Multi-Dimensional Pass-Through, Incidence, and the Welfare Burden of Taxation and Other External Changes in Oligopoly

Takanori Adachi
Nagoya University

Michal Fabinger
University of Tokyo

September 27, 2017

Motivating Example

- Firms are faced with a complex set of policy interventions and other external changes. Naturally, it is **multi-dimensional**.

- Example: Consumers pay unit and value-added taxes, $t \geq 0$ and $v \in [0, 1)$. Firm's profit is:

$$\pi = [(1 - v)p - t]q - c(q) = [pq - c(q)] - [tq + vpq]$$

  Now, the government plans to change the value-added part: $s > 0$ fraction of $c(q)$ is tax deductible.

$$\pi = [pq - c(q)] - [\underbrace{tq + v\{pq - s \cdot c(q)\}}_{\equiv \phi(p,q,\mathbf{T}):\ \text{additional cost}}]$$

  "**What are the effects of introducing** s, ***when*** $t > 0$ ***and*** $v > 0$ ***are already implemented*?**"

- Intervention vector, $\mathbf{T} = (t, v, s)$, is **three-dimensional**.

Research Question (1/2)

- Better not to specify a particular type of competition:

    Many industries are ***oligopolistic***, more or less

    However,

        Quantity- or Price-Setting?

        Collusive to some degree?

- "***How can one evaluate the welfare consequences of a change in such multi-dimensional environments***, taking into account ***imperfect competition***?"

Research Question (2/2)

- We provide **general formulas** for welfare evaluation in consideration of ***multi-dimensionality*** and ***oligopoly***.

  In this way, we generalize Weyl and Fabinger's (2013) analysis of **single**-*dimensional pass-through*, to include **multi-dimensional pass-through**.

  Our analysis is flexible in the sense that the degree of competition is captured by a single variable, **conduct index**, $\theta \in [0, 1]$.

  We mainly work on imperfect competition with a fixed number of firms. However, in the paper, we also allow **free entry** to endogenize the number of firms (**monopolistic competition**).

Importance of Pass-Through

- **Pass-Through**: How final prices are affected by exogenous changes to firms, $\frac{dp}{dT}$

  - **Tax** scheme
  - Emission regulations (additional **cost**)
  - Change in **exchange rate**

  ...

- "Pass-Through Renaissance," initiated by Weyl and Fabinger (2013)

  - It has increasingly been recognized as an important measure for **welfare evaluation**.
  - Clear and tractable both in theory and empirics.

Generality of Our Framework

- Our framework can be used to study policy issues under **imperfect competition** in such fields as (but not limited to):

    - Industrial Organization

    - Public Economics

    - International Economics

    - Agricultural Economics

    - Environmental/Energy Economics

    - Macroeconomics

    .....

What We Do (1/2)

- We mainly work on **two-dimensional** taxation under symmetric oligopoly, $T \in \{t, v\}$:

  (1) **Unit Tax**, $t \geq 0$ (2) **Ad Valorem Tax**, $v \in [0, 1]$,

  - Firm i's profit: $\pi_i = [p_i q_i - c(q_i)] - [t q_i + v p_i q_i]$
    (Per-firm) tax revenue: $R(q) \equiv t\,q + v\,pq$

  to characterize

  (i) Unit-tax **pass-through**: $\rho_t \equiv \frac{\partial p}{\partial t}$, where $T = t$

  and two welfare measures:

  (ii) Tax **incidence**: $I_T \equiv \frac{\frac{\partial CS}{\partial T}}{\frac{\partial PS}{\partial T}}$ for $T = t$

  (iii) Marginal Cost of Public Funds ("**welfare burden**"):

  $$MCPF_T \equiv \frac{-\frac{\partial W}{\partial T}}{\frac{\partial R}{\partial T}} \text{ for } T = t$$

- Results for ad valorem tax ($T = v$) are analogous.

What We Do (2/2)

- We then generalize our two-dimensional results under symmetric oligopoly to include:

  Multi-Dimensionality

  Asymmetric Firms

  Taxation and Other External Changes

  Firm-Specific Taxation/Changes

Related Literature

Relation to the Literature: Theory

- This paper is a generalization of

  (1) Weyl and Fabinger (2013, *JPE*)

  (2) Häckner and Herzing (2016, *JET*)

	Tax Scheme	Initial Tax Level	Model
WF ('13)	Unit Tax: t	$t = 0$	General
HH ('16)	Unit Tax : t Ad Valorem Tax: v	$(t, v) = (0, 0)$	Linear Demands Constant MC
This Paper	**Multi-Dimensional**	**Non-Zero**	**General**

Relation to the Literature: Empirics

- Our framework is in line with the "sufficient-statistics" approach (Chetty, 2009).

  (1) **Price elasticities** (own and cross; $\boldsymbol{\epsilon}$)

  (2) **Conduct** ($\boldsymbol{\theta}$)

  Estimable if the mode of competition is specified

  (3) **Pass-through** ($\boldsymbol{\rho}$)

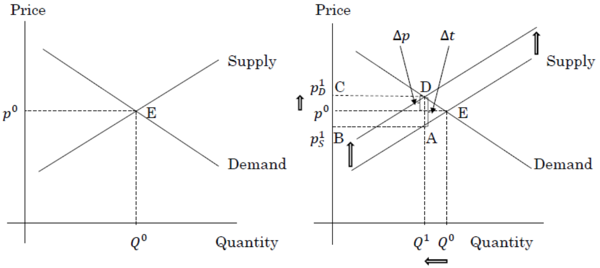
  Directly estimated if variation on the cost side is observed, or

  Indirectly estimated from the estimated $\boldsymbol{\epsilon}$ and the demand **curvatures**, and 1st- and 2nd-order cost characteristics (using our formulas)

  For example, Atkin and Donaldson (2016) study the welfare implications of changes in intra-national trade costs. See also Miller, Osborne, and Sheu (2017, *RAND*).

Quick Preview

Perfect Competition



- (i) Pass-through: $\rho_t = \frac{1}{1+\frac{\epsilon^D}{\epsilon^S}}$, where $\epsilon^D \equiv -\frac{D'p}{Q}$ and $\epsilon^S \equiv \frac{S'p}{Q}$ are the **elasticities** of demand and supply.

- (ii) Incidence: Tax Burden is devided into:

$$\Delta t = \Delta p + (\Delta t - \Delta p)$$

$$\Leftrightarrow 1 = \underbrace{\rho_t}_{\text{consumers}} + \underbrace{(1-\rho_t)}_{\text{producers}}$$

Oligopoly

- **Conduct Index** is implicitly defined from FOC:

$$\theta(q) \equiv \frac{\epsilon^D(q)}{p(q)}\left[p(q) - \frac{t + mc(q)}{1-v}\right] \in [0,1],$$

where $mc(q) \equiv c'(q)$ is the marginal cost

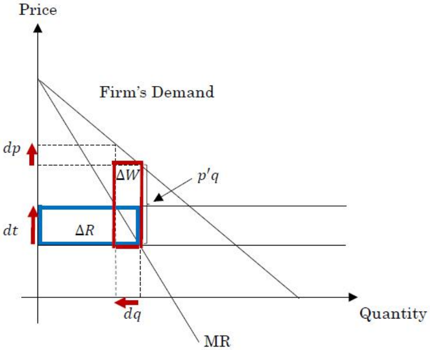
- Special cases:

  Perfect competition: $\theta = 0$
  Monopoly: $\theta = 1$
  Cournot oligopoly with n firms: $\theta = 1/n$

Comment on Multi-Product Oligopoly

- We mainly work on the case of **single-product oligopoly**. However, our analysis can be extended to the case of **multi-product oligopoly** with some more notations (see the paper's appendix).

Marginal Cost of Public Funds: Monopoly

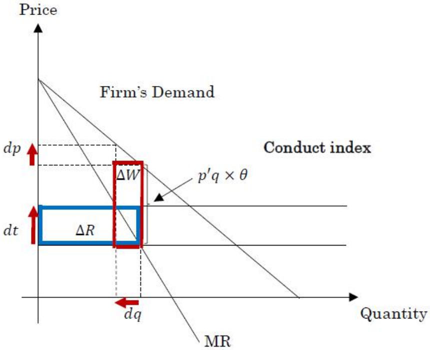


- Marginal Cost of Public Funds (MCPF):

$$MCPF_t \equiv \frac{-\Delta W}{\Delta R} = \frac{-p'q \cdot dq}{q \cdot dt}$$

$$= \frac{p'q \cdot dp/p'}{q \cdot dt} = \frac{dp}{dt} \equiv \rho_t$$

Marginal Cost of Public Funds: Oligopoly



- MCPF (with initial $t = 0$; HH 2016):

$$MCPF_t \equiv \frac{\Delta W}{\Delta R} = \frac{\theta p' q \cdot dq}{q \cdot dt}$$

$$= \frac{\theta p' q \cdot dp/p'}{q \cdot dt} = \theta \frac{dp}{dt} \equiv \theta \rho_t$$

Comparison in the Two-Dimensional Case

	Perfect, and $(t,v)=(0,0)$	Imperfect, and $(t,v)\geq(0,0)$
(i) Pass-Through	$\frac{1}{1+\frac{\epsilon_D}{\epsilon_S}}$	$\frac{\frac{1}{1-v}}{1+\frac{1-\tau}{1-v}\frac{\epsilon_D}{\epsilon_S}-\left(\frac{1}{\epsilon_D}+\frac{1}{\epsilon_S}\right)\theta+\epsilon_D q\frac{\partial(\theta/\epsilon_D)}{\partial q}}$
(ii) Incidence	$\frac{1}{\frac{1}{\rho_t}-1}$	$\frac{1}{\frac{1}{\rho_t}-(1-v)(1-\theta)}$
(iii) MCPF	0	$\frac{\frac{(1-v)\theta}{\epsilon_D}+\tau}{\frac{\frac{1}{\rho_t}+v}{\epsilon_D}-\tau}$

where

$$\tau(q)\equiv\frac{R(q)}{p(q)q}=\frac{t}{p(q)}+v$$

is the fraction of the gov's (per-firm) revenue to firm's pre-tax revenue.

(i) Pass-Through

Pass-Through

Proposition

Pass-through, ρ_t, is characterized by:

$$\rho_t = \frac{1}{1-v} \cdot \frac{1}{\left[1 + \frac{1-\tau}{1-v}\frac{\epsilon^D}{\epsilon^S}\right] + \underbrace{\left[-\left(\frac{1}{\epsilon^D} + \frac{1}{\epsilon^S}\right)\theta\right]}_{\textit{Direct Effect}} + \underbrace{\left[-\epsilon^D q \frac{\partial(-\theta/\epsilon^D)}{\partial q}\right]}_{\textit{Indirect Effect}}}.$$

Additional two terms from oligopoly:

- **Direct** effect: ρ_t becomes *larger* if the demand becomes *inelastic* (i.e., $\frac{1}{\epsilon_D}$ becomes larger), propagated by θ.
- **Indirect** effect: (i) Suppose ϵ^D is close to a constant. Then, ρ_t becomes *smaller* if $\left(-\frac{\partial q}{\partial \theta}\right)$ is *larger*. This is the case of *greater distortion*. Similar in the case of θ being close to a constant.

Relation to Weyl-Fabinger (2013)

- The original single pass-through formula by WF (2013) is:

$$\rho_t = \frac{1}{1 + \frac{\epsilon^D - \theta}{\epsilon^S} + \frac{\theta}{\epsilon^\theta} + \frac{\theta}{\epsilon^{ms}}},$$

where $\epsilon^\theta \equiv \theta/[q \cdot (\theta)']$ and $\epsilon^{ms} \equiv ms/[q \cdot (ms)']$ are the quantity elasticities of the conduct index, and the marginal consumer's surplus, respectively.

- If two-dimensional $(t, v) \geq (0, 0)$ is considered, their formula is reformulated as:

$$\rho_t = \frac{1}{1 - v} \cdot \frac{1}{1 + \frac{\frac{1-\tau}{1-v}\epsilon^D - \theta}{\epsilon^S} + \frac{\theta}{\epsilon^\theta} + \frac{\theta}{\epsilon^{ms}}},$$

which is equivalent to our formula.

(ii) Incidence

Incidence

Proposition

Incidence of unit tax,

$$\frac{1}{I_t} \equiv \frac{\frac{\partial PS}{\partial t}}{\frac{\partial CS}{\partial t}}.$$

is characterized by:

$$\frac{1}{I_t} = \frac{1}{\rho_t} - (1 - v)(1 - \theta).$$

Proof (1/3)

- The effects of an increase in unit tax dt on the producer surplus can be decomposed into the following five parts:

$$dPS = [\underbrace{(-qdt)}_{(1)<0} + \underbrace{(1-v)pdq}_{(2)<0}] + [\underbrace{(1-v)qdp}_{(3)>0} + \underbrace{(-mcdq)}_{(4)>0} + \underbrace{(-tdq)}_{(5)>0}]$$

(1) (Direct) **loss** from an increase in unit tax; the tax increase multiplied by output q

(2) (Indirect) **loss** from a reduction in production; multiplied by the ad valorem tax adjusted unit price $(1-v)p$

(3) (Direct) **gain** from the associated price increase, mitigated by $(1-v)$ due to the ad valorem tax, multiplied by q

(4) (Indirect) **gain** from cost saving by output reduction dq

(5) (Indirect) **gain** from unit tax saving by dq

Proof (2/3)

- By rewriting:

$$dPS = [\underbrace{-qdt}_{(1)<0} + \underbrace{(1-v)qdp}_{(3)>0}] + [(1-v)p - \underbrace{(mc+t)}_{Marginal\ Cost}]dq$$

- Now, in symmetric equilibrium, the marginal cost $mc + t$ is equal to the marginal revenue $(1-v)p(1-\frac{\theta}{\epsilon^D})$, which implies

$$dPS = [\underbrace{-qdt}_{(1)<0} + \underbrace{(1-v)qdp}_{(3)>0}] + [(1-v)p]\left(\frac{\theta}{\epsilon^D}\right)dq.$$

- Under perfect competition: (2) = (4)+(5), and only (1) and (3) survive.

Proof (3/3)

- However, under imperfect competition, the marginal cost is less than $(1-v)p$: (2) $>$ (4)+(5). The third term expresses this difference (2)$-$[(4)+(5)].

- Now recall: $dp = \rho_t dt$ and $(\frac{p}{\epsilon^D})dq = -qdp = -q\rho_t dt$. Thus,

$$dPS = [\underbrace{-qdt}_{(1)<0} + \underbrace{(1-v)qdp}_{(3)>0}] + [(1-v)p]\left(\frac{\theta}{\epsilon^D}\right)dq$$

$$= [-qdt + (1-v)q\rho_t dt] - (1-v)q\theta\rho_t dt$$

$$= [-1 + (1-v)\rho_t - (1-v)\theta\rho_t]qdt$$

$$= [\underbrace{-1}_{(1)<0} + \underbrace{(1-v)(1-\theta)\rho_t}_{(3)-\{(2)-[(4)+(5)]\}\gtrless 0}]qdt$$

Comment on dPS

- On the other hand, $dCS = -\rho_t(qdt)$. Thus, while it is always the case that $dCS < 0$, it is possible that $dPS > 0$.

- Finally,

$$\frac{1}{I_t} \equiv \frac{dPS}{dCS} = \frac{-1 + (1 - v)(1 - \theta)\rho_t}{-\rho_t} = \frac{1}{\rho_t} - (1 - v)(1 - \theta)$$

(iii) Marginal Cost of Public Funds (MCPF)

Marginal Cost of Public Funds

Proposition

Define the marginal welfare cost of raising the government's revenue by unit tax t, $MCPF_t$, by:

$$MCPF_t \equiv \frac{-\frac{\partial W}{\partial t}}{\frac{\partial R}{\partial t}}.$$

Then, it is characterized by:

$$MCPF_t = \frac{\frac{(1-v)\theta}{\epsilon^D} + \tau}{\frac{\frac{1}{\rho_t}+v}{\epsilon^D} - \tau},$$

where $\tau(q) \equiv R(q)/[p(q)q] = t/p(q) + v$ is the fraction of the government's per-firm revenue to the firm's pre-tax revenue.

Proof (1/4)

- Under oligopoly, the effects of an increase in unit tax dt on social welfare is written as: $dW = (p - mc)dq$.

- Thus, $(p - mc)$ serves as a measure for welfare change.

- It is decomposed into two parts:

  (1) Surplus from imperfect competition: $\frac{(1-v)p\theta}{\epsilon^D}$
  (2) Tax payment: $t + vp$

- Thus,

$$MCPF_t = \frac{-dW}{dR} = \frac{-p[\frac{(1-v)\theta}{\epsilon^D} + \underbrace{\left(\frac{t}{p} + v\right)}_{\equiv \tau}]dq}{dR}$$

Proof (2/4)

- Next, the effects of an increase in unit tax dt on the tax revenue are:

$$dR = \underbrace{qdt}_{(1)>0} + \underbrace{vqdp}_{(2)>0} + \underbrace{(t + vp)dq}_{(3)<0}$$

(1) (Direct) **gain**, multiplied by the output q

(2) (Indirect) **gain**, due to the associated price increase, multiplied by vq

(3) (Indirect) **loss** from the output reduction for both unit tax revenue and ad valorem tax revenue

Proof (3/4)

- Now recall again: $dp = \rho_t dt$ and $(\frac{p}{\epsilon^D})dq = -qdp$. Thus,

  (1) $qdt = \frac{q}{\rho_t}dp = -\frac{p}{\epsilon^D \rho_t}dq$

  (2) $vqdp = -(\frac{vqp}{q\epsilon^D})dq = -(\frac{vp}{\epsilon^D})dq$, which implies that

$$dR = \underbrace{-\left(\frac{p}{\epsilon^D \rho_t}\right) dq}_{(1)>0} + \underbrace{\left[-\left(\frac{vp}{\epsilon^D}\right) dq\right]}_{(2)>0} + \underbrace{(t + vp)dq}_{(3)<0}$$

$$= p\left[\underbrace{\left(-\frac{1}{\epsilon^D \rho}\right)}_{(1)\,>0} + \underbrace{\left(-\frac{v}{\epsilon^D}\right)}_{(2)\,>0} + \underbrace{\left(\frac{t}{p} + v\right)}_{(3)\,<0}\right] dq$$

Proof (4/4)

- Now, in the per-price term,

$$MCPF_t = \frac{-dW}{dR} = \frac{-p\left[\frac{(1-v)\theta}{\epsilon^D} + \tau\right] dq}{dR}$$

$$= \frac{p\left[\frac{(1-v)\theta}{\epsilon^D} + \tau\right] dq}{p\left[\frac{1}{\epsilon^D \rho_t} + \frac{v}{\epsilon^D} - \tau\right] dq}$$

$$= \frac{\underbrace{\frac{(1-v)\theta}{\epsilon^D} + \tau}_{\textit{Welfare Loss expressed by the Profit Margin}}}{\underbrace{\frac{\frac{1}{\rho_t} + v}{\epsilon^D}}_{\textit{Gain in Revenue}} + \underbrace{(-\tau)}_{\textit{Loss in Revenue}}}$$

Recap

- Under symmetric oligopoly, we have provided concise yet general formulas for (i) pass-through, (ii) tax incidence, and (iii) the marginal cost of public funds:

	Perfect, and $(t, v) = (0, 0)$	Imperfect, and $(t, v) \geq (0, 0)$
(i) Pass-Through	$\frac{1}{1+\frac{\epsilon^D}{\epsilon^S}}$	$\frac{\frac{1}{1-v}}{1+\frac{1-\tau}{1-v}\frac{\epsilon^D}{\epsilon^S}-\left(\frac{1}{\epsilon^D}+\frac{1}{\epsilon^S}\right)\theta+\epsilon^D q\frac{\partial(\theta/\epsilon^D)}{\partial q}}$
(ii) Incidence	$\frac{1}{\frac{1}{\rho_t}-1}$	$\frac{1}{\frac{1}{\rho_t}-(1-v)(1-\theta)}$
(iii) MCPF	0	$\frac{\frac{(1-v)\theta}{\epsilon^D}+\tau}{\frac{\frac{1}{\rho_t}+v}{\epsilon^D}-\tau}$

Rest of the Slides

- What if the mode of competition (price or quantity) is specified?

  $\rightarrow$ Conduct Index is now expressed by the **first-order demand characteristics**.

  Pass-through is characterized by up to the **second-order demand (and supply) characteristics**.

- Generalization to

  Multi-Dimensionality

  Firm Heterogeneity

Pass-Through Expressions under Price and Quantity Competition

Price Elasticities

- Recall that $\epsilon^D(p) \equiv -pq'(p)/q(p) > 0$ is the *price elasticity of the **industry demand***.

- Additionally, we define the ***own*** *price elasticity of the firm's demand* by

$$\epsilon_F(p) \equiv -\left(\frac{p}{q(p)}\right)\frac{\partial q_j(\mathbf{p})}{\partial p_j}|_{\mathbf{p}=(p,\ldots,p)},$$

  and the ***cross*** *price elasticity* by

$$\epsilon_C(p) \equiv (n-1)\left(\frac{p}{q(p)}\right)\frac{\partial q_{j'}(\mathbf{p})}{\partial p_j}|_{\mathbf{p}=(p,\ldots,p)},$$

  for any distinct pair of indices j and j'.

- These are related by $\epsilon_F = \epsilon^D + \epsilon_C$.

Demand Curvature

- We also define the ***curvature*** *of the industry's direct demand* by $\alpha(p) \equiv -p\, q''(p)/q'(p)$.

- α is *positive* (*negative*) if and only if the industry demand is *convex* (*concave*).

Pass-Through under Price Competition

Proposition

Under **price** *competition, the unit-tax and the ad valorem tax pass-through rates are characterized by:*

$$\rho_t = \frac{1}{1-v} \cdot \frac{1}{1+\frac{(1-\alpha/\epsilon_F)\epsilon^D}{\epsilon_F}+\left(\frac{1-\tau}{1-v}-\frac{1}{\epsilon_F}\right)\left(\frac{\epsilon^D}{\epsilon^S}\right)},$$

and

$$\rho_v = \frac{1}{1-v} \cdot \frac{1}{\frac{1}{1-1/\epsilon_F}+\frac{(1-\alpha/\epsilon_F)\epsilon^D}{\epsilon_F-1}+\left(\frac{1-\tau}{1-v}\ \frac{\epsilon_F}{\epsilon_{F-1}}-\frac{1}{\epsilon_{F-1}}\right)\left(\frac{\epsilon^D}{\epsilon^S}\right)},$$

respectively.

Proof for ρ_t

- First, recall that

$$\rho_t = \frac{1}{1-v}\cdot\frac{1}{\underbrace{\left[(1-\theta/\epsilon^D) + \left(\theta/\epsilon^D\right)'\epsilon^D q\right]}_{\text{revenue increase}} + \underbrace{\left[\frac{1-\tau}{1-v}\epsilon^D - \theta\right]\frac{1}{\epsilon^S}}_{\text{cost savings}}}.$$

- Then, with $\theta = \epsilon^D/\epsilon_F$, $1-\theta/\epsilon^D = 1 - 1/\epsilon_F$, $\left(\theta/\epsilon^D\right)'\epsilon^D q = \left(1+\epsilon^D - \alpha\epsilon^D/\epsilon_F\right)/\epsilon_F$, it is rewritten as:

$$\rho_t = \frac{\frac{1}{1-v}}{\underbrace{\left[\left(1-\frac{1}{\epsilon_F}\right) + \frac{1+\epsilon^D - \alpha\epsilon^D/\epsilon_F}{\epsilon_F}\right]}_{\text{revenue increase}} + \underbrace{\left[\frac{1-\tau}{1-v} - \frac{1}{\epsilon_F}\right]\left(\frac{\epsilon^D}{\epsilon^S}\right)}_{\text{cost savings}}}.$$

Quantity Elasticities

- Define $\eta^D(q) = 1/\epsilon^D\,(p)\,|_{q(p)=q}$.

- We also define the ***own*** *quantity elasticity of the firm's inverse demand* by

$$\eta_F(q) \equiv -\left(\frac{q}{p(q)}\right)\frac{\partial p_j(\mathbf{q})}{\partial q_j}|_{\mathbf{q}=(q,\ldots,q)},$$

  and the the ***cross*** *quantity elasticity* by

$$\eta_C(q) \equiv (n-1)\left(\frac{q}{p(q)}\right)\frac{\partial p_{j'}(\mathbf{q})}{\partial q_j}|_{\mathbf{q}=(q,\ldots,q)},$$

  for any distinct pair of indices j and j'.

- These are related by $\eta_F = \eta^D + \eta_C$.

Inverse Demand Curvature

- We also define the ***curvature*** *of the industry's inverse demand* $\sigma(q) \equiv -q\, p''(q)/p'(q)$.

- σ is *positive* (*negative*) if and only if the industry's inverse demand is *convex* (*concave*),

Pass-Through under Quantity Competition

Proposition

Under **quantity** *competition, the unit-tax and the ad valorem tax pass-through rates are characterized by:*

$$\rho_t = \frac{1}{1-v} \cdot \frac{1}{1+\frac{\eta_F}{\eta^D} - \sigma + \left(\frac{1-\tau}{1-v} - \eta_F\right)\left(\frac{1}{\eta^D \epsilon^S}\right)},$$

and

$$\rho_v = \frac{1}{1-v} \cdot \frac{1-\eta_F}{1+\frac{\eta_F}{\eta^D} - \sigma + \left(\frac{1-\tau}{1-v} - \eta_F\right)\left(\frac{1}{\eta^D \epsilon^S}\right)},$$

respectively.

Proof for ρ_t

- Recall again that

$$\rho_t = \frac{1}{1-v} \cdot \frac{1}{\underbrace{[(1-\theta/\epsilon^D) + \left(\theta/\epsilon^D\right)' \epsilon^D q]}_{\text{revenue increase}} + \underbrace{\left[\frac{1-\tau}{1-v}\epsilon^D - \theta\right] \frac{1}{\epsilon^S}}_{\text{cost savings}}}.$$

- Then, $\theta = \eta_F/\eta^D$ implies $\left(1/\epsilon^S - \eta^D\right)\theta = [(1/\epsilon^D\eta^D) - 1]\eta_F$ and $\left(\theta\eta^D\right)'(q/\eta^D) = \left(1 + \eta^D - \sigma\eta^D/\eta_F\right)(\eta_F/\eta^D)$. Thus, it is rewritten as

$$\rho_t = \frac{\frac{1}{1-v}}{\underbrace{\left[(1-\eta_F) + \frac{1+\eta^D - \sigma\eta^D/\eta_F}{\eta^D}\eta_F\right]}_{\text{revenue increase}} + \underbrace{\left[\frac{1-\tau}{1-v}\frac{1}{\epsilon^S\eta^D} - \frac{\eta_F}{\epsilon^S\eta^D}\right]}_{\text{cost savings}}}.$$

Parametric Example: Linear Demands

- Functional form:

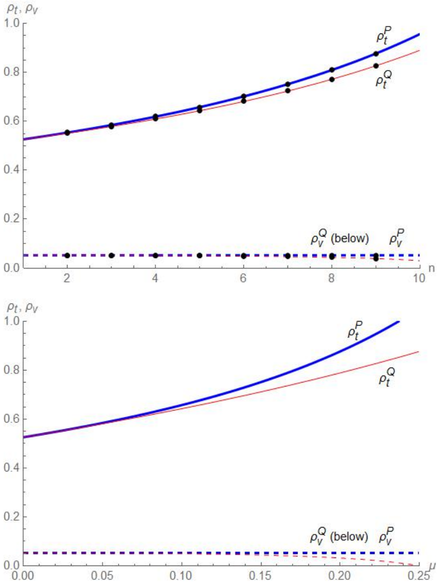
$$q_j(p_1, ..., p_n) = b - \lambda p_j + \mu \sum_{j' \neq j} p_{j'},$$

  where $b > mc$ and $\mu \in [0, \lambda/(n-1))$ measures the degree of *substitutability*.

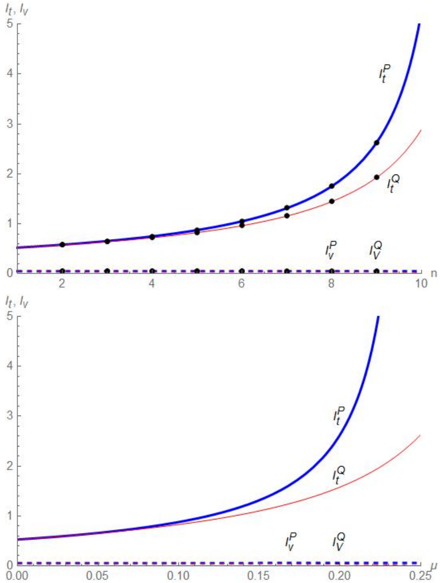
- To focus on n and μ, we set: $b = 1$, $mc = 0$, and $\lambda = 1$.

- When we change n, we set $\mu = 0.1$. When we change μ, we set $n = 5$.

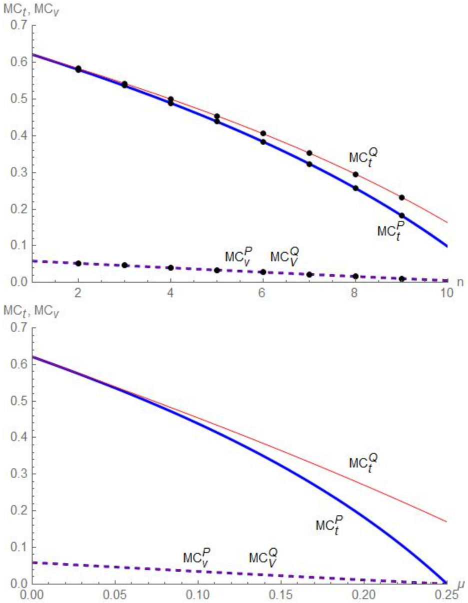
Pass-Through



Incidence (dCS/dPS)



Marginal Cost of Public Funds



Generalization

Generalization

- Up to now, the firm's profit is:

$$\pi = [(1-v)p - t]q - c(q) = [pq - c(q)] - [\underbrace{tq + vpq}_{\equiv \phi(p,q,\mathbf{T})}]$$

- How to proceed:

  (1) First, consider **Multi-Dimensionality**, maintaining the symmetry assumption:

$$\phi(p, q, \underbrace{\mathbf{T}}_{\dim = L}) \Leftarrow tq + vpq$$

$$\pi = [pq - c(q)] - \phi(p, q, \mathbf{T})$$

  (2) Then, incorporate **Firm Heterogeneity**:

$$\pi_i = [p_i q_i - c_i(q_i)] - \phi_i(p_i, q_i, \mathbf{T})$$

Multi-Dimensionality

Set-up

- Now, the additional cost the firm has to pay is written as:

$$\phi(p, q, \mathbf{T})$$

where $\mathbf{T} \equiv (T_1, ..., T_\ell, ..., T_L)$ is an L-**dimensional** vector of policy/shock parameters.

- Then, the firm's profit is written as:

$$\pi = [pq - c(q)] - \phi(p, q, \mathbf{T}).$$

Motivating Example (again)

- The government considers a new tax scheme which makes $s > 0$ fraction of the cost $c(q)$ tax deductible.

  "**What are the effects of introducing** s, ***when*** $t > 0$ ***and*** $v > 0$ ***are already implemented*?**"

  - The firm's profit is written as:

$$\pi = (p - t)q - c(q) - v[pq - s \cdot c(q)]$$

$$= [pq - c(q)] - [\underbrace{tq + v\{pq - s \cdot c(q)\}}_{\equiv \phi(p,q,\mathbf{T}):\ \text{additional cost}}]$$

  - Intervention vector, $\mathbf{T} = (t, v, s)$, is **three-dimensional**.

FOC (1/2)

- Again, the conduct index θ is implicitly defined by:

$$\left[(1-\tau) - (1-\nu)\left(\frac{\theta}{\epsilon^D}\right)\right] p = mc,$$

where

$$\tau(p, q, \mathbf{T}) \equiv \frac{1}{p}\frac{\partial \phi}{\partial q}(p, q, \mathbf{T})$$

is the (**first-order**) **quantity sensitivity** of the (per-firm) tax revenue, and

$$\nu(p, q, \mathbf{T}) \equiv \frac{1}{q}\frac{\partial \phi}{\partial p}(p, q, \mathbf{T})$$

is its (**first-order**) **price sensitivity**.

FOC (2/2)

- Note that

$$\left[(1-\tau)-(1-\nu)\left(\frac{\theta}{\epsilon^D}\right)\right]p = mc$$

is the generalization of

$$[1-(\underbrace{v+\frac{t}{p}}_{=\tau})-(1-\underbrace{v}_{=\nu})\left(\frac{\theta}{\epsilon^D}\right)]p = mc$$

in our two-dimensional case of taxation above.

Multi-Dimensional Pass-Through (1/2)

- Now, we define the **pass-through rate vector** by:

$$\tilde{\boldsymbol{\rho}} \equiv \left(\frac{\partial p(\mathbf{T})}{\partial T_1}, \dots, \underbrace{\frac{\partial p(\mathbf{T})}{\partial T_\ell}}_{\equiv \tilde{\rho}_\ell}, \dots, \frac{\partial p(\mathbf{T})}{\partial T_L}\right)$$

and the **pass-through quasi-elasticity vector** by:

$$\boldsymbol{\rho} \equiv (\rho_1, \dots, \rho_\ell, \dots, \rho_L), \qquad \rho_\ell \equiv \frac{q}{\frac{\partial \phi}{\partial T_\ell}(p, q, \mathbf{T})}\ \tilde{\rho}_\ell$$

Multi-Dimensional Pass-Through (2/2)

Proposition

Each ℓ-th element of $\boldsymbol{\rho}$ is characterized by

$$\rho_\ell = \frac{pq}{\phi_\ell}\left[\tau_\ell - \left(\frac{\theta}{\epsilon^D}\right)\nu_\ell\right]\rho_{(0)},$$

where

$$\frac{1}{\rho_{(0)}} = \left[(1-\kappa) + \epsilon^D \tau_{(2)} + (1-\tau)\left(\frac{\epsilon^D}{\epsilon^S}\right)\right]$$

$$+\left[\left(\nu - \kappa + \frac{\nu_{(2)}}{\epsilon^D}\right) - (1-\nu)\left(\frac{1}{\epsilon^D} + \frac{1}{\epsilon^S}\right)\right]\theta + (1-\nu)\epsilon^D q \frac{\partial(\theta/\epsilon^D)}{\partial q},$$

with $\kappa \equiv \frac{\partial^2 \phi}{\partial p\, \partial q}$, $\tau_{(2)} \equiv \frac{q}{p}\frac{\partial^2 \phi}{\partial q^2}$, *and* $\nu_{(2)} \equiv \frac{p}{q}\frac{\partial^2 \phi}{\partial p^2}$.

For $\ell = t$ (Unit Tax Pass-Through: 1/2)

- If $\phi(p, q, \mathbf{T}) = tq + vpq$, then

$$\rho_t = \frac{pq}{\partial\phi/\partial t}\left[\tau_t - \left(\frac{\theta}{\epsilon^D}\right)\nu_t\right]\rho_{(0)} = \rho_{(0)}$$

because $\partial\phi/\partial t = q, \tau_t = 1/p$, and $\nu_t = 0$. Then,

$$\frac{1}{\rho_{(0)}} = [(\underbrace{1-\kappa}_{=1-v}) + \underbrace{\epsilon^D \tau_{(2)}}_{=0} + (1-\tau)\left(\frac{\epsilon^D}{\epsilon^S}\right)]$$

$$+ \left[\underbrace{\nu - \kappa + \eta\nu_{(2)}}_{=0} - (\underbrace{1-\nu}_{=1-v})\left(\frac{1}{\epsilon^D} + \frac{1}{\epsilon^S}\right)\right]\theta + (\underbrace{1-\nu}_{=1-v})\epsilon^D q \frac{\partial(\theta/\epsilon^D)}{\partial q}$$

$$= (1-v)\left\{\left[1 + \frac{1-\tau}{1-v}\left(\frac{\epsilon^D}{\epsilon^S}\right)\right] - \left(\frac{1}{\epsilon^D} + \frac{1}{\epsilon^S}\right)\theta + \epsilon^D q \frac{\partial(\theta/\epsilon^D)}{\partial q}\right\}$$

because $\kappa \equiv \frac{\partial^2\phi}{\partial p\,\partial q} = v$, and $\tau_{(2)} \equiv \frac{q}{p}\frac{\partial^2\phi}{\partial q^2} = 0 = \nu_{(2)} \equiv \frac{p}{q}\frac{\partial^2\phi}{\partial p^2}$.

For $\ell = t$ (Unit Tax Pass-Through: 2/2)

- Thus, it coincides with:

$$\rho_t = \frac{\frac{1}{1-v}}{\left[1 + \frac{1-\tau}{1-v}\frac{\epsilon^D}{\epsilon^S}\right] + \underbrace{\left[-\left(\frac{1}{\epsilon^D} + \frac{1}{\epsilon^S}\right)\theta\right]}_{\textit{Direct Effect}} + \underbrace{\left[-\epsilon^D q \frac{\partial(-\theta/\epsilon^D)}{\partial q}\right]}_{\textit{Indirect Effect}}},$$

as we saw above.

Proposition: Welfare Measures

	Two-Dimensional, (t, v)	**Multi-Dimensional**
Incidence	$\frac{1}{\frac{1}{\rho_t}-(1-v)(1-\theta)}$	$\frac{1}{\frac{1}{\rho_\ell}-(1-\nu)(1-\theta)}$
MCPF	$\frac{\frac{(1-v)\theta}{\epsilon^D}+\tau}{\frac{\frac{1}{\rho_t}+v}{\epsilon^D}-\tau}$	$\frac{\frac{(1-\nu)\theta}{\epsilon^D}+\tau}{\frac{\frac{1}{\rho_\ell}+\nu}{\epsilon^D}-\tau}$

where $\nu \equiv \frac{1}{q}\frac{\partial \phi}{\partial p}(p, q, \mathbf{T})$ and $\tau \equiv \frac{1}{p}\frac{\partial \phi}{\partial q}(p, q, \mathbf{T})$.

Including Other Changes Than Taxes

Exchange Rate and Technology

- Up to now, the additional cost consists of taxation only:

$$\pi = [pq - c(q)] - \phi(p, q, \mathbf{T}),$$

which means that the firm's additional cost contributes to welfare as the government's revenue:

$$dW = dCS + dPS + dR.$$

- However, our $\phi(p, q, \mathbf{T})$ can be extended to include changes in exchange rates and production costs:

$$\phi(p, q, \mathbf{T}) = \underbrace{\tilde{\phi}(p, q, \mathbf{T})}_{\text{tax}} + \underbrace{[\phi(p, q, \mathbf{T}) - \tilde{\phi}(p, q, \mathbf{T})]}_{\text{others}}$$

Example

- Firm uses some imported inputs for production. Then,

$$\begin{aligned}\pi &= [(1-v)p - t]q - [(1-a) + a\cdot e]c(q) \\ &= [pq - c(q)] - [\underbrace{tq + vpq + a\cdot(1-e)c(q)}_{\equiv \phi(p,q,\mathbf{T})}],\end{aligned}$$

  where a measures the ratio of imported inputs and $e > 0$ is the exchange rate.

- Intervention vector, $\mathbf{T} = (t, v, e)$, is **three-dimensional**.

Welfare Measures

- Define

$$g_\ell \equiv \frac{\frac{1}{q} \cdot \frac{\partial \widetilde{\phi}}{\partial T_\ell}(p, q, \mathbf{T})}{\frac{\partial \phi}{\partial T_\ell}(p, q, \mathbf{T})}$$

as the fraction of an increase in additional cost (ϕ) to the firm that is collected by the government in the form of taxes ($\tilde{\phi}$). Then,

	Two-Dimensional, with Taxation Only	Multi-Dimensional, **also with Other Changes**
Incidence	$\frac{1}{\frac{1}{\rho t}-(1-v)(1-\theta)}$	$\frac{1}{\frac{1}{\rho t}-(1-v)(1-\theta)}$
MCPF	$\frac{\frac{(1-v)\theta}{\epsilon^D}+\tau}{\frac{\frac{1}{\rho t}+v}{\epsilon^D}-\tau}$	$\frac{\frac{(1-\nu)\theta+\frac{1-g_\ell}{\rho_\ell}}{\epsilon^D}+\tau}{\frac{\frac{g_\ell}{\rho_\ell}+\nu}{\epsilon^D}-\tau}$

Incorporating Firm Heterogeneity

Definitions (1/2)

- We allow for the tax function

$$\phi_i\left(p_i, q_i, \mathbf{T}\right)$$

to depend explicitly on the identity of the firm. Similar for the sensitivities $\tau_i\left(p_i, q_i, \mathbf{T}\right)$, $\nu_i\left(p_i, q_i, \mathbf{T}\right)$, etc.

- The marginal cost $mc_i\left(q_i\right)$ of firm i is also allowed to depend on the identity of the firm. We denote its elasticity by

$$\epsilon_i^S\left(q_i\right) \equiv \frac{mc_i\left(q_i\right)}{q_i\, mc_i'\left(q_i\right)}.$$

Definitions (2/2)

- Instead of the conduct index, we define the **pricing strength index**

$$\psi_i(\mathbf{q}),$$

implicitly from FOC:

$$[1 - \tau_i - \psi_i(\mathbf{q})(1 - \nu_i)]\, p_i(\mathbf{q}) = mc_i(q_i),$$

where $\tau_i = \tau_i(p_i(\mathbf{q}), q_i, \mathbf{T})$ and $\nu_i = \nu_i(p_i(\mathbf{q}), q_i, \mathbf{T})$.

- In the case of symmetric firms, this definition reduces to:

$$\psi_i = \frac{\theta}{\epsilon^D}.$$

Multi-Dimensional Pass-Through (1/3)

- First, we define the $(n \times d)$ **pass-through matrix** $\tilde{\boldsymbol{\rho}}$ with columns $\tilde{\boldsymbol{\rho}}_\ell \equiv \partial \mathbf{p}/\partial T_\ell$ and whose (i,ℓ) element is:

$$\tilde{\rho}_{i\,\ell} = \frac{\partial p_i}{\partial T_\ell}$$

- Notice here that our framework can easily be extended to include **firm-specific taxation/shocks**:

$$\tilde{\rho}_{i\,\ell_j} = \frac{\partial p_i}{\partial T_{\ell_j}}$$

is the effect of firm j's specific shock in the ℓ-th instrument on firm i's price.

Multi-Dimensional Pass-Through (2/3)

Proposition

The pass-through matrix $\tilde{\boldsymbol{\rho}} = (\tilde{\boldsymbol{\rho}}_1, ..., \tilde{\boldsymbol{\rho}}_\ell, ..., \tilde{\boldsymbol{\rho}}_L)$ *is characterized by:*

$$\tilde{\boldsymbol{\rho}}_\ell = \mathbf{b}^{-1} \,.\, \iota_\ell,$$

where matrix $\mathbf{b}$ *is an* $(n \times n)$ *matrix whose* (i,j) *element is*

$$b_{ij} = \left[(1-\kappa_i)\delta_{ij} + \epsilon^D_{ij}\tau_{(2)i} + (1-\tau_i)\left(\frac{\epsilon^D_{ij}}{\epsilon^S_i}\right)\right]$$

$$+\left\{\left[(\nu_i - \kappa_i)\epsilon^D_{ij} + \nu_{(2)i}\delta_{ij}\right] - (1-\nu_i)\left(\delta_{ij} + \frac{\epsilon^D_{ij}}{\epsilon^S_i}\right)\right\}\psi_i - (1-\nu_i)\psi_i\Psi_{ij},$$

where δ_{ij} *is the Kronecker delta, and*

$$\Psi_{ij} = \frac{p_i}{\psi_i}\frac{\partial \psi_i(\mathbf{q}(\mathbf{p}))}{\partial p_j}, \quad \epsilon^D_{ij} = -\frac{p_i}{q_i}\frac{\partial q_i(\mathbf{p})}{\partial p_j}, \qquad (contd)$$

Multi-Dimensional Pass-Through (3/3)

Proposition

(contd) and ι_ℓ is an n-dimensional vector defined for each T_ℓ, whose (i,1) element is:

$$\iota_{i\,\ell} \equiv p_i \cdot \left[\frac{\partial \tau_i}{\partial T_\ell} - \psi_i \frac{\partial \nu_i}{\partial T_\ell}\right].$$

- Then, we can define the $(n \times d)$ **pass-through quasi-elasticity matrix** $\boldsymbol{\rho}$ whose whose (i,ℓ) element is:

$$\rho_{i\,\ell} = \frac{q_i}{\frac{\partial}{\partial T_\ell}\phi_i}\ \tilde{\rho}_{i\,\ell}.$$

Welfare Measures: Takeaway

- We have developed **general formulas** for welfare evaluation under **imperfect competition** in consideration of the **multi-dimensionality** of taxation and other external changes.

	Perfect comp	Two-Dimensional, Symmetric Oligopoly	**Multi-Dimensional, Firm Heterogeneity, Also with Other Changes**
Incidence	$\frac{1}{\frac{1}{\rho t}-1}$	$\frac{1}{\frac{1}{\rho t}-(1-v)(1-\theta)}$	$\frac{1}{\frac{1}{\rho_{i\,\ell}}-(1-\nu_i)(1-\psi_i\,\epsilon^{\rho}_{i\,\ell})}$
MCPF	0	$\frac{\frac{(1-v)\theta}{\epsilon^D}+\tau}{\frac{\frac{1}{\rho t}+v}{\epsilon^D}-\tau}$	$\frac{\frac{(1-\nu_i)\,\psi_i+\frac{1-g_{i\,\ell}}{\rho_{i\,\ell}}}{\epsilon^{\rho}_{i\,\ell}}+\tau_i}{\frac{\frac{g_{i\,\ell}}{\rho_{i\,\ell}}+\nu_i}{\epsilon^{\rho}_{i\,\ell}}-\tau_i}$

where $\epsilon^{\rho}_{i\,\ell} \equiv \epsilon^D_i.\widetilde{\boldsymbol{\rho}}_\ell/\widetilde{\rho}_{i\,\ell} = \epsilon^D_i.\boldsymbol{\rho}_\ell/\rho_{i\,\ell}$.

Possible Extensions

- **Vertical Relationships / Two-Sided Platforms**

  Tremblay (2017): *Taxation on a Two-Sided Platform*

- Macroeconomics?

  General-Equilibrium Effects

  Dynamics (incorporating **adjustment**/**menu costs**)

  "How does imperfect competition matter to the determination of the aggregate price level?"