NON-RENEWABLE RESOURCES, EXTRACTION TECHNOLOGY, AND ENDOGENOUS GROWTH*

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Abstract

We develop a theory of innovation in non-renewable resource extraction and economic growth. Firms increase their economically extractable reserves of non-renewable resources through R&D investment in extraction technology and reduce their reserves through extraction. Our model allows us to study the interaction between geology and technological change, and its effects on prices, total output growth, and the resource intensity of the economy. The model accommodates long-term trends in non-renewable resource markets – namely stable prices and exponentially increasing extraction – for which we present data extending back to 1792. The paper suggests that over the long term, development of new extraction technologies balances the increasing demand for non-renewable resources. (JEL codes: O30, O41, Q30)

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1 Introduction

In his seminal paper, Nordhaus (1974) estimates that the crustal abundance of non-renewable resources is sufficient to continue consumption for hundreds of thousands of years. He also emphasizes that prohibitively high extraction costs make a very large share of mineral deposits not recoverable. Proven reserves – those non-renewable resources that are economically recoverable with current technologies – are a far smaller share.

However, innovation in extraction technology helps overcome scarcity by turning mineral deposits into economically recoverable reserves (Nordhaus, 1974; Simon, 1981, and others). There is empirical evidence for such technological change across a broad variety of non-renewable resources (see, e.g., Managi et al., 2004; Mudd, 2007; Simpson, 1999).

In the literature on growth and natural resources, models rarely consider technological change in extraction. Scarcity is primarily overcome by technological change involving the efficient use of resources and substitution of capital for non-renewable resources (see Groth, 2007; Aghion and Howitt, 1998). These models typically predict decreased non-renewable resource extraction, and increasing prices in the long run, which is not in line with empirical evidence of increasing production and non-increasing prices (see Krautkraemer, 1998; Livernois, 2009; Von Hagen, 1989).

This paper develops a theory of technological change in non-renewable resource extraction in an endogenous growth model. Modeling technological change in resource extraction in a growth model is technically challenging because it adds a layer of dynamic optimization to the model. We boil down the investment and extraction problem to a static problem, which makes our model both simple enough to solve and rich enough to potentially connect to long-run data as a next step.

To our knowledge, our model is the first that allows the study of the interaction between technological change and geology, and its effects on prices, total output growth, and its use in the economy. Learning about these effects is important for making predictions of long-run development of resource prices and for understanding the impact of resource production on aggregate output. For example, distinguishing between increasing and constant resource prices in the long run is key to the results of a number of recent papers on climate economics (Acemoglu et al., 2012; Golosov et al., 2014; Hassler and Sinn, 2012; van der Ploeg and Withagen, 2012).

We add an extractive sector to a standard endogenous growth model of expanding varieties and directed technological change by Acemoglu (2002), such that aggregate output is produced from non-renewable resources and intermediate goods.

Modeling the extractive sector has four components: First, we assume that there is a continuum of deposits of declining grades. The quantity of the non-renewable resource is distributed such that it increases exponentially as the ore grades of deposits decrease, as a local approximation to Ahrens (1953, 1954) fundamental law of geochemistry. Although we recognize that non-renewable resources are ultimately finite in supply, we make the assumption that the underlying resource quantity goes to infinity for all practical economic purposes as the grade of the deposits approaches zero. Without innovation in extraction technology, the extraction cost is assumed to be infinitely high.

Second, we build on Nordhaus' (1974) idea that reserves are akin to working capital or inventory of economically extractable resources. Firms can invest in grade specific extraction technology to subsequently convert deposits of lower grades into economically extractable reserves. We assume that R&D investment exhibits decreasing returns in making deposits of lower grades extractable, as historical evidence suggests. Once converted into a reserve, the firm that developed the technology can extract the resource at a fixed operational cost.

Third, new technology diffuses to all other firms. As each new technology is specific to a deposit of a certain grade, it cannot be used to extract resources from deposits of lower grades. However, all firms can build on existing technology when they invest in developing new technology for deposits of lower grades. The idea is that firms can, for example, use the shovel invented by another firm but have a cost to train employees to use it for a specific deposit of lower grade. As technology diffuses, firms only maximize current profits in their R&D investment decisions in equilibrium.

Finally, the non-renewable resource is a homogeneous good. Despite a fully competitive resource market in the long run, firms invest in extractive technology because it is grade specific. Most similar to this understanding of innovation is Desmet and Rossi-Hansberg (2014). We abstract from other possible features like uncertainty about deposits, negative externalities from resource extraction, recycling, and short-run price fluctuations.

Our model accommodates historical trends in the prices and production of major non-renewable resources, as well as world real GDP for which we present data extending back to 1792. It implies a constant resource price equal to marginal cost over the long run. Extraction firms face constant R&D costs in converting one unit of the resource into a new reserve. This is due to the offsetting interaction between technological change and geology: (i) new extraction technology exhibits decreasing returns in making deposits of lower grades extractable; (ii) the resource quantity is geologically distributed such that it increases exponentially as the grade of its deposits decreases.

The resource price depends negatively on the average crustal concentration of the resource. For example, our model predicts that iron ore prices are on average lower than copper prices, because iron is more abundant (5 percent of crustal mass) than copper (0.007 percent). The price is also negatively affected by the average effect of technology in terms of making lower grade deposits extractable. For example, the average effect might be larger for deposits that can be extracted in open pit mines (e.g. coal) than for deposits requiring underground operations (e.g. crude oil). This implies that coal prices are lower than crude oil price in the long term.

The resource intensity of the economy, defined as the resource quantity used to produce one unit of aggregate output, is positively affected by the average geological abundance and the average effect of extraction technology, while the elasticity of substitution has a strong negative effect. If the resource and the intermediate good are complements, the resource intensity of the economy is relatively high, while it is significantly lower in the case of the two being substitutes. As the resource intensity is constant in equilibrium, firms extract the non-renewable resource at the same rate as aggregate output.

Aggregate output growth is constant on the balanced growth path. Our model predicts that a higher abundance of a particular resource or a higher average effect of extractive technology in terms of lower grades positively impact aggregate growth in the long run.

The extractive sector features only constant returns to scale. In contrast to the intermediate goods sector, where firms can make use of the entire *stock* of technology for production, firms in the extractive sector can only use the *flow* of new technology to convert deposits of lower grades into new reserves. Earlier developed technologies are grade specific and the related deposits are exhausted. The stock of extraction technology therefore grows proportionally to output, while technology in the intermediate goods sectors increases at the same rate as aggregate output.

The paper contributes to a literature that mostly builds on the seminal Hotelling (1931) optimal depletion model. Heal (1976) introduces a non-renewable resource, which is inexhaustible, but extractable at different grades and costs. Extraction costs increase with cumulative extraction, but then remain constant when a "backstop technology" (Heal, 1976, p. 371) is reached. Slade (1982) adds exogenous technological change in extraction technology to the Hotelling (1931) model and predicts a U-shaped relative price curve. Cynthia-Lin and Wagner (2007) use a similar model with an inexhaustible non-renewable resource and exogenous technological change. They obtain a constant relative price with increasing extraction.

There are three papers, to our knowledge, that like ours include technological change in the extraction of a non-renewable resource in an endogenous growth model. Fourgeaud et al. (1982) focuses on explaining sudden fluctuations in the development of non-renewable resource prices by allowing the resource stock to grow in a stepwise manner through technological change. Tahvonen and Salo (2001) model the transition from a non-renewable energy resource to a renewable energy resource. Their model follows a learning-by-doing approach as technological change is linearly related to the level of extraction and the level of productive capital. It explains decreasing prices and the increasing use of a non-renewable energy resource over a particular time period before prices increase in the long term. Hart (2012) models resource extraction and demand in a growth model with directed technological change. The key element in his model is the depth of the resource. After a temporary "frontier phase" with a constant resource price and consumption rising at a rate only close to aggregate output, the economy needs to extract resources from greater depths. Subsequently, a long-run balanced growth path is reached with constant resource consumption and prices that rise in line with wages.

In Section 2, we document stylized facts on the long-term development of non-renewable resource prices, production, and world real GDP. We also provide evidence for the major assumptions of our model regarding geology and technological change. Section 4 presents the growth model and its extractive sector. In Section 5 we draw conclusions.

2 Stylized facts

2.1 Prices, Resource Production, and Aggregate Output over the Long Term

Annual data for major non-renewable resource markets going back to 1792 indicate that real prices are roughly trend-less and that worldwide primary production as well as world real GDP grow roughly at a constant rate.

Figure 1 presents data on the real prices of five major base metals and crude oil. Real prices exhibit strong short-term fluctuations. At the same time, the growth rates of all prices are not significantly different from zero (see Table 1 in the appendix). The real prices are, thus, trend-less. This is in line with evidence over other time periods provided by Krautkraemer (1998), Von Hagen (1989), Cynthia-Lin and Wagner (2007), Stuermer (2016) and references therein. The real price for crude oil exhibits structural breaks, as shown in Dvir and Rogoff (2010). Overall, the literature is certainly not conclusive (see Pindyck, 1999; Lee et al., 2006; Slade, 1982; Jacks, 2013; Harvey et al., 2010), but we believe the evidence is sufficient to take trend-less prices as a motivation for our model.

Figure 2 shows that the world primary production of the examined non-renewable resources and world real GDP approximately exhibit constant positive growth rates since 1792. A closer statistical examination confirms that the production of non-renewable resources exhibits significantly positive growth rates in the long term (see table 2 in the appendix).[1]

The crude oil production follows this pattern up to 1975. Inclusion of the time period from 1975 until 2009 reveals a statistically significant negative trend and, therefore, declining growth rates over time due to a structural break in the oil market (Dvir and Rogoff, 2010; Hamilton, 2009). In the case of primary aluminum production, we also find declining growth rates over time and hence, no exponential growth of the production level. This might be attributable to the increasing importance of recycling (see data by U.S. Geological Survey, 2011a).

Insert Figure 1 about here.

Insert Figure 2 about here.

Overall, we take these stylized facts as motivation to build a model that exhibits trend-less resource prices and constant growth in the worldwide production of non-renewable resources and in world aggregate output.

2.2 Geological Abundance of Non-Renewable Resources

We update earlier computation of the total abundance (or quantity) of non-renewable resources by Nordhaus (1974). Table 4 shows the ratios of the quantities of reserves, resources, and geological abundance with respect to annual mine production for several

[1]As our model does not include population growth, we run the same tests for the per capita data as a robustness check. The results are roughly in line with the results described above. See table 3 in the appendix.

important non-renewable resources.[2] It provides evidence supporting the validity of Nordhaus' statement that "the future will not be limited by sheer availability of important materials"(Nordhaus, 1974, p. 23) As most metals are recyclable, the extractable stock in the techno-sphere even increases (Wellmer and Dalheimer, 2012).

We also add numbers for hydrocarbons. Even though conventional oil resources may be exhausted someday, resources of unconventional oil, natural gas, and coal, which could substitute for conventional oil in the long run, are abundant. Aguilera et al. (2012) state that conventional and unconventional resources "are likely to last far longer than many now expect" (p. 59). Rogner (1997) concludes that "fossil energy appears almost unlimited" (p. 249) given a continuation of historical technological trends.

Insert table 4 about here.

2.3 Geological Distribution of Non-Renewable Resources

Non-renewable resources are not uniformly concentrated in the earth's crust, reflecting variations in geochemical processes over time. Ahrens (1953, 1954) states in the fundamental law of geochemistry that the elements exhibit a log-normal grade-quantity distribution in the earth's crust, as he postulates a decided positive skewness.

Geologists do not fully agree on a log-normal distribution, especially regarding very low concentrations of metals, which might be mined in the distant future. Skinner

[2] Table 5 in the appendix illustrates that the assumption of exponentially increasing extraction of non-renewable resources does not alter the overall conclusion of table 4.

(1979) and Gordon et al. (2007) propose a discontinuity in the distribution due to the so-called "mineralogical barrier," the approximate point below which metal atoms are trapped by atomic substitution.

Gerst (2008) concludes in his geological study of copper deposits that he can neither confirm nor refute these two hypotheses. However, based on worldwide data on copper deposits over the past 200 years, he finds evidence for a log-normal relationship between copper production and ore grades. Mudd (2007) analyzes the historical evolution of extraction and grades of deposits for different base metals in Australia. He finds that production has increased at a constant rate, while grades have consistently declined.

We recognize that there remains uncertainty about the geological distribution, especially regarding hydrocarbons with their distinct formation processes. However, we believe that it is reasonable to assume that a non-renewable resource is distributed according to a log-normal relationship between the grade of deposits and quantity.

2.4 Technological Change in the Extractive Sector

Technological change in resource extraction offsets the depletion of economically extractable reserves of non-renewable resources (Simpson, 1999, and others). Hence, reserves are drawn down by extraction, but increase by technological change in extraction technology. The reason for this phenomenon is that non-renewable resources such as copper, aluminum, and hydrocarbons are extractable at different costs due to varying grades, thickness, depths, and other characteristics of mineral deposits. Technological change makes deposits economically extractable that, due to high costs, have not been previously extractable (see Simpson, 1999; Nordhaus, 1974, and others).

There is ample empirical and narrative evidence for this phenomenon (see for example Lasserre and Ouellette, 1991; Mudd, 2007; Simpson, 1999; Wellmer, 2008). For example, Radetzki (2009) and Bartos (2002) describe how technological changes in mining equipment, prospecting, and metallurgy have gradually made possible the extraction of copper from lower grade deposits. Figure 4 shows that copper reserves[3] have increased by more than 700 percent over the last couple of decades. As a consequence, the average ore grades of copper mines, for example, have decreased from about twenty percent 5,000 years ago to currently below one percent (Radetzki, 2009). Figure 3 illustrates this development using the example of U.S. copper mines.

Gerst (2008) and Mudd (2007) come to similar results for worldwide copper mines and the mining of different base-metals in Australia. The evidence also shows that decreases in average mined ore grades have slowed as technological development progressed. Under the assumption that global R&D investment has stayed constant or increased in real terms, this suggests that there are decreasing returns to R&D in terms of making mining from deposits of lower grades economically feasible.

Insert Figure 3 about here.

Insert Figure 4 about here.

We observe similar developments for hydrocarbons. Using the example of the offshore oil industry, Managi et al. (2004) show that technological change has offset the

[3]Reserves are those resources for which extraction is considered economically feasible (U.S. Geological Survey, 2011c).

cost-increasing degradation of resources. Crude oil has been extracted from ever deeper sources in the Gulf of Mexico, as Figure 5 in the appendix shows. Furthermore, technological change and high prices have made it profitable to extract hydrocarbons from unconventional sources, such as light tight oil, oil sands, and liquid natural gas (International Energy Agency, 2012). As a result, oil reserves have doubled since the 1980s (see figure 6 in the appendix).

Overall, empirical evidence suggests that technological change offsets resource depletion by increasing economically extractable reserves. History shows that average ore grades of mines declined while technological development progressed. Evidence suggests that the effect of technological change has slowed down in terms of making deposits of lower grades economically extractable.

3 Innovation and Extraction Technology in Partial Equilibrium

We first set up and analyze an extractive sector in a partial equilibrium to explain key concepts of our theory.

3.1 Extractive Firms

We consider an extractive industry with a large number of infinitely small firms. The non-renewable resource R is located in a continuum of deposits of declining grades $O \in (0, 1)$. Grade O refers to a measure of quality of mineral deposits, for example, ore grades in the case of metals. We use continuous time to facilitate interpretation of the necessary conditions and the analysis of equilibrium dynamics.

3.1.1 Market Entry

Firm j can freely enter the market if it develops a patent for a new extraction technology (or machine variety) j at cost $\frac{1}{\eta_R}$.[4] The new technology allows the firm to ultimately produce the resource $R(j)$ and to derive profits $\pi_R(j)$. Firms enter the market until the value of entering, namely profits, equals market entry cost. The free entry condition is thus[5]

$$\frac{1}{\eta_R} = \pi_{Rt} \ . \tag{1}$$

3.1.2 Reserves

The patent for new machine variety j is specific to deposits of certain grades O. It allows the firm to claim ownership of all of the non-renewable resource in the respective deposits and to declare them its new reserves $X_t(j) \geq 0$.

Reserves are defined as deposits of the non-renewable resource in the ground, which can be extracted at a constant extraction cost $\phi > 0$. We assume that the marginal extraction cost for deposits not classified as reserves are infinitely high, $\phi = \infty$. Firms start with $R(0) = 0$. After market entry, an extractive firm j's stock of reserves S

[4]This notation is chosen for consistency with the general equilibrium model, where the cost of innovation is $\frac{1}{\eta_R}$ in terms of final output. Please note also that we use j to denominate both, new machine varieties and firms, because each firm can only produce one new machine variety in line with Acemoglu (2002).

[5]In line with the standard approach in endogenous growth theory we assume that a firm decides in a first step whether to innovate and to enter the market. Firms calculate the net present value of innovation. Only if the NPV exceeds (or equals) the cost of developing the innovation, it enters. Once the firm is in the market, it maximizes profits.

evolves according to:

$$\dot{S}_t(j) = -R_t(j) + X_t(j) \qquad S_t(j) \geq 0, X_t(j) \geq 0, R_t(j) \geq 0, X(j) + S(j) \geq R(j) \ . \ (2)$$

A change in the resource quantity in reserves $\dot{S}_t(j)$ is the result of two flows: New reserves $X_t(j)$ are created due to the development of new extraction technology that converts deposits of certain ore grades O into reserves. Second, reserves decline due to extraction and marketing of $R_t(j)$.

3.1.3 Profits

The profit of firm j from developing one new technology j is given as

$$\pi_R(j) = p_R R(j) - \phi R(j) - \psi \ , \tag{3}$$

where p_R is the resource price, ψ is the cost to produce one machine from machine variety j, and it must hold that $X(j) + S(j) \geq R(j)$. The technology can only be used once so that the NPV is given by current profits, $V = \pi_R$.

Note that $X_t(j)$ is a function of the geological function $Q(O)$ and the extraction technology function $O(N)$, which we explain in the following.

3.2 Extraction Technology

In the technology function (see figure 8), each deposit of a certain grade O is associated with a unique state of the accumulated extraction technology N_R. Technology is hence non-rival, but excludable because it applies only to deposits of specific grades. Firms obtain a patent, which allows them to exercise monopoly power and to claim ownership of all of the non-renewable resource in the respective deposits. Knowledge diffuses immediately to other firms in line with standard assumptions in endogenous growth theory. We assume that firms do not face uncertainty about technology outcomes.

Based on stylized facts, we assume that technological development makes deposits economically extractable, but that there are decreasing returns in terms of ore grades O:

$$O(N_R) = e^{-\mu N_R},\ \mu \in \mathbb{R}_+ \ \ N_R \in (0, \infty)\ . \tag{4}$$

The curve starts with deposits of close to a 100 percent ore grade, which represents the state of the world several thousand years ago. The functional form is roughly in line with historical data for U.S. copper mining over the last 100 years. We assume this relationship holds continuous in a time-frame that is relevant to economic decision making. We presume that ore grades only get closer to zero in the long term.

The marginal effect of extraction technology on ore grade that becomes economically extractable declines as grades decrease. μ is the curvature parameter of the extraction technology function. If, for example, μ is high, the average effect of new technology on converting deposits to reserves in terms of grades is relatively high.

Insert Figure 8 about here.

3.3 Geology

The geology function describes the distribution of the non-renewable resource in the geological environment. Firms fully know about this distribution. We assume that the non-renewable resource R is located in a continuum of deposits of declining grades $O \in (0,1)$. Grade O refers to a measure of quality of mineral deposits, for example, ore grades in the case of metals. Mineral deposits exhibit a multitude of characteristics that affect extraction cost, but ore grade seemed to be the most important of these characteristics. Grade may also point to different types of deposits of hydrocarbons. For example, we could say that conventional crude oil is extracted from high grade deposits, while unconventional crude oil is produced from low grade deposits. Grade $O = 1$ corresponded to a deposit, which is extractable without any technology. For example, 7,000 years ago humans picked up high-grade copper nuggets from the ground. A deposit of grade zero does not contain any of the non-renewable resource.

We define $Q(O)$ as the "cumulative resource quantity", that is the amount of resources available in deposits of ore grades in the interval $[O,1)$. Based on geological research, for example in Ahrens (1953, 1954), we assume that the cumulative resource quantity can be approximated by

$$Q(O) = -\delta \ln(O), \quad \delta \in \mathbb{R}_+ \;\; O \in (0,1) \; . \tag{5}$$

This functional form is in line with historical evidence that the total quantity of non-renewable resource production has been inversely proportional to the grades of the deposits. The continuous formulation is a simplifying approximation, which is reasonably realistic for the long time horizons considered here. Although we recognize that non-renewable resources are ultimately finite in supply, we assume that the underlying resource quantity goes to infinity. The functional form therefore implies that the quantity of the resource tends to infinity as the grade of deposits gets closer to zero.

Parameter δ controls the curvature of the function. If δ is high, the marginal effect on the quantity of the non-resources from shifting to deposits of lower grades is high. It implies that the average concentration of the non-renewable resource is high in the crustal mass (see also figure 7).

Insert Figure 7 about here.

3.4 What is the Marginal Effect of New Extraction Technology on Reserves?

The technological function, equation (4) and the geological function, equation (5), have offsetting effects. This leads to a constant marginal effect of new technology on new reserves.

Proposition 1 *The cumulative resource quantity develops proportionally to the level of extraction technology N_R:*

$$Q(O(N_{Rt})) = \delta\mu N_{Rt}\ . \tag{6}$$

The marginal effect of a new machine variety j on new reserves $X_t(j)$ equals:

$$X_t(j) = \frac{dQ(O(N_{Rt}))}{dN_R} = \delta\mu \ . \qquad (7)$$

As functions (5) and (4) are inverse, the relationship between investment in technology and reserves is linear. **Proof of Proposition 1**

$$\begin{aligned} Q(O(N_{Rt})) &= -\delta \ln(O(N_{Rt})) \\ &= -\delta \ln(e^{-\mu N_{Rt}}) \\ &= \mu\delta N_{Rt} \end{aligned}$$

□

The intuition is that two offsetting effects cause this result: (i) new extraction technology exhibits decreasing returns in terms of making lower grade deposits extractable; (ii) the resource quantity is geologically distributed such that it implies increasing returns in terms of resource quantities as the grade of its deposits declines.

Equation (6) depends on the shapes of the geological function and the technology function. If the respective parameters δ and μ are high, the marginal return on new extraction technology will also be high. The constant effect of technology on resources also implies that the social value of an innovation is equal to the private value. The reason is that the development does not cause an exhaustion of the resource. Future innovations are not reduced in profitability. No positive or negative spill-overs occur.

3.5 Firms' Optimization

Using profit function (3), we obtain the following optimization problem of firm j:

$$\max_{R(j)} (p_R - \phi)R(j) - \psi \text{ such that } X(j) + S(j) \geq R(j). \tag{8}$$

The maximization problem can be expressed with the following Lagrangian:

$$L = (p_R - \phi)R(j) - \psi + \lambda[X(j) + S(j) - R(j)]. \tag{9}$$

This leads to the following first order conditions:

$$(p_R - \phi)R(j) - \lambda = 0 \tag{10}$$

$$\lambda[X(j) + S(j) - R(j)] = 0 \tag{11}$$

$$\tag{12}$$

Consider the case that the constraint is not binding. Given (11), we obtain $\lambda = 0$, and from (10) follows $p_R - \phi = 0$. This is a contradiction, since the market entry condition ensures $\pi_R > 0$, which is not in line with $p_R - \phi = 0$. Therefore, the constraint must be binding and $R(j) = X(j) + S(j)$. In equilibrium, it is profit maximizing for firm j to not keep reserves, $S(j) = 0$, and we obtain $R(j) = X(j)$. If we assume stochastic technological change, extractive firms will keep a positive stock of reserves S_t to insure against a series of bad draws in R&D. Reserves will grow over time in line with aggregate growth. The result would, however, remain the same: In the long term, resource extraction equals resources in new reserves.

3.6 The Value of Innovation in Partial Equilibrium

Let us assume that firms sell the resource, which is a homogenous good, in a fully competitive market. The resource price p_R is hence equal to marginal cost in equilibrium. Why do extractive firms innovate despite full competition? What are their incentives to innovate?

In an equilibrium without innovation, the marginal extraction cost is infinitely high, $\phi = \infty$. The resource price is therefore also infinitely high, $p_R = \infty$. This results in no demand $D(\phi) = 0$.[6] No firm enters the market and firms do not make any profits.

In an equilibrium with innovation, firms innovate and enter until profits from a new machine variety j equal market entry cost for developing the technology (see the free entry condition (1)). Taking into account (7) and assuming that $\phi = 0$, the firm's profit function is given by

$$\pi_R = p_R \mu \delta - \psi. \tag{13}$$

Firms in the extractive sector invest in new extraction technology despite the fully competitive environment, because new technology is grade-specific. As benefits from innovation diffuse within one period, Firms maximize only current profits when making their technology investment decisions.

[6] The demand side of the industry is modeled with a standard demand curve $R = D(p_R)$, where p_R is the resource price and R is the demand for the non-renewable resource at this price. We assume that $D(p_R)$ is strictly decreasing, differentiable, and satisfies $D(\zeta) > 0$ for all $\zeta > 0$. These conditions ensure that there is positive demand when the price is equal to marginal cost.

Using the free market condition, the resource price equals marginal cost, $p_R = \left(\frac{1}{\eta_R} + \psi\right)\frac{1}{\mu\delta}$, which includes the machine cost, ψ, the extraction cost ϕ and the innovation cost associated with a patent, $\frac{1}{\eta_R}$. The price also reflects the linear production function between input N_R and output X in the term $\mu\delta$.

The equilibrium amount of resource production is given as the intersection of the inverse linear supply function $p_R = \left(\frac{1}{\eta_R} + \psi\right)\frac{1}{\mu\delta}$ and the demand function $D(p_R)$ as $D\left(\left(\frac{1}{\eta_R} + \psi\right)\frac{1}{\mu\delta}\right)$. The profits for one innovation are thus given as $\pi_R(j) = \frac{1}{\eta_R}$. These profits are the incentive to develop new technology j.

Boiling down a dynamic optimization problem to a static one is key to our theory. It allows us to make the model solvable and computable. At the same time, the model is rich enough to derive meaningful theoretical predictions about the relationship between technological change, geology and economic growth.

4 Innovation in Extraction Technology in an Endogenous Growth Model

We build an endogenous growth model with two sectors, an extractive sector and an intermediate goods sector, and take the framework by Romer (1986) and Acemoglu (2002) as a starting point. So far extractive firms have done both, extracting the resource and developing technologies. To adapt to the growth framework, we split up the extractive sector into extractive firms and extraction technology firms.

4.1 Setup

We consider a standard setup of an economy with a representative consumer that has constant relative risk aversion preferences:

$$\int_0^\infty \frac{C_t^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt \ . \tag{14}$$

The variable C_t denotes consumption of aggregate output at time t, ρ is the discount rate, and θ is the coefficient of relative risk aversion.

The aggregate production function combines two inputs, namely an intermediate good Z and a non-renewable resource R, with a constant elasticity of substitution:

$$Y = \left[\gamma Z^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) R^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \ . \tag{15}$$

The distribution parameter $\gamma \in (0, 1)$ indicates their respective importance in producing aggregate output Y. The elasticity of substitution is $\varepsilon > 0$. Inputs Z_t and R_t are substitutes for $\varepsilon > 1$. In this case, the resource is not essential for aggregate production (see Dasgupta and Heal, 1980). The Cobb-Douglas case is $\varepsilon = 1$. For $0 < \varepsilon < 1$ the two inputs are complements.

The budget constraint of the representative consumer is:

$$C + I + M \leq Y \ . \tag{16}$$

I denotes aggregate spending on machines and M aggregate R&D investment, where $M = M_Z + M_R$. The usual no-Ponzi game conditions apply.

4.1.1 Extractive Firms and Intermediate Goods Firms

There are two sectors, the extractive sector and the intermediate goods sector. Each sector consists of a large number of infinitely small firms that produce the respective good, and technology firms that produce the sector-specific technologies.[7] Firms in the two sectors use different types of machines to produce the non-renewable resource and the intermediate good, respectively. We assume that all machines depreciate after use.

In the extractive sector, extractive firms extract the non-renewable resource R from their reserves S:

$$R_t = X_t - \dot{S}_t \ , \qquad S_t \geq 0, X_t \geq 0, R_t \geq 0 \ , \tag{17}$$

where the resource quantity is a function of two flows: (i) inflows of new resource quantities X due to technology investment that convert mineral deposits into reserves, and (ii) changes in reserves due to extraction and marketing $\dot{S}$.

Firms can buy machines j to increase their reserves by the resource quantity X according to the following function:

$$X_t = \delta\mu \lim_{h \to 0} \frac{1}{h} \int_{N_R(t-h)}^{N_R(t)} x_R(j)^{(1-\beta)} dj \ , \tag{18}$$

where $x_R(j)$ refers to the number of machines used for each machine variety j. We

[7] We assume that the firm level production functions in both sectors exhibit constant returns to scale, so there is no loss of generality in focusing on aggregate production functions.

assume that $\beta = 0$ in the extractive sector, because extractive firms invest into technology to continue resource production. If they do not invest, extraction cost becomes infinitely high. Firms are indifferent from which deposits they extract the resource. This is different from the intermediate goods sector, where firms increase the division of labor by investing into new machine varieties that are partial complements. They hence make production of the intermediate good more efficient.

However, while machines of type j in the intermediate sector can be used infinitely often, a machine of variety j in the resource sector is grade-specific and essential to extracting the resource from deposits of certain grades D and therefore can only be used once and the range of machines used to produce resources at time t is only $\dot{N}_R$. In contrast, the intermediate good sector can use the full range of machines $[0, N_Z(t)]$ complementing labor. Under the assumption that $x_R(j) = 1$, equation 18 turns into:

$$X(t) = \delta\mu \lim_{h \to 0} \frac{1}{h} \int_{N_R(t-h)}^{N_R(t)} 1 dj \tag{19}$$

$$= \delta\mu \dot{N}_R \; . \tag{20}$$

As we assume that extractive firms hold no reserves $S = 0$ (see the intuition in earlier chapter), $R_t = X_t$. It follows that the production function of the extractive firms is

$$R_t = \delta\mu \dot{N}_{Rt}. \tag{21}$$

The intermediate good sector follows the basic setup of Acemoglu (2002). Firms produce an intermediate good Z according to the following production function:

$$Z = \frac{1}{1-\beta}\left(\int_0^{N_z} x_z(j)^{1-\beta} dj\right) L^{\beta} \ , \tag{22}$$

where $x_Z(j)$ refers to the number of machines used for each machine variety j in the production of the intermediate good, L is labor, which is in fixed supply, and β is $\in (0,1)$. This implies that machines in the intermediate good sector are partial complements.

4.1.2 Technology Firms in the Extractive Sector and in the Intermediate Goods Sector

All machines are supplied by sector-specific technology firms that each have one fully enforced perpetual patent on the respective machine variety. The price charged by these firms at time t is denoted $\chi_R(j)$ for $j \in [N_{t-h}, N_t]$ and $\chi_Z(j)$ for $j \in [0, N_Z(t)]$.

In the resource sector, machines are substitutes, and there is full competition between technology firms. Machine prices result from the market equilibrium of demand and marginal cost. Technology firms in the extractive sector have a monopoly for the use of a particular patent. All patents, however, are exchangeable, since they give access to additional homogenous resources. Notice that this does not mean that patents (or machines) have zero value: The resource firms have to buy machines to continue producing. It just does not matter, which ones. Given that machines are interchangeable, the technology monopolists are in perfect competition with each other.

In the intermediate good sector, machines are partial complements. Technology firms have some degree of market power and can set the price for machines.

Once invented, each machine in the two sectors can be produced at a fixed marginal cost $\psi_Z > 0$ and $\psi_R > 0$, respectively. Total resources devoted to machine production at time t are

$$I_t = (1-\beta)(\int_0^{N_z(t)} x_Z(j)dj) + \dot{N}_t. \tag{23}$$

The innovation possibilities frontier, which determines how new machine varieties are created, is assumed to take the following forms in the two sectors:

$$\dot{N}_R = \eta_R M_R \text{ and } \dot{N}_Z = \eta_R M_Z \ . \tag{24}$$

Technology firms can spend one unit of the final good for R&D investment M at time t to generate flow rates $\eta_R > 0$ and $\eta_Z > 0$ of new patents, respectively. It hence needs $\frac{1}{\eta_R}$ and $\frac{1}{\eta_Z}$ units of final output to develop a new machine variety in the two sectors, respectively. The economy is assumed to start at the initial technology levels $N_R(0) > 0$ and $N_Z(0) > 0$.

Firms can freely enter the market if they develop a patent for a new machine variety. They can only invent one new variety. There is no aggregate uncertainty in the research process. There is idiosyncratic uncertainty, but with many different technology firms undertaking research, (24) holds deterministically at the aggregate level.

In the extractive sector, the value of a technology firm that discovers one new variety of machines depends only on instantaneous profit. Machine prices result from the market equilibrium of demand and marginal cost.

$$V_R(j) = \pi_R(j) = (\chi_R(j) - \psi_R)x_R(j) \ , \tag{25}$$

which is equal to

$$V_R(j) = \pi_R(j) = \chi_R(j) - \psi \ , \tag{26}$$

under the assumption that each technology firm only produces one machine per patent, $x_R(j) = 1$. Machines are grade specific and can therefore only be used once. For the same reason, technology firms in the extractive sector maximize only their instantaneous profits for given prices.

The value of a technology firm in the intermediate goods sector that discovers one of the machines is given by the standard formula for the present discounted value of profits:

$$V_Z(j) = \int_t^{\infty} exp\left(-\int_t^{s} r(s')ds'\right) \pi_Z(j)ds \ , \tag{27}$$

where instantaneous profits are denoted

$$\pi_Z(j) = (\chi_Z(j) - \psi_Z)x_Z(j) \ , \tag{28}$$

where r is the market interest rate, and $x_Z(j)$ and $\chi_Z(j)$ are the profit-maximizing choices for the technology monopolist in the intermediate goods sector. The Hamilton-Jacobi-Bellman Equation version of the value function for the intermediate good sector is

$$r_t V_Z(j) - \dot{V}_Z(j) = \pi_Z(j). \tag{29}$$

Setting the price of the final good as the numeraire gives (for the derivation of the price index see the derivation of equation (12.11) in Acemoglu (2009)):

$$\left[\gamma^{\varepsilon} p_Z^{1-\varepsilon} + (1-\gamma)^{\varepsilon} p_R^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} = 1 \ , \tag{30}$$

where p_Z is the price index of the intermediate good and p_R is the price index of the non-renewable resource. Intertemporal prices of the intermediate good are given by the interest rate $[r_t]_{T=0}^{\infty}$.

4.2 Characterization of Equilibrium

We define the *allocation* in this economy by the following objects: time paths of consumption levels, aggregate spending on machines, and aggregate R&D expenditure, $[C_t, I_t, M_t]_{t=0}^{\infty}$; time paths of available machine varieties, $[N_{Rt}, N_{Zt},]_{t=0}^{\infty}$; time paths of prices and quantities of each machine, $[\chi_{Rt}(j), x_{Rt}(j)]_{j\in[0,N_{Rt}]t}^{\infty}$ and $[\chi_{Zt}(j), x_{Zt}(j)]_{j\in[0,N_{Zt}],t}^{\infty}$; the present discounted value of profits V_R and V_Z, and time paths of factor prices, interest rate and wages, $[r_t, w_t]_{t=0}^{\infty}$.

An *equilibrium* is an allocation in which all technology firms in the intermediate good sector choose $[\chi_{Zt}(j), x_Zt(j)]_{j\in[0,N_Z(t)],t}^{\infty}$ to maximize profits. Machine prices in the extractive sector $\chi_{Rt}(j)$ result from the market equilibrium, because extraction technology firms are in full competition and can only produce one machine per patent.

The evolution of $[N_{Rt}, N_{Zt}]_{t=0}^{\infty}$ is determined by free entry; the time paths of factor prices, $[r, w]_{t=0}^{\infty}$, are consistent with market clearing; and the time paths of $[C_t, I_t, M_t]_{t=0}^{\infty}$

are consistent with household maximization.

4.2.1 The Final Good Producer

The final good producer demands the intermediate good and the resource for aggregate production. Prices and quantities for both are determined in a fully competitive equilibrium. Taking the first order condition with respect to the intermediate good and the non-renewable resource in (15), we obtain the demand for the intermediate good

$$Z = \frac{Y(1-\gamma)^{\varepsilon}}{p_Z^{\varepsilon}} \, , \tag{31}$$

and the demand for the resource

$$R = \frac{Y(1-\gamma)^{\varepsilon}}{p_R^{\varepsilon}} \, . \tag{32}$$

4.2.2 Extractive Firms and Intermediate Good Firms

To characterize the (unique) equilibrium, we first determine the extractive machine prices and the number of machine varieties in the extractive sector. While machines in the intermediate good sector are partial complements, machines in the extractive sector are perfect substitutes. As a result, technology firms in the extractive sector do not have market power. The demand for machines per machine variety j is assumed to be one, because machines are grade specific.

Machine prices and the number of machine varieties are determined in a market equilibrium between extractive firms and technology firms. One unit of the resource needs $\frac{1}{\delta\mu}$ extractive machines (remember there is only one machine produced per machine variety) following (21). The representative firm faces a cost for producing R_t units of resource given by $\Omega(R_t) = R_t\chi_R(j)\frac{1}{\delta\mu}$, where χ_R is the machine price charged by the technology firms in the extractive sector. The marginal cost for producing R_t units of the resource is $\Omega'(R_t) = \chi_R(j)\frac{1}{\delta\mu}$. The inverse supply function of the resource is hence constant $p_R = \chi_R(j)\frac{1}{\delta\mu}$ (see also proposition 1). We obtain a market equilibrium at

$$p_R = \chi_R(j)\frac{1}{\delta\mu} \tag{33}$$

and

$$R_t = \frac{Y(1-\gamma)^{\varepsilon}}{(\chi_R(j)\frac{1}{\delta\mu})^{\varepsilon}} \ . \tag{34}$$

Using (21), we obtain the demand for machine varieties:

$$\dot{N} = \frac{1}{\delta\mu}\frac{Y(1-\gamma)^{\varepsilon}}{(\chi_R(j)\frac{1}{\delta\mu})^{\varepsilon}} \ . \tag{35}$$

In the intermediate goods sector, the maximization problem is static since machines depreciate fully after use. The problem can be written as

$$\max_{L,\{x_Z(j)\}j\in[0,N_{Zt}]} p_Z Z - wL - \int_0^{N_Z} \chi_Z(j)x_Z(j)dj \ , \tag{36}$$

The FOC with respect to $x_Z(j)$ immediately implies the following isoelastic demand function for machines:

$$x_{Zt}(j) = \left(\frac{p_{Zt}}{\chi_{Zt}(j)} \right)^{1/\beta} L \ , \tag{37}$$

for all $j \in [0, N_Z(t)]$ and all t,

4.2.3 Technology Firms

In the extractive sector, the demand function for extraction technologies (35) is also isoelastic, but there is perfect competition between the different suppliers of extraction technologies, as machine varieties are perfect substitutes. Because there is only one machine per machine variety, the constant rental rate χ_R that all monopolists $j \in [N_{t-h}, N_t] \lim_{h \to 0}$ charge includes the cost of machine production ψ and a mark-up that refinances R&D costs for producing the machine variety j. The rental rate is the result of a competitive market and derived from (34). It equals:

$$\chi_R(j) = (Y/R)^{\frac{1}{\varepsilon}} (1 - \gamma)\delta\mu. \tag{38}$$

In the extractive sector, $x_{Rt} = 1$ by assumption, as discussed above. The demand for machine varieties in the extractive sector (35) depends on the free entry condition, as explained below.

Substituting (37) into (28), we calculate the FOC with respect to machine prices in the intermediate good sector: $\chi_Z(j)$: $\left(\frac{p_Z}{\chi_Z(j)} \right)^{\frac{1}{\beta}} L - (\chi_Z(j) - \psi) p_Z^{\frac{1}{\beta}} \frac{1}{\beta} \chi_Z(j)^{\frac{1}{\beta}-1} L = 0$. Hence, the solution of the maximization problem of any monopolist $j \in [0, N_Z]$ involves setting the same price in every period according to

$$\chi_{Zt}(j) = \frac{\psi}{1-\beta} \text{ for all } j \text{ and } t \ . \tag{39}$$

All monopolists in the intermediate goods sector charge a constant rental rate equal to a markup over their marginal cost of machine production, ψ. We normalize the marginal cost of machine production to $\psi \equiv (1-\beta)$ (remember that the elasticity of substitution between machines is $\epsilon \equiv \frac{1}{\beta}$), so that

$$\chi_{Zt}(j) = \chi_Z = 1 \text{ for all } j \text{ and } t \ . \tag{40}$$

In the intermediate good sector, substituting the machine prices (40) into the demand function (37) yields:

$$x_{Zt}(j) = p_{Zt}^{1/\beta} L \text{ for all } j \text{ and all } t.$$

Since the machine quantities do not depend on the identity of the machine, only on the sector that is being served, profits are also independent of machine variety in both sectors. Firms are symmetric.

In particular profits in the two sectors are

$$\pi_{Zt} = \beta p_{Zt}^{1/\beta} L, \text{ and } \pi_{Rt} = (Y/R)^{\frac{1}{\varepsilon}} (1-\gamma)\delta\mu - \psi \ . \tag{41}$$

This implies that the net present discounted value of monopolists only depends on the sector and can be denoted by V_{Zt} and V_{Rt}.

Combining the demand for machines (37) with the production function of the intermediate goods sector (22) yields the *derived* production function:

$$Z(t) = \frac{1}{1-\beta} p_{Zt}^{\frac{1-\beta}{\beta}} N_{Zt} L, \tag{42}$$

The equivalent equation in the extractive sector is (21), because there is no optimization over the number of machines by the extraction technology firms, as the demand for machines per machine variety is one.

To complete the description of equilibrium on the technology side, we need to impose the free-entry conditions

$$V_{Zt} = \frac{1}{\eta_Z}, \text{ and } V_{Rt} = \frac{1}{\eta_R} \; . \tag{43}$$

Like in the intermediate sector, markups are used to cover technology expenditure in the extractive sector.[8] Combining (38) and (25), we obtain that the profit of technology

[8] While technology firms in the intermediate good sector set machine prices in monopolistic competition, technology firms in the extractive sector are price takers due to perfect competition. In the intermediate good sector, technology firms optimize over the price-quantity combination for their machine. The free entry condition ensures that optimal profits from a machine variety are just enough to refinance the R&D cost. In the extractive sector, technology firms face full competition. Therefore, the free entry condition ensures that profits from producing a new machine variety must equal the cost for developing the patent. As soon as firm set a price $\chi_R > \frac{1}{\eta_R} + \psi$ they make pure profits. In this situation, a competitor would enter and would offer a price between χ_R and $\frac{1}{\eta_R} + \psi$, thus stealing the firm's business. As soon as the firm sets a price $\chi_R < \frac{1}{\eta_R} + \psi$, the firm's profit π_R are not sufficient to cover the investment of $\frac{1}{\eta_R}$. Knowing this the firm will not enter. Squeezed between the threat of making a loss and the threat of being undercut by a competitor, the firm chooses the only possible and hence optimal price $\chi_R = \frac{1}{\eta_R} + \psi$. Despite different market structures, both types of firms set profit maximizing prices and enter the market freely such that profit equal exactly to market entry cost.

firms from developing one new machine variety is:

$$V_R(j) = \pi_R(j) = \chi_R(j) - \psi = (Y/R)^{\frac{1}{\varepsilon}} (1-\gamma)\delta\mu - \psi \; . \tag{44}$$

To compute the equilibrium quantity of machines and machine prices, we first rearrange (44) with respect to R and consider the free entry condition. We obtain

$$R_t = \frac{Y(1-\gamma)^{\varepsilon}}{\left(\left(\frac{1}{\eta_R} + \psi\right)\frac{1}{\delta\mu}\right)^{\varepsilon}} \; . \tag{45}$$

We insert (45) into (38) and obtain the equilibrium machine price.

$$\chi_R(j) = \frac{1}{\eta_R} + \psi \; . \tag{46}$$

4.2.4 Equilibrium Prices

Prices of the non-renewable resource and the intermediate good are derived from the marginal product conditions of the final good technology, (15), which imply

$$p \equiv \frac{p_R}{p_Z} = \frac{1-\gamma}{\gamma}\left(\frac{R}{Z}\right)^{-\frac{1}{\varepsilon}} \tag{47}$$

$$= \frac{1-\gamma}{\gamma}\left(\frac{\delta\mu\dot{N}_R}{\frac{1}{1-\beta}p_L^{\frac{1-\beta}{\beta}}N_Z L}\right)^{-\frac{1}{\varepsilon}} \tag{48}$$

There is no derived elasticity of substitution in analogy to Acemoglu (2002), because there is only one fixed factor, namely L in the extractive sector. In the extractive sector, resources are directly produced from the machines. The first line of this expression simply defines p as the relative price between the intermediate good and the non-renewable resource, and uses the fact that the ratio of the marginal productivities of the two goods must be equal to this relative price. The second line substitutes from (42) and (21) There are no relative factor prices in this economy like in Acemoglu (2002), because there is only one fixed factor in the economy, namely L in the intermediate goods sector.

Equation (46) implies the following proposition regarding the equilibrium resource price:

Proposition 2 *The resource price depends negatively on the average crustal concentration of the non-renewable resource and the average effect of extraction technology:*

$$p_R = \left(\frac{1}{\eta_R} + \psi\right)\frac{1}{\delta\mu}\ , \tag{49}$$

where ψ reflects the marginal cost of producing the machine and η_R is a markup that serves to compensate technology firms for R&D cost. The resource price equals marginal production costs due to perfect competition in the resource market.

The intuition is as follows: If, for example, δ increases, the average crustal concentration of the resource increases (see equation (5)) and the price decreases. If μ rises, the average effect of new extraction technology on converting deposits of lower grades to reserves increases (see equation (4)). This implies lower prices. The resource price level also depends negatively on the cost parameter of R&D development η_R.

4.2.5 Resource Intensity of the Economy

Substituting equation (49) into the resource demand equation (32), we obtain the ratio of resource consumption to aggregate output.

Proposition 3 *The resource intensity of the economy is positively affected by the average crustal concentration of the resource and the average effect of extraction technology:*

$$\frac{R}{Y} = (1-\gamma)^{\varepsilon}\left[(\frac{1}{\eta_R}+\psi)\frac{1}{\delta\mu}\right]^{-\varepsilon} . \tag{50}$$

The resource intensity of the economy is negatively affected by the elasticity of substitution $(1-\gamma)^{\varepsilon}\left[(\frac{1}{\eta_R}+\psi)\frac{1}{\delta\mu}\right]^{-\varepsilon} < 1$ *and positively otherwise.*

4.2.6 Directed Technological Change

Finally, household optimization implies

$$\frac{\dot{C}_t}{C_t} = \frac{1}{\theta}(r_t - \rho), \tag{51}$$

and

$$\lim_{t\to\infty}\left[\exp\left(-\int_0^t r(s)ds\right)(N_{Zt}V_{Zt} + \dot{N}_{Rt}V_{Rt})\right] = 0, \tag{52}$$

which uses the fact that $N_{Zt}V_{Zt} + \dot{N}_{Rt}V_{Rt}$ is the total value of corporate assets in this economy. In the resource sector, only *new* machine varieties produce profit.

We define the BGP equilibrium as an equilibrium path where consumption grows at the constant rate g^* and the relative price p is constant. From (30) this definition implies that p_{Zt} and p_{Rt} are also constant.

Let V_Z and V_R be the BGP net present discounted values of new innovations in the two sectors. Then (29) and the free entry condition of extraction technology firms imply that

$$V_Z = \frac{\beta p_Z^{1/\beta} L}{r^*}, \text{ and } V_R = \chi_R(j) - \psi \; , \tag{53}$$

where r^* is the BGP interest rate, while p_Z is the BGP price of the intermediate good and $\chi_R(j)$ is the BGP machine price in the extractive sector.

The greater is V_R relative to relative to V_Z, the greater are the incentives to develop machines in the extractive sector, N_R, rather than developing machines in the intermediate goods sector N_Z. Taking the ratio of the two equations in (53) and including the equilibrium machine price (46) yields

$$\frac{V_R}{V_Z} = \frac{\chi_R(j) - \psi}{\frac{1}{r}\beta p_Z^{\frac{1}{\beta}} L} = \frac{\frac{1}{\eta_R}}{\frac{1}{r}\beta p_Z^{\frac{1}{\beta}} L} \; . \tag{54}$$

This expression highlights the effects on the direction of technological change

1. The price effect manifests itself because V_R/V_Z is decreasing in p_Z. The greater is the intermediate good price, the smaller is V_R/V_Z and thus the greater are the incentives to invent technology complementing labor. Since goods produced by the relatively scarce factor are relatively more expensive, the price effect favors

technologies complementing the scarce factor. The resource price p_R does not affect V_R/V_Z due to perfect competition among extraction technology firms and a flat supply curve.

2. The market size effect is a consequence of the fact that V_R/V_Z is decreasing in L. The market for the intermediate good technology is the workers that use and work with this technology. Consequently an increase in the supply of labor translates into a greater market for the technology complementing labor. The market size effect in the intermediate good sector is defined by the exogenous factor labor. There is no equivalent in the extractive sector, because there is no exogenous factor of production.

3. Finally, the cost of developing one new machine variety in terms of final output also influences the direction of technological change. If the parameter η increases, the cost goes down, the relative profitability V_R/V_Z decreases, and therefore the incentive to invent extraction technology declines.[9]

[9] The above discussion is incomplete, however, since the intermediate good price is endogenous. Combining (48) with (54) the relative profitability of the technologies becomes

$$\frac{V_R}{V_Z} = \frac{\frac{1}{\eta_R}}{\frac{1}{r}\beta \left(p_R \frac{\gamma}{1-\gamma} \left(\frac{\delta\mu\dot{N}_R}{\frac{1}{1-\beta} p_Z^{\frac{1-\beta}{\beta}} N_Z L} \right)^{\frac{1}{\varepsilon}} \right)^{\frac{1}{\beta}} L} \tag{55}$$

Rearranging equation (30) we obtain

$$p_Z = \left(\gamma^{-\varepsilon} - \left(\frac{1-\gamma}{\gamma} \right)^{\varepsilon} p_R^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} . \tag{56}$$

Combining (56) and (49), we can eliminate relative prices, and the relative profitability of technologies

4.2.7 The Growth Rate on the Balanced Growth Path

Finally, we turn to the solution of the model. Adding the extractive sector to the standard model by Acemoglu (2002), changes the interest part of the Euler equation, $g = \theta^{-1}(r - \rho)$.[10] Instead of two exogenous production factors, the interest rate r in our model only includes labor, but adds the resource price.

The consumer earns wages from working in the intermediate goods sector and earns interest on investing in technology M_Z. The budget constraint thus is $C = wL + rM_Z$. Maximizing utility in equation (14) with respect to consumption and investments yields the first order conditions $C^{-\theta}e^{-\rho t} = \lambda$ and $\dot{\lambda} = -r\lambda$ so that the growth rate of consumption is

$$g_c = \theta^{-1}(r - \rho) \ . \tag{60}$$

This will be equal to output growth on the balanced growth path. We can thus solve for the interest rate and obtain $r = \theta g + \rho$. The free entry condition for the

becomes:

$$\frac{V_R}{V_Z} = \frac{\frac{1}{\eta_R}}{\frac{1}{r}\beta\left(\left(\gamma^{-\varepsilon} - \left(\frac{1-\gamma}{\gamma}\right)^{\varepsilon}\left(\left(\frac{1}{\eta_R} + \psi\right)\frac{1}{\mu\delta}\right)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}\right)^{\frac{1}{\beta}} L} \ . \tag{57}$$

This does not depend on ε in an interpretable way. Using the two free-entry conditions (43) and assuming that both of them hold as equalities, we obtain the following BGP *technology market clearing* condition:

$$\eta_Z V_Z = \eta_R V_R. \tag{58}$$

Combining 58 with 55, we obtain the following BGP ratio of relative technologies and solving for $\frac{\dot{N}_R}{N_Z}$ yields:

$$\left(\frac{\dot{N}_R}{N_Z}\right)^{*} = \left(\left(\frac{r}{\eta_Z \beta L}\right)^{\beta}\frac{1-\gamma}{\gamma p_R}\right)^{\varepsilon}\frac{L p_Z^{\frac{1-\beta}{\beta}}}{(1-\beta)\delta\mu} \tag{59}$$

where the asterisk $(*)$ denotes that this expression refers to the BGP value. The relative productivities are determined by both prices and the supply of labor.

[10]There is no capital in this model, but agents delay consumption by investing in R&D as a function of the interest rate.

technology firms imposes that profits from investing in patents must be zero. Revenue per unit of R&D investment is given by V_Z, cost is equal to $\frac{1}{\eta_Z}$. Consequently, we obtain $\eta_Z V_Z = 1$. Making use of equation (53), we obtain $\frac{\eta_Z \beta p_Z^{\frac{1}{\beta}} L}{r} = 1$. Solving this for r and substituting it into equation (60) we obtain:

$$g = \theta^{-1}(\beta \eta_Z L p_Z^{\frac{1}{\beta}} - \rho) \ . \tag{61}$$

Together with equations (56) and (49), this yields the growth rate on the balanced growth path. □

We characterize this growth rate of the economy, as follows:

Proposition 4 *Consider the directed technological change model described above. Suppose that*

$$\begin{aligned} \beta \left[(1-\gamma)_R^{\varepsilon}(\eta_R R)^{\sigma-1} + \gamma_Z^{\varepsilon}(\eta_Z L)^{\sigma-1}\right]^{\frac{1}{\sigma-1}} &> \rho, \text{ and} \\ (1-\theta)\beta \left[\gamma_R^{\varepsilon}(\eta_R R)^{\sigma-1} + \gamma_Z^{\varepsilon}(\eta_Z L)^{\sigma-1}\right]^{\frac{1}{\sigma-1}} &< \rho. \end{aligned} \tag{62}$$

If $(1-\gamma)^{\varepsilon}(\eta_R \delta\mu)^{1-\varepsilon} < 1$ *the economy cannot produce. Otherwise, there exists a unique BGP equilibrium in which the relative technologies are given by* (59)*, and consumption and output grow at the rate*

$$g = \theta^{-1}\left(\beta \eta_Z L \left[\gamma^{-\varepsilon} - \left(\frac{1-\gamma}{\gamma}\right)^{\varepsilon}\left(\frac{1}{\eta_R \delta\mu} + \frac{\psi}{\delta\mu}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}\frac{1}{\beta}} - \rho\right) \ . \tag{63}$$

If the inequality $(1-\gamma)^{\varepsilon}(\eta_R \delta\mu)^{1-\varepsilon} < 1$ holds, then the substitution between the intermediate goods and the resource is low and investments into extraction have a very small yield in terms of resources. The effect that economic growth is impossible if the resource cannot be substituted by other production factors is known as the "limits to growth" effect in the literature (see Dasgupta and Heal, 1979, p. 196 for example). When the effect occurs, growth is *limited* in models with a positive initial stock of resources, because the initial resource stock can only be consumed in this case. In our model growth is *impossible*, because there is no initial stock and the economy is not productive enough to generate the necessary technology. When the inequality does not hold, the economy is on a standard balanced growth path.

It can also be verified that there are simple transitional dynamics in this economy whereby starting with technology levels $N_R(0)$ and $N_Z(0)$, there always exists a unique equilibrium path, and it involves the economy monotonically converging to the BGP equilibrium of (63) like in Acemoglu (2002).

4.2.8 Technology Growth

We derive the growth rates of technology in the two sectors from equations (21), (32), and (49). The stock of technology in the intermediate goods sector grows at the same rate as the economy.

Proposition 5 *The stock of extraction technology grows proportionally to output according to:*

$$\dot{N}_R = (1-\gamma)^{\varepsilon} Y \left(1/\eta_R + \psi\right)^{-\varepsilon} (\delta\mu)^{\varepsilon-1} .$$

In contrast to the intermediate goods sector, where firms can make use of the entire *stock* of technology, firms in the extractive sector can only use the *flow* of new technology to convert deposits of lower grades into new reserves. Previously developed technology cannot be employed because it is grade specific, and deposits of that particular grade have already been depleted. Note also that firms in the extractive sector need to invest a larger share of total output to attain the same rate of growth in technology in comparison as firms in the intermediate goods sector.

The effects of the two parameters δ from the geological function and μ from the extraction technology function on $\dot{N}_R$ depend on the elasticity of substitution ε. As in Acemoglu (2002), there are two opposing effects at play: The first is a price effect, meaning that technology investments are directed towards the sector, which is scarce. The second is a market size effect, meaning that technology effects are directed more to the larger sector.

If the goods of the two sectors are complements ($\varepsilon < 1$), the price effect dominates. An increase in δ or μ makes resources cheaper, but the technology growth rate in resources decelerates, because the freed sources are invested into the complementary sector. If the goods of the two sectors are substitutes ($\varepsilon > 1$), the market size effect dominates. An increase in δ or μ makes resources cheaper and causes an acceleration in the technology growth rate in resources, because more of the now cheaper good is demanded.

4.3 Discussion

We discuss the assumptions made in section 4, the comparison to other models with non-renewable resources, and the ultimate finiteness of the resource.

How would other functional forms of the geological function in equation (5) affect the predictions of the model? First, if D is discontinuous with an unanticipated break at d_0, at which the parameter changes to $\delta' \in \mathbb{R}_+$, there would be two balanced growth paths: one for the period before, and one for the period after the break. Both paths would behave according to the model's predictions. As an illustration, assume that $\delta' > \delta$. According to proposition 1, the amount of resources obtained per unit of investment into extraction technology would decrease. Since the amount of the resource used for production equals the amount of resources extracted, $R = X$, the change to δ' would also reduce the amount of resources used in production. Following equation (49), the resource price would increase.

Second, as long as the functions in equation (5) and (4) are inverse to each other, the result of constant returns to extraction technology investments described in proposition 1 is preserved.

Third, if the two functions are not inverse, resource prices follow a simple intuition: If the increase in resources at lower ore grades do not compensate the decreasing returns in technology, then resources become more expensive over time and a scarcity rent as in the model of Hotelling (1931) occurs. Note that such a scarcity rent has not yet been found empirically (see e.g. Hart and Spiro, 2011). If the increase in resources at lower ore grades more than offsets the decreasing returns in technology, then resources become increasingly abundant, so that their price has a downward trend over time. Our model can be generalized to this case, since the condition that resource prices equal marginal resource extraction cost would extend to this case. Prices cannot be below marginal extraction cost, since firms would make negative profits. Due to competitive pressure, they can also not be above marginal cost.

How does our model compare to other models with non-renewable resources? We make the convenient assumption that the quantity of non-renewable resources is for all practical economic purposes infinite. As a consequence, resource availability does not limit growth. Substitution of capital for non-renewable resources, technological change in the use of the resource, and increasing returns to scale are therefore not necessary for sustained growth as in Groth (2007) or Aghion and Howitt (1998). If the resource were finite in our model, the extractive sector would behave in the same way as in standard models with a sector based on Hotelling (1931). As Dasgupta and Heal (1980) point out, in this case the growth rate of the economy depends strongly on the degree of substitution between the resource and other economic inputs. For $\varepsilon > 1$, the resource is non-essential; for $\varepsilon < 1$, the total output that the economy is capable of producing is finite. The production function is, therefore, only interesting for the Cobb-Douglas case.

Our model suggests that the non-renewable resource can be thought of as a form of capital: if the extractive firms invest in R&D in extraction technology, the resource is extractable without limits as an input to aggregate production. This feature marks a distinctive difference from models such as the one of Bretschger and Smulders (2012). They investigate the effect of various assumptions about substitutability and a decentralized market on long-run growth, but keep the assumption of a finite non-renewable resource. Without this assumption, the elasticity of substitution between the non-renewable resource and other input factors is no longer central to the analysis of limits to growth.

Some might argue that the relationship described in proposition 1 cannot continue to hold in the future as the amount of non-renewable resources in the earth's crust is ultimately finite. Scarcity will become increasingly important, and the scarcity rent will be positive even in the present. However, for understanding current prices and consumption patterns, current expectations about future developments are important. Given that the quantities of available resources indicated in table 4 are very large, their ultimate end far in the future should not affect behavior today. The relationship described in proposition 1 seems to have held in the past and looks likely to hold for the foreseeable future. Since in the long term, extracted resources equal the resources added to reserves due to R&D in extraction technology, the price for a unit of the resource will equal the extraction cost plus the per-unit cost of R&D and hence, stay constant in the long term. This may explain why scarcity rents cannot be found empirically.

5 Conclusion

This paper examines interaction between geology and technology and its impact on the resource price, total output growth, and the resource intensity of the economy. We argue that economic growth causes the production and use of a non-renewable resource to increase at a constant rate. Marginal production costs stay constant in the long term. Economic growth enables firms to invest in extraction technology R&D, which makes resources from deposits of lower grades economically extractable. We help explain the long-term evolution of non-renewable resource prices and world production for more than 200 years. If historical trends in technological progress continue, it is possible that non-renewable resources are, within a time frame relevant for humanity, practically inexhaustible.

Our model makes strong simplifying assumptions, which render our model analytically solvable. However, we believe that a less simple model would essentially provide the same results. There are four major simplifications, which should be examined in more detail in future extensions. First, there is no uncertainty in R&D development, and therefore no incentive for firms to keep a positive amount of the non-renewable resource in their reserves. If R&D development is stochastic as in Dasgupta and Stiglitz (1981), there would be a need for firms to keep reserves.

Second, our model features perfect competition in the extractive sector. We could obtain a model with monopolistic competition in the extractive sector by introducing explicitly privately-owned deposits. A firm would need to pay a certain upfront cost or exploration cost in order to acquire a mineral deposit. This upfront cost would give technology firms a certain monopoly power as they develop machines that are specific to a single deposit.

Third, extractive firms could face a trade-off between accepting high extraction costs due to a lower technology level and investing in R&D to reduce extraction costs. A more general extraction technology function would provide the basis to generalize this assumption.

Fourth, our model does not include recycling. Recycling has become more important for metal production over time due to the increasing abundance of recyclable materials and the comparatively low energy requirements (see Wellmer and Dalheimer, 2012). Introducing recycling into our model would further strengthen our argument, as it increases the economically extractable stock of the non-renewable resource.

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Appendix 1 Tables

		Aluminum	Copper	Lead	Tin	Zinc	Crude Oil
Range		1905-2009	1792-2009	1792-2009	1792-2009	1824-2009	1862-2009
Constant	Coeff.	-1.774	0.572	0.150	1.800	1.072	8.242
	t-stat.	(-0.180)	(0.203)	(0.052)	(0.660)	(0.205)	(0.828)
Lin.Trend	Coeff.	0.008	0.009	0.016	0.001	0.014	-0.021
	t-stat.	(0.137)	(0.428)	(0.714)	(0.069)	(0.357)	(-0.317)
Range		1905-2009	1850-2009	1850-2009	1862-2009	1850-2009	1850-2009
Constant	Coeff.	-1.299	0.109	-0.268	2.439	1.894	7.002
	t-stat.	(-0.200)	(0.030)	(-0.073)	(0.711)	(0.407)	(1.112)
Lin.Trend	Coeff.	0.008	0.020	0.030	-0.004	0.013	-0.021
	t-stat.	(0.137)	(0.518)	(0.755)	(-0.109)	(0.267)	(-0.317)
Range		1900-2009	1900-2009	1900-2009	1900-2009	1900-2009	1900-2009
Constant	Coeff.	-0.903	-1.428	-0.490	1.068	2.764	-1.974
	t-stat.	(-0.239)	(-0.332)	(-0.102)	(0.269)	(0.443)	(-0.338)
Lin.Trend	Coeff.	0.008	0.055	0.054	0.010	0.010	0.100
	t-stat.	(0.137)	(0.820)	(0.713)	(0.168)	(0.099)	(1.106)
Range		1950-2009	1950-2009	1950-2009	1950-2009	1950-2009	1950-2009
Constant	Coeff.	2.269	1.556	-3.688	-0.061	-0.515	3.445
	t-stat.	(0.479)	(0.240)	(-0.505)	(-0.011)	(-0.062)	(0.354)
Lin.Trend	Coeff.	-0.055	0.041	0.198	0.049	0.103	0.090
	t-stat.	(-0.411)	(0.225)	(0.958)	(0.307)	(0.441)	(0.326)
Range		1875-1975	1875-1975	1875-1975	1875-1975	1875-1975	1875-1975
Constant	Coeff.	-0.549	1.323	0.370	3.719	1.136	-1.111
	t-stat.	(-0.088)	(0.266)	(0.081)	(0.812)	(0.176)	(-0.176)
Lin.Trend	Coeff.	-0.003	0.011	0.030	-0.012	0.051	0.094
	t-stat.	(-0.033)	(0.135)	(0.383)	(-0.152)	(0.468)	(0.875)

Notes: The table presents coefficients and *t*-statistics for regressions of the growth rates on a constant and a linear trend.***, **, and * indicate significance at the 1%, 2.5% and 5% level, respectively.

Table 1: Tests of the stylized fact that the growth rates of real prices of mineral commodities equal zero and do not follow a statistically significant trend.

		Aluminum	Copper	Lead	Tin	Zinc	Crude Oil	World GDP
Range		1855-2009	1821-2009	1802-2009	1792-2009	1821-2009	1861-2009	1792-2009
Constant	Coeff.	48.464	4.86	16.045	4.552	30.801	35.734	0.128
	t-stat.	*** 3.810	*** 2.694	*** 3.275	* 2.231	** 2.58	*** 4.365	0.959
Lin.Trend	Coeff.	-0.221	-0.006	-0.087	-0.016	-0.174	-0.182	0.018
	t-stat.	** -2.568	-0.439	** -2.294	-0.999	* -1.975	*** -3.334	*** 16.583
Range		1855-2009	1850-2009	1850-2009	1850-2009	1850-2009	1861-2009	1850-2009
Constant	Coeff.	48.464	5.801	6.032	3.569	5.579	25.198	0.995
	t-stat.	*** 3.810	*** 3.461	***3.371	* 2.185	*** 3.774	*** 4.81	*** 5.49
Lin.Trend	Coeff.	-0.221	-0.018	-0.038	-0.015	-0.021	-0.182	0.019
	t-stat.	** -2.568	-1.007	-1.938	-0.833	-1.308	*** -3.334	*** 9.797
Range		1900-2009	1900-2009	1900-2009	1900-2009	1900-2009	1900-2009	1900-2009
Constant	Coeff.	19.703	5.965	2.980	2.844	4.44	9.883	2.004
	t-stat.	*** 5.498	*** 2.651	* 2.043	1.361	* 2.225	*** 6.912	*** 7.8
Trend	Coeff.	-0.178	0.035	-0.019	-0.015	-0.018	-0.083	0.018
	t-stat.	*** 3.174	-0.995	-0.853	-0.464	-0.592	***-3.711	***4.549
Range		1950-2009	1950-2009	1950-2009	1950-2009	1950-2009	1950-2009	1950-2009
Constant	Coeff.	10.781	5.043	13.205	0.051	5.675	9.897	4.729
	t-stat.	*** 7.169	*** 4.979	*** 2.936	0.028	*** 4.619	*** 9.574	*** 12.89
Lin.Trend	Coeff.	-0.171	-0.057	-0.48	0.04	-0.078	-0.196	-0.028
	t-stat.	*** -3.999	-1.978	-1.553	0.768	* -2.255	*** -6.64	*** -2.724
Range		1875-1975	1875-1975	1875-1975	1875-1975	1875-1975	1875-1975	1875-1975
Constant	Coeff.	50.75	6.307	3.851	3.762	4.384	12.272	1.244
	t-stat.	*** 4.846	** 2.543	1.938	1.664	* 2.032	*** 4.060	*** 5.509
Lin.Trend	Coeff.	-0.53	-0.024	-0.018	-0.026	-0.005	-0.072	0.027
	t-stat.	*** -2.974	-0.566	-0.536	-0.66	-1.26	-1.403	***7.045

Notes: The table presents coefficients and t-statistics for regressions of the growth rates on a constant and a linear trend. ***, **, and * indicate significance at the 1%, 2.5% and 5% level, respectively.

Table 2: Tests for the stylized facts that growth rates of world primary production and world real GDP are equal to zero and trendless.

		Aluminum	Copper	Lead	Tin	Zinc	Crude Oil	World GDP
Range		1855-2009	1821-2009	1802-2009	1792-2009	1821-2009	1861-2009	1792-2009
Constant	Coeff.	48.301	5.474	20.57	4.427	30.7	35.689	0.032
	t-stat.	*** 3.824	*** 3.06	*** 3.845	* 2.181	** 2.584	*** 4.379	0.276
Lin.Trend	Coeff.	-0.229	-0.018	-0.125	-0.023	-0.182	-0.19	0.01
	t-stat.	*** -2.677	-1.367	*** -3.025	-1.457	* -2.071	*** -3.499	*** 11.066
Range		1855-2009	1850-2009	1850-2009	1850-2009	1850-2009	1861-2009	1850-2009
Constant	Coeff.	48.301	5.399	5.629	3.179	5.18	24.681	0.628
	t-stat.	*** 3.824	*** 3.254	***3.169	1.961	*** 3.541	*** 4.733	*** 4.052
Lin.Trend	Coeff.	-0.229	-0.027	-0.047	-0.024	-0.03	-0.19	0.01
	t-stat.	*** -2.677	-1.523	** -2.442	-1.348	-1.895	*** -3.499	*** 5.876
Range		1900-2009	1900-2009	1900-2009	1900-2009	1900-2009	1900-2009	1900-2009
Constant	Coeff.	18.595	4.985	2.028	1.903	3.473	8.869	1.071
	t-stat.	*** 5.242	* 2.241	1.41	0.918	1.763	*** 6.306	*** 4.862
Trend	Coeff.	-0.184	-0.042	-0.027	-0.023	-0.026	-0.09	0.01
	t-stat.	*** -3.315	-1.214	-1.186	-0.694	-0.404	*** -4.084	*** 3.01
Range		1950-2009	1950-2009	1950-2009	1950-2009	1950-2009	1950-2009	1950-2009
Constant	Coeff.	8.583	2.952	1.141	-1.954	3.578	7.716	2.632
	t-stat.	*** 5.742	*** 2.892	1.04	1.086	*** 2.87	*** 7.493	*** 7.444
Lin.Trend	Coeff.	-0.156	-0.044	-0.35	0.051	-0.065	-0.18	-0.016
	t-stat.	*** -3.667	-1.515	-1.129	0.997	-1.819	*** -6.14	-1.551
Range		1875-1975	1875-1975	1875-1975	1875-1975	1875-1975	1875-1975	1875-1975
Constant	Coeff.	50.004	5.854	3.413	3.317	3.942	11.789	0.834
	t-stat.	*** 4.81	** 2.386	1.738	1.480	1.851	*** 3.933	*** 4.509
Lin.Trend	Coeff.	-0.542	-0.038	-0.032	-0.039	-0.019	-0.086	0.013
	t-stat.	*** -3.06	-0.908	-0.959	-1.028	-0.517	-1.691	***4.004

Notes: The table presents coefficients and t-statistics for regressions of the growth rates on a constant and a linear trend. ***, **, and * indicate significance at the 1%, 2.5% and 5% level, respectively.

Table 3: Tests for the stylized fact that growth rates of world per capita primary production and world per capita real GDP are equal to zero and trendless.

	Reserves/ Annual production (Years)	Resources/ Annual production (Years)	Crustal abundance/ Annual production (Years)
Aluminum	139[1a]	263,000[1a]	48,800,000,000[bc]
Copper	43[a]	189[a]	95,000,000[ab]
Iron	78[a]	223[a]	1,350,000,000[ab]
Lead	21[a]	362[a]	70.000.000[ab]
Tin	17[a]	"Sufficient"[a]	144.000[ab]
Zinc	21[a]	158[a]	187.500.000[ab]
Gold	20[d]	13[d]	27,160,000[ef]
Rare earths[2]	827[a]	"Very large"[a]	n.a.
Coal[3]	129[g]	2,900[g]	} 1,400,000[6i] (Coal, Crude oil, Gas)
Crude oil[4]	55[g]	76[g]	
Gas[5]	59[g]	410[g]	

Notes: Reserves include all material which can currently be extracted. The definition of resources can be found in Section 2.4. Sources: [a]U.S. Geological Survey (2012b), [b]Perman et al. (2003), [c]U.S. Geological Survey (2011c),[d]U.S. Geological Survey (2011b),[e]Nordhaus (1974),[f]U.S. Geological Survey (2010), [g]Federal Institute for Geosciences and Natural Resources (2011) [g][i]Littke and Welte (1992). Notes: [1] data for bauxite, [2] rare earth oxide, [3] includes lignite and hard coal, [4] includes conventional and unconventional oil, [5] includes conventional and unconventional gas, [6] all organic carbon in the earth's crust.

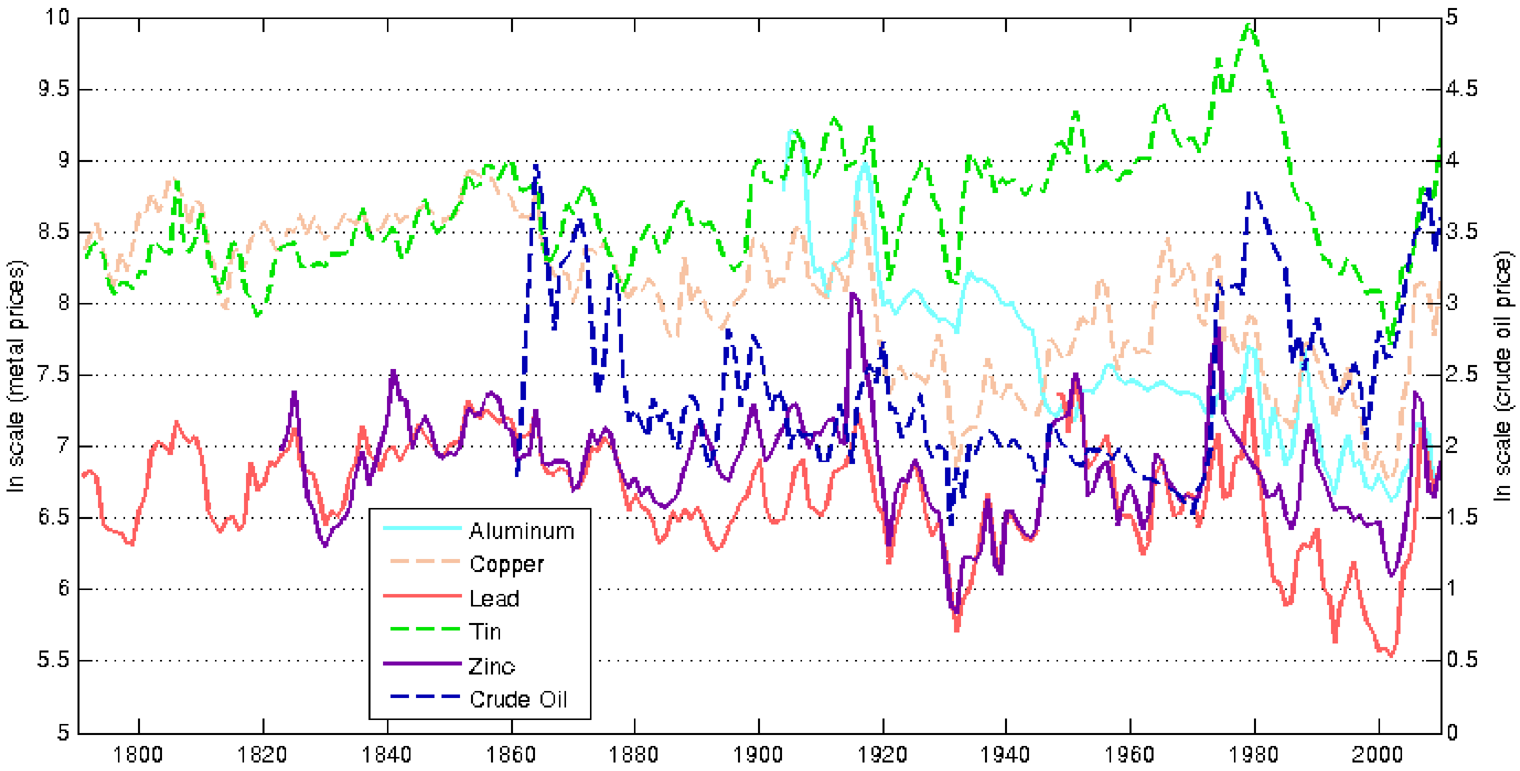
Table 4: Availability of selected non-renewable resources in years of production left in the reserve, resource and crustal mass based on current annual mine production.

	Reserves/ Annual production (Years)	Resources/ Annual production (Years)	Crustal abundance/ Annual production (Years)
Aluminum	65[1ah]	419[1ah]	838[bch]
Copper	30[ag]	77[ag]	718[abg]
Iron	44[ah]	78[ah]	744[abh]
Lead	18[ah]	181[ah]	1,907[abh]
Tin	18[ah]	n.a.	3,588[abh]
Zinc	17[ah]	74[ah]	842[abh]
Gold	18[dh]	11[dh]	2,170[efh]
Rare earths[2]	127[ah]	n.a.	n.a.
Coal[3]	65[gk]	215[gk]	}
Crude oil[4]	46[gk]	60[gk]	} 729[6j]
Natural gas[5]	41[gk]	123[gk]	}

Notes: The numbers for reserves and resources are not summable as in Table 4. We have used the following average annual growth rates of production from 1990 to 2010: Aluminum: 2.5%, Iron: 2.3%, Copper: 2%, Lead: 0.7%, Tin: 0.4%, Zinc: 1.6%, Gold: 0.6%, Rare earths: 2.6%, Crude oil: 0.7%, Natural gas: 1.7%, Coal: 1.9%, Hydrocarbons: 1.4%. Reserves include all material which can currently be extracted. The definition of resources can be found in Section 2.4. Sources: [a]U.S. Geological Survey (2012b), [b]Perman et al. (2003), [c]U.S. Geological Survey (2011c),[d]U.S. Geological Survey (2011b),[e]Nordhaus (1974),[f]U.S. Geological Survey (2010), [g]Federal Institute for Geosciences and Natural Resources (2011), [h]U.S. Geological Survey (2012a), [i]U.S. Bureau of Mines (1991), [j]Littke and Welte (1992), [k]British Petroleum (2013). Notes: [1] data for bauxite, [2] rare earth oxide, [3] includes lignite and hard coal, [4] includes conventional and unconventional oil, [5] includes conventional and unconventional gas, [6] all organic carbon in the earth's crust.

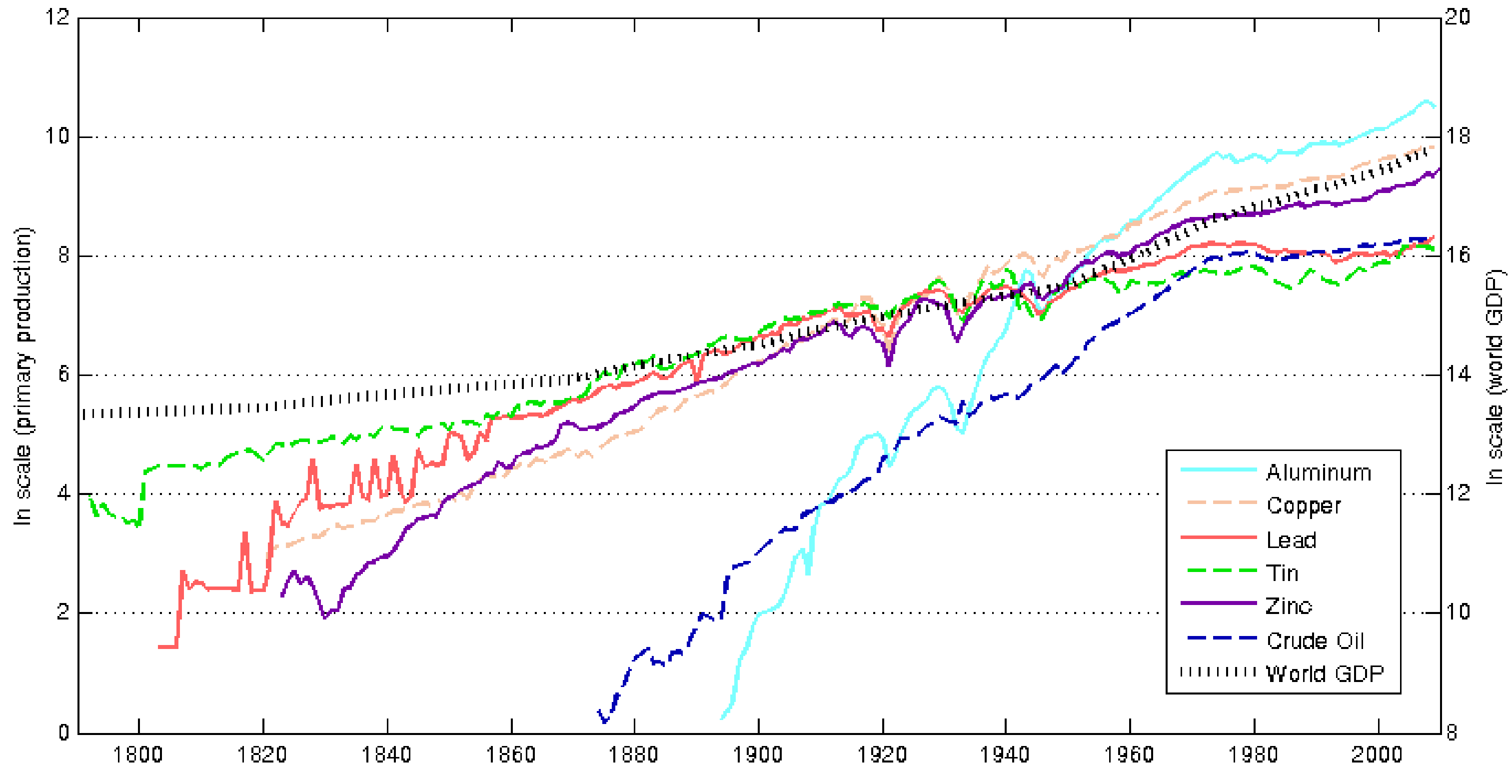
Table 5: Availability of selected non-renewable resources in years of production left in the reserve, resource and crustal mass based on an exponentially increasing annual mine production (based on the average growth rate over the last 20 years).

Appendix 2 Figures



Notes: All prices, except for the price of crude oil, are prices of the London Metal Exchange and its predecessors. As the price of the London Metal Exchange used to be denominated in Sterling in earlier times, we have converted these prices to U.S.-Dollar by using historical exchange rates from Officer (2011). We use the U.S.-Consumer Price Index provided by Officer and Williamson (2011) and the U.S. Bureau of Labor Statistics (2010) for deflating prices with the base year 1980-82. The secondary y-axis relates to the price of crude oil. For data sources and description see Stuermer (2013).

Figure 1: Real prices of major mineral commodities in natural logs.



For data sources and description see Stuermer (2013).

Figure 2: World primary production of non-renewable resources and world real GDP in logs.

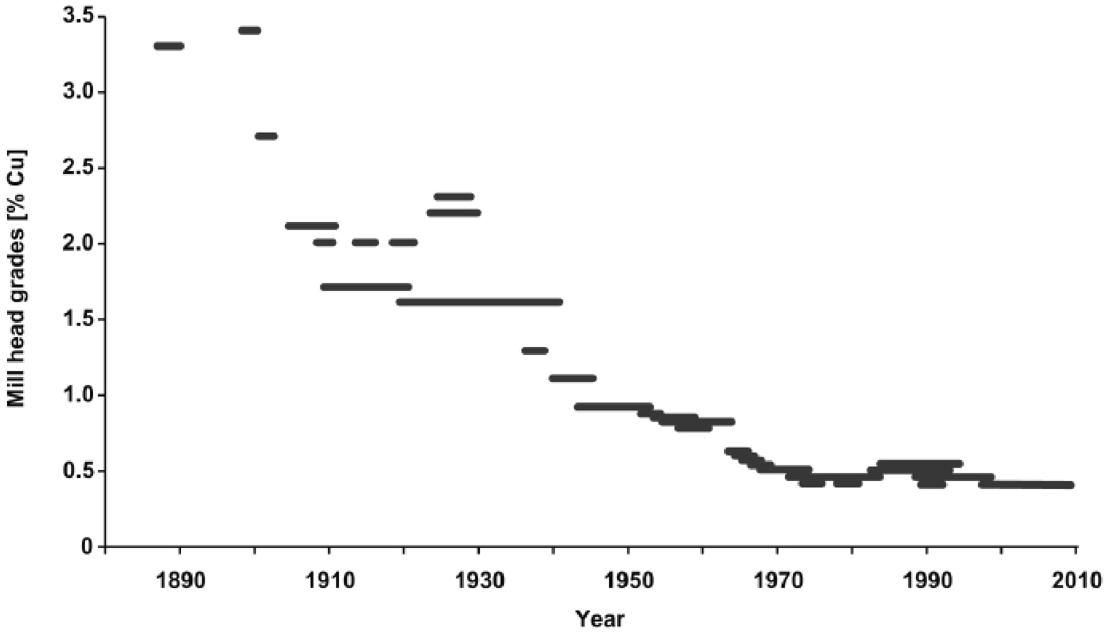


Figure 3: The historical development of mining of various grades of copper in the U.S. Source: Scholz and Wellmer (2012)

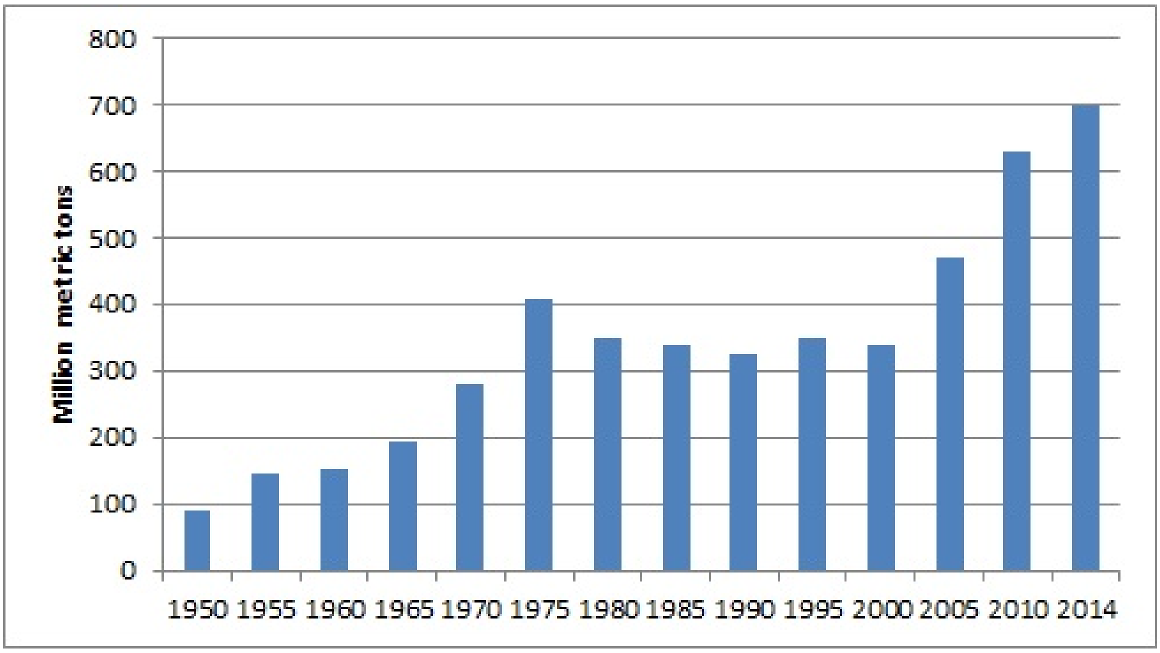


Figure 4: Historical evolution of world copper reserves from 1950 to 2014. Sources: Tilton and Lagos C.C. (2007), USGS.

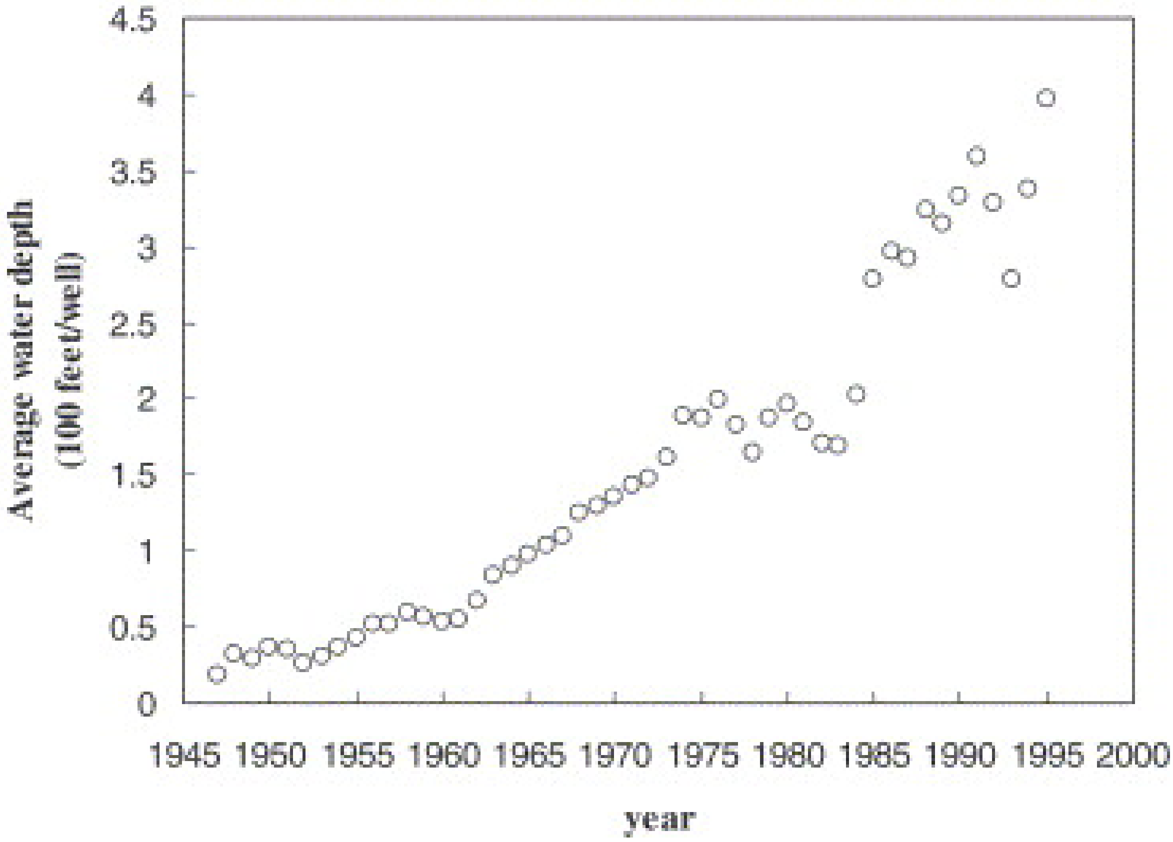


Figure 5: Average water depth of wells drilled in the Gulf of Mexico. Source: Managi et al. (2004).

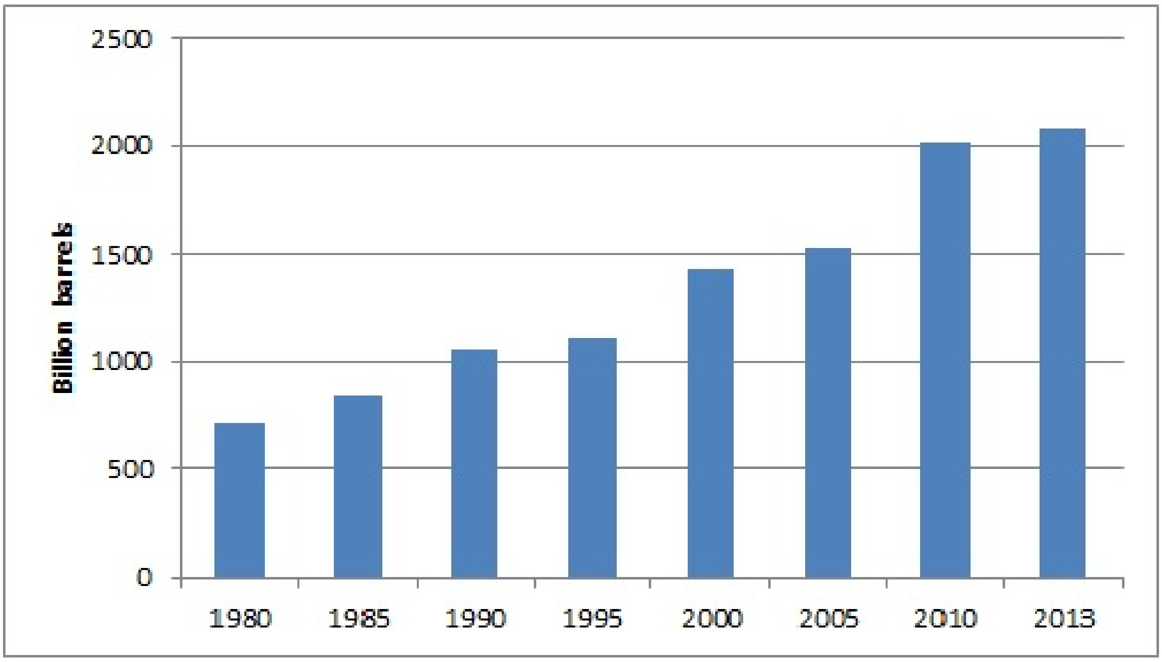


Figure 6: Historical evolution of oil reserves, including Canadian oil sands from 1980 to 2013. Source: BP, 2015.

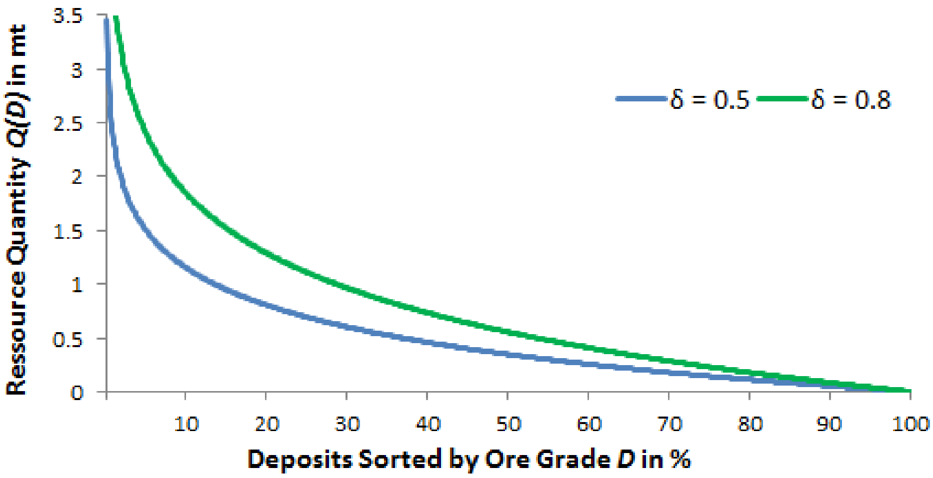


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Figure 7: Geological Function.

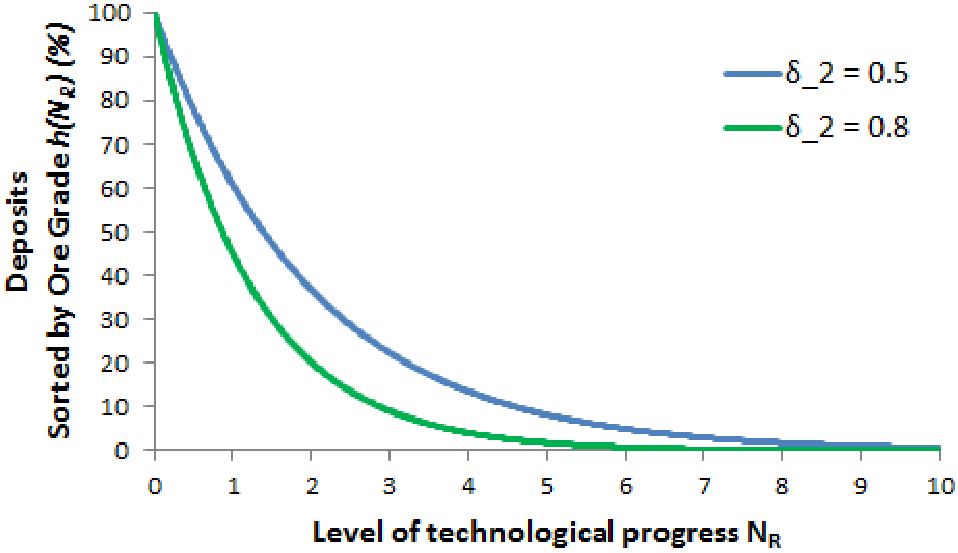

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Figure 8: Extraction Technology Function.