase the maximized value of welfare, so that $W^c \geq W^{nc}$. Moreover, we can show that a lack of commitment does not reduce welfare in this instance:

Lemma 2. (Commitment is irrelevant when the planner controls banks) *When the planner perfectly controls banks' borrowing decisions $\boldsymbol{b}$, the solutions to problems (17) and (18) coincide and satisfy $W^c = W^{nc}$.*

Intuitively, commitment provides no additional value when the planner controls the banks, because in any case, the solution with commitment involves choosing bailouts that are ex-post optimal. When the planner does not control the banks, by contrast, commitment may be beneficial because it allows the planner to distort bailouts away from ex-post optimality in order to curb moral hazard.

Lemma 2 allows us to characterize the planner's optimal borrowing choices:

Proposition 3. (Efficient leverage is independent of bank size) *The constrained efficient choice is to set $b^i = b^\star$ for almost all i, regardless of the distribution μ of bank sizes, and regardless of whether the planner can commit to a transfer policy.*

Proposition 3 shows that the planner chooses symmetric policies for all banks, and in particular, that the socially optimal borrowing level b^i is independent of bank size. This is a natural consequence of assuming identical technologies for large and small banks and constant returns to scale: A large bank has no technological reason to take more or less leverage than a small bank. This policy is in contrast to the laissez-faire equilibrium of Section 3, where large banks unambiguously chose larger borrowing levels. These equilibrium choices were motivated not by technological or contractual differences, but rather by the desire to maximize an implicit government subsidy. Since the planner internalizes that such a subsidy is socially wasteful, he does not respond to the same incentives. More generally, there may be other unmodeled reasons that may lead to efficient leverage choices that are heterogeneous. Therefore, Proposition 3 should be understood as showing that leverage disparities caused by size differences are inefficient.

Before considering ways to implement the optimal allocation, it is useful to explicitly characterize the best borrowing level $b^{\star}$. When all banks choose the same borrowing level b, we can write ex-ante welfare as

$$\bar{W}_0\left(b, t\left(s\right)\right) = V\left(b, t\left(s\right)\right) + \bar{\psi}\Pr\left[\mathcal{N}\right]\mathbb{E}\left[t\left(s\right) + u - b|\mathcal{N}\right] - \mathbb{E}\left[t\left(s\right) + \kappa\left(t\left(s\right)\right)\right],$$

where $V\left(b, t\left(s\right)\right)$ is the private value of an individual bank in Equation (8), and $\mathcal{N} = \left\{t\left(s\right) + u \geq b\right\}$ is the event of non-default. The first term is aligned with the bank's objective. The second term captures the fact that the costs of equity issuance are not fully social costs if $\bar{\psi} > 0$. The last term arises because, unlike the bank, the social planner internalizes the full cost $t\left(s\right) + \kappa\left(t\left(s\right)\right)$ of the bailout to taxpayers.

Since the welfare maximization problems with and without commitment are equivalent, we can assume that the planner is already committed to the optimal ex-post transfer $t(b^{\star}|s)$. Then the choice of $b^{\star}$ must maximize welfare taking this commitment as given, thus solving the fixed point equation

$$b^{\star} = \arg\max_{b} \bar{W}_0\left(b, t\left(b^{\star}|s\right)\right)$$

The first-order condition in this problem gives a characterization of socially optimal borrowing:

$$\frac{\partial V\left(b^{\star}, t\left(b^{\star}|s\right)\right)}{\partial b} - \bar{\psi}\Pr\left[\mathcal{N}^{\star}\right] = 0, \tag{19}$$

where $\mathcal{N}^{\star} = \left\{t\left(b^{\star}|s\right) + u > b^{\star}\right\}$ is the non-default event under the optimal policy. Intuitively, the planner considers the impact of debt choices on the value of the bank, adjusted for the part of private equity issuance costs which is explained by transfers. One might expect the planner to also consider the impact of debt choices on the ex-post transfer. Formally, Equation (19) includes the term $\mathbb{E}\left[\frac{\partial W_0}{\partial t(b^{\star}|s)}\frac{\partial t(b^{\star}|s)}{\partial b}\right]$. However, since transfers are chosen optimally, marginal changes in transfers have only a second-order welfare effect, and this term vanishes after an application of the envelope theorem.

Optimal capital requirements

Dictating banks' choices b^i is tantamount to quantity regulation. Indeed, the constrained efficient choice $b^\star$ can be achieved by imposing binding capital requirements, as included in the Basel III Accord, which limit borrowing as a fraction of risky assets (recall that in our model, borrowing choices b^i are already expressed per unit of risky capital). The following result is a direct corollary of Proposition 3.

Corollary. (Optimal capital requirements are independent of bank size) *The constrained efficient allocation can be implemented by imposing binding capital requirements that are independent of bank size.*

It follows that strategic differences between large and small banks are insufficient to justify size-dependent capital regulation. Even though the incentives to take leverage remain stronger for large banks when capital requirements are in place (the Lagrange multiplier on a large bank's capital constraint will be greater), the optimal level of capital that a regulator wishes to enforce is independent of bank size, as long as technologies exhibit constant returns to scale. As noted when introducing Proposition 3, if efficient leverage choices were to be heterogeneous, this Corollary implies that binding capital requirements should not be a direct function of bank size per se.

Naturally, this conclusion changes when policy is conducted via Pigouvian taxes, which we examine next.

Optimal Pigouvian taxes

We now assume that the social planner cannot directly control banks' borrowing quantities $\boldsymbol{b}$. Moreover, we focus on the case where the planner has no commitment power; in particular, he cannot credibly announce a date 1 bailout policy at date 0. Market participants instead expect that the date 1 bailout policy will be chosen to maximize welfare ex-post.

The socially optimal choices $b^\star$ can be implemented by taxing banks in proportion to their borrowing choices at date 0. We let τ^i denote the Pigouvian tax levied on bank i, who consequently pays $b^i\tau^i$ to the government; tax revenues are rebated to financiers as a lump sum. Therefore, the value of τ^i should be interpreted as a tax on the bank's face value of debt. Under Pigouvian taxation, the bank maximizes its value net of tax, $V(b^i, t(\boldsymbol{b}|s)) - b^i\tau^i$. As before, small banks take the bailout policy $t(\boldsymbol{b}|s)$ as fixed. Large banks, on the other hand, realize that the government lacks commitment and take into account their impact on the ex-post optimal bailout. A standard argument then leads to optimal taxes:

Proposition 4. (Optimal Pigouvian taxes) *The following Pigouvian taxes implement the social planner's choice $b^i = b^\star$ in equilibrium:*

– If i is a small bank, then set

$$\tau^i = \bar{\psi} Pr[\mathcal{N}^\star]\,. \tag{20}$$

– *If j is a large bank, then set*

$$\tau^j = \tau^i + \mathbb{E}\left[\left.\frac{\partial t\left(\boldsymbol{b}|s\right)}{\partial b^j}\right|_{\boldsymbol{b}=b^\star} \left(1-\psi \mathbf{1}\left(\mathcal{N}^\star\right)\right)\right] + (1-\phi) \int\limits_{\partial \mathcal{D}^\star} \left.\frac{\partial t\left(\boldsymbol{b}|s\right)}{\partial b^j}\right|_{\boldsymbol{b}=b^\star} \left(b^\star - t\left(b^\star|s\right)\right) dF, \qquad (21)$$

where $\partial D^\star = \{t(b^\star|s) + u = b^\star\}$ is the boundary of the optimal default region.

To implement the efficient outcome, each small bank is charged in proportion to the expected wedge between private and social costs of equity issuance. Second, large banks are charged a size top-up tax in order to account for their risk taking incentives. The ”size tax” is designed to counteract the part of large banks’ incentives that stems from their desire to maximize implicit subsidies. In particular, the second term in (21) offsets a large bank’s incentive to attract larger subsidies for its financiers, while the third term offsets the incentives generated by the fact that larger subsidies reduce the expected costs of default. Note that these two terms are equivalent to the final two terms in the large bank’s first-order condition (11), evaluated at the optimal choice $b^\star$.

Policy implications

The prescription of the optimal policy is as follows. If large banks are present, regulators need to be wary of significant and amplified increases in risk taking incentives. If binding capital requirements can be imposed to correct all externalities, then constant returns to scale still guarantee that there is no need to make capital requirements size-dependent. However, if regulators choose to use price instruments to combat systemic risk, then a top-up tax in line with Equation (21) should be charged to large banks. We now discuss some further implications of our results, which concern (i) the interaction between financial policy and antitrust tools that can directly affect the bank size distribution, and (ii) cases in which the implementation of quantity or price controls is imperfect.

Prudential and antitrust policy Consider again the thought experiment in which the economy starts out with only small banks, but a subset of them propose a merger in order to form a large bank. Should antitrust authorities prevent this? The traditional tests for allowing the merger would be based on concerns about market power and reductions in consumer surplus. Our model abstracts from consumer surplus, since banks capture all available surplus in equilibrium, but points towards different trade-offs.

The arguments in Section 3 imply that a merger among banks can have significant and amplified effects on risk taking when (i) banks benefit from systemic bailouts and (ii) the banking sector is unregulated. In a financial sector with weak or absent regulation, financial stability ought to be a first-order concern when assessing mergers. This line of reasoning provides partial justification for the Federal Reserve’s merger approval process under the Dodd-Frank act, which involves a mandatory financial stability assessment.[19]

Our results highlight that the case for financial stability tests depends on the strength of other financial regulations. Antitrust policy can be more lenient when financial regulation is strong. If regulators are already implementing the optimal quantity constraint $b^\star$, then the merger should be permitted, modulo considerations about consumer surplus, since risk taking will not change as a result. If regulators are imposing the optimal Pigouvian tax on small banks, as defined in Equation (20), then the merger should be permitted as long as prudential regulators are ready to respond by charging the appropriate top-up tax, defined in Equation (21), to the newly created large bank. Conversely, a stricter antitrust policy is needed when financial regulation is weak, or when banks in our model are taken to be "shadow banks" that fall outside the regulatory perimeter.

It follows that prudential and antitrust policy are substitutes. Mergers should be waved through more frequently when prudential regulation is strong. Conversely, prudential regulation – especially when implemented via Pigouvian taxes – can be more lenient when the antitrust authority does not permit banks to merge and form large, dominant entities.

One implication of our model, therefore, is that caps on bank size can be valuable when prudential regulation is imperfect. While this paper identifies an additional marginal benefit of size caps, other defensible arguments exist in favor of large financial institutions. Better diversification and risk management, economies of scale, ability to handle large projects, creation of insensitive claims a la Gorton and Pennacchi (1990) or reduction of international frictions as in Freixas and Holthausen (2005) are relevant arguments on the opposite side of the debate. Trading off these considerations will determine the optimal size distribution for the banking sector.

Discussion of second-best policy We have focused on first-best policy, that is, on the exact implementation of the constrained efficient outcome. There is a stark contrast between quantity and price controls in this case: Capital requirements ought to be independent of bank size, while optimal Pigouvian taxes call for a top-up tax on large banks. This dichotomy becomes less strict when we relax the assumption of first-best policy.

For example, consider the second-best case in which the regulator cannot push borrowing limits all the way to $b^\star$, for example due to political constraints that are generated by lobbying. In this case, there may again be a case for increased capital requirements for large banks; these would curtail large banks' leverage and, via strategic complementarities, reduce risk-taking incentives for small banks.

Alternatively, consider a model in which banks can deviate from binding capital requirements: If the regulator requires bank i to choose $b^i = \bar{b}$, then the bank can extend its borrowing to $\bar{b} + \delta$ at a cost χ per unit of capital. This cost can be interpreted as an outright bribe that entices the regulator to turn

[19] According to Section 503(d) of Dodd-Frank, the Federal Reserve must take into account whether a bank merger "would result in greater or more concentrated risks to the stability of the United States banking or financial system." Similar tests apply when banks acquire non-banking organizations.

a blind eye, or as the cost of a more indirect strategy, such as gaming regulatory risk models. If this cost is intermediate, small banks do not find it worth it to deviate from the capital requirement. Large banks, on the other hand, may still choose to deviate because they realize that doing so increases their implicit subsidy and therefore improves their valuation at date 0. Therefore, it is reasonable to assume that large banks have a stronger incentive to deviate from capital requirements. For intermediate costs χ, the second-best policy is therefore to impose a higher capital requirement on large banks in anticipation of such deviations in equilibrium.

5 Idiosyncratic risk

In our baseline environment, to simplify the exposition, we assume that bank returns are fully determined by an aggregate shock. In this Section, we assume that bank returns have both an aggregate and an idiosyncratic component.

Modified environment Formally, we preserve the rest of the assumptions, but we now assume that bank i's assets yield a random return u^i at date 1 of the form

$$u^i = v + w^i,$$

where the first component v is an aggregate shock, while the second component w^i is an idiosyncratic shock to bank i, which is independent across banks and independent of v, with mean $\mathbb{E}\left[w^i\right] = 0$, variance $\mathbb{V}ar\left[w^i\right] < \infty$, and density $h^i\left(w^i\right)$.[20]

As in the baseline model, the government receives a signal s of the common shock v, and we write $f_v\left(v|s\right)$ for the conditional density of v. Therefore, the value of the aggregate capital stock at date 1 is given by

$$\bar{u} = \int u^i d\mu\left(i\right) = v + \sum_{j\in\mathcal{L}} \mu\left(j\right) w^j,$$

where $\mathcal{L} = \{j : \mu\left(j\right) > 0\}$ denotes the set of large/non-infinitesimal banks. This setup allows for a flexible relationship between idiosyncratic risk and bank size. Intuitively, the idiosyncratic shocks w^i of small banks integrate to zero by the law of large numbers. By contrast, idiosyncratic shocks w^j to large banks, who have a strictly positive point mass $\mu\left(j\right) > 0$, directly affect aggregate output. In particular, aggregate output is more volatile when large banks are present as long as the variance of large bank's idiosyncratic return component is non-zero, since

$$\mathbb{V}ar\left[\bar{u}\right] = \mathbb{V}ar\left[v\right] + \sum_{j\in\mathcal{L}} \mu\left(j\right)^2 \mathbb{V}ar\left[w^j\right].$$

[20]If we allowed banks to choose their relative exposure to aggregate and idiosyncratic risk, banks would gravitate towards aggregate risk-taking so as to maximize expected bailouts, and in line with our arguments above, this incentive would be strongest for large banks. Holding exposures constant makes for a conservative assessment of the strategic effects we highlight.

Therefore, depending on the assumptions on how the variance of large banks, $\mathbb{V}ar\left[w^j\right]$, is determined, our formulation allows us to capture different views of how idiosyncratic risk aggregates within entities. On the one hand, one can assume, appealing to a Law of Large Numbers, that when a continuum of small banks merge to form a large bank, their idiosyncratic risk cancels out. In that case, the variance of the idiosyncratic return component for large banks becomes zero, that is, $\mathbb{V}ar\left[w^j\right] = 0$ if $j \in \mathcal{L}$. On the other hand, one can assume that the idiosyncratic return component is realized at the bank level (e.g., each bank has a CEO, and the idiosyncratic shock is driven by the CEO's individual decisions). In that case, which we refer to as the granular scenario as in Gabaix (2011), the variance of the idiosyncratic return component for otherwise comparable large and small banks should be identical.

Formally, consistently with our assumption that all banks are otherwise identical but for their size, we modulate the extent to which the idiosyncratic shocks to large bank returns of granular by defining a parameter $\zeta \in [0, 1]$, such that,

$$\zeta = \frac{\mathbb{V}ar\left[w^{\text{large}}\right]}{\mathbb{V}ar\left[w^{\text{small}}\right]} \qquad \text{(Granularity parameter)}.$$

Under this formulation, when $\zeta = 0$, idiosyncratic shocks cancel out at the bank level and a Law of Large of Numbers applies, which corresponds to the first scenario we just described. When $\zeta = 1$ instead, idiosyncratic shocks occur at the bank level, consistently with the granular scenario. Intermediate levels of ζ simply represent a combination between both extremes.

Equilibrium with idiosyncratic risk In the presence of idiosyncratic risk, the conditional c.d.f. of total asset values in bank i is given by

$$\Pr\left[u^i \leq u | s\right] = \int_0^\infty \left[\int_{-\infty}^{u-v} h^i\left(w\right) dw\right] f_v\left(v|s\right) dv \equiv F_u^i\left(u|s\right). \tag{22}$$

In Appendix B.4, we show formally that our main results remain valid under mild regularity conditions after this change of variable. Intuitively, the government has less information about individual banks' performance when there is idiosyncratic risk. In general, the impact of idiosyncratic risk on the level of bailouts is therefore ambiguous: After good aggregate news, the government is less confident that individual banks are safe, and may increase its bailout, while after bad aggregate news, the government is less confident that a bailout is necessary, since a positive measure of individual banks are certain to recover. However, for any given level of bailout, it remains true that (i) only large banks internalize the marginal impact of their leverage choices on government policy, and that (ii) leverage choices are strategic complements across banks. Therefore, large banks continue to choose greater leverage in any equilibrium, as in Proposition 1, and small banks choose greater leverage in response, as in Proposition 2. We provide further insights into how changes in the granularity parameter ζ affect our results in Section 6.3.

6 Quantitative assessment

As formally shown above, changes in industry composition have the potential to generate substantial amplification due to the presence of strategic complementarities. To gauge the strength of the complementarities and, more generally, to illustrate how bank size affects aggregate leverage, the likelihood of bank failure, and the magnitude of government bailouts in equilibrium, we illustrate the predictions of our model by selecting parameters consistent with U.S. data over the period 1990Q1 to 2013Q4.[21] Given that our model seeks to capture long-term funding decisions, we map a period in our model to a two-year time horizon. We work with the richer formulation with idiosyncratic risk developed in Section 5, since it is easier to discipline by observables.

Functional forms In order to explicitly solve the model, we must make specific functional form assumptions, which were not needed to derive our theoretical results.

First, regarding industry composition, we assume that there exists a finite number N of large banks that hold a share λ of total bank assets, and a set of small banks that hold a share $1-\lambda$ of total assets – this is the same formulation used to draw Figure 3. Figure 4 illustrates the distribution of bank's assets in the economy. Although our framework allows us to match the whole distribution of bank assets, the current formulation is more parsimonious and allows us to easily parametrize the effect of changes in banking concentration. In Section 6.4, we relax our distributional assumptions and argue that this formulation captures most of the relevant effects.

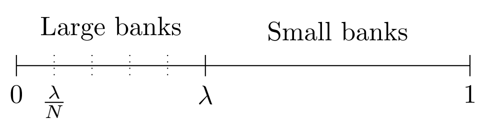


Figure 4: Distribution of banks' assets

Second, we assume that the social cost of government bailouts $\kappa(t)$ takes the following exponential-affine form

$$\kappa(t) = \frac{\kappa_1}{\kappa_2}\left(e^{\kappa_2 t} - 1\right),$$

where κ_1 and κ_2 are non-negative parameters. The parameter κ_1 represents the marginal cost of public funds for a small intervention, since $\kappa'(0) = \kappa_1$ while the parameter κ_2 is a measure of curvature, since $\frac{\kappa''(t)}{\kappa'(t)} = \kappa_2$, $\forall t$. Note that this formulation naturally guarantees that $\kappa(0) = 0$. Although we find similar results with alternative functional forms (e.g. linear or quadratic), the exponential formulation counteracts the incentives of large banks in the model to take implausibly large leverage positions, in particular when λ is sufficiently large.

[21]If not stated explicitly, any reference to measures of actual banks' performance is drawn from U.S. Call Reports data, as distributed by Drechsler, Savov and Schnabl (2016, 2017).

Finally, we assume that all random variables are normally distributed. In particular, we assume that the aggregate component v and the idiosyncratic component w^i of banks' returns are normally distributed as follows:

$$v \sim N(\mu_v, \sigma_v) \quad \text{and} \quad w^i \sim N(0, \sigma_w),$$

where $\mathbb{C}ov[v, w^i] = 0$, $\forall i$, and $\mathbb{C}ov[w^j, w^i] = 0$, $\forall i \neq j$. We also assume that the government receives a signal s with the following structure: $s = v + \varepsilon_s$, where $\varepsilon_s \sim N(0, \sigma_s)$. Therefore, after observing the signal s, the government perceives the return v to be distributed as $v|s \sim N\left(\frac{\sigma_v^2}{\sigma_v^2+\sigma_s^2}s + \frac{\sigma_s^2}{\sigma_v^2+\sigma_s^2}\mu_v, \frac{\sigma_v^2\sigma_s^2}{\sigma_v^2+\sigma_s^2}\right)$. Although assuming a Gaussian structure for returns and signals simplifies the government inference problem; its drawback is the possibility of experiencing negative payoffs. Our parametrization is such that the probability of u taking negative values is negligible. Similar results emerge assuming that returns are log-normally distributed. Since we allow for idiosyncratic risk, we are not restricted in the choice of σ_s. In particular, the government's problem remains smooth even when the aggregate state is perfectly observed with $\sigma_s = 0$.

Parameter values Table 1 summarizes the parameter choices in our baseline parametrization. We need to assign values to twelve parameters, which can be classified into three broad categories: banks' profitability, industry composition, and government related parameters. Our strategy is to select parameters to target average cross-sectional values, letting the model endogenously generate differences in behavior between large and small banks. We take a conservative stance and discipline all parameters using directly observable or already available information, except for σ_v and κ_2, which we jointly determine by matching two key statistics of our model. We adopt this approach because obtaining direct measures of the volatility σ_v of aggregate shocks to the banking sector is difficult, since such shocks (i.e. financial crises) are rare events. Bianchi (2016) and Mendicino, Nikolov and Suarez (2017) follow a similar approach by calibrating aggregate volatility parameters to match the empirical frequency of crises.

The first set of parameters determines banks' profitability and their capital structure. We set the value of deadweight losses associated with default to be $1 - \phi = 20\%$, of banks' value. This choice is consistent with the existing literature on capital structure, as shown by Davydenko, Strebulaev and Zhao (2012) and Strebulaev and Whited (2012). We set the value of $\psi = 0.15$, which is on the high end of estimates used in Hennessy and Whited (2005), but which is necessary to generate the large levels of leverage observed in the banking sector. Consistent with our agnostic view on the social vs. private costs of equity we set $\bar{\psi}$ equal to $\frac{\psi}{2}$. We set $\mu_v = 1.02$ and $\sigma_w = 0.06$ to match the average standard deviation across all banks during the period of interest in the Call Report data. In line with the findings of Gabaix (2011), and consistently with our own calculations using the Call Reports data, we set the granularity parameter to $\zeta = 1$, implying that the idiosyncratic return risk is identical across banks of all sizes. Finally, we choose $\sigma_v = 0.02$ (jointly with κ_2) to target a probability of a significant intervention, defined as the probability of receiving a transfer higher than 2%, of 10%, which corresponds to a crisis episode in 20 years, as in Mendicino, Nikolov and Suarez (2017) – see also Reinhart and Rogoff (2009), and to target an average intervention conditional on a bailout occurring of 3% of bank value, which is slightly above the median for developed countries in Laeven and Valencia (2013), but substantially below the median for emerging markets, and in line with bailouts granted to major global banks during the crisis of 2008, as reported by Hüttl and Schoenmaker (2016).

Table 1: Parameter values

Parameter	Definition	Value
	Banks' profitability and capital structure determinants	
$1-\phi$	Default Deadweight Loss	0.20
ψ, $\bar{\psi}$	Cost of Equity Issuance	0.15, 0.075
μ_v	Average ROA	1.02
σ_v	Standard Deviation ROA (aggregate)	0.02
σ_w	Standard Deviation ROA (idiosyncratic)	0.06
ζ	Granularity Parameter	1
	Industry composition	
λ	Large Banks' Share	0.5
N	Number of Large Banks	5
	Government	
κ_1, κ_2	Bailout Costs	0.13, 10
σ_s	Government's Signal Precision	0

The second set of parameters pins down the industry composition. Consistently with our description of recent U.S. data in Figure 1, we set values of $\lambda = 0.5$ and $N = 5$. Our choice of N seeks to capture the persistence in the size rankings of a handful of banks. In particular, Bank of America, Citibank, JP Morgan Chase, and Wells Fargo have persistently ranked among the top 5 banks by asset size for the whole period considered. Our benchmark choice of λ is consistent with current levels of concentration, although our central counterfactual exercise explores in detail how alternative values of λ affect the outcomes of the model.

The third set of parameters determines the magnitude of the bailout policy. Given that σ_s and σ_w cannot be separately identified when $\zeta = 1$, and to remain parsimonious, we set $\sigma_s = 0$, implying that the government perfectly observes the aggregate state. We set the value of κ_1 to match a (net) marginal cost of public funds for small interventions of 13%, which is a standard estimate in the literature (Dahlby, 2008). Finally, as described above, we choose $\kappa_2 = 10$ (jointly with σ_v) to target a probability of a significant intervention every 20 years, and an average intervention conditional on a bailout occurring of 3% of bank value.

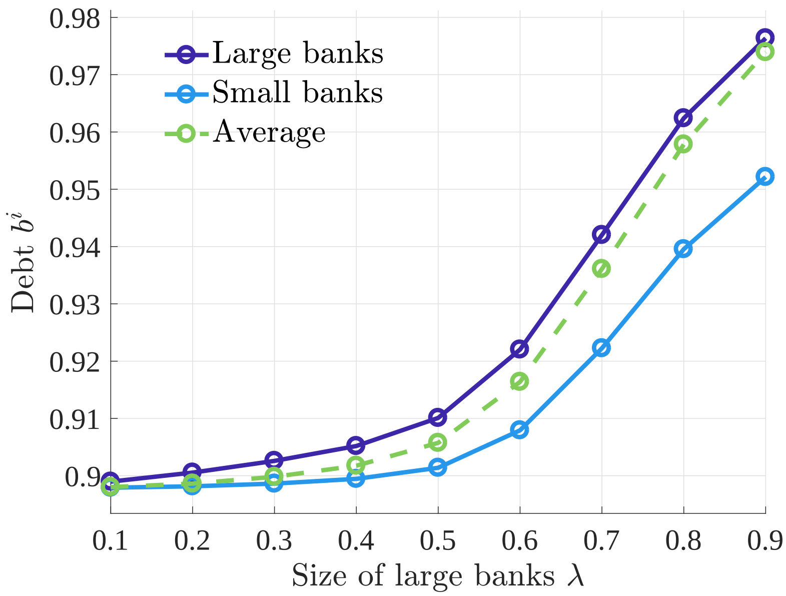


Figure 5: Borrowing choices

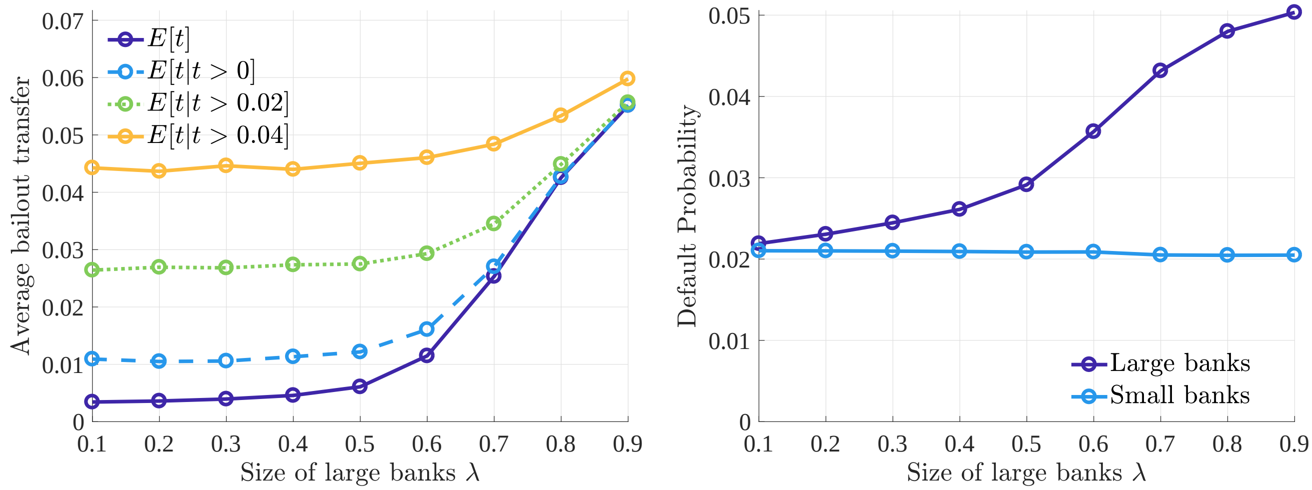
Note: Figure 5 shows the optimal equilibrium debt-to-asset ratio chosen by large banks (dark blue solid line) and by small banks (light blue solid line) for different levels of the share of assets held by large banks $\lambda \in [0.1, 0.9]$. It also shows the average debt-to-asset ratio in the economy (green dashed line). Note that, mechanically, the average debt-to-asset ratio tends to the debt-to-asset ratio of small banks when $\lambda \to 0$ and to the debt-to-asset ratio of large banks when $\lambda \to 1$.

6.1 Model results and industry concentration counterfactuals

Figures 5 and 6 summarize our results for the baseline calibration and illustrate our leading counterfactual exercise, in which we explore how changes in industry composition modify the leverage decisions for large and small banks, as well as other equilibrium outcomes. The equilibrium outcomes reported in this subsection correspond to the unregulated equilibrium described in Section 3; we turn to optimal policy below.

Our baseline parametrization generates values for average debt to assets of 0.906, with large banks choosing 0.910 and small banks 0.901. Compared to the average debt-to-asset ratios for top 5 banks and the remaining banks during the period considered, which respectively correspond to 0.935 and 0.914 in the Call Reports data, our model accounts for around half of the difference in debt-to-asset choices between large and small banks, although it somewhat understates average leverage. The difference in leverage choices was not directly targeted by our parameter choices. In particular, since large and small banks are otherwise identical, any difference in the behavior of large banks relative to small banks is due to the strategic effect that we identify in this paper.

Consistently with our theoretical results, Figure 5 shows that increases in the level of industry concentration in the form of an increase in the share of assets held by the top 5 banks are associated with a significant effect on the leverage choices of both large and small banks. Our results show that increases in industry concentration have a significant impact on system-wide borrowing, in particular for



(a) Average bailout transfer

(b) Probability of default

Figure 6: Government response and default probabilities

Note: Figure 6a shows the equilibrium level of average bailout transfers, conditional on the size the intervention. Figure 6b shows the biannual probability of default for large banks (dark blue solid line) and small banks (light blue solid line).

values of λ higher than 0.5. For instance, our model implies that an increase in concentration in which the largest 5 banks hold 70% of assets (λ moves from 0.5 to 0.7) would be associated with an increase in system-wide borrowing of 3.5 percentage points, from 90.1% to 93.6%, with the difference between large and small banks borrowing increasing slightly to around 2%. Put differently, the aggregate leverage ratio in the economy increases by around 50% (from 10.1 to 15.6) following the increase in concentration.[22]

Figure 6 shows the behavior of the government response and the default probabilities by large and small banks in equilibrium. Figure 6a shows that the magnitude of bailouts in equilibrium increases monotonically with the level of industry concentration. Intuitively, given that both large and small banks borrow more, all else equal, the probability of failure is larger, which makes more appealing for the government to bail out banks in bad aggregate states. In the baseline scenario, the magnitude of the government's transfer, conditional on a significant intervention, which we define as transfer greater than 2% of bank value, corresponds to just under 3% of bank's assets, as targeted by our calibration. The unconditional value of government guarantees is roughly 0.6% of banks' total asset value. A shift from $\lambda = 0.5$ to $\lambda = 0.7$ would be associated with a 25% increase in the magnitude of significant intervention, from 2.75% to 3.5%, and with a four-fold increase in the average expected transfer, from

[22]The leverage ratio of bank i in our model is calculated as $\frac{\text{Assets}}{\text{Equity}} = \frac{1}{1-\text{Debt/Assets}} = \frac{1}{1-b^i}$.

0.6% to roughly 2.5% of banks' value.[23]

The likelihood of default for large and small banks is respectively 1.5% and 1% per annum under the baseline parametrization. Figure 6b shows that the probability of bank failure is increasing in the level of banking concentration λ for large banks, but is decreasing in the level of concentration for small banks. While the result for large banks is intuitive, the finding for small banks is somewhat surprising, in particular given that we know from Figure 5 that small banks borrow more when λ is higher. In this case, the reduction in the probability of default is due to the increase in the ex-post transfer associated with the large bank taking on more leverage, which counteracts the direct effect of higher leverage by small banks in the probability of default.

In sum, beyond the exact numerical predictions of the model, Figures 5 and 6 jointly illustrate that, through the mechanism that we study in this paper, increases in banking concentration can generate a substantial effect on banks' choices, especially when large banks are sufficiently large.

6.2 Optimal Policy

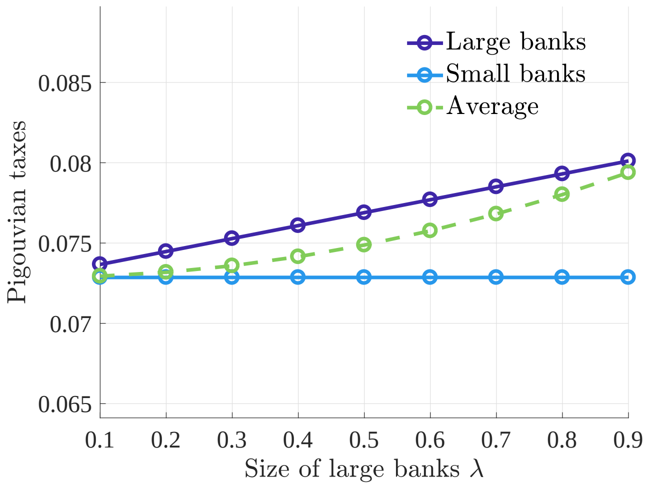
Building on the results described in Section 4, Figure 7 quantitatively assesses the magnitude of the optimal Pigouvian policy, for different values of concentration. Since we have assumed that the wedge $\bar{\psi}$ between the private and social costs of equity issuance is non-zero, the planner finds optimal to set a non-zero corrective tax for both large and small banks.

The optimal Pigouvian tax for the baseline parametrization is of the order of 7.5% of the value of the debt issues by banks, and is directly related to the value of $\bar{\psi}$, as implied by the optimal tax characterization in Equation (20).[24] In particular, large and small respectively face taxes of 7.69% and 7.29%. However, the main object of interest for us is the differential tax charged to large banks relative to small banks. In our baseline scenario, large banks face an additional 0.4% additional tax relative to small under the optimal ex-ante policy: we refer to this difference in taxes as a "size tax". The optimal size tax grows approximately linearly with λ. For instance, our model implies that an increase in concentration in which the largest 5 banks hold 70% of assets (λ moves from 0.5 to 0.7) would be associated a 50% increase in the optimal size tax, from 0.4% to about 0.6%.

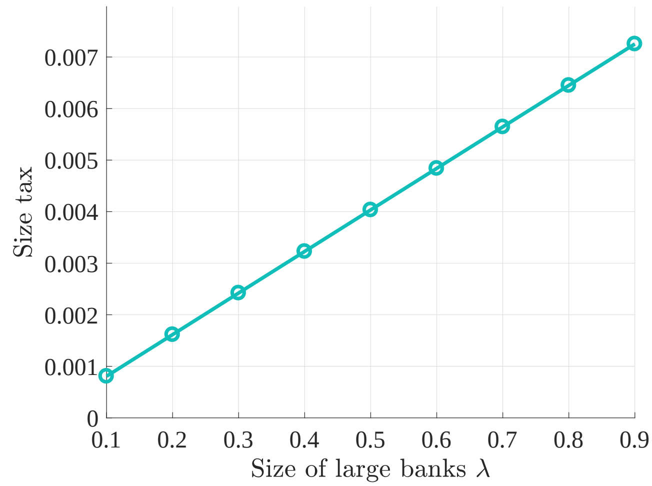
We further evaluate the constrained efficient choice for our baseline parametrization. The optimal debt-to-asset ratio is 87.4% and, consistently with Proposition 3, this choice is independent of bank size. Under perfect enforcement, the planner's choice can then be decentralized by imposing the capital requirement that $\frac{\text{Equity}}{\text{Assets}} = \frac{\text{Assets} - \text{Debt}}{\text{Assets}} \leq 12.6\%$.

[23] Our quantitative assessment also yields a direct measure of leverage multipliers, as defined in Equations (14) and (15), which characterize the amplified effect of a local increase $d\lambda$ in banking industry concentration. For our baseline calibration, the small- and large-bank multipliers are $M^S = 1.300$ and $M^L = 1.228$ respectively. The aggregate multiplier which magnifies the total equilibrium response is $\bar{M} = 1.091$.

[24] The optimal debt-to-asset choice for the planner in the baseline scenario for both large and small banks is 0.874.



(a) Optimal Pigouvian taxes



(b) Optimal size tax

Figure 7: Optimal taxes

Note: Figure 7a shows the optimal Pigouvian tax levied on large banks (dark blue solid line), small banks (light blue solid line), as well as the average tax (green solid line) in our baseline calibration. The optimal tax for small banks is uniform and independent of bank size, consistently with our results in Proposition 4. Figure 7b shows the optimal size tax, which corresponds to the difference between the taxes levied on large and small banks in Figure 7a.

6.3 Sensitivity analysis: granularity parameter

We explore the sensitivity of our results to the granularity parameter ζ, which is crucial to generating meaningful counterfactuals regarding changes in bank concentration. Figure 8 assesses the sensitivity of our quantitative results to ζ. Recall that $\zeta = 1$ corresponds to the case where idiosyncratic shocks occur at the bank level, consistent with the granular hypothesis; $\zeta \simeq 0$ corresponds to the case where large banks are perfectly diversified. It is clear that granularity is important for our findings. The high levels of leverage associated with our baseline calibration, and the strategic effects of bank size, arise only for ζ sufficiently close to 1, and are small for $\zeta < 0.5$.

Figure 9 illustrates the deeper reasons for this result by examining large banks' objective function and expected government transfers. As ζ falls, large banks face lower idiosyncratic risk and are less likely to default for given leverage choices. Hence, bailout transfers become less relevant relative to trade-off theory considerations, dampening the strategic differences between large and small banks, and inducing lower aggregate leverage in equilibrium. This occurs because the expected government transfer become less sensitive to large banks' borrowing choices, which decreases large banks' marginal benefit from borrowing, consistently with banks' optimality conditions (10) and (11).

The granular case $\zeta \simeq 1$ is empirically plausible: Minton, Stulz and Taboada (2017), for example, report a marginally *higher* standard deviation of return on assets for the largest banks in the Call Reports data – our own unreported calculations replicate their conclusions. This is consistent with estimates for large companies across industries in Gabaix (2011). Therefore, the sensitivity of our

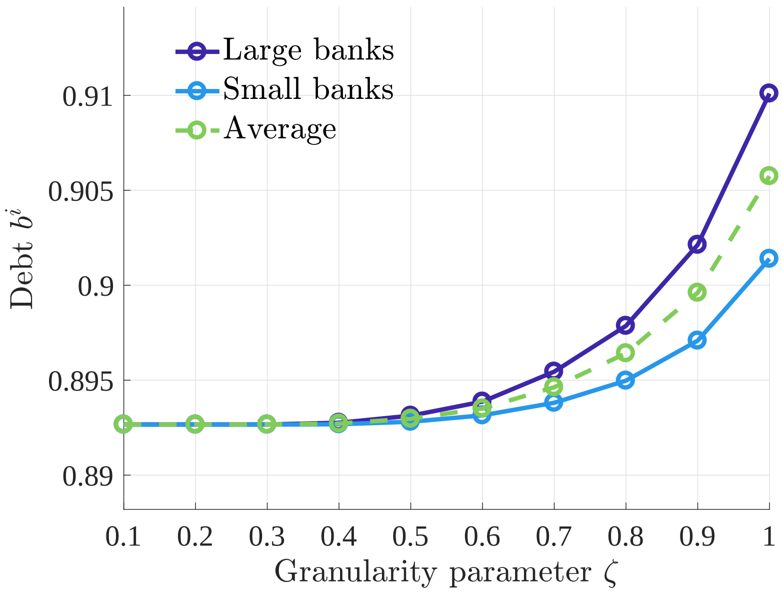


Figure 8: Sensitivity analysis: Granularity parameter ζ

Note: Figure 8 shows the effect of varying the granularity parameter ζ on banks' borrowing choices in our baseline calibration.

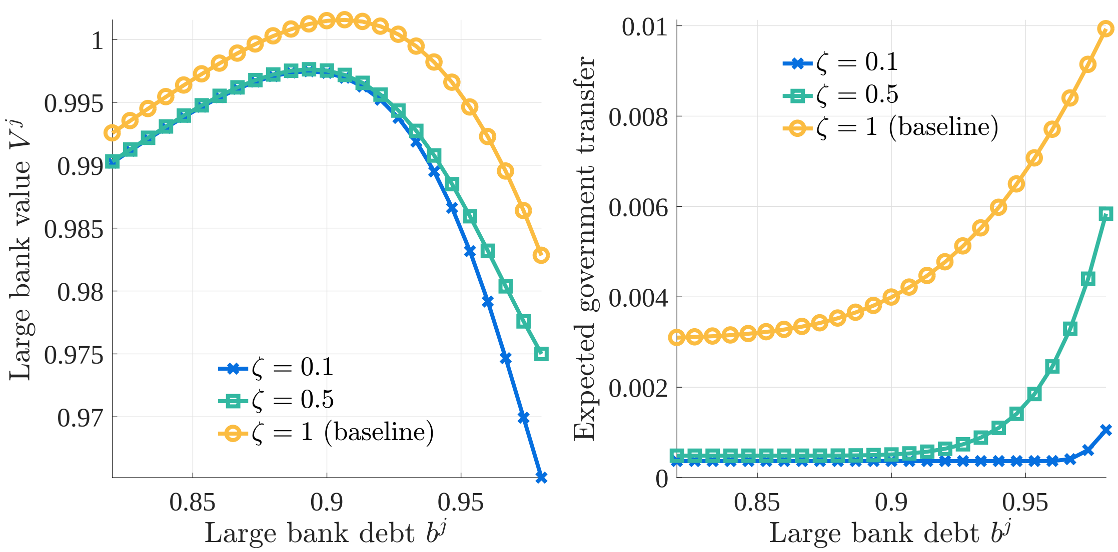


Figure 9: Large banks' incentives and the granularity parameter ζ

Note: Figure 9 illustrates the effect of varying the granularity parameter ζ on large banks' objective function. The left panel plots the value $V\left(b^j, t\left(\boldsymbol{b}|s\right)\right)$ of a large bank as a function of its borrowing b^j, for different levels of ζ, assuming that all other banks select $\boldsymbol{b}^{-j} = 0.9$. The right panel plots the corresponding expected government transfer $\mathbb{E}\left[t\left(\boldsymbol{b}|s\right)\right]$ as a function of b^j.

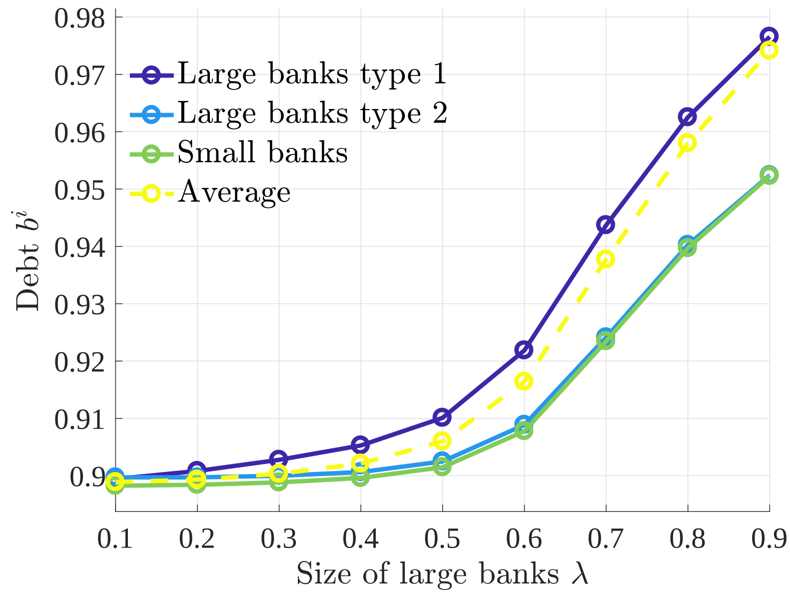


Figure 10: Sensitivity analysis: Bank size distribution

Note: Figure 10 shows the optimal equilibrium debt-to-asset ratio chosen by the 5 largest banks (type 1, dark blue solid line), the 15 additional large banks (type 2, light blue solid line) and by small banks (green solid line) for different levels of the share of assets held by type 1 large banks $\lambda \in [0.1, 0.9]$. The relative asset shares of remaining banks are held constant at the baseline level, so that type 2 large and small banks control shares $0.4(1-\lambda)$ and $0.6(1-\lambda)$ respectively. The Figure also shows the average debt-to-asset ratio in the economy (yellow dashed line).

results to ζ arises in an empirically unlikely region of the parameter space. However, our analysis highlights an interesting feature of the model. The impact of granular shocks on system-wide risk is amplified by the strategic responses of large banks. Therefore, and in addition to the failure of the law of large numbers highlighted by Gabaix (2011), granular shocks further increase aggregate volatility due to the behavioral response of large firms in equilibrium.

6.4 Sensitivity analysis: bank size distribution

Lastly, we evaluate whether our results are sensitive to our approximation of the bank size distribution. Our baseline results use the size distribution shown in Figure 4, with 5 large banks controlling 50% of aggregate assets. For a more detailed calibration, consistent with the Call Reports data (see Figure 1a), we introduce 15 additional “type 2” large banks that jointly control 20% of aggregate assets. The remaining 30% is managed by a continuum of small banks.

Figure 10 illustrates leverage choices in equilibrium. We vary the share λ of assets controlled by the 5 largest banks while holding constant the relative shares of remaining bank types. In the baseline case $\lambda = 0.5$, the largest banks choose debt-to-assets equal to 0.910, while “type 2” banks and small banks select 0.902 and 0.901 respectively. Accordingly, the Figure shows that the relationship between λ and aggregate leverage is also similar to our baseline calibration.

Intuitively, the similarity in behavior of medium-sized and small banks suggests that strategic leverage incentives quickly diminish when considering large banks outside the top 5. Since our model is designed to isolate strategic effects, there appears to be little loss in considering the stylized size distribution in Figure 4.

7 Conclusion

We have shown that the size distribution of financial institutions does matter for the ex-ante determination of leverage when bailouts are possible. Large banks, by internalizing that their actions affect the government's bailout response, find it optimal to increase their leverage in equilibrium. Their increased leverage increases the magnitude of bailouts, thereby encouraging small banks to take on more leverage. Both effects are mutually reinforcing, and generate further increases system-wide leverage in equilibrium. Hence, aggregate leverage and the magnitude of government bailout interventions are larger when large banks are present.

Our results rely on the fact that system-wide policy responses induce strategic complementarities and do not hinge on the exact nature of how banks determine their capital structure. Our findings support the notion that regulators and policymakers must pay special attention to large financial institutions, since they have a direct motive to take on more risk and also because their behavior disproportionally influences the decisions of small players in equilibrium. In this model, a regulator that closely monitors and restricts funding decisions of large financial institutions arises as a natural optimal policy.

A quantitative assessment of the model implies that further increases beyond the current levels of concentration will be associated with a substantial increase in system-wide leverage. At the current concentration levels, the optimal ex-ante corrective policy in our model can be implemented with a size tax on large bank's debt of 40 basis points (0.4%) per dollar of debt issued.

References

Acharya, V.V., and T. Yorulmazer. 2007. "Too many to fail - An analysis of time-inconsistency in bank closure policies." *Journal of Financial Intermediation*, 16(1): 1–31.

Adrian, Tobias, and Markus K Brunnermeier. 2016. "CoVaR." *American Economic Review*, 106(7): 1705–1741.

Angeletos, G-M, and Chen Lian. 2016. "Incomplete information in macroeconomics: Accommodating frictions in coordination." *Handbook of Macroeconomics*, 2: 1065–1240.

Bianchi, Javier. 2016. "Efficient Bailouts?" *American Economic Review*, 106(12): 3607–3659.

Bianchi, Javier, and Enrique Mendoza. 2017. "Optimal Time-Consistent Macroprudential Policy." *Journal of Political Economy*, Forthcoming.

Bulow, Jeremy I, John D Geanakoplos, and Paul D Klemperer. 1985. "Multimarket oligopoly: Strategic substitutes and complements." *Journal of Political Economy*, 93(3): 488–511.

Chari, VV, and Patrick J Kehoe. 2016. "Bailouts, time inconsistency, and optimal regulation: A macroeconomic view." *The American Economic Review*, 106(9): 2458–2493.

Cooper, Russell, and Andrew John. 1988. "Coordinating coordination failures in Keynesian models." *Quarterly Journal of Economics*, 103(3): 441–463.

Corbae, Dean. 2015. "Discussion of large banks, loan rate markup, and monetary policy." *International Journal of Central Banking*, 11: 179–92.

Corbae, Dean, and Pablo D'Erasmo. 2010. "A quantitative model of banking industry dynamics." *Working Paper.*

Corsetti, Giancarlo, Amil Dasgupta, Stephen Morris, and Hyun Song Shin. 2004. "Does one Soros make a difference? A theory of currency crises with large and small traders." *Review of Economic Studies*, 71(1): 87–113.

Cuciniello, Vincenzo, and Federico M Signoretti. 2015. "Large Banks, Loan Rate Markup, and Monetary Policy." *International Journal of Central Banking.*

Dahlby, Bev. 2008. "The marginal cost of public funds." *The MIT Press*, 1(8): 3–2.

Davison, Lee K, and Carlos D Ramirez. 2014. "Local banking panics of the 1920s: Identification and determinants." *Journal of Monetary Economics*, 66: 164–177.

Davydenko, Sergei A, Ilya A Strebulaev, and Xiaofei Zhao. 2012. "A market-based study of the cost of default." *The Review of Financial Studies*, 25(10): 2959–2999.

DeAngelo, Harry, and René M Stulz. 2015. "Liquid-claim production, risk management, and bank capital structure: Why high leverage is optimal for banks." *Journal of Financial Economics*, 116(2): 219–236.

Diamond, D. W., and R G. Rajan. 2001. "Liquidity Risk, Liquidity Creation and Financial Fragility: A Theory of Banking." *Journal of Political Economy*, 109(2): 287–327.

Diamond, D. W., and R G. Rajan. 2012. "Illiquid Banks, Financial Stability, and Interest Rate Policy." *Journal of Political Economy*, 120(3): 552–591.

Drechsler, Itamar, Alexi Savov, and Philipp Schnabl. 2016. "The deposits channel of monetary policy." *The Quarterly Journal of Economics.*

Drechsler, Itamar, Alexi Savov, and Philipp Schnabl. 2017. "Banking on Deposits: Maturity Transformation without Interest Rate Risk."

Drozd, Lukasz, and Ricardo Serrano-Padial. 2016. "Credit Enforcement Cycles." *Working Paper.*

Egan, Mark, Ali Hortaçsu, and Gregor Matvos. 2017. "Deposit competition and financial fragility: Evidence from the us banking sector." *American Economic Review*, 107(1): 169–216.

Farhi, Emmanuel, and Jean Tirole. 2012. "Collective moral hazard, maturity mismatch, and systemic bailouts." *The American Economic Review*, 102(1): 60–93.

Fernholz, Ricardo T, and Christoffer Koch. 2016. "Why are big banks getting bigger?" *Working Paper, Dallas Fed.*

Freixas, Xavier. 1999. "Optimal Bail-Out, Conditionality and Creative Ambiguity." *CEPR Discussion Papers.*

Freixas, Xavier, and Cornelia Holthausen. 2005. "Interbank market integration under asymmetric information." *Review of Financial Studies*, 18(2): 459.

Freixas, Xavier, and Jean-Charles Rochet. 2008. *Microeconomics of Banking, 2nd Edition.* The MIT Press.

Gabaix, Xavier. 2011. "The granular origins of aggregate fluctuations." *Econometrica*, 79(3): 733–772.

Gandhi, Priyank, and Hanno Lustig. 2015. "Size anomalies in US bank stock returns." *The Journal of Finance*, 70(2): 733–768.

Gorton, Gary, and Andrew Winton. 2003. "Financial intermediation." *Handbook of the Economics of Finance*, 1: 431–552.

Gorton, Gary, and George Pennacchi. 1990. "Financial intermediaries and liquidity creation." *Journal of Finance*, 49–71.

Gourinchas, Pierre Olivier, and Philippe Martin. 2017. "Economics of Sovereign Debt, Bailouts and the Eurozone Crisis." *Working Paper.*

Hachem, Kinda, and Zheng Michael Song. 2017. "Liquidity Rules and Credit Booms." *Working Paper.*

Hennessy, Christopher A, and Toni M Whited. 2005. "Debt dynamics." *The Journal of Finance*, 60(3): 1129–1165.

Holmstrom, Bengt, and Jean Tirole. 1998. "Private and public supply of liquidity." *Journal of Political Economy*, 106(1): 1–40.

Hüttl, Pia, and Dirk Schoenmaker. 2016. "Fiscal capacity to support large banks." *Policy Contribution, Bruegel*, 17.

Janicki, Hubert P., and Edward S. Prescott. 2006. "Changes in the Size Distribution of US Banks: 1960–2005." *Federal Reserve Bank of Richmond Economic Quarterly*, 92(4): 291.

Kacperczyk, Marcin, Jaromir Nosal, and Savitar Sundaresan. 2017. "Market Power and Informational Efficiency." *Working Paper.*

Keister, Todd. 2016. "Bailouts and financial fragility." *The Review of Economic Studies*, 83(2): 704–736.

Kelly, Bryan, Hanno Lustig, and Stijn Van Nieuwerburgh. 2016. "Too-systemic-to-fail: What option markets imply about sector-wide government guarantees." *The American Economic Review*, 106(6): 1278–1319.

Kraus, Alan, and Robert H Litzenberger. 1973. "A state-preference model of optimal financial leverage." *Journal of Finance*, 28(4): 911–922.

Laeven, Luc, and Fabian Valencia. 2013. "Systemic banking crises database." *IMF Economic Review*, 61(2): 225–270.

Laeven, Luc, Lev Ratnovski, and Hui Tong. 2014. "Bank size and systemic risk." *IMF Staff Papers*, , (14).

Mendicino, Caterina, Kalin Nikolov, and Javier Suarez. 2017. "Equity versus Bail-in Debt in Banking: An Agency Perspective." *CEPR Working Paper.*

Milgrom, Paul, and Chris Shannon. 1994. "Monotone comparative statics." *Econometrica*, 62(1): 157–180.

Minton, Bernadette A., Rene M. Stulz, and Alvaro G. Taboada. 2017. "Are Larger Banks Valued More Highly?" *NBER Working Paper.*

Modigliani, F., and M.H. Miller. 1958. "The cost of capital, corporation finance and the theory of investment." *The American economic review*, 48(3): 261–297.

Myers, Stewart C. 1984. "The capital structure puzzle." *Journal of Finance*, 39(3): 574–592.

Nosal, Jaromir B, and Guillermo Ordoñez. 2016. "Uncertainty as commitment." *Journal of Monetary Economics*, 80: 124–140.

Reinhart, Carmen M, and Kenneth Rogoff. 2009. *This time is different: Eight centuries of financial folly.* Princeton University Press.

Sakovics, Jozsef, and Jakub Steiner. 2012. "Who matters in coordination problems?" *The American Economic Review*, 102(7): 3439–3461.

Schneider, Martin, and Aaron Tornell. 2004. "Balance Sheet Effects, Bailout guarantees and Financial crises." *Review of Economic Studies*, 71(3): 883–913.

Stein, Jeremy C. 2012. "Monetary policy as financial stability regulation." *The Quarterly Journal of Economics*, 127(1): 57–95.

Stern, Gary H, and Ron J Feldman. 2004. *Too big to fail: The hazards of bank bailouts.* Brookings Institution Press.

Strebulaev, Ilya A, and Toni M Whited. 2012. "Dynamic models and structural estimation in corporate finance." *Foundations and Trends in Finance*, 6(1–2): 1–163.

Appendix

A Quotations

Breaking up big banks wouldn't really solve our problems, because it's perfectly possible to have a financial crisis that mainly takes the form of a run on smaller institutions.

(...) Breaking up big financial institutions wouldn't prevent future crises, nor would it eliminate the need for bailouts when those crises happen. **The next bailout wouldn't be concentrated on a few big companies - but it would be a bailout all the same**. I don't have any love for financial giants, but I just don't believe that breaking them up solves the key problem.

Paul Krugman. The New York Times, 04/01/2010

Most observers who study this believe that to try to break banks up into a lot of little pieces would hurt our ability to serve large companies and hurt the competitiveness of the United States. But that's not the important issue. They believe that it would actually make us less stable, because the individual banks would be less diversified and, therefore, at greater risk of failing, because they would haven't profits in one area to turn to when a different area got in trouble.

And most observers believe that **dealing with the simultaneous failure of many small institutions would actually generate more need for bailouts** and reliance on taxpayers than the current economic environment.

Lawrence Summers. Interview with Jeffrey Brown, PBS Newshour, 04/22/2010

But, while regulation must address the oversized bank balance sheets that were at the root of the crisis, the IMF is right not to focus excessively on fixing the "too big to fail" problem. **A surprising number of pundits seem to think that if one could only break up the big banks, governments would be far more resilient to bailouts**, and the whole "moral hazard" problem would be muted. **That logic is dubious**, given how many similar crises have hit widely differing systems over the centuries. **A systemic crisis that simultaneously hits a large number of medium-sized banks would put just as much pressure on governments to bail out the system as would a crisis that hits a couple of large banks.**

Kenneth Rogoff. All for One Tax and One Tax for All? Project Syndicate, 04/29/2010

B Proofs and derivations

B.1 Microfoundation for the bank's objective function

We assume in the paper that each bank chooses a capital structure that maximizes the market value of the firm. To derive this objective function, we consider a single bank, and we assume, as in the paper, that there are no conflicts of interest between equity-holders and bank managers. All lowercase variables are defined per unit of capital managed by the bank. Equity-holders maximize expected utility of consumption, $c_0 + \beta \mathbb{E}[c_1]$. Their date 0 and date 1 consumption is respectively given by

$$c_0 = w_0 - e_0$$

where w_0 is their initial wealth and e_0 is their inside equity contribution to date 0 investment (a dividend payout at date 0 implies $e_0 < 0$); and

$$c_1 = w_1 + \sigma d_1 (1 - \psi)$$

where d_1 is the final dividend paid to shareholders at date 1, w_1 is shareholders' exogenous date 1 endowment, and σ is the share of equity retained by initial equity-holders. Note that w_1 and d_1 can both be random variables.

In addition, the bank issues debt with face value b_0 and initial market value $q_0 b_0$, as well as outside equity, which is associated with a claim to a share $1-\sigma$ of final dividends, and has initial market value $\bar{e}_0$. Since the bank can make a fixed investment of one dollar per unit of capital at date 0, its budget constraint is

$$\underbrace{1}_{\text{Investment}} = \underbrace{e_0}_{\text{Inside equity}} + \underbrace{\bar{e}_0}_{\text{Outside equity}} + \underbrace{q_0 b_0}_{\text{Debt market value}}$$

The final dividend d_1 is defined as the residual claim on the bank's assets, including any transfers from the government, and therefore given by the random variable

$$d_1 = \max\{u + t - b_0, 0\}$$

The market values of debt and outside equity satisfy

$$q_0 b_0 = \beta\left[\int_{\mathcal{D}} (\phi u + t)\, dF + b_0 \int_{\mathcal{N}} dF\right] \tag{A.1}$$

$$\bar{e}_0 = (1-\sigma)\,\beta\left[(1-\psi)\int_{\mathcal{N}} (u + t - b_0)\, dF\right] \tag{A.2}$$

where $\mathcal{D} = \{u + t < b_0\}$ is the default event, and $\mathcal{N} = \mathcal{D}^c$.

Shareholders' full maximization problem, after substituting the budget to eliminate inside equity e_0, becomes

$$\max_{b_0, q_0, \bar{e}_0, \sigma} -(1 - q_0 b_0 - \bar{e}_0) + \sigma\beta(1-\psi)\int_{\mathcal{N}} (u + t - b_0)\, dF \text{ subject to (A.1) and (A.2)}$$

Substituting the market pricing constraints (A.1) and (A.2), we can further eliminate q_0 and $\bar{e}_0$ to get the equivalent unconstrained problem

$$\max_{b_0,\sigma} -1 + \beta\left[\int_{\mathcal{D}} (\phi u + t)\, dF + b_0 \int_{\mathcal{N}} dF + \underbrace{(\sigma + (1-\sigma))}_{=1}(1-\psi)\int_{\mathcal{N}} [u + t - b_0]\, dF\right]$$

Note that the share σ of equity retained by insiders is indeterminate, because outside equity is fairly priced. It is then straightforward to show that this problem reduces to

$$\max_{b_0} V(b_0, t)$$

with $V(b_0, t) = \mathbb{E}[u + t] - (1-\phi)\Pr[\mathcal{D}]\,\mathbb{E}[u|\mathcal{D}] - \psi\Pr[\mathcal{N}]\,\mathbb{E}[u + t - b_0|\mathcal{N}]$ is the market value of the bank, and corresponds exactly to Equation (8) in the paper. Therefore, the bank's initial equity-holders optimally choose the capital structure to maximize the market value of the firm.

B.2 Section 3

Government's problem

For a given signal realization s, the government optimally chooses a transfer $t(\boldsymbol{b}|s)$ at date 1 to maximize:

$$\max_{t \geq 0} W_1(\boldsymbol{b}, t|s),$$

where $W_1(\boldsymbol{b}, t|s)$ is given by

$$W_1(\boldsymbol{b}, t|s) = \int_0^\infty u dF_u(u|s) - (1-\phi)\int \left[\int_0^{b^i - t} u dF_u(u|s)\right] d\mu - \left(\psi - \bar{\psi}\right)\int \left[\int_{b^i - t}^\infty (t + u - b^i) dF_u(u|s)\right] d\mu - \kappa(t),$$

which corresponds to Equation (2) in the text. Note that the outer integrals in the second and third terms are cross-sectional integrals over banks i, while the inner ones are integrals over the possible realizations of u given the signal s.

The first-order and second-order conditions to this problem correspond to

$$\frac{\partial W_1(\boldsymbol{b}, t|s)}{\partial t} = (1-\phi)\int f_u\left(b^i - t|s\right)\left(b^i - t\right) d\mu - \left(\psi - \bar{\psi}\right)\int \left[1 - F_u\left(b^i - t|s\right)\right] d\mu - \kappa'(t)$$
$$\frac{\partial^2 W_1(\boldsymbol{b}, t|s)}{\partial t^2} = -(1-\phi)\int \left[f_u'\left(b^i - t|s\right)\left(b^i - t\right) + f_u\left(b^i - t|s\right)\right] d\mu - \left(\psi - \bar{\psi}\right)\int f_u\left(b^i - t|s\right) d\mu - \kappa''(t) \leq 0.$$

Note that the second-order condition is always satisfied with strict inequality under our regularity condition (1).[25] It immediately follows that $\lim_{t\to\infty} \frac{\partial W_1(\boldsymbol{b}, t|s)}{\partial t} < 0$, which guarantees that the optimal t is bounded above. Note further that

$$\left.\frac{\partial W_1(\boldsymbol{b}, t|s)}{\partial t}\right|_{t=0} = (1-\phi)\int f_u\left(b^i|s\right) b^i d\mu - \left(\psi - \bar{\psi}\right)\int \left[1 - F_u\left(b^i|s\right)\right] d\mu - \kappa'(0),$$

which implies that the optimal transfer $t(\boldsymbol{b}|s)$ is zero when $(1-\phi)\int f_u\left(b^i|s\right) b^i d\mu \leq \kappa'(0) + \left(\psi - \bar{\psi}\right)\int \left[1 - F_u\left(b^i|s\right)\right] d\mu$, and determined uniquely by the first-order condition otherwise.

Totally differentiating the first-order condition with respect to b^j, we have

$$\begin{aligned} 0 &= \frac{\partial^2 W_1(\boldsymbol{b}, t|s)}{\partial t^2} \times \frac{\partial t}{\partial b^j} + \frac{\partial^2 W_1(\boldsymbol{b}, t|s)}{\partial t \partial b^j} \\ &= \frac{\partial^2 W_1(\boldsymbol{b}, t|s)}{\partial t^2} \times \frac{\partial t}{\partial b^j} + \mu(j) \times \left[(1-\phi)\left(f_u'\left(b^j - t|s\right)\left(b^j - t\right) + f_u\left(b^j - t|s\right)\right) + \left(\psi - \bar{\psi}\right) f_u\left(b^j - t|s\right)\right], \end{aligned}$$

and it follows that

$$\frac{\partial t(\boldsymbol{b}|s)}{\partial b^j} = \left(-\frac{\partial^2 W_1(\boldsymbol{b}, t|s)}{\partial t^2}\right)^{-1} \mu(j)\left[(1-\phi)\left(f_u'\left(b^j - t|s\right)\left(b^j - t\right) + f_u\left(b^j - t|s\right)\right) + \left(\psi - \bar{\psi}\right) f_u\left(b^j - t|s\right)\right].$$

Note that first factor is strictly positive, given the second-order condition, which implies Equation (5).

Banks' market value

The ex-post payoffs to debtholders and shareholders are defined in Equations (6) and (7) in the text. Taking expectations under the joint distribution of returns and signals $F(u, s)$ yields the date 0 value of bonds and shares for bank i, respectively given by

$$q^i b^i = \int_{\mathcal{N}^i} b^i dF + \int_{\mathcal{D}^i} (\phi u + t(\boldsymbol{b}|s)) dF$$
$$e^i = (1-\psi)\int_{\mathcal{N}^i} (u + t(\boldsymbol{b}|s) - b^i) dF,$$

[25] Note that when $u|s$ is normally distributed, $x\frac{f_u'(x)}{f_u(x)} = \frac{x - \mu_{u|s}}{\sigma_{u|s}}\frac{1}{\sigma_{u|s}}$, so Condition (1) is satisfied whenever $\sigma_{u|s}$ is sufficiently large.

where the default and repayment regions are defined as $\mathcal{D}^i = \left\{t(\boldsymbol{b}|s) + u < b^i\right\}$ and $\mathcal{N}^i = \left\{t(\boldsymbol{b}|s) + u \geq b^i\right\}$. Adding up both, we can express the market value of the bank as follows

$$\begin{aligned}
q^i b^i + e^i &= \int_{\mathcal{N}^i} \left[u + t(\boldsymbol{b}|s) - \psi\left(u + t(\boldsymbol{b}|s) - b^i\right)\right] dF + \int_{\mathcal{D}^i} \left[u + t(\boldsymbol{b}|s) - (1-\phi)u\right] dF \\
&= \int \left(u + t(\boldsymbol{b}|s)\right) dF - (1-\phi) \int_{\mathcal{D}^i} u dF - \psi \int_{\mathcal{N}^i} \left(u + t(\boldsymbol{b}|s) - b^i\right) dF \\
&= \mathbb{E}\left[u + t(\boldsymbol{b}|s)\right] - (1-\phi) \Pr\left[\mathcal{D}^i\right] \mathbb{E}\left[u|\mathcal{D}^i\right] - \psi \Pr\left[\mathcal{N}^i\right] \mathbb{E}\left[u + t(\boldsymbol{b}|s) - b^i|\mathcal{N}^i\right], \qquad \text{(A.3)}
\end{aligned}$$

which corresponds to the definition of $V\left(b^i, t(\boldsymbol{b}|s)\right)$ in Equation (8).

Banks' leverage choices

The bank maximizes its market value. Let $\mathcal{S}$ denote the support of the signal s, and recall that $G(s)$, $s \in \mathcal{S}$, is the marginal distribution of signals. We can write the bank's value, from (A.3), more explicitly as

$$V\left(b^i, t(\boldsymbol{b}|s)\right) = \mathbb{E}[u] + \int_{\mathcal{S}} \left[t(\boldsymbol{b}|s) - (1-\phi) \int_0^{b^i - t(\boldsymbol{b}|s)} u dF_u(u|s) - \psi \int_{b^i - t(\boldsymbol{b}|s)}^{\infty} \left(u + t(\boldsymbol{b}|s) - b^i\right) dF_u(u|s)\right] dG(s).$$

Partially differentiating with respect to b^i gives

$$\begin{aligned}
\frac{\partial V}{\partial b^i} &= \int_{\mathcal{S}} \left[-(1-\phi)\left(b^i - t(\boldsymbol{b}|s)\right) f_u\left(b^i - t(\boldsymbol{b}|s)|s\right) + \psi \int_{b^i - t(\boldsymbol{b}|s)}^{\infty} dF_u(u|s)\right] dG(s) \\
&= \psi \Pr\left[\mathcal{N}^i\right] - (1-\phi) \int_{\partial \mathcal{D}^i} u dF,
\end{aligned}$$

which corresponds to Equation (10) in the text. The total effect of increasing b^i further takes into account the effect of b^i on optimal transfers, and is given by

$$\begin{aligned}
\frac{dV}{db^i} &= \frac{\partial V}{\partial b^i} + \int_{\mathcal{S}} \frac{\partial t(\boldsymbol{b}|s)}{\partial b^i} \left[1 + (1-\phi)\left(b^i - t(\boldsymbol{b}|s)\right) f_u\left(b^i - t(\boldsymbol{b}|s)|s\right) - \psi \int_{b^i - t(\boldsymbol{b}|s)}^{\infty} dF_u(u|s)\right] dG(s) \\
&= \frac{\partial V}{\partial b^i} + \mathbb{E}\left[\frac{\partial t(\boldsymbol{b}|s)}{\partial b^i}\left(1 - \psi \mathbf{1}\left(\mathcal{N}^i\right)\right)\right] + (1-\phi) \int_{\partial \mathcal{D}^i} \frac{\partial t(\boldsymbol{b}|s)}{\partial b^i} u dF,
\end{aligned}$$

which corresponds to Equation (11) in the text. The first-order conditions (10) and (11) follow by noting that $\frac{\partial t(\boldsymbol{b}|s)}{\partial b^i} = 0$ for small banks.

Proof of Lemma 1 (Effect of large bank leverage on small banks' incentives.)

Proof. Differentiating the bank's value function in (8) with respect to b^j gives

$$\frac{dV\left(b^i, t(\boldsymbol{b}|s)\right)}{db^j} = \mathbb{E}\left[\frac{\partial t(\boldsymbol{b}|s)}{\partial b^j}\left(1 - \psi \mathbf{1}\left(\mathcal{N}^i\right)\right)\right] + (1-\phi) \int_{\partial \mathcal{D}^i} \frac{\partial t(\boldsymbol{b}|s)}{\partial b^j} u dF > 0,$$

where the inequality follows from the properties of optimal bailouts. Indeed, (5) implies that $\frac{\partial t(\boldsymbol{b}|s)}{\partial b^j} \geq 0$, with strict inequality for the (positive measure) set of public signals s such that $t(\boldsymbol{b}|s) > 0$.

Differentiating again with respect to b^i gives

$$\frac{d^2 V\left(b^i, t(\boldsymbol{b}|s)\right)}{db^i db^j} = \psi \int_{\partial \mathcal{D}^i} \frac{\partial t(\boldsymbol{b}|s)}{\partial b^j} dF + (1-\phi) \int_{\partial \mathcal{D}^i} \frac{\partial t(\boldsymbol{b}|s)}{\partial b^j} \left\{1 + \frac{u f_u'(u|s)}{f_u(u|s)}\right\} dF > 0,$$

where the inequality follows from our regularity condition (1). $\square$

Proof of Proposition 1 (Large banks borrow more.)

Proof. Take any equilibrium with leverage choices $\hat{\boldsymbol{b}}$ and ex-post optimal transfer $\hat{t}(s) = t\left(\hat{\boldsymbol{b}}|s\right)$. Suppose that j is a large bank, and let $t^j\left(b^j|s\right) = t\left(\boldsymbol{b}|s\right)|_{\boldsymbol{b}^{-j}=\hat{\boldsymbol{b}}^{-j}}$ be the transfer which becomes ex-post optimal if a large bank j chooses borrowing level b^j, while all other banks play their equilibrium strategy (note that $t^j(\hat{b}^j|s) \equiv \hat{t}(s)$).

First, suppose that $\hat{b}^i > \hat{b}^j$ for a small bank i and a large bank j. Optimality implies that neither bank has a profitable deviation by copying the other's strategy. Bank i's market value in equilibrium is $V\left(\hat{b}^i, \hat{t}\left(s\right)\right)$. If bank i deviates from equilibrium by copying bank j's choice $\hat{b}^j$, the bailout policy is unchanged because bank i is small, and i's market value becomes $V\left(\hat{b}^j, \hat{t}(s)\right)$. Therefore, optimality for bank i implies

$$V\left(\hat{b}^i, \hat{t}\left(s\right)\right) \geq V\left(\hat{b}^j, \hat{t}\left(s\right)\right).$$

Bank j's market value in equilibrium is $V\left(\hat{b}^j, \hat{t}\left(s\right)\right)$. If bank j deviates to $\hat{b}^i$, then the bailout policy changes to $t^j\left(\hat{b}^i|s\right)$, and j's market value becomes $V\left(\hat{b}^i, t^j\left(\hat{b}^i|s\right)\right)$. Therefore, optimality for bank j requires that

$$V\left(\hat{b}^j, \hat{t}\left(s\right)\right) \geq V\left(\hat{b}^i, t^j\left(\hat{b}^i|s\right)\right).$$

Combining the two optimality conditions, we have $V\left(\hat{b}^i, \hat{t}\left(s\right)\right) \geq V\left(\hat{b}^i, t^j\left(\hat{b}^i|s\right)\right)$, or equivalently,

$$0 \geq V\left(\hat{b}^i, t^j\left(\hat{b}^i|s\right)\right) - V\left(\hat{b}^i, t^j\left(\hat{b}^j|s\right)\right) = \int_{\hat{b}^j}^{\hat{b}^i} \left.\frac{dV\left(\hat{b}^i, t\left(\boldsymbol{b}|s\right)\right)}{db^j}\right|_{\boldsymbol{b}^{-j}=\hat{\boldsymbol{b}}^{-j}} db^j,$$

which contradicts Lemma 1. Therefore, $\hat{b}^j \geq \hat{b}^i$.

Second, suppose that $\hat{b}^j = \hat{b}^i$. We begin by showing that the small bank makes an interior choice in equilibrium, with $0 < \hat{b}^i < \infty$. Taking limits of the derivative in (10), we get

$$\lim_{b^i \to 0} \frac{\partial V\left(b^i, \hat{t}(s)\right)}{\partial b^i} = \psi \Pr\left[\mathcal{N}^i\right] - (1-\phi) \int_{\partial \mathcal{D}^i} \left(-\hat{t}\left(s\right)\right) dF > 0.$$

Thus the small bank must choose $\hat{b}^i > 0$. Moreover, we have

$$\lim_{b^i \to \infty} \frac{\partial V\left(b^i, \hat{t}(s)\right)}{\partial b^i} = -\left(1-\phi\right) \lim_{b^i \to \infty} \int_{\partial \mathcal{D}^i} \left(b^i - \hat{t}\left(s\right)\right) dF.$$

Note that the integral is strictly positive for a given b^i if and only if $b^i > \mathbb{E}\left[\hat{t}\left(s\right)|\mathcal{D}^i\right]$. The convexity of the cost $\kappa\left(t\right)$ of bailouts, combined with our assumption that $\lim_{t \to \infty} \kappa\left(t\right) = \infty$, implies that $\hat{t}\left(s\right)$ is bounded above. Since the small bank takes $\hat{t}\left(s\right)$ as given, it follows that the integral is positive for large enough b^i, so that $\lim_{b^i \to \infty} \frac{\partial V\left(b^i, \hat{t}(s)\right)}{\partial b^i} < 0$, which establishes that the optimal choice $\hat{b}^i < \infty$. Therefore small banks are at an interior solution with $\hat{b}^i \in (0, \infty)$, and the first-order condition

$$\frac{\partial V\left(\hat{b}^i, \hat{t}(s)\right)}{\partial b^i} = 0.$$

holds with equality. The large bank's first-order condition, using the conjecture $\hat{b}^j = \hat{b}^i$, can in turn be written as

$$\frac{\partial V\left(\hat{b}^i, \hat{t}(s)\right)}{\partial b^i} + \frac{dV\left(\hat{b}^i, t\left(\boldsymbol{b}|s\right)\right)}{db^j} = 0.$$

Lemma 1 implies that the second term is strictly positive, and we obtain $\frac{\partial V\left(\hat{b}^i, \hat{t}(s)\right)}{\partial b^i} < 0$, a contradiction. $\square$

Proof of Proposition 2 (When large banks are present, small banks take more leverage.)

Proof. Let $t_0(b|s)$ denote the ex-post optimal transfer when there are only small banks who play the symmetric strategy $b^i = b$, and let $BR_0(b) = \arg\max_{b^i} V\left(b^i, t_0(b|s)\right)$ be a small bank's best response to this transfer. b is a symmetric equilibrium if $b \in BR_0(b)$. Since $\left.\frac{\partial V(b^i,t)}{\partial b^i}\right|_{b^i=0} > 0$ regardless of transfers, we know that $BR_0(0) > 0$. By Tarski's fixed point theorem, the smallest equilibrium b_0 exists, and moreover, we have $\inf BR_0(b) > b$ for all $b < b_0$.

Now take any economy with large banks, and any equilibrium (not necessarily symmetric) with leverage choices $\hat{\boldsymbol{b}}$ and ex-post optimal transfer $\hat{t}(s) = t\left(\hat{\boldsymbol{b}}|s\right)$. Let the lowest borrowing level arising in equilibrium be $b_1 \equiv \inf_i \hat{b}^i$. Proposition 1 implies that a positive measure of banks (at least all large banks) chooses to borrow strictly more than b_1. Moreover, a parallel argument to Lemma 1 implies that incentives to borrow for small banks are strictly stronger under the equilibrium transfer $\hat{t}(s)$, than under the transfer $t_0(b_1|s)$ which would obtain if everybody chose b_1:

$$\frac{\partial V\left(b^i, \hat{t}(s)\right)}{\partial b^i} > \frac{\partial V\left(b^i, t_0(b_1|s)\right)}{\partial b^i}. \tag{A.4}$$

We need to show that $b_1 > b_0$.

First, suppose that $b_1 < b_0$. Let $\tilde{b}_1 = \inf BR_0(b_1)$ be the lowest borrowing level which small banks pick if everybody else chooses b_1. We know that $\tilde{b}_1 > b_1$, because our assumption $b_1 < b_0$ implies that $BR_0(b_1) > b_1$. Since $\tilde{b}_1$ maximizes bank value given the transfer $t_0(b_1|s)$, we have

$$V\left(\tilde{b}_1, t_0(b_1|s)\right) \geq V\left(b_1, t_0(b_1|s)\right)$$

Moreover, since b_1 maximizes some (small) bank's value given the equilibrium transfer $\hat{t}(s)$, we have

$$V\left(b_1, \hat{t}(s)\right) \geq V\left(\tilde{b}_1, \hat{t}(s)\right).$$

Combining,

$$V\left(\tilde{b}_1, t_0(b_1|s)\right) - V\left(b_1, t_0(b_1|s)\right) \geq 0 \geq V\left(\tilde{b}_1, \hat{t}(s)\right) - V\left(b_1, \hat{t}(s)\right),$$

which implies

$$\int_{b_1}^{\tilde{b}_1} \left(\frac{\partial V\left(b^i, t_0(b_1|s)\right)}{\partial b^i} - \frac{\partial V\left(b^i, \hat{t}(s)\right)}{\partial b^i}\right) \geq 0,$$

contradicting (A.4).

Second, suppose that $b_1 = b_0$. Since b_0 is optimal if everybody else chooses b_0, we have the first-order condition for a small bank

$$\frac{\partial V\left(b_0, t_0\left((b_0|s)\right)\right)}{\partial b^i} = \frac{\partial V\left(b_1, t_0(b_1|s)\right)}{\partial b^i} = 0.$$

Moreover, since some small bank optimally chooses b_1 in equilibrium, we have

$$\frac{\partial V\left(b_1, \hat{t}(s)\right)}{\partial b^i} = 0 = \frac{\partial V\left(b_1, t_0(b_1|s)\right)}{\partial b^i},$$

again contradicting (A.4). □

B.3 Section 4

Proof of Lemma 2 (Commitment is irrelevant when the planner controls banks.)

Proof. Taking expectations over signals in (2) and comparing to (16) we obtain, for any (not necessarily optimal) bailout policy $t(s)$,

$$W_0(\boldsymbol{b}, t(s)) = \mathbb{E}\left[W_1(\boldsymbol{b}, t(s)|s)\right].$$

Suppose that $W^c > W^{nc}$. Then the constraint in W^{nc} must bind in some states, that is, the policy achieving W^c is not ex-post optimal for a set of signals s with positive probability measure. Call this policy $\{\boldsymbol{b}^c, t^c(s)\}$ and consider replacing the transfer policy with the ex-post best response to $\boldsymbol{b}^c$, setting $t'(s) = \arg\max_t W_1(\boldsymbol{b}^c, t|s)$ for almost all s. By ex-post optimality, $W_1(\boldsymbol{b}^c, t'|s) \geq W_1(\boldsymbol{b}^c, t^c|s)$, with strict inequality for a positive measure of signals. The policy $\{\boldsymbol{b}^c, t'(s)\}$ yields ex-ante welfare $W_0(\boldsymbol{b}^c, t'(s)) = \mathbb{E}[W_1(\boldsymbol{b}^c, t'(s)|s)] > \mathbb{E}[W_1(\boldsymbol{b}^c, t^c(s)|s)]$, contradicting optimality of $\{\boldsymbol{b}^c, t^c(s)\}$. $\square$

Proof of Proposition 3 (Efficient leverage is independent of bank size.)

Proof. Lemma 2 shows that the maximization problems with and without commitment are equivalent. We consider the problem (17) with commitment. Suppose policy $\{\boldsymbol{b}^c, t^c(s)\}$ solves this problem. Note that we may write welfare in terms of bank value, the wedge between private and social equity issuance costs, and transfers:

$$W_0(\boldsymbol{b}, t(s)) = \int \left[V(b^i, t(s)) + \bar{\psi}\Pr\left[\mathcal{N}^i\right]\mathbb{E}\left[t(s) + u - b^i\right]\right] d\mu - \mathbb{E}\left[t(s) + \kappa(t(s))\right].$$

Since $t(s)$ may be chosen independently of $\boldsymbol{b}$, the first-order condition for the optimal $\boldsymbol{b}$ is obtained by differentiating pointwise under the integral, and we obtain

$$\frac{\partial V\left(b^{c,i}, t^c(s)\right)}{\partial b^i} = \bar{\psi}\Pr\left[\mathcal{N}^i\right]$$

for all i. But under our regularity condition, the pointwise maximization problem is strictly concave in b^i so that we have $b^{c,i} = b^{c,j} = \bar{b}$ for almost all i, j. $\square$

B.4 Section 5

Equation (22) defines the distribution of total asset returns $u^i = v + w^i$ of bank i. We denote its density conditional on the public signal s by $f_u^i(u|s)$ as before. We impose the equivalent to our previous regularity condition (1) on this density: In marginal default states where $u^i = b^i - t$, we require that

$$\frac{d\log f_u^i(u|s)}{d\log u} > -1, \quad \forall s, i. \tag{A.5}$$

Note that this is generally a weaker condition than our requirement in (1) for the baseline model. Indeed, (1) holds when public signals about the aggregate state are sufficiently noisy. Since idiosyncratic risk effectively introduces noise into the government's signal of bank health, (A.5) is generally an even milder condition than (1).

In the model with idiosyncratic risk, welfare at date 1 is

$$W_1(\boldsymbol{b}, t|s) = \mathbb{E}[v|s] - (1-\phi)\int\left[\int_0^{b^i-t} u dF_u^i(u|s)\right] d\mu - \left(\psi - \bar{\psi}\right)\int\left[\int_{b^i-t}^{\infty}\left(t + u - b^i\right) dF_u^i(u|s)\right] d\mu - \kappa(t), \tag{A.6}$$

The ex-post optimal bailout policy $t(\boldsymbol{b}|s)$ can be characterized exactly as in the text. In particular, for a large bank j, we have the analogue to Equation (5):

$$\frac{\partial t(\boldsymbol{b}|s)}{\partial b^j} \overset{\text{sign}}{=} \left[f_u^{j,\prime}\left(b^j - t|s\right)\left(b^j - t\right) + f_u^j\left(b^j - t|s\right) + \frac{\psi - \bar{\psi}}{1-\phi} f_u^j\left(b^j - t|s\right)\right]\Bigg|_{u = b^j - t(\boldsymbol{b}|s)} \geq 0, \tag{A.7}$$

with strict inequality whenever $t(\boldsymbol{b}|s) > 0$. Now writing down the market value of bank i, we have

$$V\left(b^i, t(\boldsymbol{b}|s)\right) = \int_{\mathcal{S}}\left[t(\boldsymbol{b}|s) - (1-\phi)\int_0^{b^i - t(\boldsymbol{b}|s)} u dF_u^i(u|s) - \psi\int_{b^i - t(\boldsymbol{b}|s)}^{\infty}\left(u + t(\boldsymbol{b}|s) - b^i\right) dF_u^i(u|s)\right] dG(s).$$

Given this characterization, we can establish that the key qualitative properties of the bank's objective function V are as in the baseline model without idiosyncratic risk. In particular, if j is a large bank and i is a small bank, then

1. Bank j has strictly higher incentives to take leverage, other things equal:
$$\frac{dV\left(b^j|t\left(\boldsymbol{b}|s\right)\right)}{db^j} > \frac{dV\left(b^i|t\left(\boldsymbol{b}|s\right)\right)}{db^i}$$
2. Banks' leverage choices are strategic complements:
$$\frac{d^2V\left(b^i, t\left(\boldsymbol{b}|s\right)\right)}{db^i db^j} > 0.$$

The derivations of these properties are identical to those in Appendix B.2 and in the Proof of Lemma 1. We can now apply the arguments of Propositions 1 and 2 without further modification.

C Computational algorithm

1. We write $\theta = \left(\phi, \psi, \bar{\psi}, \kappa_1, \kappa_2, \lambda, N, \mu_v, \sigma_v, \sigma_{w,i}, \sigma_\epsilon\right)$ for the primitives of the model.
2. We first characterize the optimal ex-post transfer given a signal as a function of parameters
$$t\left(\boldsymbol{b}, s; \theta\right),$$
which also allows us to characterize $\frac{\partial t}{\partial b^j}\left(\boldsymbol{b}, s; \theta\right)$ as a function of the same primitives.
Conditional on s, the assets of bank i are distributed $u^i|s \sim N\left(\mu_{u|s}, \sigma^2_{u|s,i}\right)$, where $\mu_{u|s} = \omega s + (1-\omega)\mu_v$ and $\sigma^2_{u|s,i} = \mathbb{V}ar\left[v|s\right] + \mathbb{V}ar\left[w^i\right] = \omega\sigma^2_\epsilon + \sigma^2_{w,i}$, with updating weight $\omega = \frac{\sigma^2_v}{\sigma^2_v + \sigma^2_\epsilon}$.
We write $\varphi\left(x|\mu, \sigma^2\right)$ and $\Phi\left(x|\mu, \sigma^2\right)$ for the normal density and c.d.f. respectively. The first-order condition for ex-post optimal transfers is then
$$(1-\phi)\int \varphi\left(b^i - t|\mu_{u|s}, \sigma^2_{u|s,i}\right)\left(b^i - t\right)d\mu - \left(\psi - \bar{\psi}\right)\int\left[1 - \Phi\left(b^i - t|\mu_{u|s}, \sigma^2_{u|s,i}\right)\right]d\mu - \kappa'\left(t\right) \leq 0$$
with equality whenever $t > 0$.
3. We can compute bank value as follows. Note that the marginal distribution of signals is $s \sim N\left(\mu_s, \sigma^2_s\right)$, where $\sigma^2_s = \sigma^2_v + \sigma^2_\epsilon$.
The value of a bank, given b^i and signal-contingent transfer policy $t\left(s\right)$, is
$$\begin{aligned} V\left(b^i, t\right) &= \mathbb{E}\left[u + t\left(s\right)\right] - (1-\phi)\Pr\left[\mathcal{D}^i\right]\mathbb{E}\left[u|\mathcal{D}^i\right] - \psi\Pr\left[\mathcal{N}^i\right]\mathbb{E}\left[t\left(s\right) + u - b^i|\mathcal{N}^i\right] & \text{(A.8)} \\ &= \mu_u + \int \hat{V}\left(s\right)\varphi\left(s|\mu_s, \sigma^2_s\right)ds, & \text{(A.9)} \end{aligned}$$
where
$$\hat{V}(s) = t\left(s\right) - (1-\phi)\Pr\left[\mathcal{D}^i|s\right]\mathbb{E}\left[u^i|\mathcal{D}^i, s\right] - \psi\Pr\left[\mathcal{N}^i|s\right]\left\{t\left(s\right) + \mathbb{E}\left[u^i|\mathcal{N}^i, s\right] - b^i\right\}.$$
The cutoff for default given s is $u^i = b^i - t\left(s\right)$. Define the normalized cutoff
$$\upsilon^i\left(s\right) = \frac{b^i - t\left(s\right) - \mu_{u|s}}{\sigma_{u|s,i}}$$
We write $\varphi\left(z\right) \equiv \varphi(z|0,1)$ and $\Phi\left(z\right) = \Phi(z|0,1)$ for the standard normal density and c.d.f. We can then compute $\hat{V}(s)$ by noting that
$$\begin{aligned} \Pr\left[\mathcal{D}^i|s\right] &= \Pr\left[u^i \leq b^i - t\left(s\right)|s\right] = \Phi\left(\upsilon^i\left(s\right)\right) \\ \Pr\left[\mathcal{N}^i|s\right] &= 1 - \Phi\left(\upsilon^i\left(s\right)\right) \end{aligned}$$

and

$$\mathbb{E}\left[u^i|\mathcal{D}^i,s\right]=\mathbb{E}\left[u^i|u^i\leq b^i-t\left(s\right),s\right]=\mu_{u|s}-\sigma_{u|s,i}\frac{\varphi\left(v^i\left(s\right)\right)}{\Phi\left(v^i\left(s\right)\right)},$$
$$\mathbb{E}\left[u^i|\mathcal{N}^i,s\right]=\mathbb{E}\left[u^i|u^i>b^i-t\left(s\right),s\right]=\mu_{u|s}+\sigma_{u|s,i}\frac{\varphi\left(v^i\left(s\right)\right)}{1-\Phi\left(v^i\left(s\right)\right)}.$$

4. We then characterize the best response of a small bank, given the borrowing choices of other small and large banks, by solving the first-order condition

$$\frac{\partial V^i}{\partial b^i}=\psi\Pr\left[\mathcal{N}^i\right]-(1-\phi)\int\left(b^i-t\left(s\right)\right)\varphi\left(b^i-t\left(s\right)|\mu_{u|s},\sigma^2_{u|s,i}\right)\varphi\left(s|\mu_s,\sigma^2_s\right)ds=0,$$

which allows us to find bank i's optimal choice

$$b^{i\star}\left(\boldsymbol{b}^{-i};\theta\right).$$

We can then characterize the aggregate best response of all small banks, given the borrowing choices of large banks, as the solution in b^i of $b^i=b^{i\star}\left(b^i,b^L;\theta\right)$.

5. For large banks, we optimize directly the bank's objective function, to allow for possible non-concavities in the objective function of large banks. This yields large bank's optimal choice

$$b^{j\star}\left(\boldsymbol{b}^{-j};\theta\right).$$

Note that, at any interior optimum, large banks best response satisfies Equation (11). We can then characterize the aggregate best response of all large banks, given the borrowing choices of small banks, as the solution in b^j of $b^j=b^{j\star}\left(b^S,b^j;\theta\right)$.

6. Finally, we combine both responses and solve for the value of b^i for both large and small banks that jointly satisfy banks' best responses.