Bayesian Belief Update and Mispricing: Theory and Experiment

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Abstract

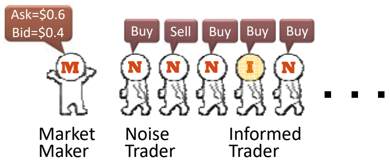
- We assume an **asymmetric information** environment.
- There are two uncertainties: **uncertainty of the asset value** and **the existence of informed traders**.
- When a market maker does not know of the existence of informed traders and informed traders do not exist, the asset price systematically deviates from the fair value, causing **asset mispricing**.
- This situation is close to the **information mirage** that Camerer and Weigelt (1991) found in their asset market experiments.
- After the market maker sufficiently updates his belief, he adequately finds the non-existence of informed traders. Then asset mispricing shrinks, and the market becomes **efficient**.

Market

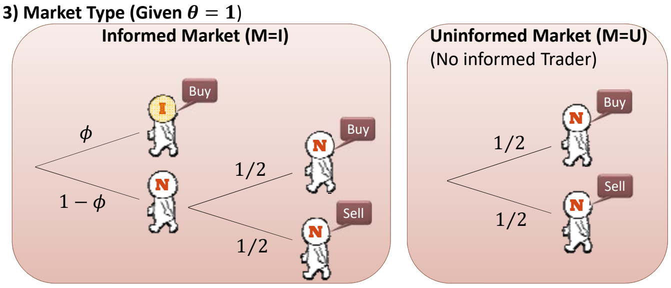
1) Risky Asset

Asset value : $\theta = \begin{cases} 1 & (prob. = 1/2) \\ 0 & (prob. = 1/2) \end{cases}$

2) Sequential Trading



3) Market Type (Given $\theta = 1$)



Belief Update

1) Market Maker's belief

$\mu_n = \text{Prob}(\theta = 1 \mid M = I, \mathcal{H}_n)$ $\quad \mathcal{H}_n = \omega_1\omega_2\cdots\omega_n \ : \ \frac{Buy}{Sell}\, history$

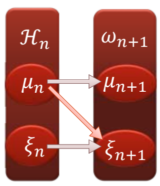
$\mathcal{E}xample \ \ \mathcal{H}_n$ =BSBBB

$\xi_n = \text{Prob}(M = I \mid \mathcal{H}_n)$ $\quad$ *Suspicion of informed traders*

2) Bayesian Update

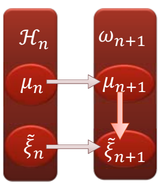
$$\mu_{n+1} = \begin{cases} \dfrac{(1+\phi)\mu_n}{1-\phi+2\phi\mu_n} & (\omega_{n+1} = B) \\ \dfrac{(1-\phi)\mu_n}{1+\phi-2\phi\mu_n} & (\omega_{n+1} = S) \end{cases}$$

$$\xi_{n+1} = \begin{cases} \dfrac{0.5+\phi(\mu_n - 0.5)}{0.5+\phi(\mu_n - 0.5)\xi_n}\cdot\xi_n & (\omega_{n+1} = B) \\ \dfrac{0.5-\phi(\mu_n - 0.5)}{0.5-\phi(\mu_n - 0.5)\xi_n}\cdot\xi_n & (\omega_{n+1} = S) \end{cases}$$



3) Quasi-Bayesian Update

$$\tilde{\xi}_{n+1} = \begin{cases} \dfrac{0.5+\phi(\mu_{n+1} - 0.5)}{0.5+\phi(\mu_{n+1} - 0.5)\tilde{\xi}_n}\cdot\tilde{\xi}_n & (\omega_{n+1} = B) \\ \dfrac{0.5-\phi(\mu_{n+1} - 0.5)}{0.5-\phi(\mu_{n+1} - 0.5)\tilde{\xi}_n}\cdot\tilde{\xi}_n & (\omega_{n+1} = S) \end{cases}$$



Theorem

1) $\xi_n \xrightarrow{p} True\ Value$ — ***Bayesian markets become efficient.***

2) $\tilde{\xi}_n \xrightarrow{p} True\ Value$ — ***Quasi-Bayesian markets become efficient.***

3) $\xi_n < \tilde{\xi}_n \quad for\ all\ n \quad (and\ all\ \mathcal{H}_n)$

4) $E[\tilde{\xi}_n]$ *has Local Maximum* — ***Temporal Miss price (Bubble) occurs.***

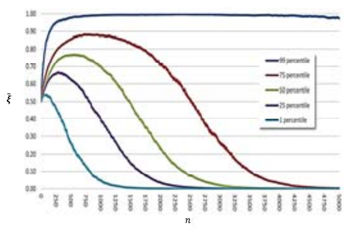


Figure 1. Transition of $\tilde{\xi}$.

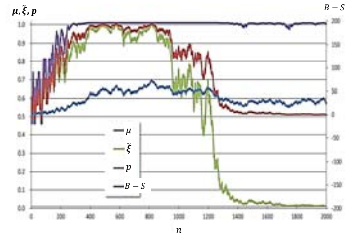
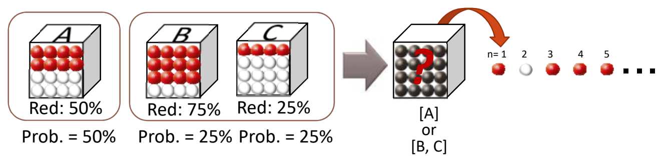


Figure 2. A typical pass of miss pricing.

Experiment



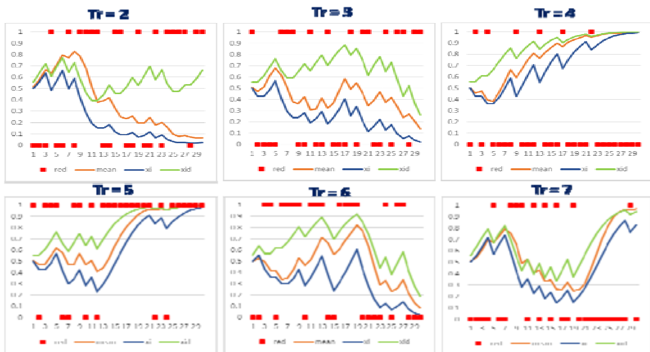


Figure 3. Transition of ξ and $\tilde{\xi}$.

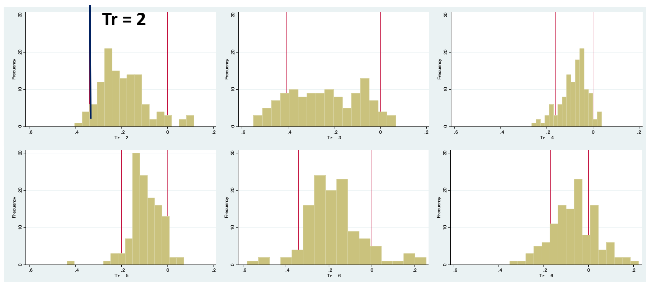


Figure 4. $D_i = \sum_{n=1}^{30}(p_{i,n} - \tilde{\xi}_n)$

Conclusion

Our results suggest that mispricing may occur when investors believe that private information exists in stock markets and trade on the basis of their own belief even if private information does not exist. For stock markets to be efficient, controlling the information flow is important for policy makers and regulators.

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