Conceal to Coordinate

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Abstract

How does a leader's incentive to coordinate her followers' behavior affect her ability to communicate? In our coordinated investment game, a manager communicates with her employees because her payoff from investment is higher when others also invest. We show that the manager must conceal information in any cheap talk equilibrium: when she chooses to invest, she only reveals that she will invest. We then explore whether the ability to commit to full disclosure is valuable. We find that both welfare and investment efficiency may be higher with partially informative cheap-talk than with full disclosure.

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Communication is key to effective leadership. A skillful leader both *informs* her team about the appropriate course of action, and *persuades* them to coordinate their behavior towards this common goal. We study how these two roles interact to jointly determine the nature of communication between a leader and her team. For instance, one might expect that when the leader's incentives are more closely aligned with those of the rest of her team, such communication is more credible and informative. We show that the very opposite result can prevail: in fact, the leader's motive to coordinate the team's actions limits her credibility to communicate informatively to her team.

We study strategic communication by an informed manager to the rest of her team in the context of a coordinated investment game. The manager and her team must decide whether to invest in a new risky project. The payoff from investing depends on the project fundamentals, which may be good or bad. The manager and her employees also have incentives to coordinate their actions: the payoff from investing to the manager (an employee) is higher when an employee (the manager, respectively) also invests.[1] Each player receives a private, noisy signal about the project fundamentals, and the manager can strategically choose to send a cheap-talk message to her team before they decide whether to invest.

We show that strategic communication features *concealment* — for signal realizations when the manager chooses to invest, her message to the team only reveals that she will invest. To see why, note that the manager's message only affects her payoffs through changing the likelihood that her employees invest. When the manager is sufficiently optimistic about fundamentals, she chooses to invest. In this case, however, she always has an incentive to distort her message to increase the chance that her employees invest, irrespective of her signal. In other words, even though their incentives are symmetric ex-ante, conditional on the manager's decision to invest, her incentive to convince her team to also invest is too strong to credibly convey any additional information.

Given the manager's limited ability to communicate credibly via cheap talk, we then ask whether she would commit to full disclosure if she could. We find that both the manager and

[1] In the analysis, we also allow for their incentives for coordination to be misaligned by letting the manager's payoff differ from the employees' by a known bias when both invest.

her team members may prefer partially-informative cheap talk to full disclosure.[2] Standard intuition suggests that the employees should prefer full disclosure, since they receive more information in this case than with cheap talk. However, unlike pure-sender receiver games, the likelihood that the manager (sender) invests also changes across the two scenarios. In fact, we show that the manager is more likely to invest with cheap talk, which increases the payoff to employees from investing. The relative impact of these offsetting effects depends on how biased the manager's payoffs are relative to her teammates. When the bias is small, the second effect dominates the first, and expected utility for the manager and the team is higher with strategic communication. However, when the bias is large, the informational cost of concealment with cheap talk is large, and so the employees' expected utility can be higher under the full disclosure scenario.

Finally, we consider how efficiency of the team's investment decision varies across these types of communication. Efficiency measures the extent to which the team's actions match the fundamental state and, as such, may be easier to measure empirically than welfare. Inefficiency in our analysis is driven by two distinct sources: under-investment when fundamentals are good and over-investment when fundamentals are bad. Facilitating communication not only leads employees to make a more informed decision, but it also helps the manager and her team better coordinate their actions. When fundamentals are good, these effects reinforce each other and improve efficiency by decreasing under-investment. When fundamentals are bad, these effects operate in opposite directions, and can decrease efficiency by increasing over-investment.

These effects interact with the endogenous nature of information revealed by strategic communication. The cheap-talk equilibrium reveals less information than a full disclosure model, but offers a coordination benefit (since the manager is more likely to invest in this case). When the manager is biased against investment, the informational disadvantage of the cheap-talk equilibrium is low, and as a result, investment efficiency is higher with cheap talk. However, when the manager is very biased in favor of investment, the informational disadvantage outweighs the coordination benefit, and overall investment efficiency is higher with full disclosure.

[2] In standard cheap-talk settings (e.g., Crawford and Sobel (1982)), commitment to perfect disclosure is generally Pareto superior to cheap-talk equilibria.

Our analysis has implications for how changes to the communication environment can affect the performance of an organization. For instance, while mandating more transparency within a firm may improve investment efficiency when the leader is excessively biased toward coordination, it can reduce both efficiency and welfare otherwise. Moreover, contrary to standard intuition, we find that aligning incentives need not improve efficiency. As we show, investment efficiency can be higher with a "reluctant" leader, who is biased against coordinated investment, than if her payoffs are unbiased relative to those of her employees.[3]

The next section briefly discusses the related literature. Section 2 introduces the model, and discusses some of the assumptions. Section 3 describes the equilibria under the no communication and the full-disclosure benchmarks. Section 4 characterizes the cheap-talk equilibria of our model, and discusses an extension to the case of spillovers. Section 5 studies welfare and investment efficiency in the three scenarios. Section 6 discusses some implications of our results and concludes. Proofs and additional results are in the Appendix.

1 Related Literature

Bolton, Brunnermeier, and Veldkamp (2013) also consider a setting where the leader of an organization communicates with her followers to encourage coordination, but the focus of their analysis is different. They study how the leader trades off encouraging coordination with flexibility and show that resoluteness in communication can help overcome the dynamic consistency problem that the leader faces. A key assumption in their analysis is that the leader can commit to a communication strategy before observing information about the underlying state. The central focus of our analysis is to study choice of communication in the absence of commitment, and whether the ability to commit to a disclosure policy is valuable. Moreover, in their model, the mission statement communicated by the leader is directly about fundamentals, while communication by the leader in our model conveys information about both fundamentals and her action. These features also distinguish our analysis from papers in the global games literature that study the effect of public information in the presence of

[3]Intuitively, the negative bias dampens the manager's incentive to over-invest in bad state and so tilts the tradeoff in favor of greater informational efficiency. This result differs from the implications of standard cheap-talk models, where eliminating the sender's bias tends to increase the efficiency of outcomes.

strategic complementarities (e.g., Morris and Shin (2002), Corsetti, Dasgupta, Morris, and Shin (2004), Angeletos and Werning (2006), Angeletos and Pavan (2007), Ozdenoren and Yuan (2008)). The tradeoff between greater informational efficiency and better coordination highlighted by Morris and Shin (2002) also arises in our model, but because the leader's cheap-talk reflects both fundamental and strategic information, the impact on welfare and investment efficiency can be different from those in the earlier papers.

Our paper is related to the large literature on cheap talk initiated by Crawford and Sobel (1982) (see Sobel (2013) for a recent survey), and more specifically, models which introduce a cheap talk stage before a game with strategic complementarities.[4] The most closely related papers include Baliga and Morris (2002), Alonso, Dessein, and Matouschek (2008), Rantakari (2008), and Hagenbach and Koessler (2010). Baliga and Morris (2002) study how adding a cheap talk stage before play affects a two player, one-sided incomplete information game with strategic complementarities and positive spillovers, and characterize sufficient conditions for full communication and no communication. Alonso et al. (2008) and Rantakari (2008) compare how centralization affects coordination and communication between divisions who wish to adapt their action to local (independent) conditions, but also coordinate with other divisions. Our setting differs from these in that it features two-sided incomplete information about a common fundamental that affects both players' payoffs.[5]

Hagenbach and Koessler (2010) consider a setting in which the players in a beauty contest game strategically choose to communicate their information about a common fundamental with each other. Although complementary, the focus of their analysis is different from ours. They study the question of which other players a player chooses to communicate with, and characterize the equilibrium strategic communication network that arises in a setting where players either disclose their information fully or not at all. In contrast, our analysis highlights how partially informative communication can arise when players choose *what* information to communicate, and how this affects welfare.

[4]Our paper is also related to the literature on single sender-multiple receiver cheap talk models (e.g., Farrell and Gibbons (1989), Newman and Sansing (1993) and Goltsman and Pavlov (2011)), although unlike these earlier papers, our focus is not on how differences across receivers' beliefs / payoffs affect communication. In contrast to this earlier literature, receivers in our model have symmetric payoffs but have private information about fundamentals, and the sender can take an action in addition to sending messages.

[5]Since ours is a model with two-sided incomplete information and correlated types, it combines the distinguishing features of Examples 2 and 3 in Baliga and Morris (2002).

Our welfare implications also distinguish us from standard cheap-talk models and the recent literature on Bayesian persuasion models. In cheap talk models, commitment to full disclosure Pareto dominates partially informative cheap-talk equilibria — the receiver is usually better off with more informative communication. Similarly, in standard models of Bayesian persuasion (e.g., Rayo and Segal (2010) and Kamenica and Gentzkow (2011)), while the sender may prefer to commit to partially informative communication, the receiver usually prefers more informative signals. In contrast, we find that *both* the sender and the receiver may prefer the less informative cheap talk equilibrium to fully informative communication. The key distinction from standard sender-receiver games is that in our model, both the sender and the receiver take actions that are strategic complements. As a result, welfare depends not only on the informativeness of the sender's messages but also on her actions.

2 Model

The payoff to investment depends on fundamentals $\theta \in \{\theta_H, \theta_L\}$, which are high with prior probability $p_0 \equiv \Pr(\theta = \theta_H)$. There are two types of players: one manager (M, "she") and N employees (indexed by $e \in E$, "he"). Each player receives a private signal of the form $x_i = \theta + \varepsilon_i$ (for $i \in \{M, E\}$), where ε_i are independent and normally distributed with mean-zero and variance σ_i^2 (i.e., $\varepsilon_i \sim N(0, \sigma_i^2)$), and where all employees are symmetrically informed, i.e., $\sigma_e = \sigma_E$ for all $e \in E$.[6] Each player must decide whether to invest ($a_i = 1$) or not ($a_i = 0$). The payoff to employee $e \in E$ from investing (i.e., $a_e = 1$) depends on the fundamental and the manager's action, and is given by

$$\theta - 1 + a_M \tag{1}$$

and his payoff from not investing (i.e., $a_e = 0$) is zero. The payoff to the manager from investing (i.e., $a_M = 1$) is given by

$$\theta - 1 + \frac{1+b}{N} \sum_{e \in E} a_e, \tag{2}$$

[6] We sometimes use index E to denote employees collectively, warranted by the symmetry of employees.

and from not investing (i.e., $a_M = 0$) is zero. The parameter b captures a potential conflict of interest between the manager and her employees. When $b > 0$ ($b < 0$), the manager is biased in favor of (against, respectively) coordinated actions relative to her employees. While we will explore the effects of a positive or negative bias on outcomes in the later sections, our focus will be on the natural benchmark case of $b = 0$ in which the payoffs to the manager are unbiased relative to those of her employees. We maintain the following assumptions on the parameters:

(A1) $\theta_L < 0 < 1 < \theta_H$: This ensures that in the benchmark case of $b = 0$, it is efficient to each player to invest in the "good" state when fundamentals are high (i.e., $\theta = \theta_H$) and to not invest in the "bad" state when fundamentals are low (i.e., $\theta = \theta_L$).

(A2) $-\theta_L > b > -1$: This ensures that the manager's bias is not too large; despite her bias, all players coordinate on investment when fundamentals are good (i.e., $\theta = \theta_H$), and no investment when fundamentals are bad (i.e., $\theta = \theta_L$).[7]

We allow the manager to send a message $m(x_M)$ about her signal to her employees before they decide whether or not to invest. Specifically, we assume that a messaging rule $\mu : \Re \to B$ is a function that takes a signal realization x_M to an element (or *message*) $m = \mu(x_M) \in B$, where B is the Borel algebra on the reals $\Re$. We consider three scenarios: (i) no communication (NC), (ii) full disclosure communication (FC), and (iii) strategic communication (SC). The no communication scenario serves as a benchmark when the players are not allowed to communicate (i.e., $\mu(x_M) = \Re$). The full disclosure communication scenario assumes that the manager commits to perfectly disclosing her signal before they decide whether to invest (i.e., $\mu(x_M) = x_M$). Finally, in the strategic communication scenario, the manager can send an arbitrary message $\mu(x_M)$ about her signal x_M to her employees after they each observe their signals, but before they decide whether to invest.

We restrict attention to a finite number of fundamental states due to tractability. In particular, updating beliefs conditional on private information and messages takes a log-linear form in our setting, as the next result highlights.[8]

[7]When $b \leq -1$, M has a higher payoff from investing when R does not invest, and the resulting investment game is one of strategic substitutability.

[8]With a continuum of states and standard distributional assumptions (e.g., normal or uniform priors),

Lemma 1. *Conditional on a signal $x_i = x$, and a message $m \in B$, posterior beliefs about θ are given by $p(x,m) \equiv \Pr(\theta = \theta_H | x_i = x, x_j \in m)$, where*

$$\log\left(\frac{p(x,m)}{1-p(x,m)}\right) = \log\left(\frac{p_0}{1-p_0}\right) + \frac{1}{\sigma_i^2}(\theta_H - \theta_L)\left(x - \frac{\theta_H+\theta_L}{2}\right) + \log\left(\frac{\Pr(x_j \in m|\theta_H)}{\Pr(x_j \in m|\theta_L)}\right). \tag{3}$$

Also, note that $\log\left(\frac{\Pr(x_j=x|\theta_H)}{\Pr(x_j=x|\theta_L)}\right) = \frac{1}{\sigma_j^2}(\theta_H - \theta_L)\left(x - \frac{\theta_H+\theta_L}{2}\right)$, *and*

$$\log\left(\frac{\Pr(c_1 < x_j \le c_2|\theta_H)}{\Pr(c_1 < x_j \le c_2|\theta_L)}\right) = \log\left(\frac{\Phi\left(\frac{c_2-\theta_H}{\sigma_j}\right) - \Phi\left(\frac{c_1-\theta_H}{\sigma_j}\right)}{\Phi\left(\frac{c_2-\theta_L}{\sigma_j}\right) - \Phi\left(\frac{c_1-\theta_L}{\sigma_j}\right)}\right).$$

As expected, the posterior belief $p(x,m)$ increases in the realization of the private signal x for a fixed message m. Moreover, the above result characterizes the posterior for two types of messages. When the message itself is a point (i.e., $m = x$), then the log-likelihood ratio is linear in the two signals (i.e., x_i and x_j). When the message is an interval (i.e., $m = (c_1, c_2]$), then the log-likelihood ratio depends on the relative probability of x_j being in the interval (i.e., $x_j \in (c_1, c_2]$) conditional on fundamentals being high vs. low.

Conditional on observing a signal x and receiving a message m, an employee's expected payoff from investment is given by

$$\pi_e(x,m) = \begin{array}{c} p(x,m)\left[\theta_H - 1 + \Pr(a_M = 1|\theta_H, m)\right] \\ +(1-p(x,m))\left[\theta_L - 1 + \Pr(a_M = 1|\theta_L, m)\right] \end{array}. \tag{4}$$

Since $\pi_e(x,m)$ is increasing in x for a fixed m, each employee optimally chooses to follow a cutoff strategy: employee r only invests when his signal is greater than or equal to a cutoff $k_e(m)$ (i.e., when $x_e \ge k_e(m)$), but not otherwise. Given the cutoff strategies of her employees, and since all employees are symmetrically informed, the manager's expected payoff from investment conditional on a signal x is given by

$$\pi_M(x,m) = \mathbb{E}\left[\theta - 1 + \tfrac{1+b}{N}\sum_{e \in E}\Pr(a_e = 1|\theta, m)\,\middle|\,x_M = x\right]$$

updating beliefs about fundamentals using both a private signal and general messages in equilibrium is less analytically tractable.

$$= \begin{array}{c} \Pr\left(\theta=\theta_H|x_M=x\right)\left[\theta_H-1+(1+b)\Pr\left(x_e\geq k_e\left(m\right)|\theta_H\right)\right] \\ +\Pr\left(\theta=\theta_L|x_M=x\right)\left[\theta_L-1+(1+b)\Pr\left(x_e\geq k_e\left(m\right)|\theta_L\right)\right] \end{array} \tag{5}$$

As with the employees, since the payoff to investing is increasing in x for a fixed m, the manager optimally chooses to follow a cutoff strategy: she will invest if and only if her signal is higher than a cutoff $k_M(m)$, i.e., $x_M \geq k_M(m)$. Finally note that since the payoff to not investing is zero for the manager and her employees, the cutoff $k_i(m)$ for each player is characterized by the indifference condition:

$$\pi_i\left(k_i\left(m\right), m\right) = 0. \tag{6}$$

We focus on pure strategy, Perfect Bayesian equilibria.[9] In particular, an equilibrium of the game with SC is characterized by a messaging rule $\mu : \Re \to B$ and cutoff strategies $\{k_M(m), k_e(m)\}$, such that: (i) the messaging rule μ is truthful (i.e., for all x_M, $x_M \in \mu(x_M)$), (ii) the messaging rule μ is optimal for player M, (iii) given a message m, it is optimal for player i to only invest when $x_i \geq k_i(m)$ (i.e., expression (6) holds), and (iv) players' beliefs satisfy Bayes' rule wherever it is well-defined. In particular, the restriction to pure-strategy, truth-telling equilibria implies that given a messaging rule μ, each possible signal realization x_M maps into only one message $\mu(x_M)$. For the games with NC and FC, an equilibrium is characterized by conditions (iii) and (iv) above, since the messaging rule is exogenously specified ($\mu(x_M) = \Re$ and $\mu(x_M) = x_M$, respectively).

2.1 Discussion of Assumptions

The assumption that players can take one of a finite number of actions does not drive our results. For instance, suppose players are risk-neutral, can choose an investment level $a_i \in [0,1]$, and the payoff to player i is given by $a_i(\theta - 1 + a_j)$. In this case, a player's optimal investment decision is characterized by the same cutoff strategy as in our benchmark specification, since each player chooses the maximum investment level ($a_i = 1$) if she chooses

[9] Since the sender's type (her signal) is continuous and unbounded, restriction to pure strategies is without loss of generality.

to invest at all.[10]

We explore alternative payoff specifications in supplementary analysis. Specifically, for the proofs of our main results, we consider the case where both M and $e \in E$ can have biased payoffs, i.e., if everyone invests, the payoffs are $(\theta + b_M, \theta + b_e)$. This does not qualitatively change the characterization of equilibria in the three scenarios. Similarly, as we show in Appendix B, the equilibria do not qualitatively change when the cost to investing alone for the manager can be different; if she invests but her employees do not, her payoffs are $(\theta - c, 0)$. Finally, Section 4.1 considers a specification with spillovers: the manager receives an incremental payoff $\frac{\nu}{N}$ when each of her employees invests, irrespective of whether she invests.

The assumptions of (i) no payoff externalities within employees (i.e., an employee's payoff does not depend on the actions of other employees), and (ii) one-sided (manager to employee) communication are made for tractability. However, such restrictions can arise naturally in many settings. Since managers usually have more influence over performance evaluations and compensation than other team members, employees naturally have a strong incentive to "follow the leader." Moreover, large teams and firms are usually organized in hierarchical structures to facilitate top-down communication, while bottom-up percolation of information is more difficult to sustain. In our setting, one can show that fully-informative, two-sided (i.e., manager to employee, employee to manager), cheap-talk communication can be sustained when the manager's payoff is unbiased (i.e., when $b = 0$), and the equilibrium investment decision is analogous to the welfare-maximizing decision we describe in Proposition 8. Unfortunately, the analysis of two-sided cheap talk communication when the manager's payoff is biased (i.e., $b \neq 0$) is not tractable in the current setting. Similarly, a complete analysis of a setting with inter-employee payoff externalities is beyond the scope of the current paper, and left for future work.

[10] We assume that if the player is indifferent between investing and not, she chooses $a_i = 1$.

3 Benchmarks

This section characterizes the equilibria in natural benchmark scenarios. These are useful in developing intuition, and also for the welfare and efficiency comparisons we make in Section 5.

3.1 No Communication

The no communication benchmark recovers a standard result from the global games literature.

Proposition 1. *Let the function $K(k; b, \sigma_i, \sigma_j)$ be defined as:*

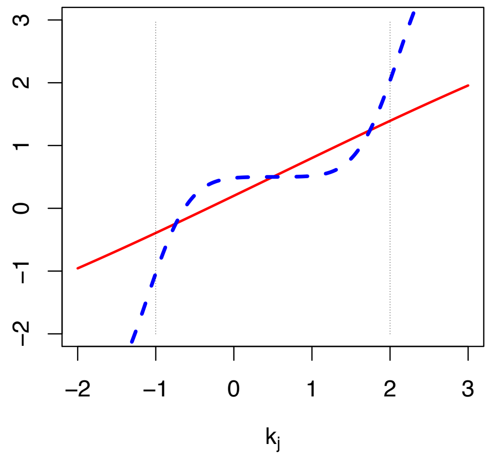
$$K(k; b, \sigma_i, \sigma_j) \equiv \frac{\theta_H + \theta_L}{2} + \frac{\sigma_i^2}{\theta_H - \theta_L} \left\{ \log \left(\frac{(1+b)\Phi\left(\frac{k-\theta_L}{\sigma_j}\right) - (\theta_L + b)}{(\theta_H + b) - (1+b)\Phi\left(\frac{k-\theta_H}{\sigma_j}\right)} \right) - \log \left(\frac{p_0}{1-p_0} \right) \right\}, \tag{7}$$

where $\Phi(\cdot)$ is the CDF of the normal distribution. Suppose the manager cannot communicate with her employees: for all x_M, we have $\mu(x_M) = \Re$. Then there exist equilibria characterized by cutoffs $k_{M,NC}$ and $k_{e,NC}$, which solve the system: $k_{M,NC} = K(k_{e,NC}; b, \sigma_M, \sigma_e)$ and $k_{e,NC} = K(k_{M,NC}; 0, \sigma_e, \sigma_M)$. For given b, θ_H and θ_L, there exist cutoffs $\bar{a}(b, \theta_H, \theta_L)$ and $\underline{a}(b, \theta_H, \theta_L)$ so that if $\frac{\sigma_M^2}{\sigma_e} < \bar{a}$ and $\frac{\sigma_e^2}{\sigma_M} < \underline{a}$, the equilibrium is unique. When $\sigma_M = \sigma_e = \sigma$, there exists a cutoff $a(b, \theta_H, \theta_L)$ such that if $\sigma < a$, the equilibrium is unique.

The best response function (7) is increasing in the other player's cutoff. This is intuitive — player j is less likely to invest when her cutoff is higher, which leads player i to respond by increasing her own cutoff. However, increasing best response functions imply that there may be multiple equilibria. As we discuss in the proof for Proposition 1, a sufficient condition for uniqueness is that the slope of the best response function (7) is less than one (i.e., $\frac{\partial K_i}{\partial k_j} < 1$). In the special case when the signals are symmetrically distributed (i.e., $\sigma = \sigma_e = \sigma_M$), the sufficient condition for uniqueness mirrors those in the earlier literature which require that private signals are sufficiently accurate (see Morris and Shin (2001), Frankel, Morris, and Pauzner (2003), and Morris and Shin (2003) for extensive discussions).

Figure 1: Best response function $K_i(k_j)$ in the No Communication Scenario

The figure plots the best response function $K_i(k_j, 0, \sigma_i, \sigma_j)$ in equation (7) when $\sigma_i = \sigma_j = 4$ (solid) and when $\sigma_j = \frac{\sigma_i}{10} = 0.4$ (dashed). The slope for the best response function is much steeper for the latter case when k_j is close to θ_H or θ_L (marked by dotted vertical lines). The other parameters are set to $p_0 = 0.5$, $b = 0$, $\theta_H = 2$, and $\theta_L = -1$.



In the general case, the sufficient conditions require not only that each player's private signal is sufficiently precise, but also that neither player's signal is too precise relative to the other's signal. If player j's signal is too precise relative to player i's, then player i's best response changes very quickly when k_j is close to either θ_H or θ_L — Figure 1 presents an example of this. This can lead to multiple solutions for the system of equations in Proposition 1, and consequently, multiple equilibria. In contrast, as we show in the next subsection, there always exists a unique equilibrium when the manager can commit to revealing her information perfectly.

3.2 Full Disclosure Communication

Suppose the manager can commit to fully disclosing her private information, i.e., $\mu(x_M) = x_M$ for all x_M. Then, conditional on x_M and his own signal x_e, an employee's posterior beliefs about $\theta = \theta_H$ are given by

$$\log\left(\frac{p}{1-p}\right) = \log\left(\frac{p_0}{1-p_0}\right) + \frac{1}{\sigma_M^2}(\theta_H - \theta_L)\left(x_M - \frac{\theta_H+\theta_L}{2}\right) + \frac{1}{\sigma_e^2}(\theta_H - \theta_L)\left(x_e - \frac{\theta_H+\theta_L}{2}\right). \quad (8)$$

Since the employee can perfectly observe the manager's signal, there is no uncertainty about whether she will invest. This implies that if M reveals a signal x_M and uses a cutoff k_M, the employee's best response is to invest only if $x_e \geq K_{FC}(x_M, k_M)$, where

$$K_{FC}(x,k) \equiv \begin{cases} \frac{\theta_H+\theta_L}{2} + \frac{\sigma_e^2}{\sigma_M^2}\left(\frac{\theta_H+\theta_L}{2} - x\right) + \frac{\sigma_e^2}{\theta_H-\theta_L}\left(\log\left(-\frac{\theta_L}{\theta_H}\right) - \log\left(\frac{p_0}{1-p_0}\right)\right) & \text{if } x \geq k, \\ \frac{\theta_H+\theta_L}{2} + \frac{\sigma_e^2}{\sigma_M^2}\left(\frac{\theta_H+\theta_L}{2} - x\right) + \frac{\sigma_e^2}{\theta_H-\theta_L}\left(\log\left(\frac{1-\theta_L}{\theta_H-1}\right) - \log\left(\frac{p_0}{1-p_0}\right)\right) & \text{if } x < k. \end{cases} \tag{9}$$

Intuitively, the employee's best response is decreasing in the manager's signal — a higher signal implies that the higher state ($\theta = \theta_H$) is more likely, and this leads r to lower his cutoff. The next result characterizes the equilibrium in this scenario in terms of the above best response function.

Proposition 2. *Let $k_{M,FC}$ be the (unique) fixed point of $x = K(K_{FC}(x,x), b, \sigma_M, \sigma_e)$, where $K(\cdot)$ is defined by equation (7), and $K_{FC}(\cdot)$ is defined by (9). If the manager can commit to revealing her information perfectly (i.e., $\mu(x_M) = x_M$ for all x_M), then the unique equilibrium is characterized by the cutoff $k_{M,FC}$ for the manager and the cutoff (function) $K_{FC}(x_M, k_{M,FC})$ for player $e \in E$.*

The result highlights how the manager's ability to communicate changes the nature of the coordination game: unlike the NC scenario, there always exists a unique equilibrium with full disclosure. The equilibrium is characterized by the manager's cutoff (i.e., k_M) given her employees' cutoff *conditional* on the information that her signal is equal to her cutoff (i.e., $x_M = k_M$). In contrast to the NC scenario, each employee faces no uncertainty about whether the manager invests. This implies that conditional on the manager's signal being equal to her cutoff (i.e., $x_M = k_M$), a higher cutoff is good news about fundamentals and so the employees' best response decreases in k_M.[11] As we show in the proof, this ensures that there always exists a unique solution to the fixed point problem in Proposition 2 that characterizes the equilibrium.

[11] This is analogous to the effect of observing the action of an earlier player in a sequential move global game (e.g., Corsetti et al. (2004)).

4 Strategic Communication

We now turn to the case where the manager can strategically choose to send an arbitrary message to her employees after observing her signal. As is common in cheap talk models, there exist multiple equilibria. However, as the following proposition describes, they are all characterized by a common feature: the manager conceals information about the realization of her signal when she invests.[12]

Proposition 3. *Let $k_{M,SC}$ be the fixed point of $x = K\left(K_e\left(x\right), b, \sigma_M, \sigma_e\right)$, where K is defined by (7), and $K_e\left(x\right)$ is given by:*

$$K_e\left(x\right) = \tfrac{\theta_H+\theta_L}{2} + \tfrac{\sigma_e^2}{\theta_H-\theta_L}\left\{\log\left(\tfrac{-\theta_L}{\theta_H}\right) - \log\left(\tfrac{p_0}{1-p_0}\right) - \log\left(\frac{1-\Phi\left(\frac{x-\theta_H}{\sigma_M}\right)}{1-\Phi\left(\frac{x-\theta_L}{\sigma_M}\right)}\right)\right\}. \tag{10}$$

In any sender-optimal strategic equilibrium, (i) the manager invests if and only if $x_M \geq k_{M,SC}$ and (ii) the messaging rule is equivalent to $\mu\left(\cdot\right)$, where for any signal $x_M \geq k_{M,SC}$, the optimal message is $\mu\left(x_M\right) = [k_{M,SC}, \infty)$.

Instead of detailing the proof of the above result, we provide some intuition for this result. First, note that the manager's message affects her payoff only through the likelihood that her employees invest. For signal realizations where the manager chooses to invest, she always has an incentive to report that her signal is higher than it actually is, since this increases the likelihood that her employees invest, but does not affect her payoff otherwise. But this implies that she cannot convey *any* additional information credibly when she chooses to invest.[13] The restriction to sender-optimal equilibria rules out equilibria in which the sender either babbles, or pools some low (no-invest) signals with high (invest) signals.[14]

[12]Note that the nature of cheap talk equilibrium is not an immediate consequence of the fact that employees have a binary action space. For instance, Chakraborty and Yilmaz (2016) consider a setting in which detailed cheap talk communication arises even though the receiver has a binary action.

[13]As we discuss in the proof, there is some indeterminacy. We show that while M can send other messages when $x_M \geq k_{M,SC}$, they must be equivalent to the message $x_M \in [k_{M,SC}, \infty)$ in terms of their impact on r's posterior beliefs. As a result, for all economically relevant implications, the messaging rules are equivalent to the one stated in the Proposition when $x_M \geq k_{M,SC}$.

[14]In either case, a sender with a high signal realization should strictly prefer to separate herself from these no-investment, low types.

Second, as we argue in the proof, it is natural that the messaging rule and the investment decisions are determined by the same cutoff. Intuitively, the message m equivalent to "M will invest" should include all signal realizations such that M chooses to invest having sent message m, but should exclude any signal realizations such that M optimally chooses not to invest having sent that message. Also, note that if M chooses to invest at a signal realization x_M, having sent message m, then she must necessarily choose to invest for all signal realizations $x > x_M$, conditional on sending message m. This, in turn, ensures that the investment and messaging rule intervals are half-lines.

Finally, the unique cutoff $k_{M,SC}$ is the solution to a fixed point problem: the manager's cutoff (i.e., k_M) is her best response to the employees' cutoff *conditional* on the information that her signal is greater than or equal to her cutoff (i.e., $x_M \geq k_M$). As in the FC scenario, the existence and uniqueness of this cutoff is guaranteed by the fact that the employees' best response (10) is decreasing in the manager's cutoff, conditional on her message. However, unlike the FC scenario, uniqueness of the cutoff $k_{M,SC}$ does not imply uniqueness of equilibria. This is because, for signal realizations where the manager does not invest (i.e., $x_M < k_{M,SC}$), she is indifferent to various messaging rules. This naturally gives rise to two extreme equilibria, which can be characterized by how informative the manager's message is about her signal in this region. We describe these in the following result.

Proposition 4. *(i) The least informative strategic equilibrium is characterized by the messaging rule:*

$$\mu\left(x_M\right) = \begin{cases} \left(-\infty, k_{M,SC}\right) & \text{if } x_M < k_{M,SC} \\ \left[k_{M,SC}, \infty\right) & \text{if } x_M \geq k_{M,SC} \end{cases}, \tag{11}$$

and the cutoff $k_{M,SC}$ for the manager and the cutoff function $K_{SC}\left(m\right)$ for employee $e \in E$, where

$$K_{SC}\left(\left(-\infty, k_{M,SC}\right)\right) = \frac{\theta_H + \theta_L}{2} + \frac{\sigma_e^2}{\theta_H - \theta_L}\left\{\log\left(\frac{1-\theta_L}{\theta_H - 1}\right) - \log\left(\frac{p_0}{1-p_0}\right) - \log\left(\frac{\Phi\left(\frac{k_{M,SC}-\theta_H}{\sigma_M}\right)}{\Phi\left(\frac{k_{M,SC}-\theta_L}{\sigma_M}\right)}\right)\right\}, \tag{12}$$

$$K_{SC}\left(\left[k_{M,SC}, \infty\right)\right) = K_e\left(k_{M,SC}\right). \tag{13}$$

(ii) The most informative strategic equilibrium is characterized by the messaging rule:

$$\mu(x_M) = \begin{cases} x_M & \text{if } x_M < k_{M,SC} \\ [k_{M,SC}, \infty) & \text{if } x_M \geq k_{M,SC} \end{cases}, \tag{14}$$

and the cutoff $k_{M,SC}$ for the manager and the cutoff function $K_{SC}(m)$ for employee $e \in E$, where

$$K_{SC}(x_M) = \tfrac{\theta_H+\theta_L}{2} + \tfrac{\sigma_e^2}{\sigma_M^2}\left(\tfrac{\theta_H+\theta_L}{2} - x_M\right) + \tfrac{\sigma_e^2}{\theta_H-\theta_L}\left(\log\left(\tfrac{1-\theta_L}{\theta_H-1}\right) - \log\left(\tfrac{p_0}{1-p_0}\right) \right), \tag{15}$$

$$K_{SC}([k_{M,SC}, \infty)) = K_e(k_{M,SC}). \tag{16}$$

In the least informative equilibrium, the manager sends one of two possible messages, which correspond to whether or not she invests. In the most informative equilibrium, she reveals her signal perfectly when she chooses not to invest, but conceals the realization of her signal in the investment region. The manager is indifferent between these equilibria because her payoff from not investing is unaffected by the employees' action. As we discuss in the next subsection, this indifference plays an important role in ensuring one-sided cheap talk is partially informative in our setting.

It is worth noting that perfectly informative cheap-talk is not sustainable in our setting even when the manager's payoffs are unbiased, i.e., $b = 0$, because the sender also takes an action and actions exhibit strategic complementarity. This is in sharp contrast to a standard, sender-receiver setting where the sender does not take an action and her payoff depends only on the receiver's action. In particular, suppose the manager can send a cheap talk message to her employees, and her payoff is given by $\frac{\theta+b}{N}\sum_{e\in E} a_e$. The payoff to each employee from investing is θ and from not investing is zero. As the following result establishes, perfectly informative communication is sustainable in this case when payoffs are aligned.

Proposition 5. *Suppose the sender cannot take an action and the manager's payoffs are unbiased (i.e., $b = 0$). Then, there exists a strategic communication equilibrium in which the sender perfectly reveals her signal to the receivers.*

4.1 Spillovers

We consider an alternative specification of payoffs in this section to establish the robustness of our results, and to highlight the role of the manager's indifference when not investing in generating informative communication. The case of positive spillovers may also be a more natural assumption when modeling a manager who derives private benefits of control (e.g., an "empire builder" who gets utility when her team invests more in the project, irrespective of the project fundamentals). Suppose the payoffs to the employees are as before, but the manager's payoffs are given by

$$\theta - 1 + \frac{1+\nu}{N} \sum_{e \in E} a_e \tag{17}$$

when she invests and

$$\frac{\nu}{N} \sum_{e \in E} a_e \tag{18}$$

when she does not invest. In this case, the employees' decision to invest has a spillover on the manager's payoffs: irrespective of whether she invests, the manager receives an incremental payoff of $\frac{\nu}{N}$ when an employee invests. To ensure we are in the interesting region of the parameter range, the assumptions (A1)-(A2) generalize to the following: (i) $\theta_L < 0 < 1 < \theta_H$, (ii) $\nu > -1$ and $\nu < -\theta_L$. While an employee's incremental payoff from investing, π_e, remains the same as in the benchmark model, the manager's optimal decision is given by

$$\max_{a_M \in \{0,1\}} a_M \pi_M^1 (x, m) + (1 - a_M) \pi_M^0 (x, m), \tag{19}$$

where $\pi_M^a (x, m)$ is the payoff for action $a \in \{0, 1\}$:

$$\pi_M^1 (x, m) = \begin{array}{c} p(x) [\theta_H - 1 + (1+\nu) \Pr (x_e \geq k_e (m) | \theta_H)] \\ + (1 - p(x)) [\theta_L - 1 + (1+\nu) \Pr (x_e \geq k_e (m) | \theta_L)] \end{array}, \text{ and} \tag{20}$$

$$\pi_M^0 (x, m) = \nu \{ p(x) \Pr (x_e \geq k_e (m) | \theta_H) + (1 - p(x)) \Pr (x_e \geq k_e (m) | \theta_L) \}, \tag{21}$$

and $p(x) = \Pr (\theta = \theta_H | x_M = x)$. Her incremental payoff from investing is independent of ν, since

$$\pi_M (x, m) = \pi_M^1 (x, m) - \pi^0 (x_M, m)$$

$$= p(x)[\theta_H - 1 + \Pr(x_e \geq k_e(m)|\theta_H)] + (1 - p(x))[\theta_L - 1 + \Pr(x_e \geq k_e(m)|\theta_L)].$$

As a result, the NC and FC equilibria with a spillover are identical to the corresponding equilibria in the benchmark model with $b = 0$. However, as the following result establishes, with cheap talk, even partially informative communication is difficult to sustain.

Proposition 6. *If the employees' investment decision generates a positive spillover for the manager (i.e., $\nu > 0$), there can (effectively) be no communication in any strategic equilibrium. If the employees' investment decision generates a negative spillover for the manager (i.e., $\nu < 0$), any strategic equilibrium is equivalent to the least informative equilibrium described in Proposition 4 (with $b = 0$).*

The presence of spillovers limits the manager's ability to communicate effectively. In fact, if the spillover is positive, even if arbitrarily small, no information can be communicated in a one-sided cheap talk equilibrium. When the spillover is negative, only a partially informative cheap-talk equilibrium analogous to the least-informative SC equilibrium above survives. This is because, even if the manager decides not to invest, she has an incentive to increase (decrease) the likelihood that her employees invest when the spillover ν is positive (negative, respectively). As a result, when the spillover is positive, the manager always has an incentive to distort her message upwards, and so cannot communicate any information via cheap talk. When the spillover is negative, she has an incentive to distort her message upwards (downwards) when she chooses to invest (not invest, respectively), and so cannot convey any additional information.

The above result is also related to Baliga and Morris (2002), who establish that in special cases of their one-sided, incomplete information model, no communication is possible when there are positive spillovers. Morris and Shin (2003) informally discuss a two-player, investment game (similar to ours) where both players impose spillovers. They suggest that an argument similar to Baliga and Morris (2002) implies that fully informative cheap talk is possible when spillovers are negative, but not when spillovers are positive. Our analysis suggests that the assumption of symmetric payoffs is important for these conclusions: with one-sided spillovers, we show one-sided cheap talk cannot be informative at all with positive spillovers and is, at best, partially informative when spillovers are negative.

5 Welfare and Investment Efficiency

We explore whether the ability to commit to full disclosure is valuable in our setting, using two measures: welfare and investment efficiency. Welfare measures the expected payoff to team members, including the benefits of coordination and incremental payoff externalities to the manager (i.e., the effect of b). Investment efficiency measures how well the team's investment decision matches the project fundamentals. One might expect that both welfare and investment efficiency are higher when the manager commits to full disclosure. However, we find that neither result need hold. We show that when the bias (b) in the manager's payoff is near zero, both she and her employees may prefer the partially informative, cheap-talk equilibrium to the full disclosure equilibrium.[15] Similarly, we find that investment efficiency can be higher under the cheap talk equilibrium, especially when the manager is biased against coordinated investment (i.e., b is negative).

5.1 Communication and Strategic Uncertainty

A key difference of our model relative to standard, sender-receiver games is that receivers (i.e., employees) face not only fundamental uncertainty, but also strategic uncertainty. Specifically, without communication, employees are uncertain about whether the manager will invest, and this discourages them from investing (due to complementarities in the investment decision). In contrast, with communication, the employees can infer perfectly whether the manager invests, and this decrease in strategic uncertainty leads them to invest more. Anticipating this response, the manager also invests more aggressively in the communication equilibria (i.e., her investment threshold is higher under NC than under SC).

Next, note that employees do not face strategic uncertainty in either the SC or FC scenarios, since they can perfectly infer whether the manager invests in either case. However, in the SC scenario, each employee conditions on the information that the manager's signal is higher than her cutoff, while in the FC scenario, he conditions on the realization of the signal itself. For any cutoff k chosen by the manager, the information that her signal is

[15] This is in contrast to standard cheap-talk models, where both parties usually prefer to commit to full disclosure, and to Bayesian persuasion models, where the receiver prefers full disclosure.

greater than or equal to her cutoff (i.e., $x_M \geq k$) makes the employee more optimistic about fundamentals than the information that her signal is equal to her cutoff (i.e., $x_M = k$) — this is because the distribution of the signal, parameterized by θ, satisfies the monotone likelihood ratio property. As we show in the proof of the next result, this implies that, for any cutoff k chosen by the manager, an employee's best response to $x_M = k$ in the FC scenario is always higher than his response to $x_M \geq k$ in the SC scenario. But this, in turn, implies that the manager faces less strategic uncertainty about the employees' investment decision under the SC scenario, which leads her to invest more aggressively (i.e., $k_{M,FC} \geq k_{M,SC}$). These observations are summarized in the following result.

Proposition 7. *The manager is more likely to invest under strategic communication than under the no-communication or forced communication scenarios:*

$$k_{M,NC} \geq k_{M,SC} \text{ and } k_{M,FC} \geq k_{M,SC}. \tag{22}$$

The comparison between the full disclosure (FC) and cheap-talk (SC) scenarios highlights a novel tradeoff between fundamental uncertainty and strategic uncertainty. On the one hand, because communication is more informative under full disclosure, fundamental uncertainty is lower in this case. On the other hand, because of the complementarity in investment decisions, strategic uncertainty (for the manager) is lower in the SC scenario. The next two subsections characterize how this tradeoff affects welfare and investment efficiency in our model.

5.2 Welfare

In order to compute welfare, we define the expected utility for player $i \in \{M, E\}$, U_i, as the unconditional expected payoff over realizations of x_i:

$$U_i = \mathbb{E}\left[a_i(x_i)\left(a_j(\theta + b_i) + (1 - a_j)(\theta - 1)\right)\right], \tag{23}$$

where $b_e = 0$ and $b_M = b$. As a baseline, we first characterize the investment decision rule which maximizes welfare (i.e., the sum $U_M + \frac{1}{N}\sum_{e \in E} U_e$).

Proposition 8. *Conditional on signals x_M and x_e, the investment rule that maximizes $U_M + \frac{1}{N}\sum_{e\in E} U_e$ is given by: both M and R invest if and only if*

$$\sigma_e^2 x_M + \sigma_M^2 \sum_{e\in E} x_e \geq \left(N\sigma_M^2 + \sigma_e^2\right) \tfrac{\theta_H+\theta_L}{2} + \tfrac{\sigma_e^2\sigma_M^2}{\theta_H-\theta_L} \left(\log\left(-\tfrac{b+2\theta_L}{b+2\theta_H}\right) - \log\left(\tfrac{p_0}{1-p_0}\right)\right) \equiv K_M. \quad (24)$$

The above decision rule, which we refer to as the welfare-maximizing investment decision, represents the recommendation of a social planner who maximizes welfare conditional on all private signals. Relative to the NC, FC and SC scenarios, the decision rule is different for two reasons. First, it is informationally more efficient: players' decisions are determined by the optimal use of all private signals.[16] Second, it accounts for the externality that each player's action has on the other player's payoffs. Since it provides an upper bound on the welfare that may be achieved in our setting, it serves as a natural benchmark for comparison.[17]

In general, the welfare outcomes in the NC, FC and SC scenarios are worse than under the above investment rule. However, as the sender's private signal becomes infinitely precise, welfare with communication (i.e., under FC and SC) approaches this benchmark, while welfare under the no communication benchmark is strictly lower. This observation is summarized by the following result.

Proposition 9. *With full disclosure and strategic communication (i.e., in the FC and SC equilibria), welfare is maximized when the manager's private signal becomes infinitely precise (i.e., when $\sigma_M \to 0$) irrespective of the bias b. In the no communication equilibrium (i.e., the NC equilibrium), welfare may not be maximized even when the manager's signal is infinitely precise.*

Moreover, when the manager's signal is noisy, welfare is still higher in the cheap-talk equilibrium than in the no communication equilibrium.

[16] In contrast, even in the most informative of the other three scenarios, while employees conditions on their signals and the manager's signals, the manager can only condition on her own signal.

[17] We do not claim this outcome is achievable using an optimally designed mechanism. Our analysis is concerned with situations in which players have no commitment power, and as such, cannot commit to using an optimal mechanism.

Proposition 10. *Expected utility for the manager and for the employees is higher under the least-informative strategic communication equilibrium than it is under no communication.*

Since more information is communicated to employees under SC than under NC, the intuition from standard strategic communication games suggests that the above result may be immediate. However, an important difference from pure sender-receiver games is that the sender's investment strategy is also different across the two scenarios — specifically, as Proposition 7 suggests, the manager is more likely to invest under SC than under NC. These effects reinforce each other when comparing the NC and SC equilibria, and as a result, expected utility is higher under SC for the manager and her employees. However, the two effects offset each other when comparing the FC and SC scenarios: employees receive more information under FC, but the manager is more likely to invest under SC. This implies that, in contrast to standard cheap-talk models, expected utility need not always be higher under commitment to full disclosure.

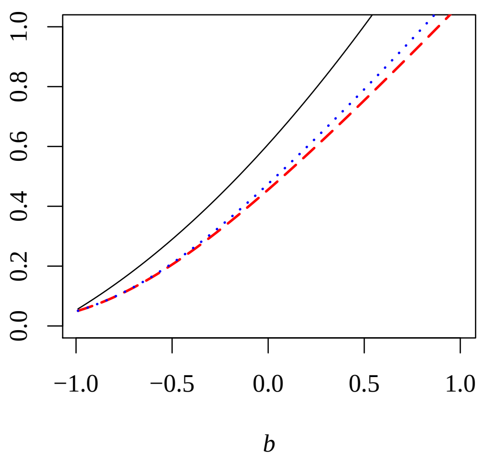
Unfortunately, analytically characterizing the players' expected utility under full disclosure communication is not tractable. Instead, we numerically compute the expected utility in the FC and SC equilibria for various ranges of parameter values. While we have explored the robustness of these results for other parameter values, we report the results based on a benchmark parametrization, where the values are set to the following unless otherwise specified: $p_0 = 0.5$, $\theta_H = 2$, $\theta_L = -1$, $\sigma_M = \sigma_e = 4$, $N = 1$. Figure 2 plots the expected utility for the manager (U_M) and an employee (U_e) as a function of b for this parametrization.

The plots suggest that, somewhat surprisingly, expected utility for both players can be higher with strategic communication than with full disclosure. Specifically, for the parameter regions plotted, we find that this is always true for the manager, and true for the employee when the bias b is close to zero. In other words, when the incentives to coordinate are better aligned ex-ante (b is close to zero), neither the manager nor the employees prefer to commit to full disclosure by the manager. However, when incentives are not well aligned (i.e., b is very positive or very negative), the expected utility for employees may be higher under full disclosure.

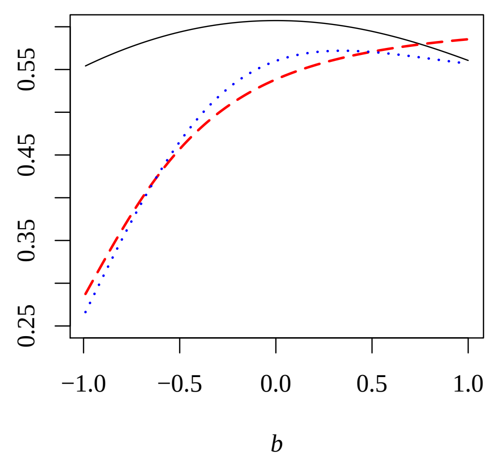
In interpreting these results, recall the two offsetting effects on an employee's expected utility: (i) more information is communicated to him under full-disclosure but (ii) the man-

Figure 2: Expected utility as a function of b

The figure plots expected utility U_M for the manager and expected utility U_e for an employee $e \in E$ as a function of the bias parameter b for the full disclosure communication equilibrium (dashed), the least informative strategic communication equilibrium (dotted), and the welfare maximizing investment decision (solid). The benchmark parameter levels are set to: $p_0 = 0.5$, $\theta_H = 2$, $\theta_L = -1$, $N = 1$ and $\sigma_M = \sigma_e = 4$.



(a) U_M vs. b



(b) U_e vs. b

ager is more likely to invest with strategic communication. The employee's expected utility is higher with SC if the informational disadvantage is smaller than the benefit from more investment. Conditional on knowing whether M will invest, a message is more valuable to $e \in E$ when it is more informative about fundamentals around his cutoff (see Yang (2015) for a discussion of this in the context of flexible information acquisition). When the manager's bias is small, their cutoffs are close, and so the message with SC is quite valuable to the employee. In this case, even though the FC equilibrium is more informative overall, the information advantage over SC is not very large. As a result, the second effect dominates, and expected utility tends to be higher for SC.

However, if the bias is very positive, the manager's cutoff is much lower than the employee's, and so her message in SC is not very valuable to him. In this case, the informational advantage of FC dominates, and expected utility is higher for FC. Similarly, in the least-informative SC equilibrium, signals below the investment cutoff are also concealed, and so expected utility can be lower than in the FC (and the most-informative SC equilibrium) when the bias is extremely negative (and consequently, the employee's investment cutoff is very high).

Comparing the FC and SC plots to the welfare-maximizing investment decision suggests that the largest loss in the employee's utility is when the manager is biased against coordinated investment (i.e., when b is negative). Intuitively, by allowing the manager's investment decision to depend on x_e, the welfare-maximizing decision reduces under-investment by the manager when her bias is very negative, which improves welfare. This suggests that commitment to an optimal mechanism may be most valuable when the manager is biased against investment.

5.3 Investment Efficiency

Next, we turn to investment efficiency, which measures how well the team's investment decision matches the underlying fundamental. It provides an alternative external measure of the team's overall performance, and is arguably easier to measure empirically than welfare in certain settings. Specifically, measuring investment efficiency does not rely on knowledge of the payoff externalities of the team members' actions, but can be estimated by correlating the team members' actions to the project fundamentals.

The distribution of θ implies that investment is efficient when fundamentals are high (i.e., $\theta = \theta_H$), but inefficient when fundamentals are low (i.e., $\theta = \theta_L$). For any equilibrium, this allows us to characterize two sources of distinct measures of inefficiency:[18]

1. Under-investment UI in the good state, which we measure as:

$$UI = 1 - \Pr\left(a_M = 1, \{a_e = 1\}_{e \in E} \,|\theta_H\right). \tag{25}$$

[18] These measures highlight that, in our setting, investment is not always efficient. As a result, while measures of total investment (e.g., $\Pr(a_M = 1) + \Pr(a_e = 1)$) may be interesting for other reasons, they do not capture a notion of efficiency.

2. Over-investment OI in the bad state, which we measure as:

$$OI = 1 - \Pr\left(a_M = 0, \{a_e = 0\}_{e \in E} | \theta_L\right). \tag{26}$$

Overall investment efficiency IE can then be defined as a weighted average of the two:

$$IE = -\left(p_0 UI + (1 - p_0) OI\right). \tag{27}$$

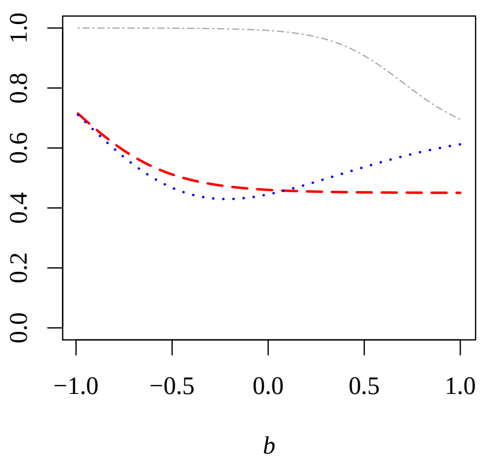
As with expected utility, comparing these efficiency measures across equilibria is not analytically tractable, and so we focus on numerical solutions across a wide range of parameter values. A common feature of the results below is the tradeoff that increased communication introduces in our setting. One the one hand, improving communication (e.g., going from the NC equilibrium to the FC or SC equilibrium) increases informational efficiency since employees have access to more information — this increases efficiency since it allows them to form more precise beliefs about the fundamental state. On the other hand, improving communication increases the ability of the players to coordinate on investment. This improves efficiency by reducing under-investment in the good state, but can decrease efficiency by increasing over-investment in the bad state.

An important feature of the SC equilibrium that distinguishes it from the NC and FC equilibria is that the amount of information communicated is endogenous. Since the manager does not fully disclose the realization of her signal in the region where she invests, SC equilibria are less informative than the FC equilibrium, but more informative than the NC equilibrium. However, since the region over which information is concealed is endogenously determined, the efficiency ranking of SC equilibria is not always between the FC and NC equilibria.

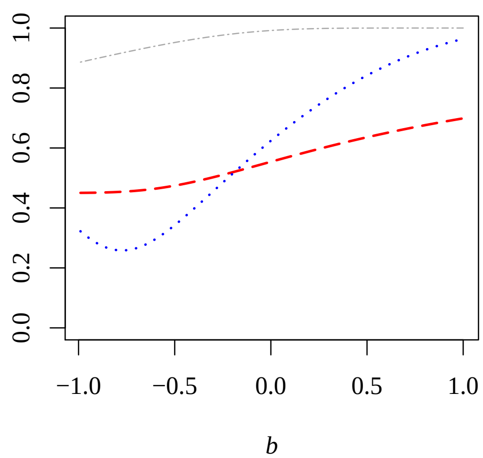
Figure 3 compares the various measures of efficiency across equilibria as the manager's bias changes. An increase in b increases the manager's payoff from investment, and so lowers her investment cutoff k_M across all equilibria, which increases the likelihood she invests for both realizations of fundamentals. When b is close to -1, the under-investment problem is very severe: there is very little to be gained from coordination, and the manager invests only if her posterior expectation of fundamentals is sufficiently close to 1 (in the limit as

Figure 3: Efficiency as a function of b

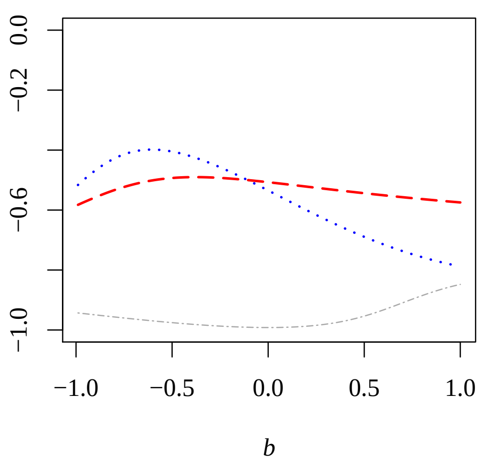
The figure plots (a) under-investment (UI) in the good state, (b) over-investment (OI) in the bad state, and (c) investment efficiency (IE) as a function of the bias parameter b for the no communication equilibrium (dot-dashed), the full disclosure communication equilibrium (dashed), and the least informative strategic communication equilibrium (dotted). The benchmark parameter levels are set to: $p_0 = 0.5$, $\theta_H = 2$, $\theta_L = -1$, $\sigma_M = \sigma_e = 4$.



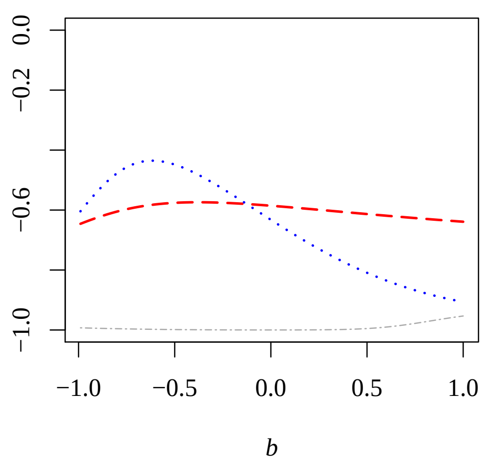
(a) Under-investment ($N = 10$)



(b) Over-investment ($N = 10$)



(c) Investment Efficiency ($N = 10$)



(f) Investment Efficiency ($N = 20$)

$b \downarrow -1$, she invests only if $\mathbb{E}[\theta|x_M] \geq 1$). In this region, the increase in efficiency due to lower under-investment dominates the decrease in efficiency due to higher over-investment, and so investment efficiency for the communication equilibria (FC and SC) initially increases in b. At the other extreme, when b is close to $-\theta_L$, the over-investment problem is very severe for both communication equilibria, because the manager is biased very strongly in favor of investment. In this region, the over-investment effect dominates and so overall investment efficiency decreases in b.

Figure 3 also highlights that the ranking across equilibria can change with b. As expected, both the under-investment and over-investment problems are more severe in the no communication equilibrium than in either communication scenario across all b. However, the relative ranking of the cheap talk equilibrium and the full disclosure scenario depend on the level of b. Recall that the SC equilibrium has an informational disadvantage relative to full disclosure (since less information is conveyed to the employees), but offers a coordination benefit (since the manager is more likely to invest). When the manager is biased against investment (i.e., b is close to -1), both under-investment and over-investment are (weakly) lower with strategic communication. As we show in the proof of the following result, the manager's threshold is close to $\frac{\theta_H+\theta_L}{2}$ in this region, which makes the cheap-talk message in the strategic communication equilibrium very informative to the employees. However, as the bias towards investment increases, the manager's investment threshold becomes lower and consequently, the cheap-talk message becomes less informative to the employees. As a result, the informational disadvantage of the SC equilibrium outweighs the coordination benefit, and overall efficiency is lower than with full disclosure.

A surprising normative implication of this effect is that aligning incentives ex-ante (i.e., setting the manager's bias $b = 0$) need not maximize investment efficiency even when coordination is valuable. For instance, the numerical analysis from Figure 3 suggests that for both communication equilibria, efficiency is maximized for a negative bias. The following result formalizes this observation more generally.

Proposition 11. *Suppose* $p_0 = \frac{1}{2}$. *Then investment efficiency in the least-informative, strategic communication equilibrium is maximized when the manager is biased against coordination. In addition, when* $\theta_H + \theta_L = 1$, *investment efficiency in the full disclosure communication equilibrium is also maximized when the manager is biased against coordina-*

tion.

Intuitively, the negative bias tilts the tradeoff from more communication in favor of greater informational efficiency by reducing the manager's incentive to co-ordinate on over-investment. As such, in settings where the manager's payoff can be chosen exogenously (e.g., compensation contracts for firm employees), it may be informationally efficient to bias her against coordination. This implication is in sharp contrast to the insights from standard cheap-talk models, where reducing the sender's bias tends to generate more informationally efficient outcomes.

Finally, panels (c) and (d) of Figure 3 illustrate the effect of increasing the number of employees (i.e., N). The plots suggest that an increase in N leads to (weakly) lower efficiency in all three scenarios. This is intuitive — increasing the number of employees makes coordination on the appropriate action for each state more difficult. Moreover, the difference in overall efficiency between the SC and FC scenarios increases with N. As such, our analysis suggests that changes in communication strategy (e.g., from cheap-talk to full disclosure) have larger effects on efficiency for larger teams.

6 Conclusions

The effectiveness of a leader is often driven by their ability to inform others and to persuade them to act towards a common goal. We show that the very incentive to coordinate actions can limit a leader's ability to convey information. We study how a manager communicates with her employees when facing an investment decision, where actions are strategic complements and all players are privately informed. We show that informative communication is difficult to sustain: in any cheap talk equilibrium, the manager must conceal some information. For signal realizations where she chooses to invest, the manager can only reveal that she will invest. Moreover, in the presence of positive spillovers, no information can be conveyed via cheap talk.

We also find that the ability to commit to full disclosure may not be valuable, even though more information is communicated than in a cheap-talk equilibrium. When the manager is not too biased in favor of (or against) investment, we find that both the manager and her employees prefer a partially informative cheap talk equilibrium to commitment to full disclosure. Moreover, when the manager is biased against investment, investment efficiency can also be higher under strategic communication.

Our analysis lends itself naturally to experimental tests (see Crawford (1998) for an early survey of experiments on cheap-talk games). For example, our results are broadly consistent with the experimental evidence in Brandts, Cooper, and Fatas (2007), who suggest that simple, one-sided communication by a manager can be most effective at improving coordination across employees.[19] Our analysis also generates a number of testable implications for organizations and firms in real-world settings, although empirically identifying measures of communication and team performance is extremely challenging. For instance, when the leader's bias towards investment is sufficiently large or when the employees' investments generate positive spillovers for the leader, forcing greater disclosure by the leader improves both efficiency and welfare. This suggests that changes in the information environment, either due to external regulations or internal governance policies, that improve the degree of communication within a firm are most beneficial for firms with weak internal governance, and firms in which "empire building" is of greater concern (e.g., family firms, founder-run startups).

However, when the leader's incentives are more closely aligned with those of her employees', such changes to the information environment may not just be ineffective; they may actually be counter-productive. Furthermore, these adverse effects are likely to be more severe for larger teams and firms. Finally, contrary to standard intuition, aligning incentives need not improve informational efficiency. In fact, a "reluctant" leader might improve efficiency of the firm's decisions: in both the full disclosure and strategic communication scenarios, investment efficiency can be higher when the leader is biased against coordinated investment than if she is unbiased.

Although stylized, our analysis is based on a widely used model of coordination. Our results suggest that allowing for cheap-talk communication in such settings can lead to

[19]The setup they consider, through related, is distinct from ours. In their game, the manager is privately informed, but does not take an action herself. They find that simply emphasizing the benefits of coordination to employees is more effective at encouraging coordination than increasing incentives.

different conclusions than in a standard, sender-receiver setting (in which only the receivers take actions). Natural next steps would be to consider greater heterogeneity in receiver (employee) preferences, two-sided communication (employee to manager communication), and inter-receiver communication (communication among employees). We hope to explore this in future work.

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Appendix

A Proofs of main results

Proof of Proposition 1. Since there is no communication, we have for $i \in \{M, E\}$,

$$\log\left(\frac{p_i}{1-p_i}\right) = \log\left(\frac{p_0}{1-p_0}\right) + \frac{1}{\sigma_i^2}\left(\theta_H - \theta_L\right)\left(x_i - \frac{\theta_H+\theta_L}{2}\right), \tag{28}$$

which implies that player i's best response is to invest only when $x_i \geq K\left(k_j; b_i, \sigma_i, \sigma_j\right)$. Note that

$$\lim_{k_j \to -\infty} K\left(k_j\right) = \frac{\theta_H+\theta_L}{2} + \frac{\sigma_i^2}{\theta_H-\theta_L}\left\{\log\left(\frac{-(\theta_L+b_i)}{(\theta_H+b_i)}\right) - \log\left(\frac{p_0}{1-p_0}\right)\right\} \equiv \underline{k}, \tag{29}$$

$$\lim_{k_j \to \infty} K\left(k_j\right) = \frac{\theta_H+\theta_L}{2} + \frac{\sigma_i^2}{\theta_H-\theta_L}\left\{\log\left(\frac{1-\theta_L}{\theta_H-1}\right) - \log\left(\frac{p_0}{1-p_0}\right)\right\} \equiv \bar{k}, \tag{30}$$

and for $b_i > -1$, we have $\frac{\partial}{\partial k}K > 0$. Since $-\theta_H < -1 < b_i < -\theta_L$, $\underline{k}$ and $\bar{k}$ are well defined and finite. The equilibrium is characterized by the fixed point of $x = H\left(x\right)$, where

$$H\left(x\right) \equiv K\left(K\left(x; b_e, \sigma_e, \sigma_M\right); b_M, \sigma_M, \sigma_e\right). \tag{31}$$

Since K is (strictly) increasing, so is H. Also, $H\left(-\infty\right) > -\infty$ and $H\left(\infty\right) < \infty$, which implies a fixed point exists. To ensure uniqueness, we require H is a contraction, or equivalently,

$$\frac{\partial}{\partial x}H\left(x\right) < 1. \tag{32}$$

A sufficient condition for this to be true is that the best response function for each player has a slope less than one, i.e., $\frac{\partial}{\partial k}K < 1$. Note that

$$\frac{\partial}{\partial k}K = \frac{\sigma_i^2}{\theta_H-\theta_L}\left\{\frac{(1+b)\phi\left(\frac{k-\theta_H}{\sigma_j}\right)}{\sigma_j\left((\theta_H+b)-(1+b)\Phi\left(\frac{k-\theta_H}{\sigma_j}\right)\right)} - \frac{(1+b)\phi\left(\frac{k-\theta_L}{\sigma_j}\right)}{\sigma_j\left((\theta_L+b)-(1+b)\Phi\left(\frac{k-\theta_L}{\sigma_j}\right)\right)}\right\} \tag{33}$$

We need to bound the above. Let

$$g(x,\theta,b) = \frac{(1+b)\phi(x)}{\sigma_j((\theta+b)-(1+b)\Phi(x))} \tag{34}$$

$$\Rightarrow g_x = \frac{(1+b)\phi'(x)((\theta+b)-(1+b)\Phi(x))+(1+b)^2(\phi(x))^2}{\sigma_j((\theta+b)-(1+b)\Phi(x))^2} \tag{35}$$

$$= \frac{(1+b)\phi(x)[-x((\theta+b)-(1+b)\Phi(x))+(1+b)\phi(x)]}{\sigma_j((\theta+b)-(1+b)\Phi(x))^2} \tag{36}$$

A necessary condition for the extremum of $g(x,\theta,b)$ is that $g_x = 0$, or equivalently,

$$\frac{\theta+b}{1+b} = \left[\frac{\phi(x)}{x} + \Phi(x)\right]. \tag{37}$$

Recall that $b > -1$, $\theta_L + b < 0$ and $\theta_H > 1$. This implies $g(x,\theta_H,b) > 0$ and $g(x,\theta_L,b) < 0$. Moreover, this implies there is a solution $x_L^*(b,\theta_L) < 0$ for $\theta = \theta_L$ and a solution $x_H^*(b,\theta_H) > 0$ for $\theta = \theta_H$. Finally, the first order condition also implies that

$$g(x^*,\theta,b) = \frac{(1+b)\phi(x)}{\sigma_j((\theta+b)-(1+b)\Phi(x))} = \frac{x^*}{\sigma_j}, \tag{38}$$

that is, $g(x,\theta_H,b)$ is maximized at $\frac{x_H^*(b,\theta_H)}{\sigma_j}$ and $g(x,\theta_L,b)$ is minimized at $\frac{x_L^*(b,\theta_L)}{\sigma_j}$. But this implies that

$$\frac{\partial}{\partial k}K = \frac{\sigma_i^2}{\theta_H-\theta_L}\left\{g\left(\frac{k-\theta_H}{\sigma_j},\theta_H,b\right) - g\left(\frac{k-\theta_L}{\sigma_j},\theta_L,b\right)\right\} \leq \frac{\sigma_i^2}{\theta_H-\theta_L}\left\{\frac{x_H^*(b,\theta_H)-x_L^*(b,\theta_L)}{\sigma_j}\right\} \tag{39}$$

Given b, θ_H and θ_L and σ_j, one can always pick σ_i^2 small enough so that $\frac{\partial}{\partial k}K_i < 1$. In particular,

$$\frac{\sigma_M^2}{\sigma_e} < \frac{\theta_H-\theta_L}{x_H^*(b_M,\theta_H)-x_L^*(b_M,\theta_L)} \equiv \bar{a}, \quad \frac{\sigma_e^2}{\sigma_M} < \frac{\theta_H-\theta_L}{x_H^*(b_e,\theta_H)-x_L^*(b_e,\theta_L)} \equiv \underline{a} \tag{40}$$

ensures that there is a unique equilibrium. □

Proof of Proposition 2. Given the belief updating in equation (8), player R's best response cutoff is given by

$$K_{FC}(x,k) = \frac{1}{2}(\theta_H+\theta_L) + \frac{\sigma_e^2}{\theta_H-\theta_L}\left(\log\left(\frac{(1+b_e)1_{\{x\leq k\}}-(\theta_L+b_e)}{(\theta_H+b_e)-(1+b_e)1_{\{x\leq k\}}}\right) - \log\left(\frac{p_0}{1-p_0}\right)\right) + \frac{\sigma_e^2}{\sigma_M^2}\left(\frac{\theta_H+\theta_L}{2} - x\right). \tag{41}$$

If the signal x coincides with the cutoff k, the above best response simplifies to

$$K_{FC}(k,k) = \frac{1}{2}(\theta_H + \theta_L) + \frac{\sigma_e^2}{\theta_H - \theta_L}\left(\log\left(\frac{-(\theta_L+b_e)}{(\theta_H+b_e)}\right) - \log\left(\frac{p_0}{1-p_0}\right) \right) + \frac{\sigma_e^2}{\sigma_M^2}\left(\frac{\theta_H+\theta_L}{2} - k\right) \equiv g(k), \tag{42}$$

and note that $g(k)$ is decreasing in k. Moreover, note that M should only invest when $x_M \geq k_M$, where

$$k_M = \frac{\theta_H+\theta_L}{2} + \frac{\sigma_M^2}{\theta_H-\theta_L}\left\{\log\left(\frac{(1+b_M)\Phi\left(\frac{g(k_M)-\theta_L}{\sigma_e}\right)-(\theta_L+b_M)}{(\theta_H+b_M)-(1+b_M)\Phi\left(\frac{g(k_M)-\theta_H}{\sigma_e}\right)}\right) - \log\left(\frac{p_0}{1-p_0}\right)\right\} \equiv H(k_M) \tag{43}$$

The equilibrium cutoff k_M is given by the solution to the fixed point $x = H(x)$. Since

$$\lim_{x\to-\infty} H(x) = \frac{\theta_H+\theta_L}{2} + \frac{\sigma_M^2}{\theta_H-\theta_L}\left\{\log\left(\frac{1-\theta_L}{\theta_H-1}\right) - \log\left(\frac{p_0}{1-p_0}\right)\right\} > -\infty, \tag{44}$$

$$\lim_{x\to\infty} H(x) = \frac{\theta_H+\theta_L}{2} + \frac{\sigma_M^2}{\theta_H-\theta_L}\left\{\log\left(-\frac{\theta_L+b_M}{\theta_H+b_M}\right) - \log\left(\frac{p_0}{1-p_0}\right)\right\} < \infty, \tag{45}$$

and $H_x < 0$, we have that a fixed point exists and is unique. □

Proof of Proposition 3. Since we focus on pure strategy messaging rules with truth telling, and consider sender optimal equilibria, the image $\mathcal{M}(\mu) = \{\mu(x_M) : x_M \in \Re\}$ for messaging rule μ is a partition or $\Re$.[20] Given a message $m \in \mathcal{M}(\mu)$ and cutoff k, player R's best response is a cutoff $K_{e,SC}(m,k)$ given by

$$K_{e,SC}(m,k) = \frac{\theta_H+\theta_L}{2} + \frac{\sigma_e^2}{\theta_H-\theta_L}\left\{\log\left(\frac{\Pr(x_M<k|x_M\in m,\theta_L)(1+b_e)-(\theta_L+b_e)}{\theta_H+b_e-\Pr(x_M<k|x_M\in m,\theta_H)(1+b_e)}\right) - \log\left(\frac{p_0}{1-p_0}\right) - \log\left(\frac{\Pr(x_M\in m|\theta_H)}{\Pr(x_M\in m|\theta_L)}\right)\right\}. \tag{46}$$

Given this best response, each message m corresponds to a cutoff $k(m)$ for player M, given by:

$$k(m) = \frac{\theta_H+\theta_L}{2} + \frac{\sigma_M^2}{\theta_H-\theta_L}\left\{\log\left(\frac{(1+b)\Phi\left(\frac{K_{e,SC}(m,k(m))-\theta_L}{\sigma_e}\right)-(\theta_L+b)}{(\theta_H+b)-(1+b)\Phi\left(\frac{K_{e,SC}(m,k(m))-\theta_H}{\sigma_e}\right)}\right) - \log\left(\frac{p_0}{1-p_0}\right)\right\}. \tag{47}$$

First we show that for any message m in equilibrium, we must have $k(m) \notin \text{int}(m)$, where

[20] Specifically, sender optimality rules out some sender types sending a message $m = \Re$, since it is not optimal for senders with high signals to be pooled with senders with low signals.

$\text{int}(m)$ denotes the interior of m. Suppose otherwise, and let $m_1 = m \cap \{x < k(m)\}$and $m_2 = m \setminus m_1$. Then, it must be that m_1 is to the left of m_2, i.e., $\limsup m_1 < \liminf m_2$ — denote this as $m_1 \prec m_2$. This implies that $K_{e,SC}(m_1, k(m)) > K_{e,SC}(m, k(m)) > K_{e,SC}(m_2, k(m))$, which in turn implies $k(m_2) < k(m) < k(m_1)$, which implies M does not invest for $x_M \in m_1$ and always invests for $x_M \in m_2$, and is strictly (weakly) better off for $x_M \in m_2$ ($x_M \in m_1$, respectively) using messages $\{m_1, m_2\}$ instead of m.

This implies that for any candidate equilibrium messaging rule μ, the corresponding messages must be such that $k(m) \notin \text{int}(m)$. This implies that we can partition the image of μ, $\mathcal{M}(\mu)$, into two subsets $\bar{F}(\mu) = \{m \in \mathcal{M}(\mu) : k(m) \leq \liminf(m)\}$ and $\underline{F}(\mu) = \mathcal{M}(\mu) \setminus \bar{F}(\mu)$. Note that $\bar{F}(\mu)$ is the set of all messages $m \in \mathcal{M}(\mu)$ where M invests. If we define $c_1(\mu) \equiv \sup\left\{x : \bar{F}(\mu) \geq x\right\}$ and $c_2(\mu) \equiv \inf\{x : \underline{F}(\mu) \leq x\}$. It is immediate to see $c_1(\mu) \leq c_2(\mu)$; otherwise, the interval $(c_2(\mu), c_1(\mu))$ does not exist in $\mathcal{M}(\mu)$. Suppose $c_1(\mu) < c_2(\mu)$. Then, in $(c_1(\mu), c_2(\mu))$, there exists a real number y such that a left neighborhood of y belongs to $\bar{F}(\mu)$ and a right neighborhood of y belongs to $\underline{F}(\mu)$. It implies that $\lim_{z\uparrow y} U_M(z, m(z)) \geq 0$ and $\lim_{z\downarrow y} U_M(z, m(z)) \leq 0$, and one of the inequalities is strict. As a result, the indifference requirement at y is violated. Hence, we have $c_1(\mu) = c_2(\mu) \equiv c(\mu)$, and $\limsup \underline{F}(\mu) \leq c(\mu)$ and $\liminf \bar{F}(\mu) \geq c(\mu)$.

Unless $\bar{F}(\mu)$ consists of a single interval, for any $m \in \bar{F}(\mu)$, there exists a $\tilde{m} \in \bar{F}(\mu)$, such that $cl(m) \cap cl(\tilde{m}) \neq \varnothing$, where $cl(m)$ denotes the closure of m. The optimality of μ requires that for any $x_M \in cl(m) \cap cl(\tilde{m})$, M is indifferent between sending the message m and $\tilde{m}$, but this implies $K_{e,SC}(m, k(m)) = K_{e,SC}(\tilde{m}, k(\tilde{m}))$. Since $m, \tilde{m} \in \bar{F}(\mu)$ (i.e., for any signals x_M, M invests given message m, $\tilde{m}$ and so $\Pr(x_M < k(m) | x_M \in m, \theta) = \Pr(x_M < k(\tilde{m}) | x_M \in \tilde{m}, \theta) = 0$) this in turn must imply

$$\frac{\Pr(x_M \in m|\theta_H)}{\Pr(x_M \in m|\theta_L)} = \frac{\Pr(x_M \in \tilde{m}|\theta_H)}{\Pr(x_M \in \tilde{m}|\theta_L)}. \tag{48}$$

But $\bar{F}(\mu)$ is a partition of the half-line $[c(\mu), \infty)$, and so for all $m \in \bar{F}(\mu)$, $\frac{\Pr(x_M \in m|\theta_H)}{\Pr(x_M \in m|\theta_L)} = t$ for some constant. This implies that

$$t = \frac{\sum_{m\in\bar{F}} \Pr(x_M \in m|\theta_H)}{\sum_{m\in\bar{F}} \Pr(x_M \in m|\theta_L)} = \frac{\Pr(x_M \in \cup_{m\in\bar{F}} m|\theta_H)}{\Pr(x_M \in \cup_{m\in\bar{F}} m|\theta_L)} = \frac{\Pr\left(x_M \in \bar{F}|\theta_H\right)}{\Pr\left(x_M \in \bar{F}|\theta_L\right)}. \tag{49}$$

This implies any candidate messaging rule μ is equivalent to a messaging rule $\tilde{\mu}$, where

$$\tilde{\mu}(x) = \begin{cases} \mu(x) & \text{if } x < k(\mu) \\ [k(\mu), \infty) & \text{if } x \geq k(\mu) \end{cases} \tag{50}$$

and M invests if and only if $x_M \geq k(\mu)$. An optimal messaging rule must satisfy player M's indifference condition:

$$\pi_M(k, [k, \infty)) = 0, \tag{51}$$

This is characterized by the solution k_{SC} to the fixed point problem $k = K(K_e(k), b_M, \sigma_M, \sigma_e)$, where

$$K_e(k) \equiv K_{e,SC}(k, [k, \infty)) \tag{52}$$

$$= \frac{\theta_H+\theta_L}{2} + \frac{\sigma_e^2}{\theta_H-\theta_L}\left\{\log\left(\frac{-(\theta_L+b_e)}{\theta_H+b_e}\right) - \log\left(\frac{p_0}{1-p_0}\right) - \log\left(\frac{1-\Phi\left(\frac{k-\theta_H}{\sigma_M}\right)}{1-\Phi\left(\frac{k-\theta_L}{\sigma_M}\right)}\right)\right\} \tag{53}$$

since for $x \in [k, \infty]$, R knows that M invests (i.e., $\Pr(a_M = 0 | x \in [k, \infty], \theta) = 0$). Note that $\lim_{x\to\infty} K_e(x) = -\infty$ and $\lim_{x\to-\infty} K_e(x) = \frac{\theta_H+\theta_L}{2} + \frac{\sigma_e^2}{\theta_H-\theta_L}\left\{\log\left(\frac{-(\theta_L+b_e)}{\theta_H+b_e}\right) - \log\left(\frac{p_0}{1-p_0}\right)\right\} \equiv \underline{k} < \infty$, and K_e is decreasing in x. But this implies

$$\lim_{x\to\infty} H(x) = \frac{\theta_H+\theta_L}{2} + \frac{\sigma_M^2}{\theta_H-\theta_L}\left\{\log\left(-\frac{\theta_L+b_M}{\theta_H+b_M}\right) - \log\left(\frac{p_0}{1-p_0}\right)\right\} \leq \infty \tag{54}$$

$$\lim_{x\to-\infty} H(x) = \frac{\theta_H+\theta_L}{2} + \frac{\sigma_M^2}{\theta_H-\theta_L}\left\{\log\left(\frac{(1+b_M)\Phi\left(\frac{\underline{k}-\theta_L}{\sigma_e}\right)-(\theta_L+b_M)}{(\theta_H+b_M)-(1+b_M)\Phi\left(\frac{\underline{k}-\theta_H}{\sigma_e}\right)}\right) - \log\left(\frac{p_0}{1-p_0}\right)\right\} \geq -\infty \tag{55}$$

and H is decreasing in x, a fixed point exists. Note that for $x_M \leq k_{SC}$, M does not invest and so is indifferent between different messaging rules μ that differ in this region. □

Proof of Proposition 5. It is sufficient to show

$$x_M = \arg\max_{m_M} \mathbb{E}\left[1_{\{x_e > k_e(m_M)\}}\theta | x_M\right], \tag{56}$$

where $\mathbb{E}[\theta | m_M, k_e(m_M)] = 0$. Note that

$$k_e(m_M) = \frac{\theta_H+\theta_L}{2} + \frac{\sigma_e^2}{\sigma_M^2}\left(\frac{\theta_H+\theta_L}{2} - m_M\right) + \frac{\sigma_e^2}{\theta_H-\theta_L}\left(\ \log\left(-\frac{\theta_L}{\theta_H}\right) - \log\left(\frac{p_0}{1-p_0}\right)\ \right)$$

The objective function is

$$\mathbb{E}\left[1_{\{x_e>k_e(m_M)\}}\theta|x_M\right]=\theta_H p\left(x_M\right)\left[1-\Phi\left(\tfrac{k_e(m_M)-\theta_H}{\sigma_e}\right)\right]+\theta_L\left(1-p\left(x_M\right)\right)\left[1-\Phi\left(\tfrac{k_e(m_M)-\theta_L}{\sigma_e}\right)\right] \tag{57}$$

where

$$\log\left(\frac{p\left(x_M\right)}{1-p\left(x_M\right)}\right)=\log\left(\frac{p_0}{1-p_0}\right)+\frac{\theta_H-\theta_L}{\sigma_M^2}\left(x_M-\frac{\theta_H+\theta_L}{2}\right).$$

The first-order condition is

$$0=\theta_H p\left(x_M\right)\phi\left(\tfrac{k_e(m_M)-\theta_H}{\sigma_e}\right)+\theta_L\left(1-p\left(x_M\right)\right)\phi\left(\tfrac{k_e(m_M)-\theta_L}{\sigma_e}\right). \tag{58}$$

Equivalently,

$$\begin{aligned}
0&=\log\left(\tfrac{p_0}{1-p_0}\right)-\log\left(-\tfrac{\theta_L}{\theta_H}\right)+\tfrac{\theta_H-\theta_L}{\sigma_M^2}\left(x_M-\tfrac{\theta_H+\theta_L}{2}\right)+\log\frac{\phi\left(\frac{k_e(m_M)-\theta_H}{\sigma_e}\right)}{\phi\left(\frac{k_e(m_M)-\theta_L}{\sigma_e}\right)} && (59)\\
&=\log\left(\tfrac{p_0}{1-p_0}\right)-\log\left(-\tfrac{\theta_L}{\theta_H}\right)+\tfrac{\theta_H-\theta_L}{\sigma_M^2}\left(x_M-\tfrac{\theta_H+\theta_L}{2}\right)+\tfrac{\theta_H-\theta_L}{\sigma_e^2}\left(k_e\left(m_M\right)-\tfrac{\theta_H+\theta_L}{2}\right) && (60)\\
&=\begin{array}{c}\log\left(\tfrac{p_0}{1-p_0}\right)-\log\left(-\tfrac{\theta_L}{\theta_H}\right)+\tfrac{\theta_H-\theta_L}{\sigma_M^2}\left(x_M-\tfrac{\theta_H+\theta_L}{2}\right)\\ +\tfrac{\theta_H-\theta_L}{\sigma_e^2}\left[\tfrac{\sigma_e^2}{\sigma_M^2}\left(\tfrac{\theta_H+\theta_L}{2}-m_M\right)+\tfrac{\sigma_e^2}{\theta_H-\theta_L}\left(\log\left(-\tfrac{\theta_L}{\theta_H}\right)-\log\left(\tfrac{p_0}{1-p_0}\right)\right)\right]\end{array} && (61)\\
&=\tfrac{\theta_H-\theta_L}{\sigma_M^2}\left(x_M-m_M\right) && (62)
\end{aligned}$$

The second-order condition is given by, noting $\phi'(x)=-x\phi(x)$,

$$\begin{aligned}
&\theta_H p\left(x_M\right)\tfrac{k_e(m_M)-\theta_H}{\sigma_e}\phi\left(\tfrac{k_e(m_M)-\theta_H}{\sigma_e}\right)+\theta_L\left(1-p\left(x_M\right)\right)\tfrac{k_e(m_M)-\theta_L}{\sigma_e}\phi\left(\tfrac{k_e(m_M)-\theta_L}{\sigma_e}\right) && (63)\\
=&\theta_H p\left(x_M\right)\tfrac{k_e(m_M)-\theta_H}{\sigma_e}\phi\left(\tfrac{k_e(m_M)-\theta_H}{\sigma_e}\right)-\theta_H p\left(x_M\right)\tfrac{k_e(m_M)-\theta_L}{\sigma_e}\phi\left(\tfrac{k_e(m_M)-\theta_H}{\sigma_e}\right) && (64)\\
=&-\theta_H p\left(x_M\right)\phi\left(\tfrac{k_e(m_M)-\theta_H}{\sigma_e}\right)\tfrac{\theta_H-\theta_L}{\sigma_e} && (65)\\
<&0, && (66)
\end{aligned}$$

where the first equality comes from the first-order condition. Therefore, the objective function is maximized at

$$m_M = x_M, \tag{67}$$

implying that truth-telling is optimal. $\square$

Proof of Proposition 6. First consider the positive spillover case, i.e., $\nu > 0$. Suppose there is an equilibrium in which M can communicate some information about x_M to R. Then there are messages m and $\tilde{m}$ such that (i) $cl\,(m) \cap cl\,(\tilde{m}) \neq \varnothing$, (ii) fixing the cutoff strategy k_M for M, $\Pr(a_e = 0|\theta, m) \neq \Pr(a_e = 0|\theta, \tilde{m})$, and (iii) for $x_M \in cl\,(m) \cap cl\,(\tilde{m})$, $\Pi_M(x_M, m) = \Pi_M(x_M, \tilde{m})$. Without loss of generality, suppose $\Pr(a_e = 0|\theta, m) > \Pr(a_e = 0|\theta, \tilde{m})$. This implies that given a signal x_M, one of the following cases must arise:

(i) M invests for m and $\tilde{m}$: But in this case,

$$\Pi_M(x_M, \tilde{m}) - \Pi_M(x_M, m) = \pi_M^1(x_M, \tilde{m}) - \pi_M^1(x_M, m) \tag{68}$$

$$= -(1+\nu)\left(\Pr(a_e = 0|\tilde{m}) - \Pr(a_e = 0|m)\right) \neq 0 \tag{69}$$

and so we have a contradiction.

(ii) M does not invest for m and $\tilde{m}$:

$$\Pi_M(x_M, \tilde{m}) - \Pi_M(x_M, m) = \pi_M^0(x_M, \tilde{m}) - \pi_M^0(x_M, m) \tag{70}$$

$$= -\nu\left(\Pr(a_e = 0|\tilde{m}) - \Pr(a_e = 0|m)\right) \neq 0 \tag{71}$$

and so we have a contradiction.

(iii) M invests for m but not for $\tilde{m}$: Since $\Pr(a_e = 0|\theta, m) > \Pr(a_e = 0|\theta, \tilde{m})$, we have $\pi_M^1(x, \tilde{m}) > \pi_M^1(x, m)$. But since M is indifferent at x_M, we have $\pi_M^0(x_M, \tilde{m}) = \pi_M^1(x_M, m)$, which implies $\pi_M^1(x_M, \tilde{m}) > \pi_M^0(x_M, \tilde{m})$, i.e., it cannot be optimal to not invest at x_M with message $\tilde{m}$, and so we have a contradiction.

(iv) M invests for $\tilde{m}$ but not for m: Since $\Pr(a_e = 0|\theta, m) > \Pr(a_e = 0|\theta, \tilde{m})$, we have $\pi_M^0(x, \tilde{m}) > \pi_M^0(x, m)$. But since M is indifferent at x_M, we have $\pi_M^1(x_M, \tilde{m}) = \pi_M^0(x_M, m)$. But this implies $\pi_M^0(x_M, \tilde{m}) > \pi_M^1(x_M, \tilde{m})$, i.e., it cannot be optimal to invest at x_M with message $\tilde{m}$, and so we have a contradiction.

This implies that in any strategic equilibrium, M effectively cannot communicate any information to R.

When the spillover is negative (i.e., $\nu < 0$), analogous arguments establish for $\Pr(a_e = 0|\theta, m) > \Pr(a_e = 0|\theta, \tilde{m})$ and $x_M \in cl(m) \cap cl(\tilde{m})$, we can only have case (iv), i.e., M invests for $\tilde{m}$ but does not invest for m. Moreover, since cases (i) and (ii) are not possible, the message m must be equivalent to $m = \{a_M = 0\}$ and the message $\tilde{m}$ must be equivalent to $\tilde{m} = \{a_M = 1\}$. Since the incremental payoff to investing $\pi_M(x_M, m)$ is independent of ν, the equilibrium in this case is equivalent to the least informative equilibrium in our main model, when $b = 0$. □

Proof of Proposition 7. Denote the (equilibrium) best response functions for the receiver R in each of the three scenarios as:

$$k_{e,NC}(x) = \frac{\theta_H+\theta_L}{2} + \frac{\sigma_e^2}{\theta_H-\theta_L}\left\{\log\left(\frac{(1+b_e)\Phi\left(\frac{x-\theta_L}{\sigma_M}\right)-(\theta_L+b_e)}{(\theta_H+b_e)-(1+b)\Phi\left(\frac{x-\theta_H}{\sigma_M}\right)}\right) - \log\left(\frac{p_0}{1-p_0}\right)\right\} \tag{72}$$

$$k_{e,FC}(x) = \frac{\theta_H+\theta_L}{2} + \frac{\sigma_e^2}{\theta_H-\theta_L}\left(\log\left(-\frac{\theta_L+b_e}{\theta_H+b_e}\right) - \log\left(\frac{p_0}{1-p_0}\right)\right) + \frac{\sigma_e^2}{\sigma_M^2}\left(\frac{\theta_H+\theta_L}{2} - x\right) \tag{73}$$

$$k_{e,SC}(x) = \frac{\theta_H+\theta_L}{2} + \frac{\sigma_e^2}{\theta_H-\theta_L}\left\{\log\left(-\frac{\theta_L+b_e}{\theta_H+b_e}\right) - \log\left(\frac{p_0}{1-p_0}\right) - \log\left(\frac{1-\Phi\left(\frac{x-\theta_H}{\sigma_M}\right)}{1-\Phi\left(\frac{x-\theta_L}{\sigma_M}\right)}\right)\right\} \tag{74}$$

$$k_M(x) = \frac{\theta_H+\theta_L}{2} + \frac{\sigma_M^2}{\theta_H-\theta_L}\left\{\log\left(\frac{(1+b_M)\Phi\left(\frac{x-\theta_L}{\sigma_e}\right)-(\theta_L+b_M)}{(\theta_H+b_M)-(1+b_M)\Phi\left(\frac{x-\theta_H}{\sigma_e}\right)}\right) - \log\left(\frac{p_0}{1-p_0}\right)\right\}, \tag{75}$$

Note that

$$\begin{aligned} &k_{e,NC}(x) - k_{e,SC}(x) \\ &= \frac{\sigma_e^2}{\theta_H-\theta_L}\left\{\underbrace{\log\left(\frac{(1+b_e)\Phi\left(\frac{x-\theta_L}{\sigma_M}\right)-(\theta_L+b_e)}{(\theta_H+b_e)-(1+b)\Phi\left(\frac{x-\theta_H}{\sigma_M}\right)}\right) - \log\left(-\frac{\theta_L+b_e}{\theta_H+b_e}\right)}_{\geq 0} + \underbrace{\log\left(\frac{1-\Phi\left(\frac{x-\theta_H}{\sigma_M}\right)}{1-\Phi\left(\frac{x-\theta_L}{\sigma_M}\right)}\right)}_{\geq 0}\right\} \geq 0 \end{aligned} \tag{76}$$

and

$$k_{e,FC}(x) - k_{e,SC}(x) = \frac{\sigma_e^2}{\sigma_M^2}\left(\frac{\theta_H+\theta_L}{2} - x\right) + \frac{\sigma_e^2}{\theta_H-\theta_L}\log\left(\frac{1-\Phi\left(\frac{x-\theta_H}{\sigma_M}\right)}{1-\Phi\left(\frac{x-\theta_L}{\sigma_M}\right)}\right) \tag{77}$$

$$= \frac{\sigma_e^2}{\sigma_M^2} \frac{\sigma_M^2}{\theta_H - \theta_L} \log \left[e^{\frac{\theta_H - \theta_L}{\sigma_M^2}\left(\frac{\theta_H + \theta_L}{2} - x\right)} \left(\frac{1 - \Phi\left(\frac{x - \theta_H}{\sigma_M}\right)}{1 - \Phi\left(\frac{x - \theta_L}{\sigma_M}\right)} \right) \right] \tag{78}$$

$$= \frac{\sigma_e^2}{\theta_H - \theta_L} \left\{ \log \left(\frac{\phi\left(\frac{x - \theta_L}{\sigma_M}\right)}{1 - \Phi\left(\frac{x - \theta_L}{\sigma_M}\right)} \right) - \log \left(\frac{\phi\left(\frac{x - \theta_H}{\sigma_M}\right)}{1 - \Phi\left(\frac{x - \theta_H}{\sigma_M}\right)} \right) \right\} \geq 0 \tag{79}$$

Since $k_{M,NC} = k_M \left(k_{e,NC}\left(k_{M,NC}\right)\right)$, $k_{M,FC} = k_M \left(k_{e,FC}\left(k_{M,FC}\right)\right)$, and $k_{M,SC} = k_M \left(k_{e,SC}\left(k_{M,SC}\right)\right)$, we must have $k_{M,NC} \geq k_{M,SC}$ and $k_{M,FC} \geq k_{M,SC}$. □

Proof of Proposition 8. First, note that it is never optimal to have only some of the players invest. If this was the preferred outcome, the total payoff must be higher than if both players invest and if both players do not invest. Let $U = U_M + \frac{1}{N}\sum_{e \in E} U_e$.

- If all players invest, $U = 2\theta + b$. If no player invests, then $U = 0$. As such, if all players invest / do not invest together, $U^* = \max\{0, 2\theta + b\}$.
- If the manager does not invest, but all employees do, then $U = \theta - 1$. To have $U > U^*$, we need either (i) $\theta > 1$, which implies $\theta - 1 < 2\theta + b$ (since $b > -1$), or (ii) $-\theta - 1 > b$, which contradicts $b > -1$. This implies $U \leq U^*$.
- If the manager and $n < N$ of the employees invest, then $U = \left(1 + \frac{n}{N}\right)\theta + \frac{n}{N}(1 + b) - 1$. To have $U > U^*$, we need either (i) $\left(1 + \frac{n}{N}\right)\theta + \frac{n}{N}(1 + b) - 1 > 0$, but this contradicts $U > 2\theta + b$, or we need (ii) $\left(1 + \frac{n}{N}\right)\theta + \frac{n}{N}(1 + b) - 1 > 2\theta + b$, which implies $\theta < 0$, but this contradicts $\left(1 + \frac{n}{N}\right)\theta + \frac{n}{N}(1 + b) - 1 > 0$.

This implies that M recommends investment, conditional on observing x_M and $\{x_e\}_{e \in E}$, when

$$\log\left(\frac{p_0}{1 - p_0}\right) + \frac{1}{\sigma_M^2}(\theta_H - \theta_L)\left(x_M - \frac{\theta_H + \theta_L}{2}\right) + \frac{1}{\sigma_e^2}(\theta_H - \theta_L)\sum_{e \in E}\left(x_e - \frac{\theta_H + \theta_L}{2}\right) \geq \log\left(-\frac{b + 2\theta_L}{b + 2\theta_H}\right), \tag{80}$$

or equivalently,

$$\sigma_e^2 x_M + \sigma_M^2 \sum_{e \in E} x_e \geq \left(N\sigma_M^2 + \sigma_e^2\right)\frac{\theta_H + \theta_L}{2} + \frac{\sigma_e^2 \sigma_M^2}{\theta_H - \theta_L}\left(\log\left(-\frac{b + 2\theta_L}{b + 2\theta_H}\right) - \log\left(\frac{p_0}{1 - p_0}\right)\right) \equiv K_M \tag{81}$$

which gives us the result. □

Proof of Proposition 9. Given the expressions in equations (7), (9), (10), Proposition 4, and equation (24) we can show the following:

(i) For the NC, FC and SC equilibria, the sender's cutoff in the limit is given by

$$\lim_{\sigma_M \to 0} k_M = \tfrac{\theta_H+\theta_L}{2} \equiv k_M^0. \tag{82}$$

(ii) In the NC equilibrium,

$$\lim_{\sigma_M \to 0} k_{e,NC} = \lim_{\sigma_M \to 0} \tfrac{\theta_H+\theta_L}{2} + \tfrac{\sigma_e^2}{\theta_H-\theta_L}\left\{\log\left(\tfrac{(1+b_e)\Phi\left(\frac{k_M-\theta_L}{\sigma_M}\right)-(\theta_L+b_e)}{(\theta_H+b_e)-(1+b_e)\Phi\left(\frac{k_M-\theta_H}{\sigma_M}\right)}\right)-\log\left(\tfrac{p_0}{1-p_0}\right)\right\} \tag{83}$$

$$= \tfrac{\theta_H+\theta_L}{2} + \tfrac{\sigma_e^2}{\theta_H-\theta_L}\left\{\log\left(\tfrac{1-\theta_L}{\theta_H+b_e}\right)-\log\left(\tfrac{p_0}{1-p_0}\right)\right\} \equiv k_{e,NC}^0. \tag{84}$$

(iii) In the FC equilibrium,

$$k_{e,FC}^0\left(x_M\right) \equiv \lim_{\sigma_M \to 0} K_{FC}\left(x_M, k_M\right) = \begin{cases} +\infty & \text{if } x_M < \frac{\theta_H+\theta_L}{2} \\ 0 & \text{if } x_M = \frac{\theta_H+\theta_L}{2} \\ -\infty & \text{if } x_M > \frac{\theta_H+\theta_L}{2} \end{cases}. \tag{85}$$

(iv) In the least informative SC equilibrium,

$$k_{e,SC}^0\left(m_M\right) \equiv \lim_{\sigma_M \to 0} K_{SC}\left(m_M\right) = \begin{cases} -\infty & \text{if } m_M = [k_{M,SC}, \infty), \\ +\infty & \text{if } m_M = (-\infty, k_{M,SC}) \end{cases} \tag{86}$$

and in the most informative SC equilibrium,

$$k_{e,SC}^0\left(m_M\right) \equiv \lim_{\sigma_M \to 0} K_{SC}\left(m_M\right) = \begin{cases} -\infty & \text{if } m_M = [k_{M,SC}, \infty), \\ +\infty & \text{if } m_M < k_{M,SC}, \text{ and } m_M = x_M < \frac{\theta_H+\theta_L}{2} \\ 0 & \text{if } m_M < k_{M,SC}, \text{ and } m_M = x_M = \frac{\theta_H+\theta_L}{2} \\ -\infty & \text{if } m_M < k_{M,SC}, \text{ and } m_M = x_M > \frac{\theta_H+\theta_L}{2} \end{cases}. \tag{87}$$

(v) For the welfare maximizing investment decision, both M and R invest if and only if

$$\lim_{\sigma_M \to 0} \sigma_e^2 x_M + \sigma_M^2 \sum_{e \in E} x_e \geq \lim_{\sigma_M \to 0} K_M, \tag{88}$$

or equivalently, $x_M \geq \frac{\theta_H + \theta_L}{2}$.

This implies that, as the precision of the sender's signal becomes infinite, the states of the world in which there is investment in the SC and FC equilibria coincide with those in which there is investment under the welfare maximizing investment decision. However, for the NC equilibrium, this is not the case and consequently, welfare is lower. □

Proof of Proposition 10. First consider player M's utility under SC vs. NC. The equilibrium expected utility is given by

$$U_M(k_e) = \max_k \mathbb{E}^\theta \left[(\theta - 1) \Pr(x_M \geq k) + (b_M + 1) \Pr(x_M \geq k) \Pr(x_e \geq k_e) \right]. \tag{89}$$

By the envelope theorem,

$$\tfrac{\partial}{\partial k_e} U_M = \mathbb{E}^\theta \left[-\tfrac{1}{\sigma_e} (b_A + 1) \Pr(x_M \geq k) \phi\left(\tfrac{k_e - \theta}{\sigma_e} \right) \right] \leq 0, \tag{90}$$

which implies $U_{M,SC} = U_M(k_{e,SC}) \geq U_M(k_{e,NC}) = U_{M,NC}$. Next consider player R's utility under SC vs. NC. Let

$$V(k_M) = \max_k \mathbb{E} \left[(\theta - 1) \Pr(x_e \geq k, x_M < k_M) + (\theta + b_e) \Pr(x_M \geq k_M, x_e \geq k) \right] \tag{91}$$

and note that $U_{e,NC} = V(k_{M,NC})$. Moreover, since $k_{M,NC} > k_{M,SC}$, the envelope theorem implies $V(k_{M,NC}) < V(k_{M,SC})$. Finally, note that

$$U_{e,SC} = \max_{k_0,k_1} \mathbb{E} \left[(\theta - 1) \Pr(x_e \geq k_0, x_M < k_{M,SC}) + (\theta + b_e) \Pr(x_M \geq k_{M,SC}, x_e \geq k_1) \right] \tag{92}$$

$$\geq \max_k \mathbb{E} \left[(\theta - 1) \Pr(x_e \geq k, x_M < k_{M,SC}) + (\theta + b_e) \Pr(x_M \geq k_{M,SC}, x_e \geq k) \right] \tag{93}$$

$$= V(k_{M,SC}) > U_{e,NC}. \tag{94}$$

Hence, both R and M prefer the SC equilibrium to the NC equilibrium. □

Proof of Proposition 11. Since $p_0 = \frac{1}{2}$, we have $IE(b) = -\frac{1}{2}(UI(b) + OI(b))$. Let $k_{e,1} \equiv K_{SC}([k_{M,SC}, \infty))$, $k_{e,0} \equiv K_{SC}((-\infty, k_{M,SC}))$ and $k_M \equiv k_{M,SC}$, and we treat them as univariate functions of b. Given the expressions for $k_{e,1}$ and $k_{e,0}$, we have

$$k'_{e,1}(b) = \frac{\sigma_e^2}{\sigma_M(\theta_H - \theta_L)} \left[\frac{\phi\left(\frac{\theta_H - k_M}{\sigma_M}\right)}{\Phi\left(\frac{\theta_H - k_M}{\sigma_M}\right)} - \frac{\phi\left(\frac{\theta_L - k_M}{\sigma_M}\right)}{\Phi\left(\frac{\theta_L - k_M}{\sigma_M}\right)} \right] k'_M(b), \text{ and} \tag{95}$$

$$k'_{e,0}(b) = -\frac{\sigma_e^2}{\sigma_M(\theta_H - \theta_L)} \left[\frac{\phi\left(\frac{k_M - \theta_H}{\sigma_M}\right)}{\Phi\left(\frac{k_M - \theta_H}{\sigma_M}\right)} - \frac{\phi\left(\frac{k_M - \theta_L}{\sigma_M}\right)}{\Phi\left(\frac{k_M - \theta_L}{\sigma_M}\right)} \right] k'_M(b) \tag{96}$$

which implies that

$$\frac{\partial IE(b)}{\partial b} = \begin{array}{c} -\frac{1}{2\sigma_M}\phi\left(\frac{\theta_H - k_M}{\sigma_M}\right)\left[\Phi\left(\frac{\theta_H - k_{e,1}}{\sigma_e}\right)\right]^N k'_M \\ -\frac{N\sigma_e}{2\sigma_M(\theta_H - \theta_L)}\phi\left(\frac{\theta_H - k_{e,1}}{\sigma_e}\right)\left[\Phi\left(\frac{\theta_H - k_{e,1}}{\sigma_e}\right)\right]^{N-1}\Phi\left(\frac{\theta_H - k_M}{\sigma_M}\right)\left[\frac{\phi\left(\frac{\theta_H - k_M}{\sigma_M}\right)}{\Phi\left(\frac{\theta_H - k_M}{\sigma_M}\right)} - \frac{\phi\left(\frac{\theta_L - k_M}{\sigma_M}\right)}{\Phi\left(\frac{\theta_L - k_M}{\sigma_M}\right)}\right] k'_M \\ +\frac{1}{2\sigma_M}\phi\left(\frac{k_M - \theta_L}{\sigma_M}\right)\left[\Phi\left(\frac{k_{e,0} - \theta_L}{\sigma_e}\right)\right]^N k'_M \\ -\frac{N\sigma_e}{2\sigma_M(\theta_H - \theta_L)}\phi\left(\frac{k_{e,0} - \theta_L}{\sigma_e}\right)\left[\Phi\left(\frac{k_{e,0} - \theta_L}{\sigma_e}\right)\right]^{N-1}\Phi\left(\frac{k_M - \theta_L}{\sigma_M}\right)\left[\frac{\phi\left(\frac{k_M - \theta_H}{\sigma_M}\right)}{\Phi\left(\frac{k_M - \theta_H}{\sigma_M}\right)} - \frac{\phi\left(\frac{k_M - \theta_L}{\sigma_M}\right)}{\Phi\left(\frac{k_M - \theta_L}{\sigma_M}\right)}\right] k'_M \end{array} \tag{97}$$

Consider b such that $k(b) = \frac{\theta_H + \theta_L}{2}$. In this case, we have

$$\frac{\theta_H - k}{\sigma_i} = \frac{k - \theta_L}{\sigma_i}, \tag{98}$$

implying that

$$\Phi\left(\frac{\theta_H - k}{\sigma_M}\right) = \Phi\left(\frac{k - \theta_L}{\sigma_M}\right), \tag{99}$$

$$\phi\left(\frac{\theta_H - k}{\sigma_i}\right) = \phi\left(\frac{k - \theta_L}{\sigma_i}\right) = \phi\left(\frac{k - \theta_H}{\sigma_i}\right) = \phi\left(\frac{\theta_L - k}{\sigma_i}\right). \tag{100}$$

Moreover, with the same value of b,

$$\begin{aligned} k_{e,1}(k) + k_{e,0}(k) &= \theta_H + \theta_L + \frac{\sigma_e}{\theta_H - \theta_L}\left\{ \log\left(-\frac{\theta_L}{\theta_H}\right) + \log\left(\frac{1 - \theta_L}{\theta_H - 1}\right) - \log\left(\frac{\Phi\left(\frac{\theta_H - k}{\sigma_M}\right)}{\Phi\left(\frac{\theta_L - k}{\sigma_M}\right)}\right) - \log\left(\frac{\Phi\left(\frac{k - \theta_H}{\sigma_M}\right)}{\Phi\left(\frac{k - \theta_L}{\sigma_M}\right)}\right) \right\} \\ &= \theta_H + \theta_L, \end{aligned} \tag{101}$$

which implies that

$$\Phi\left(\frac{\theta_H - k_{e,1}(k)}{\sigma_e}\right) = \Phi\left(\frac{k_{e,0}(k) - \theta_L}{\sigma_e}\right) \text{ and } \phi\left(\frac{\theta_H - k_{e,1}(k)}{\sigma_e}\right) = \phi\left(\frac{k_{e,0}(k) - \theta_L}{\sigma_e}\right). \tag{102}$$

Combining the observations above, we conclude that when $k_M(b) = \frac{\theta_H + \theta_L}{2}$, we have $\frac{\partial IE(b)}{\partial b} = 0$. Now, when $p_0 = \frac{1}{2}$ and $b = 0$, we know $k_{M,NC} = \frac{\theta_H + \theta_L}{2}$, and by Proposition 7, we have that $k_{M,SC} < \frac{\theta_H + \theta_L}{2}$. Since $k_{M,SC}(b)$ is decreasing in b, it must be that $k_{M,SC}(b) = \frac{\theta_H + \theta_L}{2}$, and consequently, $\frac{\partial IE(b)}{\partial b} = 0$, for some $b < 0$.

Turning to FC, let

$$K_{FC,1}(x) = \frac{\theta_H + \theta_L}{2} + \frac{\sigma_e^2}{\sigma_M^2}\left(\frac{\theta_H + \theta_L}{2} - x\right) + \frac{\sigma_e^2}{\theta_H - \theta_L}\left(\log\left(-\frac{\theta_L}{\theta_H}\right) - \log\left(\frac{p_0}{1 - p_0}\right)\right) \tag{103}$$

$$K_{FC,0}(x) = \frac{\theta_H + \theta_L}{2} + \frac{\sigma_e^2}{\sigma_M^2}\left(\frac{\theta_H + \theta_L}{2} - x\right) + \frac{\sigma_e^2}{\theta_H - \theta_L}\left(\log\left(\frac{1 - \theta_L}{\theta_H - 1}\right) - \log\left(\frac{p_0}{1 - p_0}\right)\right). \tag{104}$$

$K_{FC,1}$ is the best response of employees when the manager invests, while $K_{FC,0}$ is when the manager does not. Since investment efficiency is[21]

$$-\frac{1}{2}\left[1 - \int_{k_{M,FC}}^{\infty} \Phi\left(\frac{\theta_H - K_{FC,1}(x)}{\sigma_e}\right)^N \frac{1}{\sigma_M}\phi\left(\frac{x - \theta_H}{\sigma_M}\right) dx\right] - \frac{1}{2}\left[1 - \int_{-\infty}^{k_{M,FC}} \Phi\left(\frac{K_{FC,0}(x) - \theta_L}{\sigma_e}\right)^N \frac{1}{\sigma_M}\phi\left(\frac{x - \theta_L}{\sigma_M}\right) dx\right],$$

its derivative with respect to b is

$$\begin{aligned}\frac{\partial IE(b)}{\partial b} = &-\frac{1}{2}\Phi\left(\frac{\theta_H - K_{FC,1}\left(k_{M,FC}\right)}{\sigma_e}\right)^N \frac{1}{\sigma_M}\phi\left(\frac{k_{M,FC} - \theta_H}{\sigma_M}\right) k_{M,FC}'(b) \\ &+ \frac{1}{2}\Phi\left(\frac{K_{FC,0}\left(k_{M,FC}\right) - \theta_L}{\sigma_e}\right)^N \frac{1}{\sigma_M}\phi\left(\frac{k_{M,FC} - \theta_L}{\sigma_M}\right) k_{M,FC}'(b)\end{aligned}$$

If $\theta_H + \theta_L = 1$,[22] we have

$$K_{FC,1}\left(\frac{\theta_H + \theta_L}{2}\right) + K_{FC,0}\left(\frac{\theta_H + \theta_L}{2}\right) = \theta_H + \theta_L,$$

similar to $k_{e,1}(k) + k_{e,0}(k) = \theta_H + \theta_L$ for SC. For b that satisfies $k_{M,FC}(b) = \frac{\theta_H + \theta_L}{2}$, therefore,

[21] Still we assume $p_0 = \frac{1}{2}$.
[22] This is the benchmark for our numerical examples.

we again have

$$\frac{\partial IE(b)}{\partial b} = 0.$$

By the same logic as in the SC case, we conclude that the efficiency is maximized at $b < 0$ due to the fact that $k'_{M,FC}(b) < 0$. □

B Misaligned cost of investment

In this section, we consider an alternative specification which introduces an asymmetric cost of failing to coordinate. Suppose the payoffs are given by the following table:

$S \setminus R$	Invest ($a_e = 1$)	Not Invest ($a_e = 0$)
Invest ($a_M = 1$)	$\theta + b$, θ	$\theta - c$,0
Not Invest ($a_M = 0$)	0, $\theta - 1$	0,0

As before, when both players invest, M receives $\theta + b$ while R receives θ. Moreover, in this case, when M invests but R does not, M receives $\theta - c$. Assumptions (A1)-(A3) generalize to the following: (i) $\theta_L < 0 < c < \theta_H$, (ii) $b > -c$, and (iii) $b < -\theta_L$. While R's incremental payoff from investing π_e is still given by 4, the incremental payoff to M from investing is now given by

$$\pi_M(x_M, m) = \begin{array}{c} p(x_M, \Re)\left[(\theta_H + b) - (b + c)\Pr(a_e = 0|\theta_H, m)\right] \\ +(1 - p(x_M, \Re))\left[(\theta_L + b) - (b + c)\Pr(a_e = 0|\theta_L, m)\right] \end{array}. \tag{105}$$

The above payoffs imply that player M's best response function is given by

$$K_M(k) = \frac{\theta_H + \theta_L}{2} + \frac{\sigma_M^2}{\theta_H - \theta_L}\left\{\log\left(\frac{(b_M + c)\Phi\left(\frac{k - \theta_L}{\sigma_e}\right) - (\theta_L + b_M)}{(\theta_H + b_M) - (b_M + c)\Phi\left(\frac{k - \theta_H}{\sigma_e}\right)}\right) - \log\left(\frac{p_0}{1 - p_0}\right)\right\}. \tag{106}$$

The parameter restrictions imply that $\lim_{k\to\infty} K_M(k) < \infty$ and $\lim_{k\to-\infty} K_M(k) > -\infty$ and $\frac{\partial}{\partial k} K_M > 0$. This ensures that the proofs of Propositions 1, 2, and 3 apply immediately to this case, since the fixed point conditions (i.e., $x = H(x)$) that characterize the equilibria inherit analogous existence and uniqueness properties.