Forward Guidance and Heterogeneous Beliefs*

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November 2017

First draft: March 2014

Abstract

Central banks' announcements that future rates are expected to remain low for some time could signal either a weak macroeconomic outlook – which is bad news – or a more accommodative policy stance – which is good news. We use the Survey of Professional Forecasters to show that, when the Fed gave date-based forward guidance between 2011Q3 and 2012Q4, these two interpretations coexisted despite a consensus that rates would stay low for long. We rationalize these facts in an otherwise standard New-Keynesian model where agents: (i) are uncertain about the length of the trap, (ii) have different priors on the commitment ability of the central bank, and (iii) perceive central bank announcements of expected rates as accurate. This heterogeneity of beliefs introduces a trade-off in forward guidance policy: leveraging on the optimism of those who believe the central bank can commit comes at the cost of inducing excess pessimism in non-believers. When pessimistic views prevail, forward guidance can even be detrimental.

Keywords: signaling channel, disagreement, optimal policy, zero lower bound, survey forecasts.

JEL Classification: E31, E52, E65.

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1 Introduction

> The FOMC has not been clear about the purpose of its forward guidance. Is it purely a transparency device, or is it a way to commit to a more accommodating future policy stance to add more accommodation today?
>
> Charles I. Plosser, March 6, 2014.

Overview. When facing the Zero Lower Bound (ZLB) on its nominal policy rate, a central bank can still affect current allocations by making statements about future policy rates, indicating that they will remain very low for a significant length of time.[1] In the aftermath of the Great Recession, several central banks implemented such forward guidance policies with somewhat mixed success: they succeeded in lowering expected future interest rates[2] but their resulting impact on the macroeconomy seemed limited or even sometimes lead agents to expect future contractions.[3] As the quote of Charles Plosser suggests and Woodford (2012) emphasizes, one possible reason is that a central bank's announcement that future interest rates will remain low for a given period of time is ambiguous: it is consistent with anticipations of bad economic fundamentals – forward guidance is then said to be *Delphic* – or with anticipations of an expansionary monetary policy – forward guidance is then said to be *Odyssean*.[4] In this paper, we show that these two interpretations emerged behind an agreement on future policy rates as the Fed engaged into date-based forward guidance announcements between 2011Q3 and 2012Q4, and we study how such heterogeneous interpretations affect the design of forward guidance policies.

We provide empirical and theoretical support for the idea that forward guidance can be ineffective when an announcement of a period of low interest rates, although believed, it is not clearly understood as a commitment to future accommodation at the end of the trap. First, we document that, at the time of date-based forward guidance announcements of the Fed, individuals broadly agreed that interest rates will stay low for long. At the same time, agents had different views on the macroeconomic outlook related to different interpretations of the same announcements. Optimistic views were associated to expectations of a more accommodative stance whereas pessimistic views were associated to expectations of a longer period of weak fundamentals. Second, we rationalize these facts in an otherwise standard New-

[1] See Krugman (1998), Eggertsson and Woodford (2003) or, more recently, Werning (2012).

[2] See e.g. Swansson and Williams (2014).

[3] See e.g. Campbell et al. (2012), Del Negro et al. (2015) or Campbell et al. (2016).

[4] This terminology has been introduced by Campbell et al. (2012). See also Ellingsen and Söderström (2001) for a seminal contribution where monetary policy decisions are either related to new information about the economy or to changes in the preferences of the central banker.

Keynesian model where agents are uncertain about the length of the trap, do not observe the commitment ability of the central bank, and view central bank's announcements of expected rates as accurate – i.e. based on precise information. We show that an agreement on future rates can coexist with a disagreement on the type of policy conducted. Third, we analyze the trade-off in forward guidance communication induced by such disagreement: leveraging on the optimism of agents who interpret the announced policy as Odyssean comes at the cost of generating excess pessimism – i.e an expectation that the trap is longer than the one the central bank expects – of the agents who interpret the announced policy as Delphic. We show that, with too many Delphic agents, forward guidance can even be detrimental: by generating excess pessimism, instead of a stimulus, it may lead to lower current aggregate demand.

Overall, our story can rationalize the evidence in the literature that forward guidance had been extremely powerful in lowering expected future interest rates, although it had limited macroeconomic impact and even sometimes signaled future contractions. This implies that, in contrast to frequent policy discussions, gauging the efficiency of forward guidance announcements by merely looking at the reaction of expected future policy rates can be misleading as agents may disagree on the meaning of such a low future interest rate path.

Empirics. We start by documenting new facts on the heterogeneity of individual expectations in the US Survey of Professional Forecasters (SPF). Disagreement about 1-year and 2-year ahead short-term interest rate forecasts dropped to historically low levels after the Fed switched to date-based forward guidance in August 2011. At that time, the FOMC announced that its policy rate would remain "at exceptionally low levels [...] at least through mid 2013." This date-based forward guidance was therefore coincident with an exceptional coordination of views about the path of future short-term interest rates over the next 2 years. In addition, forecasters having significantly more optimistic revisions of their macroeconomic forecasts exhibited revisions of interest rates forecasts that were not significantly different from the ones of other forecasters.

We then compare these revisions of interest rate forecasts with the counterfactual revisions implied by a normal-time Taylor rule estimated using individual SPF forecasts on a pre-crisis period. We show that optimistic revisions of the macroeconomic outlook were associated with interest rate expectations below the counterfactual rate. On the contrary, pessimistic revisions were associated with interest rate expectations below or in line to the counterfactual rate. These patterns of correlations are consistent with optimists interpreting forward guidance announcements as Odyssean, i.e. as a promise of future accommodation hence better macroeconomic conditions, while pessimists interpret it as Delphic, i.e. as a signal of worse macroeconomic conditions. In sum, the disagreement on the future macroeconomic variables coincides with different interpretations of the same policy announcement on future rates.

Importantly, this correlation between views on fundamentals and views on monetary policy emerges with date-based forward guidance announcements and it does not pertain to the zero lower bound period as such. In the period that goes from 2009Q1 – when policy rates started to be at the ZLB – to 2011Q2 – the last quarter before the first date-based forward guidance announcement – revisions in inflation and interest rates forecasts were positively correlated both for optimists and pessimists. This correlation turns negative after date-based forward guidance announcements for optimistic agents only. This finding reinforces our view that fixed-date forward guidance induced a switch in the correlation between expectations on policy and expectations on fundamentals *for some but not all* agents. Before these forward guidance announcements, changes in future interest rates were mostly interpreted as Delphic i.e. related to either optimistic or pessimistic perceived changes in future fundamentals. This latter piece of evidence is consistent with a large body of literature showing that, in normal times, monetary policy decisions reveal information about the future state of the economy (Romer and Romer, 2000; Gürkaynak et al., 2005; Nakamura and Steinsson, Forthcoming), that agents understand the normal time policy reaction function of the central bank (Carvalho and Nechio, 2014; Andrade et al., 2016), and that forward guidance policy implemented before 2011 were on average interpreted as Delphic (Campbell et al., 2012; Del Negro et al., 2015; Campbell et al., 2016).

We also analyse individual answers in the Michigan Survey of Consumers over the same period and find that households' expectations exhibit patterns that are comparable to the ones observed in the SPF.

Theory. Our second contribution is to introduce an otherwise standard New-Keynesian model to study the trade-off induced by heterogeneous interpretation of forward guidance communication. In our model, a discount factor shock forces the economy to the ZLB for an uncertain number of periods. Both the central bank and the agents receive a private signal on the length of the trap. Based on its own information the central bank forms an expectation about the number of periods during which it will keep interest rates at zero. This expectation is publicly obsevable. We show that the agreement on future rates is a monotonic function of the perceived precision of the information held by the central bank. However, although all agents observe the same announcement and believe that the authority has superior information, they can still disagree on the reasons that lead to the same expected rate path depending on their prior on the commitment ability of the central bank.

Specifically, as the commitment ability is not observable, agents can agree to disagree about the policy actually conducted. Odyssean agents – who believe in the commitment ability of the central bank – see the announcement as including some periods of extra accommodation, contingent to any possible realization of the length of the trap. By contrast, Delphic agents – who do not believe in the commitment ability – consider there will be no period of extra accommodation at the end of the trap. As a result, Delphic agents can misinterpret the announcement as signal of a longer trap. This confusion can only arise once the ZLB binds in which case policy rates will be at zero for some period irrespective of the central bank commitment ability.

At the equilibrium, and consistent with the patterns observed in survey data, agents can agree on the path of future interest rates but still disagree about the potential future accommodation at the end of the trap leads them to disagree on future aggregate demand and inflation. Optimistic agents who interpret the policy as Odyssean anticipate higher future inflation and consumption while pessimistic agents who interpret the policy as purely Delphic expect lower future inflation and consumption. Moreover, the different interpretations of forward guidance induce different actions which offset each other: optimists consume more today in anticipation of a boom tomorrow; pessimists consume less today in anticipation of a recession tomorrow. These offsetting choices hamper the efficiency of date-based forward guidance even in situation where the private sector agrees on a path of low future interest rates.

Optimal policy. Finally, we use the model to study how heterogeneous understandings of forward guidance policy affects its optimality. We show that the optimal number of periods of extra accommodation is a hump-shaped function of the fraction of Delphic agents: a few of these agents may lead the central bank to reinforce its forward guidance by pushing higher consumption of Odyssean agents. Yet, when a sufficiently large fraction of agents are Delphic, additional periods of policy accommodation have a negative effects on current macroeconomic conditions, thus making the implementation of an Odyssean forward guidance inefficient and even detrimental compared to the implementation of a pure Delphic announcement. This may lead a central bank, who has commitment ability, to engage into Delphic rather than Odyssean forward guidance. The main reason is that these Delphic agents may become overly pessimistic, i.e. an Odyssean announcement can make them believe that the trap is longer than it actually is. This triggers a direct effect – these agents consume less because of worse expected fundamentals – but also a general equilibrium effect – optimists also consume relatively less because pessimists negatively weight on aggregate demand. In contrast, if the fraction of Odyssean agents is sufficiently large, Odyssean forward guidance can stimulate their optimism and push consumption and inflation expectations higher by committing to low rates for longer.

Our results have two main policy implications. First, in the spirit of Woodford (2012), they stress the key role of additional policies, such as central bank communication or balance sheet policies, in alleviating the heterogeneity of private agents' perception.[5] This is crucial to make forward guidance effective and to avoid its potential detrimental impact. Identifying how to avoid the ambiguity of forward guidance announcements is beyond the scope of the present paper although the issue is briefly discussed in the conclusion. Second, our results also emphasize that looking at the reaction of expected future interest rates is not enough to gauge whether forward guidance has been effective. Our empirical analysis provide, as a by product, simple alternative survey based criteria to qualify if agents predominantly interpreted the policy as good news.

Related literature. The increasing reliance of central banks to forward guidance has recently stimulated an intense debate on the forward-looking properties of New Keynesian models. Carlstrom et al. (2012) and Del Negro et al. (2015) underline that standard DSGE models predict incredibly high macroeconomic impact of changes in future rates even if such change will occur in the far future: a result that Del Negro et al. (2015) dubbed the "forward guidance puzzle". Some explanations of this puzzle rely on some form of "discounting" in the aggregate Euler equation stemming for the limitation of general equilibrium effects due to borrowing constraints (McKay et al., 2016), imperfect information (Wiederholt, 2014; Angeletos and Lian, 2016), bounded rationality – alone (García-Schmidt and Woodford, 2015; Gabaix, 2016) – or in conjunction with incomplete markets (Farhi and Werning, 2017). This literature calls for a wide revision of the benchmark model which has consequences for the predictions on how a New-Keynesian economy reacts to changes in future fundamentals also in normal times.

We focus instead on a specific distortion that the ZLB imposes on communication policy. We provide an explanation of the lack of effectiveness of forward guidance announcements that remains inside the logic of the benchmark model and strictly pertains to the presence of the ZLB. Our key point is that the commitment ability of the policy maker, which is necessary for the implementation of the optimal policy at the ZLB, is only observable ex-post and therefore, ex-ante, agents can agree to disagree on the central bank having such ability. Hence, the ambiguity in the interpretation of credible announcements emerges once the ZLB binds, which is what we document in our empirical analysis.

The fact that we find sizeable Delphic interpretation of forward guidance announcements – i.e. that monetary policy conveys information about the future state of the economy to the private sector – connects our paper to the literature emphasizing the signaling channel

[5] See the conclusion for further discussion and references.

of monetary policy (e.g. Ellingsen and Söderström, 2001). Such a channel was found to be empirically relevant (see Romer and Romer, 2000; Gürkaynak et al., 2005; Nakamura and Steinsson, Forthcoming, among others). Campbell et al. (2012) extend these results in a sample that includes the Great Recession and FG policies.[6] More recently, Melosi (2016) and Nakamura and Steinsson (Forthcoming) investigate the effects of this channel in general equilibrium models on the dynamic of inflation expectations or on the response of macroeconomic variables to policy announcements.

In contrast to these papers, in our model imperfect signaling originates from the possibility of endogenous – rather than exogenous – deviations from a normal-time Taylor rule (i.e. the deviations provided for by the implementation of Odyssean Forward Guidance at the ZLB). In addition, our analysis complements this literature by analyzing the dispersion of individuals' macroeconomic forecasts, rather than aggregate measures, at the time of forward guidance announcements. Looking at the cross-sectional dimension of the impact of forward guidance announcements is key in understanding the trade-off that it generates: leveraging on the optimism of commitment believers comes at the cost of inducing excess pessimism of non-believers.

Our findings also relate to the recent literature studying survey data, and in particular their cross-sectional heterogeneity, to characterize the formation of macroeconomic expectations (e.g. Mankiw et al., 2003; Coibion and Gorodnichenko, 2012, 2015; Andrade and Le Bihan, 2013; Andrade et al., 2016). In particular, Carvalho and Nechio (2014) and Andrade et al. (2016) provide evidence that – in normal times – individual (respectively households and professional forecasters) views on future inflation and activity influence the perception of future short term interest rates in a way that is consistent with the reaction function of the Fed. Engen et al. (2014) provide survey evidence that it took time for the Fed to convince private agents that its unconventional policies implied a long period of low future interest rates.

We contribute to this literature by documenting that, at the time of date-based forward guidance, agents believed and understood future policy actions (i.e. future interest rates), but this coordination was not sufficient to ensure the success of the policy because only a fraction of agents interpreted the policy as signaling a deviation from the normal-time Taylor rule. In addition, we underline that looking at the reaction of expected future interest rates is not enough to gauge whether such communication has been effective. A byproduct of our empirical analysis is to provide simple alternative survey based criteria to qualify if agents predominantly interpreted the policy as good news.

[6] See also Del Negro et al. (2015) and Campbell et al. (2016) for similar evidence on the more recent period.

Several papers investigate the role of imperfect information at the ZLB. Kiley (2014) analyses how sticky information may solve several policy paradoxes appearing with the ZLB in the New Keynesian model. Bianchi and Melosi (2015a) emphasize that transparency on future deviations from active stabilization is welfare improving as it anchors medium to long-run macroeconomic expectations. Bianchi and Melosi (2015b) study the consequences of the private sector's uncertainty on whether a stabilizing monetary/fiscal policy mix regime will be abandoned when the economy reach the ZLB. Michelacci and Paciello (2016) investigate the consequences of ambiguity on future policy actions for the effects of monetary policy when agents are ambiguity-averse. Wiederholt (2014) argues that forward guidance can be detrimental as it triggers a coordination failure by providing information about an inefficient shock – a principle that parallels the analysis of the social value of information by Angeletos et al. (2016). Gaballo (2016) and Amador and Weill (2010) provide two examples on how learning externalities may make a partial release of public information welfare detrimental.

Our approach differs from other incomplete information approaches as uncertainty concerns neither the current allocation nor future policy actions (in line with empirical properties of surveys). In our model, agents see the expected path of policy actions conveyed by forward guidance announcements as highly accurate and, hence, they agree on future nominal interest rates.[7] Moreover, in our model, the inefficiency of forward guidance communication goes beyond the simple revelation of an inefficient shock as it may induces *excess* pessimism, that is, expectations that the trap will be longer than the central bank expects.

Finally, our paper contributes to the literature on optimal monetary policy at the ZLB. Krugman (1998), Eggertsson and Woodford (2003) and Werning (2012) study the optimal policy at the ZLB in an infinite horizon model, emphasizing the associated commitment problem (see also Jung et al. (2005), Adam and Billi (2007) and Nakov (2008)). Recently, Bilbiie (2016) characterized the optimal policy using closed-form solutions in alternative formulations of the liquidity traps. Coibion et al. (2012) determines the optimal inflation target in the presence of occasional liquidity traps. In contrast to these studies, the distinctive feature of our setup is that we allow for heterogeneous beliefs about the type of policy conducted by the central bank. Bodenstein et al. (2012) investigate quantitatively how imperfect credibility of future policy rate announcements lowered the impact of forward guidance policies. They conclude that imperfect credibility should be compensated by extending the period of low interest rate providing future monetary stimulus.

[7] Farhi and Werning (2017) provide an another mechanism consistent with this empirical property. In Farhi and Werning (2017), agents do not fully understand the consequences of this period of low interest rates (as any other general equilibrium implication), while, in our model, they disagree on its meaning.

In contrast, we show that the impact of forward guidance may not only be muted compared to the full commitment case, but that it can even be detrimental when the type of policy that is implemented is not shared broadly enough among agents. In particular, despite agents perfectly see announced future rates, forward guidance cannot be indefinitely reinforced without hurting welfare because the trade-off generated by disagreement becomes particularly negative as the time horizon increases.

The paper is organized as follows. We present the stylized facts specific to forward guidance in Section 2. We introduce a New-Keynesian model with heterogenous beliefs in Section 3 and we characterize the optimal monetary policy in Section 4. Section 5 concludes by discussing the policy implications of our results.

2 Survey-based stylized facts on date-based forward guidance

In this section, we present new facts on the cross-sectional dispersion of forecasts observed in surveys of expectations during the period when the Fed conducted a date-based forward guidance policy.

On August 8, 2011, the FOMC stated that "The Committee currently anticipates that economic conditions [...] are likely to warrant exceptionally low levels for the federal funds rate at least through mid-2013". Following this first announcement of date-based forward guidance, the Fed extended the horizon twice. On January 25, 2012 the horizon was extended to "... at least through late 2014". And on September 13, 2012, it was extended to "...at least through mid 2015". The FOMC had already implemented forward guidance before Summer 2011; starting its December 2008 meeting the Fed issued "open-ended" statements and referred to a period of "exceptionally low interest rates" that will last "for some time" or "for a long period of time". So the horizon during which interest rates would be kept at zero was less precise. Date-based forward guidance ended on December 12, 2012, when the FOMC moved to a "state-based" forward guidance whereby it committed to keep policy rates at zero as long as inflation and unemployment rates did not reach specific numerical thresholds (and inflation expectations remained anchored) and the reference to a specific date for keeping rates at exceptionally low levels was dropped altogether.

As we detail below, this period of date-based forward guidance coincided with striking patterns in the heterogeneity of forecasts observed in the US Survey of Professional Forecasters (SPF). Professional forecasters agreed that interest rates will stay low for long. At the same time, they had two different views on the future macroeconomic outlook: some revised future inflation and consumption growth upward while some others revised downward. As we show, optimistic revisions were associated to an increase in the degree of monetary policy accommodation. By contrast, pessimistic revisions were associated to no increase or a decrease in such accommodation. Moreover, such different interpretations of the same future path of policy rate did not prevail before date-based forward guidance announcements.

In addition, although data characteristics prevents us to conduct exactly the same analysis than for the SPF, individual households' expectations observed in Michigan Survey of Consumers feature comparable patterns over the same period and the heterogeneity in expectations translated in heterogeneous decisions. Before describing in detail how the cross-section of forecasts evolved when the FOMC started to conduct date-based forward guidance, we first document what happened to the average of individual forecasts in the US SPF at that time.

2.1 Average expectations did not react much in the US SPF

Figure 1 displays the evolution of the average of individual short term interest rate forecasts 1 quarter, 1 year and 2 years ahead. Three specific subperiods are highlighted: 2008Q4-2011Q2 which corresponds to the time when the US economy reached the ZLB and the Fed conducted its "open-ended" forward guidance; 2011Q3-2012Q3 which corresponds to the "date-based" forward guidance period; and 2012Q4-2013Q2 which corresponds to a "state-based" forward guidance policy.

This figure shows that 1-quarter ahead short-term interest rate forecasts reached levels close to zero in 2009 that is when the US economy hit the ZLB. 1-year and 2-year ahead short-term interest rate forecasts were already low when date-based forward guidance policy started, but they went further down during that period to finally reach levels close to zero and comparable to the 1-quarter ahead forecasts.

As Figure 2 illustrates, over the same date-based forward guidance period, 1-quarter, 1-year and 2-year ahead consumption growth (resp. inflation) forecasts only slightly decreased (resp. increased). This makes our first fact.

Fact 1. *Date-based forward guidance was coincident with a drop in the mean forecasts of the short-term nominal interest rates to historically low (and close to zero) levels up to 2 years, a limited increase in the average forecast of inflation and a limited decrease in the average forecast of consumption growth.*

These patterns are reminiscent of results stressed in previous studies documenting the reaction of macroeconomic expectations to various forward guidance announcements (Campbell et al., 2012; Del Negro et al., 2015; Campbell et al., 2016): such policy lowered expected future short-term interest rates but the reaction of inflation, output or consumption growth were much smaller and sometimes negative. One reading is that forecasters had a Delphic interpretation of forward guidance: announcements of future low interest rates were interpreted as signalling worse future macroeconomic conditions.

2.2 Forecasters agreed that short-term interest rates will remain low for long

We now analyse the cross-sectional dispersion of professionals' forecasts. Figure 3 presents the evolution of the interquartile range in the cross-section distribution of individual 1-quarter, 1-year and 2-year short-term nominal interest rate forecasts since 2002.

This figure reveals that, in addition to the lowering in average interest rate forecasts documented above, the date-based forward guidance period was associated with a sharp drop in the dispersion of short-term nominal interest rate forecasts. In particular, this dispersion on medium-term (1-year and 2-year) nominal interest rate forecasts dropped to their lowest value since 2002 (and actually since at least 1982 when the survey started to collect such interest rate forecasts). So, the commitment to keep interest rates at zero until an explicit date was associated with an exceptional coordination of opinions about future interest rate on an horizon broadly consistent with the length of the initial date-based forward guidance announcement. By contrast, the "open-date" announcements used from 2008Q4 to 2011Q2 were only associated with a strong coordination of opinions on 1-quarter interest rate forecasts.[8]

We can summarize this paragraph's findings as follows:

Fact 2. *When "date-based" forward guidance started, professional forecasters' disagreement on future short-term interest rates 1-year and 2-year ahead declined sharply and reached an historical low.*

To summarize, Fact 2 together with Fact 1 indicate that, during the date-based forward guidance period, forecasters revised their short-term interest rate forecasts to eventually agree on a period of low interest rates over the next two years. Therefore, the policy announcement has been perceived as credible. Note that this does not necessarily imply that agents also agree for more long term horizons. We next investigate to which extent such agreement on interest rates reflected an agreement on the future macroeconomic outlook.

[8]As this can be observed in Figure 3, this drop in medium-term interest rate forecasts was already initiated during the "open-ended" forward guidance episode for the 1-year horizon (-50% of disagreement for the peak of 2008) but accelerated during the "date-based" forward guidance episode (-75% during the period). For the 2-year horizon, the drop mainly happened during the "date-based" episode (-22% during the "open-date" period and -67% during the "date-based" period).

2.3 Forecasters interpreted the same path of future interest rates differently

We now document how agents revised their expectations about macroeconomic conditions, i.e. inflation and consumption growth. We show that, despite that forecasters all revise similarly their expectations on interest rates, optimistic revisions were associated with an Odyssean interpretation of the announced path for future interest rates while pessimistic revisions were in line with a Delphic interpretation of the same path.

Heterogeneous revisions of inflation and consumption forecasts. We first investigate how professional forecasters updated their medium-term (2 years ahead) forecasts of consumption growth, inflation and short term interest rates in the SPF released for 2011Q4, 2012Q1 and 2012Q4, i.e. after each of the three date-based forward guidance announcements.[9]

Table 1 presents the average revisions of these 2-year forecasts for three different subgroups of forecasters. "Optimists" who have both revisions of inflation and consumption growth above the average revision across forecasters observed at that date. "Pessimists" who have both revisions of inflation and consumption growth that are below the average. And "Pessimists and others" which are forecasters that are not optimists.

By construction, optimists have higher revisions of inflation and consumption forecasts than pessimists. However, Table 1 reveals that, on average, every group revised similarly (downward or non-significantly differently from zero) their interest rate forecasts. This happened despite revisions of inflation and consumption forecasts were *significantly* different when comparing the average of the two groups. These results also applies, although with a lower degree of significance, when comparing optimists with the rest of forecasters as a whole.

Different perceptions of future monetary policy. That optimists (resp. pessimists) revised their inflation and consumption forecasts upward (resp. downwards) when revising interest rates downward is consistent with the anticipation that lower future interest rates correspond to a more accommodative monetary policy (resp. weaker fundamentals) in the

[9] Specifically, we look at individual revisions of forecasts between two subsequent surveys observed after each of the "date-based" forward guidance announcements (which were made on August 9, 2011; January 27, 2012 and September 13, 2012): *(i)* revisions between the the 2011Q3 survey (conducted between July 29 and August 8, 2011) and the 2011Q4 survey (conducted between October 27 and November 8, 2011); *(ii)* revisions between the 2011Q4 survey and the 2012Q1 survey (conducted between January 27 and February 7, 2012); *(iii)* revisions between the 2012Q3 survey (conducted between July 27 and August 7, 2012) and the 2012Q4 survey (conducted between October 26 and November 6, 2012). The relatively low frequency of the SPF data prevents from observing the reaction of individual forecasts on a specific day. See (Del Negro et al., 2015) for an attempt that uses monthly data.

Forecast revisions	Optimists	Pessimists	Pessimists and others
2011Q4			
Share of individuals	19%	29%	81%
Consumption	.32 (.28) [**/#]	-.20 (.19)	-.05 (.41)
Inflation	.19 (.22) [**/#]	-.22 (.14)	-.12 (.55)
Nominal rates	-.41 (.46)	-.38 (.30)	-.42 (.44)
Revisions of shadow Taylor-rate	.35 (.25) [***/###]	-.37 (.14)	-.16 (.37)
2012Q1			
Share of individuals	22%	23%	78%
Consumption	.79 (.33) [***/##]	.13 (.24)	.19 (.24)
Inflation	.48 (.29) [***/###]	-.26 (.29)	-.12 (.30)
Nominal rates	-.37 (.55)	-.04 (.08)	-.04 (.07)
Revisions of shadow Taylor-rate	.86 (.55) [**/#]	-.17 (.31)	.05 (.35)
2012Q4			
Share of individuals	36%	24%	64%
Consumption	.20 (.19) [***/###]	-.26 (.22)	-.21 (.26)
Inflation	.19 (.23) [***/#]	-.32 (.32)	-.17 (.36)
Nominal rates	-.04 (.15)	.02 (.02)	-.02 (.06)
Revisions of shadow Taylor-rate	.23 (.30) [***/##]	-.36 (.27)	-.27 (.26)
Corr(rev. inflation, rev. rates)			
2009Q1-2011Q3	.41 (.07)	.15 (.07)	.24 (.07)
2011Q4-2012Q4	-.26 [†††] (.20)	.38 (.25)	.22 (.15)

Table 1: Average revisions of 2 years ahead forecasts across groups of forecasters.

This table reports the cross-section average of individual revisions for consumption, inflation and short-term nominal interest rates forecasters 2 years ahead for different groups of forecasters. Forecasters are classified as 'Optimists' when both their inflation and consumption revisions are above the cross-sectional mean at the specified date, as 'Pessimists' when both inflation and consumption revisions are below the cross-sectional mean. The group 'Pessimists and others' gathers all forecasters excluding the optimists. Standard deviations are reported in parenthesis. T-stats significance levels are reported in brackets: ***, **, * indicate that the average revisions of optimists are significantly different from the average revision of pessimists at the 1%, 5% and 10% levels; ###, ##, # indicate that the average revisions of optimists are significantly different from the average revision of pessimists and others at the 1%, 5% and 10% levels; †††, ††, †indicate that correlation between optimists' revisions in inflation and interest rates after date-based forward guidance announcement is significantly different from the same correlation before forward guidance announcements at the 1%, 5% and 10% levels.

future.

We provide evidence that this is indeed the case by comparing the observed individual revisions in nominal rate to the individual revisions of their subjective shadow Taylor rate implied by each forecasters' revision in inflation and consumption growth expectations. More precisely, we compute by how much each forecaster should have revised their 2-year forecasts of short term-interest rate, if they were to infer these forecasts from their pre-ZLB perceived Taylor rule.[10] A lower revision in the nominal rate compared with the revision in the shadow Taylor rate corresponds to a policy accommodation (i.e. the expectation of a accommodative monetary policy shock) with respect to normal times. Conversely, a larger revision in the nominal rate than the revision in the shadow Taylor rate corresponds to a policy tightening (i.e. the expectation of a restrictive monetary policy shock) with respect to normal times.

Table 1 reports the average and the standard deviation of this revision in the shadow Taylor rate for optimistic and pessimistic forecasters.

We obtain that agents making pessimistic revisions of fundamentals have revised their expected shadow rate downward by the same extent or by even more than their expected nominal interest rates. The result suggests that, for these forecasters, future monetary policy was perceived as either just in line with what the normal time reaction function of the central would predict, or even tighter compared to this with normal time rule. This is consistent with pessimistic revisions being associated with a Delphic view of future monetary policy.

In contrast, we obtain that agents making optimistic revisions have revised their expected shadow rate by less than their expectations of nominal interest rates, with the difference being statistically significant. In relative terms, this then implies that these agents were expecting more policy accommodation, consistently with an Odyssean view of future monetary policy. Importantly, agents expected the policy accommodation to take place within the horizon of the policy announcement as they anticipated a rise in the shadow Taylor rate at the 2-year horizon, consistently with a lift-off.

This leads to the following fact:

Fact 3. *Two groups of forecasters coexisted during the date-based forward guidance period despite they had a similar downward revision in expected interest rates. Optimistic revisions of inflation and consumption forecasts were associated with an expectation of more policy accommodation, consistent with an Odyssean interpretation of the announcements. By contrast, pessimistic revisions in inflation and consumption forecasts were associated with an expectation of equal or less policy accommodation, consistent with a Delphic interpretation of the*

[10] We use the panel of individual forecasts in the SPF to estimate a perceived Taylor rule using data prior to 2008Q3. The postulated rule includes an interest rate smoothing term and reactions to an inflation and to an output growth gaps. The target inflation and potential growth are obtained from individual 10 year ahead forecasts. The estimation features individual fixed effects. We then infer revisions in the individual shadow Taylor rate using their estimated perceived Taylor rules and their individual revisions in macro-economic forecasts.

same announcements.

It is interesting to remark that the fraction of optimistic forecasters was non-negligible during the date-based forward guidance period and that this fraction has varied over time: it has increased from roughly 20% to 36% with the later announcement of September 2012. This suggests some forward guidance announcements were more effective than others in convincing a significant fraction of the population that it intended to provide further accommodation.[11] Del Negro et al. (2015) provide complementary evidence that the interpretation of forward guidance announcements changed over time. Finally, despite some evolution of optimists and pessimists, it is important to note that the fraction of pessimists remained sizeable for the three specific dates considered. So how to interpret the path of future interest rate announced remained ambiguous for each of these three announcements

The specificity of the Date-Based Forward Guidance period. The difference between revisions in expected interest rates and revisions in shadow Taylor rates suggests that FG announcements modified the relation between expected interest rates and expected fundamentals for optimistic agents.

This is confirmed by the results shown in the bottom panel of Table 1. For optimists, the correlation between revisions of inflation forecasts and interest rate forecasts is negative for the dates corresponding to date-based forward guidance, again consistent with an Odyssean understanding of such announcements. In contrast, this correlation was positive until 2011Q3 for all groups of forecasters and it remained positive for non-optimists after date-based forward guidance, consistent with a Delphic interpretation of the announcements.

Overall, our evidence suggests that, with the date-based forward guidance period, the expectation of lower interest rates led to more optimistic revisions both on fundamentals and future monetary policy stance for some agents. This is a key difference with normal time evidence, showing that, on average, expectations of lower interest rates signal worse macroeconomic outlook (see e.g. Campbell et al. (2012), Nakamura and Steinsson (Forthcoming)) and that agents understand the mapping between policy rates and fundamentals (see e.g.

[11] Interestingly, the Federal Reserve press release of August 9, 2011: said *"The Committee currently anticipates that economic conditions – including low rates of resource utilization and a subdued outlook for inflation over the medium run – are likely to warrant exceptionally low levels for the federal funds rate at least through mid-2013.*. The Federal Reserve press release of September 13, 2012 stated *To support continued progress toward maximum employment and price stability, the Committee expects that a highly accommodative stance of monetary policy will remain appropriate for a considerable time after the economic recovery strengthens. In particular, the Committee [...] currently anticipates that exceptionally low levels for the federal funds rate are likely to be warranted at least through mid-2015.* The wording "for a considerable time after the economic recovery strengthens" probably helped to clarify the intention of the FOMC.

Carvalho and Nechio (2014) and Andrade et al. (2016)).

Disagreement about macroeconomic forecasts when the Fed started date-based forward guidance. The combination of similar views about future interest rates together with disparate views on future inflation and consumption that started with date-based forward guidance has some consequences on the disparities in the levels of individual 2 years ahead forecasts. More precisely, compared to historical standards, the beginning of date-based forward guidance is associated with a significant excess disagreement about these medium-term forecasts.

This is illustrated by Figure 4 which displays the residuals from a regression of the disagreement about 2-year ahead forecasts of consumption (resp. inflation) on the disagreement about 2-year ahead forecasts of short-term nominal interest rates controlling for the disagreement about 1-quarter ahead consumption and inflation forecasts and estimated on a pre-crisis sample (1982Q2-2008Q4). The beginning of the date-based forward guidance policy is again a striking outlier. Before August 2011 the residuals are not significantly different from zero. Disagreement about future inflation stays in the range of what its correlation with disagreement on 1 quarter inflation forecasts and disagreement on interest rate would predict. By contrast, disagreement on future consumption and inflation becomes significantly higher than its predicted value starting August 2011. Controlling for short-term disagreement, disagreement about medium-term inflation started to be much higher than what disagreement about future interest rates would imply at the time the date-based forward guidance started to be conducted.

A concern is that this excess disagreement results from an increase in uncertainty. Yet, our control for disagreement on short-term inflation and consumption forecasts, which are often used as a proxy for uncertainty, should partly capture such an increase. As we document in an Appendix, there is no clear evidence that date-based forward guidance coincided with an increase in uncertainty as measured with five alternative measures of macroeconomic uncertainty introduced recently. Furthermore, one still observes excess disagreement in 2-year consumption and inflation forecasts during the date-based forward guidance period if one includes such alternative measures as additional controls in the regression exercise.

2.4 Further evidence using household surveys

We also investigate how expectations of US households evolved when date-based forward guidance was conducted by exploiting the Michigan Survey of Consumers. More specifically, we analyse households' expectations about *(i)* the evolution of interest rates over the next 12 months (increase, stay constant, decrease), *(ii)* the evolution of prices over the next 12 months (average inflation rate), *(iii)* whether it is a good time to buy durables (good, neutral, bad) and *(iv)* the expected overall aggregate business conditions over the next 12 months (good, neutral, bad).[12]

We start with households' expectations of future interest rates. Figure 5 plots the share of respondents anticipating that interest rates will stay constant over the next 12 months over a 2002-2014 sample. The chart underlines that this share jumped to levels above 50% during the date-based forward guidance period.[13] So the majority of households in the Michigan Survey of Consumers interpreted forward guidance announcements as indicating that interest rate will not increase (at least) over the next year.[14]

In a second step, we analyze the heterogeneity of expectations across surveyed households. We split the sample of respondents expecting stable or lower interest rates into three categories: optimists if they expect better aggregate business conditions and have inflation expectations above the average; pessimists if they expect worsening business conditions and have inflation expectations below average; and others. Table 2 reports the average expectations of each of these groups observed in the month following the three date-based forward guidance announcements of August 2011, January 2012 and September 2012. The results reveal that among households who anticipated stable or lower interest rates, the ones who expected higher inflation and better economic conditions also considered that the time was more favorable to purchase durable goods. By contrast, pessimists expect lower inflation and

[12]Each month, about 500 households are surveyed. The sample is designed to be representative of the US population. 60% of individuals that are first time respondents to the survey. Due to this repeated cross-section structure it is not possible to compute revisions of forecasts between 2 subsequent survey rounds for the whole sample of household surveyed. 40% of households that are interviewed twice but with a 6 months period between the two interviews. In the appendix, we analyse individual revisions based on this subsample. Moreover, several questions asked to households call for qualitative rather than quantitative answers. These data limitations prevent us to conduct an analysis as detailed as the one we can conduct with the SPF. The other way around, the question on current durable good consumption in the Michigan survey as no equivalent in the SPF. As Bachmann et al. (2015) emphasize, consumption of durable goods follows total consumption so that answers to this question is a good proxy for the current total consumption decisions.

[13]This surge in the proportion of households expecting that interest rate will not increase is mainly driven by households expecting interest rates will stay constant which reaches an all-time high (above 50%) during the date-based forward guidance. This contrasts with the 2009 episode where the high share of households foreseeing that interest rate will not increase is mostly due to a large majority of people expecting a drop in future interest rates.

[14]This complements the results of Carvalho and Nechio (2014). They show that, in normal times, some households in the Michigan survey understand monetary policy: they adjust their interest rate expectations in a way that is consistent with a Taylor rule and their views on the macroeconomic outlook. We find that a substantial share of households reports interest rate forecasts consistent with the date-based forward guidance policy implemented at the ZLB.

a smaller fraction among them consider that it is time to purchase durable goods.[15] In the appendix, we document that this pattern also holds if one looks at revisions of forecasts for the households that are surveyed twice.

	Optimists	Pessimists	Pessimists and others
	Averages observed in 2011m9		
Fraction of respondents	5%	50%	95%
Good times for durable	.50	.27	.25
Inflation	6.64	1.77	3.51
	Averages observed in 2012m2		
Fraction of respondents	13%	28%	87%
Good times for durable	.55	.30	.36
Inflation	5.50	1.37	3.10
	Averages observed in 2012m10		
Fraction of respondents	15%	30%	85%
Good times for durable	.46	.24	.29
Inflation	7.34	1.95	3.37

Table 2: Average of forecasts across groups of households.

This table computes the cross-sectional mean for current durable consumption (qualitative answers) and expected inflation over next 12 months (quantitative answers) when forecasters are sorted according to their expected business conditions and nominal interest rate over next 12 months. Pessimists expected lower inflation than the cross-sectional mean and had a negative view of the business/financial conditions over the next 12 months. Others include all households except optimists. All forecasters considered expect constant or decreasing nominal interest rates over the next 12 months.

Overall, we get the following stylized fact:

Fact 4. *During the period of date-based forward guidance, among households expecting interest rates not to increase over the next 12 months, the ones anticipating better economic conditions*

[15] Note that observing a fraction of optimistic households who declare to consume more when they anticipate higher inflation and better economic conditions does not contradict the results in Bachmann et al. (2015) who find that, *on average*, during the ZLB period, households who report higher inflation expectations in the Michigan survey are likely to consume less. Moreover, in the Appendix, we also report results that are very similar to theirs over the date-based forward guidance episode. Namely, we drop the expected inflation criteria in the definition of optimistic households and consider the larger group of agents who expect an improvement in future activity. The results show that this broader class of optimistic households is more likely to purchase durable goods but also expect lower inflation than the average household. These optimists behave like in Werning (2012)'s model of forward guidance in which such policy increases consumption today by generating expectations of a boom in activity tomorrow. Finally while this broader class of optimists by definition accounts for a larger fraction of the sample of households surveyed, it does not represent the whole sample: again optimists coexisted with pessimists during the period of date-based forward guidance.

and higher inflation were more likely to purchase durable goods than the ones anticipating worse economic conditions and lower inflation.

To sum up, the period of date-based forward guidance is characterized by striking patterns in the cross-section distribution of professional forecasters or households expectations. Some of them had optimistic expectations associated with the belief of future policy accommodation – Odyssean interpretation of forward guidance. Some other had pessimistic expectations associated with the belief that the policy announcement was a signal of bad news on future fundamentals – Delphic interpretation of forward guidance – (Fact 3). Such an heterogeneity of individual forecasts was coincident with a consensus (or a lower disagreement) on the path of future interest rate (Fact 2), and a mild response of average survey expectations of these two variables (Fact 1). Finally, such heterogeneous readings of future low interest rates implied different perception on whether it is a good time to purchase durable goods, which other studies have shown is correlated to current consumption decisions (Fact 4).

In the next Section, we develop a model that replicates these stylized facts.

3 A model of disagreement on policy

In this section, we extend a standard New-Keynesian model with a zero lower bound (ZLB) in the spirit of Eggertsson and Woodford (2003) by allowing for heterogenous beliefs. Our aim is to model a situation in which agents may disagree on the type of policy conducted by the central bank although they agree on future interest rates. We show that this is the case when agents are uncertain about fundamentals have different priors on the commitment ability of the central bank, but view central bank's announcements of expected rates as accurate.

When agents perceive a central bank's announcement of expected future rates to be based on information of relative high precision (relative to their private information) then their disagreement about expected rates must unambiguously decrease. However, different priors on the commitment ability of the central bank - and so about the type of policy implemented - creates heterogeneity in the interpretation of the public announcement as a signal on the central bank's expectation about the state of the economy. In particular, an announcement that policy rates will be low for longer can be either interpreted optimistically as a signal of a more accommodative future stance (i.e. Odyssean forward guidance), or pessimistically as a signal of weaker future fundamentals (i.e. Delphic forward guidance).

This ambiguity in signaling is a peculiar outcome of times where the ZLB binds. In fact, only in such a case, the authority can improve on its normal time Taylor rule by committing to a future deviation from it. On the contrary, away from the ZLB, there cannot be ambiguity as incentives to deviate disappear in normal times.

In the remainder of this section, we first present the key equations and assumptions of the modeling framework. We then characterize different equilibria, stressing the possibility of disagreement on future policy. We finally derive some positive implications and relate them to the facts of the empirical section. The optimality of the monetary policy at the ZLB in the presence of heterogenous beliefs is discussed in Section 4.

3.1 NK-economy with heterogeneous beliefs

Our model is a standard New-Keynesian model extended to account for heterogeneous beliefs. To streamline the presentation, we directly discuss the three key equations of the model – the IS curve, the New-Keynesian Phillips curve and the monetary policy rule – expressed in log-linear deviations from the steady-state. The detailed microfoundations underlying these equations as well as the detailed derivations are postponed to Appendix D.

Consumption equation. There is a unit mass of atomistic agents indexed by $i \in [0, 1]$; they are homogeneous in any respect, except that they may hold different beliefs. Their consumption decisions comply with the standard Euler equation (expressed in log-linear deviations from the steady-state):

$$c_{i,t} = -\gamma^{-1}(E_{i,t}[\xi_{t+1}] - \xi_t + r_t - E_{i,t}[\pi_{t+1}]) + E_{i,t}[c_{i,t+1}], \tag{1}$$

where $c_{i,t}$ denotes the consumption of agent i at time t, γ is the inverse of the inter-temporal elasticity of substitution $E_{i,t}[\cdot]$ represents the conditional expectation of agent i at date t, ξ_t is a common shock hitting at date t the discount factor in agents' utility function, r_t is the nominal interest rate and π_{t+1} is inflation at date $t + 1$.

Heterogeneity in beliefs can potentially induce different individual consumption paths, and hence different wealth profiles. In the micro-foundations of our model, detailed in Appendix D, we include a risk sharing mechanism that endogenously make agents equalize their wealth as soon as they agree on the future course of the economy. As a result, in our setup differences in wealth can only be temporary; that is, agents have the same steady state level of consumption. This assumption allow us to isolate the effect of heterogeneity of beliefs on the effectiveness of monetary policy, abstracting from potential long-run inequality effects. Similar mechanisms have been proposed in the New Keynesian literature to deal with long-run inequality (see for example Curdia and Woodford (2010) or Wiederholt (2014) among others). More generally, one could just assume that wealth will be equalized at some point in time after the trap, none of the results of this paper hinges on details of long-run risk sharing.

Shock. As it is standard in the literature of optimal policy at the ZLB, we focus on a particular sequence of discount factor shocks that drags the economy in a liquidity trap, i.e. a situation where the natural rate of interest is below its steady state for a number of periods.[16] Formally, at time 0 Nature draws a series of shocks $\{\xi_\tau\}_{t=0}^{\infty}$ from a commonly known prior distribution to the households' discount factor $\mathcal{P}$ such that $\xi_\tau - \xi_{\tau+1}$ takes value $-\xi$ with $\tau = 0, ...T-1$ and zero afterward. This generates a *trap* of *length* $T \in \mathbb{N}$ which starts at time $t = 0$ and ends in $t = T$, which is the first period out of the trap. None of the following results hinge on the particular form of the prior distribution; we will come back later on this point in due course.

We assume that agents perfectly observe the size of the discount factor shock at time $t = 0$, but they are uncertain about its persistence, i.e. the length of the trap T. Note that, from the point of view of agents, the end of the trap has the feature of a "news" on a future shock. As such it can be only assessed *ex post*. We describe how agents form expectations on the length of the trap T later in this section.

Phillips Curve. The optimal choices of firms and households lead to a New-Keynesian Phillips curve which links the current aggregate inflation to current aggregate consumption and to the average expectation of future inflation across different individuals i. Namely, we get (see Appendix D for details):

$$\pi_t = \kappa c_t + \beta \int_0^1 E_{i,t}[\pi_{t+1}]di, \tag{2}$$

with κ the slope of the Phillips curve and β the agents' discount factor. Three conditions are needed for the derivation to hold: i) agents perfectly observe the current allocation; ii) firms' shares are held by agents in equal proportion; and iii) agents participate to the same labor market. In particular, the second condition makes sure that firms maximize profit using the same stochastic discount factor and that the relevant expectation for pricing is the average one across agents' type. We show in the Appendix D.5 that our results are robust to changes in these assumptions.

[16] Similar insights can be obtained in the context of endogenously low natural rate of interest as in Eggertsson and Krugman (2012) or Eggertsson and Mehrotra (2014) as soon as agents can disagree on future monetary policies.

Monetary policy. The central bank sets a path of nominal interest rate $\{r_t\}_{t\geq 0}$. The objective of the monetary authority and the derivation of its optimal policy are addressed in Section 4. At this stage, it is sufficient to note that the monetary authority could potentially offset any change in real interest rates by appropriately setting r_t so that the term appearing in brackets in (1) equals zero at any time. In this case, there are fluctuations neither in consumption nor in inflation so that the resulting allocation entails the unconstrained first-best.

However, nominal interest rates face a zero lower bound (ZLB) so that they cannot go negative. This constraints policy actions, so that, when a negative discount shock is large enough, the best the monetary authority can do is to maintain interest rates at zero for a given period of time, *i.e.* to set the interest rate in deviation from steady-state to $r_t = -\log R$ with $-\log R = \log \beta$ the (real) natural rate of interest in steady-state.

Therefore, for our purposes and without loss of generality (given the nature of the shock considered), we restrict our attention to the following policy representation:

$$\begin{aligned} r_t &= -\log R \text{ for } t \leq T_{zlb} \\ &= \phi \pi_t, \text{ otherwise.} \end{aligned} \tag{3}$$

with $\phi > 1$ ensuring determinacy. T_{zlb} represents a lift-off date for interest rates, which is the key policy choice of the authority.

A *conventional* monetary policy would conduct to a lift-off date satisfying $T_{zlb} = T$. The monetary authority provides the maximal stimulus during the trap and then raises interest rates once out of the trap. In line with the standard *Taylor principle*, this policy ensures reaching the steady state as soon as possible.

At the ZLB, however, following such a conventional monetary policy may not be the closest to social optimum. As shown by Krugman (1998), Eggertsson and Woodford (2003) and Werning (2012), the constrained first-best policy then prescribes to keep policy rates at zero for longer than required by a strict application of the Taylor principle. Indeed, the authority can stimulate current consumption by promising to keep short-term rates at zero for some periods after the trap ends that a liftoff date $T_{zlb} \geq T$. This policy generates an expansionary stimulus after the end of the trap, i.e. boosts inflation and consumption in the future. In turn, the expectation of a future boom stimulates current consumption and reduces the impact of the crisis.

As is well-known, this *unconventional* policy is time-inconsistent. Once the recovery occurs, inflation is no longer socially desirable and the authority is tempted to renege on her promise and to set $r_t = \phi \pi_t$ from T onward as prescribed by a Taylor rule satisfying the Taylor principle. Therefore, implementing this second-best policy at the ZLB requires a central bank to commit to tolerate inflation above the target (zero in our case) for some periods in the future. We then define the following.

Definition 1. *Given a length of the trap T, the authority implements an Odyssean policy when setting a lift-off date $T_{zlb} = \mathcal{F}_o(T) > T$, whereas it implements a Delphic policy when setting a lift-off date $T_{zlb} = \mathcal{F}_p(T) = T$. Only if the authority has commitment ability, it can implement an Odyssean policy.*

We borrow this terminology from Campbell et al. (2012). When the authority implements an *Odyssean* policy, the liftoff date includes a commitment to a period of extra accommodation that depends on the actual length of the trap (i.e. the authority ties its hands as Odysseus before he meets the sirens). Conversely, when the authority implements a *Delphic* policy, the liftoff date just reflects its expected length of the trap (in this sense, it directly reveals the expectation of the authority). We shall assume $\mathcal{F}_o(\cdot)$ being invertible.

Note that having the ability to commit is a necessary but not sufficient condition for an authority to be willing to implement an Odyssean policy. There are cases in which an authority with commitment ability may find optimal to not implement any commitment (or equivalently to commit to zero periods of extra accommodation). We will discuss this issue in Section 4 when discussing the optimal policy. For the moment, we will take the type of policy as given and look at how agents react to it.

Information of the Central Bank. The central bank receives a signal of the actual length of the trap T_{cb}, which is the true realization of the trap with a probability p_{cb} and it is a non-informative signal (i.e. it is an independent draw from $\mathcal{P}$) otherwise. So the central bank expected length of the trap is given by $E_{cb,0}[T] = p_{cb}T_{cb} + (1 - p_{cb})E[T|\mathcal{P}]$. The central bank then forms an expectation on its future policy actions $E_{cb,0}[T_{zlb}]$ as

$$E_{cb,0}[T_{zlb}] = p_{cb}\mathcal{F}_\rho(T_{cb}) + (1 - p_{cb})E[\mathcal{F}_\rho(T)|\mathcal{P}] \tag{4}$$

where $\rho \in \{o, p\}$ denotes the type of policy implemented: $\rho = o$ in case of an Odyssean policy and $\rho = p$ in case of a Delphic policy (bear in mind this the *p*essimistic case). As just said above, in this section we take ρ as given.

It is worth to remark that implementing an Odyssean policy does not require that the authority fixes a path of interest rates once forever no matter what. In an optimal Ramsey plan under uncertainty the authority commits to a period of extra accommodation as a function of the realized end of the trap. In this sense, an authority who implements an Odyssean policy *expects* setting policy rates at zero for a number of periods, but should the realized length of the trap be longer (shorter), the periods of extra accommodation will be longer (shorter). Hence, $E_{cb,0}[T_{zlb}]$ is the *rational expectation* about the number of periods with zero policy rates formed by an authority depending on the type of policy implemented and its information on the length of the trap – it is never a deterministic prediction.

Information of households. Each agent receives a private signal about the length of the trap. Together with the prior, this represents the knowledge of an agent absent any announcement from the central bank.

Let us call T_i the signal received by agent i, which is the actual T extracted by Nature with a probability p_i and non-informative otherwise.

In addition, each agent also observes a central bank's announcement about its future expected actions, i.e. agents observe the expectation of the lift-off date announced by the authority, $E_{cb,0}[T_{zlb}]$. Note that we do not model the choice to make an announcement, we just assume $E_{cb,0}[T_{zlb}]$ is publicly observable.[17]

Yet, agents do not observe neither ρ nor T_{cb}. In particular, during the trap, both the Odyssean and the Delphic types of policy require keeping interest rates at the ZLB for a number of periods, so that one cannot be distinguished from the other.[18] Therefore, provided there is no evidence that contradicts their beliefs, agents can agree to disagree about both the type of authority and the length of the trap, which are only revealed when the trap actually ends and the effective policy is observed.

We define two types of agents based on their prior beliefs on the type of policy pursued by the authority.

Definition 2. *Let be $\rho_i \in \{o, p\}$ the belief of agent i on the type of policy implemented. A fraction of agents $\alpha \in [0, 1]$ believe that the policy implemented is Delphic – that is, $\rho_i = p$ for each $i \in [0, \alpha]$ – the rest of the population believe that the policy implemented is Odyssean – that is $\rho_i = o$ for each $i \in (\alpha, 1]$.*

This definition involves no loss of generality. In particular, Delphic agents can also expect some future policy accommodation. As we will see, the important ingredient is that – everything else being equal – Delphic agents revise their expectations about the length of the trap by more than Odyssean agents do, so that their inflation and consumption expectations fall.

[17] See the discussion on the optimality of making announcements at the beginning of next section.

[18] Our assumption of heterogeneous beliefs about the type of central bank's policy is consistent with a pooling equilibrium with agents having heterogeneous priors taken as exogenous. To endogeneize such beliefs, one needs to enrich the model with strategic communication as in Bassetto (2015) or pre-ZLB-signaling as in Bhattarai et al. (2014) or Barthélemy and Mengus (2016). This goes beyond the scope of the paper and we discuss these additional policies in the conclusion.

Similarly, note that we consider degenerate individual expectations: agents believe the policy is either Delphic or Odyssean, without any uncertainty. Although theoretically this specification can be equivalently interpreted as a case in which all agents put an equal probability α on the authority being Delphic, the heterogeneous belief interpretation connects with the evidence documented in Section 2.

Equilibrium definition *For given prior distribution $\mathcal{P}$ on the length of the trap, probabilities and signals $\{p_i, p_{cb}, T_{cb}, \{T_i\}_{i\in[0,1]}\}$, a type of policy $\rho \in \{o, p\}$ and a profile of agents beliefs on the type of policy $\{\rho_i\}_{i\in[0,1]}$ characterized by α, an equilibrium at time 0 is:*

(i) *an expectation of the authority on the lift-off date $E_{cb,0}[T_{zlb}]$,*

(ii) *for each individual i, a set of beliefs on $\{T, T_{zlb}\}$ and $\{\{c_{j,\tau}\}_{j\in[0,1]}, \pi_\tau\}_{\tau=1}^{\infty}$ which are conditional to $\{E_{cb,0}[T_{zlb}], \rho_i, T_i, \mathcal{P}\}$,*

(iii) *a current rate of inflation and a profile of current individual consumptions, $\{\{c_{i,0}\}_{i\in[0,1]}, \pi_0\}$, that satisfy (1)-(3) given the set of individual beliefs.*

We focus on equilibria as in Eggertsson and Woodford (2003) or Werning (2012) where the economy exits the trap when the shock ends. In general, many equilibria can exist including self-fulfilling liquidity traps as in Mertens and Ravn (2014).

3.2 Characterization of equilibria with heterogeneous beliefs

Let us investigate how agents form beliefs and how these beliefs shape the allocation and, in particular, inflation and consumption expectations.

3.2.1 Agents' beliefs on the lift-off date and the length of the trap

To form expectations on the lift-off date $E_{i,0}[T_{zlb}]$ and on the length of the trap $E_{i,0}[T]$, agents use both the public and their private signals, conditionally on their priors on the type of policy conducted by the monetary authority. Let us further clarify this formation and how this translates into disagreement.

To establish a benchmark, let us analyze the case where agents have only private signals. To this purpose, let us suppose that $p_{cb} = 0$, i.e. agents believe that the announcement of the central bank is based on non-informative signals. Based on their private signal T_i, agents' belief on the length of the trap is $E_{i,0}[T] = p_i T_i + (1 - p_i)E[T|\mathcal{P}]$ and on the lift-off date is $E_{i,0}[T_{zlb}] = p_i \mathcal{F}_{\rho_i}(T_i) + (1 - p_i)E[\mathcal{F}_{\rho_i}(T)|\mathcal{P}]$.

Remark: no-announcement benchmark. *Absent any announcement, beliefs on the length of the trap and beliefs on the type of the monetary policy are in principle independent: there is nothing that tightens these two dimensions of agents' beliefs.*

Let us suppose now that agents believe that the public signal $E_{cb,0}[T_{zlb}]$ is informative, i.e. $p_{cb} > 0$. Agents' inference on the length of the trap will vary with their prior on the type of policy implemented. Given the same public announcement $E_{cb,0}[T_{zlb}]$, agents will see the announcement as signaling a length of the trap T_{ρ_i} that they compute inverting the believed $\mathcal{F}_\rho$ map defined by (4) according to

$$T_{\rho_i} = \mathcal{F}_{\rho_i}^{-1}\left(\frac{E_{cb,0}[T_{zlb}] - (1-p_{cb})E[\mathcal{F}_{\rho_i}(T)|\mathcal{P}]}{p_{cb}}\right),$$

where $\rho_i \in \{o, p\}$.

Lemma 1. *The fact that $\mathcal{F}_o(T) > \mathcal{F}_p(T) = T$ for any T is a sufficient condition to conclude that*

$$T_o < T_p, \tag{5}$$

that is, the signal about the length of the trap inferred from the announcement of the central bank is shorter for Odyssean than for Delphic agents.[19]

Therefore, agents form their beliefs using Bayes' Law based on the informational content of both a public and a private signal. These two signals coincide only when they are either both true or both wrong, an event that agent can recognize. Let p_x be the probability that the private and the public signals, T_i and T_{ρ_i}, are the same non-informative signal. Then the probability of the two simply being equal is given by $p_x + p_{cb}p_i$. We shall also require the natural condition $p_x \leq (1-p_{cb})(1-p_i)$; that is, the case in which both signals are equal and non-informative is less likely than both signals are simply non-informative. (Note that, because of this, when $p_{cb} \to 1$ then $p_x \to 0$.) We then obtain the following lemma:

Lemma 2. *The expected lift-off date of agent $i \in [0, 1]$ is given by:*

$$E_{i,0}[T_{zlb}] = \kappa_{cb}\mathcal{F}_{\rho_i}(T_{\rho_i}) + \kappa_i\mathcal{F}_{\rho_i}(T_i) + (1-\kappa_{cb}-\kappa_i)E[\mathcal{F}_{\rho_i}(T)|\mathcal{P}] \qquad \text{if} \quad T_i \neq T_{\rho_i}, \tag{6}$$

$$E_{i,0}[T_{zlb}] = \kappa_x\mathcal{F}_{\rho_i}(T_{\rho_i}) + (1-\kappa_x)E[\mathcal{F}_{\rho_i}(T)|\mathcal{P}] \qquad \text{if} \quad T_i = T_{\rho_i}, \tag{7}$$

whereas the expected length of the trap is given by:

$$E_{i,0}[T] = \kappa_{cb}T_{\rho_i} + \kappa_iT_i + (1-\kappa_{cb}-\kappa_i)E[T|\mathcal{P}] \qquad \text{if} \quad T_i \neq T_{\rho_i}, \tag{8}$$

$$E_{i,0}[T] = \kappa_xT_{\rho_i} + (1-\kappa_x)E[T|\mathcal{P}] \qquad \text{if} \quad T_i = T_{\rho_i}, \tag{9}$$

[19] To prove the Lemma it is sufficient to use the fact $E[\mathcal{F}_o(T)|\mathcal{P}] > E[\mathcal{F}_p(T)|\mathcal{P}]$.

with

$$\kappa_{cb} = \frac{p_{cb}(1-p_i)}{1-p_{cb}p_i-p_x}, \quad \kappa_i = \frac{p_i(1-p_{cb})}{1-p_{cb}p_i-p_x}, \quad \kappa_x = \frac{p_{cb}p_i}{p_{cb}p_i+p_x}.$$

where p_x, that satisfies $p_x \leq (1-p_{cb})(1-p_i)$.

The lemma is proved by straightforward application of Bayes' Law with κ_{cb}, κ_i and κ_x denoting the probability that: only the public signal is truthful conditional on the two signals being different, only the private signal is truthful conditional on the two signal being different and both signals being truthful when both are equal, respectively.

Let us now characterize how the agents' beliefs as described in the Lemma above translate into disagreement. To simplify the exposition and enlighten our main point, we will focus on the situation where p_{cb} converges to 1, that is, the central bank expectation is perceived to be infinitely accurate, i.e. the truth. First of all we note that:

Corollary 1. *At the limit where $p_{cb} \to 1$ we have $E_{i,0}[T_{zlb}] = E_{cb,0}[T_{zlb}] = T_{zlb}$ for each i, that is, there is perfect agreement on expected interest rates.*

The corollary is proved by the simple observation that $\lim_{p_{cb}\to 1} \kappa_{cb} = \lim_{p_{cb}\to 1} \kappa_x = 1$ and $\lim_{p_{cb}\to 1} \kappa_i = 0$. Corollary 1 makes clear that the perceived accuracy of central bank announcements is what makes our model replicate an agreement on future interest rates. This model prediction is supported by Fact 2 documented in the previous section.

However, although agents end up all believing the announcement of the central bank, they will tend to revise differently their expectation on the length of the trap depending on their prior beliefs on the type of policy conducted. In particular we have:

Corollary 2. *At the limit where $p_{cb} \to 1$ we have $E_{i,0}[T] = T_p$ for each $i \in [0, \alpha]$ and $E_{i,0}[T] = T_o$ for each $i \in (\alpha, 1]$ with $T_{zlb} = T_p > T_o$, that is, Odyssean agents are more optimistic than Delphic agents about the length of the trap. Differently said, Odyssean agents expect a policy accommodation from period T_o to period T_{zlb} while Delphic agents do not expect any accommodation.*

This second corollary is therefore consistent with a peculiar correlation in individual expectations: forecasters who are optimistic on the state of the economy are also the ones who expect a policy accommodation – i.e. a negative deviation from the rate implied by a normal-time Taylor rule – while pessimistic forecasters did not. This model prediction is supported by Fact 3 documented in the previous section.

3.2.2 Agents' beliefs on consumption and inflation

Our main objective now is to discuss how the resulting heterogeneity in the length of the trap transmits to expected consumption and inflation paths. We stick to our simplifying assumption of highly accurate announcements ($p_{cb} \to 1$) and show that the resulting equilibrium has features in line with the all the facts described in Section 2. Given our results above, it is clear that in this particular case optimist (resp. pessimist) agents and Odyssean (Delphic) agents are indeed the same. We will use the two terms equivalently.

We first analyse the two polar cases where agents are all pessimists ($\alpha = 1$) or all optimists ($\alpha = 0$) and then present the effects of heterogeneity. In doing so, we take the central bank's policy (T_{zlb}) as exogenous. In Section 4, we will study the optimal policy for a given α.

Case $\alpha = 1$. When all agents believe the policy is of the Delphic type, they all interpret the lift-off date as the expected end date of the trap. In such a case, agents have homogeneous beliefs that the trap will last for T_{zlb} periods and that the authority will keep interest rates at zero from $t = 0$ until $T_{zlb} - 1$ included.

The expected current consumption is given for each i by:

$$\text{for } t \in (0, T_{zlb} - 1): \qquad E_{i,0}[c_{i,t}] = \gamma^{-1}(\log R - \xi + E_{i,0}[\pi_{t+1}]) + E_{i,0}[c_{i,t+1}], \tag{10a}$$

$$\text{for } t \geq T_{zlb}: \qquad E_{i,0}[c_{i,t}] = 0 \tag{10b}$$

where the expected inflation path is determined by the Phillips curve (2).

Figure 6 illustrates the path for aggregate consumption and inflation (green lines) in an example where the length of the trap is 12 quarters, the central bank announces it will keep interest rates at zero until $T_{zlb} = 12$, and everybody understands that this lift-off date corresponds to the end of the trap. In that case, consumption and inflation increase monotonically and reach their steady state values after 12 quarters.

Case $\alpha = 0$. When instead all agents believe the policy is of the Odyssean type, they all interpret that the time until lift-off implies some periods of extra accommodation where the interest rate will be at zero after the end of the trap. In such a case, agents have homogeneous beliefs that the trap will last less than T_{zlb} periods, i.e. $E_{i,0}[T] = T_o < T_{zlb}$ (in fact, with $p_{cb} \to 1$ and $\alpha = 0$, we have $T_o = T$ and $T_{zlb} = \mathcal{F}_o(T)$), and that the authority will maintain interest rates at zero from $t = 0$ to $T_{zlb} - 1$ included, even though this implies that inflation is above its steady state level between T_o and $T_{zlb} - 1$.

The expected current consumption is given for each i by:

$$\text{for } t \in [0, T_o - 1]: \qquad E_{i,0}[c_{i,t}] = \gamma^{-1}(\log R - \xi + E_{i,0}[\pi_{t+1}]) + E_{i,0}[c_{i,t+1}], \tag{11a}$$

$$\text{for } t \in [T_o, T_{zlb} - 1]: \qquad E_{i,0}[c_{i,t}] = \gamma^{-1}(\log R + E_{i,0}[\pi_{t+1}]) + E_{i,0}[c_{i,t+1}], \tag{11b}$$

$$\text{for } t \geq T_{zlb}: \qquad E_{i,0}[c_{i,t}] = 0, \tag{11c}$$

where, again, the inflation path is determined by the Phillips curve (2). In fact, an Odyssean policy amounts to stimulate current consumption promising lower short-term rates once the trap ends.

Figure 6 illustrates the resulting path for aggregate consumption and inflation (blue lines) in an example where the length of the trap is again 12 quarters but the central bank announces it will the liftoff date is $T_{zlb} = 17$. In contrast to the previous case, consumption and inflation converge to the steady state non-monotonically. In particular, once the trap is over at date $t = 12$, low interest rates generate a boom, which induces more current consumption. The steady state is reached later than in the previous case, but now the path remains on average closer to the steady state. Therefore, this policy can, through an optimal choice of T_{zlb}, deliver higher welfare than what following a conventional Taylor rule would imply.

However, as we already mentioned, once the trap ends at time $t = 12$, the boom is no longer socially desirable and the authority is tempted to renege on its promise and to set $r_t = \phi\pi_t$ onward, which corresponds to the time-consistent solution with perfect stabilization at steady state, after the end of the trap. Therefore, the second-best policy at the ZLB requires a commitment ability to solve for this time-inconsistency problem.

Case with heterogeneous beliefs, $0 < \alpha < 1$. We now describe a case where, although agents have homogeneous beliefs about the interest rate path and observe the current allocation, they still disagree on the length of the trap to the extent they disagree on the type of policy that is conducted by the monetary authority.

In the case at hand, all agents believe in the announcement that the lift-off date is a certain T_{zlb}. As already discussed, this leads to agreement on the interest rate path for this given period of time. Also they entertain different beliefs on the reasons why the interest rate will be at zero for a given period of time. Agents may also observe the current distribution of beliefs but they have no reason to update their own opinion as the event on which they disagree (the policy commitment after the trap) did not realise yet; in this sense, they agree to disagree.

However, it is common knowledge that at time $E_{o,0}[T] = T_o$, which is, according to optimists, the date at which the trap ends, only one of the two types will be right. In case the trap is over at T_o optimists will be right and pessimists will be wrong. Otherwise, the opposite occurs. In either case, the heterogeneity of beliefs cannot last beyond T_o. All this is common knowledge among agents.

Such temporary disagreement has an impact on the current and expected allocations as established by the following proposition.

Lemma 3 *At the limit $p_{cb} \to 1$, for a given $\alpha \in [0, 1]$, the expected path for individual consumption is given respectively by (11) for the optimists, and (10) with for the pessimists, where the inflation path is for each agent i according to:*

$$E_{i,0}[\pi_t] = \kappa E_{i,0}\left[\int_0^1 c_{j,t} di\right] + \beta E_{i,0}\left[\int_0^1 E_{j,t}[\pi_{t+1}] di\right], \tag{12}$$

where

$$\text{for } t \in [0, T_o - 1]: \quad E_{i,0}[c_{j,t}] = E_{j,0}[c_{j,t}] \quad \text{and} \quad E_{i,0}[E_{j,t}[\pi_{t+1}]] = E_{j,0}[\pi_{t+1}], \tag{13a}$$

$$\text{for } t \geq T_o: \quad E_{i,0}[c_{j,t}] = E_{i,0}[c_{i,t}] \quad \text{and} \quad E_{i,0}[E_{j,t}[\pi_{t+1}]] = E_{i,0}[\pi_{t+1}], \tag{13b}$$

where $j \in [0, 1]$.

The interpretation of beliefs is intuitive. Each type understands that until date T_o no information can lead the other type to change her beliefs. Hence, in the short run, agents agree on the path of both inflation and consumption and they only disagree for periods after the date T_o, when optimists expect the end of the trap. At that date, as the truth finally unfolds, each type expects that the other will conform to her own expectations. After that date, optimists believe that monetary policy will engineer a boom resulting in higher inflation and higher consumption, and that pessimists will finally update their view. Conversely, pessimists expect that the economy will still be experiencing the negative shock and that optimists will update their views instead. In sum, disagreement on the type of policy conducted by the authority tends to yield disagreement over medium-term inflation and consumption expectations, whereas it will have no impact on short-term expectations.

Figure 7 plots the dynamics of consumption and inflation when beliefs on the nature of policy are heterogeneous. In this example, the trap lasts again 12 quarters but 75 per cent of households are convinced that the monetary policy is Odyssean and the other 25 per cent believes that the policy is Delphic ($\alpha = .25$). With this fraction of optimists and pessimists, the optimal policy of the central bank is still to implement an Odyssean forward-guidance. Yet, to compensate for the presence of pessimists, it keeps interest rates at zero for more periods after the end of the trap (6 periods instead of 5 when $\alpha = 0$) and announces a later date of liftoff (at date $t = 18$ instead of 17 when $\alpha = 0$). This results in a larger and longer boom at the end of the trap.

Figure 7 also illustrates that the heterogeneity of beliefs about the effects of policy at the end of the trap induces different current individual actions despite every agent agrees on future allocations until the optimist lift-off date T_o: optimists consume more in the short run as they expect higher consumption and inflation than pessimists after T_o. Optimists expect pessimists to consume less than themselves in the short run as they know that pessimists do not share their beliefs. But they expect pessimists to revise their beliefs at date T_o and, then, to consume more in the future, catching up with optimists. This expected revision of pessimists' beliefs contributes to the optimists' anticipation of a future boom. Symmetrically, pessimists expect optimists to consume more than themselves in the short run, but they also expect them to revise their expectations downward at date T_o, pushing the economy to a new recession and a longer trap.

Let us also note that disagreement in the model concerns medium-term macroeconomic outcomes but agents agree on short-run variables. This echoes our findings that disagreement over 2 years ahead inflation and consumption increased well beyond its predicted values by 1 quarter ahead disagreement. Such heterogeneous beliefs about the future state of the economy affect date-0 behavior, in particular:[20]

Corollary 3. *Agents who expect higher future consumption are also consuming more at date $t = 0$.*

This corollary parallels Fact 4 on the Michigan survey, that is, households that anticipate better future economic conditions are also more likely to consider that the time is good to purchase durable goods.

Finally, Figure 7 illustrates that, when beliefs are heterogeneous, date-based forward guidance is not as effective as in a world where all agents are optimistic, i.e. all agents consider that the central bank is pursuing an Odyssean policy. Pessimists attenuate the current impact of future monetary accommodation. More generally, the higher the fraction of pessimists, the lower the impact of a date-based forward guidance policy on the macro-economy. In fact, when the fraction of pessimists is high enough, the impact can become even negative, as pessimists misinterpret extra periods of accommodation for a more severe recession. We will discuss this issue formally in the next section. But, so far, what we obtained leads to the following claim:

Corollary 4. *(Forward Guidance Puzzle) An announcement that coordinates agents' agreement on an expected interest rate path may have limited or negative effects on current*

[20]This may or may not involve the expectation of future higher inflation. As in Werning (2012) clarifies, it is sufficient that agents expect higher future consumption. See next paragraph for more details.

and expected aggregate economic activity and inflation.

This corollary captures how Fact 1 – forecasts of aggregate consumption and inflation have not reacted substantially or even with the wrong sign over the date-based forward guidance period – can coexist with a coordination on a long period of zero interest rates as stated by Fact 2. More generally, the mechanism that we emphasize can potentially explain why the effects US forward guidance policy on the economy has been much weaker than predicted by state of the art New Keynesian models as Del Negro et al. (2015) underlined (see Appendix D.4 for further analysis).

In the end, the model that we have built replicates the main stylized facts outlined in Section 2. We next investigate how this potential heterogenous understanding of a forward guidance announcement may change the optimal design of monetary policy at the ZLB.

Heterogeneous beliefs and rigid prices. To give some further understanding on how heterogeneity affects current aggregate demand, we extend our analysis to the case where prices are rigid over the whole period, as in Werning (2012). This implies that "the expected inflation channel" of forward guidance is off. Figure 13a plots the dynamics of aggregate consumption and inflation in an illustrative example where the trap lasts for 12 periods. The results looks pretty similar to our benchmark case. The only difference is that agents do not disagree on current real rates since inflation and nominal interest rates are both pegged. This leads then to share the same *growth rate* of consumption. However, they still disagree on future real interest rates and, thus, on future consumption, so that they have different *levels* of current consumption. Hence, even if disagreement has limited effects on inflation expectations or if these expectations are not affecting current households decisions, it would still dampen the effects of an Odyssean policy as long as agents form different expectations on future consumption.

4 Optimal policy with heterogeneous beliefs

In this section, we take the point of view of a central bank that knows the true length of the trap and is willing to implement Odyssean forward guidance. We assume that central bank's information is accurate and this is common knowledge (i.e. $p_{cb} \to 1$)[21] We want to discuss how the coexistence of optimists and pessimists affects the design of an optimal Odyssean forward guidance. In other words, we will discuss how the Odyssean policy function $\mathcal{F}_\rho(\mathcal{T})$

[21] In this sense, we abstract from the optimal communication problem of the central bank (see Bassetto, 2015, among others).

varies with α; let us indicate this map by $T_{zlb}(\alpha, T)$.[22] It is worth remarking that we do not evaluate the desirability of announcements as such, but we study how, conditional on making an announcement, the central bank's optimal policy may change.

We also assume that central bank's announcements are perceived as highly accurate (i.e. $p_{cb} \to 1$)[23] and we discuss how the coexistence of optimists and pessimists affects the design of an optimal Odyssean forward guidance. In other words, we will discuss how the Odyssean policy function $\mathcal{F}_\rho(\mathcal{T})$ it varies with α; let us indicate this map by $T_{zlb}(\alpha, T)$.[24]

To determine optimal policy, conditional to its own belief on the length of the trap T, the authority chooses a lift-off date according to (3) that maximizes the expected utility of agents, taking as given agents' optimal consumption, pricing decisions and beliefs. To avoid creating additional multiplicity of equilibria, we shall assume that the central banker knows α and does not need to infer it.[25]

In Appendix D.6, we proceed similarly to Gali (2008) to approximate the resulting welfare objective by a quadratic function. In the special case where marginal utility is equally elastic to consumption and labor – i.e., agents' coefficient of relative risk aversion γ equals the inverse of the Frisch elasticity of labor supply ψ – this functions is:

$$\mathbb{W} = \int_0^1 \mathbb{W}_i di \equiv -\varpi\theta^{-1} \int_0^1 \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \left(\lambda c_{i,t}^2 + \pi_t^2\right)\right] di, \tag{14}$$

which looks like the textbook welfare approximation typical of New-Keynesian models with homogeneous beliefs.[26]

This particular specification is useful to obtain analytical result on the form of the optimal policy for a given length of the trap T and a fraction of pessimists α. This result is described in the proposition below. In the next paragraph, we quantitatively extend these conclusions to cases where $\gamma \neq \psi$.

[22] We still consider α as given. We provide a short discussion on how a monetary authority could affect the fraction of pessimists in Conclusion.

[23] We then abstract from the optimal communication problem of the central bank (see Bassetto, 2015, among others).

[24] We still consider α as given. We provide a short discussion on how a monetary authority could affect the fraction of pessimists in Conclusion.

[25] Given that the central bank observes α, it can commit to implement an Odyssean forward guidance whenever it is optimal. This way, we prevent the emergence of a coordination failure between the central bank and optimists. Indeed, if optimists behaved as if they were pessimists, then it would be optimal for the Odyssean central bank not to implement its Odyssean policy. In turn, this makes self-fulfilling optimists behaving like pessimists. Note that this kind of equilibrium is not a consequence of the heterogeneity of beliefs, but it can also emerge in the classical framework of Eggertsson and Woodford (2003).

[26] Our derivation in the special case is also close to the one of Bilbiie (2008) who looks to a case in which agents have limited asset market participation.

Proposition 1. *In the special case where the coefficient of risk aversion equals the inverse of the Frisch elasticity, there exist two values $\underline{\alpha}$ and $\bar{\alpha}$ where $\underline{\alpha} < \bar{\alpha}$ such that, for a given T, the optimal policy $T_{zlb}(\alpha, T)$ is:*

- *increasing in α, i.e. $T_{zlb}(0, T) < T_{zlb}(\alpha, T)$, for $\alpha < \underline{\alpha}$;*

- *decreasing in α, i.e. $T_{zlb}(\alpha', T) > T_{zlb}(\alpha'', T)$, for $\underline{\alpha} < \alpha' < \alpha'' < \bar{\alpha}$;*

- *equal to T, i.e. $T_{zlb}(0, T) > T_{zlb}(\alpha, T) = T$, for $\alpha > \bar{\alpha}$.*

Proof. See Appendix C. □

Proposition 1 states that the optimal number of periods of accommodation after the end of the trap is a non-monotonic function of the share of pessimists. This is because a coordination of beliefs on an extended period of low interest rates can be detrimental when misunderstood.

Indeed, Proposition 1 states the existence of an "*excess pessimism*" channel of forward guidance in presence of heterogeneous beliefs. By fixing a lift-off date beyond the end of the trap, the central bank induces pessimists to consume less as if the lift-off date is the end of the trap. This occurs because pessimists interpret the policy as a signal that the trap is longer than the one that it actually is.[27] As a result, the central bank can be better off not implementing an Odyssean forward guidance policy, no matter whether it is willing and able to commit to it.

On the other hand, when only a small share of agents misunderstand the Odyssean forward guidance, the central bank is better off opting for this policy. In such a case, a further drop in pessimists' consumption (as they wrongly interpret additional periods of low interest rate as a sign of a longer trap) can be more than compensated by an increase in optimists' consumption. Thus, when the fraction of pessimists is sufficiently low, the optimal policy calls for increasing the period of low interest rates T_{zlb}.

Our "*excess pessimism*" channel is different from the channel through which the revelation of inefficient shocks may generate a drop in welfare, as emphasized by Angeletos et al. (2016) and Wiederholt (2014). These papers investigate the effect of a release of information, we instead show how the optimal monetary policy changes conditional to making an announcement. In our case the drop of welfare signaled by an Odyssean policy may go beyond its perfect information level as agents may become excessively pessimists; this is why Delphic forward guidance maybe optimal although the authority has commitment ability.

[27] The "excess pessimism" channel is not the only channel at work, but the most important. See Appendix D.7 for details.

Numerical illustration. Here, we present numerical simulations illustrating how results obtained in the special case where $\psi = \gamma$ extend to the general case $\psi \neq \gamma$.

Figure 8 plots the number of periods of extra accommodation as a function of the fraction of pessimists. We contrast the optimal policies after a large shock ($\xi = -0.01$) in the upper panel and after a small shock ($\xi = -0.007$) in the lower panel. In both cases we consider a shock lasting for 20 periods.[28]

In each panel, there are three types of curves: solid, dashed and dotted. The solid line corresponds to the optimal policy when $\lambda = 0$. This is a limit case when the authority only cares about inflation. In this case, the relation is hump-shaped as described in Proposition 1: the presence of pessimists forces the central bank to extend its monetary stimulus, until the contractionary effects that are growing with the share of pessimist outweight the benefits of additional stimulus. Then, the central bank starts reducing the length of its stimulus and eventually reaches a point where it prefers not to implement Odyssean forward guidance.

The dotted and dashed lines represent the optimal policy when $\lambda = 50$ (a case where the policy maker's loss function puts a large weight on the variance of output gap) and $\psi = \gamma$ or $\psi = \gamma/4$ respectively. The two curves illustrate that the optimal length of extra accommodation becomes a monotonically decreasing function of α for a sufficiently high ratio γ/ψ. This illustrates that, when deviating from the condition $\psi = \gamma$, additional welfare cost terms appear due to heterogeneity and these terms reduce the incentive of the authority to generate disagreement by further reinforcing Odyssean forward guidance.

Finally, let us comment on how policy reactions vary with the size of the shock. *Ceteris paribus*, with larger shocks, the contractionary effect of pessimists increases. For sufficiently low fractions of pessimists, the optimal number of extra-accommodation after the end of trap increases when the shock is larger. Yet, the threshold value of (the fraction) of pessimists beyond which the central bank prefers not to implement Odyssean forward guidance decreases when the size of the shock increases.

5 Conclusion

In this paper, we have shown a form of disagreement among professional forecasters and households on future monetary policy in the period when the Fed implemented a date-based forward guidance policy. We also showed that such a form of disagreement may lead forward

[28]Figure 13 presents similar results obtained with alternative specifications of the Phillips curve. In Panel (a) we shut down the NK Phillips curve setting redetermined prices. In Panel (b) we assume that firms are owned by optimists, that is, we set firms' expectations equal to optimist expectations (instead of being equal to the average expectations as in the standard case).

guidance announcements to be detrimental. The core of our analysis relies on the assumption that agents are unsure about the nature of announcements, whether they are Odyssean, i.e. a signal of a commitment to future accommodation, or Delphic, i.e. a signal that the economy will be forced at the ZLB by future fundamentals. In our benchmark model, policies are constrained by the ZLB during the trap. As a result, a pure Delphic or an Odyssean policy implies similar policy rates until the end of the trap, thus sustaining contrasting interpretations by private agents. A natural question is: how could then a central banker conducting Odyssean policy credibly signal the type of its policy by changing interest rates?

Before the end of the trap, signaling with policy rates may result impossible. On the one hand, credibility is hampered by the fact that a Delphic type could easily replicate the same signal without bearing the "time inconsistency" costs of forward guidance. On the other hand, signaling using interest rates would imply raising the current nominal rate, which may have extremely costly effects in comparison with the benefits of forward guidance. More generally, Barthélemy and Mengus (2016) show that signaling Odyssean forward guidance could only take place before the liquidity trap begins.

Signaling instruments other than rates may be available such as communication, transparency on central banks' beliefs (e.g. by releasing forecasts) or unconventional monetary policy instruments.[29]

One way to limit the fraction of pessimists would be to communicate on her type of policy. As argued by Woodford (2012), the announcement of a clear commitment by the central banker can be a way to make costly *ex post* deviations from the central bank's commitment ("to cause embarrassment" to borrow Woodford's words) and, so, to convince pessimists to change their views on the type of policy. Yet, such announcements can be also made by Delphic central bankers. And again, the latter will not bear the cost of reneging it while it pockets the *ex-ante* gains related to increasing the proportion of optimists. In the end, communication on commitment is plagued by cheap talk problems: to the extent that it costs nothing more to the Delphic central banker, such communication provides no information on types of policy. Such cheap talk aspect of forward guidance announcements is developed in Bassetto (2015).

One other way is to increase the proportion of optimists is to communicate on fundamentals and to try to coordinate agents on shorter liquidity traps than the horizon of the zero interest rate policy. This can be achieved by releasing forecasts of macro-economic variables, as

[29]These include quantitative easing or loan policies such as the targeted long-term refinancing operation (TLTRO) implemented by the ECB. Some evidence of the signaling effect of Asset Purchasing Program has been documented by Coenen et al. (2017).

frequently done by central banks, or by committing to temporarily overshoot the inflation target (the Fed, the Bank of England and more recently the Bank of Japan have made such announcements). Yet, this policy can also be mimicked by Delphic central bankers and it involves only some unpalatable costs when deviating *ex post* from the commitment.

Finally, quantitative policies and the purchase of long maturity bonds at very low rates, or supplying liquidity at long horizons at zero interest rates amount to "putting your money where your mouth is". It can provide a strong signal on the central bank's willingness not to raise policy rates in the future. Indeed, such policies can imply a cost to the central bank in case it deviates from its commitment: a rise in interest rate may lead to a depreciation of purchased assets and so to capital losses to the central bank (see Bhattarai et al., 2014, for an investigation of this mechanism). Yet, such a signaling device hinges on the central bank's aversion for capital losses and the extent to which it cannot be rescued by the fiscal authority in case of negative equity.

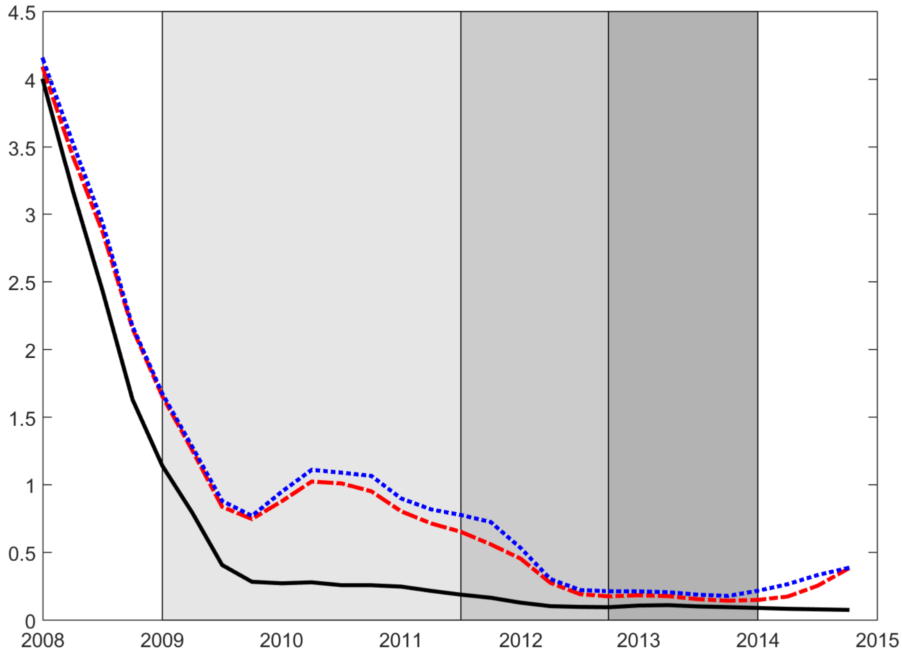
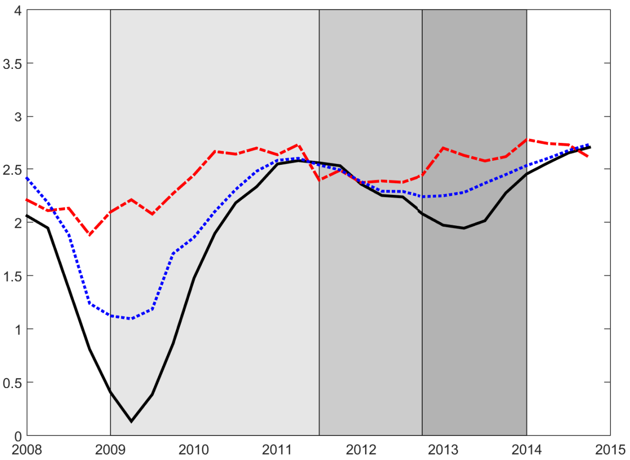


Figure 1: Average of individual short-term interest rate forecasts.

The chart displays the evolution of a moving average over the last 4 quarters of the average of individual forecasts of the 1-quarter (black line), 1-year (red line), and 2-year (blue line) ahead individual mean point forecasts for 3-month T-Bill interest rate. The shaded areas correspond to the periods of the ZLB and "open-date" forward guidance, "date-based" forward guidance and the "state-contingent" forward guidance.

(a) Consumption



(b) Inflation

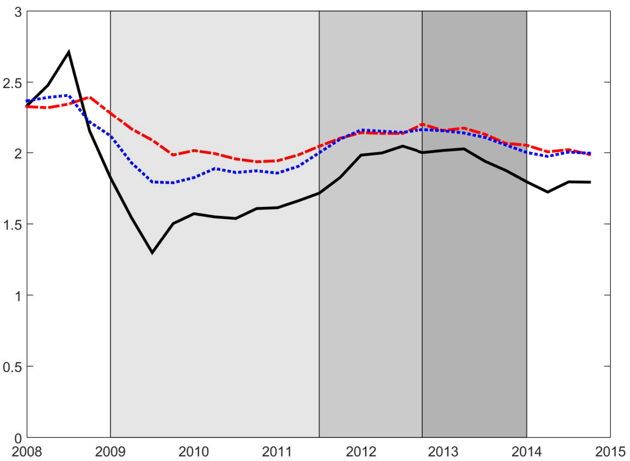


Figure 2: Average of individual consumption growth and inflation forecasts.

The figure shows the evolution of a moving average over the last 4 quarters of the average of individual forecasts of 1-quarter (black line), 1-year (red line), and 2-year (blue line) ahead individual mean point forecasts for real consumption growth and CPI inflation. The shaded areas correspond to the periods of the ZLB and "open-date" forward guidance, "date-based" forward guidance and the "state-contingent" forward guidance.

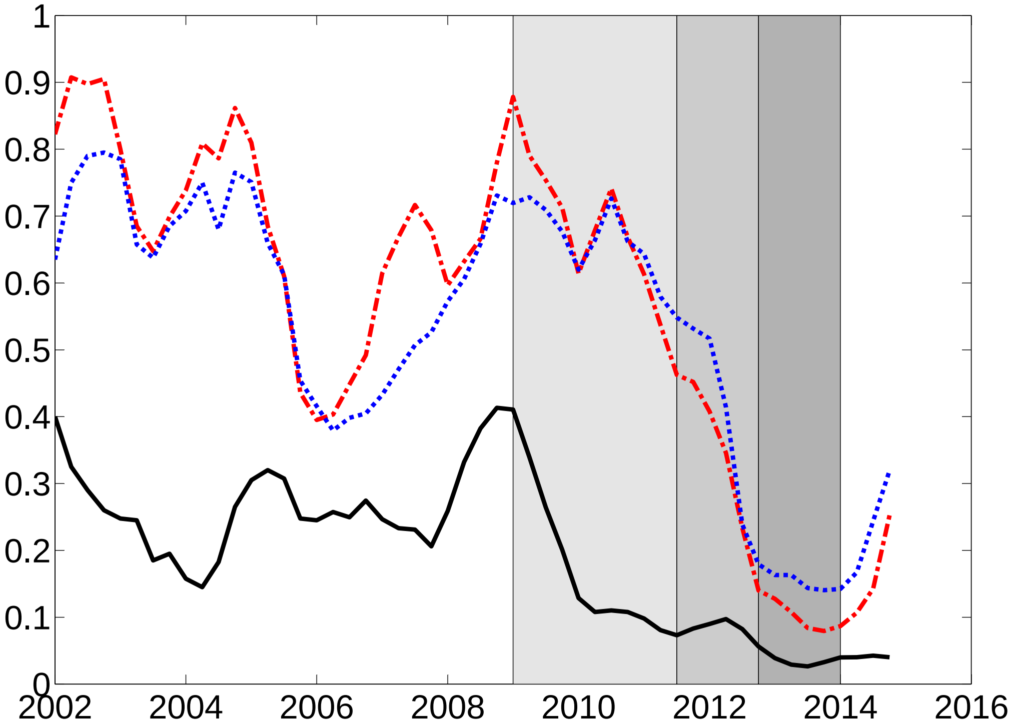
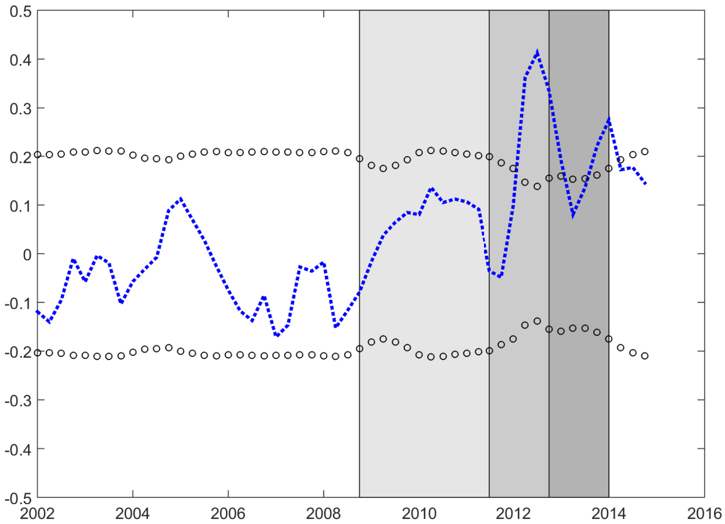


Figure 3: Disagreement about future short-term interest rates.

The chart displays the evolution of a moving average over the last 4 quarters of the 75/25 inter-quantile range in the distribution of 1-quarter (black line), 1-year (red line), and 2-year (blue line) ahead individual mean point forecasts for 3-month T-Bill interest rate. The shaded areas correspond to the periods of the ZLB and "open-date" forward guidance, "date-based" forward guidance and the "state-contingent" forward guidance.

(a) Consumption - 2 years ahead



(b) Inflation - 2 years ahead

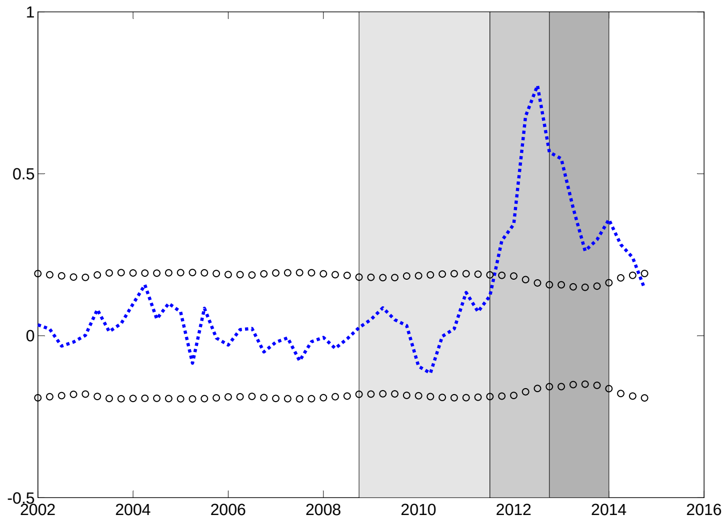


Figure 4: Excess disagreement about future consumption and inflation .

The Figure plots the residuals of a regression of the (log) disagreement on (1-year and 2-year ahead) inflation forecasts on the (log) disagreement on (1-year and 2-year ahead) short-term interest rate and disagreement on 1-quarter ahead inflation forecast. The regression is estimated on a pre-crisis sample (1982Q2-2008Q4). Black circles give the bands of a 95% confidence interval that take into account autocorrelation and heteroskedasticity of the residuals. The shaded areas correspond to the periods of the ZLB and "open-date" forward guidance, "date-based" forward guidance and the "state-contingent" forward guidance.

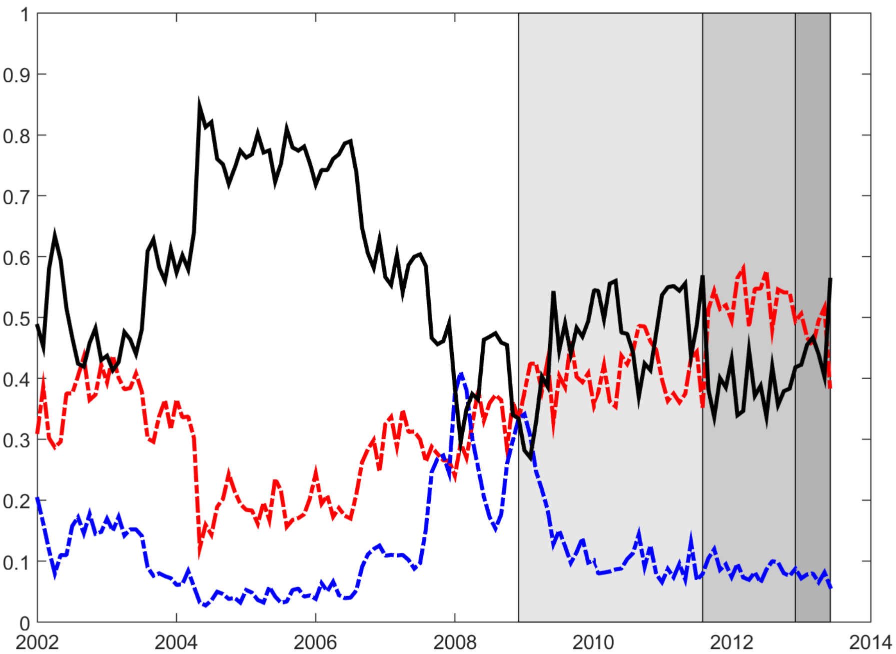


Figure 5: Interest rate expectations in the Michigan survey of households.

The chart displays the evolution of the share of respondents to the survey who thought that over the next 12 months, interest rates will increase (dark line), stay constant (red line) or decline (blue line).

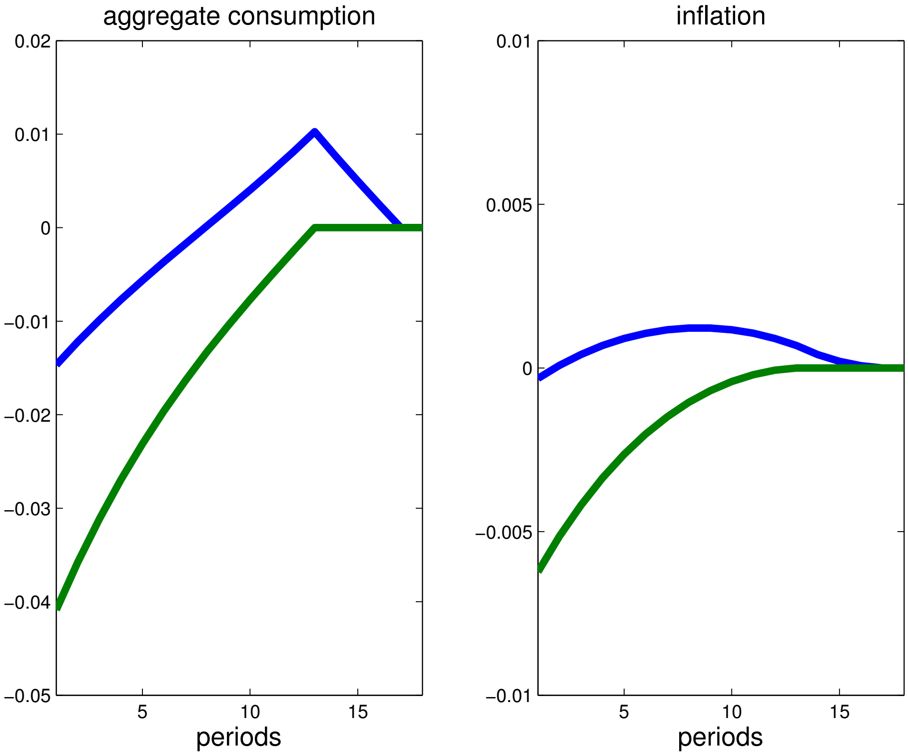


Figure 6: The effect of Delphic (green) and Odyssean (blue) policies.

We consider a shock ($\xi = -0.01$) on the discount rate that lasts 12 quarters and implies a drop of consumption of 4% at impact in the absence of Odyssean forward guidance, which provides for 4 extra quarters of accommodation. We calibrate the reaction to inflation at $\phi = 1.5$. The discount factor β is such that the annual real interest rate equals 2% and the utility function is assumed to be CRRA $u(c) = c^{1-\gamma}/(1-\gamma)$ with $\gamma = 2$. The probability not to reset prices is .85, and the slope of the Phillips' curve is then .027. We use $\psi = 2$ to compute the optimal policy, so as to have $\gamma = \psi$ as in our benchmark case.

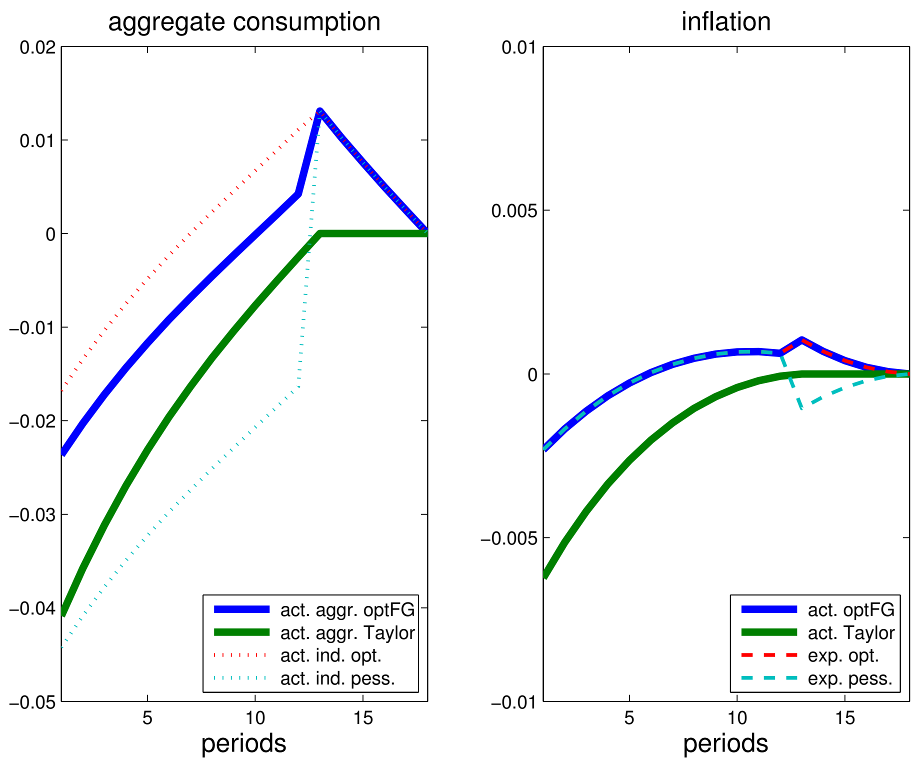
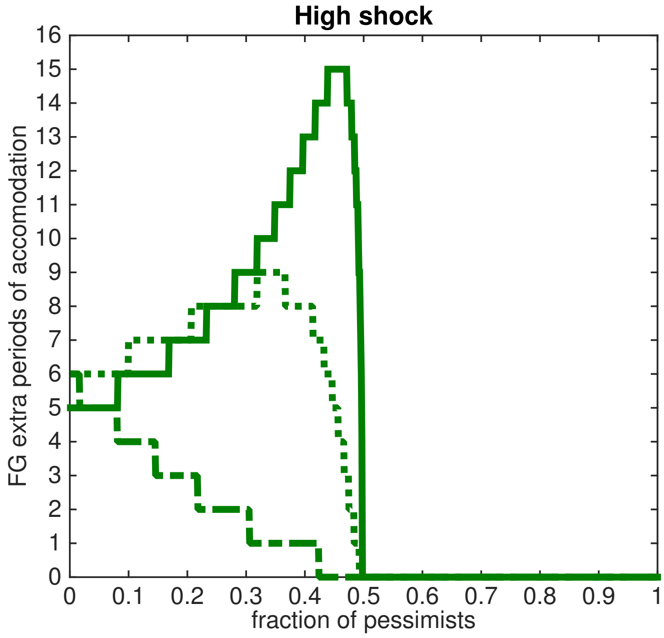


Figure 7: The effect of Odyssean (blue) with a fraction $\alpha = .25$ of pessimists.

We consider a shock ($\xi = -0.01$) on the discount rate that lasts 12 quarters and implies a drop of consumption of 4% at impact in the absence of Odyssean forward guidance, which provides for 4 extra quarters of accommodation. We calibrate the reaction to inflation at $\phi = 1.5$. The discount factor β is such that the annual real interest rate equals 2% and the utility function is assumed to be CRRA $u(c) = c^{1-\gamma}/(1-\gamma)$ with $\gamma = 2$. The probability not to reset prices is .85, and the slope of the Phillips' curve is then .027. We use $\psi = 2$ to compute the optimal policy, so as to have $\gamma = \psi$ as in our benchmark case. The plain blue line is the actual path of inflation/aggregate consumption in the case of the optimal odyssean policy. The plain green line is the actual path of inflation/aggregate consumption in the case of a delphic policy. The red doted line is the expected path of inflation / individual consumption of optimists. The blue dotted line is the expected path of inflation / individual consumption of pessimists.



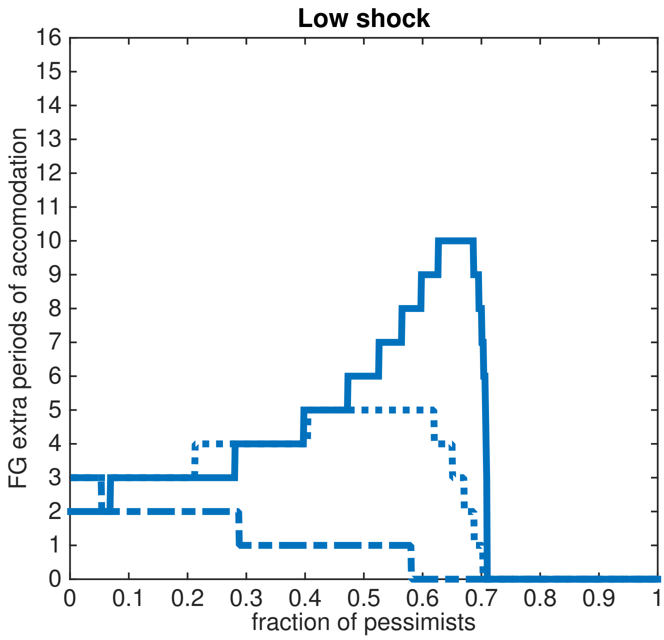


Figure 8: We plot the optimal $T_{zlb}(\alpha, 20) - 20$ for $\xi = -.007$ (lower panel) and $\xi = -.01$ (upper panel) for: $\lambda = 0$ with a solid line; $\lambda = 50$ and and $\gamma/\psi = 1$ with a dotted line; $\lambda = 50$ and and $\gamma/\psi = 4$ with a dashed line. λ is the weight on the average volatility of working hours in the loss function of the central bank. We calibrate the reaction to inflation at $\phi = 1.5$. The discount factor β is such that the annual real interest rate equals 2%, The probability not to reset prices is .85, and the slope of the Phillips' curve is then .027.

A Additional evidence: Household survey

Evidence with a broader definition of optimists households Only a few households in the Michigan survey have expectations that are consistent with what New-Keynesian models predict the impact of forward guidance policy should be: they foresee stable or lower interest rates, more inflation, a boom in future activity and therefore want to consume more today. That their number is quite limited is consistent with the evidence in Bachmann et al. (2015). These authors showed using data from the Michigan survey that, on average, when the US economy was at the ZLB, households who expected higher inflation expectations also considered the time as less favorable to consume.

Werning (2012) showed that forward guidance does not require that agents expect higher future inflation (hence lower future real interest rates) to have a positive impact on consumption today. Such policy can be effective if agents only expect a future boom. This suggests to conduct the same analysis than before but with a broader definition of optimistic and pessimistic households that does not depend on their inflation expectations. More specifically we sort households into two categories only: optimists if they expect better future conditions and pessimists otherwise. Table 3 shows the average of macro expectations of these two groups observed at the dates following date-based forward guidance announcements. Three comments can be made. First, two views (an optimistic and a pessimistic ones) about the macroeconomic outlook prevailed within the group of households who foresaw stable or lower interest rates. Second, there is a now substantial number, sometimes a majority, of optimistic households, who have expectations consistent with the effects of forward guidance as emphasized in Werning (2012): after the forward guidance announcements they expected better future economic conditions and were likely to consume more today. Third, and again consistent with Bachmann et al. (2015)'s results aforementioned, households who expected better economic conditions in the future and consumed more after date-based forward guidance, were also expecting lower inflation on average.

	Optimists	Pessimists
Averages observed in 2011m9		
Fraction of respondents	14%	86%
Good times for durable	.64	.21
Inflation	2.95	3.90
Averages observed in 2012m2		
Fraction of respondents	60%	40%
Good times for durable	.56	.19
Inflation	2.45	3.99
Averages observed in 2012m10		
Fraction of respondents	68%	32%
Good times for durable	.48	.17
Inflation	2.81	4.69

Table 3: Average of forecasts across groups of households.

This table computes the cross-sectional mean for current durable consumption (qualitative answers) and expected inflation over next 12 months (quantitative answers) when forecasters are sorted according to their expected business conditions and nominal interest rate over next 12 months. Optimistic forecasters had a positive view of the business/financial conditions over the next 12 months. Pessimists had a negative view of the same business/financial conditions. All forecasters considered expect constant or decreasing nominal interest rates over the next 12 months.

Evidence using revisions in households' expectations Some households in the Michigan survey are sampled twice with a 6 month interval. We exploit this panel dimension to control for individuals' fixed effects. Table 4 below shows that optimists households have both higher revision of inflation expectations and willingness to buy durable goods after FG announcements.

	Optimists	Pessimists
Averages observed in 2011m9		
Fraction of respondents	4%	58%
Good times for durable	.00	-.19
Inflation	-1.75	-2.71
Averages observed in 2012m2		
Fraction of respondents	14%	28%
Good times for durable	.32	.25
Inflation	-1.24	-2.29
Averages observed in 2012m10		
Fraction of respondents	9%	31%
Good times for durable	.11	.03
Inflation	3.25	-1.55

Table 4: Average of forecasts revisions across groups of households surveyed twice.

This table computes the cross-sectional mean for revisions in current durable consumption (qualitative answers) and revisions in expected inflation over next 12 months (quantitative answers) for forecasters surveyed twice (with a 6 months interval). Forecasters are sorted according to their expected business conditions and nominal interest rate over next 12 months (at the time of the second survey). Pessimists expected lower inflation than the cross-sectional mean and had a negative view of the business/financial conditions over the next 12 months. Others include all households except optimists. All forecasters considered expect constant or decreasing nominal interest rates over the next 12 months. All forecasters considered expect constant or decreasing nominal interest rates over the next 12 months.

B Additional evidence: patterns in various measures of macroeconomic uncertainty

In this paragraph, we investigate whether forward guidance has had effects on another channel than just the first moments namely a reduction in uncertainty. Figure 9 plots three different recent measures of uncertainty between 2002 and 2016: the CBOE financial market volatility index (VIX), the macroeconomic uncertainty measure developed by Jurado et al. (2015) (JLN), the economic policy uncertainty measure developed by Baker et al. (2016) (BBD). Figure 10 shows two additional measures that are derived from subjective probability distribution observed in the survey of professional forecasters: the probability of a drop in the level of real GDP in 4 quarters (REC) and the conditional variance of inflation 4 quarters ahead (VIN). A first observation is that, consistent with e.g. Bianchi and Melosi (2015b), macroeconomic uncertainty increased as the economy hit the ZLB and the usual monetary policy stabilisation instrument has been lost. Yet, when the Fed switched to date-based forward guidance, there is no clear common pattern in the three measures of uncertainty. The index by Jurado et al. (2015) remained almost unaffected, while economic policy uncertainty measure and the VIX both peaked around the time of the first announcement. In sum, this evidence is not consistent with a systematic reduction of uncertainty due to date-based forward guidance announcements.[30]

We also checked that the fact that date-based forward guidance is associated with an increase in disagreement about medium-run forecasts of consumption growth and inflation, illustrated in Figure 4, does not primarily result from variations in macroeconomic uncertainty.

We regressed the disagreement about 2-year ahead forecasts of consumption (resp. inflation) on the disagreement about 2-year ahead forecasts of short-term nominal interest rates estimated on a pre-crisis sample, controlling for the disagreement about 1-quarter ahead consumption and inflation forecasts as previously, as well as for four different measures of uncertainty: the JLN measure of macroeconomic uncertainty, the BBD measure of economic policy uncertainty, and the 2 SPF based measures REC and VIN.

Figures 11 and 12 display the residuals from these regressions. They show that the beginning of the date-based forward guidance policy is again a striking outlier: controlling for fundamental uncertainty, disagreement about future inflation should have been significantly lower given how much agents agreed on future short-term interest rates. So, changes in uncer-

[30] Note, in contrast, that state-contingent forward guidance has had a large negative impact on the economic policy uncertainty measure by Baker et al. (2016). As a result, the reduction in uncertainty channel seems to be more relevant for state-contingent than for date-based forward guidance.

tainty are not the main explanation for why the normal time correlation between disagreement about future interest rates and disagreement about future fundamentals disappears at the time of forward guidance.

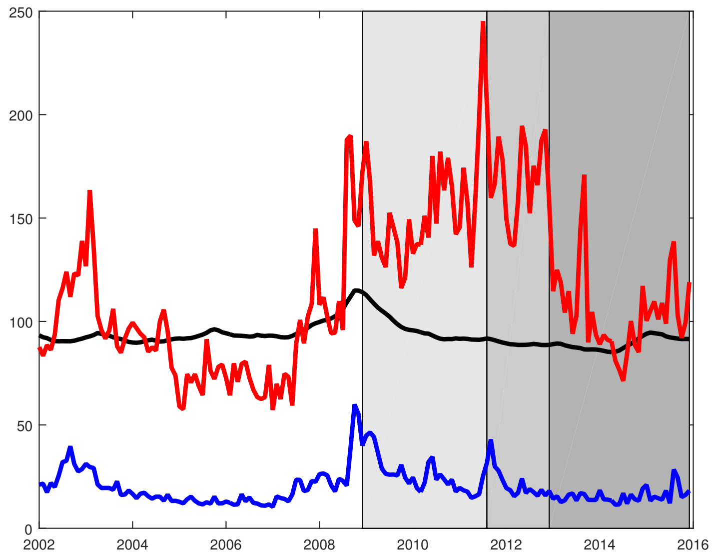


Figure 9: Measures of uncertainty.

The chart displays the evolution of 3 different measures of uncertainty: the CBOE financial market volatility index (VIX, blue line), the macroeconomic uncertainty measure developed by Jurado et al. (2015) (JLN, dark line), the economic policy uncertainty measure developed by Baker et al. (2016) (BBD, red line).

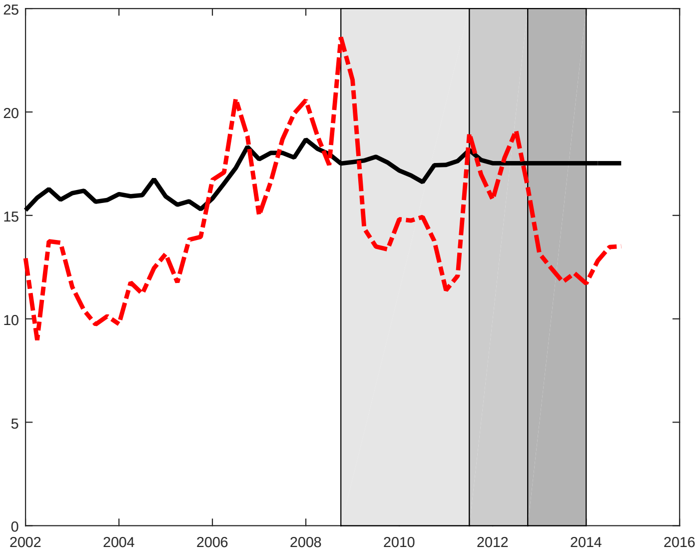


Figure 10: Survey based measures of uncertainty.

The chart displays the evolution of 2 different measures of uncertainty based on survey of professional forecasters: the cross-sectional average of the probability of a recession (REC, red line) and the cross-sectional average of the conditional variance of inflation 1-year ahead derived from the individual subjective probability distribution forecasts (VIN, dark line).

(a) Consumption - 2 years ahead - EPU

(b) Inflation - 2 years ahead - EPU

(c) Consumption - 2 years ahead - JLN

(d) Inflation - 2 years ahead - JLN

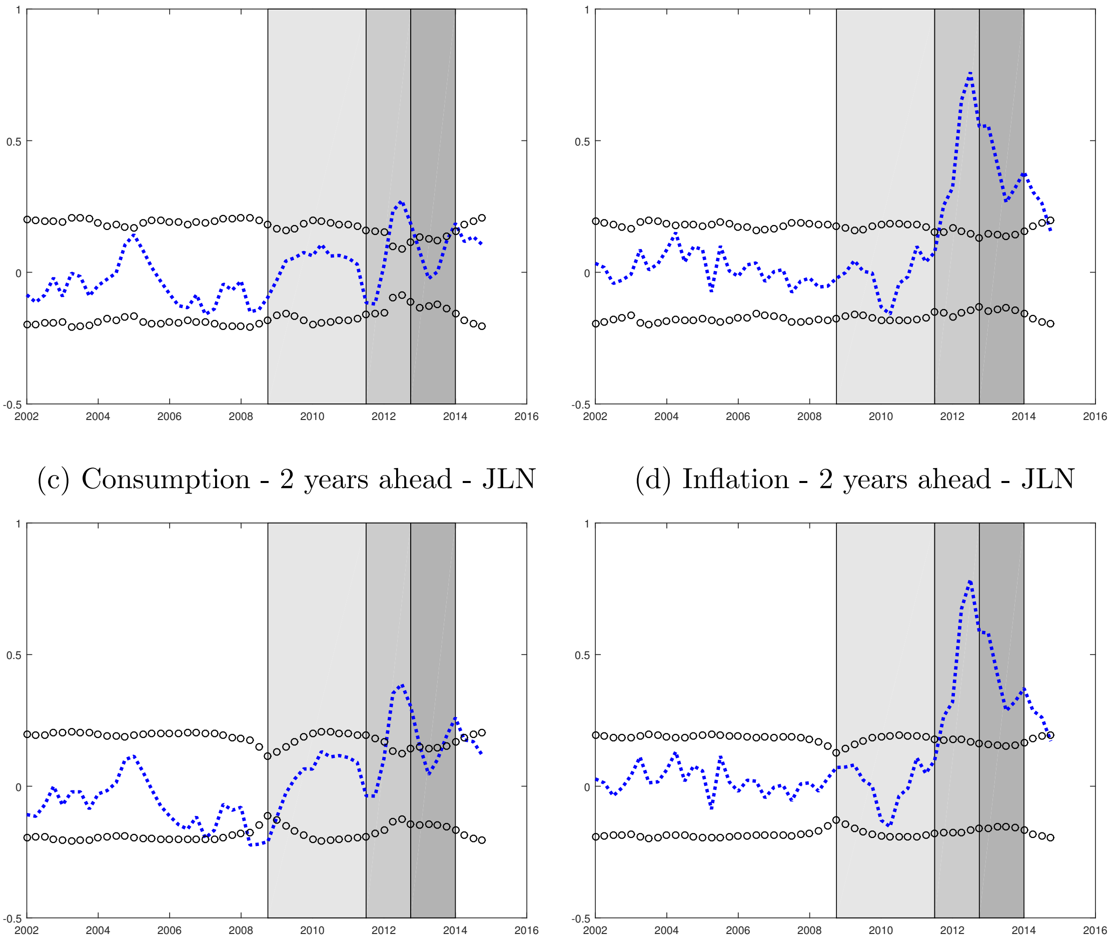


Figure 11: Excess disagreement about future consumption and inflation, controlled by uncertainty.

The Figure plot the residuals of a regression of the (log) disagreement on (1-year and 2-year ahead) inflation and consumption forecasts on the (log) disagreement on (1-year and 2-year ahead) short-term interest rate and disagreement on 1-quarter ahead inflation forecast and the uncertainty measure (JLN : Jurado et al. (2015) and EPU : economic policy uncertainty measure developed by Baker et al. (2016)). The regression is estimated on a pre-crisis sample (1982Q2-2008Q4). Black circles give the bands of a 95% confidence interval that take into account autocorrelation and heteroskedasticity of the residuals. The shaded areas correspond to the periods of the ZLB and “open-date” forward guidance, “date-based” forward guidance and the “state-contingent” forward guidance.

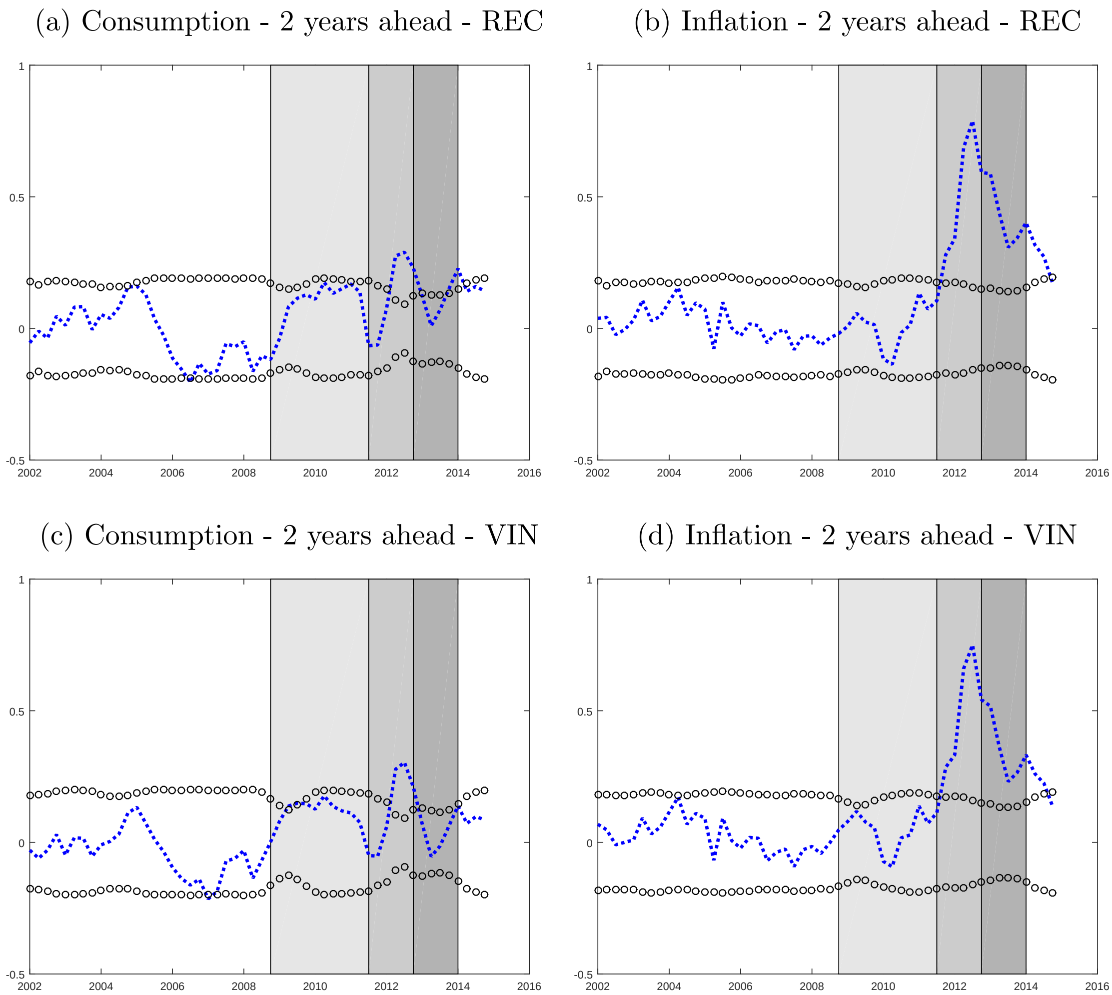


Figure 12: Excess disagreement about future consumption and inflation, controlled by uncertainty.

The Figure plot the residuals of a regression of the (log) disagreement on (1-year and 2-year ahead) inflation and consumption forecasts on the (log) disagreement on (1-year and 2-year ahead) short-term interest rate and disagreement on 1-quarter ahead inflation forecast and two measures of uncertainty derived from the SPF: REC and VIN. The regression is estimated on a pre-crisis sample (1982Q2-2008Q4). Black circles give the bands of a 95% confidence interval that take into account autocorrelation and heteroskedasticity of the residuals. The shaded areas correspond to the periods of the ZLB and "open-date" forward guidance, "date-based" forward guidance and the "state-contingent" forward guidance.

C Proof of Proposition 1

To enlighten the main intuition behind the proof, we firstly only consider a one-period trap that hits at time 0, in the case $\lambda = 0$. Let us then denote by $FG(k) = \sum_{t>0} \beta^t \pi_t^2$ when there is k of periods of Odyssean forward guidance. $FG(k)$ is increasing in k and does not depend on α. The last two properties are general all the periods after the end of the trap, irrespective of its length and the value λ. The reson is that for $t > T$ agents will not disagree and anticipate that at time 0. For the sake of notational convenience, let us denote by $i = p$ the pessimist type and by $i = o$ the optimist type.

Inflation and consumption at time 0 are given by

$$\begin{aligned}
\pi_0 &= \left(\alpha\left(\beta\pi_{p,1} + \kappa c_{p,0}\right) + (1-\alpha)\left(\beta\pi_{o,1} + \kappa c_{o,0}\right)\right), \\
c_0 &= \alpha c_{p,0} + (1-\alpha) c_{o,0} \\
c_{o,0} &= c_{o,1} - \gamma^{-1}\left(\rho_l - \pi_{o,1}\right) \\
c_{p,0} &= c_{p,1} - \gamma^{-1}\left(\rho_l - \pi_{p,1}\right)
\end{aligned}$$

where $\pi_{i,1}$ is a short notation for the expectation of agent i about inflation at time 1 and $\rho_l = -\log R - \xi < 0$. Bear in mind that $c_{i,1}$ and $\pi_{i,1}$ do not depend on α as agents consistently expect homogeneous beliefs are restored after that date.

Let us investigate the conditions for which for $k > k'$ we can have $\pi_0^2(k) + FG(k) \leq \pi_0^2(k') + FG(k')$, i.e. forward guidance for k period is not less efficient of a forward guidance for k' periods. First note that

$$\frac{\partial c_0(k)}{\partial \alpha} = c_{p,0}(k) - c_{o,0}(k) = \gamma^{-1}(\pi_{p,1}(k') - \pi_{o,1}(k')) + c_{p,1}(k') - c_{o,1}(k'))$$

as $\partial c_{p,0}/\partial\alpha = \partial c_{o,0}/\partial\alpha = 0$, and so

$$\begin{aligned}
\frac{\partial c_0(k)}{\partial \alpha} &< \frac{\partial c_0(k')}{\partial \alpha} \\
\frac{\partial \pi_0(k)}{\partial \alpha} &= \beta\left(\pi_{p,1}(k) - \pi_{o,1}(k)\right) + \kappa\left(c_{p,0}(k) - c_{o,0}(k)\right) < \frac{\partial \pi_0(k')}{\partial \alpha} < 0.
\end{aligned}$$

given the facts:

i) $\pi_{p,1}(k) < \pi_{p,1}(k') < \pi_{p,1}(0) = 0$, $\pi_{o,1}(k) > \pi_{o,1}(k') > \pi_{o,1}(0)$,

ii) $c_{p,1}(k) < c_{p,1}(k') < c_{p,1}(0) = 0$, $c_{o,1}(k) > c_{o,1}(k') > c_{o,1}(0)$.

The derivative of $\Pi(k, k', \alpha) = \pi_0^2(k) - \pi_0^2(k')$ with respect to α is:

$$\frac{\partial \Pi(k,k',\alpha)}{\partial \alpha} = 2\left(\pi_0(k)\frac{\pi_0(k)}{\partial \alpha} - \pi_0(k')\frac{\pi_0(k')}{\partial \alpha}\right) \tag{15}$$

whereas, $\Phi(k, k') = FG(k') - FG(k) < 0$. By substitution we get:

$$\pi_0 \frac{\pi_0}{\partial \alpha} = \alpha \left(\left(\beta + \kappa \gamma^{-1} \right) (\pi_{p,1} - \pi_{o,1}) + \kappa (c_{p,1} - c_{o,1}) \right)^2 + \\ + \left(\left(\beta + \kappa \gamma^{-1} \right) \pi_{o,1} + \kappa c_{o,1} - \gamma^{-1} \rho_l \right) \left(\left(\beta + \kappa \gamma^{-1} \right) (\pi_{p,1} - \pi_{o,1}) + \kappa (c_{p,1} - c_{o,1}) \right)$$

where the term

$$\left(\left(\beta + \kappa \gamma^{-1} \right) (\pi_{p,1}(k) - \pi_{o,1}(k)) + \kappa (c_{p,1}(k) - c_{o,1}(k)) \right),$$

is smaller than

$$\left(\left(\beta + \kappa \gamma^{-1} \right) (\pi_{p,1}(k') - \pi_{o,1}(k')) + \kappa (c_{p,1}(k') - c_{o,1}(k')) \right),$$

for the facts i) and ii) above. As a result, when $\alpha = 0$, the derivative $\partial \Pi(k, k') / \partial \alpha$ is negative. In addition, $\partial \Pi(k, k') / \partial \alpha$ is a linear and increasing function of α.

Therefore, let us consider a situation in which $\Pi(k, k', \alpha) > \Phi(k, k')$ - i.e. forward guidance for k' is preferred to k, with $k > k'$, in the absence of pessimists. As α increases in the range $(0, 1)$, the inequality can switch sign either never or twice, given that by construction $\Pi(k, k', 1) > 0$ (all agents are Delphic). In particular, the upper threshold $\bar{\alpha}$ is such that $\Pi(1, 0, \bar{\alpha}) = \Phi(1, 0)$.

Let us go back now to the case $\lambda > 0$. In this case the relevant inequality becomes

$$(1 - \alpha) \left(c_{o,0}^2(k) - c_{o,0}^2(k') \right) + \alpha \left(c_{p,0}^2(k) - c_{p,0}^2(k') \right) + \Pi(k, k', \alpha) \leq \hat{\Phi}(k, k')$$

where $\hat{\Phi}(k, k')$, which preserves the properties of $\Phi(k, k')$, has been extended accordingly. As before with $k > k'$, we have facts i) and ii). To show that the additional term

$$c_{o,0}^2(k) - c_{o,0}^2(k') + \alpha \left(c_{p,0}^2(k) - c_{o,0}^2(k) \right) - \alpha \left(c_{p,0}^2(k') - c_{o,0}^2(k') \right)$$

is also increasing in α, notice that $0 > c_{p,0}(k') > c_{p,0}(k)$ and $c_{o,0}(k) > c_{o,0}(k') > 0$ implies

$$c_{o,0}(k) > c_{p,0}(k') + c_{o,0}(k')$$

so that

$$(c_{p,0}(k) + c_{o,0}(k)) (c_{p,0}(k) - c_{o,0}(k)) > (c_{p,0}(k') + c_{o,0}(k')) (c_{p,0}(k') - c_{o,0}(k'))$$

can be easily shown given that $c_{p,0}(k) - c_{o,0}(k) > c_{p,0}(k') - c_{o,0}(k')$ from facts ii). Nevertheless, the additional term is positive at $\alpha = 0$. This implies that whereas all the qualitative feature of our analysis equally hold considering $\lambda > 0$, a longer forward guidance are ceteris paribus more efficient at low α.

Let us look at how the reasoning can be extended to multiple periods in the liquidity trap. Without loss of generality, let us go back to the simple case $\lambda = 0$. We add a period $t = -1$ that takes place just before period 0, then the reasoning can be extended recursively. We have then to compare:

$$1/\beta\pi_{-1}^2(k) - 1/\beta\pi_{-1}^2(k') + \Pi(k, k', \alpha) \leq \Phi(k, k').$$

Notice that the additional term is typically positive, so ceteris paribus, with a longer trap a longer forward is needed for low α. The derivative with respect to α of the additional terms $\Pi_{-1}(k, k', \alpha)$ is:

$$\frac{\partial \Pi_{-1}(k, k', \alpha)}{\partial \alpha} = \beta^{-1}\left(2\pi_{-1}(k)\frac{\partial \pi_{-1}(k)}{\partial \alpha} - 2\pi_{-1}(k')\frac{\partial \pi_{-1}(k')}{\partial \alpha}\right),$$

which has the same structure than (15) and can be expressed similarly as a linear combination of future actual aggregate consumption and inflation. In particular, we can show

$$\begin{aligned}
\frac{\partial c_{-1}(k)}{\partial \alpha} &= \frac{\partial c_0(k)}{\partial \alpha} + \gamma^{-1}\frac{\partial \pi_0(k)}{\partial \alpha} < \frac{\partial c_{-1}(k')}{\partial \alpha} < 0 \\
\frac{\partial \pi_{-1}(k)}{\partial \alpha} &= \kappa\frac{\partial c_{-1}(k)}{\partial \alpha} + \beta\frac{\partial \pi_0(k)}{\partial \alpha} < \frac{\partial \pi_{-1}(k')}{\partial \alpha} < 0
\end{aligned}$$

using previous relations. Therefore, $\partial\pi_{-1}^2(k)/\partial\alpha$ is a linear downward sloping function of α. Given this result, we can then extend recursively the analysis to an arbitrarily number of periods.

D Detailed model derivation

In this appendix, we micro-found the linear model that we use in the core of this paper.

D.1 Environnment

The economy is populated by a continuum of households, firms and the central bank. Time is discrete and indexed by $t \in \{0, ...\infty\}$.

Household. The household family is constituted by a continuum of agents of mass one indexed by $i \in [0, 1]$. Each agent decides how much to work, consume and save in order to maximally contribute to the household welfare:

$$U = \int_0^1 \sum_{t=0}^{\infty} \beta^t e^{\xi_t}\left(\frac{C_{i,t}^{1-\gamma} - 1}{1-\gamma} - \frac{L_{i,t}^{1+\psi}}{1+\psi}\right) di, \tag{16}$$

where $C_{i,t}$ and $L_{i,t}$ are respectively consumption and labor supply of agent i in period t. The parameter $\beta \in (0, 1)$ is a discount factor, the parameter $\gamma > 0$ is the inverse of the inter-temporal elasticity of substitution, and the parameter $\psi \geq 0$ is the inverse of the Frisch elasticity of labor supply. The variable ξ_t is a preference shock discussed below.

Each agent manages a portfolio representing a fraction of the household wealth. Between periods t and $t+1$, agent i deals with the following flow budget constraint:

$$B_{i,t} = R_{t-1}B_{i,t-1} + W_t L_{i,t} + D_t - P_t C_{i,t} + Z_{i,t}, \tag{17}$$

where $B_{i,t}$ are bond holdings of the agent between periods $t-1$ and t, R_{t-1} is the gross nominal interest rate on bond holdings between periods $t-1$ and t, W_t is the nominal wage rate in period t, D_t is the difference between nominal profits received and nominal lump-sum taxes paid, by each agent in period t (we assume here diffuse ownership), and P_t is the price of the final good in period t. The agent can borrow (formally, bond holdings can be negative), but the household is not allowed to run a Ponzi scheme. Finally, the term $Z_{i,t}$ denotes a nominal intra-household transfer by agent i.

Intra-Household risk sharing. Each period is divided into three stages. In the first stage, current shocks hit and agents observe them. At this stage agents form their beliefs on the state of the world. In the second stage of each period, agents can implement a feasible transfer plan in which each agent $i \in [0,1]$ at date t contributes by an amount $Z_{i,t}$ and such that:

$$\int_0^1 Z_{i,t} di = 0. \tag{18}$$

only if every agent agrees on it. Without loss of generality,[31] we assume that when no unanimity is reached, then no transfers are made; in such a case each agent owns the wealth resulting from her own portfolio management. Let us therefore introduce the following formal definition.

Definition 3. *An implementable transfer plan at time t is a feasible transfer plan $\{\hat{Z}_{i,t}\}_{i=0}^1$ such that*

$$E_{t,i}[U_t|\{\hat{Z}_{i,t}\}_{i=0}^1] \geq E_{t,i}[U_t|\{Z_{i,t}\}_{i=0}^1],$$

for each $i \in [0,1]$ and each feasible transfer plan $\{Z_{i,t}\}_{i=0}^1$.

In the last stage, once intra-household wealth transfers are carried out, each agent decides on her own labor supply and consumption, based on their own individual beliefs and taking other agents' decisions as given. The crucial assumption we are making here is that agents cannot commit on future transfers: each period they decide under discretion. We also assume that the whole mechanism is common knowledge.

[31] To explain why is without loss of generality, we need to introduce a bit more structure. See footnote 32 below.

Firms. Production is implemented in the context of a standard monopolistic competition environment. The final good is produced by competitive firms using the technology: $Y_t = (\int Y_{j,t}^{(\theta-1)/\theta} dj)^{\theta/(\theta-1)}$. Y_t denotes output of the final good and $Y_{j,t}$ denotes input of intermediate good j. The parameter θ is the elasticity of substitution between intermediate goods. Final good firms have perfect information and fully flexible prices. Profit maximization of firms producing final goods implies the following demand function for intermediate good j:

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{\theta} Y_t, \tag{19}$$

where $P_{j,t}$ is the price of intermediate good j and P_t is the price of the final good. Furthermore, the zero profit condition of firms producing final goods implies $P_t = (\int P_{j,t}^{1-\theta} dj)^{1/(1-\theta)}$. Each intermediate good j is produced by a monopolist using the linear technology:

$$Y_{j,t} = L_{j,t}, \tag{20}$$

where $Y_{j,t}$ is output and $L_{j,t}$ is labor input of this monopolist.

Monopolists producing intermediate goods are subject to a price-setting friction as in Calvo (1983). Each monopolist can optimize its price with probability $1 - \chi$ in any given period. Finally, we assume that firms' stocks are held by households in equal shares.

D.2 Intra-household risk-sharing

In this subsection, we derive our result on endogenous risk sharing. Disagreement has major consequences for the dynamics of intra-family transfers. At the second stage of each period, agents need to decide on the wealth transfers. In the absence of disagreement, this would optimally result in an even distribution of wealth. Yet, the type of policy will be revealed only at a future date, let us sat $\hat{T}$ (in the simplified version of our model at $p_{cb} \to 1$ such date is T_o). Before that date, agents have different beliefs on the future course of the economy and so on which transfer plan maximizes family welfare; this prevents transfers from happening before the truth unfolds. In any case, all agents anticipate that they will share their wealth in the future as soon as they have evidence on which they cannot disagree any longer. This implies that no transfer plans can be implemented before date $\hat{T}$.

The following proposition states this formally.

Proposition 2. *Consider the case of heterogeneous beliefs before $\hat{T}$, then the only equilibrium sequence of implementable plans of transfers $\{\{Z_{i,t}^*\}_0^1\}_{t=0}^{\infty}$ is the one providing for $\{Z_{i,t}^*\}_0^1 = 0$ at each $t \neq \hat{T}$ and $\{Z_{i,\hat{T}}^*\}_0^1$ such that*

$$U_c(C_{i,t}) = U_c(C_{j,t}) \text{ for } t = \hat{T}, \tag{21}$$

namely, the marginal utility of consumption is equal across agents at the time $\hat{T}$ when heterogeneity in beliefs vanishes, which implies $B_{i,t} = B_{j,t}, \forall(i,j) \in [0,1]^2$ for $t \geq \hat{T}$.

Proof. The proof is organized in five steps. *First step.* Consider an economy with homogeneous agents at the date $T_{zlb}+1$ just after the end of the zero-rate period (no matter how it gets fixed), so that the steady state can be restored. Because of Ricardian equivalence holds, the present value of their life-utility is the same irrespective of the stock of bonds they hold at that time, which is a legacy of the realized states of the words. Therefore, because of the permanent income hypothesis, the level of homogenous individual consumption $C_{T_{zlb}+1} = \bar{C}$ is pin down only by the forward evolution of the economy that will remain at steady state. *Second step.* At time $\hat{T}$ (given the nature of our shock $\hat{T} \leq T_{zlb}+1$), as soon as agents become homogeneous, they would agree on a plan of transfers $\{Z^*_{i,t}\}^1_0$ such that $B_{o,t} = B_{p,t}$, that is, their stock bonds is equalized. In fact, as a consequence, consumption is equalized and so $U_{C_{o,T}} = U_{C_{p,T}}$, that is, social welfare is maximized. After that period, irrespective of whether or not the economy is already at steady state (preference shock does not hit), individual consumption will converge to $C_{T_{zlb}+1} = \bar{C}$ because of what argued in the first step. *Third Step.* Consider now the sequence of transfers $\{\{Z^*_{i,t}\}^1_0\}^\infty_{t=0}$, since step two and three are common knowledge, there is only one equilibrium consumption path associated to each state of the word as described in the proposition. *Fourth step.* Different transfers plans, which modify agents' path of consumption, imply, because of the permanent income hypothesis, different level of consumption at steady state. Given that agents anticipate step 2, no plan of this kind can be implemented. In other words, agents anticipate that at time t they will agree to equalize their wealth so that $\bar{C}$ will be their steady state consumption that in turn determines the unique consumption path described at step three. *Fifth step.* Among all the transfer plans that can engineer an equalization in the stock of bonds at time $\hat{T}$ onwards, $\{Z^*_{i,t}\}^1_0$ is the only one that is implementable because before time T agents disagree on the actual transfer that will equalize bonds holding at time $\hat{T}$ as they expect different real interest rates paths, after time $\hat{T}$ they agree on no transfers. □

As no transfers are made during the period of the trap, the two types of agents then consume according to their beliefs, managing the share of wealth that they hold at the beginning of the trap.

It is worth to remark that proposition 2 relies on the assumption that households cannot commit to future transfers. As a consequence, agents of each type anticipate that, whatever their financial position, intra-household wealth will be equalized at a future date, when the truth will eventually unfold. Before that date, intra-family transfers, even if they were implemented,[32] cannot change agents' perceptions of their permanent income, and so cannot

[32] With different opinions about which plan achieves the first best, agents cannot implement any transfer plan. However, this rule has the mere role of selecting a unique feasible plan when agents disagree. That is, another backup rule would not change the results. For example, we could have equally assumed that, when

affect current consumption-saving choices. In other words, as they expect wealth to be equalized in the future – even though not at the same level – but anticipate different paths of real interest rates, pessimists and optimists select different paths of consumption. If different transfers are implemented, pessimists and optimists both modify their portfolio choices, keeping consumption paths unmodified and anticipating future transfers.

Finally, once we obtain 2, we can log-linearize our model around the unique steady state where the ZLB is not binding.

D.3 Aggregate Behavior and the New-Keynesian Phillips Curve

Following standard steps, we can write down the log-linearized versions of optimality conditions as:

$$c_{i,t} = -\tfrac{1}{\gamma}\left(E_{i,t}\xi_{t+1} - \xi_t + r_t - E_{i,t}\pi_{t+1}\right) + E_{i,t}c_{i,t+1}, \tag{22}$$

$$\gamma c_{i,t} + \psi l_{i,t} = w_t - p_t \tag{23}$$

Notice that that $\xi_t < 0$ in the trap and $\xi_t = 0$ out of the trap. This means that an exit form the trap, say at time $t+1$, implies $\xi = E_{i,t}\xi_{t+1} - \xi_t > 0$. So, the term $\xi = E_{i,t}\xi_{t+1} - \xi_t$ is positive at the time of reverting to normal times and equals 0 otherwise. As a result, the Euler equation (22) implies that consumption decreases at the beginning of the liquidity trap before it gradually increases during the trap.

Aggregate behavior. Assuming that ξ can be anticipated a period in advance and by solving forward, we obtain that individual consumption equals:

$$c_{i,t} = -\frac{1}{\gamma}E_{i,t}\left[\sum_{\tau=t}^{\infty}(r_\tau - \pi_{\tau+1} + \xi_{\tau+1} - \xi_\tau)\right]$$

and aggregate consumption equals:

$$c_t = -\frac{1}{\gamma}E_t\left[\sum_{\tau=t}^{\infty}(r_\tau - \pi_{\tau+1})_{\tau+1} + \xi_{\tau+1} - \xi_\tau\right]$$

Notice that as long as agents do not disagree on the size of the shock (this is the case as they observe it), but only on the future date on which it will unfold, it enters as a fix wedge in the IS curve. This wedge will disappear only at the optimistic date when agents will discover the truth.

agents disagree, a dictator decides on their transfers. Given that the dictator cannot enforce future transfers (no commitment), agents commonly know that, from some future date onward, they will agree again, and so, their wealth will be equalized. In this case, the dictator's transfers cannot affect the perceived permanent income of an agent, and so cannot change agents consumption-saving plans.

New-Keynesian Phillips Curve. The optimal price setting for producer j is given by:

$$x_{j,t} = (1-\chi\beta)\, E_t^j \left[\sum_{\tau=t}^{\infty} (\chi\beta)^{\tau-t}\, w_\tau \right]$$

as standard in the sticky price literature. Aggregating over producers yields:

$$x_t = (1-\chi\beta)\, w_t + \chi\beta \int E_{i,t} x_{i,t+1} \mathrm{di},$$

which is a standard realtion. We obtain the New-keynesian Phillips Curve in the presence of heterogeneous beliefs as follows. By defining $\Delta_t \equiv \int E_{i,t} x_{i,t+1}\mathrm{di} - E_t x_{t+1}$, we can write x_t recursively as:

$$x_t = (1-\chi\beta)\, w_t + \chi\beta E_t x_{t+1} + \chi\beta\Delta_t$$

At the same time, $x_t = \frac{p_t - \chi p_{t-1}}{1-\chi}$ and so, we can write

$$p_t - \chi p_{t-1} = (1-\chi)\,(1-\chi\beta)\, w_t + \chi\beta E_t\,(p_{t+1} - \chi p_t) + (1-\chi)\chi\beta\Delta_t$$

Thus, by noticing that $\pi_t = p_t - p_{t-1}$, we obtain:

$$\pi_t = \frac{(1-\chi)\,(1-\chi\beta)}{\alpha}\,(w_t - p_t) + \beta E_t \pi_{t+1} + (1-\chi)\beta\Delta_t$$

By definition, $\Delta_t \equiv \int E_{i,t} x_{i,t+1}\mathrm{di} - E_t x_{t+1}$ and $x_{i,t}$ is a function of current and future wages (w_τs). As a result, we can rewrite Δ_t as follows:

$$\Delta_t = (1-\chi\beta) \sum_{\tau=0}^{\infty} (\chi\beta)^{\tau} \int E_{i,t} \left(w_{t+\tau+1} - \int E_{i,t+1}\left[w_{t+\tau+1}\right] \mathrm{di} \right) \mathrm{di}$$

which equals 0 in this case, yielding the New Keynesian Phillips Curve

$$\pi_t = \frac{(1-\chi)\,(1-\chi\beta)}{\chi}\,(w_t - p_t) + \beta E_t \pi_{t+1}, \tag{24}$$

which is identical to the one under homogeneous beliefs. This result crucially relies on the assumption that producers observe all current variables, wage included, and that there is a unique labor market. As a result, it is common knowledge that there will be no aggregate forecast error on the wage neither at present nor at a future date, which makes Δ nil.

D.4 Forward Guidance Puzzle

In our model, disagreement on aggregate consumption and inflation arises if and only if the share of pessimists is sufficiently low ($\alpha \in (0, \bar{\alpha})$), in which case the central bank implements an Odyssean forward guidance. As already mentioned, such a presence of heterogeneity can help to explain why the empirical effect of Odyssean forward guidance is usually found to be much lower than what the theory under homogeneous beliefs predicts (see Carlstrom et al., 2012; Del Negro et al., 2015, among others).

Here, we give further insights on our explanation to the puzzle. Similarly to McKay et al. (2016) with discounting, our mechanism limits the effects of future shocks on current aggregate consumption via the Euler equation. To highlight the mechanism, suppose the simplest case of a trap of one period and with two periods of Odyssean forward guidance, i.e. $T_{zlb} = 2$, before monetary policy fully stabilizes inflation and output starting in period $t = 2$: $c_{o,2} = c_{p,2} = \pi_2 = 0$.

In period $t = 1$, individual consumption and inflation equal:

$$c_{o,1} = \gamma^{-1}(\log R) > 0 \text{ and } c_{p,1} = \gamma^{-1}(\log R - r)$$
$$\pi_{o,1} = \kappa\gamma^{-1}(\log R) > 0 \text{ and } \pi_{p,1} = \kappa\gamma^{-1}(\log R - r)$$

Using the Euler equation for each household, we obtain date-0 consumption levels:

$$c_{o,0} = c_{o,1} - \gamma^{-1}(\log R - r - \pi_{o,1})$$
$$c_{p,0} = c_{p,1} - \gamma^{-1}(\log R - r - \pi_{p,1})$$

This allows us to write the aggregate Euler equation.

$$c_0 = (1-\alpha)c_{o,1} - \gamma^{-1}(\log R - r - \pi_{o,1}) + \alpha(c_{p,1} + \gamma^{-1}(\pi_{p,1} - \pi_{o,1})).$$

Finally, using the fact that $c_{p,1}$ and $\pi_{p,1}$ are expected to be negative by pessimists, aggregate consumption c_0 satisfies:

$$c_0 \leq (1-\alpha)c_{o,1} - \gamma^{-1}(\log R - r - (1-\alpha)\pi_{o,1}).$$

$(1-\alpha)$ appears in the aggregate Euler equation as the limit of the effect of forward guidance, as measured by optimists' beliefs. Therefore, the presence of pessimists affects the effectiveness of Odyssean forward guidance by reducing the aggregate impact of the future boom on current aggregate consumption.

D.5 Alternative assumptions on the NKPC

In this appendix, we investigate the transmission channels of our mechanism in the NK model and more precisely the precise role played by the NKPC. We have already consider a case where firms' shares are not traded and another one where prices are fully rigid. We now allow firms' shares to be traded. As optimists have a higher valuation of future firms' profits, they are the only stockholders in the economy, thus implying that firms share the same views as the optimists about future economic conditions. We show that even in this situation our main results still hold, as the pessimists still weight on current consumption. This can be observed in Figure 13b, which uses same conventions and parameters of the pictures before. This happens even though pessimists are forecasting inflation in the short run taking as given firms' optimistic views on future outcomes: pessimists' expectation of a recession is sufficient to drive down their current consumption and, then, to dampen the effects of forward guidance.

D.6 The welfare function

To determine optimal policy, the central bank's problem is to maximize the expected utility of agents:

$$U = \int_0^1 \sum_{t=0}^{\infty} \beta^t e^{\xi_t} \left(\frac{C_{i,t}^{1-\gamma} - 1}{1-\gamma} - \frac{L_{i,t}^{1+\psi}}{1+\psi} \right) di, \tag{25}$$

where $C_{i,t}$ and $L_{i,t}$ are respectively consumption and labor supply of agent i in period t. The parameter $\beta \in (0,1)$ is a discount factor, the parameter $\gamma > 0$ is the inverse of the inter-temporal elasticity of substitution, and the parameter $\psi \geq 0$ is the inverse of the Frisch elasticity of labor supply. The variable ξ_t is a preference shock as discussed above.

We show that proceeding similarly to Gali (2008) (pag.87), we can approximate the per period utility of each agent around a steady state as:

$$\mathbb{W}_i \equiv \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t U_{i,t} - U\right] \simeq \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t U_c C \left(c_{i,t} + \frac{1-\gamma}{2} c_{i,t}^2\right) + U_l L \left(l_{i,t} + \frac{1+\psi}{2} l_{i,t}^2\right)\right]. \tag{26}$$

For the sake of notational convenience, let us denote by $i = p$ the pessimist type and by $i = o$ the optimist type. The next step is to use the fact $L_t = Y_t \int (P_{j,t}/P_t)^{-\theta} di$ to derive

$$(1-\alpha) l_{o,t} + \alpha l_{p,t} = (1-\alpha) c_{o,t} + \alpha c_{p,t} + d_t \tag{27}$$

where the last price dispersion term is derived from as a direct implication of the Calvo assumption (for a proof see Woodford (2003) p.399) as being proportional to the square of inflation π_t^2. Given the first order condition on labor supply, and in particular because of the assumption of homogeneous labor market (23), we have that $\gamma c_{o,t} + \psi l_{o,t} = \gamma c_{p,t} + \psi l_{p,t}$ that is:

$$l_{p,t} - l_{o,t} = -\frac{\gamma}{\psi}\left(c_{p,t} - c_{o,t}\right). \tag{28}$$

Therefore we can rewrite

$$\begin{aligned} l_{o,t} + \alpha(l_{p,t} - l_{o,t}) &= c_{o,t} + \alpha(c_{p,t} - c_{o,t}) + d_t, \\ l_{p,t} + (1-\alpha)\left(l_{o,t} - l_{p,t}\right) &= c_{p,t} + (1-\alpha)\left(c_{o,t} - c_{p,t}\right) + d_t, \end{aligned}$$

or

$$
\begin{aligned}
l_{o,t} &= c_{o,t} + \alpha\left(1+\frac{\gamma}{\psi}\right)(c_{p,t}-c_{o,t}) + d_t, \\
l_{p,t} &= c_{p,t} + (1-\alpha)\left(1+\frac{\gamma}{\psi}\right)(c_{o,t}-c_{p,t}) + d_t.
\end{aligned}
$$

In the special case $\gamma = \psi$ we can show that

$$\alpha l_{p,t}^2 + (1-\alpha) l_{o,t}^2 = \alpha c_{p,t}^2 + (1-\alpha) c_{o,t}^2$$

since

$$\alpha\left(c_{p,t} + 2(1-\alpha)(c_{o,t}-c_{p,t})\right)^2 + (1-\alpha)\left(c_{o,t} + 2\alpha(c_{p,t}-c_{o,t})\right)^2 = (1-\alpha)c_{o,t}^2 + \alpha c_{p,t}^2.$$

Therefore, as in Gali (2008), we get

$$\mathbb{W}_i = -\varpi\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\left((1+\psi)l_{i,t}^2 - (1-\gamma)c_{i,t}^2 + \theta\pi_t^2\right)\right] \tag{29}$$

where ϖ is a positive constant, so that finally social welfare can be approximated by $\mathbb{W} = \int_0^1 \mathbb{W}_i di$. In the special case, $\gamma = \psi$, (29) becomes

$$\tilde{\mathbb{W}}_i = -\varpi\theta^{-1}\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\left(\lambda c_{i,t}^2 + \pi_t^2\right)\right] \tag{30}$$

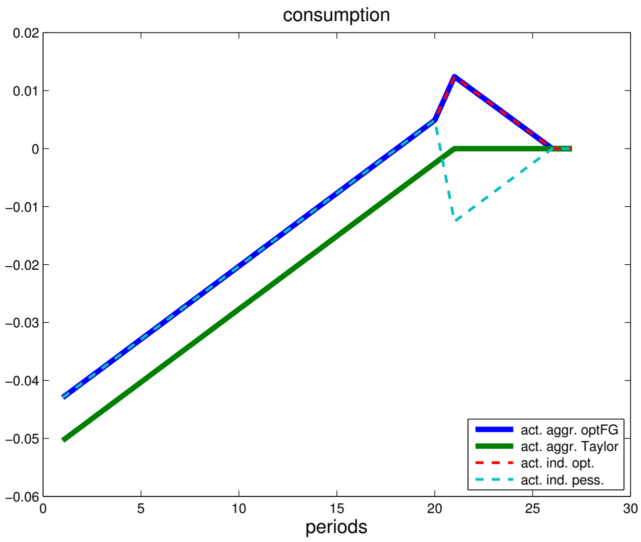
with $\lambda = 2\gamma/\theta$, which is identical to the case with homogeneous agents.

D.7 Identifying welfare effects.

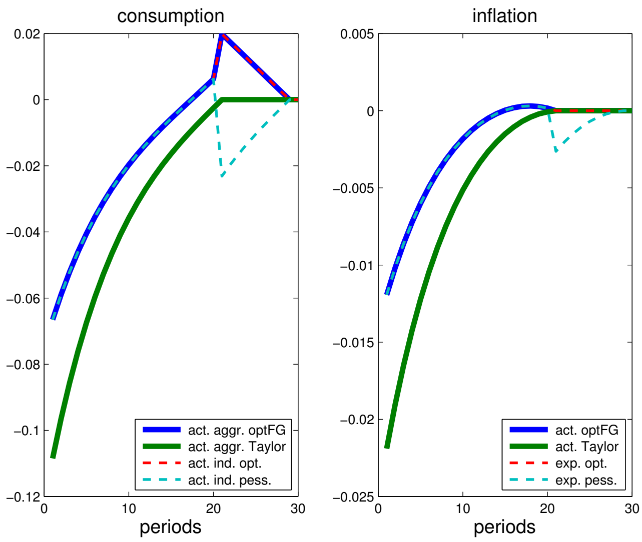
In this paragraph, we further investigate what drives our results on the hump-shaped optimal policy as well as what are the relevant effect in terms of welfare. In particular, there are two additional channels can also affect the central bank's optimal policy. First, once disagreement vanishes the boom engineered by the central bank will concern all agents, even though the *ex-ante* benefits from the future boom are dampened by the presence of heterogeneous beliefs; we refer to this effect as a "costly boom" channel. Second, the monetary authority's objective function also includes additional cross-product terms which relate to the inequality generated by disagreement; we refer to this effect as a "inequality" channel. These additional channels do not qualitatively alter the effects of the "bad news" channel as their effects are small.

This is shown by Figure 14a that reports the optimal period of extra-accommodation for the benchmark model as well as for the case where the boom is not "paid in full". The latter means that, to turn off the "costly boom" channel, we exogenously assume that pessimists do not catch up with optimists once the truth unfolds but go directly to the steady state. This implies that the boom negatively weights on welfare only to the extend that it was anticipated by optimists. We obtain in both cases a similar pattern for the optimal policy. Intuitively, when the boom is not paid in full, the monetary authority can afford a longer period of accommodation, as this can be observed in the graph. In the end, only the "bad news" channel, which is present in both cases, is necessary to obtain qualitatively our results. We do not report the effect of the "inequality" channel that was negligible in all calibrations that we have explored.

Where the hump-shaped pattern is then coming from? To explore this issue, we turn off the NK Phillips curve by assuming that prices are pre-determined for all periods at date -1, as in Werning (2012). Figure 14b plots the resulting optimal policy and compares it to our benchmark. Importantly, there is no hump-shaped pattern anymore. The main intuition for this observation is that the NK Phillips curve introduces non-linear effects of the extra-accommodation, as future monetary easing not only increases future consumption but also future inflation, thus having a double effects on current consumption, through both lower real interest rate and the expectation of higher consumption.



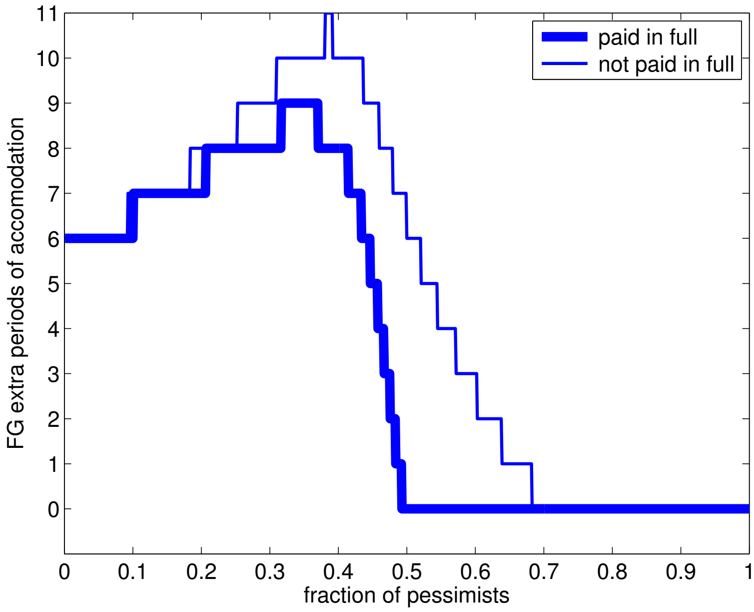
(a) Fully predetermined prices.



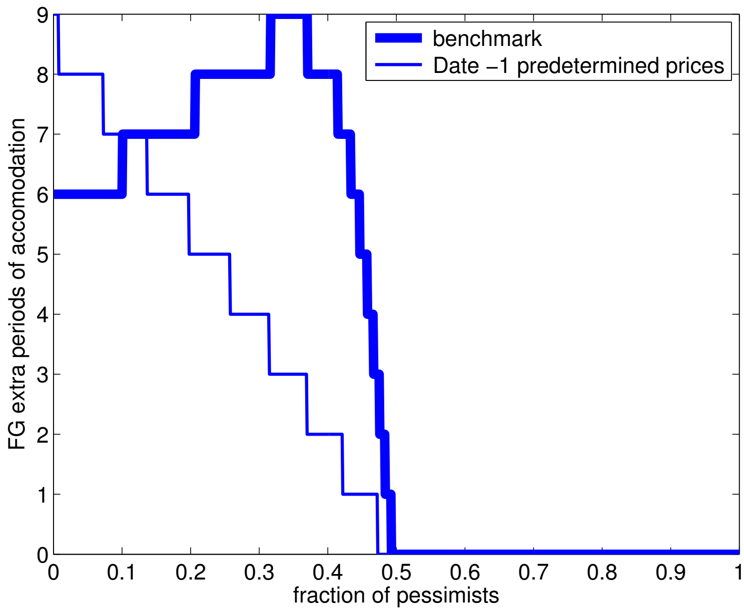
(b) Endogenous stockholdings.

Figure 13: The effect of Odyssean (blue) with a fraction $\alpha = .25$ of pessimists with alternative NK Phillips curve specifications.

We consider a shock ($\xi = -0.01$) on the discount rate that lasts 12 quarters and implies a drop of consumption of 4% at impact in the absence of Odyssean forward guidance, which provides for 4 extra quarters of accommodation. We calibrate the reaction to inflation at $\phi = 1.5$. The discount factor β is such that the annual real interest rate equals 2% and the utility function is assumed to be CRRA $u(c) = c^{1-\gamma}/(1-\gamma)$ with $\gamma = 2$. The probability not to reset prices is .85, and the slope of the Phillips' curve is then .027.



(a) Optimal policy when the boom is not "paid in full".



(b) Optimal policy with pre-determined prices at date -1.

Figure 14: We plot the optimal $T_{zlb}(\alpha, 20) - 20$ for $\xi = -.01$ with $\gamma/\psi = 1$;We calibrate the reaction to inflation at $\phi = 1.5$. The discount factor β is such that the annual real interest rate equals 2%, The probability not to reset prices is .85, and the slope of the Phillips' curve is then .027.

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