Public Goods, the Distribution of Income and International GDP Comparisons

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Abstract: I present a model wherein the aggregate provision of a class of public goods affects the ability to market successfully private goods. In this model, the provision of public goods may result from an optimization of purely economic factors, rather than, per ***Samuelson***, being set by an exogenous political process. I next evaluate boundary conditions for the distribution of income between the public and private sectors. Extensions of the model enable us to examine conditions under which it is possible to decide democratically to transfer income to those who provide neither public nor private goods. Another extension of the model enables us to evaluate the conditions under which other, non-market-enhancing types of public goods may be provided. Finally, I discuss some potential research strategies for evaluating whether or not a common economically determined optimum value for the share of public goods production of GDP might be found for all countries or classes of countries.

Introduction:

I examine some conditions under which **Samuelson's** (1954)[1] conclusions with respect to public goods do not hold and use a spreadsheet model that incorporates these conditions to evaluate several resulting income distributions. Samuelson demonstrated that under perfect competition, etc., it is not possible to calculate an optimal level of public goods production and, moreover, that public goods will tend to be under-produced as individuals have incentives to understate their true preferences for public goods. These findings have been the textbook standard ever since. Critical to his findings is the assumption, explicit in his formal mathematics, that an individual's ability to ***produce*** either public or private goods is independent of the overall level of public goods production. That may well be true. A brief scan of economic history (for example**, Heilbroner**, 1953, Chapter ii)[2], however, reveals that the ability ***to market*** successfully private goods, and thus take best advantage of the benefits of ***specialization*** (**Smith**, 1776)[3], is a function of both the quantity and quality of governance. I define governance as the specialized provision of traditional public goods such as national defense, justice, transportation infrastructure and sound currency—the public goods that reduce the costs and risks of bringing private goods to market. A spreadsheet model incorporates this interaction between the quantity and efficacy of the market-enhancing public goods provided and the subsequent ability to reap through trade the productivity benefits of specialization. Seeking illumination rather than verisimilitude, I use the model to calculate ranges of outcomes over which public goods might be provided, and, in many cases, calculate an optimum level of traditional public goods production. I then lay out some conditions under which it is beneficial to those who provide public goods to restrict the production of public goods (by limiting entry into the public goods production sector) and to enforce a non-egalitarian income distribution. I find that non-egalitarian outcomes can occur under either autocratic decree or majority rule. I next investigate conditions under which non-market-enhancing public goods might be created. Depending on their utility, such goods may not be produced at all, have a single or multiple equilibria, or be over-produced. Finally, I note that there is no compelling reason to believe that the current 200 or so existing nations, spread as they are across different stages of development, should have identical relationships between governance, income distribution and markets. In the absence of such a common characteristic, I am not surprised that, thus far, studies using international comparisons to determine what an optimal level of government as a share of GDP might be have exhibited generally less than robust results.

Market Enhancing and Other Public Goods

Traditional public goods have some commonalities with term life insurance. The average person who buys term life does so not because he wants to die, but because the next best alternative is to allocate a disproportionate share of current income as precautionary savings, leading to suboptimal lifetime utility. Just as the average person would prefer not to “win” the life insurance policy payoff, he would prefer those outcomes where, even though he has been taxed to pay for them, he minimizes his interactions with the police and the court system. Unlike term life insurance though, societies which pay for additional quantities of traditional public goods can reduce directly the chance that they will be actively invoked. A beat policeman who is watchful and a night court judge who wields an active gavel of justice can reduce the risks of street crime and make it safer to go to the market. Well maintained public roads can increase the odds that perishable goods sent to market will arrive on time and in good condition. The modeling approach described below implements this concept by tying an increase in the total amount of governance-related public goods to an increase in the share of produced private goods that an individual can deliver successfully to the marketplace. The immediate societal trade-off occurs because, with specialization, each individual who is producing traditional public goods is one who is not producing private goods. Other types of public goods, such as chalk drawings of cute kittens on the walls of the public market place, can exist and potentially provide utility to all, but these public goods do not affect the ability to market private goods. I will evaluate the demand for the latter type of public goods after I have explored models in which just traditional public goods and private goods exist. At that point I can then measure the opportunity cost of other types of public goods in relation to the combined effects of traditional public goods and private goods production foregone.

The Basic Model: Egalitarian distribution

This modeling approach owes an enormous debt of gratitude to the methods of **C. J. Bliss** (1975)[4]. Neither capital, interest rates nor money exist explicitly in my models, but my models, like many of those Bliss considered with respect to capital, optimize output given a limited set of simple production choices. The initial production possibility frontier I consider consists of the sum of choices made by each increment of 1/10 of a percentage point of the workforce. Each increment can produce one of four production possibilities involving three different goods. The goods are traditional public goods, avocados and beer. The production possibilities available for each increment are (1tpg, 0a, 0b); (0tpg, 1a, 1b); (0tpg, Xa, 0b); or (0tpg, 0a, Yb). Thus one increment can produce either (1) one unit of traditional public goods (2) 1 unit each of avocados and beer (3) X units of Avocados or (4) Y units of beer. If Xa or Yb is chosen, then the producing increment must bring a quantity of X or Y to the central market in order to trade for the other good. I assume that the quantity of a private good that arrives at the market and is available to be traded is a function of the total quantity of traditional public goods provided and lies between 0 and 100 percent of the quantity initially shipped to the market.[1] (See footnote 1 references for ***Clower and Leijonhuvfud***[5] and also ***Hewett***[6]). The efficiency of marketability is assumed to be a function of the total number of increments that produce traditional public goods. In the first illustration of this approach, each increment of the workforce knowingly has exactly the same preferences and capabilities as every other increment of the workforce. I assume initially that utility is purely a function of the quantity of private goods consumed and so traditional public goods only affect utility in so far as their provision increases the ability to consume private goods. I assume that all production that is delivered to market is divided among all increments so that each increment consumes exactly the same amount of A and B, whether they originally produced A, B, or traditional public goods. I assume that if the outcome of a production period does not result in an optimum level of consumption per capita, that increments will adjust their production choices (learning exists) and will attempt to choose

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[1] The concept of notional versus effective demand is widespread among Keynesians of various persuasions; less frequently discussed is the concept of notional versus effective supply, though see **Clower** and **Leijonhuvfud** (1975). Those who retain their initial skepticism as to the possibility of widespread deadweight loss occurring between the time of production and the delivery of same to the market stemming, in part, from a systemic failure of both appropriate governance and legal infrastructure should cast their memories back to the Soviet Union's economy in the 1970s and 1980s and consider the challenges it faced with respect to its various economic reform initiatives (**Hewett**, 1988, for example).

a better outcome the next production period, so that if an optimum level of consumption exists for a given set of technologies, it generally will be found.

Logically, increments will only send goods to market if by doing so they can improve on the level of utility available under autarchy, which is set by definition as one unit of avocados and one unit of beer. If this is the optimal level of production, then all increments will choose it and there will be no provision of traditional public goods. Private goods producers will thus only choose to specialize if the benefits from specialization and exchange exceed the combined losses due to (1) losses in transit due to insufficient public goods provision and (2) the taxes in kind at the market place that compensate the producers of public goods. If an increment chooses to specialize in avocados production, it will minimize its transport and exchange losses by withholding a certain amount of avocados from the market—equaling its expected own consumption of avocados—as only those avocados which arrive at the market are subject to being taxed (in kind, of course, since there is no money, only exchange). Since all increments have the same production possibilities, preferences, and knowledge, exchange will take place if the technical characteristics of the interaction between public goods production and private goods specialization permit consumption per increment to exceed base utility at the (0tpg,1a, 1b) threshold.

I now look at a simple example. To make the spreadsheet easy to manipulate, I assume, for illustrative purposes, that the governance function is in the form of the cumulative distribution of a normal distribution, with u = u0 and b =b0, where u0 and b0 have reasonable values. I first examine the example where u0=0.2 and b0=0.1. Thus if 20 percent of the population is engaged in providing traditional public goods, then the fraction of the workforce engaged in traditional public goods production is 0.2 and the fraction of the workforce engaged in private goods production is 0.8. The cumulative value of this normal distribution at 0.2 is 0.5, so for every 10 units of avocados or beer sent to the market, 10*0.5 or five will arrive. If 30 percent of the population provides traditional public goods, then the cumulative value of this normal distribution at 0.3 would be 0.84, so that 8.4 units of avocados or beer would arrive to market for every 10 units sent, though only 70 percent of the population will be engaged in producing private goods. As seen in the spreadsheet, if the specialized producers of avocados can produce 20 units and beer producers can produce 15 units, then the maximum utility, assuming a simple utility function of a*b, is achieved with 319 producers of traditional public goods—providing a marketing efficiency of 0.883—681 increments of private goods divided equally between brewers and avocado growers. A tax rate of 0.4837 on goods sent to market results in society-wide egalitarian consumption per increment of 6.26 avocados and 4.7 beers. If no public goods were required to produce private goods, and the market place was costless to operate[2], then per increment consumption of avocados and beer would have been 10 and 7.5, respectively. (See Figure 1 on associated spreadsheet)

There are three obvious types of technological progress available for this type of model. The mean of the normal distribution for the efficiency of public goods can be reduced, which will increase the proportion of labor available to produce private goods, the standard deviation of the distribution for the efficiency of public goods can be reduced or the coefficients of specialized production can be increased. Increasing the latter will not change the optimum share of labor involved in producing traditional public goods. Assuming a market place optimum exists, then reducing the standard deviation of the efficiency of public goods production should decrease the optimum number of public goods providers until the standard deviation reaches a number close to zero. In this instance, as b0 approaches zero, then the optimum share of increments providing public goods approaches a constant equal to the mean of the normal distribution, which leads to the situation speculated on by **Samuelson**, in which society only needs to calculate and impose the correct value of non-distorting taxes in order to pay for the minimally required amount of traditional public goods. This process operates similarly to how Einsteinian relativity converges to the Newtonian laws of motion at the normal scale of human observation. In a **Samuelson** world, where the standard deviation of the government efficiency function is very close to 0, then if the mean of the government efficiency function was 0.2, then there would be 200 providers of traditional public goods and 800 providers of private goods so that the per increment consumption of avocados and beer would be 8 and 6, respectively.

Non-Egalitarian Distribution: Authoritarian Decision Making

I next look at distributions of income that can occur if the tax rate is fixed before the optimization process, rather than tax rates being set as a result of the optimization process. As any reader familiar with economic history will have noticed, the egalitarian income distribution imposed upon the initial modeling effort is not one which has been common throughout human history. In point of fact, throughout many eras of human history, specialization in either producing private goods or the provision of public goods, though a chicken and egg conundrum, coincided with large percentages of the population tilling the fields or herding the herds and a much smaller group providing protection against bandits and invaders. Those who provided the latter service did so by learning to ride in war chariots, putting on armor and riding on sturdy steads, or leading professional gunpowder armies. As a result, their opinions as to the optimal level of taxes and the resulting distribution of income came to carry and sustain more weight than those held by the masses who specialized in scything wheat or milking cows--throughout history, peasant revolts armed with scythes or milk stools had poor success rates.

[2] This is situation is equivalent functionally to u0 being at least several standard deviations less than zero.

In both the next and the following example, I thus set tax rates exogenously. In these situations we find that it behooves those who set the tax rates to limit entry into the set of decision makers. Given a free choice between mucking out the stables or living in the best room in a castle most people will choose the latter, and if entry is not limited, that room will get crowded, quickly.

In the next example, I set the tax rate first and then assume that there are two classes of citizens in the society, the public goods providers, who also oversee the market place and collect the taxes on marketed goods, and the producers of private goods. I assume that all private goods available to a class are distributed equally within the class. Using the same technology and the same tax rate as the first example, I find that public goods producers will enjoy a maximum utility if they can limit the number of public goods producers to 202. If they can keep private goods producers from becoming public goods producers, then there will be 798 producers of private goods. The public goods producers will each enjoy 7.7 and 5.6 units of avocados and beer, respectively, per time period while the private goods producers will receive 4.2 and 3.1 units of avocados and beer. While this is a better deal for the public goods providers than would be possible under the egalitarian distribution of 6.3 avocados and 4.7 beers, the private goods providers are worse off. The overall per increment consumption of avocados and beer in the non-egalitarian distribution is 4.9 and 3.7, respectively, for a dead weight loss to society as a whole due to the insufficient provision of public goods of about 22 percent.

Non-Egalitarian Distribution: Democratic Decision Making

Now let us examine a non-egalitarian income distribution stemming from a democratic decision making process having three classes of increments. In modern societies, governments often spend non-trivial sums on income redistribution with a stated purpose of ameliorating the conditions of the aged or the infirm or those otherwise judged incapable of producing either public or private goods. I can approximate some of the stress points of this type of income redistribution within the modeling framework above by assuming that leisure is an option for some, if there can be assembled a majority coalition that includes the producers of public goods and sufficient increments from among the rest of the population. This new class of increments consists of those who will opt out of the production process if they are guaranteed an income in goods sufficiently in excess of the subsistence level of private goods production, the production possibility available under autarchy (0tpg, 1a, 1b). The total utility enjoyed by those offered the leisure option is the sum of the utility derived from leisure plus the utility derived from their allocation of consumption goods. This total utility must be greater than the utility enjoyed by the producers of private goods, else those offered leisure would choose to work.

A coalition must be at least 50 percent of the increments plus one increment in order to be voted in democratically by a majority of the population. The maximum size of the coalition is constrained by three conditions: First, the majority coalition must be able to restrict membership in the public goods producing and leisure-enjoying groups, otherwise private goods producers will attempt to switch groups. Second, each additional increment in the leisure class[3] is one less increment producing private goods, thus one less increment contributing to the tax base, and one additional increment that will have to be offered the basic bundle of consumption goods. Once the leisure class becomes so numerous that public goods producers would be better off under an egalitarian distribution where no increment could choose leisure, they should seek to move to that distribution. The final constraint is that if the tax rate is too high, the producers of private goods will choose autarchy.

We can check to see whether or not democratically selected non-egalitarian distributions are feasible for any paired set of tax rates and leisure class consumption bundles. Let us look at an example featuring the same technological constraints as the previous examples and assume that those offered leisure will choose it if they can also enjoy a minimum goods-based utility, the value of which can be set in the spreadsheet. In this example I set the value at 1.5, so that leisure consumers enjoy both the utility derived from leisure and the utility derived from a volume of private goods that is 50 percent higher than they would have had under the autarchic production choice. I next choose a tax rate on goods delivered to market of 75 percent and find that a democratically decided, non-egalitarian distribution is feasible with 222 public goods producing increments and between 279 and 313 leisure consuming increments (see Figure Three for an illustration of the distribution of utility at the minimum sustainable democratic coalition). Such non-egalitarian coalitions may not be politically sustainable if those who consume leisure and a free consumption goods bundle are actually able to work off the books, so to speak. If so, they can keep both their free consumption bundle, plus the entirety of either consumption good they choose to produce through specialization and will do so if their resulting utility exceeds the utility provided by leisure combined with a subsistence-level-plus allocation of private goods. I can envision few tensions that will put more stress on the bonds that bind societies together than a widespread awareness among the Peters of the world, that not only are they doing exactly the same work as the Pauls of the world, but that they are taxed on their efforts while the Peters not only evade taxes, but get free beer and avocados as well.

Egalitarian Distribution: Other Public Goods

I return to the egalitarian distribution in order to evaluate the impact of other public goods, such as drawings of happy kittens chalked onto the walls of the public market place, on overall

[3] Clearly used in a manner other than that used by Veblen in his classic.

utility. This is the equivalent of adding the production function (0tpg, 1opg, 0a, 0b) into the set of available production choices, and adjusting the other production choices accordingly. In this formulation, the utility derived from private goods is still the product of the number of units of avocados times the number of units of beer consumed per increment, and all increments enjoy the same consumption of private and public goods no matter what they produce. All increments have the same utility and production possibilities and perfect knowledge of all other increments' utility functions and production possibilities. Total utility is the sum of goods utility plus the utility each increment derives from other public goods. The direct utility of the latter depends only on the total quantity of other public goods produced and both the quantity of and the total utility of other public goods produced is independent of the quantity of traditional public goods produced. I assume that the incremental utility from each additional unit of other public goods is non-negative. The opportunity cost to produce a unit of other public goods is either one less increment of production of traditional public goods or one less increment of private goods production. I can determine which increment is sacrificed by re-optimizing the production mix between market enhancing public goods and private goods for every numerical value of increments of other public goods production. Tax rates vary at each level of total public goods production as taxes are also optimized endogenously. To illustrate this concept, still using u=0.2 and b=0.1 for the effectiveness of governance function, I see that optimum number of increments providing private goods production falls more rapidly than do the number of increments producing market enhancing public goods. See Figure 4.

I find that there may or may not be a non-zero optimal level(s) of production for other public goods. I can easily construct a utility function for other public goods that is insufficient to offset the decline in utility caused by the joint reduction of private goods production and traditional public goods. I can also imagine a utility function under which the incremental utility of an additional unit of other public goods is always non-negative, on its own, but varies in a sine-wave like fashion, creating the possibility of a single or multiple optimal production points. Finally, if the utility provided by other public goods exceeds the utility lost through the non-production of private goods, the situation could turn into the utility equivalent of a capital consuming growth model whereby the demand schedule for other public goods explodes until the level of private goods consumption falls below subsistence levels. (See Figure 5). Thus, even if all increments in society have exactly the same production possibilities, equivalent knowledge, and utility functions, there may or not be an achievable, sustainable equilibrium that includes a non-zero amount of production of other public goods. This finding stems only from analyzing the opportunity costs of producing other public goods, rather than from the impulse for consumers to be free riders of same.

Implications for International Comparisons

Having grown up professionally by trying to understand Soviet economic performance, I spent many hours examining ways to compare macroeconomic performance across countries. I believe the approach to evaluating public goods described above has some implications for these types of international economic comparisons. First, there clearly is an optimal level of producing traditional public goods in the context of the model that I proposed—it would be where u0 is sufficiently less than or equal to zero and b0 is close to zero, which would reflect a world where all individuals behave as angels, markets are costless, transportation is free, etc. In the real world, not only do all of these assumptions fail to hold but some countries are more burdened by the vicissitudes of history and geography than are others. Thus I cannot think of a compelling reason to believe that the current 200 or so existing nations, spread as they are across different stages of development and enjoying different national endowments, should have identical relationships between governance, income distribution, markets and private goods production possibilities. In the absence of such a common characteristic, I am not surprised that thus far even the most careful studies using international comparisons to determine what an optimal level of government as a share of GDP might be have exhibited generally less than robust results[7]. This observation is not intended to cast any aspersions at any study or the efforts of any researcher, as scholars in this field have made exceptional contributions to various disciplines within economics, in part by working out how to fit sometimes deliberately opaque national accounts data into a common framework, adjusting for price controls, inflation rates, levels of trade dependence, etc., just to assemble data sufficiently robust to attempt such international comparisons.

If we are to be able to stand just a little higher on the shoulders of those who came before us, what data will we need, and how should we go about using it? For a truly thorough analysis we would like to have data on after tax income per private sector worker, though for countries, such as the U.S., that hire a lot of government contractors, the public/private sector division is already problematic. For the potential explanatory variables, we are likely to need detailed, annual time series data, by country, that include aggregate (local, intermediate, and national) amounts of public spending broken down by end-use, on a per private sector worker basis: traditional governance, income transfers, capital spending, interest payments, and other spending, as well a national capital stock series. This effort will require a number of country specialists, as each country tends to have unique national income accounting conventions, in part because it is always tempting for politicians to reclassify what are actually income transfers as some other type of expenditures in the national books. We will also need the total number of government workers in order to make the arithmetically trivial calculation of the share of the total workforce engaged in governance. The category of other types of spending is likely to be an issue for some countries, especially those with state-owned or quasi-governmental entities that produce natural resources, such as crude oil or natural gas. With possibly less than a handful of exceptions, such entities are not famous for operating as efficiently as private sector counterparts in other countries, and in some resource producing countries, these state energy producers are responsible for a substantial share of national income. Such countries are probably best left out of any attempt to determine the optimum share of government.

For just an initial exploration of this approach, we might consider looking at the residuals of a standard time series cross-sectional regression where GDP by various countries is modeled as a function of labor and capital, and then see if the residuals are correlated with changes in the share of taxes to GDP (or alternatively, if data exist, taxes minus income transfers as a share of GDP). For example, assume country X has a tax share of GDP ratio of 0.24 in year one and 0.25 in year two. If we test for an assumed cumulative distribution function where u0=.1 and b0=.1, then we would expect to see total production change as follows:

Share of Taxes	Cumulative Distribution assuming u0=0.1 and b0=0.1	Non Tax Share	Total Production	Natural Log of Production	Difference in Natural Log of Totals
0.24	0.919243341	0.76	0.698624939	-0.358641249	
0.25	0.933192799	0.75	0.699894599	-0.356825528	0.001815721

We can assemble the values for the differences of natural log totals just described, for a given set of assumed u0 and b0 and regress these values against the residuals of the GDP equation. We can then construct another set of log differences around different values of u0 and b0. Through iteration, we can test to determine if there exist sets of plausible u0's and b0's that are statistically significantly correlated with the residuals of the GDP equation. While there is no a priori reason that I can think of that there should exist such a universal, optimum share of public goods to GDP across all countries, or even categories of countries, this process should determine whether further searches for such might prove fruitful.

Concluding Observations

Finally, let us consider **Bastiat**[8] (1850) and the how his arguments with respect to broken windows might apply to the provision of public goods, or to phrase the question another way, "Can there be a free lunch?" Traditional public goods, by their nature, are largely unseen. If the above mental peregrinations have any utility, it is to suggest that if there is an insufficient provision of traditional public goods, and society taxes shopkeepers enough extra to pay for an additional unit of public goods, in this case an extra constable, this may dissuade hooligans from throwing bricks through windows in the first place. Alternatively, if there is a surfeit of traditional public goods production so that there is already an excessive number of underutilized bureaucrats, every dollar spent on additional, traditional public goods production is money wasted through foregone production of private goods. Money spent on other types of public goods remains subject to the utility maximization conditions discussed above, given my set of assumptions, or as described by Samuelson, using his, as the reader prefers.

Having spent well over three decades working in either the military or the federal civilian government sector, the vast majority of my career spent as a professional economist, I have sometimes pondered how to measure, on the scale for the heroism of assumptions, how heroic it was for the national income accounting conventions to assume that I was compensated at a level that approximated my marginal product. After all, I learned that **Samuelson** had shown that people have every incentive to hide their true demand for public goods, and that the best that we could hope for was to have a non-distorting tax structure to pay for the volume of public goods selected through the political process. Under the different set of assumptions described above, however, it can be possible to calculate an optimal level of traditional public goods endogenously within a purely economic model, without reference to an exogenous political parameter.

Admittedly there is a vast difference between showing how under some highly restrictive assumptions that an optimal level of traditional public goods might be found and making use of such a finding in the real world. That said, it seems plausible to me that if anything like the conditions described above hold that we are more likely to approach a societally optimal and sustainable level of traditional public goods production and income transfers if we as a profession can improve our understanding of the potential tradeoffs between public and private goods production across countries and across decades.

Appendix: Calculating optimal marketing volumes under taxes and dead weight losses

If there are just taxes, and no dead weight loss, and we assume for simplicity that one unit of avocados trades for one unit of beer, then the distribution of avocados by end use is as follows:

Total Avocados = Avocados kept for home Consumption + Avocados sent to Market

Avocados sent to Market = Avocados claimed for taxes plus Avocados Exchanged for Beer

To keep this illustration simple, we assume beer and avocados trade one for one. The ratio of Avocados taxed to Avocados exchanged is (Avocados Sent *tax rate)/Avocados Sent *(1-Tax Rate), so

$$\frac{\text{A taxed}}{\text{A Exchanged}} = \frac{\text{Tax Rate}}{\text{(1-Tax Rate)}}$$

And A Taxed = A Exchanged * (t/(1-t))

Thus Total Quantity of Avocados produced is equal to A Kept + A exchanged + A taxed

Substituting in various identities: Total Avocados = 2 A Kept + t/(1-t) * A kept

So A Kept = Total Avocados / (2+t/(1-t)) So if Total Avocados =400, and the tax rate is 1/3, then A kept = 400 / (2+1/2) = 160 , Avocados Sent = 400-160 = 240, of which 1/3rd or 80 is taxed and 160 are exchanged for beer.

If there are dead weight losses, then we still have Total Avocados = Avocados kept for home Consumption + Avocados sent to Market, where Avocados sent to market incur transport losses of (1-g) times Avocados sent, where g is the cumulative value of the government efficiency function. So Avocados sent to Market = A sent*(1-g) + A sent * g. A sent * g is split between avocados taxed and avocados marketed, which are equal to A taxed = A sent *g * tax rate and A sent * g * (1-tax rate). Again

$$\frac{\text{A taxed}}{\text{A Exchanged}} = \frac{\text{Tax Rate}}{\text{(1-Tax Rate)}}$$

So A again A Taxed = A Exchanged * (t/(1-t)). Again, assuming for simplicity of the example that A and B trade one for one, then the total amount of avocados arriving at the market is equal to A kept + A kept (t/(1-t)). The ratio of A sent to A arrived is 1/g. So A sent = (A kept+ A kept*(t/(1-t)))/g. Thus total Avocados are distributed as follows. Total Avocados = A kept + (A kept+ A kept*(t/(1-t)))/g . If Total Avocados =400, t=1/3 and g=0.9, then Total Avocados = 1 A kept + A kept *(1 + ½)/0.9= 1 A kept + (1.66666)* A kept = 2.66666 *A kept, so A kept =

400/2.66666 =150, A sent= 400 -150= 250, dead weight loss =(1-.9)*250=25, Avocados arrived at market =250-25=225, of which 1/3 of 225 goes to taxes, which equal 75 Avocados, leaving 150 avocados to be traded for beer. So the total distribution of avocados is 150 kept for household consumption, 250 sent to market, of which 25 are lost in transit and 225 arrive, of which 75 are taxed to support public goods producers and 150 are exchanged at the market one-for-one for units of beer.

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