Multi-Dimensional Pass-Through, Incidence, and the Welfare Burden of Taxation in Oligopoly*

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Abstract

This paper studies welfare consequences of unit and ad valorem taxes in oligopoly with general demand, non-constant marginal costs, and a generalized type of competition. We present formulas providing connections between marginal cost of public funds, tax incidence, unit tax pass-through, ad valorem tax pass-through, and other economic quantities of interest. First, in the case of symmetric firms, we show that there exists a simple, empirically relevant set of sufficient statistics for the marginal cost of public funds, namely the *pass-through* and the *industry demand elasticity*. Specializing to the case of price or quantity competition, we show how marginal cost of public funds and pass-through are expressed using *elasticities* and *curvatures* of demand and inverse demand. Second, we present a generalization with the tax revenue function specified as a general function parameterized by a vector of tax parameters. We analyze *multi-dimensional* pass-through, generalizing the results of Weyl and Fabinger (2013), and show that it is crucial for evaluating welfare changes in response to changes in taxation. Finally, we argue that our results are carried over to the case of heterogeneous firms and other extensions.

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1 Introduction

The marginal cost of public funds, i.e. the marginal social welfare loss associated with raising additional tax revenue, is a crucial characteristic that a policymaker needs to take into account when designing an optimal system of taxes.[1] However, it is somewhat surprising that relatively little is known about the general mechanism of how tax levies are passed through to final prices as well as their welfare consequences in oligopoly, a ubiquitous feature of competition in the real-world economy. In this paper, we aim to contribute to the general understanding of the welfare consequences of taxation in oligopolistic markets with a general (first-order) type of competition (of single-product or multi-product firms), with a possibly non-constant marginal cost, and with non-zero initial levels of unit and ad valorem taxes. Specifically, we establish connections between welfare measures, namely the marginal cost of public funds for unit taxes and ad valorem taxes, and variables that are easily interpretable from an empirical standpoint, namely the *pass-through* of these taxes (i.e. the marginal change of prices induced by a tax rate change).[2] In particular, we show that with a general type of competition, there exists a simple set of sufficient statistics that determines the marginal cost of public funds of unit and ad valorem taxes, namely *pass-through* of these taxes and the *industry demand elasticity* (in addition to easily observable taxation levels).[3]

[1]In the absence of other considerations, the marginal cost of public funds should be equalized across markets in order to maximize social welfare.

[2]The usefulness of pass-through in welfare analysis has been verified by related studies such as Cowan (2012); Miller, Remer, and Sheu (2013); Weyl and Fabinger (2013); Gaudin and White (2014); MacKay, Miller, Remer, and Sheu (2014); Adachi and Ebina (2014a,b); Chen and Schwartz (2015); Gaudin (2016); Cowan (2016); Alexandrov and Bedre-Defolie (2017); and Mrázová and Neary (2017). See also Ritz (2017) for an excellent survey of theoretical studies on pass-through and pricing under imperfect competition. As an antitrust analysis, Froeb, Tschantz, and Werden (2005) study to theoretically compare the price effects when no synergies in cost reduction realize when they are passed through as a form of price reduction. See also Alexandrov and Koulayev (2015) for discussions on the role of pass-through in antitrust analysis

[3]The sufficient-statistics approach to connecting structural and reduced-form methods, as advocated by Chetty (2009), has been successful in empirical economics. For example, in the study by Atkin and Donaldson (2016), the pass-through rate provides a sufficient statistic for welfare implications of intra-national trade costs in low-income countries, without the need for a full demand estimation. Similarly, Ganapati, Shapiro, and Walker (2017) examine the welfare effects of input taxation, where a unit tax is levied on the input. These effects are related to the effects of unit taxes on output, but not identical. See also Fabra and Reguant (2014); Shrestha and

This result is a part of a larger set of relationships that link economic quantities of interest. We derive succinct formulas that relate the marginal cost of public funds to pass-through of taxes of the same type. We also establish a relationship that connects pass-through of unit taxes and pass-through of ad-valorem taxes in the same market. Further, we derive convenient expressions for values of unit and ad valorem pass-through that are valid under a general type competition and have not appeared in the previous literature. In addition, specializing to price (differentiated Bertrand) or quantity (pure or differentiated Cournot) competition, we show how the marginal cost of public funds and pass-through are expressed using *elasticities* and *curvatures* of demand and inverse demand, and provide illustrative examples. Our results also apply without change to symmetric oligopoly with multi-product firms. Throughout the analysis, we allow for non-zero levels of unit and ad valorem taxes. However, we also discuss some additional simplifications that appear when instead the initial level of taxes is zero.

Furthermore, we generalize our results to a significantly more general specification of taxation that involves multiple tax parameters. We define two different types of pass-through vectors: the *pass-through rate vector* and the *pass-through quasi-elasticity vector*. We study their properties and show that they are crucial for evaluating welfare changes in response to changes in taxation. Special cases involve not only unit and ad valorem taxation, but also exogenous competition discussed by Weyl and Fabinger (2013), as well as, for example, value-added tax, under which the firm can deduct a portion of its costs from its profit for taxation purposes. Another type of generalization we discuss is the case of changes in both production costs and taxes. It turns out that this generalization is very straightforward. This allows us to consider other economic situations, such as cost changes due to exchange rate movements or movements in the world prices of commodities, within a single general

Markowitz (2016); Duso and Szücs (2016); Stolper (2016); and Hong and Li (2017) for studies with the same spirit. In contrast, Kim and Cotterill (2008) is among the first studies that structurally estimate cost pass-through in differentiated product markets, followed by Bonnet, Dubois, Villas-Boas, and Klapper (2013); Bonnet and Réquillart (2013); Campos-Vázquez and Medina-Cortina (2015); Griffith, Nesheim, and O'Connell (2015); Miller, Remer, Ryan, and Sheu (2016); Conlon and Rao (2016); and Miller, Osborne, and Sheu (2017).

framework.

From both theoretical and empirical standpoints, it is desirable to be able to understand the welfare properties of oligopolistic markets with a general type of competition. In real-world situations, firms' behavior may not simply be categorized into either the idealized price competition or the idealized quantity competition. Price competition does not allow for any friction in scaling production levels up or down, yet in reality there tend to be substantial frictions, such as those related to financial constraints or the labor market. Quantity competition implies that the firm will not be able to increase production levels when its competitor suddenly decides to increase prices. In reality, such adjustment is probably feasible, since capacity utilization is typically less than complete, and even if the firm is operating at full capacity, boosting production levels is possible by overtime work or by hiring temporary workers. Moreover, firms may behave, to some extent, in a collusive way. Although the realities of competition by firms may be complicated, it is possible to capture their essence by working with a general type of competition, using the *conduct index*.[4]

Besides working with a general type of competition, it is also useful to relax the assumption of constant marginal costs that often appears in the literature. Production technologies often have non-trivial structure, and so does the internal organization of the firm. For example, if a firm decides to operate at a larger scale, it may take advantage of technological and logistical economies of scale, but at the same time, it may face more severe principal-agent problems as top managers have to delegate responsibilities to lower-level managers. The interplay between these forces can lead to a non-trivial dependence of the marginal cost of production on the scale of the operation.

This paper is related to the inspiring study by Häckner and Herzing (2016), which motivates parts of this work. In the special case of linear demand, and constant marginal cost, Häckner and Herzing (2016, p. 147) explain that as long as

[4]For details of this approach, see Bresnahan (1989) and Weyl and Fabinger (2013). It has been successfully applied also to more general situations, such as selection markets (Mahoney and Weyl 2017) or supply chains (Gaudin 2017).

the initial level of taxes is zero, the marginal cost of public funds for unit taxation equals $MC_t = \theta\rho_t$, where ρ_t is the unit-tax pass-through rate (the marginal effect of unit taxes on prices), and θ, usually referred to as the conduct "parameter," measures the industry's competitiveness (for example, in the case of monopoly $\theta = 1$, while under perfect competition $\theta = 0$). For ad valorem taxes, Häckner and Herzing (2016) provide a similar formula. They show, however, that if we let the initial level of taxes be non-zero, those formulas are no longer valid. For this reason, they are forced to analyze the magnitude of the marginal cost of public funds on a case-by-case basis using explicit solutions to specific models.

This situation represents a puzzle. If there are simple formulas for the marginal cost of public funds that were valid at zero taxes, is there no compact generalization of these expressions in the case of non-zero taxes? If there is no such generalization, that would be an obstacle to empirical work, since we would have to make additional modeling assumptions before obtaining empirical estimates of the marginal cost of public funds. Our paper provides a solution to this problem. In particular, Proposition 1 presents formulas for the marginal cost of public funds that are valid even when the initial level of (both ad valorem and unit) taxes is non-zero. They are a bit longer than $MC_t = \theta\rho_t$, but still very manageable. They also represent a starting point for the topics discussed in the rest of the paper. These results with a non-zero initial taxes being allowed, which are differentiated from Weyl and Fabinger (2013) and Häckner and Herzing (2016), should be useful if one needs to evaluate the marginal cost of taxation when some tax has been already implemented.

The welfare cost of taxation has been extensively studied at least since Pigou (1928). The majority of the studies simply assume perfect competition (with zero initial taxes).[5] As is widely known, under perfect competition, unit tax and ad valorem tax are equivalent, and whether consumers or producers bear more is determined by the relative elasticities of demand and supply (Weyl and Fabinger

[5]See, e.g., Vickrey (1963), Buchanan and Tullock (1965), Johnson and Pauly (1969), and Browing (1976) for early studies. A study of unit and ad valorem taxation under imperfect competition with homogenous products dates back to Delipalla and Keen (1992). See Auerbach and Hines (2002) and Fullerton and Metcalf (2002) for comprehensive surveys for this field.

(2013)). The initial attempt to relax the assumption of perfect competition started with an analysis of homogeneous-product oligopoly under quantity competition, i.e., Cournot oligopoly. Notably, Delipalla and Keen (1992), Skeath and Trandel (1994), and Hamilton (1999) compare ad valorem and unit taxes in such a setting. Then, Anderson, de Palma, and Kreider (2001a) extend these results importantly to the case of differentiated oligopoly under price competition. In particular, Anderson, de Palma, and Kreider (2001a) find that whether the after-tax price for firms and their profits rise by a change in ad valorem tax depends importantly on the ratio of the curvature of the firm's own demand (ε_m in their notation, and α_F in our notation below) to the elasticity of the market demand ε_{DD} in their notation, and ϵ in our notation).

We extend Anderson, de Palma, and Kreider's (2001a) setting and results in a number of important directions. First, we consider the general mode of competition, captured by the conduct index, including both quantity and price competition. Second, we provide a complete characterization of tax burdens that enables one to quantitatively compare consumers' burden with producers' burden, whereas Anderson, de Palma, and Kreider's (2001a) focus only on the effective prices for consumers and producers' profits. Third, while Anderson, de Palma, and Kreider's (2001a) assume constant marginal cost, we allow non-constant marginal cost and show how this generalization makes a difference in our general formulas. Fourth, we further generalize the initial tax level. When they analyze the effects of a unit tax, Anderson, de Palma, and Kreider (2001a) assume that ad valorem tax is zero, and vice versa. In contrast, we allow non-zero initial taxes in both dimensions. Finally, and importantly, we generalize these results to the case of a very general type of taxation, as well as to production cost changes. This opens up the possibility to study a wider range of interventions/taxes and to derive convenient sufficient statistics for characteristics, including welfare characteristics, of the markets of interest.

In the next section, we study the problem of oligopoly with a general type of competition. In Section 3, we specialize to the case of price or quantity competition. Section 4 generalizes the results from unit and ad valorem taxation to much more flexible taxation parameterized by d different tax parameters and discusses the implications of these general results. Section 5 contains a discussion of heterogeneous firms. Section 6 generalizes our previous results to the case of changes in both production costs and taxes. Section 7 concludes.

2 Taxation and Welfare in Symmetric Oligopoly

We study oligopolistic markets with n symmetric firms and a general (first-order) mode of competition and the resulting symmetric equilibria.[6] Our discussion applies to single-product firms as well as to *multi-product* firms if intra-firm symmetry conditions are satisfied, as discussed in Appendix D. For simplicity of exposition, we use terminology corresponding to single-product firms here, and later we discuss how to interpret the results in the case of multi-product firms.

The demand for firm j's product $q_j = q_j(p_1, ..., p_n) \equiv q_j(\mathbf{p})$ depends on the vector of prices $\mathbf{p} \equiv (p_1, ..., p_n)$ charged by the individual firms. The demand system is symmetric and the cost function $c(q_j)$ is the same for all firms. We assume that $q_j(\cdot)$ and $c(\cdot)$ are twice differentiable and conditions for the uniqueness of equilibrium and the associated second-order conditions are satisfied. We denote by $q(p)$ the per-firm industry demand corresponding to symmetric prices: $q(p) \equiv q_j(p, ..., p)$. The elasticity of this function, defined as $\epsilon(p) \equiv -pq'(p)/q(p) > 0$ and referred to as the *price elasticity of industry demand,* should not be confused with the elasticity of the residual demand that any of the firms faces.[7] We also use the notation $\eta(q) = 1/\epsilon(p)|_{q(p)=q}$ for the reciprocal of this elasticity as a function of q. For the corresponding functional values, when we do not need to specify explicitly their dependence on either q or p, we use η interchangeably with $1/\epsilon$.

[6] Although for brevity we speak of a general mode of competition, we consider only "first-order" competition, in the sense of the firms making decisions based on marginal cost and marginal revenue. This excludes, for example, the possibility of each producer being composed of two vertically related firms where the upstream firm sets prices for a relationship-specific intermediate good, as in the usual double-marginalization setting.

[7] The elasticity ϵ here corresponds to ϵ_D in Weyl and Fabinger (2013, p. 542). Note that $q'(p) = \partial q_j(\mathbf{p})/\partial p_j + (n-1)\partial q_j(\mathbf{p})/\partial p_{j'}|_{\mathbf{p}=(p,...,p)}$ for any two distinct indices j and j'. We will define the firm's elasticity and other related concepts in Section 3.

We introduce two types of taxation: a unit tax t and an ad valorem tax v, with firm j's profit being $\pi_j = (1-v)p_j(q_j)q_j - tq_j - c(q_j)$. At symmetric quantities the government tax revenue per firm is $R(q) \equiv tq + vp(q)q$, and we denote by $\tau(q)$ the fraction of firm's pre-tax revenue that is collected by the government in the form of taxes: $\tau(q) \equiv R(q)/pq = v + t/p(q)$. We define the *conduct index* $\theta(q)$ as

$$\theta(q) = \frac{1}{\eta(q)\,p(q)}\left(p(q) - \frac{t + mc(q)}{1-v}\right), \tag{1}$$

where $mc(q) \equiv c'(q)$ is the marginal cost of production, and we denote by θ its functional value at the equilibrium quantity.[8] This is also understood as the *elasticity-adjusted Lerner index*: the mark-up rate $[p-(t+mc)/(1-v)]/p$ should be adjusted by the industry-wide elasticity to reflect the competitiveness in the industry, where $(t+mc)/(1-v)$ is interpreted as the effective marginal cost.[9] We emphasize here that once the conduct index is introduced, one is able to describe oligopoly in a synthetic manner, without specifying whether it is price or quantity setting, or whether it exhibits strategic substitutability or complementarity.

Note here that this specification of taxation scheme is a case of two-dimensional pass-through instruments, and it is a special case of multi-dimensional pass-through instruments. For example, if the parameter z in the cost function (i.e., $mc(q;z)$), capturing the cost reduction due to an exogenous technology improvement, is also considered, the "policy mix" is three-dimensional: (v, t, z). In Section 5, we introduce a framework of multi-dimensional pass-through.

[8]More precisely, $\theta(q)$ is defined to be a function independent of the cost side of the economic problem such that the symmetric equilibrium condition may be written in the form of Equation (1). Our conduct index corresponds to what is known as "conduct parameter" in the empirical indusrtial organization literature, where it is supposed to be constant as a target of estimation (see, e.g., Bresnahan 1989; Genesove and Mullin 1998; Nevo 1998; and Corts 1999). In this paper, we opt for the term "conduct index" to make it explicit that it is a variable. Note that this definition does not exclude $\theta(q) > 1$, although in most interesting cases it lies in $[0, 1]$.

[9]Accordingly, one can write the modified Lerner rule under (v, t) as

$$\frac{p - \frac{t+mc}{1-v}}{p} = \eta\theta,$$

which implies the restriction on θ: $\theta \leq \epsilon$.

2.1 The marginal cost of public funds

The marginal welfare cost MC_t or MC_v of raising government revenue by the unit tax t or the ad valorem tax v, i.e. the marginal cost of public funds associated with such a tax, is defined as

$$MC_t \equiv -\left(\frac{\partial R}{\partial t}\right)^{-1}\frac{\partial W}{\partial t}, \qquad MC_v \equiv -\left(\frac{\partial R}{\partial v}\right)^{-1}\frac{\partial W}{\partial v},$$

where W is the social welfare per firm, which includes consumer surplus, producer surplus, and government tax revenue. We define the *unit tax pass-through rate* ρ_t and the *ad valorem tax pass-through semi-elasticity* ρ_v as:[10]

$$\rho_t = \frac{\partial p}{\partial t}, \qquad \rho_v = \frac{1}{p}\frac{\partial p}{\partial v}.$$

Consider an infinitesimal change in the unit tax, with the initial tax level (t, v). As mentioned in the introduction, in the special case of zero initial taxes, linear demand, and constant marginal cost, Häckner and Herzing (2016, p. 147) show that $MC_t = \theta\rho_t$ and $MC_v = \theta\rho_v$, noting that at non-zero initial taxes the formula no longer applies. In the absence of such formula, they were forced to study the marginal cost of public funds on a case-by-case basis, for different specifications of demand and cost.

However, there are mainly two deficiencies in using $\theta\rho_t$ as a measure of the marginal cost of public funds when a unit tax is raised (the argument for $\theta\rho_v$ is analogous). First, the expression is simply proportional to θ, but when v is large, the firms sell at prices that are too high from the social perspective not because of a lack of competitiveness, but because the tax effectively raises their perceived cost. When v is large, we would expect the marginal cost of public funds to be less sensitive to θ, for a given value of ρ_t. Second, the expression $\theta\rho_t$ does not explicitly feature the level of the unit tax t. However, a situation where t is large and mc small is very different from a situation where t is small and mc large, even if the equilibrium prices and quantities are the same. In the former case, raising additional tax revenue is

[10]Note that Häckner and Herzing (2016) use the symbol ρ_v for the ad valorem tax pass-through rate $\partial p/\partial v$, which corresponds to $p\rho_v$ in our notation.

quite harmful, since firms' production cuts will not substantially decrease the total technological (i.e., pre-tax) cost of production. In the latter case, raising additional tax revenue is less harmful since it leads to reduced total technological cost. Based on this intuition, we would expect the marginal cost of public funds to be an increasing function of t.[11]

Thus, we are led to find a generalization of the formula $MC_t = \theta\rho_t$ and $MC_v = \theta\rho_v$ that would be applicable even at non-zero initial taxes. It turns out that it is possible to identify a formula with precisely these properties, as the following proposition shows.

Proposition 1. ***Marginal cost of public funds for unit and ad valorem taxations.*** *Under symmetric oligopoly with a possibly non-constant marginal cost, the marginal cost of public funds associated with a unit tax may be expressed as*

$$MC_t = \frac{(1-v)\,\theta + \epsilon\,\tau}{\frac{1}{\rho_t} + v - \epsilon\,\tau},$$

and the marginal cost of public funds associated with an ad valorem tax may be expressed as

$$MC_v = \frac{(1-v)\theta + \epsilon\tau}{\frac{1}{\rho_v} + v - \epsilon\tau}.$$

Proof. See Appendix A. □

Figure 1 documents that these expressions for the marginal cost of public funds MC_t and MC_v evaluated at realistic values of taxes and other economic variables are very different from the values of the expressions $\theta\rho_t$ and $\theta\rho_v$ (discussed above) that would be equal to MC_t and MC_v if taxes were zero.

The intuition behind Proposition 1 for the case of unit taxation can explained as follows. The argument for ad valorem taxation is analogous. First, the firm's per-output profit margin is decomposed into two parts: (1) tax payment, $t+vp = p\tau$ and (2) surplus from imperfect competition, $(1-v)p\eta\theta$. Under imperfect competition,

[11] In the sense of making the change $t \to t + \Delta t$, and simultaneously $c(q) \to c(q) - q\Delta t$ in order to keep q, θ, and ρ_t at some fixed values.

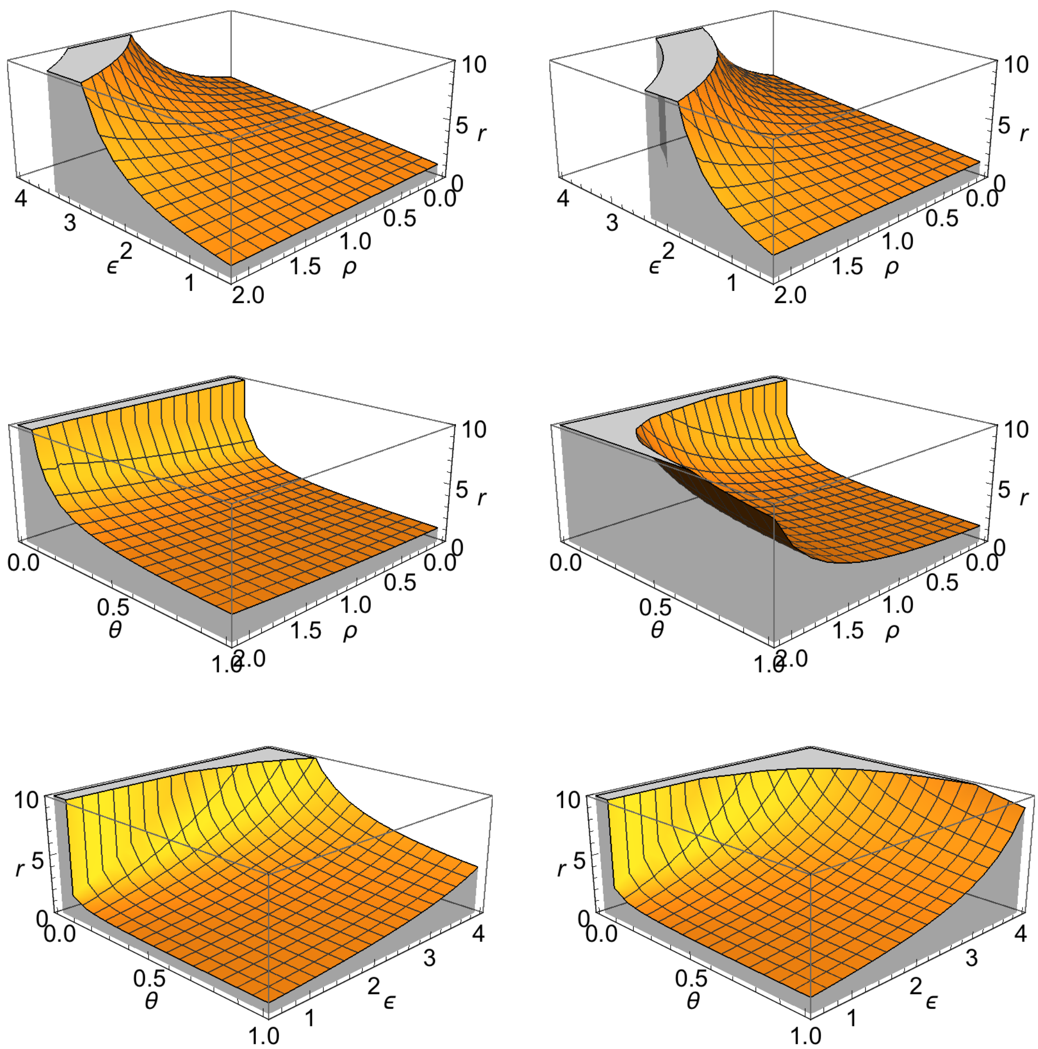

Figure 1: The ratio of the actual marginal cost of public funds MC and the naive expression $\theta\rho$ discussed just before Proposition 1, plotted as a function of combinations of the conduct index θ, the pass-through ρ, and the industry demand elasticity ϵ. The figures on the left correspond to infinitesimal changes in unit taxation: ρ stands for ρ_t and MC stands for MC_t. The numerical values were chosen to be $t = 0$, $v = 0.2$, $\tau = 0.2$. The figures on the right correspond to infinitesimal changes in ad valorem taxation: ρ stands for ρ_v and MC stands for MC_v. The numerical values were chosen to be $t/p = 0.2$, $v = 0$, $\tau = 0.2$. The top figures correspond to $\theta = 0.3$, the middle figures correspond to $\epsilon = 2$, and the bottom figures correspond to $\rho = 1$.

the effects of an increase in unit tax, dt, on the social welfare can be decomposed into two parts:

$$dW = \underbrace{p\,dq}_{(1)<0} + \underbrace{(-mc\,dq)}_{(2)>0},$$

where term (1) corresponds to the loss incurred to consumer surplus, whereas term (2) to the gain from cost savings associated with the output reduction. Thus, *the firm′s per-output profit margin serves as a measure for welfare change*. On the other hand, the effects of an increase in unit tax, dt, on the tax revenue are:

$$dR = \underbrace{q\,dt}_{(1)>0} + \underbrace{vq\,dp}_{(2)>0} + \underbrace{(t+vp)dq}_{(3)<0},$$

where term (1) expresses (direct) gain, multiplied by the output q, and term (2) shows (indirect) gain, due to the associated price increase, multiplied by vq, whereas term (3) is the part that exhibits (indirect) loss from the output reduction for both unit tax revenue and ad valorem tax revenue. Now recall that $dp = \rho_t dt$ and $p\eta dq = -qdp$. Thus, $qdt = qdp/\rho_t = -(p\eta/\rho_t)dq$ and $vqdp = -(vqp/q)\eta dq = -(vp\eta)dq$, which implies that

$$dR = -(p\eta/\rho)dq - (vp\eta)dq + (t+vp)dq = [\underbrace{(-p\eta/\rho_t)}_{(1)>0} + \underbrace{(-vp\eta)}_{(2)>0} + \underbrace{(t+vp)}_{(3)<0}].$$

Now, in the per-price term, the denominator and the numerator in MC_t are expressed as follows:

$$MC_t = \frac{\underbrace{(1-v)\eta\theta + \tau}_{\text{welfare loss expressed by the profit margin}}}{\underbrace{\left(\frac{1}{\rho_t} + v\right)\eta}_{\text{revenue gain}} + \underbrace{(-\tau)}_{\text{revenue loss}}}.$$

2.2 Incidence and pass-through

We next define the incidence I_t of unit taxation as the ratio of changes dCS in (per-firm) consumer surplus and changes dPS in (per-firm) producer surplus induced by an infinitesimal increase dt in the unit tax t. The incidence I_v is defined analogously. Then, we obtain the following succinct formulas for the incidence of taxation at non-zero unit and ad valorem taxes,

Proposition 2. ***Incidence of taxation.*** *Under symmetric oligopoly with a general type of competition and with a possibly non-constant marginal cost, the incidence of unit taxes I_t and of ad valorem taxes I_v is given by*

$$\frac{1}{I_t} = \frac{1}{\rho_t} - (1 - v)(1 - \theta),$$

$$\frac{1}{I_v} = \frac{1}{\rho_v} - (1 - v)(1 - \theta).$$

Proof. See Appendix B. □

In the case of zero ad valorem tax, the expression for I_t reduces to Weyl and Fabinger's (2013, p.548) Principle of Incidence 3. The intuitive reasoning can be provided as follows. First, the effects of an increase in unit tax, dt, on the producer surplus can be decomposed into the following five parts:

$$dPS = [\underbrace{(-q\,dt)}_{(1)<0} + \underbrace{(1-v)p\,dq}_{(2)<0}] + [\underbrace{(1-v)q\,dp}_{(3)>0} + \underbrace{(-mc\,dq)}_{(4)>0} + \underbrace{(-t\,dq)}_{(5)>0}],$$

where term (1) shows the (direct) loss from an increase in unit tax: the tax increase multiplied by the output q, and term (2) is another (indirect) loss from a reduction in production, multiplied by the ad valor em tax adjusted unit price $(1 - v)p$, whereas term (3) corresponds to the (direct) gain from the associated price increase, mitigated by $(1 - v)$, due to the ad valorem tax, multiplied by the output q, and finally terms (4) and (5) are (indirect) gains from cost savings by the output reduction, dq, and from unit tax saving by the output reduction, dq, respectively. Note here that the equation above is rewritten as

$$dPS = [\underbrace{-q\,dt}_{(1)<0} + \underbrace{(1-v)q\,dp}_{(3)>0}] + [(1-v)p - \underbrace{(mc+t)}_{\text{marginal cost}}]dq.$$

Now, in symmetric equilibrium, the marginal cost, $mc + t$, is equal to the marginal benefit, $(1 - v)p[1 - \eta\theta]$, which implies

$$dPS = [\underbrace{-q\,dt}_{(1)<0} + \underbrace{(1-v)q\,dp}_{(3)>0}] + [(1-v)p]\eta\theta\,dq.$$

Under perfect competition, part (2) is equal to the sum of parts (4) and (5), and thus only parts (1) and (3) survive. However, under imperfect competition, the marginal cost is less than $(1 - v)p$, thus part (2) is greater than the sum of parts (4) and (5). The third term in the equation above now expresses the difference between part (2) and the sum of parts (4) and (5). Now, recall that $dp = \rho_t dt$ and $p\eta dq = -qdp$. Thus,

$$dPS = [-q\,dt + (1-v)q\rho_t\,dt] - (1-v)q\theta\,dp = [-q\,dt + (1-v)q\rho_t\,dt] - (1-v)q\theta\rho_t\,dt$$

$$= [-1 + (1-v)\rho_t - (1-v)\theta\rho_t]q\,dt = [\underbrace{-1}_{(1)<0} + \underbrace{(1-v)(1-\theta)\rho_t}_{(3)-\{(2)-[(4)+(5)]\}\gtrless 0}]q\,dt.$$

On the other hand, $dCS = -\rho_t(qdt)$. Thus, while it is always the case that $dCS < 0$, it is possible that $dPS > 0$.[12]

Next, we show how ρ_t and ρ_v are related in the following proposition.

Proposition 3. ***Relationship between pass-through of ad valorem and unit taxes.*** *Under symmetric oligopoly with a possibly non-constant marginal cost, the pass-through semi-elasticity ρ_v of an ad valorem tax may be expressed in terms of the*

[12]One can also define *social incidence* by $SI_t \equiv dW/dPS$ and SI_v in association with a small change in t and v, respectively. Hereafter, we focus on MC_t and MC_v as measures of welfare burden in society, and I_t and I_v as measures of loss in consumer welfare. We provide general formulas for social incidence in the context of multi-dimensional pass-through after Section 4.

unit tax pass-through rate ρ_t, the conduct index θ, and the industry demand elasticity ϵ as

$$\rho_v = \left(1 - \frac{\theta}{\epsilon}\right)\ \rho_t. \tag{2}$$

Proof. See Appendix C. □

To understand this proposition intuitively, note that Δt and Δv must satisfy:

$$\frac{t + \Delta t + mc}{1 - (v + \Delta v)} = \frac{t + mc}{1 - v}.$$

Thus, the relative Δt that must be offset by a reduction $-\Delta v$ equal to $(t+mc)/(1-v)$: $\Delta t = -(t + mc)\Delta v/(1 - v)$, which, together with $\rho_t dt + \rho_v p\, dv = 0$, leads to $(t + mc)\rho_t/[(1 - v)p] = \rho_v$. Now, recall the Lerner rule:

$$1 - \underbrace{\frac{t + mc}{(1 - v)p}}_{\text{per-price marginal cost}} = \eta\theta,$$

which implies that $(1 - \eta\theta)\rho_t = \rho_v$, as Proposition 3 claims. Now, $\theta/\epsilon = 1 - \rho_v/\rho_t$ implies that $\rho_v \le \rho_t \le (1 - 1/\epsilon)\, \rho_v$.

This formula provides another look of the well-known result that unit tax and ad valorem tax are equivalent in the welfare effects under perfect competition: if $\theta = 0$, then $\rho_t = \rho_v$. However, under imperfect competition, ρ_t is always greater than ρ_v. This also provides another look of Anderson, de Palma, and Kreider's (2001b) result that unit taxes are welfare-inferior to ad valorem taxes, when a policy maker faces the choice of whether a unit tax solely or an ad valorem tax solely is employed.[13]

By using this proposition, we claim that MC_t and MC_v can be expressed without the degree of competitiveness, θ, which is a complex measure of both readily observable concepts such as the number of firms and less clearly observable concepts such as how much the industry is collusive.

[13]Under the cannonical mode of quantity competition (i.e., Cournot competition), where firms' products are perfect substitutes so that $p = P(q_1, ..., q_n) = P(q_1 + \cdots + q_n)$, and in symmetric equilibrium $P(nq) = p(q)$, Auerbach and Hines' (2002, p.1396) Equation (6.13) is identical to Equation (2) above. Proposition 3 above shows that this equation is a general property that holds irrespective of competition mode. We thank Germain Gaudin for pointing this out.

Proposition 4. ***Sufficient statistics for marginal costs of public funds.*** *Under symmetric oligopoly with a possibly non-constant marginal cost, the unit pass-through rate ρ_t, the ad valorem pass-through semi-elasticity ρ_v, and the elasticity ϵ of industry demand (together with the tax rates and the fraction τ of the firm's pre-tax revenue collected by the government in the form of taxes) serve as sufficient statistics for the marginal cost of public funds both with respect to unit taxes and ad valorem taxes. In particular:*

$$MC_t = \frac{(1-v+\tau)\rho_t - (1-v)\rho_v}{1+(v-\epsilon\tau)\rho_t}\,\epsilon,$$

$$MC_v = \frac{(1-v+\tau)\,\rho_t - (1-v)\rho_v}{1+(v-\epsilon\tau)\,\rho_v}\,\frac{\rho_v}{\rho_t}\epsilon.$$

Proof. Proposition 3 allows us to express the conduct index θ as $\theta = (1-\rho_v/\rho_t)\epsilon$. Substituting this into the relationships in Proposition 1 gives the desired result. $\square$

Recall from Proposition 1 that

$$MC_t = \frac{\underbrace{(1-v)\eta\theta+\tau}_{\text{welfare loss expressed by the profit margin}}}{\underbrace{\left(\frac{1}{\rho_t}+v\right)\eta}_{\text{revenue gain}} + \underbrace{(-\tau)}_{\text{revenue loss}}}.$$

Now, Proposition 4 states that it is also understood as

$$MC_t = \frac{\underbrace{(1-v)\left(1-\frac{\rho_v}{\rho_t}\right)+\tau}_{\text{welfare } loss \text{ expressed by the profit margin}}}{\underbrace{\left(\frac{1}{\rho_t}+v\right)\eta}_{\text{revenue gain}} + \underbrace{(-\tau)}_{\text{revenue loss}}}.$$

Of course, it is true that θ is expressed by the empirical measures such as $\theta = (1-\rho_v/\rho_t)\epsilon$. For example, in the case of the assumption of Cournot competition, researchers often may observe the number n of firms and conclude that the value of conduct index is $\theta = 1/n$. However, even in the case of homogeneous products, the "true" conduct may be higher than $1/n$ due to such reasons as collusion.[14] Proposition 4 above circumvents this difficulty in estimating MC_t and MC_v.[15] Conversely, one would be able to estimate θ using the proposition above once ϵ, ρ_t, and ρ_v are estimated. In the next section, we provide another formulas with the explicit use of the *second-order* measures of the demand and the supply by assuming that the industry is described as a pure form of price or quantity competition.

As the last result presented in this section, the next proposition shows how the two forms of pass-through are characterized.

Proposition 5. ***Pass-through under general symmetric oligopoly.*** *Under symmetric oligopoly with a general mode of competition and with a possibly non-constant marginal cost:*

$$\rho_t = \frac{1}{1-v}\ \frac{1}{1-(\eta+\chi)\,\theta+\epsilon q\,(\theta\eta)'+\frac{1-\tau}{1-v}\epsilon\chi},$$

where the derivative is taken with respect to q and $\chi \equiv mc'q/mc$ is the "quantity elasticity of the marginal cost." Further,

$$\rho_v = \frac{\epsilon-\theta}{(1-v)\,\epsilon}\ \frac{1}{1-(\eta+\chi)\,\theta+\epsilon q\,(\theta\eta)'+\frac{1-\tau}{1-v}\epsilon\chi}.$$

Proof. Here, we provide a proof as well as intuitive arguments. Consider the comparative statics with respect to a small change dt in the per-unit tax t. Following Weyl and Fabinger (2013, p.538), we define $ms \equiv -p'q$: this is the negative of marginal consumer surplus. Then, the Learner condition becomes:

$$\underbrace{p-\frac{t+mc}{1-v}}_{\text{markup}} = \underbrace{\theta\,ms}_{\text{CS}},$$

[14]See Miller and Weinberg (2017) for an empirical study of the possibility of oligopolistic collusion in a different manner from directly estimating the conduct parameter.

[15]Similarly, the incidence with a unit tax is expressed as

$$\frac{1}{I_t} = \frac{1}{\rho_t} - (1-v)\left[(1-\epsilon)+\frac{\rho_v}{\rho_t}\epsilon\right],$$

and analogously for the case of an ad valorem tax.

where CS is consumer surplus for the inframarginal consumers. Importantly, θms measures how much consumer surplus rises for a small increase in output, and it is largest under monopoly. Now consider a small change in unit tax expressed by $dt > 0$. Then, in equilibrium,

$$dp - \frac{dt + dmc}{1 - v} = d(\theta\, ms)$$

$$\Leftrightarrow \underbrace{(1 - v)[\underbrace{dp}_{>0} - \underbrace{d(\theta\, ms)}_{<0}]}_{\text{change in marginal benefit}} = \underbrace{\underbrace{dt}_{>0} + \underbrace{dmc}_{<0}}_{\text{change in effective marginal cost}}$$

Thus, using $dt = dp/\rho_t$, the equation is rewritten as

$$\rho_t = \frac{1}{\underbrace{(1 - v)\left[dp + (-d\left(\theta\, ms\right))\right]}_{(1)>0\text{: revenue increase}} + \underbrace{(-dmc)}_{(2)>0\text{: cost savings}}} dp.$$

Now, consider term (1). Note first $d(\theta ms) = (\theta ms)' dq$ so that $d(\theta ms) = -q\epsilon(\theta ms)' dp/p$ because by definition $dq = -q\epsilon dp/p$. Here, for a small increase $dt > 0$,

$$\underbrace{d(\theta\, ms)}_{<0} = \underbrace{-q\epsilon}_{>0} (\theta\, ms)' \underbrace{\frac{dp}{p}}_{>0}$$

so that $(\theta\, ms)' > 0$. By definition, $ms \equiv -p'q = \eta p$. Thus, $d(\theta ms) = -q\epsilon(\theta\eta p)' dp/p$. Now note that $(\theta\eta p)' = (\theta\eta)' p + (\theta\eta) p'$. Thus,

$$d(\theta\, ms) = -q\epsilon \left[(\theta\eta)' p + (\theta\eta) p'\right] \frac{dp}{p}$$

$$\Leftrightarrow d(\theta\, ms) = -q\epsilon(\theta\eta)' dp + (-q\epsilon(\theta\eta) p' dp/p)$$

$$\Leftrightarrow d(\theta\, ms) = [\theta\eta - q\epsilon(\theta\eta)'] dp > 0.$$

Next, consider term (2). A change in the marginal cost, dmc, is expressed in terms of dp by $dmc = -[(1-v)\,\theta\eta + 1 - \tau]\chi\epsilon\, dp < 0$. To see this, note first that $dmc = \chi mc \cdot (dq/q) = -(\chi\epsilon\, mc)\,(dp/p)$. Then, mc in this expression can be eliminated rewriting $p - \theta\, ms = (mc + t)\,/\,(1-v) \Rightarrow mc = (1-v)\,(p + \theta q p') - t = (1-v)\,(1-\theta\eta)\,p - t$, which leads to $dmc = -[(1-v)\,(1+\theta\eta) - t/p]\chi\epsilon\, dp$. Then, in terms of the per-unit revenue burden, $\tau \equiv v + t/p$, that is, $dmc = -[(1-v)\,(1-\theta\eta) - \tau + v]\chi\epsilon\, dp = -[-(1-v)\,\theta\eta + 1 - \tau]\chi\epsilon\, dp$. Finally, using the expressions for dmc and $d(\theta ms)$,

$$\rho_t = \frac{dp}{(1-v)\,[dp - d\,(\theta\, ms)] - dmc} = \frac{1}{\underbrace{(1-v)\,[(1-\theta\eta) + (\theta\eta)'\,\epsilon q]}_{\text{revenue increase}} + \underbrace{(1-\tau)\,\epsilon\chi - (1-v)\,\theta\chi}_{\text{cost savings}}}.$$

$$\Leftrightarrow \rho_t = \frac{1}{1-v}\;\frac{1}{\underbrace{[(1-\theta\eta) + (\theta\eta)'\,\epsilon q]}_{\text{revenue increase}} + \underbrace{\left[-\theta + \frac{1-\tau}{1-v}\epsilon\right]\chi}_{\text{cost savings}}}.$$

$\square$

Remark 1: Relationship to Weyl and Fabinger (2013)

It can be verified that our formula for ρ_t above is a generalization of Weyl and Fabinger's (2013, p.548) Equation (2):

$$\rho = \frac{1}{1 + \frac{\epsilon_D - \theta}{\epsilon_S} + \frac{\theta}{\epsilon_\theta} + \frac{\theta}{\epsilon_{ms}}},$$

where $\epsilon_\theta \equiv \theta/[q \cdot (\theta)']$, $\epsilon_{ms} \equiv ms/[ms' q]$ ($ms \equiv -p'q$ is defined in the proof of Proposition 5 just above), and ϵ_D and ϵ_S here are our ϵ and $1/\chi$, respectively. First, the denominator in our formula is rewrirren as:

$$1 - (\eta + \chi)\,\theta + \epsilon q\,(\theta\eta)' + \frac{1-\tau}{1-v}\epsilon\chi$$

$$= 1 + \frac{\frac{1-\tau}{1-v}\epsilon_D - \theta}{\epsilon_S} + \frac{\theta}{\epsilon_\theta} + \theta \cdot \left(-\frac{1}{\epsilon_D} + \eta' \epsilon_D q \right)$$

because

$$(\theta\eta)'\epsilon q = (\theta'\eta + \theta\eta')\epsilon q = \left[\frac{\theta}{q\epsilon_\theta}\eta + \theta\eta' \right] \epsilon q = \frac{\theta}{\epsilon_\theta} + \theta\eta'\epsilon q.$$

Next, since $\eta = -qp'/p$, it is verified that $\eta' = -\{p'p + qpp'' - q[p']^2\}/p^2$, implying that

$$\eta'\epsilon_D q = \frac{p'p + qpp'' - q[p']^2}{p^2} \cdot \frac{p}{p'q} \cdot q = \frac{1}{\epsilon_D} + \left(1 + \frac{p''}{p'} q \right),$$

where $1 + p''q/p$ is replaced by $1/\epsilon_{ms}$ because $ms \equiv -p'q$ and thus $ms' = -(p''q + p')$. Then, it is readily verified that

$$1 - (\eta + \chi)\,\theta + \epsilon q\,(\theta\eta)' + \frac{1-\tau}{1-v}\epsilon\chi = 1 + \frac{\frac{1-\tau}{1-v}\epsilon_D - \theta}{\epsilon_S} + \frac{\theta}{\epsilon_\theta} + \frac{\theta}{\epsilon_{ms}}.$$

In summary, Weyl and Fabinger's (2013, p.548) original Equation (2) is generalized to

$$\rho = \frac{1}{1-v} \frac{1}{1 + \frac{\frac{1-\tau}{1-v}\epsilon_D - \theta}{\epsilon_S} + \frac{\theta}{\epsilon_\theta} + \frac{\theta}{\epsilon_{ms}}}$$

with non-zero initial ad valorem tax, which is equivalent to our formula for ρ_t:

$$\rho_t = \frac{1}{1-v} \frac{1}{1 + \frac{1-\tau}{1-v}\epsilon\chi - (\eta + \chi)\,\theta + \epsilon q\,(\theta\eta)'}.$$

Remark 2: Comparison of Perfect and Oligopolistic Competition

One can further interprete the formula for ρ_t in comparison to the case of perfect competition (with zero initial taxes), when the unit tax pass-through rate is given by (see Weyl and Fabinger (2013, p.534)): $\rho_t = 1/(1 + \epsilon\chi)$.

The first term of the additional terms in the denominator of ρ_t in Proposition 5), $-(\eta + \chi)\,\theta$, shows that as the demand becomes *inelastic* (i.e., η becomes larger, although η cannot be too large; recall the restriction, $\eta < 1/\theta$) or the supply becomes *inelastic* (i.e., χ becomes larger), the denominator becomes smaller, that is, the pass-through rate becomes larger, but this effect is mitigated by the degree of competitiveness, θ: this effect becomes smaller, and hence the pass-through becomes *larger* as the degree of competitiveness becomes closer to perfect competition. This is the *direct* effect of θ on on the pass-through rate, via the first-order characteristics of demand and supply, captured by η and χ, respectively

The second term, $\epsilon q\,(\theta\eta)' = -\epsilon q\,(-\theta\eta)'$, shows the *indirect* effect in the following sense: suppose that η is close to a constant. Then, $-\epsilon q\,(-\theta\eta)' = -q\,(-\theta)'$, which implies that a larger $(-\theta)' \equiv -\partial\theta/\partial q > 0$ works to *raise* the pass-through rate. This situation is consistent with the case when $-\partial q/\partial\theta$ is small; the effect of imperfect competition on the output reduction is small, implying less distortion, an important feature if the degree of competitiveness is close to perfect competition. If, instead, θ is close to a constant, then the second term is now $-\epsilon q\,(-\theta\eta)' = -\epsilon q\theta\,(-\eta)' = \theta(\eta+1/\epsilon_{ms})$. Thus, the additional terms become $-\,(\eta-\chi)\,\theta+\epsilon q\,(\theta\eta)' = \theta(1/\epsilon_{ms}-\chi)$. The effect captured by $-\theta\chi$ is similar to the argument above. Now, as $1/\epsilon_{ms}$ becomes smaller, the pass-through rate is also *larger*. Note that $1/\epsilon_{ms} = (\Delta ms/ms)/(\Delta q/q)$ measures how quick the marginal surplus lowers as a response to a decrease in output q. Thus, a lower $1/\epsilon_{ms}$ is associated with less distortion. Overall, Weyl and Fabinger's (2013, p. 548) Equation (2) and our formula for ρ_t show how the industry's competitiveness directly and indirectly lowers the pass-through rate ρ_t to the level with perfect competition.

Let us also point out that the *exchange rate pass-through* can be included naturally in our framework.[16] Suppose that domestic firms in a country of interest use some imported inputs for production. For concreteness, let us specify the profit function of firm j as $\pi_j = [(1-v)p_j - t]q_j - (1+a\,e)c(q_j)$, where the constant coefficient a measures the importance imported inputs and $e > 0$ is the exchange rate. Notice that the firm's profit is rewritten as $\pi_j = (1+ae)\left[\left(\frac{1-v}{1+ae}p_j - \frac{t}{1+ae}\right)q_j - c(q_j)\right]$. Since the first factor on the right-hand side is constant, the firm will behave as if its profit

[16] See, e.g., Feenstra (1989); Feenstra, Gagnon, and Knetter (1996); Yang (1997); Campa and Goldberg (2005); Hellerstein (2008); Gopinath, Itskhoki, and Rigobon (2010); Goldberg and Hellerstein (2013); Auer and Schoenle (2016); and Chen and Juvenal (2016) for empirical studies of exchange rate pass-through.

function was simply $\tilde{\pi}_j = \left[(1-\tilde{v})p_j - \tilde{t}\right] q_j - c(q_j)$, with $\tilde{v} \equiv (v+ae)/(1+ae)$ and $\tilde{t} \equiv t/(1+ae)$. By utilizing the explicit expressions for the derivatives $\partial\tilde{v}/\partial e = (a-v)/(1+ae)^2$ and $\partial\tilde{t}/\partial e = -at/(1+ae)^2$, one can analyze the effect of a change in the exchange rate e on social welfare. Note that this is simply interpreted as the *cost pass-through* as well (see the references in Footnote 16 for empirical studies). Alternatively, one may use the results of Section 6 to study consequences of exchange rate movements.

3 Taxation and Welfare under Specific Types of Competition

In this section, we show that for price competition and quantity competition in differentiated oligopoly, our general expressions of the marginal cost of public funds and pass-through lead to expressions in terms of demand primitives such as the *elasticities* and the *curvatures*, and the marginal cost elasticity χ defined above. This simplification becomes possbile mainly because the conduct index can be expressed by the elasticities and the inverse elastisties (see Subsection 3.2 below). We also provide parametric examples for these results by assuming constant marginal costs (i.e., $\chi = 0$). The question of whether quantity- or price-setting firms are more appropriate depends on the nature of competition. As Riordan (2008, p. 176) argues, quantity competition is a more appropriate model if one depicts a situation where firms determine the necessary capacity for production. However, price-setting firms are more suitable if firms in the industry of focus can quickly adjust to demand by changing their prices. Although the real-world case of competition is not as clear-cut as this, as we have emphasized in Introduction, we argue below that it is possible to provide another useful characterization for the marnal costs of public funds and the pass-through rates by specifying the mode of competition.

3.1 Elasticities and curvatures of demand and inverse demand

Direct demand. Following Holmes (1989, p. 245), we define the *own price elasticity of the firm's demand* by $\epsilon_F(p) \equiv -(p/q(p))\ \partial q_j(\mathbf{p})/\partial p_j|_{\mathbf{p}=(p,\dots,p)}$ and the *cross price elasticity* by $\epsilon_C(p) \equiv (n-1)(p/q(p))\ \partial q_{j'}(\mathbf{p})/\partial p_j|_{\mathbf{p}=(p,\dots,p)}$ for any distinct pair of indices j and j'. These are related to the industry demand elasticity $\epsilon(p)$ by $\epsilon_F = \epsilon + \epsilon_C$.[17] This equation simply means that the percentage of consumers who cease to purchase firm j's product in response to its price increase is decomposed into (i) those who no longer purchase from any of the firms (ϵ) and (ii) those who switch to (any of) the other firms' products (ϵ_C). Thus, ϵ_F measures the *firm's own competitiveness*: it is decomposed into the industry elasticity and the degree of rivalness. In this sense, these three price elasticities characterize "first-order competitiveness," which determines whether the equilibrium price is high or low, but one of them is not independently determined from the other two elasticities.

Next, we define the *curvature of the industry's direct demand* $\alpha(p) \equiv -p\,q''(p)/q'(p)$, as well as the *own curvature* $\alpha_F(p)$ *of the firm's direct demand* and the *cross curvature* $\alpha_C(p)$ *of the firm's direct demand*:[18]

$$\alpha_F(p) \equiv -p\left(\frac{\partial q_j(\mathbf{p})}{\partial p_j}\right)^{-1}\frac{\partial^2 q_j(\mathbf{p})}{\partial p_j^2}|_{\mathbf{p}=(p,\dots,p)},$$

$$\alpha_C(p) \equiv -(n-1)\,p\left(\frac{\partial q_j(\mathbf{p})}{\partial p_j}\right)^{-1}\frac{\partial^2 q_j(\mathbf{p})}{\partial p_j\,\partial p_{j'}}|_{\mathbf{p}=(p,\dots,p)},$$

where j and j' is an arbitrary pair of distinct indices. These curvatures satisfy $\alpha = (\alpha_F + \alpha_C)(\epsilon_F/\epsilon)$. They are related to the elasticity of $\epsilon_F(p)$ by $p\,\epsilon_F'(p)/\epsilon_F(p) = 1 + \epsilon(p) - \alpha_F(p) - \alpha_C(p)$.[19] Thus, α is *positive* (*negative*) *if and only if the*

[17] Holmes (1989) shows this for two symmetric firms, but it is straightforward to verify this relation more generally. See the equation in Footnote 7 above.

[18] The curvature $\alpha_F(p)$ here corresponds to $\alpha(p)$ of Aguirre, Cowan, and Vickers (2010, p. 1603).

[19] This relationship can be verified as follows. The elasticity of the function $\epsilon_F(p)$ equals the sum of the elasticities of the three factors it is composed of:

$$\frac{1}{\epsilon_F(p)}p\frac{d}{dp}\epsilon_F(p) = \frac{1}{p}p\frac{d}{dp}p + q(p)\,p\frac{d}{dp}\frac{1}{q(p)} + \left(\frac{\partial q_j(\mathbf{p})}{\partial p_j}\right)^{-1}|_{\mathbf{p}=(p,\dots,p)}p\frac{d}{dp}\left(\frac{\partial q_j(\mathbf{p})}{\partial p_j}|_{\mathbf{p}=(p,\dots,p)}\right).$$

The first elasticity on the right-hand side equals 1, the second elasticity equals $\epsilon(p)$, and the third *industry demand is convex* (*concave*), and α_F is positive (negative) if and only if the demand as a function of firm j's own price is convex (concave). Hence, both α and α_F measure the degree of convexity in the demand function for an industry-wide price change and for an individual firm's price change, respectively.

Note that $\partial(\partial q_j/\partial p_j)/\partial p_{j'}$ in α_C measures the effects of firm j's price change on how many consumers rival j' loses if it raises its price. If this is negative (positive), then firm j' loses more (less) consumers by its own price increase for a higher value of p_j. Thus, because $\partial q_j/\partial p_{j'}$ is *positive* in the expression for α_C, a higher α_C also indicates more competitiveness in the industry. It is also expected that the industry is more competitive if α and α_F are higher. In effect, the equilibrium price is characterized by ϵ_F. However, a policy change around equilibrium is also affected by the curvatures, which measure "second-order competitiveness" around the equilibrium. However, Proposition 6 below shows that α is the only curvature that determines the pass-through rates.

Inverse demand. We define the *own quantity elasticity of the firm's inverse demand* $\eta_F(q) \equiv -(q/p(q))\, \partial p_j(\mathbf{q})/\partial q_j|_{\mathbf{p}=(p,\ldots,p)}$ and the the *cross quantity elasticity* $\eta_C(q) \equiv (n-1)(q/p(q))\ \partial p_{j'}(\mathbf{q})/\partial q_j|_{\mathbf{p}=(p,\ldots,p)}$. These satisfy $\eta_F = \eta + \eta_C$. This identity means that as a response to firm$j$'s increase in its output, the industry as a whole reacts by lowering firm j's price (η). However, each individual firm (other than j) reacts to this firm j's output increase by reducing its own output. This counteracts the initial change in the price ($\eta_C < 0$), and thus a percentage reduction in the price for firm j (η_F) is smaller than η**,** which does not take into account strategic reactions. Note here that $1/\eta_F$, not η_F, measures the industry's competitiveness. Thus, these three quantity elasticities characterize "first-order competitiveness," which determines whether the equilibrium quantity is high or low.

We define the *curvature of the industry's inverse demand* $\sigma(q) \equiv -q\, p''(q)/p'(q)$,

elasticity equals $-\alpha_F(p) - \alpha_C(p)$, since

$$p\frac{d}{dp}\frac{\partial q_j(\mathbf{p})}{\partial p_j}|_{\mathbf{p}=(p,\ldots,p)} = p\frac{\partial^2 q_j(\mathbf{p})}{\partial p_j^2}|_{\mathbf{p}=(p,\ldots,p)} + (n-1)\,p\frac{\partial^2 q_j(\mathbf{p})}{\partial p_j \partial p_{j'}}|_{\mathbf{p}=(p,\ldots,p)}.$$

as well as the *own curvature* $\sigma_F(q)$ *of the firm's inverse demand* and the *cross curvature* $\sigma_C(q)$ *of the firm's inverse demand* by:

$$\sigma_F(q) \equiv -q\left(\frac{\partial p_j(\mathbf{q})}{\partial q_j}\right)^{-1}\frac{\partial^2 p_j(\mathbf{q})}{\partial q_j^2}|_{\mathbf{q}=(q,\ldots,q)},$$

$$\sigma_C(q) \equiv -(n-1)\,q\left(\frac{\partial p_j(\mathbf{q})}{\partial q_j}\right)^{-1}\frac{\partial^2 p_j(\mathbf{q})}{\partial q_j\,\partial q_{j'}}|_{\mathbf{q}=(q,\ldots,q)},$$

for an arbitrary pair of distinct indices j and j'. These curvatures represent an oligopoly counterpart of monopoly $\sigma(q)$ in Aguirre, Cowan, and Vickers (2010, p. 1603). They satisfy the relationship $\sigma = (\sigma_F + \sigma_C)(\eta_F/\eta)$. They are related to the elasticity of $\eta_F(q)$ by $q\,\eta_F'(q)/\eta_F(q) = 1 + \eta\,(q) - \sigma_F\,(q) - \sigma_C\,(q)$.[20] Now, σ is *positive* (*negative*) if and only if the industry's inverse demand is *convex* (*concave*), and σ_F is positive (negative) if and only if the inverse demand as a function of firm j's own output is convex (concave). Here, concavity, not convexity, is related to a sharp reduction in price in response to an increase in firm j's output. Thus, $-\sigma$ and $-\sigma_F$ measure "second-order competitiveness" of the industry, which characterizes the responsiveness of the equilibrium output when a policy is changed.[21]

Note here that $\partial(\partial p_j/\partial q_j)/\partial q_{j'}$ in σ_C measures the effects of firm j's output increase on the extent of rival ($j\,'$)'s price drop if it increases its output. If this is negative (positive), then firm $j\,'$ expects a huge (little) drop in its price by increasing its output for a higher value of q_j. Because $\partial p_j/\partial q_{j'}$ is negative in the expression for σ_C, a *lower* σ_C or a higher $-\sigma_C$ indicates more competitiveness in the industry. In sum, while $1/\eta_F$ characterizes competitiveness that determines the level of the equilibrium quantity, $-\sigma$, $-\sigma_F$, and $-\sigma_C$ determine competitiveness that characterizes the responsiveness of the equilibrium output by a policy change. However, similar to the case of price competition, Proposition 7 below shows that σ is the only curvature that determines the pass-through rates.

[20] In analogy with Footnote 19, the elasticity of the function $\eta_F\,(q)$ is the sum of the elasticities of the three factors it is composed of, which are equal to 1, $\eta\,(q)$, and $-\sigma_F\,(q) - \sigma_C\,(q)$.

[21] Homogeneous-product Cournot competition is a very simple special case, where $\theta = 1/n$, $\eta = n\,\eta_F$, and $\sigma_C = (n-1)\,\sigma_F$.

3.2 Expressions for pass-through

In the case of price competition, the conduct index θ is $\theta = \epsilon/\epsilon_F = 1/(\eta\epsilon_F)$, which is verified by comparing the firm's first-order condition with Equation (1). The marginal cost of public funds and the incidence are obtained by substituting these expressions into those of Propositions 1 and 2.

Proposition 6. *Pass-through under price competition.* *Under symmetric oligopoly with price competition and with a possibly non-constant marginal cost:*

$$\rho_t = \frac{1}{1-v} \frac{1}{1 + \frac{(1-\alpha/\epsilon_F)\epsilon}{\epsilon_F} + \left(\frac{1-\tau}{1-v} - \frac{1}{\epsilon_F}\right)\epsilon\chi},$$

$$\rho_v = \frac{1}{1-v} \frac{1}{\frac{1}{1-1/\epsilon_F} + \frac{(1-\alpha/\epsilon_F)\epsilon}{\epsilon_F - 1} + \left(\frac{1-\tau}{1-v}\frac{\epsilon_F}{\epsilon_F-1} - \frac{1}{\epsilon_F-1}\right)\epsilon\chi}.$$

Proof. Since in the case of price setting $\theta = \epsilon/\epsilon_F = 1/(\eta\epsilon_F)$, we have $(\eta+\chi)\theta = (1+\epsilon\chi)/\epsilon_F$ and $(\theta\eta)'\epsilon q = \epsilon q \frac{d}{dq}(\theta\eta) = \epsilon q \frac{d}{dq}(\epsilon_F^{-1}) = -\epsilon_F^{-2}\epsilon q \frac{d}{dq}\epsilon_F = \epsilon_F^{-2} p \frac{d}{dp}\epsilon_F = (1+\epsilon-\alpha\epsilon/\epsilon_F)/\epsilon_F$, where in the last equality we utilize the expression for the elasticity of $\epsilon_F(p)$ and $\alpha_F + \alpha_C = \alpha\epsilon/\epsilon_F$ from Subsection 3.1. Substituting these into the expression for ρ_t in Proposition 5 gives

$$\rho_t = \frac{1}{1-v} \frac{1}{1 - \frac{1}{\epsilon_F}(1+\epsilon\chi) + \frac{1}{\epsilon_F}\left(1+\epsilon-\frac{\alpha\epsilon}{\epsilon_F}\right) + \frac{1-\tau}{1-v}\epsilon\chi},$$

which is equivalent to the expression for ρ_t in the proposition. Since for price setting $\theta = \epsilon/\epsilon_F$, the relationship in Proposition 3 implies $\rho_v = (\epsilon-\theta)\rho_t/\epsilon = (\epsilon_F - 1)\rho_t/\epsilon_F$, which leads to the desired expression for ρ_v. $\square$

The intuition for ρ_t is given as follows. First, recall from Proposition 5 that

$$\rho_t = \frac{1}{1-v} \frac{1}{\underbrace{[(1-\theta\eta)+(\theta\eta)'\epsilon q]}_{\text{revenue increase}} + \underbrace{\left[\frac{1-\tau}{1-v}\epsilon - \theta\right]\chi}_{\text{cost savings}}}.$$

Then, with $\theta = \epsilon/\epsilon_F$, $1-\theta\eta = 1-1/\epsilon_F$, $(\theta\eta)'\epsilon q = (1+\epsilon-\alpha\epsilon/\epsilon_F)/\epsilon_F$, the equality above is rewritten as

$$\rho_t = \frac{1}{1-v}\frac{1}{\underbrace{\left[\left(1-\frac{1}{\epsilon_F}\right)+\frac{1+\epsilon-\alpha\epsilon/\epsilon_F}{\epsilon_F}\right]}_{\text{revenue increase}}+\underbrace{\left[\frac{1-\tau}{1-v}-\frac{1}{\epsilon_F}\right]\epsilon\chi}_{\text{cost savings}}}.$$

$$= \frac{1}{1-v}\frac{1}{\underbrace{\left[1+\frac{(1-\alpha/\epsilon_F)\epsilon}{\epsilon_F}\right]}_{\text{revenue increase}}+\underbrace{\left[\frac{1-\tau}{1-v}-\frac{1}{\epsilon_F}\right]\epsilon\chi}_{\text{cost savings}}}.$$

To further facilicate the understanding the connection of this result for to Proposition 5, consider the case of zero intial taxes ($t = v = \tau = 0$). Then, Proposition 5 claims that

$$\rho_t = \frac{1}{1+\epsilon\chi-\theta\chi+[-\eta\theta+\epsilon q\,(\theta\eta)']},$$

whereas Proposition 6 shows that

$$\rho_t = \frac{1}{1+\epsilon\chi-\theta\chi+[-\frac{1}{\epsilon}\cdot\frac{\epsilon}{\epsilon_F}+\frac{1+(1-\alpha/\epsilon_F)\epsilon}{\epsilon_F}]}$$

$$= \frac{1}{1+\epsilon\chi-\theta\chi+\left(1-\frac{\alpha}{\epsilon_F}\right)\theta}$$

because $\theta = \epsilon/\epsilon_F$. Here, the direct effect from $-\eta\theta$ is canceled out by the part of the indirect effect from $\epsilon q\,(\theta\eta)'$. The new term, which appears as the fourth term in the denominator, shows *how the industry's curvature affects the pass-through rate*: as the demand curvature becomes larger (i.e., as the industry's demamd becomes more convex), then the pass-through rate becomes higher, although this effect is mitigated by the degree of competitiveness, θ.

Next, in the case of quantity competition, the conduct index θ is given by $\theta = \eta_F/\eta$, which is, again, verified by comparing the firm's first-order condition with Equation (1). Again, the marginal cost of public funds and the incidence are obtained by substituting these expressions into those of Propositions 1 and 2.

Proposition 7. ***Pass-through under quantity competition.*** *Under symmetric oligopoly with quantity competition and with a possibly non-constant marginal cost:*

$$\rho_t = \frac{1}{1-v}\ \frac{1}{1+\frac{\eta_F}{\eta}-\sigma+\left(\frac{1-\tau}{1-v}-\eta_F\right)\frac{\chi}{\eta}},$$

$$\rho_v = \frac{1}{1-v}\ \frac{(1-\eta_F)}{1+\frac{\eta_F}{\eta}-\sigma+\left(\frac{1-\tau}{1-v}-\eta_F\right)\frac{\chi}{\eta}}.$$

Proof. In the case of quantity setting, $\theta = \eta_F/\eta$, so $(\eta+\chi)\,\theta = (1+\chi/\eta)\,\eta_F$ and $(\theta\eta)'\,\epsilon q = q\,(\eta_F)'/\eta = (1+\eta-\sigma\eta/\eta_F)\,\eta_F/\eta$, where in the last equality we utilize the expression for the elasticity of $\eta_F\,(q)$ and $\sigma_F+\sigma_C = \sigma\eta/\eta_F$ from Subsection 3.1. Substituting these into the expression for ρ_t in Proposition 5 gives

$$\rho_t = \frac{1}{1-v}\ \frac{1}{1-(1+\frac{1}{\eta}\chi)\eta_F+\frac{1}{\eta}\left(1+\eta-\frac{\sigma\eta}{\eta_F}\right)\eta_F+\frac{1-\tau}{1-v}\frac{1}{\eta}\chi},$$

which is equivalent to the expression for ρ_t in the proposition. Since $\theta = \eta_F/\eta$, Proposition 3 implies $\rho_v = (\epsilon-\theta)\,\rho_t/\epsilon = (1/\eta-\eta_F/\eta)\,\rho_t\eta = (1-\eta_F)\,\rho_t$, which can be used to verify the expression for ρ_v. $\square$

The intuition for ρ_t is similar to the case of price competition. Recall again that

$$\rho_t = \frac{1}{1-v}\ \frac{1}{\underbrace{[(1-\theta\eta)+(\theta\eta)'\,\epsilon q]}_{\text{revenue increase}}+\underbrace{\left[\frac{1-\tau}{1-v}\epsilon-\theta\right]\chi}_{\text{cost savings}}}.$$

Then, $\theta = \eta_F/\eta$ implies $(1/\epsilon_S-\eta)\,\theta = [(1/\epsilon_S\eta)-1]\eta_F$ and $(\theta\eta)'\,(q/\eta) = q\,(\eta_F)'/\eta = (1+\eta-\sigma_F-\sigma_C)\,(\eta_F/\eta)$. Thus, the equality above is rewritten as

$$\rho_t = \frac{1}{1-v}\ \frac{1}{\underbrace{\left[(1-\eta_F)+\frac{1+\eta-\sigma\eta/\eta_F}{\eta}\eta_F\right]}_{\text{revenue increase}}+\underbrace{\left[\frac{1-\tau}{1-v}\frac{1}{\epsilon_S\eta}-\frac{\eta_F}{\epsilon_S\eta}\right]}_{\text{cost savings}}}$$

$$= \frac{1}{1-v}\ \frac{1}{\underbrace{\left[1+\frac{\eta_F-\sigma\eta}{\eta}\right]}_{\text{revenue increase}}+\underbrace{\left[\frac{1-\tau}{1-v}-\eta_F\right]\frac{1}{\epsilon_S\eta}}_{\text{cost savings}}}.$$

To further facilicate the understanding the connection of this result for to Proposition 5, consider the case of zero intial taxes ($t = v = \tau = 0$) again. Then, Proposition 7 shows that

$$\rho_t = \frac{1}{1 + \epsilon\chi - \theta\chi + [-\eta \cdot \frac{\eta_F}{\eta} + \left(1 + \frac{1}{\eta} - \frac{\sigma}{\eta_F}\right)\eta_F]}$$

$$= \frac{1}{1 + \epsilon\chi - \theta\chi + \left(1 - \frac{\sigma}{\theta}\right)\theta}$$

because $\theta = \eta_F/\eta$. Here, the term $(1 - \sigma/\theta)\,\theta$ demonstates the *effects of the industry′s inverse demand curvature*, σ, *on the pass-through rate*: as the inverse demand curvature becomes larger (i.e., as the industry's inverse demamd becomes more convex), then the pass-through rate becomes higher. Interestingly, in contrast to the case of price competition, this effect is not mitigated by the degree of competitiveness, θ.

Lastly, monopolistic competition, another important class of the mode of competition, may be obtained by taking the large n limit. As discussed in Weyl and Fabinger (2013, pp. 544-546), in the case of quasi-linear utility of the form $U(\int u\,(q_i)\,di) - p_i q_i di$, it may be shown that $\theta = U'u''/(U''(u')^2 + U'u'')$. With the most typical specification $u\,(q) = q^\beta$, $U\,(x) = x^\gamma$, this leads to a constant value of conduct index: $\theta = (1 - \beta)\,/\,(1 - \beta\gamma)$. Then $\eta = \eta_F\;(1 - \beta\gamma)\,/\,(1 - \beta)\,.$

3.3 Simple parametric examples

Below, we provide two parametric examples with n symmetric firms and constant marginal cost: $\chi = 0$. In this case, the pass-through expressions are simplified to

$$\rho_t = \frac{1}{(1 - v)\left[1 + \left(1 - \frac{\alpha}{\epsilon_F}\right)\theta\right]}, \qquad \rho_v = \frac{\epsilon_F - 1}{\epsilon_F\left\{(1 - v)\left[1 + \left(1 - \frac{\alpha}{\epsilon_F}\right)\theta\right]\right\}}$$

under price competition, where $\theta = \epsilon/\epsilon_F$, and

$$\rho_t = \frac{1}{(1 - v)\left[1 + \left(1 - \frac{\sigma}{\theta}\right)\theta\right]}, \qquad \rho_v = \frac{1 - \eta_F}{(1 - v)\left[1 + \left(1 - \frac{\sigma}{\theta}\right)\theta\right]}$$

under quantity competition, where $\theta = \eta_F/\eta$.

One is the case wherein each firm faces the following *linear demand*, $q_j(p_1, ..., p_n) = b - \lambda p_j + \mu \sum_{j' \neq j} p_{j'}$, where $b > mc$ and $\lambda > (n-1)\mu \geq 0$, implying that all firms produce substitutes and μ measures the degree of *substitutability* (firms are effectively monopolists when $\mu = 0$).[22,23] Under symmetric pricing, the industry's demand is thus given by $q(p) = b - [\lambda - (n-1)\mu]p$. The inverse demand system is given by

$$p_j(q_j, \mathbf{q}_{-j}) = \frac{\lambda - (n-2)\mu}{(\lambda + \mu)\,[\lambda - (n-1)\mu]}(b - q_j) + \frac{\mu}{(\lambda + \mu)\,[\lambda - (n-1)\mu]}\left[\sum_{j' \neq j}(b - q_{j'})\right],$$

implying that $p(q) = (b-q)/[\lambda - (n-1)\mu]$ under symmetric production. Obviously, both the direct and the indirect demand curvatures are zero: $\alpha = 0 = \sigma$. Thus, the pass-through rates are simply given by

$$\rho_t = \frac{1}{(1-v)\,(1+\theta)}, \qquad \rho_v = \frac{\epsilon_F - 1}{\epsilon_F(1-v)\,(1+\theta)}$$

under price competition, where $\theta = [\lambda - (n-1)\mu]/\lambda$, and $\epsilon_F = \lambda(p/q)$ (where p and q are the equilibrium price and output under price setting), and

$$\rho_t = \frac{1}{(1-v)\,(1+\theta)}, \qquad \rho_v = \frac{1 - \eta_F}{(1-v)\,(1+\theta)}$$

under quantity competition, where $\theta = [\lambda - (n-2)\mu]/(\lambda + \mu)$ and $\eta_F = \{[\lambda - (n -$

[22] These linear demands are derived by maximizing the representative consumer's net utility, $U(q_1, ..., q_n) - \sum_{j=1}^{n} pq_j$, with respect to $q_1, ...,$ and q_n. See, e.g., Vives (1999, pp. 145-6) for details.

[23] In our notations below, the demand in symmetric equilibrium is given by $q_j(p_j, p_{-j}) = b - \lambda p_j + \mu(n-1)p_{-j}$, whereas it is written as

$$q_j(p_j, p_{-j}) = \frac{\alpha}{1 + \gamma(n-1)} - \frac{1 + \gamma(n-2)}{(1-\gamma)[1 + \gamma(n-1)]}p_j + \frac{\gamma(n-1)}{(1-\gamma)[1+\gamma(n-1)]}p_{-j}$$

in Häckner and Herzing's (2016) notations, where $\gamma \in [0, 1]$ is the parameter that measures substitutability between (symmetric) products. Thus, if our (b, λ, μ) is determined by $b = \alpha/[1+\gamma(n-1)]$, $\lambda = [1+\gamma(n-2)]/\{(1-\gamma)[1+\gamma(n-1)]\}$, and $\mu = \gamma/\{(1-\gamma)[1+\gamma(n-1)]\}$, given Häckner and Herzing's (2016) (α, γ), then our results below can be expressed by Häckner and Herzing's (2016) notations as well. Note here that our formulation is more flexible in the sense that the number of the parameters is three. This is because the coefficient for the own price is normalized to one: $p_j(q_j, q_{-j}) = \alpha - q_j - \gamma(n-1)q_{-j}$, which is analytically innocuous, and Häckner and Herzing's (2016) γ is the normalized parameter.

$2)\mu](q/p)\}/\{(\lambda+\mu)[\lambda-(n-1)\mu]\}$ (where q and p are the equilibrium output and price under quanty setting).

Now, from Propostions 1 and 2, the marginal costs of public funds and the incidences are given by

$$MC_t = \frac{(1-v)\,\theta + \epsilon\,\tau}{1+(1-v)\theta - \epsilon\,\tau}, \qquad MC_v = \frac{(1-v)\theta + \epsilon\tau}{\frac{(1-v)(1+\theta)}{\epsilon_F - 1} + v - \epsilon\tau}$$

$$I_t = \frac{1}{2(1-v)[1-(n-1)(\mu/\lambda)]}, \qquad I_v = \frac{\epsilon_F - 1}{(1-v)[2-\epsilon_F(1-\theta)]}$$

under price competition, with $\epsilon = [\lambda-(n-1)\mu](p/q)$ is additionally provided, where p and q are the equilibrium price and output under price setting, and

$$MC_t = \frac{(1-v)\,\theta + \frac{1}{\eta}\,\tau}{1+(1-v)\theta - \frac{1}{\eta}\,\tau}, \qquad MC_v = \frac{(1-v)\theta + \frac{1}{\eta}\,\tau}{\frac{(1-v)(1+\theta)}{1-\eta_F} + v - \frac{1}{\eta}\,\tau}$$

$$I_t = \frac{\lambda+\mu}{2(1-v)[\lambda-(n-2)\mu]}, \qquad I_v = \frac{1-\eta_F}{(1-v)[\eta_F + (2-\eta_F)\theta]}$$

under quantity competition, with $1/\eta = [\lambda-(n-1)\mu](p/q)$ is additionally provided, where p and q are the equilibrium price and output under quantity setting. Thus, it suffices to solve for the equilibrium price and output under both settings to compute the pass-through rate and the marginal cost of public funds for all four cases.

Table 1 (a) summarizes the key variables that determine the pass-through rates and the marginal costs of public funds. It is verified that under both price and quantity competition, $\partial\theta/\partial n < 0$ and $\partial\theta/\partial\mu < 0$. To focus on the roles of these two parameters, n and μ, which directly affect the degree of competition, we employ the following simplication to compute the ratio p/q in equilibrium: $b = 1$, $mc = 0$, and $\lambda = 1$. Then, the equilibrium price and output under price competition are

$$p = \frac{1+\frac{t}{1-v}}{2-(n-1)\mu}, \qquad q = \frac{1-[1-(n-1)\mu]\frac{t}{1-v}}{2-(n-1)\mu},$$

and thus

(a) Linear Demands	
Price setting	Quantity setting
$\epsilon = [\lambda - (n-1)\mu]\left(\frac{p}{q}\right)$	$\eta = \frac{1}{\lambda-(n-1)\mu}\left(\frac{q}{p}\right)$
$\epsilon_F = \lambda\left(\frac{p}{q}\right)$	$\eta_F = \frac{\lambda-(n-2)\mu}{(\lambda+\mu)[\lambda-(n-1)\mu]}\left(\frac{q}{p}\right)$
$\theta = \epsilon/\epsilon_F = 1-(n-1)\left(\frac{\mu}{\lambda}\right)$	$\theta = \eta_F/\eta = \frac{\lambda-(n-2)\mu}{\lambda+\mu}$
$\alpha = 0$	$\sigma = 0$

(b) Logit Demands	
Price setting	Quantity (market share) setting
$\epsilon = \beta(1-ns)p$	$\eta = \frac{1}{\beta(1-ns)p}$
$\epsilon_F = \beta(1-s)p$	$\eta_F = \frac{1-(n-1)s}{\beta(1-ns)p}$
$\theta = \epsilon/\epsilon_F = \frac{1-ns}{1-s}$	$\theta = \eta_F/\eta = 1-(n-1)\,s$
$\alpha = \frac{(2ns-3)ns}{1-ns}p$	$\sigma = \frac{1-2ns}{1-ns}$

Table 1: Elasticities, Conduct Indices, and Curvatures

$$\frac{p}{q} = \frac{1}{1-[1-(n-1)\mu]\frac{t}{1-v}}\left(1+\frac{t}{1-v}\right),$$

implying that

$$\epsilon = \frac{[1-(n-1)\mu]\left(1+\frac{t}{1-v}\right)}{1-[1-(n-1)\mu]\frac{t}{1-v}}, \qquad \epsilon_F = \frac{1+\frac{t}{1-v}}{1-[1-(n-1)\mu]\frac{t}{1-v}}.$$

Similarly, the equilibrium price and output under quantity competition are

$$p = \frac{\frac{1-(n-2)\mu}{1-(n-1)\mu}+(1+\mu)\frac{t}{1-v}}{2-(n-3)\mu}, \qquad q = (1+\mu)\frac{1-[1-(n-1)\mu]\frac{t}{1-v}}{2-(n-3)\mu},$$

and thus

$$\frac{p}{q} = \frac{1}{1-[1-(n-1)\mu]\frac{t}{1-v}}\left(\frac{1-(n-2)\mu}{(1+\mu)[1-(n-1)\mu]}+\frac{t}{1-v}\right),$$

implying that

$$\eta = \frac{1-[1-(n-1)\mu]\frac{t}{1-v}}{\frac{1-(n-2)\mu}{1+\mu}+[1-(n-1)\mu]\frac{t}{1-v}}, \qquad \eta_F = \frac{1-[1-(n-1)\mu]\frac{t}{1-v}}{1+\frac{(1+\mu)[1-(n-1)\mu]}{1-(n-2)\mu}\frac{t}{1-v}}.$$

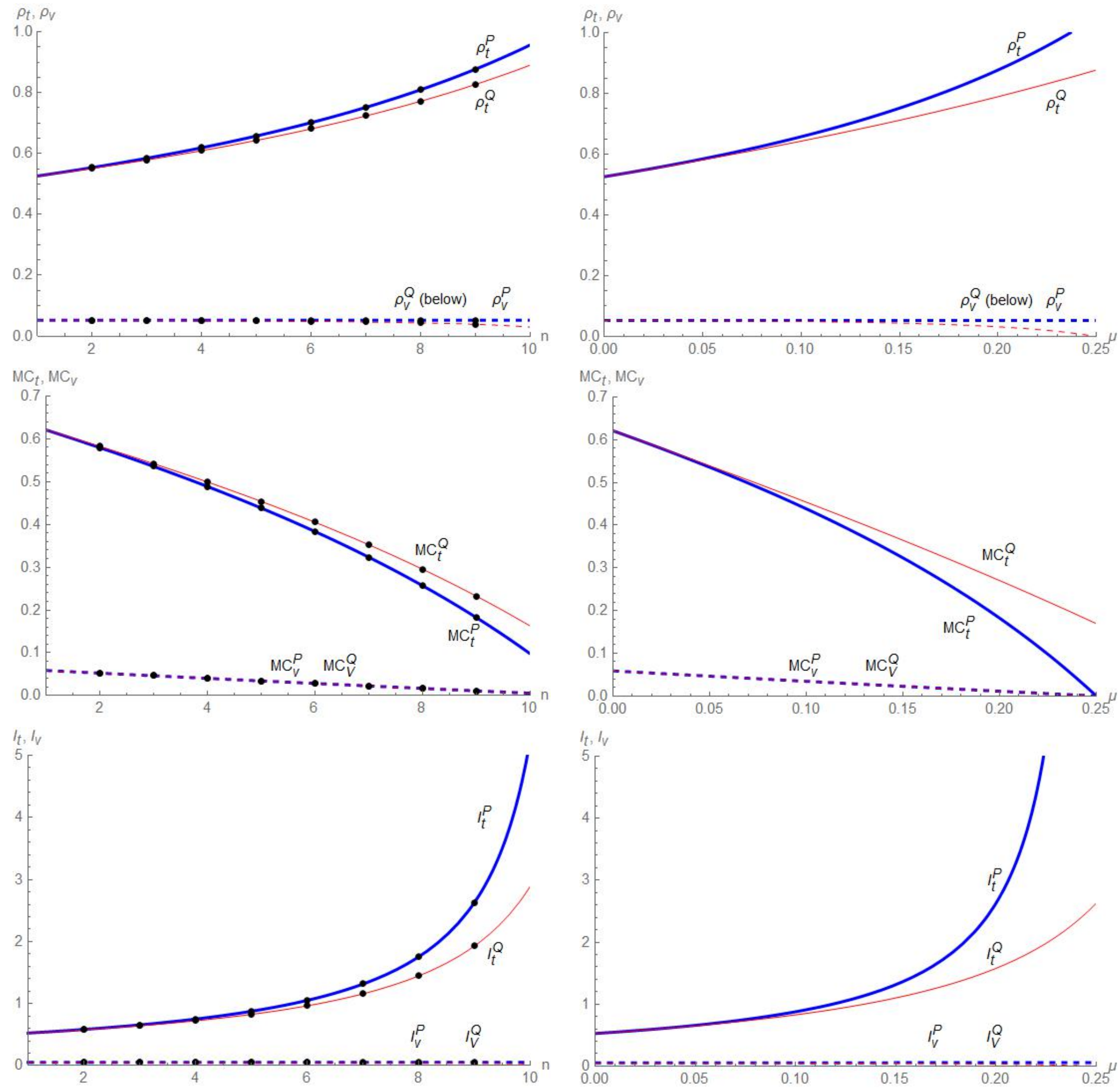

Figure 2: Pass-through rates (top), marginal costs of public funds (middle), and incidence (bottom) with linear demands. The horizontal axes on the left and the right penels are the number of firms (n) and the substitutability parameter (μ), respectively.

The top two panels in Figure 2 illustrates how ρ_t and ρ_v behave as the number of firms (n; the left) or the sustainability parameter (μ; the left) increases, with the superscript denoting price (P) or quantity (Q) setting. Similarly, the middle and the bottom panels draw MC_t and MC_v, and I_t and I_v, respectively. It is observed that the ad valorem tax pass-through rates are close to zero because in this case both ϵ_F and η_F are close to 1. As competition becomes fiercer, both ρ_t^P and ρ_t^Q become larger. although the discrepancy also becomes larger. In the case of linear demands, the difference in the mode of competition does not yield a significant difference in each of the three measures. As is verified by Anderson, de Palma, and Kreider (2001b), the ad valorem tax is more efficient than the unit tax: the dashed lines in the two middle panels lie below the solid lines. This ranking is related inversely to the pass-through and the incidence: as the pass-through or the incidece becomes larger, the marginal cost of public funds becomes smaller.

The next parametric demand is *logit demand*. Each firm $j = 1, ..., n$ faces the following *market share*: $s_j(\mathbf{p}) = \exp(\delta - \beta p_j)/[1 + \sum_{j'=1,\ldots,n} \exp(\delta - \beta p_{j'})] \in (0, 1)$, where δ is the (symmetric) product-specific utility and $\beta > 0$ is the *responsiveness* to the price.[24] We use s_j and s, instead of q_j and q, respectively, following the customary notation in the empirical industrial organization literature, to mean the market share. We define $s_0 = 1 - \sum_{j=1,\ldots,n} s_j < 1$ as the share of all outside goods. Table 1 (b) summarizes the key variables that determine the pass-through rates and the marginal costs of public funds. We need to numerically solve for the equilibrium price and market share under both settings to compute the pass-through rate, the marginal cost of public funds, and incidence for all four cases. To focus on the two parameters, β and n, we assume that $\delta = 1$ and $mc = 0$. Because $\partial s_j(\mathbf{p})/\partial p_j|_{\mathbf{p}=(p,\ldots,p)} = -\beta s(1 - s)$, the first-order conditions for the symmetric equilibrium price and the market share satisfy $p - t/(1 - v) = 1/[\beta(1 - s)]$

[24]Here, $q_j(p_1, ..., p_n)$ is derived by aggregating individuals who prefer product j the most over the population (the total number of individuals is normalized to one): individual i's net utility from consuming j is given by $u_{ij} = \delta - \beta p_j + \varepsilon_{ij}$, whereas $u_{i0} = \varepsilon_{i0}$ is the net utility from consuming nothing, and $\varepsilon_{i0}, \varepsilon_{i1}, ..., \varepsilon_{in}$ are independently and identically distributed according to the Type I extreme distribution for all individuals. See, e.g., Anderson, de Palma, and Thisse (1992, pp. 39-45) for details.

and $s = \exp(1 - \beta p)/[1 + n \exp(1 - \beta p)]$. If p and s are solved numerically, then ϵ, ϵ_F, θ and α can also be computed.[25] Next, we consider the inverse demands under quantity competition. Then, as in Berry (1994), firm j's inverse demand is given by $p_j(\mathbf{s}) = [\delta - \log(s_j/s_0)]/\beta$, which implies that $\partial p_j(\mathbf{s})/\partial s_j|_{\mathbf{s}=(s,\ldots,s)} = -[1-(n-1)s]/[\beta s(1-ns)]$. Thus, the first-order conditiond for the symmetric equilibrium price and the market share satisfy $p - t/(1-v) = [1-(n-1)s]/[\beta(1-ns)]$ and $p = [1 - \log(s/[1-ns])]/\beta$. Then, as above, η, η_F, θ and σ are computed by numerically solving the first-order conditions for p and s. Interestingly, it is verified that in symmetric equilibrium under share setting, $\partial p/\partial n = 0$: the equilbrium price is the same irrespective of the number of firms, whereas the individual market share is decreasing in the number of firms: $\partial s/\partial n < 0$. On the other hand, both the equilibrium price and market share are decreasing in the price coefficient, β.

Figure 3 illustrates the pass-through rates, the marginal costs of public funds, and the incidences as in Figure 2 (the superscript S denotes "market share setting"). Now, on the right panels is measured the price coefficient β on the horizontal axes. Overall, as in the case of linear demands, an increase in the ad valorem tax has a small impact on these measures for each of n and β, whereas an increase in the unit tax has a large effect. However, there are important differences between linear and logit demands. First, the unit tax pass-through under share competition, ρ_t^S is *decreasting* in the number of firms. To understand this, compare the difference in the denominators of $\rho_t^P = 1/\{(1-v)\,[1 + (1 - \alpha/\epsilon_F)\theta]\}$ and $\rho_t^S = (1-v)\,[1 + \theta - \sigma]$: as θ decreases (i.e., as competition becomes fiercer), the the second and the third terms in the denominator of ρ_t^P vanish, and thereby ρ_t^P increases as n increases. However, $(\theta - \sigma)$ increases faster than the decrease in θ, and thus ρ_t^S decreases. This difference in the denominators also reflects in the fact that I_t^S is decreasing in n. Naturally, MC_t^S is decreasing in n as in the linear demands because $1/\rho_t^S$ becomes larger (see the formulas in Proposition 1). Second, while the pass-through rates

[25] It can be verified that $s_j(\cdot;\mathbf{p}_{-j})$ is convex as long as $s_j < 1/2$ because $\partial^2 s_j/\partial p_j^2 = -\beta(\partial s_j/\partial p_j)(1-2s_j) > 0$. However, the second-order condition is always satisfied because $\partial^2 \pi_j/\partial p_j^2 = -\beta s_j < 0$. In symmetric equilibrium with $\delta = 1$ and $mc = 0$, the largest market share is attained as $1/(n+1)$ when the equilibrium price is zero, which implies that the market share of the outside goods s_0 is no less than each firm's market share: $s_0 > s$.

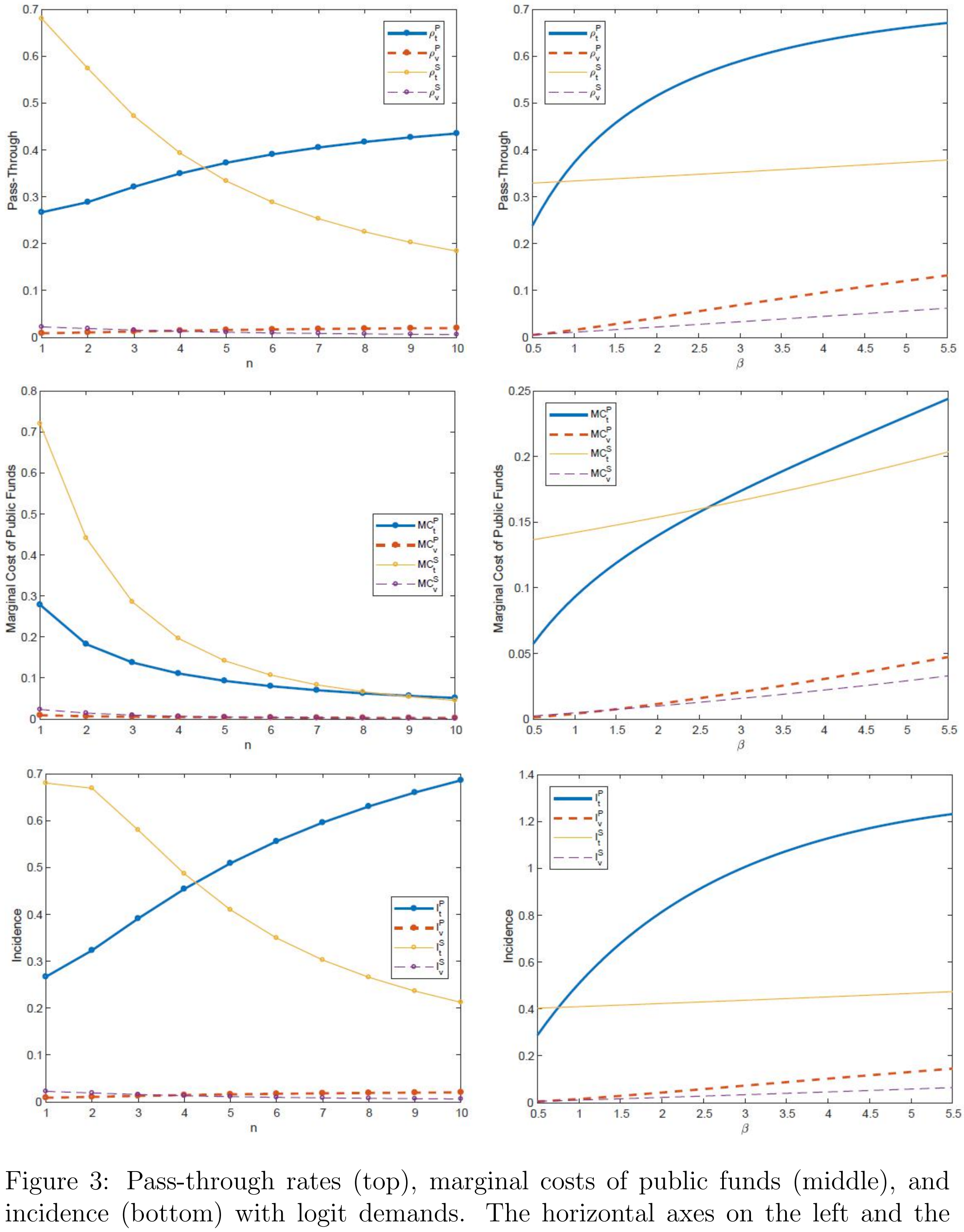

Figure 3: Pass-through rates (top), marginal costs of public funds (middle), and incidence (bottom) with logit demands. The horizontal axes on the left and the right penels are the number of firms (n) and the price coefficient (β), respectively.

and the incidences increase as β increases, the marginal costs of public funds are also increasing in contrast to the case of linear demands. The reason is that the shirking effects of θ on MC are weaker than the effect from an increase in ϵ: the industry's demand becomes elastic quickly as consumers become more sensitive to a price increase.

4 Multi-Dimensional Pass-Through Framework

In this section, we generalize our previous results to a significantly more general specification of taxation that involves multiple tax parameters. We define two different types of pass-through vectors: the *pass-through rate vector* and the *pass-through quasi-elasticity vector*. We study their properties and show that they play a central role in evaluating welfare changes in response to changes in taxation.

4.1 Pass-through, conduct index, and welfare: A general discussion

4.1.1 Generalized pass-through and tax sensitivities

Consider a tax structure under which a firm's tax payment is expressed as $\phi(p, q, \mathbf{T})$, where $\mathbf{T} \equiv (T_1, ..., T_d)$ is a d-dimensional vector of tax parameters[26] so that the firm's profit in symmetric equilibrium is written as $\pi = pq - c(q) - \phi(p, q, \mathbf{T})$. Note that the argument so far is a special case of two dimensional pass-through: $\phi(p, q, \mathbf{T}) = tq + vpq$, where $\mathbf{T} = (t, v)$. The components of the (per-firm) tax revenue gradient vector $\nabla\phi(p, q, \mathbf{T})$ are

$$\phi_{T_\ell}(p, q, \mathbf{T}) \equiv \frac{\partial \phi(p, q, \mathbf{T})}{\partial T_\ell}.$$

Here, as in other parts of the paper, we use the symbol ∇ for the d-dimensional gradient with respect to $\mathbf{T}$. The arguments p and q in $\phi(p, q, \mathbf{T})$ are treated as fixed for the purposes of taking this gradient. We denote by $\mathbf{f}$ a vector components

[26]To be precise, $\phi(p, q, \mathbf{T})$ represents a simplified notation for a function $\phi(p, q, T_1, ..., T_d)$ with $d + 2$ arguments.

$\phi_{T_\ell}(p,q,\mathbf{T})/q$. We denote the equilibrium price function[27] by $p^\star(\mathbf{T})$ and its gradient, the *pass-through rate vector*, by $\tilde{\boldsymbol{\rho}} \equiv \nabla p^\star(\mathbf{T})$. Further, we use the components of the $\mathbf{f}$ and $\tilde{\boldsymbol{\rho}}$ to define the *pass-through quasi-elasticity vector* as

$$\boldsymbol{\rho} \equiv \left(\rho_{T_1}, ..., \rho_{T_d}\right), \qquad \rho_{T_\ell} \equiv \frac{\tilde{\rho}_{T_\ell}}{f_{T_\ell}} = \frac{q}{\phi_{T_\ell}(p,q,\mathbf{T})}\,\frac{\partial p^\star}{\partial T_\ell}.$$

Note that the components of $\boldsymbol{\rho}$ are all dimensionless. We define the (*first-order*) *price sensitivity* ν of the tax revenue and the (*first-order*) *quantity sensitivity* τ of the (per-firm) tax revenue as follows:

$$\nu(p,q,\mathbf{T}) \equiv \frac{1}{q}\phi_p(p,q,\mathbf{T}), \qquad \tau(p,q,\mathbf{T}) \equiv \frac{1}{p}\phi_q(p,q,\mathbf{T}).$$

Their derivatives are

$$\nu_{T_\ell}(p,q,\mathbf{T}) \equiv \frac{\partial \nu(p,q,\mathbf{T})}{\partial T_\ell}, \qquad \tau_{T_\ell}(p,q,\mathbf{T}) \equiv \frac{\partial \tau(p,q,\mathbf{T})}{\partial T_\ell}.$$

The analogous definitions for *second-order sensitivities* are:

$$\nu_{(2)}(p,q,\mathbf{T}) \equiv \frac{p}{q}\,\frac{\partial^2\phi(p,q,\mathbf{T})}{\partial p^2},\ \tau_{(2)}(p,q,\mathbf{T}) \equiv \frac{q}{p}\,\frac{\partial^2\phi(p,q,\mathbf{T})}{\partial q^2},\ \kappa(p,q,\mathbf{T}) \equiv \frac{\partial^2\phi(p,q,\mathbf{T})}{\partial p\,\partial q}.$$

The first-order and second-order sensitivities are dimensionless, just like the components of $\boldsymbol{\rho}$. In this section, we keep the same definition of the elasticities ϵ and η as before.

4.1.2 Generalized conduct index

We introduce the conduct index θ as a function independent of the cost-side of the oligopoly game such that in equilibrium the following condition holds:

$$[1 - \tau - (1-\nu)\,\eta\,\theta]\,p = mc. \tag{3}$$

In the case of unit and ad valorem taxation, this definition reduces to the conduct index defined earlier (Equation (1)). In principle, there are many possible definitions that agree with the earlier definition in the case of unit and ad valorem taxation. However, we find the specification of Equation (3) particularly convenient.

[27]Unlike the inverse demand function $p(q)$, the function $p^\star(\mathbf{T})$ takes the vector of taxes as arguments and its functional value is the price in the resulting equilibrium.

4.1.3 Relative size of the components of pass-through vectors

We now establish the following relationship.

Proposition 8. *The pass-through rates and quasi-elasticities satisfy*[28]

$$\frac{\tilde{\rho}_{T_\ell}}{\tilde{\rho}_{T_{\ell'}}} = \frac{\tau_{T_{\ell'}} - \nu_{T_{\ell'}}\eta\,\theta}{\tau_{T_\ell} - \nu_{T_\ell}\eta\,\theta}, \qquad \frac{\rho_{T_\ell}}{\rho_{T_{\ell'}}} = \frac{f_{T_{\ell'}}}{f_{T_\ell}}\,\frac{\tau_{T_\ell} - \eta\,\theta\,\nu_{T_\ell}}{\tau_{T_{\ell'}} - \eta\,\theta\,\nu_{T_{\ell'}}}.$$

Proof. Consider an infinitesimal tax change such that the equilibrium price (and therefore quantity) does not change: $\tilde{\boldsymbol{\rho}} \cdot d\mathbf{T} = 0$. Let us choose $d\mathbf{T}$ to have just two non-zero components: dT_ℓ and $dT_{\ell'}$. This implies

$$\frac{\tilde{\rho}_{T_\ell}}{\tilde{\rho}_{T_{\ell'}}} = -\frac{dT_{\ell'}}{dT_\ell}. \tag{4}$$

Since Equation (3) must hold both before and after the tax change, it must be the case that $1 - \tau - (1-\nu)\,\eta\theta$ does not change, and in turn

$$(-\tau_{T_\ell} + \nu_{T_\ell}\eta\theta)\,dT_\ell + \left(-\tau_{T_{\ell'}} + \nu_{T_{\ell'}}\eta\theta\right)dT_{\ell'} = 0.$$

Substituting for $dT_{\ell'}$ from this equation into Equation (4) and using the definition of pass-through quasi-elasticities leads to the desired result. □

Since the components have known proportions, we can write them using a common factor $p\rho_{(0)}$ as

$$\begin{aligned} \tilde{\rho}_{T_\ell} &= (\tau_{T_\ell} - \nu_{T_\ell}\eta\,\theta)\,p\rho_{(0)}, \\ \rho_{T_\ell} &= \frac{p}{f_{T_\ell}}(\tau_{T_\ell} - \nu_{T_\ell}\eta\,\theta)\,\rho_{(0)}. \end{aligned} \tag{5}$$

4.1.4 Absolute size of the components of pass-through vectors

Proposition 9. *The value of the factor $\rho_{(0)}$ introduced above is given by the formula:*

$$\frac{1}{\rho_{(0)}} = 1 - \kappa + \epsilon\tau_{(2)} + (1-\tau)\epsilon\chi + \left[\nu - \kappa + \eta\nu_{(2)} + (\omega - \eta - \chi)(1-\nu)\right]\theta, \tag{6}$$

[28]If the denominators are zero, the fractions become ill-defined. In that case, of course, the statement does not apply.

where $\omega \equiv q\,(\eta\theta)'/\,(\eta\theta)$*, with the prime denoting a derivative with respect to the quantity* q*.*

Proof. The same type of reasoning as in the proof of Proposition 5 is useful here. In particular, comparative statics of Equation (3) with respect to a tax T_ℓ leads to the desired result after utilizing the definitions above and eliminating marginal cost using, again, Equation (3). The calculation is a bit tedious, but completely straightforward. □

4.1.5 Welfare changes and their relationship to pass-through vectors

Now, we establish the general formulas for the marginal cost of public fund and incidence in the multi-dimensional pass-through framework. Welfare component changes in response to an infinitesimal change in taxes can be found as follows. The (per-firm) consumer surplus change in response to an infinitesimal change dT_ℓ in the tax T_ℓ is

$$dCS = -qdp = -q\tilde{\rho}_{T_\ell} dT_\ell,$$

which means that in vector notation, $\frac{1}{q}\nabla CS = -\tilde{\boldsymbol{\rho}}$. The change in (per-firm) producer surplus is

$$dPS = d\,(pq - c\,(q) - \phi(p,q,\mathbf{T})) = \left[\phi_{T_\ell}(p,q,\mathbf{T}) - (1-\nu)\,(1-\theta)\,\tilde{\rho}_{T_\ell}\right] dT_\ell,$$

where we utilize Equation (3) to eliminate marginal cost. In vector notation, this is $\frac{1}{q}\nabla PS = (1-\nu)\,(1-\theta)\,\tilde{\boldsymbol{\rho}} - \mathbf{f}$, since $\mathbf{f} = \frac{1}{q}\nabla\phi(p,q,\mathbf{T})$. The change in tax revenue is

$$dR = \phi_p(p,q,\mathbf{T})dp + \phi_q(p,q,\mathbf{T})dq + \phi_{T_\ell}(p,q,\mathbf{T})dT_\ell = \left[\phi_{T_\ell}(p,q,\mathbf{T}) - (\epsilon\tau - \nu)\,\tilde{\rho}_{T_\ell}\right] dT_\ell.$$

In vector notation, $\frac{1}{q}\nabla R = \mathbf{f} - (\epsilon\tau - \nu)\,\tilde{\boldsymbol{\rho}}$. Finally, for the change in social welfare, we have

$$dW = (p - mc)\,dq = \left[\epsilon\tau + \theta\,(1-\nu)\right]\tilde{\rho}_{T_\ell} dT_\ell.$$

In vector notation, $\frac{1}{q}\nabla W = -\left[\epsilon\tau + \theta\,(1-\nu)\right]\tilde{\boldsymbol{\rho}}$.

Note that the welfare components $CS\,(\mathbf{T})$, $PS\,(\mathbf{T})$, $R\,(\mathbf{T})$, and $W\,(\mathbf{T}) = CS\,(\mathbf{T}) + PS\,(\mathbf{T}) + R\,(\mathbf{T})$ are all treated as functions of taxes only and represent the equilibrium outcomes. This is different from the tax revenue function $\phi(p, q, \mathbf{T})$, which has also p and q as arguments and which is specified by the government irrespective of the equilibrium. We summarize these findings in the following proposition.

Proposition 10. *The tax gradients of consumer surplus, producer surplus, tax revenue, and social welfare with respect to the taxes all belong to a two-dimensional vector space spanned by* $\mathbf{f}$ *and* $\tilde{\boldsymbol{\rho}}$. *The precise linear combinations of* $\mathbf{f}$ *and* $\tilde{\boldsymbol{\rho}}$ *are*

$$\frac{1}{q}\nabla CS = -\tilde{\boldsymbol{\rho}},$$

$$\frac{1}{q}\nabla PS = (1-\nu)(1-\theta)\,\tilde{\boldsymbol{\rho}} - \mathbf{f},$$

$$\frac{1}{q}\nabla R = \mathbf{f} + (\nu - \epsilon\tau)\,\tilde{\boldsymbol{\rho}},$$

$$\frac{1}{q}\nabla W = -\left[(1-\nu)\,\theta + \epsilon\tau\right]\tilde{\boldsymbol{\rho}}.$$

These relationships, considered component-wise, immediately imply the following results for welfare change ratios, and generalize Propositions 1 and 2.[29]

Proposition 11. *The marginal cost of public funds of a tax* T_ℓ, $MC_{T_\ell} = (\nabla W)_{T_\ell} / (\nabla R)_{T_\ell}$, *is*

$$MC_{T_\ell} = \frac{(1-\nu)\,\theta + \epsilon\tau}{\frac{1}{\rho_{T_\ell}} + \nu - \epsilon\tau}.$$

The incidence of this tax, $I_{T_\ell} = (\nabla CS)_{T_\ell} / (\nabla PS)_{T_\ell}$, *equals:*

$$I_{T_\ell} = \frac{1}{\frac{1}{\rho_{T_\ell}} - (1-\nu)(1-\theta)}.$$

Similarly, the social incidence, $SI_{T_\ell} = (\nabla W)_{T_\ell} / (\nabla PS)_{T_\ell}$, *equals:*

$$SI_{T_\ell} = \frac{(1-\nu)\,\theta + \epsilon\tau}{\frac{1}{\rho_{T_\ell}} - (1-\nu)(1-\theta)}.$$

[29]Remember that the T_ℓ component of the vector $\mathbf{f}$ is $\phi_{T_\ell}(p, q, \mathbf{T})/q = \tilde{\rho}_{T_\ell}/\rho_{T_\ell}$.

4.2 Pass-through, conduct index, and welfare: Special cases

The results of the previous subsection contain our results for ad valorem and unit taxes as special cases, but offer much greater generality, since the taxes (government interventions) may be specified in a very flexible way. In fact, Weyl and Fabinger's (2013) results under symmetric oligopoly can be interpreted as special cases of the present results. In particular, Weyl and Fabinger's (2013) analysis considers either unit taxes or exogenous competition (an exogenous quantity supplied to the market). The case of unit taxes are clearly included in the present results, which has motivated this paper. At the same time, it turns out that the case of exogenous competition is included as well. The reasoning is as follows.

Consider a tax $T_1 = \tilde{q}$ of the form: $\phi\,(p, q, \tilde{q}) = \tilde{q}\,p + c(q - \tilde{q}) - c(q)$. Then, the firm's profit is given by:

$$pq - c\,(q) - \phi\,(p, q, \tilde{q}) = p\,(q - \tilde{q}) + c(q - \tilde{q}).$$

The firm, therefore, has the same profit function as in the case of exogenous competition $\tilde{q}$ in Weyl and Fabinger (2013). Proposition 11 (specialized to constant marginal cost and zero initial $\tilde{q}$) then implies the social incidence result in Principle of Incidence 3 in Weyl and Fabinger (2013, p. 548).

Similarly, the relationships between pass-through of unit taxes and of exogenous competition are implied by the general result of Proposition 8 for the tax specification $T_1 = t$, $T_2 = \tilde{q}$,

$$\phi\,(p, q, t, \tilde{q}) = tq + \tilde{q}\,p + c(q - \tilde{q}) - c(q).$$

To obtain the absolute size of the two types of pass-through, one can straightforwardly use Proposition 9.

5 Heterogeneous Firms

In this section, we extend our results to the case of n heterogeneous firms (i.e. asymmetric firms), where each firm i controls a strategic variable σ_i, which could be, for example, the price or quantity of its product. We allow for the tax function $\phi_i\,(p_i, q_i, \mathbf{T})$ to depend explicitly on the identity of the firm; we write $f_{i\,T_\ell}\,(p_i, q_i, \mathbf{T}) = \frac{1}{q_i}\frac{\partial}{\partial T_\ell}\phi_i\,(p_i, q_i, \mathbf{T})$ for its derivative with respect to tax T_ℓ. Similarly, the sensitivities $\tau_i\,(p_i, q_i, \mathbf{T})$, $\nu_i\,(p_i, q_i, \mathbf{T})$, etc., now also have the firm index i. The marginal cost $mc_i\,(q_i)$ of firm i is also allowed to depend on the identity of the firm, and we denote its elasticity $\chi_i\,(q_i) \equiv q_i\, mc_i'\,(q_i)\,/mc_i\,(q_i)\,.$

5.1 Pricing strength index and pass-through

We define the *pricing strength index* $\psi_i\,(\mathbf{q})$ of firm i to be a function independent of the cost side of the economic problem such that the first-order condition for firm i is:

$$\{1 - \tau_i\,(p_i\,(\mathbf{q})\,, q_i, \mathbf{T}) - \psi_i\,(\mathbf{q})\,[1 - \nu_i\,(p_i\,(\mathbf{q})\,, q_i, \mathbf{T})]\}\, p_i\,(\mathbf{q}) = mc\,(q_i)\,.$$

In the special case of symmetric firms, this definition reduces to $\psi_i = \eta\,\theta$.

We wish to express the pass-through rate in terms of these pricing strength indices. Specifically, the pass-through rate is an $n \times d$ matrix $\tilde{\boldsymbol{\rho}}$ with rows $\tilde{\boldsymbol{\rho}}_{T_\ell} \equiv \partial\mathbf{p}/\partial T_\ell$ and elements $\tilde{\rho}_{i\,T_\ell} = \partial p_i/\partial T_\ell$. It is shown that the pass-through rate equals

$$\tilde{\boldsymbol{\rho}}_{T_\ell} = \mathbf{b}^{-1}\,.\,\boldsymbol{\iota}_{T_\ell}, \tag{7}$$

where the factors on the right-hand side are defined as follows. The matrix $\mathbf{b}$ is an $n \times n$ matrix, independent of the choice of T_ℓ, with elements

$$\begin{aligned} b_{ij} \;&=\; \left[1 - \kappa_i - \left(1 - \nu_i - \nu_{(2)i}\right)\psi_i\right]\delta_{ij} - (1 - \nu_i)\psi_i\Psi_{ij} \\ &+\; \left\{\tau_{(2)i} + \nu_i\psi_i - \kappa_i + [1 - \tau_i - (1 - \nu_i)\,\psi_i]\chi_i\right\}\epsilon_{ij} \end{aligned}$$

where

$$\epsilon_{ij} = -\,\frac{p_i}{q_i}\,\frac{\partial q_i\,(\mathbf{p})}{\partial p_j}, \qquad \Psi_{ij} = \frac{p_i}{\psi_i}\frac{\partial\psi_i\,(\mathbf{q}\,(\mathbf{p}))}{\partial p_j}.$$

For each tax T_ℓ, $\boldsymbol{\iota}_{T_\ell}$ is an n-dimensional vector with components

$$\iota_{i\,T_\ell} = p_i\,\frac{\partial\tau_i\,(p_i, q_i, \mathbf{T})}{\partial T_\ell} - p_i\,\psi_i\,\frac{\partial\nu_i\,(p_i, q_i, \mathbf{T})}{\partial T_\ell}.$$

In the case of symmetric firms and at symmetric prices, the pass-through rate expression in Equation (7) agrees with the expression represented by Equations (5) and (6) in Section 4.[30]

To generalize the notion of pass-through quasi-elasticity to the case of heterogeneous firms, we define the pass-through quasi-elasticity matrix $\boldsymbol{\rho}$ as an $n \times d$ matrix with elements

$$\rho_{i\,T_\ell} = \frac{1}{f_{i\,T_\ell}\,(p_i, q_i, \mathbf{T})}\,\frac{\partial p_i}{\partial T_\ell},$$

and with rows denoted $\boldsymbol{\rho}_{T_\ell}$.

5.2 Welfare changes

In the following, for each i, $\boldsymbol{\epsilon}_i$ is an n-dimensional vector with its j-th component equal to ϵ_{ij}. For the tax gradients of welfare components corresponding to individual firms we obtain:

$$\frac{1}{q_i}\nabla CS_i = -\mathbf{e}_i.\tilde{\boldsymbol{\rho}},$$

$$\frac{1}{q_i}\nabla PS_i = (1-\nu_i)\left(\mathbf{e}_i - \psi_i\,\boldsymbol{\epsilon}_i\right).\tilde{\boldsymbol{\rho}} - \mathbf{f}_i,$$

$$\frac{1}{q_i}\nabla R_i = \left(\nu_i\,\mathbf{e}_i - \tau_i\,\boldsymbol{\epsilon}_i\right).\tilde{\boldsymbol{\rho}} + \mathbf{f}_i,$$

$$\frac{1}{q_i}\nabla W_i = -\left[\tau_i + \psi_i\ (1-\nu_i)\right]\boldsymbol{\epsilon}_i.\tilde{\boldsymbol{\rho}}.$$

The corresponding gradients of total welfare components are then obtained by adding up contributions from individual firms. For example, $\nabla CS = \sum_{i=1}^{n} \nabla CS_i$. Denoting the total quantity as $Q \equiv \sum_{i=1}^{n} q_i$, this means that $\frac{1}{Q}\nabla CS$ is a weighted average of $-\mathbf{e}_i.\tilde{\boldsymbol{\rho}}$, with the weights proportional to q_i. Similarly for the other welfare components. This generalizes Proposition 10 above.

We can also consider ratios of welfare changes corresponding to some tax T_ℓ:

$$MC_{i\,T_\ell} = \frac{\left[\tau_i + (1-\nu_i)\ \psi_i\right]\,\epsilon^{\rho}_{i\,T_\ell}\,\rho_{i\,T_\ell}}{1 + (\nu_i - \tau_i\,\epsilon^{\rho}_{i\,T_\ell})\,\rho_{i\,T_\ell}},$$

[30]To confirm this agreement, note that at symmetric prices, $\sum_{j=1}^{n} \Psi_{ij} = -\epsilon\omega$. Note also that $\epsilon_{ii}\,(\mathbf{p})\,|_{\mathbf{p}=(p,\ldots,p)} = \epsilon_F(p)$, and for $j \neq i$, $\epsilon_{ij}\,(\mathbf{p})\,|_{\mathbf{p}=(p,\ldots,p)} = -\frac{1}{n-1}\,\epsilon_C(p)$.

$$I_{i\,T_\ell} = \frac{\rho_{i\,T_\ell}}{1 - (1-\nu_i)\,(1 - \psi_i\,\epsilon^{\rho}_{i\,T_\ell})\,\rho_{i\,T_\ell}},$$
$$SI_{i\,T_\ell} = \frac{[\tau_i + (1-\nu_i)\;\psi_i]\;\epsilon^{\rho}_{i\,T_\ell}\,\rho_{i\,T_\ell}}{1 - (1-\nu_i)\,(1 - \psi_i\,\epsilon^{\rho}_{i\,T_\ell})\,\rho_{i\,T_\ell}},$$

where $\epsilon^{\rho}_{i\,T_\ell} \equiv \boldsymbol{\epsilon}_i.\tilde{\boldsymbol{\rho}}_{T_\ell}/\tilde{\rho}_{i\,T_\ell} = \boldsymbol{\epsilon}_i.\boldsymbol{\rho}_{T_\ell}/\rho_{i\,T_\ell}$. The ratios of the corresponding total welfare changes will be weighted averages of these firm-specific ratios. The weights correspond to the sizes of the denominators times q_i. For example, MC_{T_ℓ} will lie between $\min_i MC_{i\,T_\ell}$ and $\max_i MC_{i\,T_\ell}$. Similarly for the other ratios. This generalizes Proposition 11.

5.3 Conduct index and welfare changes

For heterogeneous firms, we define the conduct index of firm i as

$$\theta_i = -\frac{\sum_{j=1}^{n} \{p_j\,[1-\tau_j\,(p_j, q_j, \mathbf{T})] - mc\,(q_j)\}\,\frac{dq_j}{d\sigma_i}}{\sum_{j=1}^{n} [1-\nu_j\,(p_j, q_j, \mathbf{T})]\,q_j\,\frac{dp_j}{d\sigma_i}}.$$

In the special case of only unit taxation, this definition reduces to Weyl and Fabinger's (2013) Equation 4. In the special case of symmetric firms the definition reduces to our Equation (3) with $\theta_i = \theta$.

The conduct index θ_i is closely connected to the pricing strength index ψ_i, but not as closely as it would be in the case of symmetric oligopoly. Using the definitions of the indices, it may be shown that

$$\theta_i = -\frac{\sum_{j=1}^{n} (1-\nu_j)\,\psi_j\,p_j\,\frac{dq_j}{d\sigma_i}}{\sum_{j=1}^{n} (1-\nu_j)\,q_j\,\frac{dp_j}{d\sigma_i}}.$$

For symmetric oligopoly, this equation reduces simply to $\theta = \epsilon\psi$.

The conduct index is used to express welfare component changes in response to infinitesimal changes in taxes. The relationships are a bit more complicated than in the case of using the pricing strength index and can be expressed as follows. We define the price response to an infinitesimal change in the strategic variable σ_k of firm j as

$$\zeta_{ij} = \frac{dp_i}{d\sigma_j}.$$

Since the vectors ζ_{i1}, ζ_{i2}, ... , ζ_{in} form a basis in the n-dimensional vector space to which $\tilde{\rho}_{i\,T_\ell}$ for a given ℓ belongs, we can write $\tilde{\rho}_{i\,T_\ell}$ as a linear combination of them for some coefficients $\lambda_{i\,T_\ell}$:

$$\tilde{\rho}_{i\,T_\ell} = \sum_{j=1}^{n} \lambda_{j\,T_\ell} \zeta_{ij}.$$

For changes in consumer and producer surplus we get:

$$\frac{dCS}{dT_\ell} = -\sum_{i=1}^{n} q_i \tilde{\rho}_{i\,T_\ell} = -\sum_{j=1}^{n} \left(\sum_{i=1}^{n} q_i \zeta_{ij} \right) \lambda_{j\,T_\ell},$$

$$\frac{dPS}{dT_\ell} = -\sum_{i=1}^{n} f_{i\,T_\ell}\left(p_i, q_i, \mathbf{T}\right) - \sum_{j=1}^{n} \hat{\zeta}_j \left(1 - \theta_j\right) \lambda_{j\,T_\ell},$$

where we used the notation

$$\hat{\zeta}_j \equiv \sum_{i=1}^{n} \left[1 - \nu_i\left(p_i, q_i, \mathbf{T}\right)\right] q_i\, \zeta_{ij}.$$

These surplus change expressions represent a generalization of the surplus expressions in Weyl and Fabinger's (2013) Section 5.

5.4 Aggregative games

In the case of oligopoly in the form of aggregative games, where all other firms' actions are summarized as a aggregator in each firm's profit,[31] we can manipulate the above formulas for pricing strength and conduct indices further. We identify the firm's strategic variable σ_i with an action $a_i \equiv \sigma_i$ the firm can take, which contributes to an aggregator $A = \sum_{i=1}^{n} a_i$. The prices and quantities are functions of just two arguments: $p_i\left(A, a_i\right)$ and $q_i\left(A, a_i\right)$. Their derivatives that take into account the dependence of A on the action of firm i are $\frac{dq_j}{d\sigma_i} = {q_j}^{(0,1)}\left(A, a_i\right) + {q_j}^{(1,0)}\left(A, a_i\right)$, $\frac{dp_j}{d\sigma_i} = {p_j}^{(0,1)}\left(A, a_i\right) + {p_j}^{(1,0)}\left(A, a_i\right)$. The firm's first-order condition is

$$0 = \left({p_i}^{(0,1)}\left(A, a_i\right) + {p_i}^{(1,0)}\left(A, a_i\right)\right) q_i\left(A, a_i\right) \left(\nu_i\left(p_i\left(A, a_i\right), q_i\left(A, a_i\right), \mathbf{T}\right) - 1\right) +$$

$$\left(q^{(0,1)}(A, a_i) + {q_i}^{(1,0)}(A, a_i)\right) \left(mc\left(q_i\left(A, a_i\right)\right) + p_i\left(A, a_i\right)\left(\tau_i\left(p_i\left(A, a_i\right), q_i\left(A, a_i\right), \mathbf{T}\right) - 1\right)\right),$$

[31] For a recent treatment of aggregative oligopoly games, see Anderson, Erkal, and Piccinin (2016). Here we consider a setup consistent with their Section 2.

which gives us a relatively simple expression for the pricing strength index:

$$\psi_i\left(A,a_i\right)=-\frac{q_i\left(A,a_i\right)}{p_i\left(A,a_i\right)}\;\frac{{p_i}^{(0,1)}\left(A,a_i\right)+{p_i}^{(1,0)}\left(A,a_i\right)}{{q_i}^{(0,1)}\left(A,a_i\right)+{q_i}^{(1,0)}\left(A,a_i\right)}.$$

The expression for the conduct index also simplifies:

$$\theta_i=\sum_{j=1}^{n}w_j\;\frac{\gamma_j\left(A,a_i\right)}{\gamma_j\left(A,a_j\right)},$$

where w_i is a normalized version of unnormalized "weights" $\tilde{w}_j$,

$$w_i\equiv\frac{\tilde{w}_i}{\sum_{j=1}^{n}\tilde{w}_j},\qquad \tilde{w}_j\equiv\left(1-\nu_j\right)q_j\left(A,a_j\right)\,\left({p_j}^{(0,1)}\left(A,a_j\right)+{p_j}^{(1,0)}\left(A,a_j\right)\right),$$

and

$$\gamma_j\left(A,a_i\right)\equiv{q_j}^{(0,1)}\left(A,a_i\right)+{q_j}^{(1,0)}\left(A,a_i\right).$$

These simplified formulas would be used for further analysis of pass-through and welfare in aggregative oligopoly games.

6 Pass-Through and Welfare under Production-Cost and Taxation Changes

In the previous sections, we have studied changes in taxation, but not changes in production costs. Here we generalize our main results to incorporate both taxation and production costs. The additional cost to the firm is denoted $\phi\left(p,q,\mathbf{T}\right)$ as before, but the tax bill of firm i, denoted $\tilde{\phi}\left(p,q,\mathbf{T}\right)$, is different, in general. Here $\mathbf{T}$ is a vector of interventions (by the government or by external circumstances), which may or may not include traditional taxes. We recover the previous case of only taxation by setting $\tilde{\phi}\left(p,q,\mathbf{T}\right)=\phi\left(p,q,\mathbf{T}\right)$. If all of the additional cost to the firm comes from the production side, we have $\tilde{\phi}\left(p,q,\mathbf{T}\right)=0$. In general, $\phi\left(p,q,\mathbf{T}\right)-\tilde{\phi}\left(p,q,\mathbf{T}\right)$ is the production part of the additional cost $\phi\left(p,q,\mathbf{T}\right)$.

6.1 Symmetric firms

In addition to the notation used in the previous section, we define $\tilde{\mathbf{f}}=\frac{1}{q}\nabla\tilde{\phi}(p,q,\mathbf{T})$. First, we obtain a generalization of the formulas for the tax gradients of welfare components in Proposition 10. The equilibrium outcome depends only on the additional cost $\phi(p, q, \mathbf{T})$ and not on its split between taxes and production costs. For this reason, the formulas for consumer and producer surplus will be unchanged. The government revenue and therefore also total social welfare will depend on $\tilde{\phi}(p, q, \mathbf{T})$, of course. In the formula for the gradient of government revenue, $\mathbf{f}$ will be replaced by $\tilde{\mathbf{f}}$, and the formula for social welfare will be adjusted to reflect this difference. Hence the generalization of the results in Proposition 10 is:

$$\frac{1}{q}\nabla CS = -\tilde{\boldsymbol{\rho}},$$

$$\frac{1}{q}\nabla PS = (1-\nu)(1-\theta)\,\tilde{\boldsymbol{\rho}} - \mathbf{f},$$

$$\frac{1}{q}\nabla R = \tilde{\mathbf{f}} + (\nu - \epsilon\tau)\,\tilde{\boldsymbol{\rho}},$$

$$\frac{1}{q}\nabla W = -[(1-\nu)\,\theta + \epsilon\tau]\tilde{\boldsymbol{\rho}} + \tilde{\mathbf{f}} - \mathbf{f}.$$

We further define $g_{T_\ell} \equiv \tilde{f}_{T_\ell}/f_{T_\ell}$, which represents the fraction of an increase in additional cost (ϕ) to the firm (due to a change in the tax parameter T_ℓ) that is collected by the government in the form of taxes ($\tilde{\phi}$). In other words, g_{T_ℓ} is the government's share in increases of the additional costs induced by marginal changes in T_ℓ. If ϕ is a pure tax, then $g_{T_\ell} = 1$, and if ϕ is a pure production cost with no tax tax component, then $g_{T_\ell} = 0$. By taking ratios of the components of the tax gradients above, we obtain a generalization of Proposition 11: The marginal cost of public funds associated with intervention T_ℓ, $MC_{T_\ell} = (\nabla W)_{T_\ell} / (\nabla R)_{T_\ell}$, is

$$MC_{T_\ell} = \frac{1 - g_{T_\ell} + [(1-\nu)\,\theta + \epsilon\tau]\,\rho_{T_\ell}}{g_{T_\ell} + (\nu - \epsilon\tau)\,\rho_{T_\ell}}.$$

The incidence of this intervention, $I_{T_\ell} = (\nabla CS)_{T_\ell} / (\nabla PS)_{T_\ell}$, equals:

$$I_{T_\ell} = \frac{\rho_{T_\ell}}{1 - (1-\nu)(1-\theta)\,\rho_{T_\ell}}.$$

Similarly, the social incidence, $SI_{T_\ell} = (\nabla W)_{T_\ell} / (\nabla PS)_{T_\ell}$, equals:

$$SI_{T_\ell} = \frac{1 - g_{T_\ell} + [(1-\nu)\,\theta + \epsilon\tau]\,\rho_{T_\ell}}{1 - (1-\nu)(1-\theta)\,\rho_{T_\ell}}.$$

6.2 Heterogeneous firms

The adjustments to our formulas needed to generalize the results of Subsection 5.2 are analogous to the case of symmetric firms we just discussed. For each firm i, we define $\tilde{\mathbf{f}}_i = \frac{1}{q}\nabla\tilde{\phi}_i(p, q, \mathbf{T})$. For the welfare gradients, we obtain:

$$\frac{1}{q_i}\nabla CS_i = -\mathbf{e}_i.\tilde{\boldsymbol{\rho}},$$

$$\frac{1}{q_i}\nabla PS_i = (1-\nu_i)\left(\mathbf{e}_i - \psi_i\,\boldsymbol{\epsilon}_i\right).\tilde{\boldsymbol{\rho}} - \mathbf{f}_i,$$

$$\frac{1}{q_i}\nabla R_i = \left(\nu_i\,\mathbf{e}_i - \tau_i\,\boldsymbol{\epsilon}_i\right).\tilde{\boldsymbol{\rho}} + \tilde{\mathbf{f}}_i,$$

$$\frac{1}{q_i}\nabla W_i = -\left[\tau_i + \psi_i\ (1-\nu_i)\right]\boldsymbol{\epsilon}_i.\tilde{\boldsymbol{\rho}} + \tilde{\mathbf{f}}_i - \mathbf{f}_i.$$

Similarly, for each firm i, we define $g_{i\,T_\ell} \equiv \tilde{f}_{i\,T_\ell}/f_{i\,T_\ell}$. For the firm-specific welfare change ratios, we obtain:

$$MC_{i\,T_\ell} = \frac{1 - g_{i\,T_\ell} + (\tau_i + (1-\nu_i)\ \psi_i)\,\epsilon^{\rho}_{i\,T_\ell}\,\rho_{i\,T_\ell}}{g_{i\,T_\ell} + (\nu_i - \tau_i\,\epsilon^{\rho}_{i\,T_\ell})\,\rho_{i\,T_\ell}},$$

$$I_{i\,T_\ell} = \frac{\rho_{i\,T_\ell}}{1 - (1-\nu_i)\,(1 - \psi_i\,\epsilon^{\rho}_{i\,T_\ell})\,\rho_{i\,T_\ell}},$$

$$SI_{i\,T_\ell} = \frac{1 - g_{T_\ell} + (\tau_i + (1-\nu_i)\ \psi_i)\,\epsilon^{\rho}_{i\,T_\ell}\,\rho_{i\,T_\ell}}{1 - (1-\nu_i)\,(1 - \psi_i\,\epsilon^{\rho}_{i\,T_\ell})\,\rho_{i\,T_\ell}}.$$

We see that the generalization to production cost changes is very straightforward. These more general formulas may be applied to a range of economic situations such as cost changes due to exchange rate movements or movements in the world prices of commodities.

7 Concluding Remarks

In this paper, we characterize the welfare burden of taxation and the tax incidence in oligopoly with a general specification of competition, demand and cost. For symmetric oligopoly, we first derive formulas for marginal welfare losses from unit and ad valorem taxation, MC_t and MC_v, using the unit tax pass-through rate ρ_t

and the ad valorem tax pass-through semi-elasticity ρ_v (Proposition 1) as well as the formulas for tax incidence, I_t and I_v (Proposition 2). We then show that ρ_v can be expressed in terms of ρ_t (Proposition 3). These relationships are used to derive sufficient statistics for MC_t and MC_v (Proposition 4). The pass-through is also characterized, generalizing Weyl and Fabinger's (2013) formula (Proposition 5). In the case of price or quantity competition, we explain how ρ_t and ρ_v can be written only in terms of the demand elasticities, the demand curvatures, and the marginal cost elasticity (Propositions 6 and 7). We have discussed the relationships to other quantities of interest, as well as illustrative special cases. We also show that these results have a very natural generalization to a general specification of the tax revenue function as a function parameterized by a vector of tax parameters (Propositions 8, 9, 10, and 11). We further discuss an extension of our analysis to the case of asymmetric oligopoly, where the firms face different costs and possibly also different taxes.[32,33] In addition, we provide a generalization of our results to the case of changes in both production costs and taxes. Finally, we also examine oligopolistic competition with firm entry.

It would be possible to extend the analysis to the case of supply chains (see Peitz and Reisinger 2014). Other possible directions include the case of two-sided platform competition (White and Weyl 2016 and Tremblay 2017) or the case of the interactive effects of taxation for multiple imperfectly competitive product markets.[34] In addition, our methodology could be utilized to study other important

[32]By allowing (constant) asymmetric marginal costs, Anderson, de Palma, and Kreider (2001b) show that under quantity competition with homogeneous products (i.e., Cournot competition), ad valorem taxation is still preferable to unit taxation, although they were not able to verify if the same conclusion held under quantity competition with product differentiation. However, Anderson, de Palma, and Kreider (2001b) discuss a specific demand system (with perfectly inelastic individual demand) under which unit taxation is preferable to ad valorem taxation if the required tax revenue is sufficiently high. We conjecture that one could obtain further generalization by allowing the conduct index θ to be firm-specific. See also Zimmerman and Carlson (2010) for a parametric analysis of asymmetric firms.

[33]Interestingly, Tremblay and Tremblay (2016) study tax incidence in an asymmetric duopoly where one firm competes in price and the other firm competes in quantity, focusing on unit taxation. The pass-through rates can be different for the two identical firms (in terms of demand and cost): the quantity-competing firm has a higher pass-through rate than the price-competing firm has. This is in contrast with the result that the pass-through rate under price competition is generally higher under quantity competition.

[34]Among many others, Ballard, Shoven, and Whalley (1985) study this issue for perfectly com-

issues of pricing in general such as welfare effects of oligopolistic third-degree price discrimination (Adachi and Fabinger 2017). One may also study, for example, advertising pass-through (Draganska and Vitorino 2017): the firm's demand can be modeled as $q_j = q_j(p_1, ..., p_n; a_1, ..., a_n)$, where a_j is firm j's investment in advertising. Free-riding, because of the spillover effect, may be more or less serious depending on the conduct index. Furthermore, it would be of interest to develop flexible, but analytically solvable examples along the lines of Fabinger and Weyl (2016).

Appendix

A. Proof of Proposition 1

Using Equation (1) to substitute for mc, we first obtain a useful expression for the markup: $p - mc = t + pv + p(1-v)\eta\theta$. Now consider an infinitesimal change dt in the unit tax that induces a change dp in the equilibrium price and a change dq in the equilibrium quantity. These are related by $dt = dp/\rho_t = -\eta\, p\, dq/\,(q\,\rho_t)$. The corresponding change in social welfare per firm is $dW = (p - mc)\, dq = t\, dq + vp\, dq + (1-v)p\eta\theta\, dq$, and the change in tax revenue per firm is $dR = (t + vp)\, dq + vq\, dp + q\, dt = (t + vp)\, dq - vp\eta\, dq - \eta p\, dq/\rho_t$. Combining these relationships gives the result

$$MC_t = -\frac{dW}{dR} = -\frac{t + vp + (1-v)p\eta\theta}{t + vp - vp\eta - \frac{1}{\rho_t}p\eta} = \frac{(1-v)\eta\theta + \frac{t}{p} + v}{\frac{1}{\rho_t}\eta + v\eta - \frac{t}{p} - v} = \frac{(1-v)\theta + \epsilon\tau}{\frac{1}{\rho_t} + v - \epsilon\tau}.$$

Next, consider an infinitesimal change dv in the ad valorem tax that induces a change dp in the equilibrium price and a change dq in the equilibrium quantity, related by $dv = dp/(p\rho_v) = -\eta\, dq/(q\rho_v)$. The change in social welfare per firm is again $dW = (p - mc)\, dq = t\, dq + vp\, dq + (1-v)p\eta\theta\, dq$. The change in tax revenue per firm can be written as $(t + vp)\, dq + vq\, dp + pq\, dv = (t + vp)\, dq - vp\eta\, dq - p\eta\, dq/\rho_v$. Combining these relationships leads to the result

$$MC_t = -\frac{dW}{dR} = -\frac{t + vp + (1-v)p\eta\theta}{t + vp - vp\eta - \frac{1}{\rho_v}p\eta} = \frac{(1-v)\eta\theta + \frac{t}{p} + v}{\frac{1}{\rho_v}\eta + v\eta - \frac{t}{p} - v} = \frac{(1-v)\theta + \epsilon\tau}{\frac{1}{\rho_v} + v - \epsilon\tau}.$$

petitive markets.

B. Proof of Proposition 2

The impact of a change dt in the tax t on consumer surplus (per firm) is $dCS = -qdp = -q\rho_t dt$. The impact on producer surplus is

$$dPS = d[(1-v)\,pq - c\,(q) - tq] = -q\,dt + (1-v)\,p\,dq + (1-v)\,qdp - mc\,dq - t\,dq,$$

$$\Leftrightarrow dPS = -qdt + (1-v)\,q\rho_t dt + [(1-v)\,p - mc - t]dq.$$

Substituting for mc from Equation (1) as $mc = (1-v)\,(1-\eta\theta)\,p - t$ gives

$$dPS = -qdt + (1-v)\,q\rho_t dt + (1-v)\,\eta\theta pdq = -qdt + (1-v)\,q\rho_t dt - (1-v)\,\theta qdp,$$

$$\Leftrightarrow dPS = -qdt + (1-v)\,q\rho_t dt - (1-v)\,\theta q\rho_t dt = -[1-(1-v)\,(1-\theta)\,\rho_t]q\,dt.$$

The reciprocal of the incidence ratio is

$$\frac{1}{I_t} = \frac{dPS}{dCS} = \frac{(1-v)\,(1-\theta)\,q\rho_t - q}{-q\rho_t} = \frac{1}{\rho_t} - (1-v)\,(1-\theta)\,.$$

Similarly, for infinitesimal changes in ad valorem taxes we proceed analogously. The change in consumer surplus is $dCS = -qdp = -qp\rho_v dv$. For the change in producer surplus we have

$$dPS = d\,((1-v)\,pq - c\,(q) - tq) = -pq\,dv + (1-v)\,p\,dq + (1-v)\,qdp - mc\,dq - t\,dq.$$

Manipulating the last four terms on the right-hand side in the same way as before leads to

$$dPS = -pq\,dv + (1-v)\,p\,dq + (1-v)\,qdp - mc\,dq - t\,dq,$$

$$dPS = -pq\,dv + (1-v)\,qp\rho_v dv - (1-v)\,\theta qp\rho_v dv = [(1-v)\,(1-\theta)\,\rho_v - 1]\,qp\,dv.$$

The reciprocal of the incidence ratio then becomes

$$\frac{1}{I_t} = \frac{dPS}{dCS} = \frac{(1-v)\,(1-\theta)\,\rho_v q - q}{-q\rho_v} = \frac{1}{\rho_v} - (1-v)\,(1-\theta)\,.$$

C. Proof of Proposition 3

Let us consider a simultaneous infinitesimal change dt and dv in the taxes t and v that leaves the equilibrium price (and quantity) unchanged, which requires the effective marginal cost $(t + mc) / (1 - v)$ in Equation (1) to remain the same. This implies the comparative statics relationship

$$\frac{\partial}{\partial t}\left(\frac{t+mc}{1-v}\right) dt + \frac{\partial}{\partial v}\left(\frac{t+mc}{1-v}\right) dv = 0 \Rightarrow \frac{dt}{1-v} + \frac{t+mc}{(1-v)^2}\, dv = 0 \Rightarrow dt = -\frac{t+mc}{1-v}\, dv.$$

Note that here we do not need to take derivatives of mc even though it depends on q, simply because by assumption the quantity is unchanged. The total induced change in price, which generally would be expressed as $dp = \rho_t dt + \rho_v p\, dv$, must equal zero in this case, implying the result

$$\rho_t dt + \rho_v p\, dv = 0 \Rightarrow -\frac{t+mc}{1-v}\rho_t dv + \rho_v p\, dv = 0 \Rightarrow \rho_v = (1 - \eta\theta)\, \rho_t \Rightarrow \rho_v = \frac{\epsilon - \theta}{\epsilon}\rho_t.$$

D. Oligopoly with Multi-Product Firms

Here, we argue that the results obtained in Sections 2 and 3 can be extended to the case of multi-product firms just by a reinterpretation of the same formulas (without modifying them).[35] Assume there are n_p product categories, and the demand for firm j's k-th product is given by $q_{jk} = q_{jk}(\mathbf{p}_1, \mathbf{p}_2, .., \mathbf{p}_n)$, where $\mathbf{p}_j = (p_{j1}, ..., p_{jk}, ..., p_{jK})$ for each $j = 1, 2, ..., n$.[36] The firms are symmetric, and for each firm, the product it produces are also symmetric. The firm's profit per product is

$$\pi_j = \frac{1}{n_p}\sum_{k=1}^{n_p}\left((1-v)\, p_{jk} q_{jk} - t q_{jk} - c(q_{jk})\right).$$

We work with an equilibrium in which any firm j sets a uniform price p_j for all of its products: $p_{jk} = p_j$, and consequently sells an amount q_j of each of them: $q_{jk} = q_j$.[37]

[35] Lapan and Hennessy (2011) study unit and ad valorem taxes in multi-product Cournot oligopoly. Alexandrov and Bedre-Defolie (2017) also study cost pass-through of multi-product firms in relation to the LeChatelier–Samuelson principle.

[36] See, e.g., Nocke and Schutz (2016) for a recent treatment of multi-product oligopoly.

[37] For brevity, we do not explicitly discuss the standard conditions for the existence and uniqueness of non-cooperative Nash equilibria of the different underlying oligopoly games.

In this case, the profit per product equals $\pi_j = (1-v)\, p_j q_j - t q_j - c(q_j)$, which is formally the same as for single-product firms. For this reason, we can identify the prices p_j and quantities q_j of Section 2 with the prices p_j and quantities q_j introduced here in this paragraph. The discussion in Section 2 was general and applies to this case of symmetric oligopoly with multi-product firms as well. We can use the same definitions for the variables of interest, including the industry demand elasticity ϵ and the conduct index θ.

The definitions and results for the cases of price competition and quantity competition discussed Section 3 are also applicable here. It may be useful to translate some of the most important variables of that discussion into product-level variables. For derivatives of the direct demand system, we introduce the notation[38]

$$\begin{array}{cccc} & \xi_1 \equiv \frac{\partial q_{jk}}{\partial p_{jk}}, & \xi_{0,1} \equiv \frac{\partial q_{jk}}{\partial p_{jk'}}, & \\ \xi_2 \equiv \frac{\partial q_{jk}}{\partial p_{jk}^2}, & \xi_{1,1} \equiv \frac{\partial q_{jk}}{\partial p_{jk} \partial p_{jk'}}, & \xi_{0,2} \equiv \frac{\partial q_{jk}}{\partial p_{jk'}^2}, & \xi_{0,1,1} \equiv \frac{\partial q_{jk}}{\partial p_{jk'} \partial p_{jk''}}, \\ \tilde{\xi}_2 \equiv \frac{\partial q_{jk}}{\partial p_{jk} \partial p_{j'k}} & \tilde{\xi}_{1,1} \equiv \frac{\partial q_{jk}}{\partial p_{jk} \partial p_{j'k'}}, & \tilde{\xi}_{0,2} \equiv \frac{\partial q_{jk}}{\partial p_{jk'} \partial p_{j'k'}}, & \tilde{\xi}_{0,1,1} \equiv \frac{\partial q_{jk}}{\partial p_{jk'} \partial p_{j'k''}}, \end{array}$$

where the derivatives are evaluated at the fully symmetric point, where any p_{jk} equals the common value p. For specific choices of the demand system, these derivatives can be closely related. For example, if the substitution pattern between two goods produced by two different firms does not depend on the identity of the goods, then $\tilde{\xi}_2 = \tilde{\xi}_{0,2} = \tilde{\xi}_{1,1} = \tilde{\xi}_{0,1,1}$. In terms of these derivatives, we can write

$$\begin{aligned} \epsilon_F &= -\tfrac{p}{q} \left(\xi_1 + (n_p - 1)\, \xi_{0,1} \right), \\ \epsilon &= -\tfrac{p}{q} \left(\xi_1 + (n_p - 1)\, \xi_{0,1} + (n-1)\, \tilde{\xi}_1 + (n-1)\,(n_p - 1)\, \tilde{\xi}_{0,1} \right), \\ \alpha_F &= \tfrac{p^2}{q\, \epsilon_F} \left(\xi_2 + (n_p - 1) \left(\xi_{1,1} + \xi_{0,2} + (n_p - 2)\, \xi_{0,1,1} \right) \right), \\ \alpha_C &= (n-1)\, \tfrac{p^2}{q\, \epsilon_F} \left(\tilde{\xi}_2 + (n_p - 1)\, (\tilde{\xi}_{1,1} + \tilde{\xi}_{0,2} + (n_p - 2)\, \tilde{\xi}_{0,1,1}) \right). \end{aligned}$$

These can be substituted into the results of Proposition 6 to find the pass-through and the marginal cost of public funds under price competition.

[38] In this notation, the first subscript counts the derivatives with respect to the relevant price with index k, the second subscript counts the derivatives with respect to the price with index k' distinct from k, and the third subscript counts derivatives respect to the price with index k'' distinct from both k and k'. Further, ξ corresponds to derivatives with respect to prices charged by the same firm j, while $\tilde{\xi}$ corresponds to derivatives with respect to prices charged by firm j and some other firm j'.

For the inverse demand system the analogous definitions are

$$
\begin{array}{cccc}
& \zeta_1 \equiv \frac{\partial q_{jk}}{\partial p_{jk}}, & \zeta_{0,1} \equiv \frac{\partial q_{jk}}{\partial p_{jk'}}, & \\
\zeta_2 \equiv \frac{\partial q_{jk}}{\partial p_{jk}^2}, & \zeta_{1,1} \equiv \frac{\partial q_{jk}}{\partial p_{jk} \partial p_{jk'}}, & \zeta_{0,2} \equiv \frac{\partial q_{jk}}{\partial p_{jk'}^2}, & \zeta_{0,1,1} \equiv \frac{\partial q_{jk}}{\partial p_{jk'} \partial p_{jk''}}, \\
\tilde{\zeta}_2 \equiv \frac{\partial q_{jk}}{\partial p_{jk} \partial p_{j'k}} & \tilde{\zeta}_{1,1} \equiv \frac{\partial q_{jk}}{\partial p_{jk} \partial p_{j'k'}}, & \tilde{\zeta}_{0,2} \equiv \frac{\partial q_{jk}}{\partial p_{jk'} \partial p_{j'k'}}, & \tilde{\zeta}_{0,1,1} \equiv \frac{\partial q_{jk}}{\partial p_{jk'} \partial p_{j'k''}}.
\end{array}
$$

The relations

$$
\begin{aligned}
\eta_F &= -\tfrac{q}{p} \left(\zeta_1 + (n_p - 1)\, \zeta_{0,1} \right), \\
\eta &= -\tfrac{q}{p} \left(\zeta_1 + (n_p - 1)\, \zeta_{0,1} + (n-1)\, \tilde{\zeta}_1 + (n-1)\,(n_p - 1)\, \tilde{\zeta}_{0,1} \right), \\
\sigma_F &= \tfrac{q^2}{p\,\eta_F} \left(\zeta_2 + (n_p - 1) \left(\zeta_{1,1} + \zeta_{0,2} + (n_p - 2)\, \zeta_{0,1,1} \right) \right), \\
\sigma_C &= (n-1)\, \tfrac{q^2}{p\,\eta_F} \left(\tilde{\zeta}_2 + (n_p - 1) \left(\tilde{\zeta}_{1,1} + \tilde{\zeta}_{0,2} + (n_p - 2)\, \tilde{\zeta}_{0,1,1} \right) \right).
\end{aligned}
$$

can be substituted into the results of Proposition 7 to find the pass-through and marginal cost of public funds under price competition.

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