The Effects of Capital Requirements on Good and Bad Risk Taking*

N. Aaron Pancost† and Roberto Robatto‡

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Abstract

We identify a new channel by which deposit insurance and capital requirements affect welfare in general equilibrium. In the model, entrepreneurs have access to socially valuable projects whose return is subject to idiosyncratic, uninsurable risk. This risk reduces the entrepreneurs' investment and gives rise to a demand for safe assets, which are supplied by banks. If the government provides subsidized deposit insurance, banks pay a higher risk-adjusted return on deposits and entrepreneurs increase their investment in the socially-valuable projects. The optimal capital requirement trades off this effect with the moral hazard induced by deposit insurance.

*Financial support from the Wisconsin Alumni Research Foundation (WARF) is gratefully acknowledged.
†University of Texas at Austin, McCombs School of Business. Email address: aaron.pancost@mccombs.utexas.edu
‡University of Wisconsin-Madison, Wisconsin School of Business. Email address: robatto@wisc.edu

1 Introduction

After the 2007-2008 financial crisis, the revision of the regulatory framework of financial intermediaries has become a central topic under discussion by regulators and academics. The Basel III accord has tightened the restrictions imposed on banks by previous regulation and has introduced new tools to reduce the likelihood and depth of financial crises. One of the key sets of rules at the center of this debate is capital requirements, namely, limits on the fraction of debt that banks can use to finance their investment.

According to the Modigliani and Miller (1958) theorem, it is irrelevant whether a firm is financed with debt, equity, or a mix of the two. Admati and Hellwig (2013) use this result as a starting point for their analysis of bank regulation. They argue that capital requirements should be raised substantially to eliminate the moral hazard induced by government guarantees and the implicit too-big-to-fail subsidies, and some regulators have made a case for similar rules.[1] The typical argument against this suggestion is that banks are "special" because their liabilities are valued not only for their pecuniary return but also for their "liquidity" value, violating a key Modigliani-Miller assumption. Under this view, it is not desirable to impose large capital requirements that reduce the supply of banks deposits.

In this paper, we propose an alternative channel that reduces the desirability of high capital requirements. This channel is related to the role of banks as suppliers of safe assets and is different—though complementary—to the typical transaction role of deposits. In our model, the demand for safe assets comes from entrepreneurs who have access to projects that are socially valuable but whose return is subject to idiosyncratic, uninsurable risk. It is this idiosyncratic risk that gives raise to a demand for safe assets—or, more precisely, for assets whose return does not correlate with the entrepreneurs' projects.

A crucial result of our model is that the return on safe assets interacts with entrepreneurs' willingness to pursue risky, socially-valuable projects. The idiosyncratic shocks to the return on entrepreneurs' projects create volatility in entrepreneurs' own wealth. Because of this

[1] See the Minneapolis Plan discussed by Kashkari (2016).

volatility, entrepreneurs reduce their investment in the projects in comparison to an economy without idiosyncratic shocks. In this context, a higher return on safe assets provides a source of stable income that reduces the volatility of entrepreneurs' wealth. This effect encourages entrepreneurs to take on more risk, increasing their demand for labor and the scale of their idiosyncratic projects. Vice-versa, a lower return on safe assets reduces entrepreneurial activity. We call this channel "good risk taking" because the projects are socially valuable but not fully exploited due to the lack of insurance against idiosyncratic risk.

We think of the entrepreneurs in our model as small, privately-owned firms. Such firms are not included in standard datasets, such as Compustat, because they do not issue equity or file with the SEC; as such, they are under-represented in the finance literature relative to publicly-traded firms. However, privately-owned firms account for more than half of total domestic non-government employment in the US.[2] Thus any force affecting employment in this sector can have large effects on aggregate output and employment.

We highlight a new cost of capital requirements: they reduce the good risk taking of entrepreneurs. If the government provides subsidized deposit insurance, the subsidy increases the risk-adjusted return on deposits. Since deposits are safe assets demanded by entrepreneurs for insurance purposes, the higher return promotes good risk taking by entrepreneurs according to the channel explained above. As a result, capital requirements that limit deposit insurance disbursements and reduce the return on deposits also reduce good risk taking, and produce a negative impact on growth and welfare. The optimal level of capital requirements trades off this effect with the benefit of reducing the "bad" risk taking of banks, that is, the moral hazard of deposit insurance.

Our general message is that financial regulation has an impact on the real economy by changing the risk-taking capacity of the non-financial sector, and that this channel should be accounted for when studying optimal regulation. Because of this motivation, we focus on deposit insurance and capital requirements due to their primary role in banking regulation,

[2]We arrive at this figure by dividing total employment at firms in the Compustat database by total private employment from the BLS each year and taking the average from 1950–2016.

and we abstract from other policies that might enhance entrepreneurs' risk-taking but are unrelated to the financial sector. In this sense, we follow a common approach in the literature that studies capital requirements and motivates deposit insurance because of its role in preventing runs, as in Diamond and Dybvig (1983), but does not model explicitly this feature. More generally, deposits insurance in our model can also be interpreted as any government guarantee on bank debt, such as the Temporary Liquidity Guarantee Program set up by the Federal Deposit Insurance Corporation (FDIC) in 2008.

We first derive our results using some simplifying assumptions that keep our model tractable and allow us to isolate our channel from other effects. In particular, even if idiosyncratic shocks create heterogeneity across entrepreneurs' wealth, the equilibrium in our model depends only on average wealth, and the other moments of the distribution are irrelevant. We also assume that the government finances the shortages of the deposit insurance funds with taxes that do not create any distortions in entrepreneurs' decisions. We then plan to extend the paper by providing a quantitative analysis and assessing the magnitude of the good risk-taking channel.

2 Literature Review

This paper is part of a growing literature that studies capital requirements using macroeconomic models with a financial sector. Several papers employ quantitative models, such as Begenau (2016), Begenau and Landvoight (2017), Corbae and D'Erasmo (2014), Christiano and Ikeda (2013), Davydiuk (2017), Dempsey (2017), Gertler, Kiyotaki and Prestipino (2016), Nguyễn (2014), and van den Heuvel (2008).

A related paper by Elenev, Landvoight and van Nieuwerburgh (2017) studies the effect of increasing the price of mortgage guarantee offered by government-sponsored enterprises (GSEs). While they focus on a different policy, we share with them the idea that regulation and subsidies to the financial sector can have an impact on wealth distribution.

Our approach for modeling entrepreneurs' risk builds on Quadrini (2017), who also emphasizes the role of bank liabilities for insurance purposes.[3] However, his focus is different because he studies how various forces affect banks' risk taking and crises, without focusing on entrepreneurs' risk taking.

Our paper is also related to the literature that studies financial intermediaries as suppliers of safe assets, such as Diamond (2016), Magill, Quinzii and Rochet (2016), and Stein (2012). This literature builds on the ideas of Dang et al. (2017) and Gorton and Pennacchi (1990), in which the debt of banks is riskless to enhance its liquidity value or to overcome an informational friction. Bank debt is valuable in our model for a related but slightly different reason: there is a demand for securities that are uncorrelated with the idiosyncratic risk of entrepreneurs. Policies that increase the increase the supply of such securities also increase entrepreneurs' risk-taking capacity.

3 Model

3.1 Environment

Time is discrete and infinite and there is a single good that can be consumed or used for investment. There are four types of players in the economy: agents (entrepreneurs), banks, laborers, and the government.

3.1.1 Agents

Agents have log utility and discount the future at a rate $\beta < 1$. Agents live forever and and choose consumption to maximize their expected discounted stream of utility:

$$U^i = E \sum_{t=0}^{\infty} \beta^t \log c_t^i. \tag{1}$$

[3]The approach used by Quadrini (2017) at modeling entrepreneurs' risk builds on Arellano, Bai and Kehoe (2011).

Agents hire workers and produce output equal to $z_{t+1}^i l_t^i$, where z_{t+1}^i is the firm's productivity and l_t^i is their labor input. z_{t+1}^i is an idiosyncratic productivity shock that is realized after agents choose their labor input l_t^i. Agents can invest in deposits d_t^i that pay an interest rate R_t^d, and in bank's equity n_t^i that earns a (stochastic) return R_{t+1}^E; their budget equation is

$$c_t^i + d_t^i + n_t^i = x_t^i,$$

where x_t^i is their total wealth at time t. Agent i's wealth x_t^i evolves as

$$x_{t+1}^i = (1 - \tau_{t+1}) \left[\left(z_{t+1}^i - w_t \right) l_t^i + R_t^d d_t^i + R_{t+1}^E n_t^i \right]$$

where τ_{t+1} is a tax levied by the government to pay back depositors at failed banks.

Agents choose their investments in bonds, equity, and labor before knowing their own idiosyncratic productivity draw z_{t+1}^i or the realization of the return on equity R_{t+1}^E. However, the government provides full deposit insurance, so deposits are safe and R_t^d is known in advance. The wage w_t is also known at time t when l_t^i is chosen.

A note on timing and notation: all variables indexed with a t subscript are known to agents at the beginning of time t when they make decisions. Thus for the agent the only unknowns are their own productivity z_{t+1}^i, the return on equity R_{t+1}^E, and the tax rate τ_{t+1}. These random variables depend on future aggregate productivity A_{t+1}, described below. Agent's idiosyncratic output $z_{t+1}^i l_t^i$ depends on productivities and occurs at the beginning of period $t + 1$, immediately after the shocks are realized.

The above assumptions lead to the following Bellman equation for agents:

$$
\begin{aligned}
V_t(x_t) = \max_{c_t, d_t, n_t, l_t \geq 0} \; & \log c_t + \beta E_t V_{t+1}(x_{t+1}) \\
s.t. \; & \\
& c_t + d_t + n_t = x_t \\
& x_{t+1} = (1 - \tau_{t+1}) \left[(z_{t+1} - w_t) l_t + R_t^d d_t + R_{t+1}^E n_t \right]
\end{aligned}
$$

where the t subscript on the value function incorporate all aggregate information at time t, including the distribution of wealth x_t^i across agents. As we show below in Proposition 1, the wealth distribution will not enter into agents' optimal decisions, a key result that keeps our model tractable. This result stems from our assumption of log utility, and from the fact that the agents' problem exhibits constant returns to scale.

The assumption that the government taxes agents in proportion to agents' wealth x_t^i, rather than lump-sum or on project income $\left(z_{t+1}^i - w_t\right) l_t^i$, allows us to isolate our channel from other effects. The combination of log utility and proportionality to wealth implies that agents' investment and labor-demand choices are independent of the level of the tax, as we show below, similar to the way lump-sum taxes work in other classes of models.[4] Moreover, a tax on project income would have a direct effect on agents' labor demand and would smooth the return on entrepreneurs' projects – the tax would be paid by entrepreneurs with successful projects, but not by those with unsuccessful projects. This channel would create a further effect that reduces entrepreneurs risk above and beyond that of deposit insurance and capital requirements. Because our goal is to understand the effects of capital requirements themselves, we have chosen to pay for deposit insurance in the model with a non-distortionary tax on wealth. We conjecture that a tax on income would lead to even stronger effects.

[4]We do not use lump-sum taxes because they would make agents' decisions dependent on their own wealth, thereby eliminating the tractability advantages of log utility.

3.1.2 Banks

Banks live for a single period; that is, a bank set up at time t is liquidated at the beginning of time $t+1$. At the beginning of period t they have total equity n_t and have access to a decreasing-returns-to-scale technology which produces output equal to $A_{t+1}e^{\sigma\varepsilon}k_t^{\alpha}$, where A_{t+1} is aggregate productivity, ε is a bank-specific idiosyncratic shock realized at $t+1$, and k_t is the physical capital invested in by the bank. Banks finance any investment in k_t beyond their own equity capital n_t by borrowing from agents at R_t^d. Banks face a capital requirement that limits their ability to raise deposits; formally, their equity ratio n_t/k_t must be weakly larger than some number ζ.

The banker's problem is

$$
\begin{aligned}
&\max_{k_t,d_t} \; E_t \int \left\{A_{t+1}e^{\sigma\varepsilon}k_t^{\alpha} - R_t^d d_t\right\}^{+} dF(\varepsilon) \\
&\quad \text{s.t.} \\
&\qquad k_t = d_t + n_t \qquad\qquad (2)\\
&\qquad \frac{n_t}{d_t+n_t} \geq \zeta \\
&\qquad n_t \text{ given}
\end{aligned}
$$

where the constraint in ζ reflects the capital requirement, $\{\cdot\}^{+} = \max\{\cdot, 0\}$ is the positive part of bank's profits, $F(\cdot)$ is the CDF of banks' idiosyncratic productivity shocks, and the expectation is taken over the distribution of A_{t+1}. Banks that receive a low value of the productivity shock ε, such that $A_{t+1}e^{\sigma\varepsilon}k_t^{\alpha} < R_t^d d_t$, do not pay back their depositors; these deposits are guaranteed by the government.

When firms invest in bank's equity, they invest in a mutual fund that diversifies over the idiosyncratic shocks ε. Thus investments in bank equity are exposed to the aggregate risk in A_{t+1} and to the fraction of banks that default in equilibrium. The realized return on equity is given by

$$R^E_{t+1} = \frac{1}{n_t} \int \left\{ A_{t+1} e^{\sigma\varepsilon} k_t^\alpha - R_t^d d_t \right\}^+ dF(\varepsilon). \tag{3}$$

Equation (3) implies that $\underline{\varepsilon}_{t+1}$, the highest value of ε at which banks default on their depositors, is implicitly defined as a function of A_{t+1}:

$$R_t^d d_t = A_{t+1} e^{\sigma \underline{\varepsilon}_{t+1}} k_t^\alpha. \tag{4}$$

3.1.3 Government

The government taxes agents' wealth x^i_{t+1} at a rate τ_{t+1} in order to ensure that depositors at failed banks at the beginning of $t+1$ are made whole. The government seizes output at failed banks (who return zero to their equity-holders) to partially defray the expenses of paying back depositors, but its recovery efforts are subject to a deadweight loss that is quadratic in the amount of output to be collected.

The total amount of tax to be collected is

$$T_{t+1} = \int \left\{ \underbrace{R_t^d d_t}_{\text{owed to depositors}} - \underbrace{A_{t+1} e^{\sigma\varepsilon} k_t^\alpha}_{\text{collected from banks}} \right\}^+ dF(\varepsilon) + \underbrace{\frac{\lambda}{2} \left[\int_{-\infty}^{\underline{\varepsilon}_{t+1}} A_{t+1} e^{\sigma\varepsilon} k_t^\alpha dF(\varepsilon) \right]^2}_{\text{deadweight loss}}$$

where the parameter λ in the second term indexes the extent of deadweight losses in the economy. Then the tax rate on wealth τ_{t+1} satisfies

$$\tau_{t+1} = \frac{T_{t+1}}{\int \left[\left(z^i_{t+1} - w_t \right) l^i_t + R^d_t d^i_t + R^E_{t+1} n^i_t \right] di} \tag{5}$$

where the denominator is aggregate pre-tax wealth across all agents i. τ_{t+1} depends on the realization of the aggregate shock A_{t+1}, which affects both the realized return on equity and the fraction of banks that default in equilibrium.

3.1.4 Laborers

Laborers live for a single period and are hand-to-mouth; they choose consumption c_t and labor supply l_t to solve

$$\max_{c_t, l_t} \; c_t - \nu_1 \frac{l_t^{1+\frac{1}{\nu_2}}}{1+\frac{1}{\nu_2}} \tag{6}$$

subject to the budget constraint

$$c_t = w_t l_t.$$

3.2 Equilibrium Definition

Given exogenous stochastic processes for the aggregate productivity A_t and z_t^i, equilibrium is a collection of firm policies, bank policies, and government taxes such that

1. Agents' choices for labor demand l_t^i, deposits d_t^i, equity investment n_t^i, and consumption c_t^i maximize their utility (1);
2. Banks' choices for capital k_t and deposits d_t solve their problem (2),
3. Bank profits are returned to agents holding bank equity through the return on equity given in equation (3);
4. The government taxes agents in proportion to their wealth and uses the proceeds to pay depositors at failed banks according to equation (5).
5. Laborers' labor supply is consistent with maximizing their utility (6).
6. The wage w_t and the return on deposits R_t^d clear the labor and deposit markets, respectively.

4 Results

We begin our analysis of the model by showing that entrepreneurs' choices can be easily aggregated despite the heterogeneity in their wealth (Section 4.1). We then present our main result about the effects of capital requirements on the good risk-taking channel of entrepreneurs (Section 4.2); to clarify the exposition, we set the deadweight loss of default to zero (i.e., $\lambda = 0$) so that we can ignore the more standard bad risk-taking channel of banks. We then consider a version of the model where both the good and bad risk-taking channels are at work and capital requirements trade off these two effects (Section 4.3).

4.1 Entrepreneurs' and laborer' choices, and aggregation

The following proposition greatly simplifies the analysis by allowing us to aggregate easily across entrepreneurs.

Proposition 1. *Agent i's optimal choices are given by*

$$\begin{aligned}
c_t^i &= (1-\beta)\,x_t^i \\
l_t^i &= \phi_t \beta x_t^i \\
n_t^i &= y_t \beta x_t^i \\
d_t^i &= (1-y_t)\,\beta x_t^i
\end{aligned}$$

where ϕ_t and y_t are independent of x_t^i and satisfy the following first-order conditions:

$$0 = E_t\left\{\frac{z_{t+1}^i - w_t}{\Delta_{t+1}^i}\right\} \tag{7}$$

$$0 = E_t\left\{\frac{R_{t+1}^E - R_t^d}{\Delta_{t+1}^i}\right\} \tag{8}$$

where

$$\Delta_{t+1}^{i} \equiv \left(z_{t+1}^{i} - w_t\right)\phi_t + y_t R_{t+1}^{E} + \left(1 - y_t\right) R_t^{d}. \tag{9}$$

Proof. See Appendix A. □

A key result of Proposition 1 is that the tax rate τ_{t+1} does not enter agents' first-order conditions. In addition, the total savings of entrepreneurs, $\int \left[d_t^i + n_t^i\right] di = d_t + n_t$ are a constant fraction β of aggregate wealth $X_t \equiv \int x_t^i di$, so that total investment by banks $k_t = d_t + n_t$ is also a constant fraction of entrepreneurial wealth in equilibrium. Thus, for a given X_t, the risk-shifting activity of banks due to deposit insurance affects their default probability and the deposit and equity returns, but not the size of their balance sheet.

In the remainder of the paper, y_t is the fraction of agents' savings βx_t^i devoted to bank equity, and ϕ_t is their labor demand as a proportion to their savings.

Because agents' choices are all proportional to their individual wealth x_t^i, aggregates don't depend on the distribution of x_t^i across agents. Thus aggregate labor demand is given by

$$\begin{aligned} L_t &= \int l_t^i di \\ &= \int \phi_t \beta x_t^i di \\ &= \phi_t \beta X_t, \end{aligned} \tag{10}$$

and aggregate wealth next period is

$$X_{t+1} = \left(1 - \tau_{t+1}\right)\left[\left(\bar{z}_{t+1} - w_t\right)\phi_t + y_t R_{t+1}^{E} + \left(1 - y_t\right) R_t^{d}\right] \beta X_t \tag{11}$$

where $\bar{z}_{t+1}$ is the average idiosyncratic shock across agents.

The first-order condition to the laborer's problem (6) implies that the labor supply curve is given by

$$w_t = \nu_1 \left(L_t\right)^{\frac{1}{\nu_2}}. \tag{12}$$

4.2 Capital requirements and good risk taking

In this section, we provide our main results about the effects of capital requirements on good risk taking by entrepreneurs. To clarify the exposition, we focus on the simple case in which aggregate productivity is constant (i.e., $A_t = A$ for all t) and there are no deadweight losses associated with bank default (i.e., $\lambda = 0$). Because of the assumption of constant A_t, we can focus on a steady state in which all aggregate variable are constant as well.

We first present the effects of capital requirements on aggregate wealth X_t and agents' consumption in Proposition 2, and then we turn to welfare in Proposition 3. We distinguish between two cases: constant z^i_{t+1} and stochastic z^i_{t+1}.

Proposition 2. *Suppose $\lambda = 0$ and $A_{t+1} = A$ is not random. Then if z^i_{t+1} is a known constant for all t and all i, changing the capital requirement ζ has no effect on aggregate wealth X_t and agents' consumption. On the other hand, if z^i_{t+1} is random, increasing ζ when the capital requirement constraint binds reduces aggregate wealth X_t and agents' consumption in steady-state.*

Proof. See Appendix A. □

Proposition 2 characterizes the real effects of capital requirements, in the way they affect agents' idiosyncratic risk-taking. Capital requirements force banks to hold more equity, which reduces their default probability in equilibrium. But because $\lambda = 0$, the level of bank default is irrelevant: a reduction in bank defaults reduces the amount that the government must tax agents to pay for deposit insurance, but this reduction in the tax is exactly offset by a reduction in the return to deposits (and to bank equity).

Thus if agents face no idiosyncratic risk, capital requirements have no real effects. However, when agents face idiosyncratic productivity shocks, the reduction in the return on deposits has the effect of increasing the risk agents face: the risks in their idiosyncratic labor income have not changed, but the share of their income from safe bank deposits has gone down. Because agents are risk-averse, they reduce their labor demand in response. This lowers aggregate output and steady-state wealth, making agents worse off.

Figure 1 illustrates the basic mechanism in the model. The top panel plots the equilibrium in the deposit market, where the red line and blue lines plot equations (3) and (8), respectively, in the $\left(R^d, R^E\right)$ plane. Because there is no aggregate risk, equation (8) implies that $R^E = R^d$ and therefore the red line is a 45-degree line in the $\left(R^E, R^d\right)$ plane. On the other hand, by equation (3) it must be that R^E is decreasing in R^d from the bank's perspective, which gives the blue curves in the top panel of Figure 1.

Increasing the capital requirement ζ does not affect the link between R^E and R^d implied by entrepreneurs' first-order condition (8), but it changes the return on equity payed by banks. Thus, the red line in the top panel of Figure 1 is unchanged, whereas the blue line shifts from the solid to the dotted one. To see this, rewrite equation (3) as

$$\begin{aligned} R^E &= \frac{1}{n} \int \left\{ A e^{\sigma \varepsilon} k^{\alpha} - R^d d \right\}^{+} dF\left(\varepsilon\right) \\ &= \frac{1}{\zeta} \int_{\underline{\varepsilon}}^{\infty} \left[A e^{\sigma \varepsilon} k^{\alpha - 1} - \left(1 - \zeta\right) R^d \right] dF\left(\varepsilon\right) \end{aligned}$$

where the second line plugs in the constraints $n/k = \zeta$ and $d = k - n = \left(1 - \zeta\right) k$. Taking the derivative of R^E with respect to ζ yields

$$\frac{\partial R^E}{\partial \zeta} = -\frac{1}{\zeta^2} \int_{\underline{\varepsilon}}^{\infty} \left[A e^{\sigma \varepsilon} k^{\alpha - 1} - R^d \right] dF\left(\varepsilon\right) \tag{13}$$

where there is no $\partial \underline{\varepsilon} / \partial \zeta$ term because the integrand in equation (3) is zero (by definition) at $\underline{\varepsilon}$. The sign of the derivative in equation (13) can be either positive or negative, as shown in

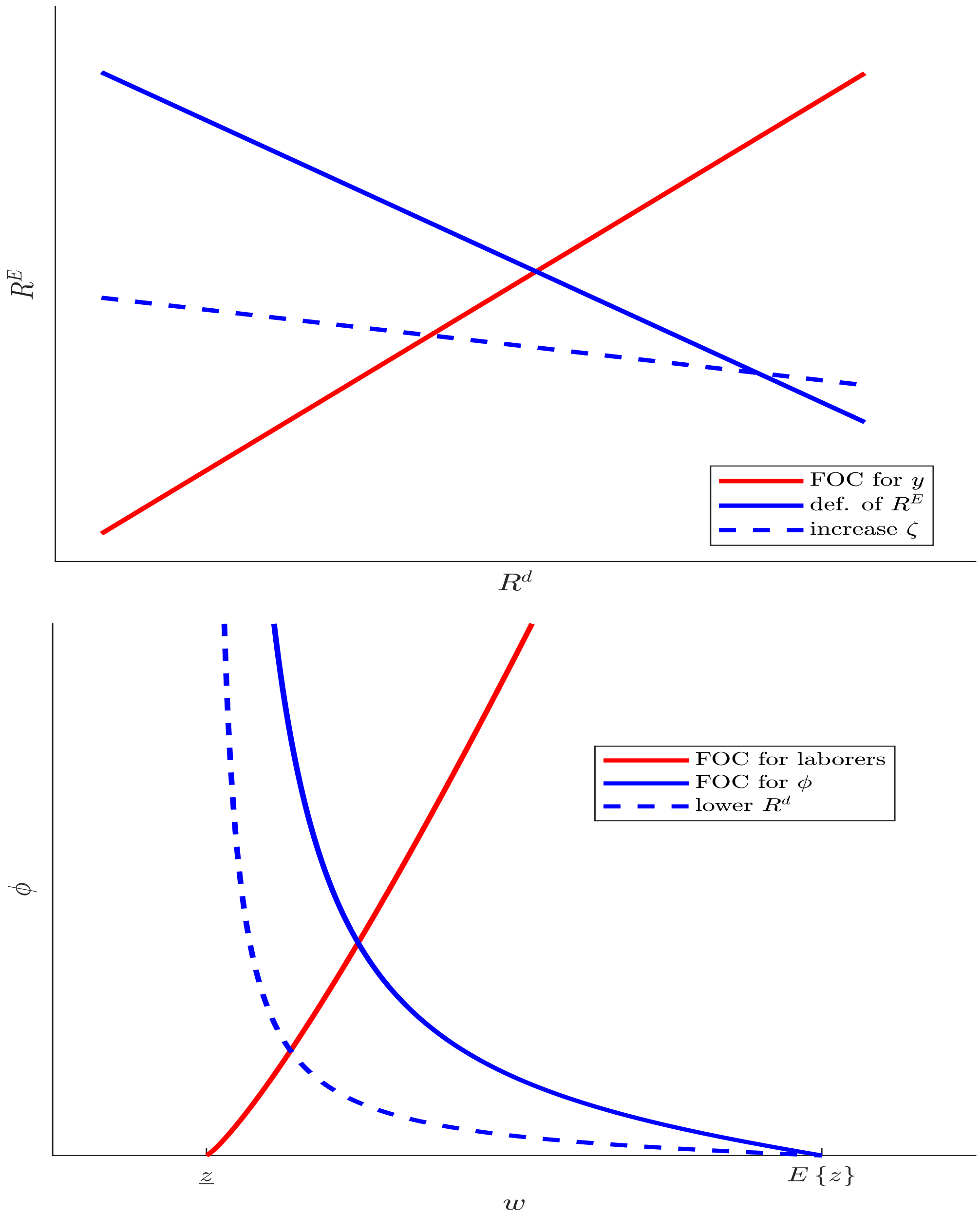

Figure 1. Equilibrium in the Asset and Labor Markets
The top panel plots the equilibrium in the asset market. The x- and y-axes are the returns on deposits R^d and equity R^E, respectively. The red line plots equation (8) and the blue lines plot equation (3) for two binding values of ζ. The dotted blue line represents a higher, binding value of ζ than the solid blue line. The bottom panel plots the equilibrium in the labor market. The red line plots the labor-supply curve (12), while the blue lines plot the labor-demand curve (7) for a fixed value of X. The solid blue line plots labor demand for one value of R^d, while the dotted line plots labor demand for a lower value of R^d.

the top panel of Figure 1, but it must be negative for values of R^d close to R^E. This is the content of Proposition 2. Intuitively, raising the capital requirement increases the number of equity holders who must be paid out of bank profits, lowering R^E, but it also reduces the number of depositors who must be paid R^d. If R^d were very high, this second effect could dominate and increasing ζ would increase R^E; but this cannot occur in equilibrium where $R^E = R^d$.

The bottom panel of Figure 1 plots the equilibrium in the labor market. By equation (10), labor demand depends both on aggregate agent wealth X_t as well as their proportional labor choice ϕ, which solves equation (7) when $R^E = R^d$ by equation (8). It is straightforward to show that $\phi = \infty$ if $w \leq \underline{z}$, the lowest possible value of z, and that $\phi = 0$ when $w = E\{z\}$, so that in equilibrium w must lie between these two values. In addition, in equilibrium ϕ scales with R^d, so that a decrease in R^d (such as the one plotted in the top panel of Figure 1) lowers agents' proportional labor demand ϕ in equilibrium. In turn, a lower value of ϕ, combined with the fact that $\lambda = 0$ and thus no output is saved from the reduction in bank defaults, implies a reduced steady-state value of X_t and thus lower labor demand and agents' consumption.

Next, we show that tightening the capital requirement constraint reduces welfare. Before presenting the results, we explain how we measure welfare, given that there are two dimensions of heterogeneity: different wealth among entrepreneurs, and two classes of agents (i.e., entrepreneurs and laborers).

Entrepreneurs are heterogenous because of the effects of productivity shocks on their wealth accumulation. We define the welfare of entrepreneurs as their value function $V(x)$ evaluated at the average wealth in the economy in steady state, $x = X$. Formally, this measure of welfare would arise in an economy in which all agents have the same wealth at $t = 0$, $x_0^i = X_0$, so that $\int V(x_0^i)di = V(X_0)$, and X_0 is initialized at the steady-state level. Thus, this measure of welfare does not account for the effects of policy on the distribution of wealth. A more general measure of welfare can be derived by extending the model to obtain a well-defined stationary distribution of entrepreneurs' wealth, and by integrating $V(x_0^i)$ with respect to such a stationary distribution. If tightening capital requirements increases the dispersion of wealth among entrepreneurs, our conclusions would be reinforced.

The fact that our model includes two classes of agents – entrepreneurs and laborers – does not affect the welfare results of this section. We show that the welfare of both entrepreneurs and laborers decreases when capital requirements are binding and they are tightened.

The next proposition formalizes the effects of capital requirements on welfare through the good-risk taking channel, when we shut down the bad risk-taking channel by setting $\lambda = 0$.

Proposition 3. *Suppose $\lambda = 0$ and $A_{t+1} = A$ is not random. Then if z_{t+1}^i is a known constant for all t and all i, changing the capital requirement ζ has no effect on welfare of agents and laborers. On the other hand, if z_{t+1}^i is random, increasing ζ when the capital requirement constraint binds reduces the welfare of both agents and laborers.*

Proof. See Appendix A. □

The results of Proposition 3 follows from those of Proposition 2. With no idiosyncratic risk, Proposition 2 shows that capital requirements have no real effects, and thus welfare must be unchanged. If instead agents are subject to idiosyncratic risk, the reduction in steady-state value of X_t reduces welfare. Given that our measure of welfare is equivalent to $V(X_0)$ with X_0 initialized at the steady-state level of X_t, welfare moves one-for-one with X_t.

We close this section by providing a numerical example to highlight the effects of capital requirements on the good risk-taking channel and welfare. To clarify the results of the simulation, we set $\nu_2 = +\infty$. This implies that laborers have linear disutility of labor and, using (6) and (12), that their welfare is constant and equal to zero in equilibrium, independent of the capital requirements. As a result, the only welfare in this economy that we need to consider is that of entrepreneurs.[5] The other parameters are given in Table 1.[6]

We solve the model numerically, assuming that A is constant, ε is a standard normal random variable, and that z^i_{t+1} is i.i.d. and can take two values, z^H with probability p and $z^L < z^H$ with probability $1-p$. See Appendix B for details on the computation of the model solution.

Figure 2 plots the consumption-equivalent welfare of entrepreneurs and the average value of wealth X_t as a function of the capital requirements ζ. The consumption-equivalent welfare is the value of ω that solves

$$E\sum_{t=0}^{\infty}\beta^t \log c_t^{\zeta} = E\sum_{t=0}^{\infty}\beta^t \log\left[(1+\omega)\, c_t^{0}\right] \tag{14}$$

where c_t^{ζ} denotes consumption in the economy with a capital requirement ζ and c_t^0 is consumption in an economy without capital requirements. Thus ω is the percentage by which one would have to increase average consumption in order to make an agent in the non-regulated economy indifferent to staying there or living in a world with capital requirement set to ζ.

If ζ is not too large, the capital requirement constraint is not binding and welfare is the same as in the economy with no regulation. When ζ is greater than the ratio n_t/k_t that arises without regulation, the capital requirement constraint is binding and the welfare of entrepreneurs decreases.

Figure 3 illustrates why welfare decreases with capital requirements in the model without deadweight losses from default. As capital requirements increase, they reduce the return on

[5]If ν_2 is finite, we can still consider only the welfare of entrepreneurs if we provide a lump-sum tax or transfers to laborers to keep their welfare constant in response to a change in ζ. The results are qualitatively identical.

[6]Even though this is just a numerical example, we want to comment on our choice of α. Since the production function of banks does not include labor, the parameter α can be interpreted as the degree of decreasing return to scale for banks' assets. With this in mind, our choice is in line with the value of 0.85 used by Midrigan and Xu (2014). In addition, plugging equation (26) into (30) and rearranging reveals that the default probability of banks must be greater than $1-\alpha$. Thus to keep the default probability at a low level, we keep α relatively high.

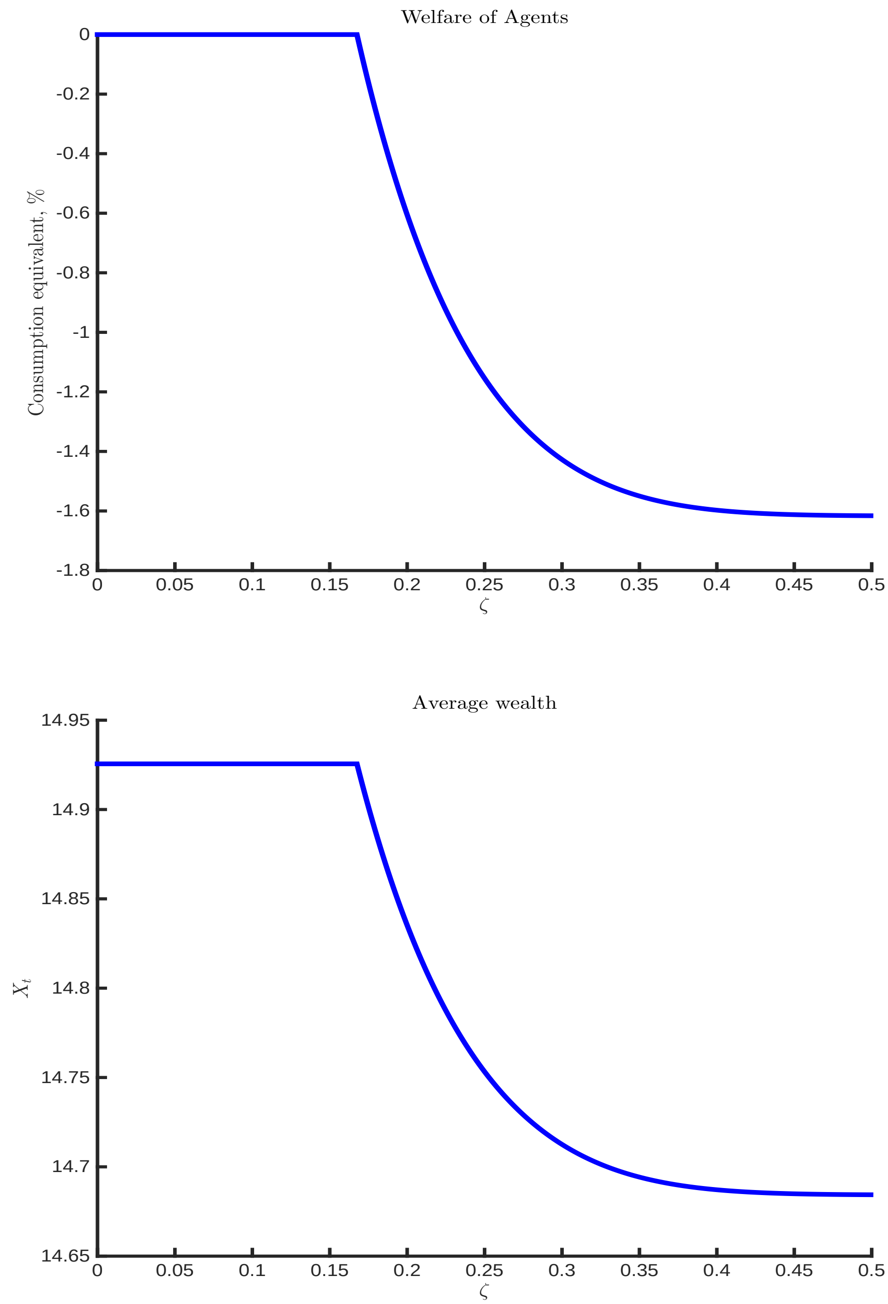

Figure 2. Welfare ($\lambda = 0$)
The top panel plots the welfare of agents and bottom panel plots the average level of wealth X_t as we vary the capital requirement ζ when $\lambda = 0$. Welfare is defined as the consumption equivalent ω with respect to an economy with no capital requirements (i.e., with $\zeta = 0$), as defined in equation (14).

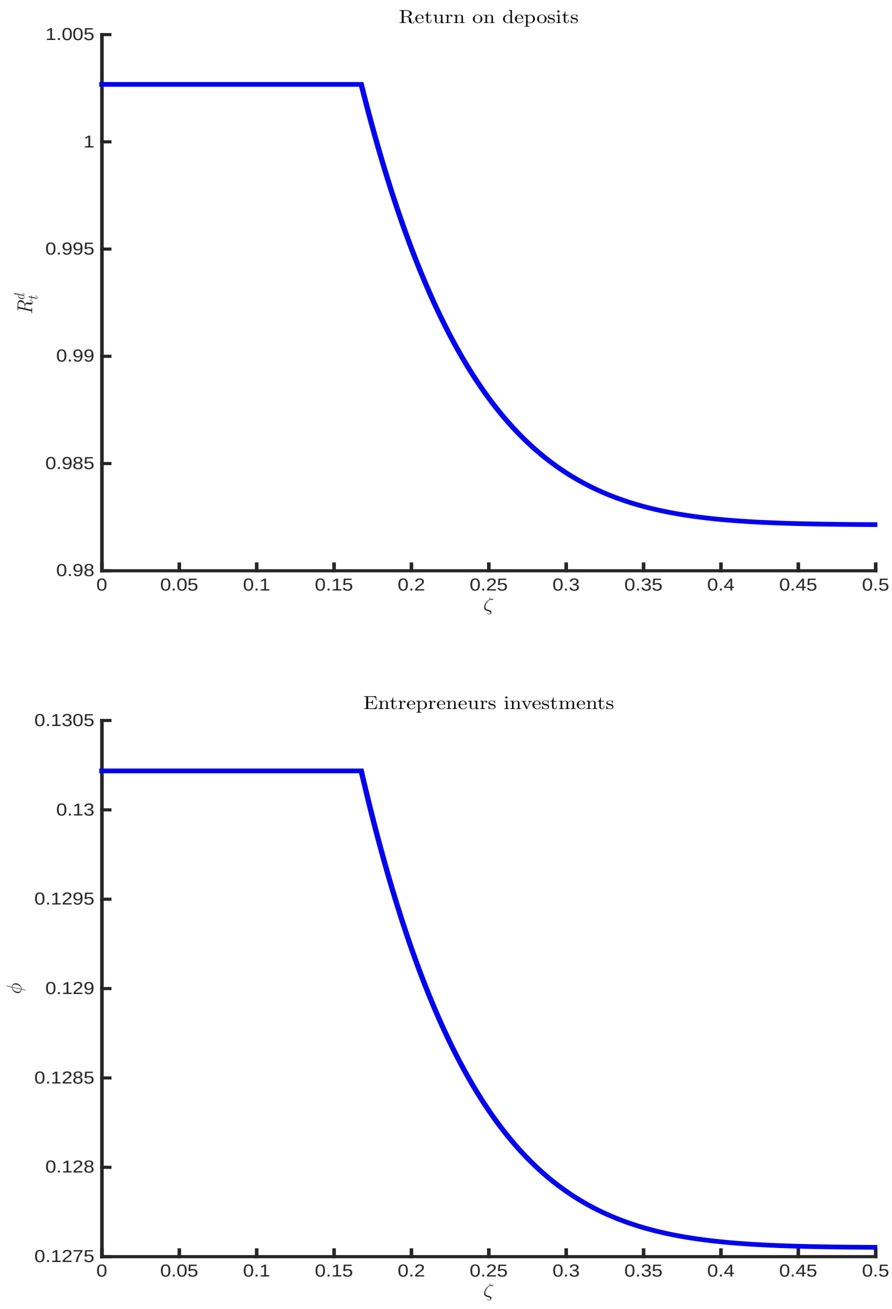

Figure 3. Return on deposits and entrepreneurs investments in the projects as a function of capital requirements in the model with $\lambda = 0$. The top panel plots R^d and the bottom panel plots ϕ.

Parameters	Value
β	0.98
α	0.95
ν_1	0.7
ν_2	∞
A	1.1
σ	0.20
z^H	4
z^L	0
$\Pr\left\{z = z^H\right\}$	0.25
λ	$\{0, 0.004\}$

Table 1. Numerical Example Parameter Values
The table reports the parameter values used in Figures 2, 3, and 4.

deposits R^d (top panel), which reduces entrepreneurs' labor demand in equilibrium (bottom panel), thus reducing aggregate output.

4.3 Good and bad risk taking

In what follows we assume that $\lambda > 0$, so that bank defaults entail a deadweight loss and capital requirements can be valuable in reducing this deadweight loss. Then the optimal capital requirement balances a reduced deadweight loss with the costs associated with lower labor demand from agents. We maintain the assumption that A_t is constant.

Figure 4 plots the consumption-equivalent welfare of entrepreneurs and the average value of wealth X_t as a function of the capital requirements ζ, as in Figure 2, but for $\lambda = 0.004$ instead of $\lambda = 0$. In this economy, bank defaults entail a loss of output that is convex in the output at defaulted banks. We think of this convexity as reflecting inefficiency in the government's ability to collect output at defaulted banks and return it to depositors: without capital requirements, or when ζ is very low, there are too many bank failures and the government is overwhelmed. As ζ rises and the amount of assets to be collected at failed banks falls, the government is able to collect more at each bank and the marginal deadweight loss falls.

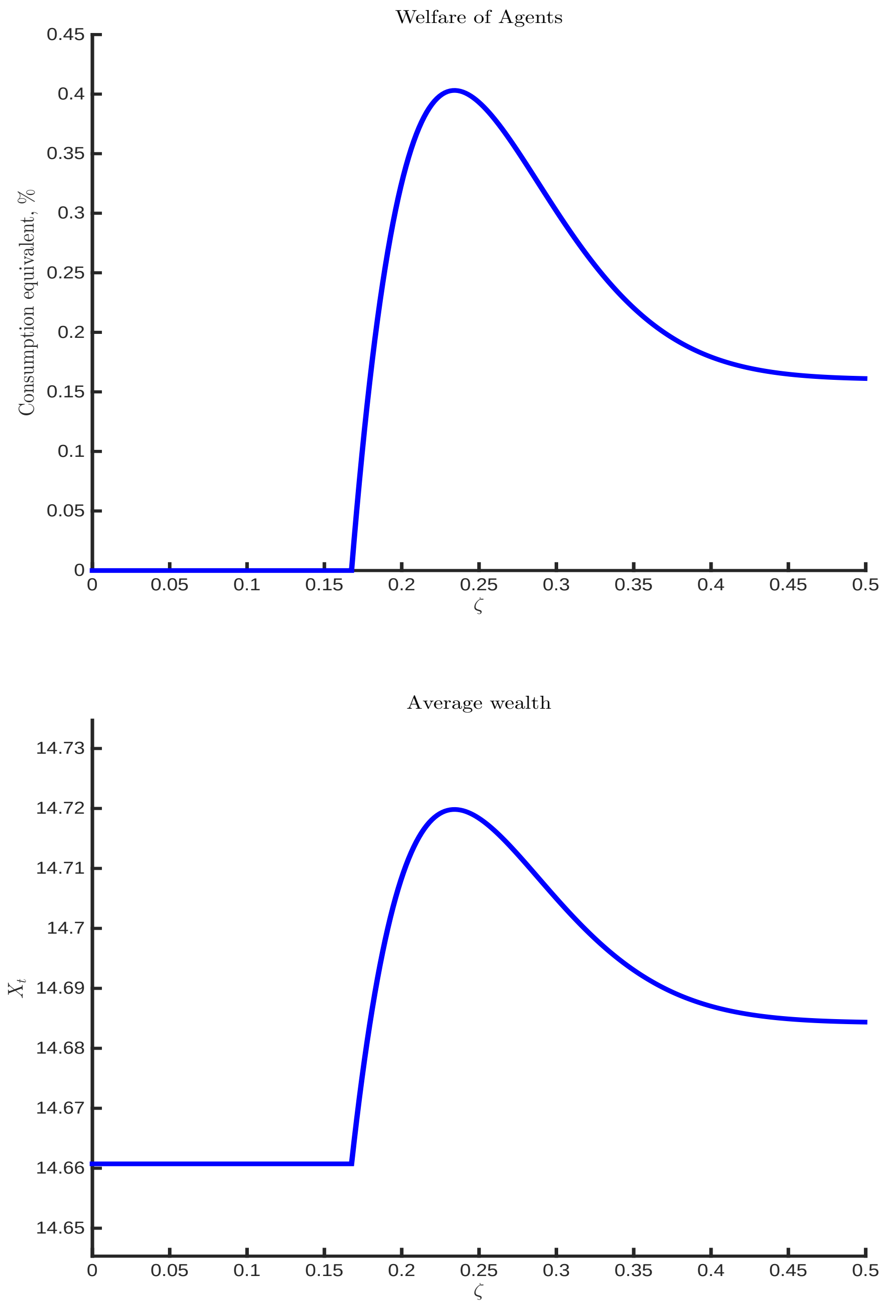

Figure 4. Welfare ($\lambda > 0$)
The top panel plots the welfare of agents and bottom panel plots the average level of wealth X_t as we vary the capital requirement ζ when $\lambda = 0$. Welfare is defined as the consumption equivalent ω with respect to an economy with no capital requirements (i.e., with $\zeta = 0$), as defined in equation (14).

Convexity in the government's ability to collect defaulted assets implies that the marginal benefit of capital requirements is very high for low values of ζ, but this marginal benefit decreases for higher value of ζ. On the other hand, higher ζ decreases the returns to banking and lowers entrepreneurs' labor demand, as discussed in Section 4.2. Optimal capital requirement regulation balances these two forces, and leads to an optimal level of ζ such that the capital requirement constraint is binding but banks are not entirely equity financed, as shown in Figure 4.

5 Conclusion

We propose a new channel through which capital requirements affect the aggregate economy by changing the volatility of the savings of entrepreneurs. Higher capital requirements reduce the returns on banks liabilities, which increase the volatility of entrepreneurs' savings and induce them to reduce their labor demand in equilibrium, lowering output and consumption. Optimal capital requirements balance the reduction in output from this "good" risk-taking channel with a reduced deadweight loss from bank default, a standard "bad" risk-taking channel.

Our analysis thus far has been qualitative, but we plan to extend the paper to assess quantitatively the magnitude of the good risk-taking channel. We have purposefully constructed the model to maintain tractability for any stochastic process for the productivities A_{t+1} and z^i_{t+1}, including making A_{t+1} persistent. This is important because, as shown for example by Davydiuk (2017), optimal capital requirements may be time-varying either because first-best investment varies with the state of the business cycle, or because the extent of excessive bank risk-taking does (or both).

A Proofs

Proof of Proposition 1

To ease notation we remove all t subscripts, and put a prime $'$ on variables that are random at time t. The agent's problem can written recursively as

$$\begin{aligned} V(x;Z) = \max_{c,d,n,l\geq 0} \; & \log c + \beta E\left\{V\left(x';Z'\right)\right\} \\ & s.t. \\ & c+d+n = x \\ & x' = \left(1-\tau'\right)\left[\left(z'-w\right)l + R^d d + R^{E\prime} n\right] \end{aligned} \tag{15}$$

where $Z = \left[A, \left\{x^j\right\}_j\right]'$ is a vector containing aggregate productivity A and the wealth of all agents j, and the expectation is taken over the distribution of z' and Z'. The variables w and R^d are known when the agent chooses c, d, n, and l, but $R^{E\prime}$ and τ' depend on the realized values of Z'.

We guess and verify that the value function takes the form $V(x;Z) = b\log x + f(Z)$ for some function f and constant b. Rewrite the agent's choice variables in terms of $s = d+n$, $n = ys$, and $\phi = l/s$, so that $d = (1-y)s$ and $\log x' = \log(1-\tau') + \log \Delta' + \log s$, where Δ' is defined in equation (9). Then plugging in the guess for the form of the value function, the first-order condition for s yields

$$\begin{aligned} \frac{1}{x-s} &= \beta b E \frac{1}{s} \\ \therefore \; s &= \frac{\beta b}{1+\beta b} x \end{aligned}$$

from which it follows from the budget constraint that $c = \frac{1}{1+\beta b}x$. This leads directly to the first-order conditions for ϕ and , y, equations (7) and (8), which are independent of x. To verify the guess, plug the optimal policies and the guess into equation (15) to obtain

$$\begin{aligned} b\log x + f(z,Z) &= \log\left[(1-\beta)x\right] + \beta E\left\{b\log\left(1-\tau'\right) + b\log\Delta' + b\log[\beta x] + f\left(z',Z'\right)\right\} \\ &= \log x + \beta b\log x + \text{terms independent of x} \end{aligned}$$

so that $b = 1+\beta b$ and therefore $b = \frac{1}{1-\beta}$, which verifies the guess and completes the proof.

Proof of Proposition 2

First suppose that z_{t+1}^i is a known constant for all t. If z is not random, then equation (27) reduces to $z = w$, and agents are completely indifferent to any level of ϕ_t. Total labor is then pinned down by equation (12) in order to make the wage equal the value of z.

Wealth evolution becomes

$$x_{t+1}^i = (1-\tau_{t+1})\left[yR_{t+1}^E + (1-y_t)R_t^d\right]\beta x_t^i. \tag{16}$$

Since $\lambda = 0$, equation (5) can be rearranged to yield

$$\begin{aligned} 1-\tau_{t+1} &= 1 - \frac{\int\left\{R_t^d d_t - A_{t+1}e^{\sigma\varepsilon}k_t^\alpha\right\}^+ dF(\varepsilon)}{\left[y_t R_{t+1}^E + (1-y_t)R_t^d\right]\beta\int x_t^i di} \qquad (17) \\ \therefore (1-\tau)\left[y_t R_{t+1}^E + (1-y_t)R_t^d\right] &= y_t R_{t+1}^E + (1-y_t)R_t^d - \frac{1}{\beta X_t}\int\left\{R_t^d d_t - A_{t+1}e^{\sigma\varepsilon}k_t^\alpha\right\}^+ dF(\varepsilon) \\ &= y_t R_{t+1}^E + (1-y_t)R_t^d - R^d(1-y)F\left(\underline{\varepsilon}_{t+1}\right) \\ &\quad + A_{t+1}(\beta X_t)^{\alpha-1}\int_{-\infty}^{\underline{\varepsilon}_{t+1}} e^{\sigma\varepsilon}dF(\varepsilon). \end{aligned}$$

where $\underline{\varepsilon}_{t+1}$ is defined in equation (4) and we have used the fact that in the aggregate $k_t =$

$s_t = \beta X_t$ and $d_t = (1 - y_t)\,\beta X_t$. Meanwhile, we have using equation (3) that

$$y_t R^E_{t+1} + (1 - y_t)\,R^d_t = A_{t+1}\,(\beta X_t)^{\alpha-1} \int_{\underline{\varepsilon}_{t+1}}^{\infty} e^{\sigma\varepsilon} dF\,(\varepsilon) - \left[1 - F\left(\underline{\varepsilon}_{t+1}\right)\right](1 - y_t)\,R^d_t + (1 - y_t)\,R^d_t$$
$$= A_{t+1}\,(\beta X_t)^{\alpha-1} \int_{\underline{\varepsilon}_{t+1}}^{\infty} e^{\sigma\varepsilon} dF\,(\varepsilon) + F\left(\underline{\varepsilon}_{t+1}\right)(1 - y_t)\,R^d_t, \tag{18}$$

which plugging into equation (17) yields

$$(1 - \tau)\left[y_t R^E_{t+1} + (1 - y_t)\,R^d_t\right] = A_{t+1}\,(\beta X_t)^{\alpha-1} \int_{-\infty}^{\infty} e^{\sigma\varepsilon} dF\,(\varepsilon)$$

so that equation (16) becomes

$$x^i_{t+1} = \left[A_{t+1}\,(\beta X_t)^{\alpha-1} \int_{-\infty}^{\infty} e^{\sigma\varepsilon} dF\,(\varepsilon)\right]\beta x^i_t.$$

This implies that both individual and aggregate wealth growth, and through it the dynamics of consumption and welfare, are unaffected by any policy that changes y or $\underline{\varepsilon}_{t+1}$. Thus a capital requirement that changes the return on the agents' wealth by affecting $\underline{\varepsilon}_{t+1}$ (equation 18) has an exactly offsetting effect on taxes levied by the government to repay depositors at failed banks.

Now suppose that z^i_{t+1} is random. In this case, wealth evolution is given by

$$x^i_{t+1} = (1 - \tau_{t+1})\left[\left(z^i_{t+1} - w_t\right)\phi_t + y R^E_{t+1} + (1 - y_t)\,R^d_t\right]\beta x^i_t. \tag{19}$$
$$= \left[\left(z^i_{t+1} - w_t\right)\phi_t + A_{t+1}\,(\beta X_t)^{\alpha-1} \int_{-\infty}^{\infty} e^{\sigma\varepsilon} dF\,(\varepsilon)\right]\beta x^i_t$$

where the last line plugs in equations (3) and (5) after including the agent's idiosyncratic project income $\left(z^i_{t+1} - w_t\right)\phi_t$. It is apparent from equation (19) that equity investment y_t and the default threshold $\underline{\varepsilon}_{t+1}$ do not affect the growth rate of wealth directly. Instead, we will show that increasing ζ when the capital requirement binds reduces steady-state X_t and the growth rates of individual wealth.

To show this, by equation (19) it suffices to show that the effect of increasing ζ is to reduce $\left(z_{t+1}^{i}-w_{t}\right)\phi_{t}$. This immediately reduces the growth rate of x_{t}^{i}, since individual agents take X_t as given, but it also reduces steady-state X_t because $\alpha<1$; this can be seen by integrating equation (19) over i and solving for steady-state X.

The rest of the proof proceeds as follows: first, we show that increasing ζ reduces $\underline{\varepsilon}_{t+1}$ when the capital requirement binds; then, we show that reducing $\underline{\varepsilon}_{t+1}$ reduces agent's labor project income $\left(z_{t+1}^{i}-w_{t}\right)\phi_{t}$.

Now because A_{t+1} is a constant, it must be that R_{t+1}^{E} and R_{t}^{d} are also constants; and from equation (8) it must be that $R^{E}=R^{d}$. Thus plugging equation (4) into equation (18) and rearranging yields

$$
\begin{aligned}
yR^{E}+(1-y)\,R^{d}=R&=\frac{A_{t+1}e^{\sigma\underline{\varepsilon}_{t+1}}\left(\beta X_{t}\right)^{\alpha-1}}{1-\zeta}\\
&=A_{t+1}\left(\beta X_{t}\right)^{\alpha-1}\int_{\underline{\varepsilon}_{t+1}}^{\infty}e^{\sigma\varepsilon}dF\left(\varepsilon\right)+F\left(\underline{\varepsilon}_{t+1}\right)A_{t+1}e^{\sigma\underline{\varepsilon}_{t+1}}\left(\beta X_{t}\right)^{\alpha-1}\\
\therefore\quad\frac{e^{\sigma\underline{\varepsilon}_{t+1}}}{1-\zeta}&=\int_{\underline{\varepsilon}_{t+1}}^{\infty}e^{\sigma\varepsilon}dF\left(\varepsilon\right)+F\left(\underline{\varepsilon}_{t+1}\right)e^{\sigma\underline{\varepsilon}_{t+1}} \qquad (20)
\end{aligned}
$$

where we have assumed that $y_t=\zeta$ binds. Totally differentiating equation (20) yields

$$
\begin{aligned}
\frac{\sigma e^{\sigma\underline{\varepsilon}_{t+1}}}{1-\zeta}d\underline{\varepsilon}_{t+1}+\frac{e^{\sigma\underline{\varepsilon}_{t+1}}}{(1-\zeta)^{2}}d\zeta&=-e^{\sigma\underline{\varepsilon}_{t+1}}f\left(\underline{\varepsilon}_{t+1}\right)d\underline{\varepsilon}_{t+1}+\sigma e^{\sigma\underline{\varepsilon}_{t+1}}F\left(\underline{\varepsilon}_{t+1}\right)d\underline{\varepsilon}_{t+1}+f\left(\underline{\varepsilon}_{t+1}\right)e^{\sigma\underline{\varepsilon}_{t+1}}d\underline{\varepsilon}_{t+1}\\
&=\sigma e^{\sigma\underline{\varepsilon}_{t+1}}F\left(\underline{\varepsilon}_{t+1}\right)d\underline{\varepsilon}_{t+1}\\
\therefore\quad\frac{d\underline{\varepsilon}_{t+1}}{d\zeta}&=\left[\sigma\left(1-\zeta\right)^{2}\left(F\left(\underline{\varepsilon}_{t+1}\right)-\frac{1}{1-\zeta}\right)\right]^{-1}<0
\end{aligned}
$$

where $f\left(\cdot\right)\equiv F'\left(\cdot\right)$ is the pdf of ε, and the last line must be less than zero since the range of F is $[0,1]$ because it is a CDF, and $\zeta<1$. Thus increasing the capital requirement ζ reduces the default threshold $\underline{\varepsilon}_{t+1}$.

Now rewrite equation (18) as

$$\begin{aligned} y_t R^E_{t+1} + (1-y_t) R^d_t &= A_{t+1} (\beta X_t)^{\alpha-1} \int_{\underline{\varepsilon}_{t+1}}^{\infty} e^{\sigma\varepsilon} dF(\varepsilon) + F\left(\underline{\varepsilon}_{t+1}\right)(1-y_t) R^d_t \\ &= A_{t+1} (\beta X_t)^{\alpha-1} \int_{-\infty}^{\infty} \max\left\{e^{\sigma\varepsilon}, e^{\sigma\underline{\varepsilon}_{t+1}}\right\} dF(\varepsilon) \end{aligned}$$

which is clearly an increasing function of $\underline{\varepsilon}_{t+1}$. Thus increasing ζ not only lowers $\underline{\varepsilon}_{t+1}$, but this also lowers the agent's portfolio return $yR^E + (1-y) R^d$.

Because $R^E = R^d$, we can rewrite equation (7) as

$$0 = E\left\{\frac{z' - w}{(z' - w)\phi + R^d}\right\}, \tag{21}$$

which must have a unique solution for ϕ because the objective is globally concave. Thus if ϕ solves equation (21), it must be that R^d and ϕ move in the same proportion (holding w fixed). Thus a decrease in R^d is a reduction in the labor-demand curve of agents (see the lower of Figure 1). Because the labor-supply curve (12) is upward-sloping, w also falls in equilibrium but nevertheless, it must be that agents choose a lower value of ϕ_t at the higher ζ.

As before, because $\lambda = 0$ the reduction in the returns to bank equity and deposits is exactly offset by a reduction in τ_{t+1}; however, there is no force that offsets the reduction in agents' labor demand.

Proof of Proposition 3

The result that welfare does not change without idiosyncratic shocks follows as a corollary of Proposition 2. In this case, capital requirements have no real effects, and thus welfare is unchanged.

With idiosyncratic shock, recall that our measure of welfare of entrepreneurs is defined as $V(X_t)$, where X_t is the steady-state value of average wealth. From Proposition 2, X_t

decreases in response to an increase in ζ when the capital requirement constraint is binding. Since $V(\cdot)$ is strictly increasing as shown in the proof of Proposition 1, welfare of entrepreneurs decreases.

Next, consider laborers. Their welfare is given by (6). Plugging their budget constraint and the first-order condition in (12) into the objective function in (6), and rearranging, we obtain that their welfare is

$$\nu_1 \frac{l^{1+\frac{1}{\nu_2}}}{1+\nu_2}$$

which is increasing in l. As shown in Proposition 2, an increase in ζ and a binding capital requirement implies a reduction in ϕ_t and X_t, and, from Proposition 1, of l_t, so that the welfare of laborers decreases too.

B Solution Method

Rewrite equation (4) as

$$\begin{aligned} A_{t+1} e^{\sigma \underline{\varepsilon}_{t+1}} k_t^{\alpha} &= R_t^d d_t \\ \therefore \quad R_t^d &= \frac{A_{t+1} e^{\sigma \underline{\varepsilon}_{t+1}} \left(\beta X_t\right)^{\alpha-1}}{1-y_t} \\ \therefore \quad \sigma \underline{\varepsilon}_{t+1} &= \log R_t^d - \log A_{t+1} + (1-\alpha) \log \beta X_t + \log\left(1-y_t\right) \end{aligned} \tag{22}$$

where the second line uses the fact that in equilibrium $k_t = d_t + n_t = s_t = \beta X_t$ and $d_t = \left(1-y_t\right) s_t = \left(1-y_t\right) \beta X_t$. Equation (22) defines a relation between $\underline{\varepsilon}_{t+1}$ and the realized value of A_{t+1}; when A_{t+1} is higher, $\underline{\varepsilon}_{t+1}$ is lower (fewer banks default). The values of X_t, R_t^d, and y_t are set at t, before A_{t+1} and $\underline{\varepsilon}_{t+1}$ are realized; thus taking differences of the last line of equation (22) for any two values of A_{t+1}, say A^* and A, and rearranging yields

$$\underline{\varepsilon}_{t+1} = \underline{\varepsilon}_{t+1}^{*} - \frac{\log A - \log A^*}{\sigma}. \tag{23}$$

Thus to solve for an equilibrium we need only solve for the single value $\underline{\varepsilon}_{t+1}$ that solves equation (22) when $A_{t+1} = A^*$; the other default thresholds then follow from equation (23).

In what follows we assume that $\varepsilon \sim \mathbb{N}(0,1)$. Then using the fact that

$$\int_a^b e^{\sigma\varepsilon} d\Phi\{\varepsilon\} = e^{\frac{1}{2}\sigma^2}\left[\Phi\{b-\sigma\} - \Phi\{a-\sigma\}\right]$$

where $\Phi\{\cdot\}$ is the standard normal CDF, we have that equation (3) becomes

$$\begin{aligned} R^E_{t+1} &= \frac{\int_{\underline{\varepsilon}_{t+1}}^{\infty}\left[A_{t+1}e^{\sigma\varepsilon}k_t^{\alpha} - R^d_t d_t\right] d\Phi\{\varepsilon\}}{n_t} \\ &= \frac{Ae^{\frac{1}{2}\sigma^2}(\beta X_t)^{\alpha}\left(1-\Phi\left\{\underline{\varepsilon}_{t+1}-\sigma\right\}\right) - R^d_t(1-y_t)\beta X_t\left(1-\Phi\left\{\underline{\varepsilon}_{t+1}\right\}\right)}{y_t\beta X_t} \\ &= \frac{1}{y_t}\left[Ae^{\frac{1}{2}\sigma^2}(\beta X_t)^{\alpha-1}\left(1-\Phi\left\{\underline{\varepsilon}_{t+1}-\sigma\right\}\right) - R^d_t(1-y_t)\left(1-\Phi\left\{\underline{\varepsilon}_{t+1}\right\}\right)\right] \end{aligned}$$

so that

$$y_t R^E_{t+1} + (1-y_t)R^d_t = Ae^{\frac{1}{2}\sigma^2}(\beta X_t)^{\alpha-1}\left(1-\Phi\left\{\underline{\varepsilon}_{t+1}-\sigma\right\}\right) + (1-y_t)R^d_t\Phi\left\{\underline{\varepsilon}_{t+1}\right\} \quad (24)$$

and

$$R^E_{t+1} - R^d_t = \frac{1}{y_t}\left[Ae^{\frac{1}{2}\sigma^2}(\beta X_t)^{\alpha-1}\left(1-\Phi\left\{\underline{\varepsilon}_{t+1}-\sigma\right\}\right) - R^d_t\left[1-(1-y_t)\Phi\left\{\underline{\varepsilon}_{t+1}\right\}\right]\right]. \quad (25)$$

Plugging equations (22), (24) and (25) into the agents' first-order conditions (7) and (8) yields

$$0 = E_t\left\{\frac{Ae^{\frac{1}{2}\sigma^2}\left(1-\Phi\left\{\underline{\varepsilon}_{t+1}-\sigma\right\}\right) - A\frac{1-(1-y_t)\Phi\left\{\underline{\varepsilon}_{t+1}\right\}}{1-y_t}e^{\sigma\underline{\varepsilon}_{t+1}}}{(z_{t+1}-w_t)\phi_t + A(\beta X_t)^{\alpha-1}\left[e^{\frac{1}{2}\sigma^2}\left(1-\Phi\left\{\underline{\varepsilon}_{t+1}-\sigma\right\}\right) + e^{\sigma\underline{\varepsilon}_{t+1}}\Phi\left\{\underline{\varepsilon}_{t+1}\right\}\right]}\right\} \quad (26)$$

and

$$0 = E_t \left\{ \frac{z_{t+1} - w_t}{(z_{t+1} - w_t)\,\phi_t + A\,(\beta X_t)^{\alpha-1}\left[e^{\frac{1}{2}\sigma^2}\left(1 - \Phi\left\{\underline{\varepsilon}_{t+1} - \sigma\right\}\right) + e^{\sigma\underline{\varepsilon}_{t+1}}\Phi\left\{\underline{\varepsilon}_{t+1}\right\}\right]} \right\}. \tag{27}$$

The tax rate τ_{t+1} from equation (5) becomes

$$\begin{aligned}
\tau_{t+1} &= \frac{\int_{-\infty}^{\underline{\varepsilon}_{t+1}}\left[R_t^d d_t - Ae^{\sigma\varepsilon}k_t^\alpha\right]d\Phi\left\{\varepsilon\right\} + \frac{\lambda}{2}\left[\int_{-\infty}^{\underline{\varepsilon}_{t+1}} Ae^{\sigma\varepsilon}k_t^\alpha d\Phi\left\{\varepsilon\right\}\right]^2}{\left[(\bar{z} - w_t)\,\phi_t + y_t R_{t+1}^e + (1 - y_t)\,R_t^d\right]s_t} \\
&= \frac{R_t^d\,(1 - y_t)\,\beta X_t\Phi\left\{\underline{\varepsilon}_{t+1}\right\} - Ae^{\frac{1}{2}\sigma^2}(\beta X_t)^\alpha\,\Phi\left\{\underline{\varepsilon}_{t+1} - \sigma\right\} + \frac{\lambda}{2}\left[Ae^{\frac{1}{2}\sigma^2}(\beta X_t)^\alpha\,\Phi\left\{\underline{\varepsilon}_{t+1} - \sigma\right\}\right]^2}{\left[(\bar{z} - w_t)\,\phi_t + y_t R_{t+1}^e + (1 - y_t)\,R_t^d\right]\beta X_t} \\
&= A\,(\beta X_t)^{\alpha-1}\frac{e^{\sigma\underline{\varepsilon}_{t+1}}\Phi\left\{\underline{\varepsilon}_{t+1}\right\} - e^{\frac{1}{2}\sigma^2}\Phi\left\{\underline{\varepsilon}_{t+1} - \sigma\right\} + \frac{\lambda}{2}A\,(\beta X_t)^\alpha\left[e^{\frac{1}{2}\sigma^2}\Phi\left\{\underline{\varepsilon}_{t+1} - \sigma\right\}\right]^2}{(\bar{z} - w_t)\,\phi_t + A\,(\beta X_t)^{\alpha-1}\left[e^{\frac{1}{2}\sigma^2}\left(1 - \Phi\left\{\underline{\varepsilon}_{t+1} - \sigma\right\}\right) + e^{\sigma\underline{\varepsilon}_{t+1}}\Phi\left\{\underline{\varepsilon}_{t+1}\right\}\right]}
\end{aligned} \tag{28}$$

where $\bar{z}$ is the average realized value of z_{t+1}^i across agents, so that the denominator in the first line of equation (28) is realized pre-tax wealth, and the last line plugs in equations (22) and (24).

Finally, the banker's first-order condition from solving (2), if the capital constraint does not bind, is given by

$$\begin{aligned}
0 &= E_t\left\{\alpha A_{t+1}e^{\frac{1}{2}\sigma^2}\left(1 - \Phi\left\{\underline{\varepsilon}_{t+1} - \sigma\right\}\right)k_t^{\alpha-1} - R_t^d\left(1 - \Phi\left\{\underline{\varepsilon}_{t+1}\right\}\right)\right\} \\
\therefore \quad R_t^d &= \alpha e^{\frac{1}{2}\sigma^2}\left[\beta X_t\right]^{\alpha-1}\frac{E_t\left\{A_{t+1}\left(1 - \Phi\left\{\underline{\varepsilon}_{t+1} - \sigma\right\}\right)\right\}}{E_t\left\{\left(1 - \Phi\left\{\underline{\varepsilon}_{t+1}\right\}\right)\right\}}
\end{aligned} \tag{29}$$

where the expected value is taken at time t over the distribution of A, and the second line uses the fact that in equilibrium $k_t = d_t + n_t = s_t = \beta X_t$. If the capital constraint does bind, then equation (29) no longer applies (it contains a Lagrange multiplier that appears nowhere else) and instead we have that $y_t = \zeta$.

To solve for an equilibrium in this economy, fix a value of $\underline{\varepsilon}^*_{t+1}$. Then the values of $\underline{\varepsilon}_{t+1}$ in the other N realized states are pinned down by equation (23). Then so long as the capital requirement does not bind, plug equation (29) into equation (22) to get

$$\begin{aligned}\sigma\underline{\varepsilon}_{t+1} = \log\alpha - \log A + \frac{1}{2}\sigma^2 + \log E_t\left\{A\left(1-\Phi\left\{\underline{\varepsilon}_{t+1}-\sigma\right\}\right)\right\} \\ - \log E_t\left\{\left(1-\Phi\left\{\underline{\varepsilon}_{t+1}\right\}\right)\right\} + \log\left(1-y_t\right)\end{aligned} \tag{30}$$

from which we can compute y_t. Then for any value of X_t, we can solve equation (27) for ϕ_t; given this value, we verify that the first-order condition (26) holds. We search over $\underline{\varepsilon}^*_{t+1}$ on a grid from -4 to 4 until all three equilibrium conditions (22), (26), and (27) hold.

If the capital constraint binds, that is the y_t we compute from this procedure is lower than ζ, then we fix $y_t = \zeta$ rather than using equation (30). Then we proceed as before, solving equation (27) for ϕ_t (given X_t) and then verifying that the first-order condition (26) holds.

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