A Semi-Nonparametric Estimator for Random Coefficient Logit Demand Models

Zhentong Lu, Xiaoxia Shi and Jing Tao

SUFE, Wisconsin-Madison and Washington-Seattle

January 7, 2018

Introduction

- the workhorse model in demand estimation for differentiated products: BLP random coefficient (RC) logit model
  - really neat idea to solve the price endogeneity problem with rich preference heterogeneity (represented by RCs)
  - standard BLP estimator: nested fixed-point GMM

Introduction

- the workhorse model in demand estimation for differentiated products: BLP random coefficient (RC) logit model
  - really neat idea to solve the price endogeneity problem with rich preference heterogeneity (represented by RCs)
  - standard BLP estimator: nested fixed-point GMM

- in this paper, we propose an alternative two-step estimator for the model
  - obtain estimates of fixed coefficients with little computational costs
  - allow nonparametric specification of RCs
  - obtain new results on some theoretical issues

Setup

- in this presentation, for expositional simplicity, we consider a *single* market with $J + 1$ competing products $\{0, 1, ..., J\}$, where 0 refers to the "outside option"

Setup

- in this presentation, for expositional simplicity, we consider a *single* market with $J+1$ competing products $\{0, 1, ..., J\}$, where 0 refers to the "outside option"
- standard random utility model

$$u_{ij} = \delta_j + X_{2,j}^{'} \upsilon_i + \varepsilon_{ij}$$

  - mean utility: $\delta_j = \alpha + X_{1,j}^{'} \beta + \xi_j$, $\delta_0 = 0$ and $\delta \equiv (\delta_1, ..., \delta_J)$
  - heterogeneity: random coefficients $\upsilon_i \sim F(\cdot)$, ε_{ij} i.i.d. type I extreme value

Setup

- in this presentation, for expositional simplicity, we consider a *single* market with $J+1$ competing products $\{0, 1, ..., J\}$, where 0 refers to the "outside option"
- standard random utility model

$$u_{ij} = \delta_j + X_{2,j}^{'} \upsilon_i + \varepsilon_{ij}$$

  - mean utility: $\delta_j = \alpha + X_{1,j}^{'} \beta + \xi_j$, $\delta_0 = 0$ and $\delta \equiv (\delta_1, ..., \delta_J)$
  - heterogeneity: random coefficients $\upsilon_i \sim F(\cdot)$, ε_{ij} i.i.d. type I extreme value

- aggregating individual optimal choices $\Rightarrow$ aggregate demand (market share) system

$$\begin{aligned} s_j &= \sigma_j(\delta, X_2; F) \\ &= \int \frac{\exp\left(\delta_j + X_{2,j}^{'} \upsilon_i\right)}{1 + \sum_{k=0}^{J} \exp\left(\delta_k + X_{2,k}^{'} \upsilon_i\right)} dF(\upsilon_i), \ \forall j \end{aligned} \quad (1)$$

Setup

- in this presentation, for expositional simplicity, we consider a *single* market with $J+1$ competing products $\{0, 1, ..., J\}$, where 0 refers to the "outside option"
- standard random utility model

$$u_{ij} = \delta_j + X_{2,j}^{'} \upsilon_i + \varepsilon_{ij}$$

  - mean utility: $\delta_j = \alpha + X_{1,j}^{'} \beta + \xi_j$, $\delta_0 = 0$ and $\delta \equiv (\delta_1, ..., \delta_J)$
  - heterogeneity: random coefficients $\upsilon_i \sim F(\cdot)$, ε_{ij} i.i.d. type I extreme value
- aggregating individual optimal choices $\Rightarrow$ aggregate demand (market share) system

$$\begin{aligned} s_j &= \sigma_j(\delta, X_2; F) \\ &= \int \frac{\exp\left(\delta_j + X_{2,j}^{'} \upsilon_i\right)}{1 + \sum_{k=0}^{J} \exp\left(\delta_k + X_{2,k}^{'} \upsilon_i\right)} dF(\upsilon_i), \ \forall j \end{aligned} \qquad (1)$$

- we want to estimate $\theta \equiv (\alpha, \beta, F)$ using aggregate data $(s_j, X_{1,j}, X_{2,j})$

The BLP Idea

- invert the demand system (1) (see Berry (1994) and Berry, Gandhi, and Haile (2013))

$$\delta_j = \sigma_j^{-1}\left(s, X_2; F\right), \ \forall j$$

The BLP Idea

- invert the demand system (1) (see Berry (1994) and Berry, Gandhi, and Haile (2013))

$$\delta_j = \sigma_j^{-1}\left(s, X_2; F\right), \ \forall j$$

- impose IV assumption $E\left[\xi_j \middle| Z_j\right] = 0$

The BLP Idea

- invert the demand system (1) (see Berry (1994) and Berry, Gandhi, and Haile (2013))

$$\delta_j = \sigma_j^{-1}\left(s, X_2; F\right), \ \forall j$$

- impose IV assumption $E\left[\xi_j \middle| Z_j\right] = 0$
- construct a GMM estimator, with a *parametric* F (e.g., normal)

$$\arg\min_{\theta} \left\| \frac{1}{J} \sum_{j=1}^{J} Z_j^{'} \left[\sigma_j^{-1}\left(s, X_2; F\right) - \alpha - X_{1,j}^{'}\beta\right] \right\|$$

Challenges

- the inverse demand $\sigma_j^{-1}(\cdot)$ must be solved numerically (i.e., BLP contraction mapping)
  - computational issues have aroused research interests, e.g., Knittel and Metaxoglou (2012), Dubé, Fox, and Su (2012), Lee and Seo (2015)

Challenges

- the inverse demand $\sigma_j^{-1}(\cdot)$ must be solved numerically (i.e., BLP contraction mapping)
  - computational issues have aroused research interests, e.g., Knittel and Metaxoglou (2012), Dubé, Fox, and Su (2012), Lee and Seo (2015)
- many endogenous variables (s is J-dimensional), in addition to endogenous product characteristics (Berry and Haile (2014))

$$E\left[\sigma_j^{-1}(s, X_2; F) - \alpha - X_{1,j}^{'}\beta \middle| Z_j\right] = 0$$

Challenges

- the inverse demand $\sigma_j^{-1}(\cdot)$ must be solved numerically (i.e., BLP contraction mapping)
  - computational issues have aroused research interests, e.g., Knittel and Metaxoglou (2012), Dubé, Fox, and Su (2012), Lee and Seo (2015)
- many endogenous variables (s is J-dimensional), in addition to endogenous product characteristics (Berry and Haile (2014))

$$E\left[\sigma_j^{-1}(s, X_2; F) - \alpha - X_{1,j}^{'}\beta \middle| Z_j\right] = 0$$

- nontrivial interdependence of $(X_{1,j}, X_{2,j})$ across j (Berry, Linton, and Pakes (2004))

Our Approach: Transform to Partial Linear Form

- exploit a separability property of the random coefficient logit model

$$\int \frac{\exp\left(\delta_j + X_{2,j}^{'}\upsilon\right)}{1+\sum_{k=1}^{J}\exp\left(\delta_k + X_{2,k}^{'}\upsilon\right)} dF\left(\upsilon\right) = \exp\left(\delta_j\right) \cdot \int \frac{\exp\left(X_{2,j}^{'}\upsilon\right)}{1+\sum_{k=1}^{J}\exp\left(\delta_k + X_{2,k}^{'}\upsilon\right)} dF\left(\upsilon\right)$$

Our Approach: Transform to Partial Linear Form

- exploit a separability property of the random coefficient logit model

$$\int \frac{\exp\left(\delta_j + X_{2,j}' \upsilon\right)}{1 + \sum_{k=1}^{J} \exp\left(\delta_k + X_{2,k}' \upsilon\right)} dF(\upsilon) = \exp(\delta_j) \cdot \int \frac{\exp\left(X_{2,j}' \upsilon\right)}{1 + \sum_{k=1}^{J} \exp\left(\delta_k + X_{2,k}' \upsilon\right)} dF(\upsilon)$$

- taking log on both sides of demand equation,

$$\log(s_j) = \alpha + X_{1,j}' \beta + \tilde{\psi}_J(X_{2,j}) + \xi_j,$$

where

$$\tilde{\psi}_J(X_{2,j}) \equiv \log\left[\int \frac{\exp\left(X_{2,j}' \upsilon\right)}{1 + \sum_{k=1}^{J} \exp\left(\delta_k + X_{2,k}' \upsilon\right)} dF(\upsilon)\right]$$

Normalization

- normalize by the outside share

$$\log\left(\frac{s_j}{s_0}\right) = \alpha + X_{1,j}^{'}\beta + \psi_J(X_{2,j}) + \xi_j$$

where

$$\psi_J(X_{2,j}) = \tilde{\psi}_J\left(X_{2,j}\right) - \tilde{\psi}_J\left(0\right) = \log\left[\frac{\int \frac{\exp\left(X_{2,j}^{'}\upsilon\right)}{1+\sum_{k=1}^{J}\exp\left(\delta_k + X_{2,k}^{'}\upsilon\right)}dF(\upsilon)}{\int \frac{1}{1+\sum_{k=1}^{J}\exp\left(\delta_k + X_{2,k}^{'}\upsilon\right)}dF(\upsilon)}\right]$$

Normalization

- normalize by the outside share

$$\log\left(\frac{s_j}{s_0}\right) = \alpha + X_{1,j}^{'}\beta + \psi_J(X_{2,j}) + \xi_j$$

where

$$\psi_J(X_{2,j}) = \tilde{\psi}_J\left(X_{2,j}\right) - \tilde{\psi}_J\left(0\right) = \log\left[\frac{\int \frac{\exp\left(X_{2,j}^{'}\upsilon\right)}{1+\sum_{k=1}^{J}\exp\left(\delta_k + X_{2,k}^{'}\upsilon\right)} dF(\upsilon)}{\int \frac{1}{1+\sum_{k=1}^{J}\exp\left(\delta_k + X_{2,k}^{'}\upsilon\right)} dF(\upsilon)}\right]$$

- now we have a partial linear form, except that $\psi_J(\cdot)$ is a *random* function
  - we treat the *limit* of $\psi_J(\cdot)$, $\psi(\cdot)$, as an unknown function and apply sieve approximation as $J \to \infty$
  - comparing to simple logit, we can see that random coefficients imply the nonlinear terms of $x_{2,j}$

A Two-Step Semi-Nonparametric Estimator

- first step: estimate (α, β, ψ) in the partial linear model
  - approximate ψ by a linear sieve $\psi_{k_{1,J}}(X_{2,j}) \equiv \sum_{\ell=1}^{k_{1,J}} b_\ell p_\ell(X_{2,j})$, where $\{p_\ell(\cdot) : \ell = 1, ..., k_{1,J}\}$ are basis functions
  - sieve GMM based on $E[\xi_j | Z_j] = 0 \Leftrightarrow E\left[\xi_j \cdot \mathbf{I}^{\zeta_J}(z_j)\right] = 0$, where $\mathbf{I}^{\zeta_J}(\cdot)$ is a ζ_J-dimensional vector of basis functions

A Two-Step Semi-Nonparametric Estimator

- first step: estimate (α, β, ψ) in the partial linear model
  - approximate ψ by a linear sieve $\psi_{k_{1,J}}(X_{2,j}) \equiv \sum_{\ell=1}^{k_{1,J}} b_\ell p_\ell(X_{2,j})$, where $\{p_\ell(\cdot) : \ell = 1, ..., k_{1,J}\}$ are basis functions
  - sieve GMM based on $E[\xi_j | Z_j] = 0 \Leftrightarrow E\left[\xi_j \cdot \mathbf{I}^{\zeta_J}(z_j)\right] = 0$, where $\mathbf{I}^{\zeta_J}(\cdot)$ is a ζ_J-dimensional vector of basis functions

- second step: estimate F nonparametrically via sieve MD

$$\arg\min_{F_{k_{2,J}}} \frac{1}{J} \sum_{j=1}^{J} \left\{ \log\left(\frac{s_j}{s_0}\right) - \log\left[\frac{\int \frac{\exp\left(\hat{\delta}_j + X_{2,j}^{\prime}\upsilon\right)}{1+\sum_{k=1}^{J}\exp\left(\hat{\delta}_k + X_{2,k}^{\prime}\upsilon\right)} dF_{k_{2,J}}(\upsilon)}{\int \frac{1}{1+\sum_{k=1}^{J}\exp\left(\hat{\delta}_k + X_{2,k}^{\prime}\upsilon\right)} dF_{k_{2,J}}(\upsilon)}\right]\right\}^2$$

  - $\hat{\delta}_j = \hat{\alpha} + X_{1,j}^{\prime}\hat{\beta} + \hat{\xi}_j$ is obtained from the first stage estimation
  - $F_{k_{2,J}}$ is sieve approximation to F

Remarks

- computationally lighter than standard BLP nested fixed point GMM estimator
  - no fixed-point computation and the estimates of fixed coefficients (α, β) could be obtained with little computational cost, similar to Salanie and Wolak (2016)

Remarks

- computationally lighter than standard BLP nested fixed point GMM estimator
  - no fixed-point computation and the estimates of fixed coefficients (α, β) could be obtained with little computational cost, similar to Salanie and Wolak (2016)
- the "many endogenous variable" does *not* show up in our estimation equation

$$\log\left(\frac{s_j}{s_0}\right) = \alpha + X_{1,j}^{'}\beta + \psi\left(X_{2,j}\right) + \xi_j$$

  - the endogeneity issue has been "taken care of" automatically because $\psi_J(\cdot) \to \psi(\cdot)$ as $J \to \infty$

Remarks (Cont'd)

- relaxing the parametric assumptions on RCs can be important: the shape of RC determines the substitution patterns, i.e., cross-product elasticities

Remarks (Cont'd)

- relaxing the parametric assumptions on RCs can be important: the shape of RC determines the substitution patterns, i.e., cross-product elasticities
- result: under very mild assumptions, normal RCs imply that all the cross-product elasticities vanishes at the same rate $O\left(J^{-1}\right)$ as $J \to \infty$
  - the vanishing cross-elasticities means that "local competition" disappears, which may not be realistic (effectively "IIA property")
  - intuition: the tail of normal RC is too thin to offset the effects of the logit error

Remarks (Cont'd)

- relaxing the parametric assumptions on RCs can be important: the shape of RC determines the substitution patterns, i.e., cross-product elasticities
- result: under very mild assumptions, normal RCs imply that all the cross-product elasticities vanishes at the same rate $O\left(J^{-1}\right)$ as $J \to \infty$
  - the vanishing cross-elasticities means that "local competition" disappears, which may not be realistic (effectively "IIA property")
  - intuition: the tail of normal RC is too thin to offset the effects of the logit error
- thus, flexible/nonparametric RCs are important for generating realistic substitution patterns

Data Structure and Asymptotic Framework

- data structure: a large cross-section of products in a single market
  - practically relevant: national market (e.g., BLP auto data); products defined at disaggregate level, e.g., scanner data/online shopping data at SKU level
  - theoretically, it is useful to understand identification/estimation issues within a single market, as Berry, Linton, and Pakes (2004) and Armstrong (2016)

Data Structure and Asymptotic Framework

- data structure: a large cross-section of products in a single market
  - practically relevant: national market (e.g., BLP auto data); products defined at disaggregate level, e.g., scanner data/online shopping data at SKU level
  - theoretically, it is useful to understand identification/estimation issues within a single market, as Berry, Linton, and Pakes (2004) and Armstrong (2016)
- major challenge: product characteristics $\{X_j : j = 1, ..., J\}$ are interdependent in a non-trivial way due to firm's strategic interactions (e.g., price/advertising strategies)

Key Assumptions

Assumption

For each J*, there exists a* σ*-field* $\mathcal{C}$ *such that, conditional on* $\mathcal{C}$*,* $\{(X_j, Z_j) : j = 1, ..., J\}$ *are independent across* j*.*

- the interdependence of (X_j, Z_j) across j are captured by the "common shock" $\mathcal{C}$
- in the paper, we provide more primitive/verifiable conditions and compare with Berry, Linton, and Pakes (2004)'s approach in handling the interdependence

Key Assumptions

Assumption

For each J, there exists a σ-field $\mathcal{C}$ such that, conditional on $\mathcal{C}$, $\{(X_j, Z_j) : j = 1, ..., J\}$ are independent across j.

- the interdependence of (X_j, Z_j) across j are captured by the "common shock" $\mathcal{C}$
- in the paper, we provide more primitive/verifiable conditions and compare with Berry, Linton, and Pakes (2004)'s approach in handling the interdependence

Assumption

The unobserved product characteristics ξ_j are independent across j conditional on $\{Z_j\}_{j=1}^{J}$ and satisfy $E[\xi_j | Z_j] = 0$ a.s.

- identical to the standard assumptions imposed on the unobserved characteristic ξ as in Berry, Linton, and Pakes (2004)

Asymptotic Results

- first stage: suppose that the above assumptions, standard identification assumption for partial linear IV models, as well as appropriate LLN and CLT results hold, we have

$$\sqrt{J} V_J^{-1/2} \begin{pmatrix} \hat{\alpha} - \alpha \\ \hat{\beta} - \beta \end{pmatrix} \to_d N(\mathbf{0}, I),$$

where V_J achieves the semi-parametric efficiency bound in the limit.

Asymptotic Results

- first stage: suppose that the above assumptions, standard identification assumption for partial linear IV models, as well as appropriate LLN and CLT results hold, we have

$$\sqrt{J}V_J^{-1/2}\begin{pmatrix}\hat{\alpha}-\alpha\\ \hat{\beta}-\beta\end{pmatrix}\rightarrow_d N\left(\mathbf{0},I\right),$$

  where V_J achieves the semi-parametric efficiency bound in the limit.

- second stage: sieve MLE with generated regressor (see Newey (1994))
  - consistency: $d_{LP}\left(\hat{F}_J,F\right)\xrightarrow{p}0$, where $d_{LP}(\cdot,\cdot)$ is the Lévy-Prokhorov metric
  - similar to the idea in Fox, Kim and Yang (2016)

Monte Carlo Simulations: DGP

- a single market with J inside products
  - exogenous characteristic: $X_j \sim U[0, \bar{x}]$
  - unobserved characteristic: $\xi_j \sim N(0, .5^2)$
  - endogenous price/marginal cost:
    $p_j = mc_j = \gamma_1 X_j + \gamma_2 W_j + \xi_j + \zeta_j$
    - exogenous cost shifter $W_j \sim U[0, \bar{w}]$ and a shock $\zeta_j \sim N(0, .1^2)$
    - assumed market structure: single-product firms, perfect competition

Monte Carlo Simulations: DGP

- a single market with J inside products
  - exogenous characteristic: $X_j \sim U[0, \bar{x}]$
  - unobserved characteristic: $\xi_j \sim N\left(0, .5^2\right)$
  - endogenous price/marginal cost:
    $p_j = mc_j = \gamma_1 X_j + \gamma_2 W_j + \xi_j + \zeta_j$
    - exogenous cost shifter $W_j \sim U[0, \bar{w}]$ and a shock $\zeta_j \sim N\left(0, .1^2\right)$
    - assumed market structure: single-product firms, perfect competition

- market share is generated via simulation

$$s_j = \frac{1}{R}\sum_{i=1}^{R} \frac{\exp\left(\delta_j + \upsilon_i p_j\right)}{1 + \sum_{k=1}^{J} \exp\left(\delta_k + \upsilon_i p_k\right)}$$

  - mean utility: $\delta_j = \alpha + X_j\beta + \xi_j$, and $\alpha \sim U[-12, -8]$ is a "common shock"
  - random coefficient: $\upsilon_i \sim F$ with R draws

Estimation: Implementation Details

- first stage: two-stage (sieve) GMM
  - sieve approximation $\psi_{k_{1,J}}$: cubic splines/power series
  - instrument function (of x and ω): cubic splines/power series

Estimation: Implementation Details

- first stage: two-stage (sieve) GMM
  - sieve approximation $\psi_{k_{1,J}}$: cubic splines/power series
  - instrument function (of x and ω): cubic splines/power series
- second stage: sieve MD with F approximated by $F_{k_{2,J}}$
  - sieve I: generate random draws from $F_{k_{2,J}}$, as suggested by Fosgerau and Mabit (2013)
    - draw $u \sim U[0, 1]$ and stick into cubic splines/power series
    - in effect, this strategy approximates the inverse CDF F^{-1}
  - sieve II: approximate F by the probability weights on a grid of υ, as suggested by Train (2016)
    - pre-specify a grid of υ: $\upsilon_1, ..., \upsilon_S$
    - weight on each grid point υ_s is a logit probability $\frac{\exp\left[\varphi_{k_{2,J}}(\upsilon_s)\right]}{\sum_{t=1}^{S}\exp\left[\varphi_{k_{2,J}}(\upsilon_t)\right]}$, where $\varphi_{k_{2,J}}$ is a linear sieve to be estimated

Results: F is Normal

Table: Monte Carlo Results: Fixed Coefficients

			$J = 50$	100	200	400
β	SN	RtMSE	.0610	.0396	.0268	.0179
		Bias	-.0039	-.0011	-.0012	4.58E-4
	BLP	RtMSE	.0499	.0352	.0246	.0172
		Bias	-.0047	-.0019	-.0024	-3.97E-4
α	SN	RtMSE	.0946	.0629	.0429	.0297
		Bias	-.0052	-.0015	-.0018	5.63E-4
	BLP	RtMSE	.0567	.0401	.0284	.0208
		Bias	-6.79E-4	-.0012	-.0013	-.0013

Results: F is Normal

Table: Monte Carlo Results: Mean of Random Coefficient

Estimator	J	50	100	200	400
SN-I	RtMSE	.0758	.0518	.0385	.0298
	Bias	.0034	-.0039	-.0013	-8.92E-4
SN-II	RtMSE	.0795	.0521	.0360	.0244
	Bias	-4.46E-4	-.0046	-1.38E-5	.0016
SN-Para	RtMSE	.0585	.0498	.0461	.0445
	Bias	-.0258	-.0343	-.0353	-.0380
BLP	RtMSE	.0478	.0359	.0304	.0261
	Bias	-.0094	-.0166	-.0157	-.0175

Results: F is Normal

Table: Monte Carlo Results: Std. Dev. of Random Coefficient

Estimator	J	50	100	200	400
SN-I	RtMSE	.0778	.0569	.0507	.0441
	Bias	-.0090	-.0022	-.0033	-.0036
SN-II	RtMSE	.1020	.0667	.0519	.0426
	Bias	-.0287	-.0077	-.0041	-.0030
SN-Para	RtMSE	.0808	.0520	.0380	.0262
	Bias	-.0049	.0014	5.77E-4	.0025
BLP	RtMSE	.0693	.0459	.0345	.0246
	Bias	-.0030	9.96E-4	4.87E-4	.0010

Results: F is Mixed Normal

Table: Monte Carlo Results: Fixed Coefficients

			50	100	200	400
β	SN	RtMSE	.0580	.0387	.0266	.0178
		Bias	-.0040	-9.16E-4	-.0013	2.68E-4
	BLP	RtMSE	.0500	.0353	.0246	.0171
		Bias	-.0043	-.0016	-.0022	-3.10E-4
	BLP-Mis	RtMSE	.0499	.0353	.0246	.0171
		Bias	-.0047	-.0018	-.0024	-4.83E-4
α	SN	RtMSE	.0678	.0451	.0302	.0216
		Bias	.0019	.0010	.0012	6.39E-4
	BLP	RtMSE	.0563	.0403	.0289	.0211
		Bias	-.0036	-.0020	-6.23E-4	3.01E-4
	BLP-Mis	RtMSE	.0556	.0400	.0287	.0209
		Bias	-3.17E-4	5.40E-4	6.91E-4	8.25E-4

Results: F is Mixed Normal

Table: Monte Carlo Results: Mean of Random Coefficient

Estimator	J	50	100	200	400
SN-I	RtMSE	.0542	.0377	.0299	.0217
	Bias	.0046	-.0028	-.0025	-.0033
SN-II	RtMSE	.0561	.0393	.0312	.0223
	Bias	.0095	-6.02E-4	-.0017	-.0037
SN-Para	RtMSE	.0495	.0339	.0265	.0192
	Bias	.0057	-.0022	-.0027	-.0037
BLP	RtMSE	.0469	.0336	.0266	.0207
	Bias	-.0036	-.0105	-.0101	-.0116
BLP-Mis	RtMSE	.0456	.0303	.0228	.0164
	Bias	.0047	-.0026	-.0025	-.0043

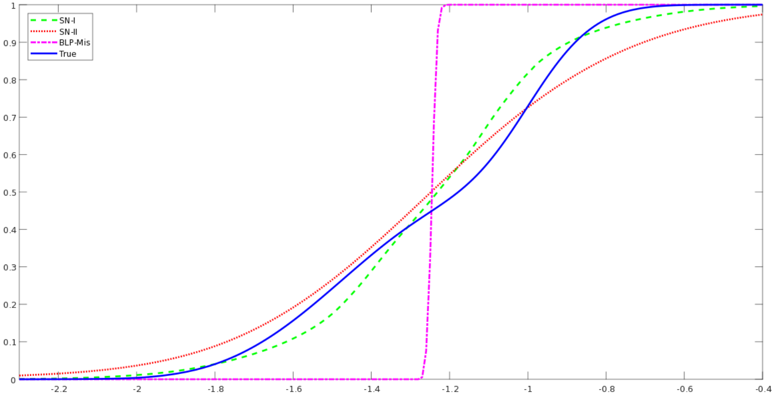
Results: F is Mixed Normal

Table: Monte Carlo Results: Std. Dev. of Random Coefficient

Estimator	J	50	100	200	400
SN-I	RtMSE	.1279	.0826	.0564	.0365
	Bias	-.0268	-.0136	-.0134	-.0088
SN-II	RtMSE	.1303	.0877	.0640	.0412
	Bias	-.0341	-.0215	-.0210	-.0130
SN-Para	RtMSE	.1311	.0844	.0569	.0358
	Bias	-.0261	-.0118	-.0110	-.0069
BLP	RtMSE	.1216	.0807	.0577	.0391
	Bias	-.0200	-.0092	-.0091	-.0075
BLP-Mis	RtMSE	.3949	.4087	.4189	.4257
	Bias	-.2598	-.2802	-.2970	-.3064

Results: F is Mixed Normal

Figure: Monte Carlo Results: CDF of Random Coefficient



Revisiting BLP Auto Data

	BLP		SN		
Fixed Coefficient	Logit	RC-Logit	First Step		
HP/Weight (log)	1.38 (.23)	.69 (.12)	1.56 (.20)		
Weight (log)	1.77 (.46)	.02 (.35)	2.19 (.56)		
Size (log)	1.05 (.58)	3.44 (.43)	2.25 (.55)		
Dollar per Miles (log)	.03 (.12)	−.31 (.11)	−1.37 (.33)		
A/C	1.25 (.14)	.57 (.08)	.42 (.12)		
Power Steering	.40 (.09)	.17 (.07)	.27 (.10)		
Automatic	.43 (.08)	.30 (.07)	.45 (.08)		
FWD	.16 (.06)	.22 (.06)	.44 (.08)		
Constant	−3.63 (.30)	−3.05 (.46)	−3.90 (1.03)		
Random Coefficient			Second Step		
on Price (Log)			Para.	I	II
Mean	−3.77 (.23)	−2.89 (.29)	-3.31	-3.24	-3.19
Std. Dev.	-	.46 (.14)	.61	.44	.36
Ave. No. of Prod. per Mkt.		110.85			
No. of Mkt.		20			

ARMSTRONG, T. B. (2016): “Large Market Asymptotics for Differentiated Product Demand Estimators with Economic Models of Supply,” *Working Paper*.

BERRY, S. (1994): “Estimating discrete-choice models of product differentiation,” *The RAND Journal of Economics*, pp. 242–262.

BERRY, S., A. GANDHI, AND P. HAILE (2013): “Connected substitutes and invertibility of demand,” *Econometrica*, 81(5), 2087–2111.

BERRY, S., O. LINTON, AND A. PAKES (2004): “Limit theorems for estimating the parameters of differentiated product demand systems,” *Review of Economic Studies*, 71(3), 613–654.

BERRY, S. T., AND P. A. HAILE (2014): “Identification in differentiated products markets using market level data,” *Econometrica*, 82(5), 1749–1797.

DUBÉ, J.-P., J. T. FOX, AND C.-L. SU (2012): “Improving the numerical performance of static and dynamic aggregate discrete choice random coefficients demand estimation,” *Econometrica*, 80(5), 2231–2267.

KNITTEL, C. R., AND K. METAXOGLOU (2012): “Estimation of Random Coefficient Demand Models: Two Empiricists’ Perspective,” .

Lee, J., and K. Seo (2015): "A computationally fast estimator for random coefficients logit demand models using aggregate data," *The RAND Journal of Economics*, 46(1), 86–102.

Newey, W. K. (1994): "The asymptotic variance of semiparametric estimators," *Econometrica: Journal of the Econometric Society*, pp. 1349–1382.