Health & Working Time: A Macroeconomic Perspective on the American Puzzle

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Abstract

Today, Americans work substantially more than Europeans and are in much poorer health despite greater medical expenditure. We provide another rationale for the American mortality disadvantage around age 60 by relying on the negative effect on health of long hours of work. To do so, we introduce health capital in an exogenous growth model with elastic labor supply impacting its depreciation rate. We remain agnostic as to why Americans work more than Europeans, but model the difference with preferences for leisure for convenience. Longer hours of work make individuals devote a larger fraction of their resources to health care which may not be sufficient to offset the extra depreciation of their health capital stock, provided the returns to medical investments are not high enough. We then calibrate the model for the US to assess how much of the difference in both mortality rates and health care expenditure come from excess labor supply. We build a counterfactual using the hours of work in UK in 2015. In the baseline counterfactual, the US will spend as much as less 2.6% of GDP in medical expenditures and will experience 143 deaths per 100,000 people less, that is respectively one half and one quarter of the actual deviations with the UK.

JEL Classification: I1, J22, O41

Keywords: Labor Supply, Working Time, Life Expectancy, Health Capital, Health Expenditure, Economic Growth

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"Mutual emulation and desire for a greater gain prompted them [workers] to over-work themselves, and to hurt their health by excessive labour"

- Adam Smith, The Wealth of Nations, 1776

1 Introduction

The growing gap in terms of health outcomes between the US and other industrialized countries (see the comprehensive report by the National Research Council and Institute of Medicine (2013)) receives more and more attention. The panel noted that the United States, despite spending much more money on health care than any other developed country, has seen its relative standing in the world fall for the last forty years. It was ranked 16th in 1960 in terms of life expectancy at birth for male and females combined but fell steadily down to the 36th rank in 2015. Such a bad performance of the US remains a puzzle for medical and social scientists, in spite of growing effort to provide potential explanations. Case and Deaton (2015, 2017) document an increase in midlife mortality rates for non-Hispanic whites between 1999 and 2015 they attribute mostly to what they label 'deaths of despair' (suicide, alcohol- and drug-related mortality, and also shed light on a worrying opioids epidemics). Earlier, several studies also pointed out the role of cigarette smoking. Cumming, Preston and Cohen (2011) wrote[1] that *"during 1950-2003, gains in life expectancy at age 50 were 2.1 years lower among U.S. women compared with the average of the other nine countries examined in this study (5.7 vs. 7.8 years gained, respectively); Preston and colleagues (2010) estimate that smoking accounts for 42 percent of this shortfall. [...] Thus although smoking clearly helps account for the lagging performance of the United States, it is only one of many factors affecting trends in life expectancy."* What is even more surprising is that this gap in health outcomes between the US and other advanced countries has been growing in spite of a comparative surge in the share of American GDP devoted to medical expenditure. Indeed, although health care spending in the US and in Europe were quite similar at the beginning of the 1980s (around 7% of GDP for Europe, 8% for the US), Americans now devote more than 16% of their GDP on medical consumption, compared to roughly 11% on average for Western Europe. Although it is well documented that the cost of medical care is higher in the US than in other industrialized countries, it cannot account for the whole deviation. The US still remains the leader for innovative medical treatments and, as pointed out by Deaton (2013)[2], one of the characteristics of the U.S. health-care system is

[1] p. 82
[2] p. 138

that innovations tend to be introduced very quickly. It is then puzzling that being ahead in terms of medical care is not enough to compensate for the relative poor health of Americans.

This paper proposes an additional factor to those mentioned above that may explain both the higher mortality at mature ages and the higher share of medical expenditures. This global explanation relies on a third well-documented trend: Americans have been working much more than Europeans for quite some time now. At the beginning of the 1980s, Americans and Europeans worked approximately the same number of hours, but today Americans work about as much as they did in the 1980s, whereas Europeans work substantially less (c.f. Blundell, Bozio and Laroque (2011) for empirical evidence). Macroeconomists (Prescott (2004), Blanchard (2004), Ohanian, Raffo and Rogerson (2008)) have paid attention to this fact without yet reaching a consensus about the cause of this divergence, which some view as a result of different cultural preferences, while other favor taxation or labor market institutions. For the sake of convenience, we do not introduce taxes and model differences in hours worked via differences in preferences for leisure.

We introduce health capital a la Grossman in a neoclassical growth model with elastic labor supply to highlight a possible link between hours of work, the share of health care expenditure and mortality rates of workers. Health is therefore viewed as a stock that can be increased via medical investment (usually the consumption of medical commodities), but that depreciates over time. Our crucial assumption is that the rate of depreciation of health capital is a positive function of working time. In other word, working is bad for one's health. Before reviewing below the ample empirical evidence that allow to justify such an assumption, let us mention that a few attempts at estimating a production function of health with leisure time as an input have been made: Sickles and Yazbeck (1998), based on US time series data, find that both medical commodities and leisure time actually make a significantly positive contribution to health, and that leisure might actually contribute more than medical consumption. Recently, Du and Yagihashi (2017) find that the elasticity of substitution between goods and health-enhancing time in producing health may very well be lower than one.

We therefore solve an optimal control problem with two state variables, capital and health capital and three control variables, consumption, labor time and medical expenditure (or health investment). Focusing on the long-run equilibrium, we show that lower preferences for leisure that make individuals work more also lead to a higher steady state share of GDP devoted to health care. This share of medical expenditure increases to offset the extra-depreciation of health capital brought on by a higher number of hours worked. The effect on the long-run health capital stock, however, is ambiguous and depends essentially on the efficiency of the medical technology with which individuals make health investments. We solve analytically the model at the steady state and prove that there is a cut-off point of returns to health investments lying between 0 and 1 below which a higher number of hours worked at the steady state associated with lower preferences for leisure leads to a deterioration of the steady state health capital stock.

We then calibrate the model to replicate some basic facts of the US economy in 2015 including mortality rates of males in the 55 - 64 age group, using the Human Mortality Database.[3] Because the potential effect of long hours of work can only have long-term effects, we focus on the deterioration of health conditions only thirty or thirty-five years after the change of working regime, that is, on health conditions of workers of a certain age, today in 2015. In addition, the focus of our analysis cannot be life expectancy (at birth) but age-specific mortality, with rates defined as the number of deaths in a population of a given age per 100,000 people at risk. Next, we use data on working hours in the UK in 2015 to build a counterfactual where the only changing parameter is the taste for leisure. More specifically, we re-calibrate preferences for leisure so that labor supply in the model economy calibrated to the US matches that of the UK to assess the effect of a reduction of working time in the US on medical expenditure and mortality rates. In the baseline counterfactual, the US works (by construction) the same number of hours as the UK, saves 2.6 percentage points in health expenditure as a share of GDP and experiences 143 deaths per 100,000 people less, that is respectively one half and one quarter of the actual difference with the UK. We choose the UK to build the counterfactual for mainly three reasons. First, the country experienced a fall in hours of work per worker (the intensive margin, contrary to the US) which has been documented by Blundell, Bozio and Laroque (2011), using nationally representative detailed microdata over a long period of time. Second, the UK is a country close to the US in many cultural factors such as eating patterns and obesity prevalence. For instance, the US and the UK are on the same league in terms of obesity, with respectively 28 percent and 25 percent of their adult population suffering from that disease (Case and Deaton 2017). Third, the UK experienced a fall on mortality rates at the end of the working life around age 60 in the last fifteen years. However, our goal is not to explain or decompose the growing gap between the US and the UK in mortality rates for senior peoples as well as the gap in health expenditures. The comparison is purely illustrative. We mainly wish to display

[3]https://mortality.org

the consequences in terms of health expenditure and outcomes for the US economy, would Americans work as much as the British.

Our growth model relies crucially on the assumption that to some extent, working too much is bad for one's health. We follow here the traditional view of modern macroeconomics that a microeconomic phenomenon can have macroeconomic upshots and review below the channels through which hours of work may be related to health conditions. These channels may operate either at the individual level or at the collective level through externalities (congestion and pollution for example), and the consequences of the former would be internalized by an informed and rational decision maker. Many people seem aware of the danger associated to long hours of work as suggested by the European Working Conditions Survey, which reveals that around 30% of workers in the European Union think that their health is at risk because of their work, and the share of employees who agree with this statement increases with the number of hours worked. Indeed, the EU enacted provisions on labor hours in 1993, limiting the work week to a maximum of 48 hours and mandating break time for at least 11 hours a day. Related to that, European working time rules must be kept after Brexit, say medical leaders in a very recent article in the BMJ.[4]

There is also vast empirical evidence that working hours may have a detrimental impact on one's health that gives credit to the quote of Adam Smith in epigraph. This evidence stems from various disciplines such as biomedical sciences and public health. Sparks et al (1997) conduct a meta-analysis of the literature on the length of the work-week and various health symptoms and find small, but significant positive mean correlations between physiological and psychological health symptoms and hours of work. White and Beswick (2003) focus on the relationship between working hours and fatigue, health and safety, and work-life balance and suggest a positive relationship between working hours and fatigue and cardiovascular disorders, and a negative relationship between hours worked and physical health. In addition to that, strong evidence suggests that people perceive that working long hours leads to poor work-life balance, which might be detrimental to mental health. Bannai & Tamakoshi (2014) provide a survey of the literature on long working hours (more than 40 hours a week, or 8 hours a day) and conclude that such long hours are associated with depressive state, anxiety, poor sleep condition and coronary heart diseases. Furthermore, whereas it is well documented that Americans work longer hours than Europeans, it is much less known that they also work at "strange" hours, that is at night and on week

[4] BMJ 2017; 359, doi: https://doi.org/10.1136/bmj.j5917 (Published 21 December 2017)

end (Hamermesh & Stancanelli, 2015), potentially causing more harmful stress. Bassanini & Caroli (2015) argue instead that instead of excess labor supply, it is the lack of control over one's hour of work that is detrimental to workers' health.

There are also empirical evidence on the contribution of leisure to good health: Pressman et al (2009) find a general positive relationship between a wide range of leisure activities (some examples are walking or cycling, exercising, sleeping well, going on holidays, engaging in social interactions, or simply having hobbies) and various health benefits, such as lower blood pressure, waist circumference, body mass index, lower levels of stress and depression, better physical function and mood, better sleep etc.

It should be noted however that this empirical literature may be subject to some identification concerns as an exogenous variation in working hours may be difficult to find. The work of Christopher Ruhm (2000, 2003, 2005) is important here to provide a solution to the lack of identification. Using aggregate data to study the effect of economic recessions on alcohol consumption, Ruhm (1995) finds evidence that alcohol consumption actually declines during economic downturns and increases during expansions. Freeman (1999) confirms Ruhm's findings, and Ruhm and Black (2002), using individual-level data, reach the same conclusion that overall, drinking is pro-cyclical, and suggest that any stress-induced increases in drinking during recessions are more than offset by declines resulting from changes in economic factors such as lower incomes. Ruhm (2005) provides further evidence that changes in lifestyles could be behind improvements in physical health during recessions and shows that smoking and excess weight also decline during short-lasting economic downturns, while physical activity during leisure-time increases. A potential explanation is that the decline in hours of work during a recession increases the non-market time available for lifestyle investments and could be the reason behaviors become healthier. At this stage, it may be important to quote again Cumming, Preston and Cohen (2011) who state[5] *"Thus, differences in the prevalence of obesity continue to explain about 20-35 percent of the shortfall in U.S. life expectancy relative to countries with superior levels."* On physical activity, the authors cannot be affirmative but the prevalence of physical inactivity is the lowest in the US among the panel of countries, 20%, against 5% in Sweden for example[6].

Another way to provide a clear-cut identification strategy is to rely on a control experiment - a case-control study - with a control and treatment group. Sokejima and Kagamimori

[5] p. 55

[6] Figure 4.4 p. 67

(1998) provide results showing a U-shape relationship between the working hours and the risk of acute myocardial infarction. Furthermore, the increase in working hours seems to be an additional factor of risk. It therefore appears that, beyond fatigue and general poorer health conditions, the risk of heart diseases is clearly positively related to long hours of work, and we keep that in mind when we compare mortality rates between countries in the next section. The adverse consequences of working on health may also go through externalities such as pollution or transport accident. Comparing European countries during and before the Great Recession, Tapia Granados and Ionides (2017) found that a one-percentage-point increase in the unemployment rate is associated with a one percent lower mortality rate for respiratory illnesses as well as reductions in mortality for cardiovascular disease and heart conditions, which are known to be sensitive to air pollution. In countries where the recession was more severe - the Baltic States, Spain, Greece and Slovenia - respiratory disease mortality fell by 16 percent during 2007-2010, compared with just a 3.2 percent decline in the four years preceding the recession. Cutler, Huang and Lleras-Muney (2016) found that the strongest link between economic fluctuations and mortality is indeed pollution[7]. As much as two-thirds of the adverse effect of contemporaneous economic fluctuations may be a result of increased pollution. The consequences of externalities would not be internalized, unless Pigouvian instruments are implemented. The model of this paper does not take externalities into account and the model offers no room for an inefficient equilibrium. However, it is important to keep them in mind since mortality rates can be impacted.

The outline of the paper is as follows. The first section provides some empirical evidence for the three stylized facts connected by the model and discuss both alternative and complementary explanations. The second section presents the growth model and solves it at the steady state. The third section presents the calibration. The last section concludes and proofs are gathered in the appendix.

2 Stylized Facts

The aim of this section is to describe the facts between which we draw a causal link in our model. We start with the cause: the differential pattern of working time. We document next the first effect according to our story, the diverging trend in some age-specific mortality rates. Lastly, we focus on the second upshot: the diverging pattern regarding health spending. We start by a global comparison between the US and Western Europe to focus next more specifically on the narrow comparison between the US and the UK, which we use as a

[7] NYT *"How a Healthy Economy Can Shorten Life Spans"*, OCT. 16, 2017

benchmark country in the calibration of our model. Note that by Western Europe, we consider eight countries that seem representative of the region: Austria, Belgium, France, Germany, Italy, the Netherlands, Spain and the UK. What is striking is that the divergence started approximately at the same time, thus fueling the idea that there might be a causal link between the three stylized facts.

2.1 The Overworked American

The United States departed from Western Europe's trend around 1975-1980. The International Labor Organization's report, *Working Time Around the World*, notes that the twentieth century was characterized by a long process of reduction of working time around the developed world. However, it appears that the US put an halt on this process at the end of the 1970s and the beginning of the 1980s and stopped the collective effort to reduce working time, while European countries continued to do so. Today, Americans work about as much as they did in the 1980s while Europeans work substantially less, as illustrated in Figure 3.

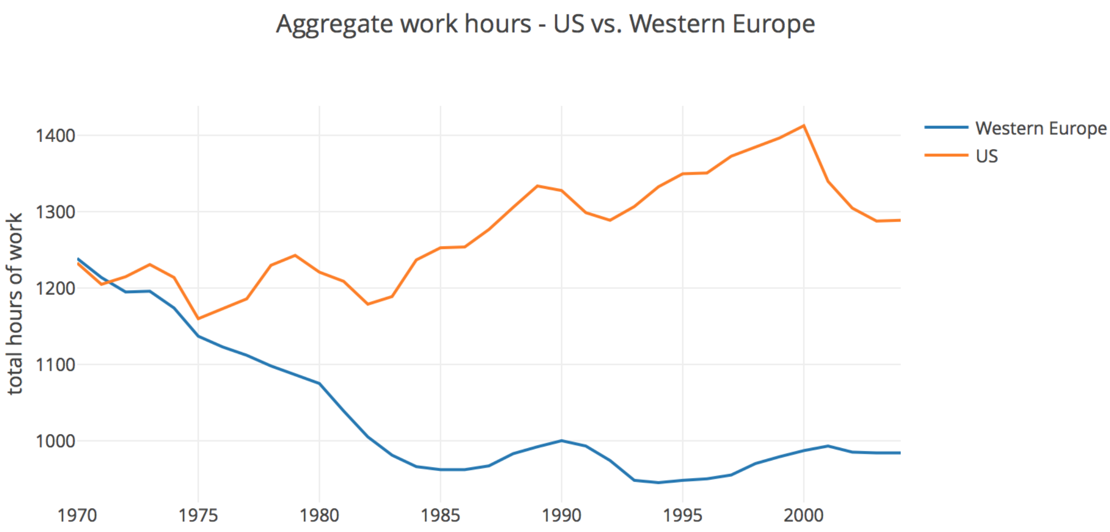
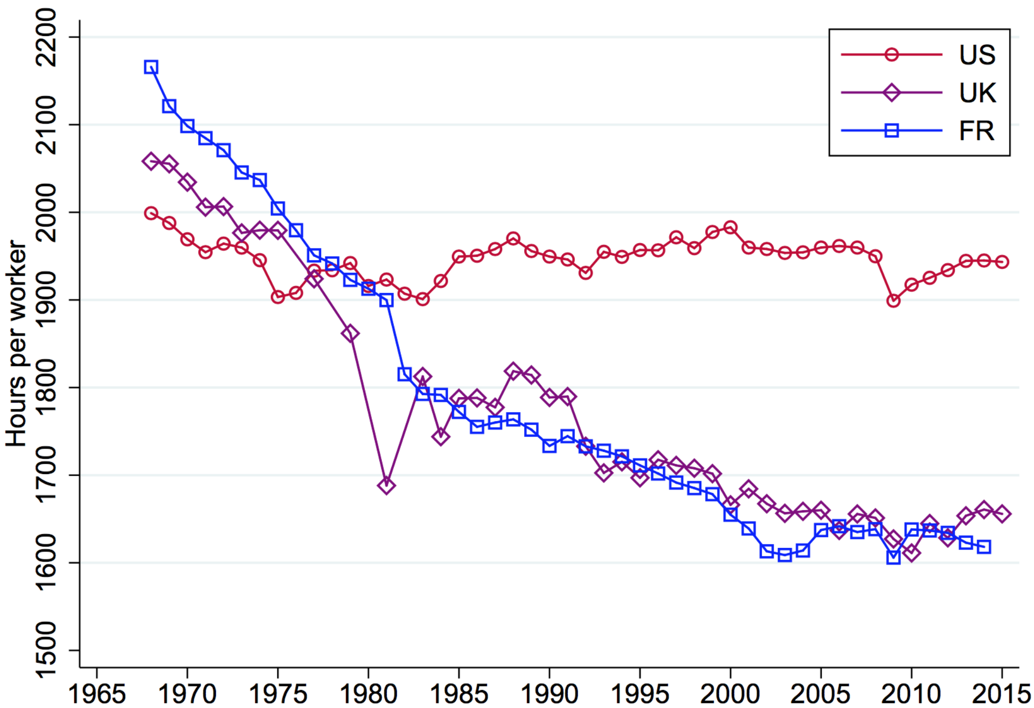


Figure 1. Source: Ohanian, Raffo & Rogerson (2008)

Such differences in aggregate hours could come from differences in hours worked per worker, clearly the relevant variable for individual health, but also from differences in employment rates across countries, or simply from demographics. Blanchard (2004) compares the cases of the US and France and decompose the change in hours worked per capita between 1970 and 2000 into different components: the change in hours worked per worker, the change in the employment rate, the change in the participation rate, and the change in the ratio of the population of working age to total population. It appears the decline in hours worked per capita in France (and by extension, in Western Europe) mostly comes from a decrease in hours worked per worker. Recent work by Bick et al (2016) reaches the same conclusion as a significant part of the difference between the US and Western Europe (but also Scandinavia) is driven by lower weekly hours of work per employed worker.

Focusing on the comparison between the US and the UK, the study by Blundell, Bozio and Laroque (2011) is very useful because, using microeconomic data (Family expenditure survey (FES) and Labour Force Survey (LFS) for the UK and Current Population Survey (CPS) for the US) over 40 years, it confirms Blanchard's result for France and shows that the UK pattern for hours of work is quite similar to that of France, a feature that has been overlooked. Indeed in 2015, in average, the American worker works about 300 hours more than the British worker as can be seen in the figure below, taken directly from Bozio (2017). It is not a small discrepancy and it is quite remarkable that the divergence took place in the late seventies. It can arguably have an impact on the health of workers 30 or 35 years later, that is, around 2010-2015.



Source: Blundell, Bozio & Laroque (2011), extended by Bozio (2017)

There is an ongoing debate on the reasons for the divergence of working hours between the US and Western Europe. On the one hand, Prescott (2004) argues that it is all about taxes: higher taxes in Europe lower the opportunity cost of leisure and make agents willing to work less. Differences in taxes between the US and Europe would therefore explain the whole of the difference in hours worked. However, his analysis has been criticized for relying on unrealistically large values of the micro-elasticity of labour supply. On the other hand, Alesina et al (2005) emphasized the role of trade unions in the reduction of working time in Europe. Furthermore, they argue that large declines of hours worked in unionized sectors may have triggered a reduction in hours worked in other sectors via a social multiplier effect. This idea of a social multiplier is related to the hypothesis that Europeans may have a cultural predilection for leisure. This is argued by Blanchard (2004) who writes that Europeans used productivity gains since the 1980s to increase leisure rather than income, while the US did the opposite. Underlying this argument is an heterogeneity in preferences for leisure across countries, and especially between Western Europe and the US. In line with Prescott, Ohanian, Raffo & Rogerson (2008) revisited this issue by resorting to the neoclassical growth model and argue that differences in taxes on income and expenditure explain much of the variation in hours worked both over time and across countries. However they note that the model tends to underpredict hours in countries with smaller changes in hours, such as the United States. This leaves room for other explanation such a change in preference or composition of the population. Bargain et al (2012) analyze the role of preferences heterogeneity in welfare comparisons of rich countries and their findings, which control for country-specific consumption-leisure preferences, tend to support the view that cultural differences play an important role for welfare. For the sake of convenience and tractability, we do not introduce taxes in the model and by construction, differences in labor supply are explained, ceteris paribus, by differences in preferences.

2.2 The American Health Disadvantage

We begin with by discussing the growing health disadvantage using a widely-used indicator such as life expectancy at birth before focusing on more precise indicators such as age-specific mortality rate and disease-specific mortality rates. Note that even if it is not our focus here, the consequences of long hours of work can be also seen on youth health, through less child-care, or on old health by less tending by relatives.

The improvement of life expectancy has slowed down in the US starting from 1980, as can be clearly seen in Figure 2. In 1980, average life expectancy at birth in the United States was 77.5 years for women and 70 for men, approximately the same as the average for Western Europe, around 74 years for the whole population. There has been considerable progress since then and life expectancy at birth for the US population is now 79 years. However, such progress needs to be put in perspective as life expectancy in Western Europe increased much faster and now reaches roughly 82 years. This slow-down of US life expectancy improvement relative to other rich countries is consistent for both men and women, although it is more pronounced for women.

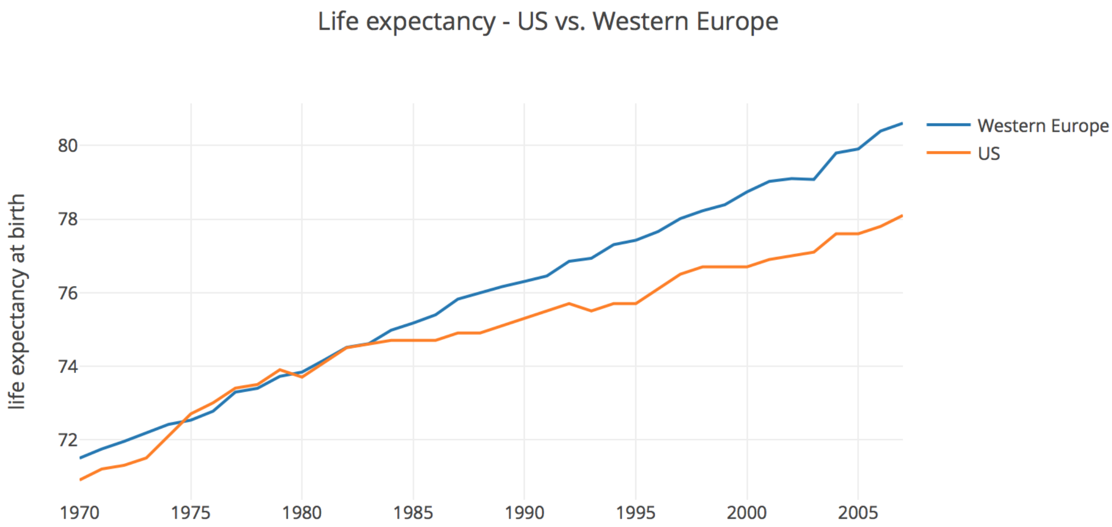


Figure 2. Source: OECD

Various measures of self-reported health status and biological markers of disease confirm the rough picture painted by aggregate life expectancy. Americans across the socioeconomic distribution report a higher disease burden: 30% higher for lung disease and myocardial infarction, 60% higher for all heart disease and stroke, and twice as high for diabetes (Banks et al, 2006). Furthermore, the disadvantage is pervasive and affects all groups up to age 75 for multiple diseases, biological and behavioral risk factors, and injuries. More specifically, the US fares worse than its counterparts in nine health domains: adverse birth outcomes, injuries and homicides, adolescent pregnancy and STDs, HIV and AIDS, drug-related mortality, obesity and diabetes, heart diseases, chronic lung diseases, disability, and overall, Americans who reach age 50 are in poorer health due to several risk factors such as smoking, obesity, diabetes (National Research Council and Institute of Medicine, 2013).

The first potential explanation that comes to mind is that the surge in income and wealth inequality observed in the US since the early 1980s was accompanied by an increase in health inequality that drove down the national average. However, it appears that the US health disadvantage relative to peer countries persists even when the US data are limited to non-Hispanic whites or upper-income populations. The disadvantage, although more pronounced among lower socio-economic groups that often lack health insurance, is pervasive and is still present among higher socio-economic groups (Martinson et al, 2011a; Avendano et al., 2009, 2010). Americans who are white, relatively wealthy and insured, are still in poorer health than their Europeans counterparts. Furthermore, despite the lack of access, the US health care system is actually quite performing: it makes use of the most advanced medical techniques and has the highest survival rates for many cancers for example. Inequality is of course an important part of the story, but such a pervasive health disadvantage in the US rules it out as the sole explanation.

We now focus on more specific data regarding mortality rates for specific age group, using the Human Mortality Database. Figure 3 displays the ratio of mortality rates UK/US for each age between 40 and 80 for three periods, at the beginning of the divergence between Britain and the States 1975-1979. Several observations are in order. First, at 40, the relative US disadvantage was already present forty years ago and if any, it is nowadays a little bit reduced. Second, twenty years later, in 1995-1999, the gap has worsen by a constant whatever the age. Third, for the more recent period, 2010-2014, the situation deteriorates considerably for the age group 45-60. The deaths of despair plays obviously a role here but it is interesting that the slope of the gap seems to widen at older age. We just notice that these American workers have worked 300 hours more than the British workers every year during 30-35 years and therefore argue that the potential negative effect of long working hours on health should appear already. When calibrating the model to the data, we therefore choose as a health indicator mortality rates of males aged 55-64 in 2015, to assess whether our theory can account for the discrepancy between the US and the UK.

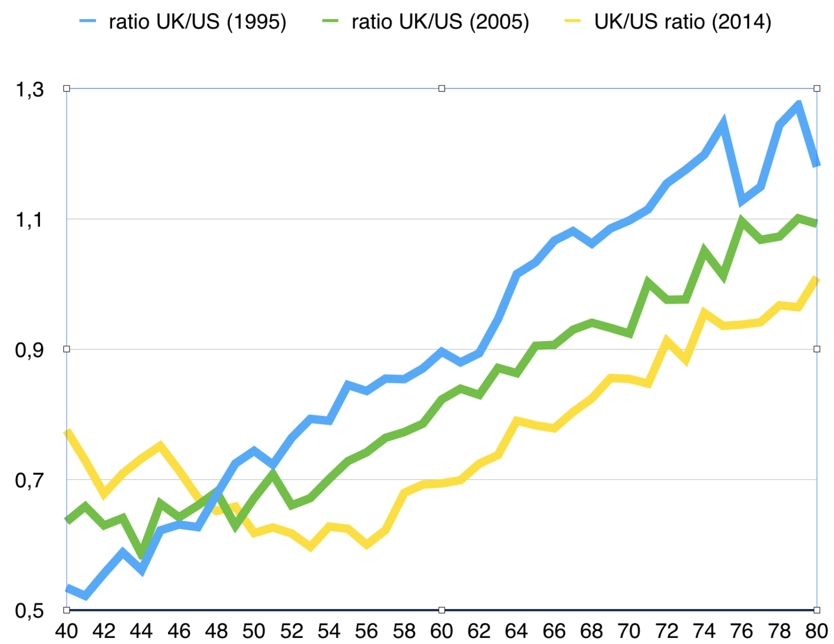
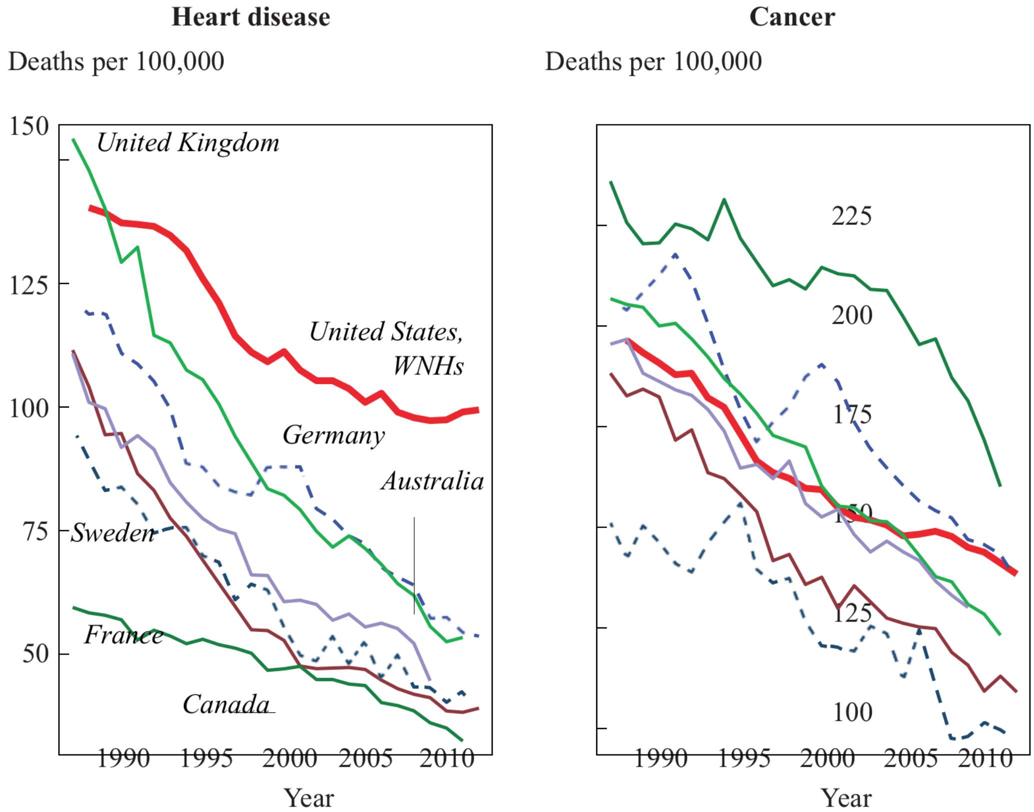


Figure 3: Mortality rates by age (UK/US). Source: HMD

Finally, we look at the disease and age-specific mortality rates gathered by Case and Deaton (2017) that we report here for the sake of completeness. It is clear that for cancer, the US are in the middle of the pack in terms of trend, which is absolutely not the case for heart disease. Recall that according to medical studies, long hours of work may be an additional factor of risk for cardiovascular diseases. In particular, the difference of patterns with the UK is stringent despite obesity-prevalence rates quite similar. The pace of the decline slowed from 2000 onwards and then completely stopped from 2010. Case and Deaton (2017) questions a possible statistical bias due to underreporting of deaths attributed to diabetes. Because, there might be a bias, we choose not to calibrate the model on these data.



Heart Disease and Cancer Mortality by Country for Age 50?54, 1989?2014.

Source: Case & Deaton (2017)

2.3 The Rise in Health Expenditure

At the beginning of the 1980s, Americans already devoted a larger fraction of their resources on health care than European, but the difference was of around one percentage point. The gap has been growing ever since and is now close to six percentage points.

There are several potential explanations for the rise in health expenditure in rich countries. On the one hand, focusing on the demand side, Hall & Jones (2007) argue that the increase in medical spending that comes along with development is optimal and results from the fact that health is a superior good. They build a model where the marginal utility of consumption diminishes faster than that of life extension, which makes people devote a larger fraction of their income of health care as the economy gets richer. Their model even makes the bold prediction that the share of health expenditure in the US could very well reach 30% of GDP by the middle of the century. On the other hand, the supply side explanation relies on technological change: Newhouse (1992) argues that the invention of new and expensive medical technologies causes health spending to rise. According to Cutler (1995), technology accounts for 49% of the growth in real health care spending per capita between 1940 and 1990. Suen (2005) builds a model where medical innovations raise the marginal product of medical care in producing health. The upshots are an increase of both the duration of life and per-period health expenditure. However, although both explanations justify an increasing share of health spending along economic development, they fail to account for cross-country differences, and especially for why the US is spending so much more on health care that its Europeans counterparts, for such mixed results.

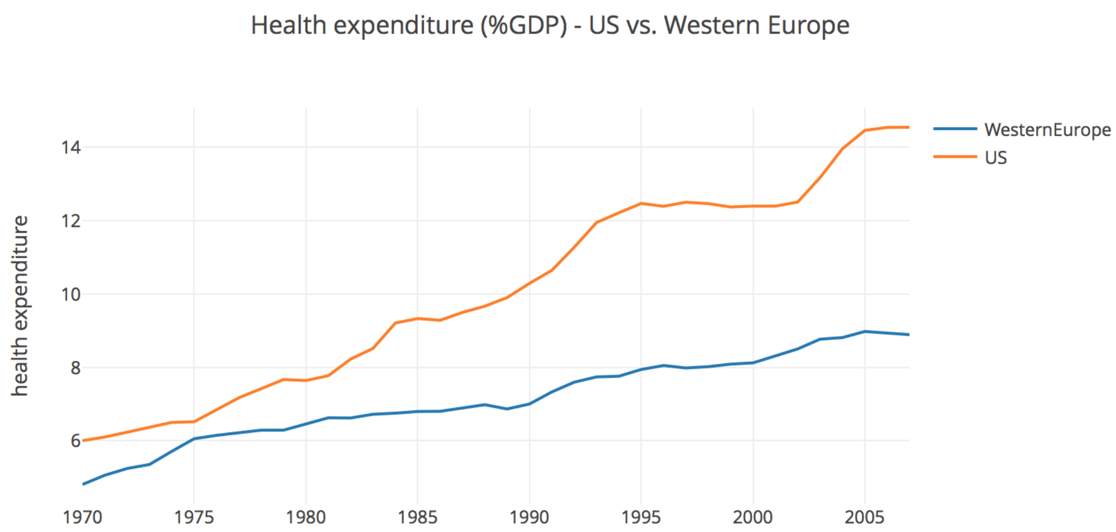


Figure 3. Source: OECD

Another explanation for the growing difference in health spending between the US and European countries is the difference in the prices of health care goods and services. According to OECD studies (c.f. OECD, 2011; Lorenzoni, Belloni and Sassi, 2014) that compute the relative price of health care for the US and five countries (Canada, Germany, France, the Netherlands, and Switzerland), the average health expenditure using general PPP deflator of these 5 countries is only 54% of the US. When we correct for the relative price of health care, this share amounts to 67%, meaning that the relative price of health care is 24% higher in the US than in the average of these five countries, a value used in the calibration. Note that the difference in the relative health care price explains almost 30% of the expenditure gap, meaning that according to this study, the remaining unexplained part reflects higher use of health care and difference in quality. He & Huang (2013) propose an explanation close to ours as to why Americans spend so much more: if one adopts a portfolio view of health investment where both medical commodities and health-enhancing leisure time serve as inputs in production of health, then the greater labor supply of Americans may account for some of the extra-spending on health care. They also highlight the role of the relative price of health care on labor supply decision, but they do not account for the effect of labor supply on health status itself.

In the next section, we are building a model of a representative-agent economy that allows us to discuss the potential inter- and intra-temporal trade-offs between health and working time.

3 The Model

We construct a standard neoclassical Ramsey growth model with infinitely-lived identical individuals and endogenous labor supply and add a second type of capital: health capital. Time is continuous and for the sake of simplicity, we assume no population growth. We begin by describing the supply side of the economy with firms, before turning to households' behavior.

3.1 Firms

Firms produce the sole final good of the economy that can be either used for consumption, medical care or saved as investment. Let us consider the following simple Cobb-Douglas production function, common to each firm according to which output $Y(t)$ is produced:

$$Y(t) = A(t)K(t)^{\alpha}[l(t)N(t)]^{1-\alpha} \tag{1}$$

Where $K(t)$ is the stock of capital, $l(t)$ is individual labor supply and $N(t)$ is the number of workers. $A(t)$ is total factor productivity. We remain in an exogenous growth framework and are not interested in the effect of technological progress, we therefore assume $A(t)$ is constant and normalize it to one for simplicity. The function can be rewritten in per capita terms:

$$\frac{Y(t)}{N(t)} = y(t) = k(t)^{\alpha} l(t)^{1-\alpha} \tag{2}$$

Let us denote the rental rate of capital as $R(t)$. We assume the capital stock depreciates at the constant rate $\delta > 0$, therefore the net rate of return to an individual that owns a unit of capital is $R(t) - \delta$. Since households can also receive interests at rate $r(t)$ on funds lent to other households, and since both loans and capital are perfect substitutes as stores of values, we have that $r(t) = R(t) - \delta \Leftrightarrow R(t) = r(t) + \delta$. The firm therefore chooses capital and labor inputs, taking $w(t)$ and $r(t)$ as given, to maximize profit $\pi(t)$:

$$\pi(t) = K(t)^{\alpha}[l(t)N(t)]^{1-\alpha} - [r(t) + \delta]K(t) - w(t)[l(t)N(t)] \tag{3}$$

The familiar first order conditions arise naturally:

$$r(t) = \alpha k(t)^{\alpha-1} l(t)^{1-\alpha} - \delta \tag{4}$$

$$w(t) = (1-\alpha) k(t)^{\alpha} l(t)^{-\alpha} \tag{5}$$

3.2 Households

Individuals derive utility from consumption of the final good $c(t)$ and their current health status $h(t)$ which corresponds to their stock of health capital at time t, and derive disutility from individual labor $0 \leq l(t) \leq 1$ they supply each period. Notice that individuals are infinitely-lived so that there is no mortality in the model. Health therefore influences households behavior only via its impact on their utility only. We consider a simple period log-utility function with constant marginal disutility of labor of the following form:

$$u[c(t), h(t), l(t)] = \nu \log[c(t)] + (1-\nu)\log[h(t)] - \phi \cdot l(t) \tag{6}$$

where ν is the relative taste for consumption (hence $(1-\nu)$ is the relative taste for health) and ϕ can be interpreted as preferences for leisure. Individuals therefore seek to maximize overall utility U:

$$U = \int_0^{\infty} u[c(t), h(t), l(t)] e^{-\rho t} dt \tag{7}$$

where $\rho > 0$ is the rate of time preference.

Individuals hold assets which may take the form of ownership claims on capital or as loans. The two forms of assets are assumed to be perfect substitutes as stores of value so they must pay the same real rate of return $r(t)$. We denote assets per person as $a(t)$. Individuals are competitive and take as given the interest rate $r(t)$ and the wage rate $w(t)$, paid per unit of labor services. The total income received by each individual is therefore the sum of labor income, $w(t)l(t)$, and asset income, $r(t)a(t)$. They use it to consume and purchase medical care $m(t)$ at the relative exogenous price p, and use the rest to accumulate more assets. The individual budget constraint therefore takes the following form:

$$\dot{a}(t) = w(t)l(t) + r(t)a(t) - c(t) - p \cdot m(t) \tag{8}$$

We now introduce another type of capital from which agents derive direct utility: health capital. We loosely follow Grossman (1972) who conceptualized the idea that health could be viewed as a stock of capital that can be increased via investment in medical care but which also depreciates along the lifecycle. Our main assumption is that the rate of depreciation of health capital δ_h is a positive function of individual labor supply $l(t)$, or work effort.

$$\dot{h}(t) = m(t)^\sigma - \delta_h[l(t)]h(t) \tag{9}$$

Where σ characterizes the returns to scale of the medical technology - or the efficiency of health investments - and is assumed to be lower than one. Medical expenditure are therefore subject to diminishing returns, as seems to be the case empirically. We also assume that the relationship between the rate of depreciation of health capital and individual work effort is convex, as the damage of hours of work on health may become more severe as individuals work more. We therefore define $\delta_h[l(t)] = z \cdot l(t)^\gamma$ where $\gamma > 1$ and z is a scaling parameter.

Looking further at this equation, we can find the general solution for the health capital stock as a function of medical expenditure and labor supply:

$$h(t) = h_0 e^{-L(t)} + \int_0^t m(s)^\sigma e^{-L(s)} ds$$

where $L(t) = \frac{z}{\gamma+1} \int_0^t l(s)^{1+\gamma} ds$ and $L(s) = \frac{z}{\gamma+1} \int_s^t l(\tau)^{1+\gamma} d\tau$. Individuals therefore start with an initial stock of health capital h_0, which remains constant over time in the absence of both work and health expenditure. If individuals supply some labor and do not make medical investments, their health stock will progressively tend to zero. On the other hand, health expenditure increase the health capital stock and counteract the negative effect of labor supply. However, the effect of medical expenditure in period s is discounted by the amount of work between periods s and t. Hence, hours worked in the past have an effect on health expenditure ever since.

The households' problem is therefore to choose consumption, medical expenditure and labor supply (three control variables) and assets and health capital (two state variables) to maximize lifetime utility (2), subject to both constraints (3) and (4). We set up the following present-value Hamiltonian:

$$\mathcal{H} = u[c(t), h(t), l(t)]e^{-\rho t} + \lambda(t)[w(t)l(t) + r(t)a(t) - c(t) - p \cdot m(t)] + \mu(t)[m(t)^{\sigma} - z \cdot l(t)^{\gamma} h(t)]$$

The first-order conditions are as follow:

$$\frac{\partial \mathcal{H}}{\partial c(t)} = 0 \quad \Leftrightarrow \quad u_c(.)e^{-\rho t} = \lambda(t) \tag{10}$$

$$\frac{\partial \mathcal{H}}{\partial m(t)} = 0 \quad \Leftrightarrow \quad \sigma m(t)^{\sigma-1}\mu(t) = p\lambda(t) \tag{11}$$

$$\frac{\partial \mathcal{H}}{\partial l(t)} = 0 \quad \Leftrightarrow \quad -u_l(.)e^{-\rho t} = \lambda(t)w(t) - \mu(t)\gamma \cdot z \cdot l(t)^{\gamma-1}h(t) \tag{12}$$

$$\dot{\lambda}(t) = -\frac{\partial \mathcal{H}}{\partial a(t)} \quad \Leftrightarrow \quad \dot{\lambda}(t) = -r(t)\lambda(t) \tag{13}$$

$$\dot{\mu}(t) = -\frac{\partial \mathcal{H}}{\partial h(t)} \quad \Leftrightarrow \quad \dot{\mu}(t) = -u_h(.)e^{-\rho t} + \mu(t)z \cdot l(t)^{\gamma} \tag{14}$$

Since there are two state variables, $a(t)$ and $h(t)$, there are two additional transversality conditions:

$$\lim_{t \to \infty} [\lambda(t)a(t)] = 0 \tag{15}$$

$$\lim_{t \to \infty} [\mu(t)h(t)] = 0 \tag{16}$$

Equations (10) and (13) combined yields the familiar Euler equation:

$$\frac{\dot{c(t)}}{c(t)} = r(t) - \rho \tag{17}$$

which tells us individuals are willing to postpone consumption from the present to the future if and only if the real interest rate is greater than the rate of time discount.

Equation (12) characterizes the labor-leisure choice: individuals must be indifferent between one additional unit of leisure or work. The left-hand side is the marginal utility of leisure and must be equal to the marginal utility brought by an additional unit of work on the right hand side. The net gains from an additional unit of work is the extra earnings one gets minus the greater depreciation induced on health. Using equations (10) and (11) to get rid of both shadow prices $\mu(t)$ and $\lambda(t)$, we get:

$$-\frac{u_l(t)}{u_c(t)} = w(t) - \gamma\frac{p}{\sigma}m(t)^{1-\sigma}z \cdot l(t)^{\gamma-1}h(t) \tag{18}$$

where the left hand side is now the marginal rate of substitution between leisure and consumption, or how much one values leisure in terms of consumption. In standard models, it is just equal to the wage rate $w(t)$, but here since individual labor supply has a negative effect on one's health, the extra depreciation brought by an additional unit of labor lowers the net benefits of working.

Equation (11) indicates that households equate the marginal benefit of medical expenditure $m(t)$ to the shadow price of capital. This marginal benefit is the marginal product of medical expenditure in health investment times the shadow price of health capital $\mu(t)$ and is subject to diminishing returns. Differentiating with respect to time and using equation (13), we obtain:

$$\frac{\dot{m(t)}}{m(t)} = \frac{1}{1-\sigma}\left[r(t) + \frac{\dot{\mu(t)}}{\mu(t)}\right]$$

Now using equation (14) to substitute for $\frac{\dot{\mu(t)}}{\mu(t)}$, we get:

$$\frac{\dot{m(t)}}{m(t)} = \frac{1}{1-\sigma}\left[r(t) + z \cdot l(t)^{\gamma} - \frac{\sigma}{p}m(t)^{\sigma-1}\frac{u_h(t)}{u_c(t)}\right] \tag{19}$$

where $\frac{u_h(.)}{u_c(.)}$ is the marginal rate of substitution between health and consumption, or the value of an additional unit of health in terms of consumption. The growth rate of medical expenditure therefore increases with the real interest rate because it makes individuals better off if they save one dollar to spend it on health care tomorrow. The opportunity cost of purchasing health care today increases with the real interest rate. The depreciation rate of health capital $z \cdot l(t)^{\gamma}$ also increases the growth rate of medical expenditure because the fraction of health capital that depreciates will have to be offset next period. Finally, the growth rate of health expenditure decreases with the value of an additional dollar spent on health care because the higher this value, the more willing are individuals to make the investment in the present period.

The transversality condition for assets arises naturally from the equilibrium conditions. Take the differential equation (13) and integrate to get:

$$\lambda(t) = \lambda(0)\exp\left(-\int_0^t r(v)dv\right) \tag{20}$$

And plugging that back into (15) yields:

$$\lim_{t\to\infty}\left[a(t)\exp\left(-\int_0^t r(v)dv\right)\right]=0 \tag{21}$$

3.3 Equilibrium

We can now combine the behavior of households and firms who make optimizing choices taking the real interest rate $r(t)$ and the wage rate $w(t)$ as given to study the competitive market equilibrium. First, note that the economy is closed so all debts must cancel at each period and assets per person $a(t)$ are just equal to the capital stock per worker $k(t)$. Now, if we take households' budget constraint (8), replace $a(t)$ by $k(t)$ and substitute for the values of $r(t)$ and $w(t)$ given by equations (4) and (5), we obtain the resource constraint of the economy:

$$\dot{k(t)}=y(t)-c(t)-p\cdot m(t)-\delta k(t) \tag{22}$$

This, together with households' optimization conditions (the Euler equation (17), the health capital accumulation equation (9), the equation for medical expenditure (19) and the trade-off between leisure and consumption (18) plus the two transversality conditions (11) and (12), the firms' first order conditions (15) and (16) and market clearing conditions characterize the economy's equilibrium.

3.4 Steady State

We now turn to the existence of a steady state: a particular solution of the equilibrium where labor supply $l(t)$ is constant and the other variables of the economy grow at a constant rate (possibly zero, especially in the absence of technological progress). First, let us denote by $g_i^\star$ the steady state growth rate of variable i. Taking the Euler equation (17) and differentiating with respect to time while assuming that $g_c^\star=(\dot{c}/c)^\star$ is constant, we see that the real interest rate $r^\star$ must also be constant at the steady state. Looking at the first order condition for the firms, this yields $g_k^\star=g_l^\star$. Since at the steady state, labor supply is constant, this means that $g_k^\star=g_l^\star=0$. Now, differentiate the health capital accumulation equation with respect to time while assuming $\dot{h}/h$ is constant at the steady state to get $g_h^\star=\sigma g_m^\star$. Now do the same with equation (19) for medical expenditure and substitute for the previous result to get $g_c^\star=g_m^\star$. Finally, differentiate equation (18) describing the trade off between leisure and consumption and make again use of the previous results to obtain:

$$\frac{\phi}{\nu}c^\star g_c^\star=-\gamma\frac{p}{\sigma}h^\star {m^\star}^{1-\sigma}z{l^\star}^{\gamma-1}g_c^\star$$

Since $\frac{\phi}{\nu}c^\star \neq -\gamma\frac{p}{\sigma}h^\star {m^\star}^{1-\sigma} z{l^\star}^{\gamma-1}$ from equation (16) unless the wage rate $w^\star$ is null, a possibility we rule out, we get that the only steady state that exists is characterized by:

$$g_y^\star = g_k^\star = g_h^\star = g_c^\star = g_m^\star = g_l^\star = 0$$

Therefore, the steady state solution can be found by setting $\dot{k} = \dot{c} = \dot{m} = \dot{h} = 0$. This together with the leisure-consumption trade off gives us the following system of five equations and five unknowns:

$$\begin{aligned}
\dot{k} &= k^\alpha l^{1-\alpha} - c - p \cdot m - \delta k = 0 &(23)\\
\dot{h} &= m^\sigma - z \cdot l^\gamma \cdot h = 0 &(24)\\
\dot{c} &= \alpha k^{\alpha-1} l^{1-\alpha} - \delta - \rho = 0 &(25)\\
\dot{m} &= \frac{1}{1-\sigma}\left[\alpha k^{\alpha-1} l^{1-\alpha} - \delta + z \cdot l^\gamma - \frac{\sigma}{p} m^{\sigma-1}\left(\frac{1-\nu}{\nu}\right)\left(\frac{c}{h}\right)\right] m = 0 &(26)\\
\frac{\phi}{\nu}c &= (1-\alpha)k^\alpha l^{-\alpha} - \gamma\left(\frac{p}{\sigma}\right) m^{1-\sigma} z \cdot l^{\gamma-1} h &(27)
\end{aligned}$$

First, the Euler equation gives us the steady state (per capita) capital-to-labor ratio:

$$\left(\frac{k}{l}\right)^\star = \left(\frac{\alpha}{\delta+\rho}\right)^{\frac{1}{1-\alpha}}$$

This gives us output as a function of labor supply at the steady state:

$$y^\star = \left(\frac{\alpha}{\delta+\rho}\right)^{\frac{\alpha}{1-\alpha}} l^\star \tag{28}$$

Plus the capital-to-labor ratio in the capital accumulation equation to get:

$$\left(\frac{\alpha}{\delta+\rho}\right)^{\frac{\alpha}{1-\alpha}}\left[\frac{(1-\alpha)\delta+\rho}{\delta+\rho}\right] l^\star = c^\star + p \cdot m^\star \tag{29}$$

Turning to equation (26) and substituting for the real interest rate:

$$\frac{\sigma}{p}\left(\frac{1-\nu}{\nu}\right)\left(\frac{c}{h}\right)^\star (m^\star)^{\sigma-1} = \rho + z \cdot {l^\star}^\gamma \tag{30}$$

Substituting for the steady state health capital stock $h^\star = {m^\star}^\sigma/(z \cdot {l^\star}^\gamma)$ yields:

$$\left(\frac{c}{m}\right)^\star = \frac{p}{\sigma}\left(\frac{\nu}{1-\nu}\right)\left[1 + \frac{\rho}{z \cdot {l^\star}^\gamma}\right] \tag{31}$$

This equation tells us that the consumption to medical expenditure ratio is a decreasing function of labor supply at the steady state: the more agents work, the larger fraction of their resources they devote to health care. Now, we can use (29) and (31) to get expressions for consumption and medical expenditure in terms of labor supply:

$$c^\star = \left[\frac{(1-\alpha)\delta+\rho}{\rho+\delta}\left(\frac{\alpha}{\delta+\rho}\right)^{\frac{\alpha}{1-\alpha}}\right]\frac{\nu(z\cdot l^{\star^\gamma}+\rho)l^\star}{z[\sigma(1-\nu)+\nu]l^{\star^\gamma}+\nu\rho} \tag{32}$$

$$m^\star = \frac{1}{p}\left[\frac{(1-\alpha)\delta+\rho}{\rho+\delta}\left(\frac{\alpha}{\delta+\rho}\right)^{\frac{\alpha}{1-\alpha}}\right]\frac{\sigma(1-\nu)z\cdot l^{\star^{1+\gamma}}}{z[\sigma(1-\nu)+\nu]l^{\star^\gamma}+\nu\rho} \tag{33}$$

Now that we have an expression for medical expenditure as a function of labor supply, we can express the steady state health capital stock as follows:

$$h^\star = \frac{1}{z}\left[\left(\frac{1}{p}\right)\frac{(1-\alpha)\delta+\rho}{\rho+\delta}\left(\frac{\alpha}{\delta+\rho}\right)^{\frac{\alpha}{1-\alpha}}\right]^\sigma\left[\frac{\sigma(1-\nu)}{z[\sigma(1-\nu)+\nu]l^{\star^\gamma}+\nu\rho}\right]^\sigma l^{\star^{(1+\gamma)\sigma-\gamma}} \tag{34}$$

Finally, the last variable we are interested in is the share of income that is devoted to health care expenditure:

$$\left(\frac{p\cdot m}{y}\right)^\star = \left[\frac{(1-\alpha)\delta+\rho}{\rho+\delta}\right]\frac{\sigma(1-\nu)z\cdot l^{\star^\gamma}}{z[\sigma(1-\nu)+\nu]l^{\star^\gamma}+\nu\rho} \tag{35}$$

We now have the full solution expressed in terms of individual labor supply. It will be useful in the next sub-section as we intend to see how differences in hours worked affect a country's health capital stock and health expenditure. We can now plug the values of $c^\star$ and $m^\star$ given by equations (32) and (33) into equation (31) to solve for $l^\star$. This gives us the following equation:

$$\phi(l^\star)^{1+\gamma}+[(1-\nu)(\gamma-\sigma\chi)-\nu\chi](l^\star)^\gamma+\phi\frac{\rho}{z}(l^\star)-\nu\chi\frac{\rho}{z}=0 \tag{36}$$

where $\chi = \frac{(1-\alpha)(\delta+\rho)}{(1-\alpha)\delta+\rho} < 1$.

However, one cannot find an analytical solution for $l^\star$ to this equation. We therefore assume it is unique and use the implicit function theorem to study how labor supply vary with the exogenous parameters of the model, especially with preferences for leisure. We show later in the paper that for $\gamma = 2$, the specification we use for calibration, there is indeed a unique real solution to this equation.

3.5 Comparative Statics

The aim of this paper is to study how the steady state health capital stock and the share of health care expenditure vary with labor supply. Previously, we argued that differences in hours worked across countries might be the result of differences in preferences for leisure. We therefore study how preferences for leisure affect individuals' labor supply decision, and we then investigate the effects of differences in hours worked on the steady state health capital stock and share of medical expenditure.

3.5.1 Labor Supply

The issue we want to address is the following: how do two identical economies with various preferences vis-a-vis leisure will differ? As can be seen from the analytical expressions of the different variables of the model, preferences for leisure ϕ appear directly in that of labor supply only, and indirectly in those of other variables through labor supply. We therefore investigate how labor supply varies with such preferences and we then study how the resulting differences in labor supply will affect the whole economy.

Claim: *Lower preferences for leisure lead to a higher number of hours worked.*

We can then turn to the other variables of interest, namely the steady state health capital stock and the share of GDP devoted to health care.

3.5.2 Medical Expenditure as a Share of GDP

Turning to the fraction of total resources that are devoted to health care, we immediately see from equation (35) that this share is a function of preferences for leisure solely from labor supply. How does this share vary with labor supply then?

$$\frac{\partial(p \cdot m/y)^\star}{\partial l(.)^\star} = \left[\frac{(1-\alpha)\delta+\rho}{\rho+\delta}\left(\frac{\alpha}{\delta+\rho}\right)^{\frac{\alpha}{1-\alpha}}\right] \frac{\gamma\sigma(1-\nu)\nu\rho z l(.)^{\star^{\gamma-1}}}{([\sigma(1-\nu)+\nu]zl(.)^{\star^{\gamma}}+\nu\rho)^2} > 0$$

Proposition 1 *Medical expenditure as a share of GDP increases with the number of hours worked.*

Any additional hours of work (and hence any additional unit of labor income an agent gets) can be used to increase both consumption and medical spending, but also increases the rate of depreciation of the health capital stock. A higher number of hours worked therefore raises the opportunity cost of health investments, and households' optimization implies that the marginal rate of substitution of health capital in terms of consumption should also increase: household should shift increase their consumption relative to their health. However, at the steady state, health capital stock is directly negatively affected by a higher work effort, which increases the marginal utility of health capital and hence gives incentives to agents to increase medical investment relative to consumption. In other words, individuals use the proceeds of their extra labor income to increase both consumption and health care expenditure, but because they have to offset the extra depreciation of their health capital, they increase medical spending more than consumption.

3.5.3 Health Capital Stock

We now turn to analyzing the response of the health capital stock following a change in individual labor supply.

$$\frac{\partial h^\star}{\partial l(.)^\star} = B\left[\frac{l^{\star(1+\gamma)(\sigma-1)}}{([\sigma(1-\nu)+v]zl^{\star\gamma}+\nu\rho)^2}\right]\left[(1+\gamma)\sigma-\gamma-\sigma\gamma\frac{[\sigma(1-\nu)+\nu]zl^{\star\gamma}}{[\sigma(1-\nu)+\nu]zl^{\star\gamma}+\nu\rho}\right]$$

Where $B=\gamma\left[\frac{\sigma(1-\nu)}{p}\cdot\frac{(1-\alpha)\delta+\rho}{\rho+\delta}\left(\frac{\alpha}{\delta+\rho}\right)^{\frac{\alpha}{1-\alpha}}\right]^\sigma$.

The sign of this derivative is ambiguous. Specifically, the sign of the derivative is just the sign of $\left[(1+\gamma)\sigma-\gamma-\sigma\gamma\frac{[\sigma(1-\nu)+\nu]zl^{\star\gamma}}{[\sigma(1-\nu)+\nu]zl^{\star\gamma}+\nu\rho}\right]$. It is easy to show that at the steady state, the health capital stock is a non-monotonous function of labor supply and that there is a unique threshold $\bar{l}$ below which the health capital stock increases with labor supply and above which it starts to decrease.

Lemma 1 *If* $\frac{\gamma}{1+\gamma}<\sigma<\left(\frac{\gamma}{1+\gamma}\right)\left(1+\frac{z}{\nu\rho}\right)$*, there exists a unique* $l=\bar{l}\in]0;1[$ *that maximizes the steady state health capital stock.*

Such a threshold $\bar{l}$ is therefore the unique number of hours worked that maximizes the health capital stock, and its value depends crucially on the returns to health investments σ. When the returns to health investments are low such that $\sigma<\underline{\sigma}=\frac{\gamma}{1+\gamma}$, the health capital stock always decreases with labor supply at the steady state. On the other hand, when the returns to health investments are above an upper threshold value, the steady state health capital stock is an increasing function of labor supply. It is easy to show that under some condition, such a threshold is greater than one. More specifically, if one assumes $\gamma>1+\frac{\nu\rho}{z}$, then $\sigma_2>1$ (as will be the case for the calibrated model). As a result, under decreasing returns to scale, the steady state health capital stock will never be a strictly increasing function of labor supply. Therefore, if $\sigma\in]\underline{\sigma};1]$, then at the steady state the health capital stock is a non-monotonous function of labor supply: it first increases for low values of $l^\star$ because the additional labor income is more than enough to offset the extra work-related depreciation, but it starts to decline at some point because of the convex increase of the depreciation rate and the diminishing returns to medical care.

The question that arises then is, if $\bar{l} \in]0;1[$ when is the actual number of hours worked at the steady state $l^\star$ below or above the threshold? This ultimately determines whether a local variation in labor supply at the steady state leads to an increase or a decrease of the health capital stock. It is possible to show that it all depends on the value of the returns to health investment since both the threshold $\bar{l}$ and steady state labor supply $l^\star$ are functions of σ.

Proposition 2 *Under decreasing returns to scale in health investments, $\sigma \in]0;1[$:*

- *if $\lim_{\sigma \to 1} l^\star < \lim_{\sigma \to 1} \bar{l}$, then there exists a unique $\sigma^\star \in]0;1[$ for which $\partial h^\star / \partial l^\star = 0$, below which $\partial h^\star / \partial l^\star < 0$ and above which $\partial h^\star / \partial l^\star > 0$*
- *if $\lim_{\sigma \to 1} l^\star > \lim_{\sigma \to 1} \bar{l}$ then $\partial h^\star / \partial l^\star < 0$*

Since we cannot obtain an explicit expression for labor supply at the steady state, we have to consider both cases where $\lim_{\sigma \to 1} l^\star < \lim_{\sigma \to 1} \bar{l}$ and $\lim_{\sigma \to 1} l^\star > \lim_{\sigma \to 1} \bar{l}$. In the calibrated model, we will see that the former will prevail so that at the steady state, the relation between the health capital stock and labor supply is indeed non-monotonic. The impact of a variation of labor supply on health will therefore depend on the returns to health investment: with a greater number of hours worked, individuals will use the extra labor income to increase both their consumption and medical expenditure but for low values of σ, an additional dollar spent on health care yields gains that are insufficient to offset the extra depreciation of the health capital stock. On the contrary, provided the returns to health investments are high enough, a greater number of hours worked allows individuals to more than offset the work-induced depreciation and as a result, increase their steady state health capital stock.

4 Calibration

We now calibrate the model to the US economy in order to investigate the effect of a potential reduction of the number of hours worked on the share of health care expenditure and the health capital stock, at the steady state. Some parameters of the model are calibrated as is standard in the literature. We take the rate of interest to be equal to 4%, which allows us to set $\rho = 0.04$. We also know that at the steady state, investment is just equal to the depreciation of capital. With an investment-to-output ratio of 0.2 and a capital-to-output ratio of 2.5, this gives us $\delta = 0.08$. Finally, turning to the share of capital in the production function and obtain $\alpha = 0.3$, in line with standard calibrations of growth models.

4.1 Model-specific parameters

We still have a bunch of parameters to pin down: the returns to health investments σ, the relative taste for consumption ν, preferences for leisure ϕ, the slope of the depreciation rate of health capital relative to labor supply γ and a scaling parameter z. The relative price of health care p will be taken from OECD data.

Let us start with the rate of depreciation of health capital, δ_h. Empirical estimates of such a rate are scarce: Scholz and Seshadri (2010) calibrate it around 5.6%, while Lawver (2012) set it between 0 and 5% for individuals experiencing no change in health status between two periods. We therefore take $\delta_h = 2.5\%$ as a benchmark and will investigate later how the calibration results are affected by different values of δ_h. We then set $\gamma = 2$ to ensure there is a unique real root to equation (36) and calibrate the scaling parameter z accordingly.

We remain with three parameters to set (ν, ϕ and σ), but only two data moments to match: the number of hours worked per worker, which we transform into a fraction of the sole unit of time individuals are endowed with by dividing it by 365*16 as is common in the literature (which gives $l^{\star}_{us} = 0.33$), and the share of GDP devoted to health care expenditure. Empirical estimates of the returns to health investments are also scarce but the evidence points toward decreasing returns. We therefore set $\sigma = 0.8$ as a benchmark value which we change later for some robustness checks. We can now look at the expression (35) for health expenditure as a share of GDP as a function of labor supply and solve it for the relative taste of consumption ν, using $\sigma = 0.8$ and $l^{\star}_{us} = 0.33$.

Finally, we are left with preferences for leisure ϕ. This is straightforward, we simply solve equation (36) for ϕ, using $l^{\star}_{us} = 0.33$ and the value of the parameters already calibrated.

4.2 The relation between the health capital stock and mortality rates

There is no mortality in our theoretical model and the health capital stock has no direct connexion to any health indicator. We want to establish a relation between such a stock and the health indicator we find to be the most relevant for our exercise: the survival probability of men aged between 55 et 64 years. We argue that using a logistic function that would transform the health capital stock into such a probability is relevant since it would yield a value between 0 and 1, be convex for low value of health capital indicating originally large gains from medical investments and become concave as the health capital stock increases, indicating decreasing returns. Let therefore be T the survival probability between age 55 and 64: $T = (1 - m_{55-59})(1 - m_{60-64}) = 0.978184$ in the US in 2015 and define:

$$T = \frac{T_0}{T_0 + (1 - T_0)e^{-\psi \cdot h^\star}}$$

Where T_0 is the survival probability when the health capital stock is zero. To pin down the value of T_0, we interpret it as the health status in the absence of any medical investment and set it equal to the probability of surviving at age 55-64 before any progress in medicine was made. We therefore go as far back as possible in the data, which brings us to France in the early 19th century, where life expectancy did not start to increase at the pace it did in the 20th century and one may argue that the accumulation of health capital had not started yet. We therefore take mortality rates in France between 1816 and 1819 as a benchmark and hence set $T_0 = (1 - m_{55-59})(1 - m_{60-64}) = 0.934$. Using the value of $h^\star$ given by the model once every other parameter is found, we can the calibrate ψ, the steepness of the logistic function.

We thus have assigned a value to each parameter, the next table summarizes:

Parameter		Target	US
α	Capital share	Capital/output ratio	0.3
δ	Capital rate of depreciation	Investment/output ratio	0.08
ρ	Discount factor	Interest rate	0.04
γ	Health capital depreciation	Chosen	2
σ	Returns to health investment	Chosen	[0.7 ; 0.9]
z	Scaling parameter	US rate of depreciation	[0.09 ; 0.46]
p	Relative price of health care goods	OECD data	1.24
T_0	Survival probability without health capital	Mortality rates 1810	0.934
ψ	Steepness of the logistic function	Survival probability (age 55 - 64)	0.077
ν	Relative preferences for consumption	Share of health expenditure	0.542
ϕ	Preferences for leisure	Hours worked	0.235

4.3 A counterfactual: what if Americans worked as much as Europeans?

We next conduct a simple exercise to assess the effect of a reduction of the average working time in the US on the American share of health care expenditure and on Americans' mortality rates. More specifically, we ask ourselves what would happen if Americans worked as much as Europeans. We remain agnostic concerning the potential reasons for which Americans work today substantially more than Europeans, but we model such a difference via different preferences for leisure. Our strategy is therefore straightforward: we re-calibrate US preferences for leisure so that it matches not American but European labor supply, and then solve the model for steady state health expenditure as a share of GDP and the health capital stock (and hence mortality rates), all else equal. Our theory predicts that the share of medical expenditure should decrease, and the health of Americans improve, provided the returns to health investment are not too strong as seems to be the case empirically.

To provide an illustration, we choose the UK as our European country of reference and calibrate American preferences for leisure to match labor supply in the UK ($l^{\star}_{uk} = 0.28$). Solving the model, we obtain $(p \cdot m/y)^{\star} = 13.87\% < 16.5\%$. Hence, would Americans work as much as Britons, their share of health care expenditure would be around 2.6 percentage points lower.

Turning to mortality rates now, the steady state health capital stock is now greater than before, which translates into lower mortality rates (or a greater survival probability equivalently). More precisely, $T^{\star}_{us} = 0.979616$ which is greater than the actual probability $T_{us} = (1 - m_{55-59})(1 - m_{60-64}) = 0.978184$. Such a reduction of working time would therefore increase the survival probability by roughly 0.0014, meaning that around one hundred and forty deaths per thousand of people of age 55-64 would be avoided. When looking at the actual population of that age category of around 10 millions, this amounts to 29 thousands less deaths per year. Of course, such numbers should not be taken as face value given the number of factors impacting individuals' mortality that are not taken into account by our model, but rather as an empirical illustration of our qualitative theoretical results.

4.4 Deviation from benchmark parameter values

We now conduct the same exercise, using different values of both the rate of depreciation of health capital δ_h and the returns to medical investments σ.

Specification		Health expenditure[1]	Mortality[2]
	$\sigma = 0.7$	3.3	144
$\delta_h = 1\%$	$\sigma = 0.8$	3.3	49
	$\sigma = 0.9$	3.3	-46
	$\sigma = 0.7$	2.6	226
$\delta_h = 2.5\%$	$\sigma = 0.8$	2.6	143
	$\sigma = 0.9$	2.6	60
	$\sigma = 0.7$	2	300
$\delta_h = 5\%$	$\sigma = 0.8$	2	232
	$\sigma = 0.9$	2	160
[1]: Reduction in the share of health expenditure (p.p.); [2]: Lives saved per hundred thousand people			

As we can see, different values of σ do not alter the results for the share of health expenditure, only the difference in mortality explained by a greater labor supply. Intuitively, the greater the returns to medical investment, the smaller the explicative power of excess labor supply for mortality. Indeed, the extra-depreciation induced by a greater number of hours worked is more easily offset by additional medical spending. Notice that when the rate of depreciation is calibrated to be equal to 1% and $\sigma = 0.9$, the effect of a decrease in working time in the US would actually be negative. We recover the case when $\sigma > \sigma^\star$ and labor supply at the steady state lies below the threshold that maximizes the health capital stock and the loss of labor income following a reduction in hours of work would actually be harmful for health.

Regarding the effect of the rate of depreciation of health capital, we see on the one hand that the impact of a reduction of hours of work increases with δ_h when it comes to mortality, which is trivial: a higher rate of work-related depreciation will generate a greater improvement in mortality following a reduction of working hours. On the other hand, a greater rate of depreciation of health capital implies more substantial medical expenditure, all else equal; thereby reducing the effect of a lower labor supply on the share of resources devoted to health care.

5 Conclusion

This paper is ultimately about the determinants of health at a macroeconomic level, which may be a combination of individual decisions and societal factors. Research on the link between economic development and health has gained considerable attention over the last two decades, with different focus. The two-way links between health and economic growth in the long-run have been studied by development-oriented theorists such as Fogel (1994), Boucekkine, de la Croix & Licandro (2003), Chakraborty (2004), while macroeconomists such as Rhum (2000, 2005) have focused on the cyclicality of health in developed economies and highlighted the pro-cyclical nature of mortality which is found to increase during economic booms. We add to this literature by investigating the role of societal differences regarding working time on long-run levels of health and health expenditure in rich countries and provide steady state expressions for both variables as a function of labor supply. Broadly, this calls for further research on the production of health at a macroeconomic level, which inputs should enter such a production function, and on the relationship between population health and mortality rates.

In this paper, we document three distinct trends of the US economy that have been occurring at a macro level: the slowdown of improvements in life expectancy, the surge in medical expenditure and the halt that was put to the reduction of working time. Looking at Western Europe during the same period, we clearly see the US as an exception among rich countries: over the last 40 years, Americans have worked more and have been less healthy despite substantial medical expenditure. Those trends have been studied separately quite intensively, but we draw a causal link between them. In our model differences in hours of work between the US and Western European countries (which we model through differences in preferences for convenience, but could also be due to difference in taxation or in labor market institutions) may also cause differences in mortality if we assume that labor supply depreciates individuals' health capital stock. American workers therefore use the extra income they earn by working more to purchase goods as well as medical care, but since their greater labor supply depreciates their health capital stock further, they devote a larger fraction of their income to health care to offset the extra depreciation. Provided the medical technology available to them is not efficient enough, the surge in medical expenditure may not be sufficient to repair the health capital stock, which results in higher mortality rates for American workers. When calibrated to the US economy, the model offers values of plausible magnitudes but cannot account for the whole difference between the US and Europe. Given the multitude of factors impacting those variables, this is rather convincing.

There are of course many potential and competing explanations to the American health disadvantage, but none seems able to fully account for the gap between the US and its rich counterparts today. The answer is probably a combination of several factors: increasing inequalities in the US, racial and ethnic disparities, deficiencies within the health care system, inflation of medical goods and services, etc. In recent years, mortality of middle-aged non-Hispanic whites has even increased, as documented by Case and Deaton (2015, 2017), with a surge in what they label "deaths of despair" among the low-educated white population. We provide yet another theory to shed light on a mechanism that has not been as investigated as others, the fact that a pressured work schedule might be detrimental to one's physical as well as mental health. This negative effect of long work hours on health probably adds up to the existing explanations of the American health disadvantage.

One interesting feature of the model is that the share of total resources devoted to health care tends to increase with the number of hours worked in the economy. We here provide a novel demand-side explanation of why the share of medical expenditure in the US increased more substantially than that of European countries. We argue that the fraction of time individuals dedicate to market activities can be viewed as the utilization rate of their health stock, and when this rate is increased through a greater labor supply, individuals will bear the extra costs of depreciation and spend more on health care. However, curative medicine may be less efficient than a preventive lifestyle and if the returns on such medical investments are not high enough, individuals may renounce to a few years of life.

We also show that, at the steady state, the relationship between the health capital stock and labor supply may very well be non-monotonous. Provided the returns to medical investments are not to weak (too strong), in which case the relationship would be strictly negative (positive), there is a unique number of hours worked that maximizes the steady state health capital stock. Consequently, an economy whose preferences, tax system or institutions are such that labor supply lies below such a threshold could improve the health of its population by increasing working hours and using the extra labor income to make health investments. Conversely, a society with preferences for leisure such that the steady state number of hours worked is greater than the threshold is actually hurting its health with excess labor supply and could benefit from a reduction of working time. Notice however that both cases, where labor supply is either too low or too high to maximize the health capital stock, remain optimal: in the former case, individuals' preferences are such that they give up on some health and consumption to enjoy more leisure, whereas in the latter they increase consumption at the expense of both health and leisure.

A natural extension of the model would be to include some sort of externalities to economic activity that might be detrimental to health. A usual suspect is of course pollution, which is believed to be one of the reasons why mortality happens to increase during economic expansions. One may also think about traffic congestion, an obvious byproduct of booms that also causes deaths on the road during good times, or lifestyle changes such as alcohol and tobacco consumption that appear to be linked with economic activity and impact individuals' health. Adding externalities that agents fail to internalize would allow for a suboptimal decentralized equilibrium as agents do not take some effects of their decisions into account, and provide the basis for welfare analysis and hence public policy guidance.

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Appendix

Proof of Proposition 2

We prove proposition 2 in three steps.

Lemma 1

There exists a unique $l = \bar{l} \in]0;1[$ for $\sigma \in]\sigma_1;1]$ that maximizes the steady state health capital stock.

Proof:

$$\frac{\partial h^\star}{\partial l^\star} = 0 \Leftrightarrow (1+\gamma)\sigma - \gamma - \sigma\gamma \frac{[\sigma(1-\nu)+\nu]z l^{\star\gamma}}{[\sigma(1-\nu)+\nu]z l^{\star\gamma} + \nu\rho} = 0$$

After some algebra we get:

$$l^\star = \left\{ \left(\frac{\nu\rho}{z}\right) \left[\frac{(1+\gamma)\sigma - \gamma}{(\gamma-\sigma)[\sigma(1-\nu)+\nu]} \right] \right\}^{\frac{1}{\gamma}} = \bar{l}$$

It follows that $(\partial h^\star/\partial l^\star) > 0$ when $l^\star < \bar{l}$, $(\partial h^\star/\partial l^\star) = 0$ when $l^\star = \bar{l}$ and $(\partial h^\star/\partial l^\star) < 0$ when $l^\star > \bar{l}$

Lemma 2

If $\frac{\gamma}{1+\gamma} < \sigma < \left(\frac{\gamma}{1+\gamma}\right)\left(1 + \frac{z}{\nu\rho}\right)$, then $0 < \bar{l} < 1$ and health capital stock is a non-monotonic function of labor supply at the steady state.

Proof:

$\bar{l} < 0 \Leftrightarrow \sigma < \frac{\gamma}{1+\gamma}$ and a sufficient condition under which $\bar{l} > 1$ is $\sigma > \frac{\gamma}{1+\gamma}\left(1 + \frac{z}{\nu\rho}\right)$

Lemma 3

There exists a unique $\sigma^\star \in]\sigma_1; 1]$ for which $l(\sigma)^\star = \bar{l}(\sigma)$

Proof:

Rearrange $l^\star(\sigma) = \bar{l}(\sigma)$ such that:

$$[\sigma(1-\nu)+\nu] \cdot l(\sigma)^\gamma = \left(\frac{\nu\rho}{z}\right) \cdot \frac{(1+\gamma)\sigma - \gamma}{\gamma - \sigma}$$

Note that we drop the star subscript for the sake of clarity. Define $f(\sigma) = [\sigma(1-\nu)+\nu] \cdot l(\sigma)^\gamma$ and $g(\sigma) = \left(\frac{\nu\rho}{z}\right) \cdot \frac{(1+\gamma)\sigma-\gamma}{\gamma-\sigma}$

We now want to prove both functions intersect only once for $\sigma \in [\sigma_1; 1]$.

First, let us study $f(\sigma)$ and notice that $0 < f(\sigma_1) < f(1)$ since $l(\sigma) > 0$. Then:

$$f'(\sigma) = l(\sigma)^\gamma \left\{ 1 - \nu + \gamma[\sigma(1-\nu) + \nu] \frac{l'(\sigma)}{l(\sigma)} \right\} > 0$$

since $l'(\sigma) > 0$, and:

$$f''(\sigma) = \gamma l(\sigma)^\gamma \left\{ 2(1-\nu) \frac{l'(\sigma)}{l(\sigma)} + [\sigma(1-\nu) + \nu] \cdot \left(\frac{l''(\sigma)}{l(\sigma)} + (\gamma - 1) \left(\frac{l'(\sigma)}{l(\sigma)} \right)^2 \right) \right\} > 0$$

since $l''(\sigma) > 0$. Therefore, $f(\sigma)$ is strictly positive, increasing and convex for $\sigma \in]\sigma_1; 1]$.

Let us now turn to $g(\sigma)$. First, notice that $g(\sigma_1) = 0 < g(1) = \frac{\nu\rho}{z(\gamma-1)}$. Then:

$$g'(\sigma) = \left(\frac{\nu\rho}{z} \right) \cdot \left(\frac{\gamma}{\gamma - \sigma} \right)^2 > 0$$

and:

$$g''(\sigma) = 2 \left(\frac{\nu\rho}{z} \right) \cdot \frac{\gamma^2}{(\gamma - \sigma)^3} > 0$$

We now know that: $f(\sigma_1) > g(\sigma_1) = 0$ and that the two functions are both strictly increasing and convex in σ. Therefore, they intersect only once for $\sigma \in]\sigma_1; 1]$ if and only if $f(1) < g(1)$, which concludes the proof. Proposition 2 then follows from the those lemmas.