Backward Discounting

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Motivation

- Propose a new theory of time-inconsistent preferences based on two central ingredients
  - Agents explicitly consider past outcomes in current lifetime utility
  - Agents explicitly consider utility of future selves when making current decisions
- Novel predictions with empirical support
- Use the model to analyze standard consumption savings problem, as well as other applications
  - Addictive behaviour, evolutionary fitness, elections, social discounting

Motivation

Why consider *backward discounting*?

- Backward discounting + weight on future selves$\Rightarrow$ sharp form of time inconsistency...
  - U-shaped profile of rates of impatience
  - Hyperbolic models yield monotone profile
- Key point - Can't be reduced to model with purely geometric discounting

Model

Standard setting

- Time $\in [0, T]$
- Consumption stream for agent, $\{c_t\}_{t=0}^{T}$
- $u(c)$ instantaneous utility function

Time-0 value

$$\int_0^T d(s)u(c_s)ds$$

- $d(s)$ - effective discount factor

Backward Discounting

Postulate 1 - agents discount future streams, as well as past streams, in current utility

- Date t lifetime utility

$$\int_0^t e^{-\rho_b(t-s)} u(c_s) ds + \int_t^N e^{-\rho_f(s-t)} u(c_s) ds$$

- ρ_f, ρ_b forward and backward discount rates resp.

Different Selves

Postulate 2 - agents explicitly place weight on lifetime utility of future selves

- Today, focus on simple two-weight version, as well as $\rho_b = \rho_f$
- Place weight α on current self t, $1 - \alpha$ on some future self $T < N$
  - T will interpreted as shadow parent, or retirement self
  - In paper, allow for very general weighting schemes - weight placed on all selves, past selves, allowing weights to be time-varying, etc
- Adjusted t-self lifetime utility:

$$\alpha \int_0^N e^{-\rho|t-s|} u(c_s) ds + (1-\alpha) \int_0^N e^{-\rho|T-s|} u(c_s) ds$$

Rates of Impatience

- Formally, define

$$i(t,s) = \lim_{\epsilon \to 0} \ln \left[\frac{d(t,s)}{d(t,s+\epsilon)} \right] = -\frac{d_s(t,s)}{d(t,s)}$$

- $i(t,s)$ - local rate of impatience at s from the date t viewpoint
    - Standard model - $i(t,s) = \rho$
    - Hyperbolic discounting - $i(t,s)$ decreasing in s

Rates of Impatience

At all pre-retirement ages $t < T$, and for $s \in [t, T)$,

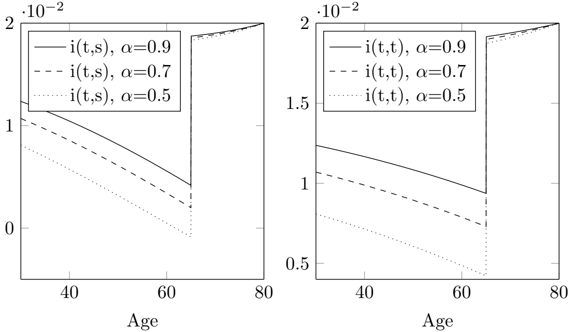
$$i(t,s) = \left[\frac{\rho_f \alpha e^{-\rho_f (s-t)} - \rho_b (1-\alpha) e^{-\rho_b (T-s)}}{\alpha e^{-\rho_f (s-t)} + (1-\alpha) e^{-\rho_b (T-s)}} \right]$$

For $s \geq T$,

$$i(t,s) = \left[\frac{\rho_f \alpha e^{-\rho_f (s-t)} + \rho_f (1-\alpha) e^{-\rho_f (T-s)}}{\alpha e^{-\rho_f (s-t)} + (1-\alpha) e^{-\rho_f (T-s)}} \right]$$

- For $s \in [t, T)$, conflict between t and T selves
- T self values dates increasingly in s, converse for t self

Proposition 1



(a) $i(t, s)$, various s (b) $i(t, t)$, various t

Figure 1: Local and Instantaneous Rates of Impatience for $t = 30$, $\rho_f = \rho_b = 0.02$, $\beta = 0.3$, $\omega = 0.001$ and Various Values of α.

Testable Implications

Theorem 1

1. *For* $t < T$, $i(t, s)$ *is decreasing in* s *for* $s \in (t, T]$
2. *For each* $t < T$, $i(t, s)$ *jumps up as* s *crosses* T
3. $i(t, t)$ *is decreasing in* t*, and jumps up as* t *crosses* T
4. *For* $t > T$, $s > t$, $i(t, s) = \rho$

- (1) - standard present-bias time-inconsistency
- (3), (4), (5) - past retirement age, conflict between different selves disappear, return to standard geometric discounting
- Plan to make sacrifices in middle age, enjoy post-retirement

Testable implications - Evidence

Novel model predictions

- Increased patience across immediate choices into middle age, decreases post-retirement
  - Harrison et al 2002, Read et al 2004
- Younger people discount hyperbolically, older discount geometrically
  - Read et al 2004, Green et al 1994

Consumption-Savings

Embed model into standard consumption-savings problem

- $u(c) = \ln c$
- Flow income y_s per period (no uncertainty)
- Constant interest rate r on borrowing/lending
- A_s denotes total wealth in period s
  - If $F_s =$ financial wealth, and $M_s = \int_s^N e^{-r(\tau - s)} y_\tau d\tau$ the present value of future income earnings, then $A_s = F_s + M_s$
  - A_s evolves according to $\dot{A}_s = rA_s - c_s$

Planned Consumption

Naive agent

- At each date t, agent solves date t problem, assuming future selves will honor current plan
  - Commitment versus equilibrium solutions. Look for solution to time 0 problem (details)

Proposition 2

Theorem 2

The optimal consumption profile at date 0 satisfies

$$c_t(A) = \left[\frac{\alpha e^{-\rho t} + (1-\alpha) e^{-\rho |T-t|}}{\alpha e^{-\rho t} a_t + (1-\alpha) p_t} \right] A \equiv \lambda_t A \tag{1}$$

$$a_t = \rho^{-1} \left[(\rho - 1) e^{-\rho(N-t)} + 1 \right] \tag{2}$$

$$\begin{aligned} p_t &= \rho^{-1} e^{-\rho(t-T)} [(\rho - 1) e^{-\rho(N-t)} + 1] && \textit{for } t > T \\ &= \rho^{-1} \left\{ [(\rho - 1) e^{-\rho(N-T)} + 1] + [1 - e^{-\rho(T-t)}] \right\} && \textit{for } t < T \end{aligned} \tag{3}$$

$$\tag{4}$$

Benchmarking

- To compare solution to standard model, set $\alpha = 1$
  - $\bar{\lambda}_t = \frac{1}{a_t}$
- Now form the ratio $\theta_t = \frac{\lambda_t}{\bar{\lambda}_t}$
  - If $\theta_t < 1$, then planned saving greater than standard

Theorem 3

For $t < T$, $\theta_t < 1$. For $t \geq T$, $\theta = 1$. Furthermore, there exists $\hat{\alpha} \in (0, 1]$ such that if $\alpha \leq \hat{\alpha}$, θ_t always increases in t; while if $\alpha > \hat{\alpha}$, θ_t first decreases and then increases in t.

- Before retirement, agent saves more than in standard model. Afterwards, same.
- For high enough weight on shadow parent, agent does bulk of saving in middle age.

Equilibrium Consumption

Sophisticated agent

- Solution takes time-inconsistency into account, i.e. time t agent takes into account decisions of future agents
- Standard approach in discrete time - model problem as a game, in which separate agents at each instant t make consumption choices, solve via backward induction
- But how to model game in continuous time?
  - Hard to interpret - each agent controls an instant, choice of c affects nothing
- Take a novel approach...

Equilibrium Consumption

Sophisticated agent

- Break $[0, N]$ into sub-intervals length Δ. Assume one agent controls each, acts as "mini-planner"
  - Suppose agent controlling $[t, t+\Delta)$ chooses $\{c_s\}_{s=t}^{t+\Delta}$ under constraint that $A_{t+\Delta} = \hat{A}$
  - Solve for optimal control $\{c_t(A_s, s : \hat{A})\}_{s=t}^{t+\Delta}$ as above - let $U_t(A, \hat{A})$ denote value of optimal control to this agent
- Induces a standard game with finitely many players. Solve via backward induction. Looks like $J_t(A) = \max_{\hat{A}} U_t(A, \hat{A}) + e^{-\rho\Delta} J_{t+\Delta}(\hat{A})$
- Combine $\{c_t(A_s, s : \hat{A})\}_{s=t}^{t+\Delta}$ and optimal $\hat{A}$ to solve for rates of consumption at t - $c_t^{\Delta}(A)$,
- Define equilibrium of original game to be profile obtained by $\lim_{\Delta \to 0} c_t^{\Delta}(A)$

Equilibrium Consumption

Theorem 4

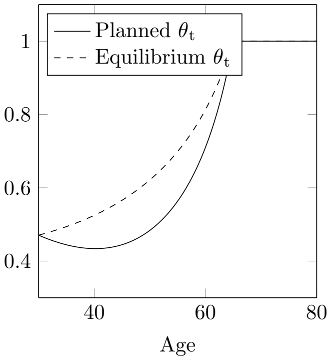
The equilibrium consumption profile satisfies

$$c_t^*(A) = \left[\frac{\alpha + (1-\alpha)e^{-\rho|T-t|}}{\alpha a_t + (1-\alpha)p_t}\right] A_t \equiv \lambda_t^* A \qquad (5)$$

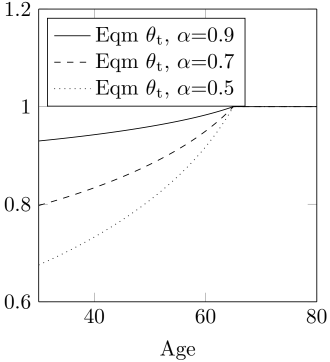
where a_t, p_t *satisfy* (2) *and* (3).

Theorem 5

1. $\theta_t, \theta_t^* < 1$ *for each* $t < T$.
2. *For dates* $t \geq T$, $\theta_t, \theta_t^* = 1$.
3. *In both the planning and equilibrium problems,* θ_t, θ_t^* *are increasing in* α.
4. $\theta*_t > \theta_t$ *for each* $t < T$. *Furthermore,* θ_t^* *rise monotonically over time, whereas* θ_t *may be U-shaped*



(a) Planned and Equilibrium θ_t, $\rho = 0.05$, $t = 30$, $N = 80$, $T = 65$, $r = 0.03$ and $\alpha = 0.5$.



(b) Equilibrium θ_t, varying α

Predictions

Predictions

- Countries with greater inter-generational linkages have higher savings rates
  - Eye-balling data on East-Asian countries vs other OECD seems in line
  - 1980-2013 savings rates: Japan, S Korea, China around 30, whereas UK, US, France, Germany around 15
  - Interest rates much lower in former countries
- Naive vs sophisticated
  - Naive show U-shaped savings rates, sophisticates monotone
  - Testable?

Summary

- Model of time-preferences, in which agents
  - Backward discount
  - Weight on future utilities
- Generates novel implications with empirical support
- Embed preferences into standard life-cycle model
- Going forward...
  - Infinite horizon, uncertainty
  - Policy implications (designing α)

Planned Consumption

Naive agent

- At each date t, agent solves date t problem, assuming future selves will honor current plan
  - Commitment versus equilibrium solutions. Look for solution to time 0 problem (▸ details)
- Define value functions $V(A,t), W(A,t)$ as

$$V(A,t) = \int_t^N e^{-\rho(s-t)} \ln(c_s) ds, \quad W(A,t) = \int_t^N e^{-\rho|T-s|} \ln(c_s) ds$$

where $\{c_s\}$ is the optimal plan

Planned Consumption

- Sup value of time t problem, viewed from time 0 is

$$e^{-\rho t}\alpha V(A,t) + (1-\alpha)W(A,t)$$

- Use this to write time t problem in standard form

$$0 = \sup_{c_t} \alpha e^{-\rho t}\big[\ln c_t + \dot{A}_t V_A(A,t) + V_t(A,t) - \rho V(A,t)\big]$$
$$+ (1-\alpha)\big[e^{-\rho|T-t|}\ln c_t + \dot{A} W_A(A,t) + W_t(A,t)\big]$$

where $\dot{A}_t = rA_t - c_t$

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