Expectation and Duration at the Effective Lower Bound

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[1] The views expressed here do not represent those of the Chicago Fed or the Federal Reserve System.

Introduction

I study the relative effects of duration exposures and short-rate expectations in a structural model of the yield curve.

- Important for understanding unconventional monetary policy - forward guidance and QE
- Previous models of this type ignore the ELB
  - Vayanos and Vila, 2009; Greenwood and Vayanos, 2014
- I incorporate the ELB using a shadow-rate structure.
  - Kim and Singleton, 2012; Krippner, 2012; Wu and Xia, 2015

Qualitatively:

- Effects of changes in bond supply on term premia are attenuated at ELB.
- Forward guidance at the ELB has effects on term premia that it does not have during normal times.

Quantitatively:

- The model matches the yield data well, including event-studies on unconventional policy.
- The Fed's unconventional policy mostly operated by changing the anticipated short-rate path, not by reducing duration exposures.

Model setup

Following Vayanos-Vila and others, arbitrageurs solve

$$\max_{x_t(\tau)\forall\tau} \mathrm{E}_t\left[dW_t\right] - \frac{a}{2}\mathrm{var}_t\left[dW_t\right] \tag{1}$$

subject to

$$dW_t = \int_0^T x_t\left(\tau\right)\frac{dP_t^{(\tau)}}{P_t^{(\tau)}}d\tau + r_t\left(W_t - \int_0^T x_t\left(\tau\right)d\tau\right) \tag{2}$$

where W_t is wealth, $x_t\left(\tau\right)$ is bond holdings at maturity τ, $P_t^{(\tau)}$ is the bond price at maturity τ, and r_t is the short rate.

Model setup

FOC:

$$E_t\left[dp_t^{(\tau)}\right] = r_t + \underbrace{a\int_0^T x_t\left(\tau'\right)\text{cov}_t\left[dp_t^{(\tau)}, dp_t^{(\tau')}\right]d\tau'}_{\text{Risk premium}} + \underbrace{J_t^{(\tau)}}_{\text{Jensen}} \qquad (3)$$

Can also solve for yields through the usual relationship.

The government supplies bonds $s_t\left(\tau\right)$. Equilibrium is determined by

$$s_t\left(\tau\right) = x_t\left(\tau\right) \qquad (4)$$

Levels of $s_t\left(\tau\right)$ that increase the portfolio variance raise required returns (and therefore yields).

State Variables: Shadow rate

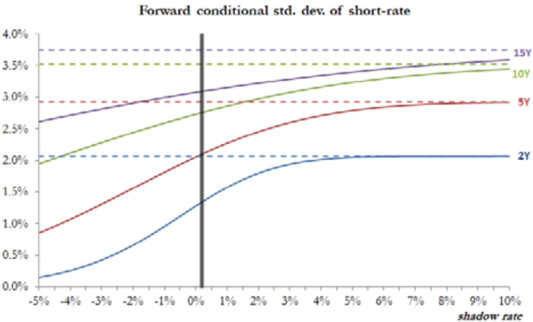
The short rate follows

$$r_t = \max\left[\widehat{r}_t, b\right] \tag{5}$$

where b is the ELB and

$$\widehat{r}_t = \mu_{\widehat{r}}(1 - \phi_{\widehat{r}}) + \phi_{\widehat{r}}\widehat{r}_{t-1} + e_t^{\widehat{r}} \qquad e_t^{\widehat{r}} \sim Niid\left(0, \sigma_{\widehat{r}}\right) \tag{6}$$

ELB dampens interest-rate uncertainty:



State Variables: Bond supply

Following Greenwood, Hanson, and Stein (2015), reduce bond supply to a single factor:

$$s_t(\tau) = \zeta + \left(1 - \frac{2\tau}{T}\right)\beta_t \tag{7}$$

$$\beta_t = \phi_\beta \beta_{t-1} + e_t^\beta \qquad e_t^\beta \sim Niid\left(0, \sigma_\beta\right) \tag{8}$$

Maturity distribution moves in a see-saw pattern in response to shocks to β_t.

(The shape of the distribution is not of major importance.)

State Variables: Bond supply

The WAM of outstanding debt is

$$WAM_t \equiv v \frac{\int_0^T \tau s_t(\tau)\, d\tau}{\int_0^T s_t(\tau)_t\, d\tau} = vT\left(\frac{1}{2} - \frac{1}{6\zeta}\beta_t\right) \tag{9}$$

where v is the length of one period, in years.

Outstanding 10y equivalents are

$$\%\Delta 10YE_t \equiv \frac{\frac{v}{10}\int_0^T \tau s_t(\tau)\, d\tau}{\frac{v}{10}\int_0^T \tau s_{t-1}(\tau)\, d\tau} = -\frac{\Delta\beta_{t+h}}{3\zeta - \beta_t} \tag{10}$$

Calibration and solution

	Bond supply				Short rate				Risk aversion
	T	ϕ_β	σ_β	ζ	$\mu_{\hat{r}}$	$\phi_{\hat{r}}$	$\sigma_{\hat{r}}$	b	a
[1] Shadow-rate model	60	0.98	0.20	0.31	5.0%	0.98	0.78%	0.17%	0.15

- Using data since 1971, I match:
  - the annual autocorrelation of Treasury WAM
  - the unconditional mean and std. dev. of the 3M and 10Y yield
  - the unconditional correlation between the 3M and 10Y yield
  - the mean 3M yield during the ELB period
- Model is solved numerically.

Evidence on the model's fit

- The model matches the basic features of yields observed at the ELB:
  - Matches the 10Y slope average to within 0.1%.
  - Matches the 10Y slope std. dev. to within 0.3%.
- Affine model predicts negative short rates, very steep slopes, and excessive volatility.
- Away from the ELB, shadow-rate and affine models perform similarly.

- Model matches regression results on the effects of bond supply (extending Greenwood-Vayanos, 2014).
  - E.g., using 10Y yield as dependent variable:

	Coef. on WAM		Coef. on 2Y yield	
	above ELB	at ELB	above ELB	at ELB
Data	0.19	0.06	0.8	2.3
Model	˜0.12	˜0.08	˜0.7	>2.0

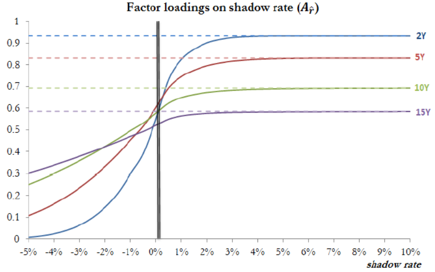
(Model results are generally within 1 s.e. of regressions.)

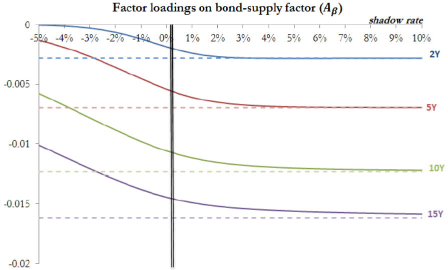
Factor loadings in the shadow-rate model

For arbitrary state values, we have

$$y_t^{(\tau)} \approx C_t^{(\tau)} + A_{\hat{r},t}^{(\tau)} \hat{r} + A_{\beta,t}^{(\tau)} \beta \quad (11)$$

- In an affine model, $A_{\hat{r},t}^{(\tau)}$ and $A_{\beta,t}^{(\tau)}$ are constant (and the equation is exact).
- In the nonlinear model, they are state-dependent.

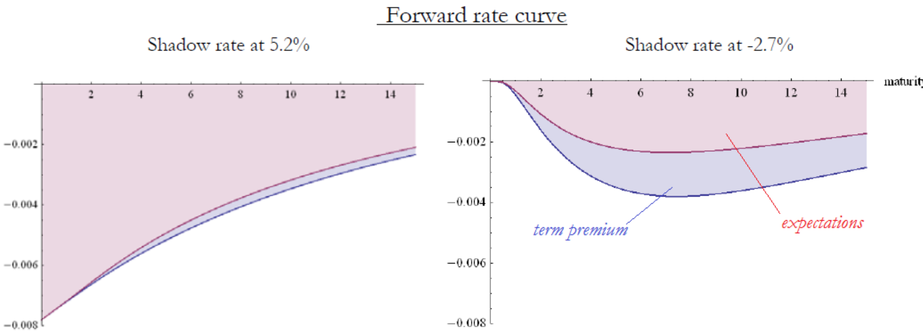




- The sensitivity to both factors is *quantitatively* attenuated by the ELB.
- The $A_{\hat{r}}^{(\tau)}$ loadings change *qualitatively*, reversing their order across maturities.

Effects of shadow-rate shock on yield curve components

Impact of a one-standard-deviation shock to $\widehat{r}_t$ from different initial values:



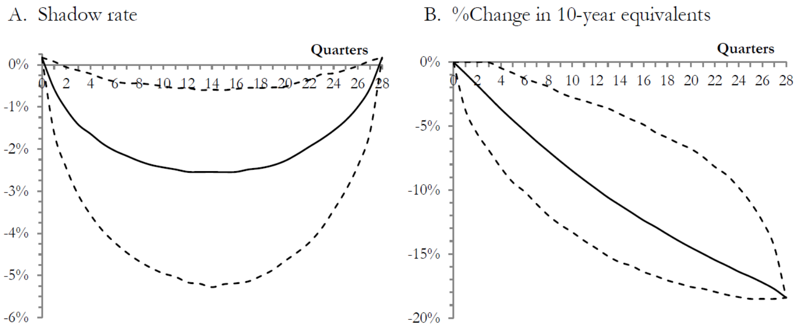
- At the ELB:
  - Overall effects are smaller.
  - Effects are increasing, not decreasing, across maturities.
  - Effects on the term premium are important.

Assessing unconventional monetary policy

To study the effects of actual Fed policy in this model, I calculate shocks that correspond to what the Fed actually did:

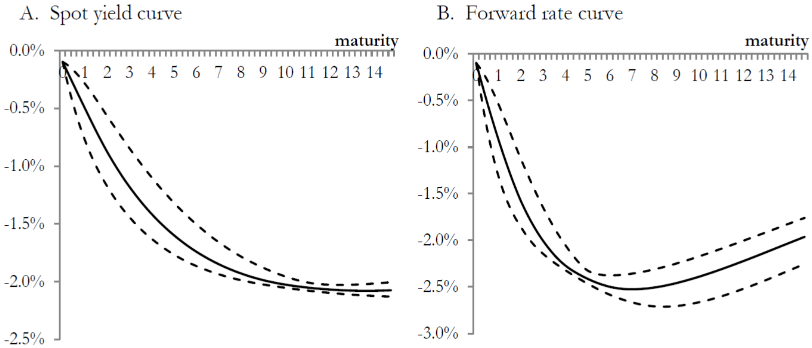
- Shadow rate shocks - kept r_t at the ELB for 7 years.
- Fed balance sheet shocks - removed 18% of government-backed duration.
    - These are assumed to be less persistent than the β_t shocks above ($\phi = 0.96$), but this makes little difference.

Consider a set of trajectories that are consistent with these observations:



Cumulative yield-curve responses in model sims

Adding up the yield-curve surprises (pseudo event study):



- Magnitude is roughly consistent with the cumulative effects of unconventional policy implied by event studies.
- Model captures the "hump shaped" forward-curve response noted by Rogers et al. (2014) and others.

Decomposition of yields w/r/t unconventional policy shocks

	Shadow-rate shocks		Fed balance-sheet shocks		
Maturity [1]	Expectations component [2]	Term premium component [3]	Term premium component [4]	Interaction [6]	Total [7]
2 years	-63	-12	-9	5	-79
	(-93, -36)	(-14, -10)	(-10, -8)	(4, 6)	(-109, -49)
5 years	-98	-45	-21	8	-156
	(-117, -73)	(-47, -42)	(-22, -18)	(6, 11)	(-174, -127)
10 years	-114	-63	-34	7	-202
	(-121, -101)	(-70, -54)	(-36, -29)	(5, 11)	(-205, -194)
15 years	-111	-63	-40	6	-208
	(-113, -105)	(-75, -53)	(-44, -35)	(4, 9)	(-213, -201)

- Shadow-rate shocks account for over 80% of the effects of unconventional policy on long-term yields.
- About 1/3 of this effect comes from the effects on term premia through reduced volatility.

Conclusion

- Simple no-arbitrage model of bond portfolio choice w/shadow rate.
- Captures both forward guidance/signaling and duration channel of QE.
- At the ELB, things change dramatically:
  - Effects of both types of shocks are attenuated by the ELB.
  - Forward guidance has effects on term premia at the ELB that don't exist elsewhere.
- Consequently, the effects of unconventional monetary policy at the ELB may not be well described by
  - Empirical estimates from pre-ELB data
  - Theoretical models that assume linearity
- Simulations suggest that communications about future short rates were far more important for yields than was duration removal during the ELB period.