Currency Manipulation*

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Abstract

We propose a novel, risk-based transmission mechanism for the effects of currency manipulation: policies that systematically induce a country's currency to appreciate in bad times lower its risk premium in international markets and, as a result, lower the country's risk-free interest rate and increase domestic capital accumulation and wages. Currency manipulations by large countries also have external effects on foreign interest rates and capital accumulation. Applying this logic to policies that lower the variance of the bilateral exchange rate relative to some target country ("currency stabilization"), we find that a small economy stabilizing its exchange rate relative to a large economy increases domestic capital accumulation and wages. The size of this effect increases with the size of the target economy, offering a potential explanation why the vast majority of currency stabilizations in the data are to the U.S. dollar, the currency of the largest economy in the world. A large economy (such as China) stabilizing its exchange rate relative to a larger economy (such as the U.S.) diverts capital accumulation from the target country to itself, increasing domestic wages, while decreasing wages in the target country.

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Differences in real interest rates across developed economies are large and persistent; some countries have lower real interest rates than others for decades rather than years. These long-lasting differences in interest rates correlate with differences in capital-output ratios across countries, and account for the majority of excess returns on the carry trade, a trading strategy where international investors borrow in low interest rate currencies, such as the Japanese yen, and lend in high interest rate currencies, such as the New Zealand dollar (Lustig, Roussanov, and Verdelhan, 2011; Hassan and Mano, 2015; Hassan, Mertens, and Zhang, 2015).

A growing literature studying these "unconditional" differences in currency returns argues that they may be attributable to heterogeneity in the stochastic properties of exchange rates: Currencies with low interest rates pay lower returns because they tend to appreciate in bad times and depreciate in good times, providing a hedge to international investors and making them a safer investment (Lustig and Verdelhan, 2007; Menkhoff et al., 2013). This literature has explored various potential drivers of heterogeneity of the stochastic properties of countries' exchange rates, ranging from differences in country size (Martin, 2012; Hassan, 2013) and financial development (Maggiori, 2013) to trade centrality (Richmond, 2015) and differential resilience to disaster risk (Farhi and Gabaix, 2015).[1] The common theme across these papers is that whatever makes countries different from each other results in differential sensitivities of their exchange rates to various shocks, such that some currencies tend to appreciate systematically in "bad" states of the world (when the price of traded goods is high). Currencies with this property then pay lower expected returns and have lower risk-free interest rates.

In this paper, we argue that this risk-based view of differences in currency returns provides a novel way of thinking about the effects of currency manipulation: Interventions in currency markets that change the stochastic properties of exchange rates should also change the expected returns on currencies and other assets. In particular, policies that induce a country's currency to appreciate in bad times should lower domestic interest rates, lower the cost of capital for the production of nontraded goods, and, as a result, increase capital accumulation. Moreover, if these interventions are large enough, that is, if the country manipulating its exchange rate is large relative to the world, its policies will affect interest rates and capital accumulation in other countries, potentially diverting capital accumulation from other countries to itself. Policies that change the variances and covariances of exchange rates should thus, via their effect on interest

[1] Other papers in this literature have studied heterogeneity in the volatility of shocks affecting the nontraded sector (Tran, 2013), factor endowments (Ready, Roussanov, and Ward, 2013; Powers, 2015), risk aversion in combination with country size (Govillot, Rey, and Gourinchas, 2010), and differences in exposure to long-run risk (Colacito et al., 2016).

rates and asset returns, affect the allocation of capital across countries.

After making this argument in its most general form, we illustrate its implications with an application to currency stabilization. Table 1 shows that 88% of countries (representing 47% of world GDP) stabilize their currency relative to some target country (Reinhart and Rogoff, 2004). Such policies specify a target currency (two-thirds of them the U.S. dollar) and set an upper bound for the volatility of the real or nominal exchange rate relative to that target country. A conventional "hard" peg may set this volatility to zero, while "soft" pegs (including moving bands, crawling pegs, stabilized arrangements, and managed floats), may officially or unofficially specify a band of allowable fluctuations around some mean. The common feature of all of these policies is that they manipulate the variances and covariances of exchange rates by changing the states of the world and the extent to which they appreciate and depreciate, without necessarily manipulating the level of exchange rates.

Table 1: 2010 Exchange Rate Arrangements Based on Reinhart and Rogoff (2004, 2011)

	% of Countries	% of GDP
Panel A	*Exchange rate arrangement*	
Floating	3%	34%
Stabilized	88%	47%
soft peg	47%	32%
hard peg	41%	15%
Currency union	9%	19%
Panel B	*Target currencies of stabilizations*	
Dollar	67%	80%
Euro	27%	19%

Notes: Classification of exchange rate regimes as of 2010 according to Reinhart and Rogoff (2004, 2011). All data are available on Carmen Reinhart's website at `www.carmenreinhart.com/data/browse-by-topic/topics/11`.

We analyze the effects of such currency stabilizations on interest rates, capital accumulation, and wages within a generic model of exchange rate determination. In the model, households consume a bundle of a freely traded good and a country-specific nontraded good. The nontraded good is produced using capital and labor as inputs. In equilibrium, the real exchange rate fluctuates in response to country-specific (supply) shocks to productivity in the production of nontraded goods, (demand) shocks to preferences, and (monetary) shocks to the inflation rates of national currencies.

As a stand-in for the various potential sources of heterogeneity in the stochastic properties of countries' exchange rates studied in the literature mentioned above, we add heterogeneity in country size to this canonical setup, as in Hassan (2013). That is, we assume that all shocks are common within countries and some countries account for a larger share of world GDP than others. This heterogeneity in country size generates differences in the stochastic properties of countries' exchange rates, because shocks that raise the price of consumption in a larger country spill over more into the world-market price of traded goods than those that raise the price of consumption in small countries. As a result, the currencies of larger countries tend to appreciate in "bad" times (when the world-market price of traded goods is high) and represent a better hedge against worldwide consumption risk. Because of these hedging properties, the currencies of large countries pay lower expected returns and have lower risk-free interest rates in equilibrium. Lower interest rates in turn lower the cost of capital in these countries, prompting them to increase capital-output ratios and pay higher wages in equilibrium (Hassan, Mertens, and Zhang, 2015).

Within this economic environment, we study the positive and normative effects of a class of policies that lower the variance of one "stabilizing" country's exchange rate relative to a "target" country's currency, while leaving the mean of the exchange rate unaffected. We largely focus our discussion on policies that stabilize the *real* exchange rate but also generalize our main results to nominal stabilizations in a setting where prices are sticky.

To stabilize its real exchange rate, the stabilizing country's government adopts a set of policies that alter the state-contingent plan of imports and exports of traded goods. In particular, when the target country appreciates, the stabilizing country matches that appreciation by reducing traded goods consumption, which raises domestic marginal utility and the price of domestic consumption. Similarly, when the stabilizing country suffers a shock that increases domestic marginal utility and would ordinarily result in an appreciation, it imports additional traded goods to lower domestic marginal utility. We show that the stabilizing country's government can use monetary policy to achieve these goals if prices are sticky, for example by announcing a fixed nominal exchange rate. More generally, it can use a set of state-contingent taxes financed by an independent source of wealth ("currency reserves").

We first consider the case in which the stabilizing country is small and thus only affects its own price of consumption. A small country that stabilizes its exchange rate relative to a larger country inherits the stochastic properties of the larger country's exchange rate. The stabilized exchange rate now tends to appreciate when the price of traded goods in world markets is high, making the stabilizing country's currency a better hedge against consumption risk and lowering its risk-free interest rate and the expected return on its currency. Similarly, investments in the stabilizing country's capital stock now become more valuable, increasing its capital-output ratio and raising wages within the country.

To sustain the stabilization, the stabilizing country ships additional traded goods to the rest of the world in states of the world when the target country appreciates. If the target country is large, then these tend to be states when the world-market price of traded goods is high. As a result, stabilizing relative to a larger target country generates an insurance premium, which lowers the cost of implementing the stabilization. If the target country is sufficiently large, this insurance premium may be so large that the stabilization generates positive revenues and the stabilizing country accumulates, rather than depletes, reserves.

However, this revenue-generating effect diminishes when the stabilizing country itself becomes larger, because the stabilization exaggerates the variation in the stabilizing country's own demand for traded goods, increasing its price impact. For example, in states of the world in which the stabilizing country has high marginal utility, and would ordinarily appreciate relative to the target country, it must now import even more traded goods than it would have without the stabilization to prevent appreciation. When the stabilizing country is large enough to affect the equilibrium price of traded goods, the stabilization thus induces an unfavorable change in the state-contingent prices of traded goods. The larger the stabilizing country, the more reserves are required to maintain the policy.

Our model also allows us to solve for the effects of the stabilization on the target country. A country that becomes the target of a stabilization imposed by a country that is large enough to affect world prices (or the target of multiple stabilizations imposed by a measure of small countries) experiences a rise in its risk-free interest rate, a decrease in its capital-output ratio, and a decrease in wages. The reason is that the stabilizing country, in order to sustain its stabilization, supplies additional traded goods to the world market whenever the target country appreciates. This activity dampens the impact of the target country's shocks on the world-market price of traded goods and reduces their spillover to the world market. The lower this impact, the lower the co-movement between the price of traded goods and the target country's exchange rate. Hence, the currency of the target country becomes a less attractive hedge for international investors, raising its risk-free interest rate.

In various robustness checks we show that this broad set of positive conclusions arises regardless of whether variation in exchange rates are driven primarily by supply, demand, or monetary shocks, and regardless of whether financial markets are complete or segmented within countries. Moreover, we show that in the presence of sticky prices, stabilizing the nominal exchange rate with monetary policy has the same positive implications as stabilizing the real exchange rate.

We also examine the welfare effects of currency stabilizations for a special case of our model, where markets are complete and exchange rates vary exclusively as a result of supply shocks. Stabilizations relative to a larger target country affect the stabilizing country's welfare by increasing the level of consumption (due to higher capital accumulation and accumulation of reserves) but also its variance (because the stabilizing country effectively provides consumption insurance to the target country). Surprisingly, we find that even in this frictionless world, the former effect can dominate, raising welfare due to a valuation effect: If households can only trade bonds indexed to each country's consumption in international markets, the allocation of bonds that de-centralizes the Pareto-efficient allocation under freely floating exchange rates is home-biased, in the sense that households in the stabilizing country hold proportionately more of their own bonds. Upon announcement of the stabilization policy, interest rates on domestic bonds fall relative to all other countries, appreciating their value in world markets and effectively re-distributing wealth towards the stabilizing country. If risk-aversion is sufficiently high, this effect dominates and makes stabilization relative to a larger target country an optimal non-cooperative policy. Paradoxically, small countries are better positioned to take advantage of this effect than large counties, as the cost of stabilization increases with the size of the stabilizing country. (However, we interpret these normative results with caution as it is unclear how they generalize once we allow for incomplete markets and monetary frictions.)

Taken together, we believe our results provide a novel way of thinking about currency manipulation in a world in which risk premia affect the level of interest rates. First, by manipulating exchange rates, policymakers may be able to manipulate the allocation of capital across countries. Second, although we can only show currency stabilization to be optimal under fairly restrictive conditions, our model also suggests that policymakers might have political motives to engage in it if their objective is to increase wages, increase capital accumulation, or raise revenue. Third, whatever the motive, stabilizations relative to larger countries appear to be cheaper to implement and more effective on all dimensions than those to smaller countries, potentially explaining why almost all stabilizations in the data are relative to the euro or dollar. Fourth, the costs of stabilizing increase with the size of the country implementing the policy, offering a potential explanation why most large developed countries do not stabilize their exchange rates. Finally, our model speaks to the external effects of stabilizations on the target country, providing a notion of what it means to be at the center of the world's monetary system: Countries that stabilize relative to a common target divert capital accumulation from the target while dampening the effects of shocks emanating from the target on the world economy.[2]

This latter point also offers an interesting perspective on the large public debate over the Chinese exchange rate regime. U.S. policymakers have often voiced concern that China may be undervaluing its exchange rate and that this undervaluation may be bad for U.S. workers and good for Chinese workers. The official Chinese response to these allegations has been that China is merely stabilizing the exchange rate and not systematically distorting its level. The implication of our analysis is that, even if this assertion is accurate, the mere fact that China is stabilizing its currency to the dollar may divert capital accumulation from the U.S. to China, a policy that is likely to be bad for U.S. workers. However, at the same time, China effectively provides consumption insurance to the U.S., making the overall effect on U.S. welfare ambiguous.

We make three main caveats to our interpretation. First, we focus on differences in country size as the source of differences in interest rates in our model only in the interest of parsimony. Variations of the model where differences in interest rates also arise as a result of differences in trade centrality, financial development, or some of the other microfoundations mentioned above should yield similar results and interpretations—with the United States typically emerging as the most systemic country for the world economy. Second, in our model, currency manipulation transmits itself only through its effects on trade flows. That is, the frictions that allow the government to manipulate exchange rates are between rather than within countries. We do not consider richer models, where currency manipulations could also operate by changing allocations within countries, such as the sectoral allocation of labor or the distribution of wealth across households. Third, although we show currency stabilizations to be optimal non-cooperative policies under some conditions, we do not consider strategic interactions or optimal retaliations.

A large literature studies the effects of monetary stabilization and exchange rate pegs in the presence of nominal frictions.[3] Closely related are Kollmann (2002), Bergin and Corsetti (2015), Ottonello (2015), and Auclert and Rognlie (2014), where currency pegs affect markups and the level of production through their effects on nominal rigidities. Another, largely empirical literature investigates the effects of currency stabilizations on the level of trade flows.[4] We add

[2]In this sense, our paper also relates to a growing literature that argues for a special role of the U.S. dollar in world financial markets. See for example Gourinchas and Rey (2007), Lustig et al. (2011), Maggiori (2013), and Miranda-Agrippino and Rey (2015).

[3]One strand of the literature analyzes optimal monetary policy in small open economies with fixed exchange rates (Kollmann, 2002; Parrado and Velasco, 2002; Gali and Monacelli, 2005), while another deals with the choice of the exchange rate regime in the presence of nominal rigidities (Helpman and Razin, 1987; Bacchetta and van Wincoop, 2000; Devereux and Engel, 2003; Corsetti, Dedola, and Leduc, 2010; Schmitt-Grohé and Uribe, 2012; Bergin and Corsetti, 2015) or collateral constraints (Ottonello, 2015; Fornaro, 2015).

[4]See Hooper and Kohlhagen (1978), Kenen and Rodrik (1986), and Frankel and Rose (2002).

to this literature in two ways. First, we study a novel effect of currency stabilization on risk premia that operates even in a frictionless economy in which money is neutral, adding to other effects of currency stabilization described in this literature. Second, we are able to study how the (internal) effects of currency stabilization vary with the choice of the target currency and how these policies affect the target country.

More broadly, our paper also relates to a large literature on capital controls.[5] Similar to the work by Costinot, Lorenzoni, and Werning (2014), who argue that capital controls may be thought of as a manipulation of intertemporal prices, we show that currency stabilizations and other policies altering the stochastic properties of exchange rates may be thought of as a manipulation of state-contingent prices. The key difference between the two concepts is that capital controls affect allocations through market power and rents, while currency manipulation affects allocations through risk premia, even when the country manipulating its exchange rate has no effect on world market prices. In addition, our work shows that, under some conditions, currency stabilizations relative to a large target country may be optimal non-cooperative policies, even within a frictionless neoclassical model.

Finally, our paper relates to a growing empirical literature that argues that "unconditional" differences in currency returns may be attributable to heterogeneity in the stochastic properties of exchange rates.[6] The theoretical side of this literature has explored various potential drivers of heterogeneity of the stochastic properties of countries' exchange rates. We add to this literature by showing that this class of model implies that exchange rate manipulations affect allocations through their effect on currency risk premia.

The remainder of this paper is structured as follows: Section 1 outlines the effects of currency manipulation on risk premia in their most general form. Section 2 analyzes the effects of stabilizations of the real exchange rate in the context of a simple international real business cycle model. Section 3 generalizes the results from this analysis to stabilizations of the nominal exchange rate when prices are sticky. Section 4 considers more general economic environments where exchange rates are driven by monetary or preference shocks. Section 5 concludes.

[5]See for example Calvo and Mendoza (2000), Jeanne and Korinek (2010), Bianchi (2011), Farhi and Werning (2012, 2013), Schmitt-Grohé and Uribe (2012) and Korinek (2013).

[6]See for example Lustig and Verdelhan (2007), Campbell, Serfaty-De Medeiros, and Viceira (2010), Lustig et al. (2011), Menkhoff et al. (2012), David, Henriksen, and Simonovska (2016), and Verdelhan (2015).

1 Effects of Currency Manipulation in Reduced-form

We begin by deriving the main insights of our analysis in their most general form. Consider a class of models in which the utility of a representative household in each country n depends on its consumption of a final good that consists of a country-specific nontraded good and a freely traded good. In this class of models, we may write the price of the final good in country n in reduced form as

$$p^n = a\lambda_T - bx^n, \tag{1}$$

where p^n is the log of the number of traded goods required to purchase one unit of the final good in country n, $\lambda_T \sim N(0, \sigma^2_{\lambda_T})$ is the log shadow price of traded goods in the world market, b is a constant greater than zero, and $x^n \sim N(0, \sigma^2_x)$ is a normally distributed shock to the log price of consumption in country n. We may think of this shock interchangeably as the effect of a country-specific supply, demand, or monetary shock; in other words, it is a stand-in for any factor that affects the price of consumption in one country more than in others. The higher x^n, the lower is the price of domestic consumption.

The real exchange rate between two countries is the relative price of their respective final goods. The log real exchange is thus

$$s^{f,h} = p^f - p^h.$$

The risk-based view of differences in currency returns applies some elementary asset pricing to this expression. Using the Euler equation of an international investor, one can show that the log expected return to borrowing in country h and to lending in country f is

$$r^f + \Delta\mathbb{E}s^{f,h} - r^h = cov\left(\lambda_T, p^h - p^f\right), \tag{2}$$

where r^n is the risk-free interest rate in country n.[7] This statement means that a currency that tends to appreciate when the shadow price of traded goods is high pays a lower expected return and, if $\Delta\mathbb{E}s^{f,h} = 0$, also has a lower risk-free interest rate. That is, a currency that appreciates in bad times (when traded goods are expensive) provides a hedge against worldwide consumption risk and must pay lower returns in equilibrium.

Equations (1) and (2) are the main ingredients of risk-based models of unconditional dif-

[7] Where $\Delta\mathbb{E}s^{f,h}$ is defined as the logarithm of the ratio of the countries' expected real price changes. See Appendix A.1 for a formal derivation in the context of our general equilibrium model in section 2.

ferences in interest rates across countries, where different approaches model differences in the stochastic properties of p^n as the result of heterogeneity in country size, the volatility of shocks, trade centrality, financial development, factor endowments, etc.

We make a simple point relative to this literature: If there is merit to this risk-based view of currency returns, policies that alter the covariance between a country's exchange rate and the shadow price of traded goods can alter interest rates, currency returns, and the allocation of capital across countries. In particular, a country that adopts a policy that increases the price of domestic consumption in states of the world where λ_T is high can lower its risk-free interest rate relative to all other countries in the world.

As an example, consider a "manipulating" country (indexed by m) that levies a tax on domestic consumption of traded goods that is proportional to the realization of λ_T, such that

$$p^m = a\lambda_T - bx^m + \pi\lambda_T,$$

where π is some positive constant. The tax increases the tendency of p^m to appreciate when λ_T is high and thus, according to (2), lowers its interest rate relative to all other countries in the world by $\pi\sigma^2_{\lambda_T}$.

If interest rates play a role in allocating capital across countries (as is the case in our fully specified model), manipulations of the stochastic properties of exchange rates can thus divert capital investment to the country that conducts the manipulation, and, more broadly, alter the equilibrium allocation of capital across countries.

The remainder of this paper fleshes out this argument in the context of a general equilibrium model of exchange rate determination and applies it to one of the most pervasive policies in international financial markets: currency stabilization.

2 Stabilizing the Real Exchange Rate

We begin by studying the effect of stabilizing the real exchange rate in the most parsimonious environment where markets are complete, money is neutral, and the allocation of capital across countries, as well as the stochastic properties of real exchange rates, are determined solely as a function of productivity shocks (Backus and Smith, 1993). Within this canonical model, one country, labeled the stabilizing country, deviates from the competitive equilibrium by stabilizing its real exchange rate relative to a target country.

Our purpose in beginning our analysis in this frictionless environment is merely to lay bare the main mechanisms as clearly and concisely as possible and to contrast them with the existing literature. We emphasize that none of our main insights depend on market completeness, and that they continue to hold when we add more realistic frictions to the model that address some of the well-known empirical shortcomings of the international real business cycle model. We consider stabilizations of the nominal exchange rate, monetary frictions, preference shocks, and other generalizations in the following sections.

2.1 Economic Environment

There are two discrete time periods, $t = 1, 2$. There exists a unit measure of households $i \in [0, 1]$, partitioned into three subsets Θ^n of measure θ^n. Each subset represents the constituent households of a country. We label these countries $n = \{m, t, o\}$ for the stabilizing (manipulating), target, and outside country, respectively. Households make an investment decision in the first period. All consumption occurs in the second period.

Households derive utility from consuming a consumption index composed of a country-specific nontraded good, $C_{N,2}$, and a traded good, $C_{T,2}$, where

$$C_2(i) = C_{T,2}(i)^{\tau}\ C_{N,2}(i)^{1-\tau}$$

and $\tau \in (0, 1)$. Each household exhibits constant relative risk aversion according to

$$U(i) = \frac{1}{1-\gamma}\mathbb{E}\left[(C_2(i))^{1-\gamma}\right], \tag{3}$$

where $\gamma > 0$ is the coefficient of relative risk aversion.

At the start of the first period, each household receives one traded good and one unit of a capital good. Traded goods can be stored for consumption in the second period and are freely shipped internationally. Capital goods can only be freely shipped in the first period when they are invested for use in the production of nontraded goods in the second period.

Households produce their country-specific nontraded good using a Cobb-Douglas production technology that employs capital and labor. Each household supplies one unit of labor inelastically and purchases capital in international markets in the first period. The per capita output of nontraded goods is

$$Y^n_{N,2} = \exp(\eta^n)\,(K^n)^{\nu}$$

where $0 < \nu < 1$ is the capital share in production, K^n is the *per capita* stock of capital in country n and η^n is a country-specific productivity shock realized at the start of the second period,

$$\eta^n \sim N\left(-\frac{1}{2}\sigma_N^2, \sigma_N^2\right).$$

At the end of the first period, a complete set of Arrow-Debreu securities is traded, completing financial markets. (When studying the welfare effects of currency stabilization in section 2.5 we also consider alternative decentralizations where households instead trade country-specific bonds or stocks in international markets.) Throughout, we use the traded consumption good as the numéraire, such that all prices and returns are accounted for in the same units. To simplify the derivation, we also assume that households receive a country-specific transfer, κ^n, before trading begins, which decentralizes the allocation corresponding to the Social Planner's problem with unit Pareto weights.

Because all households within a given country are identical and consumption only occurs in the second period, we write their consumption bundle as (C_T^n, C_N^n) and henceforth drop the household index i as well as the time subscript t whenever appropriate.

Currency Stabilization The stabilizing country's government has the ability to levy a state-contingent tax on the domestic price of all securities paying consumption goods, $Z(\omega)$, and can pay a lump-sum transfer of traded goods to each household in its country, $\bar{Z}$. We can write the budget constraint of a given household in the stabilizing country as

$$\int Z(\omega)Q(\omega)\left(P^m(\omega)C^m(\omega) - P_N^m(\omega)Y_N^m(\omega)\right)g(\omega)d\omega + 1 + \kappa^m + q(K^m - 1) - \bar{Z} \leq 0, \qquad (4)$$

where $Q(\omega)$ is the price of a state-contingent security that pays one traded good in state ω, $g(\omega)$ is the density function, $P^m(\omega)C^m(\omega)$ is the number of traded goods needed to finance domestic consumption in state ω, $P_N^m(\omega)Y_N^m(\omega)$ is the value, again in terms of traded goods, of domestic nontraded goods production, and q is the first-period price of a unit of capital.

The government's sole objective is to decrease fluctuations of its country's log real exchange rate with the target country by a fraction $\zeta \in (0, 1]$ relative to the freely floating regime, without distorting the conditional mean of the log real exchange rate. Denoting the real exchange rate that would arise under a free floating exchange rate regime with an asterisk, we can write these policy objectives as

$$var\left(s^{t,m}\right) = (1-\zeta)^2 var\left(s^{t,m*}\right) \qquad (P1)$$

and

$$\mathbb{E}\left[s^{t,m}|\{K^n\}\right] = \mathbb{E}\left[s^{t,m*}|\{K^n\}\right]. \tag{P2}$$

We refer to $\zeta \in (0,1]$ as a stabilized real exchange rate and $\zeta = 1$ as a "hard" peg.

The government thus has two policy instruments to achieve two objectives, using the state contingent tax to achieve (P1) and the lump-sum transfer to simultaneously achieve (P2). The per capita cost of implementing this stabilization policy is the cost of the lump-sum transfer, less the net revenues from the state-contingent tax. We can write it as

$$\Delta Res = \bar{Z} - \int \left(Z(\omega) - 1\right) Q(\omega) \left(P^m(\omega) C^m(\omega) - P_N^m(\omega) Y_N^m(\omega)\right) d\omega. \tag{5}$$

To focus on stabilizations of the exchange rate that do not distort the mean, we begin by assuming that the government finances this cost using an independent supply of traded goods (currency reserves) that absorbs any surpluses or deficits generated by the taxation scheme (ΔRes).

We show below that, under a range of relevant parameters, the cost of currency stabilization is negative, such that the policy is implementable even if the government has no access to currency reserves. We also show that all the main insights of our analysis below continue to hold if for some reason the government's implementation of the policy is only partially credible. When we analyze the welfare effects of exchange rate stabilization in section 2.5, we assume that $\Delta Res = 0$, so that the cost of the stabilization is fully borne by the households in the stabilizing country. Under this assumption, we allow the stabilization to distort the expected real exchange rate (and thus violate (P2)).

The market clearing conditions for traded, nontraded, and capital goods are

$$\int_{i \in [0,1]} C_{T,2}(i,\omega) di = 1 + \theta^m \Delta Res, \tag{6}$$

$$\int_{i \in \theta^n} C_{N,2}(i,\omega) di = \theta^n Y_{N,2}^n(\omega), \tag{7}$$

and

$$\sum_n \theta^n K^n = 1. \tag{8}$$

The economy is in an equilibrium when all households maximize utility taking prices and taxes as given, firms maximize profits, and goods markets clear.

2.2 Solving the Model

Appendix A.3 derives the conditions of optimality characterizing the equilibrium allocation. The first-order conditions with respect to C_T^n equate the shadow price of traded consumption across the target and outside countries

$$\tau \left(C^n(\omega)\right)^{1-\gamma} \left(C_T^n(\omega)\right)^{-1} = \Lambda_T(\omega),\ n = o, t. \tag{9}$$

In the stabilizing country, the state-contingent tax that implements the currency stabilization appears as a wedge on that shadow price

$$\tau \left(C^m(\omega)\right)^{1-\gamma} \left(C_T^m(\omega)\right)^{-1} = Z(\omega)\Lambda_T(\omega). \tag{10}$$

In all countries, marginal utilities with respect to $C_{N,2}^n$ define the shadow prices of nontraded goods

$$(1-\tau) \left(C^n(\omega)\right)^{1-\gamma} \left(C_N^n(\omega)\right)^{-1} = \Lambda_N^n(\omega). \tag{11}$$

In addition, we derive households' optimal demand for capital by taking first-order conditions with respect to K^n. Using the fact that competitive markets imply $P_N^n(\omega) = \Lambda_N^n(\omega)/\Lambda_T(\omega)$, we get

$$K^n = \frac{\nu}{\Psi_T q}\mathbb{E}\left[\Lambda_N^n Y_N^n\right], \tag{12}$$

where q is the equilibrium price of one unit of capital and $\Psi_T = \mathbb{E}\left[\Lambda_T(\omega)\right]$ is the shadow price of a traded good in the first period, prior to the realization of shocks. This Euler equation defines the level of capital accumulation in country n as a function of first-period prices and the expected (utility) value of its nontraded goods, $\mathbb{E}\left[\Lambda_N^n Y_N^n\right]$. This latter term will differ across countries and reflect any precautionary motives for capital accumulation, including those that arise as a function of the stochastic properties of the country's exchange rate.

Importantly, (12) holds in all countries, including the stabilizing country, because the stabilizing government's intervention alters the state-contingent valuation of nontraded output and nontraded consumption in an offsetting way (as shown in (4)). In equilibrium, the two effects cancel each other such that (12) holds in all countries (see Appendix A.3 for a formal derivation).

Finally, the (redundant) first-order conditions with respect to the consumption index C^n pin down the shadow prices of overall consumption in each country

$$\left(C^n(\omega)\right)^{-\gamma} = \Lambda^n(\omega). \tag{13}$$

The real exchange rate between two countries h and f equals the ratio of these shadow prices,

$$S^{f,h}(\omega) = \Lambda^f(\omega)/\Lambda^h(\omega).$$

In equilibrium, the resource constraints (6)-(8) and the conditions of optimality (9)-(12) jointly determine define the endogenous variables $\{C_N^n, C_T^n, K^n, \Lambda_N^n\}_{n\in\{p,t,o\}}$, Λ_T, and q. To study the model in closed form, we log-linearize it around the deterministic solution — the point at which the variances of shocks are zero ($\sigma_{N,n} = 0$) and all firms have a capital stock that is fixed at the deterministic steady-state level. To simplify the exposition, we thus ignore the feedback effect of differential capital accumulation on the size of risk premia, studying the *incentives* to accumulate different levels of capital across countries, while holding the capital stock fixed. Appendix A.14 shows that all propositions in section 2 continue to hold when we allow for this feedback effect. Throughout, lowercase variables continue to refer to natural logs.

2.3 The Freely Floating Regime

We begin by showing that, in the absence of currency manipulation, the model predicts that large countries should have lower real interest rates (Hassan, 2013) and accumulate higher capital-output ratios (Hassan et al., 2015). If $\zeta = 0$, equilibrium consumption of traded goods is given by

$$c_T^{n*} = \frac{(1-\tau)(\gamma-1)}{(1-\tau)+\gamma\tau}\left(\bar{y}_N - y_N^n\right), \tag{14}$$

where $\bar{y}_N = \sum_n \theta^n y_N^n$ is the average log per-capita output of nontraded goods across countries. The expression shows that households use shipments of traded goods to insure themselves against shocks to the output of nontraded goods. If $\gamma > 1$, households receive additional traded goods whenever they have a lower-than-average output of nontraded goods, and vice versa.[8]

This risk-sharing behavior generates a shadow price of traded goods of the form,

$$\lambda_T^* = -(\gamma-1)(1-\tau)\sum_n \theta^n y_N^n, \tag{15}$$

[8]The condition $\gamma > 1$ (more generally, γ multiplied with the elasticity of substitution between traded and nontraded goods > 1) ensures that the cross-partial of marginal utility from traded consumption with respect to the nontraded good is negative, that is, the relative price of a country's nontraded good falls when its supply increases. As most empirical applications of international asset pricing models find a relative risk aversion significantly larger than one and an elasticity of substitution around one, most authors assume that this condition holds (see Coeurdacier (2009) for a detailed discussion). We show in section 4 that this condition is not needed if variation in exchange rates is driven predominantly by monetary or preference shocks.

where each country's weight is proportional to its size: shocks to the productivity of larger countries affect a larger measure of households and thus tend to spill over to the rest of the world in the form of higher shadow prices of traded goods. If $\gamma > 1$, the shadow price of traded goods falls with the average output of nontraded goods across countries. Thus, λ_T tends to be low in good states of the world when countries on average experience positive productivity shocks in their nontraded sectors.

The real exchange rate between two arbitrary countries f and h is

$$s^{f,h*} = \lambda^{f*} - \lambda^{h*} = \frac{\gamma(1-\tau)}{(1-\tau)+\gamma\tau}\left(y_N^h - y_N^f\right),$$

showing that the currency of the country with lower per-capita output of nontraded goods appreciates because its consumption index is expensive relative to that in other countries.

Inspecting λ_T^* and $s^{f,h*}$ shows that currencies of larger countries are "systemic" in the sense that they tend to appreciate when the shadow price of traded goods is high: Whenever a country suffers a low productivity shock, its real exchange rate appreciates. For a given percentage decline in productivity, this appreciation occurs independently of how large the country is (note that $s^{f,h*}$ is independent of θ). However, a shock to a larger country has a larger impact on the shadow price of traded goods (λ_T). It then immediately follows from (2) that larger countries have a lower risk-free rate:

$$r^{f*} + \Delta\mathbb{E}s^{f,h*} - r^{h*} = cov\left(\lambda_T^*, p^{h*} - p^{f*}\right) = \frac{(\gamma-1)\gamma(1-\tau)^2}{1+(\gamma-1)\tau}\left(\theta^h - \theta^f\right)\sigma_N^2. \tag{16}$$

To see that these differences in interest rates across countries translate into differential incentives to accumulate capital, we can rearrange the Euler equation for capital accumulation (12) and derive an expression that links difference in capital to differences in interest rates[9]

$$k^{f*} - k^{h*} = \frac{\gamma}{\tau(\gamma-1)^2}\left(r^{h*} - \Delta\mathbb{E}s^{f,h*} - r^{f*}\right). \tag{17}$$

It is efficient to accumulate more capital in the larger country because a larger capital stock in a larger country represents a good hedge against global consumption risk. Households around the world fear states of the world in which the large country receives a low output from its nontraded sector, because larger countries transmit these shocks to the rest of the world through a higher shadow price of traded consumption. Although households cannot affect the realization

[9]For a derivation, see Appendix A.5.

of productivity shocks, they can partially insure themselves against low output in the nontraded sector of large countries by accumulating more capital in these countries. This precautionary behavior raises expected output in the nontraded sector and dampens the negative effects of a low productivity shock.

2.4 Effects of Currency Stabilization

Under freely floating exchange rates, larger (more systemic) countries thus have lower risk-free rates and higher capital-output ratios. With this result in mind, we now analyze how a country can influence interest rates and the allocation of capital by stabilizing its currency.

While the policy objectives (P1) and (P2) with $\zeta < 1$ can in principle be achieved with a range of different nonlinear policies, such as intervening only in response to shocks smaller or larger than some critical value, we focus our discussion on the unique linear policy that entails a proportional intervention in each state. The advantage of focusing on this case is simply that it preserves the Gaussian structure of the problem and thus lends itself to closed-form solutions. In section 2.6 we discuss issues that arise when the government cannot credibly commit to stabilizing shocks larger or smaller than some critical value and show that our main conclusions do not change in that case.

The following lemma characterizes the unique linear form of state contingent taxes that implements the exchange rate stabilization:

Lemma 1

A tax on all assets paying off consumption goods in the stabilizing country of the form

$$z(\omega) = \zeta \frac{1-\tau}{\tau}\left(y_N^m - y_N^t\right)$$

implements a real exchange rate stabilization of strength ζ*.*

The cost of the stabilization, ΔRes*, equals the change in the world-market cost of traded goods consumed by households in the stabilizing country,*

$$\Delta Res = \int Q(\omega) C_T^m(\omega) d\omega - \int Q^*(\omega) C_T^{m*}(\omega) d\omega. \tag{18}$$

Proof. See Appendix A.6. ■

To build intuition for the effects of this state-contingent tax, it is useful to solve for the change in the equilibrium consumption of traded goods by domestic households relative to the freely floating regime:

$$c_T^m - c_T^{m*} = \zeta \frac{(1-\tau)(1-\theta^m)}{\tau\left((1-\tau)+\gamma\tau\right)} \left(y_N^t - y_N^m\right). \tag{19}$$

When the target country receives a relatively bad productivity shock ($y_N^t < y_N^m$), its price of consumption appreciates. To mirror this increase, the stabilizing country raises taxes on traded goods, reduces its consumption of traded goods relative to the freely floating regime, and thus raises its own marginal utility.[10] Conversely, when the stabilizing country receives a relatively bad shock, its price of consumption would ordinarily increase. To offset this increase and prevent its currency from appreciating, the government subsidizes imports of traded goods, resulting in even higher imports of traded goods than under the freely floating regime.

We start by analyzing the effect of this stabilization policy on allocations, prices, and currency reserves in the stabilizing country. Afterwards, we analyze its impact on the target country.

2.4.1 Internal Effects of Currency Stabilization

The most obvious effect of currency stabilization is that the price level in the stabilizing country becomes more correlated with the price level in the target country.

$$\lambda^m = \lambda^{m*} + (1-\theta^m)\zeta \frac{\gamma(1-\tau)}{1+(\gamma-1)\tau} \left(y_N^m - y_N^t\right).$$

The real exchange rate stabilization increases the weight of the target country's shock in the stabilizing country's price level, while also decreasing the weight of its own shock. That is, λ^m starts behaving more like λ^t. Because larger countries tend to appreciate in bad times, a stabilization relative to a larger country ($\theta^t > \theta^m$) naturally also makes the stabilizing country appreciate in bad times, that is, stabilization increases the covariance between the stabilizing country's price level, λ^m, and the shadow price of traded goods, λ_T, similar to the intervention considered in section 1. As a result, a risk-free asset that pays one unit of the stabilizing country's consumption bundle with certainty becomes a better hedge against consumption risk, increasing its value in the world market, and lowering the stabilizing country's risk-free interest rate.

Similarly, the stabilization also increases the effect of the target country's shock on the world-

[10] Note that the state-contingent tax drives a wedge between the domestic and world-market prices of traded goods so that the relative prices of non-traded goods are no longer a sufficient statistic for the real exchange rate.

market value of the stabilizing country's output of nontraded goods

$$p_N^m + y_N^m = (p_N^{m*} + y_N^{m*}) + \zeta \frac{(1-\tau)\left(\theta^m + (\gamma-1)\tau\right)}{\tau\left(1+(\gamma-1)\tau\right)} \left(y_N^m - y_N^t\right),$$

increasing its covariance with the shadow price of traded goods, and thus increasing the value of capital installed in the stabilizing country.

Proposition 1

If $\gamma > 1$, a country that stabilizes its real exchange rate relative to a target country sufficiently larger than itself lowers its risk-free interest rate, increases capital accumulation, and increases the average wage in its country relative to the target country.

Proof. The interest rate differential with respect to the target country is

$$r^m + \Delta\mathbb{E}s^{m,t} - r^t = r^{m*} + \Delta\mathbb{E}s^{m,t*} - r^{t*} - \zeta \frac{\gamma(1-\tau)^2 \left((\theta^t - \theta^m)(\gamma-1)\tau + 2\theta^m(1-\zeta)\right)}{\tau(1+(\gamma-1)\tau)} \sigma_N^2.$$

See Appendix A.7 for details and the corresponding proof for capital accumulation, which requires that the target country be sufficiently large. ■

Aside from these effects on interest rates and capital accumulation, the stabilization policy also affects the level of currency reserves. From (18), we already know that the cost of implementing the stabilization is simply the cost of altering the state-contingent purchases of traded goods in world markets. Moreover, we also know that the stabilization induces the stabilizing country to sell additional traded goods in response to adverse productivity shocks in the target country, and to buy additional traded goods in response to adverse productivity shocks at home. If the target country is larger than the stabilizing country, traded goods are more expensive in the states in which it sells than in the states in which it buys. In this case, the stabilization induces the stabilizing country to provide insurance to the world market, pocketing an insurance premium.

Proposition 2

If $\gamma > 1$ and the stabilizing country is small, $\theta^m = 0$, then the cost of stabilization globally decreases with the size of the target country and locally increases with the size of the stabilizing country. Additionally, the cost of stabilization (ΔRes) is negative if and only if

$$\theta^t > \frac{\zeta + (\gamma-1)\tau}{(\gamma-1)^2\tau^2}.$$

Proof. See Appendix A.8. ■

If the target country is sufficiently large relative to the stabilizing country and risk aversion is sufficiently high, this insurance premium can be so large that the stabilization generates revenues for the government, accumulating rather than depleting currency reserves.

When the stabilizing country itself is large ($\theta^m > 0$), its purchases and sales of traded goods also affect the equilibrium shadow price of traded goods, λ_T. This price impact generally increases the cost of stabilization. The reason is that stabilization effectively increases the volatility of shipments of traded goods to the rest of the world. In states where the stabilizing country has a bad productivity shock, it imports more traded goods than it ordinarily would have. In states where the target country has a bad productivity shock, it exports more than it ordinarily would have. The more price impact the stabilizing country has, the more costly it therefore is to maintain the stabilization.

2.4.2 External Effects of Currency Stabilization

If the stabilizing country is large ($\theta^m > 0$), its actions also affect consumption and prices in the rest of the world. The shadow price of traded goods is

$$\lambda_T = \underbrace{-(1-\tau)(\gamma-1)\sum_n \theta^n y_N^n}_{=\lambda_T^*} + \frac{\theta^m(1-\tau)}{\tau}\zeta\left(y_N^t - y_N^m\right).$$

The second term on the right hand side shows that if the stabilizing country is large, stabilization dampens the effect of the target country's shocks on the shadow price of traded goods, reducing the extent to which its shocks spill over to the rest of the world, and making it *less* systemic. As a result, the currency stabilization decreases the covariance between the target country's real exchange rate and λ_T. It follows immediately that becoming the target of a stabilization raises the target country's interest rate rises and lowers its capital accumulation.

Proposition 3

If $\gamma > 1$, a country that becomes the target of a stabilization of any strength $\zeta > 0$ imposed by a large country experiences a rise in its risk-free interest rate, a fall in capital accumulation, and a fall in average wages relative to all other countries. If the stabilizing country is smaller than the target country ($\theta^m < \theta^t$), the stabilization lowers the volatility of consumption in the target country.

Proof. The interest rate differential between the target and outside country is

$$r^t + \Delta\mathbb{E}s^{t,o} - r^o = \left(r^{t*} + \Delta\mathbb{E}s^{t,o*} - r^{o*}\right) + \zeta\frac{\theta^p(1-\tau)^2\gamma}{\tau\left(1+(\gamma-1)\tau\right)}\sigma_N^2.$$

See Appendix A.9 for details and the remainder of the proof. ■

Currency stabilization can thus divert capital from the target country to the stabilizing country even though it has no effect on the level of the real exchange rate. This finding is particularly interesting because it sheds new light on recent public controversies, for example between Chinese and U.S. officials, which usually focuses on the idea that an under-valuation of the Chinese real exchange rate favors Chinese workers at the expense of U.S. workers. By contrast our results suggest, that even a currency stabilization that manipulates the variance but not the level of the real exchange rate can have this effect.

On the flip side, currency stabilization by a large country decreases the volatility of consumption in the target country because it dampens the effect of the target country's shock on the shadow price of traded goods. Effectively, the stabilizing country thus provides consumption insurance to the target country.

Becoming the target of a stabilization is thus both good and bad — it lowers the level of consumption in the target country by diverting capital accumulation but also decreases its variance. We show in appendix A.10 that, in the absence of valuation effects from the announcement of the peg, the positive effect of insurance provision outweighs the detrimental effect on the level of consumption, such that, on balance, stabilizations increase welfare in the target country under these conditions. However, we argue below that this result can easily be reversed if the announcement of the peg also changes the relative wealth of different countries.

2.5 Rationale for Stabilization

Next, we study why a country might decide to stabilize its currency. The existing literature has shown that currency stabilization can be a second-best policy response in the presence of monetary and other frictions. Perhaps surprisingly, we show that even in the absence of such frictions, stabilization relative to a larger country may increase welfare in the stabilizing country because of valuation effects.[11] After analyzing these welfare consequences we also consider alternate motives that may make stabilization politically opportune even under conditions where it

[11] Given any budget constraint, we define the valuation effect as the (log) change in the value of the household's traded consumption from its value in the Pareto-efficient allocation under freely floating exchange rates.

decreases the classical welfare criterion.

2.5.1 Welfare

So far we have defined a currency stabilization as reducing the variance of the log real exchange rate (P1) while not distorting its level (P2). Achieving both objectives simultaneously requires that the government has the ability to add or subtract resources from the economy by accumulating or depleting currency reserves. For the purposes of assessing the welfare effects of currency stabilization, we now drop objective (P2) and assume that instead the government rebates the cost of stabilizing the exchange rate back to households using a lump-sum tax ($\Delta Res = 0$). That is, households in the stabilizing country directly bear the financial cost of stabilizing the exchange rate, shifting the level of their traded consumption in all states of the world, and thus also affecting the level of their real exchange rate. Closing the model in this way does not interfere with the intuition of the positive results derived above but increases the complexity of the solution of the model, so that we relegate most of the mathematical details to the appendix.

We have already seen that a stabilization relative to a larger country can increase capital accumulation and generate revenue, so that it increases the level of consumption in the stabilizing country once we consolidate the government and household budget constraints. However, stabilization also increases the variance of consumption because the stabilizing country effectively provides insurance to the world market against shocks that affect the target country. This increase in the volatility of consumption reduces expected utility. If the stabilizing country is small and if the announcement of the stabilization has no effect on the world-market value of the stabilizing country's assets (that is, there is no valuation effect of the announcement), we can show that the latter effect dominates and the exchange rate stabilization decreases the welfare of households within the stabilizing country (see Appendix A.10).

This result is not surprising because it follows almost directly from the first theorem of welfare economics: Under freely floating exchange rates and complete markets, the stabilizing country already provides the Pareto-optimal amount of insurance to other countries. Because small countries have no effect on world prices of state-contingent claims to traded goods, and there are no frictions to heal, increasing this provision of insurance does not increase welfare, even if the stabilizing country gets compensated for providing it (the negative ΔRes). However, this intuitive result can be overturned if the stabilization has an additional effect on the relative wealth of the stabilizing country.

Up to this point, we have assumed that households in the stabilizing country own only state-contingent claims to traded goods and the rights to the output of non-traded goods in their country, as specified in (4), at the time the stabilization policy is announced. Under this assumption, the announcement of a stabilization by a small country has no effect on its relative wealth, because it must consume all of its non-traded goods in equilibrium and it has no effect on the world market prices of state-contingent claims to traded goods. However, maybe a more realistic approach to implementing the allocations described above is to assume that households trade country-specific bonds in international markets, instead of state-contingent claims. If households can trade bonds denominated in each of the three countries' consumption bundles, (4) becomes

$$\int Z(\omega)Q(\omega)\left(P^m(\omega)C^m(\omega)-\left[\sum_{l=m,t,o}B_l^m P^l(\omega)\right]\right)g(\omega)d\omega+q(K^m-1)-\bar{Z}\leq 0, \tag{20}$$

where B_l^m is the number of bonds denominated in country l's consumption bundle held by each household in country m. With this formulation, financial markets are still (first-order) complete, in the sense that the number of assets traded spans the number of shocks, such that all allocations described in sections 2.3 and 2.4 continue to arise in equilibrium. The only difference is that, with this alternative decentralization, the assets owned by households in the stabilizing country appreciate in relative value upon the announcement of the stabilization policy.

Proposition 4

If households in the stabilizing country directly bear the cost of implementing stabilization ($\Delta Res = 0$) and if all households own the requisite number of risk-free bonds denominated in the consumption bundles of the stabilizing, target, and outside countries that decentralize the Pareto-efficient allocation of consumption under freely floating exchange rates at the time of the announcement of the stabilization policy, then there exists a $\overline{\gamma}$ such that for $\gamma > \overline{\gamma}$ stabilizing relative to a larger target country strictly increases the welfare of households in the stabilizing country.

Proof. See Appendix A.11. ■

The intuition for this result is simple: The Pareto-efficient allocation of bonds across countries is home-biased in the sense that households in each country buy some of the other countries' bonds, but disproportionately invest in their own bond, giving them the means to buy relatively more traded goods in world markets when their own consumption bundle appreciates (as dictated by (14)). Naturally, the value (in terms of first-period traded goods) of each of these bonds appreciates in response to a fall in the country's risk-free interest rate. If the stabilizing country is small, the announcement of the stabilization relative to a larger country lowers its own interest rate, and thus raises the value of its own bond, but does not affect the valuation of the other two bonds.[12] Because the stabilizing country holds a disproportionate amount of its own bond and all bonds are in zero net supply, the announcement of the peg transfers additional resources to the stabilizing country. This valuation effect is larger when the effect of stabilization on risk premia is larger, and thus increases with γ.

The same result holds if, instead of bonds, households trade stocks (claims to the ownership of capital) in international markets, because the stabilization again disproportionately increases the value of the portfolio held by the stabilizing country under the freely floating regime. In these, arguably more realistic settings, stabilization can thus re-distribute wealth towards the stabilizing country and raise its welfare, making stabilization an optimal non-cooperative policy.

Importantly, this rise in welfare through stabilization is, for a given set of parameters, easier to achieve for a small country than for a large country. As we have already discussed above, the stabilization policy for larger countries manipulates state-contingent prices of traded goods in the wrong direction, which increases the cost of implementing the stabilization.

In this sense, our frictionless model produces a potential rationale based on the traditional welfare criterion for the patterns of stabilizations we see in the data, where small countries tend to stabilize relative to the largest economy in the world (the U.S. dollar), but large developed countries tend not to engage in stabilization.

2.5.2 Political Economy

Maybe as relevant in practice as these welfare considerations, our model can also provide a political economy rationalization for the same patterns: A large literature argues that policymakers trying to win elections have an interest in raising wages (e.g. if the median voter is a worker, Persson and Tabellini (2002)) and often prefer generating revenue through central bank or currency board operations over direct taxation, even if these are distortionary, because these operations are less visible to the public and easier to control (Cukierman et al., 1992; Bates, 2005). Currency stabilizations relative to the largest economy in the world achieve both of these objectives and may thus be politically attractive. For example, a stabilization relative to the largest economy in

[12] In this sense, even a small country has market power over its own price of consumption. Another way of understanding this result is to think about the representation of the stabilizing country's bond as a portfolio of state-contingent claims to traded goods. Although its actions have no effect on the world-market prices of these state-contingent claims, the stabilization policy alters the composition of this portfolio (the states in which the bond pays of well), adding more of the expensive state-contingent claims that pay off when the target country appreciates, thus raising the world-market value of the stabilizing country's risk-free bond.

the world may be optimal even in the absence of valuation effects if policymakers in a stabilizing country maximize a function of the form

$$EU^n + \mu_1 K^n - \mu_2 \Delta Res,$$

where μ_1 and μ_2 are constants that may reflect the political influence of workers, externalities from capital accumulation, or a motive for generating revenues in a way that avoids direct taxation of households or firms.

2.6 Partially Credible Stabilizations and Floating Bands

A major issue in the study of policies that manipulate the first moment of exchange rates (under- or over-valuations), is the depletion of reserves and the credibility of such manipulations in the face of potential speculative attacks (Krugman, 1979; Garber and Svensson, 1995). By contrast, we have already shown that stabilizations of the real exchange rates relative to a large target country may generate, rather than deplete, reserves, assuaging some potential concerns about the policy's credibility. Nevertheless, it is worth considering the effects of only partially credible stabilizations. Suppose the government, either by choice or necessity, abandons the stabilization in a subset of states $\Omega_{-s} \subset \Omega$ (where Ω is the set of all possible states).

Assuming that the government continues to stabilize state-by-state within $\Omega_s = \Omega \backslash \Omega_{-s}$, and that this limited stabilization continues to leave the mean of the real exchange rate undistorted (e.g. the partition of Ω into Ω_s and Ω_{-s} is symmetric around the mean), we can show that[13]

$$var(s^{m,t}) = \left(\text{Prob}\left[\omega \in \Omega_s\right](1-\zeta)^2 + \text{Prob}\left[\omega \in \Omega_{-s}\right]\right) var\left[s^{*m,t}|\Omega_{-s}\right] < var(s^{m,t*})$$

and

$$r^m + \Delta\mathbb{E}[s^{m,t}] - r^t = -\left(\text{Prob}\left[\omega \in \Omega_s\right](1-\zeta) - \text{Prob}\left[\omega \in \Omega_{-s}\right]\right) cov\left[\lambda_T, s^{*m,t}|\Omega_s\right].$$

In contrast with partially credible manipulations of the level of the real exchange rate, partially credible manipulations of its variance are thus still effective: They reduce the variance of the real exchange rate and affect interest rates and other outcomes in the same way as characterized above — only less so than a fully credible stabilization. In this sense, we may simply think of partially credible stabilizations as "weaker" credible stabilizations.

[13] See Appendix A.12 for a formal derivation.

Additionally, the two expressions above also directly describe the effects of a variety of non-linear stabilization policies, such as floating bands, that allow a freely floating exchange rate between some upper and lower limit and intervene state-by-state only when the real exchange rate departs this band.

Similarly, Appendix A.13 shows that our analysis above also extends directly to stabilizations relative to a basket of currencies, where stabilizing relative to a basket of currencies has effects akin to a stabilization relative to a (hypothetical) country with a weighted average size of the basket's constituents.

3 Nominal Stabilization and Monetary Policy

In the previous section, we have characterized the internal and external effects of a stabilization of the *real* exchange rate implemented with state-contingent taxes (or capital controls). In the data, most governments instead appear to implement stabilizations of the nominal exchange rate using monetary policy. In this section, we study the relationship between real and nominal stabilizations and show how monetary policy can implement such stabilizations in the presence of nominal rigidities.

To this end, we extend our model by assuming that the prices of traded goods are rigid in local currency. A large body of empirical work documents such rigidity, which creates a wedge in the prices of traded goods across borders, that is, failures in the law of one price (Mussa, 1986; Engel, 1999; Cavallo et al., 2014).

Parallel to our definition of a real exchange rate stabilization above, we define a stabilization of the *nominal* exchange rate of strength $\tilde{\zeta}$ as a set of policies that decreases the variance of the log nominal exchange rate between the stabilizing and target countries, $var\left(\tilde{s}^{t,m}\right) = (1-\tilde{\zeta})^2 var\left(\tilde{s}^{t,m*}\right)$, while keeping the conditional mean of the log nominal exchange rate unchanged, $\mathbb{E}\left[\tilde{s}^{t,m}|\{K^n\}\right] = \mathbb{E}\left[\tilde{s}^{t,m*}|\{K^n\}\right]$. Throughout, tildes denote nominal prices, where $\tilde{s}^{t,m}$ is the log nominal exchange rate between the currencies of countries t and m.

If prices are rigid, monetary policy can affect real allocations. Assume that all goods consumed in a given country must be purchased using the domestic currency,

$$\tilde{P}_T^n P^n C^n = \mathcal{M}^n$$

where $\tilde{P}_T^n$ is the price of one unit of the traded good in terms of the currency of country n

and $\mathcal{M}^n$ is the supply of the domestic currency that can be changed at the discretion of the country's central bank. In addition, households in each country trade state-contingent securities with their central bank that pay off in units of their own currency, so that they cannot use financial markets to hedge against inflation. The central banks in turn trade a complete set of state-contingent securities with each other, completing international financial markets except for households' susceptibility to domestic inflation (Alvarez et al., 2002). If all central banks adjust their respective $\mathcal{M}^n$ to neutralize the effects of the nominal rigidity in their own country, we thus recover the same allocation that arises under freely floating exchange rates in the frictionless model above.[14]

While the central banks of the target and outside countries follow this neutralizing policy, the central bank of the stabilizing country deviates and instead uses its control of $\mathcal{M}^n$ to stabilize its real exchange rate relative to the target country by driving a wedge between the shadow price of traded goods at home and abroad.

Proposition 5

If the price of the traded good is rigid in terms of the stabilizing country's currency,

1. *a nominal stabilization implements a real stabilization of equal strength* $\zeta = \tilde{\zeta}$

2. *the central bank of the stabilizing country implements a real and nominal exchange rate stabilization of strength* ζ *by setting the log money supply,*

$$\log\left[\mathcal{M}^m\right] = \frac{(\gamma-1)(1-\tau)}{1+(\gamma-1)\tau}\left(\bar{y}_N - y_N^m\right) + \zeta\frac{(1-\theta^m)(1-\tau)}{\tau(1+(\gamma-1)\tau)}\left(y_N^t - y_N^m\right) - \log[\tau]$$

Proof. See Appendix B. ■

The first part of the proposition follows directly from the nature of the monetary friction. Because the price of the traded good is fully rigid, the price of consumption in terms of traded goods (the real price level, P^n) always moves in the same direction as the price of consumption in domestic currency (the nominal price level $\tilde{P}_T^n P^n$). As a result, the nominal exchange rate moves in lockstep with the real exchange rate.

The second part of the proposition is also intuitive: when the target country appreciates ($y_N^t < y_N^m$), the central bank in the stabilizing country decreases the money supply to increase the value of the domestic currency and match the nominal appreciation. This reduction in the money supply then has the same real effects as a state-contingent tax that drives a wedge between

[14] See Appendix B for formal details of this extension of the model.

the price of traded goods at home and abroad: It raises the real price of traded goods relative to nontraded goods in the stabilizing country, prompting domestic households to consume fewer traded goods whenever the target country appreciates. The stabilizing country thus again exports additional traded goods whenever the target country appreciates and vice versa.

If households need domestic currency to buy consumption goods and prices are sufficiently sticky to give the central bank some leverage over real allocations, we thus conclude that stabilizations of the real exchange rate can be implemented with a simple rule that commits the central bank's control of the money supply to enforce a nominal stabilization. That is, even if prices are only partially rigid, a nominal peg, where the central bank commits to exchanging currency at a predetermined rate, implements some real exchange rate stabilization, entailing all the effects on real allocations discussed in the previous section.

4 Segmented Markets and Preference Shocks

So far, we have based our analysis of currency stabilization on a conventional international real business cycle model, where markets are complete and productivity shocks are the only drivers of variation in real exchange rates (Backus and Smith, 1993). Although an important benchmark, this framework has a number of well-known empirical shortcomings. First, it predicts a perfectly negative correlation between appreciations of the real exchange rate and aggregate consumption growth — a currency appreciates when the country's aggregate consumption falls. Second, the model predicts that consumption should be more correlated across countries than output, whereas the opposite is true in the data (Backus, Kehoe, and Kydland, 1994). Third, real exchange rates and terms of trade seem much too volatile to be rationalized exclusively by real (productivity) shocks alone (Chari, Kehoe, and McGrattan, 2002). As a result, many authors have argued for incomplete markets models that allow for an effect of monetary shocks on equilibrium real exchange rates, or models with demand shocks.

In this section, we argue that the intuition and all positive results from our analysis of exchange rate stabilizations in section 2 continue to hold in a more general class of models where real exchange rates fluctuate due to inflation shocks, market incompleteness, and preference shocks.

We again depart from the model in section 2, where prices are fully flexible. We now assume that households in each country experience preference shocks as in Pavlova and Rigobon (2007):

$$U(i) = \frac{1}{1-\gamma}\mathbb{E}\left[(\exp(\chi^n)C_2(i))^{1-\gamma}\right], \tag{21}$$

where χ^n is a common shock to households' demand for consumption goods in country n,

$$\chi^n \sim N\left(-\frac{1}{2}\sigma_\chi^2, \sigma_\chi^2\right).$$

Motivated by the fact that many households in the U.S. and in other developed economies own savings accounts or bonds but do not own stocks, foreign bonds, or other more sophisticated financial instruments that could hedge their portfolios against inflation (Giannetti and Koskinen, 2010; Nechio, 2010), we also allow for segmented markets: a measure $1-\phi$ of "inactive" households within each country is excluded from international financial markets (Alvarez et al., 2002). The remaining measure ϕ of households within each country continues to trade a complete set of state-contingent securities as in our frictionless model. Label these households as "active." Inactive households do not receive endowments or own any claims to productive assets. Instead, they own only a nominal bond that pays off one unit of the country's nominal consumer price index. We can write this payment to inactive households as $P_2^n e^{-\mu^n}$, where μ^n is a shock to the growth rate of the nominal price of one unit of the traded good in the currency of country n,

$$\mu^n \sim N\left(-\frac{1}{2}\tilde{\sigma}^2, \tilde{\sigma}^2\right).$$

Active households own all productive assets within the country and are short the nominal bonds owned by inactive households. They maximize their utility (21) subject to the constraint

$$\begin{aligned}&\int Q(\omega)\left(P^n(\omega)C^n(\omega) + \frac{1-\phi}{\phi}P^n(\omega)e^{-\mu^n}\right)g(\omega)d\omega \qquad (22)\\ \leq &\frac{1}{\phi}\left(1+q-qK^n + \int Q(\omega)P_N^n(\omega)\exp(\eta^n)\left(K^n\right)^\nu g(\omega)d\omega + \kappa^n\right),\ n=t,o\end{aligned}$$

where $(1-\phi)/\phi$ is the number of inactive households per active household in each country and endowments are adjusted by a factor $1/\phi$ because active households now own proportionally more productive assets per capita; κ^n again denotes the transfer that decentralizes the allocation corresponding to the social planner's problem with unit Pareto weights under freely floating exchange rates.[15] Inactive households maximize (21) subject to the constraint

$$\hat{C}_T^n(i,\omega) + P_N^n(\omega)\hat{C}_N^n(i,\omega) \le P_2^n(\omega)e^{-\mu^n}, \tag{23}$$

where hats denote the consumption of inactive households. The inactive household's problem is thus simply to split the real payoff of their nominal bond (after paying the inflation tax) between traded and nontraded consumption. As before, the government of the stabilizing country stabilizes its exchange rate with the target country using state-contingent taxes.

The punchline is that currency stabilization in this much richer model of exchange rate determination works in the same way as in our simple model with productivity shocks. Solving the model yields

$$\begin{aligned}\lambda^m =& -\frac{(1-\phi)\gamma^2\tau}{\phi\left(\phi(1-\tau)+\gamma\tau\right)}\bar{\mu} - \frac{(1-\phi)(1-\tau)\gamma}{\phi(1-\tau)+\gamma\tau}\mu^m - \frac{\gamma\tau(\gamma-1)}{\gamma\tau+(1-\tau)\phi}\bar{\chi} - \frac{(1-\tau)(\gamma-1)\phi}{\gamma\tau+(1-\tau)\phi}\chi^m \\ &+ (1-\theta^m)\zeta\frac{\gamma(1-\tau)(1-\phi)}{\phi(1-\tau)+\gamma\tau}\left(\mu^m-\mu^t\right) + (1-\theta^m)\,\zeta\frac{(\gamma-1)(1-\tau)\phi}{\gamma\tau+(1-\tau)\phi}\left(\chi^m-\chi^t\right)\end{aligned}$$

and

$$\begin{aligned}\lambda_T =& -\gamma\left(\frac{1-\phi}{\phi}\right)\bar{\mu} - (\gamma-1)\bar{\chi} \\ &+ \zeta\frac{\theta^m(1-\tau)}{\gamma\tau}\left(\gamma(1-\phi)\left(\mu^t-\mu^m\right) + \phi(\gamma-1)\left(\chi^t-\chi^m\right)\right),\end{aligned}$$

where $\bar{\mu} = \sum_n \theta^n\mu^n$ and $\bar{\chi} = \sum_n \theta^n\chi^n$ are the weighted sums of inflation and preference shocks in all countries, respectively, and we suppress notation relating to productivity shocks to save space.

The first lines in both expressions show the price of country m's consumption index and the shadow price of traded goods in the freely floating regime. Both inflation and preference shocks generate a relationship between exchange rates and the shadow price of traded goods identical to that in section 2: Shocks that depreciate a country's currency prompt it to export more traded goods and lower λ_T in proportion to the country's size.

First consider preference shocks: Preference shocks move exchange rates by shifting the level of utility derived from each unit of consumption. A high preference shock reduces the marginal utility of households' consumption and depreciates the country's price of consumption. Again, risk-sharing with households in other countries then compels domestic households to ship traded

[15] The budget constraint in the stabilizing country is analogous and shown in Appendix C.2.

goods to the rest of the world, transmitting part of the shock to other countries.

Inflation shocks affect exchange rates by shifting resources within a given country away from inactive households, who are excluded from financial markets (and thus are irrelevant for prices in international markets), and towards active households whose marginal utilities price assets in international markets. A positive inflation shock to the price of traded goods in terms of the domestic currency thus acts as an "inflation tax" on inactive households: The higher the inflation shock, the less their nominal bonds are worth and the less these households are able to consume. Since inflation shocks have no bearing on the real resources available for consumption, this reduction of inactive households' wealth shifts resources towards the country's active households such that they receive more traded and nontraded goods, which depreciates the domestic price of consumption. At the same time, risk-sharing compels the active households to ship some of the additional traded goods to active households in other countries, thus transmitting part of the inflation shock to active households in other countries via the shadow price of traded goods.

The second line in both expressions above shows that, again, a currency stabilization makes the stabilizing country's price of consumption behave more like the target country's price, and that the stabilization lowers the weight of the target country's shock in λ_T, while simultaneously increasing the weight of the stabilizing country's shock. This change in the size of spillovers in the shadow price of traded goods again results from the fact that the stabilizing government raises taxes on the domestic consumption of traded goods whenever the target country appreciates (see Appendix A.6 for details). Active households in the stabilizing country thus ship additional traded goods to the rest of the world whenever the target country appreciates,

$$c_T^m - c_T^{m*} = \zeta \Xi_T^m \left(\gamma(1-\phi) \left(\mu^t - \mu^m \right) + \phi(\gamma - 1) \left(\chi^t - \chi^m \right) \right), \tag{24}$$

where Ξ_T^m is a positive constant shown in Appendix C.5.

It follows directly that all of our positive predictions about the effects of currency stabilizations carry over to this richer model

Proposition 6

In the model with market segmentation, inflation shocks, preference shocks, and productivity shocks with $\gamma > 1$,

1. *a country that stabilizes its real exchange rate relative to a target country sufficiently larger than itself lowers its risk-free rate, increases capital accumulation, and increases the average wage in its country relative to the target country*

2. *if the stabilizing country is small ($\theta^m = 0$), the cost of the stabilization decreases with the size of the target country.*

3. *if a country becomes the target of a stabilization imposed by a large country ($\theta^m > 0$), its risk-free interest rate rises relative to the rest of the world, capital accumulation falls, and average wages fall relative to all other countries.*

Proof. See Appendix C.6. ■

In addition to reinforcing the main insights from our analysis of the frictionless model, this richer model also improves the quantitative implications of the model along the three dimensions outlined above: The combination of market segmentation, inflation shocks, and preference shocks loosens or even reverses the negative correlation between appreciations of the real exchange rate and aggregate consumption growth, lowers the correlation of aggregate consumption across countries, and increases the volatility of real and nominal exchange rates (Alvarez et al., 2002; Pavlova and Rigobon, 2007; Kollmann, 2012). All of our conclusions from section 2 thus carry over to this empirically viable model of exchange rate determination.

Beyond this particular model, we believe that the results stated in Proposition 6 are quite general and hold in a wide range of models where currency manipulation transmits itself through a wedge on the price of traded goods. As noted in the introduction, more general models could also allow governments to stabilize exchange rates by manipulating additional wedges on allocations within countries, such as the sectoral allocation of labor or the distribution of wealth across households. Within this broader class of models, it is possible to construct examples where stabilization of the real exchange rate is achieved by reducing rather than increasing exports in response to an appreciation by the target country. In those examples, stabilizations relative to larger countries continue to lower domestic interest rates and increase capital accumulation, but some of the other implications highlighted above may not generalize. In this sense, the first statement in Proposition 6 is the most general, while the second and third statements rely on the, we believe plausible, assumption that interventions in currency markets have their primary effect on allocations through their effect on trade and the prices of traded goods.

5 Conclusion

The majority of countries in the world, accounting for just under half of world GDP, stabilize their real or nominal exchange rate relative to a target currency. Almost all of these stabilizations target the U.S. dollar, with few exceptions that all target the euro. Although currency stabilizations are possibly the most pervasive form of currency market interventions, existing theories give relatively little guidance on the effects of such stabilizations, regarding what might be special about the U.S. dollar as a target currency, and how these stabilizations might affect the target country.

Building on a growing literature that views risk premia as the main driving force behind large and persistent differences in interest rates across developed economies, we propose a novel, risk-based transmission mechanism for the effects of interventions in currency markets: Policies that systematically induce a country's currency to appreciate in bad times lower its risk premium in international markets, lower the country's risk-free interest rate, and increase domestic capital accumulation and wages.

We show that stabilizing a country's real exchange rate relative to a larger target economy is precisely such a policy. Moreover, stabilization relative to a larger economy is cheaper than one relative to a smaller economy in that it generates (rather than depletes) central bank reserves, offering a potential explanation why the vast majority of currency stabilizations in the data target the U.S. dollar, the currency of the largest economy in the world.

We also find that larger countries must expend more resources than smaller countries when implementing a stabilization. This finding may explain why larger economies tend to either stabilize relative to baskets of currencies or freely float their exchange rates. Our model further predicts that a large economy (such as China) stabilizing its exchange rate relative to a larger economy (such as the U.S.) diverts capital accumulation from the target country to itself, increasing domestic wages while decreasing wages in the target country—thus offering a novel perspective on the ongoing controversy about Chinese interventions in foreign exchange markets.

Importantly, we show that in the presence of rigid prices, stabilizations of the real exchange rate in our model map directly to the kinds of stabilizations of nominal exchange rates we observe in the data and can be implemented simply by announcing a set of nominal exchange rates and converting currency at these preannounced rates.

Taken together, we believe that our paper provides a novel way of thinking about the effects of currency stabilization. Along with highlighting potential welfare and political rationales for stabilizing, we give an account of the costs and benefits of important choices for the stabilization regime, such as the choice of target country, the effects of hard pegs versus floating bands, and stabilizations relative to a single country versus a basket of currencies.

Our work leaves open at least three avenues for future research. First, careful empirical work will be needed to identify the effect of currency manipulation in the data and disentangle the effects of altered risk premia from effects that may transmit themselves through more conventional channels, such as facilitating trade with the target country and establishing credibility for monetary policy. A prerequisite to making progress on these questions will be to identify (and control for) stabilizations that also involve manipulating the mean of the real exchange rate—a contentious political issue that has not been satisfactorily resolved in the empirical literature. Second, although many models have argued for risk premia as the main drivers of cross-sectional differences in interest rates, all of these papers, including our own, rely on standard preferences and thus generally imply that risk premia are quantitatively small. Recent work by Govillot et al. (2010), Colacito et al. (2016), and David et al. (2016) makes progress in this dimension by studying dynamic models with heterogeneous countries and recursive preferences. However, the literature is still far from rationalizing the large differences in mean returns across currencies we see in the data in a microfounded quantitative model. Finally, our analysis has focused exclusively on a simple problem where a single country unilaterally imposes an exchange rate stabilization, taking as given the policies of other countries. In analogy to a large literature on strategic interactions in trade policy (Bagwell and Staiger, 1999; Ossa, 2011), our prediction that exchange rate policy alters the equilibrium allocation of factors of production may also serve as the basis of a multilateral theory of strategic interactions in the choice of exchange rate regime.

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Appendix

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A Appendix to Section 2

A.1 Interest Rate Differentials

If financial markets are complete, prices outside of the manipulating country must coincide with ratios of shadow prices; and the shadow price of traded goods is the unique stochastic discount factor pricing Arrow-Debreu securities that pay one unit of the traded good in state ω in the world. We denote the shadow price of traded goods in state ω as $\Lambda_T(\omega)$ in the second period and we denote the shadow price of traded goods in the first period (prior to the realization of shocks) as Ψ_T.

To derive equation (2), we apply the consumption Euler equation (the expectation of the stochastic discount factor times the return) to the risk-free real interest rate in country n,

$$\mathbb{E}\left[\frac{\Lambda_2^f}{\Psi_T}\right] R^f = \mathbb{E}\left[\frac{\Lambda_2^h}{\Psi_T}\right] R^h = 1.$$

Λ_2^n/Ψ_T is the stochastic discount factor corresponding to marginal utilities of consumption (see Appendix A.3 for a derivation within an international real business cycle model) where the shadow price of traded goods in the first period, Ψ_T, is equal in all countries. Since we use the traded good as the numeraire in the model, we write $\Lambda_2^n = \Lambda_T P_2^n$ where P_2^n denotes country n's price level in the second period. The price in the first period is equal to one.

We perform the following calculations:

$$\begin{aligned}
& \mathbb{E}\left[\frac{\Lambda_T}{\Psi_T} P_2^f\right] R^f = \mathbb{E}\left[\frac{\Lambda_T}{\Psi_T} P_2^h\right] R^h \\
\Leftrightarrow \quad & \mathbb{E}\left[\exp\left[-\psi_T + \lambda_T + p_2^f + r^f\right]\right] = \mathbb{E}\left[\exp\left[-\psi_T + \lambda_T + p_2^h + r^h\right]\right] \\
\Leftrightarrow \quad & -\psi_T + \mathbb{E}\left[\lambda_T + p_2^f\right] + \frac{1}{2}\text{var}\left(\lambda_T\right) + \frac{1}{2}\text{var}\left(p_2^f\right) + \text{cov}\left(\lambda_T, p_2^f\right) + r^f \\
& = -\psi_T + \mathbb{E}\left[\lambda_T + p_2^h\right] + \frac{1}{2}\text{var}\left(\lambda_T\right) + \frac{1}{2}\text{var}\left(p_2^h\right) + \text{cov}\left(\lambda_T, p_2^h\right) + r^h
\end{aligned}$$

We cancel out ψ_T and var (λ_T) from both sides of the previous equation. The remaining variables are all second period variables. Hence, we drop the time subscripts below

$$\begin{aligned}
& \mathbb{E}\left[p^f\right] + \frac{1}{2}\text{var}\left(p^f\right) + \text{cov}\left(\lambda_T, p^f\right) + r^f = \mathbb{E}\left[p^h\right] + \frac{1}{2}\text{var}\left(p^h\right) + \text{cov}\left(\lambda_T, p^h\right) + r^h \\
\Leftrightarrow \quad & r^f + \mathbb{E}\left[p^f - p^h\right] + \frac{1}{2}\text{var}\left(p^f\right) - \frac{1}{2}\text{var}\left(p^h\right) - r^h = -\text{cov}\left(\lambda_T, p^f - p^h\right) \\
\Leftrightarrow \quad & r^f + \log\left(\mathbb{E}\left[P^f\right] / \mathbb{E}\left[P^h\right]\right) - r^h = -\text{cov}\left(\lambda_T, p^f - p^h\right)
\end{aligned}$$

We define

$$\Delta\mathbb{E}\left[s^{f,h}\right] = \log\left(\mathbb{E}\left[P^f\right] / \mathbb{E}\left[P^h\right]\right).$$

With this definition

$$r^f + \Delta\mathbb{E}\left[s^{f,h}\right] - r^h = -\text{cov}\left(\lambda_T, p^f - p^h\right).$$

A.2 Deriving the Price Index

The cost of one unit of consumption in country n is defined as

$$P^n = \arg\min C_T^n + P_N^n C_N^n \text{ s.t. } \left(C_T^n\right)^\tau \left(C_N^n\right)^{1-\tau} = 1 \tag{25}$$

for all households i. First order conditions imply

$$C_N = \frac{1}{P_N}\frac{1-\tau}{\tau}C_T$$

We solve for C_T^n by plugging this expression into $\left(C_T^n\right)^\tau \left(C_N^n\right)^{1-\tau} = 1$,

$$C_T^n = \left(\frac{\tau}{1-\tau}P_N^n\right)^{1-\tau}.$$

We plug this solution for traded consumption along with the solution for nontraded consumption back into equation (25) to derive the optimal price index,

$$P^n = \frac{\left(P_N^n\right)^{1-\tau}}{\tau^\tau(1-\tau)^{1-\tau}}.$$

The total value of consumption for households in country n is

$$P^n C^n = \left(\frac{\left(P_N^n\right)^{1-\tau}}{\tau^\tau(1-\tau)^{1-\tau}}\right)\left(\left(C_T^n\right)^\tau \left(C_N^n\right)^{1-\tau}\right) = \frac{C_T^n}{\tau}.$$

Similarly, we use the expression $P_N^n = \frac{1-\tau}{\tau}\frac{C_T^n}{C_N^n}$ to show that

$$C_T^n + P_N^n C_N^n = \frac{C_T^n}{\tau} = P^n C^n.$$

A.3 Equilibrium Conditions of Model in Section 2

In this subsection, we provide additional details about the model in section 2 and formally derive its equilibrium conditions. To avoid solving the optimization problem separately for households in the stabilizing country and households outside of the stabilizing country, we use generalized notation that allows the governments in all countries to impose state contingent taxes, $Z^n(\omega)$, and lump sum transfers, $\bar{Z}^n$, on their domestic households. However, the governments in the target and outside countries do not use these instruments. They set their state contingent taxes to one, $Z^t(\omega) = Z^o(\omega) = 1$, and the lump sum transfer to zero, $\bar{Z}^t = \bar{Z}^o = 0$. The government in the stabilizing country chooses the state contingent tax and the lump sum tax to stabilize its exchange rate with the target country as specified in section 2.

Households maximize their utility (3) subject to their budget constraint (4). Using the definition of the price index derived in Appendix A.2, we rewrite the budget constraint (4) with generalized notation,

$$\begin{aligned}&\int Z^n(\omega)Q(\omega)\left(C_T^n(\omega) + P_N^n(\omega)C_N^n(\omega)\right)g(\omega)d\omega \\ &\le \int Z^n(\omega)Q(\omega)P_N^n(\omega)\exp(\eta^n)\left(K^n\right)^\nu g(\omega)d\omega + 1 + \kappa^n + q(1-K^n) + \bar{Z}^n.\end{aligned} \tag{26}$$

To reiterate, the government in the stabilizing country uses reserves to fund the exchange rate stabilization, such that $\bar{Z}^m$ equalizes the marginal utility of wealth between the stabilizing country and the rest of the world (P2). Hence, given the appropriate transfer $\bar{Z}^m$, we can let Ψ_T denote the Lagrange multiplier on the budget constraint in all countries.

The first order conditions are

$$\tau\left(C^n\right)^{1-\gamma}\left(C_T^n\right)^{-1} = \Psi_T Q(\omega)Z^n(\omega) \tag{27}$$

$$(1-\tau)\left(C^n\right)^{1-\gamma}\left(C_N^n\right)^{-1} = \Psi_T Q(\omega)Z^n(\omega)P_N^n(\omega). \tag{28}$$

We define $\Lambda_T(\omega) = \tau\left(C^n(\omega)\right)^{1-\gamma}\left(C_T^n(\omega)\right)^{-1}$ to be the shadow price of traded goods in the target and outside countries. Setting $Z^n(\omega) = 1$ in the target and outside countries, we show that $Q(\omega) = \Lambda_T(\omega)/\Psi_T$. Applying the definition of $\Lambda_T(\omega)$ to the equation (27) for each country yields equations (9) and (10).

Let $\Lambda_N^n(\omega) = (1-\tau)\left(C^n(\omega)\right)^{1-\gamma}\left(C_N^n(\omega)\right)^{-1}$ represent the marginal utility of nontraded consumption. We apply this definition to the left hand side of equation (28) to derive equation (11). Additionally, we apply the definition of $\Lambda_N^n(\omega)$ to the right hand side of equation (28) to derive an expression for the relative price of nontraded goods,

$$P_N^n(\omega) = \frac{\Lambda_N^n(\omega)}{Z^n(\omega)\Lambda_T(\omega)}.$$

The first order condition with respect to capital accumulation is

$$\Psi_T q = \Psi_T \int Z^n(\omega)Q(\omega)P_N^n(\omega)e^{\eta^n}\nu\left(K^n\right)^{\nu-1}g(\omega)d\omega.$$

To derive equation (12), we multiply both sides of the previous equation by K^n, substitute $Y_N^n = \exp(\eta^n)\left(K^n\right)^{\nu}$, apply the definitions of $Q(\omega)$ and $P_N^n(\omega)$ and simplify,

$$K^n = \frac{\nu}{q}\int Z^n(\omega)\frac{\Lambda_T(\omega)}{\Psi_T}\frac{\Lambda_N^n(\omega)}{Z^n(\omega)\Lambda_T(\omega)}Y_N^n(\omega)g(\omega)d\omega = \frac{\nu}{q\Psi_T}\int \Lambda_N^n(\omega)Y_N^n(\omega)g(\omega)d\omega.$$

A.4 Log-linearized System of Equations

This subsection derives the log-linearized first order conditions for the model in section 2. To reiterate, equation (6) defines the resource constraint for traded goods. Equation (7) defines the (three) resource constraints for nontraded goods in each country, and equation (8) defines the resource constraint for capital goods. Equations (9), (10) and (11) define the three first order conditions with respect to traded consumption and the three first order conditions with respect to nontraded consumption. Finally, equation (12) defines the three Euler equations for capital investment in each country. In total, we derive a system of 14 equations. We log-linearize the model around the deterministic solution — the point at which the variances of all shocks are zero ($\sigma_{N,n} = 0$). At this point, the capital stock of all firms is fixed at a level $K^n = 1$, $Y_N^n = 1$ and $C_T = 1$.

The log-linear first order conditions for the target and outside countries are

$$(1-\gamma)\left(\tau c_T^n + (1-\tau)c_N^n\right) - c_T^n + \log\tau = \lambda_T$$
$$(1-\gamma)\left(\tau c_T^n + (1-\tau)c_N^n\right) - c_N^n + \log(1-\tau) = \lambda_N^n$$

The log-linear first order conditions for the stabilizing country are

$$(1-\gamma)\left(\tau c_T^m + (1-\tau)c_N^m\right) - c_T^m + \log\tau = \lambda_T + z$$
$$(1-\gamma)\left(\tau c_T^m + (1-\tau)c_N^m\right) - c_N^m + \log(1-\tau) = \lambda_N^m$$

where z is the log-linear expression for the state-contingent tax. The log-linear approximation of equation (12) is

$$\psi_T + \log(q) + k^n = \log[v] + \mathbb{E}\left[\lambda_N^n + y_N^n\right] + \frac{1}{2}var\left(\lambda_N^n + y_N^n\right).$$

Finally, the log-linear resource constraints are

$$c_N^n = \eta^n + \nu k^n = y_N^n,$$
$$\sum_{n=m,t,o} \theta^n c_T^n = 0,$$
$$\text{and} \sum_{n=m,t,o} \theta^n k^n = 0.$$

This set of 14 equations allows us to solve for the following 14 unknowns $\{k^n, c_N^n, c_T^n, \lambda_N^n\}_{n=m,t,o}$, λ_T and $\log(q) + \psi_T$. We solve for these endogenous variables in terms of the state variables $\{y_N^n\}$ for $n = m, t, o$. Note that the output of nontraded goods in each country, $y_N^n = \eta^n + \nu k^n$, sufficiently describes the state space in the second period. We do not need to keep track of the productivity shocks and the capital stock of each country separately.

We present the solution to the log-linear model in section 2 below. The following six equations define traded consumption and nontraded consumption,

$$c_T^m = \frac{(\gamma-1)(1-\tau)}{1+(\gamma-1)\tau}\left(\bar{y}_N - y_N^m\right) - \frac{1-\theta^m}{1+(\gamma-1)\tau}z$$
$$c_T^t = \frac{(\gamma-1)(1-\tau)}{1+(\gamma-1)\tau}\left(\bar{y}_N - y_N^t\right) + \frac{\theta^m}{1+(\gamma-1)\tau}z$$
$$c_T^o = \frac{(\gamma-1)(1-\tau)}{1+(\gamma-1)\tau}\left(\bar{y}_N - y_N^o\right) + \frac{\theta^m}{1+(\gamma-1)\tau}z$$
$$c_N^n = y_N^n \quad \forall n$$

The following four equations define the shadow prices of traded and nontraded consumption.

$$\lambda_T = -(1-\tau)(\gamma-1)\bar{y}_N - \theta^m z + \log[\tau]$$
$$\lambda_N^m = -\frac{(\gamma-1)^2(1-\tau)\tau}{1+(\gamma-1)\tau}\bar{y}_N - \frac{\gamma}{1+(\gamma-1)\tau}y_N^m + \frac{\tau(1-\theta^m)(\gamma-1)}{1+(\gamma-1)\tau}z + \log[1-\tau]$$
$$\lambda_N^t = -\frac{(\gamma-1)^2(1-\tau)\tau}{1+(\gamma-1)\tau}\bar{y}_N - \frac{\gamma}{1+(\gamma-1)\tau}y_N^t - \frac{\theta^m\tau(\gamma-1)}{1+(\gamma-t)\tau}z + \log[1-\tau]$$
$$\lambda_N^o = -\frac{(\gamma-1)^2(1-\tau)\tau}{1+(\gamma-1)\tau}\bar{y}_N - \frac{\gamma}{1+(\gamma-1)\tau}y_N^o - \frac{\theta^m\tau(\gamma-1)}{1+(\gamma-t)\tau}z + \log[1-\tau]$$

Finally, we use the three log-linear Euler equations, the expressions for λ_N^n and the log-linear resource constraint for capital to solve for the incentives to accumulate capital as well as the first period price of capital q. When $K^n = 1$, we have $y_N^n = \eta^n + \nu k^n = \eta^n$, $\mathbb{E}[y_N^n] = 0$. Hence, $var[y_N^n] = \sigma_N^n$.

It is also useful to keep track of the shadow prices of overall consumption in each country, since these shadow prices define the real exchange rates. We use the log-linear expression $\lambda^n = -\gamma\left(\tau c_T^n + (1-\tau)c_N^n\right)$ along with the solutions for traded and nontraded consumption, derived above.

$$\lambda^m = -\frac{(\gamma-1)(1-\tau)\gamma\tau}{1+(\gamma-1)\tau}\bar{y}_N - \frac{\gamma(1-\tau)}{1+(\gamma-1)\tau}y_N^m + \frac{(1-\theta^m)\gamma\tau}{1+(\gamma-1)\tau}z$$
$$\lambda^t = -\frac{(\gamma-1)(1-\tau)\gamma\tau}{1+(\gamma-1)\tau}\bar{y}_N - \frac{\gamma(1-\tau)}{1+(\gamma-1)\tau}y_N^t + \frac{\theta^m\gamma\tau}{1+(\gamma-1)\tau}z$$
$$\lambda^o = -\frac{(\gamma-1)(1-\tau)\gamma\tau}{1+(\gamma-1)\tau}\bar{y}_N - \frac{\gamma(1-\tau)}{1+(\gamma-1)\tau}y_N^o + \frac{\theta^m\gamma\tau}{1+(\gamma-1)\tau}z$$

A.5 Derivation of Equation (17)

To derive (17), we rearrange the Euler equation for capital accumulation (12) and obtain an equation for the differential incentives to accumulate capital, which is analogous to (2): take logs of both sides of equation (12), substitute $\lambda_N^n = p_N^n + \lambda_T$, and take differences across countries to obtain

$$k^{f*} - k^{h*} = \frac{1}{2}var\left(p_N^f + y_N^f\right) - \frac{1}{2}var\left(p_N^h + y_N^h\right) + cov\left(p_N^f + y_N^f - p_N^h - y_N^h, \lambda_T\right) \tag{29}$$

where we interpret $p_N^f + y_N^f$ as the value of nontraded output in terms of traded goods, or as the payoff of a unit of stock in the nontraded sector of country f. The solution of the model yields

$$p_N^{f*} + y_N^{f*} = \frac{(1-\tau)(\gamma-1)}{1+(\gamma-1)\tau}\left(\bar{y}_N - y_N^f\right).$$

It shows that differences in the payoff of stocks in the nontraded sector behave in the same way as exchange rates: when country f suffers a low productivity shock, its currency appreciates and the value of its firm's output in terms of traded goods increases. If country f is large, the same adverse productivity shock also raises λ_T, inducing a positive covariance between λ_T and the value of the firm's output.

Thus, larger countries not only have lower interest rates but also have incentives to accumulate higher capital-output ratios. Approximating around the deterministic steady where $k^n = 1$, we can show that the right-hand side of (29) simplifies to

$$k^{f*} - k^{h*} = \frac{(\gamma-1)^3(1-\tau)^2\tau}{1+(\gamma-1)\tau}\left(\theta^f - \theta^h\right)\sigma_N^2.$$

To derive (17), we combine this equation with (16).

A.6 Proof of Lemma 1

First, we solve for the state contingent taxes that implement the real exchange rate stabilization in the model in section 2, and then we derive an expression for the cost of the peg. We search for a state contingent tax of the form $Z(\omega) = \left(Y_N^m(\omega)/Y_N^t(\omega)\right)^a$ and we solve for the coefficient a that stabilizes the real exchange rate. In logs, this state contingent tax is

$$z = a\left(y_N^t - y_N^m\right).$$

We plug this expression for z into the log-linear solution of the model derived in Appendix A.4 and solve for the log real exchange rate,

$$s^{m,t} = \frac{\gamma(1-\tau)}{1+(\gamma-1)\tau}\left(y_N^t - y_N^m\right) + a\frac{\gamma\tau}{1+(\gamma-1)\tau}\left(y_N^m - y_N^t\right).$$

The first term on the right hand side is real exchange rate without intervention, $s^{m,t*}$, and the second term shows the effects of the stabilization policy. We choose a such that $s^{m,t} = (1-\zeta)s^{m,t*}$,

$$\frac{\gamma(1-\tau)}{1+(\gamma-1)\tau}\left(y_N^t - y_N^m\right) + a\frac{\gamma\tau}{1+(\gamma-1)\tau}\left(y_N^m - y_N^t\right) = (1-\zeta)\frac{\gamma(1-\tau)}{1+(\gamma-1)\tau}\left(y_N^t - y_N^m\right)$$
$$\Leftrightarrow \quad a = \zeta\frac{(1-\tau)}{\tau}$$

Next, we derive an expression for the cost of stabilization, ΔRes. Equation (5) shows that the lump-sum transfer $\bar{Z}$ is comprised of the net revenues from the state contingent tax and ΔRes. We plug equation (5) in the budget constraint of households in the stabilizing country given by equation (4) and simplify the result. This allows us to eliminate $Z(\omega)$ and $\bar{Z}$ from the budget constraint,

$$\int Q(\omega)\left(P^m(\omega)C^m(\omega)\right)d\omega \leq 1 + \int Q(\omega)P_N^m(\omega)\exp(\eta^m)\left(K^m\right)^\nu d\omega + \kappa^m + \Delta Res$$

We use the market clearing condition for nontraded goods, $C_N^m(\omega) = Y_N^m(\omega)$, to simplify the expression further

$$\int Q\left(\omega\right)C_T^m(\omega)d\omega \leq 1 + \kappa^m + \Delta Res \tag{30}$$

Finally, we need an expression for the lump-sum transfer that decentralizes the Social Planner's problem with unit Pareto weights, κ^m, in the freely floating exchange rate economy. In the freely floating exchange rate economy, there is no government intervention of any kind ($\Delta Res = 0$). Hence, we rearrange equation (30) and use asterisks to denote the equilibrium consumption in the freely floating exchange rate economy,

$$\kappa^m = \int Q^*(\omega)C_T^{m*}(\omega)d\omega - 1$$

The lump-sum transfer is the difference between the value of the household's traded consumption and the household's endowment of traded goods. We plug this expression for κ^m into equation (30) to derive the expression for ΔRes in Lemma 1.

A.7 Proof of Proposition 1

Using equation (2) and the solution of the model in Appendix A.4, we can write the interest rate differential between the stabilizing country and the target country as

$$
\begin{aligned}
r^m + \Delta\mathbb{E}s^{m,t} - r^t &= cov\left(\lambda_T, p^t - p^m\right) \\
&= \left(r^{m*} + \Delta\mathbb{E}s^{m,t*} - r^{t*}\right) - \zeta\frac{(1-\tau)^2\gamma\left(2\theta^m(1-\zeta) + \left(\theta^t - \theta^m\right)(\gamma-1)\tau\right)}{\tau\left(1+(\gamma-1)\tau\right)}\sigma_N^2.
\end{aligned}
$$

When the stabilizing country is smaller than the target country, $\theta^m < \theta^t$, the right-hand side of this expression implies the exchange rate stabilization decreases the risk free rate in the stabilizing country relative to the risk free rate in the target country.

We calculate the payoff of stocks in the nontraded sector and the use equation (29) to calculate the differential incentives to accumulate capital,

$$
k^m - k^t = k^{m*} - k^{t*} + \zeta\left(\frac{(\gamma-1)^2(1-\tau)^2\left((1-2\theta^m)(1-\zeta) + (\theta^t - \theta^m)(\gamma-1)\tau\right)}{\left(1+(\gamma-1)\tau\right)^2}\right)\sigma_N^2.
$$

The last term of the right hand side of this expression shows that incentives to accumulate capital in the stabilizing country increase relative to the target country as long as

$$
\theta^t > \theta^m + \frac{(1-2\theta^m)(1-\zeta)}{\tau(\gamma-1)}.
$$

Because firms are competitive, wages are given by the marginal product of labor. $w^n = (1-\nu)\exp\left(\eta^n\right)\left(K^n\right)^\nu$. Since the marginal product of labor rises with the level of capital accumulation, the exchange rate stabilization increases wages in the stabilizing country relative to all other countries.

A.8 Proof of Proposition 2

Equation (18) shows the cost of the stabilization is the difference in the value of traded consumption between the free floating exchange rate economy and stabilized exchange rate economy. Appendix A.3 shows that the price of a state contingent claim is

$$
Q(\omega) = \frac{\Lambda_T(\omega)}{\Psi_T}.
$$

We begin by deriving a second order log-linear approximation of the log value of traded consumption,

$$v_T = \log \mathbb{E}\left[\exp\left(\lambda_T + c_T^m - \psi_T\right)\right] = \mathbb{E}\left[\lambda_T + c_T^m - \psi_T\right] + \frac{1}{2} var\left[\lambda_T + c_T^m - \psi_T\right],$$

In order to evaluate this expression, we must solve for ψ_T.

Lemma 2

Ψ_T is the shadow price of a traded good prior to the realization of shocks and is given by,

$$\Psi_T = \mathbb{E}\left[\Lambda_T\right]$$

Proof. We integrate over the budget constraints (4) of all households in the economy,

$$\int_0^1 \int_\Omega Q(\omega) P^n(\omega) C(i,\omega) g(\omega) d\omega di = \int_0^1 \left(1 + q(1 - K^n) + \int_\Omega P_N^n(\omega) Y_N^n(\omega) g(\omega) d\omega + \kappa^n\right) di$$

where households are indexed by i.

Next, we plug in the resource constraint for capital, (8), the definition of the price index from Appendix A.2, the resource constraint for nontraded goods, (7) and simplify the right hand side,

$$\int_0^1 \int_\Omega Q(\omega) C_T(i,\omega) g(\omega) d\omega di = \int_0^1 1 di + \sum_n \theta^n \kappa^n + \theta^m \Delta Res = 1 + \theta^m \Delta Res,$$

The second equality comes from recognizing $\sum_n \theta^n \kappa^n = 0$ because the κ^n represent transfers between countries. Next, we swap the order of integration on the left hand side and use the market clearing condition given by equation (6),

$$\int_\Omega \left(Q(\omega) \int_0^1 C_T^n(i,\omega) di\right) g(\omega) d\omega = (1 + \theta^m \Delta Res) \int_\Omega Q(\omega) g(\omega) d\omega = 1 + \theta^m \Delta Res.$$

Dividing through by $1 + \theta^m \Delta Res$ shows $\int_\Omega Q(\omega) g(\omega) d\omega = \mathbb{E}\left[Q\right] = 1$.

We take expectations of the left and right hand sides of the following equation: $Q(\omega) = \Lambda_T(\omega)/\Psi_T$. This yields

$$\mathbb{E}\left[Q\right] = 1 = \mathbb{E}\left[\Lambda_T\right] \frac{1}{\Psi_T} \quad \Rightarrow \quad \Psi_T = \mathbb{E}\left[\Lambda_T\right],$$

which is the desired result. ■

Hence, $\psi_T = \mathbb{E}\lambda_T + \frac{1}{2}var\left(\lambda_T\right)$. We plug in equations (15) and (19) into υ_T and set $\theta^m = 0$. The change in the log value of traded consumption is

$$\upsilon_T - \upsilon_T^* = \frac{\left((\zeta + (\gamma-1)\tau) - \tau^2(1-\gamma)^2\theta^t\right)(1-\tau)^2\zeta\sigma_N^2}{\tau^2\left(1+(\gamma-1)\tau\right)^2}$$

This expression is decreasing in the size of the target country, and becomes negative if and only if the target country is large enough.

We also evaluate the derivative of $\upsilon_T - \upsilon_T^*$ with respect to the size of the stabilizing country at the point where the stabilizing country is small,

$$\frac{\partial(\upsilon_T - \upsilon_T^*)}{\partial\theta^m} = \zeta\frac{(\gamma-1)(1-\tau)^2\left(\theta^t + 2\zeta + 2(1+\theta^t)(\gamma-1)\tau\right)}{\tau\left(1+(\gamma-1)\tau\right)^2}\sigma_N^2 > 0$$

Hence, the cost of the stabilization increases locally with the size of the pegging country.

A.9 Proof of Proposition 3

Using equation (2) and the solution of the model in Appendix A.4, we can write the interest rate differential between the target and outside country as

$$r^t + \Delta\mathbb{E}s^{t,o} - r^o = cov\left(\lambda_T, p^o - p^t\right) = \left(r^{t*} + \Delta\mathbb{E}s^{t,o*} - r^{o*}\right) + \zeta\frac{\theta^m(1-\tau)^2\gamma}{\tau\left(1+(\gamma-1)\tau\right)}\sigma_N^2$$

which implies the exchange rate stabilization increases the risk free rate in the target country relative to the risk free rate in the outside country.

We calculate the payoff of stocks in the nontraded sector and the use equation (29) to calculate the differential incentives to accumulate capital,

$$k^t - k^o = k^{t*} - k^{o*} - \frac{\theta^m(\gamma-1)^2(1-\tau)^2}{\left(1+(\gamma-1)\tau\right)^2}\zeta\sigma_N^2$$

The last term on the right hand side shows that incentives to accumulate capital in the target country decrease relative to the outside country.

Because firms are competitive, wages are given by the marginal product of labor. Since the marginal product of labor rises with the level of capital accumulation, the exchange rate stabilization decreases wages in the target country relative to all other countries.

Next, we show that if the stabilizing country is smaller than the target country, $\theta^m < \theta^t$, then the stabilization lowers the volatility of consumption in the target country. The log-linear approximation of household consumption in the target country is, $c^t = \tau c_T^t + (1-\tau)c_N^t$. We use the expression for traded consumption derived in Appendix A.4 and the expression for the state-contingent tax derived in Appendix A.6 to derive the volatility of overall consumption in the target country,

$$var\left(c^t\right) = var\left(c^{t*}\right) - \zeta \frac{2\theta^m(1-\tau)^2\left(1-\theta^m\zeta + \left(\theta^t - \theta^m\right)(\gamma-1)\tau\right)}{\left(1+(\gamma-1)\tau\right)^2}\sigma_N^2.$$

Therefore, $var\left(c^t\right)$ decreases when a country stabilizes its exchange rate relative to the target country as long as the stabilizing country is smaller, $\theta^t > \theta^m$.

A.10 Welfare of Small Country in Absence of Valuation Effects

Equation (18) implies that if $\Delta Res = 0$, then the present value of the household's traded consumption after the stabilization program must equal the present value of the household's traded consumption in the freely floating exchange rate economy,

$$\int Q(\omega)C_T^m(\omega)g(\omega)d\omega = \int Q^*(\omega)C_T^{m*}(\omega)g(\omega)d\omega. \tag{31}$$

Households in the stabilizing country maximize their utility (3) subject to their budget constraint (4). Let Ψ_T^m denote the Lagrange multiplier on their budget constraint. The first order condition with respect to traded consumption in the stabilizing country is

$$\tau\left(C^m(\omega)\right)^{1-\gamma}\left(C_T^m(\omega)\right)^{-1} = \Psi_T^m Z(\omega)Q(\omega). \tag{32}$$

The first order condition with respect to nontraded consumption in the stabilizing country is

$$(1-\tau)\left(C^m(\omega)\right)^{1-\gamma}\left(C_N^m(\omega)\right)^{-1} = \Psi_T^m Q(\omega)Z(\omega)P_N^m(\omega). \tag{33}$$

Since the government in the stabilizing country no longer uses reserves to maintain the level of the log real exchange rate, the marginal utility of an additional unit of wealth in the stabilizing country (Ψ_T^m) may differ from the marginal utility of wealth in the rest of the world (Ψ_T). Recall, Appendix A.3 showed that $Q(\omega) = \Lambda_T(\omega)/\Psi_T$.

We derive a system of ten equations, which allows us to solve for ten endogenous variables as a function of the Lagrange multipliers (Ψ_T, Ψ_T^m) and the shocks to the nontraded sector. These ten endogenous variables are $\{C_N^n, C_T^n, \Lambda_N^n\}_{n\in\{m,t,o\}}$ and Λ_T. The system of equations is comprised of the resource constraints given by equations (6) - (7), the conditions of optimality for the target and outside countries given by equations (9) and (11), and the conditions of optimality for the stabilizing country given by equation (32) and (33). After solving for these variables, we derive two additional equations which allow us to solve for the Lagrange multipliers Ψ_T and Ψ_T^m.

The log-linear first order conditions for the target and outside countries remain unchanged from Appendix A.4. The log-linear first order conditions for the stabilizing country are

$$(1-\gamma)\left(\tau c_T^m + (1-\tau)c_N^m\right) - c_T^m + \log[\tau] = \psi_T^m + \lambda_T + z - \psi_T$$
$$(1-\gamma)\left(\tau c_T^m + (1-\tau)c_N^m\right) - c_N^m + \log[1-\tau] = \lambda_N^m$$

where z is the log-linear expression for the tax given by Lemma 1. $\lambda_N^m = \psi_T^m - \psi_T + \lambda_T + z + p_N^m$ is defined to be the marginal utility of non-traded consumption in the stabilizing country. Finally, the log-linear resource constraints are

$$c_N^n = \eta^n + \nu k^n = y_N^n,$$
$$\sum_{n=m,t,o} \theta^n c_T^n = \sum_{n=m,t,o} \theta^n y_T^n = \log[1].$$

We use this system of equations to solve for $\{c_N^n, c_T^n, \lambda_N^n\}_{n\in\{m,t,o\}}$ and λ_T in terms of the nontraded output in each country and the two Lagrange multipliers.

Now, we derive two equations to solve for the Lagrange multipliers. First, by following the proof in Appendix A.8, we show $\Psi_T = \mathbb{E}\left[\Lambda_T\right]$. Second, we use equation (31), which enforces the condition $\Delta Res = 0$. The log-linear approximations of these two equations are

$$\psi_T = \mathbb{E}\left[\lambda_T\right] + \frac{1}{2} var\left[\lambda_T\right],$$
$$\mathbb{E}\left[\lambda_T - \psi_T + c_T^m\right] + \frac{1}{2} var\left[\lambda_T - \psi_T + c_T^m\right] = \mathbb{E}\left[\lambda_T^* - \psi_T^* + c_T^{m*}\right] + \frac{1}{2} var\left[\lambda_T^* - \psi_T^* + c_T^{m*}\right].$$

We plug in the solutions for traded consumption, nontraded consumption and λ_T into this system of equations and solve for ψ_T and ψ_T^m. Unfortunately, these expressions are too long to be displayed in this appendix.

Households in the stabilizing country shift the level of their traded consumption in each state of the world,

$$c_T^m - c_T^{m*} = \zeta \frac{(1-\tau)(1-\theta^m)}{1+(\gamma-1)\tau}\left(y_N^t - y_N^m\right) + \frac{(1-\theta^m)}{1+(\gamma-1)\tau}\left(\psi_T - \psi_T^m\right) \quad (34)$$

Note that we recover equation (19) whenever $\psi_T^m = \psi_T$. We use this expression for traded consumption and plug in the expressions for ψ_T and ψ_T^m to derive the volatility of overall consumption in the stabilizing country,

$$\begin{aligned} var\left(c^m\right) &= var\left(\tau c_T^m + (1-\tau)c_N^m\right) \\ &= var\left(c^{m*}\right) + \zeta \frac{2(1-\theta^m)(1-\tau)^2\left((1-\theta^m)\zeta - 1 + \left(\theta^t - \theta^m\right)(\gamma-1)\tau\right)}{\left(1+(\gamma-1)\tau\right)^2}\sigma_N^2, \end{aligned}$$

The exchange rate stabilization increases the volatility of consumption in the stabilizing country whenever,

$$\theta^t > \theta^m + \frac{1-\left(1-\theta^m\right)\zeta}{(\gamma-1)\tau}.$$

A corollary of this result is that a small country ($\theta^m = 0$) that imposes a hard peg always increases the volatility of consumption of its households.

We study the effect of exchange rate stabilization on expected utility by calculating $(1-\gamma)U(i)$. As $(1-\gamma)U(i)$ increases, expected utility decreases. We plug in the solution for traded consumption (34) and the resource constraint for nontraded goods into the log-linear expression for total consumption, $c^m = \tau c_T^m + (1-\tau)c_N^m$. For household i in the stabilizing country, we calculate

$$\begin{aligned} \frac{d}{d\zeta}\log\left[(1-\gamma)U(i)\right] &= \frac{d}{d\zeta}\left[(1-\gamma)\mathbb{E}\left(c^m\right) + \frac{(1-\gamma)^2}{2}var\left(c^m\right)\right] \\ &= \frac{(\gamma-1)(1-\tau)^2\left(\left(1-\left(\theta^m\right)^2\right)\zeta + \theta^m\left(1+\theta^t-\theta^m\right)(\gamma-1)\tau\right)}{\tau\left(1+(\gamma-1)\tau\right)}\sigma_N^2 > 0. \end{aligned}$$

Hence

$$\frac{dU(i)}{d\zeta}\frac{1}{U(i)} > 0$$

Since $U(i) = \frac{1}{1-\gamma}\left(C^m\right)^{1-\gamma} < 0$, we multiply both sides of the previous inequality by $U(i)$ and we show $\frac{dU(i)}{d\zeta} < 0$. Stabilizing the real exchange rate decreases expected utility in the stabilizing country.

A.11 Proof of Proposition 4

This proof proceeds in three steps. First, we formally define the valuation effect and show how large the valuation effect needs to be such that household welfare increases in the stabilizing country. Next, we re-derive the conditions of optimality for the stabilizing country and the system of equations we use to solve for the endogenous variables in the economy when households in the stabilizing country face the budget constraint (20). Finally, we show that when households own the requisite number of risk-free bonds that decentralize the Pareto-efficient allocation of consumption under freely floating exchange rates at the time of the announcement of the stabilization policy, then there exists a $\overline{\gamma}$ such that for $\gamma > \overline{\gamma}$, stabilizing relative to a larger target country strictly increases the welfare of households in the stabilizing country.

Step 1. Given any budget constraint, we define the valuation effect as the (log) change in the value of the household's traded consumption from its value in the Pareto-efficient allocation under freely floating exchange rates. Let δ_T denote this *valuation effect*,

$$\int Q(\omega)C_T^m(\omega)g(\omega)d\omega = e^{\delta_T}\int Q^*(\omega)C_T^{m*}(\omega)g(\omega)d\omega \tag{35}$$

When δ_T equals zero, this equation reduces to equation (34). However, by allowing δ_T to vary, we can ask how large do the valuation effects from the stabilization policy need to be in order for household welfare to increase in the stabilizing country.

Often, it is easier to calculate the change in the household's total consumption, rather than just the traded component. Hence, we also define the change in the total value of the household's consumption bundle, δ_C,

$$\int Q(\omega)P^m(\omega)C^m(\omega)g(\omega)d\omega = e^{\delta_C}\int Q^*(\omega)P^{m*}(\omega)C^{m*}(\omega)g(\omega)d\omega \tag{36}$$

We begin by deriving consumption in the economy when we allow for arbitrary valuation effects. We calculate first order conditions following Appendix A.10. In fact, the first order conditions for traded and nontraded consumption in all countries, and the resource constraints for traded and nontraded goods are unaffected by any valuation effects. Valuation effects only shift the level of consumption. We use the same system of ten equations from Appendix A.10 to solve for the following ten endogenous variables as a function of the Lagrange multipliers and the shocks to the nontraded sector: $\{C_N^n, C_T^n, \Lambda_N^n\}_{n\in\{m,t,o\}}$ and Λ_T. Since the first order conditions are unaffected, the solutions to these endogenous variables as a function of y_N^n, ψ_T and ψ_T^m are unaffected as well.

However, valuation effects do change the solutions for the Lagrange multipliers Ψ_T and Ψ_T^m. We log-linearize the system of two equations comprised of $\Psi_T = \mathbb{E}[\Lambda_T]$ and equation (36) to solve for the Lagrange multipliers ψ_T and ψ_T^m. The log-linear approximation of equation (36) is,

$$\mathbb{E}\left[\lambda_T - \psi_T + p^m + c^m\right] + \frac{1}{2} var\left[\lambda_T + p^m + c^m\right] = \mathbb{E}\left[\lambda_T^* - \psi_T^* + p^{m*} + c_T^{m*}\right] + \frac{1}{2} var\left[\lambda_T^* + p^{m*} + c_T^{m*}\right] + \delta_C$$

where we have used the identity $Q(\omega) = \Lambda_T(\omega)/\Psi_T$. Appendix A.2 shows that $p^m = (1-\tau)p_N^m - \log\left[\tau^\tau(1-\tau)^{1-\tau}\right]$, and the expression for p_N^m is shown in Appendix A.10. We plug the solutions for ψ_T and ψ_T^m into our expressions for traded consumption to write c_T^m as a function of δ_C and nontraded output.

This allows us to solve for a cutoff value for δ_C as well as the valuation effect, δ_T, such that any valuation effect that is greater than the cutoff value will increase expected utility in the stabilizing country.

Lemma 3

In the model of section 2, a small country that announces a stabilization policy can increase its household's expected utility if the valuation effects from the stabilization policy are large enough,

$$\delta_T \geq \frac{\zeta^2(1-\tau)^2}{\tau^2\left(1+(\gamma-1)\tau\right)}\sigma_N^2$$

Proof. We directly solve for the change in $\log\left[(\gamma-1)U(i)\right]$ for household i in the stabilizing country. The consumption of traded goods in the stabilizing country is given by (34) and the consumption of nontraded goods is given by the resource constraint for nontraded goods, $y_N^m = c_N^m$. We plug in the solutions for ψ_T and ψ_T^m, which we derived above using $\Psi_T = \mathbb{E}\left[\Lambda_T\right]$ and equation (36). We calculate

$$\log\left[(1-\gamma)U(i)\right] = (1-\gamma)\mathbb{E}\left(\tau c_T^m + (1-\tau)c_N^m\right) + \frac{(1-\gamma)^2}{2} var\left(\tau c_T^m + (1-\tau)c_N^m\right)$$

The change in the expected utility from stabilization is,

$$\log\left[(1-\gamma)U(i)\right] - \left(\log\left[(1-\gamma)U(i)\right]\big|_{\zeta=0}\right) = -(\gamma-1)\tau\delta_C + \frac{\zeta^2(\gamma-1)(1-\tau)^2}{\tau\left(1+(\gamma-1)\tau\right)}\sigma_N^2$$

Since the coefficient in front of δ_C is negative, the expected utility of households in the stabilizing country is strictly increasing in δ_C. When δ_C is large enough, the right hand side of the equation is negative, which means the stabilization policy has increases the expected utility of the household.

Given any δ_C, we calculate the valuation effect δ_T using equation (35),

$$\delta_T = \mathbb{E}\left[\lambda_T - \psi_T + c_T^m\right] + \frac{1}{2}var\left[\lambda_T + c_T^m\right] - \mathbb{E}\left[\lambda_T^* - \psi_T^* + c_T^{m*}\right] - \frac{1}{2}var\left[\lambda_T^* + c_T^{m*}\right] = \delta_C$$

Since households consume a Cobb-Douglas aggregate of traded and nontraded goods, they will spend a constant fraction of their wealth on each type of good. A δ_C percent increase in household wealth will result in an equivalent percentage change in the amount the household spends on traded consumption. Hence, the cutoff valuation effect must be the same as the cutoff for δ_C. ■

Step 2. Next, we decentralize the household's problem in section 2 in the freely floating exchange rate economy using bonds and calculate the valuation effects from the stabilization program. We allow households to trade the risk-free bonds denominated in the consumption bundles of each country. These risk-free bonds pay $P^n(\omega) = \Lambda^n(\omega)/\Lambda_T(\omega)$ units of the traded good in the second period in state ω. The portfolio of real bonds held by households in country n pays

$$\sum_{l=m,t,o} B_l^n P^{l*}(\omega)$$

in state ω in the second period. B_l^n is the number of country l bonds held in the portfolio. Households in the stabilizing country face the budget constraint given by equation (20).

Since households bear the full cost of the exchange rate stabilization policy, the lump-sum transfer from the government is equal to the sum of state contingent tax revenues,

$$\overline{Z} = \int \left(Z(\omega) - 1\right) Q(\omega) \left(C_T^m(\omega) + P_N^m(\omega) C_N^m(\omega) - \sum_{l=m,t,o} B_l^m P^l(\omega) \right) g(\omega) d\omega.$$

We use this expression for $\overline{Z}$ to simplify the stabilizing country's household's budget constraint,

$$\int Q(\omega) \left(C_T^m(\omega) + P_N^m(\omega) C_N^m(\omega)\right) g(\omega) d\omega = \sum_{l=m,t,o} B_l^m \int Q(\omega) P^l(\omega) g(\omega) d\omega.$$

Hence, the total value of the household's consumption must equal the total value of the household's bond portfolio.

We solve for the portfolio weights for a household in the stabilizing country in the freely floating exchange rate economy by equating the portfolio payoff with the cost of household consumption in each state of the world.

Lemma 4

In the model in section 2, households in the freely floating exchange rate equilibrium hold long positions in their own country's bond and hold short positions in other countries' bonds,

$$B_n^n = \frac{(1-\theta^n)(\gamma-1)\tau}{\gamma} \text{ and } B_l^n = -\frac{\theta^l(\gamma-1)\tau}{\gamma} \text{ for } l \neq n$$

Proof. We log-linearize the payoff of the bond portfolio,

$$\sum_{l=m,t,o} B_l^n \frac{1}{\tau} p^{l*}.$$

We solve for the number of bonds held in the household's portfolio in the stabilizing country by equating the first derivatives of the portfolio payoff with the first derivatives of the total value of consumption, $p^{n*} + c^{n*}$, with respect to the vector of nontraded output in each country. ■

Step 3. The final step is to calculate the valuation effect when households hold the portfolio of risk-free bonds that we solved for in Lemma 4. The value of the household's bond portfolio in country n in terms of traded goods in the first period is

$$V_{Portfolio}^n = \sum_{l=m,t,o} B_l^n V_{Bond}^l \text{ where } V_{Bond}^l = \int Q(\omega) P^l(\omega) g(\omega) d\omega$$

where $Q(\omega) = \Lambda_T(\omega)/\Psi_T$. At the deterministic steady state, $V_{Portfolio}^n = V_{Bond}^n = 1/\tau$. Hence, the log-linear approximation of the value of the household's portfolio is

$$v_{Portfolio}^n + \log[\tau] = \sum_{l=m,t,o} B_l^n \left(v_{Bond}^l + \log[\tau]\right),$$

where the log value of the bond is,

$$v_{Bond}^l = \mathbb{E}\left[\lambda_T - \psi_T + p^l\right] + \frac{1}{2} var\left[\lambda_T - \psi_T + p^l\right].$$

Hence, the change in the value of the household's portfolio from announcing a real exchange rate stabilization of strength ζ, which is also the change in the total value of the household's consumption (δ_C), is

$$\delta_C = \sum_{l=m,t,o} B_l^n \left[v_{Bond}^l - \left(v_{Bond}^l|_{\zeta=0}\right)\right]$$

A small stabilizing country does not affect prices in the rest of the world. Hence, the only change in the value of the bond portfolio comes from the change in v^m_{Bond}. The valuation effect is,

$$\delta_T = \delta_C = \frac{\zeta(\gamma-1)(1-\tau)^2\left(\zeta(1+\gamma\tau)-\tau+\theta^t(\gamma-1)\tau^2\right)}{\left(1+(\gamma-1)\tau\right)^2}\sigma_N^2.$$

This valuation effect is larger than the cutoff derived in Lemma 3 whenever γ is large enough,

$$\begin{aligned}
&\delta_T \geq \frac{(1-\tau)^2}{\tau^2\left(1+(\gamma-1)\tau\right)}\sigma_N^2 \\
\Leftrightarrow &\tau^2\left(\zeta+\theta^t\tau\right)\gamma^2-\tau\left(\zeta(1-\tau)+(1+\zeta)\tau^2+2\theta^t\tau^3\right)\gamma-\zeta\left(1-\tau+\tau^2\right)+\tau^3\left(1+\theta^t\tau\right)\geq 0
\end{aligned}$$

The left hand side of this second inequality is quadratic in γ and is positive whenever γ is large enough.

A.12 Partial exchange rate stabilization

This sections formalizes the effects of partial exchange rate stabilization. In a first step, we use the partition to write the variance of exchange rates in the freely floating regime as

$$\begin{aligned}
\mathrm{var}[s^{*m,t}] &= \int_\Omega \left(s^{*m,t}-\mathbb{E}[s^{*m,t}|\{K_n\}]\right)^2 g(\omega)d\omega \\
&= \int_{\Omega_s} \left(s^{*m,t}-\mathbb{E}[s^{*m,t}|\{K_n\}]\right)^2 g(\omega)d\omega + \int_{\Omega_{-s}} \left(s^{*m,t}-\mathbb{E}[s^{*m,t}|\{K_n\}]\right)^2 g(\omega)d\omega \\
&= \mathrm{Prob}\left[\omega\in\Omega_s\right]\mathrm{var}\left[s^{*m,t}|\Omega_s\right]+\mathrm{Prob}\left[\omega\in\Omega_{-s}\right]\mathrm{var}\left[s^{*m,t}|\Omega_{-s}\right]
\end{aligned} \tag{37}$$

since the conditional mean in the two subregions of the state space are identical. By the same token, partial stabilization delivers a variance of the exchange rate of

$$\begin{aligned}
\mathrm{var}[s^{m,t}] &= \mathrm{Prob}\left[\omega\in\Omega_s\right]\mathrm{var}\left[s^{m,t}|\Omega_s\right]+\mathrm{Prob}\left[\omega\in\Omega_{-s}\right]\mathrm{var}\left[s^{m,t}|\Omega_{-s}\right] \\
&= \mathrm{Prob}\left[\omega\in\Omega_s\right]\mathrm{var}\left[(1-\zeta)(s^{*m,t}-\mathbb{E}[s^{*m,t}|\{K_n\}])|\Omega_s\right]+\mathrm{Prob}\left[\omega\in\Omega_{-s}\right]\mathrm{var}\left[s^{*m,t}|\Omega_{-s}\right] \\
&= \mathrm{Prob}\left[\omega\in\Omega_s\right](1-\zeta)^2\mathrm{var}\left[s^{*m,t}|\Omega_s\right]+\mathrm{Prob}\left[\omega\in\Omega_{-s}\right]\mathrm{var}\left[s^{*m,t}|\Omega_{-s}\right] \\
&< \mathrm{var}\left[s^{*m,t}\right]
\end{aligned} \tag{38}$$

With exchange rate stabilization of strength ζ, the interest rate differential given by equation

(2) becomes

$$\begin{aligned} r^m + \Delta\mathbb{E}[s^{m,t}] - r^t &= -\text{cov}\left[\lambda_T, s^{m,t}\right] \\ &= -\text{cov}\left[\lambda_T, (1-\zeta)s^{*m,t}\right] \\ &= -(1-\zeta)\text{cov}\left[\lambda_T, s^{*m,t}\right]. \end{aligned}$$

The effects of partial stabilization for interest rate differentials work in the same direction. Again using the fact that the conditional means are identical, we decompose the covariance into the following terms:

$$\begin{aligned} r^m + \Delta\mathbb{E}[s^{m,t}] - r^t = -\text{cov}\left[\lambda_T, s^{m,t}\right] &= -\int_{\Omega_s} \left(\lambda_T - \mathbb{E}\left[\lambda_T|\{K_n\}\right]\right)\left(s^{m,t} - \mathbb{E}\left[s^{m,t}|\{K_n\}\right]\right) g(\omega)d\omega \\ &= -\text{Prob}\left[\omega \in \Omega_s\right] \int_{\Omega_s} \left(\lambda_T - \mathbb{E}\left[\lambda_T|\{K_n\}\right]\right)\left(s^{m,t} - \mathbb{E}[s^{m,t}|\{K_n\}]\right) g_s(\omega)d\omega \\ &\quad - \text{Prob}\left[\omega \in \Omega_{-s}\right] \int_{\Omega_{-s}} \left(\lambda_T - \mathbb{E}\left[\lambda_T|\{K_n\}\right]\right)\left(s^{m,t} - \mathbb{E}[s^{m,t}|\{K_n\}]\right) g_{-s}(\omega)d\omega \\ &= -\text{Prob}\left[\omega \in \Omega_s\right] \int_{\Omega_s} \left(\lambda_T - \mathbb{E}\left[\lambda_T|\Omega_s, \{K_n\}\right]\right)\left(s^{m,t} - \mathbb{E}[s^{m,t}|\{K_n\}]\right) g_s(\omega)d\omega \\ &\quad - \text{Prob}\left[\omega \in \Omega_s\right]\left(\mathbb{E}\left[\lambda_T|\Omega_s, \{K_n\}\right] - \mathbb{E}\left[\lambda_T|\{K_n\}\right]\right) \int_{\Omega_s} \left(s^{m,t} - \mathbb{E}[s^{m,t}|\{K_n\}]\right) g_s(\omega)d\omega \\ &\quad - \text{Prob}\left[\omega \in \Omega_{-s}\right] \int_{\Omega_{-s}} \left(\lambda_T - \mathbb{E}\left[\lambda_T|\Omega_{-s}, \{K_n\}\right]\right)\left(s^{m,t} - \mathbb{E}[s^{m,t}|\{K_n\}]\right) g_{-s}(\omega)d\omega \\ &\quad - \text{Prob}\left[\omega \in \Omega_{-s}\right]\left(\mathbb{E}\left[\lambda_T|\Omega_{-s}, \{K_n\}\right] - \mathbb{E}\left[\lambda_T|\{K_n\}\right]\right) \int_{\Omega_{-s}} \left(s^{m,t} - \mathbb{E}[s^{m,t}|\{K_n\}]\right) g_{-s}(\omega)d\omega \\ &= -\text{Prob}\left[\omega \in \Omega_s\right] \int_{\Omega_s} \left(\lambda_T - \mathbb{E}\left[\lambda_T|\Omega_s, \{K_n\}\right]\right)\left(s^{m,t} - \mathbb{E}[s^{m,t}|\{K_n\}]\right) g_s(\omega)d\omega \\ &\quad - \text{Prob}\left[\omega \in \Omega_{-s}\right] \int_{\Omega_{-s}} \left(\lambda_T - \mathbb{E}\left[\lambda_T|\Omega_{-s}, \{K_n\}\right]\right)\left(s^{m,t} - \mathbb{E}[s^{m,t}|\{K_n\}]\right) g_{-s}(\omega)d\omega \\ &= -\text{Prob}\left[\omega \in \Omega_s\right]\text{cov}\left[\lambda_T, s^{m,t}|\Omega_s\right] - \text{Prob}\left[\omega \in \Omega_{-s}\right]\text{cov}\left[\lambda_T, s^{m,t}|\Omega_{-s}\right] \end{aligned}$$

where $g_s(\omega) = \dfrac{g(\omega)}{\text{Prob}\left[\omega \in \Omega_s\right]}$ and $g_{-s}(\omega) = \dfrac{g(\omega)}{\text{Prob}\left[\omega \in \Omega_{-s}\right]}$. The second-to-last step follows from the fact that the conditional means are identical and thus $\mathbb{E}\left[s^{m,t} - \mathbb{E}\left[s^{m,t}|\{K_n\}\right]|\Omega_s\right] = 0$.
With partial exchange rate stabilization, we get

$$\begin{aligned} r^m + \Delta\mathbb{E}[s^{m,t}] - r^t &= -\text{cov}\left[\lambda_T, s^{m,t}\right] \\ &= -\text{Prob}\left[\omega \in \Omega_s\right]\text{cov}\left[\lambda_T, s^{m,t}|\Omega_s\right] - \text{Prob}\left[\omega \in \Omega_{-s}\right]\text{cov}\left[\lambda_T, s^{m,t}|\Omega_{-s}\right] \\ &= -\text{Prob}\left[\omega \in \Omega_s\right](1-\zeta)\text{cov}\left[\lambda_T, s^{*m,t}|\Omega_s\right] - \text{Prob}\left[\omega \in \Omega_{-s}\right]\text{cov}\left[\lambda_T, s^{*m,t}|\Omega_{-s}\right]. \end{aligned}$$

Rearranging the last equation to

$$r^m + \Delta\mathbb{E}[s^{m,t}] - r^t = -\text{cov}\left[\lambda_T, s^{m,t}\right] = -\text{cov}\left[\lambda_T, s^{*m,t}\right] + \zeta\text{Prob}\left[\omega \in \Omega_s\right]\text{cov}\left[\lambda_T, s^{*m,t}|\Omega_s\right],$$

we see that the effects of partial stabilization are a milder version of currency stabilization discussed previously. In fact, partial stabilization of strength ζ in a subset of the state space corresponds to currency stabilization of strength $\zeta\text{Prob}\left[\omega \in \Omega_s\right]\text{cov}\left[\lambda_T, s^{*m,t}|\Omega_s\right]$.

A.13 Stabilization Relative to a Basket of Currencies

Our analysis above also extends directly to stabilizations relative to a basket of currencies. Consider a country that wishes to stabilize its real exchange rate with the basket

$$p^b = (1-w)p^t + wp^o$$

where w is the basket's weight on the outside country and $1-w$ the weight on the target country. Using (2) it is then easy to show that stabilizing relative to a basket of currencies has effects akin to a stabilization relative to a (hypothetical) country with a weighted average size of the basket's constituents:

$$r^m + \Delta\mathbb{E}s^{m,o} - r^o = \left(r^{t*} + \Delta\mathbb{E}s^{t,o*} - r^{o*}\right) - \zeta\frac{\gamma(1-\tau)^2\left((\bar{\theta}-\theta^m)(\gamma-1)\tau + \theta^m(2-w-2\bar{w}\zeta)\right)}{\tau\left(1+(\gamma-1)\tau\right)}\sigma_N^2$$

where $\bar{\theta} = w\theta^t + (1-w)\theta^o$ is the weighted average size of the basket's constituents and $\bar{w} = 1-(1-w)w$ is a positive constant less than one.

Although clearly a less effective means of lowering domestic interest rates than stabilizations relative to the largest economy in the world, stabilizing relative to a basket may be appealing for some countries, because it reduces price impact. When stabilizing relative to a basket, the stabilizing country's exports are less sensitive to shocks affecting only one of the two other countries, decreasing the volatility of its exports and thus lowering the stabilization's impact on world-market prices. For a large country, stabilizing relative to a basket may thus be cheaper to implement than stabilizing relative to the largest economy in the world, providing a potential rationale for China's recent moves towards targeting a basket of currencies rather than the US dollar.

A.14 Results with Feedback between Risk Premia and Capital Accumulation

In this section, we show that propositions from section 2 continue to hold when we allow for feedback between risk premia and capital accumulation. As in section 2, there exists a unit of capital in the world which can be allocated across the three countries. Allowing for endogenous capital accumulation changes the expected level of consumption. However, it does not affect the conditional covariance of consumption across countries. Hence, all results that depend on the covariances between asset payoffs and the shadow price of traded goods are not affected.

To solve for the endogenous capital stock, we return to the system of four equations that defined incentives for capital investment with an exogenous capital stock. These are the first order conditions for capital in each country (12) and the resource constraint for capital accumulation,

$$
\begin{aligned}
k^n &= \log[v] - \log[q] - \log[\psi_T] + \mathbb{E}\left[\lambda_N^n + y_N^n\right] + \frac{1}{2} var\left[\lambda_N^n + y_N^n\right] \qquad \forall n \\
0 &= \sum_n \theta^n k^n.
\end{aligned}
$$

We plug in the solution for λ_N^n from the model in section 2 to write λ_N^n as a function of the y_N^ns and then plug in $y_N^n = \eta + \nu k^n$ to write all the y_N^ns in terms of the capital stock and the productivity shock. We solve this system of four equations for k^m, k^t, k^o and q.

The final expression for log differences in capital accumulation is more complicated when we allow for endogenous capital accumulation. However, the qualitative results remain unchanged. In a freely floating exchange rate economy ($\zeta = 0$), the larger country still accumulates a higher capital per capita stock

$$
k^{m*} - k^{t*} = \frac{(\gamma-1)^3(1-\tau)^2\tau}{(1+(\gamma-1)\tau)\,(1+(\gamma-1)(1-\tau)\nu+(\gamma-1)\tau)} \left(\theta^m - \theta^t\right)\sigma_N^2
$$

By comparing this expression to the expression in Appendix (A.5), we observe the realized difference in capital accumulation is *smaller* than the differential incentives to accumulate capital. Capital flows towards countries with higher incentives to accumulate capital and away from countries with lower incentives to accumulate capital. The final capital allocation reflects this movement.

When capital is allowed to adjust endogenously, a country that imposes a hard exchange rate peg on a target country equalizes the per capita capital stock across the two countries,

$$\left[k^m - k^t|_{\zeta=1}\right] = k^{m*} - k^{t*} + \frac{(\gamma-1)^3(1-\tau)^2\tau\left(\theta^t-\theta^m\right)\sigma_N^2}{(1+(\gamma-1)\tau)\left(1+(\gamma-1)(1-\tau)\nu+(\gamma-1)\tau\right)} = 0.$$

Hence, a smaller country increases its domestic capital stock by stabilizing its real exchange rate relative to a larger target country.

B Appendix to Section 3

Section 3 extends the model in section 2 to allow for nominal rigidities in the traded goods sector. For convenience, we re-iterate the definitions and formulas introduced in section 3. Recall that $\tilde{P}_T^n$ denotes the nominal price of the traded good in country n's currency and that all goods consumed in a given country must be purchased using domestic currency. The central bank in each country controls the domestic money supply,

$$\tilde{P}_T^n P^n C^n = \tilde{P}_T^n C_T^n + \tilde{P}_T^n P_N^n C_N^n = \mathcal{M}^n,$$

where $\tilde{P}_T^n P^n$ is the nominal price of a unit of final consumption and $\tilde{P}_T^n P_N^n$ is the nominal price of a unit of nontraded consumption. The central bank in country n sets the domestic money supply $\mathcal{M}^n$.

In section 3, households trade a complete set of state contingent claims with their central banks denominated in local currency. Let $\tilde{Q}^n(\omega)$ be the stochastic discount factor denominated in the currency of country n. Households in country n face the following budget constraint,

$$\int \tilde{Q}^n(\omega)\left(\tilde{P}_T^n(\omega)P_N^n(\omega)C_N^n(i,\omega)+\tilde{P}_T^n(\omega)C_T^n(i,\omega)\right)g(\omega)d\omega = \quad (39)$$
$$\int \tilde{Q}^n(\omega)\left(\tilde{P}_T^n(\omega)P_N^n(\omega)Y_N^n(\omega)+\tilde{P}_T^n(\omega)\right)g(\omega)d\omega + \kappa^n$$

In section 3, the nominal price of the traded good is rigid, $\tilde{P}_T^n$. This friction allows the central bank in each country to control the relative price of nontraded goods and the consumption of traded goods by choosing $\mathcal{M}^n$.

We begin by deriving first order conditions for the households' problem in each country. Households maximize utility (3) subject to their budget constraint. The first order conditions with respect to traded consumption and nontraded consumption are:

$$\tau \left(C^n(\omega)\right)^{1-\gamma} \left(C_T^n(\omega)\right)^{-1} = \tilde{P}_T^n \tilde{Q}^n(\omega) \tag{40}$$

$$(1-\tau) \left(C^n(\omega)\right)^{1-\gamma} \left(C_N^n(\omega)\right)^{-1} = \tilde{P}_T^n \tilde{Q}^n(\omega) P_N^n(\omega) \tag{41}$$

These six first order conditions ($n = m, t, o$) along with the three resource constraints for non-traded goods, the resource constraint for traded goods and the money supply equation in each country comprise a system of 13 equations. Finally, to remain consistent with the setup in section 2, where governments in the target and outside countries do not alter the incentives to consume traded goods, we equalize the marginal utility of traded goods in the target and outside countries,

$$\tilde{P}_T^t \tilde{Q}^t = \tilde{P}_T^o \tilde{Q}^o.$$

This creates a system of 14 equations. These 14 equations implicitly define the following 14 endogenous variables: $\left\{C_T^n, C_N^n, \tilde{Q}^n, P_N^n\right\}_{n=m,t,o}$, $\mathcal{M}^t$ and $\mathcal{M}^o$. The state variables are the output of nontraded goods, Y_N^n, the nominal prices of the traded goods and the money supply in the stabilizing country.

Note that we must solve for $\mathcal{M}^t$ and $\mathcal{M}^o$ as a function of monetary policy in the stabilizing country, $\mathcal{M}^m$. Since there is no labor supply decision, the quantity of goods supplied in the economy cannot adjust to changes in monetary policy. Households can only respond to changes in the relative price of nontraded goods by importing or exporting traded goods. However, the total supply of traded goods in the model remains fixed at one. Hence, the relative prices of the nontraded goods in the target and outside countries are partially determined by the monetary policy decision in the stabilizing country.

To derive closed form solutions, we log-linearize this system of equations around the deterministic steady state used in section 2. In order to simplify expressions, we set $\tilde{P}_T^n = 1$. The log-linear system of equations is,

$$
\begin{aligned}
-(1-\tau)(\gamma-1)\tau c_N^n-\tau(1+(\gamma-1)\tau)c_T^n+\log\left[\tau^\tau\right] &= \tau(\tilde{q}^n+\tilde{p}_T^n) \quad \forall n\\
-(1-\tau)(\gamma-1)\tau c_T^n-\tau(1+(\gamma-1)\tau)c_N^n+\log\left[(1-\tau)^{1-\tau}\right] &= (1-\tau)(\tilde{q}^n+\tilde{p}_T^n+p_N^n) \quad \forall n\\
y_N^n &= c_N^n \quad \forall n\\
\sum_n \theta^n c_T^n &= 0\\
\tau\left(\tilde{p}_T^n+c_T^n\right)+(1-\tau)\left(\tilde{p}_T^n+p_N^n+c_T^n\right) &= \log[\mathcal{M}^n]+\log\left[(1-\tau)^{1-\tau}\tau^\tau\right] \quad \forall n\\
\tilde{p}_T^t\tilde{q}^t &= \tilde{p}_T^o\tilde{q}^o
\end{aligned}
$$

The consumption of traded goods in the stabilizing country is

$$c_T^m=\log[\mathcal{M}^m]-\tilde{p}_T^m+\log\left[\tau\right].$$

We derive the monetary policy function of the stabilizing country's central bank by setting $\tilde{p}_T^m=0$ and choosing $\log[\mathcal{M}^m]$ to match the consumption of traded goods given by equation (19).

C Appendix to Section 4

In this section, we provide additional details about the model in section 4 and formally derive its equilibrium conditions. Similar to Appendix A.3, we avoid solving the optimization problem separately for households in the stabilizing country and households outside of the stabilizing country by using generalized notation that allows the governments in all countries to impose state contingent taxes, $Z^n(\omega)$, and lump sum transfers, $\bar{Z}^n$, on their domestic households.

C.1 Equilibrium Consumption of Inactive Households

Inactive households in country n maximize utility, defined in equation (21), in each state of the world by splitting their wealth $\exp(-\mu^n)P^n(\omega)$ optimally between traded and nontraded consumption,

$$
\begin{aligned}
\max_{\hat{C}_T^n(\omega),\hat{C}_N^n(\omega)} &\frac{1}{1-\gamma}\left(\exp\left(\chi^n\right)\left(\hat{C}_T^n\left(\omega\right)\right)^\tau\left(\hat{C}_N^n\left(\omega\right)\right)^{1-\tau}\right)^{1-\gamma}\\
\text{s.t. } &\hat{C}_T^n(\omega)+P_N^n(\omega)\hat{C}_N^n(\omega)\leq\exp(-\mu^n)P^n(\omega),
\end{aligned}
$$

where hats indicate consumption by inactive households. We solve this problem by setting up a Lagrangian and taking first-order conditions with respect to $\hat{C}^n_T(\omega)$ and $\hat{C}^n_N(\omega)$. Inactive households then optimally consume the following bundle of traded and nontraded goods,

$$\hat{C}^n_T(\omega) = \exp(-\mu^n)\tau P^n(\omega), \quad \hat{C}^n_N(\omega) = \exp(-\mu^n)\frac{(1-\tau)P^n(\omega)}{P^n_N(\omega)}.$$

C.2 Equilibrium Consumption of Active Households

Active households own all the productive assets within the country and are short the nominal bonds owned by inactive households. They maximize their utility (21) subject to their budget constraint (stated for the target and outside countries in (22)),

$$\begin{aligned} &\int Z^n(\omega)Q(\omega)\left(P^n(\omega)C^n(\omega) + \frac{1-\phi}{\phi}P^n(\omega)e^{-\mu^n}\right)g(\omega)d\omega \\ \le&\frac{1}{\phi}\left(1+q-qK^n + \int Z^n(\omega)Q(\omega)P^n_N(\omega)\exp(\eta^n)\left(K^n\right)^{\nu}g(\omega)d\omega + \kappa^n + \bar{Z}^n\right). \end{aligned} \tag{42}$$

To reiterate, the governments in the target and outside countries do not manipulate the state contingent prices of traded goods ($Z^n(\omega) = 1$ and $\bar{Z}^n = 0$ for $n = t, o$). In the stabilizing country, the government uses reserves to fund the exchange rate stabilization as specified in section 2, so that $\bar{Z}^m$ equalizes the marginal utility of wealth between the stabilizing country and the rest of the world (P2). Hence, given the appropriate transfer $\bar{Z}^m$, we can let Ψ_T denote the Lagrange multiplier on the budget constraint in all countries.

The first order conditions of the active household's problem are

$$\tau\exp((1-\gamma)\chi^n)\left(C^n\right)^{1-\gamma}\left(C^n_T\right)^{-1} = \Psi_T Q(\omega)Z^n(\omega) \tag{43}$$

$$(1-\tau)\exp((1-\gamma)\chi^n)\left(C^n\right)^{1-\gamma}\left(C^n_N\right)^{-1} = \Psi_T Q(\omega)Z^n(\omega)P^n_N(\omega) \tag{44}$$

We define $\Lambda_T(\omega) = \tau\exp((1-\gamma)\chi^n)\left(C^n(\omega)\right)^{1-\gamma}\left(C^n_T(\omega)\right)^{-1}$ to be the shadow price of traded goods in the target and outside countries. Setting $Z^n(\omega) = 1$ in the target and outside countries, we show that $Q(\omega) = \Lambda_T(\omega)/\Psi_T$ still holds.

Let $\Lambda^n_N(\omega) = (1-\tau)\exp((1-\gamma)\chi^n)\left(C^n(\omega)\right)^{1-\gamma}\left(C^n_N(\omega)\right)^{-1}$ represent the marginal utility of nontraded consumption. We plug this definition into the left hand side of equation 44 to derive an expression for the relative price of nontraded goods,

$$P^n_N(\omega) = \frac{\Lambda^n_N(\omega)}{Z^n(\omega)\Lambda_T(\omega)}.$$

The first order condition with respect to capital accumulation is

$$\Psi_T q = \Psi_T \int Z^n(\omega)Q(\omega)P_N^n(\omega)e^{\eta^n}\nu\left(K^n\right)^{\nu-1}g(\omega)d\omega.$$

We multiply both sides of this equation by K^n, substitute $Y_N^n = \exp(\eta^n)\left(K^n\right)^\nu$, apply the definitions of $Q(\omega)$ and $P_N^n(\omega)$ and simplify,

$$K^n = \frac{\nu}{q}\int Z^n(\omega)\frac{\Lambda_T(\omega)}{\Psi_T}\frac{\Lambda_N^n(\omega)}{Z^n(\omega)\Lambda_T(\omega)}Y_N^n(\omega)g(\omega)d\omega = \frac{\nu}{q\Psi_T}\int \Lambda_N^n(\omega)Y_N^n(\omega)g(\omega)d\omega. \tag{45}$$

C.3 Log-linearized System of Equations

The following subsection derives the log-linearized first order conditions for the generalized model in section 4. Equation (6) defines the resource constraint for traded goods. Equation (7) defines the (three) resource constraints for nontraded goods in each country, and equation (8) defines the resource constraint for capital goods. Equations (43), (44) define the three first order conditions with respect to traded consumption and the three first order conditions with respect to nontraded consumption. Equation (45) defines the three Euler equations for capital investment in each country. In total, we derive a system of 14 equations. We log-linearize the model around the deterministic solution — the point at which the variances of all shocks are zero ($\sigma_{\chi,n}, \sigma_{N,n}, \tilde{\sigma} = 0$). At this point, the capital stock of all firms is fixed at a level $K^n = 1$, $Y_N^n = 1$ and $C_T = 1$.

The log-linear first order conditions for the target and outside countries are

$$(1-\gamma)\chi^n + (1-\gamma)\left(\tau c_T^n + (1-\tau)c_N^n\right) - c_T^n + \log\tau = \lambda_T$$
$$(1-\gamma)\chi^n + (1-\gamma)\left(\tau c_T^n + (1-\tau)c_N^n\right) - c_N^n + \log(1-\tau) = \lambda_N^n$$

The log-linear first order conditions for the stabilizing country are

$$(1-\gamma)\chi^m + (1-\gamma)\left(\tau c_T^m + (1-\tau)c_N^m\right) - c_T^m + \log\tau = \lambda_T + z$$
$$(1-\gamma)\chi^m + (1-\gamma)\left(\tau c_T^m + (1-\tau)c_N^m\right) - c_N^m + \log(1-\tau) = \lambda_N^m$$

where z is the log-linear expression for the tax given by Lemma 1. The log-linear approximation of equation (45) is

$$\psi_T + \log(q) + k^n = \log[v] + \mathbb{E}\left[\lambda_N^n + y_N^n\right] + \frac{1}{2}var\left(\lambda_N^n + y_N^n\right).$$

We use the results from Appendices A.2 and C.1 to write the consumption of inactive households as a function of the relative price of nontraded goods. Additionally, we use the expression for $P_N^n(\omega)$ derived in Appendix C.2 to write the price of nontraded goods as a function of Lagrange multipliers. Hence, the log-linear resource constraints are

$$\phi c_N^n + (1-\phi)\left(-\mu^n - \tau\left(\lambda_N^n - \lambda_T - \log\left(\frac{1-\tau}{\tau}\right)\right)\right) = \eta^n + \nu k^n = y_N^n$$

$$\sum_{n=m,t,o} \theta^n \left[\phi c_T^n + (1-\phi)\left(-\mu^n - (1-\tau)\left(\lambda_N^n - \lambda_T - \log\left(\frac{1-\tau}{\tau}\right)\right)\right)\right] = \sum_{n=m,t,o} \theta^n y_{T,1}^n = 1$$

$$\sum_{n=m,t,o} \theta^n k^n = 1$$

This set of fourteen equations allows us to solve for the following fourteen unknowns $\{k^n, c_N^n, c_T^n, \lambda_N^n\}_{n=m,t,o}$, λ_T and $\log(q) + \psi_T$. We solve for these endogenous variables in terms of the following nine state variables $\{y_N^n, \mu^n, \chi^n\}$ for $n = m, t, o$.

C.4 State Contingent Taxes and the Cost of the Peg in the Model in Section 4

First, we solve for the state contingent taxes that implement the real exchange rate stabilization in the model in section 4, and then we derive an expression for the cost of the peg. We recover the results in Appendix A.6 by removing the market segmentation ($\phi = 1$), by setting the monetary shocks to zero ($\mu^n = 0$) and by setting the preference shocks to zero ($\chi^n = 0$).

In the model in section 4 with monetary shocks, segmented markets and preference shocks, we search for a state contingent tax of the form

$$Z(\omega) = \left(\frac{Y_N^m}{Y_N^t}\right)^{a_1} \left(\frac{\exp(-\mu^t)}{\exp(-\mu^m)}\right)^{a_2} \left(\frac{\exp(\chi^m)}{\exp(\chi^t)}\right)^{a_3}.$$

In logs, this state contingent tax is

$$z = a_1\left(y_N^t - y_N^m\right) + a_2\left(-\mu^t + \mu^m\right) + a_3\left(\chi^t - \chi^m\right).$$

We follow the procedure from Appendix A.6 to derive the coefficients a_1, a_2 and a_3 that stabilize the exchange rate. The following lemma summarizes these results.

Lemma 5

In the model in section 4, where real exchange rates fluctuate in response to inflation shocks, *market segmentation, and preference shocks, a tax on all assets paying off consumption goods in the stabilizing country of the form*

$$z(\omega) = \frac{\zeta(1-\tau)}{\tau\left(\tau+\phi(1-\tau)\right)}\left(y_N^m - y_N^t\right) + \frac{(1-\tau)(1-\phi)}{\tau\left(\tau+\phi(1-\tau)\right)}\left(\mu^m - \mu^t\right) + \frac{(\gamma-1)(1-\tau)\phi}{\gamma\tau\left(\tau+\phi(1-\tau)\right)}\left(\chi^m - \chi^t\right)$$

implements a real exchange rate stabilization of strength ζ*.*

Next, we derive the cost of the stabilization. We start with the budget constraint of the active household in the stabilizing country given by equation (42), and we identify the components of the lump-sum transfer, $\bar{Z}$. The lump-sum transfer $\bar{Z}$ consists of the lump-sum rebate of the state contingent tax revenues, κ_{Tax}^m, and an additional transfer from government reserves, ΔRes, which keeps the conditional expected level of the exchange rate constant.

Following the same procedure as in Appendix A.6, we can show that the cost of the peg is,

$$\Delta Res = \int Q\left(\omega\right)\left(\phi C_T^m(\omega) + (1-\phi)\hat{C}_T^m(\omega)\right)d\omega - \int Q\left(\omega\right)\left(\phi C_T^{m*}(\omega) + (1-\phi)\hat{C}_T^{m*}(\omega)\right)d\omega$$

In the model in section 4, the cost of the peg is the change in the value of the stabilizing country's total consumption of traded goods from active and inactive households.

C.5 Traded Consumption in the Model in Section 4

Equation (24) shows that traded consumption in the stabilizing country in the generalized model in section 4 is,

$$c_T^m - c_T^{m*} = \zeta \Xi_T^m \left(\gamma(1-\phi)(\mu^t - \mu^m) + \phi(\gamma-1)(\chi^t - \chi^m)\right)$$

where Ξ_T^m is a positive constant,

$$\Xi_T^m = \frac{(1-\theta^m)\left(\tau+(1-\tau)\phi\right)^2 + (1-\tau)(1-\phi)\left(\gamma\tau+(1-\tau)\phi\right)\left(1+(1-\tau)\right)(1-\tau)\phi}{\gamma\tau\left(\phi+(1-\tau)\phi\right)\left(\gamma\tau+\phi(1-\tau)\right)\left(\tau+(1-\tau)\phi\right)} > 0$$

C.6 Proof of Proposition 6

We first prove results for the internal effects of a real exchange rate stabilization. Using equation (2) and the solution of the model from Appendix C.3, we can write the interest rate differential between the stabilizing country and the target as

$$r^m + \Delta\mathbb{E}s^{m,t} - r^t = \left(r^{m*} + \Delta\mathbb{E}s^{m,t*} - r^{t*}\right) - \zeta\frac{\gamma(1-\tau)^2\left((\theta^t-\theta^m)\tau(\gamma-\phi)+2\phi\theta^m(1-\zeta)\right)}{\tau\phi\left(\gamma\tau+(1-\tau)\phi\right)}\sigma_N^2$$
$$- \zeta\frac{\gamma(1-\tau)(1-\phi)^2\left((\theta^t-\theta^m)\gamma\tau+2\phi\theta^m(1-\zeta)(1-\tau)\right)}{\tau\phi\left(\gamma\tau+(1-\tau)\phi\right)}\tilde{\sigma}^2$$
$$- \zeta\frac{\phi(1-\tau)(1-\gamma)^2\left((\theta^t-\theta^m)\gamma\tau+2\phi\theta^m(1-\zeta)(1-\tau)\right)}{\tau\gamma\left(\gamma\tau+(1-\tau)\phi\right)}\sigma_\chi^2,$$

which implies the exchange rate stabilization decreases the risk free rate in the stabilizing country relative to the risk free rate in the target country as long as the target country is larger than the stabilizing country, $\theta^t > \theta^m$.

We show that the relative incentives to accumulate capital in the stabilizing country increase with the size of the target country,

$$\frac{d}{d\theta^t}\left[\left(k^m - k^t\right) - \left(k^{m*} - k^{t*}\right)\right] = \frac{\zeta(\gamma-1)(1-\tau)^2\tau(\gamma-\phi)^2}{\left(\phi+(1-\phi)\tau\right)\left(\gamma\tau+(1-\tau)\phi\right)}\sigma_N^2$$
$$+ \frac{\zeta(\gamma-1)\gamma(1-\tau)\tau(\gamma-\phi)(1-\phi)^2}{\left(\phi+(1-\phi)\tau\right)\left(\gamma\tau+(1-\tau)\phi\right)}\tilde{\sigma}^2 + \frac{\zeta(\gamma-1)^3(1-\tau)\tau(\gamma-\phi)\phi^2}{\gamma\left(\phi+(1-\phi)\tau\right)\left(\gamma\tau+(1-\tau)\phi\right)}\sigma_\chi^2 > 0.$$

Thus, there exists a country size θ_{min} such that stabilizing the real exchange rate relative to any country larger than θ_{min} will increase the incentives to accumulate capital in the stabilizing country. Because firms are competitive, wages are given by the marginal product of labor. Hence, the exchange rate stabilization increases wages in the stabilizing country relative to all other countries.

To analyze the effect of exchange rate stabilization on the cost of the peg, we calculate changes in the log value of traded consumption in the stabilizing country. The log linear approximation of the total traded consumption in the stabilizing country from active and inactive households is: $\phi c_T^m + (1-\phi)\hat{c}_T^m$. We calculate the log value of total traded consumption

$$v_T = \mathbb{E}\left[\lambda_T - \psi_T + \phi c_T^m + (1-\phi)\hat{c}_T^m\right] + \frac{1}{2}var\left[\lambda_T - \psi_T + \phi c_T^m + (1-\phi)\hat{c}_T^m\right]$$

When the stabilizing country is small ($\theta^m = 0$), the cost of the stabilization decreases as the target country gets larger,

$$\frac{d}{d\theta^t}\left(v_T - v_T^*\right) = -\zeta\frac{(1-\tau)(1-\phi)^2(\gamma-\phi)}{\left(\phi+(\gamma-\phi)\tau\right)^2}\tilde{\sigma}^2 - \zeta\frac{(1-\tau)^2(\gamma-\phi)^2}{\left(\phi+(\gamma-\phi)\tau\right)^2}\sigma_N^2 - \zeta\frac{(\gamma-1)^2(1-\tau)(\gamma-\phi)\phi^2}{\gamma\left(\phi+(\gamma-\phi)\tau\right)^2}\sigma_\chi^2 < 0.$$

Hence, it is cheaper to stabilize relative to a larger country.

Next, we prove results for the external effects of a real exchange rate stabilization. Using equation (2) and the solution of the model from Appendix C.3, we can write interest rate differential between the target country and the outside country as

$$r^t + \Delta\mathbb{E}s^{t,o} - r^o = \left(r^{t*} + \Delta\mathbb{E}s^{t,o*} - r^{o*}\right) + \frac{\zeta\theta^m\gamma(1-\tau)^2}{\tau\left(\gamma\tau+\phi(1-\tau)\right)}\sigma_N^2 + \frac{\zeta\theta^m\gamma(1-\tau)^2(1-\phi)^2}{\tau\left(\gamma\tau+\phi(1-\tau)\right)}\tilde{\sigma}^2 + \frac{\theta^m\zeta(\gamma-1)^2(1-\tau)^2\phi^2}{\gamma\tau\left(\gamma\tau+(1-\tau)\phi\right)}\sigma_\chi^2,$$

which implies the exchange rate stabilization increases the risk free rate in the target country relative to the risk free rate in the outside country.

The differential incentives to accumulate capital in the target country relative to the outside country is given by

$$k^t - k^o = k^{t*} - k^{o*} - \frac{\theta^m\zeta(1-\tau)^2(\gamma-\phi)^2}{\left(\gamma\tau+(1-\tau)\phi\right)^2}\sigma_N^2 - \frac{\theta^m\gamma\zeta(1-\tau)(\gamma-\phi)(1-\phi)^2}{\left(\gamma\tau+(1-\tau)\phi\right)^2}\tilde{\sigma}^2 - \frac{\theta^m(\gamma-1)^2\zeta(1-\tau)(\gamma-\phi)\phi^2}{\gamma\left(\gamma\tau+(1-\tau)\phi\right)^2}\sigma_\chi^2.$$

Incentives to accumulate capital in the target country decrease relative to the outside country. Since the marginal product of labor rises with the level of capital accumulation, the exchange rate stabilization decreases wages in the target country relative to all other countries.