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## **Interest Rate Conundrums in the Twenty-First Century**

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### **Abstract**

A large literature argues that long-term interest rates appear to react far more to high-frequency (for example, daily or monthly) movements in short-term interest rates than is predicted by the standard expectations hypothesis. We find that, since 2000, such high-frequency “excess sensitivity” remains evident in U.S. data and has, if anything, grown stronger. By contrast, the positive association between low-frequency changes (such as those seen at a six- or twelve-month horizon) in short- and long-term interest rates, which was quite strong before 2000, has weakened substantially in recent years. As a result, “conundrums”—defined as six- or twelve-month periods in which short rates and long rates move in opposite directions—have become far more common since 2000. We argue that the puzzling combination of high-frequency excess sensitivity and low-frequency decoupling between short- and long-term rates can be understood using a model in which (i) shocks to short-term interest rates lead to a rise in term premia on long-term bonds and (ii) arbitrage capital moves slowly over time. We discuss the implications of our findings for interest rate predictability, the transmission of monetary policy, and the validity of high-frequency event study approaches for assessing the impact of monetary policy.

Key words: interest rates, conundrum, monetary policy transmission

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# 1 Introduction

The sensitivity of long-term interest rates to movements in short-term interest rates is a central feature of the term structure and plays a key role in the transmission of monetary policy to the real economy. Despite its importance, there is considerable disagreement about the extent to which long-term rates respond to short rates. For example, this basic disagreement features prominently in the ongoing debate about the extent to which easy monetary policy contributed to the 2003–2007 boom in house prices ([Bernanke, 2010](#); [Taylor, 2010](#)).

In this paper, we document an important and previously unrecognized fact about the term structure of interest rates: the positive association between low- frequency changes (say, at a 6-month or 12-month horizon) in short- and long- term interest rates was quite strong before 2000, but has weakened substantially in recent years. In contrast, the association between high- frequency changes (say, at a daily or 1-month horizon) in short- and long-term interest rates has strengthened.<sup>1</sup>

A stark example of such low-frequency decoupling is the period after June 2004 when the Federal Reserve raised its short-term policy rate, but longer-term yields and forward rates actually declined. This was famously described by then Fed Chairman Alan Greenspan as a “conundrum,” and has been discussed in many papers, including [Backus and Wright \(2007\)](#). One simple way to summarize our key finding is that we show that this 2004 episode was by no means unique, and that “conundrums”—defined as 6- or 12-month periods where short and long-term rates move in opposite directions—have become far more commonplace since 2000. For instance, since 2000, one- and ten-year yields have moved in the same direction in 61% of all 6-month periods. By contrast, from 1971 to 1999, the corresponding figure was 83%, and the difference is statistically significant ( $t = 4.0$ ).

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<sup>1</sup>Throughout the paper, we take the short-rate to be the 1-year nominal Treasury yield rather than the overnight federal funds rate targeted by the Federal Reserve.

We construct a simple model featuring term premium shocks and slow moving capital that can help explain this key stylized fact. In our model, a set of risk-averse investors can either invest in short- or long-term default-free bonds. Because there is always a risk that the prices of long-term bonds will unexpectedly decline tomorrow, risk-averse investors are only willing to hold long-term bonds if they offer an expected return in excess of the riskless rate on short-term bonds—i.e., a term premium. While monetary policy pins down the rate on short-term bonds (i.e., short-term bonds are available in perfectly elastic supply), long-term bonds are available in a given supply that varies stochastically over time. Since shocks to the supply of long-term bonds must be absorbed by risk-averse investors in equilibrium, supply shocks affect term premia on long-term bonds as in [Vayanos and Vila \(2009\)](#) and [Greenwood and Vayanos \(2014\)](#).

From a textbook expectations hypothesis perspective, movements in short rates should have only a minor impact on long-term rates, unless shocks to short rates are extremely persistent. Judged relative to this expectations hypothesis benchmark, many studies have found that the sensitivity of long rates to short rates is surprisingly high at daily and intra-daily frequencies, a finding has been referred to as the “excess sensitivity” puzzle ([Gürkaynak et al., 2005](#); [Hanson and Stein, 2015](#)). To match this high-frequency excess sensitivity, we assume that shocks to the net supply of long-term bonds are positively correlated with shocks to short-term interest rates. This implies that increases in short-term rates are associated with increases in the term premium component of long-term rates, generating excess sensitivity at high frequencies. We discuss several distinct amplification mechanisms that have been highlighted in the recent literature that can rationalize this reduced-form assumption, including investors who have a tendency to “reach for yield” in response to declines in short rates, mortgage convexity hedging flows, and over-extrapolative investors.

To match the low-frequency decoupling between short- and long-term interest rates, our second key assumption is that capital is slow-moving as in [Duffie \(2010\)](#): the previously mentioned supply shocks encounter a short-run demand curve that is a good deal steeper than the long-run demand curve. This slow-moving capital dynamic implies that excess sensitivity is greatest when measured at short horizons, enabling our model to match the key stylized fact we have emphasized. And, we show that the model can be used to make sense of the greater high-frequency excess sensitivity and lower-frequency decoupling between short- and long-term rates that we observe since 2000. Specifically, this pattern emerges naturally if shocks to short-term nominal rates have become slightly less persistent (say, because long-run inflation expectations have become better anchored) and if the kinds of supply-based amplification mechanisms noted above have grown in importance over time (say, because the mortgage market has become a larger part of the overall market for long-term bonds).

We also show that the post-2000 combination of high-frequency excess sensitivity and long-frequency decoupling between short- and long-term rates implies profitable trading strategies, whereby investors short the slope of the yield curve when short-rates have recently risen. We finally discuss implications of our findings for affine term structure models.

Our stylized fact has implications for the transmission of monetary policy. [Stein \(2013\)](#) points out that the excess sensitivity of long-term yields—whereby shocks to short rates move term premia in the same direction—will strengthen the real effects of monetary policy relative to a world where the expectation hypothesis holds. He refers to this as the “recruitment” transmission channel of monetary policy. Although we also find excess sensitivity at high-frequencies, our findings suggest that the “recruitment” transmission channel may not be quite as strong as [Stein \(2013\)](#) suggests because the resulting shifts in term premia tend to be quite transitory and, as such, should have only limited effects on aggregate demand.

Our stylized fact also has implications for how one should interpret event study evidence based on high-frequency changes. Economic news comes out in a lumpy manner, and the change in long-term interest rates around a particular piece of news is often used as a convenient measure of the long-run impact of that shock. However, if the impact on long-term rates tends to quickly wear off, then the shock's short-term and long-term impact will be quite different; and the event study approach will necessarily capture the short-term impact. As a recent example, in the week ending March 3, 2017, several Federal Reserve officials made comments that were widely interpreted as signaling a high likelihood of a rate hike at the upcoming FOMC meeting. Not surprisingly, short-term interest rates rose on the news. But in that week, 10-year yields and even 30-year forward rates rose over 15 basis points. It seems extremely unlikely that this news raised expected rates decades into the future by 15 basis points. The interpretation that would be most consistent with the stylized fact documented in this paper is that, in addition to raising short rates, the news gave a *temporary* boost to term premia. In that case, the event study methodology would give a misleading estimate of the longer-run impact of this news on long-term bond yields.

**Related literature** [Gürkaynak et al. \(2005\)](#) argue that the high sensitivity of daily changes in long-term *nominal* rates to daily changes in short-term *nominal* rates may reflect the fact that long-run inflation expectations are largely unmoored and are being continuously revised in light of incoming news. In other words, [Gürkaynak et al. \(2005\)](#) argue that excess sensitivity may work through a simple expectations hypothesis channel: the puzzle can be resolved once one allows for very persistent shocks to inflation expectations. However, as shown by [Beechey and Wright \(2009\)](#), [Abrahams et al. \(2016\)](#) and [Hanson and Stein \(2015\)](#), in the post-2000 period, the excess sensitivity of long-term nominal rates primarily reflect the excess sensitivity of long-term *real* rates to short-term nominal rates and not the sensitivity

of long-term break-even inflation. To the extent that one doubts that expected future real rates at distant horizons fluctuate meaningfully from day to day, this finding casts doubt on an expectations hypothesis explanation for excess sensitivity.

Thus, [Hanson and Stein \(2015\)](#) argue that excess sensitivity works through term premia on long-term bonds: shocks to short rates temporarily move term premia in the same direction. Specifically, they argue that the excess sensitivity of long-interest rates is due to shifts in the demand for long-term bonds from “yield-oriented” investors who target a certain level of yields in their investment portfolios and extend the maturity of their investments as rates decline. However, a variety of distinct supply and demand mechanisms may contribute to excess sensitivity. For instance, [Hanson \(2014\)](#) and [Malkhozov et al. \(2016\)](#) argue that negative shocks to short-term rates induce mortgage refinancing waves that lead to temporary decline in the duration of outstanding fixed-rate mortgages —i.e., a temporary reduction in the effective supply of long-term bonds. Relatedly, [Domanski et al. \(forthcoming\)](#) and [Shin \(2017\)](#) point to an amplification mechanism where insurers and pension funds need to match the duration of their assets and liabilities and consequently demand more long duration assets following a reduction in the level of interest rates. Our model shows that these kinds of supply and demand mechanisms imply that long-term interest rates tend to “over-react” to movements in short rates at high frequencies, but that this over-reaction is far less pronounced at the lower frequencies that should be of greatest interest to monetary policymakers.

The plan for the remainder of the paper is as follows. In [Section 2](#), we document our basic stylized fact. In [Section 3](#), we construct a simple reduced-form time-series model of interest rates that can match this stylized fact: the key takeaway here is that lagged changes in the level of the term structure are negatively associated with future changes in the slope.

Section 4 develops the simple model that we use to explain and interpret our key stylized fact. Section 5 discusses implication for trading strategies and affine term structure models. Finally, Section 6 concludes.

## 2 Main Finding: Yield decoupling at low frequencies

This section presents our main empirical finding: the association between low-frequency (6 months or longer) changes in short- and long-term interest rates was quite strong before 2000, but has weakened substantially in recent years; by contrast, the association between high-frequency (1 month or short) changes in short- and long- term interest rates has actually strengthened in recent years. We begin by documenting this basic fact in the U.S. We then repeat this same exercise for Canada, Germany, and the U.K. and show that the patterns in those countries are broadly in line with what we find in the U.S.

### 2.1 Yield decoupling in the US

We obtain historical data on the nominal and real Treasury yield curve from [Gürkaynak et al. \(2007\)](#); [Gürkaynak et al. \(2010\)](#). We focus on continuously compounded 10-year zero-coupon rates and 10-year instantaneous forward rates. We also decompose nominal yields into real yields and inflation compensation, defined as the difference between nominal and real yields. Our sample starts in 1971, which is when reliable data on 10-year nominal yields become available, and ends in 2016. As discussed in more detail below, we focus on the pre- and post-2000 samples. For real yields and inflation compensation, we only study the post-2000 sample, as data on TIPS start in 1999. All data are measured as of the end of the reference period (such as the last trading day of each month).

In standard monetary models, the central bank sets overnight nominal interest rates, and other rates move in response to expectations of the path of overnight rates. A large literature

argues that central banks in the U.S. and abroad have increasingly relied on communication—explicit signaling about the future path of overnight rates—as an active policy instrument (Gurkaynak et al., 2004; Lucca and Trebbi, 2009). To capture news about the near-term path of monetary policy that would not impact the current overnight rate, we follow the recent literature (Campbell et al., 2012; Gertler and Karadi, 2015; Hanson and Stein, 2015) and take the short-rate to be the 1-year Treasury rate. Using 1-year rates also limits any potential distortions stemming from the 2008–2015 period when overnight U.S. rates were stuck at the zero lower bound. By contrast, 1-year yields continued to fluctuate over the 2008–2015 period (Swanson and Williams, 2014).

To illustrate our key stylized fact, we begin by regressing changes in 10-year yields or forward rates on changes in 1-year yields. Specifically, we estimate regressions of the form:

$$y_{t+h}^{(10)} - y_t^{(10)} = \alpha_h + \beta_h(y_{t+h}^{(1)} - y_t^{(1)}) + \varepsilon_{t,t+h} \quad (2.1)$$

and

$$f_{t+h}^{(10)} - f_t^{(10)} = \alpha_h + \beta_h(y_{t+h}^{(1)} - y_t^{(1)}) + \varepsilon_{t,t+h}, \quad (2.2)$$

where  $y_t^{(n)}$  is the  $n$ -year zero-coupon rate on day  $t$  and  $f_t^{(n)}$  is the  $n$ -year-ahead instantaneous forward rate. Panel A in Table 1 reports estimated slope coefficients in (2.1) for zero-coupon nominals yields, real yields, and inflation compensation yields using daily data ( $h = 1$ ) and using monthly data with  $h = 1, 3, 6, 12$ —i.e., we report coefficients for daily, monthly, quarterly, semi-annual, and annual changes in yields. The results are shown for the pre-2000 and post-2000 subsamples separately. We base this sample split on a number of break-date tests that we will discuss shortly. Panel B reports the corresponding slope coefficients in (2.2) using changes in instantaneous forwards as the dependent variable.

At a daily frequency, there has been only modest change in the regression coefficients

between the pre-2000 and post-2000 subsamples. For instance, from panel B, the coefficient for daily changes in 10-year forward rates is  $\beta_h = 0.39$  in the pre-2000 subsample and  $\beta_h = 0.48$  in the post-2000 subsample. If anything, the coefficients for daily changes have risen somewhat in recent years. The coefficient on the 10-year zero yield from Panel A has increased from  $\beta_h = 0.56$  in the pre-2000 subsample to  $\beta_h = 0.85$  in the post-2000 subsample.

The magnitude of  $\beta_h$  at a daily frequency is a long-standing puzzle in the macroeconomic and finance literature. Short-term interest rates, are pinned down by current and expected monetary policy, and shocks to monetary policy and macroeconomic data are empirically short-lived. Because of the transitory nature of these shocks, long-term forwards should not be very responsive. [Gürkaynak et al. \(2005\)](#) ascribe the excess sensitivity to the possibility that inflation expectations may be unanchored. [Beechey and Wright \(2009\)](#) and [Hanson and Stein \(2015\)](#) find that much of the excess sensitivity owes to the response of real rates rather than inflation compensation. Consistent with this, we find that the majority of the response of instantaneous 10-year nominals can be accounted for by the response of instantaneous real rates in the post-2000 sample (panel B).

But looking down the rows, Table 1 also shows a new fact that has not previously been documented. The regression coefficients at lower frequencies are much lower in the post-2000 subsample than in the pre-2000 subsample. For example, the coefficient for yearly changes in 10-year forward rates is  $\beta_h = 0.39$  in the pre-2000 subsample (the same as for daily changes) but  $\beta_h = -0.19$  in the post-2000 subsample. Similarly, for the 10-year yields the coefficient at a yearly frequency is  $\beta_h = 0.56$  in the pre-2000 subsample and  $\beta_h = 0.18$  in the post-2000 subsample. More generally, in the post-2000 subsample, the size of the coefficient  $\beta_h$  clearly declines as the frequency over which yield changes are calculated. In terms of a decomposition between real yields and inflation compensation, we see that the majority of

the decline in  $\beta_h$  as a function of  $h$  is accounted for by the real yield component for the 10-year zero rate, and is equally split between real and inflation compensation components for the 10-year forward.

In summary, panels A and B of Table 1 show that, prior to 2000, there was strong tendency for short- and long-term interest rates to rise and fall together at both high- and low-frequencies. While the high-frequency relationship has persisted and has even grown stronger since 2000, at lower frequencies, there is little relationship between movements in short- and long-term rates in the recent period. In other words, the excess sensitivity puzzle appears to be only present at high-frequencies but not at lower-frequencies in recent years. Put differently, events such as “Greenspan’s conundrum” (as discussed by [Backus and Wright, 2007](#), for example)—the period following June 2004 when the Federal Reserve raised its short-term policy rate and longer-term yields and forward rates declined—have grown increasingly common since 2000. For instance, since 2000, one- and ten-year yields have moved in the same direction in 61% of all 6-month periods. By contrast, from 1971 to 1999, the corresponding figure was 83%, and the difference is statistically significant ( $t = 3.91$ ).

We use two other approaches to document our key stylized fact. The first is to estimate equations (2.1) and (2.2) using ten-year rolling windows. The estimated slope coefficients for yearly changes ( $h = 12$  with monthly data) are shown in Figure 1 for 10-year yields and forward rates. The coefficient declines substantially in more recent windows. The second approach is to test for a structural break in equations (2.1) and (2.2), allowing for a break date that is not known *a priori*. We use the test of [Andrews \(1993\)](#) who conducts a Chow test ([Chow, 1960](#)) at all possible break dates, and then takes the maximum of these test statistics. Figure 2 plots the Wald test statistic for each possible break date in equations (2.1) and (2.2). The strongest evidence for a break is in 2000 or 2001 in both equations (2.1)

and (2.2). Figure 2 shows the Andrews (1993) critical values for a null of no structural break and the break is clearly statistically significant.<sup>2</sup>

Before turning to the international evidence, it is important to note that the break around 2000 in Figure 2 is not a result of the period from December 2008 to December 2015 where the federal funds rate was stuck at the zero lower bound. Even if the sample period is ended in 2008, a break is still found around 2000. More specific to the results in Table 1, if the sample ends in December 2008, we find a daily  $\beta_h = 0.77$  and a monthly  $\beta_h = 0.20$ , which are essentially indistinguishable from the numbers in Table 1.

## 2.2 International Evidence

Our primary focus is on the U.S., but to better understand plausible economic mechanisms it is useful to consider whether this low-frequency yield decoupling is observed in other industrialized economies. In this subsection, we briefly explore international evidence on low-frequency yield-curve decoupling for the U.K., Germany and Canada. We obtain yield curve data from each country’s central bank website.

Table 2 reports estimates of equation (2.1) for the UK in panel A as well as Germany, and Canada in panel B. For the UK, estimates are broken out into real yields and inflation breakevens (panel A). Data for Germany starts in 1972, with exception of daily frequency which is only available around 2000. For the U.K. and Canada, data is available (both at daily and monthly frequency) starting in 1985-86.

The evidence for the UK is remarkably similar to the US evidence in Table 1. Before 2000, the daily coefficient ( $\beta_h = .44$ ) and the yearly coefficient ( $\beta_h = .38$ ) are very similar. After

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<sup>2</sup>In this paper, we are comparing pre-2000 and post-2000 data based on the estimated slope coefficients in equations (2.1) and (2.2) for yearly changes. There may be breaks in the slope coefficients for higher frequency changes at other dates, and for example, Thornton (forthcoming) argues that there is a break in the relationship between monthly changes in ten-year yields and monthly changes in the federal funds rate somewhat earlier in the sample.

2000, the daily sensitivity increases ( $\beta_h = .80$ ), and the yearly sensitivity drops ( $\beta_h = .29$ ). Because we have data on real yields prior to 2000 in the U.K., we can decompose the change in  $\beta_h$  into its real and inflation compensation components. As shown in Table 2, the inflation compensation component of  $\beta_h$  is quite stable across sample periods and frequency  $h$ . Thus, most of the changes in  $\beta_h$  are accounted for variations in the real yield components.

Moving to the bottom panel, we again observe very similar patterns to those in the U.S. and U.K. for Germany and Canada. In the pre-2000 sample,  $\beta_h$  is stable across frequencies. Post-2000 we observe a higher sensitivity at higher frequency and a decay in  $\beta_h$  at lower frequency.

For the remainder of this paper, we revert to considering only U.S. data.

### 3 Lead-Lag relationships in the yield curve

In this section, we pinpoint the time series properties of the term structure of interest rates that are needed to account for the low-frequency decoupling between short- and long-term rates documented above. In examining term structure dynamics, it is useful and customary to study the dynamics of principal components, especially level, slope, and curvature (Litterman and Scheinkman, 1991). Defining level as the 1-year zero coupon yield ( $L_t \equiv y_t^{(1)}$ ), the slope as the 10-year yield less the 1-year yield ( $S_t \equiv y_t^{(10)} - y_t^{(1)}$ ), and the curvature as the 5.5-year yield less the average of the 1- and 10-year yields ( $C_t = y_t^{(5.5)} - \frac{y_t^{(1)} + y_t^{(10)}}{2}$ ), the puzzle described in this paper can be restated as the observation that, since 2000, changes in level and slope have become *negatively* associated at low frequency, but not at high frequencies. The slope coefficient in equation (2.1) can be rewritten as:

$$\beta_h = 1 + \frac{\sum_{j=-h+1}^{h-1} (h - |j|) \text{Corr}(\Delta L_t, \Delta S_{t+j})}{\sum_{j=-h+1}^{h-1} (h - |j|) \text{Corr}(\Delta L_t, \Delta L_{t+j})} \sqrt{\frac{\text{Var}(\Delta S_t)}{\text{Var}(\Delta L_t)}}, \quad (3.1)$$

and so the decline in  $\beta_h$  at low, but not high, frequencies must logically mean that there was a shift in the autocorrelation of  $\Delta L_t$ , that the cross-correlation between changes in level and future changes in slope dropped and/or that the cross-correlation between changes in slope and future changes in levels has dropped. It turns out that both of these cross-correlations have declined. This can be seen from Figure 3, which plots the cross correlation between the slope (difference between 10- and 1-year Treasury yields) and the level (1-year yield) of the yield curve. The contemporaneous correlation is less negative post-2000, meaning that yields are more likely to move in lockstep. In contrast, cross-correlations between changes in level and slope are consistently negative post-2000, resulting in lower values of the  $\beta_h$  coefficient at low frequency.

### 3.1 Predicting level, slope and curvature

As another way of looking at the evolution of yield curve dynamics, we consider predictive regressions for the level, slope, and curvature of the yield curve. Most term structure models are Markovian with respect to the filtration given by current state variables, meaning that the conditional mean of future yields depends only on today's state variables. However, our key finding—that the correlation between changes in short- and long-term rates has declined at low-frequencies, even though the two remain highly correlated at a daily frequency—suggests that it may be useful to include additional lags when forecasting yields. We therefore consider

the following system of predictive regressions:

$$\begin{aligned}
L_t &= \delta_{0L} + \delta_{1L}L_{t-1} + \delta_{2L}S_{t-1} + \delta_{3L}C_{t-1} \\
&+ \delta_{4L}(L_{t-1} - L_{t-h}) + \delta_{5L}(S_{t-1} - S_{t-h}) + \delta_{6L}(C_{t-1} - C_{t-h}) + \varepsilon_{L,t}
\end{aligned} \tag{3.2}$$

$$\begin{aligned}
S_t &= \delta_{0S} + \delta_{1S}L_{t-1} + \delta_{2S}S_{t-1} + \delta_{3S}C_{t-1} \\
&+ \delta_{4S}(L_{t-1} - L_{t-h}) + \delta_{5S}(S_{t-1} - S_{t-h}) + \delta_{6S}(C_{t-1} - C_{t-h}) + \varepsilon_{S,t}
\end{aligned} \tag{3.3}$$

$$\begin{aligned}
C_t &= \delta_{0C} + \delta_{1C}L_{t-1} + \delta_{2C}S_{t-1} + \delta_{3C}C_{t-1} \\
&+ \delta_{4C}(L_{t-1} - L_{t-h}) + \delta_{5C}(S_{t-1} - S_{t-h}) + \delta_{6C}(C_{t-1} - C_{t-h}) + \varepsilon_{C,t},
\end{aligned} \tag{3.4}$$

where  $L_t$ ,  $S_t$  and  $C_t$  denote the level and slope at the end of month  $t$ , respectively, as defined above. All of these regressions include lags of level, slope, and curvature, as is standard, but also lagged changes of level, slope, and curvature. Several other authors have considered the use of lags in term structure models, including [Duffee \(2013\)](#), [Feunou and Fontaine \(2014\)](#), [Cochrane and Piazzesi \(2005\)](#), [Joslin et al. \(2013\)](#) and [Monfort and Pegoraro \(2013\)](#), although none of these have discussed the relationship to yield curve conundrums.

Table 3 reports estimates of various restricted forms of equations (3.2), (3.3), and (3.4) for  $h = 2$  and  $h = 12$  and for both the pre-2000 and post-2000 subsamples. All specifications include the first lag of level, slope and curvature. We include specifications omitting all lagged changes ( $\delta_4 = \delta_5 = \delta_6 = 0$ ), omitting lagged changes in slope and curvature ( $\delta_5 = \delta_6 = 0$ ), omitting lagged changes in curvature ( $\delta_6 = 0$ ) and including all predictors. Based on the AIC or BIC, the model with one lag of level, slope, and curvature and lagged changes in level is selected in the post-2000 subsample, while no lagged changes are needed in the pre-2000 subsample. In the post-2000 subsample, the lagged change in level is a highly significant negative predictor of the slope. Increases in the level of yields predict subsequent yield curve flattening, much more so than in the past.

The model in equations (3.2)-(3.4) can match the key stylized fact we documented above. These equations can be written jointly as a restricted vector autoregression in  $Y_t = (L_t, S_t, C_t)'$  of the form:

$$Y_t = \mu + A_1 Y_{t-1} + A_2 Y_{t-h} + \varepsilon_t. \quad (3.5)$$

Let  $\Gamma_{ij}(k)$  denote the  $ij$ th element of  $E(Y_t Y_{t-k}')$ . Given the estimated parameters from equations (3.2) and (3.3), we can work out  $\Gamma_{ij}(k)$  and hence obtain the model-implied values of  $\beta_h$  in equation (2.1) as:

$$\begin{aligned} \beta_h &= \frac{\text{Cov}(L_t - L_{t-h}, S_t - S_{t-h}) + \text{Var}(L_t - L_{t-h})}{\text{Var}(L_t - L_{t-h})} \\ &= 1 + \frac{2\Gamma_{12}(0) - \Gamma_{12}(h) - \Gamma_{12}(-h)}{2(\Gamma_{11}(0) - \Gamma_{11}(h))}. \end{aligned} \quad (3.6)$$

At an annual frequency, Table 1 reported estimated values of  $\beta_h$  in equation (2.1) of 0.56 and 0.18 for the pre-2000 and post-2000 subsamples, respectively. The last row of Table 3 includes the model-implied values of  $\beta_h$  in equations (3.2)-(3.4) with  $h = 12$ , in the pre-2000 and post-2000 subsamples, with various restrictions. We can see that, as a matter of statistical description, the model in equations (3.2)-(3.4) can match the basic stylized fact that we document in this paper, as long as the VAR includes lagged changes in level. If the VAR did not include lagged changes in level ( $\delta_4 = \delta_5 = \delta_6 = 0$ ), the model-implied values of  $\beta_h$  would be 0.51 and 0.50 for the pre-2000 and post-2000 subsamples, respectively, and so would be nowhere near what we observe in the data.

### 3.2 Specific episodes

The fact that the slope depends on lagged changes in level is important for explaining some recent episodes. We estimated equation (3.3) over the post-2000 sample with  $h = 12$ , and then re-estimated the equation restricting the coefficients on lagged changes to zero

( $\delta_{4S} = \delta_{5S} = \delta_{6S} = 0$ ). We then simulated the path of the ten-year yield for one year starting in May 2004 holding the level and curvature at their actual values, and using the residuals from the unrestricted estimation of equation (3.3), but setting the parameters to their estimated values in the *restricted* regression. The top panel of Figure 4 plots the actual value of one-year and ten-year yields over this period, and the values of the ten-year yield under this alternative scenario. This was the original “conundrum” period, as ten-year yields fell while short rates rose. However, had the slope not responded to lagged changes in the term structure, we can see that ten-year yields would actually have risen.

The bottom panel of Figure 4 does a similar exercise of simulating the path of the ten-year yield for one year starting in December 2007 under the alternative scenario that the slope does not respond to lagged changes. This was a period in which short rates were cut sharply, but ten-year yields actually rose for a while, in something of a conundrum in reverse. We can see that had the slope not responded to lagged changes, ten-year yields would have fallen sharply.

## 4 Explanations

In this section, we construct a simple model that is useful for explaining the key stylized fact we have documented in this paper: short-term and long-term yields no longer move strongly together at low frequencies, although they continue to move together at high frequencies. From an expectations hypothesis perspective, changes in short rates should have only a minor impact on long-term forward rates, unless there are highly persistent shocks to short rates. Thus, from this perspective, the puzzle is not the fact that, in recent years, short- and long-term yields do not move strongly together at low frequencies: basic term structure theory clearly tells us that this should be the case. Rather, the true puzzle is why long rates

were so sensitive to movements in short rates at all frequencies in the pre-2000 sample and why, at high frequencies, long rates continue to exhibit such strong sensitivity to this day. Authors including [Gürkaynak et al. \(2005\)](#) and [Hanson and Stein \(2015\)](#) have discussed this puzzling “excess sensitivity” of long rates to movements in short rates. However, to the best of our knowledge, we are the first to document the breakdown of such excess sensitivity at lower frequencies in the post-2000 sample.

In our model, time is discrete and infinite. A set of risk-averse investors can either hold long-term, perpetual bonds or in short-term bonds, both of which are default-free. Short-term bonds are available in perfectly elastic supply and the interest rate on short-term bonds from  $t$  to  $t + 1$ , denoted  $r_t$ , follows an exogenous stochastic process. Long-term bonds are available in a given net supply that must be absorbed by the investors in our model. In the model, this net supply  $s_t$  varies over time and also follows an exogenous stochastic process.

As in [Vayanos and Vila \(2009\)](#) and [Greenwood and Vayanos \(2014\)](#), shifts in the net supply of bonds impact the term premium on long-term bonds. The first key assumption in our model is that shocks to the net supply of long-term bonds are positively correlated with shocks to short-term interest rates. This assumption, which we discuss in detail below, implies that increases in short-term rates are associated with increases in term premia, generating “excess sensitivity” of long rates to movements in short rates. The second key assumption, following [Duffie \(2010\)](#), is that capital is slow-moving, so these supply shocks encounter a short-run demand curve that is a good deal steeper than the long-run demand curve. This slow-moving capital dynamic implies that excess sensitivity is greatest when measured at short horizons, enabling our model to match the key stylized fact we have emphasized throughout. Formally, our model is a close cousin of the model in [Greenwood et al. \(2016\)](#), who incorporate slow-moving capital effects into a model of the term structure.

As we discuss in more detail below, our model can match our key stylized fact—i.e., that the fact that  $\beta_h$  has fallen post-2000 for large  $h$  at the same time it has risen for small  $h$ —if (i) shocks to short-term nominal rates have become slightly less persistent and (ii) the kinds of supply-based amplification mechanisms that we emphasize have grown in importance.

## 4.1 Long-term bond

The long-term bond is a perpetuity that pays a coupon of  $K$  each period. Let  $P_{L,t}$  denote the price of this long-term bond at time  $t$ , so the return on long-term bonds from  $t$  to  $t + 1$  is:

$$1 + R_{L,t+1} = \frac{P_{L,t+1} + K}{P_{L,t}}.$$

To generate a tractable linear model, we use a [Campbell and Shiller \(1988\)](#) log-linear approximation to the return on this perpetuity. Specifically, defining  $\theta \equiv 1/(1 + K) < 1$ , the 1-period log return on the perpetuity is

$$r_{L,t+1} \equiv \ln(1 + R_{L,t+1}^L) \approx \overbrace{\frac{1}{1 - \theta}}^D y_t - \overbrace{\frac{\theta}{1 - \theta}}^{D-1} y_{t+1}, \quad (4.1)$$

where  $y_t$  is the log yield-to-maturity at time  $t$  and  $D = 1/(1 - \theta) = (K + 1)/K$  is the Macaulay duration when the bond is trading at par, obtained using the [Campbell and Shiller \(1988\)](#) log-linearization.<sup>3</sup>

Let  $r_t$  denote the interest rate on short-term bonds from  $t$  to  $t + 1$  and  $rx_{t+1} \equiv r_{L,t+1} - r_t$  denote the excess return on long-term bonds over short-term bonds from  $t$  to  $t + 1$ . Then, iterating equation (4.1) forward and taking expectations, the yield on long-term bonds is given by:

$$y_t = (1 - \theta) \sum_{j=0}^{\infty} \theta^j E_t [r_{t+j} + rx_{t+j+1}]. \quad (4.2)$$

---

<sup>3</sup>This log-linear approximation for default-free coupon-bearing bonds appears in Chapter 10 of [Campbell et al. \(1996\)](#).

Naturally, the long-term yield is the sum of (i) an expectations hypothesis component— $(1 - \theta) \sum_{j=0}^{\infty} \theta^j E_t [r_{t+j}]$ —that reflects expected future short rates and (ii) a term premium component— $(1 - \theta) \sum_{j=0}^{\infty} \theta^j E_t [rx_{t+j+1}]$ —that reflects expected future excess returns on long-term bonds.

## 4.2 Market participants

There are two types of risk-averse investors in the model, all with identical risk tolerance  $\tau$ , who differ solely in the frequency with which they can rebalance their portfolios.

The first group of investors are fast-moving arbitrageurs who are free to adjust their holdings of the long-bond and the riskless short-term bond each period. Fast moving investors are present in mass  $q$  and we denote their demand for long-term bonds at time  $t$  by  $b_t$ . Fast-moving arbitrageurs have mean-variance preferences over 1-period portfolio log returns. Thus, the demand their for long-term bonds is given by

$$b_t = \tau \frac{E_t [rx_{t+1}]}{Var_t [rx_{t+1}]}, \quad (4.3)$$

where

$$rx_{t+1} = r_{L,t+1} - r_t = \frac{1}{1 - \theta} y_t - \frac{\theta}{1 - \theta} y_{t+1} - r_t$$

is the excess return on long-term bonds from  $t$  to  $t + 1$ .

The second group of investors is a set of slow-moving investors who can only adjust their holdings of long-term and short-term bonds every  $k$  periods. Slow-moving investors are present in mass  $1 - q$ . A fraction  $1/k$  of these slow-moving investors is active each period and can reallocate their portfolios. However, they must then maintain this same portfolio allocation for the next  $k$  periods. As in [Duffie \(2010\)](#), this is a reduced form way to model the frictions that limit the speed of capital flows. Since they only rebalance their portfolios every  $k$  periods, slow-moving generalist investors have mean-variance preferences over their

$k$ -period *cumulative* portfolio excess return. Thus, the demand for long-term bonds from the subset of slow-moving investors who are active at time  $t$  is given by

$$d_t = \tau \frac{E_t [rx_{t \rightarrow t+k}]}{Var_t [rx_{t \rightarrow t+k}]},$$

where

$$rx_{t \rightarrow t+k} = \sum_{i=1}^k rx_{t+i} = \sum_{i=0}^{k-1} (y_{t+i} - r_{t+i}) - \frac{\theta}{1-\theta} (y_{t+k} - y_t)$$

is the  $k$ -period cumulative excess return on long-term bonds from  $t$  to  $t+k$ .

### 4.3 Risk factors

Investors in long-term bonds face two different types of risk. First, they are exposed to *interest rate risk*: they will suffer a capital loss on their long-term bond holdings if short-term rates unexpectedly rise. Second, they are exposed to *supply risk*: we assume that there are shocks to the supply of long-term bonds that impact long-term bond yields, holding fixed the expected future path of short-term interest rates—i.e., these supply shocks impact the term premium on long-term bonds.

We make the following concrete assumptions about the evolution these two risk factors:

**Short-term interest rates:** Short-term bonds are available in perfectly elastic supply. At time  $t$ , investors learn that short-term bonds will earn a log riskless return of  $r_t$  between time  $t$  and  $t+1$ . We assume that  $r_t$  follows an exogenous AR(1) process:

$$r_{t+1} = \bar{r} + \rho_r (r_t - \bar{r}) + \varepsilon_{r,t+1}, \tag{4.4}$$

where  $Var_t [\varepsilon_{r,t+1}] = \sigma_r^2$ . We think of the short-term rate as being determined outside the model by monetary policy.

**Supply:** We assume that the long-term bond is available in an exogenous, time-varying *net supply*  $s_t$  that must be held in equilibrium by fast arbitrageurs and slow-moving investors. This *net supply* equals the *gross supply* of long bonds *minus the demand* for long-term bonds from any unmodeled agents who participate in the bond market. Formally, we assume that  $s_t$  follows an AR(1) process:

$$s_{t+1} = \bar{s} + \rho_s (s_t - \bar{s}) + \varepsilon_{s,t+1} + C\varepsilon_{r,t+1}, \quad (4.5)$$

where  $Var_t [\varepsilon_{s,t+1}] = \sigma_s^2$  and  $Cov_t [\varepsilon_{s,t+1}, \varepsilon_{r,t+1}] = 0$ .

In the simple case where  $Var_t [\varepsilon_{s,t+1}] = 0$  so shocks to bond supply are entirely driven by shocks to short-term interest rate, one can show that

$$s_t = \bar{s} + C \left[ (r_t - \bar{r}) - (\rho_r - \rho_s) \sum_{j=0}^{\infty} \rho_s^j (r_{t-j-1} - \bar{r}) \right].$$

Thus, for instance, when  $\rho_s < \rho_r$ , bond supply depends on the difference between the current level of short rates and a geometrically-decaying, weighted-average of past short rates.

#### 4.4 Shocks to supply and the short rate

Crucially, we assume that  $C$  in equation (4.5) is positive, so that shocks to short rates are positively associated with shocks to the net supply of the long-term bonds. Our assumption that  $C > 0$  can be seen as a reduced-form way of capturing a variety of distinct supply-and-demand mechanisms that help explain why negative shocks to short-term rates are associated with declines in the term premium on long-term bonds. Mechanisms of this sort include:

**The “mortgage convexity” channel.** According to the mortgage convexity channel (Hanson, 2014; Malkhozov et al., 2016), negative shocks to short-term interest rates induce mortgage refinancing waves that lead to temporary declines in the duration of outstanding fixed-rate mortgages—i.e., a temporary reduction in the effective gross supply of long-term

bonds. As a result, declines in short-term interest rates are associated with temporary declines in the term premium on long-term bonds.

**The “reaching-for-yield” channel.** According to the reaching for yield channel ([Hanson and Stein, 2015](#)), negative shocks to short-term interest rates boost the demand for long-term bonds from “yield-oriented investors.” The idea is that, for either frictional or behavioral reasons, these yield-oriented investors care about the *current yield* on their portfolios over and above their expected portfolio return. Because expected mean reversion in short rates means that the yield curve is steeper when short rates are low, yield-oriented investors’ demand for long-term bonds is greater when short rates are low. Holding fixed the *gross supply* of long-term bonds, this means that the *net supply* of long-term bonds that must be held by fast arbitrageurs and slow-moving investors is lower when short rates are low. As a result, term premia on long-term bonds are low when short rates are low. More generally, low short rates could increase investor risk appetites, thereby depressing term premia, through a variety of channels ([Maddaloni and Peydró, 2011](#); [Becker and Ivashina, 2015](#); [Di Maggio and Kacperczyk, 2017](#); [Drechsler et al., 2014](#)).

**Asset and liability management by insurers and pensions.** [Domanski et al. \(forthcoming\)](#) and [Shin \(2017\)](#) point to a related amplification mechanism that stems from the desire of insurers and pensions to match the duration of their assets and liabilities. They argue that, as interest rates decline, the duration of insurers’ and pensions’ liabilities tends to increase more than the duration of their long-term bond holdings. As a result, the demand for long-term bonds from insurers and pensions rises following a decline in the level of interest rates. As in the reaching for yield channel, this means that the net supply of long-term bonds that must be held by fast arbitrageurs and slow-moving investors declines

when short-rates are low, thereby depressing term premia.

**A behavioral over-extrapolation mechanism.** According to this channel hinted at by Cieslak and Povala (2015) and Piazzesi et al. (2015), there is a set of biased investors who over-estimate the persistence of short-term interest rates. As a result, negative shocks to short rates lead this set of biased investors to demand more long-term bonds relative to rational investors who properly estimate the persistence of short rates. Again, this means that the net supply of long-term bonds that must be held by fast arbitrageurs and slow-moving investors declines when short-rates are low.

**Transitory flight-to-quality flows.** A final possibility is that there can be bad macro-financial news that leads the Federal Reserve to cut short-term rates and that, at the same time, induces transitory increases in the demand for long-term nominal bonds—i.e., flight to quality flows.

## 4.5 Equilibrium yields

There is a mass  $q$  of fast-moving arbitrageurs, each with demand  $b_t$ , and a mass  $(1 - q)k^{-1}$  of active slow-moving investors, each with demand  $d_t$ . These investors must accommodate the *active supply*, which is the total supply  $s_t$  of long-term bonds less any supply held off the market by inactive slow-moving investors,  $(1 - q)k^{-1} \sum_{j=1}^{k-1} d_{t-j}$ . Thus, the market-clearing condition for long-term bonds is

$$\underbrace{qb_t}_{\text{Fast demand}} + \underbrace{(1 - q)k^{-1}d_t}_{\text{Active slow demand}} = \underbrace{s_t}_{\text{Total bond supply}} - \underbrace{(1 - q)(k^{-1} \sum_{i=1}^{k-1} d_{t-i})}_{\text{Inactive slow holdings}}. \quad (4.6)$$

We conjecture that equilibrium yields  $y_t$  and the demands of active slow-moving investors  $d_t$  are linear functions of a state vector,  $\mathbf{x}_t$ , that includes the steady-state deviations of the

short-term interest rate, the net supply of bonds, and holdings of bonds by inactive slow-moving investors. Formally, we conjecture that long-term yields are

$$y_t = \alpha_0 + \alpha_1' \mathbf{x}_t \quad (4.7)$$

and that the demands of active slow-moving investors are

$$d_t = \delta_0 + \delta_1' \mathbf{x}_t, \quad (4.8)$$

where the  $(k + 1) \times 1$  dimensional state vector,  $\mathbf{x}_t$ , is given by

$$\mathbf{x}_t = [r_t - \bar{r}, s_t - \bar{s}, d_{t-1} - \delta_0, \dots, d_{t-(k-1)} - \delta_0]'. \quad (4.9)$$

These assumptions imply that the state vector follows a VAR(1) process

$$\mathbf{x}_{t+1} = \mathbf{\Gamma} \mathbf{x}_t + \epsilon_{t+1}, \quad (4.10)$$

where the transition matrix  $\mathbf{\Gamma}$  depends on slow-moving investor demands.

It can be shown that equilibrium yields take the form:

$$y_t = \underbrace{\left\{ \bar{r} + \left( \frac{1 - \theta}{1 - \rho_r \theta} \right) (r_t - \bar{r}) \right\}}_{\text{Expected future short rates}} \quad (4.11)$$

$$+ \underbrace{\left[ (q\tau)^{-1} V^{(1)} (\bar{s} - (1 - q) \delta_0) \right]}_{\text{Unconditional term premia}}$$

$$+ \underbrace{\left[ (q\tau)^{-1} V^{(1)} \left( \begin{array}{c} \frac{1 - \theta}{1 - \theta \rho_s} (s_t - \bar{s}) \\ - (1 - \theta) (1 - q) k^{-1} \sum_{i=0}^{\infty} \theta^i E_t [\sum_{j=0}^{k-1} (d_{t+i-j} - \delta_0)] \end{array} \right) \right]}_{\text{Conditional term premia}},$$

where  $V^{(1)} = \text{Var}_t [r x_{t+1}]$  is the equilibrium variance of 1-period excess returns on bonds. Solving the model involves numerically finding a solution to a system of  $2k$  polynomial equations in  $2k$  unknowns.

## 4.6 Matching the main finding

Consider the slope coefficient from a regression of  $y_{t+h} - y_t$  on  $r_{t+h} - r_t$  in the model:

$$\beta_h = \frac{Cov[y_{t+h} - y_t, r_{t+h} - r_t]}{Var[r_{t+h} - r_t]}. \quad (4.12)$$

This is the model counterpart of the regression coefficient in equation (5.9). Our main finding is that, empirically, this coefficient declines in the post-2000 sample at low frequencies (high  $h$ ) but actually rises somewhat at high frequencies (low  $h$ ).

Given a solution to the model, it is straightforward to work out the model-implied regression coefficients  $\beta_h$ . Specifically, since  $\mathbf{x}_t$  follows a VAR(1) in the model, if we let  $\mathbf{V} = Var[\mathbf{x}_t]$ , we have  $vec(\mathbf{V}) = (I - \mathbf{\Gamma} \otimes \mathbf{\Gamma})^{-1}vec(\mathbf{\Sigma})$ . We also have  $Cov[\mathbf{x}_{t+j}, \mathbf{x}'_t] = \mathbf{\Gamma}^j \mathbf{V}$  and  $Cov[\mathbf{x}_t, \mathbf{x}'_{t+j}] = \mathbf{V}(\mathbf{\Gamma}')^j$ , so that  $Var[\mathbf{x}_{t+h} - \mathbf{x}_t] = 2\mathbf{V} - \mathbf{\Gamma}^h \mathbf{V} - \mathbf{V}(\mathbf{\Gamma}')^h$ . Thus, letting  $\mathbf{e}_r$  denote the vector with a one in the first position and zeros everywhere else, we have

$$\beta_h = \frac{Cov[y_{t+h} - y_t, r_{t+h} - r_t]}{Var[r_{t+h} - r_t]} = \frac{Cov[\alpha'_1(\mathbf{x}_{t+h} - \mathbf{x}_t), (\mathbf{x}_{t+h} - \mathbf{x}_t)' \mathbf{e}_r]}{Var[(\mathbf{x}_{t+h} - \mathbf{x}_t)' \mathbf{e}_r]} = \frac{\alpha'_1(2\mathbf{V} - \mathbf{\Gamma}^h \mathbf{V} - \mathbf{V}(\mathbf{\Gamma}')^h) \mathbf{e}_r}{\mathbf{e}_r(2\mathbf{V} - \mathbf{\Gamma}^h \mathbf{V} - \mathbf{V}(\mathbf{\Gamma}')^h) \mathbf{e}_r}. \quad (4.13)$$

## 4.7 Special case without slow-moving capital

The model can be solved using pencil and paper in the special case in which there is no slow-moving capital (i.e., if either  $q = 1$  or  $k = 1$ ). We briefly explore this special case because it helps build intuition about the behavior of  $\beta_h$ . In this special case, the equilibrium yield on long-term bonds is

$$y_t = \overbrace{\left[ \bar{r} + \frac{1 - \theta}{1 - \rho_r \theta} (r_t - \bar{r}) \right]}^{\text{Expected future short rates}} + \overbrace{\tau^{-1} V^{(1)} \left[ \bar{s} + \frac{1 - \theta}{1 - \rho_s \theta} (s_t - \bar{s}) \right]}^{\text{Term premium}}, \quad (4.14)$$

where  $V^{(1)}$  is the smaller root of the following quadratic equation:

$$0 = \left[ \left( \frac{1}{\tau} \frac{\theta}{1 - \rho_s \theta} \sigma_s \right)^2 + \left( \tau^{-1} \frac{\theta C}{1 - \rho_s \theta} \sigma_r \right)^2 \right] \times (V^{(1)})^2 \quad (4.15)$$

$$+ \left[ 2 \left( \frac{\theta}{1 - \rho_r \theta} \sigma_r \right) \left( \tau^{-1} \frac{\theta C}{1 - \rho_s \theta} \sigma_r \right) - 1 \right] \times V^{(1)} + \left[ \left( \frac{\theta}{1 - \rho_r \theta} \sigma_r \right)^2 \right].$$

And, the model-implied regression coefficient is

$$\beta_h = \frac{Cov[y_{t+h} - y_t, r_{t+h} - r_t]}{Var[r_{t+h} - r_t]} \quad (4.16)$$

$$= \frac{\frac{1-\theta}{1-\rho_r\theta} \times Var[r_{t+h} - r_t] + \tau^{-1}V^{(1)}\frac{1-\theta}{1-\rho_s\theta} \times Cov[s_{t+h} - s_t, r_{t+h} - r_t]}{Var[r_{t+h} - r_t]}$$

$$= \frac{\frac{1-\theta}{1-\rho_r\theta} \times 2\frac{1-\rho_r^h}{1-\rho_r^2}\sigma_r^2 + \tau^{-1}V^{(1)}\frac{1-\theta}{1-\rho_s\theta} \times \frac{2-\rho_s^h-\rho_r^h}{1-\rho_s\rho_r}C\sigma_r^2}{2\frac{1-\rho_r^h}{1-\rho_r^2}\sigma_r^2}.$$

Suppose that  $C \geq 0$  and  $\rho_s \leq \rho_r$ , then, inspecting equation (4.16), it easy to see that:

(1)  $\beta_h$  is constant that is independent of  $h$  if (i)  $C = 0$  or (ii)  $C > 0$  and  $\rho_s = \rho_r$ ; and (2)  $\beta_h$  is a decreasing function of  $h$  if  $C > 0$  and  $\rho_s < \rho_r$ .

## 4.8 Calibration

We consider a illustrative calibration in which each period corresponds to a month. We assume the following parameters:

- **Persistence:**  $0.78 = \rho_s < \rho_r = 0.98$ . This implies that shocks to the short rate have a half-life of 2.9 years and that shocks to the net supply of long-term bond have a half-life of approximately 3 months.
- **Slow-moving capital:**  $q = 0.5$  and  $k = 18$ . Thus, 50% of the investors are slow-moving and only rebalance their portfolios every 18 months.
- **Supply process:**  $C = 70.5 > 0$  and  $\sigma_s^2 = 0$ . This means that supply shocks induced by shocks to the short-rate are the only reason term premia vary in the model.

- **Other parameters:**  $\sigma_r^2 = 0.0008$  (chosen to match the volatility of short-rates),  $\tau = 50$ , and  $\theta = 119/120$ , so the duration of the perpetuity is  $D = 1/(1 - \theta) = 120$  months—i.e., 10 years.

Figure 5 plots the model-implied coefficients  $\beta_h$  in equation (4.12) against horizon ( $h$ ) in months. We show this for the baseline calibration and in an alternate calibration where  $\rho_r = 0.99$  (implying that shocks to the short rate have a half-life of 5.7 years) and  $C = 20$ , with all other parameters unchanged. In both cases,  $\beta_h$  is a declining function of  $h$ . But the decline is much more pronounced in the baseline calibration.

The assumption that shocks to the net supply of long-term bonds are positively correlated with shocks to the short rate (i.e.,  $C > 0$ ) drives the excess sensitivity of long-term yields to short-term yields in our model. Furthermore, the assumption that  $C > 0$  is necessary to match the decline in  $\beta_h$  as a function of  $h$ . Indeed, if  $C = 0$ , then  $\beta_h = [(1 - \theta) / (1 - \rho_r \theta)]$ —i.e., the coefficient is simply the expectations hypothesis term in (4.11)—which is independent. This can be seen clearly in the special case without slow-moving capital as shown in equation (4.16).

Assuming that  $C > 0$  so there is excess sensitivity, two features of our model can help match the finding that  $\beta_h$  is a declining function of  $h$ .

First, if  $\rho_s < \rho_r$ , so that supply shocks are less persistent than short-rate shocks, then  $\beta_h$  will be a declining function of  $h$  even in the absence of slow-moving capital. This can be seen analytically by inspecting equation (4.16) in the case without slow-moving investors. Intuitively, if the supply shocks induced by innovations to short rates are more transient than the underlying shocks to short rates, then term premia will react more in the short-term than in the long-term. Thus, there will be greater excess sensitivity in the short-term than over the longer term. For instance, if shocks to net supply are driven by the mortgage convexity

channel, then it is natural to suppose that the resulting supply shocks are highly transient in nature (Hanson, 2014) implying that  $\rho_s \ll \rho_r$ .

Second, when there is slow-moving capital (i.e.,  $k > 1$  and  $q < 1$ ), the short-run demand curve for bonds is steeper than the long-run demand curve. As a result, so long as  $C > 0$ ,  $\beta_h$  will decline with  $h$  even if  $\rho_r = \rho_s$ . For instance, simple interpretations of the “reaching-for-yield” channel (Hanson and Stein, 2015) would suggest that  $\rho_s \approx \rho_r$ . Thus, in order for this channel to explain our results, we would need to appeal to slow-moving capital effects (Duffie, 2010; Greenwood et al., 2016), which imply that these induced supply shocks move the market along a short-run demand curve that is a good deal steeper than the long-run demand curve.

In summary, for  $\beta_h$  to be a steeply declining function of  $h$ , we need (i)  $C > 0$  and (ii) either  $\rho_s < \rho_r$  or slow-moving investors (i.e.,  $k > 1$  and  $q < 1$ ). In practice, we believe that both  $\rho_s < \rho_r$  and slow-moving capital likely play some role in explaining why  $\beta_h$  is a declining function of  $h$ . Furthermore, we find that these mechanisms reinforce one another so that it is easiest to match the decline in  $\beta_h$  using calibrations, such as our baseline calibration, that feature both mechanisms.

In Figure 6, we show the model-implied impulse response functions following a one-time shock to short rates in month  $t = 13$ . By definition, the long-term bond yield is the sum of a expectations hypothesis component and a term premium component  $y_t = eh_t + tp_t$ , where  $eh_t = \bar{r} + [(1 - \theta) / (1 - \rho_r \theta)] (r_t - \bar{r})$ . Thus, the term spread (i.e., the slope factor) is  $ts_t = y_t - r_t = tp_t + (eh_t - r_t)$ , where  $(eh_t - r_t) = -\theta [(1 - \rho_r) / (1 - \rho_r \theta)] (r_t - \bar{r})$ . We show the impulse responses for short- and long-rates, the term premium, and the term spread in Figure 6.<sup>4</sup>

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<sup>4</sup>Consistent with the decomposition of  $\beta_h$  in equation (3.1), raising  $C$  or increasing the degree of slow-moving capital raises  $Corr(\Delta r_t, \Delta ts_t)$  in the model, giving rise to high-frequency excess sensitivity. These same forces lower  $Corr(\Delta r_t, \Delta ts_{t+j})$  for  $j > 0$  or  $j < 0$ , explaining low-frequency decoupling.

The initial shock to short rates at  $t = 13$  leads to a rise in term premia. This means that, relative to the expectations hypothesis benchmark, long-term rates are excessively sensitive to movements in the short rate. However, this effect tends to wear off over time, explaining our stylized fact. Nonetheless, the rise in short-rates still causes the yield curve to flatten. This is because the normal flattening due to the expectations hypothesis outweighs the steepening due to the rise in term premia.

## 4.9 Post-2000 versus pre-2000 behavior

We have shown that, prior to 2000,  $\beta_h$  was roughly constant as a function of horizon ( $h$ ). However, since 2000,  $\beta_h$  declines significantly with horizon. Our model points to several potential explanations for this change. First and foremost, this could be explained by an increase in  $C$  around 2000. Specifically,  $C$  may have risen because the growth of the MBS market has made the mortgage convexity channel more important over time (Hanson, 2014). Alternately, the low absolute level of rates since 2000 may have increased the tendency of investors to focus on yield rather than expected return, strengthening the reaching for yield channel. Or it may be that transitory flight-to-quality flows have grown increasingly important. As documented by many authors including Campbell et al. (forthcoming) and Campbell et al. (2015), the correlation between stock and bond returns flipped from being positive to negative around 1998. And the increased frequency of flight-to-quality flows associated with episodes of financial instability is a potential interpretation of the change in the stock-bond correlation.

Second, the impact of any increase in  $C$  on the decoupling of long- and short-rates at low frequencies would be amplified if we think that the importance of slow-moving capital effects has also risen over time. This also seems plausible. Specifically, foreign official holdings of Treasury securities have increase significantly since 2000, and these official holders tend to

be quite slow-moving.

Third, a small decline in  $\rho_r$  may have reduced the overall level of  $\beta_h$  somewhat since 2000. One natural conjecture here is that  $\rho_r$  has declined somewhat because long-run inflation expectations have been far more anchored since 2000 than they were in the 1980s and 1990s. For instance, [Gürkaynak et al. \(2005\)](#) argued that excess sensitivity reflects the fact that long-run inflation expectations are not well anchored and are being continuously revised in light of incoming news. In other words, excess sensitivity works through the expectations hypothesis: the “puzzle” is resolved once we recognize that  $\rho_r$  is much higher than we think because long-run inflation expectations are largely unmoored. While this may be a plausible partial explanation for excess sensitivity in the 1970s and 1980s, it strikes us as something of a stretch in recent years since long-run inflation expectations are so well anchored. And, as shown by [Hanson and Stein \(2015\)](#), in the post-2000 period, the excess sensitivity of long-term nominal rates to movements in short-term nominal rates primarily reflect the excess sensitivity of long-term real rates to short-term nominal rates.

Our alternative calibration in [Figure 5](#) features a higher value of  $\rho_r$  and a lower value of  $C$ . This could be thought of as representing the pre-2000 period, while the baseline calibration represents the post-2000 period. Less persistent interest rates and a higher value of  $C$  can, at least qualitatively, account for the rotation in the plot of  $\beta_h$  against the horizon  $h$  that we see in the data around 2000.

## 5 Trading strategies and affine term structure models

This section first describes trading strategies that exploit the predictability described in [Section 3](#). We then show how standard Markovian affine term structure models have a hard time fitting the fact that the sensitivity of long to short rates,  $\beta_h$ , declines at lower

frequencies in recent years.

## 5.1 Trading strategies

**Level and Slope Mimicking Portfolios** We can also recast our findings as a finding about bond return predictability. To do so, we form bond portfolios that locally mimic changes in the level and slope factors—i.e., in response to small changes in yields. The 1-month return on  $n$ -year zero coupons bonds in month  $t$  is defined as  $R_t^{(n)} = (P_t^{(n-1/12)} - P_{t-1}^{(n)})/P_{t-1}^{(n)}$ . Because  $L_t \equiv y_t^{(1)}$  and  $S_t \equiv y_t^{(10)} - y_t^{(1)}$ , we have  $R_t^{(10)} \approx -10 \times (\Delta L_t + \Delta S_t)$  and  $R_{t+1}^{(1)} \approx -1 \times \Delta L_t$  for a small changes in yields. (This approximation is exact in continuous time.) We then follow [Joslin et al. \(2014\)](#) and construct factor-mimicking portfolios with a weight  $w_i$  on bonds with yield  $n_i$ . The excess returns on these portfolios are:

$$RX_t = \frac{\sum_i w_i (R_t^{(n_i)} - R_t^{(1/12)})}{|\sum_i w_i|},$$

where  $R_t^{(1/12)}$  is the riskless return on 1-month bills.<sup>5</sup> The level-mimicking portfolio has a weight  $-1$  on the 1-year bond and no weight on any other bonds. The level-mimicking portfolio has a monthly excess return of:

$$RX_t^{LEVEL} = -1 \times (R_t^{(1)} - R_t^{(1/12)}) \approx \Delta L_t + R_t^{(1/12)}.$$

The slope-mimicking portfolio has a weight 1 on the 1-year bond and  $-0.1$  on the 10-year bond. The slope-mimicking portfolio has an excess return of:

$$RX_t^{SLOPE} = \frac{1}{0.9} \times (R_t^{(1)} - R_t^{(1/12)}) - \frac{0.1}{0.9} \times (R_t^{(10)} - R_t^{(1/12)}) \approx \frac{\Delta S_t}{0.9} - R_t^{(1/12)}.$$

---

<sup>5</sup>[Joslin et al. \(2014\)](#) also consider excess returns on such factor-mimicking portfolios, although they were using principal components whereas we define level and slope from fixed points on the yield curve.

We then consider the following predictive regressions:

$$\begin{aligned}
RX_t^{LEVEL} &= \delta_{0L} + \delta_{1L}L_{t-1} + \delta_{2L}S_{t-1} + \delta_{3L}C_{t-1} \\
&\quad + \delta_{4L}(L_{t-1} - L_{t-1-h}) + \delta_{5L}(S_{t-1} - S_{t-h}) + \delta_{6L}(C_{t-1} - C_{t-h}) + \varepsilon_{L,t} \quad (5.1)
\end{aligned}$$

and

$$\begin{aligned}
RX_t^{SLOPE} &= \delta_{0S} + \delta_{1S}L_{t-1} + \delta_{2S}S_{t-1} + \delta_{3S}C_{t-1} \\
&\quad + \delta_{4S}(L_{t-1} - L_{t-1-h}) + \delta_{5S}(S_{t-1} - S_{t-h}) + \delta_{6S}(C_{t-1} - C_{t-h}) + \varepsilon_{S,t}, \quad (5.2)
\end{aligned}$$

where  $RX_t^{LEVEL}$  and  $RX_t^{SLOPE}$  denote the monthly excess returns on level- and slope-mimicking portfolios as defined above. We report the results from estimating these predictive regressions in Table 4.

**Trading strategies** The results in Table 4 are entirely consistent with those in Table 3. This is as expected because the dependent variables in equations (3.2) and (3.3) are the future level and slope respectively, whereas the dependent variables in equations (5.1) and (5.2) are approximately the future changes in level and slope. Specifically, in the post-2000 sample, lagged changes in level are highly significant predictors of excess returns on the slope-mimicking portfolio.<sup>6</sup> Our basic finding is that an increase in the level of rates has been followed by subsequent yield curve flattening. As another way of assessing this, we consider trading strategies in which every month, the investor decides to take either a

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<sup>6</sup>Many researchers have examined bond return predictability by considering the excess returns on 10-year or other long-term bonds. By construction, the excess return on a 10-year bond is a linear combination of the excess returns on our level- and slope- mimicking portfolios:  $R_t^{(10)} - R_t^{(1/12)} = -9 \times RX_t^{SLOPE} - 10 \times RX_t^{LEVEL}$ . From Table 5, in the post-2000 sample, the excess return on the level-mimicking portfolio depends positively on  $L_{t-1} - L_{t-h}$  but the excess return on the slope-mimicking portfolio depends negatively on  $L_{t-1} - L_{t-h}$ . The two effects partially cancel out when predicting 10-year excess returns, although the net effect (not shown) is estimated to be positive. Thus, we prefer to work with level- and slope- mimicking portfolios as it makes the dependence on lagged changes in levels more stark, and these are clearly tradeable portfolios.

long or short position in the slope-mimicking portfolio<sup>7</sup>. We assume that the investor takes a long (short) position in the slope-mimicking portfolio from month  $t - 1$  to month  $t$  if  $L_{t-1} < L_{t-1-h}$  ( $L_{t-1} > L_{t-1-h}$ ). Alternatively, we could assume that the investor takes a position in the slope-mimicking portfolio from month  $t - 1$  to month  $t$  that is proportional to  $-(L_{t-1} - L_{t-1-h})$ . And then we compute the annualized Sharpe ratios of these trading strategies for different choices of  $h$ , over the post-2000 sample. Table 5 shows implied Sharpe ratios between 0.4 to 0.7.

## 5.2 Affine Term Structure Model

Affine models of the term structure are a widely used tool for understanding bond yields. A standard discrete-time affine term structure model (Duffee, 2002; Duffie and Kan, 1996) starts from the assumption that there is a state vector  $X_t$  that follows a VAR(1):

$$X_t = \mu + \Phi X_{t-1} + \Sigma \varepsilon_t, \quad (5.3)$$

where the error term is Gaussian with mean zero and identity variance-covariance matrix. The short-term interest rate is  $r_t = \delta_0 + \delta'_1 X_t$ . Meanwhile, the pricing kernel is:

$$M_{t+1} = \exp(-r_t - \lambda'_t \varepsilon_{t+1} - \frac{1}{2} \lambda'_t \lambda_t), \quad (5.4)$$

where  $\lambda_t = \lambda_0 + \lambda_1 X_t$ . After extensive, but well-known algebra, it follows that the price of an  $n$ -period zero-coupon bond is:

$$P_t(n) = E_t(\Pi_{i=1}^n M_{i+1}) = \exp(A_n + B'_n X_t), \quad (5.5)$$

---

<sup>7</sup>There is a huge literature on momentum and contrarian strategies in equity markets. The literature on such strategies in the bond market is much smaller, but Durham (2013) considers returns on certain bond momentum strategies.

where  $A_n$  is a scalar and  $B_n$  is an  $m \times 1$  vector that satisfy the recursions:

$$A_{n+1} = -\delta_0 + A_n + B_n' \mu^* + \frac{1}{2} B_n' \Sigma \Sigma' B_n \quad (5.6)$$

$$B_{n+1} = \Phi^{*'} B_n - \delta_1, \quad (5.7)$$

where  $\mu^* = \mu - \Sigma \lambda_0$  and  $\Phi^* = \Phi - \Sigma \lambda_1$ , starting from  $A_1 = -\delta_0$  and  $B_1 = -\delta_1$ . The yield on an  $n$ -period zero-coupon bond is in turn given by:

$$y_t(n) = -\frac{1}{n} \log(P_t(n)) = -\frac{A_n}{n} - \frac{B_n'}{n} X_t. \quad (5.8)$$

Let  $\Gamma(j) = E((X_t - E(X_t))(X_{t-j} - E(X_{t-j})))'$  be the autocovariance function of the state vector, which can be obtained from the equations:

$$vec(\Gamma(0)) = (I - \Phi \otimes \Phi)^{-1} vec(\Sigma \Sigma'), \quad (5.9)$$

$$\Gamma(j) = \Phi^j \Gamma(0) \text{ for } j \geq 1.$$

The population coefficient in a regression of  $h$ -month changes in 120-month yields on  $h$ -month changes in 12-month yields is:

$$\frac{E[(y_{t+h}(120) - y_t(120))(y_{t+h}(12) - y_t(12))]}{E[(y_{t+h}(12) - y_t(12))^2]} = \frac{B_{120}' [2\Gamma(0) - \Gamma(h) - \Gamma(h)'] B_{12}}{10 B_{12}' [2\Gamma(0) - \Gamma(h) - \Gamma(h)'] B_{12}} \quad (5.10)$$

We can fit this model using the first  $K$  principal components of yields as factors, and applying the estimation methodology of [Adrian et al. \(2013\)](#). We do this in the pre-2000 and post-2000 samples separately. We take the estimated parameters, and work out the model implied regression coefficients using equations (5.9) and (5.10). These are shown in panel A of Table 6. As can be seen, the model fails to match the low-frequency decoupling of yields seen in the data.

An alternative version augments the state variable to include not just  $K$  principal components of yields, but also  $L$  additional lags of these principal components. These lags are

treated as unspanned factors. This requires that if the first  $K$  elements of the state vector are the current principal components, all but the first  $K$  elements of  $\delta_1$  are equal to zero, and the upper right  $K \times KL$  block of  $\Phi^*$  is a matrix of zeros. This implies that the lags are important for expected future yields, but are not reflected in the yield curve today. It's a rather unusual model, but has been considered in [Joslin et al. \(2013\)](#), and is a non-Markovian model. Again, the parameters can be estimated, with the restriction of the lags being unspanned factors, as described in [Adrian et al. \(2013\)](#). The model implied regression coefficients can then be deduced. These are shown in panel B of Table 6. The model is able to get reasonably close to matching the regression coefficients at both frequencies and in both samples.

Our conclusion is that the affine term structure model needs lagged principal components to match the low-frequency decoupling of yields in recent years. A large number of static principal components does not do the job. These findings are consistent with the lead-lag relations between changes in level and slope that we documented in the previous section.

## 6 Conclusions

The excess sensitivity of changes in long rates to changes in short rates is a long-standing puzzle about the term structure of interest rates. In this paper, we have documented that this puzzle has disappeared since 2000 when looking at low-frequency changes. As a result, low-frequency decoupling between long and short rates—the phenomenon that former Federal Reserve Chairman Greenspan called a conundrum—has become increasingly common. At the same time, we find that high-frequency excess sensitivity has actually become somewhat stronger since 2000.

From an expectations hypothesis perspective, the puzzle is not the weak relationship between short- and long-term rates observed recently at low frequencies. Instead, the puzzle

is why this relationship was previously so strong at low frequencies and why it still remains so strong at high frequencies. We have proposed a simple model that can explain these stylized facts. In the model, the expectations hypothesis does not hold because risk-averse investors demand a term premium to compensate them for the risk of holding long-term bonds. And, naturally, the term premium is increasing in the net supply of long-term bonds that investors must hold. As a shorthand for a variety of amplification mechanisms—including “reaching for yield”, mortgage convexity hedging flows, and asset and liability management by insurers—we assume that shocks to the short rate lead to an increase in the net supply of long-term bonds. As a result, shocks to short rates move term premia in the same direction, giving rise to excess sensitivity at high frequencies. Our model also incorporates slow moving capital, which makes the demand curve for bonds steeper in the short run than in the long run. This allows the model to capture the surprising frequency-specific sensitivity of long rates to short rates that we observe in recent data.

Our findings have important implications for the transmission of monetary policy and for event-study methodology. The excess sensitivity of long-term yields reinforces the effects of monetary policy, but in recent years this channel of monetary policy transmission has been fairly short-lived. That, in turn, makes it harder for monetary policy to influence aggregate demand, and requires the central bank to move monetary policy more aggressively to achieve the same effect. The event-study methodology studies high-frequency responses of asset prices in windows where the nature of the underlying shock is easy to identify. However, part of the high-frequency response of long rates to shocks to short rates represents term premium movements that appear to wear off systematically over time. Consequently, it is important to remember that the event-study methodology only measures high-frequency responses and that the effects may often be more muted at the lower frequencies that are

typically of greatest concern to macroeconomists and policymakers.

**Table 1: Regression of changes in long-term rates on short-term rates.** This table reports the estimated slope coefficients from equations (2.1) and (2.2) for each reported sample. The dependent variable is the change in the 10-year yield or forward rate, either nominal, real or their difference (IC, or inflation compensation). The independent variable is the change in the 1-year nominal yield in all cases. Changes are considered with daily data, and with monthly data using monthly ( $h = 1$ ), quarterly ( $h = 3$ ), semi-annual ( $h = 6$ ) and annual ( $h = 12$ ) horizons. We report Newey-West standard errors in brackets, using a lag truncation parameter of  $1.5 \times (h - 1)$  (rounded to the nearest integer). Significance:  $*p < 0.1$ ,  $**p < 0.05$ ,  $***p < 0.01$ .

**Panel A:** 10-year zero coupon yields and IC

	(1) Nominal	(2) Nominal	(3) Real	(4) IC
Daily	0.56*** [0.02]	0.85*** [0.03]	0.53*** [0.03]	0.33*** [0.02]
Monthly	0.46*** [0.04]	0.64*** [0.12]	0.38*** [0.10]	0.28*** [0.10]
Quarterly	0.48*** [0.04]	0.42*** [0.07]	0.21** [0.10]	0.25* [0.13]
Semi-annual	0.50*** [0.04]	0.31*** [0.07]	0.19** [0.08]	0.15 [0.10]
Yearly	0.56*** [0.05]	0.18*** [0.04]	0.12** [0.06]	0.09* [0.05]
Sample	1971-1999	2000-2015	2000-2015	2000-2015

**Panel B:** 10-year instantaneous forward yields and IC

	(1) Nominal	(2) Nominal	(3) Real	(4) IC
Daily	0.39*** [0.03]	0.48*** [0.04]	0.30*** [0.03]	0.17*** [0.03]
Monthly	0.29*** [0.04]	0.23 [0.15]	0.17** [0.08]	0.06 [0.10]
Quarterly	0.31*** [0.05]	0.03 [0.09]	0.08 [0.05]	-0.05 [0.05]
Semi-annual	0.33*** [0.06]	-0.07 [0.07]	0.02 [0.05]	-0.10** [0.04]
Yearly	0.39*** [0.07]	-0.19*** [0.05]	-0.03 [0.05]	-0.16*** [0.04]
Sample	1971-1999	2000-2015	2000-2015	2000-2015

**Table 2: Regression of changes in long-term international rates on short-term rates.** This table reports the estimated slope coefficients from equations (2.1) and (2.2) for UK, German and Canadian interest rates on each reported sample. Daily data for Germany are not available pre-1997. The dependent variable is the change in the 10-year yield or forward rate, either nominal, real or their difference (IC, or inflation compensation). The independent variable is the change in the 1-year nominal yield in all cases. Changes are considered with daily data, and with monthly data using monthly ( $h = 1$ ), quarterly ( $h = 3$ ), semi-annual ( $h = 6$ ) and annual ( $h = 12$ ) horizons. We report Newey-West standard errors in brackets, using a lag truncation parameter of  $1.5 \times (h - 1)$  (rounded to the nearest integer). Significance: \* $p < 0.1$ , \*\* $p < 0.05$ , \*\*\* $p < 0.01$ .

**Panel A: UK 10-year zero-coupon yields and inflation compensation**

	(1)	(2)	(3)	(4)	(5)	(6)
	Nominal	Nominal	Real	Real	IC	IC
Daily	0.44*** [0.04]	0.80*** [0.04]	0.14*** [0.01]	0.58*** [0.04]	0.29*** [0.04]	0.22*** [0.02]
Monthly	0.47*** [0.06]	0.54*** [0.14]	0.19*** [0.05]	0.10 [0.26]	0.28*** [0.09]	0.44*** [0.13]
Quarterly	0.49*** [0.08]	0.43*** [0.10]	0.23*** [0.04]	0.04 [0.17]	0.26*** [0.10]	0.39*** [0.10]
Semi-annual	0.45*** [0.09]	0.39*** [0.08]	0.22*** [0.05]	0.06 [0.10]	0.23** [0.11]	0.32*** [0.06]
Yearly	0.38*** [0.06]	0.29*** [0.07]	0.16** [0.06]	0.05 [0.07]	0.22*** [0.08]	0.24*** [0.03]
Sample	1985-1999	2000-2015	1985-1999	2000-2015	1985-1999	2000-2015

**Panel B: German and Canadian 10-year nominal zero coupon yields**

	(1)	(2)	(3)	(4)
	DE	DE	CAN	CAN
Daily		0.65*** [0.03]	0.42*** [0.03]	0.71*** [0.03]
Monthly	0.34*** [0.06]	0.48*** [0.11]	0.46*** [0.05]	0.49*** [0.07]
Quarterly	0.41*** [0.04]	0.44*** [0.07]	0.51*** [0.05]	0.36*** [0.05]
Semi-annual	0.41*** [0.04]	0.40*** [0.08]	0.50*** [0.07]	0.25*** [0.05]
Yearly	0.43*** [0.04]	0.32*** [0.10]	0.43*** [0.08]	0.09* [0.05]
Sample	1972-1999	2000-2015	1986-1999	2000-2015

**Table 3: Estimates of predictive equations for level, slope and curvature.** This table reports the estimated slope coefficients in equations (3.2), (3.3) and (3.4) over August 1971-December 2000 and January 2001-December 2016 subsamples, with  $h = 12$ . White standard errors, are included. Dependent variables are monthly changes in level and slope. The table also shows AIC and BIC values (to be minimized) for each possible specification of the system of three equations. Lastly, the implied coefficient  $\beta_h$  in equation (2.1) corresponding to each possible specification of the system is reported.

	Pre-2000						Post-2000					
	Est	SE	Est	SE	Est	SE	Est	SE	Est	SE	Est	SE
Dependent Variable: Level												
$L_{t-1}$	0.97	0.03	0.96	0.03	0.95	0.03	0.96	0.03	0.98	0.01	0.98	0.01
$S_{t-1}$	-0.08	0.13	-0.08	0.15	-0.09	0.12	-0.07	0.02	-0.01	0.02	-0.01	0.02
$C_{t-1}$	0.26	0.49	0.35	0.53	0.30	0.44	0.20	0.09	0.13	0.09	0.14	0.09
$L_{t-1} - L_{t-h}$			0.03	0.02	0.06	0.03	0.03	0.07	0.07	0.02	0.07	0.02
$S_{t-1} - S_{t-h}$					0.08	0.06	0.05	0.11	-0.01	0.02	-0.01	0.03
$C_{t-1} - C_{t-h}$					0.07	0.32			0.01	0.02	0.01	0.07
Dependent Variable: Slope												
$L_{t-1}$	0.01	0.02	0.02	0.02	0.02	0.02	0.00	0.01	-0.02	0.01	-0.02	0.01
$S_{t-1}$	1.03	0.09	1.03	0.10	1.02	0.08	1.01	0.04	0.93	0.04	0.93	0.03
$C_{t-1}$	-0.23	0.33	-0.31	0.35	-0.27	0.30	-0.21	0.12	-0.12	0.12	-0.12	0.12
$L_{t-1} - L_{t-h}$			-0.03	0.01	-0.04	0.02	-0.03	0.02	-0.10	0.02	-0.11	0.03
$S_{t-1} - S_{t-h}$					-0.01	0.04	0.00	0.08	-0.02	0.03	-0.01	0.05
$C_{t-1} - C_{t-h}$					-0.05	0.23			-0.04	0.10	-0.04	0.10
Dependent Variable: Curvature												
$L_{t-1}$	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01	-0.01	0.01	-0.01	0.01
$S_{t-1}$	0.07	0.03	0.07	0.03	0.06	0.03	0.02	0.01	0.00	0.02	0.00	0.02
$C_{t-1}$	0.74	0.11	0.71	0.12	0.75	0.10	0.84	0.04	0.88	0.04	0.88	0.04
$L_{t-1} - L_{t-h}$			-0.01	0.01	-0.01	0.01	-0.01	0.01	-0.03	0.01	-0.02	0.01
$S_{t-1} - S_{t-h}$					0.00	0.01	0.02	0.03	0.00	0.01	0.02	0.02
$C_{t-1} - C_{t-h}$					-0.06	0.09			-0.05	0.05	-0.05	0.05
AIC	-9584		-9260		-9259		-6187		-6242		-6237	
BIC	-9538		-9203		-9190		-6145		-6192		-6178	
Implied $\beta_h$	0.51		0.47		0.53		0.50		0.23		0.22	

**Table 4: Estimates of predictive equations for level and slope-mimicking portfolio excess returns** This table reports the estimated slope coefficients in equations (5.1) and (5.2) over August 1971-December 2000 and January 2001-December 2016 subsamples. White standard errors, are included. Dependent variables are monthly excess returns on level- and slope-mimicking portfolios.

	Pre-2000		Post-2000	
	Estimate	SE	Estimate	SE
Dependent Variable: $xrt_t^{LEVEL}$ .				
$L_{t-1}$	-0.05	0.03	-0.02	0.01
$S_{t-1}$	-0.01	0.11	-0.01	0.02
$C_{t-1}$	0.11	0.41	0.02	0.08
$L_{t-1} - L_{t-h}$	0.05	0.03	0.05	0.02
$S_{t-1} - S_{t-h}$	0.04	0.11	-0.01	0.03
$C_{t-1} - C_{t-h}$	0.04	0.30	0.00	0.07
Dependent Variable: $xrt_t^{SLOPE}$ .				
$L_{t-1}$	0.02	0.02	-0.02	0.01
$S_{t-1}$	0.01	0.09	-0.09	0.04
$C_{t-1}$	-0.09	0.30	0.02	0.13
$L_{t-1} - L_{t-h}$	-0.02	0.02	-0.10	0.03
$S_{t-1} - S_{t-h}$	0.02	0.08	-0.01	0.05
$C_{t-1} - C_{t-h}$	-0.03	0.23	-0.03	0.11

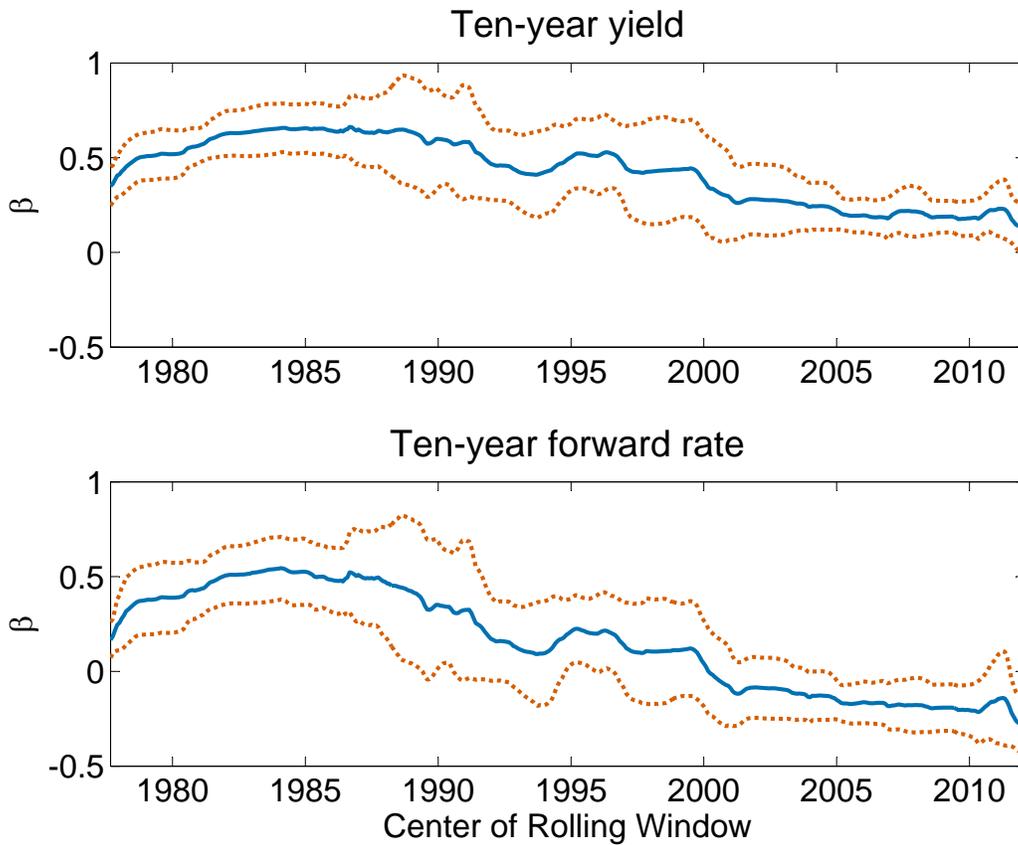
**Table 5: Sharpe ratios for slope-mimicking portfolios** This table reports the annualized Sharpe ratios since 2000 of the strategy of going long (short) the slope-mimicking portfolio if the level (fell) rose over the previous  $h$  months and also the strategy of taking a position in the slope-mimicking portfolio that is proportional to  $-(L_{t-1} - L_{t-h})$ . The position is rebalanced each month. Annualized Sharpe ratios are computed as the sample average monthly excess returns multiplied by  $\sqrt{12}$  and divided by the standard deviation of those monthly excess returns.

Investment in slope-mimicking portfolio	$h = 1$	$h = 3$	$h = 6$	$h = 12$
$1(L_{t-1} - L_{t-h} < 0)$	0.43	0.62	0.45	0.44
$-(L_{t-1} - L_{t-h})$	0.54	0.63	0.70	0.66

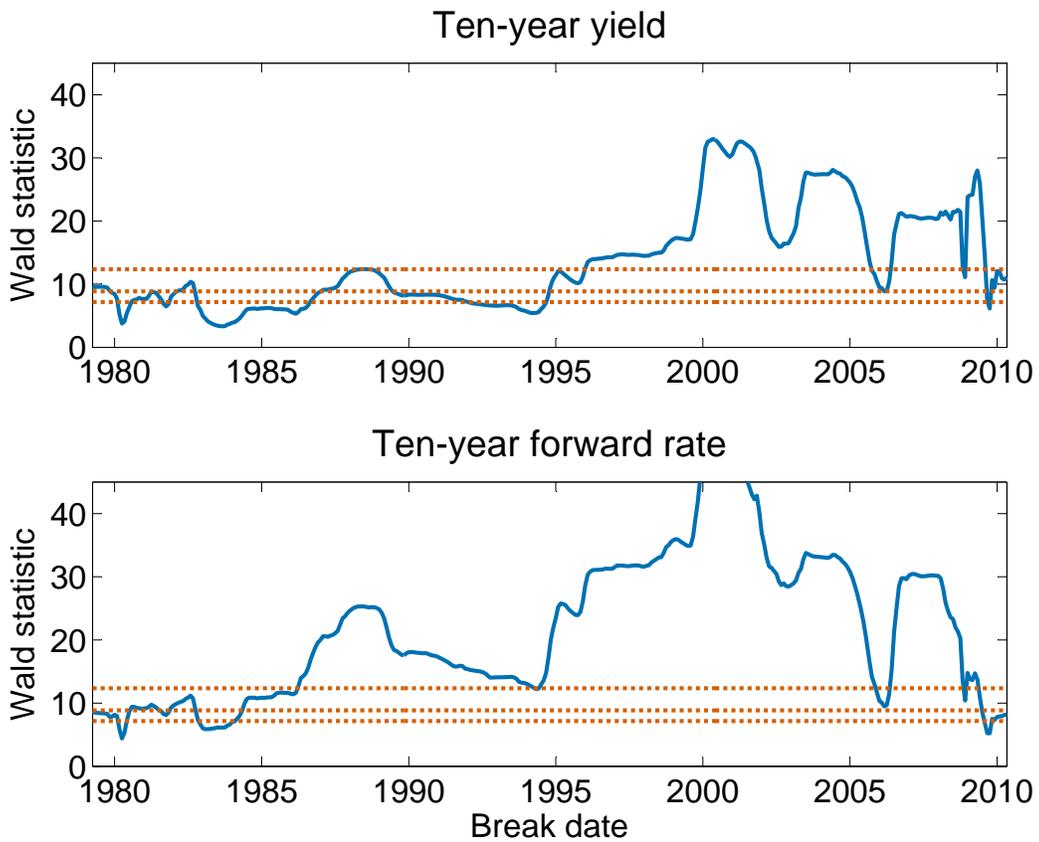
**Table 6: Affine Term Structure Model-Implied coefficients in regression of monthly/yearly changes in 10-year yields on changes in 1-year yields** This table reports the slope coefficients in equation (5.10) corresponding to the parameters in an affine term structure model estimated as proposed by [Adrian et al. \(2013\)](#) over August 1971-December 2000 and January 2001-December 2016 subsamples. The term structure model uses  $K$  principal components of yields as state variables in panel A, and adds  $L$  additional lags of these principal components (for a total of  $K + LK$  state variables) in panel B. As memo items the results of the regressions using actual yields are included—these are simply transcribed from Table 1.

		Pre-2000		Post-2000	
		Monthly	Yearly	Monthly	Yearly
Panel A: ATSM with $K$ principal components of yields as factors					
$K = 2$		0.42	0.49	0.83	0.75
$K = 3$		0.46	0.52	0.79	0.61
$K = 4$		0.47	0.52	0.74	0.57
$K = 5$		0.47	0.52	0.71	0.51
Panel B: ATSM with $L$ lags as additional unspanned factors					
$K = 2$	$L = 1$	0.42	0.50	0.83	0.54
$K = 3$	$L = 1$	0.46	0.52	0.79	0.46
$K = 2$	$L = 2$	0.42	0.50	0.85	0.44
$K = 3$	$L = 2$	0.46	0.53	0.81	0.37
$K = 2$	$L = 3$	0.42	0.48	0.84	0.41
$K = 3$	$L = 3$	0.46	0.51	0.81	0.40
$K = 2$	$L = 5$	0.42	0.50	0.85	0.32
$K = 3$	$L = 5$	0.46	0.54	0.81	0.35
Memo: Estimates in data (from Table 1)		0.46	0.56	0.64	0.18

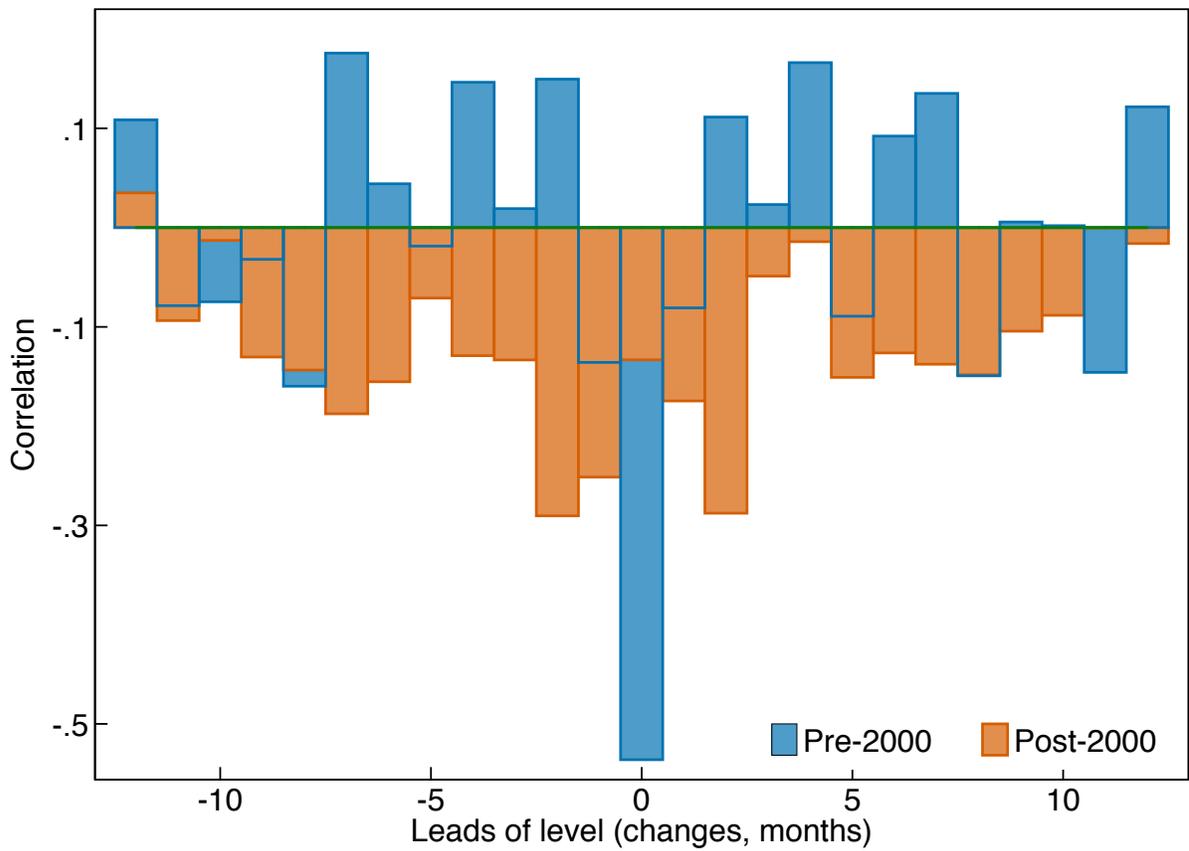
**Figure 1: Rolling Regression Estimates of Equations (2.1) and (2.2)** This figure plots rolling estimates of the slope coefficients in equations (2.1) and (2.2) with one-year changes (monthly data with  $h=12$ ) using 10-year rolling windows for estimation. 95% confidence intervals are included (red dashed lines), formed using Newey-West standard errors with a lag truncation parameter of  $1.5 \times (h - 1)$ . Results are plotted against the midpoint of the rolling window.



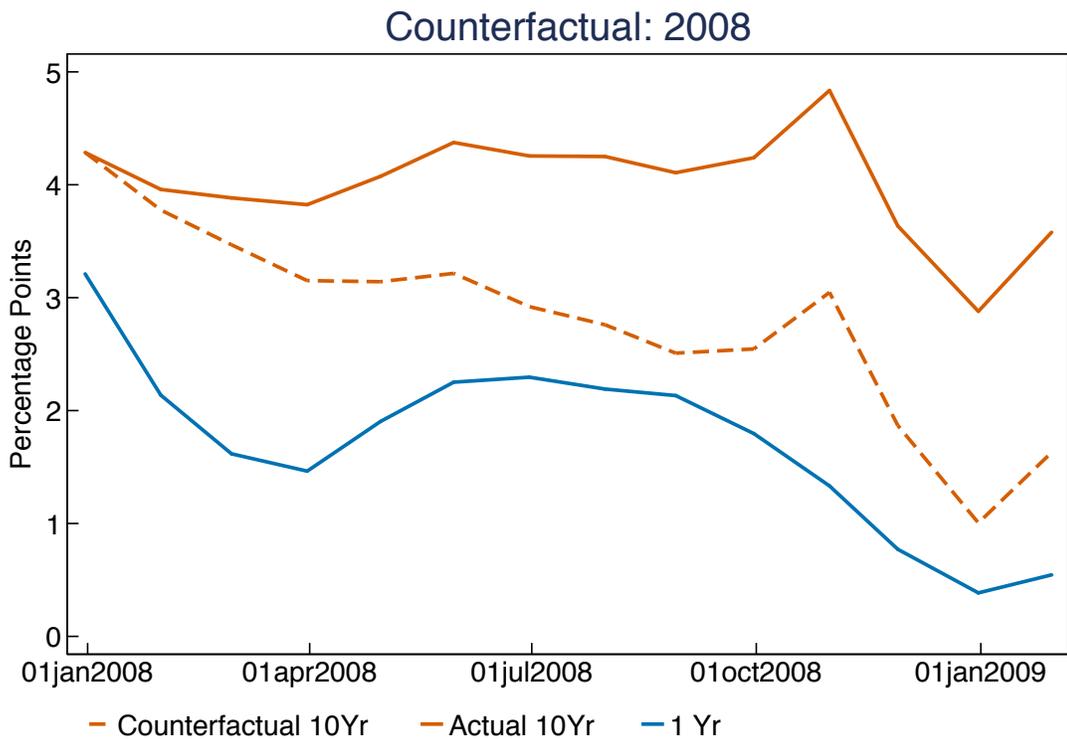
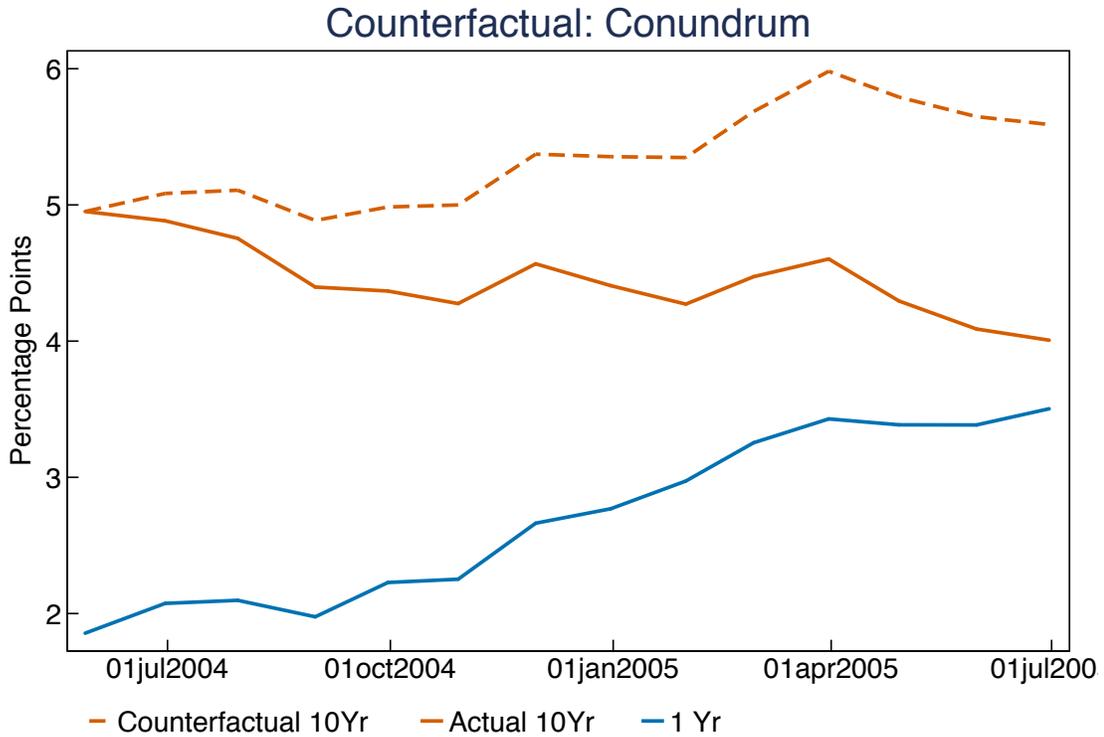
**Figure 2: Break Tests for Equations (2.1) and (2.2)** This figure plots the Wald test statistic for each possible break date in equations (2.1) and (2.2) with one-year changes (monthly data with  $h=12$ ) from a fraction 15% of the way through the sample to 85% of the way through the sample. The horizontal red dashed lines denote 10%, 5% and 1% critical values for the maximum of these Wald statistics, from the critical values of Andrews (1993). The overlapping forecasts are accounted for using Newey-West standard errors with a lag truncation parameter of  $1.5 \times (h - 1)$ .



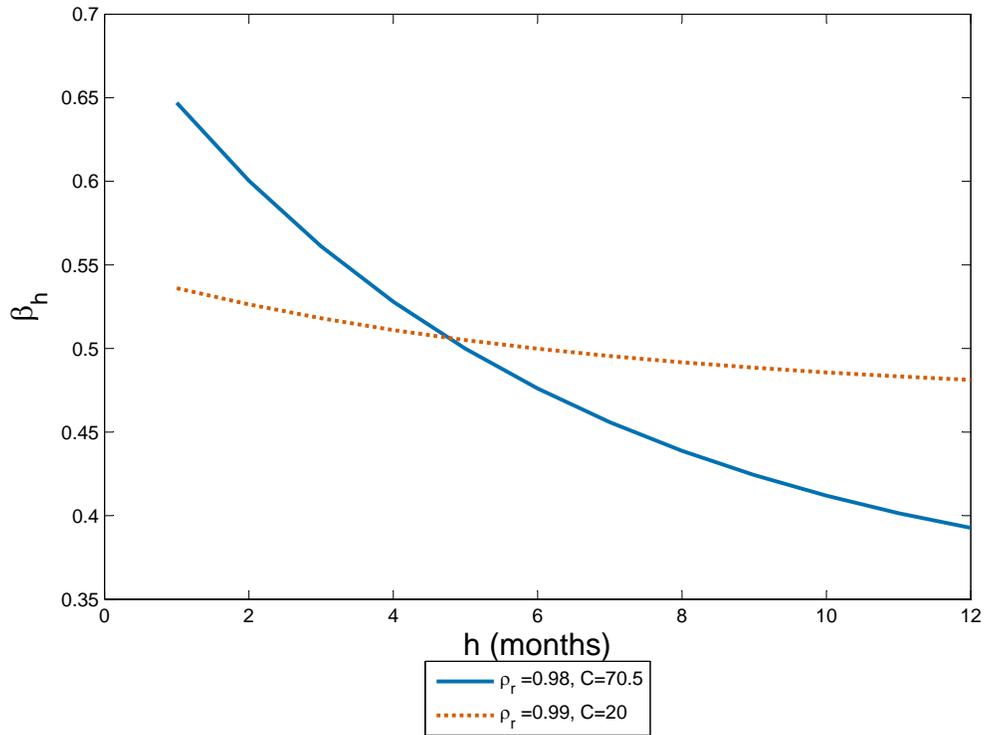
**Figure 3: Cross-Correlation of Changes in Level and Slope.** Level is the 1-year zero coupon Treasury yield; slope is the 10-year less the 1-year yield. The cross-correlation at lag 0 is the contemporaneous correlation of daily changes in level and slope. Correlations at negative lags (on the left of the figure) denote correlations between monthly changes in level and *future* changes in slope.



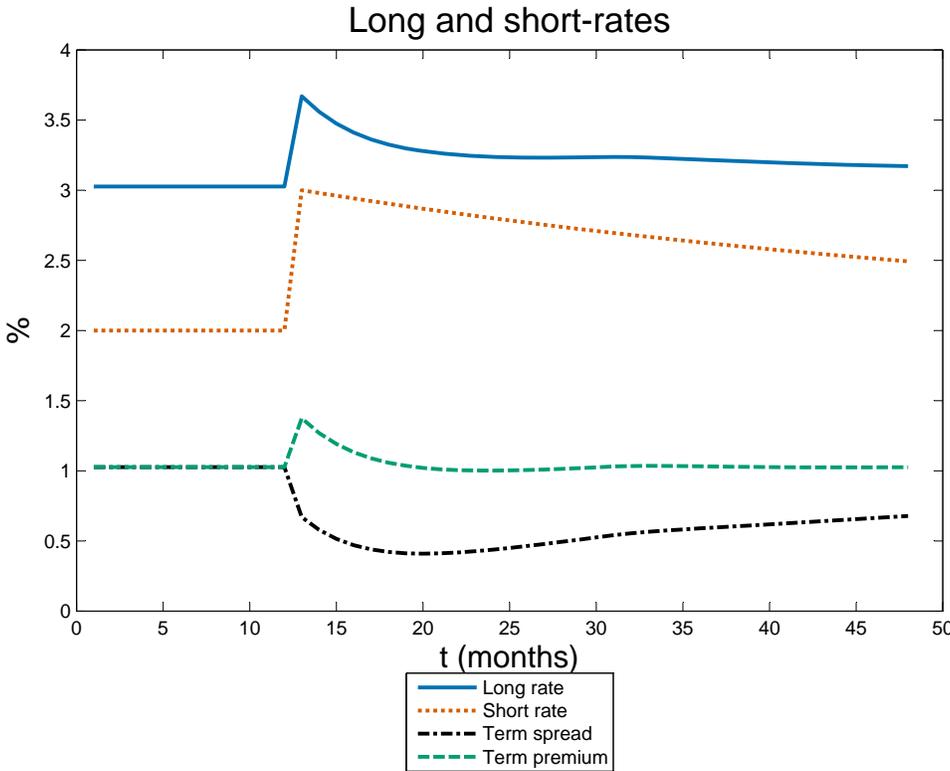
**Figure 4: Counterfactual paths of ten-year yields in selected episodes** This figure plots one- and ten-year yields in the original “conundrum” period and in 2008, along with counterfactual ten-year yields generated from restricting the slope to depend on lags of level and slope, but not also on lagged changes, as described in the text.



**Figure 5: Model-implied coefficients  $\beta_h$  in equation (4.12) against horizon ( $h$ ) in months.** We show this for the baseline calibration and in an alternate calibration where persistence of short rates is greater, and there is lower feedback from short rates to supply. In both cases,  $\beta_h$  is a declining function of  $h$ . But the decline is much more pronounced in the baseline calibration.



**Figure 6: Model-implied impulse response functions over time** For the baseline model calibration, we show the response of interest rates following a one-time shock to short-term interest rates.



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