Countercyclical bank liquidity, procyclical capital and their interactions in an equilibrium analysis

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Abstract

I use a dynamic model to analyse the impacts of several Basel regimes on banking equilibrium behaviours. I find that high capital ratios will discourage banks from retaining sufficient liquidity. However, when banks hoard excess liquidity, this effect is insignificant. Fire sale loss is higher in booms due to the pronounced countercyclical liquidity holding behaviour. An effective liquidity requirement is desirable when the capital requirement is strict enough, such as Basel III. As for social welfare, Basel III is the best regulatory regime among the rules considered. I also find an inverted U-shaped relationship between social welfare and liquidity requirements, indicating the existence of an optimal level of liquidity requirement.

Key words: Capital requirement, liquidity requirement, Basel III Accords

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1. Introduction

The new Basel III Accord is now being setting up and is expected to be fully implemented by 2019. Besides capital regulations, this accord has introduced a set of requirements to address banks' liquidity risk. The Liquidity Coverage Ratio (LCR) and the Net Stable Funding Ratio (NSFR) are intended to mitigate short- (30-day) and longer-term liquidity mismatches respectively[2]. Moreover, a harsher capital requirement has been put in place, including a conservation buffer (at 2.5%), a countercyclical buffer (at a range of 0-2.5%) and an additional capital buffer from 1% to 2.5% exclusively for the global systemically important banks (SIBs)[3]. In all, Basel III has amended the capital requirement profoundly and introduced new liquidity requirements to make the banking system worldwide more immune from different sources of risks.

This paper mainly discusses the effects of the new Basel III regulation on banking operations to reveal its improvements to the stability and social welfare of the banking system. We develop a two-period dynamic model in which banks are able to choose their capital and liquidity holdings simultaneously to maximize their shareholder wealth. Yet, banks are unable to reissue equities except at the beginning of the first period and no more deposits can be absorbed. The equilibrium of this model is achieved at a steady state capital ratio that will result in the long term zero shareholder net worth. The optimal liquidity holding ratio is determined given this steady state capital-holding ratio. Banks are operated within business cycles (featured by booms and recessions) and the average loan default rates determine the financial situation. Liquidity shocks will happen at the first period which will trigger fire sales to loans once banks fail to effectively manage liquidity shocks, while credit risks will occur at the second period. Banks are subject to capital requirements and liquidity requirements, which are designed by the government to mitigate credit and liquidity risks respectively.

As in Acharya *et al.* (2010) and Holmstrom & Tirole (2001), we conduct two separate analyses in which banks are able to adjust liability side and asset side to maximize shareholder net worth when facing with liquidity shocks. Our baseline analysis is conducted when banks are only able to adjust their liability side by trading off between long-term and short-term debt. Short-term debt is costless but will cause liquidity shocks when debtholders withdraw at the first period, while long-term debtholders will stay with the banks until the second period but the long-term debt will incur an interest payment. The alternative analysis is under the assumption that banks can only alter liquid and illiquid asset holdings but cannot determine the composition of deposits. Liquid assets is safe but give no return to the banks, while illiquid assets will yield a positive expected return but cannot be recalled until the loan investment activities are concluded. From these two analyses we can acknowledge the

[2] From BCBS (2013), LCR will help to ensure that banks have an adequate stock of high-quality liquid assets (HQLA) that can be converted easily and immediately into cash to meet liquidity needs for 30 calendar day liquidity stress scenarios, while NSFR supplements the LCR to have a time horizon of one year.
[3] See BCBS (2011) for more details.

different capital and liquidity holding behaviours when banks are subject to different liquidity risk management strategies.

Our result suggests that banks' capital holding behaviours are significantly different in the two analyses. For our baseline analysis, capital holdings and capital buffers are nearly the same for booms and recessions, while the capital buffers are exceptionally higher in booms than recessions for our alternative analysis. This insight implies that banks might behave differently regarding liquidity risk management when they follow different adjustment strategies. We have also demonstrated an excess liquidity holding behaviour when in recessions for our alternative analysis, which stands with Acharya & Merrouche (2012) and Acharya *et al.* (2010). However, banks seems to overlook liquidity shocks in booms and thus face a higher fire sale loss and probability of default. Capital holdings will affect on banks' liquidity holding behaviours, and specifically a relatively high capital ratio will discourage banks from holding liquidity. This effect is more pronounced under our baseline analysis. When banks hold excess liquidity, as in recessions for the alternative analysis, capital holding might not affect liquidity holding behaviours due to their reluctance to lend when the excepted investment return is comparatively low. Lastly, we show that liquidity requirement is necessary for mitigating fire sales and is more desirable when the overall capital requirement is stringent enough to discourage banks from hoarding liquidity, for example the Basel III. There exists an inverse U-shaped relationship between social welfare and liquidity requirements, implying the existence of optimal liquidity requirement.

Our contributions are as follows. First, we have theoretically demonstrated an equilibrium result for banks' dynamic capital and liquidity holding behaviours. Most literature, like Diamond *et al.* (2011), Acharya *et al.* (2010), Hugonnier & Morellec (2017), all somehow fails to consider the co-existence of capital buffers and liquidity buffers during the business cycle. The majority of them just assume the capital holding is fixed at the capital requirement ratio, and thus neglect the existence of capital buffers and its interaction with liquidity holding behaviours. Second, we have mathematically shown the impacts of capital holdings on the liquidity hoarding behaviours and have illuminated a way to link the capital requirement and liquidity requirement. Third, to our knowledge, this is the first study to analyse the impacts of the new Basel III Accords to the banking capital and liquidity holding behaviour and have compared its improvements with the earlier regulation. We have thus observed some shortcomings of Basel I and Basel II, and have shown evidence of the superiority of Basel III.

Overall, our study are designed especially for regulation-makers to be aware about banks' cyclical capital and liquidity holding behaviours. We firstly suggest that when a capital requirement is strict enough, an additional liquidity requirement is desirable to mitigate banks' reluctance to hoard sufficient liquidity. Then, we have found a significant interaction between banks' capital and liquidity holdings, and thus have theoretically linked the joint impacts of capital and liquidity requirements.

Moreover, our model indicates that liquidity requirements are more beneficial in booms when banks are less incentivized to hoard liquidity in order to avoid fire sale loss. Lastly, our paper contributes to the literature by suggesting that risk-based capital requirement (Basel II) is suboptimal by allowing a lower capital requirement in financial booms. Our result, thus justifies that a countercyclical capital buffer, proposed by Basel III, is reliable by increasing social welfare. As a result, our paper deserves more attentions regulators with the aim to further improve the benefits of Basel Accords.

The rest of our paper is organized as follows. Section 2 summarizes some key and related literature we are following. Section 3 introduces the participants of the baseline model, and Section 4 shows the equilibrium results. Section 5 builds an alternative analysis. Section 6 demonstrates some extensions for our analysis, and Section 7 gives some insights and discussions regarding our results. Section 8 concludes our paper. The Appendix shows the proofs of our propositions.

2. Related Literature

As for theoretical analysis, the relevant literature normally focuses on the dynamic equilibrium of the banking regulation when banks are subject to capital and liquidity requirements. Repullo and Suarez (2012) build a dynamic model which considers the capital requirements only. They show a higher capital buffer in booms than in recessions for Basel I and Basel II regulations, while Basel II is more procyclical than Basel I but makes banks safer and is superior in social welfare. Shleifer and Vishny (2010) prove that capital requirements help to limit balance sheet expansions in good times and fire sales in bad times, and a higher capital requirement in good times would be socially optimal to avoid economy bubbles. They further claim that procyclical capital requirements would reduce the volatility of real activity and as a result improve the efficiency of resource allocation. For liquidity requirements, Ratnovski (2009) links the liquidity requirements to banking transparency and concludes liquidity requirements will compromise banks' transparency choices, imperfectly to insure banks in case of liquidity shocks. In addition, Ratnovski (2013) identifies a herd behaviour in liquidity risk management and maintains that together with an efficient capital requirement and transparent economies will result in a better liquidity regulation solution. Nicolo *et al.* (2014) have studied a quantitative impact of banking regulations and have found an inverted U-shaped relationship between capital requirements and social welfare. They further incorporate the liquidity requirements and conclude that liquidity requirement will unambiguously reduce social welfare. With respect to this result, we suspect they have underestimated the significance of liquidity shocks and thus disapproved the validation of liquidity requirements.

Gertler and Kiyotaki (2015) develop a dynamic model that allows for liquidity mismatch and bank runs. They argue that a bank run will significantly contract the economy activity and an anticipation of a run will have harmful effects even if the run does not occur. Uhlig (2010) points out that the governmental purchase will help to alleviate financial crises. Diamond and Rajan (2001) argue that banks with a fragile capital structure will commit to creating liquidity and thus with the introduction of capital requirement will reduce liquidity creation. Furthermore, Diamond and Rajan (2005) maintain that liquidity and solvency interact with each other but they fail to decompose these risks apart, but instead propose a robust sequence of interventions. Hugonnier and Morellec (2017) establish a dynamic model within which banks are subject to liquidity and leverage requirements and face taxation, securities issuance costs and default costs. The authors conclude that combining liquidity and leverage requirement reduces both the likelihood of default and the magnitude of bank losses in case of bankruptcy. Similarly, Gorton and Winton (2016) prove that bank capital reduces the probability of bankruptcy, but is a poor hedge against liquidity shocks. Rochet and Vives (2004) maintain that there exists an equilibrium that a solvent bank cannot find liquidity assistance in the market, and thus derive a policy implication as the form of the Lender of Last Resort (LOLR). Moreover, Acharya *et al.* (2010) and Diamond & Rajan (2011) consider the effects of fire sales and thus link the liquidity shocks with the fire sales, which is the basis for our analysis.

Other theoretical literature considers the existence of interbank market, which is not modelled for our analysis, when banks dealing with liquidity risks. Ashcraft *et al.* (2011) theoretically and empirically evidence that when fed funds rate volatility is high (an indication for an unstable interbank lending market), banks will retain a precautionary holding of reserves and will be reluctant to lend. Freixas and Jorge (2008) use a model with asymmetric information in the interbank market and they maintain the imperfection of interbank market will affect the credit market, helping to justify *liquidity effect* and *magnitude effect*. Freixas *et al.* (2011) show that it is optimal for central banks to lower the interbank rates to facilitate liquidity risk management, and fails to intervene in the interbank markets will increase the fragility of banking system. Castiglionesi and Wagner (2013) establish a model that incorporates interbank insurance. They prove even if the liquidity support from government breaks down, the insurance among banks will still be efficient and the efficiency can be restored through government guarantees. Allen *et al.* (2009), through using a model when banks trade a long term and safe asset, also maintain that the government intervention will reduce the uncertainty of liquidity demand and thus stimulate the interbank market. Acharya and Skeie (2011) show that leveraged banks are more likely to hoard precautionary liquidity and in extreme interbank markets might completely freeze. Heider *et al.* (2015) similarly highlight the importance of policy intervention and fails to accomplish it will result in a higher interbank market interest rate and an excess liquidity hoarding behaviour.

Empirical studies regarding banking capital and liquidity requirements are more documented, and our results stands with the majority of the literature. For the capital requirements alone, Aiyar *et al*. (2014) discover that a 100 basis point increase in the requirement is associated with a reduction in the growth rate of credit at 5.5 percentage points. Similarly, Behn *et al*. (2016) argue that 0.5% points increase in capital charge could result in 2.1%-3.9% points decrease in loan lending, suggesting a sizeable effect of the cyclical capital regulation. Berger and Bouwman (2013) study the effect of bank capital during financial crises using the US market data and reveal that capital helps small banks to survive and increase their market share at all times, while capital only enhances medium and large banks' performance during banking crises. With respect to banks' liquidity risk management, the evidence of the reduction in credit supply prevails. Acharya and Merrouche (2012) collect the data from UK large settlement banks around the subprime crisis of 2007-2008 and demonstrate that a 30% increase in liquidity demand during the financial crisis. This evidence highlights a precautionary liquidity hoarding during financial crisis, because of the freeze of the interbank market. Dagher and Kazimov (2015) investigate the impacts of liquidity shocks through wholesale funding on credit supply during financial crises using US loan applications data from 1992 to 2010. They figure out that banks that rely more on wholesale funding curtailed their lending more significantly although this effect is only pronounced during financial crises periods. Cornett *et al.* (2011) use US data from 2006 to 2009 to investigate banks' liquidity risk management behaviours when the overall liquidity dried up. They show that during the financial crisis of 2007-2009 the efforts to manage liquidity risk is more likely to cause a decline in credit supply. This evidence is also supported by Antoniades (2016) who uses US micro-level data on mortgage loan applications during the financial crisis from 2007 to 2009 and convey that banks which were more exposed to liquidity risks would more sharply contract their credit supply. Similarly, Ippolito *et al.* (2016) evidence that banks that are more exposed to liquidity risks will actively grant less credit lines to firms that run frequently during financial crises. Fecht *et al.* (2011) take bank size into consideration and maintain that small banks are more prone to be vulnerable to squeezes due to a higher liquidity price. Some literature has also documented the role of capital holdings on liquidity risk management. Berger and Bouwman (2009) have studied the liquidity creation within US banks and have evidenced a positive relationship between capital and liquidity creation for large banks, while negative for small banks. Banerjee and Mio (2017) reveal that banks can adjust the composition of both assets and liabilities, and during the liquidity crises they will be inclined to hold more liquid assets and non-financial deposits and to reduce the amount of intra-financial loans and short-term wholesale funding. Distinguin *et al.* (2013) consider the role of liquidity on capital buffers and highlight that banks will decrease the regulatory capital ratios when faced with higher illiquidity.

3. The Model

Consider an economy with three dates: Time 0, Time 1 and Time 2, during which the economy situation is assumed to be invariant, while potential liquidity shocks are unanticipated. The participants of this economy are entrepreneurs, depositors, banks and government; no participants discount the future. Entrepreneurs borrow the money from the banks to undertake projects. Depositors lend their money to the banks to perform as short-term debt holders or long-term debt holders, conditional on their preferences. Banks channel funds from depositors to entrepreneurs. Government insures bank deposits and imposes capital and liquidity requirements.

3.1 Timeline

At time 0, all participants enter the market. Banks lend the loans to the entrepreneurs and will obtain a return at time 2 if the project succeeds. Banks, at the same time, fund the loans in the form of short-term debt and long-term debt based on their net worth maximization. Long-term debt will mature at time 2. However, short-term debt will mature at time 1 and depositors are free to stay or leave the banks at that time. Thus the banks might face the danger of not being able to restore sufficient short-term debt to honour the projects, causing a liquidity shortfall. As a result, the bank has to sell partial (or full) of their loans to outside buyers, triggering fire sales (Walther, 2016). Until time 2, the actual return of the projects is realized, depositors will be paid the loan interests if the banks survive.

There are two kind of economy situations: booms and recessions. Each economy situation experiences a possibility of liquidity shocks, which occur at time 1. Although the actual return of the project will only be available at time 2, a liquidity shock at time 1 might cause a possibility of bankruptcy before the project return is realized. Project return is featured by economy situations and we treat it as a proxy for credit risk. Liquidity risk is solely caused by the depositors, and this sort of risk cannot be controlled by the banks[4].

3.2 Entrepreneurs

For simplicity and in order to highlight liquidity issues, we assume the entrepreneurs will use the money to undertake long-term projects that starts at time 0 and will yield a return of $1 + a$ to the bank in time 2 if succeed. Each project requires one unit investment; but once the project fails, it will only obtain $1 - \lambda$. All projects operating from time 0 to time 2 have a probability of failure p_m, where m denotes the economy situation. The aggregate failure rate $x_m \in [0,1]$ is a random variable describing the proportion of the failed project. As in Repullo & Suarez (2012) it satisfies

$$p_m = E(x_m) = \int_0^1 x_m \, dF_m(x_m) \tag{1}$$

where $F_m(x_m)$ indicates cumulative distribution function (CDF) of the project performance. For simplicity, we assume the project performance will not be realized until time 2.

[4] According to Albuquerque and Schroth (2015), the aggregate liquidity shocks are more likely in recessions, and they argue liquidity shocks are more often when the overall economy situation declines. However, they also propose a scenario that liquidity shock might also be severe in booms as depositors will withdraw their money more frequently to invest in other projects that are more profitable. On the other hand, during financial crises, depositors tend to keep their deposits in the banks as the returns in alternative projects are equally low. We thus introduce this force by assuming the liquidity shocks are not purely related to economy situations.

3.3 Depositors

Banks will distribute their financing decision with long-term and short-term debts for maximization of their shareholders' net worth. As in Walther (2016), depositors prefer liquidity due to a potential investment opportunity at time 1. Thus, if the banks intend to attract long-term debt, they have to pay interest to the long-term debt holders to compensate for their opportunity costs for not investing at time 1. However, short-term debt holders are free to leave and invest in alternative projects at time 1, and thus they will not be compensated by this interest payment. As we have addressed before, nobody discounts the future and the deposits are fully insured by the government, the short-term deposit rate is zero.

3.4 Banks and fire sales

Banks are operated by the shareholders, whose required return is δ, in order to maximize their net worth, and are required to abide by the capital requirements and liquidity requirements (proposed by Basel III). For the capital requirements, each unit of loans should be honoured by at least γ unit of equity, where $0 < \gamma < 1$. Banks thus have to hoard capital at $k \geq \gamma$ if they are permitted to invest in one unit of loans. For the liquidity requirements, proposed by Basel III Accord (BIS 2010), we introduce one tool for liquidity calibration: NSFR requirement[5]. The NSFR (*Net stable funding ratio*) works as follows:

$$NSFR = \frac{ASF}{RSF} \geq 1 \tag{3}$$

In Equation (3), *ASF* stands for *available stable funding*, denoting the weighted sum of bank liabilities, within which the illiquid liabilities have low weights. *RSF* means *required stable funding*, summing up bank assets according to their weights and illiquid assets are assigned with high weights. In order to customize NSFR into our analysis, we follow Nicolo *et al*. (2014) to adopt the worst-case scenario for its calculation.

At time 1, short-term debts mature, and a portion of *d* of the original short-term debts will not be re-deposited, causing liquidity shock[6]. To fill up the gap, banks have to take fire sales to sell partial (or full) of their long-term projects. Since the projects are long-term, it is impossible to reduce the lending to the entrepreneurs as the projects are still in process. Thus, undertaking fire sales is the only way out. At the same time, the projects will be revalued based on the fire sale activities undertaken by the

[5] This is an indication for long-term liquidity requirement, ideally for one year. We adopt this indication because it better suits our analysis that focuses on the long-term investment activities.

[6] To keep our model simple but reflect the reality, the long-term debt holders will not contribute to the liquidity shock by early withdrawing at time 1 since the deposits are fully insured and depositors are compensated for the liquidity preference by the long-term debt interest rate.

banks. The motivation for assuming so is to analyse the effects of unanticipated liquidity shock on the banking behaviours. Banks will liquidate partial or all loans to the outsiders. The loan liquidation value due to fire sales is uniformly distributed between 0 and 1, as in Diamond and Rajan (2011). However, the outsiders are inefficient investors who cannot optimally operate the project and thus will yield no return at time 2. Clearly, a deadweight loss a will be caused if fire sales happen. What is more, the whole society will treat fire sales a stigma for the banks and will decrease the asset value of the banks further[7].

3.5 Government

The government is responsible for insuring the deposits in case of bankruptcy, and is expected to set up the capital requirements and liquidity requirements (for Basel III) and to ensure the banks follow the regulations. Additionally, the government will pay for a proportion c of the asset size of the failed banks, which is incurred by the externalities of the banks' failure. Thus, the overall social welfare is the sum of the revenue (or costs) obtained by entrepreneurs, banks, depositors and the government.

4. Equilibrium Analysis

In this section, we will set up our baseline model to identify banks' optimal capital holdings and liquidity preferences under different banking regulations. Banks are under collateral constraints (capital requirements) and liquidity conditions (must repay the no-returned short-term debt holders); failure to do so will make them bankrupt. Without loss of generality, we assume the long-term project will only default at time 2, while the banks might also fail once they cannot cope with the liquidity shock at time 1. Once the banks fail, the projects will be sold by the government and pay the depositors in full as a role of deposit insurer.

4.1 Fire sales and fire sale price

In order to deal with liquidity shock, banks have to undertake fire sales to obtain cash to repay the early withdrawn depositors. The price of the fire sale assets is linearly decreasing with the amount that is sold. That is, the price of fire sales $p = (1 - z)$, where z is the amount that is fire sold (Diamond & Rajan, 2011).

Proposition 1. *Suppose the loan is originally of unit size, and if the banks have to pay off the liquidity shock (including the interest payment) at d, they must sell a portion of z. We can obtain $z = 1 - \sqrt{1 - 2d}$.*

Proof: See Appendix

[7] The effects of this valuation will make banks' market value lower than its book value due to fire sale prices, and this effects will be discussed in next section.

4.2 Banks' capital and liquidity choice

Entrepreneurs enter the market at time 0 to borrow one unit-size loan from the banks and promise to repay the banks $1 + a$ at time 2 if the projects succeed. At time 1, liquidity shock will arise, at a higher or lower level. As a response, banks have to take fire sales in order to make up the liquidity shock.

At time 1, a portion of short-term debt holders d_{t1},[8] is withdrawn, and thus cause the liquidity shock at $G_s(d_{t1}) = d_{t1}(1 - l_m - k_m)$. Similar to Freixas *et al*. (2011) and Allen & Gale (2009), we disregard sunspot-triggered bank runs throughout our paper and assume a portion of the short-term debt holders are 'impatient', who will need to consume at time 1 and will never return to the banks. Calculation of shareholders' net worth, $k'_m(d_{t1})$, is determined as follows. After fire sales, banks have sold $z_s(d_{t1})$, where $s = b, n$ standing for bad and normal liquidity shock respectively, unit of the assets, and following Proposition 1, banks' asset value is $[1 - z_s(d_{t1})]p = [1 - z_s(d_{t1})]^2$. The amount of the debt banks are responsible for is $l_m(1 + r_m) + (1 - d_{t1})(1 - l_m - k_m)$. The term r_m is the interest rate quoted and is payable to the long-term debt holders. It is determined by $r_m = j(1 - p_m)$. In addition, the fire sold asset is paid for the liquidity shock, and thus $z_s(d_{t1}) = 1 - \sqrt{1 - 2G_s(d_{t1})}$, where $G_s(d_{t1}) = d_{t1}(1 - l_m - k_m)$. Accordingly, the net worth of the shareholders $k'_m(d_{t1})$ at time 1, conditional on d_{t1}, are

$$k'_m(d_{t1}) = [1 - z_s(d_{t1})]^2 - l_m(1 + r_m) - (1 - d_{t1})(1 - l_m - k_m)$$

After simplifying the above equation, we can yield

$$k'_m(d_{t1}) = k_m - d_{t1}(1 - l_m - k_m) - l_m r_m \tag{4}$$

Thus, the banks will fail if $k'_m(d_{t1}) < 0$, equivalent to $d_{t1} > \widehat{d_m}$

where

$$\widehat{d_m} = \frac{k_m - l_m r_m}{1 - l_m - k_m} \tag{5}$$

From Equation (5), we can notice that $\widehat{d_m}$ is strictly increasing with k_m. Additionally, for a given k_m, $\frac{d\widehat{d_m}}{dl_m} = \frac{k_m - (1 - k_m) r_m}{(1 - l_m - k_m)^2}$. It is straightforward to show that when $k_m > \frac{r_m}{1 + r_m}$, $\frac{d\widehat{d_m}}{dl_m} > 0$ will always hold,

[8] Liquidity shock should be more accurately expressed as $d_{s,t1}$ but for simplicity we drop the state subscript s.

which means once the capital holdings is above a particular level, a higher l_m means a higher $\widehat{d_m}$, which means a lower probability of default.

Proposition 2. *When capital holdings is above* $\frac{r_m}{1+r_m}$ *, retaining higher liquidity assets will effectively reduce the probability of default. However, when banks hold sufficiently low capital, namely if* $k_m < \frac{r_m}{1+r_m}$ *, holding more liquidity assets will increase banks' probability of default due to the long-term loan rate payment. Thus, increasing liquidity holdings with a higher interest rate* r_m *will increase banks' probability of default once banks' capital holdings is relatively low.*

Secondly, we consider the situation where the banks' net worth is positive but have to reduce the asset size to meet the capital requirement, for example selling a portion of the projects at the fire sale price to the outsiders. Owing to lack of skills, the portion of the projects sold to the outsiders will not yield a return to the economy at time 2.

To satisfy the capital requirements, the remaining equity should be at least $\gamma_m[1 - z_s(d_{t1})]$. To put in mathematical way, we can obtain

$$k'_m(d_{t1}) = k_m - d_{t1}(1 - l_m - k_m) - l_m r_m \geq \gamma_m[1 - z_s(d_{t1})\,]$$

Proposition 3. *The banks will trigger the capital requirement constraint once* $d_{t1} > \widehat{d_{mm}}$*, where*

$$\widehat{d_{mm}} = \frac{k_m - l_m r_m - \gamma_m^2 - \gamma_m\sqrt{\gamma_m^2 - 2(k_m - l_m r_m) + 1}}{1 - l_m - k_m} \tag{8}$$

Compared with Equation (5), a higher capital requirement will directly reduce $\widehat{d_{mm}}$*, lending to a higher probability of being credit rationed. It is straightforward to show that* $\frac{d\widehat{d_{mm}}}{dl_m} = \frac{\widehat{d_{mm}} - r_m\left[1 + \frac{\gamma_m}{\sqrt{\gamma_m^2 - 2(k_m - l_m r_m) + 1}}\right]}{1 - l_m - k_m}$*, and we can notice that, similar to Proposition 2, increasing liquidity holdings seems ineffective in preventing banks from being credit rationing if banks' capital holdings is sufficiently low or the loan rate* r_m *is relatively high.*

Proof: See Appendix

The net present value for the shareholders of the banks with capital k_m and liquidity choice l_m, and faces an investment of one unit of long-term asset is

$$v_m(k_m,\ l_m) = \frac{1}{1+\delta} E_{t1}\left[v_{sm}(d_{t1}) + \min(l_m r_m, k_m)\frac{d_s - \widehat{d_m}}{d_s}\right] - k_m \tag{10}$$

where

$$v_{sm}(d_{t1}) = \begin{cases} \pi_{sm}(d_{t1}), & if\ d_{t1} \leq \widehat{d_{mm}} \\ \pi'_{sm}(d_{t1}), & if\ \widehat{d_{mm}} < d_{t1} < \widehat{d_m} \\ 0, & if\ d_{t1} > \widehat{d_m} \end{cases} \tag{11}$$

is the conditional equity value at time 1, and

$$\pi_{sm}(d_{t1}) = \frac{1}{1+\delta}\int_0^1 \max\{[1 - z_s(d_{t1})\ ][1 + a - x_{t2}(\lambda + a) - \mu] - l_m(d_{t1} + r_m) - (1 - d_{t1})(1 - k_m), 0\}\, dF_m(x_{t2}) \tag{12}$$

The calculation of $\pi'_{mm'}$ is shown in the Appendix.

$$\pi'_{sm}(d_{t1}) = \frac{1}{1+\delta}\int_0^1 \max\{\frac{k'_m(d_{t1})}{\gamma_m}[\gamma_m + z_s(d_{t1})\ + a - x_{t2}(\lambda + a) - \mu], 0\} dF_m(x_{t2}) \tag{13}$$

Equations (10) to (13) indicate that bank shareholders' net worth depends on the liquidity shock d_{t1} occurred at time 1 and project's default rate x_{t2} at time 2. Given the fact that banks will only pay the long-term interest $l_m r_m$ at time 2, and are not required to pay the interest if they fail at time 1, the valuation equation will be revised accordingly. Thus, Equation (10) adds the value of $\min(l_m r_m, k_m)\,(d_m - \widehat{d_m})/d_m$ [9] in case that banks retain less capital that cannot cover interest payment and thus fail at time 1. For Equation (12), if no credit rationing is required at time 1, banks' remaining asset invested in the projects is $1 - z_s(d_{t1})$, x_{t2} of which will yield $1 - \lambda$, and $1 - x_{t2}$ of which will obtain $1 + a$, and a proportional cost of managing the loans μ will be occurred when loans mature. After deducting the long-term debt $l_m(1 + r_m)$ and short-term debt $(1 - d_{t1})(1 - l_m - k_m)$, the banks will obtain the remaining value, if positive, as their income. If the banks trigger the capital requirement constraint, they have to sell the portion that is above the capital requirement to the outsiders to obtain the money and keep it until time 2 to repay the debt holders, shown by $\pi'_{sm}(d_{t1})$, the proof of which is illustrated in the Appendix. Shareholders operate the banks aiming to maximize $v_m(k_m,\ l_m)$ subject to the capital requirement $k_m > \gamma_m$ and liquidity requirement (Basel III).

[9] The reason for presenting this term is that the liquidity shocks are uniformly distributed.

4.3 Equilibrium

To define an equilibrium, we assume that banks are under perfect competition, which means banks' net worth is zero under their optimal capital and liquidity decision. When the net worth is negative, the banks will never choose to undertake their activities; while if the net worth is positive, the banks will have an incentive to expand their lending. Accordingly, we have the following

$$v_m(k_m^*, l_m^*) = 0 \tag{14}$$

and

$$l_m^* = \arg\max v_m(l_m, k_m^*) \tag{15}$$

where $0 < l_m < 1 - k_m$

The equilibrium indicates that banks will firstly consider the capital holdings at time 0 and then determine the liquidity holdings to maximize shareholders' net worth. The following proposition proves the existence of at least an equilibrium.

Proposition 4. *There exists at least one value of k_m^*, at $k_m^* = 0$,which satisfies equations (14) and (15). If there exist more than one value, we choose the smallest one as it is suboptimal for banks to hoard more capital once the capital requirements are satisfied.*

Proof: See Appendix

Proposition 4 can be interpreted as that if there are more than one solution which leads to equilibrium zero income of the banks, the smallest value might be the economical solution to the banks as they will have no incentives to hold more capital which is well above the capital constraints. We adopt this assumption and thus treat the smallest one as our solution. For the optimal solution of liquidity holdings l_m, we have the following proposition 5 for its corner solutions.

Proposition 5.

1) A corner solution at $l_m = 1 - k_m$ will be possible if the long-term interest rate r_m is zero or sufficiently low[10]. However, this solution is also possible once banks are certain to fail at time 1, due to low capital holdings or extremely high long-term rate.

2) When a corner solution at $l_m = 0$ exists, it indicates that the probability of not triggering credit rationing condition at time 1 is strictly positive.

[10] Unfortunately, we cannot give an analytical solution for the threshold of $r_{m'}$, but we can give the results in our extension analysis.

3) Moreover, a higher capital retained by the banks will discourage banks from hoarding liquidity (holding long-term debt).

Proof: See Appendix

Banks will finance with more long-term debt if the interest rate r_m is low enough as the cost of retaining the long-term debt is small. However, when banks notice they will be likely to fail at time 1, there might also exist a corner solution at $l_m = 1 - k_m$. If a proper capital requirement is in effect, banks will not be definitely to fail at time 1, given $\widehat{d_m} > 0$, and for this situation an unnecessary liquidity hoarding is no longer optimal. On the other hand, banks might be reluctant to hoard liquidity if they have already retained a sufficiently high capital k_m or the liquidity shock d_m is low enough. In addition, when the liquidity shock is less likely to happen, banks will also be unwilling to raise liquidity holdings in order to lower their interest payment cost.

4.4 Parameter values

Firstly, we determine some key parameters we will use for our analysis.

< Insert Figure 1 here >

Table 1 summarizes the key parameters for our baseline calibration and the main source to obtain their value. Probabilities of default p_l and p_h are adopted from Repullo & Suarez (2012) in which they choose to equalize the average Tier 1 capital requirements of Basel II to Basel I regime, set at 4%. Thus, our estimates are $p_l = 0.010$ and $p_h = 0.036$ for the average probabilities of default in booms and recessions respectively. To make our analysis consistent, we also adopt the value of state-invariant correlation $\rho = 0.174$ from them as ρ has considered the probabilities of default in each state[11].

For the estimates for long-term deposit premium, Calem, Covas and Wu (2013) use the U.S. data from 2006 to 2008 and divide this timespan into two sub-time sections (breaking point at 2007 Q3) to term them as before and after financial collapse. They estimated the deposit cost ranges 6.6% to 46.2% to the total deposits. For estimation, we calculate the difference between 75% and 25% percentiles before collapse ($11.9\% - 9.6\% = 2.3\%$) to assume it as the difference from long-term debt rates to the short ones. This value is the same for the after collapse sample ($11.6\% - 9.3\% = 2.3\%$). Ben-David *et al*. (2017) uses the U.S. data from 2007Q1 to 2012Q3 and give the average 60-month CD rates is at 2.752% (while 24-month CD rates 2.144%), compared with the rates in money markets 0.751%. Using this source, the long-term deposit premium ranges from 1.393% to 2.001%. Additionally, from the database of FDIC, they report rate caps for 60-month CD are 1.64% and 1.67%

[11] Refer to Repullo & Suarez (2012) for more details.

respectively for non-jumbo and jumbo deposits[12]. For a cap value, we adopt 2.40% for our baseline analysis.

Parameter λ is adopted from the Basel II ‘foundation Internal Ratings-Based (IRB) approach’ to determine the loss given default (LGD). To keep in line with Repullo and Suarez (2012), we use value $\lambda = 0.45$ for assuming banks' loan loss if the loan fails. The success loan rates a is adopted from the FDIC Statistics. The ratio of *Domestic office loans* and *Foreign office loans* from 2007 to 2015 has average value at around 3.29%[13]. Since the promised income from the failed loans is not included, we might underestimate the overall loan return if we directly adopt this value. Repullo & Suarez (2012) use 4% (an approximation for 3.97%) for one-period loan return. Since we are estimating the long-term loan rates that are set higher to compensate for increased uncertainty, we thus use a higher 4.5% to minimize this error and simply times two to reflect the fact the loans lasts for two periods in our analysis. Thus, our choice of success loan rate stands at $a = 4.5\% \times 2 = 9.0\%$.

The liquidity shock d_m is from existing literature. Albuquerque *et al.* (2015) collect U.S. disclosed-value acquisitions cases ranging from 35% to 90% between January 1990 and December 2010. They plot a distribution graph regarding the liquidity shock and discover that a liquidity shock at 0 to 0.10 happens at the probability of around 0.67, and a shock from 0.10 to 0.20 at around 0.15[14]. These two level of liquidity shock attribute to approximately 82% of the total shocks, and we thus adopt 0.05 and 0.10, the average value of the liquidity shock ranges, as our baseline worst-case shock for the normal and bad liquidity shock respectively. Additionally, from Antoniades (2016), he uses U.S. data from 2006 to 2009 and treats core deposit funding and unused commitments as the liquidity risk to the commercial banks. The liquidity risk ranges from 0.012 to 0.071, and thus justifies our choice of $d_n = 0.05$ and $d_b = 0.10$.[15]

To keep in line with success loan return, the cost of managing loans is also estimated from FDIC datasets. We estimate the *Total Interest Expense* and *Additional Noninterest Expense* using the average value from 2007 to 2015. The sum of these expenses is around 2.38% of the total asset. Since the loans will last for two period, we simply multiply it by two and adopt an approximate value at $\mu = 4.5\%$. This amount of expense is payable once the loans are successful till time 2, and thus the loan spread is about 450 basis points ($= 9.0\% - 4.5\%$) (without considering the cost of paying interests to the long-term debt) once the loans mature. Moreover, for our alternative analysis, we adopt $\mu' =$

[12] These values were updated on January 16, 2018 and are available at https://www.fdic.gov/regulations/resources/rates/#two

[13] The data can be accessed from www5.fdic.gov/sdi, and we use the All Commercial Banks National data for analysis.

[14] This can be found in Figure 3 of Albuquerque *et al.* (2015).

[15] As we assume the liquidity shock is uniformly distributed, the average value using our calibration is 0.025 and 0.075, very close to Antoniades (2016). The value originates from the *Liquidity risk* column of Table 3 of the paper and is calculated manually.

5.8% to represent the unit cost of managing loans in order to equalize the absolute value of loan managing costs for both analyses.

The probabilities of bad liquidity shock in each state is hard to be estimated given the fact there is no appropriate variable to describe the likelihood of liquidity shock during the business cycle. He *et al*. (2012) adopt the estimates from Bao *et al*. (2011) to assign a liquidity shock intensity at one to model the illiquidity of bond market. Furthermore, Chen *et al*. (2017) give the baseline parameters of liquidity shock intensity at 0.70 and 1 for good and bad aggregate economy states based on the Bond turnover rate (TRACE), and is adjusted to fit for Poisson intensity. Although the illiquidity in corporate bond markets cannot perfectly reflect the level of liquidity in banking system, Antoniades (2016) documents that corporate bond issuers, mainly the firms, might transform the illiquidity to the banks through rapid drawdowns. Based on this argument, we thus use the estimates from the corporate bond markets as the higher bound for our analysis and assign a smaller (larger) weight for booms (recessions) to feature the transmission probability through these two markets and reflect the facts that banking system is less vulnerable than bond market, especially in booms (Antoniades, 2016). Thus, we assign $q_{bl} = 0.36$ and $q_{bh} = 0.90$ for our baseline analysis. Additionally, for the ease of comparison for the results, we introduce unconditional probabilities of booms and recessions $\phi_l = 0.643$ and $\phi_h = 1 - \phi_l = 0.357$ as quoted in Repullo & Suarez (2012).

For the shareholder required return δ, we simply follow Repullo & Suarez (2012) to make our estimation of previous parameters consistent. Repullo & Suarez (2012) assume the shareholders invest equally in Tier 1 and Tier 2 capital and will require a same return for these equities. They document that Iacoviello (2005) who gives an estimation for the required return spreads, between entrepreneurs and lenders, conservatively at 4%, while Van den Heuvel (2008) proposes a spread around 3.16%. Thus, for our baseline analysis, we simply adopt the rates of return at 4% and follow Repullo & Suarez (2012) to consider existence of Tier 1 and Tier 2 capital to make our estimation at $\delta = 2 \times 0.04 = 0.08$. Bankruptcy costs c captures the additional social costs due to bankruptcy. Hennessy & Whited (2007) use a sample data from U.S. from 1988 to 2001, and have estimated that bankruptcy costs equals to 0.104. As discussed by Nicolo *et al.* (2014), Hennessy & Whited (2007) use the data from nonfinancial firms, and thus this should be viewed as a lower bound for bankruptcy costs caused by financial institutions. Repullo (2013) adopt a higher bankruptcy cost at $c = 0.20$ to calculate the dynamic optimal capital requirements. Thus, to keep in line with the above estimation, we use $c = 0.20$ for our analyses.

Finally, the estimation for the diminishing return to scales is hard as there is little empirical literature discussing this effect, especially for our interest that considers the returns when risky and risk-free investments are both available. Lopez-de-Silanes *et al.* (2015) document that 10% of the investments does not return any profits and 1 in 4 has an internal rate of return above 50%. Thus, to customize with our analysis, we adopt $w = 0.026$ which indicates around 14% ($= \frac{0.026}{2}/0.09$) of the investment returns is lost when banks invest all the assets as risky assets, which is nearly the same as 10% as indicated by Lopez-de-Silanes *et al.* (2015), and this value is better to fit other variables in our model.

4.5 Capital and liquidity requirements

Capital requirements are set up to mitigate credit risks that arise in time 2. We consider four capital requirement regimes: *Laissez-faire regime*, Basel I, Basel II and Basel III. Following BCBS (2010) and Repullo & Suarez (2012), we focus on Tier 1 capital requirements, which essentially includes common equity. In the *Laissez-faire regime*, we set up $\gamma_h = \gamma_l = 0$. In the *Basel I regime*, we set up $\gamma_h = \gamma_l = 4\%$, which is line with the minimum Tier 1 capital requirements on all nonmortgage credit to the private sector documented by Basel Accord of 1988. The capital requirements for *Basel II regime* is divided by two the overall requirements of Tier 1 + Tier 2 capital determined by the Basel II formula, under an explicit value-at-risk with a confidence level of 99.9%, which is given by Equation (16)

$$\gamma_m = \Phi\left[\frac{\Phi^{-1}(p_m) + \sqrt{\rho(p_m)}\Phi^{-1}(0.999)}{\sqrt{1-\rho(p_m)}}\right] \tag{16}$$

where

$$\rho(p_m) = 0.12\left(2 - \frac{1-e^{-50p_m}}{1-e^{-50}}\right) \tag{17}$$

The term $\rho(p_m)$ captures the correlation of firms in cross-section analysis, and we will use a unique weighted value to parameterize it as a constant ρ as we focus on time-series dimension. Thus, utilizing equation (16) and (17), and based on the value of p_m and ρ quoted in Table 1, we have $\gamma_l = 3.2\%$ and $\gamma_h = 5.5\%$ for *Basel II regime*.

Basel III regime requirements have introduced an additional conservation buffer and a countercyclical buffer which are at 2.5% and 0-2.5% respectively, in the form of common equity and ideally Tier 1 capital (BCBS 2011). Accordingly, following *Basel II regime* and BCBS (2011) we adopt the requirements at $\gamma_l = 7\% = (3.2\% + 2.5\% + 1.3\%$[16]) and $\gamma_h = 8\% = (5.5\% + 2.5\%)$ as our *Basel III regime* requirements.

[16] 1.3% is the average value we adopted of the countercyclical buffer requirements exclusively for Basel III regime ranging from 0 to 2.5%.

To comply with the liquidity requirements (issued in Basel III), banks will have to hold liquidity holdings in line with NSFR requirement. Following Equation (3) and Walther (2016), we have the following equation, which we will adopt for the liquidity requirement

$$\frac{ASF}{RSF} = \frac{k_m + (1 - d_s)l_m}{[d_s(1 - k_m - l_m) + l_m r_m]} \geq C \tag{18}$$

In the numerator of Equation (18), capital receives a weight 100%, while the short-term debt receive zero weight. Although long-term debt l_m will not cause the liquidity shock, according to Walther (2016), it not quite as 'stable' as capital k_m because this debt cannot be pledged as capital once banks trigger the capital constraints. We will assign a weight of $1 - d_s$ for the long-term debt to reflect the worst-case scenario, where d_s represents the largest possible liquidity shock. In the denominator, the project is of unit size and the short-term debt will receive a weight of d_s as if the worst case happens. Then, the long-term interests will also attribute to liquidity shock and is included in the denominator as well. The parameter C is the required liquidity requirement by the government, and in Basel III Accord it is 100%. To customize this requirement to our model, we adopt $C = 10$ for our baseline analysis, and then compare C in different value to investigate the impact of different level of liquidity requirements. As in Equation (3), the ratio of *ASF* to *RSF* should be no less than one, and Equation is determined accordingly. After solving Equation (18), we can obtain following

$$l_m \geq \widetilde{l_m} = \frac{C(1 + k_m)d_s - k_m}{1 - d_s - C(r_m - d_s)} \tag{19}$$

As in Basel III, banks' liquidity should be $\max(l_m^*, \widetilde{l_m})$, where l_m^* is the optimal liquidity holdings, without liquidity requirements. From Equation (19), we can notice that with higher capital holdings banks might be required less liquidity holdings, while a higher interest rate r_m will raise the liquidity requirements when keep others constant.

4.6 Quantitative Result

This section summarizes the calculation of some variables that will be derived as our baseline results in next section. Fire sale loss measures the deadweight loss to the economy at time 1 when the market loan value is lower than its book value. Specifically, the *Shareholder Fire Sale Loss* (*SFSL*) can be written as

$$SFSL_{sm} = \int_0^{\widehat{d_m}} G_s(d_{t1})dH_s(d_{t1}) + \max\{[k_m - l_m r_m][1 - H_s(\widehat{d_m})], 0\} \tag{20}$$

where $H_s(d_{t1}) = \frac{d_{t1}}{d_s}$ when $0 \leq d_{t1} \leq d_s$; $H_s(d_{t1}) = 0$ when $d_{t1} < 0$; $H_s(d_{t1}) = 1$ when $d_{t1} > d_s$. The first term of Equation (20) describes the deadweight loss to the shareholders if the banks survive from the liquidity shock, while the latter calculates the loss once the banks fail, i.e. the difference of capital holdings and loan rate payments. The maximization function helps to exclude negative value when $k_m < l_m r_m$. In other words, the term *SFSL* captures shareholders' loss in time 1 due to reduced loan market value caused by fire sales.

As for bankruptcy probabilities, we have identified two sources of risks that will attribute to banks' failure. The probability of bankruptcy in time 1, due to liquidity risk, is

$$BRL_{sm} = 1 - H_s(\widehat{d_m}) \tag{21}$$

The probability of bankruptcy in time 2, due to credit risk, can be written as

$$BRC_{sm} = \int_0^{\widehat{d_{mm}}} [1 - F_m(\overline{d_{t1}})] dH_s(d_{t1}) + \int_{\widehat{d_{mm}}}^{\widehat{d_m}} [1 - F_m(\overline{\overline{d_{t1}}})] \, dH_s(d_{t1}) \tag{22}$$

where

$$\overline{d_{t1}} = \frac{[1 - z_s(d_{t1})](1 + a - \mu) - l_m(d_{t1} + r_m) - (1 - d_{t1})(1 - k_m)}{[1 - z_s(d_{t1})](\lambda + a)}$$

$$\overline{\overline{d_{t1}}} = \frac{\gamma_m + z_s(d_{t1}) + a - \mu}{\lambda + a}$$

and

$$F_m(x_{t2}) = \Phi\left[\frac{\sqrt{1-\rho}\,\Phi^{-1}(x_{t2}) - \Phi^{-1}(p_m)}{\sqrt{\rho}}\right] \tag{23}$$

Equation (21) captures the bankruptcy probability when banks fail at time 1, while equation (22) summarizes the likelihood of bank failure at time 2, conditional on the realization of liquidity shock at time 1. Equation (23) is adopted from Gordy (2003) and Repullo & Suarez (2012) who utilize a Value-at-Risk foundation following the *single common risk factor* model. The subscript $m = l, h$ representing low default risk periods (booms) and high default risk periods (recessions). $\Phi[*]$ is the CDF of a standard normal random variable and $\rho \in (0,1)$ stands for the state-invariant correlation which is calibrated in Table 1, and p_m is the average probability of default in each state.

< Insert Table 2 here >

4.6.1 Capital buffers

The steady state capital holdings are generally high in high default state h, except for Basel I that capital holdings are slightly higher in low default state l at 6.2% than in high default state h at 5.4%. This exception can be attributed to the slightly higher capital requirements in state l at 4% that makes the resulting capital holdings higher than *Laissez-faire* (requirements at 0%) *and Basel II* (requirements at 3.2%). In addition, the capital requirements in state h is also at 4% which makes banks find it is optimal to hold more capital in state l to benefit from the high income period. Accordingly, the capital holdings is higher in state l than in state h under *Basel I*.

As for capital buffers, they are nearly the same at around 2.4% for *Laissez-faire, Basel I and Basel II*, and is approximately at 3.5% for *Basel III*. This reveals that when banks can adjust the liability side by choosing long-term and short-term debt, capital buffers are not significantly different from state l and state h, partially due to the facts that banks are more vulnerable under this situation as they have to undertake fire sales regardless of the realization of the liquidity shocks. As a result, the credit rationing conditions are more likely to be triggered; consequently, capital requirements are more decisive for this situation and capital buffers are nearly the same regardless of the capital requirement level.

4.6.2 Liquidity holdings

Liquidity holdings are all relatively high for all the situations, with the smallest value at 69.0% for state l under *Laissez-faire regime*. As addressed before, banks are more vulnerable from liquidity shocks and fire sales for this analysis, they are inclined to finance with long-term debt to minimize the loss from the liquidity shocks and the resulting fire sales.

For *Laissez-faire, Basel I and Basel II regimes*[17], the liquidity holdings are all lower in state l than in state h. This fact might be explained by that liquidity shocks are less likely in booms (state l), and banks are not willing to hold more liquidity than in recessions (state h). The liquidity holdings are nearly the same for *Basel I* and *Basel II regimes*, which is at around 80%, because the capital requirements are essentially the same for these two regimes (4% for *Basel I* and around $4.35\% = (3.2\% + 5.5\%)/2$ for *Basel II*), and capital holdings are not significantly different for these two regimes. Thus, the liquidity holdings l_s^* are not heavily affected by capital holdings k_s^*, and the results from Proposition 5 are thus not pronounced. For *Laissez-faire regime*, liquidity holdings are much higher in state h (at 88.0%) than in state l (at 69.0%). This result in a way demonstrates banks' reluctance to hoard liquidity in booms, given the fact that their capital holdings are nearly the same at

[17] Since Basel III in our analysis considers liquidity requirements, it is not appropriate to compare it with other regime directly, and thus we will not discuss its liquidity holdings in this part. We will leave this discussion for Table 4.

2.2% for both two states, making banks suffer from fire sales more in booms. For all regimes, the liquidity holdings are increasing in state l (from 69.0% in *Laissez-faire regime* to 79.4% in *Basel II regime*), but are decreasing in state h (from 88.0% in *Laissez-faire regime* to 81.0% in *Basel II regime*). This is partially because that the capital is relatively expensive in booms and thus banks are willing to hoard more liquidity to preserve their capital. On the other hand, it seems not costly to hold relatively high equity in recessions, and they reduce the liquidity holdings to some extent when the capital requirement increases from 0% to 5.5% as indicated by *Basel II regime*. For *Basel III regime*, it introduces liquidity requirements, and thus the liquidity holdings is exceptionally high in state l at 90.4%, but slightly lower in state h at 70.7%. This result indicates that liquidity requirements seems more effective in state l, but less powerful in state h during which state the banks' incentives to preserve capital is relatively low.

4.6.3 Expected shareholder fire sale loss

Overall, the fire sale losses are lowest in *Basel III* regime (at 0.31%), which achieves the preliminary aim of *Basel III regime* to increase banks' immunity when faced with liquidity shocks. The shareholder fire sale losses are relatively small under *Laissez-faire regime*, due partially to the fact that liquidity holdings (at 88.0%) are highest in state h, which makes its losses the lowest at 0.23% when compared with other regimes. Moreover, its capital holdings is the lowest (at around 2.3%), and to some degree limits its fire sale loss, this can be proved by the facts that fire sale loss is lower (at 0.47%) in booms than the average loss for *Basel I* and *Basel II* (at around 0.53%), even if its liquidity holdings is the lowest at 69.0%.

4.6.4 Probability of bankruptcy

Due to the significantly low capital holdings, banks under *Laissez-faire regimes* are much more likely to fail at the first period (time 1), due to liquidity risks, which is highly at 68.79%. However, the probability of default is zero for *Basel I, Basel II* and *Basel III regimes,* which means a higher capital holding will effectively make banks immune from liquidity shocks, given the fact that liquidity holdings for these three regimes are not significantly different from the *Laissez-faire* regime. For the second period (time 2), the bankruptcy are principally caused by the credit risks of the loan investment. It is not surprising that probability of default is lowest under *Basel III regime* at around 0.07%, and that figure is 0.51% and 0.26% for *Basel I* and *Basel II regimes* respectively. The probability of default is slightly lower under *Laissez-faire regime* at 0.46% than that of *Basel I regime* due partially to the fact that probability of default for *Laissez-faire regime* is extremely high in first-period and no further bankruptcy could be triggered in the second period and thus its probability of default is slightly lower. Overall, the probability of default for all periods (sum of the probabilities of the first and second period) is exceptionally high for *Laissez-faire regime* at 69.25%, and the figure drops significantly to 0.51% for *Basel I regime* and 0.26% for *Basel II regime*, it is the lowest at

0.07% for *Basel III regime*. This result confirms the capital requirements are necessary to significantly reduce the probability of bankruptcy when banks are facing with liquidity shocks and can only determine financing structures by choosing long-term and short-term debt.

4.7 Externality to social welfare

This section presents an additional analysis for banks' regulation on social welfare. This section selects banks' additional fire sale loss due to reduced market value of the projects, and the social welfare increase from banks' loan investments. Accordingly, *Additional Fire Sale Loss* (*AFSL*), which excludes the loss to shareholders that has been captured by *SFSL*, can be written as

$$AFSL_{sm} = \int_0^{\widehat{d_{mm}}} z_s(d_{t1})dH_s(d_{t1}) + \int_{\widehat{d_{mm}}}^{\widehat{d_m}} \left[1 - \frac{k'_m(d_{t1})}{\gamma_m}\right] dH_s(d_{t1}) + \left[1 - H_s(\widehat{d_m})\right] \tag{24}$$

The first term of Equation (24) captures the productivity loss when capital constraints are not binding, while the second term is the loss of reduced loan investment due to credit rationing, and the last term summarizes the loss when banks fail. Additionally, social welfare during the overall investment periods (s, m) can be calculated as

$$W_{sm} = E_{sm} + GI_{sm} + FC_{sm} \tag{25}$$

where

$$E_{sm} = (1 - p_m)(1 - AFSL_{sm})b \tag{26}$$

are the expected payoff of the entrepreneurs if the projects are owned by the banks, and the parameter $b = a$ features a non-pledge-able return to distinguish the deadweight loss to the projects if they are sold to outsiders who are unable to efficiently operate the projects,

$$GI_{sm} = \int_0^{\widehat{d_{mm}}} \Pi_{sm}(d_{t1})\, dH_s(d_{t1}) + \int_{\widehat{d_{mm}}}^{\widehat{d_m}} \Pi'_{sm}(d_{t1})\, dH_s(d_{t1}) + \int_{\widehat{d_m}}^{d_s} k'_m(d_{t1})\, dH_s(d_{t1}) + l_m r_m{}^{18} \tag{27}$$

are the (negative) payoffs to the government as its role of a deposit insurer and the (positive) payoffs to the long-term depositors once the banks survive from time 1 and proceed to undertake projects until time 2, where

[18] The term $l_m r_m = \int_{\widehat{d_m}}^{d_s} l_m r_m\, dH_s(d_{t1}) + l_m r_m H_s(\widehat{d_m})$, covering the long-term debt interest benefits for banks (or government) when banks fail at time 1 and thus is waived for interest payment and for depositors when banks survive from time 1.

$$\Pi_{sm}(d_{t1}) = \int_0^1 \min\{0, [1 - z_s(d_{t1})][1 + a - x_{t2}(\lambda + a) - \mu] - l_m(d_{t1} + r_m) - (1 - d_{t1})(1 - k_m)\} \, dF_m(x_{t2})$$[19]

(28)

$$\Pi'_{sm}(d_{t1}) = \int_0^1 \min\left[0, \left\{\frac{k'_m(d_{t1})}{\gamma_m}[\gamma_m + z_s(d_{t1}) + a - x_{t2}(\lambda + a) - \mu]\right\}\right], dF_m(x_{t2})$$

(29)

denoting the expected (negative) payoffs to the government if the banks fail at time 2, conditional on the realization of d_{t1} at time 1, while

$$FC_{sm} = -c[BRL_{sm} + BRC_{sm}]$$

(30)

are the (negative) payoffs due to social costs of bankruptcy, and BRL_{sm} are BRC_{sm} are determined in Equations (21) and (22) respectively. To explain Equation (25), it summarizes the sum of payoffs to government and entrepreneurs. Since in our equilibrium analysis bank shareholders net worth is zero, their payoffs are thus dropped out. Equation (26) describes the expected non-pledge-able return to the entrepreneurs if the projects are financed by the banks until time 2. The motivation for assuming so is to highlight the social costs of credit rationing and fire sales to outsiders. Equation (27) calculates the payoffs to the government and depositors (principally the long-term debt holders), and equation (28) and (29) compute the expected negative payoffs to the government when banks fail at time 2, conditional on the liquidity risk realization at time 1. Finally, equation (30) is the expected social costs due to bankruptcy. Table 3 gives the quantitative results for the above variables under different banking regulations.

< Insert Table 3 here>

4.7.1 Expected additional fire sale loss

Besides shareholders' loss, fire sales will also impair the social income through banking system. The results shown in Table 3 indicates that the economy suffers more under *Laissez-faire regime*, and the overall additional fire sale loss is highly at 72.69%, which means only a portion of 27.31% of invested loan will obtain return to the society. The losses are strictly decreasing from 14.47% for *Basel I* regime, to 6.14% for *Basel II regime* and merely 0.32% under *Basel III regime*. Thus, Basel III is proved to be the most efficient one in minimizing social welfare loss. The loss for *Basel I regime* is significantly higher in state h (recessions) at 29.16% than in state l (booms) at 6.31%. This is because the capital holdings in recessions is merely at 5.4% as shown in Table 2 for recessions, which significantly increases the probability of being credit rationing (the second component for Equation

[19] Note that we do not discount Equations (28) and (29) with $1/(1 + \delta)$ as what we have done in Equation (12) because GI_{sm} measures the loss of the government instead of the shareholders, and thus obtaining the results without discounting.

(24)), and thus the additional fire sale losses are much higher in recessions. On the other hand, the expected loss are higher in booms (8.07%) than in recessions (2.67%) for *Basel II regime* due partially to the fact that capital holdings are lower in booms (5.4%) than in recessions (7.9%). The loss is the lowest under *Basel III* regime because of its resulting highest capital holdings for both states and generally the highest liquidity holdings among other regimes.

4.7.2 Expected social welfare

For *Laissez-faire regime*, the expected social welfare is at -10.03%, which means banking system cannot obtain positive return to the economy, and it is advocated to impose capital requirements to obtain a positive income. This negative return is largely due to the high probability of default at 69.25% and the high additional fire sale loss at 72.69% that cause a significant negative externality to the economy. However, the social welfare is positive at 9.31% for *Basel I*, 10.13% for *Basel II* and 10.97% for *Basel III regime* respectively, and confirms that *Basel III* is more efficient to other regimes in improving social welfare. The expected social welfare are higher in booms than in recessions for all regimes, except for *Basel II regime* whose expected value is are 10.07% for booms and is lower than that of recessions at 10.23%. This can be attributed to a lower capital holding in booms (5.4%) which significantly increase the probability of default and probability of being credit rationed. This result confirms that it is suboptimal to allow a lower capital requirement in booms (only at 3.2%), and thus a higher capital requirement is preferred especially in booms (state l).

4.7.3 Liquidity requirements

For our baseline analysis, we simply adopt $C = 10$ as shown in Equation (18). To better acknowledge the impacts of liquidity requirements on banking system and social welfare improvement, we compare them under different strictness of liquidity requirement by changing the value of C, and we also demonstrate the equilibrium results for *Basel III regime* without liquidity requirements. Table 4 below shows the results.

< Insert Table 4 here >

Table 4 shows that capital holdings are nearly the same for state l at around 9.1% and 10.3% in state h, although the capital holdings are slightly higher in our baseline results around 9.2% and 11.3% for state l and state h respectively. This is caused by the fact that liquidity requirements force banks to hold more liquidity (compared with the scenario when $C = 0$) and banks accordingly find it is profitable to retain a higher capital to preserve the higher costs of retaining more liquidity. However, when liquidity requirements are too tough, when $C = 20$ and above, this effect is offset by the comparatively high costs of keeping excess liquidity and thus it is not worthy to hold capital at a high ratio.

When no liquidity requirement is imposed, banks' liquidity holdings are only at 55.1% and 66.1% respectively for booms and recessions. When compared with Table 2, the effects by Proposition 5 is more pronounced that liquidity holdings reduces significantly from around 80% when under *Basel II regime*. Consequently, the expected shareholder fire sale loss and additional fire sale loss are at 1.18% and 6.80%, higher than that of *Basel II regime* at 0.52% and 6.14% respectively. The expected social welfare are only at 9.42%, lower than *Basel II* at 10.13%. This result reveals liquidity requirement is necessary and is not merely the substitute for capital requirement to limit banks' loss from liquidity shocks. When liquidity requirements are set at $C = 20$ and $C = 30$ the expected social welfare increases to the maximum at 11.24% and decreases to 10.80% when $C = 30$. Thus, there demonstrates an inverted U-shaped relationship between social welfare and strictness of liquidity requirements. Although with the increase of liquidity requirements the shareholder fire sale loss decreases to nearly 0.00% when $C = 30$, the additional fire sale loss raises dramatically from 0.08% ($C = 20$) to 1.50% ($C = 30$). With the increase of liquidity holdings, the payment of interest increases, and thus reduces banks' net income and the probability of facing credit rationing increases accordingly. Overall, the result from Table 4 suggests the existence of an optimal liquidity requirement regarding the maximization of social welfare.

5. Alternative Analysis

In previous section, we focus on the situation where banks are able to adjust their financing by choosing the portion of long-term debt (l_m)[20] and short-term debt ($1 - l_m - k_m$). However, according to Acharya *et al.* (2010) and Holmstrom & Tirole (2001), it is rather realistic for banks to achieve this process. We relax this assumption to focus on the situation that banks can only manipulate their asset side by choosing liquid and risky asset. As in Acharya *et al.* (2010) and without loss of generality, banks will collect deposits from short-term debt holders who will possibly leave banks at time 1.

5.1 Model Setup

At time 0, banks will setup the capital holdings K_m and finance the rest of the liability in the form of short-term debt at $1 - K_m$. Banks use this one unit of liability to invest in risky assets (loans), which will due until time 2, at the ratio of $1 - L_m$ which will yield a return of $1 + a_{L_m}$ to the bank, subject to default risk. As in Holmstrom and Tirole (2001), due to diminishing returns to scales, the return of the loans is decreasing in $1 - L_m$, and is given by

$$a_{L_m} = a - w\frac{1 - L_m}{2} \tag{31}$$

[20] To distinguish from our benchmark model, we use the uppercase to denote these counterparties.

The rest, at the portion of L_m, is invested in liquid assets that will yield no return, but only the principal, to the banks. Without loss of generality, we assume liquid assets are risk-free but they represent an opportunity cost to the banks because the assets obtain no return. Similar to benchmark model, the risk-free rate is normalized to zero and banks have to sell their risky assets (loans) once their available funding cannot repay the non-returned debt holders.

At time 1, a liquidity shock happens, a portion of D_{t1} debt holders leave the banks, causing the shock at the level of $D_{t1}(1-K_m)$. The total resources available with the banks are L_m. At time 2 banks will continue to invest in liquid assets if liquid assets are in surplus after being paid for liquidity shocks.

To work out banks' shareholders net worth, we start with the situation where fire sales are not necessary. Banks' funding which is available to fill up with the liquidity gap is simply L_m, where banks could obtain at time 1 when liquid assets come due. Thus, after paying to the debt holders, the remaining available funding is $L'_m = L_m - D_{t1}(1-K_m)$. Accordingly, banks will have to undertake fire sales when available funding has been consumed, equivalent to $D_{t1} > \widehat{D_m}$, where

$$\widehat{D_m} = \frac{L_m}{1-K_m} \tag{32}$$

When banks have to rely on fire sales to repay the debt holders, assume they have to sell $Z_s(D_{t1})$ of the loans, where $s = b, n$, to fill up the gap. Similar to our benchmark model, fire sale price is $P_s = (1-Z_s)$. Based on Proposition 1, banks' loans (market) value is $(1-L_m)[1-Z_s(D_{t1})]^2$, where $Z_s(D_{t1}) = 1-\sqrt{1-2[D_{t1}(1-K_m)-L_m]/(1-L_m)}$. In this case, we refer to $G_s(D_{t1}) = D_{t1}(1-K_m)-L_m$ as the net liquidity shock that triggers the fire sales. Thus, banks will fail if their assets' value cannot support the debt. After paying to the non-returned debt holders, the remaining debt banks is responsible for is $(1-D_{t1})(1-K_m)$. Thus, the net worth, when fire sales is triggered, is

$$K'_m(D_{t1}) = (1-L_m)[1-Z_s(D_{t1})]^2 - (1-D_{t1})(1-K_m)$$

After using the definition of $Z_s(D_{t1})$ and $G_m(D_{t1})$, the above equation can be simplified to

$$K'_m(D_{t1}) = K_m + L_m - D_{t1}(1-K_m) \tag{33}$$

Thus, banks will fail if $K'_m(D_{t1}) < 0$, equivalent to $D_{t1} > \widehat{\widehat{D_m}}$, where

$$\widehat{\widehat{D_m}} = \frac{K_m + L_m}{1-K_m} \tag{34}$$

Making differentiation to $\widehat{\widehat{D_m}}$ with respect to K_m and L_m, we can notice that they are all increasing to $\widehat{\widehat{D_m}}$. Thus, contradicting to Proposition 2, when banks could adjust their asset side and given certain K_m, liquidity holdings L_m will unambiguously reduce banks' probability of default. Banks' capital holdings should satisfy the capital requirement, and failure to satisfy it will result in credit rationing. Based on the capital requirement given before, we can simply conclude that banks will not trigger the capital constraint if $K_m'(D_{t1}) \geq \gamma_m[1 - Z_s(D_{t1})](1 - L_m)$.

Proposition 6. *The banks will trigger the capital requirement constraint once $D_{t1} > \overline{D_m}$, where*

$$\overline{D_m} = \frac{K_m + L_m - \gamma_m^2(1 - L_m) - \gamma_m\sqrt{\gamma_m^2(1 - L_m)^2 + (1 - L_m)(1 - L_m - 2K_m)}}{1 - K_m} \tag{35}$$

From the definition of $\widehat{D_m}$, $\widehat{\widehat{D_m}}$ and $\overline{D_m}$, we can notice that liquidity holding will unambiguously increase the thresholds of the above terms and thus reduce the probability of default, credit rationing and fire sales. Different from the conclusion of our baseline model that thresholds might be negatively related to liquidity holdings, demonstrated in proposition 2 and 3, we can attribute this difference to the fact that long-term debt rates will not affect banks' income. Accordingly, a higher liquidity holding will help to fight against liquidity shock while this is at the cost of giving up the return obtained by investing in risky projects.

For banks who can manipulate their investment composition, there are three scenarios. First, when liquidity shock $D_{t1} < \widehat{D_m}$, no fire sales is necessary and banks can operate their loan in full until time 2. Banks will reinvest $L_m' = L_m - D_{t1}(1 - K_m)$ into the liquid asset, after liquidity shock is paid off. When liquidity shock is $\widehat{D_m} \leq D_{t1} < \overline{D_m}$, fire sales is possible. Banks have to sell a portion of their loans to receive funding to fill up with the liquidity shock. When liquidity shock is $\overline{D_m} \leq D_{t1} \leq \widehat{\widehat{D_m}}$, banks are subject to capital requirements and have to sell an additional amount of their loans at the fire sale price to the outsiders in order to satisfy the capital requirement. When liquidity shock $D_{t1} > \widehat{\widehat{D_m}}$, banks fail due to insufficient capital to honour the debt.

5.2 Shareholder net worth and equilibrium

Similar to our benchmark model, banks' shareholders net worth is subject to the liquidity shock D_{t1} occurred in time 1 and credit risk X_{t2} realized at time 2, where X_{t2} is the fraction of the failed loans and is conditional on the financial situation of time 2. Banks' shareholders net worth can be summarized as follows

$$V_m(K_m, L_m) = \frac{1}{1 + \delta} E_{t1}[V_{sm}(D_{t1})] - K_m$$

(36)

where

$$V_{sm}(D_{t1}) = \begin{cases} \Pi_{sm}(D_{t1}), & if \;\; D_{t1} < \widehat{D_m} \\ \Pi'_{sm}(D_{t1}), & if \;\; \widehat{D_m} \leq D_{t1} < \overline{D_m} \\ \Pi''_{sm}(D_{t1}), & if \;\; \overline{D_m} \leq D_{t1} \leq \widehat{\widehat{D_m}} \\ 0, & if \;\; D_{t1} > \widehat{\widehat{D_m}} \end{cases}$$

(37)

is the conditional shareholders' market value[21] at time 1, where

$$\Pi_{sm}(D_{t1}) = \frac{1}{1+\delta}\int_0^1 \max\{K_m + (1-L_m)\big[a_{L_m} - X_{t2}\big(\lambda + a_{L_m}\big) - \mu'\big], 0\}\, dF_m(X_{t2})$$

(38)

where a_{L_m} is defined in Equation (31).

Equation (38) indicates banks' income at time 2 if no fire sales are triggered in time 1. Banks have the total size of the loan at $1 - L_m$, and $1 - X_{t2}$ of which yields the return $1 + a_{L_m}$, X_{t2} of which will yield the return $1 - \lambda$, and plus the return by investing in liquid asset $L_m - D_{t1}(1 - K_m)$. Once minus the remaining debt $(1 - D_{t1})(1 - K_m)$, and the management cost of risky asset μ', banks will retain the remaining, if positive, as their return.

In addition,

$$\Pi'_{sm}(D_{t1}) = \frac{1}{1+\delta}\int_0^1 \max\{(1-L_m)[1 - Z_s(D_{t1})][1 + a_{L_m} - X_{t2}\big(\lambda + a_{L_m}\big) - \mu'] - (1 - D_{t1})(1 - K_m), 0\}\, dF_m(X_{t2})$$

(39)

Equation (39) gives banks shareholders' net worth once fire sales is triggered but capital constraint is not binding. The remaining loan, after fire sales, is at the amount of $(1 - L_m)[1 - Z_s(D_{t1})]$. This amount will yield a return, conditional on the loan performance X_{t2}, at $1 + a_{L_m} - X_{t2}\big(\lambda + a_{L_m}\big)$, and after minus the outstanding debt $(1 - D_{t1})(1 - K_m)$ together with management cost μ' is the shareholders' income, if positive.

Proposition 7. *If banks trigger the capital requirement constraint at time 1, their shareholder net worth is given by Equation (40)*

$$\Pi''_{sm}(D_{t1}) = \frac{1}{1+\delta}\int_0^1 \max\left\{\frac{K'_m(D_{t1})}{\gamma_m}\big[\gamma_m + Z_s(D_{t1}) + a_{L_m} - X_{t2}\big(\lambda + a_{L_m}\big) - \mu'\big], 0\right\} dF_m(X_{t2})$$

[21] Because at time 1 shareholders' equity is lent to the firm and thus cannot be recalled until time 2, their equity cannot be priced directly, but is determined by the market price of the loan (considering fire sale price) net of the debt. Thus, we call the market value of the shareholder in this situation.

(40)

Proposition 8.

1) A corner solution at $L_m = 1$ *seems impossible because investing in full in liquid asset will yield no return and thus expected zero return by running the banks.*

2) Banks might hold excess liquidity buffers, when $L_m > d_s(1 - K_m)$ *with which the liquidity shock can be smoothed thoroughly. Under these scenarios, the probability of fire sales at time 1 is zero, and capital holdings* K_m *will not discourage banks from hoarding liquidity* L_m.

3) Capital holdings will influence liquidity holdings when fire sales are possible if $L_m < d_s(1 - K_m)$, *and the influence is more significant, to discourage liquidity hoardings with a high capital holding, when credit rationing and bankruptcy are possible.*

4) When a corner solution at $L_m = 0$ *is possible, the probability of facing fire sales at time 1 is strictly positive.*

To interpret Proposition 8, we can notice that banks will hold liquidity buffers when $L_m > D_s(1 - K_m)$ which makes fire sales impossible. This is because that the overall expected investment returns are low, for example in recessions, and thus banks will reduce the lending to risky investment due to the reduced expected return. Since banks already cut down the lending as the risky assets, a higher capital holding will not discourage banks from hoarding excess liquidity because increasing investment will not yield positive revenue to the banks. However, when banks fail to hoard excess liquidity, typically in booms, they might find holding a higher capital is very expensive and thus they will neglect potential liquidity shocks by reducing liquidity holdings further to yield a higher return. This effect is more significant when credit rationing and bankruptcy are more likely to happen which reduces banks' income more profoundly. Equivalently, credit rationing and bankruptcy are more likely to happen when banks reduce liquidity holdings.

5.3 Equilibrium Analysis

Same as the benchmark model, the equilibrium of the banking behaviours is defined as follows

$$V_m(K_m^*, L_m^*) = 0 \tag{41}$$

and

$$L_m^* = \arg\max V_m(L_m, K_m^*) \tag{42}$$

where $0 < L_m < 1$

The rationale of Equations (41) and (42) is similar to (14) and (15), while the range of L_m is redefined from 0 to 1, which respectively means banks only invest in long-term projects or only in short-term projects.

5.4 Quantitative Results

Similar to benchmark model, the following equations are computed to be reported for analysis. For the *Shareholder Fire Sale Loss* (*SFSL*), it can be written as, and to differentiate from our baseline analysis, we use capital letters to denote the subscripts respectively

$$SFSL_{SM} = \int_{\widehat{D_m}}^{\widehat{\widehat{D_m}}} G_s(D_{t1}) dH_s(D_{t1}) + K_m \left[1 - H_s(\widehat{\widehat{D_m}})\right] \tag{43}$$

where the first term denotes the expected fire sale loss to the shareholders when the banks survive from the liquidity shock, and the second term is the deadweight loss to the shareholders once the banks fail at time 1. Thus, banks will lose all the capital holdings if they cannot suffer from the liquidity shock, and it is determined by multiplying K_m with the probability of default $1 - H_s(\widehat{\widehat{D_m}})$. The probability of default due to liquidity risk is

$$BRL_{SM} = 1 - H_s(\widehat{\widehat{D_m}}) \tag{44}$$

The probability of default due to credit risk is

$$BRC_{SM} = \int_0^{\widehat{D_m}} [1 - F_m(\overline{D_{t1}})] dH_s(D_{t1}) + \int_{\widehat{D_m}}^{\overline{D_m}} [1 - F_m(\overline{\overline{D_{t1}}})] dH_s(D_{t1}) + \int_{\overline{D_m}}^{\widehat{\widehat{D_m}}} [1 - F_m(\overbrace{D_{t1}})]\, dH_s(D_{t1}) \tag{45}$$

where

$$\overline{D_{t1}} = \frac{K_m + (a_{L_m} - \mu')(1 - L_m)}{(1 - L_m)(\lambda + a_{L_m})}$$

$$\overline{\overline{D_{t1}}} = \frac{(1 - D_{t1})(1 - K_m) - (1 + a_{L_m} - \mu')(1 - L_m)[1 - Z_s(D_{t1})]}{(1 - L_m)(\lambda + a_{L_m})[1 - Z_s(D_{t1})]}$$

$$\overbrace{D_{t1}} = \frac{\gamma_m + Z_s(D_{t1}) + a_{L_m} - \mu}{\lambda + a_{L_m}}$$

The above equations capture the default thresholds due to credit risk and is conditional on three scenarios summarized by Equation (37), and is caused by the realization of liquidity risk D_{t1} that happens at time 1. Additionally, the liquidity requirements exclusive to Basel III can be summarized as

$$\frac{ASF}{RSF} = \frac{K_m + (1 - D_s)L_m}{D_s(1 - K_m)} \geq C'^{22} \tag{46}$$

where ASF and RSF are *available stable funding* and *required stable funding* respectively as indicated by Equation (3). The numerator denotes the weighted available stable funding to which capital and liquid assets are given weights 1 and $1 - D_s$[23] respectively, and the ratio C' is introduced to customize for our analysis. After rearranging Equation (46) we can obtain that

$$L_m \geq \frac{C'D_s(1 - K_m) - K_m}{1 - D_s} \tag{47}$$

The results are given by Table 5. We adopt $C' = 2$ for Basel III analysis, and compare the results as a separate study when $C' = 0, 5, 8$, the result of which is shown in Table 7.

< Insert Table 5 here >

5.4.1 Capital buffers

Table 5 shows that capital buffers are generally higher in state l than in state h, which is different from what we have obtained from our benchmark analysis, but keeps in line with Repullo & Suarez (2012). The reason behind this is that banks are less exposed to liquidity shocks as they can hold liquid assets to significantly reduce to fire sale loss compared with our benchmark analysis, and thus are more flexible to be operated to deal with risks. Accordingly, it is more profitable for banks to hold more capital in state l to preserve more returns, but less favourable to retain a higher capital in state h. Capital holdings are nearly the same in state l (at the value of 5.7%), which indicates that banks are more profitable and thus more willing to keep capital at a higher ratio. On the other hand, capital holdings strictly increases from 2.2% to 4.4% in state h when capital requirements become stricter, and the capital buffers are nearly zero (around 0.3% except for *Laissez-faire* regime at 3.4%). This implies that banks are not inclined to hold excess capital in recessions when the overall investment return is low.

5.4.2 Liquidity holdings

[22] Since the liquidity L_m in this case is different to what is referred to in our benchmark analysis l_m, we thus replace the required ratio with C' to avoid misunderstanding.

[23] D_s ,where $s = n, b$, stands for the worst-case liquidity shock and whose counterparty is d_s in the baseline analysis.

Liquidity holdings in state l and state h demonstrates a significant difference that is high at around 40% in state h, while merely at around 7% in state l. Moreover, this liquidity holding behaviour is also different from our benchmark model. This result suggests that banks have different strategies regarding liquidity hoardings when they can respectively adjust liquid assets and illiquid liabilities. It is straightforward to show that bank liquidity is countercyclical, making it significantly higher during recessions, which is capable to make them totally immune from liquidity shocks whose worst case risks is at 0.10, while the liquidity holdings is inefficiently low in booms. In other words, banks will hold excess liquidity buffers in recessions. This result stands with Acharya & Merrouche (2012) and Acharya *et al.* (2010) regarding banks' precautionary hoarding of liquidity during financial crisis. Liquidity holdings stabilizes at around 7.0% when in booms (state l), except for the cases when under *Basel III regime* that liquidity holding doubles at 14.1%. For recessions (state h), banks' equilibrium liquidity holdings raises from 35.7% when under *Laissez-faire regime*, to 40.1% and 42.8% for *Basel I* and *Basel II regimes* respectively. The highest level of liquidity holdings stands at 47.2% under *Basel III regime*, which means nearly half of the assets are in the form as liquid assets. This confirms with Cornett *et al.* (2011) that managing liquidity crisis in financial distress leads to a decline in lending as loans.

5.4.3 Expected shareholder fire sale loss

As addressed in Part 5.4.2, banks will not suffer from liquidity shocks in state h because the excess liquidity holdings. Thus, shareholders' fire sale loss is zero when in recessions. When in booms, the fire sales is possible when the bad liquidity shocks occurs, namely when $S = b$. Fire sale loss is slightly lower under *Basel I* regime at 0.05% because of a marginal higher liquidity holding at 7.3%, while the loss is at 0.07% for *Laissez-faire* and *Basel II regime*. *Basel I regime* is better at the respect to shareholder fire sale loss is because its higher capital requirement in booms at 4.0%, which motivates banks to hold a higher liquidity ratio to preserve their revenue. However, *Basel III regime* is the most efficient one that results in no shareholder fire sale loss during all states.

5.4.4 Probability of bankruptcy

Probabilities of default at the first period is zero for all regimes, which indicates banks are able to survive from liquidity shocks even when no capital requirements are in effect. This is slightly different from our benchmark analysis that banks under *Laissez-faire regime* suffers from a probability of 68.79% of default. The reason is that banks are less exposed to liquidity shocks and fire sales for our alternative analysis which makes banks more likely to survive in first period. For the second period, the probability of default increases significantly and is much higher in booms than in recessions (except for *Basel III* regime). The average probabilities of bankruptcy is at around 25.81% when in booms, while only 2.92% for recessions. This result is in line with Part 5.4.2 and Part 5.4.3 that less liquidity holdings in booms makes banks suffers more from fire sales. Similar to Part 5.4.3, the unconditional probability of default is higher under *Basel II* than *Basel I* due to the lower capital requirements under *Basel II regime*. However, with the introduce of *Basel III regime*, the probability of default reduces dramatically to 0.23% compared with around 7% for other regulation regimes.

5.5 Externality to social welfare

We use this section to discuss the external effects of the banking operations under different regimes. Similar to our benchmark analysis, the following terms are introduced for analysis. The *Additional Fire Sale Loss* (*AFSL*) calibrating the costs to the economy due to banking externality is as follows

$$AFSL_{SM} = \int_{\widehat{D_m}}^{\overline{D_m}} Z_s(D_{t1})\, dH_s(d_{t1}) + \int_{\overline{D_m}}^{\widehat{\widehat{D_m}}} \left[1 - \frac{K'_m(D_{t1})}{\gamma_m(1-L_m)}\right] dH_s(D_{t1}) + \left[1 - H_s(\widehat{\widehat{D_m}})\right] \tag{48}$$

The first term of Equation (48) denotes the expected fire sale loss once banks have used up all the liquidity L_m to pay for the shock. The second term is the expected loss because of reduced lending as the result of credit rationing. The term $K'_m(D_{t1})/\gamma_m$ is scaled by $1/(1-L_m)$ to adjust for the unit of investment is at $1-L_m$. The third term summarizes the total loss of investment when banks fail at time 1. Social welfares during the overall investment periods (S, M) is determined as

$$W_{SM} = E_{SM} + GI_{SM} + FC_{SM} \tag{49}$$

where

$$E_{SM} = (1-L_m)(1-p_m)(1-AFSL_{sm})b_{L_m} \tag{50}$$

are the expected payoff to the entrepreneurs once the projects are developed by the banks and succeed until time 2. As in Equation (26), the parameter $b_{L_m} = a_{L_m}$ captures the impacts of deadweight loss once the projects are sold (in full or at a portion) to outsiders and we then multiply it by $1-L_m$ to reflect the facts that the rest portion of the investments L_m is short-term and riskless which will not yield return to the entrepreneurs (or the economy),

$$GI_{SM} = \int_0^{\widehat{D_m}} \widehat{\Pi}_{sm}(D_{t1})\, dH_s(D_{t1}) + \int_{\widehat{D_m}}^{\overline{D_m}} \widehat{\Pi}'_{sm}(D_{t1})\, dH_s(D_{t1}) + \int_{\overline{D_m}}^{\widehat{\widehat{D_m}}} \widehat{\Pi}''_{sm}(D_{t1})\, dH_s(D_{t1}) + \int_{\widehat{\widehat{D_m}}}^{D_s} K'_m(D_{t1})\, dH_s(D_{t1}) \tag{51}$$

are the (negative) payoffs to the government as the role of deposit insurer once the banks fail at time 1 (the forth term) and at time 2 (the first three terms). Moreover, in Equation (51),

$$\widehat{\Pi}_{sm}(D_{t1}) = \int_0^1 \min\{0, K_m + (1 - L_m)[a_{L_m} - X_{t2}(\lambda + a_{L_m}) - \mu]\} \, dF_m(X_{t2})$$

(52)

$$\widehat{\Pi}'_{sm}(D_{t1}) = \int_0^1 \min\{0, (1 - L_m)[1 - Z_s(D_{t1})][1 + a_{L_m} - X_{t2}(\lambda + a_{L_m}) - \mu] - (1 - D_{t1})(1 - K_m)\} \, dF_m(X_{t2})$$

(53)

$$\widehat{\Pi}''_{sm}(D_{t1}) = \int_0^1 \min\left\{0, \frac{K'_m(D_{t1})}{\gamma_m}[\gamma_m + Z_s(D_{t1}) + a_{L_m} - X_{t2}(\lambda + a_{L_m}) - \mu]\right\} dF_m(X_{t2})$$

(54)

are the realizations of the losses to the government conditional on the liquidity shocks, and

$$FC_{SM} = -c[BRL_{SM} + BRC_{SM}]$$

(55)

are the bankruptcy costs due to banks' failure where BRL_{SM} and BRC_{SM} are determined by Equation (44) and (45) respectively. Similar to our benchmark analysis, the expected social welfare is given by Equation (49) for different financial situations (booms and recessions) when faced with liquidity shocks (normal and bad liquidity shock).

< Insert Table 6 here >

The expected additional fire sale loss shows a similar result as in shareholder fire sale loss that *Basel I regime* is marginally superior (at 0.06%) to *Laissez-faire* and *Basel II* regime at 0.08%. *Basel III regime* is the most efficient one that causes no fire sale loss to the economy. For social welfare analysis, it is straightforward to show that *Basel III regime* is the most superior one yielding 5.86% to the economy. This result keeps in line with our benchmark analysis that *Basel III regime* contributes the most to the social welfare. However, a slight difference from our baseline analysis is that the welfare of *Basel II regime* (4.90%) is lower than that of *Basel I* (5.02%). This higher revenue is attributed to the higher income at 5.52% (*Basel I*) than 5.30% (*Basel II*) in booms, because of a higher capital requirement under *Basel I regime* for booms.

Moreover, in order to reveal the effects of liquidity requirements on banking behaviours and their impacts to the economy, we continue our analysis to compare with different level of liquidity requirements under *Basel III regime*. Table 7 gives the results.

< Insert Table 7 here >

From Table 7, we can also notice an inverse U-shaped relationship between social welfare and liquidity requirements. The higher expected social welfare is achieved when $C' = 2$ at the value of 5.86%. When liquidity requirements becomes harsher, the social welfare reduces dramatically to

1.17% when $C' = 8$. This is because a harsher liquidity requirement will induce banks to hold more liquid assets and thus reduce the expected investment income. Unlike our baseline analysis, when no liquidity requirements are imposed to *Basel III* regime, banks are not inclined to reduce liquidity holdings significantly, and it stays at 8.9% and 45.3% respectively, which will not introduce additional fire sale loss compared with *Basel I* and *Basel II regime*. The social welfare is higher (at 5.81%) than that of *Basel II regime* (4.90%). This result indicates that banks are less sensitive to a higher capital requirement and capital holding when determining liquidity holdings. Capital holdings decreases with the increase in the level of liquidity requirements, partially because of a reduced investment scale, an increase in L_S^*. Accordingly, a higher capital buffer is caused by a reduction in risky assets and thus a less amount of capital is required to honour the risky asset investment. Fire sale loss, including *shareholders'* and *additional*, is reduced to nearly zero, except for $C' = 0$ the loss of which is 0.04%. Overall, this table shows that too soft or too strict liquidity requirement will reduce the overall expected social welfare, although this effect is less pronounced compared with our benchmark analysis within which the liquidity holdings are more profoundly affected by a harsher capital requirement.

6. Extension

In this section, we will graphically show some extensional results to better acknowledge some insights behind our models. We will demonstrate the relationship between liquidity holdings and capital holdings for both of our models and will help to verify some of our Propositions as argued before. In addition, we have considered several additional analyses for some key parameters. We will show the results for the benchmark model and alternative model individually.

6.1 Benchmark model

< Insert Figure 1 here >

Figure 1 proves the Proposition 5 (3) that a high capital requirement will discourage banks from holding liquidity. When capital holdings is nearly at the capital requirement level, namely when $k_m = \gamma_m$, banks will almost hold long-term debt, making short-term debt holdings zero. However, when the capital holdings increases, the corresponding liquidity holding reduces, and when capital holdings is above 0.14 (0.12) for recessions (booms) banks will not hold any liquidity, exposing themselves totally to liquidity shocks.

< Insert Figure 2 here >

Figure 2 shows the responses of the optimal liquidity holdings as the function of long-term deposit rate. For comparison, we presents two scenarios when $\Delta = 0.02$ and $\Delta = 0.04$. When capital buffers are at Δ$= 0.02$, liquidity holdings starts at $l_m = 1 - k_m$ when $j = 0.00$. With the increase of j, liquidity holdings generally decreases, although with slight variations when $j < 0.03$. The reason is that capital buffers is relatively small and thus banks might be able to trade off between the cost of holding capital and long-term debt interest payment when the deposit rate is relatively small. However, when j increases further, the cost of holding liquidity is high and accordingly banks will not hold any long-term debt, resulting in $l_m = 0$. Moreover, when the deposit rate is sufficiently high, banks are incentivized to hold excess liquidity which makes them definitely to fail. This is proved by the figure that a jump exists when $j > 0.09$. These mentioned results justify the Proposition 5 (1) that a corner solution at $l_m = 1 - k_m$ is possible when r_m is sufficiently low or extremely high.

To further investigate the impact of a higher capital holding, we compare the results when Δ$= 0.04$ under which situation the costs of holding capital is higher. We can thus notice that liquidity holdings are strictly decreasing with the increase in deposit rate j, which contradicts to the case when Δ$= 0.02$ that a marginal increase of liquidity holdings is present in the process of j increase. This contradiction can be attributed to the fact that a higher capital holding is so costly that making trading off unprofitable and unnecessary, thus banks will not slightly increase liquidity holdings for some value of j as revealed when Δ$= 0.02$. Moreover, Proposition 5 (3) is also proved by the fact that liquidity holdings are generally lower when Δ$= 0.04$ than the case Δ$= 0.02$, if under the same deposit rate j.

6.2 Alternative model

< Insert Tables 3 and 4 here >

Figure 3 stands with Proposition 8 (2) and (3) which discusses the relationship between capital and liquidity holdings under different scenarios. For booms, banks will not hold excess liquidity, with the largest value $L_m = 0.094$ when $K_m = 0.06$. Thus, according to Proposition 8(3), the capital holdings will profoundly affect the liquidity holding behaviours, that is with the increase in capital holdings, the resulting liquidity holdings decreases quasi-linearly to $L_m = 0.00$ when $K_m = 1.00$, under which situation banks are fully financed with equity. This trends stands with our benchmark model, however, when in recessions it is opposite. When in recessions, the minimum liquidity holdings is $L_m = 0.481$ for $K_m = 0.06$, which means banks will hold excess liquidity and thus reduce loan investment. For this scenario, liquidity holdings marginally increases to $L_m = 0.490$ when $K_m \geq 0.08$, and the liquidity remains at this value even when $K_m = 1.00$. This result contradicts to the trend as in booms and our benchmark model's, but confirms with Proposition 8(2) that although capital holdings will affect hoarding liquidity, this effect is insignificant when an excess liquidity is retained. Figure 4 presents the impact of diminishing return to scales w. When w is sufficiently small, the diminishing effects is not pronounced and thus banks are not motivated to hold more liquidity, while when the value increases the liquidity holdings increases to $L_m = 0.702$ and $L_m = 0.853$ for booms and recessions respectively if $w = 0.09$. Moreover, this effect is more pronounced for recessions when the expected return is relatively low and thus banks will be more reliant on the diminishing returns to retain some revenue.

7. Discussion

This section summarizes some aspects that are derived from our analysis and implications to the policy makers, together with some potential extensions for further analysis. This includes the capital requirements for booms, the risk-based capital requirements for Basel II, and the introduction of the countercyclical capital buffer by Basel III. Some extensions is conducted, including the relaxation of equity markets that allows banks to accumulate equities in the first period or a bailout policy undertaken by the government or central banks to inject capital to distressed banks. In addition, a consideration of interbank market will help to better reflect the reality.

7.1 Countercyclical capital buffers

As we have addressed in Table 3, for our benchmark analysis, that the expected social welfare in booms is 10.07% for *Basel II regime*, lower than that of 10.20% under *Basel I regime*. Additionally, for our alternative analysis, from Table 5 and Table 6 the expected probability of default is higher for *Basel II regime* and the expected social welfare is lower under *Basel II regime* compared with the *Basel I* one. This is all caused by a lower capital requirement under *Basel II regime* in booms, which is risk-adjusted, at merely 3.2%, even lower than 4.0% required by *Basel I regime* for booms. Moreover, as shown in Table 5 and Table 6, in booms, banks will hold much less liquidity, than in recessions, which makes fire sales possible. This result implies that during booms banks will be more likely to overlook potential crises once liquidity risks and fire sales are considered. This insight thus suggests that it is not optimal to require less capital in booms due to its lower credit risk. Thus, the drawback of *Basel II regime* is recognized, and the proposed countercyclical buffers is highly recommended to mitigate potential cyclical behaviours.

7.2 Access to equity market and capital injection

When banks have access to equity market which allows them to build up capital to avoid credit rationing, our results might be, to some degree, altered. As pointed out by Repullo and Suarez (2012), if the access is possible, banks might not be likely to hold capital buffers. They have also introduced an extension analysis for assuming a state-based probability of accessing equity in the first period in order to capture capital market imperfection. Moreover, with the consideration of governmental bailout policy might change our results as well. Since banks will be confident of a bailout policy especially for large banks due to the Too-Big-To-Fail assumption, we conjecture this consideration will make banks hold less capital buffers. However, as in Acharya *et al.* (2010), a financial support to distressed banks conditional on their liquid asset holdings will increase banks' incentives to hold liquidity. An extension for this consideration will be better to reflect banks' risk-taking behaviours and shed light on banks' bailout policy implications.

7.3 Interbank market

Our paper fails to consider the existence of interbank market which could help to facilitate liquidity support within banks. A potential extension for interbank market could be formed by incorporating interbank market rates and its changes during the business cycle. Following Freixas *et al.* (2011) and Diamond and Rajan (2012), we anticipate a countercyclical interbank interest rate, making a lower rate during crises and a higher rate in booms. Moreover, as suggested by the results from Acharya *et al.* (2010), a higher equilibrium liquidity holding can be expected due to the potential gains from acquiring distressed banks. A further study regarding interbank market might help to capture this effect and reveal some insights for banks' capital and liquidity holdings during the business cycle.

8. Conclusion

This paper presents a two-period model to reveal the impacts of capital and liquidity requirements on the banking behaviours over the business cycle. Our model is featured by the fire sale losses caused by the liquidity shocks and have revealed banks' capital and liquidity holding behaviours and their interactions with each other. We have considered two separate situations when banks are able to adjust the asset and liability sides and helped to reveal banks' different risk management behaviours when subject to different adjustment strategies. Banks are more reliant on liquidity holdings to deal with liquidity shocks when they can determine the composition of deposits, and capital holdings will heavily affect liquidity holding behaviours. When banks can choose between liquid and illiquid assets, we have verified an excess liquidity holding behaviour as revealed by some existing literature and have attributed this to the overall low investment return and diminishing return to investment that motivates banks to hoard liquidity. On the contrary, banks will hold much less liquidity in booms which will trigger fire sales and thus a liquidity requirement is necessary to help resolve this issue. We have also identified that a high capital holding will generally discourage banks to hoard liquidity in booms although this effect is less pronounced compared with the first situation. In addition, for both situations, we have identified an inverse U-shaped relationship between liquidity requirement and social welfare, suggesting the existence of an optimal liquidity requirement. We conclude our paper with some potential extensions which worthy attention for future studies. Firstly, future models could consider the access to equity markets, which allows banks to re-issue equities in the first period, or the potential bailout policy, which permits capital injection to distressed banks. Secondly, adding the effects of interbank market will better reflect the reality.

Appendix

Proof of Proposition 1

Assume the loan is of unit face value. Suppose an amount of debt with face value of $d < 1$ needs to be paid, and they have to liquidate the loans to repay, they will have to liquidate z unit loan, which satisfies

$$\int_{(1-z)}^{1} x\,dx = d$$

Using the law of integral, the above equation is equivalent to $\frac{1}{2}[1-(1-z)^2] = d$ (*).

Then the market value of the remaining loan, at the amount of $1-z$, is $(1-z)^2$. Rearrange (*), we can obtain that $z = 1 - \sqrt{1-2d}$.

Proof of Proposition 3

After simplifying the equation

$$k_m - d_{t1}(1 - l_m - k_m) - l_m r_m \geq \gamma_m[1 - z_s(d_{t1})]$$

using the definition of $z_s(d_{t1})$, we can get

$$k_m - G_s(d_{t1}) - l_m r_m \geq \gamma_m\sqrt{1 - 2G_s(d_{t1})}$$

The roots of the above equation are

$$G_s(d_{t1})_1 > k_m - l_m r_m - \gamma_m^2 + \gamma_m\sqrt{\gamma_m^2 - 2(k_m - l_m r_m) + 1} \tag{A1}$$

and

$$G_s(d_{t1})_2 < k_m - l_m r_m - \gamma_m^2 - \gamma_m\sqrt{\gamma_m^2 - 2(k_m - l_m r_m) + 1}$$

For the first root, obtained by Equation (A1), we can have following

$$G_s(d_{t1})_1 > k_m - l_m r_m$$ [24]

which is equivalent to Equation (5) indicating the bankruptcy, accordingly this root is ruled out.
Thus, the banks will not trigger the capital constraints if

$$d_{t1} < \widehat{d_{mm}} = \frac{k_m - l_m r_m - \gamma_m^2 - \gamma_m\sqrt{\gamma_m^2 - 2(k_m - l_m r_m) + 1}}{1 - l_m - k_m}$$

which is the results of Proposition 3.

Calculation of $\pi'_{sm}(d_{t1})$

If the capital requirement is triggered, the banks have to sell (a portion of) the assets to the outsiders and obtain the money to repay the debt holders. Suppose banks have to hold the assets only at the amount of $\frac{k'_m(d_{t1})}{\gamma_m}$, and after the fire sales, banks own $1 - z_s(d_{t1})$, thus the amount the banks have to sell is $1 - z_s(d_{t1}) - \frac{k'_m(d_{t1})}{\gamma_m}$ at the price of $p_s(d_{t1}) = [1 - z_s(d_{t1})]$. Thus, the total cash banks obtained from selling assets (not fire sales) is

$$\left[1 - z_s(d_{t1}) - \frac{k'_m(d_{t1})}{\gamma_m}\right][1 - z_s(d_{t1})] = 1 - 2G_s(d_{t1}) - \frac{k'_m(d_{t1})}{\gamma_m}\sqrt{1 - 2G_s(d_{t1})}$$

This cash is kept by the banks until time 2 to repay the debt holders. Recall that the total debt at time 2 is $l_m(1 + r_m) + (1 - d_{t1})(1 - l_m - k_m)$. Since the total asset held by the banks until time 2 is $\frac{k'_m(d_{t1})}{\gamma_m}$, the net worth of the banks' shareholders, as the function of d_{t1} and x_{t2} is

[24] This transformation indicates that in our calibration, when banks are subject to credit rationing, $k_m < 0.5$.

$$\pi'_{sm}(d_{t1}) = \frac{1}{1+\delta}\int_0^1 \max\{\frac{k'_m(d_{t1})}{\gamma_m}[1+a-x_{t2}(\lambda+a)] + 1 - 2G_m(d_{t1}) - \frac{k'_m(d_{t1})}{\gamma_m}\sqrt{1-2G_s(d_{t1})} - l_m(1+r_m) - (1 - d_{t1})(1-l_m-k_m), 0\}dF_m(x_{t2})$$

Using the definition of $k'_m(d_{t1})$ in Equation (4), and after simplification, we can obtain $\pi'_{sm}(d_{t1})$ as in Equation (13).

$$\pi'_{sm}(d_{t1}) = \frac{1}{1+\delta}\int_0^1 \max\{k'_m(d_{t1}) + \frac{k'_m(d_{t1})}{\gamma_m}[1+a-x_{t2}(\lambda+a)-\mu] - \frac{k'_m(d_{t1})}{\gamma_m}\sqrt{1-2G_s(d_{t1})}, 0\}\, dF_m(x_{t2})$$

Proof of Proposition 4

Following the theorem of maximum, the function $v_m[l_m(k_m), k_m]$, where $l_m(k_m)$ is the optimal choice of l_m given k_m, is continuous in k_m. We thus have

$$\frac{dv_m}{dk_m} = \frac{\partial v_m}{\partial l_m}\frac{dl_m}{dk_m} + \frac{\partial v_m}{\partial k_m}$$

By the theorem of envelope, if $l_m(k_m)$ is interior, the first term is zero. If $l_m(k_m)$ is at the corner, we have $dl_m/dk_m = 0$, and thus the first term is also zero. Overall, we have $dv_m/dk_m = \partial v_m/\partial k_m$. Accordingly, we have following properties regarding $\frac{dv_m}{dk_m}$,

$$\frac{dv_m}{dk_m} = \frac{\partial v_m}{\partial k_m} = \frac{1}{(1+\delta)^2}\left[\int_0^{\widehat{d_{mm}}}\int_0^{\dddot{x}_{t2}}\left\{\frac{d_{t1}}{1-z_s(d_{t1})}[1+a-x_{t2}(\lambda+a)-\mu] + 1 - d_{t1}\right\}dF_m(x_{t2})d(d_{t1}) + \int_{\widehat{d_{mm}}}^{\widehat{d_m}}\frac{d_{t1}}{\gamma_{m'}}\int_0^{\ddot{x}_{t2}}\left\{\gamma_{m'} + z_s(d_{t1}) + a - x_{t2}(\lambda+a) - \mu - \frac{k'_m(d_{t1})}{1-z_e(d_{t1})}\right\}dF_m(x_{t2})d(d_{t1})\right] - 1 \gtreqless 0$$

where $\dddot{x}_{t2} = \frac{[1-z_s(d_{t1})](1+a-\mu) - l_m(d_{t1}+r_m) - (1-d_{t1})(1-k_m)}{[1-z_s(d_{t1})](\lambda+a)}$ and $\ddot{x}_{t2} = \frac{\gamma_m + z_s(d_{t1}) + a - \mu}{\lambda + a}$

Unfortunately, we cannot obtain an analytical solution to the above equation. However, we can confirm that when $k_m = 0$, $v_m[l_m(k_m), k_m] = 0$ and $\frac{dv_m}{dk_m} = -1 < 0$. Thus, when banks' capital holdings are sufficiently low (not equal to zero), banks' net income is negative. Additionally, $v_m[l_m(k_m), k_m] < 0$ if $k_m = 1$, because the return a is not significant, in our calibration, to make a positive income if banks are fully equity financed. Overall, from the theorem of maximum and $l_m(k_m)$ is continuous in k_m, there might exist other solutions to satisfy the condition $v_m[l_m^*(k_m), k_m^*] = 0$. However, if there exist multiple solutions, we adopt the smallest solution because it is suboptimal to hoard more equity once the capital requirements are satisfied.

Proof of Proposition 5

For any given capital holdings k_m, banks will adopt a liquidity holding l_m to maximize shareholders' net worth. By the Weierstrass theorem, there should exist a liquidity decision given the fact that $v_m(k_m, l_m)$ is continuous in l_m for a given capital holding k_m. The function $v_m(k_m, l_m)$ can be written as, where q_{sm} represents the transition probabilities for $s = n, b$ and $m = l, h$

$$v_m(k_m, l_m) = q_{bm}v_{bm}(k_m, l_m) + q_{nm}v_{nm}(k_m, l_m)$$

where

$$v_{sm}(k_m, l_m) = \frac{1}{1+\delta}\left[\int_0^{\widehat{d_{mm}}}\pi_{sm}(d_{t1})d(d_{t1}) + \int_{\widehat{d_{mm}}}^{\widehat{d_m}}\pi'_{sm}(d_{t1})d(d_{t1}) + \min(l_m r_m, k_m)\frac{d_m - \widehat{d_m}}{d_m}\right] - k_m$$

Using the definition of $\widehat{d_{mm'}}$ in (8), $\widehat{d_m}$ in (5), $\pi_{sm}(d_{t1})$ in (12) and $\pi'_{sm}(d_{t1})$ in (13), we can establish the following properties of function $v_{sm}(k_m, l_m)$. First, we consider the situation when r_m is zero, banks would like to retain as much liquidity as they can because holding long-term debt is costless and will prevent them from liquidity shocks. Thus, banks will raise liquidity position at $1 - k_m$, causing a corner solution where all the debt is long-term. Then we will consider the scenario where r_m is non-zero.

(1) For $l_m \geq \frac{k_m}{r_m}$, we have $\widehat{d_{mm}} < \widehat{d_m} \leq 0$, so

$$\frac{\partial v_{sm}}{\partial l_m} = 0$$

When k_m is small or r_m is relatively high, $\frac{k_m}{r_m} \le l_m < 1 - k_m$ can be easily satisfied, and thus banks will fail at time 1 as the capital they retain cannot cover the interest payment. Since banks are indifferent in choosing liquidity holdings under this circumstance, a corner solution at $l_m = 1 - k_m$ seems plausible.

(2) For $\frac{k_m-\gamma_m}{r_m} \le l_m < \frac{k_m}{r_m}$, we have $\widehat{d_{mm}} \le 0 < \widehat{d_m}$, so

$$\frac{\partial v_{sm}}{\partial l_m} = \frac{1}{1+\delta}\left\{\int_0^{\widehat{d_m}} \frac{d\pi'_{sm}(d_{t1})}{dl_m} d(d_{t1}) + r_m - \frac{r_m}{d_m}\left[\widehat{d_m} + l_m \frac{k_m(1+r_m)-r_m}{(1-l_m-k_m)^2}\right]\right\}$$
$$= \frac{1}{1+\delta}\left\{\frac{1}{(1+\delta)\gamma_m}\int_0^{\widehat{d_m}}\int_0^{\dot{x_{t2}}}\Big\{(d_{t1}-\gamma_m)[\gamma_m + z_s(d_{t1}) + a - x_{t2}(\lambda+a) - \mu]\right.$$
$$\left. - k'_m(d_{t1})[\frac{d_{t1}}{1-z_s(d_{t1})}]\Big\} dF_m(x_{t2})\, d(d_{t1}) - \frac{r_m}{d_m}\left[(\widehat{d_m} - d_m) + l_m \frac{k_m(1+r_m)-r_m}{(1-l_m-k_m)^2}\right]\right\} \gtreqless 0^{25}$$

where $\dot{x_{t2}} = \frac{\gamma_m + z_s(d_{t1}) + a - \mu}{\lambda + a}$

We can infer that with a higher capital holding k_m, $-\frac{r_m}{d_m}\left[(\widehat{d_m} - d_m) + l_m \frac{k_m(1+r_m)-r_m}{(1-l_m-k_m)^2}\right]$ can be negative, together with the fact that $z_s(d_{t1})$ decreases with k_m, and $k'_m(d_{t1})[\frac{d_{t1}}{1-z_s(d_{t1})}]$ might also be large enough to make $\frac{\partial v_{sm}}{\partial l_m} < 0$. That is, the integrand of the above equation strictly decreases with k_m. Thus, a higher capital holding k_m will discourage banks to finance with long-term debt. In our calibration, $\frac{k_m-\gamma_m}{r_m}$ can be easily proved to be strictly positive if with capital requirements, but it cannot be guaranteed to be less than $1 - k_m$, especially when r_m is low enough. Thus, a corner solution, at $l_m = 1 - k_m$, could be possible if with an appropriate capital holding. However, a corner solution at $l_m = 0$ is impossible in this scenario.

(3) For $\dot{l_m} \le l_m < \frac{k_m-\gamma_m}{r_m}$, where $\dot{l_m}$ is the smaller root of $\widehat{d_{mm}} = d_s$, we have $0 < \widehat{d_{mm}} \le d_s$, so

$$\frac{\partial v_{sm}}{\partial l_m} = \frac{1}{1+\delta}\left\{\frac{1}{(1+\delta)\gamma_m}\int_0^{\widehat{d_m}}\int_0^{\dot{x_{t2}}}\Big\{(d_{t1}-\gamma_m)[\gamma_m + z_s(d_{t1}) + a - x_{t2}(\lambda+a) - \mu]\right.$$
$$\left. - k'_m(d_{t1})[\frac{d_{t1}}{1-z_s(d_{t1})}]\Big\} dF_m(x_{t2})\, d(d_{t1}) - \frac{r_m}{d_m}\left[(\widehat{d_m} - d_m) + l_m \frac{k_m(1+r_m)-r_m}{(1-l_m-k_m)^2}\right]\right\}$$
$$+ \frac{1}{(1+\delta)^2}\int_0^{\widehat{d_{mm}}}\int_0^{\widehat{x_2}} \{[z_s(d_{t1}) + a - x_{t2}(\lambda+a) - \mu]\frac{d_{t1}}{1-z_s(d_{t1})} - r_m\} dF_m(x_{t2}) d(d_{t1}) \gtreqless 0$$

where $\widehat{x_2} = \frac{[1-z_s(d_{t1})](1+a-\mu) - l_m(d_{t1}+r_m) - (1-d_{t1})(1-k_m)}{[1-z_s(d_{t1})](\lambda+a)}$

For our analysis, $\dot{l_m}$ can be positive or negative, depending on the value of d_s. When d_m is high enough, $\dot{l_m}$ might be negative, and thus a corner solution at $l_m = 0$ is possible. Another corner solution $l_m = 1 - k_m$ can also be possible if r_m is low enough. Similar to our analysis in scenario (2), the first term can be negative when k_m is high enough. We next consider the second term $\frac{\partial v'_{mm}}{\partial l_m} = \frac{1}{(1+\delta)^2}\int_0^{\widehat{d_{mm}}}\int_0^{\widehat{x_2}}\{[z_s(d_{t1}) + a - x_{t2}(\lambda+a) - \mu]\frac{d_{t1}}{1-z_s(d_{t1})} - r_m\} dF_m(x_{t2}) d(d_{t1})$.

i) Since r_m is positive, we first neglect it for simplicity, but this will not change our results as $\frac{\partial v_{sm}}{\partial l_m}$ cannot be larger than the scenario when r_m is considered. Thus, we have the following

$$\frac{\partial v_{sm}}{\partial l_m} < \frac{1}{(1+\delta)^2}\int_0^{\widehat{d_{mm}}}\int_0^{\widehat{x_2}} d_{t1}[\frac{z_s(d_{t1}) + a - x_{t2}(\lambda+a) - \mu}{1-z_s(d_{t1})}] dF_m(x_{t2}) d(d_{t1}) \tag{A3}$$

Using the definition of $\widehat{x_2}$, we can obtain the maximum of $z_s(d_{t1}) + a - x_{t2}(\lambda+a) - \mu$ is $a + z_s(d_{t1}) - \mu \gtreqless 0$, while the minimum is $z_s(d_{t1}) - 1 + \frac{l_m(d_{t1}+r_m)+(1-d_{t1})(1-k_m)}{1-z_s(d_{t1})} = \frac{-[1-z_s(d_{t1})]^2 + l_m(d_{t1}+r_m)+(1-d_{t1})(1-k_m)}{1-z_s(d_{t1})} = \frac{-k'_m(d_{t1})}{1-z_s(d_{t1})} < 0$[26]. Thus, there

[25] Since we are more interested in the corner solution of liquidity holdings at 0 and $1 - k_m$, we simply skip the detailed calculation of $\frac{\partial v_{mm'}}{\partial l_m}$.

[26] This inequality can be obtained by using the definition of $k'_m(d_{t1})$ in Equation (4).

must exist a $\dddot{k_m}$ that makes (A3) equal to zero. When $k_m^* > \dddot{k_m}$, we can notice that $\frac{\partial v_{mm}}{\partial l_m} < 0$, and thus indicating banks are not willing to retain long-term debt.

ii) Then, when we consider the existence of r_m, we can confirm $\frac{\partial v_{mm}}{\partial l_m} < 0$ will be more likely to be satisfied given r_m is positive and thus an even lower $\dddot{k_m}$ will more likely to raise banks' reluctance to hold long-term debt.

Thus, the above two terms will be negative if with high k_m. Similar to our conclusion in (2), a high enough capital holding k_m will discourage banks to hold long-term debt.

(4) For $l_m < \dot{l_m}$, we have $d_s < \widehat{d_{mm'}} < \widehat{d_m}$, so

$$\frac{\partial v_{sm}}{\partial l_m} = \frac{1}{1+\delta}\int_0^{d_s} \frac{d\pi_{mm}(d_{t1})}{dl_m} d(d_{t1}) = \frac{1}{(1+\delta)^2}\int_0^{d_s}\int_0^{\widehat{x_2}} \{[z_s(d_{t1}) + a - x_{t2}(\lambda + a) - \mu]\frac{d_{t1}}{1 - z_s(d_{t1})} - r_m\} dF_m(x_{t2}) d(d_{t1}) \tag{A2}$$

Similar to (3), $\dot{l_m}$ can be positive when d_s is small enough. However, if $\dot{l_m}$ is negative, this scenario is ruled out. When positive, a corner solution at $l_m = 0$ might be possible if $\frac{\partial v_{mm}}{\partial l_m} < 0$; however, a corner solution at $l_m = 1 - k_m$ will also be possible if r_m is low enough and if $\frac{\partial v_{mm}}{\partial l_m} > 0$. Similar to our conclusion in (3), $\frac{\partial v_{mm}}{\partial l_m}$ can be negative if with a high enough capital holding k_m.

Overall, a corner solution at $l_m = 1 - k_m$ will be possible if the long-term interest rate r_m is zero or sufficiently low, but in this case, the probability of credit rationing or bankruptcy is uncertain because this can be possible in the above four scenarios. In all scenarios, banks will have incentives to hold less long-term debt if they have already retain a high enough capital holding k_m. However, when banks retain no liquidity the probability of no credit rationing is strictly positive, as this solution cannot be possible when $\widehat{d_{mm}} \leq 0 < \widehat{d_m}$.

Proof of Proposition 6

Banks under capital requirements will not bind capital constraint if $K_m'(D_{t1}) \geq \gamma_{m'}[1 - Z_s(D_{t1})](1 - L_m)$. Using the definition of $K_m'(D_{t1})$ in (19). The condition can be rewritten as

$$K_m'(D_{t1}) = K_m + L_m - D_{t1}(1 - K_m) \geq \gamma_m[1 - Z_s(D_{t1})](1 - L_m)$$

where $Z_s(D_{t1}) = 1 - \sqrt{1 - 2[D_{t1}(1 - K_m) - L_m]/(1 - L_m)}$.

The root of the above equation are

$$D_{t1_1} > \frac{K_m + L_m - \gamma_m^2(1 - L_m) + \gamma_m\sqrt{\gamma_m^2(1 - L_m)^2 + (1 - L_m)(1 - L_m - 2K_m)}}{1 - K_m}$$

and

$$D_{t1_2} < \frac{K_m + L_m - \gamma_m^2(1 - L_m) - \gamma_m\sqrt{\gamma_m^2(1 - L_m)^2 + (1 - L_m)(1 - L_m - 2K_m)}}{1 - K_m} \tag{A4}$$

In our calibration, the minimum value of D_{t1_1} is $\frac{K_m + L_m}{1 - K_m} = \widehat{\widehat{D_m}}$, which indicates banks' bankruptcy and thus the first root is ruled out. Accordingly, banks will not face the capital requirement constraint if $D_{t1} < \overline{D_m}$ which is given by Equation (A4). Banks will face the capital requirement constraints once the liquidity shock exceeds that level.

Proof of Proposition 7

Similar to our benchmark model, once banks bind the capital requirement constraint, they have to sell a portion of their loans at the fire sale price $P_s = (1 - Z_s)$. After selling a portion of $Z_s(D_{t1})$, banks have to retain the loan only at the amount of $\frac{K_m'(D_{t1})}{\gamma_m}$. Banks will have to sell (a portion of) loans to the outsiders at the amount of $[1 - Z_s(D_{t1})](1 - L_m) - \frac{K_m'(D_{t1})}{\gamma_m}$ at the price of $P_s(D_{t1}) = [1 - Z_s(D_{t1})]$, and this funding will be paid to the debt holders, at the amount of $\{[1 - Z_s(D_{t1})](1 -$

$L_m) - \frac{K_m'(D_{t1})}{\gamma_m}\}[1 - Z_s(D_{t1})]$. This funding is equivalent to, using the definition of $Z_s(D_{t1})$, $\left\{1 - \frac{2[D_{t1}(1-K_m)-L_m]}{1-L_m}\right\}(1 - L_m) - \frac{K_m'(D_{t1})}{\gamma_m}[1 - Z_s(D_{t1})]$. Thus, the remaining debt is at the amount of $(1 - D_{t1})(1 - K_m) - 1 + L_m + 2[D_{t1}(1 - K_m) - L_m] + \frac{K_m'(D_{t1})}{\gamma_m}[1 - Z_s(D_{t1})]$. Accordingly, banks' net income $\Pi_{sm}''(D_{t1})$, conditional on X_{t2}, is

$$\Pi_{sm}''(D_{t1}) = \frac{1}{1+\delta}\int_0^1 \max\left\{\frac{K_m'(D_{t1})}{\gamma_m}\left[1 + a_{L_m} - X_{t2}\left(\lambda + a_{L_m}\right) - \mu\right] - (1 - D_{t1})(1 - K_m) + 1 - L_m \right. \\ \left. - 2[D_{t1}(1 - K_m) - L_m] - \frac{K_m'(D_{t1})}{\gamma_m}[1 - Z_s(D_{t1})], 0\right\} dF_m(X_{t2})$$

The above equation, after simplification using the definition of $K_m'(D_{t1})$ in (20), is equivalent to Equation (27).

Proof of Proposition 8

Similar to Proposition 5, banks shareholder net worth function can be summarized as follows

$$V_m(K_m, L_m) = q_{bm}V_{bm}(K_m, L_m) + q_{nm}V_{nm}(K_m, L_m)$$

where

$$V_{sm}(K_m, L_m) = \frac{1}{1+\delta}\left[\int_0^{\widehat{D_m}} \Pi_{sm}(D_{t1})d(D_{t1}) + \int_{\widehat{D_m}}^{\overline{D_m}} \Pi_{sm}'(D_{t1})d(D_{t1}) + \int_{\overline{D_m}}^{\widehat{\widehat{D_m}}} \Pi_{sm}''(D_{t1})d(D_{t1})\right] - K_m$$

Using the definition of above terms in corresponding equations, we can derive the following properties of $V_{sm}(K_m, L_m)$. From the definition of $\widehat{D_m}$, $\overline{D_m}$ and $\widehat{\widehat{D_m}}$, they are all strictly positive, and thus we are more interested in their upper bound D_s.

1) For $L_m > D_s(1 - K_m)$, we have $\widehat{D_m} > D_s$, which means fire sales will never happen, so

$$\frac{\partial V_{sm}}{\partial L_m} = \frac{1}{(1+\delta)^2}\left\{[\lambda + a - (1 - L_m)w]\int_0^{\overline{X_{t2}}} X_{t2}\, dF_m(X_{t2}) - [a - (1 - L_m)w - \mu']F_m(\overline{X_{t2}})\right\} \gtreqless 0 \tag{A5}$$

where $\overline{X_{t2}} = \min\left[\max\left(\frac{K_m + (a_{L_m} - \mu')(1-L_m)}{(\lambda + a_{L_m})(1-L_m)}, 0\right), 1\right]$ to make $F_m(\overline{X_{t2}})$ definable.

To make $a_{L_m} > 0$, and from (18) and our calibration, we can note that $(1 - L_m)w < 2a < \lambda + a$. We can thus identify that when $w > \frac{a}{1-L_m}$, $\frac{\partial V_m(K_m,L_m)}{\partial L_m} > 0$, that is, banks will find it optimal to hold more liquid asset; however, when w is sufficiently low, $\frac{\partial V_m(K_m,L_m)}{\partial L_m} < 0$ because holding liquid asset will not yield any return and thus banks will not hold excess liquid asset. This scenario helps to explains banks' liquidity hoarding behaviours, due partially to diminishing return to investment which makes excess investing in risky assets suboptimal especially when the overall expected investment return is significantly low like in recessions. When hoarding excess liquidity, capital holdings seems not directly influence the liquidity hoardings as there are no K_m in the integrand of Equation (A5) to introduce a negative sign to Equation (A5).

2) For $\widetilde{L_m} < L_m < D_s(1 - K_m)$, where $\widetilde{L_m}$ is the bigger root of $\overline{D_m} = D_s$, we have $\widehat{D_m} < D_s < \overline{D_m}$, which means fire sales is possible but credit rationing will not happen, so

$$\begin{aligned}
\frac{\partial V_{sm}}{\partial L_m} = {} & \frac{1}{(1+\delta)^2}\int_0^{\widehat{D_m}}\left\{[\lambda + a - (1 - L_m)w]\int_0^{\overline{X_{t2}}} X_{t2}\, dF_m(X_{t2}) - [a - (1 - L_m)w - \mu']F_m(\overline{X_{t2}})\right\} d(D_{t1}) \\
& + \frac{1}{(1+\delta)^2\sqrt{(1 - L_m)[1 + L_m - 2D_{t1}(1 - K_m)]}}\int_{\widehat{D_m}}^{D_s}\int_0^{\overline{\overline{X_{t2}}}}\left\{(1 - L_m)[1 + L_m - 2D_{t1}(1 - K_m)]\left(\frac{w}{2} - \frac{w}{2}X_{t2}\right) + [1 \right. \\
& \left. + a_{L_m} - X_{t2}\left(\lambda + a_{L_m}\right) - \mu'][D_{t1}(1 - K_m) - L_m]\right\} dF_m(X_{t2})\, d(D_{t1}) \\
= {} & \frac{1}{(1+\delta)^2}\int_0^{\widehat{D_m}}\left\{[\lambda + a - (1 - L_m)w]\int_0^{\overline{X_{t2}}} X_{t2}\, dF_m(X_{t2}) - [a - (1 - L_m)w - \mu']F_m(\overline{X_{t2}})\right\} d(D_{t1}) \\
& + \frac{1}{(1+\delta)^2\sqrt{(1 - L_m)[1 + L_m - 2D_{t1}(1 - K_m)]}}\int_{\widehat{D_m}}^{D_s}\int_0^{\overline{\overline{X_{t2}}}}\left\{\frac{w}{2}(1 - L_m)(1 + L_m)(1 - X_{t2}) \right. \\
& - L_m\left[\left(1 + a - w\frac{1 - L_m}{2}\right) - X_{t2}\left(\lambda + a - w\frac{1 - L_m}{2}\right) - \mu'\right] \\
& \left. + D_{t1}(1 - K_m)\left[1 + a - X_{t2}(\lambda + a) - \mu' - \frac{3}{2}(1 - L_m)w(1 - X_{t2})\right]\right\} dF_m(X_{t2})\, d(D_{t1}) \gtreqless 0
\end{aligned} \tag{A6}$$

where $\overline{\overline{X_{t2}}} = \min\left[\max\left(\frac{(1-D_{t1})(1-K_m)-(1+a_{L_m}-\mu\prime)(1-L_m)[1-Z_s(D_{t1})]}{(1-L_m)(\lambda+a_{L_m})[1-Z_s(D_{t1})]},0\right),1\right]$ which is set to make $F_m(\overline{\overline{X_{t2}}})$ definable.

To investigate the capital holdings K_m to liquidity holding L_m, we can notice from Equation (A6) that K_m exists only in the coefficient $\frac{1}{(1+\delta)^2\sqrt{(1-L_m)[1+L_m-2D_{t1}(1-K_m)]}}$ which is strictly positive in our calibration, and the part $D_{t1}(1-K_m)\left[1+a-X_{t2}(\lambda+a)-\mu'-\frac{3}{2}(1-L_m)w(1-X_{t2})\right]$. It is straightforward to show that $1+a-X_{t2}(\lambda+a)-\mu'-\frac{3}{2}(1-L_m)w(1-X_{t2})>1-\lambda-\mu'-\frac{3}{2}w=0.453>0$ in our calibration, and $D_{t1}(1-K_m)$ is also positive, which means although capital holdings might affect banks' liquidity holdings, a sufficiently high capital holding might not directly discourage banks from hoarding liquidity holdings.

3) For $D_s(1-K_m)-K_m<L_m<\widetilde{L_m}$, we have $\widehat{D_m}<\overline{D_m}<D_s<\widehat{\widehat{D_m}}$, which means banks will not fail but are subject to credit rationing, so

$$\begin{aligned}\frac{\partial V_{sm}}{\partial L_m} = \frac{1}{(1+\delta)^2}\int_0^{\widehat{D_m}}&\left\{[\lambda+a-(1-L_m)w]\int_0^{\overline{X_{t2}}}X_{t2}\,dF_m(X_{t2})-[a-(1-L_m)w-\mu']F_m(\overline{X_{t2}})\right\}d(D_{t1})\\&+\frac{1}{(1+\delta)^2\sqrt{(1-L_m)[1+L_m-2D_{t1}(1-K_m)]}}\int_{\widehat{D_m}}^{\overline{D_m}}\int_0^{\overline{\overline{X_{t2}}}}\left\{(1-L_m)[1+L_m-2D_{t1}(1-K_m)]\left(\frac{w}{2}-\frac{w}{2}X_{t2}\right)\right.\\&\left.+\left[1+a_{L_m}-X_{t2}\left(\lambda+a_{L_m}\right)-\mu'\right][D_{t1}(1-K_m)-L_m]\right\}dF_m(X_{t2})d(D_{t1})\\&+\frac{1}{\gamma_m}\int_{\overline{D_m}}^{D_s}\int_0^{\widehat{X_{t2}}}\left\{\left[\gamma_m+Z_s(D_{t1})+a_{L_m}-X_{t2}\left(\lambda+a_{L_m}\right)-\mu'\right]\right.\\&\left.+\frac{K_m'(D_{t1})}{2}\left[w(1-X_{t2})-\frac{1-D_{t1}(1-K_m)}{(1-L_m)^2\sqrt{(1-L_m)[1+L_m-2D_{t1}(1-K_m)]}}\right]\right\}dF_m(X_{t2})\,d(D_{t1})\gtreqless 0\end{aligned} \tag{A7}$$

where $\widehat{X_{t2}} = \min\left[\max\left(\frac{\gamma_m+Z_s(D_{t1})+a_{L_m}-\mu}{\lambda+a_{L_m}},0\right),1\right]$ to ensure $F_m(\widehat{X_{t2}})$ is definable. For the term $\frac{K_m'(D_{t1})}{2}\left[w(1-X_{t2})-\frac{1-D_{t1}(1-K_m)}{(1-L_m)^2\sqrt{(1-L_m)[1+L_m-2D_{t1}(1-K_m)]}}\right]$ from Equation (A7), we can notice it can be negative when K_m is high, and thus a high capital holding is likely to discourage banks from hoarding liquidity.

4) For $L_m<D_s(1-K_m)-K_m$, we have $\widehat{D_m}<\overline{D_m}<\widehat{\widehat{D_m}}<D_s$, so

$$\begin{aligned}\frac{\partial V_{sm}}{\partial L_m} = \frac{1}{(1+\delta)^2}\int_0^{\widehat{D_m}}&\left\{[\lambda+a-(1-L_m)w]\int_0^{\overline{X_{t2}}}X_{t2}\,dF_m(X_{t2})-[a-(1-L_m)w-\mu']F_m(\overline{X_{t2}})\right\}d(D_{t1})\\&+\frac{1}{(1+\delta)^2\sqrt{(1-L_m)[1+L_m-2D_{t1}(1-K_m)]}}\int_{\widehat{D_m}}^{\overline{D_m}}\int_0^{\overline{\overline{X_{t2}}}}\left\{(1-L_m)[1+L_m-2D_{t1}(1-K_m)]\left(\frac{w}{2}-\frac{w}{2}X_{t2}\right)\right.\\&\left.+\left[1+a_{L_m}-X_{t2}\left(\lambda+a_{L_m}\right)-\mu'\right][D_{t1}(1-K_m)-L_m]\right\}dF_m(X_{t2})d(D_{t1})\\&+\frac{1}{\gamma_m}\int_{\overline{D_m}}^{\widehat{\widehat{D_m}}}\int_0^{\widehat{X_{t2}}}\left\{\left[\gamma_m+Z_s(D_{t1})+a_{L_m}-X_{t2}\left(\lambda+a_{L_m}\right)-\mu'\right]\right.\\&\left.+\frac{K_m'(D_{t1})}{2}\left[w(1-X_{t2})-\frac{1-D_{t1}(1-K_m)}{(1-L_m)^2\sqrt{(1-L_m)[1+L_m-2D_{t1}(1-K_m)]}}\right]\right\}dF_m(X_{t2})\,d(D_{t1})\gtreqless 0\end{aligned}$$

Similar to scenario (3), a high capital holding K_m will discourage banks to hold liquidity holdings L_m.

Overall, since we cannot guarantee $\widetilde{L_m}$ is strictly positive, thus a corner solution at $L_m=0$ might be possible in scenarios (2), (3) and (4). Thus, when a corner solution at $L_m=0$ is obtained, we can infer that the probability of experiencing fire sales is strictly positive. When banks hoard excess liquidity holdings, as in scenario (1), capital holdings will not influence liquidity holdings, while when under scenario (2), (3) and (4) capital holdings will affect liquidity holdings although this effect is more pronounced under scenario (3) and (4) where credit rationing and bankruptcy is possible.

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Table 1

Parameter calibration

Description	Parameters	Value	Main Source(s)
Probability of default in booms	p_l	0.010	Repullo & Suarez (2012)
Probability of default in recessions	p_h	0.036	Repullo & Suarez (2012)
State-invariant correlation	ρ	0.174	Repullo & Suarez (2012)
Long-term deposit rate premium	j	0.024	Ben-David *et al*. (2017), FDIC Statistics
Loss given default	λ	0.45	Basel II IRB approach (2004)
Success loan rate	a	0.09	FDIC Statistics, Repullo & Suarez (2012)
Worst-case normal liquidity shock	d_n	0.05	Albuquerque *et al.* (2015), Antoniades (2016)
Worst-case bad liquidity shock	d_b	0.10	Albuquerque *et al.* (2015), Antoniades (2016)
Cost of managing loans	μ	0.045	FDIC Statistics
Unit cost managing loans	μ'	0.058	FDIC Statistics
Prob. of bad liquidity shock in booms	q_{bl}	0.36	Chen *et al.* (2017), Antoniades (2016)
Prob. of bad liquidity shock in recessions	q_{bh}	0.90	Chen *et al.* (2017), Antoniades (2016)
Unconditional probability of booms	ϕ_l	0.643	Repullo & Suarez (2012)
Shareholder required return	δ	0.08	Iacoviello (2005), Van den Heuvel (2008)
Bankruptcy costs	c	0.20	Hennessy & Whited (2007), Repullo (2013)
Diminishing scales to return	w	0.026	Lopez-de-Silanes (2015)

Table 2

Equilibrium banking decisions: capital holdings, capital buffers, liquidity holdings, shareholder fire sale loss, probability of bankruptcy under different regulatory regimes (all values in %)

	Laissez-faire	Basel I	Basel II	Basel III
Capital holdings in state s				
k_l^*	2.2	6.2	5.4	9.2
k_h^*	2.3	5.4	7.9	11.3
Capital buffers in state s				
$\Delta_l = k_l^* - \gamma_l$	2.2	2.2	2.2	3.5[27]
$\Delta_h = k_h^* - \gamma_h$	2.3	1.4	2.4	3.3
Liquidity holdings in state s				
l_l^*	69.0	78.2	79.4	90.4
l_h^*	88.0	79.1	81.0	70.7
Expected shareholder fire sale loss				
Unconditional, in booms	0.47	0.53	0.52	0.01
Booms $(s,m) = (b,l)$	0.51	0.78	0.76	0.02
Booms $(s,m) = (n,l)$	0.45	0.39	0.38	0.01
Unconditional, in recessions	0.23	0.73	0.52	0.86
Recessions $(s,m) = (b,h)$	0.23	0.77	0.55	0.90
Recessions $(s,m) = (n,h)$	0.19	0.39	0.28	0.45
Unconditional, all periods	0.38	0.60	0.52	0.31
Probability of bankruptcy				
First-period, in booms	68.08	0.00	0.00	0.00
Booms $(s,m) = (b,l)$	80.54	0.00	0.00	0.00
Booms $(s,m) = (n,l)$	61.07	0.00	0.00	0.00
First-period, in recessions	70.06	0.00	0.00	0.00
Recessions $(s,m) = (b,h)$	72.78	0.00	0.00	0.00
Recessions $(s,m) = (n,h)$	45.56	0.00	0.00	0.00
First-period, Unconditional	68.79	0.00	0.00	0.00
Second-period, in booms	0.02	0.01	0.02	0.01
Booms $(s,m) = (b,l)$	0.04	0.01	0.02	0.01
Booms $(s,m) = (n,l)$	0.01	0.01	0.02	0.01
Second-period, in recessions	1.24	1.42	0.69	0.17
Recessions $(s,m) = (b,h)$	1.34	1.44	0.73	0.17
Recessions $(s,m) = (n,h)$	0.33	1.24	0.35	0.17
Second-period, Unconditional	0.46	0.51	0.26	0.07
All periods, unconditional	69.25	0.51	0.26	0.07

This table reports the results from numerically solving for the equilibrium of the model with the parameters described in Table 1. The Row *Expected shareholder fire sale loss* labelled as 'Unconditional, in booms (recessions)' represents the weighted average of the fire sales loss conditional on the likelihood of the liquidity shocks in each period. The notation s and m is the state variables for liquidity shocks and credit risks. The term b and n stands for bad liquidity shocks and normal liquidity shock respectively, and l and h represents low default periods (booms) and high default periods (recessions). The term 'Unconditional, all periods' is calculated as the expected value of the loss for all periods when the likelihood of booms $\phi_l = 0.643$ as shown in Table 1. Similarly, the row named as *Probability of bankruptcy* reports the probability of default in first period (due to liquidity risks) and in second period (due to credit risks) respectively. All the value termed as *Unconditional* in this row is calculated in a same way as the previous one.

[27] Note that $3.5 = 2.2 + 1.3$, where 1.3 is the countercyclical buffer required by the Basel III.

Table 3

Equilibrium social welfare to banking regulation: expected additional fire sale loss and expected social welfare (all values in %)

	Laissez-faire	Basel I	Basel II	Basel III
Expected additional fire sale loss				
Unconditional, in booms	70.99	6.31	8.07	0.01
Booms $(s, m) = (b, l)$	82.31	11.93	15.00	0.02
Booms $(s, m) = (n, l)$	64.62	3.15	4.17	0.01
Unconditional, in recessions	75.75	29.16	2.67	0.86
Recessions $(s, m) = (b, h)$	77.95	30.13	2.93	0.91
Recessions $(s, m) = (n, h)$	55.90	20.44	0.28	0.45
Unconditional, all periods	72.69	14.47	6.14	0.32
Expected social welfare				
Unconditional, in booms	-9.91	10.20	10.07	11.40
Booms $(s, m) = (b, l)$	-13.83	9.70	9.46	11.05
Booms $(s, m) = (n, l)$	-7.70	10.48	10.42	11.60
Unconditional, in recessions	-10.24	7.70	10.23	10.20
Recessions $(s, m) = (b, h)$	-10.97	7.62	10.21	10.20
Recessions $(s, m) = (n, h)$	-3.65	8.42	10.36	10.23
Unconditional, all periods	-10.03	9.31	10.13	10.97

Similar to Table 2, the term 'Unconditional' is calculated as the weighted average conditionally for different scenarios with the weights shown in Table 1. The term *Expected additional fire loss* calibrates the reduced banking lending scale due to fire sales and credit rationing and is calculated based on Equation (24), and *Expected social welfare* measures net social income because of banks' investments and is determined by Equation (25).

Table 4

Equilibrium Basel III regulations: capital holdings, capital buffers, liquidity holdings, shareholder fire sale loss, additional fire sale loss and expected social welfare under different liquidity requirements *C* (all values in %)

	$C = 0$	Baseline results $C = 10$	$C = 20$	$C = 30$
Capital holdings in state s				
k_l^*	9.1	9.2	9.2	9.0
k_h^*	10.6	11.3	10.3	10.1
Capital buffers in state s				
$\Delta_l = k_l^* - \gamma_l$	3.4	3.5	3.5	3.3
$\Delta_h = k_h^* - \gamma_h$	2.6	3.3	2.3	2.1
Liquidity holdings in state s				
l_l^*	55.1	90.4	90.4	90.9
l_h^*	66.1	70.7	86.6	89.8
Expected shareholder fire sale loss				
Unconditional, in booms	1.22	0.01	0.01	0.00
Booms $(s, m) = (b, l)$	1.79	0.02	0.02	0.00
Booms $(s, m) = (n, l)$	0.90	0.01	0.01	0.00
Unconditional, in recessions	1.11	0.86	0.15	0.00
Recessions $(s, m) = (b, h)$	1.17	0.90	0.16	0.00
Recessions $(s, m) = (n, h)$	0.58	0.45	0.08	0.00
Unconditional, all periods	1.18	0.31	0.06	0.00
Expected additional fire sale loss				
Unconditional, in booms	8.29	0.01	0.01	2.33
Booms $(s, m) = (b, l)$	15.62	0.02	0.02	2.35
Booms $(s, m) = (n, l)$	4.17	0.01	0.01	2.32
Unconditional, in recessions	4.13	0.86	0.19	0.00
Recessions $(s, m) = (b, h)$	4.52	0.91	0.20	0.00
Recessions $(s, m) = (n, h)$	0.58	0.45	0.08	0.00
Unconditional, all periods	6.80	0.32	0.08	1.50
Expected social welfare				
Unconditional, in booms	9.20	11.40	11.40	10.86
Booms $(s, m) = (b, l)$	9.85	11.05	11.05	10.86
Booms $(s, m) = (n, l)$	8.83	11.60	11.60	10.86
Unconditional, in recessions	9.82	10.20	10.95	10.68
Recessions $(s, m) = (b, h)$	9.79	10.20	10.95	10.68
Recessions $(s, m) = (n, h)$	10.11	10.23	10.96	10.68
Unconditional, all periods	9.42	10.97	11.24	10.80

Table 4 shows the different equilibrium results under different level of liquidity requirements. The term C is defined by Equation (18). For the ease of comparison, the first column shows the results when liquidity requirements are not imposed to the banks when under Basel III regime that merely sets up $\gamma_l = 7.0\%$ and $\gamma_h = 8.0\%$. The second column shows our baseline results when $C = 10$, and third and fourth columns show the results when $C = 20$ and $C = 30$ respectively.

Table 5

Equilibrium banking decisions for alternative analysis: capital holdings, capital buffers, liquidity holdings, shareholder fire sale loss, probability of bankruptcy under different regulatory regimes (all values in %)

	Laissez-faire	Basel I	Basel II	Basel III
Capital holdings in state s				
K_l^*	5.7	5.7	5.7	6.1
K_h^*	2.2	2.8	3.3	4.4
Liquidity holdings in state s				
L_l^*	7.0	7.3	7.0	14.1
L_h^*	35.7	40.1	42.8	47.2
Capital buffers in state s				
$\Delta_l = K_l^*/(1 - L_l^*) - \gamma_l$	6.1	2.1	2.9	1.4[28]
$\Delta_h = K_h^*/(1 - L_h^*) - \gamma_h$	3.4	0.7	0.3	0.3
Expected shareholder fire sale loss				
Unconditional, in booms	0.11	0.08	0.11	0.00
Booms $(S, M) = (b, l)$	0.31	0.23	0.31	0.00
Booms $(S, M) = (n, l)$	0.00	0.00	0.00	0.00
Unconditional, in recessions	0.00	0.00	0.00	0.00
Recessions $(S, M) = (b, h)$	0.00	0.00	0.00	0.00
Recessions $(S, M) = (n, h)$	0.00	0.00	0.00	0.00
Unconditional, all periods	0.07	0.05	0.07	0.00
Probability of bankruptcy				
First-period, in booms	0.00	0.00	0.00	0.00
Booms $(S, M) = (b, l)$	0.00	0.00	0.00	0.00
Booms $(S, M) = (n, l)$	0.00	0.00	0.00	0.00
First-period, in recessions	0.00	0.00	0.00	0.00
Recessions $(S, M) = (b, h)$	0.00	0.00	0.00	0.00
Recessions $(S, M) = (n, h)$	0.00	0.00	0.00	0.00
First-period, Unconditional	0.00	0.00	0.00	0.00
Second-period, in booms	9.32	8.17	9.32	0.02
Booms $(S, M) = (b, l)$	25.81	22.63	25.81	0.02
Booms $(S, M) = (n, l)$	0.04	0.04	0.04	0.02
Second-period, in recessions	5.19	2.92	1.80	0.61
Recessions $(S, M) = (b, h)$	5.19	2.92	1.80	0.61
Recessions $(S, M) = (n, h)$	5.19	2.92	1.80	0.61
Second-period, Unconditional	7.84	6.30	6.63	0.23
All periods, unconditional	7.84	6.30	6.63	0.23

Table 5 shows the equilibrium results when banks can manage liquid asset holdings to deal with liquidity shocks. The notation S and M are the states variable designed to represent liquidity shocks and credit risks. Other notations are same as in Table 2. The results are presented in a similar way to Table 2. The term capital buffers $\Delta_s = K_s^*/(1 - L_s^*) - \gamma_s$ is scaled by $1 - L_s^*$ because the liquid assets are safe and are not required to be financed with capital. Thus, the capital retained by the banks is only used for honouring risky assets which is at the amount of $1 - L_s^*$.

[28] Note that $1.4 = 0.1 + 1.3$ where 1.3 is the countercyclical buffer that is required by Basel III.

Table 6

Alternative analysis, equilibrium social welfare to banking regulation: expected additional fire sale loss and expected social welfare (all values in %)

	Laissez-faire	Basel I	Basel II	Basel III
Expected additional fire sale loss				
Unconditional, in booms	0.12	0.10	0.12	0.00
Booms $(S, M) = (b, l)$	0.34	0.27	0.34	0.00
Booms $(S, M) = (n, l)$	0.00	0.00	0.00	0.00
Unconditional, in recessions	0.00	0.00	0.00	0.00
Recessions $(S, M) = (b, h)$	0.00	0.00	0.00	0.00
Recessions $(S, M) = (n, h)$	0.00	0.00	0.00	0.00
Unconditional, all periods	0.08	0.06	0.08	0.00
Expected social welfare				
Unconditional, in booms	5.30	5.52	5.30	6.70
Booms $(S, M) = (b, l)$	1.99	2.61	1.99	6.70
Booms $(S, M) = (n, l)$	7.16	7.15	7.16	6.70
Unconditional, in recessions	3.95	4.12	4.17	4.34
Recessions $(S, M) = (b, h)$	3.95	4.12	4.17	4.34
Recessions $(S, M) = (n, h)$	3.95	4.12	4.17	4.34
Unconditional, all periods	4.82	5.02	4.90	5.86

Table 6 presents the impacts of banking regulations to the economy and is organized as in Table 3. The term *Expected additional fire sale loss* is calculated based on Equation (48) and *Expected social welfare* is from Equation (49).

Table 7

Equilibrium Basel III regulations: capital holdings, capital buffers, liquidity holdings, shareholder fire sale loss, additional fire sale loss and expected social welfare under different liquidity requirements *C'* (all values in %)

	$C' = 0$	Baseline results $C' = 2$	$C' = 5$	$C' = 8$
Capital holdings in state *s*				
K_l^*	6.4	6.1	4.3	1.5
K_h^*	4.6	4.4	4.2	1.2
Liquidity holdings in state *s*				
L_l^*	8.9	14.1	48.5	85.9
L_h^*	45.3	47.2	48.6	86.5
Capital buffers in state *s*				
$\Delta_l = K_l^*/(1 - L_l^*) - \gamma_l$	1.3	1.4	2.6	4.9
$\Delta_h = K_h^*/(1 - L_h^*) - \gamma_h$	0.4	0.3	0.2	0.9
Expected shareholder fire sale loss				
Unconditional, in booms	0.00	0.00	0.00	0.00
Booms $(S, M) = (b, l)$	0.01	0.00	0.00	0.00
Booms $(S, M) = (n, l)$	0.00	0.00	0.00	0.00
Unconditional, in recessions	0.00	0.00	0.00	0.00
Recessions $(S, M) = (b, h)$	0.00	0.00	0.00	0.00
Recessions $(S, M) = (n, h)$	0.00	0.00	0.00	0.00
Unconditional, all periods	0.00	0.00	0.00	0.00
Expected additional fire sale loss				
Unconditional, in booms	0.06	0.00	0.00	0.00
Booms $(S, M) = (b, l)$	0.16	0.00	0.00	0.00
Booms $(S, M) = (n, l)$	0.00	0.00	0.00	0.00
Unconditional, in recessions	0.00	0.00	0.00	0.00
Recessions $(S, M) = (b, h)$	0.00	0.00	0.00	0.00
Recessions $(S, M) = (n, h)$	0.00	0.00	0.00	0.00
Unconditional, all periods	0.04	0.00	0.00	0.00
Expected social welfare				
Unconditional, in booms	6.68	6.70	4.25	1.23
Booms $(S, M) = (b, l)$	6.05	6.70	4.25	1.23
Booms $(S, M) = (n, l)$	7.04	6.70	4.25	1.23
Unconditional, in recessions	4.25	4.34	3.99	1.06
Recessions $(S, M) = (b, h)$	4.25	4.34	3.99	1.06
Recessions $(S, M) = (n, h)$	4.25	4.34	3.99	1.06
Unconditional, all periods	5.81	5.86	4.16	1.17

Table 7 shows the equilibrium results for Basel III when under no capital requirements ($C' = 0$), baseline model ($C' = 2$) and two more scenarios when $C' = 5$ and $C' = 8$, and the value is adopted to fit for Equation (47). The results is presented in a similar way as in Table 4.

Figure 1

Benchmark model capital holdings and liquidity holdings

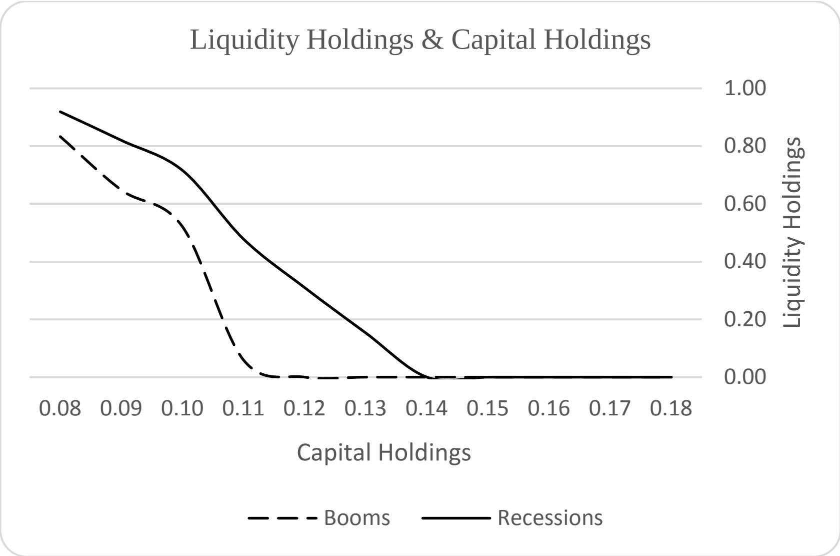


Figure 1 depicts the relationship between capital holdings and liquidity holdings within booms and recessions. The horizontal axis is the capital holdings k_m, while the vertical axis the liquidity holdings l_m. Capital requirements are set at $\gamma_m = 0.07$ for booms and $\gamma_m = 0.08$ for recessions. Other parameters are fixed at the baseline value, for example $j = 0.024$. To obtain the results and to keep other conditions the same, we rule out the equilibrium condition $v_m(k_m^*, l_m^*) = 0$ in order to analyse banks' individual liquidity holding behaviours. This treatment also applies to the rest of the Figures in this section.

Figure 2

Liquidity holdings and long-term deposit rate[29]

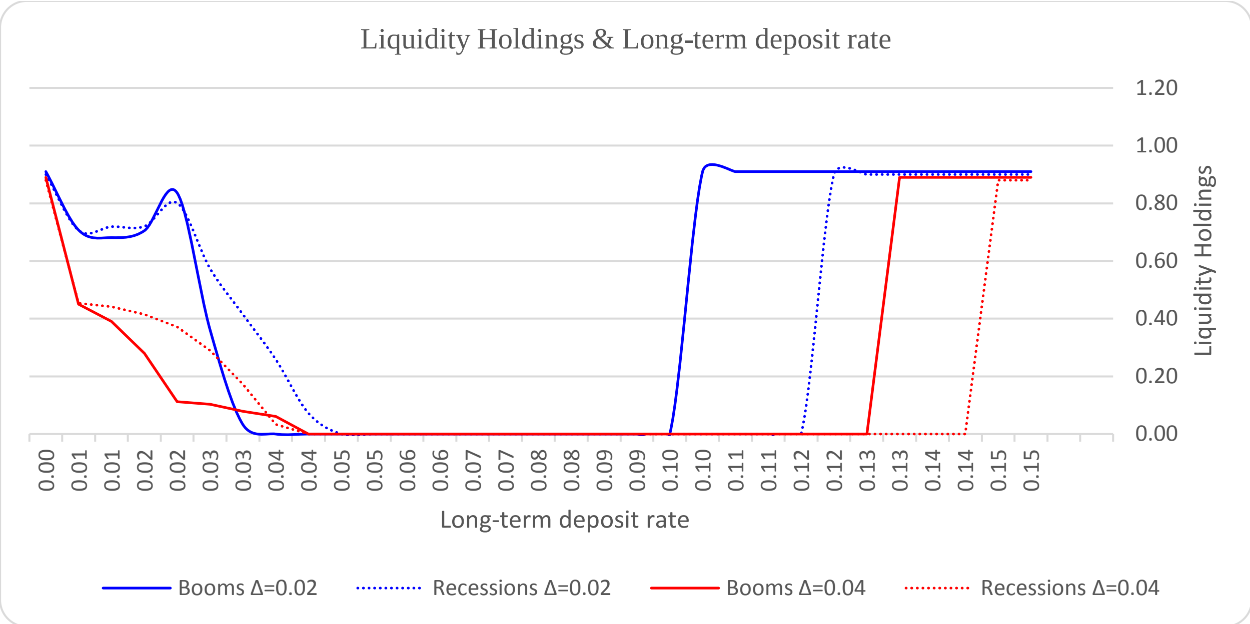
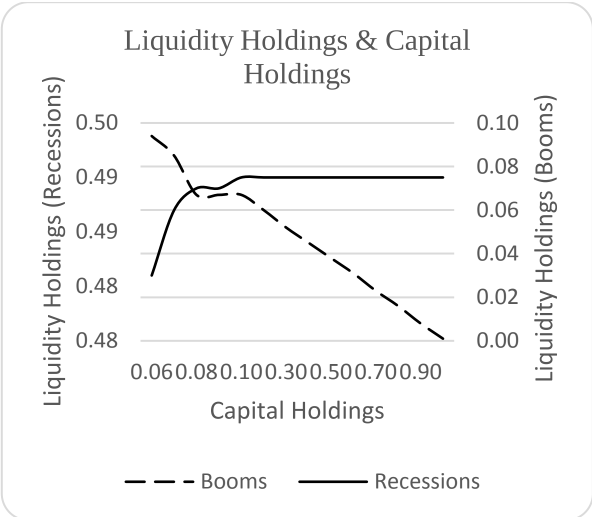


Figure 2 shows the trends of liquidity holdings as a function of long-term deposit rate j. Capital requirements are set at $\gamma_m = 0.07$ and $\gamma_m = 0.08$ respectively for booms and recessions. Other parameters are kept at the baseline value. To compare the effects of capital buffers on the liquidity holdings, or the impacts of a higher capital holding on the liquidity holdings, we present two scenarios when capital buffers are $\Delta = 0.02$ and $\Delta = 0.04$, which means the capital holdings are 0.09 and 0.10 when $\Delta = 0.02$ and are at the value of 0.11 and 0.12 when $\Delta = 0.04$.[30]

[29] The thresholds that make liquidity holdings jump from zero to $1 - k_m$ are $j_h = 0.116$ and $j_l = 0.100$ for the case $\Delta = 0.02$, and are $j_h = 0.142$ and $j_l = 0.126$ when $\Delta = 0.04$. All of these values are exactly at the points of j that makes $k_m = l_m r_m = l_m j(1 - p_m)$ at which points features the thresholds of bankruptcy.

[30] In an unreported Figure, we also conduct the calculation when $\Delta = 0.00$, that is banks hold no capital buffers at all for booms. The corresponding liquidity holdings are fixed at $1 - k_m$, which means banks will not hold any short-term debt. This is partially because this fragile capital structure will be prone to make them more vulnerable to liquidity shocks, and thus banks will hold much liquidity as them can to avoid further loss. However, for recessions the trend is very similar to the case when $\Delta = 0.04$, that is the liquidity holdings is strictly decreasing with the increase of j.

Figures 3 and 4



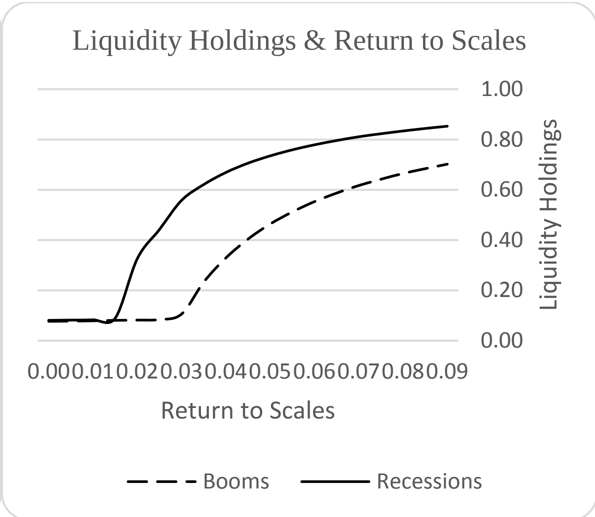


Figure 3 Alternative model liquidity holdings and capital holdings

Figure 4 Alternative model liquidity holdings and return to scales

Figure 3 reports the relationship between liquidity holdings (vertical axis) and capital holdings (horizontal axis). The liquidity holding value for recessions are shown at the left hand side, while booms are shown at the right hand side. Capital requirements are set at $\gamma_m = 0.07$ for booms and $\gamma_m = 0.08$ for recessions. Other parameters are kept constant with the baseline analysis, for example $w = 0.026$. Figure 4 reports the liquidity holdings (vertical axis) as the response to return to scales w (horizontal axis). For recessions, the capital requirement and capital holding are set at $\gamma_m = 0.08$ and $k_m = 0.08$ respectively, while for booms, they are set at $\gamma_m = 0.07$ and $k_m = 0.07$.