Modelling Competitive Mortgage Termination Option Strategies: Default vs Restructuring and Prepayment vs Defeasance *

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Abstract

We build a two-stage competing risk model for pricing four types of early termination options written on commercial mortgages: default vs restructuring and prepayment vs defeasance as two pairs of competitions. It is the first study to consider restructuring as a "competitor" with default. The key feature of our model is to introduce collateral underlying property market supply constraints into a property price process which would determine values of early termination options. Our simulations find out greater probability to restructure mortgages by reducing interest and extending maturity and to prepay in cash. We also prove that tightening property supply constraints pushes up values of default, restructuring and prepayment by pricing their analogous options: default (a series of compound European Call on Put options), mortgage restructuring (an exchange option between mortgages with different cash flow structures), prepayment in cash (a series of compound European Call on Call options), and defeasance (an exchange option of more liquid assets with less liquid ones) in different scenarios. Therefore, we suggest controlling property supply constraints as an alternative risk management measure for mortgage markets.

JEL: G11, G13, G21, R33

Keywords: Mortgage Default, Restructuring, Prepayment, Defeasance, Property Supply Constraints

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1 Introduction

Owing to financial institutions in the US face tightening requirements of risk-based capital for multifamily mortgages and heightening risk-based capital charges on high-volatility commercial real estate loans [1] starting from 2015 due to implementation of the Basel Accord III, risk exposure in commercial mortgages has been receiving more and more attentions among financial institutions. Unlike residential mortgages, more complicated designs in commercial mortgages bring more types of early termination options - default, restructuring, prepayment and defeasance - that quantify mortgage risks. Enormous impacts on credit stability of financial institutions varying by types of early termination option in terms of loss exposure could be foreseen because commercial mortgage outstanding accounts for more than 15% of bank credits (equivalent to more than 45% of overall mortgages). This is also one of reasons for a radical reform in regulations on commercial mortgages.

Theoretical models of mortgage default are mainly built up from changes in utility and consumption along life-cycle of borrowers in residential mortgages (Campbell and Cocco 2015 [8]; Geradi, Rosen and Willen 2010 [20]; Piskorski and Tchistyi 2010 [36] and 2011 [37]). Due to different nature of demand in leverage for investing commercial real estate, theoretical framework of early terminations in commercial mortgages - whatever they are terminated by default or full prepayment - is rooted from the Black & Scholes option pricing models[6]. In the extant literature, decisions to default and prepay are regarded as a compound of European put options and a call option respectively (Vandell 1995[42], Ambrose, Buttimer and Capone 1997[2]) [2]. Interactions between default and prepayment risk are investigated as a pair of competing risk (Deng, Quigley and Van Order 2000 [16]; Clapp, Deng and An 2006 [13]; Pennington-Cross 2010 [34]). However, boundary conditions of default partially overlap with the condition that prepayment options are not exercised, thus we would argue that default and prepayment are not direct "competitors". Moreover, we suggest two pairs of perfect substitutes as competing risk - (1) default and restructuring, and (2) full prepayment and defeasance[3] in which payments are settled by submitting Treasury securities; and hence develop a two-stage option pricing theoretical model for obtaining boundary conditions in each pair of substitutes in the first stage and finding numerical solutions to option values in the second stage.

[1] They are defined as loans for the purposes of acquisition, construction and development with 80% higher loan-to-value ratio and contributed capital from a borrower would account for less than 15% of the project's value according to the Mortgage Bankers Association.

[2] Kau, Keenan, Muller and Epperson suggested treating a mortgage contract as a compound option, however did not explain the principle in depth (1987[25], 1992[26] and 1995[24])

[3] Defeasance is an exclusive covenant in a commercial mortgage contract that residential mortgage borrowers are not provided.

Alternative mortgage products which have been launched to markets with growing existence offer flexibility to borrowers by altering payment schedules. The complex features are similar to restructuring, but those covenants are written on original contracts during origination. The paradox - alternative home mortgage products are concluded as useful tools for enhancing benefits of borrowers through smoothing consumption but entail considerable default risk which has caused the mortgage crisis (Cocco 2010 [14]; LaCour-Little and Yang 2010 [28]) - perplexes impacts of deferred amortization and leverage effects. Although we would not consider alternative mortgage products in this paper, understanding decisions on mortgage restructuring coming from same rationales of borrowers who wish to revise mortgage schedules in certain circumstance could also offer insights into designs of alternative mortgage products. Moreover, mortgage modification or restructuring could create value for both borrowers and lenders by enhancing efficiency (Agarwal et al 2011 [1]; Piskorski and Tchistyi 2011), thus our model in the second stage aims to find out values of restructuring relative to mortgage default.

As interest rates will be adjusted after such a long quantitative easing period, it would be interesting to discover how borrowers would choose between full prepayment in cash and defeasance. Because defeasance is the unique covenant in commercial mortgage contracts, it is never modelled in residential mortgages. So far, defeasance options are conceptualized by Dierker, Quan and Torous (2005 [17]) as exchange options between mortgages and riskless debt; and their values are simulated by American option pricing models under the assumption of complete markets (Varli and Yildirim 2015 [43]).

In this paper, we would contribute to extant literature about pricing early termination options with three new approaches. First, we develop a two-stage competing risk model for depicting two-round decision paths in which we revise analogies of options. Three outcomes (i.e. continue scheduled payments, deny obligations, or redeem collateral properties) in the first stage are considered and boundary conditions for each outcome would be computed. In the second stage, within separate boundary conditions for redeeming collateral and denying obligations, we model decision paths of channels to execute the first round decision. For denying obligations, we would theorize competitions between default which is analogous to a series of compound European call on put options and restructuring which is treated as an exchange option between mortgages with different cash flow structures in order to find out their boundary conditions. For redeeming collateral, competitions about forms of settlements - defeasance which is regarded as an exchange option of more liquid assets with less liquid assets and full prepayment in cash which is analogous to a series of compound European call on call options - would be modelled for computing values of each option and hence obtaining related boundary conditions. Second, it is the first study to treat mortgage restructuring as a direct competitor of default and model the competition in the second stage. Third, unlike conventional property price stochastic process, we would introduce constraints driven by collateral underlying property supply elasticity into the mean reverting stochastic process so as to enhance pricing models for early termination options of commercial mortgages by considering more realistic property market dynamics.

Apart from developing a theoretical model, we simulate option values and net benefits of four types of early mortgage terminations by Monte Carlo method. We find out the greatest value in mortgage restructuring, in which interests are reduced and maturity is extended, in the first pair of competing options and prepayment in cash in the second pair. Furthermore, we would hypothesize that tightening underlying collateral property supply constraints increases values of early termination options. Our simulated results verify this hypothesis, except for defeasance options. Since we simulate our pricing models with 1000 paths in this research, increasing in number of replications (at least 10000 times) will be carried out in order to check accuracy of simulation results.

The paper is organized as follows. Next section provides literature reviews related to early termination options. Section 3 presents our theoretical model. Section 4 describes the setup of simulation followed by results and analysis in section 5. Subsequently, we conduct scenario analysis and particularly discuss impacts of LTV ratios, property supply elasticity and mortgage restructuring rates in section 6. We draw the final conclusion and brief future research direction in last section.

2 Literature Review

In the extant literature related to pricing default options (except for Kau, Keenan, Muller and Epperson treating a mortgage contract as a compound option), mortgage default is regarded as a compound of European put options - at each mortgage payment date, one put option with a strike price which is determined by contemporaneous net equity values is virtually written on a mortgage contract during origination and could be exercised only on a payment date. A borrower, as an option owner, will exercise an option (i.e. go bust) if property values are smaller than mortgage values or insolvency occurs. However, this analogy ignores actual linkages between options and hence could misstate values of default options. Thus, our work would refer to the idea of Kau, Keenan, Muller and Epperson to revise the analogy to a series of compound European call on put options and the corresponding principle would be discussed in latter section.

The dynamic model for analyzing decisions of home mortgage borrowers which include options to default, prepay or refinance a loan constructed by Campbell and Cocco (2015 [8]) emphasizes the role of negative home equity for mortgage default and the differential impact of heterogeneous characteristics of borrowers, for instance labour income growth and risk on termination decisions. The study evidences that higher labour income risk leads to higher probabilities of default and cash out prepayment for both adjustable or fixed rate mortgage borrowers and even higher if labour income risks are correlated with real interest rates. Labour income growth would not bring significant changes in probabilities of default, cash out and refinancing since effects of lower "incentives to save" cancel off benefits from improving affordability. Apart from borrowers, heterogeneity of originators and special servicers could determine mortgage default particularly for commercial mortgages. Originators would adjust credit spreads to mortgage rates according to their financial conditions - those facing slumps in their stock prices in quarters would levy higher credit spreads that turn out higher probabilities of default (Titman et al 2010[41] and Black et al 2012[7]). Special servicers would make initial workout strategic decisions that alter the likelihood of mortgage default (Chen et al 2012[10]). These reflect important roles of originators and special servicers in managing mortgage risks. Since heterogeneity of collateral underlying property supply constraints is never discussed, our work would address its role in determining early termination options.

Kau, Keenan, Muller and Epperson (1987[25], 1992[26] and 1995[24]) build generalized models for default and prepayment of fixed rate mortgages by using extended Black & Scholes model in which stochastic interest rates are assumed. They suggested treating a mortgage contract as a compound options, however did not specify the types of compound options. Moreover, they set up different boundary conditions and calculate differences in mortgage values in different scenarios that are used to deduce values of prepayment and default. They find that prepayment values are greater than default values in general. We follow them to construct models with the setup of stochastic interest rates, but use a direct approach to compute values of early termination options by formulas of valuation of compound options based on our revised analogies. Similar findings are also obtained.

Unlike conventional competing risk analysis between prepayment and default (Ambrose and Sanders 2003 [4], Ciochetti et al 2002 [11] and 2003 [12], Deng, Quigley and Van Order 2000 [16], Grovenstein et al 2005 [22], Seslen and Wheaton 2010 [38] and An et al 2013 [5]), the dynamic model built by Campbell and Cocco does not consider interactions of risk between prepayment and default, but separately capture conditions of prepayment in terms of negative interest rates and accumulated positive home equity values, and conditions of default in terms of negative home equity. Our model would be in line with this approach to obtain separate boundary conditions for denying mortgage obligations and early paying up loans, instead assume a competing relationship between both situations.

Studies of mortgage restructuring or modifications (Agarwal et al 2011 [1], Ghent 2011 [21], Piskorski, Seru and Vig 2010 [35]) focus on likelihood of redefault which implies inefficiency of restructuring. Ghent explained that lack of information leads to wrong decisions on modifications. We would argue that values created by restructuring shall be quantified in order to ensure modification plans as suitable as they could be, and hence theorize mortgage restructuring with the analogy of an exchange option between mortgages with different cash flow structures as well as estimate option values. Piskorski and Tchistyi (2010[36] and 2011[37]) characterize optimal dynamic mortgage design and find out that mortgage modification could create values for both borrowers and lenders in optimal conditions. The likelihood of mortgage modification is represented as an inverted U shaped function of borrowers' combined loan-to-value (LTV) ratio in their theoretical models. To further extend, we would suggest other determinants for estimating option values of modification in addition of LTV ratio.

Options of prepayment are usually regarded as simple call options, since a borrower considers full prepayment in cash when market interest rates with respect to refinancing are lower than mortgage rates. However, we would argue that the feature of fixed exercise prices is not the most appropriate due to time-varying net equity values of collateral. A series of European call on call options is suggested as more appropriate. Defeasance firstly theorized by Dierker, Quan and Torous (2005) is treated as an exchange option between mortgages and riskless debt since mortgages are replaced with Treasury securities which replicate cash flow structures of mortgages so as to redeem collateral properties once defeasance options are exercised. Indeed, a borrower would switch a settlement method from "paying cash" to "pledging with Treasury securities". Therefore, we restate defeasance as an exchange option between less liquid assets and more liquid assets.

Varli and Yildirim (2015) simulated values of default options, prepayment penalty and defeasance options for participating mortgages by employing American options[4] that is unique work to estimate values of three types of options. Defeasance options have the highest values and option values of prepayment are much greater than default. Changes in LTV ratio stimulate the greatest fluctuations in defeasance

[4]Participating mortgages allow borrowers to obtain below-market interest rates in return for a percentage of future appreciation and / or net operating income of collateral properties (Varli and Yildirim 2015).

options comparing with prepayment and default. We apply different analogy of options instead of American options they suggested and also consider mortgage restructuring as an option for pricing mortgage risk.

3 Theoretical Framework

We attempt to scrutinize the exercise of mortgage termination options in the two-step decision making process and develop theories to estimate values of each termination option (i.e. default, mortgage restructuring, defeasance, and full prepayment in cash), and subsequently consider executing costs as well as net benefits which are crucial for explaining delayed exercises of options. In the first stage, three options of mortgage termination are defined as:

1. Continue scheduled payments
2. Deny obligations
3. Earlier pay up mortgages

In order to capture key features of property market dynamics related to property supply constraints into the pricing framework of mortgage termination options, we incorporate a mean reverting process of property values in our option framework. After finding out conditions of exercising these three options, in the second step we model the likelihood of occurrence of two pairs of competing options:

1. Default and mortgage restructuring
2. Full prepayment in cash and defeasance

Each option (series of compound options in some cases) in each pair is priced simultaneously and consequently the net benefit function of exercising each option is derived.

3.1 The First Stage

A mortgage underwriter decides mortgage interest rates, LTV ratios, duration of a mortgage and other mortgage terms based on credit assessments on borrowers. An interest rate (r_t) is assumed as a mean reverting stochastic process with a non-negative boundary developed by Cox, Ingersoll and Ross (1985[15]) as follow:

$$dr = \gamma_r \cdot (\mu_r - r)dt + \sigma_r \sqrt{r} dZ_r \tag{1}$$

γ_r : Speed of reversion of r
μ_r : Drift on r
σ_r : Volatility of r
Z_r : Z-Wiener process of r

A lender approves mortgage applications depending on rental income flows and physical conditions of collateral properties. In existing literature, property values were assumed to follow same stochastic process of a dividend paying stock price with diffusion that was suggested by Merton (1973[32]). However, since values of service flow are considered in house price dynamics in which a steady rate would be reached, mean reversion is modelled in a house price process (Ambrose and Capone 1998[3], Titman, Tompaidis and Tsyplakov 2004[40]). Commercial real estate are income-producing properties, thus we follow to assume the mean reverting stochastic process with a non-negative boundary for property values (p_t)[5]:

$$dp = \gamma_p \cdot (\mu_p - p)dt + \sigma_p \sqrt{p} dZ_p \tag{2}$$

γ_p : Speed of reversion of p
μ_p : Drift on p
σ_p : Volatility of p
Z_p : Z-Wiener process of p

Furthermore, Wheaton's stock-flow model (1999[44]) illustrates cyclical movements of property prices driven by cyclicality of long run equilibrium state and short run disequilibrium where search and bargaining leads to a delay of market responses to shocks. This movement is equivalent to a mean reverting process where adjustment to equilibrium is captured (Capozza et al 2004[9]). Fluctuations in property prices are limited by property supply constraints. The degree of fluctuation is quantified by speed of reversion (γ_p) and volatility (σ_p). The necessary condition for market oscillations (or over- and under- building) is related to supply elasticity and demand elasticity as shown in Wheaton's model. In other words, speed of reversion (γ_p) would be determined by demand and supply elasticity. We regard that oscillations explained by Wheaton could be interpreted with the principle of wave physics, and thus apply formula related to harmonic oscillators of motions for setting up the function of speed of reversion in real estate markets.
Since real estate cycle normally demonstrates similar oscillation paths which are

[5] We follow Kau et al (1990, 1992, 1995) to assume "zero" correlation coefficient between interest rate movement and property price movement, therefore the equation of property price process does not involve correlation between these two movements.

generated by "under-damping" harmonic oscillators[6], we use related harmonic motion models to construct the function of speed of reversion with demand and supply elasticity. Firstly, we construct a function of displacement ($x(t)$) which is a distance to equilibrium as equation 3 (where $\alpha = \sqrt{|\omega_0^2 - \beta^2/4|}$, A $= x_0$, and B $= x_0\beta/(2\alpha)$) and subsequently take the first and second derivatives for writing an equation of an under-damped harmonic oscillator. Lo and Mueller (2010[29]) applied harmonic oscillator to measure uncertainty in financial markets. Also, Kulesza and Belej (2015[27]) explained property market dynamics by under-damped harmonic oscillator.

$$x(t) = e^{-\beta t/2}[A\cos(\alpha t) + B\sin(\alpha t)] \tag{3}$$

$$\begin{aligned} x'(t) &= \sin(\alpha t)e^{-\beta t/2}(-A\alpha - \frac{\beta B}{2}) + \cos(\alpha t)e^{-\beta t/2}(B\alpha - \frac{\beta A}{2}) \\ x''(t) &= \cos(\alpha t)e^{-\beta t/2}(\frac{A\beta^2}{4} - A\alpha^2 - B\alpha\beta) + \sin(\alpha t)e^{-\beta t/2}(A\alpha\beta - B\alpha^2 + \frac{B\beta^2}{4}) \end{aligned}$$

In our model, a displacement refers to disequilibrium in property values where equilibrium is reached at the time that property supply equals demand[7]. β represents demand adjustment per radian of oscillation, and ω denotes property demand. Demand elasticity (DE) would determine strength of transmission of exogenous demand forces into a property market, and hence we introduce it as a transmission scale into an oscillation model. The more elastic the demand, the higher the transmission. Therefore, a demand driving force is described as F·FG·SP·DE· $\cos(\omega t)$ where F is an exogenous demand driver(i.e. employment base), FG is employment growth and SP is space per worker. On the other side, we would reflect supply elasticity (SE) in the model by setting an initial displacement to disequilibrium which could be negatively correlated with supply elasticity, since greater degree in disequilibrium is observed in more constrained markets. Thus, parameter A would be rewritten as follow.

$$A = \begin{cases} (1 - SE) \times m & \text{if } 0 \leq \text{SE} < 1 \\ -(1 - \frac{1}{SE^{0.25}}) \times m & \text{if SE} \geq 1 \end{cases}$$

Parameter B should also change correspondingly as B = A$\beta/(2\alpha)$. An oscillation is described as equation 4 where m represents property supply. We interpret speed of

[6]Under-damping oscillation illustrates that an object swings back and forth with decreasing fluctuation until coming to a stop.

[7]To be precise, demand equals supply after deducting frictional and structural vacancy at equilibrium.

reversion (γ_p) as the first derivative of displacement which can be written as equation 5 by transforming equation 4. The equation demonstrates that speed of reversion positively relates with demand elasticity and decreases with supply elasticity, since smaller A and B would be used for less constrained markets.

$$F \cdot FG \cdot SP \cdot DE \cdot cos(\omega t) = mx'' + m\beta x' + m\omega_0^2 x \tag{4}$$

$$\gamma_p = \frac{1}{\beta m}(F \cdot FG \cdot SP \cdot DE \cdot \cos(\omega t) - mx'' - m\omega_0^2 x) \tag{5}$$

Figure 1 demonstrates the oscillation in property market based on our setup.

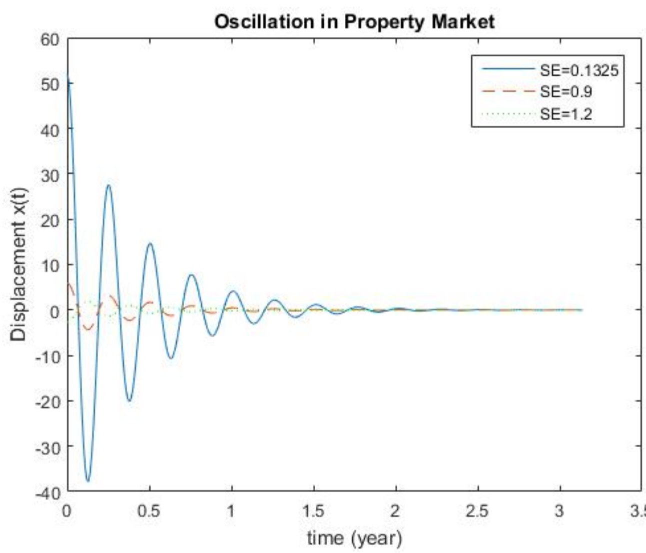

Figure 1: Under-damped Oscillation in Property Markets

Regarding volatility of property values (σ_p), the study by Paciorek (2013 [33]) evidenced strong positive relationships between house price volatility and supply constraints. Therefore, we assume a negative exponential function of supply elasticity for property price volatility which is written as equation 6. In which, the multiplier ($c_1 \in \mathbf{R}^+$) would be chosen based on historic volatility instead of an arbitrary term.

$$\sigma_p = c_1 \times e^{-SE} \tag{6}$$

Combining equations 5 and 6 with 2, we rewrite the property value process in terms of supply and demand elasticity as shown in equation 7 [8].

[8]Speed of reversion is explained by both demand and supply elasticity. Expanding the equation to exhibit supply elasticity would cause a tedious expression, therefore we keep a simpler expression in which substitutions of supply elasticity into x and x'' are skipped.

$$dp = \frac{1}{\beta m}(F \cdot FG \cdot SP \cdot DE \cdot \cos(\omega t) - mx'' - m\omega_0^2 x)(\mu_p - p)dt + c_1 \times e^{-SE}\sqrt{p}dZ_p \quad (7)$$

Assuming that a fixed-rate, constant payment mortgage is originated, initial loan size, monthly mortgage payments and unpaid principal can be calculated as follow:

$$L_0 = LTV_0 \times p_0 \quad (8)$$

$$M = L_0 \cdot \frac{h_m(1+h_m)^n}{(1+h_m)^n - 1} \quad (9)$$

As

$$L_0 = M\sum_{i=1}^{n} \frac{1}{(1+h_m)^i}$$

$$U_j = (1+h_m)U_{j-1} - M = L_0(1+h_m)^j - M\frac{(1+h_m)^j - 1}{h} \quad (10)$$

L_0 : Initial loan amount
LTV_0 : Initial LTV ratio
p_0 : Initial property value
M : Monthly constant mortgage payment
h_m : Monthly fixed rate
n : Amortization period in terms of month
U_j : Unpaid principal on payment date j after a payment is made

To study early mortgage termination, we firstly identify terminal conditions of promised mortgage payments which negate conditions of early termination. Following Kau (1987[25], 1992[26]), these can be separated into “before maturity” and “at maturity”.
At a payment date ($j < n$),

$$PM(r_j, t(j), j) = M + PM(r_j, t(j), j+1) \quad (11)$$

At maturity ($j = n$),

$$PM(r_j, t(n), n) = M + U_n \quad (12)$$

Where $PM(r_j, t(j), j)$ represents the value at time t of promised mortgage payments from j to n at a discounted interest rate r_j

We would argue that two early termination options - prepayment and denial of obligations, are virtually written on a mortgage during origination. In our model, both options are treated as the first decision making and thus mortgage values are computed as the present values of promised mortgage payments minus the values of both options.

$$G(r_j, p_t, t, j) = PM(r_j, t(j), j) - EP(r_j, p_t, t, j) - DO(r_j, p_t, t, j) \tag{13}$$

$EP(r_j, p_t, t, j)$: option value of earlier paying up
$DO(r_j, p_t, t, j)$: option value of denying obligations

Both options are written on same collateral properties with same expiration date but different strike prices. Each option is in European style with one-month duration. The key condition of early paying up mortgages depends on market interest rates relative to mortgage rates. If we assume that paid-in capital of a borrower is refinanced by a new mortgage at contemporaneous LTV ratio of an original mortgage, another condition - collateral values shall have moderately appreciated since origination of an original mortgage - will exist. When market interest rates are lower than original mortgage rates, a borrower would prepay by refinancing with new loan ($L_{new,0}$=$U_{old,n}$ + $M_{old,n}$). Prepayment can help to save interest payments. That means interests newly charged ($\sum_{t=1}^{N-n} U_{new,t} \times r_m$) would be lower than remaining interest payments of original mortgages ($\sum_{t=n}^{N} U_{old,t} \times h_m$). To keep it simple, the condition of prepayment option in terms of interest rate is that interest rates (r_m) are lower than original monthly mortgage rate, h_m. Furthermore, collateral value should also be sufficiently high. Comparing with initial property value p_0, property value greatly appreciates. LTV ratio ($U_{old,t}/p_0$) would be reduced as remaining mortgage balance drops. We assume new loan can be under-written based on contemporaneous LTV ratio (calculated with p_0). The strike price of prepayment option in terms of property value is $\frac{U_T+M}{LTV(p=p_0)}$.
Unlike prepayments, the strike price equals the sum of remaining mortgage balance and a mortgage payment (i.e. U_T + M) when a borrower considers to exercise an option of denying obligations. We summarize both option values in the below equations.

$$DO_{(r_j,p_t,t,j)} = e^{-r(T-t)} E_{p_t,t}[max(0, (U_T + M) - p_T)] \tag{14}$$

$$EP_{(r_j,p_t,t,j)} = e^{-r(T-t)} E_{p_t,t}[max(0, p_T - \frac{U_T + M}{LTV(p = p_0)})|r_m < h_m] \tag{15}$$

3.2 The Second Stage

After finding conditions of denying obligations and early paying up mortgages, we set up two pairs of competing options in the second stage. Once having decided to deny obligations, a borrower would move to the second step - compare two choices (**default and mortgage restructuring**) and subsequently execute the best option. On the other hand, if a borrower decides to early pay up mortgages, he will further consider whether **full prepayment in cash or defeasance** will bring him the greatest benefit. In a nutshell, conditional expected values, which indicate corresponding conditional probabilities, are measured for four options which are grouped into two pairs in the second stage.

3.2.1 The First Pair: Default vs Mortgage Restructuring

Denying obligations is defined as rejecting to pay original scheduled payments. A borrower who goes default would surrender collateral and would not repay remaining mortgage balance in the future. Following Epperson et al (1985[18]) viewed mortgages as the compound of European put options, we would argue that the series of compound European call on put options is more appropriate to describe decisions to default because of better match for decision paths of payments. Option values represent the gross benefit how much a borrower gains through default. Since enormous implicit opportunity costs driven by inaccessibility of new borrowings after bankruptcy declaration shall not be ignored, we would further compute net benefit. Alternatively, a borrower considers negotiating a restructure in which a modified payment schedule is officially provided and original repayment obligations would be superseded. Analogously, mortgage restructuring can be viewed as an exchange option between mortgages with different cash flow structures. In contrast to default, a borrower shall pay explicit restructuring fees. Our approach to depict decision paths in this pair of competition consists of two steps - (1) estimate gross benefit of exercising an option in monetary term; and (2) compute net benefit after estimating related costs.

A. Mortgage Default As A Series of Compound European Call On Put Options

Mortgage default, in general, can be viewed as a series of default options which are expired on every payment date until scheduled mortgage payments cease. In our analogy, a borrower is regarded as the European call on put option buyer since a mortgage is originated. Intuitively, he does not go default on the first payment date. The first payment is treated as the strike price of call option. Through an exercise of the first call option, signified by settling the first mortgage payment, the put option written on a collateral property is received in return. The put option would be matured at the second payment date with the strike price which equals the sum of a monthly mortgage payment and remaining mortgage balance after the payment is made at maturity of the option. An exercise of the put option indicates mortgage default. In the case that a borrower settles the second payment, the put option is not exercised but another European call on put option is activated by exercising the new call option. The generic payoff function of a default option is written as equation 16. Recursive decision paths would be drawn and the only difference is the strike prices varying over maturity date in terms of time value of a property. Recursion would be ended until a mortgage is terminated.

$$PAYOFF_{D,j} = max[(0, PUT(p_{j+1}, (U_{j+1} + M_j), j+1) - M_j] \tag{16}$$

$PAYOFF_{D,j}$: Payoff of default option (equivalent to European call on put option)
j : Exercise date of call option in call on put option (i.e. mortgage payment date)
$j+1$: Exercise date of put option in call on put option (i.e. one month after exercising the call option)
PUT : Put option value
p_{j+1} : Underlying property value at mortgage payment date j+1
$U_{j+1} + M_j$: Remaining mortgage balance at date j+1 plus mortgage payment on date j (i.e. strike price of the put option)
M_j : Mortgage payment on date j (i.e. strike price of the call option)
Numerical solution of the default option can be yielded by the pricing formula of the European call on put option (Hull 2012[23]).

$$\begin{aligned} DEFAULT_0 = (U_{j+1} + M_j)e^{-r(j+1)} & BN(-\alpha_2, -\beta_2; \sqrt{\frac{j}{j+1}}) \\ -pe^{-q(j+1)} & BN(-\alpha_1, -\beta_1; \sqrt{\frac{j}{j+1}}) \\ & -e^{-rj} M_j \cdot N(-\alpha_2) \end{aligned} \tag{17}$$

q: Rental yield
BN : Cumulative bivariate normal distribution
N : Normal distribution

Where

$$
\begin{aligned}
\alpha_1 &= \frac{(r - q + \frac{\sigma^2}{2}) \cdot j}{\sigma\sqrt{j}} \\
\alpha_2 &= \alpha_1 - \sigma\sqrt{j} \\
\beta_1 &= \frac{ln(\frac{p}{U_{j+1}+M_j}) + (r - q + \frac{\sigma^2}{2}) \cdot (j+1)}{\sigma\sqrt{j+1}} \\
\beta_2 &= \beta_1 - \sigma\sqrt{j+1}
\end{aligned}
$$

B. Mortgage Restructuring As An Exchange Option

Mortgage restructuring is mainly conducted by modifying three important mortgage characteristics: (1) a reduction in interest rates, (2) an extension of maturity, and (3) an adjustment of principal by capitalizing unpaid interests or partial write-down of original principal. Except for the write-down, other types of restructure act as a replacement of an original mortgage, which a borrower cannot fulfill obligations, with a new mortgage which its payment schedule could provide a buffer against financial stress of a borrower. In a nutshell, for all these cases, mortgage restructuring shall be viewed as an exchange option between mortgages with different cash flow structures. Only write down of principal is treated as an option on debt-equity swap. Due to its uncommon occurrence, we merely develop the option framework of mortgage restructuring in line with an exchange option for main cases. Margrabe (1978[31]) developed the pricing model of exchange option between two company stocks in which exchange option values are critically determined by correlation between underlying company stocks and positions of original stocks. Similar to mortgage restructuring, we hypothesize that cash flow structures built with original scheduled payments, being the least favourable to a borrower, necessitate an alternation. Negative correlation between two different cash flow structures optimizes the benefit of mortgage restructuring. Exchange option values can be simplified as cumulative difference between cash flows on each payment date. We construct three scenarios to summarize valuation approaches.

Case 1: Reduction in Interest Rates, But Principal And Maturity Remain Unchanged

In this case, we examine reduction in mortgage rates. Each original mortgage payment (MP_1) is fixed at $\hat{a}$. The restructured mortgage will be charged at $\hat{b_1}$ that is lower than $\hat{a}$ as assumed. By intuition, only when total values of restructured mortgage payments are lower than total values of original mortgage payments, a borrower is motivated to renegotiate mortgage contracts. The below graph illustrates cash flow differences between original and restructured mortgages.

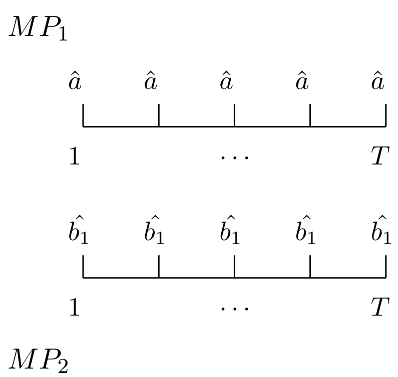

Figure 2: Comparison of cash flow structures between mortgages (a reduction in interest rates)

To compute values of an exchange option between mortgages, we firstly set an equation showing differences between mortgage payments (equation 18).

$$g(t) = MP_1(t) - MP_2(t) \tag{18}$$

Since a borrower has to decide if he should immediately exercise this "exchange option" once after being informed about modified terms, option values would equal payoff.

$$PAYOFF_{MR} = max(V_{M1} - V_{M2}, 0) \tag{19}$$

$PAYOFF_{MR}$: Payoff of a mortgage restructuring option
V_{M1} : Value of promised payments in an original mortgage contract at an exercise date
V_{M2} : Value of promised payments after restructure at an exercise date
The difference in values of promised payments between mortgages would be calculated in terms of time value of money by Riemann sum approach.

$$
\begin{aligned}
V_{M1} - V_{M2} &= \int_0^T [MP_1(t) - MP_2(t)]e^{-rt}dt \\
&= \int_0^T (\hat{a} - \hat{b_1})e^{-rt}dt \\
&= \frac{\hat{a} - \hat{b_1}}{r}(1 - e^{-rT})
\end{aligned}
$$

Equation 20 describes values of restructuring options assuming same maturity of both mortgages.

$$
MRes = max[0, \frac{\hat{a} - \hat{b_1}}{r}(1 - e^{-rT})] \tag{20}
$$

Case 2: Changes in Interest Rates and Maturity, But Principal Remains Unchanged

A reduction in interest rates and / or an extension of maturity are common approaches to restructure mortgages. The restructured one could be a constant payment mortgage and its mortgage payments (MP_2) are fixed at $\hat{b_2}$. Comparing with Case 1, the only difference is that a borrower in case 2 repays with lower fixed payments ($\hat{b_2} < \hat{b_1} < \hat{a}$) in longer time (i.e. $T_2 > T_1$). For the period after an original mortgage is matured, only restructured mortgage payments remain. In general, values of these kinds of restructure could be computed in the similar approach as adopted in Case 1. The graph illustrates difference in cash flow structures. The option value equals $\frac{1}{r}(\hat{a} - \hat{b_2} + \hat{b_2}e^{-rT_2} - \hat{a}e^{-rT_1})$.

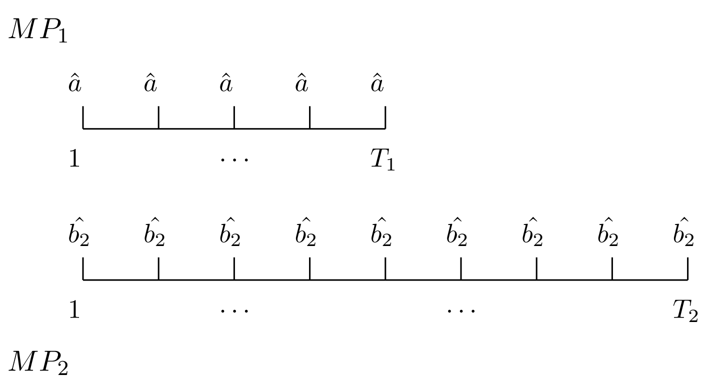

Figure 3: Comparison of cash flow structures between mortgages (a reduction in interest rates and an extension of maturity)

From the perspective of borrowers, mortgage restructuring options shall be optimized by repayment schedules which match with their insolvent financial situation. Correlation of cash flow structures between original and restructured mortgages ($\hat{\alpha}$) would determine optimization. To maximize option values, initial payment schedules between mortgages shall significantly differ.

Case 3: Changes In Principal, Interest Rates and Maturity

Capitalization of unpaid interests or partial write down of principal would alter an amount of principal. Due to rare occurrence of a write down, we would omit modeling it and discuss capitalization only. In some cases, a lender may allow a borrower postponing interest payments through negotiating modified mortgage terms, for example, unpaid interest payments are capitalized into principal of a restructured mortgage. This would increase a borrower's burden by levying more interests in total, but simultaneously extending maturity lowers average burden on each payment date particularly during an initial period. As a result, a reduction in initial mortgage payments and a gradual increment in a later stage are expected. In addition, re-appraisal of collateral during restructuring marks down collateral values. It is striking to note that LTV ratio of a restructured mortgage is even higher than that of an original mortgage. This translates into - "mortgage restructuring's benefit by increasing leverage" in theory. However, a borrower does not, in fact, receive extra credit facilities because a depreciation in collateral values offsets higher leverage effect.

$$\text{leverage benefit} = (LTV_{M2} \text{ - } LTV_{M1}) \cdot \text{depreciated collateral values}$$

Illustrated in Figure 3, mortgage payments, assuming a constant payment mortgage, of a restructured one (MP_2) are expected to be lower than an original mortgage (MP_1). As the maturity is extended from T_1 (original) to T_2 (restructured), a higher initial ratio of interest payments to mortgage payments ($\frac{I_{2,t}}{I_{2,t}+P_{2,t}} > \frac{I_{1,t}}{I_{1,t}+P_{1,t}}$) is expected. In other words, initial principal repayments of a restructured mortgage are lower ($P_{2,t} > P_{1,t}$) until reaching a break-even time t_+ where $I_{2,t_+}+P_{2,t_+} = I_{1,t_+} + P_{1,t_+}$ and lower speed of principal repayments for a restructured one is expected. Despite altering principal, we could still use the same approach to quantify mortgage restructuring option values by solving equation 20.

Apart from constructing pricing functions which are derived from differences in cash flow structures between both mortgages, we, referring to Margrabe's theorem, offer closed form solutions for assumed stochastic processes of mortgage prices in order to conduct robustness checks on our pricing models. The underlying assets are

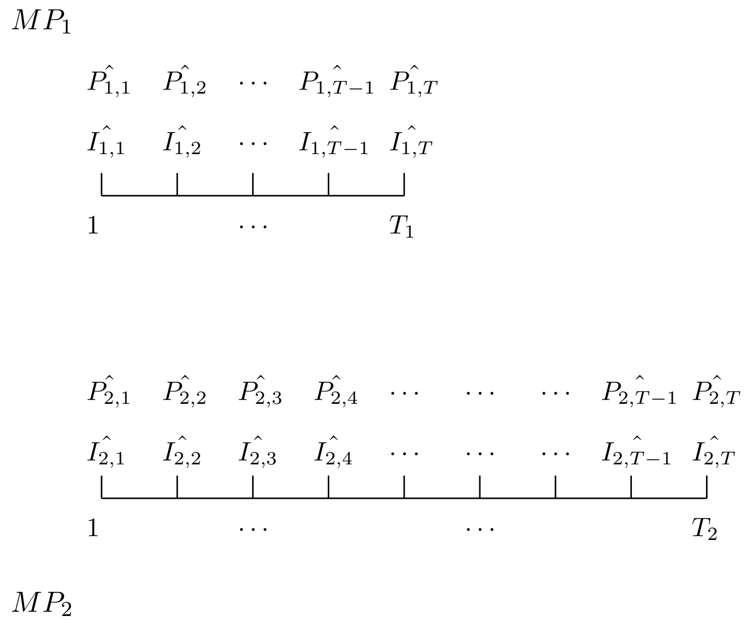

Figure 4: Comparison of cash flow structures between mortgages (changes in principal, interest rates and maturity)

mortgages.

$$dM_i = \mu_i M_i dt + \sigma_i M_i dW_i \qquad i = 1, 2 \tag{21}$$

M_1 : Original mortgage price
M_2 : Restructured mortgage price
W_1 and W_2 : Risk neutral measure Wiener process (M_1 and M_2 are correlated, $dW_1 dW_2 = \rho dt$)
Incorporating the terminal condition (MRes(T,M_1,M_2) = max [M_1 - M_2, 0]), restructuring option values can be presented as equation 22 which follows Margrabe's pricing function.

$$MRes = M_1 e^{(\mu_1 - r)(T-t)} N(d_+) - M_2 e^{(\mu_2 - r)(T-t)} N(d_-) \tag{22}$$

$$\begin{aligned} d\pm &= \frac{ln(M_1/M_2) + (\mu_1 - \mu_2 \pm \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} \\ \sigma &= \sqrt{\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2\rho} \end{aligned}$$

Values of default options and mortgage restructuring options quantify gross benefits of early termination respectively. To estimate net benefits, we shall consider costs that a borrower would bear for each option type. For decisions to default, there are no explicit costs but opportunity costs are involved. As a borrower goes default, bankruptcy shall be declared and in consequence he is forbidden from new borrowings. In short, levered investment opportunities could not be taken until the end of bankruptcy period. We define it as “bankruptcy costs” which would be determined by duration of bankruptcy and future capital return. Regarding mortgage restructuring, we can simply subtract modification fees from option values so as to find out net benefits. An option with greater net benefits implies that a borrower should exercise it. In case of same level of net benefits, it is indifferent between default and mortgage restructuring.

3.2.2 The Second Pair: Full Prepayment In Cash vs Defeasance

Early paying up mortgages can be settled with lump-sum cash payments or defeasance. Defeasance means a replacement of a mortgage with Treasury securities which could replicate cash flow structures of that mortgage. Prepayment is viewed as an American call option in existing literature since interest rates are regarded as a single determinant of prepayment and a mortgage rate is viewed as an “exercise price”. However, we would argue that European call on call options would be more appropriately analogous to “full prepayment in cash” as it helps matching cutoff time and time-varying strike prices along a payment schedule. Firstly, prepayment prior to a scheduled payment date cannot reduce interest costs because a same amount of scheduled interest payments shall be paid normally in monthly basis. In light of this, we assume that a borrower would consider to prepay only at payment dates. Therefore, a prepayment option would never be early exercised if an exercise date is set on a payment date; and hence prepayment could be treated as European style options. Secondly, a vanilla call option, where its strike price is fixed, cannot deal with time-varying strike prices until mortgages are terminated. In contrast, European compound “call on call” options can be designed with different strike prices. Thus, a new analogy is proposed. Same as the first pair, we would find out gross benefits of each decision by option pricing and subsequently net benefits by taking costs into account.

A. Full Prepayment In Cash As A Series of Compound European Call On Call Options

Similar to mortgage default, we assume that a borrower does not prepay at the first payment date. At origination, a borrower has already virtually bought European call on call options. The first mortgage payment is a strike price of the call option in this compound option. Also, the call option is exercised when the first payment is done and the second call option written on a collateral property at a strike price (i.e. $\frac{U_T+M}{LTV(p=p_0)}$) is immediately received in return under the condition that $r_m < h_m$. The compound option is mature on the second mortgage payment date. If a borrower continues to settle a scheduled payment, the second call option is not exercised but another European call on call option is purchased. Otherwise, a mortgage is early terminated whatever capital is sourced from. In a nutshell, a borrower exercises the first call option depending on expected values of the second call option. Except for the first European call on call option which is purchased at origination, we assume that the first call option embedded in each compound option is immediately exercised once being purchased. Thus, option values could be estimated at each mortgage payment date. Equation 23 describes option payoff which is derived from the above principle in the condition that interest rates are lower than the original mortgage rate.

$$PAYOFF_{PP,j} = max[(0, CALL(p_{j+1}, \frac{U_T + M}{LTV(p = p_0)}, j + 1) - M_j]|r_m < h_m \quad (23)$$

$PAYOFF_{PP,j}$: Payoff of a prepayment option (equivalent to European call on call option)
j : Exercise date of the first call option in call on call option (i.e. mortgage payment date)
$j + 1$: Exercise date of the second call option (i.e. one month after exercising the first call option)
CALL : Values of the second call option
p_{j+1} : Underlying property values at mortgage payment date j+1
$\frac{U_T+M}{LTV(p=p_0)}$: Strike price of the second call option
M_j : Mortgage payment on date j+1 (i.e. strike price of the first call option)
Changes in prepayment option values are positively proportional to changes in underlying collateral property values and this can be proved by the following pricing formula for European call on call options in which numerical solutions are provided (Hull 2012[23]).

$$PREPAY_0 = -\frac{U_T + M}{LTV(p = p_0)} e^{-r(j+1)} BN(\alpha_2, \beta_2; \sqrt{\frac{j}{j+1}})$$
$$+pe^{-q(j+1)} BN(\alpha_1, \beta_1; \sqrt{\frac{j}{j+1}})$$
$$-e^{-rj} M_j \cdot N(\alpha_2) \qquad (24)$$

BN : Cumulative bivariate normal distribution
N : Normal distribution

Where

$$\begin{aligned} \alpha_1 &= (r - q + \frac{\sigma^2}{2}) \cdot j\sigma\sqrt{j} \\ \alpha_2 &= \alpha_1 - \sigma\sqrt{j} \\ \beta_1 &= \frac{ln(\frac{p}{PM_{j+1}}) + (r - q + \frac{\sigma^2}{2}) \cdot (j+1)}{\sigma\sqrt{j+1}} \\ \beta_2 &= \beta_1 - \sigma\sqrt{j+1} \end{aligned}$$

In order to compare our analogy with American call options suggested in existing literature, we could apply equation 24 and a pricing function of American call options to simulate differences in prepayment option values between both analogies.

B. Defeasance As An Exchange Option

The distinction between prepayment in cash and defeasance is the settlement method. Obviously, prepayment and defeasance are settled respectively by cash and by submitting Treasury securities which are required to replicate patterns of scheduled mortgage payments (Dierker et al 2005[17]). Dierker and his co-authors emphasized defeasance as an exchange option of a risky mortgage with riskless debt from the perspective of a lender, where liquidity benefit was also conceptualized but out of an option pricing model. We have same vein of the analogy, however, from the perspective of a borrower, since he is a decision maker to exercise a defeasance option. Therefore, the benefit from defeasance shall be given to a borrower. In fact, defeasance would be viewed as an exchange option of more liquid assets with less liquid assets when he switches the settlement method from "paying in cash" to "pledging with Treasury securities".

An exchange option involves liquidity premium which is given to a security holder as a compensation for holding non-cash assets. Luttmer (1996[30]) and Fortaine and

Garcia (2012[19]) estimate liquidity premium by exponential decay of liquidity factor (i.e. discounted values of aggregate future benefits over holding horizon of more liquid Treasury securities) with bond age. We would refer their pricing approach to coupon bonds in which liquidity premium is taken into account to price a basket of scheduled mortgage payments and a basket of Treasury securities as shown in equation 25.

$$P_i(Liq_t, Z_t) = \sum_{t=1}^{T} D_t \cdot M_t + \zeta_i(Liq_t, Z_t) \quad i = \text{mortgages or treasury securities} \tag{25}$$

P_i : Price of remaining mortgage or treasury securities pledged
Liq_t : Liquidity factor at time t
Z_t : Characteristics of mortgage or treasury securities e.g. scheduled interest and principal payments
T : Maturity date of mortgage
D_t : Vector of discount factors
M_t : Vector of scheduled mortgage payments (equivalent to coupons and principal of Treasury securities)
ζ : Liquidity premium of mortgage or Treasury securities

As the sum of discounted mortgage payments equals discounted values of coupons and principal of Treasury securities, their price differences rely on liquidity premium which is gained by Treasury securities. As a result, price of Treasury securities is greater than price of a mortgage by liquidity premium values. By applying Margrabe's exchange option pricing model and incorporating the terminal condition [Defease(T, Cash, Treasury Securities) = max(ζ of Treasury securities,0)], we conclude an exercise of a defeasance option as a combination of sales of Treasury securities and purchases of mortgages assuming that cash is uncorrelated with Treasury securities when $r_m < h_m$. The value function is written as equation 26.

$$Defease = (Mort + \zeta_{treasury})e^{(\mu_{treasury}-r)(T-t)}N(d_+) - (Mort)e^{(\mu_{mort}-r)(T-t)}N(d_-) \tag{26}$$

$$\begin{aligned} d\pm &= \frac{ln((Mort + \zeta_{treasury})/Mort) + (\mu_{treasury} - \mu_{mort} \pm \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} \\ \sigma &= \sqrt{\sigma_1^2 + \sigma_2^2} \end{aligned}$$

Same as the first pair, option values of prepayment and defeasance reflect gross benefits separately. To estimate net benefits, we shall take related costs into account. For this pair, prepayment penalty may be charged for full prepayment in cash and alternatively defeasance fees are involved. Net benefits of defeasance is subtraction of defeasance fee from its option values. For full prepayment in cash, prepayment penalty would be deducted from option values in order to yield net benefits. If net benefits of defeasance are greater than prepayment, a borrower will choose defeasance to end up a mortgage or vice versa.

To consolidate our theoretical framework, we summarize boundary conditions in the first and second stages of the model for each option in Table 1.

[Insert Table 1 Here]

4 Parameterization

We model two-stage decision paths for mortgage borrowers, in which conditions of denying obligations and early paying up are found out in the first stage with simulated interest rate and property price processes under different assumptions, and four early termination options would be priced with the consideration of the first-stage conditional likelihood in the second stage.

4.1 Simulated interest rate process

We collect mortgage rate data between July 1993 and December 2015 from the Federal Reserve and estimate average values and standard deviation of rates. Both are applied in the Cox, Ingersoll and Ross model as the drift and volatility. Due to long period of unchanged target rates by the Federal Reserve since the Global Financial Crisis and long term cautious monetary policies, we assume interest rates would move steadily. That means reversion of rates would proceed at moderate speed that is expected to be greater than reversion of property prices and thus 0.2 is assumed.

4.2 Simulated property price process

Since our models are expected to analyze early termination of commercial mortgages, we use average office values among 42 metropolitan statistical areas (MSA hereafter) from the second quarter of 1993 to the fourth quarter of 2015 which are sourced from CBRE Econometric Advisors to generate property price process. As we argue that real estate cycle as a main feature should be considered in the process, we also apply non-negative mean reverting Cox, Ingersoll and Ross model to the property price process.

Unlike interest rate process, we would argue that more complicated dynamics in property prices could be illustrated by the principle of under-damped harmonic oscillators. It is because both oscillation driven by external force and diminishing amplitudes are exhibited. Therefore, we apply the related wave oscillation model to set up a function of speed of reversion in property markets. For this issue, we gather office using employment sourced from Moody's Analytic, property supply as market mass, net accumulated absorption from CBRE Econometric Advisors, demand and supply elasticity which are estimated in our other research. Combining with the assumptions of driving and angular frequencies (i.e. 45 and 15 degree), dumping factor as well as space per worker, we simulate the impulse of employment force to office space and hence compute speed of reversion.

4.3 Mortgage Terms

The model is expected to adopt in any types of mortgages. In our studies, we begin with 10-year 5% constant payment mortgages (CPM) and 10-year 5% interest only mortgages (IOM) since they are common types of commercial mortgage contracts. Assuming initial loan-to-value ratios (i.e. 70% for CPM and 50% for IOM), we take the recent values of 10000 square foot property for setting up a mortgage size.

Regarding defeasance, Treasury securities are pledged for settling remaining mortgage payments. We quantify Treasury liquidity premium by bid-ask spread of Treasury notes from July 2011 to December 2015 which are sourced from the Wall Street Journal in order to estimate the benefits of defeasance. Since we would expect that the situation in short term would follow recent trends, we parametrize liquidity premium based on the period of January 2014 to December 2015.

4.4 Execution Costs

In the second stage, we estimate each type of execution costs so as to find out net benefits which determine final decisions how to early terminate mortgage contracts by borrowers.

4.4.1 Bankruptcy Costs

Overall bankruptcy costs could be estimated by opportunity costs in terms of future capital return during bankruptcy period that borrowers would forgo due to unable access to funds. We check about 1000 bankruptcy records since October 1979 provided in UCLA-Lo Pucki Bankruptcy Research Database and estimate bankruptcy duration as the length of period from filing to confirming effective plans. On average, bankruptcy would last for 17 to 18 months.

We proxy post-trough capital return by total return in office markets among 42 MSAs which are sourced from CBRE Econometric Advisors. Since trough is seen in 2009, we take average annual returns from 2010 to 2011 as post-trough capital returns so as to estimate bankruptcy costs.

4.4.2 Modification Fees

Modification fees are normally charged by servicers. The charges on sub-prime loans is the highest compared with alt-A loans and prime loans, i.e. 50 basis points of outstanding principal balance (Thompson (2011)[39]). We add this charge as restructuring costs.

4.4.3 Prepayment Penalty

Based on over 10,000 securitized mortgages which contain covenants of prepayment penalty sourced from Bloomberg, we found lenders mostly charge at a rate of 9% for 9.5 months. In our model, we set up the penalty with this rate on remaining principal balance.

4.4.4 Defeasance Fees

In practice, defeasance costs consist of Treasury securities costs and transaction costs. However, we would treat net differences between securities costs and loan principal balance as a part of defeasance costs instead of entire securities costs, since we regard Treasury securities as an exchange of remaining mortgage payments. We would use net difference to compute values of defeasance options. Our baseline mortgage size is about $2 million. We refer to quotation from the company "Commercial Defeasance LLC".

Table 2 summarizes parameter values used in the baseline case.

[Insert Table 2 Here]

5 Simulation

5.1 Methodology

We develop a "R Coding" function program for adopting Monte Carlo simulation approach to estimate option values in each stage of our model as well as net benefits which indicate final decisions of borrowers. Simulation is replicated 1000 times in which 1000 paths for each data generating process are applied.

5.2 Baseline Results

5.2.1 Comparison With Analogies of Options

Before presenting baseline results, we would investigate how big difference could make if we switch analogies for early termination options. We take a 10-year interest only mortgage as an example and compute values of default and prepayment by different analogies. For each option, exercise price is loan size (i.e. $100), interest rate and rental yield are set at 5% and 7% respectively. Initial LTV ratio is 70%. We set up three scenarios by different volatility level of property prices (i.e. 10%, 25% and 40%). Figures 5 to 10 demonstrate paths of option values by types of options. Mortgage default described as a European call on put option and American put option would have similar patterns of theoretical payoff for put option values when volatility becomes smaller. Comparing with an American put option and a European put option which are estimated by binomial method and Black & Scholes method respectively, a call on put option payoff lines are more curved. Once property prices exceed $150, a call on put option has the lowest values among three options when volatility equals 40%. Regarding analogy of prepayment, a call on call option also has similar patterns of theoretical payoff for call option values, whereas American and European call options do not have. Values of a call on call option are lower than both American and European call options. That means different analogies cause discrepancy in estimating values of mortgage default or prepayment.

[Insert Figure 5 Here]
[Insert Figure 6 Here]
[Insert Figure 7 Here]
[Insert Figure 8 Here]
[Insert Figure 9 Here]
[Insert Figure 10 Here]

5.2.2 First Stage

Tables 3 and 4 summarize baseline results. We assume LTV ratios at the levels of 70% and 50% for CPM and IOM respectively. We compare the results between models with or without our suggested property price process. In general, standard geometric Brownian motion which does not have reversion brings much greater fluctuation in property prices. In the first stage, assumptions of standard geometric Brownian motion for property price process yield the greatest values of denying obligations and early paying up mortgages. That means early mortgage termination under these assumptions are more valuable particularly for early paying up mortgages. Furthermore, similarly if we assume that property prices follow our suggested process, prepayment in the first stage is more valuable than denial options. Values of early paying up mortgages are at least four times greater than values of denial options. Thus, we would be more cautious in prepayments which is driven by changes in interest rates.

[Insert Table 3 Here]
[Insert Table 4 Here]

5.2.3 Second Stage

We investigate how borrowers decide a settlement method after triggering to deny obligations or early pay up their mortgages in the second stage, using different property price processes. For the first pair of competing options in which borrowers decide to deny obligations, there are three main types of restructuring or simply default to choose. Referring to Table 3, mortgage restructuring by reducing interest rates and extending maturity(case 2) is the best method to deny obligations instead of mortgage default. The highest values of restructuring options in case 2 under each assumption indicates that reducing mortgage rates and extending maturity could maximize gross benefits of borrowers. Capitalization of unpaid interest in case 3 leads to mortgage restructuring unfavourable for CPM borrowers since principal balance would be raised. Even a reduction of interest rates to 2.5% and an extension of maturity for 60 months could not give a turnaround. For interest only mortgages, case 1 is the least favourable as the option values turn to negative. That means the value of constant payment mortgage with lower interest rate is higher than interest only mortgage with lower interest rate. Among restructuring approaches, the case 2 is still the best. Furthermore, if property prices follow standard geometric Brownian motion, value of mortgage default is higher than value of restructuring options. However, we still need to consider execution costs before concluding which approach the borrower choose ultimately.

For the second pair of competing options (full prepayment in cash vs defeasance), we analyze which option borrowers would choose once having decided to early pay up mortgages. Prepayment in cash is much more favourable than defeasance as gross benefits of prepayment in cash is much greater than defeasance. Including interest only mortgages written on standard geometric Brownian motion of property price process, less favourable offers from defeasance would be attributed to low Treasury liquidity premium which leads to insignificant liquidity gain through pledging Treasury securities instead of settling in cash. However, our findings do not match with similar valuation of early termination options in participating mortgages by Varli and Yildirim (2015[43]) that defeasance values of that kind of mortgages are higher than prepayment. Inconsistency exists because different types of mortgages are investigated and American option pricing models are used in their study.

To make a final decision, execution costs driven by exercising any early termination options should be considered. Table 4 summarizes execution costs and net benefits of each type of options. Among four types of early termination options, borrowers bear the largest costs for default and full prepayment in cash. For default, opportunity costs driven by bankruptcy are treated as implicit execution costs, in contrast, other costs are explicit. Since estimated costs are significantly determined by future capital return, bankruptcy costs could not be precisely measured. Therefore, we should be cautious about interpreting this cost in figures. In contrast, relatively minimal modification fees for administrative costs are counted for executive costs of restructuring. Surprisingly, defeasance cost turns negative for interest only mortgages. This indicates that net gain is yielded when replicating cash flow pattern of interest only mortgages with Treasury securities at discounted prices. Less spending is needed for purchase of no-coupon discounted Treasury securities, thereby borrowers pay less comparing to prepayment in cash.

Regarding net benefits, all assumptions reach consistent conclusion for deciding to deny obligations - borrowers would choose restructuring by reducing interest rates and extending maturity rather than default. As there are huge and implicit executive costs for mortgage default, net losses are obtained for default. Regarding early paying up mortgages, interest only mortgages with LTV ratio at 50% written on standard geometric Brownian process would conclude that defeasance is more preferable than prepayment in cash. This is consistent with conclusion drawn by Varli and Yildirim. Other scenarios obtain much higher net benefits of prepayments in cash than defeasance. Thus, relatively inconsistent conclusion is drawn for early paying up mortgages.

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6 Scenario Analysis and Discussion

6.1 LTV Ratio and Fixed Rated Mortgage Rates

In order to examine the effects of mortgage design on early termination options, we analyze other scenarios varied by mortgage terms - (1) LTV ratios: 30%, 50%, 70% and 90%; (2) fixed mortgage rates: 5% and 7%.

Significant leverage effects on values of four early termination options are exhibited in Table 5, moreover significance varies by type of early termination and by market assumptions. Increases in LTV ratios lead to vigorously increase values of denying obligations but mixed impacts are found in values of early paying up mortgages. For both constant payment mortgages and interest only mortgages, greatest fluctuations in option values are shown in mortgage default and restructuring. In particular for constant payment mortgages, when LTV ratios rise from 30% to 90% (keeping mortgage rates unchanged), option values of restructuring by reducing interest rates and extending maturity (i.e. case 2) are more than 5890 folded. Defeasance values moderately increase with LTV ratios. In contrast, leverage effects on prepayment is less obvious. Mixed impact of LTV changes is exhibited for constant payment mortgages with 5% mortgage rates and interest only mortgages with 5% mortgage rates. In other scenarios, prepayment values slightly decrease with LTV ratios.

Upward adjustments in fixed mortgage rates exert obvious positive impacts on option values of mortgage default and all restructuring cases where the options are written on constant payment mortgages and mortgage default as well as the second and third restructuring cases where the options are written on interest only mortgages. In addition, negative impacts on prepayment values for CPM and IOM are exhibited while positive impacts on defeasance values for CPM and IOM are found.

Table 6 summarizes the related impacts on net benefits. The impacts are similar to those on option values. After considering execution costs, leverage effects on mortgage default become much stronger but are a little weakened for restructuring (except for the third case for constant payment mortgages with mortgage rates at 5%). Net loss is escalated by at least 3700 times if CPM or IOM borrowers choose default when LTV ratios increase from 30% to 90%. Therefore, borrowers would prefer restructuring by reducing interest rates and extending maturity as the highest net benefits can be obtained. Furthermore, leverage effects cause significant fluctuations in net benefits of prepayment and defeasance. Prepayment is more favourable for CPM borrowers in general. However, if LTV ratios reach 90%, net benefits of defeasance will be greater than prepayment in cash for interest only mortgages. For constant payment mortgages, an increase in original mortgage rates brings positive impacts on net benefits of restructuring and defeasance and negative impacts on default and prepayment. For interest only mortgages, positive and moderate impacts are exhibited on the second restructuring case and negative impacts are exerted on other options. We, therefore, conclude that lenders could pay attention on LTV ratios that might bring dramatic changes in their credit facilities.

[Insert Table 5 Here]
[Insert Table 6 Here]

6.2 The Impact Of Collateral Underlying Property Supply Constraints On Mortgage Terminations

In order to investigate how vigorous collateral underlying property supply constraints would affect values of early mortgage termination options in conjunction with demand elasticity, we conduct sensitivity tests on supply elasticity in four different levels of demand elasticity (i.e. -0.2: very inelastic; -0.6: inelastic; -1: unitary elastic; and -3: elastic). For supply elasticity, we classify five categories - 0 (perfectly inelastic), 0.2 (very inelastic), 0.6 (inelastic), 1 (unitary elastic) and 3 (elastic). Based on our research about estimation of office supply elasticity in the US, all office markets are supply inelastic. Therefore, we mainly focus on scenarios with inelastic supply.

Tables 7 and 8 exhibit the effects of tightening supply constraints on values and net benefits of early mortgage termination options. Under any assumptions, loosening supply constraints (i.e. increase in supply elasticity) would lower option values (except for defeasance) and reduce net losses or net benefits gained at any levels of demand elasticity. However, changes in demand elasticity do not bring significant difference in option values and net benefits. As shown in Figures 11 to 15, for constant payment mortgages, when supply becomes elastic, option values of mortgage default and restructuring turn to “zero" and values of prepayment in cash moderately drop. Similar features are found in scenarios for interest only mortgages (exhibited in Figures 16 to 20). In particular, loss of the first restructuring case is reduced by loosening supply constraints. However, defeasance values are less affected. This may be attributed to different dynamics related to Treasury liquidity premium. In general, the simulation results get in line with economic rationales - inelastic supply would boost larger fluctuations in collateral property prices and hence leads to greater probability of default, restructuring and prepayments.

Furthermore, figures in bold in Table 8 indicate that net benefits of prepayments for interest only mortgages could dramatically drop to the level which is lower than net benefits of defeasance if supply elasticity reaches the level of +1 (i.e. the collateral property supply is elastic). In a nutshell, defeasance is more favourable for IOM borrowers who pledge with properties in such supply elastic markets when they decide to prepay. In other words, controlling supply constraints could be an alternative risk management tool to alter IOM borrowers' decision which may reduce mortgage risks for lenders.

[Insert Table 7 Here]
[Insert Table 8 Here]
[Insert Figure 11 Here]
[Insert Figure 12 Here]
[Insert Figure 13 Here]
[Insert Figure 14 Here]
[Insert Figure 15 Here]
[Insert Figure 16 Here]
[Insert Figure 17 Here]
[Insert Figure 18 Here]
[Insert Figure 19 Here]
[Insert Figure 20 Here]

6.3 The Impact Of Restructuring Rates On Mortgage Restructuring Options

To understand profit maximization driven by lender's restructuring plan, we analyze 27 scenarios in different level of restructuring rates (1%, 2% and 4%) for constant payment mortgages and interest only mortgages respectively as shown in Tables 9 and 10. In general, increases in restructuring rates for any restructuring cases reduce option value and net benefits for both constant payment mortgages and interest only mortgages. For CPM, keeping the second and third restructuring rates constant, increase in the first restructuring rates from 1% to 2% reduces option values by 18%-27% while shifting the rates from 2% to 4% (also 100% increment) significantly lowers option values by 64%-68%. Similar levels of reduction are shown on net benefits. The changes lead to the first case sometimes more favourable than the second case, particularly setting the second restructuring rates at 4% that means higher interest payments are charged in total throughout the extended maturity. However, the third restructuring case is always least favourable. For IOM, whatever restructuring rates change among three restructuring cases, the second case is always most favourable and the first case is least favourable in terms of option values and net benefits. In other words, extending maturity brings greatest benefits to IOM borrowers but can be compensated by changing restructuring rates for CPM borrowers.

[Insert Table 9 Here]
[Insert Table 10 Here]

6.4 The Impact Of Treasury Liquidity Premium And Treasury Yield On Defeasance Options

In baseline scenario, option values of defeasance are much lower than prepayment in cash. This implies borrowers would choose prepaying mortgages in cash. Although borrowers would be charged prepayment penalty, penalty may not fully compensate re-investment risk for lenders. Defeasance in which cash flow patterns of mortgages are replicated can avoid re-investment risk. To make defeasance more attractive than prepayment in cash to borrowers, we examine the impact of Treasury liquidity premium and Treasury yield on defeasance options. Regarding adjustments in Treasury liquidity, we conduct sensitivity tests on steady Treasury liquidity premium, its volatility and speed of reversion in Table 11. In general, adjustments in Treasury liquidity premium are not sufficient to narrow the difference in values between prepayment and defeasance. Under any mortgage assumptions, we set up Treasury liquidity premium related parameters about 1.25 to 132 times higher than our baseline condition (steady rate: 32-132 times higher, volatility: 2-3 times higher, reverting speed: 1.25-2.5 times faster) and hence option values are 5200 and 6400 folded at most for CPM and IOM respectively. Huge escalations of option values still cannot help to rise up net benefits which almost remain unchanged. Interestingly, replicating cash flow patterns of interest only mortgage results in net gain because Treasury securities are priced at a discount but CPM has net loss. In other words, borrowers pay less than par value of mortgages. However, net gain is still lower than net benefits of prepayment. Thus, altering Treasury liquidity premium cannot stimulate borrowers changing their decision.

Table 12 summarizes the impact of Treasury yield on defeasance. Similar to Treasury liquidity, we set up related parameters of volatility and reverting speed which are nearly double of our baseline values. However, upward adjusting steady rates of Treasury yields results in mixed effects of option values. Positive relationships are found between option values and volatility of Treasury yield. Changes in reverting speed causes mixed impacts on defeasance value. In general, there are insignificant changes in net benefits. To conclude, it is hard to switch borrower's decision from prepayment in cash to defeasance even by tremendous changes in Treasury yields and Treasury liquidity.

However, adjusting LTV ratios can give a turnaround as we find that net benefits of defeasance are greater than that of prepayment in cash if LTV ratios for interest only mortgages reach 90% (figures in bold in Table 6). Increasing supply elasticity as shown in Table 8 could also help to reduce net benefits of prepayment in cash for IOM. That means lenders can control LTV ratios or related policymakers adjust property supply constraints to encourage IOM borrowers going defeasance if they decide to prepay, but there are no solutions for CPM yet.

[Insert Table 11 Here]
[Insert Table 12 Here]

7 Conclusion

We contribute to the research area of early mortgage termination by building a new two-stage theoretical model for pricing four types of early termination options, of which restructuring is considered as a termination strategy that is not found in extant literature. Our model features a breakthrough with three new characteristics: (1) introduce collateral underlying property-market-supply constraints into a property price process so as to capture more realistic market dynamics; (2) include mortgage restructuring as one of the competing options; and (3) set up two stages to precisely describe two competitions of early termination options in each condition. In a baseline scenario, we have found that prepayment is more valuable than denial options in the first stage. Moreover, once we switch from constant payment mortgages to interest only mortgages, values of both decisions (deny obligations or early pay up mortgages) escalate if LTV ratios remain constant. In the second stage, we conclude that mortgage restructuring by reducing mortgage rates and extending maturity would bring borrowers greater values rather than default. However, because of extremely low Treasury liquidity premium, defeasance is unfavourable relative to full prepayment in cash. Even though we vigorously adjust Treasury liquidity premium and Treasury yield, defeasance values are far below values of prepayment in cash, but adjusting LTV ratios or collateral underlying property supply constraints for interest only mortgages can help to enhance relative values of defeasance (i.e. net benefits of defeasance are greater than that of prepayment in cash). Also, net benefit gains are estimated in all three cases of restructuring and prepayment in cash for CPM and the second restructuring case and prepayment in cash for IOM. These imply the greater likelihood of restructuring mortgages in bad time and prepaying mortgages in cash during property market booms. Furthermore, we hypothesize that tightening property market supply constraints would push up values of early termination options since larger jumps or slumps in property prices would be foreseen. Our related sensitivity tests have verified our hypotheses.

Our model offers a new reference for credit markets after the financial crisis, in particular, interest rate hikes are expected, by introducing property market supply constraints into risk management for mortgage markets. We suggest loosening supply constraints would help to reduce risk in mortgage markets. Since loss exposure would be varied by types of early termination, our model could be used for developing strategies to minimize risk burden on financial institutions by attracting borrowers to exercise options in which smaller risks or less loss exposure are transferred to lenders.

In this paper, we assume that virtual option markets and property markets are complete although real transaction costs have been considered. In fact, we would argue that incomplete markets should be assumed. There are three reasons to support: discontinuous trading of underlying assets, options are non-tradable and cannot be replicated. Thus, in our companion paper, we would price four types of early termination options by an indifferent pricing approach and compare the results under different assumptions of complete and incomplete markets.

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Table 1: Summary of Boundary Conditions of Four Early Termination Options

Early Termination Type	Default	Mortgage Restructuring	Prepayment	Defeasance
First Stage:	Deny Obligation		Early Pay Up	
Boundary Condition	p(t=T) $< U_T$ + M		p(t=T) $> \frac{U_T+M}{LTV(p=p_0)} \vert r_m < h_m$	
Second Stage:				
Option Analogy	European Call on Put	Exchange Option (A)	European Call on Call	Exchange Option (B)
Cost	Bankruptcy Cost	Modification Fee	Prepayment Penalty	Defeasance Fee
Boundary Condition	NB(Default)>NB(Restructuring)	NB(Default)<NB(Restructuring)	NB(Prepay)>NB(Defease)	NB(Prepay)<NB(Defease)

Notes:
(A) Exchange option between mortgages with different cash flow structures
(B) Exchange option of more liquid assets with less liquid assets
(C) NB stands for net benefit calculated by [E(Option Values|p(t=T)) - Costs]

Table 2: Baseline Parameters

Description	Parameter	Initial	μ	σ	γ
Mortgage Rates	r	0.035	0.0611	0.0148	0.2
Mortgage Return	MR	0.05	0.06	0.02	0.2
Treasury Liquidity Premium	TP	0.0002	0.0003	0.05	0.08
Treasury Yield	TY	0.025	0.03	0.08	0.25
Property Value Process					
Property Values (USD/sqf)	p	280	172.1553	18.6535	
Computing Reverting Speed:		Value			
Volatility of Property Value Growth		0.3			
Rental Yield	q	0.07			
Property Supply (mn sqf)	m	60			
Accumulated Net Absorption (mn sqf)	ω	17			
Initial Absorption (mn sqf)	ω_0	25			
Under-damped Demand Adj (mn sqf)	β	5			
Office Using Employment (mn person)	F	0.3379			
Employment Growth	FG	0.0217			
Space Per Worker (sqf)	SP	200			
Supply Elasticity	SE	0.1325			
Demand Elasticity	DE	-0.4648			
Damping Factor		2.2105			
Driving Frequency (^{o})	ωt	45			
Angular Frequency (^{o})	αt	15			
Multiplier	c_1	21.2964			
Computed Speed of Reversion	γ_p	0.0157			
Mortgage Terms					
Initial Loan-to-Value Ratio	LTV_0	0.5			
Fixed Rate of FRM (p.a.)	h	0.05			
Maturity (Months)	n	120			
Property Size (sqf)	SZ	10000			
Mortgage Restructuring					
Restructured Mortgage Rates		0.025	(Case 1)		
		0.025	(Case 2)		
		0.025	(Case 3)		
Options Related Characteristics					
Bankruptcy Duration (Months)		17.685			
Post-Trough Return (%)		3			
Modification Fees (% of Principal)		0.005			
Prepayment Penalty Rate (%)		9			
Penalty Period (Months)		9.5			
Value Difference (Treasury Sec. & Loan in %)		-4			
Defeasance Transaction Fees (%)		0.033			

Notes:
μ: mean; σ: standard deviation; γ: reverting speed.

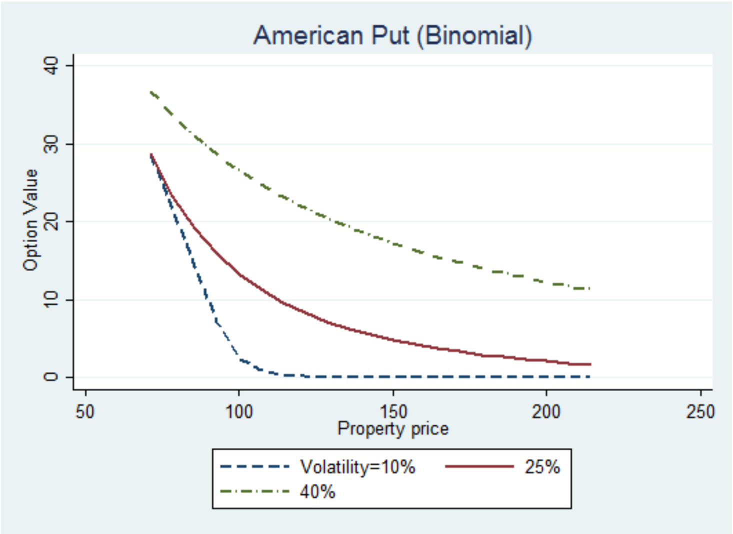

Figure 5: Comparison Of Analogies For Mortgage Default: American Put

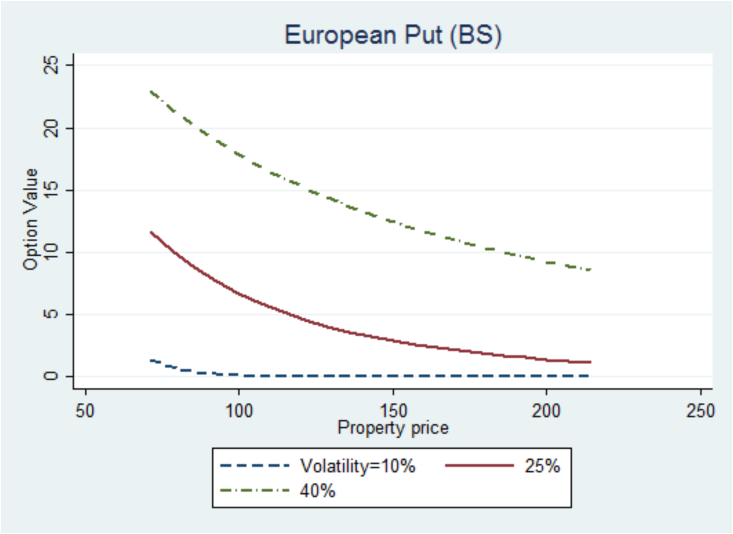

Figure 6: Comparison Of Analogies For Mortgage Default: European Put

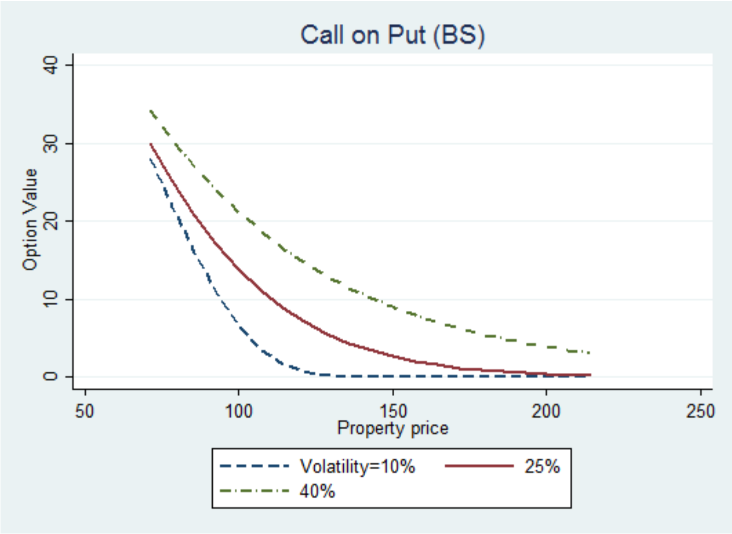

Figure 7: Comparison Of Analogies For Mortgage Default: Call on Put

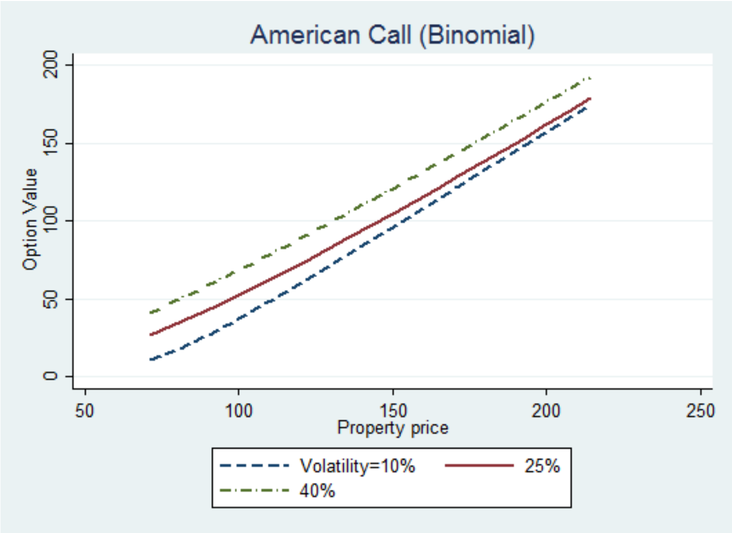

Figure 8: Comparison Of Analogies For Mortgage Prepayment: American Call

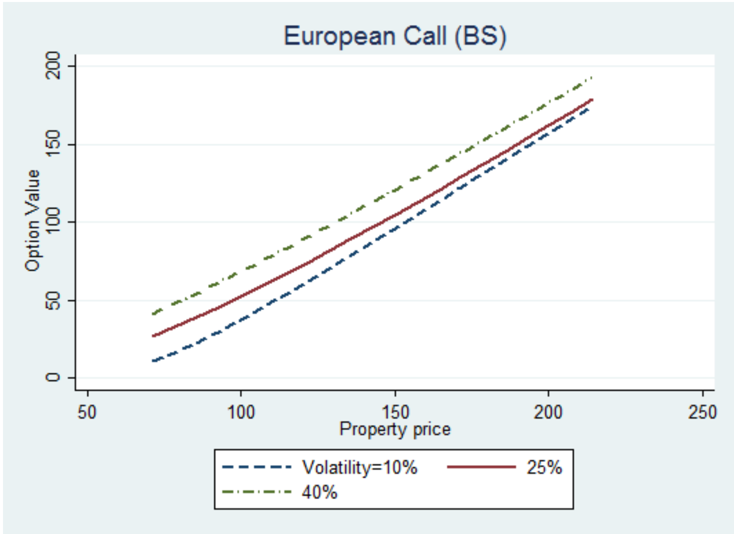

Figure 9: Comparison Of Analogies For Mortgage Prepayment: European Call

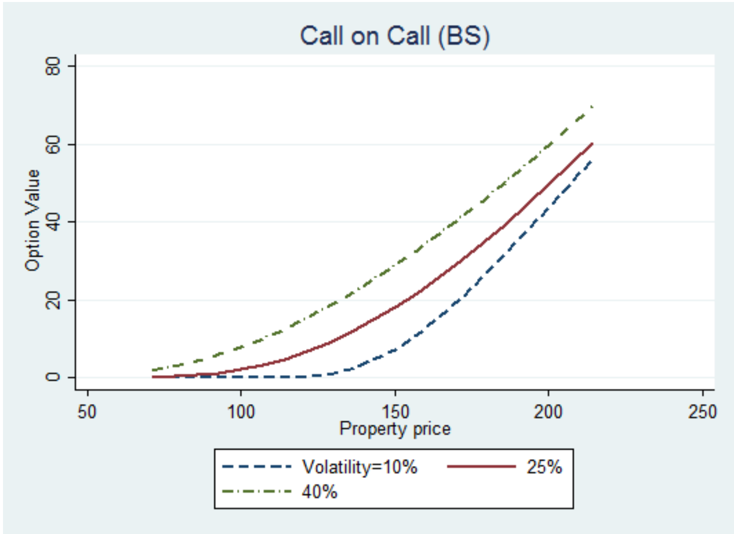

Figure 10: Comparison Of Analogies For Mortgage Prepayment: Call on Call

Table 3: Comparison Between The Setup With Suggested Property Price Process and Geometric Brownian Motion Process: Option Values

Options		**New property price process (Y/N)**					
		Y	Y	Y	Y	N	N
		CPM		IOM		CPM	IOM
		LTV=50%	70%	LTV=50%	70%	LTV=70%	LTV=50%
Denying Obligations		10.6658	78.0318	52.8719	366.7057	2273.051	2907.246
Mortgage Default		0.1148	5.6093	3.5825	108.3684	769.2288	1001.828
Mortgage Restructuring	Case 1	6.7239	43.9806	-0.0089	-0.0268	300.6206	-0.0543
	Case 2	8.1563	53.3733	40.8469	246.3774	378.6748	572.3707
	Case 3	0.4984	2.5051	0.6444	3.3895	9.6212	8.167
Early Paying Up Loan		1367.203	1358.794	1535.852	1525.089	7877.736	7725.732
Prepayment In Cash		279.7868	277.6082	334.4933	330.7832	788.8667	7.9253
Defeasance		3.5486×10^{-5}	6.2874×10^{-6}	5.2311×10^{-5}	7.3895×10^{-6}	6.0664×10^{-6}	2.2932×10^{-6}

Notes:
We set up CPM and interest only mortgages in the scenario at loan-to-value ratio = 70% and 50% respectively.
Case 1: A reduction of interest rates to 2.5%
Case 2: A reduction of interest rates to 2.5% and an extension of maturity for 60 months
Case 3: Capitalization of unpaid interest, a reduction of interest rates to 2.5%, and an extension of maturity for 60 months
If option values = 0, the option is not exercised.

Table 4: Comparison Between The Setup With Suggested Property Price Process and Geometric Brownian Motion Process: Net Benefit

Options		**New property price process (Y/N)**					
		Y	Y	Y	Y	N	N
		CPM		IOM		CPM	IOM
		LTV=50%	70%	LTV=50%	70%	LTV=70%	LTV=50%
Execution Costs							
Mortgage Default		120.6423	1912.511	812.0283	11505.47	23662.57	30236.78
Mortgage Restructuring		0.3252	2.1388	1.5872	9.6109	17.3724	22.2852
Prepayment In Cash		119.5749	166.4446	216.0576	299.4261	86.581	108.3895
Defeasance		24.8651	34.165	-20.317	-28.1566	20.4755	-10.1924
Net Benefit							
Mortgage Default		-120.5275	-1906.902	-808.4457	-11397.1	-22893.34	-29234.95
Mortgage Restructuring	Case 1	6.3987	41.8417	-1.5961	-9.6376	283.2481	-22.3396
	Case 2	7.8311	51.2345	39.2597	236.7665	361.3023	550.0855
	Case 3	0.1733	0.3663	-0.9428	-6.2213	-7.7513	-14.1182
Prepayment In Cash		160.2118	111.1636	118.4357	31.3571	702.2857	-100.4642
Defeasance		-24.8651	-34.1649	20.3171	28.1566	-20.4755	10.1924

Notes:
We set up CPM and interest only mortgages in the scenario at loan-to-value ratio = 70% and 50% respectively.
Case 1: A reduction of interest rates to 2.5%
Case 2: A reduction of interest rates to 2.5% and an extension of maturity for 60 months
Case 3: Capitalization of unpaid interest, a reduction of interest rates to 2.5%, and an extension of maturity for 60 months

Table 5: Effects Of LTV Ratios And Mortgage Rates On Values Of Early Termination Options

LTV_0	**h**	**DO**	**EP**	**Default**	**Restructure** C1	C2	C3	**Prepay**	**Defease**
Constant Payment Mortgage									
30%	5%	0.4666	1373.366	0	0.3161	0.3828	0.0291	287.2866	2.9128×10^{-6}
50%	5%	10.6658	1367.203	0.1148	6.7239	8.1563	0.4984	279.7868	3.5486×10^{-5}
70%	5%	78.0318	1358.794	5.6093	43.9806	53.3733	2.5051	277.6082	6.2874×10^{-6}
90%	5%	284.3792	1382.201	51.3611	141.0319	171.6705	7.315	278.9303	2.4664×10^{-5}
30%	7%	0.5623	1361.373	0	0.6531	0.7269	0.062	283.3886	8.1114×10^{-6}
50%	7%	11.5671	1361.981	0.129	13.2945	14.8377	0.8696	277.3221	3.6107×10^{-5}
70%	7%	84.214	1353.3617	6.1538	86.1537	96.2211	4.321	274.8917	6.3687×10^{-6}
90%	7%	304.4286	1366.206	55.0495	275.8586	308.5985	12.986	274.6954	3.2938×10^{-5}
Interest Only									
30%	5%	2.4067	1540.649	0.0017	0.0004	1.9751	0.0403	341.6812	3.2846×10^{-6}
50%	5%	52.8719	1535.852	3.5825	-0.0089	40.8469	0.6444	334.4933	5.2311×10^{-5}
70%	5%	366.7057	1525.089	108.3684	-0.0268	246.3774	3.3895	330.7832	7.3895×10^{-6}
90%	5%	1243.173	1549.598	772.1621	-0.0656	717.0187	10.0636	331.7011	3.192×10^{-5}
30%	7%	2.5551	1521.328	0.0025	0.0003	3.1773	0.093	335.444	9.0137×10^{-6}
50%	7%	53.403	1525.77	3.635	-0.0106	71.585	1.2737	330.0678	5.2311×10^{-5}
70%	7%	369.9183	1515.107	109.5637	-0.0262	431.3682	6.4626	326.1094	7.3895×10^{-6}
90%	7%	1254.453	1529.951	775.7465	-0.0592	1252.527	19.6025	325.7519	4.1549×10^{-5}

Table 6: Effects Of LTV Ratios And Mortgage Rates On Net Benefit Of Early Termination

LTV_0	h	**Default**	**Restructure**			**Prepay**	**Defease**
			C1	C2	C3		
Constant Payment Mortgage							
30%	5%	-2.074	0.3009	0.3676	0.0139	215.5584	-14.8669
50%	5%	-120.5275	6.3987	7.8311	0.1733	160.2118	-24.8651
70%	5%	-1906.902	41.8417	51.2345	0.3663	111.1636	-34.1649
90%	5%	-9767.137	134.0688	164.7074	0.352	62.9755	-44.8289
30%	7%	-2.3724	0.6362	0.7099	0.045	210.5463	-13.4624
50%	7%	-131.9174	12.9442	14.4874	0.5193	154.8815	-22.9539
70%	7%	-2065.766	83.8643	93.9317	2.0316	104.3393	-31.5236
90%	7%	-10488.26	268.428	301.168	5.5554	54.0613	-41.0046
Interest Only							
30%	5%	-12.2413	-0.0734	1.9013	-0.0335	212.5724	12.1408
50%	5%	-808.4457	-1.5961	39.2597	-0.9428	118.4357	20.3171
70%	5%	-11397.1	-9.6376	236.7665	-6.2213	31.3571	28.1566
90%	5%	-46785.16	-28.0272	689.0571	-17.898	**-56.6279**	**36.5166**
30%	7%	-12.5719	-0.0742	3.1028	0.0186	208.0328	11.9811
50%	7%	-813.1359	-1.6097	69.9859	-0.3254	115.0619	20.2182
70%	7%	-11457.27	-9.695	421.6994	-3.2062	28.0526	28.0278
90%	7%	-46895.43	-28.1067	1224.479	-8.445	**-59.6164**	**36.2382**

Table 7: Impact Of Property Supply and Demand Elasticity On Early Mortgage Termination - Constant Payment Mortgage

		Option Values						**Net Benefits**					
SE	**DE**	Default	Restructure			Prepay	Defease	Default	Restructure			Prepay	Defease
			C1	C2	C3				C1	C2	C3		
0	-3	0.7039	12.4837	15.1636	0.6976	314.5294	1.9584×10^{-5}	-257.581	11.874	14.5539	0.0879	195.8647	-25.0254
+0.2	-3	0.0622	4.9266	5.9424	0.2239	268.0823	2.4183×10^{-5}	-82.6246	4.6934	5.7091	-0.0093	148.2773	-24.57
+0.6	-3	0	0.2772	0.3313	0.0459	188.2733	8.7959×10^{-6}	-4.1274	0.2648	0.3189	0.0335	67.6452	-23.4342
+1	-3	0	0	0	0	146.8141	8.2716×10^{-8}	0	0	0	0	21.5946	-22.4602
+3	-3	0	0	0	0	9.7317	4.8179×10^{-6}	0	0	0	0	-21.8455	-1.1837
0	-1	0.8171	12.1687	14.7989	0.7547	319.9661	8.9148×10^{-6}	-249.4625	11.5712	14.2015	0.1573	200.9118	-24.9696
+0.2	-1	0.0979	4.8399	5.8485	0.3402	263.7208	4.6306×10^{-5}	-82.8941	4.6094	5.618	0.1097	144.5081	-24.5361
+0.6	-1	0	0.227	0.2708	0.0132	192.7656	1.4814×10^{-5}	-3.2222	0.2169	0.2606	0.0031	71.1402	-23.3812
+1	-1	0	0.005	0.006	0	147.0325	9.9283×10^{-6}	-0.0574	0.0048	0.0058	-0.0002	21.3769	-22.4633
+3	-1	0	0	0	0	9.405	0	0	0	0	0	-21.3745	-1.142
0	-0.6	0.6326	12.2165	14.8565	0.6188	318.6511	1.3648×10^{-7}	-243.0523	11.618	14.2579	0.0202	199.4625	-24.9915
+0.2	-0.6	0.0891	4.78	5.7816	0.3679	269.611	1.8094×10^{-5}	-81.9254	4.5523	5.5539	0.1402	149.5629	-24.7038
+0.6	-0.6	0	0.2028	0.243	0.0348	191.362	1.4751×10^{-6}	-3.1476	0.1937	0.2339	0.0257	70.1247	-23.4201
+1	-0.6	0	0	0	0	148.8542	2.7655×10^{-5}	0	0	0	0	22.2957	-22.4221
+3	-0.6	0	0	0	0	9.0543	0	0	0	0	0	-21.0264	-1.1409
0	-0.2	0.6981	12.5787	15.3007	0.9024	319.2728	7.8144×10^{-6}	-264.714	11.9607	14.6827	0.2844	200.9778	-24.8754
+0.2	-0.2	0.0653	4.6307	5.595	0.3043	266.5994	5.6867×10^{-6}	-75.5938	4.4108	5.3751	0.0844	147.1615	-24.6381
+0.6	-0.2	0.0013	0.256	0.3062	0.038	189.2759	1.3086×10^{-5}	-3.8065	0.2445	0.2948	0.0266	67.671	-23.411
+1	-0.2	0	0	0	0	146.92	1.7165×10^{-5}	0	0	0	0	21.4995	-22.4828
+3	-0.2	0	0	0	0	9.6516	3.1575×10^{-6}	0	0	0	0	-21.6741	-1.1658

Table 8: Impact Of Property Supply and Demand Elasticity On Early Mortgage Termination - Interest Only Mortgage

		Option Values						Net Benefits					
SE	**DE**	Default	Restructure			Prepay	Defease	Default	Restructure			Prepay	Defease
			C1	C2	C3				C1	C2	C3		
0	-3	9.9828	-0.0063	66.868	0.9429	368.7179	2.0541×10^{-5}	-1466.072	-2.5798	64.2945	-1.6305	155.8585	20.0163
+0.2	-3	1.7914	-0.0033	31.7307	0.3077	320.9675	3.0357×10^{-5}	-583.6573	-1.227	30.507	-0.916	104.4694	20.3585
+0.6	-3	0.0058	-0.0003	2.3056	0.06	241.9664	1.6644×10^{-5}	-34.7628	-0.0914	2.2146	-0.0311	19.2318	20.9449
+1	-3	0	0	0	0	202.4533	4.0721×10^{-7}	0	0	0	0	**-34.1395**	**22.248**
+3	-3	0	0	0	0	38.89	4.9461×10^{-6}	0	0	0	0	**-94.7357**	**12.5655**
0	-1	10.5108	-0.0032	66.9848	1.022	372.9742	1.5875×10^{-5}	-1483.353	-2.6168	64.3712	-1.5917	159.9914	20.0279
+0.2	-1	2.1435	-0.0038	30.4162	0.4363	317.6888	7.5279×10^{-5}	-566.868	-1.1912	29.2287	-0.7511	101.9525	20.2869
+0.6	-1	0.0025	0.0003	2.2167	0.0185	246.206	2.4331×10^{-5}	-31.6646	-0.088	2.1284	-0.0698	22.0297	21.0805
+1	-1	0	0	0.0085	0	202.2186	2.3556×10^{-5}	-0.0651	-0.0002	0.0083	-0.0002	**-34.6701**	**22.2759**
+3	-1	0	0	0	0	38.5104	0	0	0	0	0	**-94.4707**	**12.5049**
0	-0.6	8.9127	-0.0182	66.4544	0.8066	373.0068	2.7627×10^{-6}	-1455.692	-2.5952	63.8773	-1.7704	159.7062	20.0577
+0.2	-0.6	2.2958	-0.0039	31.2489	0.5346	323.0685	2.733×10^{-5}	-576.8517	-1.2074	30.0453	-0.6689	106.3021	20.3837
+0.6	-0.6	0.0504	0.0001	2.1804	0.0478	243.1685	5.655×10^{-6}	-32.2774	-0.0861	2.0941	-0.0385	19.7231	21.0117
+1	-0.6	0	0	0.0095	0	204.3719	5.5821×10^{-5}	-0.1869	-0.0005	0.009	-0.0005	**-34.4789**	**22.4604**
+3	-0.6	0	0	0	0	37.7467	0	0	0	0	0	**-93.1493**	**12.3088**
0	-0.2	10.0147	-0.0176	68.0085	1.2056	373.3812	1.8289×10^{-5}	-1518.676	-2.6655	65.3605	-1.4423	160.949	19.9761
+0.2	-0.2	2.3318	-0.0044	31.6093	0.4223	320.2506	1.775×10^{-5}	-583.1872	-1.2424	30.3713	-0.8157	104.4894	20.2891
+0.6	-0.2	0.0046	0.0002	2.1472	0.0505	242.7144	3.5174×10^{-5}	-30.0322	-0.0819	2.0652	-0.0316	**18.3206**	**21.1009**
+1	-0.2	0	0	0	0	202.9879	2.7333×10^{-5}	0	0	0	0	**-34.7403**	**22.3548**
+3	-0.2	0	0	0	0	38.6294	1.427×10^{-5}	0	0	0	0	**-94.3118**	**12.5012**

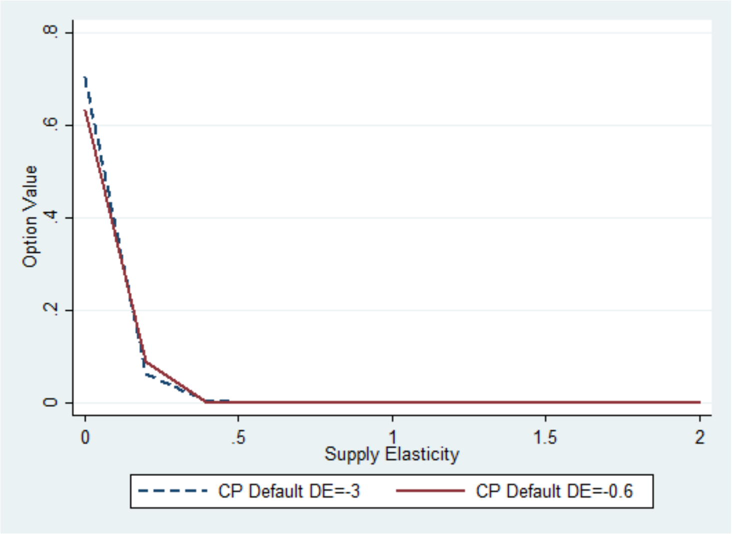

Figure 11: The Impact of Supply Elasticity on Default Options (CPM)

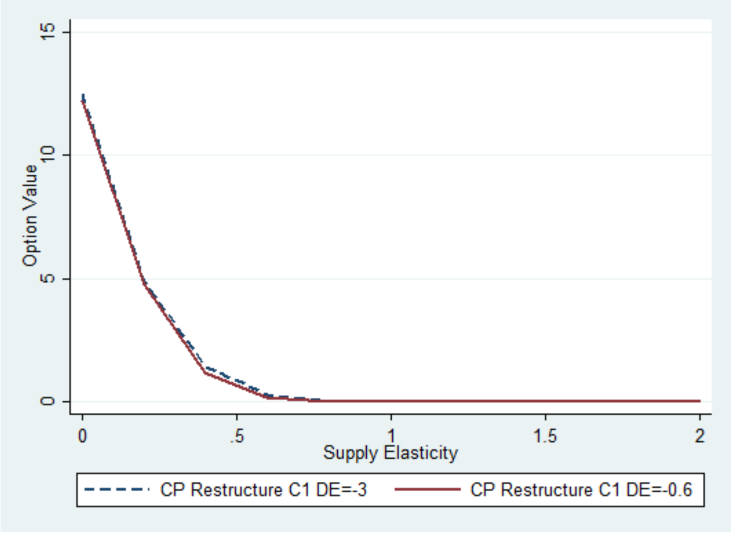

Figure 12: The Impact of Supply Elasticity on Mortgage Restructuring Options - Case 1 (CPM)

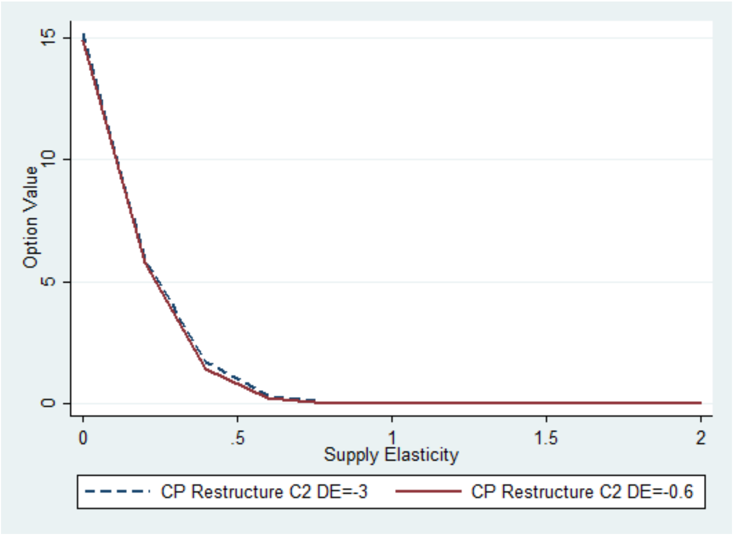

Figure 13: The Impact of Supply Elasticity on Mortgage Restructuring Options - Case 2 (CPM)

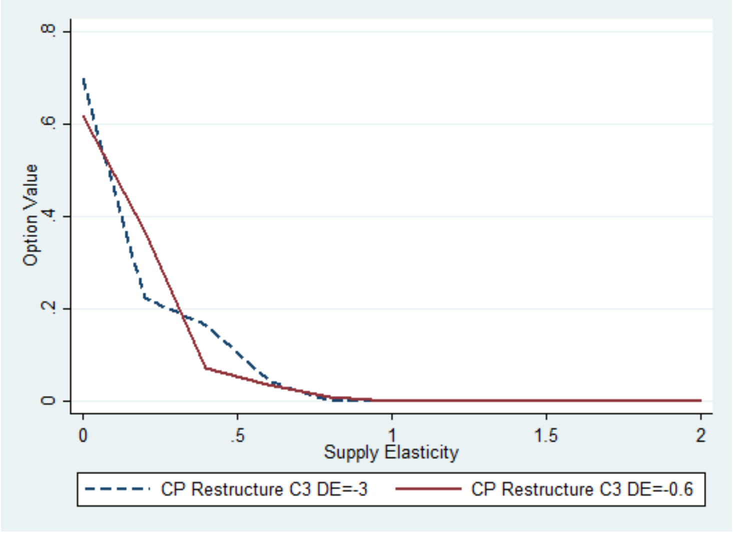

Figure 14: The Impact of Supply Elasticity on Mortgage Restructuring Options - Case 3 (CPM)

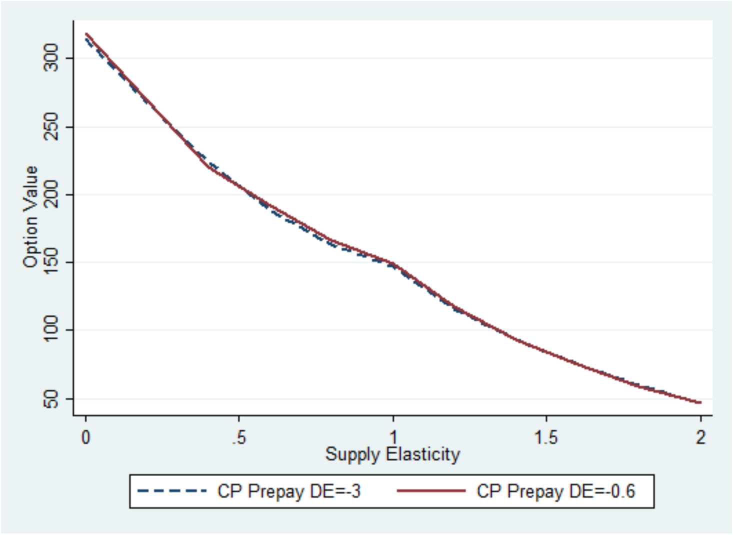

Figure 15: The Impact of Supply Elasticity on Prepayment Options (CPM)

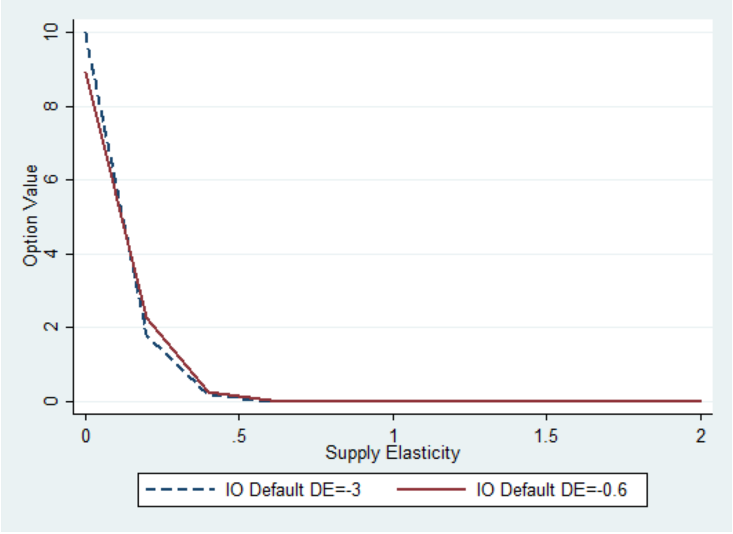

Figure 16: The Impact of Supply Elasticity on Default Options (IOM)

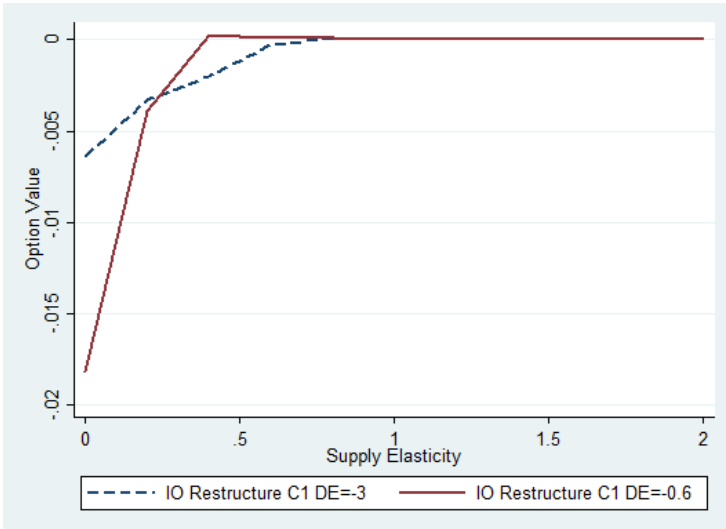

Figure 17: The Impact of Supply Elasticity on Mortgage Restructuring Options - Case 1 (IOM)

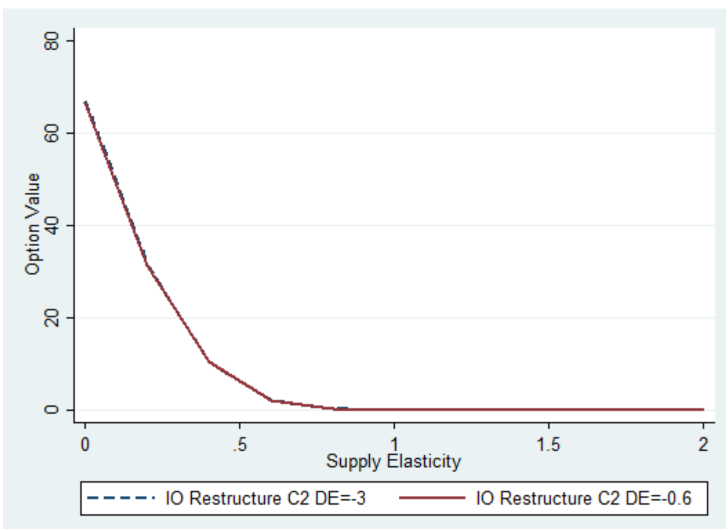

Figure 18: The Impact of Supply Elasticity on Mortgage Restructuring Options - Case 2 (IOM)

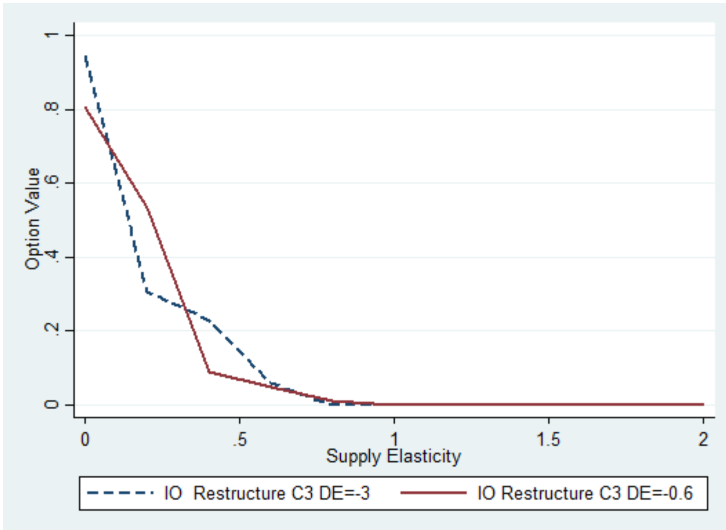

Figure 19: The Impact of Supply Elasticity on Mortgage Restructuring Options - Case 3 (IOM)

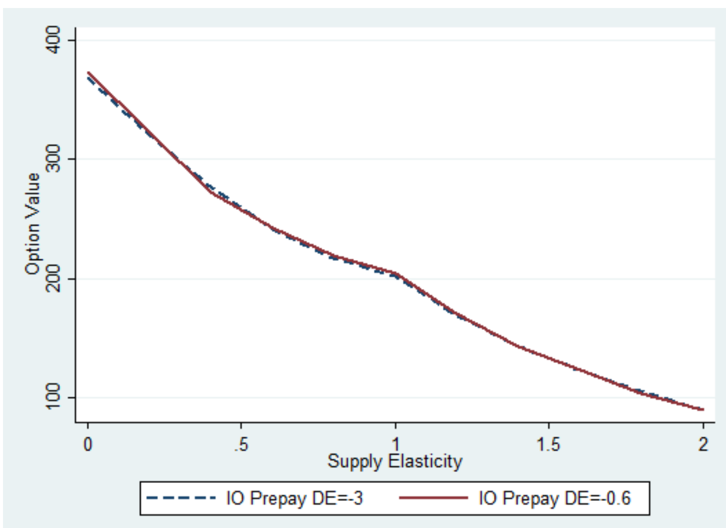

Figure 20: The Impact of Supply Elasticity on Prepayment Options (IOM)

Table 9: Impact Of Restructuring Rates On Mortgage Restructuring Options:CPM

Restructuring Rates			**Option Values**			**Net Benefit**		
C1	C2	C3	C1	C2	C3	C1	C2	C3
1%	1%	1%	10.5603	13.8763	0.8489	10.2351	13.5511	0.5237
2%	1%	1%	8.1295	14.015	0.7429	7.8032	13.6887	0.4166
4%	1%	1%	2.6653	13.5151	0.5792	2.35	13.1998	0.264
1%	2%	1%	10.7958	10.3134	0.7225	10.4652	9.9828	0.392
2%	2%	1%	8.0408	10.1046	0.5894	7.7172	9.7811	0.2659
4%	2%	1%	2.8499	10.5241	0.7745	2.5119	10.1861	0.4365
1%	4%	1%	10.6471	2.109	0.6093	10.3214	1.7834	0.2836
2%	4%	1%	7.8831	2.0393	0.6824	7.5637	1.72	0.363
4%	4%	1%	2.6437	2.0106	0.6482	2.3322	1.6991	0.3366
1%	1%	2%	10.2124	13.3809	0.5384	9.9007	13.0691	0.2267
2%	1%	2%	8.1275	14.0582	0.4192	7.7982	13.7289	0.0898
4%	1%	2%	2.7955	14.086	0.6568	2.4679	13.7584	0.3292
1%	2%	2%	10.2112	9.7505	0.4274	9.8985	9.4378	0.0115
2%	2%	2%	8.1463	10.2323	0.665	7.8193	9.9053	0.3381
4%	2%	2%	2.8605	10.4991	0.6143	2.5253	10.1639	0.2791
1%	4%	2%	10.6991	2.1166	0.5432	10.3728	1.7903	0.2169
2%	4%	2%	7.8302	2.0288	0.4199	7.515	1.7136	0.1047
4%	4%	2%	2.7938	2.1319	0.5255	2.4632	1.8014	0.195
1%	1%	4%	10.5897	13.879	0.07248	10.2661	13.5555	-0.251
2%	1%	4%	8.3676	14.4553	0.1187	8.0296	14.1173	-0.2261
4%	1%	4%	2.7656	13.9605	0.0981	2.4399	13.6348	-0.2276
1%	2%	4%	10.38	9.9232	0.0959	10.0607	9.6038	-0.2235
2%	2%	4%	7.7446	9.7286	0.1004	7.433	9.417	-0.2112
4%	2%	4%	2.6543	9.7477	0.1201	2.3425	9.436	-0.1917
1%	4%	4%	10.7061	2.0933	0.0684	10.3768	1.764	-0.2609
2%	4%	4%	8.1736	2.1354	0.1481	7.846	1.8077	-0.1796
4%	4%	4%	2.6528	2.0158	0.0944	2.3401	1.7031	-0.2183

Table 10: Impact Of Restructuring Rates On Mortgage Restructuring Options:IOM

Restructuring Rates			**Option Values**			**Net Benefit**		
C1	C2	C3	C1	C2	C3	C1	C2	C3
1%	1%	1%	-0.0061	62.7569	0.9948	-1.5933	61.1697	-0.5924
2%	1%	1%	0.0009	65.2259	0.8993	-1.6376	63.5875	-0.7391
4%	1%	1%	-0.011	62.1009	0.7127	-1.603	60.5089	-0.8793
1%	2%	1%	-0.017	49.2871	0.8099	-1.6327	47.6714	-0.8058
2%	2%	1%	-0.0129	48.6339	0.7041	-1.612	47.0348	-0.895
4%	2%	1%	-0.0187	48.7171	0.8925	-1.6206	47.1152	-0.7094
1%	4%	1%	-0.0021	18.3449	0.7262	-1.6149	16.7321	-0.8866
2%	4%	1%	-0.0053	17.8499	0.8307	-1.6205	16.2347	-0.7845
4%	4%	1%	-0.0058	17.6499	0.8153	-1.6167	16.0391	-0.7956
1%	1%	2%	0.0027	62.7641	0.7067	-1.5943	61.1672	-0.8902
2%	1%	2%	-0.0167	64.4136	0.5417	-1.6448	62.7854	-1.0864
4%	1%	2%	-0.0042	63.7912	0.8653	-1.6311	62.1642	-0.7616
1%	2%	2%	-0.0023	47.5312	0.5304	-1.5789	45.9546	-1.0462
2%	2%	2%	0.0026	50.1164	0.8627	-1.6419	48.4718	-0.7819
4%	2%	2%	-0.0002	49.5578	0.818	-1.6255	47.9325	-0.8073
1%	4%	2%	0.0029	18.8003	0.6997	-1.6355	17.1618	-0.9388
2%	4%	2%	-0.0082	17.288	0.548	-1.6002	15.696	-1.044
4%	4%	2%	-0.0227	18.2905	0.613	-1.6385	16.6747	-1.0028
1%	1%	4%	-0.0111	63.2327	0.1872	-1.6102	61.6336	-1.412
2%	1%	4%	-0.0154	63.3851	0.2192	-1.6173	61.7832	-1.3827
4%	1%	4%	-0.0079	64.0131	0.2063	-1.6207	62.4003	-1.4065
1%	2%	4%	-0.0037	48.7676	0.2442	-1.6189	47.1524	-1.371
2%	2%	4%	-0.0028	48.4169	0.2392	-1.6136	46.806	-1.3717
4%	2%	4%	-0.0027	48.2544	0.2749	-1.5996	46.6575	-1.322
1%	4%	4%	-0.0145	18.5063	0.171	-1.6427	16.8782	-1.4572
2%	4%	4%	-0.0014	17.7744	0.3329	-1.6284	16.1475	-1.294
4%	4%	4%	-0.0075	17.487	0.1904	-1.5841	15.9103	-1.3863

Table 11: Impact Of Treasury Liquidity Premium On Defeasance

μ_{TP}	σ_{TP}	γ_{TP}	**Option Values**		**Net Benefit**	
			CPM	IOM	CPM	IOM
1%	5%	5%	7.6485×10^{-5}	1.4109×10^{-4}	-24.8691	20.364
2%	5%	5%	8.4722×10^{-5}	1.2464×10^{-4}	-24.6214	20.2071
4%	5%	5%	9.074×10^{-5}	1.5314×10^{-4}	-24.5723	20.314
1%	10%	5%	0.0071	0.013	-24.6713	20.2872
2%	10%	5%	0.0089	0.0156	-24.6927	20.2291
4%	10%	5%	0.0124	0.0223	-24.7947	20.1703
1%	15%	5%	0.0514	0.0935	-24.8306	20.4106
2%	15%	5%	0.0568	0.1024	-24.775	20.3659
4%	15%	5%	0.069	0.1263	-24.6726	20.3447
1%	5%	10%	5.7571×10^{-5}	6.8444×10^{-5}	-24.4833	20.1549
2%	5%	10%	6.8183×10^{-5}	1.0426×10^{-5}	-24.8934	20.2824
4%	5%	10%	2.768×10^{-4}	4.7055×10^{-4}	-24.7154	20.1446
1%	10%	10%	0.0085	0.0153	-24.485	20.2325
2%	10%	10%	0.0123	0.0227	-24.557	20.1885
4%	10%	10%	0.0223	0.0397	-24.7052	20.3105
1%	15%	10%	0.0564	0.1052	-24.8319	20.4284
2%	15%	10%	0.0649	0.1211	-24.5038	20.2944
4%	15%	10%	0.1037	0.1936	-24.6766	20.4867
1%	5%	20%	4.638×10^{-5}	6.9127×10^{-5}	-24.9159	20.3098
2%	5%	20%	3.2452×10^{-4}	6.0511×10^{-4}	-24.5674	20.2431
4%	5%	20%	0.003	0.053	-24.6773	20.217
1%	10%	20%	0.0134	0.0239	-24.5284	20.1802
2%	10%	20%	0.0217	0.0388	-24.5759	20.2419
4%	10%	20%	0.0577	0.1064	-25.0231	20.4491
1%	15%	20%	0.0639	0.1193	-24.6156	20.3334
2%	15%	20%	0.0984	0.1829	-24.5129	20.4465
4%	15%	20%	0.1834	0.3357	-24.3135	20.4132

Table 12: Impact Of Treasury Yield On Defeasance

μ_{TY}	σ_{TY}	γ_{TY}	**Option Values**		**Net Benefit**	
			CPM	IOM	CPM	IOM
1%	5%	10%	0	0	-24.978	20.2963
2%	5%	10%	1.1479×10^{-5}	3.1625×10^{-5}	-24.6956	20.3509
4%	5%	10%	1.6898×10^{-5}	2.4779×10^{-5}	-24.7001	20.1225
1%	10%	10%	5.1364×10^{-5}	7.4139×10^{-5}	-24.9365	20.2805
2%	10%	10%	8.7313×10^{-5}	1.3581×10^{-4}	-24.8999	20.3672
4%	10%	10%	1.5655×10^{-4}	1.915×10^{-4}	-24.6969	20.2627
1%	15%	10%	0.0012	0.0022	-24.6212	20.1922
2%	15%	10%	0.0016	0.0027	-24.7153	20.2404
4%	15%	10%	0.0013	0.0022	-24.7299	20.2061
1%	5%	20%	1.1379×10^{-6}	1.8876×10^{-6}	-24.6055	20.0977
2%	5%	20%	3.2824×10^{-6}	6.8521×10^{-6}	-24.5982	20.1373
4%	5%	20%	4.6207×10^{-6}	9.3355×10^{-6}	-24.7777	20.3395
1%	10%	20%	1.1839×10^{-4}	1.6584×10^{-4}	-24.6576	20.1904
2%	10%	20%	4.2149×10^{-5}	6.362×10^{-5}	-24.6443	20.2273
4%	10%	20%	2.293×10^{-5}	8.7119×10^{-5}	-24.5612	20.1953
1%	15%	20%	0.0013	0.0023	-24.6285	20.119
2%	15%	20%	0.0013	0.0023	-24.7806	20.2517
4%	15%	20%	0.0014	0.0029	-24.5204	20.2782
1%	5%	40%	2.2516×10^{-6}	3.0932×10^{-6}	-24.5596	20.1405
2%	5%	40%	1.1708×10^{-6}	3.0724×10^{-6}	-24.7425	20.203
4%	5%	40%	4.5621×10^{-6}	7.9997×10^{-6}	-24.8866	20.3514
1%	10%	40%	2.7951×10^{-5}	3.3055×10^{-5}	-24.6354	20.2349
2%	10%	40%	2.6231×10^{-5}	6.3416×10^{-5}	-24.8248	20.2699
4%	10%	40%	3.261×10^{-5}	6.5237×10^{-5}	-25.2185	20.4033
1%	15%	40%	9.711×10^{-4}	0.0018	-24.8643	20.333
2%	15%	40%	0.0012	0.0021	-24.861	20.1467
4%	15%	40%	0.0018	0.0031	-24.6821	20.1591