Blockholder Voting*

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December 2017

Abstract

By introducing a shareholder with many votes (a blockholder) into a standard voting model, we uncover striking results. First, an unbiased blockholder may not vote with all of her shares. This is efficient because it prevents her from drowning out the information in others' votes. Second, if this blockholder announces her vote upfront, shareholders may ignore their information and vote with the blockholder to support her superior information. The results are robust to permitting information acquisition and trade. We also show that shareholders may coordinate to oppose a blockholder who is biased. Regulations discouraging abstention, strategic behavior, and/or coordination reduce efficiency.

Keywords: Blockholder, shareholder voting, corporate governance

JEL Codes: G34, D72

*We thank Patrick Bolton, Philip Bond, Susan Christoffersen, Vicente Cunat, Steven Davidoff Solomon, Ignacio Esponda, Stephen Hansen, Peter Iliev, Alexander Ljungqvist, Frédéric Malherbe, Daniel Metzger, Thomas Noe, Frank Partnoy, Miriam Schwartz-Ziv, Scott Winter, Moqi Xu, Yishay Yafeh and participants of seminars at UC Dublin, Lancaster, London FIT, London FTG, Goethe University, U. Utrecht, U. Nottingham, Tel Aviv U., EFA and the third IO Theory & Empirics Research Conference for helpful comments. Bar-Isaac thanks SSHRC (435-2014-0004) for financial support and the London Business School for its hospitality in hosting him as a visitor while much of this paper was written.

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1 Introduction

Shareholder voting has a stark difference from voting in the political context: a shareholder may have many shares and, thus, many votes. Blockholders, that is, shareholders with a large fraction (often defined as 5%) of the shares of a firm, are ubiquitous. In a sample of representative U.S. public firms, Holderness (2009) finds that 96% of the firms have at least one blockholder, and the average stake of the largest blockholder of a firm is 26%.[1] The identity of blockholders may vary substantially from activist investors to passive index funds to even the managers or directors of the firm.

Blockholders play a large role in the governance of firms. The U.S. Securities and Exchange Commission (SEC) has increasingly focused regulatory scrutiny on blockholders for their voting behavior. Investment advisers (including mutual funds) have been told to ensure that their votes are in the best interests of their clients, and they must publicly report their votes (SEC, 2003). Large investors must report their stakes and intentions when they reach 5% ownership of a firm. Nevertheless, groups of activist investors ("wolf packs") sometimes act in concert but individually avoid the 5% rule, generating both academic and regulatory debate (Coffee and Palia, 2016).

In this paper, we study blockholder voting in a standard theoretical framework and find striking results:

- *Blockholders may prefer not to vote with all of their shares, and this is efficient*: Consider the voting strategy of an unbiased blockholder, where unbiased means that the blockholder wants her vote to increase the value of the firm. Our first result is that this blockholder may not vote with all of her shares. Given that she wants to maximize the value of the firm, if she does not have precise information, she will prefer that other shareholders' information not be drowned out by her votes. The blockholder is not wasting her unvoted shares; she is acting optimally to improve the efficiency of the vote.[2] We also demonstrate that given the opportunity to trade shares, the blockholder may not trade her unvoted shares. Given that evidence suggests that many investment advisers try to satisfy SEC requirements by blindly voting their shares with the rec-

[1] He also finds that this distribution is not very different from that in the rest of the world.

[2] We define efficiency below as maximizing the probability that the vote matches the true state of the world.

ommendation of proxy advisers (Iliev and Lowry, 2015, and Malenko and Shen, 2016), our result suggests that such rules may be inefficient.

- *Other shareholders may ignore their own information to vote with an unbiased blockholder:* Our second result looks at the situation where the unbiased blockholder can observably announce its voting intentions before other shareholders vote. This could represent an activist investor or a large pension fund using public statements to communicate their position. In this case, if this blockholder is well informed, some of the shareholders may ignore their information and vote with the blockholder. It is perfectly rational to ignore their information; if the blockholder has a lot of information but not enough shares to express that information, the shareholders essentially supplement the blockholder's shares with their own. Determining which shareholders will support the blockholder requires coordination. Our result, then, suggests that shareholder coordination to support blockholder voting can improve efficiency; therefore, some concerns about coordination by groups of investors such as "wolf packs" may be overstated.

The voting model in this paper is standard, aside from the fact that there is one voter (the blockholder) with more shares than the others. All other shareholders want to improve the value of the firm and receive a signal about which of two decisions will increase value. The blockholder also receives a signal, which we assume to be more informative than the signal of an individual shareholder. A vote wins by a majority, and voters may vote strategically (which includes the possibility of abstention).

We alter the model in three ways to make it more realistic.

First, we allow the blockholder to acquire information and to trade shares. This provides a robustness check for our result that the blockholder may not vote all of its shares, as it is natural to ask why the blockholder would not trade shares that it does not vote. We show that the blockholder might face a "lemons" discount when it sells shares, as investors may infer that the blockholder did not acquire information. The blockholder will then have to retain the shares rather than sell them.

Second, we provide a rationale for the SEC rules for investment advisers, given that they seem contrary to the economic intuition from our main model. In particular, we show that forcing the blockholder to vote all of her shares could potentially enhance efficiency, as it might provide her with incentives to acquire information. However, we demonstrate that this argument relies on impediments to trade; when the blockholder can trade shares, no such new incentives arise with such a regulation.

Third, we consider the situation in which the blockholder is known to be biased. We define bias here as the blockholder supporting one side of the proposal no matter what information it receives. For example, the blockholder could support management's position because she is a manager or has business dealings with the firm that depend on good relationships with management.[3] In this case, some shareholders may ignore their information in order to counter the blockholder's bias.

Strategic voting is a key element of our model and the focus of our analysis. Allowing the blockholder and shareholders to vote strategically leads to more-informative outcomes. Strategic abstention enhances efficiency alternatively by (i) allowing the blockholder to express its information precisely (Proposition 1), (ii) allowing shareholders to step aside when the blockholder has very precise information (Proposition 1), (iii) allowing shareholders to express themselves while still countering the bias of a blockholder (Section 6), and (iv) forcing the biased blockholder to not express its bias (Proposition 6). Beyond strategic abstention, shareholders may also ignore their signal to increase informativeness when (i) they vote with the blockholder to support its precise information (Proposition 2) and when (ii) they vote against the biased blockholder to allow information to flow through the remaining votes (Proposition 5).

Our analysis is kept simple to present the effects clearly and maintain tractability. We recognize several directions in which we have kept the model uncomplicated. First, the bias (or lack thereof) of the blockholder is common knowledge. Second, the precision of all shareholders' and the blockholder's information is common knowledge. Third, we focus on the most informative equilibrium. There are good reasons to do so, as this is the equilibrium all agents in the model prefer; however, there are many other equilibria, as in all strategic voting models.

Shareholder voting is an important source of corporate governance (see, e.g. McCahery, Sautner, and Starks (2016)). Maug and Rydqvist (2009) provide evidence on aggregate

[3] For example, a mutual fund may support management to preserve business ties (see Davis and Kim (2007), Ashraf, Jayaraman, and Ryan (2012), and Cvijanovic, Dasgupta, and Zachariadis (2016)). Another example of blockholder bias is documented in Agrawal (2012), where labor union pension funds may vote for labor-friendly directors. Such bias may also arise as a result of a blockholder's holdings in other firms, as in Azar, Schmalz and Tecu (forthcoming).

voting results that shareholders vote strategically in corporate elections. Nevertheless, in order to examine whether blockholders and shareholders behave as predicted in our model, we would need voting data by shareholder. The existing data that has shareholder voting is far from satisfactory for our purposes; we describe the challenges in detail in Section 7.

In the following subsection, we review the related theoretical literature. In Section 2, we set up the model. In Section 3, we analyze the case where the blockholder is unbiased and passive in the sense that it makes no announcement. In Section 4, we have the unbiased blockholder make an announcement before the other shareholders vote, which we call the active blockholder case. In Section 5, we alter the model to allow for information acquisition by the blockholder and share trading. We also present a further examination of regulations regarding blockholder voting. In Section 6, we allow the blockholder to be biased, i.e. to prefer one choice regardless of its information and analyze how its vote affects the value of the firm. Section 7 presents empirical challenges for testing the implications of our model and a review of the empirical literature demonstrating an important role for shareholder voting in governance. Section 8 concludes. All proofs that are not in the text are in the Appendix.

1.1 Theoretical Literature

Edmans (2014) and Edmans and Holderness (2016) provide thorough surveys of the theoretical and empirical literature on blockholders. Only one theoretical paper cited is about voting - in the rest of the cited literature, blockholders make costly interventions in a firm and/or trade the firm's shares.[4] Yermack (2010) surveys the literature on shareholder voting and also does not cite any such research.

The paper in the political economy literature that began the analysis of strategic voting was Feddersen and Pesendorfer (1996). They analyze the swing voter's curse, where uninformed voters abstain in order to allow more-informed voters to sway the vote. This is the intuition for why the unbiased blockholder may not vote all of her shares in our model, i.e., her information has value, and so does the information of the remaining shareholders. We provide a more detailed comparison of this result with the work of Feddersen and Pesendorfer (1996) in the text. Our result that shareholders may ignore their information to vote

[4] The cited paper is Dhillon and Rossetto (2015), which provides a rationale for the presence of multiple mid-sized blockholders in a setting where shareholders are risk-averse and have private values.

with the unbiased blockholder and improve the outcome is also related.

A few other papers use this type of logic in contexts related to ours. Eso, Hansen, and White (2014) study empty voting[5] and find that uninformed shareholders and even biased shareholders may sell their votes (at a zero price) to informed shareholders in order to improve the outcome. Note that our analysis demonstrates that activists do not necessarily have to resort to empty voting strategies to gain votes if they have credibility. Maug (1999), Maug and Rydqvist (2009), and Persico (2004) find that informed voters/shareholders may ignore their information in response to different voting rules.[6] This result is related to our findings on biased blockholders; in these papers, the voters ignore their information to correct the bias of the voting rule, while in our paper, the shareholders may ignore their information to deal with the bias of the blockholder. Beyond differences in the questions we address and the asymmetries present in our model, our model is different from these papers in that we allow for abstention and for communication (both of which affect the results).

Malenko and Malenko (2017) look at a common value model where shareholders can acquire information and/or purchase it from a proxy advisory service. The authors demonstrate that although this proxy advisor may provide useful information, the outcome may be less efficient than in the absence of the advisor since the advisor's presence may crowd out independent investment in information acquisition. They focus on an environment where shareholders and their strategies are symmetric, which is critically different from our asymmetric environment (with a large blockholder). However, our result that regulations which require a blockholder to vote in line with its information may be inefficient is related; it is an inefficiency in information aggregation due to over-reliance on one signal (like the over-reliance on the signal of the proxy adviser).

While we study the role of information aggregation in corporate elections, there is a literature that focuses on preference aggregation. Perhaps, most relevant to our study, Harris and Raviv (1988) and Gromb (1993) study whether it is optimal to allocate uniform voting rights for all shares (one shareholder, one vote). Cvijanović, Groen-Xu, and Zachariadis (2017) examine the participation decision in a corporate setting where voters are partisan.

[5] Brav and Matthews (2011) also examine empty voting in a model of a hedge fund who may both trade shares and buy votes, while all other voting is random.

[6] Bhattacharya, Duffy, and Kim (2014) have a similar effect when symmetric voters have a different precision of receiving a signal about one state of the world versus the other.

2 Model

The model setup is similar to that in the strategic voting literature (e.g., Feddersen and Pesendorfer, 1996), with the novel departure that one voter (shareholder) has more votes (shares) than the others—the blockholder. Most of the literature relies on symmetry in order to pin down equilibria. However, in the corporate environment, asymmetry in the number of votes participants have arises naturally. We will also allow for asymmetry in strategies (for shareholders who have an equal number of shares).

There are two types of agents who own shares in the firm. There is a blockholder (B) who has $2b$ shares and $2n+1$ other shareholders (each denoted by S), each of whom owns one share. Henceforth, we use the term shareholder only to refer to one of the $2n+1$ who each hold only a single share. The assumption that the blockholder has an even and shareholders an odd number of shares allows us to disregard ties for many of the cases we study. For simplicity, we assume there is no trading of shares, although we relax this assumption in Section 5. We also assume that the blockholder does not own a majority of the shares:

Assumption A1: $n \geq b$

A proposal at the shareholder meeting[7] will be implemented if there are more votes in favor than against. If there is a tie, we will assume that each of the two possible decisions will be implemented with probability 0.5, as is standard in the literature. There are two states of the world θ, management is correct ($\theta = M$), and against management ($\theta = A$), which are both ex ante equally likely to occur. Let d denote whether management wins ($d = M$) or loses ($d = A$). Shareholders and the blockholder have common values, i.e., they both prefer the choice that maximizes the value of the firm. We will relax this assumption in Section 6, where we allow the blockholder to be biased and strictly favor one decision. The payoff per share for both the blockholder and shareholders $u(d, \theta)$ from a vote depends on the decision and the state of the world. If the decision matched the state of the world, the payoff is 1 for each share: $u(M, M) = u(A, A) = 1$. If the decision did not match the state of the world,

[7] From Yermack (2010): "In addition to director elections, shareholders may vote on such topics as the appointment of outside auditors, issuances of new shares, creation of equity-based compensation plans, amendments to the corporate charter or bylaws, major mergers and acquisitions, and ballot questions submitted in the form of advisory shareholder proposals. Shareholders may also be asked to ratify certain decisions of the board of directors, such as related-party transactions with members of management. When shareholder approval of an item such as an acquisition becomes time critical, votes may be held at special shareholder meetings called in the middle of a year."

the payoff is 0: $u(M, A) = u(A, M) = 0$.

The blockholder and shareholders receive individual imprecise signals $s \in \{m, a\}$ about what the correct state of the world is. This precision is given by $\pi_i(\theta \mid s)$, where $i \in \{B, S\}$.[8]

$$\pi_B(M \mid m) = \pi_B(A \mid a) = q$$
$$\pi_S(M \mid m) = \pi_S(A \mid a) = p$$

The probability that the blockholder infers the correct state from the signal is q, and the same probability for the shareholder is p. We assume that $q \geq p > 0.5$. This indicates that signals are informative, as their precision is above 0.5, and that the blockholder receives a more precise signal than the shareholder. The blockholder presumably receives more-precise information because she is a larger investor and possibly has (i) more contact with the firm, (ii) more infrastructure in place to gather information, and (iii) more incentives to gather information. In the analysis, we discuss what happens when the blockholder's precision varies, and we endogenize information acquisition by the blockholder in Section 5.

We allow the blockholder and shareholders to vote strategically. Given a particular signal, they may vote for M or A or abstain. Nevertheless, to gain some tractability and focus on the key trade-offs of the model, we restrict the voting behavior of the blockholder and shareholders as follows:

Assumption A2: The blockholder can vote only an even number of shares, and cannot simultaneously vote for both sides of the proposal.

Assuming that the blockholder votes with an even number of shares (like the assumption that the blockholder holds an even number of shares and other shareholders an odd number) allows us to disregard ties for many of the cases we study. This does not affect the results but simplifies the analysis and presentation. Preventing the blockholder from voting for both sides of the proposal is more a question of presentation, as what will matter in that case would be the net votes the blockholder produces for one side, which could be replicated by voting only a fraction of her votes and abstaining with the rest (which we allow).[9] We also

[8] Formally, in the statistics literature, precision is equal to the reciprocal of the variance; here, this is equal to $\frac{1}{\pi_i(1-\pi_i)}$, which is monotonic in π_i in the relevant range $\pi_i \in (\frac{1}{2}, 1]$.

[9] This choice of presentation might raise the question of whether quorum rules are met. Often, these require

assume:

Assumption A3: The blockholder and shareholders are restricted to pure strategies.

The restriction of all agents to pure strategies is in contrast to Feddersen and Pesendorfer (1996), who focus on symmetric mixed strategies. We make several remarks on this. First, for our main result, the case of the passive blockholder, the equilibrium that is most informative (which we focus on) involves symmetric pure strategies for shareholders. Therefore it is indeed comparable to Feddersen and Pesendorfer (1996). We discuss this in detail in the relevant part of the text.[10] Second, this assumption is relevant only when shareholders are indifferent among different voting strategies, and it can be understood as restricting the way they respond to indifference. In more-general games, allowing mixed strategies may be required to ensure the existence of equilibria—we will show that pure strategy equilibria exist in all the environments we consider. We rely on this assumption to reduce the number of cases we consider and for tractability. Intuitively, the informationally efficient equilibria we focus on should require (potentially asymmetric) pure strategies rather than mixed strategies to avoid costly miscoordination. Persico (2004) points out that such a restriction, which he also makes, still allows for rich strategic behavior, which we also demonstrate. Moreover, Esponda and Pouzo (2012) argue that in the voting environment we consider, pure strategy equilibria are stable, whereas mixed strategy equilibria are not.

The precision of the signals and the structure of the game is common knowledge.

Lastly, the voting setting leads naturally to multiple equilibria, even when the blockholder and shareholders are restricted to pure strategies. It is natural to focus on the equilibria that lead to the best outcome. Note that since all shareholders and the blockholder have identical preferences and differ only in their information, they all agree on what "best" means here—the equilibria that lead to the highest probability of selecting the decision that matches the state. We will refer to this as the most informative equilibrium. In Section 6, we allow the blockholder to be biased; therefore, this may no longer be the blockholder's preferred equilibrium. Nevertheless, this selection criterion will remain the preferred one of

a majority of shares to be present at a vote. Abstention by those present may be permitted. Alternatively, by voting on both sides of an issue, and thus replicating abstention, shareholders in our model can ensure compliance with such a quorum requirement.

[10]The result on the active blockholder involves an equilibrium which is not symmetric for shareholders and therefore requires coordination. Nevertheless, it should be clear that mixed strategies could lead to miscoordination, and therefore a loss of efficiency in that setting.

the shareholders, and we continue to employ it.

In the following sections, we consider two variations of the model. First, we suppose the blockholder makes no announcements before it votes, which we designate as a passive blockholder. Next, we allow the blockholder to observably announce its position before other shareholders, which we designate an active blockholder.

3 Passive blockholder

In this section, we analyze the model outlined above where both the blockholder and shareholders care about maximizing the value of the firm and move simulatenously. Looking at how much information the blockholder holds relative to shareholders is critical to understanding what the most informative equilibrium is in this context.

Consider two examples. In the first example, the blockholder has imprecise information, say, equal to the precision of an individual shareholder's information. Nevertheless, in this example, the blockholder owns 40% of the shares, while each individual shareholder holds less than 1%. If the blockholder voted with all of its shares in this scenario, the final vote will mostly reflect the blockholder's information. If instead the blockholder did not vote all of its shares, the final vote would reflect the information of all shareholders. Because the blockholder cares about maximizing the informativeness of the vote, since that will augment the likelihood of increasing firm value, it is natural to posit that the blockholder prefers not to vote all of its shares.

In the second example, the blockholder is perfectly informed, i.e., $q = 1$, but only owns 5% of the shares of a firm. Individual shareholders have imprecise information. In this case, if all individual shareholders vote, the blockholder's vote will have little impact. But if the individual shareholders could delegate the vote to the blockholder, they would prefer to do so, as it would maximize the informativeness of the vote. They can accomplish this by abstaining.

These two examples lead us to two equilibrium outcomes that might maximize informativeness. We prove in Proposition 1 that these are unique most informative equilibrium outcomes for certain parameters.

We first define an optimal vote threshold for the blockholder, $b^* := \left\lfloor \frac{1}{2} \frac{\ln q - \ln(1-q)}{\ln p - \ln(1-p)} \right\rfloor$, so that $2b^*$ is the even number of shares that most closely corresponds to the optimal vote threshold (without exceeding it),[11] accounting for the fact that the theoretical optimal weight in decision-making may not correspond to a round number. In particular, the threshold b^* depends on the relative quality of information of the blockholder and shareholders as embodied by their respective likelihood ratios; b^* is the integer such that the blockholder's information is weighted to equal the contribution of b^* one-vote shareholders' information:

$$\left(\frac{p}{1-p}\right)^{2b^*} \leq \frac{q}{1-q} < \left(\frac{p}{1-p}\right)^{2b^*+1}. \tag{1}$$

It is worth noting that b^* does not depend on the number of shareholders, captured by n. One way to understand this (and derive it) is by considering how a planner would Bayesian update the prior given the blockholder's signal and the shareholders' signals. The term b^* is the weight that the planner would assign to the blockholder's signal normalized by the weight that the planner would assign to an individual shareholder's signal.

As described below, the number of shareholders n will play a role in the equilibrium characterization since a greater number of shareholders (who in aggregate will lead to a more informative vote) mechanically have more votes and, collectively, can have more influence as n rises.

We also define a condition on parameters that determines which of two equilibrium outcomes (described below) is more informative.

$$\frac{\sum_{i=n+b+1}^{2n+1} \binom{2n+1}{i} \left(\frac{p}{1-p}\right)^i}{\sum_{i=0}^{n-b} \binom{2n+1}{i} \left(\frac{p}{1-p}\right)^i} > \frac{q}{1-q} \tag{C1}$$

Finally, we say that a shareholder or blockholder votes sincerely if they vote with their signal.

Proposition 1 *The most informative equilibrium takes the following form:*

(i) When $b \geq b^*$, all shareholders vote sincerely and the blockholder votes sincerely with $2b^*$ shares,

[11] The function $\lfloor x \rfloor$ is called the floor function. It defines the largest integer less than or equal to x.

(ii) When $b < b^*$ and Condition C1 holds, all shareholders vote sincerely and the blockholder votes sincerely with $2b$ shares, and

(iii) When $b < b^*$ and Condition C1 does not hold, the equilibrium outcome is equivalent to one in which only the blockholder votes.

The equilibria summarized above always exist.

We prove the proposition in the appendix by first observing that if we were to assume that voting was sincere rather than strategic, Nitzan and Paroush (1982) show that, in order to maximize the informativeness of a vote, a planner should weight the votes of voters with heterogeneous information according to how precise their signals are. These weights allow us to define b^*. In our model, however, shareholders need not vote sincerely. *Strategic* voting implies they may vote against their signals and/or abstain. We must therefore consider equilibrium behavior (rather than the planner's choice). In the case where the blockholder has more than $2b^*$ shares, this is a simple exercise. The results of McLennan (1998) ensure that in this common interest game, everyone voting sincerely with the optimal weights can be implemented as an equilibrium (that is, that shareholders and the blockholder would not want to deviate from voting sincerely according to these vote shares).

Now consider the case where the blockholder has fewer than $2b^*$ shares. We first demonstrate that the equilibria we propose are, in fact, equilibria. We then show that they are the most informative equilibria, where Condition C1 determines which of the two is most informative.

Hence, when the blockholder has too many shares in comparison with her information, she will not vote them all. If she were to vote them all, she would drown out useful information from the other shareholders, impairing the effectiveness of the overall vote. Because the blockholder and the shareholders have a common interest in maximizing the value of the firm, the blockholder internalizes this effect and only votes a fraction of her shares. As mentioned in the introduction, for investment advisers (such as mutual funds), the requirement to act in the best interest of investors is often interpreted as prohibiting the blockholder from acting strategically. This result demonstrates that there is an efficiency loss from such a policy.

When the blockholder has too few shares in comparison with her information, there are two equilibria that may be the most informative. One is similar to the above case - the blockholder votes all of her shares sincerely and the other shareholders also vote sincerely. In the other, only the blockholder votes. All other shareholders abstain to allow the blockholder's superior information to determine the vote. Clearly, strategic behavior is important to this equilibrium - allowing the shareholders to get out of the blockholder's way by abstaining leads to this result.

Note that in the most informative equilibrium, for all parameter values, we demonstrate that the shareholders strictly prefer to choose symmetric pure strategies. This is directly comparable to the result of Feddersen and Pesendorfer (1996) who restrict agents to symmetric mixed strategies (and do not use the criterion of selecting the most informative equilibrium), as pure strategies are a subset of possible mixed strategies. Therefore, even though we restrict agents to pure strategies for tractability throughout the paper, in this section this restriction is unimportant. Note that in Feddersen and Pesendorfer (1996) some of the equilibria involve mixing; the mixing results from the agents' counteracting the effect of partisans (in our terminology, biased voters) on the vote. In this section, we don't have any biased voters, so this type of strategy is not relevant. It will be relevant in our extension to the case of a biased blockholder.[12]

In determining which of these equilibria is most informative, the number of shareholders, n, the precision of the signals (q and p), and the number of shares the blockholder has are also important. First, consider an extreme case where there are many shareholders (n is high). In this case, Condition C1 holds, and in the extreme case where $n \to \infty$, the shareholders' collective vote will be accurate with probability approaching 1, as is well understood following Condorcet (1785). Therefore having the shareholders vote is helpful. Instead, when n is small, there is scope for noise and mistakes. In this case, Condition C1 fails and the collective vote of all shareholders is relatively inaccurate. Second, Condition C1 implies that the equilibrium outcome where shareholders abstain is more informative if shareholders' information is poor in comparison to the blockholder and the blockholder has few votes.

We formalize these statements in the following lemma.

Lemma 1 *When $b < b^*$, the most informative equilibrium involves shareholders voting sincerely the (i) higher p is, (ii) the higher n is, (iii) the higher b is and (iv) the lower q is.*

[12] Nevertheless, in that subsection, we maintain the restriction to pure strategies as in Persico (2004), for tractability.

We now study the case where the blockholder can announce its position to the other shareholders before they vote.

4 Active blockholder

Blockholders differ from small shareholders in several ways beyond holding more shares. Up until this point, the only additional difference we have assumed is that the blockholder has better information than shareholders. Here, we suppose that the blockholder can also observably announce a voting strategy before other shareholders vote. Note that the blockholder need not commit to the announced position, as its objectives are aligned with other shareholders.[13]

Of course, in this common interest game, communication before voting can lead to a better vote outcome. Communication among many dispersed shareholders is generally infeasible, and voting is the technique used to aggregate information. Nevertheless, blockholders may easily be more visible and scrutinized. Any shareholder with over 5% of the shares of a publicly traded firm must publicly disclose this to the SEC. Some blockholders are very public about their activities, such as activist investors. Activist investors make public statements about their intentions and often discuss their intentions with other investors (see Coffee and Palia, 2016).

The analysis of this sequential game proceeds by backwards induction, and the equilibrium concept we apply is Perfect Bayesian Equilibrium: the shareholders observe the blockholder's statement, drawing inferences on the blockholder's signal, and then choose their votes; the blockholder, anticipating equilibrium behavior of the shareholders in response to their signals and to its own statement, will choose its voting strategy.

Consider a blockholder with few votes but very precise information. This blockholder cannot represent all of its information in its vote. A shareholder with an imperfect signal who has seen the blockholder's voting intention may, then, prefer to ignore its information and vote in line with the blockholder (to support the blockholder's information). Indeed, some voters should mimic the blockholder's vote to make the weight of the blockholder's information correspond to the optimal threshold $2b^*$, with the remainder voting sincerely.

[13] There are, of course, other equilibria involving, for example, babbling. Sincere communication will be a feature of the most informative equilibrium.

We demonstrate that this behavior is an equilibrium and is, in fact, the most informative equilibrium.

Proposition 2 *Given an active blockholder, the unique most informative equilibrium is where the blockholder votes sincerely with* $\min[2b, 2b^*]$ *shares, and*

(i) if $b \geq b^*$*, all shareholders vote sincerely, or*

(ii) if $b < b^*$*,* $2b^* - 2b$ *shareholders vote the same way the blockholder does, and the rest of the shareholders vote sincerely.*

Therefore, having some shareholders ignore their information to support the blockholder's information maximizes the probability that the vote matches the state. The most informative equilibrium involves $2b^* - 2b$ shareholders ignoring their information; this clearly leads to a coordination problem. Moreover, the shareholders only want to coordinate because the blockholder has moved first. This suggests that leadership and coordination among shareholders may enhance efficiency.[14]

Activist investors are often very public about their positions. For example, Bill Ackman, of the Pershing Square Hedge Fund, appeared "almost daily on CNBC to take his case directly to investors" (George and Lorsch, 2014). Direct communication with other shareholders was permitted by the SEC's rule 14a-12 in 1999. Communication enters a gray area when investors have sizable stakes in the firm - an individual or group stake of 5% or more must be declared publicly to the SEC under section 13(d). The gray area is what constitutes a group, i.e., do communication and parallel actions constitute coordination or not? The purpose of this public declaration to the SEC is to make other shareholders aware of changes in control of the firm (Lu, 2016). The downside of such unobservable coordination is that trading profits are made by the insiders in a "wolf pack" while other shareholders are unaware. However, the trading profits may be needed to incentivize such behavior. Our model demonstrates that there can be benefits from having such a "wolf pack", i.e., having (i) a lead informed activist and (ii) subsequent coordination among some other investors to support the activist's position. Because we do not incorporate coordinated trading into

[14] Allowing for heterogeneity among the shareholders in the precision of their signal creates a natural coordination device: the lower-precision shareholders would be the ones who ignore their information and follow the blockholder.

the model, we cannot address the costs of such behavior.[15] A different type of coordination might be made feasible by having shareholders invest in a fund, with the fund taking the coordination role. Appel, Gormley, and Keim (2016b) find that passive mutual funds have facilitated activism by supporting activists with large blocks of shares and that this has led to changes in activist presence, tactics, and overall outcomes.

It is natural to compare outcomes and the informational efficiency of the equilibria in the passive blockholder case to that of the active blockholder case. When $b \geq b^*$, the outcome is the same. When $b < b^*$, in the case of the passive blockholder, there is no scope for shareholders to allow the blockholder's signal to carry more weight given that the blockholder's information is unobservable. This leads to an outcome where either all shareholders vote with their signal or all abstain. Instead, when the active blockholder's intention is observed, shareholders can condition their vote on the blockholder's communication, with some ignoring their information to vote with the blockholder. As each shareholder wants to maximize the value of the firm, the fact that some shareholders decide to condition their vote implies that welfare is higher in the active blockholder case. Thus, in either of these cases, there is a benefit to vocal activist blockholders.

5 Information acquisition and trading

We extend the model to allow the blockholder an opportunity to acquire information and highlight that the blockholder will underinvest in information-acquisition. This can provide a simple rationale for the SEC rules for investment advisers. Forcing the blockholder to vote all her shares may provide stronger incentives to acquire information since the consequence of having low precision information and voting incorrectly is more severe.

We then extend the model to allow the blockholder to freely trade shares before voting in a simple model of trade with endogenous share prices. The opportunity to trade can lead to an outcome that is more efficient since trading allows votes to move to where they have the largest informational value; in particular, the equilibrium will involve no abstention.

[15] Brav, Dasgupta, and Matthews (2016) provide a very different motivation for coordination by a wolf pack in a model that does not include voting. A sufficiently large bloc of shareholders can make a value-enhancing change in a firm. These activists have complementarities in their costly decision to take a stake because they receive reputation benefits from a successful activism campaign. The complementarities lead to a coordination game.

Lastly, we allow for both information acquisition and trading. The interaction reverses some of the results from the situation where only one of the choices is possible. First, the interaction provides a rationale for why the blockholder holds shares that she does not vote with. Since potential buyers cannot be sure whether the blockholder has invested in information acquisition or not, there is a possibility of a lemons discount if the blockholder trades her shares, i.e., shareholders do not believe the blockholder has acquired information and therefore discount the shares being sold. This discount may incentivize the blockholder to keep her shares even if she would abstain on them. Second, the benefit from the SEC regulation no longer arises; the blockholder could choose to sell her shares rather than be forced to vote with them all, and so the SEC rule does not provide additional incentive for information acquisition any more.

5.1 Information acquisition

We suppose that the blockholder starts out with the same precision of information as a shareholder, i.e., the precision p. The blockholder may then pay a fixed cost c to acquire information, which boosts the precision of its information to $q > p$.

If the blockholder invests, then her higher quality information, which will be incorporated at least to some extent into a vote, will lead to a better outcome and so a higher payoff. Indeed this higher payoff would be enjoyed by all shareholders, who all benefit from a vote that more accurately captures the underlying state. The blockholder owns a fraction $\frac{2b}{2n+2b+1}$ of the shares, and, so, enjoys only this fraction of any gain from a more accurate vote. Consequently, her private incentives to acquire information will be below the socially efficient level.

As we point out above, the SEC rule (SEC, 2003) that requires investment advisers to vote in the best interest of their clients can reduce the efficiency of information aggregation in voting by prohibiting strategic behavior. However, if the rule led to more information acquisition, the cumulative effect could actually *enhance* efficiency.

For this analysis, we interpret the SEC rule in our model as requiring (i) the blockholder to vote with all her shares and (ii) that the blockholder must vote sincerely, i.e., vote in line with her information. Incorporating this rule into the model implies that the value per share when the blockholder invests in information acquisition may be lower since information is less efficiently aggregated into the vote. However, this regulation also implies that the value per share when the blockholder does not invest in information acquisition is also lower. This second effect may be stronger, as the blockholder might be impounding very poor information into the vote. This can skew the blockholder's decision toward acquiring information. This will be socially beneficial when the blockholder would not have acquired information without the regulation, even though that would have maximized social welfare.

Of course, if the requirement were not enough to induce the blockholder to acquire information, then its sole effect would be to lead to an inefficient aggregation of the available information and, in this way, would be detrimental.

Note that given the results in Propositions 1 and 2, in the case where the blockholder acquires information and $b < b^*$, the requirement that the blockholder vote with all her shares does not reduce the efficiency of the vote: she would in any case choose to vote with all her shares. Thus, the SEC rule would reduce the value of not investing in information, with no impact on her value if she does invest. Consequently, this increases the blockholder's incentives to invest and welfare will be higher if she does so.[16] The case in which $b > b^*$ is more involved since, conditioning on the blockholder acquiring information, a requirement to vote with all her shares would be detrimental for the decision and, thus, for the blockholder. However, for some parameters, the drop in the quality of decision-making in case of information acquisition is lower than the drop in the quality of the decision-making without acquiring information, and so a requirement to vote all of her shares can induce the blockholder to acquire information and the social gain from doing so may outweigh the costs of inefficient use of information. Our working paper, Bar-Isaac and Shapiro (2017) provides a specific example to illustrate this possibility.

5.2 Trading Shares

We examine a simplified trading game where we assume the blockholder trades publicly. The price at which a share trades is determined by the blockholder making simultaneous observable take-it-or-leave-it offers. Consequently the price will reflect the expected value of the share. This expectation, in turn, reflects shareholders' beliefs about the quality of the blockholder's information and the blockholder's post-trade holdings which determine the

[16] Indeed, it is possible that such a regulation could lead the blockholder to over-invest in information acquisition.

extent to which information is efficiently incorporated into the vote.

We assume the blockholder can sell to new shareholders who buy one share each and have precision of information p. The blockholder can also buy shares from existing shareholders. The observability of the blockholder's offers simplifies the inference problem of the shareholders and might reflect the fact that the blockholder is likely to face limits to its ability to disguise large trades. We suppose that after trade occurs, the vote proceeds with the blockholder taking an active role and announcing her voting intention, as in Section 4.[17]

We assume the following timing:

1. The blockholder begins with an amount of shares $2b$. There are $2n+1$ existing shareholders. The blockholder and each of the shareholders have information of precision p.

2. The blockholder can pay c in order to improve the precision of its information to q. This investment decision is not observed.

3. The blockholder can simultaneously and publicly make take-it-or-leave-it offers to sell to potential new shareholders or to buy from existing shareholders. After trading, the number of shares that the blockholder holds is given by $2\widehat{b}$ and the number of shareholders is $2\hat{n}+1 = 2n+1+2b-2\widehat{b}$.

4. The blockholder announces her voting intention.

5. Votes are cast.

Note that the price at which shares trade will depend on all agents' expectations about the blockholder's quantity of shares held after trading, since this will affect the likelihood of voting correctly. The price will also reflect shareholders' beliefs about the blockholder's information precision.

We begin by considering the case of trade where information is exogenous (i.e., the cost of information acquisition $c = 0$ so that the blockholder always acquires information) as in the baseline model, before allowing for information acquisition in Subsection 5.2.2.

[17] A similar analysis would apply to the case of the passive blockholder as in Section 3 and, we focus on the active voting case for expositional ease.

5.2.1 Trading with exogenous information

The blockholder's trading decision will affect the quality of the voting decision (since it will affect the quantity of information that feeds into the decision and the way that it does so). Consequently, this affects the price at which shares trade; this will simply reflect the likelihood that the vote correctly identifies the state of the world. Note that the blockholder will also value the shares that she does not trade in the same way: each share's value corresponds to the likelihood that the vote correctly identifies the state of the world. Therefore, in the model a share has highest value in the hands of the agent who can best use it to vote informatively, and the blockholder wants to trade so as to maximize the vote's accuracy. Following Proposition 1, and noting that standard Condorcet reasoning suggests that having more shareholders voting leads, on average, to better decisions, suggests that the blockholder will trade so as to hold exactly $2b^*$ shares and not hold shares that she does not vote with; that is, she will choose to sell $2(b - b^*)$ shares if $b > b^*$.

Note that as we are examining the active blockholder, the blockholder need not buy shares to get to a voting share of $2b^*$. This is because existing shareholders will already ignore their information and vote with the active blockholder. The blockholder is thus indifferent about buying those shares, as no information is added due to their purchase. This discussion can be summarized as follows.

Proposition 3 *In case of exogenous information and trade, the blockholder will not hold shares on which it will abstain.*

As described above, the price at which shares trade will reflect the anticipated post-trade holdings, which, in turn, affect the extent to which information is efficiently aggregated through the vote.

Corollary 1 *When there is trade, shares trade at a price equal to*

$$\begin{aligned} v_I \quad : \quad &= q \sum_{i=n+b+1-2b^*}^{2(n+b-b^*)+1} \binom{2(n+b-b^*)+1}{i} p^i (1-p)^{2(n+b-b^*)+1-i} \\ &+ (1-q) \sum_{i=n+b+1+2b^*}^{2(n+b-b^*)+1} \binom{2(n+b-b^*)+1}{i} p^i (1-p)^{2(n+b-b^*)+1-i}. \end{aligned} \tag{2}$$

When $b > b^*$, all agents anticipate that in equilibrium, following trade, the blockholder will hold $2b^*$ shares, and will vote with all of them (based on her information which is of quality q). Since the blockholder starts with $2b$ shares but ends up holding $2b^*$ there will be $2(b - b^*)$ more shareholders so that the total number of shareholders will be $2(n + b - b^*) + 1$. The total number of votes, when adding those of the shareholders and blockholders together, is $2n + 2b + 1$. The share price reflects the probability that the vote matches the state; which in turn requires that a majority (of $n + b + 1$ or more) of the total number of votes match the state. This could happen either with the blockholder casting her $2b^*$ votes correctly (with probability q), in which case at least $n + b - 2b^* + 1$ out of $2(n + b - b^*) + 1$ shareholders must also vote correctly (i.e., having an accurate signal which occurs with probability p) with the remainder voting incorrectly; or with the blockholder voting incorrectly (with probability $1 - q$) and at least $n + b + 2b^* + 1$ out of $2(n + b - b^*) + 1$ shareholders voting correctly. Summing the probability of each of the possibilities leads to the expression above.

5.2.2 Trading and Information acquisition

In this subsection, we incorporate both trading and information acquisition. The interaction between these two elements can reverse some of the conclusions made above.

The blockholder does not trade its unvoted shares: Prior to trade and prior to voting, we allow for the blockholder to acquire information at a cost (stage 2 of the timing listed above). We assume that the information acquisition decision is unobservable.

Given this, we are able to explain why, if a blockholder does not vote all of its shares, she does not sell the shares with which she does not vote. The key intuition is that the blockholder may not be able to trade shares easily, as new potential shareholders may be dubious about whether the blockholder acquired information if she is trying to dump shares on the market and, hence, may have a low willingness to pay for them. Thus, the equilibrium price offered for a share may be very low and the blockholder might therefore prefer not to sell shares even if doing so would lead to a more accurate voting outcome (conditional on the information acquisition decision).

The following result establishes that an equilibrium exists where the blockholder acquires information and does not sell its shares in excess of $2b^*$.

Proposition 4 *There are parameters for which an equilibrium exists in the game defined* *above where the blockholder (i) improves her information to precision q, (ii) does not trade any shares ($2\widehat{b} = 2b$), and (iii) votes $2b^* < 2b$ shares.*

The proof specifies the equilibrium strategies and beliefs of the blockholder and the shareholders and the incentive constraints of the blockholder. It demonstrates by numerical example that such an equilibrium exists.[18]

Of course, if information acquisition were a public rather than private decision then the analysis of Section 5.2.1 would apply: new potential shareholders could condition their willingness to pay on the incorporation of more information, and the blockholder could get the fair value for the shares. When shareholders cannot see whether or not the blockholder invested in information, there is a lemons discount for traded shares. Shareholders observe shares on the market from the blockholder and assume she did not invest in information. Therefore, the blockholder does not receive full value for the shares, although she still benefits from new information being incorporated into the vote.

The SEC rule is inefficient: With trading, the beneficial role of the SEC, described in Section 5.1, is eliminated. Suppose that the blockholder did not acquire information in the absence of the SEC rule. Then, she would gain by trading all of her shares (since she has the same information precision p as other shareholders and, so, there are shareholders who would improve the value of the vote by buying the shares and voting informatively). The introduction of the SEC rule does not affect how much the blockholder receives from trading all her shares, and therefore does not affect her value when she does not invest in information acquisition. Therefore the SEC rule no longer worsens the blockholder's payoff when she does not invest in information acquisition. The rule then doesn't make information acquisition more likely and may make information aggregation less efficient (by forcing the blockholder to vote its $2b > 2b^*$ shares).[19]

[18] Note that this equilibrium is unlikely to be the most informative equilibrium. In particular, for some parameters, one could construct an equilibrium where: if after trading, the blockholder holds any amount shares other than $2b^*$, shareholders believe that the blockholder did not acquire information, and if the blockholder holds $2b^*$ shares after trading, shareholders believe the blockholder acquired information.

[19] There are, of course, several reasons that are not modeled here for why a blockholder might not want to engage in trading and, so, might suggest that the rule has bite. One is that new potential shareholders might not have useful information to incorporate into the vote, reducing their willingness to pay (e.g., liquidity traders). A second is that the blockholder is assumed to obtain the full surplus from any trade, which is unlikely to be true. Lastly, there may be practical impediments to trading. This could arise if the blockholder was an index fund that needed to trade to track the index or if the blockholder had to worry about having too much of a price impact from selling shares.

6 Biased Blockholder

So far, we have considered votes where the blockholder and shareholders have had identical objectives. In this section, we suppose that the blockholder is biased and prefers that the action M be chosen regardless of the underlying state. As discussed earlier, this bias may arise because the blockholder is part of management or directly tied to it through business dealings. We maintain that timing, preferences and the quality of signals are known by all. We analyze two situations. First, we consider the case of a biased passive blockholder who makes no announcements. Second, we consider a biased active blockholder.

6.1 Passive blockholder

When the blockholder is passive and votes at the same time as other shareholders, it is immediate that in any equilibrium the biased blockholder prefers to vote all of its shares for its preferred position: Its choice is not observed and can have no effect on the choices made by shareholders. Since more votes in favor of its preferred position increase the likelihood that this position is adopted, taking as given the behavior of shareholders, it will vote all of its $2b$ shares for M.

Multiple equilibria can arise here, as before. Our focus here is on the equilibria that are optimal from the perspective of shareholders, that is, equilibria that maximize the probability that the proposal adopted matches the underlying state - the most informative equilibria.[20]

First, suppose that no shareholder can abstain (we relax this below). In this situation, the most informative equilibrium involves $2b$ shareholders voting for A, effectively nullifying the biased blockholder's influence, while the remainder vote sincerely.

Proposition 5 *In the case of a biased passive blockholder, the most informative equilibrium with no abstention involves the blockholder voting for M with all of its $2b$ shares, $2b$ of the shareholders voting for A independently of their signals, and the remaining $2n+1-2b$ shareholders voting sincerely.*

Proposition 5 highlights that allowing shareholders to ignore their signals can lead to more-efficient outcomes. The efficiency comes from those votes canceling out the bias of

[20] Trivially, a preferred equilibrium of the biased blockholder is the one where all shareholders vote M regardless of their information.

the blockholder and allowing the sincere voting of the remaining shareholders to provide an informative vote. Note that this involves coordination among shareholders (to choose which shareholders will block the vote) as in the case with the unbiased active blockholder, but here, it is to oppose the blockholder rather than support her. This could still fit into the "wolf pack" scenario, where the blockholder is management and the coordinating shareholders are activists seeking to change management practices.

This result is closely related to the results of Maug and Rydqvist (2009), who also assume that abstention is not allowed. Maug and Rydqvist (2009) study a common value model with strategic voting where shareholders each have one vote and there may be different majority voting rules. In their equilibrium with asymmetric pure strategies (i.e. where identical shareholders may have different strategies), they find that a voting rule which is not optimal will induce a subset of shareholders to ignore their information and vote to restore the optimality of the voting rule, allowing all other shareholders to vote informatively. Here, we find a similar result: the biased blockholder, by voting for M without using information, imposes a higher threshold for A to win. A subset of shareholders ignore their information to counteract this effect.

Next, we point out that when we allow for abstention, the equilibrium can be more efficient. We use a simple example that shows how abstaining rather than voting A when observing an m-signal can allow shareholders to reflect their information to some extent while still mitigating the influence of the biased blockholder. The intuition for this is the following. Take one biased blockholder vote for M and add a vote where the shareholder abstains when observing an m-signal and votes A when observing the a-signal. If the state of the world is M, it is likely that adding the two votes together will produce one vote for M. Otherwise, it is likely that the two votes will cancel each other out. Therefore, this produces some information as compared to the case where a shareholder always votes for A and this vote completely cancels out a vote of the blockholder. The example is as follows:

Example 1 *Suppose that $n = 1$ and $b = 1$, so that the blockholder has two shares and there are three shareholders. In the unique most informative equilibrium, the three shareholders all abstain when observing an m-signal and vote A when observing the a-signal.*

6.2 Active blockholder with commitment

We now turn to consider the case of an active biased blockholder who may commit to a vote observably before the other shareholders vote.[21] It might seem that the biased blockholder will simply vote all its shares for its preferred position, so that the distinction between the passive and active cases is uninteresting, but this does not happen. With a passive blockholder, since the blockholder's vote is not observed, it cannot have an effect on shareholder behavior. Instead, with an active blockholder, the blockholder's voting choice is observable and *can* change shareholders' behavior (both directly to counteract the votes of this biased party and through the information learned from the vote), and a different outcome can arise. Indeed, we show that the most informative equilibrium has a blockholder who abstains.

If the blockholder abstains (and the shareholders draw no inference about her information) then the most informative equilibrium involves all shareholders voting sincerely. This is a good outcome for a blockholder who is perfectly informed that the state is M, as M is likely to win such a vote.

Of course, a blockholder who knew that the state is A would not be happy with such an outcome. However, voting by this blockholder might reveal that the blockholder knows that the state is A and would thus lead shareholders to update their beliefs and to support A even more strongly (indeed, when the blockholder is perfectly informed, all shareholders would vote A). To make these arguments clearly, we suppose throughout this section that the blockholder is perfectly informed regarding the underlying state; that is, $q = 1$. We show that there is an equilibrium where the biased blockholder abstains regardless of her information, and all shareholders vote sincerely.

Proposition 6 *In the game with a biased active blockholder, there is an equilibrium in which the blockholder always abstains and all shareholders vote sincerely. This is the most informative equilibrium.*

This equilibrium incorporates the information of shareholders as efficiently as possible. Although the biased blockholder is present, she has no influence whatsoever in equilibrium.

[21] We allow for commitment here, as the bias of the blockholder has eliminated the common value environment of the model, making communication cheap talk. Such commitment might best be understood as arising from reputational concerns. In the case of investment advisers, in line with SEC regulations, the blockholder's vote would be observed ex-post. For other kinds of large blockholders, inferences could be drawn from aggregate vote shares.

The biased blockholder with precise information that A is the state of the world would prefer to enter and reduce the informativeness of the vote (by having some of the shareholders dedicate themselves to canceling her vote rather than voting sincerely, and thus making the vote noisier). However, this would reveal the biased blockholder's information in the signaling game, and she must therefore abstain.

It is clear, following Condorcet (1785), that the active blockholder case, where $2n + 1$ shareholders vote sincerely, uses the shareholders' information more efficiently than the passive blockholder case, where some shareholders are not voting sincerely, in order to counter the bias of the blockholder.[22]

7 Empirical links

In this section, we first examine the empirical challenges in testing the implications of our model and then review the literature that describes the importance of shareholder voting in governance.

7.1 Empirical Challenges

While many studies use aggregate votes in corporate elections, votes by individual shareholders (including blockholders) are difficult to observe. The only available shareholder-level voting data in the U.S. had been made possible by the 2003 SEC rule discussed above. The rule makes votes by investment advisers public information; investment advisers must submit form N-PX annually.[23] This data, while very useful in several applications, is not very useful in our context, for three key reasons:

- Form N-PX only requires the fund to report, "Whether the registrant cast its vote on the matter...[and] how the registrant cast its vote (e.g., for or against proposal, or abstain; for or withhold regarding election of directors)". Therefore, it doesn't require the number of shares voted, and says nothing about what to report if a fraction of

[22] Note also that (prior to the realization of the state) the blockholder is indifferent about whether it is passive or active, since in either case, the vote is equally likely to result in M or A being chosen. Thus, whether or not it incorporates the utility of the blockholder, maximizing ex-ante welfare would favor an active blockholder.

[23] The data from this form can be downloaded from the SEC; ISS has compiled all of it for sale.

shares were not voted in the same direction as others.[24] Some papers in the finance literature only use the direction of the vote (e.g. Iliev et. al. (2015)); there are a couple of recent papers concerned with shareholder-level voting that match the N-PX with share holdings as reported (at a different date) in Form 13F or from blockholdings reported in firms' proxy statements (Cvijanović et. al. (2016, 2017)). Our main result is that a blockholder may prefer to partially abstain to reflect the precision of its information. It is not clear how a blockholder would fill out form N-PX to reflect partial abstention and it is unlikely this would be picked up in the data.[25] Moreover, for any approach currently used in the literature, any abstention data would presumably reflect abstention on all of an investment adviser's shares.

- Form N-PX is only for investment advisers. Even if this data were helpful, any results on this data may not generalize to other blockholders (Holderness (2009) states that only 29% of blockholders in the U.S. are financial, which includes mutual funds, banks, and pension funds).

- The 2003 SEC rule specifically requires investment advisers to vote in the best interest of their clients and makes the votes public. This creates a potential liability for investment advisers and changes their incentives away from maximizing the value of the firm through their vote. As we have seen, maximizing the value of the firm may involve strategic voting and abstention. These strategies may be difficult to justify to clients and regulators. Therefore this rule biases investment advisers toward voting all of their shares and would make it difficult to observe the strategic behavior we describe in the data.[26] The empirical literature has also documented a reliance of institutional

[24] A proposed SEC rule in 2010 required collecting this data, but has not been implemented. A comment on the rule by the fund industry body ICI stated, "Specifically, we oppose expanding the types of disclosure that funds are required to provide on Form N-PX to include the precise number of shares they were entitled to vote and the precise number of shares that were actually voted. This data is not required by Section 951, would be difficult to compile (and in fact, may be impossible to produce with the degree of precision required), is of no value to most fund investors, and is generally unnecessary to achieve the purposes of proxy vote disclosure." (https://www.ici.org/pdf/24721.pdf)

[25] Mutual fund families sometimes delegate a vote to the funds themselves, which may result in a "split vote" for the family, where funds vote in different directions. Technically, the result will be the same as a partial abstention. However, the underlying reasons for this behavior may be quite different.

[26] Nevertheless, there is some evidence of investment advisers abstaining. Iliev et al. (2015) note "that in 6.6% of the director election votes, the institutions did not vote." (Footnote 7).

investors on proxy advisors for information and voting recommendations, which points to how liability has influenced incentives.[27]

Given the problematic nature of this shareholder-level voting data, there are few appealing alternatives. Some shareholder-level data in non-U.S. countries exists, but this data comes with its own issues.[28] We suggest in the text that a blockholder might not sell shares that it wouldn't vote due to a lemons discount. This argument presumably wouldn't hold if we allowed the blockholder to lend its shares, which is a common practice. Lending shares that wouldn't be voted to informed shareholders would incorporate more information into the vote.[29] This, of course, assumes the share lending allocation mechanism is efficient and borrowers are informed and not biased. Aggarwal, Saffi, and Sturgess (2015) look at voting by institutional investors and the market for stock lending. They don't observe the identities of lenders or borrowers, but instead use ownership data for based on 13F filings and aggregate stock loan data for corporate shares around the time of votes. They find that supply is restricted and recalled around votes, and that recall is higher when blockkholdings are larger (which they interpret as larger incentive to monitor), and when the value of the vote is larger (firms with poor performance and governance measures). Much more granular data would be needed to conduct a specific test of our results.

7.2 The Importance of Shareholder Voting in Governance

Shareholder voting is a critical element of corporate governance. We now review the empirical literature demonstrating this and relate it to our paper.

McCahery, Sautner, and Starks (2016) find that among shareholder engagement measures, voting against management was the second most frequently employed (and the top measure where shareholders exerted "voice") by surveyed institutional investors. Furthermore, many of these investors had engaged managers publicly: 30% had aggressively questioned management on a conference call, 18% had criticized management at the annual

[27] McCahery, Sautner, and Starks (2016) and Iliev and Lowry (2014) demonstrate the use of proxy advisors for this purpose.

[28] For example, Belcredi et. al (2015) are able to hand collect shareholder-level voting data in Italy, but state that in Italy an abstention counts as a negative vote. Hamdani and Yafeh (2013) use mutual fund voting in Israel that are made public due to a regulatory requirement. Aside from some similar difficulties with the N-PX data, they indicate that most Israeli companies have a majority shareholder.

[29] For an introduction to and an empirical analysis of this market, see Christoffersen et al. (2007).

general meeting (AGM), 18% had publicized a dissenting vote, 16% had submitted shareholder proposals for the proxy statement, 15% had taken legal action against management, and 13% publicly criticized management in the media. These forms of public engagement lend support to the version of the model where the blockholder makes a public announcement before the vote.

Duan and Jiao (2014) find that voting is an important source of governance using mutual fund data; for conflictual proxy votes (defined by votes where the proxy advisory firm Institutional Shareholder Services (ISS) opposes management), the probability of mutual funds voting against management is 46.42% higher than for other proposals, while their probability of exit is 3.12% higher. Passive mutual funds, by definition, do not trade very often and, therefore, can only affect value through voice (as opposed to exit). Appel, Gormley, and Keim (2016a) find that a one standard deviation increase in ownership by passive funds leads to a 0.75 standard deviation decline in support for management proposals and an approximately 0.5 standard deviation increase in support for governance related proposals.

Brav et al. (2008) point out that activist hedge funds do not usually seek control in target firms. The median maximum ownership stake for their sample is approximately 9.1%. In their sample, activist hedge funds take many forms of public action that are not in direct conversation with management or a takeover attempt.[30] Appel, Gormley, and Keim (2016b) note that the presence of passive mutual funds has facilitated activism by supporting activists with large blocks of shares, and they show that this has led to changes in activist presence, tactics, and overall outcomes. This links directly to our results on shareholders voting with the blockholder to support its information.

8 Conclusion

Our paper extends a standard voting environment by introducing a voter who has multiple votes: the blockholder. This is natural in a corporate setting and leads to striking results. Contrary to the common wisdom promoted by regulators, we demonstrate that allowing for abstention can increase the informativeness of a vote. A blockholder who wants to maximize

[30]This includes seeking board representation without a proxy contest or confrontation with management (11.6%), making formal shareholder proposals or publicly criticizing the company (32.0%), threatening to wage a proxy fight in order to gain board representation or sue (7.6%), launching a proxy contest to replace the board (13.2%), and suing the company (5.4%).

the value of the firm and who has imprecise information but many shares would prefer not to vote all of its shares to allow other shareholders' information to be used in the vote. We also demonstrate that allowing shareholders to act strategically and coordinate can also increase the informativeness of a vote to support an unbiased blockholder's information or to counter a biased blockholder's vote.

In our analysis, we have made many simplifications in order to demonstrate the basic driving forces when a blockholder is voting. It would be of interest to extend the model to allow for a more realistic environment. Some avenues that could be pursued are examining uncertainty about the bias of the blockholder, allowing for a richer model of share trading, and looking at multiple blockholders.

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9 Omitted Proofs

9.1 Proof of Proposition 1 (Passive Blockholder voting)

Proposition 1 *The most informative equilibrium takes the following form:*

(i) When $b \geq b^*$, all shareholders vote sincerely and the blockholder votes sincerely with $2b^*$ shares,

(ii) When $b < b^*$ and Condition C1 holds, all shareholders vote sincerely and the blockholder votes sincerely with $2b$ shares, and

(iii) When $b < b^*$ and Condition C1 does not hold, the equilibrium outcome is equivalent to one in which only the blockholder votes.

The equilibria summarized above always exist.

We prove our result through a series of Lemmas.

It is intuitive that a voter with better information should have a greater influence on the outcome—a planner with direct access to the signals would update according to Bayes' rule, and more-informative signals would have a greater influence on the posterior belief. In the same way, a voter with better information should be granted more votes. Specifically, Theorem 1 of Nitzan and Paroush (1982) implies the following:

Lemma 2 ***(Application of Nitzan and Paroush, 1982)*** *If voting is sincere and weights (or vote shares) can be allocated among all voters to maximize the informativeness of the vote, then the weight of each voter should depend only on their own information (p or q). In particular, the blockholder's vote share should be $\ln\left(\frac{q}{1-q}\right)$ and each shareholder's vote share should be $\ln\left(\frac{p}{1-p}\right)$. Given that each of the shareholders has a single vote, the blockholder should have $\frac{\ln q - \ln(1-q)}{\ln p - \ln(1-p)}$ votes.*

We next examine equilibrium behavior, demonstrating that the equilibria we describe are indeed equilibria.

Lemma 3 *There is always an equilibrium where the blockholder votes sincerely with* $\min\{2b^*, 2b\}$ *votes and all shareholders vote sincerely.*

Proof. First, suppose that $b > b^*$, and consider the blockholder's problem when all informed shareholders are voting sincerely. Then, since the blockholder has aligned preferences with all other shareholders, the blockholder's problem, given the behavior of all other voters, is analogous to a planner's problem—it is immediate following Lemma 2 that sincere voting according to the vote share b^* is optimal. An identical argument suggests that in this case, it is optimal for an informed shareholder to vote sincerely in this situation. Essentially, this is the result of McLennan (1998) applied in this context.

Next, suppose that $b^* > b$ and the blockholder and all other shareholders vote sincerely. First, we consider the behavior of the blockholder. Again, Lemma 2 suggests that if it were feasible, the blockholder would want to vote sincerely with more votes than he has available. It is intuitive and simple to show that in this case, he would vote sincerely with as many votes as available.

Finally, consider the behavior of shareholders when $b^* > b$ and everyone votes sincerely. The intuition here is that the circumstances where an informed shareholder is pivotal are circumstances that primarily reflect the signals of other informed shareholders rather than the more informed signal of the blockholder. In this case, the additional information conveyed by a sincere vote by the informed shareholder leads to a more accurate decision and a more efficient outcome.

Formally, note that given the symmetry in the problem, the probability of a shareholder being pivotal when the state is A is the same as the probability of a shareholder being pivotal when the state is M. We write this as $\pi_S(piv) = \pi_S(piv \mid M) = \pi_S(piv \mid A)$. It follows that the expected utility of a shareholder voting her share for choice M over abstaining (which we denote by $\emptyset$) when observing a signal m is:

$$\begin{aligned} EU_S(M, \emptyset | m) &= p\pi_S(piv \mid M) - [\frac{1}{2}p\pi_S(piv \mid M) + \frac{1}{2}(1-p)\pi_S(piv \mid A)] \\ &= p\pi_S(piv \mid M) - \frac{1}{2}\pi_S(piv) \\ &> 0 \end{aligned} \tag{3}$$

where the RHS of the first line follows on noting: the first term represents the situtation where if the shareholder is pivotal and votes for M, the decision matches the state of the world (and the shareholder earns 1 rather than 0) only if the state is indeed M; given that the shareholder observes m, this occurs with probability p. The term in square brackets is the expected value of abstaining, noting that in this case there will be a tie, so the action is equally likely to be A or M. The second line follows on noting $\pi_S(piv) = \pi_S(piv \mid M) = \pi_S(piv \mid A)$.

Since $p > \frac{1}{2}$, the inequality holds; the shareholder prefers to vote for M rather than abstain when all other shareholders and the blockholder are voting sincerely. Similarly, the expected utility of a shareholder who observes an a signal of voting her share for choice A over abstaining is positive.

The benefit of voting against her signal rather than with it when observing a signal m is given by:

$$EU_S(A, M|m) = (1-p)\pi_S(piv \mid A) - \frac{1}{2}\pi_S(piv) < 0. \tag{4}$$

It follows that it is strictly optimal for the shareholder to vote with her signal. ■

We must also look at the case where shareholders abstain.

Lemma 4 *There is always an equilibrium where the blockholder votes sincerely with all of its votes (or* 2 *or more votes) and all shareholders abstain.*

Proof. It is immediately clear that if shareholders do not vote, then the blockholder votes sincerely (and is indifferent regarding the number of votes with which he does so). If the blockholder votes sincerely with 2 or more votes and all shareholders abstain, then a single shareholder can never be pivotal, so abstaining is a best response. ■

Taken together, Lemmas 3 and 4 establish that there is always a multiplicity of equilibria. Given the observation above, it is clear that they cannot be ranked unambiguously. Instead, their relative efficiency depends on parameter values.

The first statement in the Proposition is immediate, given the arguments in the text and above.

Note that an equilibrium in which two shareholders abstain is outcome equivalent to an equilibrium where one shareholder disregards its information and votes for M while another shareholder disregards its information and votes for A. Therefore, any equilibrium with shareholders abstaining implies that there are other equilibria with an identical outcome where shareholders cancel each others' votes out. Therefore, the candidate equilibrium where only the blockholder votes is outcome equivalent to many other equilibria.

When $b < b^*$, we have found two classes of equilibria that are candidates for being the most informative equilibrium: one which is outcome equivalent to only the blockholder voting, and the other where the blockholder votes all $2b$ shares and all shareholders vote sincerely. The first question we must answer is whether there are any more candidate equilibria to consider.

Other potential equilibria:

Consider a possible equilibrium where the blockholder votes z_B shares sincerely, z_S shareholders vote sincerely, and the remaining shareholders abstain, where $z_B < z_S < 2n+1$. From our result in Lemma 1 (which we prove in the next subsection), if this equilibrium is more informative than the one where only the blockholder votes, it will be less informative than one where all shareholders vote sincerely. This implies that it is less informative than one of the two candidates we have found. If any of the shareholders who abstain in this possible equilibrium would instead vote against their signal or vote for one alternative irrespective of their signal, it would be less informative.

The only possible candidates left are an equilibrium taking the form of (i) the blockholder sincerely voting z_B shares and less than z_B shareholders vote (sincerely or otherwise) or (ii) shareholders vote regardless of their signal $z > 2b$ net[31] shares for proposal $J = \{M, A\}$, and less than $z - 2b$ other shares from shareholders are cast sincerely. Equilibria that take the form of (i) are informationally equivalent to the equilibrium where only the blockholder votes, so we will select that equilibrium. Equilibria that take the form of (ii) are informationally inferior to the equilibrium where the blockholder votes $2b$ shares sincerely and all shareholders vote their shares sincerely. Therefore, the two candidate equilibria we summarized initially are the only two candidates for the most informative equilibrium.

Comparing the two equilibria:

An equilibrium where only the blockholder votes will lead to an outcome that matches the state with a probability (and delivers the expected utility) of q. An equilibrium where the blockholder votes all of its shares and all shareholders vote sincerely matches the state with probability:

$$q \sum_{i=n-b+1}^{2n+1} \binom{2n+1}{i} p^i (1-p)^{2n+1-i} + (1-q) \sum_{i=n+b+1}^{2n+1} \binom{2n+1}{i} p^i (1-p)^{2n+1-i} \tag{5}$$

This expression corresponds to $n+b+1$ or more votes cast in favor of the true underlying

[31] Net means that if x shareholders are voting for M regardless of their signal and y shareholders are voting for A regardless of their signal, $z = x - y > 0$ are voting for M regardless of their signal.

state. The first expression corresponds to the blockholder's signal matching the state (with probability q), with the summation indicating that $n-b+1$ or more of the $2n+1$ shareholders having signals that match the state. The second expression instead represents the likelihood that the blockholder's signal does not match the state but that $n+b+1$ or more of the shareholders have signals that do.

We now compare the probability with which each equilibrium matches the state. All shareholders voting is a more informationally efficient equilibrium when:

$$q\sum_{i=n-b+1}^{2n+1}\binom{2n+1}{i}p^i(1-p)^{2n+1-i}+(1-q)\sum_{i=n+b+1}^{2n+1}\binom{2n+1}{i}p^i(1-p)^{2n+1-i}>q$$

$$\begin{aligned}(1-q)\sum_{i=n+b+1}^{2n+1}\binom{2n+1}{i}p^i(1-p)^{2n+1-i} &> q\sum_{i=0}^{n-b}\binom{2n+1}{i}p^i(1-p)^{2n+1-i}\\ \frac{\sum_{i=n+b+1}^{2n+1}\binom{2n+1}{i}\left(\frac{p}{1-p}\right)^i}{\sum_{i=0}^{n-b}\binom{2n+1}{i}\left(\frac{p}{1-p}\right)^i} &> \frac{q}{1-q}.\end{aligned}$$

This is condition C1 in the text.

9.2 Proof of Lemma 1

Lemma 1 *When $b<b^*$, the most informative equilibrium involves shareholders voting sincerely the (i) higher p is, (ii) the higher n is, (iii) the higher b is and (iv) the lower q is.*
Proof. It is convenient to simplify expressions slightly by introducing the notation $r:=\frac{p}{1-p}$. Thus, when $b<b^*$, the most informative equilibrium involves informed shareholders voting if:

$$\frac{\sum_{i=n+b+1}^{2n+1}\binom{2n+1}{i}r^i}{\sum_{i=0}^{n-b}\binom{2n+1}{i}r^i}>\frac{q}{1-q}.$$

(i) Consider

$$\frac{d}{dr}\frac{\sum_{i=n+b+1}^{2n+1}\binom{2n+1}{i}r^i}{\sum_{i=0}^{n-b}\binom{2n+1}{i}r^i}$$
$$= \frac{\sum_{i=0}^{n-b}\binom{2n+1}{i}r^i\sum_{i=n+b+1}^{2n+1}\binom{2n+1}{i}ir^{i-1}-\sum_{i=n+b+1}^{2n+1}\binom{2n+1}{i}r^i\sum_{i=0}^{n-b}\binom{2n+1}{i}ir^{i-1}}{(\sum_{i=0}^{n-b}\binom{2n+1}{i}r^i)^2}$$

.

The denominator is positive, but the numerator is positive if and only if:

$$\frac{\sum_{i=n+b+1}^{2n+1}\binom{2n+1}{i}ir^i}{\sum_{i=n+b+1}^{2n+1}\binom{2n+1}{i}r^i} > \frac{\sum_{i=0}^{n-b}\binom{2n+1}{i}ir^i}{\sum_{i=0}^{n-b}\binom{2n+1}{i}r^i}.$$

Note that $\sum_{i=n+b+1}^{2n+1}\binom{2n+1}{i}ir^i > (n+b+1)\sum_{i=n+b+1}^{2n+1}\binom{2n+1}{i}r^i$ and $\sum_{i=0}^{n-b}\binom{2n+1}{i}ir^i < (n-b)\sum_{i=0}^{n-b}\binom{2n+1}{i}r^i$.

Given that $\frac{(n+b+1)\sum_{i=n+b+1}^{2n+1}\binom{2n+1}{i}r^i}{\sum_{i=n+b+1}^{2n+1}\binom{2n+1}{i}r^i} > \frac{(n-b)\sum_{i=0}^{n-b}\binom{2n+1}{i}r^i}{\sum_{i=0}^{n-b}\binom{2n+1}{i}r^i}$ reduces to $n+b+1 > n-b$, which is certainly true, the result follows.

(ii) Define the probability of getting the decision correct when all of the shareholders vote sincerely as.

$$F(n) = q\sum_{i=n-b+1}^{2n+1}\binom{2n+1}{i}p^i(1-p)^{2n+1-i} + (1-q)\sum_{i=n+b+1}^{2n+1}\binom{2n+1}{i}p^i(1-p)^{2n+1-i}.$$

Given that q is the probability of getting the decision correct when only the blockholder votes, we want to show that when $F(n) \geq q$, $F(n+1) > q$ to prove the result. We start by writing out $F(n+1)$.

$$\begin{aligned} F(n+1) &= q\sum_{i=n-b+2}^{2n+3}\binom{2n+3}{i}p^i(1-p)^{2n+3-i} \\ &\quad +(1-q)\sum_{i=n+b+2}^{2n+3}\binom{2n+3}{i}p^i(1-p)^{2n+3-i} \end{aligned}$$

We can write the first term of $F(n+1)$ as:

$$
\begin{aligned}
& \sum_{i=n-b+2}^{2n+3} \binom{2n+3}{i} p^i (1-p)^{2n+3-i} \\
= \; & \begin{array}{c} p^2 \sum_{i=n-b}^{2n+1} \binom{2n+1}{i} p^i (1-p)^{2n+1-i} + 2p(1-p) \sum_{i=n-b+1}^{2n+1} \binom{2n+1}{i} p^i (1-p)^{2n+1-i} \\ +(1-p)^2 \sum_{i=n-b+2}^{2n+1} \binom{2n+1}{i} p^i (1-p)^{2n+1-i} \end{array} \\
= \; & \begin{array}{c} \sum_{i=n-b+1}^{2n+1} \binom{2n+1}{i} p^i (1-p)^{2n+1-i} + p^2 \binom{2n+1}{n-b} p^{n-b} (1-p)^{n+1+b} \\ -(1-p)^2 \binom{2n+1}{n-b+1} p^{n-b+1} (1-p)^{n+b} \end{array} \\
= \; & \begin{array}{c} \sum_{i=n-b+1}^{2n+1} \binom{2n+1}{i} p^i (1-p)^{2n+1-i} \\ + \left[\binom{2n+1}{n-b} p - \binom{2n+1}{n-b+1} (1-p) \right] p^{n-b+1} (1-p)^{n+b+1} \end{array} \\
= \; & \begin{array}{c} \sum_{i=n-b+1}^{2n+1} \binom{2n+1}{i} p^i (1-p)^{2n+1-i} \\ + \frac{(2n+1)!}{(n-b)!(n+b)!} p^{n-b+1} (1-p)^{n+b+1} \left[\frac{p}{n+b+1} - \frac{1-p}{n-b+1} \right] \end{array} \\
= \; & \begin{array}{c} \sum_{i=n-b+1}^{2n+1} \binom{2n+1}{i} p^i (1-p)^{2n+1-i} \\ + \frac{(2p-1)(n+1)-b}{(n+1-b)(b+n+1)} \frac{(2n+1)!}{(n-b)!(n+b)!} p^{n-b+1} (1-p)^{n+b+1} \end{array} .
\end{aligned}
$$

This implies that the second term of $F(n+1)$ can be simplified as follows:

$$
\sum_{i=n+b+2}^{2n+3} \binom{2n+3}{i} p^i (1-p)^{2n+3-i} = \begin{array}{c} \sum_{i=n+b+1}^{2n+1} \binom{2n+1}{i} p^i (1-p)^{2n+1-i} + \\ \frac{(2p-1)(n+1)+b}{(n+1-b)(b+n+1)} \frac{(2n+1)!}{(n-b)!(n+b)!} p^{n+b+1} (1-p)^{n-b+1} \end{array} .
$$

Therefore:

$$
\begin{aligned}
F(n+1) \; = \; & \begin{array}{c} q \sum_{i=n-b+1}^{2n+1} \binom{2n+1}{i} p^i (1-p)^{2n+1-i} + (1-q) \sum_{i=n+b+1}^{2n+1} \binom{2n+1}{i} p^i (1-p)^{2n+1-i} \\ +q \frac{(2p-1)(n+1)-b}{(n+1-b)(b+n+1)} \frac{(2n+1)!}{(n-b)!(n+b)!} p^{n-b+1} (1-p)^{n+b+1} \\ +(1-q) \frac{(2p-1)(n+1)+b}{(n+1-b)(b+n+1)} \frac{(2n+1)!}{(n-b)!(n+b)!} p^{n+b+1} (1-p)^{n-b+1} \end{array} \\
= \; & \begin{array}{l} F(n) + q \frac{(2p-1)(n+1)-b}{(n+1-b)(b+n+1)} \frac{(2n+1)!}{(n-b)!(n+b)!} p^{n-b+1} (1-p)^{n+b+1} \\ +(1-q) \frac{(2p-1)(n+1)+b}{(n+1-b)(b+n+1)} \frac{(2n+1)!}{(n-b)!(n+b)!} p^{n+b+1} (1-p)^{n-b+1} \end{array} .
\end{aligned}
$$

Note that this is greater than $F(n)$ as long as the following holds.

$$
\begin{array}{c} q \frac{(2p-1)(n+1)-b}{(n+1-b)(b+n+1)} \frac{(2n+1)!}{(n-b)!(n+b)!} p^{n-b+1} (1-p)^{n+b+1} \\ +(1-q) \frac{(2p-1)(n+1)+b}{(n+1-b)(b+n+1)} \frac{(2n+1)!}{(n-b)!(n+b)!} p^{n+b+1} (1-p)^{n-b+1} \end{array} > 0
$$

This can be simplified to:

$$\frac{(2p-1)(n+1)-b}{(2p-1)(n+1)+b} + (\frac{p}{1-p})^{2b} \frac{1-q}{q} > 0$$

Note that this expression is increasing in n. Therefore, if there exists an n for which $F(n-1) < q$ and $F(n) > q$, this will imply that $F(m) > q$ for all $m > n$, which is what we need for our result. We now demonstrate that there must exist such an n.

The function $F(n)$ is well defined given our assumption that $2b < 2n+1$, and the minimum value n can take is b. At $n = b$, $F(n) = q \sum_{i=1}^{2b+1} \binom{2b+1}{i} p^i (1-p)^{2b+1-i}$, which is smaller than q. So it is not possible that $F(n) > q$ for all n. Furthermore, it is not possible that $F(n) < q$ for all n, since as n approaches infinity, $F(n) > q$. This is true, since Condorcet's Jury Theorem states that in our model, if $q = p$, $F(n) \to 1$ as $n \to \infty$, it cannot be smaller for $q > p$.[32] We have now demonstrated that there must exist an n for which $F(n-1) < q$ and $F(n) > q$.

(iii) The term $\frac{\sum_{i=n-2b+1}^{n+2b} \binom{2n+1}{i} r^i}{\sum_{i=0}^{n-2b} \binom{2n+1}{i} r^i}$ is increasing in b iff:

$$\frac{\sum_{i=n-2b-1}^{n+2b+2} \binom{2n+1}{i} r^i}{\sum_{i=0}^{n-2b-2} \binom{2n+1}{i} r^i} > \frac{\sum_{i=n-2b+1}^{n+2b} \binom{2n+1}{i} r^i}{\sum_{i=0}^{n-2b} \binom{2n+1}{i} r^i}$$

This holds iff:

$$\sum_{i=n-2b-1}^{n+2b+2} \sum_{j=0}^{n-2b} \binom{2n+1}{i} \binom{2n+1}{j} r^{i+j} > \sum_{i=n-2b+1}^{n+2b} \sum_{j=0}^{n-2b-2} \binom{2n+1}{i} \binom{2n+1}{j} r^{i+j}$$

This is immediate - the lower bound of the summation on the left hand side is lower than on the right hand side, while the upper bounds of the summation on the left hand side are greater than on the right hand side. Otherwise, all of the terms are identical and all are positive.

(iv) is immediate. ■

[32] One proof of Condorcet's Jury Theorem is in Ladha (1992).

9.3 Proof of Proposition 2 (Active Blockholder voting)

Proposition 2 *Given an active blockholder, the unique most informative equilibrium is where the blockholder votes sincerely with* $\min[2b, 2b^*]$ *shares, and*

(i) if $b \geq b^*$*, all shareholders vote sincerely, or*

(ii) if $b < b^*$*,* $2b^* - 2b$ *shareholders vote in an identical fashion to the blockholder, and the rest of the shareholders vote sincerely.*

Proof. We begin by proving that the equilibrium type (i) and equilibrium type (ii) exist. We then prove they are the most informative equilibria for given parameters.

Equilibrium type (i): If $b \geq b^*$, we need to check that there is an equilibrium where the blockholder votes sincerely with $2b^*$ shares and all shareholders vote sincerely. Because this is sequential, we first take as given that the blockholder voted $2b^*$ shares for choice X.

An individual shareholder's vote is consequential when the vote is otherwise tied. This may arise when the blockholder's signal is accurate and occurs with probability $q\frac{2n!}{(n-b^*)!(n+b^*)!}p^{n-b^*}(1-p)^{n+b^*}$ or when the blockholder's signal is inaccurate and occurs with probability $(1-q)\frac{2n!}{(n+b^*)!(n-b^*)!}p^{n+b^*}(1-p)^{n-b^*}$.

It follows that the shareholder prefers voting sincerely in opposition to the blockholder rather than voting in line with the blockholder in opposition to its signal when:

$$p\left[(1-q)\frac{2n!}{(n+b^*)!(n-b^*)!}p^{n+b^*}(1-p)^{n-b^*}\right] \geq (1-p)\left[q\frac{2n!}{(n-b^*)!(n+b^*)!}p^{n-b^*}(1-p)^{n+b^*}\right],$$

or, equivalently $\left(\frac{p}{1-p}\right)^{2b^*+1} > \frac{q}{1-q.}$, which is true from the definition of b^*, in Equation (1).[33]

Lastly, given the subsequent behavior of shareholders, the blockholder prefers to vote sincerely with $2b^*$ shares, as this maximizes the common payoff of the game (as in Nitzan and Paroush (1982)).

Equilibrium type (ii): We must prove two things.

[33] It is simple to show that this also implies that the shareholder prefers voting sincerely to abstention. Similarly, this will be the case for related arguments in this proof.

Of course, this analysis depends on inferences that shareholders draw from the blockholder's voting behavior. There are many off-equilibrium beliefs that would support this behavior. For example, it is sufficient, here and below, to assume that shareholders suppose that the blockholder's signal reflects its net votes, and beliefs are passive (i.e., the blockholder's signal is equally likely to be of either type) in case of a tie (for example, if the blockholder abstains on all votes).

First, we must demonstrate that if $2b^* - 2b$ other shareholders are voting with the blockholder and all other shareholders are voting sincerely, then a shareholder will vote sincerely.

Again, to establish this, it is useful to write down the probability that the vote is tied and the blockholder is correct. This involves $n + b$ votes with the blockholder and $n + b$ against. The latter must come from those who vote sincerely, and there are $2n - (2b^* - 2b)$ of these.[34] Thus, this probability is $q \frac{2n+2b-2b^*}{(n+b-2b^*)(n+b)} p^{n+b-2b^*}(1-p)^{n+b}$. Similarly, the probability that the vote is tied and the blockholder is wrong is $(1-q)\frac{2n+2b-2b^*}{(n+b)(n+b-2b^*)} p^{n+b}(1-p)^{n+b-2b^*}$. It follows that the shareholder prefers voting sincerely in opposition to the blockholder to voting in line with the blockholder when $p\left[(1-q)\frac{2n+2b-2b^*}{(n+b)(n+b-2b^*)} p^{n+b}(1-p)^{n+b-2b^*}\right] \geq (1-p)\left[q\frac{2n+2b-2b^*}{(n+b-2b^*)(n+b)} p^{n+b-2b^*}(1-p)^{n+b}\right]$ or, equivalently, $\left(\frac{p}{1-p}\right)^{2b^*+1} \geq \frac{q}{1-q}$, which follows from (1).

Second, we must demonstrate that if $2b^* - 2b - 1$ other shareholders are voting with the blockholder and all other shareholders are voting sincerely, then a shareholder will vote with the blockholder. Once again, we can write the probability that the vote is tied without this shareholder and the blockholder is correct. Here again, this requires $n + b$ shareholders who vote sincerely to vote in opposition to the blockholder, but in this case, there are $2n - (2b^* - 2b) + 1$ of these. This allows us to write this probability as $q\frac{2n+2b-2b^*+1}{(n+b-2b^*+1)(n+b)} p^{n+b-2b^*+1}(1-p)^{n+b}$. Similarly, the probability that the vote is tied and the blockholder is wrong is $(1-q)\frac{2n+2b-2b^*+1}{(n+b)(n+b-2b^*+1)} p^{n+b}(1-p)^{n+b-2b^*+1}$. It follows that the shareholder prefers voting in line with the blockholder over voting sincerely $(1-p)\left[q\frac{2n+2b-2b^*+1}{(n+b-2b^*+1)(n+b)} p^{n+b-2b^*+1}(1-p)^{n+b}\right] \geq p\left[(1-q)\frac{2n+2b-2b^*+1}{(n+b)(n+b-2b^*+1)} p^{n+b}(1-p)^{n+b-2b^*+1}\right]$, or, equivalently, $\frac{q}{1-q} \geq \left(\frac{p}{1-p}\right)^{2b^*}$. This again follows from the definition of b^* and the equivalent condition (1).

Lastly, given the subsequent behavior of shareholders, the blockholder prefers to vote sincerely its $2b$ shares, as this maximizes the common payoff of the game (as in Nitzan and Paroush (1982)).

Unique most informative equilibrium:

When $b > b^*$, then following the arguments in the proof of Proposition 1, it is immediate that the equilibrium in Proposition 2 is the most informative equilibrium.

When $b < b^*$, the arguments above about Equilibrium type (ii) make it clear that there

[34] Here, we suppose that $n > b^* - b$ for convenience, the case where $n \leq b^* - b$ involves all agents voting with the blockholder, and it can be established by similar arguments.

is no equilibrium where the blockholder votes all of his shares sincerely, fewer than $2b^* - 2b$ shareholders ignore their signal and vote with the blockholder, and the rest of the shareholders vote sincerely (since a shareholder who is voting sincerely would prefer to switch to vote with the blockholder).

Finally, we rule out conjectured equilibria where the blockholder does not vote sincerely or abstains. Imagine a conjectured equilibrium where the blockholder abstains and all other shareholders vote sincerely. First, this cannot be an equilibrium, as the blockholder would deviate. Second, while this is the most informative possible equilibrium where the blockholder does not vote sincerely or abstains, it is less informative than the equilibrium in Proposition 2 part (ii) because of the strength of the blockholder's information.[35]

The only possible candidates left are an equilibrium that takes the form of (a) the blockholder voting sincerely z shares and less than z shareholders vote (sincerely or otherwise) or (b) shareholders voting regardless of their signal $z > 2b$ net[36] shares for proposal $J = \{M, A\}$ and less than $z - 2b$ other shares are cast sincerely. Equilibria that take the form of (a) are informationally equivalent to the equilibrium where only the blockholder votes. While in the passive blockholder there were conditions under which this was the most informative equilibrium, when there is an active blockholder, this is not the case. This is because here, the blockholder's information will be reflected in $2b^*$ votes despite the fact that he only has $2b$ votes, e.g., when q approaches 1, all of the shareholders will be voting with the blockholder.

Equilibria that take the form of (b) are informationally inferior to the equilibrium in Proposition 2 part (ii). Therefore, the equilibrium in Proposition 2 part (ii) is the most informative equilibrium. ■

[35] We have ruled out mixed strategies by Assumption A3; however, allowing them would not change the result. Here, since there is a coordination problem with which shareholders will support the blockholder, there exist equilibria with mixed strategies. Given that these can lead to coordination failure, they will be less informative than the pure strategy equilibrium in Proposition 2 part (ii).

[36] Net means that if x shareholders are voting for M regardless of their signal and y shareholders are voting A regardless of their signal, $z = x - y > 0$ are voting for M regardless of their signal

9.4 Proof of Proposition 4 (Result on information acquisition and trading)

Proposition 4 *There are parameters for which an equilibrium exists in the game defined above where the blockholder (i) improves her information to precision q, (ii) does not trade any shares ($2\widehat{b} = 2b$) shares, and (iii) votes $2b^* < 2b$ shares.*

Proof. We specify the equilibrium strategies for the proposition: in stage 2, the blockholder invests in acquiring information; in stage 3, there is no trade and any attempt to trade will lead shareholders to believe that the blockholder made no investment in stage 2; in stage 4, the blockholder announces sincerely her voting intentions (to vote in line with her information and what the information is); in stage 5, shareholders vote with the belief that any stage 4 announcement about voting intention is truthful (and the equilibrium belief that if there is no trade the blockholder has acquired information and will vote with $2b^*$ shares, but otherwise that the blockholder has acquired no information and will not vote) and respond with behavior that would lead to the most informationally efficient equilibrium given these beliefs.

We proceed by working backwards.

Tautologically, there is nothing to characterize in stage 4, since this is a common interest game, and given the shareholder behavior in stage 5, the blockholder will be sincere and will vote the appropriate number of shares. In particular, if the blockholder had acquired information, she would, indeed, vote $2b^*$ shares in line with her information; in this case, shareholders would simply vote sincerely. In case the blockholder has made no investment, she is indeed indifferent to not voting. In both cases, it is an equilibrium for all shareholders to vote sincerely.

Period 3: In period 3, each shareholder i has a belief μ_3^i regarding the probability that the blockholder has made the investment c in precision q (and this belief might depend on what is offered at this round). Given the proposed equilibrium, beliefs are such that all shareholders believe that the blockholder invested in precision q with probability 1 (i.e., $\mu_3^i = 1$ for all i) when the blockholder does not try to buy or sell shares. Otherwise, off-the-equilibrium-path, we assume that shareholders believe the blockholder did not invest with probability 1 (i.e., $\mu_3^i = 0$ for all i).

The payoff on-the-equilibrium-path for the informed blockholder is:

$$2b[q\sum_{i=n-b^*+1}^{2n+1}\binom{2n+1}{i}p^i(1-p)^{2n+1-i}+(1-q)\sum_{i=n+b^*+1}^{2n+1}\binom{2n+1}{i}p^i(1-p)^{2n+1-i}] \tag{6}$$

This represents the value the blockholder receives from keeping all of its shares and voting $2b^*$ of them.[37] The payoff to the informed blockholder from trying to sell $2x \leq 2b - 2b^*$ shares to new shareholders off-the-equilibrium-path is:

$$\begin{aligned}
&(2b-2x)[q\sum_{i=n+x-b^*+1}^{2n+2x+1}\binom{2n+2x+1}{i}p^i(1-p)^{2n+2x+1-i} \qquad (7)\\
&+(1-q)\sum_{i=n+x+b^*+1}^{2n+2x+1}\binom{2n+2x+1}{i}p^i(1-p)^{2n+2x+1-i}]\\
&+2x\sum_{i=n+x+1}^{2n+2x+1}\binom{2n+2x+1}{i}p^i(1-p)^{2n+2x+1-i}
\end{aligned}$$

The first and second lines represent the value of the shares that the blockholder holds after trade occurs, and reflects that the probability that the vote matches the state. This probability incorporates the actual quality of the blockholder's signal and the behavior of the other shareholders (who, even though they believe that the blockholder has not invested, will all continue to vote sincerely). The value of these retained shares increases relative to the on-path case of no trading, since the probability that the vote matches the state incorporates the information of new shareholders. The third line represents the amount that the blockholder can sell its shares for. This amount represents the probability that new shareholders believe the vote will match the state given that they believe the blockholder has not invested.

Note that the blockholder would not buy shares from existing shareholders, whatever their beliefs were. Such a purchase reduces value since the blockholder would be effectively reducing the amount of information in the vote. The payoff of the shareholder from retaining

[37] Note that this is lower than $2bv_I$ (where v_I is given in Equation (2)) since the blockholder abstains with $2(b-b^*)$ shares rather than selling them to shareholders who could vote with them and incorporate their information. This is reflected in the different limits to the summation in this expression as compared to Equation (2).

her share is weakly larger (strictly if the blockholder invested in information) than the payoff of the share when it is sold to the blockholder, and therefore, an existing shareholder would refuse to sell at the blockholder's maximum offer.

Thus, the first requirement for the proposed equilibrium to exist would be to prove that the blockholder can lose out by deviating to equation (7), for all x, for some parameters. Below, we numerically show this is true for some parameters.

Now, we must specify what a blockholder who did not invest in information will do and characterize her payoffs. There are gains from trade here, as the blockholder can sell shares that will not aggregate information into the vote to new shareholders who will provide useful information for the vote. This blockholder will sell all of its shares. The blockholder makes strictly positive profit by selling all shares except for the last one, on which she is indifferent between selling and not (hence, we assume she sells). Her payoff is:

$$2b \sum_{i=n+b+1}^{2n+2b+1} \binom{2n+2b+1}{i} p^i (1-p)^{2n+2b+1-i} \tag{8}$$

Period 2: Given that there is no trade in period 3 if the blockholder invests in information, the blockholder must decide whether such an investment is worthwhile in period 2. The blockholder invests in information if:

$$(6) - c \geq (8) \tag{9}$$

Numerical simulation: There are two conditions that must hold in order for the specified equilibrium to actually be an equilibrium. The first is demonstrating that equation (6) is larger than equation (7) for any $2x$ shares less than $2b - 2b^*(q)$. The second is that equation (9) holds. We provide an example where both of these conditions have been satisfied, which we checked with numerical simulation. Setting $p = 0.51$, $q = 0.6$, $b = 6$, and $n = 11$ satisfies both conditions when the cost c is low enough (e.g., any $c \leq 5$ works). With this simulation, $b^* = 5$, so there was only one value for sharetrading to check ($x = 1$) for the first condition to hold. ∎

9.5 Proof of Proposition 5 (Biased passive blockholder)

Proposition 5 *In the case of a biased passive blockholder, the most informative equilibrium with no abstention involves the blockholder voting for M with all of its $2b$ shares, $2b$ of the shareholders voting for A independently of their signals, and the remaining $2n+1-2b$ shareholders voting sincerely.*

We begin with a lemma that describes strategies that are not optimal for shareholders. This will allow us to characterize strategies for the shareholders as weakly monotone, i.e., a shareholder has weakly monotone strategies when she is at least as likely to vote A when receiving the a-signal as when receiving the m-signal (and similarly for voting M).

Lemma 5 *Shareholder strategies are weakly monotone.*

Proof. We begin by proving that a shareholder would never vote anti-sincerely, i.e., voting M when she gets an a signal and voting A when she gets an m signal. Suppose, for contradiction, that there is a shareholder who votes anti-sincerely. Taking all other shareholders' strategies as given, we define several relevant pivotal probabilities as follows: Let $\pi^N(A)$ denote the probability that without this voter, the vote is split and the true state is A; let $\pi^M(A)$ denote the probability that without this voter, the vote is in favor of M by one vote and the true state is A; and let $\pi^A(A)$ denote the probability that without this voter, the vote is in favor of A by one vote and the true state is A. We can define $\pi^N(M)$, $\pi^A(M)$, $\pi^M(M)$ similarly.

It follows that we can write down the expected probability of getting the decision correct when this last voter votes anti-sincerely. This is:

$$\Pr(non\_piv) + \frac{1}{2}\left[(1-p)(\pi^N(A) + \pi^A(A) + \frac{1}{2}\pi^M(A)) + p\frac{1}{2}\pi^A(A)\right] \tag{10}$$
$$+\frac{1}{2}\left[(1-p)(\pi^N(M) + \pi^M(M) + \frac{1}{2}\pi^A(M)) + p\frac{1}{2}\pi^M(M)\right],$$

where $\Pr(non\_piv)$ denotes the probability of getting the decision correct when the voter is not pivotal. The first square bracket corresponds to the true state being A. In this case, the voter may observe the signal m with probability $(1-p)$, which, given that the voter votes anti-sincerely, leads him to vote for A. In this case, the shareholder chooses the right action when he is pivotal in the event of a tie or the vote is in favor of A by one vote. The vote also leads to a tie and choosing the right action with probability $\frac{1}{2}$ in the case where the vote is in favor of M by one vote. The final term in the first set of square brackets corresponds to the agent getting the signal a and voting for M, which leads to a tie and choosing the right action with probability $\frac{1}{2}$ in the case where the vote was otherwise in favor of A by one vote. The second square bracket is the analogous expression for when the true state is M.

Similarly, if instead the shareholder voted sincerely, the probability that the action chosen would match the state would be:

$$\begin{aligned}&\Pr(non\_piv) + \frac{1}{2}p(\pi^N(A) + \pi^A(A) + \frac{1}{2}\pi^M(A)) + \frac{1}{2}(1-p)\frac{1}{2}\pi^A(A) \\ &+\frac{1}{2}p(\pi^N(M) + \pi^M(M) + \frac{1}{2}\pi^A(M)) + \frac{1}{2}(1-p)\frac{1}{2}\pi^M(M).\end{aligned} \tag{11}$$

Thus the optimality of anti-sincere behavior requires that (10)$\geq$(11), or, equivalently:

$$\begin{array}{c}\pi^N(A)(1-2p) + \pi^A(A)(1-2p-\frac{1-p}{2}) + \pi^M(A)\frac{1-2p}{2} \\ +\pi^N(M)(1-2p) + \pi^M(M)(1-2p-\frac{1-p}{2}) + \pi^A(M)\frac{1-2p}{2}\end{array} \geq 0.$$

This is clearly false upon noting that $1-p>0$, $1-2p<0$ and all of the pivotal probabilities (the πs) are non-negative. Therefore, this proves that a shareholder would not vote anti-sincerely.

Next, we rule out voting M in the case where the shareholder observes an a signal and abstaining when the shareholder observes an m signal. The probability of the vote matching the state correctly in this case is:

$$\begin{aligned}&\Pr(non\_piv) + \frac{p}{4}\pi^A(A) + \frac{1-p}{4}\pi^N(A) + \frac{1-p}{2}\pi^A(A) \\ &+\frac{1}{2}p(\frac{1}{2}\pi^N(M) + \pi^M(M)) + \frac{1}{2}(1-p)(\pi^N(M) + \pi^M(M) + \frac{1}{2}\pi^A(M)). \\ = \quad & \frac{2-p}{4}\pi^A(A) + \frac{1-p}{4}\pi^N(A) + \frac{2-p}{4}\pi^N(M) + \frac{1-p}{4}\pi^A(M) + \frac{1}{2}\pi^M(M)\end{aligned} \tag{12}$$

We define two other possible strategies to compare this with. The first is voting for M and ignoring the signal. The probability that the vote matches the state in this case is:

$$\Pr(non\_piv) + \frac{1}{2}(\pi^N(M) + \pi^M(M) + \frac{1}{2}\pi^A(M)) + \frac{1}{4}\pi^A(A) \tag{13}$$

The second is abstaining for both signals. The probability that the vote matches the state in this case is:

$$\Pr(non\_piv) + \frac{1}{2}(\frac{1}{2}\pi^N(A) + \pi^A(A)) + \frac{1}{2}(\frac{1}{2}\pi^N(M) + \pi^M(M)). \tag{14}$$

The strategy we are considering is better than always voting M when equation (12) is larger than equation (13). Simplifying this relation gives us:

$$(1-p)(\pi^A(A) + \pi^N(A)) > p(\pi^N(M) + \pi^A(M)) \tag{15}$$

The strategy we are considering is better than always abstaining when equation (12) is larger than equation (14). Simplifying this relation gives us:

$$(1-p)(\pi^N(M) + \pi^A(M)) > p(\pi^A(A) + \pi^N(A)) \tag{16}$$

Adding these two conditions (equations (15) and (16)) yields:

$$(1-p)(\pi^N(M) + \pi^A(M) + \pi^A(A) + \pi^N(A)) > p(\pi^N(M) + \pi^A(M) + \pi^A(A) + \pi^N(A))$$

This cannot be true, as the pivotal probabilities are weakly positive and $1-p < p$. This implies that at least one of the strategies considered dominates the strategy of voting M in the case where the shareholder observes an a signal and abstaining when the shareholder observes an m signal.

Given symmetry, the case that the shareholder strategy of voting A when an m signal is observed and abstaining when an a signal is observed can be proved not to be optimal in an identical manner. ■

Lemma 5 states that strategies are weakly monotone: that is, a shareholder is at least as likely to vote A when receiving the a-signal as when receiving the m-signal (and similarly for voting M). Following Proposition 2 of Persico (2004), the equilibrium then involves some shareholders voting A independently of their signal and the remainder voting sincerely. It remains to characterize the number who do so. This is done in Lemma 6.

Lemma 6 *Suppose that a shareholders vote A and the rest vote sincerely, then: (i) if $a < 2b$, then the probability of the vote matching the state is higher with $a+1$ voting against and* *the rest sincere than with a voting against and the rest sincere; and (ii) if $a > 2b+1$, then the probability of the vote matching the state is higher with $a-1$ voting against and the rest sincere than with a voting against and the rest sincere. (iii) $a = 2b$ or $2b+1$ give the same probability of the vote matching the state.*

Proof. (i) We write $R(a)$ to denote the probability that the decision matches the state when a shareholders vote A independently of their signals and the remainder vote sincerely, and we write $R(a, npiv)$ to denote the probability that the decision matches the state when the last shareholder among sincere voters is not pivotal when a shareholders vote A independently of their signals and the remainder vote sincerely.

Note that since no shareholders abstain, the only possibility of being pivotal requires that without the last shareholder, the votes for M and A are evenly split (that is, using the notation from Lemma 5, $\pi^M(A) = \pi^A(A) = \pi^M(M) = \pi^M(M) = 0$). Thus, we can write

$$R(a) = R(a, npiv) + \frac{1}{2}p\pi^N(A) + \frac{1}{2}p\pi^N(M).$$

Instead, switching the last shareholder to always vote for A independently of his signal implies that

$$R(a) = R(a, npiv) + \frac{1}{2}\pi^N(A).$$

It follows that switching the last shareholder from voting sincerely to always voting A is beneficial if:

$$\frac{1-p}{p}\pi^N(A) \geq \pi^N(M). \tag{17}$$

Next, we can write down the pivotal probabilities $\pi^N(A)$ and $\pi^N(M)$ for the case where a shareholders vote for A (and the blockholder uses its $2b$ shares to vote for M). Of the $2n-a$ shareholders who vote sincerely, it must be that $n+a-b$ vote for M for the vote to be tied without the last shareholder. This implies that $\pi^N(A) = \begin{pmatrix} 2n-a \\ n-b \end{pmatrix} (1-p)^{n-b}p^{n+b-a}$ and $\pi^N(M) = \begin{pmatrix} 2n-a \\ n-b \end{pmatrix} p^{n-b}(1-p)^{n+b-a}$. Substituting these expressions into (17) yields

$$p^{2b-a-1} \geq (1-p)^{2b-a-1};$$

since $p > 1-p$; this holds as long as $2b - a - 1 \geq 0$.

(ii) can be similarly established.

(iii) The condition that it is better to have $a-1$ voting sincerely than a voting sincerely iff $p^{2b-a-1} \geq (1-p)^{2b-a-1}$ holds also for the case of $a = 2b+1$, where this condition holds with equality, implying that the cases $a = 2b$ and $a = 2b+1$ are equivalent in terms of their probabilities of getting the decision right. ■

Finally, to complete the proof of Proposition 5, it remains to check that this is an equilibrium—since it is optimal in the relaxed problem (where incentive constraints do not have to be satisfied in Lemma 6.), it is clearly the optimal (most informative) equilibrium. This is immediate upon noting that the conditions in the proof of Lemma 6 that ensure optimality also ensure that no shareholder has a strict incentive to change their voting strategy. Clearly, for any strategy of the shareholders, the biased blockholder optimizes by voting all his shares for M.■

9.6 Proof of Example 1

In the absence of abstention, following Proposition 5, the most informative equilibrium involves two of the shareholders voting A independently of their signals and the third voting sincerely. This selects the correct action with probability p.

Suppose instead that all three shareholders abstain when observing an m-signal and vote A when observing the a-signal. This selects the correct action with probability $\frac{1}{2}(p^3 + 3p^2(1-p)\frac{1}{2}) + \frac{1}{2}(p^3 + 3p^2(1-p) + \frac{1}{2}3p(1-p^2))$. The first term corresponds to the true state being A. The vote will reflect this when all three shareholders have an accurate signal or if two have an accurate signal (and the other abstains) the vote will be for A with probability $\frac{1}{2}$. The second term corresponds to the true state being M. If all three shareholders have the correct signal, they all abstain, and the blockholder's votes will ensure that M is chosen. Similarly, if there is only one vote against, and if there are two votes against, M will be implemented with probability $\frac{1}{2}$. The overall expression can be written as $\frac{p\left(3-2p^2+3p\right)}{4}$, and this is strictly greater than p in the range $p > \frac{1}{2}$. Lastly, following McLennan (1998), if this is the most informative set of strategies from a planner's perspective, it must be an equilibrium. We also demonstrate that this equilibrium is the unique most informative equilibrium (available upon request).

9.7 Proof of Proposition 6 (Biased active blockholder)

Proposition 6 *In the game with a biased active blockholder, there is an equilibrium in which the blockholder always abstains and all shareholders vote sincerely. This is the most informative equilibrium.*

Proof. The fact that the equilibrium described in the proposition exists is immediate - it can be supported by the off-equilibrium belief that any blockholder who votes knows that the state is A.

We proceed by describing the set of equilibria that may arise. An optimal equilibrium aggregates information efficiently in the second stage and it is clear the proposed equilibrium does so.

First, separating among blockholder types (either full or semi-separating) cannot arise as part of an equilibrium:

Any action that is taken only by a blockholder who knows the state is A would lead shareholders to vote A independently of their signals (or take other behaviors that guaranteed the outcome A) and, since this is the worst possible outcome from her perspective, the blockholder would prefer to take another action. Similarly, any action that is taken only by a blockholder who knows the state is M would lead shareholders to vote M independently of their signals. This is the best possible outcome for the blockholder, and so such an action would be mimicked by both types.[38]

Thus, without loss, the most informative equilibrium involves the blockholder types pooling, and again following the Condorcet result, the best outcome among any such equilibria is the one described in the statement of this Proposition. ■

[38] Assumption A3 restricts the blockholder to pure strategies; however, allowing mixed strategies here would not affect the result.

Consider fully mixed equilibria, where both blockholder types mix between different actions. For this to arise in equilibria, it must be that the ultimate decision being M in state A is independent of the blockholder behavior. In the second period, this implies that the number of shareholders who vote A independently of their signal while the remainder vote sincerely is identical irrespective of the blockholder behavior. However, it is clear that this is most efficient when the number of shareholders who vote A is 0, following the Condorcet result, and so the outcome can be no better than in Proposition 5.