Optimal Financing and Disclosure*

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Abstract

How does disclosure policy depend on the choice of financing? I study an entrepreneur who finances a project with uncertain cash flows and jointly chooses the disclosure and financing policies. I show that it is optimal to truthfully reveal whether the project's cash flows are above a threshold. This class of threshold policies is optimal for any prior belief, for any monotone security, and any increasing utility function of the entrepreneur. It can therefore serve as a benchmark. I characterize how the disclosure threshold depends on the underlying security, the prior, and the cost of investment. The financing choice of the entrepreneur is determined by a new tradeoff between the likelihood of persuading investors and relinquishing cash flow rights. Absent further frictions, the optimal security can be equity, debt, and options. If the entrepreneur can steal the cash flows after they have realized, the optimal security is debt and the threshold policy remains optimal.

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JEL Codes: G32, D82, D83

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1 Introduction

Firms that seek financing have discretion on how much information to disclose to investors. They have access to different communication channels which allow for potentially complex disclosure strategies, such as conference calls, voluntary earnings forecasts, IPO prospectuses, and annual reports. Empirical research has found evidence that all of these channels indeed convey information.[1] It is then quite difficult to imagine that a firm's disclosure can be summarized by a simple policy.[2] Yet, this complexity is not captured in models of information provision.

In this paper, I establish a canonical benchmark. If an entrepreneur must raise money to finance a discrete investment project, the optimal information policy is a threshold policy. That is, the entrepreneur truthfully reveals whether or not the cash flows of the project are above a threshold. This policy is derived without any a priori restrictions on what constitutes an admissible disclosure policy.[3] It is also robust to changes in investors prior beliefs, the underlying security issued, the utility of the entrepreneur, additional noise, and moral hazard. Because of this, it serves as a robust benchmark of how firms should provide information.

Importantly, this result rationalizes existing empirical findings. For example, Hutton et al. (2003) and Baginski et al. (2004) show that when firms voluntarily disclose earnings forecasts, they disclose to investors an interval of possible realizations. This corresponds exactly to the threshold strategy derived in this paper. Changes in the threshold can be mapped directly into empirical measures of information quality such as analyst forecast errors or price reversals, which allow the model to yield testable predictions.

Naturally, the entrepreneur's disclosure policy must depend on how it finances its investment. The value of the security offered to investors depends on the information the entrepreneur provides, and, conversely, the value of information depends on how the secu-

[1] Bowen et al. (2002) find that analysts participating in conference calls have lower forecast error, Balakrishnan et al. (2014) find earnings guidance increases share liquidity, Hanley and Hoberg (2010) show that IPO prospectuses contain information which helps increase pricing accuracy, and Ball et al. (2014) show that the Management Discussion and Analysis section of a company's annual report contains information beyond that contained in earnings forecasts.

[2] In the past, the literature has used e.g. normal priors and signals (Admati and Pfleiderer (2000)) or all-or-nothing disclosure (Dye (1985)).

[3] Except measurability.

rity issued reacts to that information. Here, I uncover a new tradeoff. As the entrepreneur promises higher payouts to investors, they become *easier to convince*. That is, they become willing to finance the project when their expectations about its cash flows are lower. The entrepreneur can then optimally use a disclosure policy which provides the investors with *worse* information.[4] At the same time, pledging payments to investors reduces the residual value to the entrepreneur. How to finance the firm, i.e. which security to issue and how much to pledge, is determined via this tradeoff. Absent further frictions, the security itself is indeterminate and the optimum can be implemented with debt, equity, call options, and many others.

A particularly appealing feature of my model is that the information provided to investors is solely determined by the firm's financing constraint, which allows me to derive clean comparative statics. When faced with higher ex-ante cash flows, a lower investment cost, or an existing security with higher payouts, the entrepreneur optimally provides worse information (i.e. a lower threshold). Interestingly, this also happens when the security is more information sensitive, in the sense of DeMarzo et al. (2005).

The intuition for the main results is as follows. In my model, an entrepreneur has private information about the project's cash flows and designs a disclosure strategy, which is a mapping from the project's realized payoffs to a potentially random message. Investors form posterior beliefs about the payoff using Bayesian updating after observing the message. The price of the security is then determined competitively. The project is financed whenever the amount raised exceeds a fixed cost of investment, which occurs only if investors believe the project's payoffs are sufficiently high. The entrepreneur has a private benefit of control, so that she prefers to finance the project even if the payoffs are low.[5] In equilibrium, the optimal disclosure policy induces two possible posteriors, an "optimistic" and a "pessimistic" one.[6] The project is financed under the first, but not under the second.

A threshold strategy is optimal because the security's payoff is increasing in the project's payoff, which implies that the optimistic posterior must put all mass above a threshold. If its support were disconnected, the entrepreneur could induce a different posterior which moves

[4]Precisely, the threshold is lower, which means that investors payoffs conditional on financing the project are lower. Thus, the information provided leaves investors worse off. In this sense, it is worse.

[5]Without this private benefit, full disclosure would be optimal.

[6]I call a posterior "more optimistic" than another if it first-order stochastically dominates it. Thus, under a more optimistic posterior, the expected payoffs to investors from any security are higher.

mass from the lower realizations to fill the gap in the support, which would increase both the entrepreneur's and investors' valuation.

The threshold is chosen so that at the optimistic posterior, the financing constraint binds. Because of Bayesian updating, investors' beliefs must follow a martingale, which implies that inducing a more optimistic posterior must reduce the probability with which it is realized. Thus, there is a tradeoff between convincing investors that the project is good and the likelihood that it is financed. Manipulating the investors' beliefs can never increase the expected payments for the security due to the martingale property. Therefore, it is optimal to induce the least optimistic posterior at which investors are willing to pay enough to cover the cost of investment.

The choice of financing is determined by the following tradeoff. Increasing the promised payoff to investors in some state decreases the entrepreneur's realized payoff whenever the project is financed. However, it relaxes the financing constraint, since investors are willing to finance the project at a less optimistic posterior. Thus, increasing the payoff of the security allows the entrepreneur to choose a disclosure policy that increases the likelihood that the project is financed, which increases the entrepreneur's expected payoff. The second effect always dominates. It is optimal to sell off as much of the project as possible, until either investors hold all claims, or until the disclosure is such that the project is financed whenever it social benefit is positive. Even though disclosure is imperfect and there is a uncertainty remaining after the message is observed, the particular security issued is irrelevant. The optimum can be implemented by equity, debt, call options, and many other securities. This is because the financing constraint must bind when the project is financed and investors are indifferent between any security that promises the same expected payoffs under the optimistic posterior.

With additional frictions, uniqueness of the security choice can be restored. I show this by adding moral hazard, in the sense of Gale and Hellwig (1985), into my setting. Specifically, the entrepreneur can steal the cash flow after it realizes and designs an incentive compatible security. Additionally she can disclose information about the project's productivity, which affects cash flows. Then, the threshold policy I find in my baseline setting remains optimal and the optimal security is debt. In other words, threshold policies are robust to adding moral hazard.[7] Conversely, the optimality of debt in Gale and Hellwig (1985) is robust to

[7]Moral hazard in the sense of Gale and Hellwig (1985) specifically.

adding disclosure, in the sense considered in this paper.

My model builds on the Bayesian persuasion approach of Kamenica and Gentzkow (2011), who study a model in which a sender manipulates the belief of a receiver by revealing information. They show that the problem can be restated as directly maximizing over distributions of posterior beliefs subject to the *Bayes plausibility constraint*, which requires the average posterior to equal the prior. The optimal value is then characterized as the concave closure of the sender's value under any given posterior. In my paper, instead of characterizing the concave closure directly, I decompose the problem into characterizing the induced posteriors for a given probability that the project is financed, and then optimizing over that probability. The first problem is an infinite dimensional linear program and its solution can be found by characterizing the dual.

A large literature in economics, finance, and accounting studies optimal disclosure (seminal theoretical work includes Verrecchia (1983), Dye (1985), Fishman and Hagerty (1989), Fishman and Hagerty (1990), Diamond and Verrecchia (1991), and Admati and Pfleiderer (2000)) between firms and investors.[8] My paper is, to the best of my knowledge, the first to characterize the optimal disclosure policy of an entrepreneur seeking to finance a project without any a priori restrictions on security issued, prior distribution, or disclosure strategy. It is also the first to point out that threshold strategies may be generically optimal when financing an investment project.

A recent related paper is Goldstein and Leitner (2015), who study the design of optimal stress tests in a setting with finitely many states. Their optimal policy assigns two scores depending on the value of bank capital. In this paper, I focus on the interactions between the financing choice and the disclosure policy, which is absent from their work.

Monnet and Quintin (2015) study the effect of information disclosure on liquidity in private equity markets. They model disclosure as choosing a message which truthfully reveals whenever the state is in a given subset of the state space and they allow the sender to choose this subset ex ante, which is more restrictive than the policy in my paper. In their paper, the project is continued whenever it is revealed that its success probability is above a threshold. However, the firm in their setting is financed with equity only, so their model stays silent on

[8] The literature is too large to be surveyed here. Excellent recent reviews of both theoretical and empirical contributions can be found in Healy and Palepu (2001), Holthausen and Watts (2001), and Beyer et al. (2010).

the interaction of financing choice and disclosure.

In contemporaneous work, Trigilia (2016) also considers a joint problem of security choice and disclosure. In his model, disclosure is costly and the entrepreneur chooses the probability with which the state is truthfully revealed. After investment has taken place, there is a moral hazard problem. Depending on the parameters, the optimal incentive compatible contract is either debt or a mix of debt and equity. In my model, debt remains optimal despite the entrepreneur providing information and the optimal disclosure policy is still a threshold strategy. The results differ because the disclosure policy in my paper is more general, which affects the optimal security. Also, in Section 6.3 I assume that the entrepreneur can disclose the project's productivity, not cash flows themselves as in Trigilia's paper.[9]

In my paper, the entrepreneur alters the beliefs of the investors in different states in order to obtain financing, subject to a condition linking the average posterior to the prior belief. In the literature on hedging (e.g. Froot et al. (1993)), the entrepreneur has a similar problem where she moves wealth across states and under a fairly priced hedge, her expected wealth must equal her wealth ex-ante. With limited liability however, hedging is less general than designing a security, so allowing the entrepreneur to hedge would not affect my results.[10]

Recently, there has been considerable empirical interest in the information content of firms' communication. Since information content is not directly measurable, existing papers have used observables such as analyst forecast errors, liquidity measures, or underpricing to quantify the degree of information. My optimal disclosure policy maps into the approaches found in Bowen et al. (2002), who use analyst forecast errors as a measure of informativeness and Kogan et al. (2010), who use return volatility. If we interpret my model as an IPO, the issue price is the average value of the security under the posterior, since the price is determined competitively. Once the realized payoff becomes known, the price must adjust. For a higher disclosure threshold, the average magnitude of price changes after issuance is lower, which is in line with Hanley and Hoberg (2010), who use the magnitude realized price changes after an IPO to measure the information content of IPO prospectuses. As I show in

[9]This is necessary. With costless disclosure, allowing the entrepreneur to disclose cash flows directly would eliminate the moral hazard problem, rendering the setting trivial and any comparison to Trigilia's work moot.

[10]If, in my model, the entrepreneur were allowed to hedge without being constrained by limited liability, she would always obtain the first-best. Specifically, she could incur a loss in states where financing the project has negative social value and use this to finance the project when the social value is positive.

7, we can directly compute how a change in threshold alters these measures of information. Then, the model can be used to generate testable implications.

The paper proceeds as follows. In Section 2, I set up the model. In Section 3, I solve the optimal disclosure policy for the simple case when the state space is binary in order to establish a simple intuition for the tradeoffs involved. Readers familiar with the Bayesian persuasion literature following Kamenica and Gentzkow (2011) may wish to skip ahead to Section 4, where I solve for the optimal disclosure policy with a continuous state space. I derive the optimal security to issue in Section 5. In Section 6, I show that the threshold strategy remains optimal for any increasing, continuous utility function of the entrepreneur and consider the case when the entrepreneur cannot fully reveal the payoff due to exogenous noise. I also study moral hazard in this section. To ease exposition, all proofs are deferred to Appendix A.

2 Model

A risk-neutral entrepreneur is endowed with a project with uncertain payoff $s \in [0,1]$. The project has a cost of investment $I \in (0,1)$, which the entrepreneur must raise by selling a security with payoffs $c(s)$ to a unit mass of risk-neutral investors. Both the investors' payoff $c(s)$ and the entrepreneur's residual $s - c(s)$ are increasing in s and $c(s)$ is continuous.[11] Both parties are protected by limited liability, so that $c(s) \in [0,s]$.

The project's payoff is private information. Ex ante, both entrepreneur and investors have the same prior over s, which admits a continuous density $\mu_0(s)$ with support $[0,1]$. The entrepreneur designs a public signal $\sigma : [0,1] \to \Delta([0,1])$ which is observable by all investors and which she can condition on the realized state. A signal σ sends a potentially random message $m \in [0,1]$ distributed with measure $\sigma(s)$.[12] Upon observing the realization, investors form a posterior belief about the state μ according to Bayes rule. For now, I only let the entrepreneur choose the disclosure policy. I will consider the joint choice of disclosure and financing in Section 5.

[11]This assumption is standard to ensure tractability in the security design literature. See e.g. Innes (1990), Nachman and Noe (1994), and Harris and Raviv (1989). Common securities such as equity, debt, and options satisfy the assumption.

[12]Here, $\Delta(S)$ is the set of all probability distributions over a set S. Following Kamenica and Gentzkow (2011), it is without loss of generality to assume that $m \in [0,1]$.

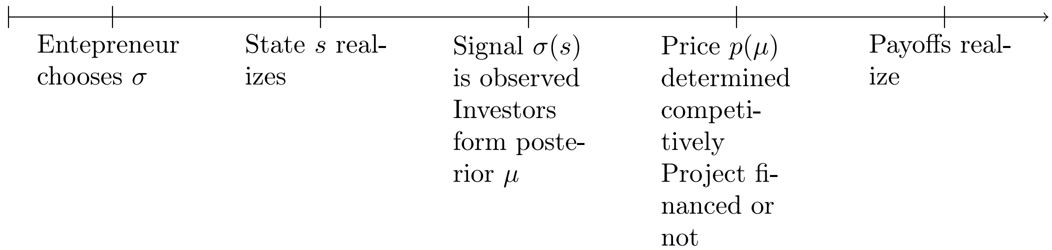

Figure 1: Timeline

Given posterior μ, the price for the security $p(\mu)$ is set competitively via the investors' zero-profit condition

$$p(\mu) = E_\mu c(s). \tag{1}$$

The project is financed whenever the amount raised exceeds the investment cost I,[13]

$$p(\mu) \geq I. \tag{2}$$

In addition to the residual payoff $s - c(s)$, the entrepreneur enjoys a private benefit of control $B > 0$ whenever the project is financed and, in case the amount raised exceeds the cost of investment, she also receives a fraction of the excess cash raised $\lambda(E_\mu c(s) - I)$, where $\lambda \in [0, 1]$ denotes a transaction cost. The entrepreneur's realized payoff is therefore

$$V(s, p) = s - c(s) + B + \lambda(p - I).$$

When $\lambda = 0$, the entrepreneur receives all additional cash while when $\lambda = 1$, the model is equivalent to the security being sold at a fixed price I.[14] Because of the private benefit, the entrepreneur would prefer to finance the project even if she knew that the payoffs are zero.[15] Whenever the project is not financed, both entrepreneur and investors receive zero. The timeline is summarized in Figure 1.

[13] With a unit mass of investors, $p(\mu)$ is simultaneously the price of the security and the amount raised.

[14] For example, the entrepreneur might bee seeking commitment from investors to buy the security by offering a fixed price, in which case the investors appropriate the surplus.

[15] This is the reason why truthful disclosure is not optimal in equilibrium.

Conditional on posterior μ, the expected payoff to the entrepreneur is

$$V(\mu) = \begin{cases} E_\mu [s - c(s) + B] + \lambda (E_\mu c(s) - I) & \text{if } E_\mu c(s) \geq I \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

while each investor's payoff is

$$W(\mu) = \begin{cases} E_\mu c(s) - I & \text{if } E_\mu c(s) \geq I \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

The entrepreneur's problem is to choose a signal to maximize her expected payoff, subject to the financing constraint (2) and competitive pricing (1). Each signal σ induces a distribution over posterior beliefs $q \in \Delta(\Delta([0,1]))$.[16] According to Kamenica and Gentzkow (2011), Prop. 1, the entrepreneur's problem of choosing the optimal message is equivalent to choosing the distribution over posteriors, subject to the *Bayes plausibility constraint*

$$E_q \mu = \mu_0. \tag{5}$$

Intuitively, given any signal σ, the investors' posterior belief must be a martingale because of Bayesian updating, which implies condition (5). Conversely, it can be shown that for any distribution of posteriors satisfying the condition, there exists a signal inducing it. The entrepreneur's problem is therefore equivalent to

$$\begin{aligned} V^*(\mu_0) &= \max_{q \in \Delta(\Delta([0,1]))} E_q V(\mu) \\ & s.t. \quad E_q \mu = \mu_0. \end{aligned} \tag{6}$$

[16] Intuitively, suppose that $s \in \{0,1\}$ and the prior probability that $s = 1$ is μ_0. Then the posterior probability that the state is 1 given message m is

$$\mu(m, \sigma) = \frac{\sigma(m|1)\mu_0}{\sigma(m|1)\mu_0 + \sigma(m|0)(1-\mu_0)},$$

which is a random variable since it depends on m. The distribution over posteriors induced by σ is

$$q(\mu|\sigma) = \sum_{m:\mu(m,\sigma)=\mu} \sigma(m|1)\mu_0 + \sigma(m|0)(1-\mu_0).$$

Throughout the paper, I will say that posterior μ' is *more optimistic* than posterior μ if μ' first-order stochastically dominates μ and *less optimistic* if the opposite holds. The interpretation is natural since if investors are "more optimistic" about the project in this sense, their expected payoffs from any security must be weakly larger.

In Sections 3 and 4, I assume the entrepreneur takes the security $c(.)$ as given and I characterize the optimal disclosure policy. In Section 5, I allow the entrepreneur to jointly choose the security and disclosure policy.

2.1 Discussion

Bayesian Persuasion assumes that once a message is drawn from the signal σ, the entrepreneur cannot alter this message.[17] We can understand this assumption as follows. The entrepreneur designs an *experiment*, which maps the true state into a random message. This experiment could be an accounting system, an external auditor, or an analyst, who collects information and writes a report which depends on the profitability of the project. The entrepreneur may be able to bias the experiment in a certain direction, say by telling the analyst to only collect positive information about the project. However, once the information is collected and the report is written, manipulating it would constitute fraud. If the entrepreneur expects this manipulation to be detected and punished with sufficient likelihood, she will never manipulate.[18] Thus, we can assume that once the message is drawn, the entrepreneur cannot alter it.

3 Illustrative Example: Binary State Space

To gain intuition, consider first the case of a binary state $s \in \{0, 1\}$. The main results developed in this section will carry over to the general case in Section 4.

To save notation, I denote the prior and posterior probability that $s = 1$ with μ_0 and μ respectively and I write $c(1) = c$, since limited liability implies that $c(0) = 0$. Given posterior μ, the price is now $p(\mu) = \mu c$ and the financing condition is

$$\mu c \geq I. \tag{7}$$

[17] If altering the message were possible and costless, this would lead to unraveling.
[18] Enforcement of this may for example come from investor scrutiny and lawsuits.

The project is financed whenever the investors' belief that the payoff is high exceeds a certain threshold, which is given by $\bar{\mu} = \frac{I}{c}$. I assume throughout this section that $I \leq c$ so that $\bar{\mu} \in (0, 1]$.[19] The entrepreneur's utility is

$$V(\mu) = \mu(1 - c + B) + (1 - \mu) B + \lambda(\mu c - I) \tag{8}$$

if $\mu \geq \bar{\mu}$ and zero otherwise.

The entrepreneur's optimization problem becomes

$$\begin{aligned} V^*(\mu_0) &= \max_q E_q \left[\mathbf{1}\{\mu \geq \bar{\mu}\} (\mu(1 - c) + B + \lambda(\mu c - I))\right] \\ s.t. \quad & E_q \mu = \mu_0 \end{aligned} \tag{9}$$

If $\mu_0 \geq \bar{\mu}$, not disclosing any information is optimal, i.e. $\mu = \mu_0$ with probability one. Intuitively, the project can be financed without providing any further information. Since the amount raised from investors is increasing in the posterior, the entrepreneur might provide disclosure in order to raise the expected amount paid for the security. However, such scheme can never generate any profit. Under Bayesian updating, the average posterior has to equal the prior, which is reflected by the Bayes plausibility condition (5), and therefore the entrepreneur can never provide a signal that alters the average value of the security from an ex-ante perspective.[20]

If $\mu_0 < \bar{\mu}$, the project is not financed if the entrepreneur does not provide any information. The entrepreneur has to induce a posterior above $\bar{\mu}$, but is constrained by Bayes plausibility. For simplicity, consider a policy that randomizes between two posteriors $\{\mu_l, \mu_h\}$ with $\mu_l < \mu_0 < \bar{\mu} < \mu_h$. Let q the the probability that μ_h is realized, which is also the probability that the project is financed. The entrepreneur's expected payoff is

$$V = q(\mu_h(1 - c) + B + \lambda(\mu_h c - I)) + (1 - q) \cdot 0 \tag{10}$$

[19]Since $I > 0$ and $c \leq 1$, $\bar{\mu} > 0$. If $I > c$, we have $\bar{\mu} > 1$ which implies the project is never financed.

[20]Importantly, this argument extends to the case with a continuous state space. By Bayes plausibility $E_q \mu = \mu_0$ and therefore for any security $c(s)$, $E_q[E_\mu c(s)] = E_{\mu_0} c(s)$.

and the Bayes plausibility constraint becomes

$$q\mu_h + (1-q)\,\mu_l = \mu_0. \tag{11}$$

If the entrepreneur wants to induce a higher posterior belief $\mu_h' > \mu_h$ or $\mu_l' > \mu_l$, the probability that this belief is realized must decrease, otherwise the constraint is violated. Thus, there is an indirect cost of inducing a higher belief, since it decreases the likelihood the project is financed.

Because the entrepreneur's payoff is zero for all $\mu_l < \bar{\mu}$, an optimal policy should choose $\mu_l = 0$ to maximize the probability that the project is financed. Then, constraint (11) becomes $q\mu_h = \mu_0$ and the entrepreneur's value is

$$V = \mu_0\,(1-c) + \lambda\,(\mu_0 c - qI) + qB.$$

The first terms measure the expected residual payoff for the entrepreneur and the excess cash raised from investors, while the last term measures the expected private benefit of control.

Increasing μ_h does not change the expected residual payoff or the expected amount of money raised from investors due to Bayes plausibility, but it decreases the probability that the project is funded and the expected private benefit.[21] Therefore, the optimal policy chooses the lowest possible μ_h and induces posteriors $\mu_l = 0$ and $\mu_h = \bar{\mu}$. That is, either investors believe with certainty that the project is bad, or their belief is the lowest one that allows the project to be financed given c. Since at $\bar{\mu}$, the financing constraint (7) binds, the entrepreneur receives no cash from investors in excess of the investment cost.

The equilibrium probability that the project is financed is then $q = \mu_0 \frac{c}{I}$, which is increasing in the prior μ_0 and the cash promised to investors, and decreasing in the cost of investment. The expected payoff to the entrepreneur is

$$V^*(\mu_0) \;\;=\;\; \mu_0\,(1-c) + \frac{\mu_0}{\bar{\mu}}B. \tag{12}$$

The formal proof is in Appendix B.1. Te argument relies on exploiting the fact that expectations are linear in probabilities. Take any distribution over posteriors q, which puts mass $q\left([\bar{\mu}, 1]\right)$ on the range of beliefs for which the project is financed and has conditional

[21] Note that $\mu_h c \geq I$ and $q = \frac{\mu_0}{\mu_h}$ imply $\mu_0 c - qI \geq 0$, so the middle term is positive.

expectation $E_q(\mu|\mu \geq \bar{\mu})$. Due to linearity of the expectations operator, this distribution achieves the same payoffs as one that puts point mass $q' = q([\bar{\mu}, 1])$ on posterior $\mu_h = E_q(\mu|\mu \geq \bar{\mu})$. It is then without loss of generality to consider a policy which puts mass on only two posteriors. Above, I have characterized the optimal policy among those putting mass on only two posteriors, which must then be optimal among all policies.

The optimal policy and value are shown in Figure 2. The blue line is $V(\mu)$ while the black dashed line shows the payoff from the optimal policy conditional on the prior. Full disclosure is generally suboptimal. Under full disclosure, the posterior is

$$\mu = \begin{cases} 1 & w.Pr. \quad \mu_0 \\ 0 & w.Pr. \quad 1-\mu_0 \end{cases}$$

and the entrepreneur's value is

$$V_{fd}(\mu_0) = \mu_0 ((1-c) + B + \lambda (c - I)) < V^*(\mu_0).$$

In Figure 2, the black dotted line shows the payoff from full disclosure.

The optimal disclosure policy and value depend on c, the amount promised to investors in the good state. If c increases, the entrepreneur gets a lower payoff if the project is financed. However, investors need to be less optimistic for the project to be financed, since they receive a higher payoff in the case of success.[22]

Formally, we have

$$\frac{dV^*(\mu_0)}{dc} = \mu_0 \frac{B-I}{I}$$

for $\mu_0 \leq \bar{\mu}$. Whenever the private benefit of control is sufficiently large, so that $B > I$, the second effect dominates. It is optimal to sell off as much of the project as possible, until either $c = 1$ or $\bar{\mu} = \mu_0$.[23] In the first case, the entrepreneur sells off the entire project and provides imperfect disclosure, while in the second case, she sells off just enough so that the project is financed with certainty without providing any information to investors. The first case occurs whenever $\mu_0 < I$ so that at $c = 1$ and $\mu = \mu_0$, the financing constraint (7) is

[22]This is because $\bar{\mu}$ is decreasing in c, so that the probability that the project is financed, $\frac{\mu_0}{\bar{\mu}}$ is increasing in c.

[23]This happens exactly when $\mu_0 = \frac{I}{c}$.

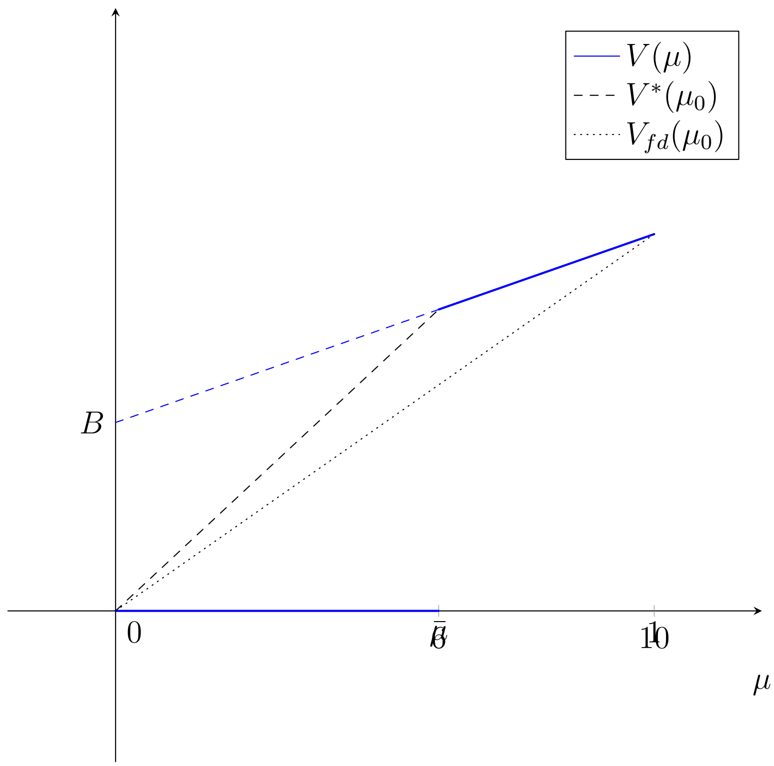

Figure 2: Optimal Value for $B > 0$

violated, while the second case occurs when $I \leq \mu_0$.

If the private benefit of control is small and $B < I$, the first effect dominates and it is optimal to sell off the smallest possible share such that the project still can be financed for some posteriors. This is achieved by setting $c = I$, so that the financing constraint holds only if $\mu = 1$. Thus, the entrepreneur raises as little financing as possible and optimally provides full disclosure.

4 Continuous State Space

While the binary case allows for studying the effect of the amount promised to investors in the good state on the entrepreneur's value and the probability that the project is financed, it does not allow to study how to optimally design securities since all securities $c(s)$ are determined by the parameter c. In this section, I return to the case $s \in [0,1]$ and I derive the optimal disclosure policy given any security $c(s)$. In Section 5, I determine the security which maximizes the entrepreneur's value.

As in the binary state case of Section 3, if the project can be financed without disclosure, then not providing any disclosure is optimal. I therefore assume $E_{\mu_0} c(s) < I$ throughout this section.

Deriving the optimal policy follows the same logic as in Section 3. In Lemma 1 below, I show that without loss of generality, the optimal policy puts all mass on two posteriors, one such that the project is financed and one such that it is not. I then characterize the optimal posteriors and show that the financing constraint must bind whenever the project is financed. I provide only heuristic arguments in this Section. The full proof is deferred to Appendix A.

Lemma 1. *Without loss of generality, the optimal policy q puts all mass on two posteriors μ_h, μ_l with $E_{\mu_l} c(s) < E_{\mu_0} c(s) < I \leq E_{\mu_h} c(s)$. μ_h and μ_l admit a pdf.*

The intuition for the result is the same as in Section 3. To simplify notation, let q denote the probability that μ_h is realized. In the following, I identify μ_0, μ_h, and μ_l with their densities $\mu_0(s)$, $\mu_h(s)$, and $\mu_l(s)$. The Bayes plausibility constraint can now be written more intuitively as

$$\mu_0(s) = q\mu_h(s) + (1-q)\mu_l(s) \tag{13}$$

for all $s \in [0,1]$, the financing condition becomes

$$\int_0^1 c(s)\,\mu_h(s)\,ds \geq I, \tag{14}$$

and the entrepreneur's problem can be written as

$$V = \max_{q,\mu_h(.),\mu_l(.)} q \int_0^1 (s - c(s) + B + \lambda (c(s) - I))\,\mu_h(s)\,ds \tag{15}$$

subject to the financing condition (14), the Bayes plausibility condition (13), and $\mu_h(s)$ and $\mu_l(s)$ being nonnegative and integrating to one.

As in the binary state case, the principal should maximize the probability that the project is financed, since no Bayes plausible distribution over posteriors can increase the ex-ante expected payoff from investors. However, simply setting $\mu_l(s) = 0$ for all s is no longer optimal with a continuous state, since via equation (13), this would imply that $q = 1$ and $\mu_h(s) = \mu_0(s)$ for all s. Then, the project would never be financed, since by assumption $E_{\mu_0} c(s) < I$.

Inspecting the entrepreneur's problem (15), we can see that holding q fixed, choosing μ_h and μ_l is an infinite-dimensional linear programming problem. Using the Bayes plausibility constraint, we can substitute

$$\mu_l(s) = \frac{\mu_0(s) - q\mu_h(s)}{1-q}. \tag{16}$$

Then, the problem reduces to choosing $\mu_h(s)$ subject to both μ_h and μ_l being probability densities. Since the problem is linear, it is intuitive to conjecture that the optimal policy is bang-bang. Either is $\mu_h(s) = 0$ or $\mu_h(s)$ is set as large as possible. Via condition (16) this implies the optimal policy takes the following form

$$\mu_h(s) = \begin{cases} \frac{\mu_0(s)}{q} & \text{for } s \in \hat{S} \\ 0 & \text{otherwise} \end{cases} \tag{17}$$

$$\mu_l(s) = \begin{cases} 0 & \text{for } s \in \hat{S} \\ \frac{\mu_0(s)}{1-q} & \text{otherwise} \end{cases}$$

$$q = \int_{\hat{S}} \mu_0(s)\, ds.$$

Here, $\hat{S} \subset [0,1]$ is the set of states for which $\mu_h(s)$ is nonzero. To determine the form of $\hat{S}$, consider the marginal contribution of including some state s in $\hat{S}$ to the entrepreneur's objective, which is is

$$s - c(s) + B + \lambda(c(s) - I) + \gamma((c(s) - I)). \tag{18}$$

The first term is the contribution to the entrepreneur's value, and the second term is the financing constraint, weighted by a Lagrange multiplier $\gamma > 0$. Since $\lambda \in [0,1]$, and both the payoff to investors $c(s)$ and the entrepreneur's residual $s - c(s)$ are increasing in s, the value is increasing in s as well. At the optimum, a state must be included in $\hat{S}$ whenever the above expression is positive. Therefore the set $\hat{S}$ is an interval. For some threshold $\hat{s}$, we have

$$\hat{S} = [\hat{s}, 1]. \tag{19}$$

Intuitively, if $\mu_h(s)$ puts weight on $[s', s'']$ and $[\hat{s}, 1]$, then we can move the mass from the lower interval and append it to the upper one. Since both the entrepreneur's and investors' values are increasing in s, this change improves the objective and relaxes the financing constraint. Thus, $\mu_h(s)$ puts mass on all states above a cutoff $\hat{s}$ while $\mu_l(s)$ puts mass only on states below the cutoff.

To find the optimal value, it remains to optimize over q. As in the binary state case, the entrepreneur wants to maximize the likelihood that the project is financed, and $V^*(q)$ is increasing in q. There is also again a tradeoff between inducing "high" beliefs and the likelihood that the project is financed, since when $\hat{s}$ increases, i.e. the high posterior only puts weight on higher values, q must decrease.

Given the functional form of μ_h, the financing constraint becomes

$$\int_{\hat{s}}^{1} \left(c\left(s\right) - I\right) \mu_0\left(s\right) ds \geq 0. \tag{20}$$

Increasing q must decrease $\hat{s}$ at the optimum, which in turn must decrease the expected payoff to investors. The optimal q is therefore the one which makes the financing constraint bind.[24]

The optimal posteriors put either all mass on $s \geq \hat{s}$ or all on $s < \hat{s}$. Since

$$\mu_h\left(s\right) = \frac{\mu_0\left(s\right)}{\int_{\hat{s}}^{1} \mu_0\left(s\right) ds}$$

is the posterior probability conditional on $s \geq \hat{s}$ and $\mu_l\left(s\right)$ is the posterior probability conditional on $s < \hat{s}$, the optimal disclosure policy can be implemented by truthfully revealing whether the state is in $[\hat{s}, 1]$ or $[0, \hat{s})$.

The argument so far has been heuristic. In the proposition below, which is proven in Appendix A, I establish that the solution above is indeed optimal. The argument relies on characterizing the dual to problem (15) and showing that the solutions coincide.

Proposition 1. *The optimal disclosure policy truthfully reveals whether $s \geq \hat{s}$ or $s < \hat{s}$. At the optimal threshold $\hat{s}$, the financing constraint in equation (20) binds.*

The threshold $\hat{s}$ can be understood as the quality of information provided. If the threshold is lower, the value of investors, conditional on financing the project, must be lower as well. In the following, I illustrate how restricting the disclosure policy ex-ante can lead to vastly different results. In both examples, full disclosure is uniquely optimal for *any* security, even though it is never optimal in the general case.

Example 1. Suppose $E_{\mu_0} c\left(s\right) < I$ and consider the following disclosure policy. The entrepreneur chooses the probability q with which the state is revealed truthfully. Specifically, the policy sends message $m = s$. With probability $1 - q$, the policy sends an uninformative

[24] I show in Appendix A that for any higher values of q, problem (15) does not admit a feasible solution since the financing constraint can never by satisfied at μ_h.

message, say $m = \emptyset$. The posterior probability conditional on message $m = s$ is

$$\mu(s|m) = \begin{cases} 1 & \text{if} \quad m = s \\ 0 & \text{if} \quad m \neq s \\ \mu_0(s) & \text{if} \quad m = \emptyset. \end{cases}$$

The financing constraint conditional on m becomes$c(m) \geq I$. Thus, the project is financed if and only if the policy sends message $m \geq \hat{s}$, where $\hat{s}$ is such that the above inequality binds. If the policy sends the uninformative message, the project is not financed. Therefore, the entrepreneur's value is

$$V = q \int_{\hat{s}}^{1} (s - c(s) + B) \mu_0(s) \, ds,$$

which is strictly increasing in q. Full disclosure, i.e. $q = 1$, is optimal because any other policy reduces the likelihood the project is financed successfully.

Example 2. Suppose again that $E_{\mu_0} c(s) < I$ and assume for simplicity that the prior is uniform. Consider the policy which with probability q reveals the state truthfully and with probability $1 - q$ sends a random message $m = s'$, which is drawn uniformly from the state space. The posterior conditional on message $m = s$ is

$$\mu(s|m) = \begin{cases} q & \text{if} \quad m = s \\ (1 - q) & \text{if} \quad m \neq s. \end{cases}$$

Conditional on message m, the financing constraint is

$$qc(m) + (1 - q) \int_0^1 c(s) \, ds \geq I.$$

There exists a cutoff $\hat{s}$ at which the financing constraint binds. The project is financed whenever the policy sends a message $m \geq \hat{s}$. The ex-ante distribution over messages is uniform and the entrepreneur's value is

$$V = \int_{\hat{s}}^{1} (s - c(s) + B) \, ds.$$

Since $\hat{s}$ is decreasing in q, full disclosure is again optimal. Intuitively, with probability $1-q$, the policy sends a random message under which in expectation, the project is not financed. The optimal policy minimizes that probability.

The optimal disclosure policy is determined entirely by the financing constraint. This allows me to derive simple comparative statics. If changing a parameter increases the value to investors, then the optimal threshold has to decrease and the entrepreneur provides worse information.

When the security becomes more information sensitive, the entrepreneur optimally provides worse information.[25] Specifically, I call a security c' more information sensitive than security c if c' crosses c form below, i.e. there exists a $\tilde{s}$ such that for $s<\tilde{s}$, $c'(s) \leq c(s)$ and for $s \geq \tilde{s}$, $c'(s) \geq c(s)$.[26] For two such securities suppose that $E_{\mu_0} c'(s) = E_{\mu_0} c(s) < I$, that is, they have both the same ex-ante payoff, but it is not so high that disclosure becomes unnecessary. Then, if the entrepreneur uses the same threshold to provide information for both securities, the value to investors is always higher under c'. This immediately implies that the optimal threshold for security s' is lower. Intuitively, since investors value information more under security c', the entrepreneur can provide worse information and they are still willing to finance the project.

Proposition 2. *If security $c'(s)$ is more information sensitive than security $c(s)$ and $E_{\mu_0} c'(s) = E_{\mu_0} c(s) < I$, then the optimal threshold under $c'(s)$ is lower than the one under $c(s)$.*

Similarly, if investors become more optimistic about the project, i.e. their prior belief increases in the sense of first-order stochastic dominance, the entrepreneur can provide worse information while still satisfying the financing constraint. Similarly, if c' promises higher payoffs to investors, then the financing constraint is relaxed. The optimal posterior therefore has to be less optimistic for the project to be financed, so $\hat{s}' < \hat{s}$ and likelihood that the project is financed increases. I summarize these results in the proposition below.

Proposition 3. *Consider two priors $\mu_0'(s)$ and $\mu_0(s)$ such that for both the project cannot be financed without disclosure, i.e. $E_{\mu_0} c(s) < I$ and $E_{\mu_0'} c(s) < I$. If μ_0' first-order stochastically dominates μ_0, then $\hat{s}' < \hat{s}$. The equilibrium probability that the project is financed is higher under μ_0'.*

[25] I am very grateful to Anton Tsoy for pointing this out.

[26] This is the definition used by DeMarzo et al. (2005) and many others.

Consider two securities $c(s)$ and $c'(s)$ with $E_{\mu_0}c(s) < I$ and $E_{\mu_0}c'(s) < I$ under the same prior μ_0. If

$$\int_{\hat{s}}^{1} \left(c'(s) - c(s)\right)\mu_0(s) > 0 \tag{21}$$

then $\hat{s}' < \hat{s}$. The probability that the project is financed is higher under $c'(s)$.

Sufficient conditions are (1) $c'(s) > c(s)$ for all s and (2) $\mu_0(s)$ is increasing in s,

$$\int_0^1 c'(s)\, ds \geq \int_0^1 c(s)\, ds,$$

and for all $x \in [0,1]$

$$C'(x) \leq C(x)$$

where $C'(x) = \frac{1}{\int_0^1 c'(s)ds}\int_0^x c'(s)\, ds$ and $C(x) = \frac{1}{\int_0^1 c(s)ds}\int_0^x c(s)\, ds$.

Intuitively, given any prior, the optimal policy discloses truthfully whether $s \geq \hat{s}$ and $\hat{s}$ is chosen to make the financing constraint bind. If μ_0' first-order stochastically dominates μ_0, the expected payoffs of investors are higher for any threshold $\hat{s}$. The entrepreneur can then optimally provide a signal that induces a less optimistic posterior and increase the probability that the project is financed. However, second-order stochastic dominance has an ambiguous effect on the optimal disclosure policy and the likelihood of financing, since it has ambiguous effect on the expected payoff to investors conditional on financing.

5 Financing Choice

In the previous section, I have shown that for any given security and any prior distribution, it is optimal to truthfully reveal whether or not the payoff is above a certain threshold, which is chosen such that that financing constraint binds whenever the project is financed. In this section, I allow the entrepreneur to jointly choose the security and the disclosure policy. I show that the financing choice is determined by a tradeoff between the likelihood of persuading investors successfully and a loss of cash flow rights. The optimal security is indeterminate without further frictions and the optimum can be implemented with equity, debt, and many other securities.

Promising additional payoffs to investors has two effects on the entrepreneur's optimal value. It lowers her realized payoff, but since investors who are promised higher payoffs are willing to finance the project at a less optimistic posterior, it also increases the likelihood that the project is financed. Whether or not it is optimal to promise higher payoffs in a certain state to investors depends on which effect dominates.

Mirroring the results in Section 3, if $B \geq I$, it is optimal to sell off as much of the project as possible, until either the project gets financed without providing disclosure, or the entrepreneur sells the entire project. Intuitively, if the benefit of control is sufficiently high, it is always worth increasing the payoff of the investors to increase the likelihood the project gets financed.

If $B < I$, there are two cases. If the cost of investment is sufficiently high, the entrepreneur again sells off the entire project, while when the cost is low, the tradeoff between obtaining financing and giving up payoffs is resolved at an interior point at which the entrepreneur retains some cash flow rights.

Given security $c(s)$, the optimal value is

$$V^* = \int_{\hat{s}}^{1} (s - c(s) + B)\,\mu_0(s)\,ds$$

and $\hat{s}$ solves

$$\int_{\hat{s}}^{1} (c(s) - I)\,\mu_0(s)\,ds = 0. \tag{22}$$

Suppose that $B < I$ and $c(s) < s$ for some s. Increasing $c(s)$ decreases the threshold $\hat{s}$, which follows from equation (22). Thus, the optimal disclosure strategy induces a less optimistic posterior and the likelihood that the project is financed increases. The total effect on the entrepreneur's value is[27]

$$\frac{dV^*(\mu_0)}{dc(s)} = \frac{\mu_0(s)}{-(c(\hat{s}) - I)}(\hat{s} - I + B)$$

for $s \geq \hat{s}$, which follows from equation (22) and the implicit function theorem.

If $\hat{s} > I - B$, the entrepreneur's value is increasing in $c(s)$ for all s, so it is optimal to increase the payout to investors for all states, until either the entire project is sold, i.e.

[27] Notice that $c(\hat{s}) < I$. If $c(\hat{s}) \geq I$, then $c(s) \geq I$ for all $s \in \hat{S}$ and the financing constraint is slack, which cannot be optimal.

$c(s) = s$ for all $s \geq \hat{s}$ and $\hat{s} > I - B$, or until the threshold $\hat{s}$ reaches $I - B$. If $\hat{s} < I - B$, then the entrepreneur's value is decreasing in $c(s)$ for all s, so it is optimal to decrease the payout until $\hat{s} = I - B$. The following Proposition provides sufficient conditions for the different cases.

Proposition 4. *For $B < I$, if selling the project implies it can be financed without disclosure, i.e. $E_{\mu_0} s \geq I$, then $\hat{s} = I - B$ for all I. If not, then for any I, there exists a $B(I)$ such that for $B > B(I)$, $\hat{s} > I - B$ and $c(s) = s$, and for $B \leq B(I)$, $\hat{s} = I - B$.*

For $B \geq I$, if there exists a security $c'(s)$ such that $E_{\mu_0} c'(s) = I$, then that security is optimal and $\hat{s} = 0$. If $E_{\mu_0} s < I$, then $c(s) = s$ and $\hat{s} > 0$.

The intuition is as follows. By designing the security, the entrepreneur can extract part of the social surplus of the project. The best possible outcome is to maximize the social surplus, which happens precisely when $\hat{s} = I - B$, i.e. the project is financed if and only if the social surplus is positive, and then extract it. If $E_{\mu_0} s \geq I$, selling off the entire project implies that it can be financed with certainty without providing any disclosure, which guarantees that $\hat{s} = I - B$ is feasible.

If this is not true, and B is large, at $\hat{s} = I - B$, the financing constraint is violated. It is then optimal to sell off as much of the project as possible since the benefit of increasing the likelihood the project gets financed outweighs the loss of promising more money to investors. If B is small, setting $\hat{s} = I - B$ does not violate the financing constraint, so it is again optimal.

Alternatively, we may interpret the results as follows. The entrepreneur can achieve the first best simply by committing to finance the project only if the social value, $\hat{s} + B - I$, is positive. Conditional on this commitment, there only remains to find a security which can guarantee investors break even. This is always possible if

$$E_{\mu_0} (s | s \geq I - B) \geq I$$

and any security which satisfies the financing constraint must be optimal since it yields the same expected value to the entrepreneur.

The result that the optimal security is indeterminate depends crucially on not restricting the set of admissible disclosure policies. If we restrict the policy ex-ante, debt may dominate equity, as the following example illustrates.

Example 3. Take the disclosure policy from example 1, i.e. with probability q, the state is disclosed truthfully, while with probability $1-q$, the policy sends an uninformative message. I have shown that full disclosure is optimal in this case. Now, I show that for this policy, debt can dominate equity. For simplicity, I assume that the prior is uniform and that $I = \frac{3}{4}$ and $B = \frac{1}{2}$. Also, I set $\lambda = 0$ to ease notation. Consider a contract offering equity share $\alpha \in [0, 1]$. The project is financed whenever a state is disclosed such that $\alpha s \geq I$, which implies $\hat{s} = \frac{I}{\alpha}$. The entrepreneur's value becomes then

$$V_E = \int_{\frac{I}{\alpha}}^{1} \left((1-\alpha) s + B \right) ds,$$

which is increasing in α given the assumptions on B and I. Thus, the optimal equity share is $\alpha = 1$. It is optimal to sell off the entire project to maximize the likelihood the project is financed. To see why debt dominates, consider a debt contract with promised return I. The project is financed whenever $s \geq I$, since for $s < I$, investors do not break even. The entrepreneur's value is therefore

$$\begin{aligned} V_D &= \int_I^1 (s - I + B) \, ds \\ &> \int_I^1 B ds = V_E. \end{aligned}$$

Debt dominates equity because it allows the entrepreneur to retain a larger residual share conditional on the same likelihood of financing.

Compare this to the optimal policy, which discloses truthfully whenever $s \geq I - B$. The optimal equity share is characterized by the financing constraint $\int_{I-B}^{1} (\alpha s - I) \, ds = 0$, which is a version of equation (22), and solves $\alpha = 2I \frac{1-(I-B)}{1-(I-B)^2}$. The optimal debt contract features risky debt, i.e. $I - B < R$, where R is the promised return, and solves $\int_{I-B}^{1} \min(s, R) \, ds = 0$. The optimal promised return can be computed as $R = 1 - \sqrt{1 + (I^2 - B^2) - 2I}$ and we can verify that the entrepreneur's value is indeed the same for equity and debt.

Similarly, it is crucial to allow for general disclosure strategies, since otherwise the entrepreneur may not be able to achieve the social optimum. The next example demonstrates this.

Example 4. Suppose given s, the message is given by $m = s + \varepsilon$, where $\varepsilon \sim N(0, v^2)$. The entrepreneur can choose the variance. This scheme can never achieve the first best. To see this, suppose that $s = I - B$. At that state, the signal has to be such that the project is financed with probability one. However, for any strictly positive variance, there is a strictly positive probability that the investor's posterior beliefs are such that the project cannot be financed for *any* security. And for $v = 0$ the project is not financed, because $s < I$.

6 Extensions

6.1 General Utility Functions

In this section, I show that the threshold strategy of Section 4 remains optimal for any increasing utility function of the entrepreneur, independently of different specifications of the transaction cost.

Specifically, suppose the entrepreneur's utility is increasing and continuous in the residual payoff of the project, and linear in the excess cash raised from issuing the security. In addition, assume that the transaction cost may depend on the amount of excess cash raised, so that with slight abuse of notation,

$$\lambda(x) \in [0, x]$$

for $x \in \mathbb{R}_+$ is the amount of excess cash that can be retained by the entrepreneur, which I assume is increasing in x. Her realized payoff now equals[28]

$$V(s, p) = u(s - c(s)) + \lambda(p - I),$$

where p is the amount raised from issuing the security. To capture the private benefit of control, I assume $u(0) > 0$. The entrepreneur's problem is

$$\begin{aligned} V &= \max_q E_q \left[\mathbf{1}\{E_\mu c(s) \geq I\} \cdot \left(E_\mu u(s - c(s)) + \lambda(E_\mu c(s) - I) \right) \right] \qquad (23) \\ s.t. &\quad E_q \mu = \mu_0 \end{aligned}$$

[28] For example, the payoffs of the project may be paid in the future while the excess cash is consumed now and the entrepreneur is risk-averse about future but not current consumption.

Again, without loss of generality, q puts all weight on two posteriors μ_h and μ_l, which must admit densities. Because of the transaction cost, the entrepreneur's utility is generally non-linear in the belief. However, the approach of Section 4 still applies because of a dominance argument. Without transaction cost, i.e. when $\lambda(x) = x$, leaving the financing condition slack is suboptimal, even though the entrepreneur captures all surplus from doing so. This must also be true with transaction costs. The particular shape of the transaction cost function λ therefore does not matter.

Proposition 5. *For any security $c(s)$ with $E_{\mu_0} c(s) < I$, any continuous, increasing utility function $u(.)$, and any continuous, increasing transaction cost function $\lambda(x) \in [0, x]$ for $x \in \mathbb{R}_+$, the threshold $\hat{s}$ solves*

$$\int_{\hat{s}}^{1} (c(s) - I)\, \mu_0(s)\, ds = 0 \tag{24}$$

and the optimal value is

$$V = \int_{\hat{s}}^{1} u(s - c(s))\, \mu_0(s)\, ds. \tag{25}$$

Thus, the threshold strategy obtained in the linear case, in Proposition 8, remains optimal. The disclosure threshold is solely determined by the financing condition (24) and therefore depends on the security, the investment cost, and the prior, but not on the utility of the entrepreneur or the particular transaction cost function.

6.2 Additional Noise

In reality, the entrepreneur may not have perfect information about the project's payoff. However, the threshold strategy of Section 4 remains optimal under natural assumptions.

Suppose the entrepreneur designs a disclosure strategy contingent on the state s, which now is correlated with the project payoff x which takes values between zero and one. x and s are jointly distributed with prior $\mu_0(x, s)$. For simplicity, I assume the conditional distribution $x|s$ admits a continuous pdf $\mu_0(x|s)$ for all s with full support. The security maps cash flows into payoffs so that $c(x) \in [0, x]$ and $c(x)$ and the residual $x - c(x)$ are both increasing, analogous to the original setup in Section 2. To capture that higher states imply higher payoffs, I assume that for $s' > s$ $\mu_0(x|s')$ first-order stochastically dominates $\mu_0(x|s)$.

Let μ denote investors' posterior on s. Unlike in the previous section, the transaction cost is linear again. The entrepreneur's payoff is

$$V(\mu) = E_\mu\left[\int_0^1 (x - c(x) + B + \lambda(c(x) - I))\,\mu_0(x|s)\,dx\right] \tag{26}$$

if the project is financed, which occurs if

$$E_\mu \int_0^1 c(x)\,\mu_0(x|s)\,dx \geq I. \tag{27}$$

Defining

$$\begin{aligned}\tilde{s}(s) &= \int_0^1 x\mu_0(x|s)\,ds \\ \tilde{c}(s) &= \int_0^1 c(x)\,\mu_0(x|s)\,ds\end{aligned}$$

we can rewrite the entrepreneur's value as

$$V(\mu) = E_\mu[\tilde{s}(s) - \tilde{c}(s) + B + \lambda(\tilde{c}(s) - I)] \tag{28}$$

and the financing condition as

$$E_\mu \tilde{c}(s) \geq I. \tag{29}$$

Since $x - c(x)$ and $c(x)$ are increasing and $x|s$ is ordered by first-order stochastic dominance, $\tilde{s}(s) - \tilde{c}(s)$ and $\tilde{c}(s)$ are both increasing in s. Thus, this setting is simply a special case of the one in Proposition 5 and the same result holds. A threshold strategy is optimal.

6.3 Moral Hazard

If the entrepreneur can steal output after the project is financed, as in Gale and Hellwig (1985), the optimal security is debt and the optimal disclosure policy remains a threshold policy. The goal of this section is to illustrate how the optimal security can be pinned down without affecting the shape of the optimal disclosure policy. That is, if I add moral hazard to the model in Section 2, a threshold policy remains optimal. Conversely, if I add disclosure to Gale and Hellwig (1985), the optimal security remains debt.[29]

I adapt the model of Gale and Hellwig (1985) to fit into my setting. The cash flow is given by $x \geq 0$, which has density $\mu_0(x|s)$. As before, $s \in [0,1]$ is the private information of the entrepreneur. For $s' > s$, $\mu_0(x|s')$ first-order stochastically dominates $\mu_0(x|s)$. After x realizes, the entrepreneur can costlessly steal output. To simplify the setting, I abstract from transaction costs and assume investors finance the project whenever their payoff is larger than the cost of investment I:

The timing is similar to Section 2, i.e. the entrepreneur chooses a security and an information policy, then the state realizes and the message is sent, then investors update their beliefs and decide whether to invest. The only difference is that (1) after x is realized, the entrepreneur decides whether to steal and (2) the security must be such that not stealing is incentive compatible. In addition to the cash flows, the optimal contract specifies whether investors monitor the entrepreneur. For simplicity, I assume this carries a fixed cost K.[30]

For any disclosure policy, the optimal security must be debt. This is for two reasons: conditional on investing, the payments to investors must be constant in the cash flow. Otherwise, the contract cannot be incentive compatible. I denote this cash flow as R. The optimal contract must also minimize the likelihood of monitoring, which implies that whenever $x < R$, investors get all the cash flows.[31] The optimal security is therefore

$$c(x) = \begin{cases} R & \text{if} \quad x \geq R \\ x - K & \text{if} \quad x < R. \end{cases}$$

But, if the security is debt, then the payoffs of entrepreneur and investors are linear in the belief μ. Specifically, for any debt contract with promised repayment R, the entrepreneur's payoff is

$$V(\mu) = E_\mu \left[\int_R^\infty (x - R)\, \mu_0(x|s)\, ds \right]$$

[29] This result is contrasting with Trigilia (2016), because I disclosure in my model is more general and costless.

[30] To put this differently: this setting is exactly the same as in Gale and Hellwig (1985), except the entrepreneur can now also choose a disclosure policy.

[31] Formally, these results follow from Prop. 2 and 3 on p. 654f of Gale and Hellwig (1985). The key is that the propositions are independent of the disclosure policy because moral hazard here is ex-post.

and the payoff of investors is

$$W(\mu) = E_{\mu}\left[\int_{0}^{R}(x-K)\,\mu_0(x|s)\,ds + R\int_{R}^{\infty}\mu_0(x|s)\,ds\right].$$

For both, the payoff functions are weakly increasing in x and therefore in s. The same argument as in Section 6.2 then implies that a threshold policy is optimal. In this sense, the threshold policies in the main section are robust to moral hazard.

Proposition 6. *With ex-post moral hazard, the optimal security is debt and the optimal disclosure policy is a threshold strategy.*

7 Empirical Implications

The optimal disclosure policy is determined by the financing constraint. As I have shown in Section 6.1, it does not depend on the particular utility function of the entrepreneur and it is robust to the transaction costs, which may be a standin for unmodeled microstructure issues. This allows me to derive clean predictions, using the comparative statics results in Propositions 2 and 3. There, I have shown that the entrepreneur provides worse information when the security becomes more information sensitive, when it provides higher payoffs, or when investors become more optimistic. She provides more valuable information when the cost of investment increases.

To translate these comparative statics into empirical predictions, it is necessary to map them into empirically operational measures of information quality. It turns out this is simple. Suppose conditional on information provided by the entrepreneur, an analyst values the security at the same time as investors. Then, after the project is financed, payoffs realize. The value of the security is now simply $c(s)$ and the analyst will update his forecasts accordingly. For a given threshold $\hat{s}$, the analyst's forecast is $m(\hat{s}) = E_{\mu_0}[c(s)\,|s \geq \hat{s}]$ and the mean squared error is

$$E_{\mu_0}\left[(c(s) - m(\hat{s}))^2\,|s \geq \hat{s}\right].$$

For a given prior, this expression can be used to calculate the exact change in forecast error, which has been used e.g. by Ball et al. (2014).

An alternative way to measure information content is via price reversals as in Hanley and Hoberg (2010). When the entrepreneur discloses information, the price is $p(\hat{s}) = E_{\mu_0}[c(s)|s \geq \hat{s}]$. However, once the project if financed and payoffs realize, rational traders should update the price of the security, to $p = c(s)$. The average price reversal is therefore

$$E_{\mu_0}[|c(s) - p(\hat{s})| |s \geq \hat{s}].$$

Finally, information content may be expressed as the conditional variance of the investor's belief. This is exactly the same expression as the mean squared error in the analyst forecast constructed above. Any of the above measures can now be used to derive predictions from my model's comparative statics.

8 Conclusion

In this paper, I study the optimal disclosure policy of an entrepreneur who needs to finance a project subject to a fixed cost. I show the optimal policy truthfully reveals whether the project's payoffs are above a threshold. The threshold, and therefore the beliefs of investors, are such that the financing constraint binds whenever the project is financed. The optimal disclosure policy is therefore determined by the security issued, the investment cost, and the prior belief, but it is independent of the particular shape of the entrepreneur's utility function or transaction costs, which may be understood as a standin for liquidity issues unmodeled in this paper. The threshold strategy also remains optimal when additional noise is present.

In disclosure models, the sender's payoff is determined solely by the distribution over induced posterior beliefs. The particular signal used is irrelevant, as long as it induces this a particular distribution. Thus, no such model can provide guidance on which means of communication firms should use, only on which beliefs they should try to induce. Given its robustness, my result hopefully constitutes a useful benchmark for empirical work trying to quantify the degree of information contained in firms' communication.

A higher threshold naturally translates into investor forecasts, be it in the sense of mean squared error, conditional variance, or price reversals. The comparative statics I provide in this paper therefore yield testable implications on how the security choice, investment cost, and investors' prior beliefs affect the equilibrium precision of investor information.

In my paper, the optimal security is determined by a novel tradeoff. Promising more cash to investors reduces the entrepreneur's residual payoff, but investors are willing to finance the project at a lower belief. This allows the entrepreneur to choose a disclosure policy which has a higher likelihood of successfully convincing investors that the project is sufficiently profitable. In the absence of further distortions, the optimal security design is indeterminate. The optimum may be implemented with equity, debt, options, and many others. In Section 6.3, I have shown that uniqueness of the optimal security can be restored when there is ex-post moral hazard. In that case, the optimal security is debt. An interesting, but yet unanswered, question is whether the optimality of debt can be restored in a traditional security design setting with adverse selection, where before the game is played the entrepreneur can disclose information.

A Proofs

A.1 Proof of Lemma 1

Consider an optimal policy q. Let $q\left(E_\mu c(s) < I\right)$ denote the measure on the set of posteriors $\{\mu : E_\mu c(s) < I\}$, and define $\mu_h = E_q\left(\mu | E_\mu c(s) \geq I\right)$ and $\mu_l = E_q\left(\mu | E_\mu c(s) < I\right)$. μ_h and μ_l are probability measures on $[0,1]$. Then,

$$\begin{aligned} V^* &= q\left(E_\mu c(s) < I\right) \cdot 0 \\ &\quad + q\left(E_\mu c(s) \geq I\right) \cdot E_q\left[E_\mu\left(s - c(s) + B\right) | E_\mu c(s) \geq I\right] \\ &= q\left(E_\mu c(s) \geq I\right) \cdot E_{\mu_h}\left(s - c(s) + B\right) \end{aligned}$$

Consider an alternative policy q' which puts measure $q\left(E_\mu c(s) < I\right)$ on posterior μ_l defined above, measure $1 - q\left(E_\mu c(s) < I\right)$ on posterior μ_h, and measure zero everywhere else. Under q' the payoff is the same as above and q' is feasible since

$$\begin{aligned} E_q(\mu) &= q\left(E_\mu c(s) < I\right) E_q\left(\mu | E_\mu c(s) < I\right) + q\left(E_\mu c(s) \geq I\right) E_q\left(\mu | E_\mu c(s) \geq I\right) \\ &= q\left(E_\mu c(s) < I\right) \mu_l + q\left(E_\mu c(s) \geq I\right) \mu_h \end{aligned}$$

and by construction

$$E_q(\mu) = \mu_0.$$

Now, I show that μ_h and μ_l admit pdfs. Note that necessarily $q \in (0,1)$. For any set $B \in \mathcal{B}([0,1])$, the Bayes plausibility constraint implies

$$\mu_0(B) = q\mu_h(B) + (1-q)\,\mu_l(B).$$

Thus, both μ_h and μ_l are absolutely continuous with respect to μ_0. Since μ_0 admits a pdf, it is is absolutely continuous with respect to the Lebesgue measure. Because absolute continuity is transitive, μ_h and μ_l must also admit a pdf.

A.2 Proof of Proposition 1

To prove the result, I first characterize the optimal disclosure policy for any given probability that the project is financed. Then, I let the entrepreneur optimize over this probability.

Proposition 7. *For a given $q \in (0,1)$, suppose that the problem*

$$V^*(q) = \max_{\mu_h(.),\mu_l(.)} q \int_0^1 (s - c(s) + B + \lambda(c(s) - I)) \mu_h(s) \, ds \tag{30}$$

subject to financing condition (14), Bayes plausibility constraint (13), and

$$\begin{aligned} \int_0^1 \mu_h(s) \, ds &= \int_0^1 \mu_l(s) \, ds = 1 \\ \mu_h(s), \mu_l(s) &\geq 0 \, \forall s \in [0,1] \end{aligned}$$

has a solution. Then, the optimal policy is given by (17) and $\hat{S} = [\hat{s}, 1]$.

The proof proceeds via a sequence of Lemmas and relies on characterizing the dual problem to (15), which I restate for the convenience of the reader below. Fix a $q \in (0,1)$. We have

$$\begin{aligned} V(q) = & \max_{\mu_h(.),\mu_l(.)} q \int_0^1 (s - c(s) + B + \lambda(c(s) - I)) \mu_h(s) \, ds \\ s.t. \quad & \int_0^1 c(s) \mu_h(s) \, ds \geq I \\ & \mu_0(s) = q\mu_h(s) + (1-q) \mu_l(s) \, \forall s \in [0,1] \\ & \int_0^1 \mu_h(s) \, ds = 1 \\ & \int_0^1 \mu_l(s) \, ds = 1 \\ & \mu_h(s) \geq 0 \, \forall s \in [0,1] \\ & \mu_l(s) \geq 0 \, \forall s \in [0,1] \end{aligned}$$

Substituting the Bayes plausibility condition we can replace the constraints on μ_l with

$\mu_h(s) \in \left[0, \frac{\mu_0(s)}{q}\right]$. Consider the relaxed problem

$$\begin{aligned} V_R(q) &= \max_{\mu_h(.)} q \int_0^1 (s - c(s) + B + \lambda(c(s) - I))\, \mu_h(s)\, ds \\ s.t. & \quad \int_0^1 c(s)\, \mu_h(s)\, ds \geq I \\ & \quad \int_0^1 \mu_h(s)\, ds \leq 1 \\ & \quad \mu_h(s) \geq 0 \forall\, s \in [0,1] \\ & \quad \mu_h(s) \leq \frac{\mu_0(s)}{q}\, \forall s \in [0,1] \end{aligned} \tag{31}$$

I restrict attention to $\mu_h \in \mathcal{L}^1([0,1])$ which satisfy the following generalization of equicontinuity: For any $\varepsilon > 0$, there exists a $\rho < 0$ such that for all $y \in \mathbb{R}$, $|y| < \rho$ and all feasible μ_h,

$$\int_0^1 |\mu_h(s+y) - \mu_h(s)|\, ds < \varepsilon. \tag{32}$$

The condition simplifies proving compactness of the set of feasible policies, which I do below in Lemma 2. The solution in Proposition 7 satisfies the additional constraint, so it is without loss of generality. Let $F(q) \subset \mathcal{L}^1([0,1])$ denote the set of set of feasible μ_h, i.e. those satisfying the constraints in Problem (31) and condition (32).

Lemma 2. *$F(q)$ is compact and convex.*

Proof. Since $\mathcal{L}^1([0,1])$ is a Banach space, $F(q)$ is compact if and only if it is totally bounded, which follows from the Kolmogorov-Reisz Theorem. See **?**, Theorem 5.[32] Convexity follows trivially since all constraints are linear in μ_h. □

[32]The theorem states that a set $F \subset \mathcal{L}^1(\mathbb{R})$ is totally bounded if and only if (1) F is bounded, (2) for all $\varepsilon > 0\ \exists R > 0$ such that $\forall \mu \in F$, $\int_{|x|>R} |\mu(x)|\, dx < \varepsilon$, and (3) condition (32) holds. The first two conditions can easily be checked.

The dual problem to (31) is given by

$$
\begin{aligned}
V_d(q) &= \min_{\alpha_1(.),\alpha_2,\alpha_3\geq 0} \int_0^1 \alpha_1(s)\frac{\mu_0(s)}{q}ds+\alpha_2-\alpha_3 I \qquad (33)\\
s.t. \quad & \int_0^1 \alpha_1(s)\mu_h(s)\,ds+\alpha_2\int_0^1 \mu_h(s)\,ds-\alpha_3\int_0^1 c(s)\mu_h(s)\,ds\\
&\geq q\int_0^1 (s-c(s)+B+\lambda(c(s)-I))\,\mu_h(s)\,ds
\end{aligned}
$$

Since $c(s)$ and $\mu_0(s)$ are continuous, the problem satisfies the conditions of Levinson (1966), Theorem 3, which guarantees that the solution of the dual equals the solution of the primal.[33]

Denote the support of μ_h as $\hat{S}\subset[0,1]$. A necessary condition for the constraint in the dual problem to hold is that for all $s\in\hat{S}$

$$\alpha_1(s)+\alpha_2-\alpha_3 c(s)\geq q\,(s-c(s)+B+\lambda(c(s)-I))\,.$$

Since the dual problem minimizes over $\alpha_1(s)$, the optimal α_1 solves

$$\alpha_1(s)=q\,(s-c(s)+B+\lambda(c(s)-I))-\alpha_2+\alpha_3 c(s)$$

for $s\in\hat{S}$ and $\alpha_1(s)=0$ otherwise. The objective therefore becomes

$$
\begin{aligned}
V_d(q) &= \int_{s\in\hat{S}} (s-c(s)+B+\lambda(c(s)-I))\,\mu_0(s)\,ds \qquad (34)\\
&\quad +\int_{s\in\hat{S}} (-\alpha_2+\alpha_3 c(s))\frac{\mu_0(s)}{q}ds+\alpha_2-\alpha_3 I\\
&= \int_{s\in\hat{S}} (s-c(s)+B+\lambda(c(s)-I))\,\mu_0(s)\,ds\\
&\quad +\alpha_3\left(\int_{s\in\hat{S}} c(s)\frac{\mu_0(s)}{q}ds-I\right)+\alpha_2\left(1-\int_{s\in\hat{S}}\frac{\mu_0(s)}{q}ds\right)
\end{aligned}
$$

At the optimal solution, both the financing and integrability constraints must bind.

Lemma 3. *$\int_0^1 \mu_h(s)\,ds\leq 1$ and $\int_0^1 \mu_h(s)\,c(s)\,ds\geq I$ cannot both be slack at the optimal solution.*

[33] In general, infinite dimensional linear programs exhibit positive duality gaps, see e.g. Reiland (1980).

Proof. Substituting $\mu_h(s)$ and combining the two inequalities implies

$$\int_{\hat{S}} (c(s) - I)\,\mu_0(s) > 0.$$

Since $c(s)$ is continuous there exist states such that $c(s) > I$ which are not part of $\hat{S}$ and have positive mass under μ_0. Then the policy cannot be optimal, since including these states in the support of μ_h increases the objective without violating any of the constraints.[34] $\square$

The optimal value of the dual problem is therefore

$$V_d(q) = \int_{s\in\hat{S}} (s - c(s) + B + \lambda(c(s) - I))\,\mu_0(s)\,ds. \tag{35}$$

Since the solution of the primal and the dual coincide, this establishes that the policy

$$\begin{aligned}
\mu_h(s) &= \begin{cases} \frac{\mu_0(s)}{q} & \text{for } s \in \hat{S} \\ 0 & \text{otherwise} \end{cases} \\
\mu_l(s) &= \begin{cases} 0 & \text{for } s \in \hat{S} \\ \frac{\mu_0(s)}{1-q} & \text{otherwise} \end{cases} \\
q &= \int_{\hat{S}} \mu_0(s)\,ds
\end{aligned} \tag{36}$$

is optimal. To prove the proposition, it remains to show that $\hat{S}$ is an interval.

Lemma 4. $\hat{S} = [\hat{s}, 1]$.

[34] Formally, there exists a set $\tilde{S}$ with $\mu_0\left(\tilde{S}\right) > 0$ such that for all $s \in \tilde{S}$, $c(s) > I$, $\hat{S} \cap \tilde{S} = \emptyset$, and

$$\int_{\tilde{S}\cup\hat{S}} \mu_h(s) \leq 1$$

and

$$\int_{\tilde{S}\cup\hat{S}} \mu_h(s)\,c(s) \geq I.$$

Having $\mu_h(s) = \frac{\mu_0(s)}{q}$ on $\hat{S} \cup \tilde{S}$ strictly improves the objective.

Proof. Substitute $q = \int_{\hat{S}} \mu_0(s)\,ds$ into equation (34), which becomes

$$V_d(q) = \int_{s\in\hat{S}} (s - c(s) + B + \lambda (c(s) - I))\,\mu_0(s)\,ds + \frac{\alpha_3}{q}\left(\int_{s\in\hat{S}} (c(s) - I)\,\mu_0(s)\,ds\right).$$

The contribution of each s to the objective is

$$(s - c(s) + B + \lambda (c(s) - I))\,\mu_0(s) + \frac{\alpha_3}{q}(c(s) - I)\,\mu_0(s)$$

and therefore $s \in \hat{S}$ if and only if

$$(s - c(s) + B + \lambda (c(s) - I)) + \frac{\alpha_3}{q}(c(s) - I) \geq 0.$$

Since both $s - c(s)$ and $c(s)$ are increasing in s, this implies $\hat{S} = [\hat{s}, 1]$. [35] $\square$

Since $q = \int_{\hat{s}}^1 \mu_0(s)\,ds$ at the optimal solution, we must have that for $q' > q$, $\hat{s}' < \hat{s}$ whenever $F(q')$ is nonempty. This concludes the proof of 7.

Having characterized the optimal policy conditional on q, it remains to optimize over q to find the entrepreneur's optimal value. Intuitively, the value $V^*(q)$ is increasing in q, so the entrepreneur maximizes the likelihood the project is financed subject to the financing constraint.

Proposition 8. *If $E_{\mu_0} c(s) < I$, the optimal value V^* satisfies*

$$V^* = \int_{\hat{s}}^1 (s - c(s) + B)\,\mu_0(s)\,ds, \tag{37}$$

the optimal policy is given by Equation (17) with $\hat{S} = [\hat{s}, 1]$ and $\hat{s}$ and q are determined by

$$\begin{aligned} 0 &= \int_{\hat{s}}^1 (c(s) - I)\,\mu_0(s)\,ds \\ q &= \int_{\hat{s}}^1 \mu_0(s)\,ds. \end{aligned}$$

[35] We can equivalently define $V_d(q) = \max_{\hat{S}\subset\mathcal{B}([0,1])} \int_{\hat{S}} \mu_0(s)(s - c(s) + B)\,ds$ subject to $\int_{\hat{S}} (c(s) - I)\,\mu_0(s)\,ds \geq 0$. The result is then a direct consequence of the Neyman-Pearson Lemma. See e.g. Dantzig and Wald (1951).

If $E_{\mu_0} c(s) \geq I$, then no disclosure is optimal.

I first show that $V^*(q)$ is indeed increasing and continuous.

Lemma 5. *$V(q)$ increasing in q, continuous, and differentiable whenever there exists a neighborhood of q on which $F(q)$ is nonempty.*

Proof. Continuity and differentiability follows from the envelope theorem in Milgrom and Segal (2002), Theorem 5, since $F(q)$ is compact and convex-valued, the objective and constraints in the primal problem (31) are continuously differentiable in q, and the maximizers are unique.

To see that $V(q)$ is increasing, take $q' > q$ such that $F(q')$ and $F(q)$ are nonempty. Then, $\hat{s}(q') < \hat{s}(q)$ and therefore

$$\begin{aligned} V(q') &= \int_{\hat{s}(q')}^{1} (s - c(s) + B + \lambda (c(s) - I)) \mu_0(s) \, ds \\ &> \int_{\hat{s}(q)}^{1} (s - c(s) + B + \lambda (c(s) - I)) \mu_0(s) \, ds. \end{aligned}$$

□

For $q = 1$, the entrepreneur's problem in 30 does not admit a solution. I now show that there exists a maximal feasible probability q such that the financing constraint binds. Later, I will show that this q is the optimal one. I denote with $\hat{s}(q)$ the cutoff for which

$$q = \int_{\hat{s}(q)}^{1} \mu_0(s) \, ds.$$

Lemma 6. *Suppose $E_{\mu_0} c(s) < I$. Then there exists a $\bar{q} \in (0,1)$, such that for $q > \bar{q}$, the problem (31) does not admit a solution. $\hat{s}(\bar{q})$ solves*

$$\int_{\hat{s}(\bar{q})}^{1} (c(s) - I) \mu_0(s) \, ds = 0.$$

Proof. Consider the problem of choosing μ_h to maximize the expected payoff to investors given q, which is

$$
\begin{aligned}
C(q) &= \max_{\mu_h(.)} \int_0^1 c(s)\,\mu_h(s)\,ds \\
s.t. &\quad \int_0^1 \mu_h(s)\,ds \leq 1 \\
&\quad \mu_h(s) \in \left[0, \frac{\mu_0(s)}{q}\right]
\end{aligned}
$$

This problem has the same solution as problem (31), and $\mu_h(s) = \frac{\mu_0(s)}{q}$ for $s \in [\hat{s}(q), 1]$ and zero otherwise. The value to investors is thus

$$
C(q) = \int_{\hat{s}(q)}^1 c(s)\,\frac{\mu_0(s)}{q} ds
$$

and for $q > \bar{q}$,

$$
C(q) < \int_{\hat{s}(\bar{q})}^1 (c(s))\,\frac{\mu_0(s)}{\bar{q}} ds = I
$$

which implies

$$
E_{\mu_h} c(s) < I.
$$

Thus, for $q > \bar{q}$, $F(q)$ is empty. □

Since $V(q)$ is increasing in q, as shown in Lemma 5, the optimal q is $\bar{q}$, for which the financing constraint binds. Together with the result in Lemma 4, this establishes the results in Proposition 1.

A.3 Proof of Proposition 2

Consider the function

$$
\delta(\hat{s}) = E\left(c'(s) - c(s)\,|s \geq \hat{s}\right).
$$

We have $\delta(0) = 0$, which follows from the assumption that $E_{\mu_0} c'(s) = E_{\mu_0} c(s)$. Using the definition of information sensitivity, for $\hat{s} < \tilde{s}$, if follows that $\delta(\hat{s})$ is weakly increasing, while for $\hat{s} \geq \tilde{s}$, it is positive. Thus, $\delta(\hat{s})$ is positive for all $\hat{s} \in [0, 1]$. Now, pick $\hat{s}$ as the optimal threshold when the security is $c(s)$. This is the threshold which satisfies

$$E(c(s)|s \geq \hat{s}) = I.$$

Let $\hat{s}'$ denote the optimal threshold under security $c'(s)$. Since δ is positive, under $c'(s)$, the financing constraint holds at threshold $\hat{s}$, i.e.

$$E(c'(s)|s \geq \hat{s}) \geq I.$$

If $\hat{s}' > \hat{s}$, this means the financing constraint is slack, which cannot be optimal. Thus, $\hat{s}' \leq \hat{s}$.

A.4 Proof of Proposition 3

In the first part of the proposition, I consider one security and two priors μ_0 and μ_0'. For both μ_0 and μ_0', the optimal thresholds $\hat{s}$ and $\hat{s}'$ are determined by

$$\begin{aligned} \int_{\hat{s}}^{1} (c(s) - I)\,\mu_0(s)\,ds &= 0 \\ \int_{\hat{s}'}^{1} (c(s) - I)\,\mu_0'(s)\,ds &= 0. \end{aligned}$$

If μ_0' first-order stochastically dominates μ_0, then

$$\int_{\hat{s}}^{1} (c(s) - I)\,\mu_0'(s) > \int_{\hat{s}}^{1} (c(s) - I)\,\mu_0(s) = 0$$

since $c(s)$ is increasing. Then, the financing constraint for μ_0' binds at $\hat{s}' < \hat{s}$.

Now, consider two securities c and c' under the same prior μ_0. If

$$\int_{\hat{s}}^{1} (c'(s) - c(s))\,\mu_0(s) > 0,$$

then $\hat{s}' < \hat{s}$ is immediate. If $\hat{s}'$ were larger, the financing constraint would be slack at c', which cannot be optimal. As for the sufficient conditions, I only prove the second one since the first is immediate. C' and C are both cumulative distribution functions and the condition simply states that C' first-order stochastically dominates C. Then, we have

$$\int_{\hat{s}}^{1} \mu_0(s)\,dC'(s) \geq \int_{\hat{s}}^{1} \mu_0(s)\,dC(s)$$

since $\mu_0(s)$ is increasing by assumption, which implies

$$\begin{aligned}\int_{\hat{s}}^{1} c'(s)\,\mu_0(s)\,ds &\geq \frac{\int_0^1 c'(s)\,ds}{\int_0^1 c(s)\,ds}\int_{\hat{s}}^{1} c(s)\,\mu_0(s)\,ds \\ &\geq \int_{\hat{s}}^{1} c(s)\,\mu_0(s)\,ds.\end{aligned}$$

A.5 Proof of Proposition 4

I first consider the case $B < I$. Substituting the financing constraint into the objective, we have

$$V = \int_{\hat{s}}^{1} (s - I + B)\,\mu_0(s)\,ds \leq \int_{I-B}^{1} (s - I + B)\,\mu_0(s)\,ds.$$

If $E_{\mu_0} s \geq I$, the upper bound is achievable by setting $\hat{s} = I - B$ for any security $c(s)$ such that

$$\int_{I-B}^{1} (c(s) - I)\,\mu_0(s)\,ds = 0.$$

If $E_{\mu_0} s < I$, $\hat{s} = I - B$ is not feasible whenever

$$\int_{I-B}^{1} (s - I)\,\mu_0(s)\,ds < 0$$

and therefore $\hat{s} > I - B$ and $c(s) = s$ at the optimal solution. Letting $B \to I$, the above integral is negative, since it converges to $E_{\mu_0} s - I < 0$, while for $B \to 0$, it converges to

$$\int_{I}^{1} (s - I)\,\mu_0(s)\,ds > 0.$$

Since the integral is decreasing in B, there exists a threshold $B(I)$ such that for $B > B(I)$ the integral is negative, and $\hat{s} > I - B$, while for $B \leq B(I)$, it is positive, so $\hat{s} = I - B$. This establishes the result.

For $B \geq I$, we have for any $c(s)$ with $E_{\mu_0} c(s) < I$,

$$\begin{aligned} V &= \int_{\hat{s}}^{1} (s - c(s) + B) \mu_0(s)\, ds \\ &= \int_{\hat{s}}^{1} (s - I + B) \mu_0(s)\, ds \\ &\leq \int_{0}^{1} (s - I + B) \mu_0(s)\, ds \\ &= \int_{0}^{1} (s - c'(s) + B) \mu_0(s)\, ds \end{aligned}$$

Here, the inequality uses $B > I$, which guarantees that the integrand is positive for all s. For any $c''(s)$ with $E_{\mu_0} c(s) > I$,

$$\begin{aligned} V &= \int_{0}^{1} (s - c''(s) + B) \mu_0(s)\, ds \\ &< \int_{0}^{1} (s - I + B) \mu_0(s)\, ds \\ &= \int_{0}^{1} (s - c'(s) + B) \mu_0(s)\, ds. \end{aligned}$$

Thus, $c'(s)$ maximizes the entrepreneur's value. If $E_{\mu_0} s < I$, for any $\hat{s} > 0$ and any security with $c(s) < s$ for some s, we have $\frac{dV}{dc(s)} > 0$. Thus, $c(s) = s$ is optimal and $\hat{s} > 0$.

A.6 Proof of Proposition 5

Because $\lambda(x) \leq x$ for $x \geq 0$, we have for any belief μ

$$V(\mu) \leq E_{\mu}\left[u(s - c(s)) + c(s) - I\right].$$

Therefore, for any disclosure policy q

$$E_q V(\mu) \leq E_q\left[\mathbf{1}\{E_{\mu} c(s) \geq I\} \cdot E_{\mu}\left[u(s - c(s)) + c(s) - I\right]\right]$$

and in particular

$$\max_{q} E_{q}V\left(\mu\right) \leq \max_{q} E_{q}\left[\mathbf{1}\left\{E_{\mu}c\left(s\right) \geq I\right\} \cdot E_{\mu}\left[u\left(s-c\left(s\right)\right)+c\left(s\right)-I\right]\right] \tag{38}$$

for any Bayes plausible q. By Proposition 8, the policy that maximizes the right hand side is a threshold policy, where the threshold is such that the financing condition 2 binds. Under this policy, the value to the entrepreneur is

$$\int_{\hat{s}}^{1}\left(u\left(s-c\left(s\right)\right)+\lambda\left(\left(\int_{\hat{s}}^{1}c\left(s\right)\mu_{0}\left(s\right)ds\right)-I\right)\mu_{0}\left(s\right)ds=\int_{\hat{s}}^{1}u\left(s-c\left(s\right)\right)\mu_{0}\left(s\right)ds,$$

which is the same as the maximal value of the upper bound in Equation (38). Therefore, this policy is optimal for the entrepreneur's original problem in Equation (23). This proves the proposition.

B Additional Results

B.1 Formal Arguments for Section 3

Proposition 9. *If $\mu_0 \geq \bar{\mu}$, the optimal policy provides no disclosure and the project is financed with probability one. If $\mu_0 < \bar{\mu}$, the optimal value of the entrepreneur's problem (9) is given by equation (12). The optimal policy induces posterior $\bar{\mu}$ with probability $\frac{\mu_0}{\bar{\mu}}$ and 0 with probability $1-\frac{\mu_0}{\bar{\mu}}$.*

Proof. Suppose $\mu_0 \geq \bar{\mu}$. For any distribution over posteriors q, we have

$$\begin{aligned}\mu_{0}\left(1-c\right)+B+\lambda\left(\mu_{0}c-I\right) &\geq E_{q}\left[\mathbf{1}\left\{\mu \geq \bar{\mu}\right\}\left(\mu\left(1-c\right)+B\right)+\lambda\left(\mu c-I\right)\right] \\ &= \int_{\mu<\bar{\mu}} 0dq\left(\mu\right)+\int_{\mu\geq\bar{\mu}}\left(\mu\left(1-c\right)+B+\lambda\left(\mu c-I\right)\right)dq\left(\mu\right)\end{aligned}$$

since

$$B\int_{\mu\geq\bar{\mu}}dq\left(\mu\right) \leq B$$

and

$$\int_{\mu\geq\bar{\mu}}\mu dq\left(\mu\right) \leq E_{q}\mu=\mu_{0}.$$

Thus, providing no information is optimal.

Suppose $\mu_0 < \bar{\mu}$. If q puts any mass on $(0, \bar{\mu})$, then there exists a q' that distributes this mass between 0 and $\bar{\mu}$. q' satisfies constraint (5) and admits a higher value. Thus, any optimal q puts zero mass on $(0, \bar{\mu})$. Let $q_0 > 0$ be the mass q puts at zero. The Bayes plausibility constraint becomes

$$\int_{\bar{\mu}}^{1} \mu dq(\mu) + q_0 \cdot 0 = \mu_0$$

and the optimal value must satisfy

$$\begin{aligned} V^* &= \int_{\bar{\mu}}^{1} (\mu(1-c) + \lambda(\mu c - I)) \, dq(\mu) + B \int_{\bar{\mu}}^{1} dq(\mu) \\ &= \mu_0 (1-c) + \lambda(\mu_0 c - I) + B \int_{\bar{\mu}}^{1} dq(\mu). \end{aligned}$$

We have

$$\int_{\bar{\mu}}^{1} dq(\mu) = (1 - q_0)$$

since q must integrate to one and

$$\int_{\bar{\mu}}^{1} \mu dq(\mu) \geq \bar{\mu} \int_{\bar{\mu}}^{1} dq(\mu) = \bar{\mu}(1 - q_0).$$

If the inequality is strict, we can find an improvement by reducing q_0 and having q put point mass $1 - q_0$ on $\bar{\mu}$. Thus, the optimal policy puts mass q_0 on 0 and $1 - q_0$ on $\bar{\mu}$ which establishes the result. □

B.2 Risk-Averse Entrepreneur

Suppose that the entrepreneur's utility function is concave, increasing, and twice continuously differentiable. Even though for any given security, the optimal disclosure policy is the same as in the linear case, the optimal security issued must change due to risk-aversion. The entrepreneur prefers the retained payoffs $s - c(s)$ to be constant, which can be achieved by either selling the project for a fixed price, or selling a call option which is exercised only if the posterior is μ_h.

To see this, consider the Lagrangian associated with maximizing over $c(s)$,[36][37]

$$\mathcal{L} = \int_{\hat{s}}^{1} u(s - c(s)) \mu_0(s) \, ds - \gamma \int_{\hat{s}}^{1} (c(s) - I) \mu_0(s) \, ds. \tag{39}$$

The first-order conditions in $c(s)$ then imply $u'(s - c(s)) = \gamma$. Thus, $c(s)$ must take the form $c(s) = s - K$ all $s \geq \hat{s}$ for some constant K. This is exactly the payoff of an option with strike price K.[38] The optimal value then takes the form

$$V(K) = u(K) \int_{\hat{s}}^{1} \mu_0(s) \, ds.$$

If the optimal strike price is interior, it can be found via the first-order condition $V'(K) = 0$ which implies[39]

$$\begin{aligned} V'(K) &= u'(K) \int_{\hat{s}}^{1} \mu_0(s) \, ds - u(K) \frac{d\hat{s}}{dK} \mu_0(\hat{s}) \\ &= \int_{\hat{s}}^{1} \mu_0(s) \, ds \cdot \left(u'(K) + u(K) \frac{1}{\hat{s} - K - I} \right) \\ &= 0 \end{aligned}$$

so that[40]

$$u'(K) = -\frac{u(K)}{\hat{s} - K - I}. \tag{40}$$

The left hand side measures the gain in payoff for the entrepreneur from increasing the strike price, while the right hand side is the loss due to the lower likelihood that the project is financed, since investors now have to be more optimistic.

[36] I ignore the requirement that $c(s)$ and $s - c(s)$ must be increasing. It will be satisfied at the solution.

[37] The method presented here is heuristic. To study the problem formally, we can formulate it as an optimal control problem where s is interpreted as time, $c(.)$ is the control, and we introduce a state $x(s) = \int_{\hat{s}}^{s} \mu_0(s)(c(t) - I) \, dt$. The initial condition is $x(\hat{s}) = 0$ and condition (24) implies the boundary condition $x(1) = 0$. The boundary $\hat{s}$ is free. Then, Pontryagin's maximum principle can be used to characterize the solution, which is the same as the one derived here. See e.g. Kamien and Schwartz (2012).

[38] Since the entrepreneur's marginal utility must be constant for $s \geq \hat{s}$, we have $\hat{s} \geq K$.

[39] V is not necessarily concave in K, but it is single-peaked, since $u'(K) + u(K) \frac{1}{\hat{s} - K - I}$ is decreasing in K. The first-order condition is thus sufficient for finding an interior maximum.

[40] Note that $\hat{s} < K + I$.

The proposition below finds sufficient conditions for the strike price to be interior. If the utility satisfies INADA and investors are not willing to finance the project without disclosure even if $K = 0$, then the optimal strike price is characterized by condition (40). Intuitively, INADA guarantees that at $K = 0$, the entrepreneur's value is strictly increasing in K. The highest possible price at which the project still gets financed if investors know that $s = 1$ is $\bar{K} = 1 - I$, but as $K \to \bar{K}$, $\hat{s} \to 1$. As the strike price becomes large, the project is financed only if the state is high, which the entrepreneur must truthfully reveal. But since for any $K < \bar{K}$, the financing constraint implies $\hat{s} < K + I$, as $\hat{s} \to 1$ and $K \to \bar{K}$, $V'(K)$ becomes negative. Therefore, the optimal value is interior.

Proposition 10. *Suppose that* $\lim_{x \downarrow 0} u'(x) = \infty$. *Then, if* $\int_0^1 (s - I)\, \mu_0(s)\, ds \leq 0$, *the optimal strike price is interior and satisfies equation (40).*

If the project can be financed without disclosure when $K = 0$, which happens when $\int_0^1 (s - I)\, \mu_0(s)\, ds > 0$, whether the strike price is interior depends on the shape of u. The corner solution $K_0 = \int_0^1 (s - I)\, \mu_0(s)\, ds$ may be optimal if $V'(K_0) < 0$. This happens when K_0 is relatively large.

References

Admati, A. R. and P. Pfleiderer (2000). Forcing firms to talk: Financial disclosure regulation and externalities. *Review of Financial Studies 13*(3), 479–519.

Baginski, S. P., J. M. Hassell, and M. D. Kimbrough (2004). Why do managers explain their earnings forecasts? *Journal of accounting research 42*(1), 1–29.

Balakrishnan, K., M. B. Billings, B. Kelly, and A. Ljungqvist (2014). Shaping liquidity: On the causal effects of voluntary disclosure. *The Journal of Finance 69*(5), 2237–2278.

Ball, C., G. Hoberg, and V. Maksimovic (2014). Disclosure, business change and earnings quality. *Available at SSRN 2260371*.

Beyer, A., D. A. Cohen, T. Z. Lys, and B. R. Walther (2010). The financial reporting environment: Review of the recent literature. *Journal of accounting and economics 50*(2), 296–343.

Bowen, R. M., A. K. Davis, and D. A. Matsumoto (2002). Do conference calls affect analysts' forecasts? *The Accounting Review 77*(2), 285–316.

Dantzig, G. B. and A. Wald (1951). On the fundamental lemma of Neyman and Pearson. *The Annals of Mathematical Statistics 22*(1), 87–93.

DeMarzo, P. M., I. Kremer, and A. Skrzypacz (2005). Bidding with securities: Auctions and security design. *The American economic review 95*(4), 936–959.

Diamond, D. W. and R. E. Verrecchia (1991). Disclosure, liquidity, and the cost of capital. *The journal of Finance 46*(4), 1325–1359.

Dye, R. A. (1985). Disclosure of nonproprietary information. *Journal of accounting research*, 123–145.

Fishman, M. J. and K. M. Hagerty (1989). Disclosure decisions by firms and the competition for price efficiency. *The Journal of Finance 44*(3), 633–646.

Fishman, M. J. and K. M. Hagerty (1990). The optimal amount of discretion to allow in disclosure. *The Quarterly Journal of Economics*, 427–444.

Froot, K. A., D. S. Scharfstein, and J. C. Stein (1993). Risk managements coordinating corporate investment and financing policies. *the Journal of Finance 48*(5), 1629–1658.

Gale, D. and M. Hellwig (1985). Incentive-compatible debt contracts: The one-period problem. *The Review of Economic Studies 52*(4), 647–663.

Goldstein, I. and Y. Leitner (2015). Stress tests and information disclosure.

Hanley, K. W. and G. Hoberg (2010). The information content of IPO prospectuses. *Review of Financial Studies 23*(7), 2821–2864.

Harris, M. and A. Raviv (1989). The design of securities. *Journal of Financial Economics 24*(2), 255–287.

Healy, P. M. and K. G. Palepu (2001). Information asymmetry, corporate disclosure, and the capital markets: A review of the empirical disclosure literature. *Journal of accounting and economics 31*(1), 405–440.

Holthausen, R. W. and R. L. Watts (2001). The relevance of the value-relevance literature for financial accounting standard setting. *Journal of accounting and economics 31*(1), 3–75.

Hutton, A. P., G. S. Miller, and D. J. Skinner (2003). The role of supplementary statements with management earnings forecasts. *Journal of Accounting Research*, 867–890.

Innes, R. D. (1990). Limited liability and incentive contracting with ex-ante action choices. *Journal of economic theory 52*(1), 45–67.

Kamenica, E. and M. Gentzkow (2011). Bayesian persuasion. *American Economic Review 101*(6), 2590–2615.

Kamien, M. I. and N. L. Schwartz (2012). *Dynamic optimization: the calculus of variations and optimal control in economics and management*. Courier Corporation.

Kogan, S., B. Routledge, J. S. Sagi, and N. A. Smith (2010). Information content of public firm disclosures and the Sarbanes-Oxley Act. *Available at SSRN 1584763*.

Levinson, N. (1966). A class of continuous linear programming problems. *Journal of Mathematical Analysis and Applications 16*(1), 73–83.

Milgrom, P. and I. Segal (2002). Envelope theorems for arbitrary choice sets. *Econometrica 70*(2), 583–601.

Monnet, C. and E. Quintin (2015). Rational opacity in private equity markets. Technical report, working paper.

Nachman, D. C. and T. H. Noe (1994). Optimal design of securities under asymmetric information. *Review of Financial Studies 7*(1), 1–44.

Reiland, T. W. (1980). Optimality conditions and duality in continuous programming II. The linear problem revisited. *Journal of Mathematical Analysis and Applications 77*(2), 329–343.

Trigilia, G. (2016). Optimal Leverage and Strategic Disclosure.

Verrecchia, R. E. (1983). Discretionary disclosure. *Journal of accounting and economics 5*, 179–194.