The Value of Performance Signals Under Contracting Constraints*

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December 26, 2017

Abstract

This paper studies the value of additional performance signals under contracting constraints, such as limited liability, monotonicity, or upper bounds to pay or incentives. We show that – contrary to the informativeness principle – informative signals may have no value, because the payment cannot be adjusted to reflect the signal realization. We derive necessary and sufficient conditions for a signal to have value under such constraints, and study how valuable signals are incorporated into the contract. Our results have implications for performance-based vesting, option repricing, pay-for-luck, and performance-sensitive debt. For example, it may be optimal to lower the strike price of an option upon a *negative* signal of effort.

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Executive contracts are typically based on multiple signals of performance. For example, Bettis et al. (2016) find that, in 2012, 70% of large U.S. firms paid their executives with performance-vesting equity, where the number of securities granted depends on performance relative to a threshold (or set of thresholds). 86% of such grants employ at least one accounting threshold, and so their value depends on factors other than the stock price – the standard "output" measure for executive contracts. Murphy's (2013) survey reports that companies use a variety of financial and non-financial performance measures when determining CEO bonuses. Additional performance signals are also used in financing contracts. Manso, Strulovici, and Tchistyi (2010) document that 40% of loans have performance pricing provisions, where the coupon rate depends on signals such as the firm's credit rating, leverage, and solvency ratios. Thus, the payment to investors depends on factors other than cash flow – the standard "output" measure for financing contracts.

The main theoretical justification for including additional performance measures is Holmström's (1979) informativeness principle. This principle states that any signal should be included in a contract if it provides incremental information about the agent's performance, over and above the information already conveyed in output. However, real-life contracts appear to *violate* the principle. Even though some contracts are based on signals other than output, many are not. Most debt does not have performance pricing provisions, and some executive equity does not exhibit performance-based vesting. Are these violations efficient? When should contracts depend on additional performance signals, which signals should be used, and how should they be incorporated into the contract? These questions are the focus of this paper.

The informativeness principle was derived assuming no contracting constraints. However, contracting constraints are an important feature of real life. Limited liability on the agent imposes a minimum (zero) on realized payments; limited liability on the principal, regulation[1] or "outrage constraints" (Bebchuk and Fried (2004)) may impose a maximum. Constraints may apply not only to the level of pay but also the level of incentives (the sensitivity of realized pay to output).[2] Thus, to apply the informativeness principle to many real-life settings, we

[1] The EU limits banker bonuses to twice the level of salary; in March 2016, Israel removed tax deductibility from banker realized pay that exceeds 35 times the salary of the lowest-paid colleague (or 2.5 million shekels, if this is lower); the UK Labour Party's 2017 election manifesto proposed an "Excessive Pay Levy" on pay exceeding certain absolute thresholds; and in November 2016, the UK government's Green Paper proposed that company pay policies stipulate a cap on realized pay.

[2] As Innes (1990) points out, if the principal could destroy output or the agent could secretly borrow, the agent cannot gain more than one-for-one from increases in output. The EU Shareholder Rights Directive stipulates that stock-based compensation should generally not exceed 50% of total variable pay, limiting the sensitivity of pay to the stock price, and some shareholder proposals aim to cap equity awards. Ertimur, Ferri, and Muslu (2011) discuss a 2004 shareholder proposal at Motorola to cap equity grants at $1 million, and a 2004 proposal at Eastman Kodak to scrap equity grants.

must first study whether it holds under contracting constraints, and if necessary extend it.

This paper derives necessary and sufficient conditions under which contracts should be based not only on output q, but also an additional performance signal s, in the presence of contracting constraints. For example, q may be the stock price and s may be accounting profits. In this setting, the principal's problem is whether to make the manager's pay dependent purely upon the stock price, as with traditional equity grants, or also upon profits, via performance-vesting equity or a profit-contingent bonus. Alternatively, s may be a stock price index of peer firms, in which case the problem is whether to engage in relative performance evaluation, or a non-accounting measure such as the number of customers.

We first consider the standard framework of risk neutrality and limited liability on the manager, originally analyzed by Innes (1990) and widely used in a number of settings (e.g. Biais et al. (2010), Clementi and Hopenhayn (2006), DeMarzo and Fishman (2007a, 2007b), DeMarzo and Sannikov (2006), and the textbook of Tirole (2006)). Similar to Innes (1990), we consider up to two additional constraints. The first is a maximum payment constraint, such as limited liability on the principal or a regulatory cap. As is standard, the optimal contract is "live-or-die" – the manager receives zero if output is below a threshold q^*, and the maximum if it exceeds it. The second is a cap on the level of incentives, such as a monotonicity constraint which requires the firm's payoff to be non-decreasing in output. The optimal contract is then an option on output with strike price q^{**}.

In either case, constraints on contracting bind everywhere except at the threshold q^* (q^{**}), and so an additional signal will only be included if it affects the optimal threshold. Under the "live-or-die" contract, changing the threshold q^* alters the payment (from 0 to q or vice-versa) only in a local neighborhood around q^*. As a result, a signal is only useful if it affects the likelihood ratio that output *equals* q^*, i.e. is informative about whether output equaling q^* is the outcome of high or low effort. If the signal suggests the manager has worked (shirked), the firm generally decreases (increases) the threshold. Under the option contract, changing the threshold q^{**} alters the payment for all $q \geq q^{**}$. Thus, a signal is only useful if it affects the likelihood ratio that output *exceeds* q^{**} – i.e. is informative about whether output exceeding q^{**} is the outcome of high or low effort.

In addition to compensation, the model with a monotonicity constraint can be applied to a financing setting, in which case the optimal contract is debt (Innes (1990)) with face value q^{**}. Our results give conditions under which the payment depends not only on output, as with a standard debt contract, but also on additional signals, as with performance-sensitive debt – if and only if these signals are informative about whether output exceeding the face value of debt is the outcome of high effort.

The results have a number of implications. Our main theoretical implication is that the informativeness principle needs to be modified under contracting constraints: a signal has value if and only if it is informative about effort at a threshold likelihood ratio – the likelihood ratio at either a single output or over a range of outputs, depending on the contracting constraint. In both cases, a signal that is informative about effort only above and/or below the threshold likelihood ratio is of no value, because the payment is bounded by either a maximum level or maximum slope constraint. As a result, a signal can be informative almost everywhere yet still have zero value. We illustrate this point with a number of real life examples where informative signals may not be incorporated in compensation and financing contracts. For example, a credit rating may be informative about effort if output is below the face value of debt (i.e. the firm defaults), since effort affects the severity of default – but the debt repayment is already maximized upon default anyway. If whether the firm actually defaults depends primarily on extraneous events (such as the bankruptcy of a major customer), then the credit rating is not informative about effort in solvency, and so has no value for the contract.

A second implication is that the value of information is non-monotonic in output. With constraints on the level of pay, the firm should only invest in additional performance signals at moderate output realizations. With also a constraint on the level of incentives, the firm should only invest in signals that change the likelihood that output exceeds a threshold. For example, a signal that redistributes probability mass either to the left or to the right of this threshold is of no value.

Moving to applied implications, that the conditions for a signal to have value may be stronger under contracting constraints can potentially explain why real-life contracts do not depend on as many signals as the original informativeness principle suggests they should, i.e. are less complex than implied by the principle.[3] For example, executive contracts typically do not depend on the firm's recovery rate in bankruptcy or the outcome of litigation against the firm, because bankruptcy and litigation typically lead to the manager being fired anyway and so he cannot be punished further. Relatedly, pay-for-luck need not be inefficient if it applies to firing decisions as found by Jenter and Kanaan (2015). On the other hand, our model does suggest that pay-for-luck is suboptimal at moderate output realizations. Indeed, we do not argue that real-life contracts are efficient. Rather, before concluding that they must be suboptimal because they violate the original informativeness principle, one must first extend

[3] Salanié (1997, p128-129) writes that "the sufficient statistic theorem indicates that the optimal wage schedule should depend on all signals that may bring information on the action chosen by the agent(...). This prediction does not accord well with experience; real-life contracts appear (...) to depend on a small number of variables only".

the informativeness principle to take into account contracting constraints and only then make an assessment.

We then extend the model to risk aversion. While the contract takes a more general form, it remains the case that informative signals have zero value if they are informative only at output levels where constraints bind. Quite separate from extending the informativeness principle, the model with risk aversion also generates the first set of sufficient conditions – limited liability, log utility, and a linear likelihood ratio – for options to be the optimal contract when the agent is risk-averse. Moreover, unlike in the risk-neutral model where the number of options is fixed (it equals the maximum implied by the slope constraint), under risk-aversion the number of options is a choice variable as the principal balances incentives with risk-sharing. The risk-averse model thus allow us to study how signals should affect the number of options granted, as the case for performance-based vesting. Despite its popularity, we are unaware of any theories that study under what conditions performance-based vesting is optimal, and what performance signals should be used. Simple intuition may suggest that the number of options should depend on a signal if it provides incremental information about effort over and above that contained in the stock price, but we show that this condition is insufficient. Surprisingly, greater vesting may be optimally triggered by signals that suggest *low* effort, and signals that trigger vesting may optimally depend on luck.

The results also have implications for option strike prices. They suggest that option repricing (which, empirically, nearly always involves a lowering of the strike price) can be justified if prompted by positive signals of CEO effort. However, surprisingly, we show that it may sometimes be optimal to lower the strike price upon signals that individually convey *bad* news about CEO effort, contrary to conventional wisdom that such practices necessarily result from rent extraction. This is because a signal provides information about effort in two ways – first, it is individually informative about effort and second, it affects the informativeness of output about effort. For example, let the signal be a credit rating, and consider a firm with high output and a low credit rating. The low credit rating individually indicates low effort. However, it also makes the high output a stronger indicator of high effort, since it is harder to achieve high output with a low credit rating and thus limited access to external finance. If this second consideration is sufficiently strong, the payment to the manager will be higher (and the strike price lower) upon a lower credit rating.

This paper is related to the theoretical literature on pay-for-performance, surveyed by Holmström (2017). In particular, Gjesdal (1982), Amershi and Hughes (1989), Kim (1995), and Chaigneau, Edmans, and Gottlieb (2016) extend the original Holmström (1979) informativeness principle, but not to settings with contracting constraints. Chaigneau, Edmans, and Gottlieb

(2017) study the effect on the optimal contract of increasing the precision of output, but do not study the introduction of additional signals and thus do not have implications for performance-sensitive debt, performance-vesting options, or option repricing. Other theories have proposed different justifications for why contracts may not depend on additional signals. Townsend (1979) and Gale and Hellwig (1985) show that, if verifying the state is costly, optimal contracts should not involve verification of – and thus be contingent upon – the state for certain realizations. Our paper shows that even freely-verifiable signals (e.g. peer performance) may optimally not be used. Allen and Gale (1992) propose that signals may not be used if they may be manipulated. A quite separate rationale is a preference for simplicity; see Gabaix (2014) for such a model in a consumer setting. In Innes (1990), the agent's wage is zero when output falls below a threshold. Even though lower outputs are associated with lower likelihood ratios, the agent's wage does not fall. In this sense, the contract does not use all the information in output due to contracting constraints, similar to why additional signals may not be used in our setting. Our main contribution is not only to point out that the original informativeness principle may fail if contracting constraints bind, but also to derive necessary and sufficient conditions for an additional signal – over and above output – to have value under such constraints.

Moving to the applied literature on pay-for-performance, Dittmann, Maug, and Zhang (2011) quantify the effect on pay and firm value of various restrictions on CEO pay – restrictions on ex-post payments, ex-ante expected pay, and specific components of pay. Their calibration differs from our optimal contracting approach. Dittmann, Maug, and Spalt (2013) calibrate the cost savings from incorporating peer performance in executive contracts and Johnson and Tian (2000) compare the incentives provided by indexed and non-indexed options. Oyer (2004), Axelson and Baliga (2009), Gopalan, Milbourn, and Song (2010), Hoffman and Pfeil (2010), and Hartman-Glaser and Hébert (2016) provide different rationalizations for pay-for-luck. Our paper suggests that pay-for-luck may be optimal for very high or very low output realizations, but suboptimal for moderate ones. Manso, Strulovici, and Tchistyi (2010) offers an explanation for performance-sensitive debt based on adverse selection; ours is based on moral hazard.

1 The Model

We consider a principal (firm) and an agent (manager). The manager is protected by limited liability and has zero reservation utility. He exerts unobservable effort of $e \in \{0, 1\}$, where $e = 0$ ("low effort") costs the manager 0, and $e = 1$ ("high effort") costs $C > 0$. As is standard, effort can be interpreted as any action that improves output but is costly to the manager, such as working rather than shirking, choosing projects that generate cash flows rather than private benefits, or not extracting rents. In this section, we assume that both the manager and firm are risk-neutral as then contracting constraints (rather than risk sharing) drive the contract, and so this is a natural framework to study the value of a signal under contracting constraints. Section 2 extends the model to risk aversion and a continuum of effort levels.

Effort affects the probability distribution of output, which is distributed over an interval $q \in [0, \bar{q}]$, where $\bar{q}$ may be $+\infty$, and of an additional signal $s \in \{s_1, ..., s_S\}$.[4] Both output and the signal are contractible. We refer to an output/signal realization (q, s) as a "state" and assume that the distribution of (q, s) conditional on any e has full support.[5]

Conditional on effort e and signal s, output q is distributed according to the probability density function ("PDF"):

$$f(q|e,s) := \begin{cases} \pi_s(q) & \text{if } e = 1 \\ p_s(q) & \text{if } e = 0 \end{cases}.$$

The marginal distribution of the signal is represented by $\phi_{e'}^{s'} := \Pr(s = s'|e = e') > 0$. Their product yields the joint distribution of (q, s) conditional on effort, which we denote $f(q, s|e)$. The marginal distribution of output is given by

$$f(q|e) = \sum_s \phi_e^s f(q|e,s). \tag{1}$$

Let

$$LR_s(q) := \frac{\phi_1^s \pi_s(q)}{\phi_0^s p_s(q)} \tag{2}$$

denote the likelihood ratio associated with output q and signal s. When the likelihood ratio depends on s, the signal is incrementally informative about effort – i.e. it provides information about effort over and above that contained in output. We assume that the output distribution satisfies the strict monotone likelihood ratio property ("MLRP"): $LR_s(q)$ is strictly increasing in q for all s.

As Holmström (1979) discusses, the principal's problem resembles a hypothesis testing problem, where the principal "tests" the null that the agent worked against the alternative that he shirked. The likelihood ratio compares the likelihood of the null to the alternative, and the problem is whether the signal s provides additional information to guide this hypothesis

[4]Working with a discrete signal space ensures that an optimal contract exists in all variations of the model that we consider. Apart from existence, however, it is straightforward to extend our results to continuous signals.

[5]The results are robust to relaxing this assumption, except that the optimal contract might not be unique. There could exist other optimal contracts that differ on a set of outputs that occur with probability zero.

test (of course, in equilibrium, the principal knows that the agent worked).

The firm has full bargaining power and offers the manager a vector of payments $\{w_s(q)\}$ conditional on the state. We assume that the gain from effort $E[q|e=1] - E[q|e=0]$ is sufficiently higher than the cost of effort C that the firm wishes to implement high effort (otherwise, the optimal contract would trivially involve a constant payment of zero). The firm thus solves the following program:

$$\min_{w_s(q)} \sum_s \int_0^{\bar{q}} w_s(q)\, \phi_1^s \pi_s(q)\, dq \tag{3}$$

$$s.t. \ \sum_s \int_0^{\bar{q}} w_s(q)\, \phi_1^s \pi_s(q)\, dq - C \geq 0 \tag{4}$$

$$\sum_s \int_0^{\bar{q}} w_s(q) \left[\phi_1^s \pi_s(q) - \phi_0^s p_s(q)\right] dq \geq C \tag{5}$$

$$w_s(q) \geq 0 \ \ \forall q, s. \tag{6}$$

It minimizes the expected payment (3) subject to the manager's individual rationality constraint ("IR") (4), incentive compatibility constraint ("IC") (5), and limited liability constraint ("LL") (6). The IC (5) and LL (6) imply that the IR (4) is automatically satisfied, and so we ignore it in the analysis that follows.

Without limited liability on the manager, the principal could implement the first best by selling the firm to him. Since the first best is achieved, any new signal automatically has zero value and so any contracting constraint must weakly increase the value of information. Thus, it is not the case that signals always have less value under contracting constraints, as intuition might suggest. We consider limited liability on the manager throughout the paper, since this constraint is relevant both for compensation and financing contracts.

1.1 Upper Bound on Payments

In this subsection, in addition to limited liability, we assume that there is a maximum payment to the manager, which can be output-dependent and is denoted $\overline{w}(q)$:

$$0 \leq w_s(q) \leq \overline{w}(q). \tag{7}$$

We assume that $\overline{w}(q)$ is nondecreasing in q. The primary application is $\overline{w}(q) = q$, i.e., limited liability on the firm. We consider the more general upper bound $\overline{w}(q)$ to allow the model to capture other contracting constraints. A finite $\overline{w}(q)$ independent of q represents a cap on ex-post payments; $\overline{w}(q)$ is increasing in q if "outrage constraints" on high pay are relaxed upon superior performance.

For each s, let q_0^s be determined by $\phi_1^s \pi_s(q_0^s) = \phi_0^s p_s(q_0^s)$ if such q_0^s exists, $q_0^s = 0$ if $\phi_1^s \pi_s(q) > \phi_0^s p_s(q)$ for all q, and $q_0^s = \bar{q}$ if $\phi_1^s \pi_s(q) < \phi_0^s p_s(q)$ for all q. By MLRP, q_0^s exists and is unique. To ensure that high effort is implementable, we assume:

$$\int_{q_0^s}^{\bar{q}} \overline{w}(q) \left[\phi_1^s \pi_s(q) - \phi_0^s p_s(q)\right] dq > C. \tag{8}$$

If (8) were not satisfied, the firm would implement low effort and the optimal contract would trivially involve a zero payment.

This upper bound on payments is not necessary for our results; Appendix B shows that, without it, informative signals may still have zero value. However, absent an upper bound, the optimal contract typically involves a very large payment in the highest likelihood ratio state, which would vastly exceed total output and thus violate a limited liability constraint on the firm, and zero payments in every other state. We thus consider an upper bound to achieve more realistic contracts.

Similar to Innes (1990), the solution involves paying the minimum amount possible (zero) when the likelihood ratio is below a threshold κ, and the maximum amount possible when it exceeds it. The threshold κ is chosen so that the IC binds (existence is shown in Appendix A); if more than one such threshold exists, we choose the largest one:

$$\kappa := \sup \left\{ \hat{\kappa} : \sum_s \int_{LR_s(q) > \hat{\kappa}} \overline{w}(q) \left[\phi_1^s \pi_s(q) - \phi_0^s p_s(q)\right] dq = C \right\}. \tag{9}$$

By MLRP, for each signal realization, the threshold for the likelihood ratio translates into a threshold for output. Lemma 1 characterizes the optimal contract:

Lemma 1 *The optimal contract with manager limited liability and an upper bound on payments is*

$$w_s(q) = \begin{cases} 0 & \text{if } q < q_s^*(\kappa) \\ \overline{w}(q) & \text{if } q > q_s^*(\kappa) \end{cases}, \tag{10}$$

where

$$q_s^*(\kappa) := \begin{cases} 0 & \text{if } LR_s(0) > \kappa \\ \bar{q} & \text{if } LR_s(\bar{q}) < \kappa \\ LR_s^{-1}(\kappa) & \text{if } LR_s(0) \le \kappa \le LR_s(\bar{q}) \end{cases} \tag{11}$$

and κ is determined by (9).

Lemma 1 yields a "live-or-die" contract: the manager receives the maximum $\overline{w}(q)$ if output exceeds a threshold q_s^* and zero otherwise. For a given signal realization s, the threshold output level q_s^* is chosen so that the likelihood ratio at this output level equals κ.[6]

In general, the output threshold will depend on the signal realization s, and so the optimal contract is contingent upon both output and the signal. Proposition 1 gives a condition expressed in terms of model primitives for when this is not the case.

Proposition 1 *The optimal contract with manager limited liability and an upper bound on payments is independent of the signal if and only if $LR_s^{-1}(\kappa)$ does not depend on s, where κ is determined by (9).*

If $LR_s^{-1}(\kappa)$ does not depend on s, define $q^* := LR_s^{-1}(\kappa)$, and we have:

$$LR_{s_i}(q^*) = LR_{s_j}(q^*) = \kappa \ \ \forall s_i, s_j. \tag{12}$$

The contract is independent of s if and only if, for every s, the output q_s^* associated with a likelihood ratio of κ is the same, i.e. $q_s^* = q^*$, and so the firm optimally sets the same threshold q^* for all signal realizations.

Proposition 1 shows that contracting constraints require us to refine the informativeness principle. Intuitively, q^* is the threshold that would be chosen in the absence of s. A signal has positive value if and only if it affects the likelihood ratio at q^* – rather than in general – as only at q^* does the firm have freedom to change the contract, by making q^* depend on the signal. When $q_s^* = q^*$ – i.e. the firm would choose not to make the threshold depend on the signal – the signal has zero value because the firm cannot use it. It cannot change the contract for $q < q^*$ because it is already paying zero, nor for $q > q^*$ because it is already paying the maximum. As a result, additional signals about effort are only valuable for intermediate output levels, not at tail output realizations. Note that the "tails" do not refer only to extreme outputs. Any output realization above or below the threshold is a "tail" realization. For example, signals which are informative at the tails, i.e., which affect the likelihood ratio above or below q^*, have zero value. While risk neutrality and limited liability is sometimes seen as an alternative to risk aversion in a contracting model (both are ways of ruling out the first-best solution of the principal selling the firm to the agent), the conditions for a signal to have value are very different.

[6]For some signal realizations, this threshold output level may be a corner solution, in which case the manager either always receives the maximum or always receives zero. If all thresholds are interior, then $q_s^* = LR_s^{-1}(\kappa)$ for all s.

In sum, if output q is a sufficient statistic for effort e given (q, s), the signal s has zero value. However, even if q is not a sufficient statistic, s still has zero value if it is informative about effort only in states at which contracting constraints bind. In turn, contracting constraints bind everywhere except for at the threshold q_s^*.

1.2 Upper Bound on Incentives

In this subsection, in addition to manager limited liability, we assume that – rather than an upper bound on payments – there is an upper bound on the sensitivity of pay to performance:

$$w_s(q + \delta\epsilon) - w_s(q) \leq \epsilon, \ \forall \ \epsilon > 0. \tag{13}$$

Constraint (13) states that, for a dollar increase in output, the payment to the manager can increase by at most $\delta \leq 1$ dollars. We consider values of δ under which high effort can be implemented by assuming:

$$\delta \left\{ \mathbb{E}[q|e=1] - \mathbb{E}[q|e=0] \right\} > C. \tag{14}$$

The primary application is a monotonicity constraint, as in Innes (1990), where $\delta = 1$: the manager cannot gain more than one-for-one with an increase in q. Innes (1990) justifies this constraint on two grounds. First, if it were violated, the manager would inject his own money to increase output, since he would gain more from his contract than the amount injected. Second, if it were violated, the firm's payoff would fall with output over some region. Thus, it would exercise its control rights to "burn" output, raising its payoff. We generalize the upper bound on pay-performance sensitivity to a general δ to capture other constraints on incentives, e.g. a maximum level of equity awards, or a maximum payment that increases with output (since higher performance makes higher pay more socially acceptable). $\delta < 1$ may also arise if the firm can engage in cash flow diversion which gives her a private benefit of $1 - \delta$, e.g. buying inputs from another company that she owns at above-market prices. Then, the slope of the contract must not exceed δ to prevent diversion.

Let

$$\overline{LR}_s(q) := \frac{\phi_1^s \int_q^{\bar{q}} \pi_s(z)dz}{\phi_0^s \int_q^{\bar{q}} p_s(z)dz} = \frac{\Pr(\tilde{q} \geq q, \tilde{s} = s|e=1)}{\Pr(\tilde{q} \geq q, \tilde{s} = s|e=0)} \tag{15}$$

denote the likelihood ratio associated with the event $(\tilde{q} \geq q, \tilde{s} = s)$, which is strictly increasing by MLRP (as shown in Appendix A). The two terms in (15) show that a signal can affect the likelihood ratio in two ways: it can either be individually informative about effort (i.e.

affect $\frac{\phi_1^s}{\phi_0^s}$), or it can affect the informativeness of output about effort $\frac{\int_q^{\bar{q}} \pi_s(z)dz}{\int_q^{\bar{q}} p_s(z)dz}$. Even if a signal is unaffected by effort and thus not individually informative about effort, it can still be informative. For example, if the manager's effort does not affect macroeconomic conditions, they may still be informative since output may be less informative about effort in booms, when all firms perform well regardless of managerial effort, than in recessions.

For each fixed κ and signal realization s, construct the threshold "strike price" as follows:

$$q_s^{**}(\kappa) := \begin{cases} 0 & \text{if } \overline{LR}_s(0) > \kappa \\ \bar{q} & \text{if } \overline{LR}_s(\bar{q}) < \kappa \\ \overline{LR}_s^{-1}(\kappa) & \text{if } \overline{LR}_s(0) \leq \kappa \leq \overline{LR}_s(\bar{q}) \end{cases}. \tag{16}$$

The threshold for the likelihood ratio κ is chosen so that the IC binds (existence is shown in Appendix A); if more than one such threshold exists, we choose the largest one:

$$\kappa := \sup\left\{\hat{\kappa} : \sum_s \int_{\overline{LR}_s(q) > \hat{\kappa}} \delta(q - q_s^{**}(\hat{\kappa})) \left[\phi_1^s \pi_s(q) - \phi_0^s p_s(q)\right] dq = C\right\} \in (0, \bar{q}). \tag{17}$$

The optimal contract given by Lemma 2 below:

Lemma 2 *The optimal contract with an upper bound on incentives is* $w_s(q) = \delta \max\{q - q_s^{**}(\kappa), 0\}$, *where* $q_s^{**}(\kappa)$ *and* κ *are determined by (16) and (17).*

Lemma 2 yields an option contract: if output exceeds q_s^{**}, the manager receives a proportion δ of the residual $q - q_s^{**}$, rather than the maximum payment as in Section 1.1. Proposition 2 gives a necessary and sufficient condition under which the contract is independent of the signal.

Proposition 2 *The optimal contract with an upper bound on incentives is independent of the signal if and only if* $\overline{LR}_s^{-1}(\kappa)$, *where* κ *is determined by (17), does not depend on* s.

If $\overline{LR}_s^{-1}(\kappa)$ does not depend on s, define $q^{**} := \overline{LR}_s^{-1}(\kappa)$, and we have:

$$\overline{LR}_{s_i}(q^{**}) = \overline{LR}_{s_j}(q^{**}) = \kappa \; \forall s_i, s_j. \tag{18}$$

The firm optimally sets the same threshold q^{**} (i.e. $q_s^{**} = q^{**} \; \forall s$) if and only if the likelihood ratio that $q \geq q^{**}$ is always κ, regardless of s.

The likelihood ratios in Propositions 1 and 2 concern different events. With a maximum payment $\overline{w}(q)$ in addition to limited liability (Proposition 1), the manager is paid $\overline{w}(q)$ if output exceeds q^*. Thus, if the firm uses the signal to vary q^*, it changes the payment only in a local neighborhood around q^* (i.e. changes it from 0 to $\overline{w}(q)$ or vice-versa). As a result, a signal is only useful if it affects the likelihood ratio at a single point $q = q^*$ – i.e. provides information on whether $q = q^*$ is more likely to have resulted from working or shirking. If signal realization s_i suggests that the manager has worked, the firm increases the payment from 0 to $\overline{w}(q)$ by reducing the threshold to $q^*_{s_i} < q^*$. If it suggests that he has shirked, the firm reduces the payment from $\overline{w}(q)$ to 0 by increasing the threshold to $q^*_{s_i} > q^*$.

With an upper bound on incentives (Proposition 2), the manager is paid $\delta(q - q^{**})$ if output exceeds q^{**}. Thus, if the firm uses the signal to vary the strike price q^{**}, this changes the payment at not only $q = q^{**}$ (as in Proposition 1) but at all $q \geq q^{**}$; it cannot change the payment at specific output levels in isolation as this would violate the upper bound on incentives. Thus, a signal has value if it affects the likelihood ratio over a whole range $q \geq q^{**}$ – i.e. provides information on whether $q \geq q^{**}$ is more likely to have resulted from working or shirking. Any signal that shifts probability mass from below to above the threshold (or vice-versa) is valuable, as it affects the likelihood that output exceeds the threshold. For example, consider $q^{**} = 5$ and $\delta = 1$. The likelihood ratio is higher for $q = 7$ than $q = 3$, and so (in the absence of a signal), the manager receives 2 if $q = 7$ and 0 if $q = 3$. If the event $(q \geq 5, s = s_i)$ indicates effort more than $(q \geq 5, s = s_j)$, i.e., given the knowledge that $q \geq 5$, s_i indicates effort more than s_j, the firm will optimally increase the payment when the signal is s_i compared to when it is s_j. To preserve monotonicity, this is achieved by setting a lower threshold for s_i than for s_j: $q^{**}_{s_i} < q^{**}_{s_j}$.

However – as with an upper bound on payments – any signal that only redistributes mass below the threshold so that it stays below the threshold, or only redistributes mass above the threshold so that it stays above the threshold, has no value. Continuing the earlier example, if $(q \geq 7, s = s_i)$ indicates effort more than $(q \geq 7, s = s_j)$, but $(q \geq 5, s = s_i)$ does not indicate effort more than $(q \geq 5, s = s_j)$, then the firm would like to increase the payment for $(q \geq 7, s = s_i)$ and keep unchanged the payment for $(q \geq 5, s = s_i)$. However, such a change would violate monotonicity, and so the firm cannot use the signal.

Despite the difference in the relevant likelihood ratios, Propositions 1 and 2 both establish similar conditions for a signal to have value. In both cases, the firm's only degree of freedom is the threshold q^* or q^{**} – under the optimal contract, the payment below the threshold is automatically zero, and the payment above is automatically the entire output or the residual. Thus, an additional signal will only be included if the firm wishes to use its realization to vary the threshold – it will not use it to change any other dimension of the contract. With an upper bound on payments in addition to limited liability, changing q^* only has local effects, and so

Proposition 1 depends on the likelihood ratio associated with $q = q^*$. With an upper bound on incentives, changing q^{**} affects payments at all higher outputs, and so Proposition 2 depends on the likelihood ratio associated with $q \geq q^{**}$.

The above result has a number of applications for compensation contracts. First, it identifies the settings in which boards should invest in additional signals of manager performance, for instance through monitoring. A signal that shifts mass locally is only useful at intermediate output levels, not tail outputs, as only then will it affect the payment. In risk management, a "smoking gun" indicates that a bad event is due to poor performance (e.g. excessive risk-taking) rather than bad luck, but the bad event will likely lead to firing anyway. For instance, investors only noticed that Enron was adopting misleading accounting practices when it was already going bankrupt. Relatedly, the threshold output can be interpreted as a performance target below which the manager is fired. Signals are then only useful if they affect this target.

Second, it implies that pay-for-luck (i.e. not obtaining signals to verify whether an output level was due to effort or luck) need not be suboptimal if it occurs at tail output realizations. Sometimes, pay-for-luck concerns very good or very bad outcomes – for example, Bertrand and Mullainathan (2001) consider how CEO pay varies with spikes and troughs in the oil price, and Jenter and Kanaan (2015) find that peer-group performance does not affect CEO firing decisions – but additional signals are only valuable for moderate outcomes. In turn, if the probability of constraints binding varies with economic conditions, then the extent of pay-for-luck will also vary with economic conditions. For example, in downturns, output is likely to be low and so managers are likely to be paid zero, regardless of the outcome of additional signals.

Proposition 2 also has implications for debt contracts. Our model can be interpreted in two ways. First, the firm offers a compensation contract to the manager, as in the above exposition. Second, the manager is an entrepreneur who raises financing from an investor, which is the exposition in Innes (1990). The optimal contract is debt, and so a signal has no value in determining the repayment schedule, which is automatically the entire output if performance is poor, and the entire promised repayment (principal plus interest) if performance is good. It has value if and only if it affects the promised repayment. In theory, this amount could depend on many signals, but in practice it is often signal-independent. Proposition 2 potentially rationalizes this practice – even if signals are informative about effort, they should not enter the contract if they are only informative in the tails.

In addition, Proposition 2 provides conditions under which the repayment *should* depend on additional signals, as in performance-sensitive debt, where the repayment is higher upon negative signals of borrower performance. This is the case if and only if the ratio of the probabilities that output exceeds the threshold under high and low effort is a function of the signal – that is, the signal is informative about effort conditional on output exceeding the threshold.

We close with three examples that apply Proposition 2 to a real-world setting. First, consider a signal which is not individually informative about effort, such as macroeconomic conditions. If the likelihood that output exceeds the threshold under high versus low effort is not a function of macroeconomic conditions, then the contract should not be contingent upon macroeconomic conditions – even if output does depend on macroeconomic conditions. For example, consider a start-up which is developing major new software. Macroeconomic conditions could affect the distribution of output both if the software is widely adopted, and if it is not adopted. However, if they do not affect the probability that output exceeds the threshold, because they do not affect the likelihood that the software will be adopted, then the contract should be independent of macroeconomic conditions.

Second, under the financing application, consider a firm that issued debt whose face value in the absence of an additional signal is q^{**}. The manager's effort affects the distribution of both output and an additional signal, the firm's credit rating, which captures the probability and severity of default. If $q < q^{**}$, the credit rating is informative about effort since effort affects the severity of default. If $q > q^{**}$, the credit rating is uninformative about effort since default can only occur due to extraneous events, such as the bankruptcy of a major customer, bank, or hedging counterparty of the firm, which is outside the manager's control. Thus, the credit rating is informative about effort only conditional upon $q < q^{**}$, but the manager's equity payoff is zero anyway. Hence, it should not be part of the contract – debt is not performance-sensitive – even though output is not a sufficient statistic for effort.

Third, under the compensation application, consider an industry with two identical firms. The signal s is the other firm's output. Suppose that output correlation is countercyclical (as found by Perez-Quiros and Timmerman (2000)): firm outputs are positively correlated when the industry is in recession, but independent otherwise. If the output distribution is such that the managers' options are out-of-the-money in a recession, then firms will not use relative performance evaluation.[7]

[7] This contrasts Theorem 7 in Holmström (1982), which yields relative performance evaluation under weaker conditions in a moral hazard setting: a contract should depend exclusively on the firm's own output if and only if firm outputs are independent.

2 Continuous Effort and Risk Aversion

This section generalizes the model to both risk aversion and a continuous effort decision, retaining previous assumptions unless otherwise specified. Effort is now given by $e \in \mathbb{R}_+$. Let $F(q|e,s)$ and $f(q|e,s)$ denote the cumulative distribution function ("CDF") and PDF of q conditional on e and s. We assume that, for each s, $F(\cdot|\cdot,s)$ is twice continuously differentiable with respect to q and e. We continue to assume MLRP, which here entails $\frac{d}{dq}[\frac{f_e(q|e,s)}{f(q|e,s)}] > 0$, where $f_e(q|e,s)$ denotes the first derivative of the PDF with respect to e. We assume that the marginal distribution of the signal ϕ_e^s is differentiable with respect to e.

The manager's utility of money is given by a strictly increasing, weakly concave, twice differentiable function u. He has outside wealth $\bar{W} > 0$ and reservation utility $\overline{u}$.[8] His cost of effort $C(e)$ is a twice differentiable, strictly increasing, and strictly convex function. Thus, given a contract $w_s(q)$ and an effort level e, his objective function is $\mathbb{E}[u(\bar{W} + w_s(q))|e] - C(e)$.

We follow Grossman and Hart (1983) and separate the principal's problem in two stages. The first stage determines the cost of implementing each effort level. Given these costs, the second stage determines which effort level to implement. We study whether the optimal contract for implementing each given effort level does not depend on signal. If those conditions hold for all effort levels, the optimal contract (with effort chosen optimally) will also not depend on the signal. To implement a given effort level $\hat{e}$, the firm chooses a function $w_s(\cdot)$, for each possible value of the signal s, to solve the following problem:

$$\min_{w_s(q)} \sum_s \phi_{\hat{e}}^s \int_0^{\overline{q}} w_s(q) f(q|\hat{e},s) dq \tag{19}$$

subject to

$$\sum_s \phi_{\hat{e}}^s \int_0^{\overline{q}} u(\bar{W} + w_s(q)) f(q|\hat{e},s) dq - C(\hat{e}) \geq \overline{u}, \tag{20}$$

$$\hat{e} \in \arg\max_e \sum_s \phi_e^s \int_0^{\overline{q}} u(\bar{W} + w_s(q)) f(q|e,s) dq - C(e), \tag{21}$$

$$w_s(q) \in [0, \overline{w}(q)]. \tag{22}$$

As in Section 1.1, the manager is subject to limited liability and there is an upper bound $\overline{w}(q)$ on payments. While an upper bound on payments will always bind for some outputs in the model of Section 1.1, with risk aversion it may not bind anywhere – intuitively, a contract

[8] With risk neutrality (Section 1) we assumed zero reservation utility, so that solving the incentive problem is costly to the principal as it involves paying the agent rents (i.e. a slack IR). With risk aversion, solving the incentive problem is costly for the principal even if the agent does not receive rents (i.e. the IR binds), since the principal must pay a premium for the risk the agent bears from receiving incentive compensation.

that involves the maximum payment for some output realizations will subject the manager's wage to significant risk. Thus, we also specifically consider the case without the upper bound: $\overline{w}(q) = +\infty \; \forall \, q$. The analog of this model for the case of risk neutrality and binary effort is in Appendix B.[9]

Following Holmström (1979), Shavell (1979) and the subsequent literature on the informativeness principle (e.g. Gjesdal (1982), Kim (1995)), we assume that the first-order approach ("FOA") is valid; see Chaigneau, Edmans, and Gottlieb (2016) for the informativeness principle without the FOA. We can thus replace the IC in (21) by the following equation:

$$\sum_s \left[\frac{d\phi_{\hat{e}}^s}{de} \int_0^{\overline{q}} u(\bar{W} + w_s(q)) f(q|\hat{e}, s) dq + \phi_{\hat{e}}^s \int_0^{\overline{q}} u(\bar{W} + w_s(q)) f_e(q|\hat{e}, s) dq \right] = C'(\hat{e}) \tag{23}$$

Let λ and μ denote the nonnegative Lagrange multipliers associated with the IR (20) and IC (21), respectively. The optimal contract is given by Lemma 3 below.

Lemma 3 *Suppose an optimal contract exists and the FOA is valid. With limited liability on the manager, the optimal contract is:*

$$w_s(q) = \max \left\{ u'^{-1} \left(1 / \left(\lambda + \mu \left[\frac{d\phi_{\hat{e}}^s / de}{\phi_{\hat{e}}^s} + \frac{f_e(q|\hat{e}, s)}{f(q|\hat{e}, s)} \right] \right) \right) - \bar{W}, 0 \right\}. \tag{24}$$

With also a maximum payment, the optimal contract is:

$$w_s(q) = \max \left\{ \min \left\{ u'^{-1} \left(1 / \left(\lambda + \mu \left[\frac{d\phi_{\hat{e}}^s / de}{\phi_{\hat{e}}^s} + \frac{f_e(q|\hat{e}, s)}{f(q|\hat{e}, s)} \right] \right) \right) - \bar{W}, \overline{w}(q) \right\}, 0 \right\}. \tag{25}$$

The contract thus involves lower payments in states associated with low likelihood ratios, but the payment cannot fall below zero, and higher payments in states associated with high likelihood ratios, but the payment cannot exceed the upper bound if imposed.

We now analyze the conditions under which the optimal contract is independent of the signal. Without the signal s, the likelihood ratio at a given value of q can be written as $LR(q) := \frac{f_e(q|\hat{e})}{f(q|\hat{e})}$. With the signal s, we define the likelihood ratio as

$$LR_s(q) := \frac{f_e(q, s|\hat{e})}{f(q, s|\hat{e})} = \frac{d\phi_{\hat{e}}^s / de}{\phi_{\hat{e}}^s} + \frac{f_e(q|\hat{e}, s)}{f(q|\hat{e}, s)}. \tag{26}$$

[9] In the risk-neutral model of Section 1 we also considered an upper bound on incentives, to rule out discontinuities that may induce either the manager or firm to manipulate output. Under risk aversion, contracts without discontinuities can be obtained without such a bound (see, e.g., Proposition 4).

As in the previous section, a signal can affect the likelihood ratio in two ways. First, it can be individually informative about effort, i.e., $\frac{d\phi_{\hat{e}}^{s}/de}{\phi_{\hat{e}}^{s}}$ depends on s. Second, it can affect the informativeness of output about effort, i.e., $\frac{f_e(q|\hat{e},s)}{f(q|\hat{e},s)}$ depends on s.

With limited liability on the manager, for each fixed κ and signal realization s, construct the threshold above which the payment is strictly positive as follows:

$$q_s^{***}(\kappa) := \begin{cases} 0 & \text{if } LR_s(0) > \kappa \\ \overline{w}(q) & \text{if } LR_s(\bar{q}) < \kappa \\ LR_s^{-1}(\kappa) & \text{if } LR_s(0) \leq \kappa \leq LR_s(\bar{q}) \end{cases} \tag{27}$$

The threshold likelihood ratio κ is chosen so that the IC binds for effort $\hat{e}$; if more than one such threshold exists, we choose the largest one:

$$\begin{aligned} \kappa := \quad & \sup\left\{\hat{\kappa} : \sum_s \left[\int_{LR_s(q)\leq\hat{\kappa}} u(\bar{W})\left[\frac{d\phi_{\hat{e}}^s}{de} f(q|\hat{e},s) + \phi_{\hat{e}}^s f_e(q|\hat{e},s)\right] dq \right.\right. \\ & \left.\left. + \int_{LR_s(q)>\hat{\kappa}} u(\bar{W} + w_s(q))\left[\frac{d\phi_{\hat{e}}^s}{de} f(q|\hat{e},s) + \phi_{\hat{e}}^s f_e(q|\hat{e},s)\right] dq\right] = C'(\hat{e})\right\} \end{aligned} \tag{28}$$

The contract in equation (24) is monotonic (via MLRP) and also continuous, since the likelihood ratio is continuous: its numerator and denominator are continuously differentiable with respect to q. However, its shape (e.g. whether it is concave, convex, or linear above q_s^{***}) depends on the shape of the utility function and likelihood ratio.

With a maximum payment in addition to limited liability, for each realization of s, define $\mathcal{M}_s$ as the set of values of q such that, with the contract described in equation (25):

$$w_s(q) = u'^{-1}\left(1/\left(\lambda + \mu\left[\frac{d\phi_{\hat{e}}^s/de}{\phi_{\hat{e}}^s} + \frac{f_e(q|\hat{e},s)}{f(q|\hat{e},s)}\right]\right)\right) - \bar{W}. \tag{29}$$

Intuitively, $\mathcal{M}_s$ is the set of output levels for which neither constraint on contracting binds.

Proposition 3 gives conditions under which the optimal contract is independent of the signal.

Proposition 3 *(i) With limited liability on the manager, the optimal contract is independent of the signal if and only if* $LR_{s_i}(q) = LR_{s_j}(q)\ \forall s_i, s_j, q \geq q^{***} := \min_s\{q_s^{***}\}$.

(ii) With also a maximum payment, the optimal contract is independent of the signal if and only if $LR_{s_i}(q) = LR_{s_j}(q)\ \forall s_i, s_j, q \in \mathcal{M}_s$.

The intuition is as follows. In both the binary and continuous effort cases, a signal has no value if and only if it does not affect the relevant likelihood ratio – $\frac{f_e(q,s|\hat{e})}{f(q,s|\hat{e})}$ with continuous effort and $\frac{\phi_1^s \pi_s(q)}{\phi_0^s p_s(q)}$ with binary effort. With risk neutrality and a maximum payment (Section 1.1), the relevant likelihood ratio is at a single intermediate output level q_s^*, because contracting constraints bind everywhere else. With risk aversion, the relevant likelihood ratio is at a range of output realizations ($q \geq q^{***}$ or $q \in \mathcal{M}_s$) – because contracting constraints do not bind at many output levels, the conditions for a signal to have value are weaker. In particular, while the manager receives zero below a threshold q_s^{***}, he does not automatically receive the maximum payment above q_s^{***}. Thus, with limited liability on the manager only, the firm can change the payment in response to the signal for any $q \geq q^{***}$; with also a maximum payment it can do so for any $q \in \mathcal{M}_s$. This result suggests that imposing constraints on executive compensation may result in more pay-for-luck, for example because of a lack of relative performance evaluation.

2.1 Option Repricing and Performance-Vesting

Thus far, we have studied *whether* informative signals have value under contracting constraints. We now turn to a second question – *how* informative signals should be incorporated into the contract when they do have value. This question requires us to be able to characterize the optimal contract. In the risk-neutral model of Section 1, the optimal contract is an option for the manager, or equivalently debt for the firm; Appendix C studies how the signal affects the contract in this case. In the risk-averse model of this section, in general the contract will be highly complex and cannot be characterized. However, Proposition 4 shows that, under the standard assumptions of log utility and normally distributed output, the optimal contract is an option – in turn allowing us to study how the signal affects the contract.

Proposition 4 *(i) With limited liability on the manager, normally distributed output q, and log utility ($u(w) = \ln w$), the optimal contract involves n_s^* options with a strike price of q_s^{***}:*

$$w_s(q) = n_s^* \max\{q - q_s^{***}, 0\}, \tag{30}$$

with $n_s^* \geq 0$ $\forall s$ and

$$q_s^{***} = -\frac{1}{b_s}\left[\frac{\lambda - \bar{W}}{\mu} + \frac{d\phi_{\hat{e}}^s/de}{\phi_{\hat{e}}^s} + a_s\right] \tag{31}$$

(ii) The number of options n_s^* received ex-post by the manager is independent of the signal if and only if $\frac{d}{dq}\frac{f_e(q|\hat{e},s)}{f(q|\hat{e},s)}$ is independent of s.

(iii) The strike price q_s^{***} is independent of the signal if and only if $\frac{f_e(q,s|\hat{e})}{f(q,s|\hat{e})} = \frac{\bar{W}-\lambda}{\mu}$ at the same value of q for all s.

In Proposition 4, the manager has n_s^* options with strike price q_s^{***}. With log utility, $u'^{-1}(1/x) = x$. With normally distributed output, the likelihood ratio is linear ($\frac{f_e(q|\hat{e},s)}{f(q|\hat{e},s)} = a_s + b_s q$ so that $LR_s(q) = \frac{d\phi_{\hat{e}}^s/de}{\phi_{\hat{e}}^s} + a_s + b_s q$; indeed, a linear likelihood ratio also arises with other distributions such as Gamma), log utility yields $w_s(q) = u'^{-1}(1/LR_s(q)) = \frac{d\phi_{\hat{e}}^s/de}{\phi_{\hat{e}}^s} + a_s + b_s q$, i.e., the contract is linear in q. To our knowledge, part (i) of Proposition 4 provides the first set of sufficient conditions for options to be the optimal contract when the manager is risk-averse.[10] Note that contracting constraints are key to the above result – without limited liability, the optimal contract would be linear in output.

While Proposition 3 studied the conditions under which a signal affects any dimension of the contract, part (ii) of Proposition 4 studies the conditions under which a signal affects specifically the number of options. Proposition 3 showed that a signal has value if it affects any component of the likelihood ratio where contracting constraints do not bind: either $\frac{d\phi_{\hat{e}}^s/de}{\phi_{\hat{e}}^s}$ (i.e. the signal is individually informative about effort) or $a_s + b_s q$ (i.e. the signal affects the informativeness of output for effort). Such a signal will, in general, alter the Lagrange multiplier μ and thus scale up or down the number of options $n_s^* = \mu b_s$ received across all signals s. However, the number of options actually received ex post may still not depend on the actual signal realization. This will only arise if b_s, rather than any other component of the likelihood ratio, depends on s – i.e. the signal realization affects the rate at which the informativeness of the stock price changes with the level of stock price. The intuition is as follows. As in any principal-agent model, pay is increasing in the likelihood ratio, and so the sensitivity of pay to output (here, the number of options $n_s^* = \mu b_s$) depends on the sensitivity of the likelihood ratio to output, $\frac{dLR_s(q)}{dq} = b_s$. If the likelihood ratio increases faster with output when s is high (i.e. if $s_i > s_j$ implies $b_{s_i} > b_{s_j}$), the contract should be steeper and more options should be granted (i.e. $n_{s_i}^* > n_{s_j}^*$).

To our knowledge, this result is the first theoretical justification of the conditions under which performance-based vesting is optimal. More broadly, it identifies conditions under which the allocation of incentives varies across states of the world – if and only if the sensitivity of the likelihood ratio to output differs across states of the world. In this case, it is optimal to

[10] Jewitt, Kadan, and Swinkels (2008) show that the contract is "option-like" with risk aversion and agent limited liability, in that incentives are zero for low output and positive for high output, but do not identify conditions under which the increasing portion of the contract is linear. Hemmer, Kim, and Verrecchia (1999) identify a linear likelihood ratio and log utility as leading to the contract having a linear portion, but did not combine them with limited liability to obtain an option contract. To our knowledge, the only existing justification of options in a moral hazard model is Innes (1990), which requires the agent to be risk-neutral. Unlike in the risk-neutral model where the manager is the residual claimant for $q \geq q^{**}$, so that the number of options is fixed at 1, under risk aversion it need not be.

concentrate incentives on states where output is more informative about effort in this sense.

Thus, using an earlier example, even if effort does not affect the probability of a recession ($\frac{d\phi_{\hat{e}}^s/de}{\phi_{\hat{e}}^s}$ is independent of s, a signal of economic conditions), more options should vest in bad (good) times if the likelihood ratio is more sensitive to the stock price – i.e. effort has a greater effect on the stock price – in bad (good) times. This example also shows that signals that trigger vesting could optimally depend on "luck" – even though economic conditions are outside the manager's control, they may still optimally affect vesting.

Note that it may be efficient for more options to vest upon *low* signals of firm performance. This result implies that the existing practice, of vesting always being triggered by good performance, may not be optimal. However, it echoes the theoretical prediction of Edmans, Gabaix, Sadzik, and Sannikov (2012) who show that more equity should be granted when firm value is low, and the empirical finding of Core and Larcker (2002) who show that more equity is granted upon poor performance (although they study stock rather than options).[11] Conversely, even if a signal (say, revenues) is individually informative about effort (i.e. $\frac{d\phi_{\hat{e}}^s/de}{\phi_{\hat{e}}^s}$ depends on revenues s), it should not affect vesting if it does not affect the sensitivity of the likelihood ratio to the stock price. The likelihood ratio given s_i could be a vertical translation of the likelihood ratio given s_j, but if both are as sensitive to q then there should not be any performance-based vesting.

Part (iii) identifies a necessary and sufficient condition under which the strike price is independent of the signal. This condition echoes that in Propositions 1 and 2, in that the likelihood ratios only need to be independent of the signal at one intermediate output level for the contract to be independent of the signal. For a manager without zero outside wealth and with a slack IR, the contract is determined by the IC. The number of options then depends on the sensitivity of the likelihood ratio to output, and the strike price is chosen such that the payment is sensitive to performance (i.e., is nonzero) if and only if the likelihood ratio is positive. If the likelihood ratio turns positive at the same value of q for all s, then the strike price is independent of s. For example, improvements in the informativeness of an accounting system will likely lead to a steeper likelihood ratio (output more closely reflecting the manager's effort) without changing the "location" of the likelihood ratio – the level of output at which the likelihood ratio is zero, i.e. at which a marginal change in effort does not change the likelihood of observing this output. If this is the case, such an improvement need not lead to the performance measure produced by the information system being optimally added to the contract.

[11]Note that, if a low signal leads to more options vesting, it may also lead to their strike price increasing, so that the agent is not better off by generating a low signal.

Contrary to intuition, it may be optimal for the strike price to be lowered upon a signal that individually indicates low effort. Consider two signal realizations, L and H, such that $\frac{d\phi_{\hat{e}}^{L}/de}{\phi_{\hat{e}}^{L}} < \frac{d\phi_{\hat{e}}^{H}/de}{\phi_{\hat{e}}^{H}}$, so that L is individually worse news about effort than H. We may have $q_L^{***} < q_H^{***}$ – the strike price is lower under the signal that individually indicates low effort.

One case is $a_L > a_H$ and $b_L = b_H$. Here, any given output q is better news about effort under L than H, in the sense that $\frac{f_e(q|\hat{e},L)}{f(q|\hat{e},L)} > \frac{f_e(q|\hat{e},H)}{f(q|\hat{e},H)}$ for all q. Thus, a high output under L is better news about effort than a high output under H. Equation (31) shows that this generates a lower strike price under L, if the difference between a_H and a_L is sufficiently large to outweigh the fact that $\frac{d\phi_{\hat{e}}^{L}/de}{\phi_{\hat{e}}^{L}} < \frac{d\phi_{\hat{e}}^{H}/de}{\phi_{\hat{e}}^{H}}$. For example, let q be firm profits and s be the number of new entrants into its industry. A low number of new entrants ($s = H$) is individually a better signal of effort than a high number ($s = L$), because it is harder to enter an industry where incumbents offer good products and have strong customer loyalty. This consideration may be outweighed by a second effect – achieving high profits in an industry with more competitors is a positive signal about effort. Thus, high profits should be rewarded more when there are more entrants. This can be achieved by setting $q_L^{***} < q_H^{***}$. Even though many firms entering the industry indicates low effort, in combination with high profits it indicates high effort, and so is associated with a lower strike price.

A second case is $b_D > b_L$, $a_D = a_L$, and $\frac{d\phi_{\hat{e}}^{D}/de}{\phi_{\hat{e}}^{D}} < \frac{d\phi_{\hat{e}}^{L}/de}{\phi_{\hat{e}}^{L}}$ where both fractions are negative so that the bracketed expression in (31) is negative for $s \in \{D, L\}$. Here, the signal affects not the level of the likelihood ratio, but the rate at which it changes with output. Both signals D ("dire") and L ("low") are individually bad news about effort, with D being worse news. Since $b_D > b_L$, output q is more informative about effort under D than L, and so the manager should be rewarded more for a high output under D than under L. This generates a lower strike price under D, if the difference between b_D and b_L is sufficiently large to outweigh the fact that $\frac{d\phi_{\hat{e}}^{D}/de}{\phi_{\hat{e}}^{D}} < \frac{d\phi_{\hat{e}}^{L}/de}{\phi_{\hat{e}}^{L}}$. For example, consider a firm whose credit rating can be downgraded by one notch but remain investment-grade ($s = L$), or downgraded to junk ($s = D$). A downgrade to junk is individually worse news about managerial effort than a one notch downgrade. Such a downgrade also restricts the firm's access to external financing; since it is now financially constrained, its performance may depend more on managerial effort (e.g. to cut costs or reallocate capital optimally across divisions). In other words, effort has a stronger effect on the distribution of q, and makes output more informative. High output following a downgrade to junk can indicate effort more than high output following a one notch downgrade. Even though a downgrade to speculative status individually indicates low effort, in combination with high output it indicates high effort, and so can be associated with a lower strike price ($q_D^{***} < q_L^{***}$).

If the effect of the signal s on the likelihood ratio of q is instead constant, we do obtain the intuitive result that a signal realization that is individually bad news about effort will be associated with a higher strike price. Overall, part (iii) provides conditions under which the strike price should depend on additional signals, which can be implemented via option indexing or option repricing. Brenner, Sundaram, and Yermack (2000) find empirically that repricing nearly always involves a lowering of the strike price, and follows poor stock price performance (both absolute and industry-adjusted). Our model suggests that a reduction in the strike price should *generally* be prompted by positive, rather than negative, signals of effort, suggesting that such practices are suboptimal.[12] However, the above examples provides conditions under which repricing is optimal, suggesting that it is not universally inefficient, contrary to concerns (e.g. Bebchuk and Fried (2004)) that it represents rewards for failure.

3 Conclusion

This paper shows that the informativeness principle must be modified in the presence of contracting constraints, in turn allowing us to understand whether and under what conditions compensation and financing contracts should depend on performance signals in addition to output. We derive necessary and sufficient conditions for a signal to have value under various contracting constraints. With risk neutrality and bilateral limited liability (or an upper bound on payments), a signal is valuable if and only if it is informative about effort at a single intermediate output. If there is also a monotonicity constraint, or an upper bound on incentives, a signal is valuable if and only if it provides information on whether beating the target performance level is more likely to have resulted from working or shirking. In both cases, contracting constraints bind almost everywhere, and so a signal can be informative about effort almost everywhere and still have zero value. If the agent is risk-averse, the principal may choose to offer incentives below the maximum to improve risk-sharing. Since contracting constraints bind in fewer places, the conditions for a signal to have value are weaker.

In addition to the theoretical contribution of new conditions for a signal to have value in the presence of contracting constraints, the results have a number of implications for real-life contracts. Starting with compensation contracts, our results offer a potential explanation as to why both pay and the firing decision do not depend on many potentially informative signals, why it may not be optimal to filter out luck, when options should be repriced, and whether options should have performance-based vesting conditions. For example, performance-based

[12] Acharya, John, and Sundaram (2000) also study the repricing of options theoretically. In their model, repricing is not undertaken to make use of additional informative signals, but instead to maintain effort incentives when options fall out of the money.

vesting is not necessarily optimal even if a signal is incrementally informative about effort; instead, it must affect the rate at which the informativeness of output changes with the level of output. Surprisingly, the strike price of an option may optimally fall, or the number of vesting options may optimally rise, upon a signal that is individually bad news about effort. Moving to financing contracts, the results suggest whether and under what conditions debt should be performance-sensitive.

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A Proofs

Proof of Lemma 1. The firm's program is:

$$\min_{\{w_s(q)\}} \sum_s \int_0^{\bar{q}} w_s\left(q\right) \phi_1^s \pi_s\left(q\right) dq$$

subject to

$$0 \leq w_s\left(q\right) \leq \overline{w}(q) \;\; \forall q \in [0, \bar{q}],$$
$$\sum_s \int_0^{\bar{q}} w_s\left(q\right) \left[\phi_1^s \pi_s\left(q\right) - \phi_0^s p_s\left(q\right)\right] dq \geq C.$$

This is an infinite-dimensional linear program, which has the following first-order conditions:

$$w_s(q) = \left\{ \begin{array}{c} \overline{w}(q) \\ 0 \end{array} \right\} \text{ if } \phi_1^s \pi_s\left(q\right) - \mu \left[\phi_1^s \pi_s\left(q\right) - \phi_0^s p_s\left(q\right)\right] \left\{ \begin{array}{c} > \\ < \end{array} \right\} 0, \tag{32}$$

for all s (where μ is the Lagrange multiplier associated with the IC), as well as the IC, which must bind:

$$\sum_s \int_{LR_s(q) \geq \frac{\mu}{\mu - 1}} \overline{w}(q) \left[\phi_1^s \pi_s\left(q\right) - \phi_0^s p_s\left(q\right)\right] dq = C. \tag{33}$$

Letting $\kappa := \frac{\mu}{\mu-1}$ and using (32), it follows that $w_s\left(q\right) = \overline{w}(q)$ if $LR_s\left(q\right) > \kappa$, and $w_s(q) = 0$ if $LR_s\left(q\right) < \kappa$. Moreover, equation (33) becomes:

$$\sum_s \int_{LR_s(q) > \kappa} \overline{w}(q) [\phi_1^s \pi_s(q) - \phi_0^s p_s(q)] dq = C. \tag{34}$$

We first show that the set of contracts satisfying these necessary conditions is not empty. Since each value of κ fully characterizes a contract through equations (10) and (11), it suffices to show that there exists a κ that solves (34). The left-hand side ("LHS") of (34) converges to $\int_{q_0^s}^{\bar{q}} \overline{w}(q) \left[\phi_1^s \pi_s(q) - \phi_0^s p_s(q)\right] dq$ as $\kappa \searrow 1$. From (8), this exceeds C. Moreover, it converges to $0 < C$ as $\kappa \nearrow +\infty$. Therefore, by the Intermediate Value Theorem, there exists κ satisfying (34).

Notice that κ orders all contracts that satisfy the necessary optimality conditions: by MLRP, a higher threshold for the likelihood ratio means that the firm pays (weakly) less in each state. Thus, if (34) has multiple solutions, the optimum is the contract associated with the highest κ, as defined in equation (9). ■

Proof of Proposition 1. From Lemma 1, there are two possible cases in which the optimal contract does not depend on the signal ($q^*_{s_1} = ... = q^*_{s_S} = q^*$): an interior solution $q^* \in (0, \bar{q})$ and a boundary solution $q^* \in \{0, \bar{q}\}$. Using the conditions from Lemma 1 for an interior solution establishes:

$$LR_{s_i}(q^*) = LR_{s_j}(q^*) = \kappa \;\; \forall s_i, s_j, \tag{35}$$

where κ is determined by (9). Using the definition of $LR_s(q)$ and rearranging yields the result stated in the proposition.

We now verify that the solution cannot be at the boundary. For a boundary solution we need either $LR_s(0) > \kappa$ for all s or $LR_s(\bar{q}) < \kappa$ for all s. In the first case, the firm always receives zero, which contradicts the optimality of implementing high effort (since the firm can always obtain strictly positive profits by paying zero in all states and implementing low effort). In the second case, the manager always receives zero, violating equation (9) as the IC is not satisfied. ■

Proof that (15) is strictly increasing in q.

We have:

$$\frac{d}{dq}\left\{\frac{\phi_1^s \int_q^{\bar{q}} \pi_s(z)dz}{\phi_0^s \int_q^{\bar{q}} p_s(z)dz}\right\} = \frac{\phi_1^s}{\phi_0^s}\frac{-\pi_s(q)\int_q^{\bar{q}} p_s(z)dz + p_s(q)\int_q^{\bar{q}} \pi_s(z)dz}{\left(\int_q^{\bar{q}} p_s(z)dz\right)^2},$$

which is positive if and only if

$$\frac{\pi_s(q)}{p_s(q)} < \frac{\int_q^{\bar{q}} \pi_s(z)dz}{\int_q^{\bar{q}} p_s(z)dz} \;\Leftrightarrow\; \int_q^{\bar{q}} \frac{\pi_s(z)}{\pi_s(q)}dz > \int_q^{\bar{q}} \frac{p_s(z)}{p_s(q)}dz \;\Leftrightarrow\; \int_q^{\bar{q}} \left[\frac{\pi_s(z)}{\pi_s(q)} - \frac{p_s(z)}{p_s(q)}\right] dz > 0,$$

which is satisfied because, for any $z > q$, MLRP guarantees that $\frac{\pi_s(z)}{p_s(z)} > \frac{\pi_s(q)}{p_s(q)}$.

Proof of Lemma 2. The proof is divided into two parts:

Step 1. Conditional on each signal realization, the optimal contract is an option.

This part adapts the argument from Matthews (2001) to show that the optimal contract gives the manager δ options with payoff $\max\{q - q_s, 0\}$ for some strike price q_s. Let $w_s(q)$ be a contract satisfying the LL, monotonicity, and the IC. Notice that there exists a unique option contract with the same expected payment conditional on each signal realization. In other words, for each s, there exists a unique q_s that solves

$$\int_0^{\bar{q}} \delta \max\{q - q_s, 0\}\pi_s(q)dq = \int_0^{\bar{q}} w_s(q)\pi_s(q)dq. \tag{36}$$

Suppose $w_s(q) \neq \delta \max\{q - q_s, 0\}$ in a set of states with positive measure. We claim that the manager's incentives to shirk are higher under $w_s(q)$ than with the option contract:

$$\int_0^{\bar{q}} w_s(q) p_s(q) dq > \int_0^{\bar{q}} \delta \max\{q - q_s, 0\} p_s(q) dq.$$

Let $v_s(q) := w_s(q) - \delta \max\{q - q_s, 0\}$. Since $v_s(q) \neq 0$ with positive probability and it has mean zero, it must be strictly positive and strictly negative in sets of states with positive probability. Moreover, because $w_s(q)$ satisfies the LL and monotonicity, there exists $k \in (0, \bar{q})$ such that $v_s(q) \geq 0$ if $q \leq k$ and $v_s(q) \leq 0$ if $q \geq k$. Then,

$$\begin{aligned} 0 &= \int_0^{\bar{q}} v_s(q)\, \pi_s(q)\, dq \\ &= \int_0^{\bar{q}} v_s(q)\, \frac{\pi_s(q)}{p_s(q)} p_s(q) dq \\ &= \int_0^{k} v_s(q)\, \frac{\pi_s(q)}{p_s(q)} p_s(q) dq + \int_k^{\bar{q}} v_s(q)\, \frac{\pi_s(q)}{p_s(q)} p_s(q) dq \\ &< \int_0^{k} v_s(q)\, \frac{\pi_s(k)}{p_s(k)} p_s(q) dq + \int_k^{\bar{q}} v_s(q)\, \frac{\pi_s(k)}{p_s(k)} p_s(q) dq \\ &= \frac{\pi_s(k)}{p_s(k)} \int_0^{\bar{q}} v_s(q)\, p_s(q) dq, \end{aligned} \tag{37}$$

where the first line uses the fact that $v_s(q)$ has mean zero under high effort; the second multiplies and divides by $p_s(q)$, the third splits the integral between the positive and negative values of $v_s(q)$; the fourth uses MLRP and the fact that the terms in the first integral are positive whereas the ones in the second integral are negative; and the last line regroups the integrals.

Thus, conditional on each signal realization s, shirking gives the manager a higher payment with the original contract than with the option. Moreover, both contracts pay the same expected amount when the manager exerts effort. We have therefore shown that substituting a non-option contract with an option allows the firm to relax the IC. Since the IC must bind at the optimum, this establishes that the original contract cannot be optimal.

Step 2. Determining the optimal strike prices.

Since any option contract satisfies the LL and monotonicity, the firm's program becomes:

$$\min_{\{q_s\}_{s=1,\ldots,S}} \sum_s \int_{q_s}^{\bar{q}} \delta\,(q - q_s)\, \phi_1^s \pi_s(q)\, dq. \tag{38}$$

subject to

$$\sum_s \int_{q_s}^{\bar{q}} \delta\,(q - q_s) \left[\phi_1^s \pi_s(q) - \phi_0^s p_s(q)\right] dq \geq C. \tag{39}$$

The necessary first-order conditions associated with this program are equation (16) and the binding IC

$$\sum_{s}\int_{\overline{LR}_s(q)>\kappa}\delta(q-q_s^{**}(\kappa))\left[\phi_1^s\pi_s(q)-\phi_0^s p_s(q)\right]dq=C, \tag{40}$$

where $\kappa:=\frac{\lambda}{\lambda-1}$ and λ is the Lagrange multiplier associated with the IC.

The remainder of the proof follows the same steps as the proof of Lemma 1. Each κ determines $q_s^{**}(\kappa)$ according to equation (16). From the Intermediate Value Theorem, there exists κ that solves equation (40): the LHS of (40) evaluated at $\kappa=0$ exceeds C (by equation (14)) and it converges to $0<C$ as $\kappa\to\infty$. Moreover, the firm's profits are ordered by κ: by MLRP, higher thresholds are associated with higher strike prices, which are cheaper. Thus, the best contract among all contracts that satisfy the necessary optimality conditions is the one associated with the largest κ, yielding (17). ■

Proof of Proposition 2. Analogous to the proof of Proposition 1. ■

Proof of Lemma 3. For now we ignore the LL constraint(s) in (22). Denoting by λ and μ the Lagrange multipliers associated respectively with (20) and (23), the first-order condition ("FOC") with respect to $w_s(q)$ in the program in (19), (20), and (23) is:

$$\phi_{\hat{e}}^s f(q|\hat{e},s)-\lambda\phi_{\hat{e}}^s u'(\bar{W}+w_s(q))f(q|\hat{e},s)-\mu u'(\bar{W}+w_s(q))\left[\frac{d\phi_{\hat{e}}^s}{de}f(q|\hat{e},s)+\phi_{\hat{e}}^s f_e(q|\hat{e},s)\right]=0$$

$$\Leftrightarrow\quad \frac{1}{u'(\bar{W}+w_s(q))}=\lambda+\mu\left[\frac{d\phi_{\hat{e}}^s/de}{\phi_{\hat{e}}^s}+\frac{f_e(q|\hat{e},s)}{f(q|\hat{e},s)}\right]. \tag{41}$$

With limited liability on the manager only, we have $\underline{m}(q)=\bar{W}$ and $\overline{m}(q)=\infty$, using the notations in Jewitt, Kadan, and Swinkels (2008). Using the FOC in (41), the same reasoning as in Proposition 1 in Jewitt, Kadan, and Swinkels (2008) applies for any given signal realization s, so that the optimal contract for a given s is defined implicitly by:

$$\frac{1}{u'(\bar{W}+w_s(q))}=\begin{cases}\lambda+\mu\left[\frac{d\phi_{\hat{e}}^s/de}{\phi_{\hat{e}}^s}+\frac{f_e(q|\hat{e},s)}{f(q|\hat{e},s)}\right] & \text{if } \lambda+\mu\left[\frac{d\phi_{\hat{e}}^s/de}{\phi_{\hat{e}}^s}+\frac{f_e(q|\hat{e},s)}{f(q|\hat{e},s)}\right]\geq\frac{1}{u'(\bar{W})},\\ \frac{1}{u'(\bar{W})} & \text{if } \lambda+\mu\left[\frac{d\phi_{\hat{e}}^s/de}{\phi_{\hat{e}}^s}+\frac{f_e(q|\hat{e},s)}{f(q|\hat{e},s)}\right]<\frac{1}{u'(\bar{W})},\end{cases} \tag{42}$$

with $\lambda\geq 0$ and $\mu>0$. Equation (42) can be rewritten as (24).

With an upper bound on payments, we have $\underline{m}(q)=\bar{W}$ and $\overline{m}(q)=\overline{w}(q)+\bar{W}$. The optimal contract for a given value s of the signal is defined implicitly by:

$$\frac{1}{u'(\bar{W}+w_s(q))}=\begin{cases}\frac{1}{u'(\bar{W}+\overline{w}(q))} & \text{if } \frac{1}{u'(\bar{W}+\overline{w}(q))}<\lambda+\mu\left[\frac{d\phi^s_{\hat{e}}/de}{\phi^s_{\hat{e}}}+\frac{f_e(q|\hat{e},s)}{f(q|\hat{e},s)}\right], \\ \lambda+\mu\left[\frac{d\phi^s_{\hat{e}}/de}{\phi^s_{\hat{e}}}+\frac{f_e(q|\hat{e},s)}{f(q|\hat{e},s)}\right] & \text{if } \frac{1}{u'(\bar{W})}\leq\lambda+\mu\left[\frac{d\phi^s_{\hat{e}}/de}{\phi^s_{\hat{e}}}+\frac{f_e(q|\hat{e},s)}{f(q|\hat{e},s)}\right]\leq\frac{1}{u'(\bar{W}+\overline{w}(q))}, \\ \frac{1}{u'(\bar{W})} & \text{if } \lambda+\mu\left[\frac{d\phi^s_{\hat{e}}/de}{\phi^s_{\hat{e}}}+\frac{f_e(q|\hat{e},s)}{f(q|\hat{e},s)}\right]<\frac{1}{u'(\bar{W})},\end{cases} \tag{43}$$

$$w_s(q)=\begin{cases}\overline{w}(q) & \text{if } \frac{1}{u'(\bar{W}+\overline{w}(q))}<\lambda+\mu\left[\frac{d\phi^s_{\hat{e}}/de}{\phi^s_{\hat{e}}}+\frac{f_e(q|\hat{e},s)}{f(q|\hat{e},s)}\right], \\ u'^{-1}\left(1/\left(\lambda+\mu\left[\frac{d\phi^s_{\hat{e}}/de}{\phi^s_{\hat{e}}}+\frac{f_e(q|\hat{e},s)}{f(q|\hat{e},s)}\right]\right)\right)-\bar{W} & \text{if } \frac{1}{u'(\bar{W})}\leq\lambda+\mu\left[\frac{d\phi^s_{\hat{e}}/de}{\phi^s_{\hat{e}}}+\frac{f_e(q|\hat{e},s)}{f(q|\hat{e},s)}\right]\leq\frac{1}{u'(\bar{W}+\overline{w}(q))}, \\ 0 & \text{if } \lambda+\mu\left[\frac{d\phi^s_{\hat{e}}/de}{\phi^s_{\hat{e}}}+\frac{f_e(q|\hat{e},s)}{f(q|\hat{e},s)}\right]<\frac{1}{u'(\bar{W})},\end{cases} \tag{44}$$

with $\lambda \geq 0$ and $\mu > 0$. Equation (44) can be rewritten as (25).

Proof of Proposition 3. With limited liability on the manager, and for each s, the optimal contract described in (24) depends on $LR_s(q) = \frac{d\phi^s_{\hat{e}}/de}{\phi^s_{\hat{e}}} + \frac{f_e(q|\hat{e},s)}{f(q|\hat{e},s)}$ for $q \geq q^{***}_s$, while it is independent of s for $q < q^{***}_s$. If $LR_{s_i}(q^{***}_{s_i}) = LR_{s_j}(q^{***}_{s_j})\ \forall s_i, s_j, q > \min_s\{q^{***}_s\}$, MLRP and the definition of q^{***}_s imply that $q^{***}_{s_i} = q^{***}_{s_j}\ \forall s_i, s_j$, i.e., there exists q^{***} such that $q^{***}_s = q^{***}\ \forall s$. Therefore, if $LR_s(q)$ does not depend on s for any $q \geq \min_s\{q^{***}_s\}$, then the payment is independent of s, otherwise it depends on s for some output realizations.

With a maximum payment in addition to limited liability, and for each s, the optimal contract described in (25) depends on $LR_s(q)$ for $q \in \mathcal{M}_s$, while it is independent of s for $q \notin \mathcal{M}_s$. Therefore, if $LR_s(q)$ does not depend on s for any $q \in \mathcal{M}_s$, then the payment is independent of s, otherwise it depends on s for some output realizations.

Proof of Proposition 4. (i) A linear likelihood ratio can be written as $\frac{f_e(q|\hat{e},s)}{f(q|\hat{e},s)} := a_s + b_s q$ with $b_s > 0\ \forall s$ due to MLRP. With log utility, $u'^{-1}(w) = \frac{1}{w}$. Thus, equation (24) can be written as

$$w_s(q) = \max\left\{\lambda + \mu\left[\frac{d\phi^s_{\hat{e}}/de}{\phi^s_{\hat{e}}} + \frac{f_e(q|\hat{e},s)}{f(q|\hat{e},s)}\right] - \bar{W}, 0\right\}. \tag{45}$$

Letting $\bar{a}_s := \lambda + \mu\left[\frac{d\phi^s_{\hat{e}}/de}{\phi^s_{\hat{e}}} + a_s\right] - \bar{W}$ and $n^*_s := \mu b_s$, we have $\lambda + \mu\left[\frac{d\phi^s_{\hat{e}}/de}{\phi^s_{\hat{e}}} + \frac{f_e(q|\hat{e},s)}{f(q|\hat{e},s)}\right] - \bar{W} = \bar{a}_s + n^*_s q$. Equation (45) can then be rewritten as

$$w_s(q) = \max\{\bar{a}_s + n^*_s q, 0\}. \tag{46}$$

Letting $q_s^{***} := -\frac{\bar{a}_s}{n_s^*} = -\frac{\frac{\lambda-\bar{W}}{\mu}+\frac{d\phi_{\hat{e}}^s/de}{\phi_{\hat{e}}^s}+a_s}{b_s}$, equation (46) can be rewritten as

$$w_s(q) = \max\{n_s^*(q - q_s^{***}), 0\} = n_s^* \max\{q - q_s^{***}, 0\}. \tag{47}$$

(ii) The number of options received by the manager for a given realization of s is $n_s^* = \mu b_s$. In addition, $\frac{d}{dq}\frac{f_e(q|\hat{e},s)}{f(q|\hat{e},s)} = b_s$. Therefore, n_s^* is independent of s if and only if $\frac{d}{dq}\frac{f_e(q|\hat{e},s)}{f(q|\hat{e},s)}$ is independent of s.

(iii) We can write the optimal contract as:

$$w_s(q) = \max\left\{\lambda + \mu\left(\frac{d\phi_{\hat{e}}^s/de}{\phi_{\hat{e}}^s} + a_s + b_s q\right) - \bar{W}, 0\right\} = \mu \max\left\{LR_s(q) + \frac{\lambda - \bar{W}}{\mu}, 0\right\}.$$

By construction, the strike price q_s^{***} is such that $w_s(q) > 0$ if and only if $q \geq q_s^{***}$. Therefore, q_s^{***} is independent of s if and only if $LR_s(q) = (\bar{W} - \lambda)/\mu$ at the same value of q for all s.

B Limited Liability on Manager Only

This Appendix considers the core model of Section 1 but without any upper bound on payments or incentives, i.e. the only constraint is limited liability on the agent. With a continuum of outputs and without limited liability on the principal, existence of an optimal contract is typically an issue.[13] We thus here assume a discrete output distribution $q \in \{q_1, ..., q_Q\}$. Let $\pi_{q,s}$ and $p_{q,s}$ denote the joint probabilities of (q, s) conditional on high and low efforts, respectively (whereas $\pi_s(q)$ and $p_s(q)$ refer to marginal distributions in the core model). To simplify the exposition, we assume full support ($\pi_{q,s} > 0$ and $p_{q,s} > 0$), although this is not needed for our results.

[13] Under discrete outputs, the optimal contract involves the principal paying only in the state with the highest likelihood ratio. With continuous outputs, this is a set of measure zero, so the contract must involve her paying in a neighborhood around that state. Without limited liability, the principal can generically improve on the contract by concentrating the payment in a smaller neighborhood, in which case an optimal contract fails to exist.

The firm solves the following program:

$$\min_{w_{q,s}} \sum_{q,s} \pi_{q,s} w_{q,s} \tag{48}$$

$$s.t. \ \sum_{q,s} \pi_{q,s} w_{q,s} - C \geq 0 \tag{49}$$

$$\sum_{q,s} \left(\pi_{q,s} - p_{q,s}\right) w_{q,s} \geq C \tag{50}$$

$$w_{q,s} \geq 0 \ \ \forall q, s. \tag{51}$$

As in Section 1, the IC and the manager's LL guarantee that the IR holds.

A signal is valuable if including it in the contract (in addition to output) reduces the firm's cost of implementing $e = 1$. Lemma 4 below states that a signal is valuable if and only if it is informative about effort (i.e. affects the likelihood ratio) in states where the payment is strictly positive. (All proofs are in Appendix A.)

Lemma 4 *Let* $\{w_{q,s}\}$ *be an optimal contract for implementing* $e = 1$ *with* $w_{q,s_i} > 0$ *and* $w_{q,s_j} > 0$ *for some* q*,* s_i*, and* s_j*. Then,* $w_{q,s_i} = w_{q,s_j}$ *only if* $\frac{\pi_{q,s_i}}{p_{q,s_i}} = \frac{\pi_{q,s_j}}{p_{q,s_j}}$.

Proof of Lemma 4. Fix a vector of payments that satisfy the IC, and consider the following perturbation:

$$w'_{q,s_i} = w_{q,s_i} + \frac{\epsilon}{\pi_{q,s_i} - p_{q,s_i}}, \quad \text{and} \quad w'_{q,s_j} = w_{q,s_j} - \frac{\epsilon}{\pi_{q,s_j} - p_{q,s_j}}.$$

This perturbation keeps the incremental benefit from effort constant and therefore preserves the IC. The LL continues to hold for $\epsilon > 0$ if $w_{q,s_j} > 0$, and for $\epsilon < 0$ if $w_{q,s_i} > 0$. The expected payment (48) increases by:

$$\left(\frac{\pi_{q,s_i}}{\pi_{q,s_i} - p_{q,s_i}} - \frac{\pi_{q,s_j}}{\pi_{q,s_j} - p_{q,s_j}} \right) \epsilon. \tag{52}$$

If the original contract entails $w_{q,s_i} = w_{q,s_j} > 0$ (i.e., a strictly positive payment for output q that does not depend on whether the signal is s_i or s_j), then such a perturbation would satisfy both the IC and LL. Thus, for this contract to be optimal, such a perturbation cannot reduce the expected payment. The term in (52) must be non-positive for all ϵ small enough:

$$\frac{\pi_{q,s_i}}{\pi_{q,s_i} - p_{q,s_i}} = \frac{\pi_{q,s_j}}{\pi_{q,s_j} - p_{q,s_j}},$$

which yields $\frac{\pi_{q,s_i}}{p_{q,s_i}} = \frac{\pi_{q,s_j}}{p_{q,s_j}}$. ■

Lemma 5 states that the payment is strictly positive only in states that maximize the likelihood ratio.

Lemma 5 *Let $\{w_{q,s}\}$ be an optimal contract for implementing $e = 1$. If $\frac{\pi_{q,s_i}}{p_{q,s_i}} < \max_{(q',s')}\left\{\frac{\pi_{q',s'}}{p_{q',s'}}\right\}$, then $w_{q,s_i} = 0$.*

Proof of Lemma 5. Let $(\check{q}, s_j) \in \arg\max_{(q'',s'')}\left\{\frac{\pi_{q'',s''}}{p_{q'',s''}}\right\}$ denote a state with the highest likelihood ratio and consider a state (q, s_i) that does not have the highest likelihood ratio:

$$\frac{\pi_{q,s_i}}{p_{q,s_i}} < \frac{\pi_{\check{q},s_j}}{p_{\check{q},s_j}}. \tag{53}$$

Consider the following perturbation, which, as in the proof of Lemma 4, keeps the incremental benefit from effort constant, thereby preserving the IC:

$$w'_{q,s_i} = w_{q,s_i} - \frac{\epsilon}{\pi_{q,s_i} - p_{q,s_i}}, \quad \text{and} \quad w'_{\check{q},s_j} = w_{\check{q},s_j} + \frac{\epsilon}{\pi_{\check{q},s_j} - p_{\check{q},s_j}}.$$

LL continues to hold for $\epsilon > 0$ if $w_{q,s_i} > 0$ and for $\epsilon < 0$ if $w_{\check{q},s_j} > 0$. The expected payment (48) increases by:

$$\left(\frac{\pi_{\check{q},s_j}}{\pi_{\check{q},s_j} - p_{\check{q},s_j}} - \frac{\pi_{q,s_i}}{\pi_{q,s_i} - p_{q,s_i}}\right)\epsilon. \tag{54}$$

From (53), the term inside the parentheses in (54) is strictly negative. Thus, the firm can reduce the expected payment by selecting $\epsilon > 0$ small enough, which does not violate the LL when $w_{q,s_i} > 0$. As a result, the solution entails zero payments in all states that do not maximize the likelihood ratio. ■

Combining these results yields Proposition 5, which states that a signal is valuable if and only if it is informative about effort in states with the highest likelihood ratio:

Proposition 5 *A signal has positive value if and only if, $\forall\, (\check{q}, s_j) \in \arg\max_{(q',s')}\{\frac{\pi_{q',s'}}{p_{q',s'}}\}$, there exists s_k such that $\frac{\pi_{\check{q},s_j}}{p_{\check{q},s_j}} \neq \frac{\pi_{\check{q},s_k}}{p_{\check{q},s_k}}$.*

A signal has positive value if and only if it affects the likelihood ratio at the output level with the maximum likelihood ratio. The firm then increases the payment at the signal where (q, s) has the highest likelihood ratio and decreases it to zero at other signal realizations. In contrast, a signal is not useful if it changes the likelihood ratio only for output levels at which the likelihood ratio is not maximized. Since the payment is zero to begin with, the firm cannot decrease it upon a low signal.

Example 1 below illustrates the result from Proposition 5:

Example 1 *Consider $q \in \{0, 1\}$, $s \in \{L, H\}$, and the following conditional probabilities:*

	$e = 1$		$e = 0$		*Likelihood Ratio*	
	$q = 0$	$q = 1$	$q = 0$	$q = 1$	$q = 0$	$q = 1$
$s = H$	$\frac{1}{8}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{3}$	2
$s = L$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{2}$	2

By Lemma 2, the optimal contract pays only in states $(1, H)$ and $(1, L)$, where the likelihood ratio is maximized. Since the likelihood ratios are equal at these two states, any payments that satisfy the IC with equality generate the same payoff to the firm:

$$\frac{w_{1,H}}{4} + \frac{w_{1,L}}{8} = C.$$

One solution is to pay a payment that does not depend on the signal:

$$w_{1,H} = w_{1,L} = \frac{8}{3}C.$$

Note, however, that q is not a sufficient statistic for e given (q, s) because the likelihood ratios at states $(0, L)$ and $(0, H)$ are different.[14]

C Effect of Signal on Debt Repayment

Section 2.1 studies *how* the signal realization affects the optimal contract in the risk-averse model when the conditions in Proposition 4, part (i) are satisfied so that the optimal contract comprises options. Here we answer this question in the risk-neutral model. Since Section 2.1 discussed options, here we use the financing interpretation of the model. We thus ask: if the condition in Proposition 2 is violated, so that performance-sensitive debt is optimal, how should

[14]It is straightforward to generalize this example to more than two outputs. To see this, let $q \in \{1, ..., Q\}$, $\pi_{N,H} = \alpha$, $\pi_{N,L} = \beta$, $p_{N,H} = \frac{\alpha}{2}$, $p_{N,L} = \frac{\beta}{2}$, and $\frac{\pi_{q,s}}{p_{q,s}} < 2$ for all $q \neq N$ and all s. Note that q is not a sufficient statistic for e given (q, s) as long as the likelihood ratio is not constant: $\frac{\pi_{q,H}}{p_{q,H}} \neq \frac{\pi_{q,L}}{p_{q,L}}$ for some q. As before, the optimal contract pays zero in all states except the ones with the highest likelihood ratios: (N, H) and (N, L). Moreover, any wage in these states that satisfies the IC with equality is optimal. In particular, paying $w_{N,H} = w_{N,L} = \frac{2C}{\alpha+\beta}$, $w_{q,H} = w_{q,L} = 0$ for $q \neq N$ is optimal.

the debt repayment depend on the signal realization? Two forces are relevant, which correspond to the two terms in the likelihood ratio (15) discussed previously. The first is what the signal realization itself implies about effort. If signal realization s_i individually indicates high (low) effort, in the sense of a high (low) $\frac{\phi_1^{s_i}}{\phi_0^{s_i}}$, then the debt repayment under this signal realization should be lower (higher) than under other signal realizations. The second force is how the signal realization affects the informativeness of the event that output exceeds a threshold. If, at any threshold, the event that output exceeds this threshold is a better (worse) indicator of high effort under s_i than under other signal realizations, in the sense that $\frac{\int_q^{\bar{q}} \pi_{s_i}(z)dz}{\int_q^{\bar{q}} p_{s_i}(z)dz}$ is higher (lower) for any q under s_i than under other signal realizations, then the debt repayment under s_i should be lower (higher) than under other signal realizations. Thus, the rewards for high effort are concentrated in the most informative states. For example, achieving a sufficiently high output may be more indicative of high effort upon an industry downturn ($s = s_i$), and in this case debt repayments should be lower in a downturn. Due to this second force, it may not be the case that a signal that individually indicates low effort leads to a higher debt repayment, as seems intuitive.

Even if the second condition does not hold one way or another, we may still be able to sign the effect of the second force. Assume that the informativeness of output exceeding a threshold under different signal realizations can be compared according to the standard single-crossing condition of the likelihood ratios $\frac{\int_q^{\bar{q}} \pi_s(z)dz}{\int_q^{\bar{q}} p_s(z)dz}$, i.e. $\frac{\int_q^{\bar{q}} \pi_{s_i}(z)dz}{\int_q^{\bar{q}} p_{s_i}(z)dz} < \frac{\int_q^{\bar{q}} \pi_{s_j}(z)dz}{\int_q^{\bar{q}} p_{s_j}(z)dz}$ for $q < \hat{q}$ and $\frac{\int_q^{\bar{q}} \pi_{s_i}(z)dz}{\int_q^{\bar{q}} p_{s_i}(z)dz} > \frac{\int_q^{\bar{q}} \pi_{s_j}(z)dz}{\int_q^{\bar{q}} p_{s_j}(z)dz}$ for $q > \hat{q}$ (where output is more informative about effort under signal realization s_i than s_j), so that the second condition does not hold. In this case, a signal realization under which output exceeding a threshold is more informative will be associated with a lower (higher) debt repayment when debt repayments across signal realizations are high (low), i.e. repayments are generally high (low).[15]

The intuition is the following. When debt repayments are generally high, the agent only gets paid following high output. Upon a signal realization where output is very informative about effort, even a moderately high output is a strong indicator of high effort, so that the agent will be paid even for moderately high output – the debt repayment falls. Upon a signal realization where output is less informative about effort, even a high output is not a strong indicator of high effort, so the agent will only be paid for very high outputs for such signal realizations.

[15]This result seems to contradict the intuition that a less volatile output should be associated with lower debt repayments. However, different signal realizations determine the allocation of incentives across different signal realizations, as opposed to the overall strength of incentives or the overall level of debt repayment.

When debt repayments are generally low, the agent gets paid for any output realization except for very low outputs. Upon a signal realization where output is very informative about effort, a very low output is a strong indicator of low effort, and so the debt repayment rises. Upon a signal realization where output is less informative about effort, a very low output is not as informative, and the agent could be paid even for very low outputs for such signal realizations. In turn, debt repayments will generally be low (high) across signal realizations when the moral hazard problem is severe (mild), i.e. the cost of effort C is high (low).[16] Thus, our results demonstrate how incentives should be allocated across different signal realizations.[17]

[16]In equation (17), the sum of integrals is decreasing in κ, taking into account the relation between κ and the thresholds q_s^{**} in equation (16), since $\overline{LR}_s(q)$ is increasing in q. An increase in C decreases the equilibrium level of κ, which results in lower thresholds according to (16).

[17]Chaigneau, Edmans, and Gottlieb (2017) study how the strength of incentives varies with the precision of output.