Contract Horizon and Turnover

Vladimir Vladimirov*

October 7, 2017

Abstract

This paper develops a model in which a principal hires agents whose fit with the firm changes over time. Agents are better informed about such changes, so the principal must decide how to elicit information that could lead to an agent's dismissal. The paper rationalizes the use of renewable fixed-term contracts as a mechanism that periodically switches from relying on severance pay to relying on firm performance on renewal dates to sort out agents. The paper's key implications focus on the determinants of contract horizon. Furthermore, it sheds light on several puzzling stylized facts concerning hiring and replacement practices.

Keywords: contract length, contract horizon, voluntary and forced turnover, renewable fixed-term contracts, asymmetric information.

JEL Classification: G30, G34, D82

*University of Amsterdam. E-mail: vladimirov@uva.nl. I thank Patrick Bolton, Arnoud Boot, Murillo Campello, Tolga Caskurlu, Jason Donaldson, Daniel Garrett, Stuart Gillan, Moqi Groen-Xu, Sebastian Gryglewicz, Michel Habib, Marina Halac, Roman Inderst, Dirk Jenter, Boyan Jovanovic, Oguzhan Karakas, Nicolas Klein, Tomislav Ladika, Chen Lin, Alexander Ljungqvist, Richard Lowery, Andrey Malenko, Frédéric Malherbe, David Martinez-Miera, Konstantin Milbradt, Martin Oehmke, Per Östberg, Daniel Paravisini, Enrico Perotti, Florian Peters, Sebastian Pfeil, Giorgia Piacentino, Joshua Pierce, Uday Rajan, Adriano Rampini, Merih Sevilir, Fenghua Song, Jan Starmans, Spyros Terovitis, Ed Van Wesep, David Yermack, as well as conference and seminar participants at the European Economic Association annual meeting (Geneva), European Winter Finance Summit, University of Amsterdam, and University of Zurich for constructive feedback.

1 Introduction

When principals evaluate agents, a first-order problem is that even high-quality agents might not always be a good fit for the organization. A good example is Ronald Boire. He was considered an "ideal chief executive for Barnes & Noble" when he was hired in 2015, but he stepped down a year later because he "was not a good fit."[1] An agent's fit reflects the complex match between her collection of skills and the organization's assets and growth options (Lazear, 2009). However, the quality of this match can change as the firm and its environment evolve. The fit might also change because of inadequate firm-specific human capital investments, family or health issues, or conflicts with colleagues or subordinates. The key problem is that agents are often better informed about such changes. This gives rise to the questions how to elicit information that could lead to an agent's dismissal and what would be the implications for the length and performance-sensitivity of contracts.

Indeed, there are still gaps in our understanding of *what characteristics determine the length of an agent's contract*. This question is of fundamental importance in executive compensation. More than 45% of executive contracts are renewable fixed-term contracts (Gillan et al., 2009), but there is barely any theory explaining their use. Mr. Boire had a three-year contract that could be renewed for another two years. However, why extend for two, and not ten years; or, given that the contract can be terminated at any time, why not simply offer straight a five-year contract? The issue of termination leads to the second overarching question, which is *what factors determine the performance-sensitivity of turnover*. Addressing this question can shed light on puzzling patterns, such as why turnover, which often happens before renewal, becomes performance-sensitive mainly close to renewal (Cziraki and Groen-Xu, 2015). It could further explain why the performance-sensitivity of turnover increases in industry-wide bad times (Jenter and Kanaan, 2015), which runs counter to the received view that agents should not be punished for bad luck outside of their control. The present paper answers these questions by linking them to the use severance promises in dynamically evolving relationships. Such promises ($10.5m in Mr. Boire's case) can make turnover seem "voluntary" by mitigating an agent's desire to avoid termination, but their relation to contract horizon has not been analyzed before.

The paper develops a model in which a principal repeatedly appoints finitely-lived agents. Once hired, an agent can increase the likelihood of being a good fit for the organization by investing in firm-specific human capital. A good fit is not only more likely to achieve high cash flows, but also be a good fit next period. However, only the agent observes the extent of her firms-specific human capital investments and how they impact her fit. The paper's main

[1] "Barnes & Noble Says CEO Boire 'Not a Good Fit' and Will Step Down," WSJ, 16 August 2016.

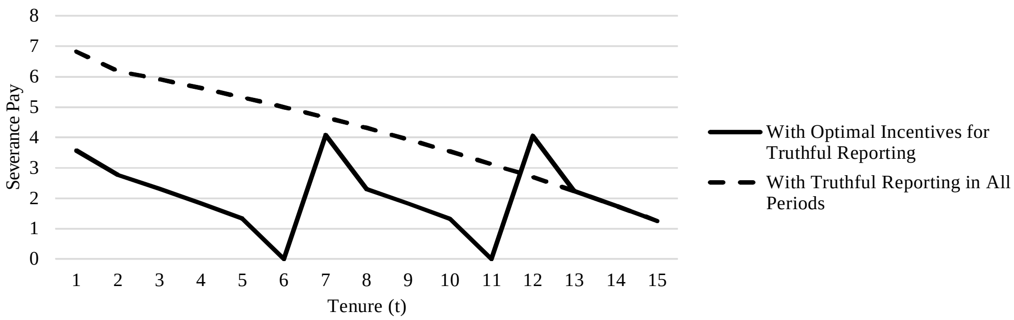

Figure 1: The severance cost of replacing an agent. The figure compares the optimal contract with one that offers incentives for truthful reporting in all periods. The dips in severance pay correspond to periods in which the board does not offer incentives for truthful reporting, but relies on firm performance. In terms of implementation, these dips correspond to the end dates of renewable fixed-term contracts.

results arise from asking whether the principal should rely on noisy performance measures to identify a deteriorating fit or on adequate incentives that the agent truthfully self-reports such deteriorations. A restriction is that the principal cannot commit not to (renegotiate to) replace an agent he comes to believe is a bad fit. Though pervasive in various settings, it is useful to interpret the model in terms of a board hiring managers. The availability of evidence in this context helps to flesh out its implications.

The first main result (illustrated in Figure 1) is that it is optimal for the board to abstain from offering incentives for truthful reporting at regular intervals. The brief intuition for the trade-offs is as follows. The problem of incentivizing a manager to admit that her fit has deteriorated is that it would lead to her dismissal. Thus, it requires offering severance pay. Longer contracts exacerbate this problem, since then managers have more to lose from premature dismissal. Offering shorter (renewable) contracts that are costless to terminate on renewal dates helps to reduce severance pay. However, on renewal dates, the board would have to rely only on noisy firm performance to judge the manager's fit.

More specifically, the advantage of incentivizing truthful reporting is that the manager's fit does not have to be evaluated based on the firm's noisy cash flow performance. This is beneficial, as a good fit today increases the likelihood of a good fit tomorrow. Thus, a good fit is retained even after a low cash flow realization. However, this tolerance for failure creates strong incentives to pretend being a good fit, as by staying with the firm, the manager can keep receiving an "efficiency" wage for her firm-specific human capital investments. These incentives to hide a bad fit are stronger, the longer the manager could stay on the job by misreporting her fit. Hence, with a contract that only relies on truthful reporting, the manager's severance package must increase in her potential remaining tenure, and it compensates her as if she would stay until the contract's end (dashed line in Figure 1).[2] What is key is that this cost is in stark mismatch with the benefits of offering incentives for truthful reporting in some future period t, as these benefits are realized only if the manager is still with the firm at this point.

In light of this cost-benefit mismatch, the advantage of *not* incentivizing truthful self-reporting is that it cuts through the manager's ability to stay with the firm by lying about her fit.[3] Specifically, it allows the board to dispense with severance when replacing a manager and, even more important, it reduces the necessary severance pay in all preceding periods. The disadvantage is that the manager's fit needs to be evaluated based on the firm's noisy cash flow performance. This results in a trade-off between minimizing the risk of making the wrong replacement or retention decision and minimizing the cost of employing the manager. This trade-off is best resolved by tying turnover to firm underperformance (rather than to truthful reporting and severance pay) at regular intervals. As illustrated by the solid line in Figure 1, this significantly reduces the need for severance pay, while exposing the board to the relatively low risk of making inefficient replacement decisions in year six and eleven, which the manager reaches only with very low probability.

Renewable fixed-term contracts offer an off-the-shelf implementation of the contract illustrated in Figure 1. Such contracts allow for turnover at any time, but their termination is costless at renewal dates. What makes them suitable is that the board can choose renewal to coincide with the periods in which it is not optimal to incentivize truthful reporting from the manager. Since the manager is not offered severance upon termination, she has no incentives for truthful reporting in such periods, and the board decides on renewal based on the firm's cash flow performance. Indeed, renewable fixed-term contracts are very common in practice (Gillan et al., 2009), and their turnover-performance sensitivity spikes close to renewal dates (Cziraki and Groen-Xu, 2015). Despite the prevalence of such contracts, however, this paper seems to be the first to try to understand their use.[4]

The paper's second contribution is to study the determinants of the length of renewable contracts, i.e., the distance between periods in which the board only relies on noisy performance rather than incentives for truthful reporting. A key factor is the manager's outside

[2] Indeed, half of S&P 1500 firms have ex ante severance agreements, which are associated with more truthful managers (Rau and Xu, 2013; Brown, 2015). Furthermore, CEOs with longer remaining tenure have higher severance agreements (Rau and Xu, 2013).

[3] With limited commitment, it is often suboptimal to screen out an agent's private information (Hart and Tirole, 1988; Laffont and Tirole, 1990; Malcomson, 2016). Note that none of the arguments requires that the manager agrees that someone else might be a better fit for the job.

[4] A renewable fixed-term contract is an implementation of a long-term contract with costless termination options. Though the literature comparing long- with short-term contracts (e.g., Hart and Tirole, 1988) is related, it has also not sought to explain the use and the determinants of the length of renewable contracts.

option.[5] Low-paying alternative employment opportunities increase the manager's reluctance to reveal information leading to her dismissal. This would necessitate offering a higher severance package, which the board could avoid by offering shorter renewable contracts. The significance of this result is that the more frequent reliance on noisy performance would lead to a stronger performance-sensitivity of turnover. Thus, Jenter and Kanaan's (2015) findings that turnover becomes more performance-sensitive in industry-wide downturns (when outside options are low) could be due to an optimal evaluation and replacement policy rather than to a lack of relative performance evaluation. Naturally, also other factors matter for contract horizon. For large firms with better growth options, there will be more at stake from having the right manager in charge. Thus, in such firms, the contract horizon will be longer and severance pay will be larger.

The manager's age also plays a key role. It offers a natural commitment to a maximum contract horizon. This is particularly important if there is scope for renegotiating her contract at renewal periods and seeking truthful reporting after all. Thus, the board will offer higher severance packages to younger managers (which is in line with the findings of Rau and Xu, 2013). Furthermore, the board will hire older managers if the managers' outside options are low (as in downturns), as then incentivizing truthful reporting is costlier.

The determinants of contract horizon shape an employment relation also in other ways. Specifically, the board might appear to prefer managers from a "select club," i.e., managers with higher outside options who are not necessarily better; and tolerate investments in general human capital, even if they come at the expense of firm-specific human capital. All of this is because a higher outside option makes it cheaper to keep the manager honest and replace her. The same intuition explains why the board would avoid damaging a departing manager's reputation. Indeed, replacing managers with a reputation for underperformance is more expensive (Goldman and Huang, 2015).

The paper's main contribution is that it rationalizes the use of renewable fixed-term contracts and analyzes some of the determinants of their length, which sheds light on a number of stylized facts. A novel element is that the principal's lack of information is not due to reduced monitoring, but to deciding not to offer severance pay that would compensate the agent for not trying to avoid termination. This helps to cut through the agent's ability to extract rent, which is magnified in a dynamic setting. These features differentiate the paper from prior work advocating that there might be benefits from laxer control (Crémer, 1995; Aghion and Tirole, 1997); from papers in which managers are better informed about their fit, but in which that fit does not change over time (Hermalin and Weisbach, 1998; Taylor,

[5] Incumbents' outside options is typically not another CEO position, and CEOs take a big pay cut in their new employment following dismissal (Fee and Hadlock, 2004; Nielsen, 2017).

2010); as well as from prior static models rationalizing the use of severance pay (Levitt and Snyder, 1997; Inderst and Mueller, 2010; Almazan and Suarez, 2003; Van Wesep, 2010; Van Wesep and Wang, 2014).

The paper's novel implications for contract horizon and the dynamic use and structure of severance agreements also distinguish it from Jenter and Lewellen (2017) and Garrett and Pavan (2012). Both papers consider dynamically changing types but take polar opposite approaches. In Jenter and Lewellen (2017), the board does not screen out managers and, thus, must rely on the firm's most recent performance to infer their productivity. By contrast, Garrett and Pavan (2012) analyze full-commitment contracts that always incentivize managers to report their private information. Their main insight is that the board becomes progressively more tolerant towards lower managerial quality. The decision of how to evaluate the manager in the present paper can be seen as optimally relying on both approaches, while relaxing the assumption of full commitment.[6] Limited commitment has been analyzed also in the literature on relational contracts, in which full revelation is also suboptimal with persistent types (Halac, 2012; Malcomson, 2016). However, this literature offers a better description of contracting "at will," as contract length and bonuses are not contractually specified, while in the present paper contract horizon and contractual incentives are the main focus.

Also related are Anderson et al. (2016) and Eisfeldt and Kuhnen (2013) in which a publicly observable shock that decreases industry returns prompts the firm to look for a manager who is better suited to the new environment. This provides one explanation for Jenter and Kanaan's (2015) findings that turnover is more likely in industry-wide bad times. Instead, in the present paper, overall turnover is not more likely, but only its forced type, as boards rely more often on firm performance to judge the manager's fit. This could help explain why Fee et al. (2015) find no evidence for a lack of relative performance evaluation, when also considering supposedly "voluntary" turnover. Related, Eisfeldt and Rampini (2008) show that CEO turnover is procyclical. However, managers in their model live for only one period, which does not allow an analysis of contract horizon.

Turnover features in the literature also as a threat to discipline managers (Stiglitz and Weiss, 1983), as well as when managers are risk averse and become too expensive to motivate or when they take a better outside option (Sannikov, 2008; Wang, 2011, 2015). Instead, the reason for turnover in the present paper is to appoint a better manager, which raises the question of whether severance pay should be offered to stimulate truthful reporting. The paper also contributes to prior work on human capital investments (Jovanovic, 1979 a,b; and Felli and Harris, 1996) by analyzing a setting in which a worker's fit changes over time and

[6] Full revelation is suboptimal also in dynamic contracting with constant types (footnote 3).

is her private information.[7]

2 Model

Consider an infinitely lived firm in which the board maximizes shareholder wealth and is in charge of hiring and replacing the firm's manager (she). The firm operates in an economy, in which every period t consists of three dates. At the first date of every period, $\tau_t = 0$, an incumbent manager can invest in firm-specific human capital. Such an investment carries a non-monetary cost c, but it increases the likelihood that her fit with the firm in the current period, $\theta_t \in \{\theta_G, \theta_N\}$, is good. Specifically, if the manager invests in firm-specific human capital, her fit is θ_G with probability e_t. With probability $1 - e_t$ or, respectively, if she does not invest in firm-specific human capital, her fit is $\theta_N < \theta_G$.

At the interim date $\tau_t = 1$, the manager learns and can report her fit, and the board can decide whether or not to replace her with a new manager. All cash flows from the period are realized at the final date $\tau_t = 2$. If the board has not already replaced the manager at the interim date, it can choose again whether or not to keep her for the next period. Cash flows are verifiable and can take values $x_t \in \{x, x + \Delta x\}$. The manager's fit $0 \leq \theta_t \leq 1$ corresponds to the likelihood of achieving the higher cash flow $x + \Delta x$, where $x, \Delta x \geq 0$. All parties are risk neutral, and the common discount factor between two neighboring periods is $\delta \in (0, 1)$.

Neither the board nor potential managers have private information when a new manager is hired. Furthermore, the managers from which the board can choose have zero wealth and are identical in all respects except their age, i.e., managers are not infinitely lived and leave the labor market once they reach their retirement age. However, the key assumptions are that the manager's investments in firm-specific human capital as well as the realizations of θ_t at the interim date $\tau_t = 1$ of every period are known only to the manager.

The probability that a manager's investment in firm-specific human capital results in a good fit in period t depends on her fit θ_{t-1} from the previous period. Specifically, there is a positive correlation with $e_t(\theta_G) > e_1 > e_t(\theta_N)$, where e_1 is the likelihood of θ_G in the manager's first (complete) period after being hired, and $e_t(\theta_{t-1})$ makes explicit the dependence on θ_{t-1}. In this Markov environment, the t-subscripts in $e_t(\theta_G)$ and $e_t(\theta_N)$ are not necessary, but they are helpful to keep track of the intertemporal forces affecting contracting. Initially, $\{e_1, e_t(\theta_G), e_t(\theta_N)\}$ and the manager's outside option, which pays $\overline{U}$

[7] Tenure limits reduce agents' ability to extract rent also in Lazear (1979), Prescott and Townsend (2006), and Hertzberg et al. (2010). The main difference from the political economy literature (Aghion and Jackson, 2016) is that turnover can occur at any time, and monetary incentives play a key role.

per period, are fixed, but Section 3.3 relaxes these assumptions.

Contracting At the beginning of the employment relation, the board offers the manager a contract $\mathbf{w} = (w_t, \Delta w_t, w_{s,t}, \psi_t^1, \psi_t^2)_{t=1}^T$. The contract components characterizing any given period t can depend on the current and past cash flow realizations $(x_i)_{i=1}^t$, as well as on the manager's reports $(\widehat{\theta}_i)_{i=0}^t$ about her fit (if there are such reports). Unless this leads to confusion, the history dependence is not made explicit but is captured only by the subscript t. In this contract, w_t stands for the manager's wage in the low cash flow state, and Δw_t stands for how much she receives in addition (i.e., her "bonus") in the case of a high cash flow realization; $w_{s,t}$ is the manager's severance pay if she is replaced at the interim date $\tau_t = 1$, *prior* to the cash flow realization $x_t \in \{x, x + \Delta x\}$ in that period, with ψ_t^1 being the probability of such an interim replacement; ψ_t^2 stands for the probability of replacing the manager at the end of the period, i.e., in $\tau_t = 2$, *after* the cash flow realization x_t.[8] While the contract does not explicitly consider a payment to the manager for leaving the firm at the beginning of a period before she obtains private information or a payment at the interim date $\tau_t = 1$ for staying with the firm, we show that such payments will not arise.

The manager is penniless and protected by limited liability, which requires that $w_t, w_{s,t} \geq 0$, and should have no incentives to destroy cash flows, i.e., $\Delta w_t \geq 0$ (Innes, 1990).[9] Contracts that satisfy these requirements are labeled as "feasible." Furthermore, it is assumed that the manager cannot be prevented from leaving the firm at any time during the employment relationship. Thus, the contract should at least compensate her for her outside employment opportunity, which would pay her $\overline{U}$ at the end of every period until her retirement in T. We assume that if the manager leaves the firm at the interim date $\tau_t = 1$ of a period, she still obtains $\overline{U}$ from her outside employment opportunity for that period.[10] Figure 2 summarizes the timing of events in each period.

Replacement and Performance Evaluation If the board replaces the incumbent at the interim date, the new manager is paid $\overline{U}$ to complete the period. In this period, her success likelihood is $\overline{\theta}$ with $\theta_N < \overline{\theta} < \theta_G$; this requires no firm-specific human capital investment; and does not give rise to private information. Then, at the beginning of the following period, the board makes the manager an offer covering the whole potential relationship. If a manager is replaced, she is not rehired.

[8] The payment to the manager at the end of the period could also be reinterpreted as the manager's severance pay if she is fired at the end of that period.

[9] We assume that the manager does not save. Though all parties are risk neutral and use the same discount factor, and cash flows are verifiable, the assumption is not innocuous. This is because a manager whose contract is terminated could otherwise offer the board a payment to be kept on the job.

[10] Assuming that the manager receives only a fraction of $\overline{U}$ leads to the same qualitative results.

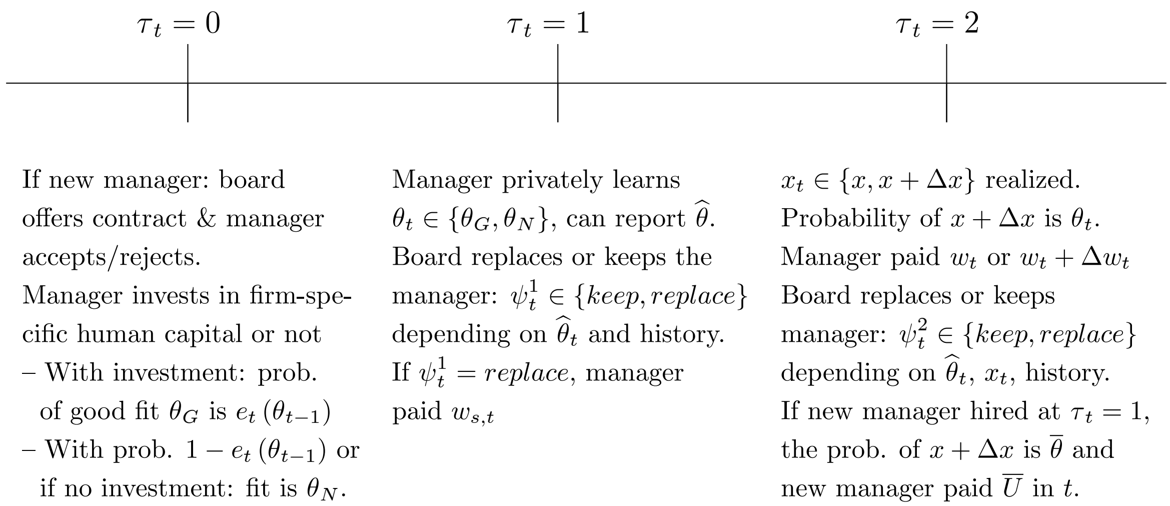

Figure 2: Timing of events in period t.

Definition 1. *An equilibrium with optimal termination is a perfect Bayesian equilibrium in pure strategies in which replacement and retention decisions are efficient from the board's perspective given its information at this point in time. In the case of full revelation, these decisions should coincide with those of the subgame perfect equilibrium of the full information game at* $\tau_t = 1$.

Following Malcomson (2016), Definition 1 restricts attention to pure strategies and rules out commitment not to renegotiate retention and replacement strategies that are suboptimal, given the board's ex post information. Following a period with revelation of the manager's fit, the only replacement strategy consistent the board's ex post information is to replace a manager if and only if her fit is θ_N. That is, $\psi_t^1 = 1$ if $\theta_t = \theta_N$, $\psi_t^1 = 0$ if $\theta_t = \theta_G$, in which case also $\psi_t^2 = 0$ regardless of x_t (unless the period coincides with the manager's retirement age). This is because, by replacing a manager whose fit is θ_N, the board can increase the likelihood of high cash flows in both the current and the subsequent period, as $\overline{\theta} > \theta_N$ and $e_1 > e_{t+1}(\theta_N)$. By contrast, it would be ex post suboptimal to replace an incumbent who is a good fit, as $\theta_G > \overline{\theta}$ and $e_{t+1}(\theta_G) > e_1$.

Because of its inability to commit not to act on its information, the board might find it optimal not to seek truthful reporting at the interim date $\tau_t = 1$ and to remain uninformed about the manager's fit.[11] We follow Aghion and Tirole (1997) and assume that, when uninformed, the board keeps the manager at the interim date. A sufficient condition for this is

$$e_t(\theta_N)\,\theta_G + (1 - e_t(\theta_N))\,\theta_N > \overline{\theta} \iff e_t(\theta_N) > \frac{\overline{\theta} - \theta_N}{\theta_G - \theta_N}. \quad (1)$$

[11] Remaining uninformed is often optimal in dynamic settings with limited commitment (Hart and Tirole, 1988; Bester and Strausz, 2001; Malcomson, 2016).

Subsequently, if the firm produces high cash flows at the end of the period, the board expects that the incumbent's likelihood of being a good fit in the following period will be higher than that of a new manager; the board assumes the opposite if the firm has produced low cash flows

$$\sum_{\theta_t \in \{\theta_B, \theta_G\}} \Pr(\theta_t | x) e_{t+1}(\theta_t) < e_1 < \sum_{\theta_t \in \{\theta_B, \theta_G\}} \Pr(\theta_t | x + \Delta x) e_{t+1}(\theta_t), \tag{2}$$

where $\Pr(\theta_t | x_t)$ is the board's posterior about the fit realization θ_t, conditional on the cash flow realization in t and the prior history. A sufficient condition for (2) to hold in terms of primitives is given in the Appendix (expression (B.17)). Thus, absent incentives for truthful reporting, we have $\psi_t^1 = 0$ and $\psi_t^2 = 0$ if $x_t = x + \Delta x$, but $\psi_t^2 = 1$ if $x_t = x$.[12]

Definition 1 and condition (2) imply that the board will not show patience for managers that are (very likely) a bad fit. This leads to a stark replacement policy that helps to abstract from issues related to learning the manager's fit over an extended period of time, which have been studied extensively in the literature.[13] Since the replacement strategies are deterministic $\psi_t^\tau \in \{0, 1\}$, they are labeled for transparency as $\{keep, replace\}$.

Finally, note that there is no truly voluntary turnover in this model. However, in practice, a smooth transition, eased by a severance package, might appear voluntary to outsiders, even if the board and the manager disagree behind the scenes as to whether a replacement could do a better job. Indeed, one could extend the model by assuming that the manager overestimates her fit or has a different vision how to run the firm than the board.[14]

3 A Multi-Period Employment Relation

Given a contract offer $\mathbf{w} = \left(w_t, \Delta w_t, w_{s,t}, \psi_t^1, \psi_t^2\right)_{t=1}^T$, let $\omega_t \in \{w_t, w_t + \Delta w_t, w_{s,t} + \overline{U}\}$ denote the firm's wage bill in period t.[15] The board's expected payoff in the first period is

$$V_1(\mathbf{w}) = \mathbb{E}\left[\sum_{i=1}^{T} \delta^{i-1} \left(q_i (x_i - \omega_i) + \widetilde{q}_i \delta V^*\right)\right], \tag{3}$$

where $\mathbb{E}$ is the expectation over the future θ_t and x_t realizations; and q_i and $\widetilde{q}_i$ are the endogenous probabilities that the incumbent manager is still with the firm in period i and,

[12] The Appendix shows that these strategies are also *ex ante* optimal, as they spur effort incentives.

[13] With continuous types and small changes to the manager's type, the board is more likely to be more patient (Garrett and Pavan, 2012). See Taylor (2010) and He et al. (2017) for models with learning.

[14] One could assume that the board believes that $\overline{\theta} > \theta_N$, while the manager believes that the board's preferred way has a success probability of only $\overline{\theta}^m < \theta_N$. Such disagreement has been motivated with heterogeneous priors and overconfidence (Goel and Thakor, 2008; Gervais et al., 2011; Huang et al., 2016).

[15] Recall that if the board replaces a manager at $\tau_t = 1$, it pays $w_{s,t}$ to the departing and $\overline{U}$ to the replacement manager to complete the period.

respectively, leaves the firm by the end of that period. V^* denotes the board's equilibrium expected payoff from hiring a new manager starting from the first complete period of that manager. Note that, since managers are ex ante identical, their information evolves independently, and time is infinite, the board's contracting problem when making an offer to a replacement manager is identical to that faced with her predecessor. Thus, in equilibrium, the board's expected payoff in (3) must be equal to V^*. For convenience, t is reset to one for every new manager, so that t could be interpreted as her tenure at the firm. The board's promise-keeping constraint implies that the manager's expected payoff at any given t during her tenure is

$$U_t\left(\theta_{t-1}, \mathbf{w}\right) = \mathbb{E}\left[\sum_{i=t}^{T} \delta^{i-t}\left(\overline{U} + q_i\left(\omega_i - c - \overline{U}\right)\right)|\theta_{t-1}\right]. \tag{4}$$

Expression (4) states that the manager can obtain $\overline{U}$ in every period until she leaves the labor market in T, but she might receive something different than $\overline{U}$ while she is employed by the firm. What is crucial to the analysis is that the manager's fit persistence implies that her payoff in t, $U_t\left(\theta_{t-1}, \mathbf{w}\right)$, depends on her fit realization in $t-1$ (because e_t depends on θ_{t-1}). There is no such prior realization when she is hired, so for period one we write $U_1\left(\mathbf{w}\right)$.

Using (4) to plug in for the manager's compensation in (3), the board's objective when hiring a manager is to choose $\mathbf{w}$ to maximize

$$\max_{\mathbf{w}} \mathbb{E}\left[\sum_{i=1}^{T} \delta^{i-1}\left(q_i\left(x_i - c - \overline{U}\right) + \widetilde{q}_i \delta V^*\right)\right] - U_1\left(\mathbf{w}\right) + \sum_{i=1}^{T} \delta^{i-1}\overline{U}, \tag{5}$$

subject to the constraints that the contract $\mathbf{w}$ is feasible, incentive compatible, and individually rational for the manager in every period. Hence, the board acts as a residual claimant and trades off maximizing the surplus generated from employing a manager with minimizing the manager's rent $U_1\left(\mathbf{w}\right) - \sum_{i=1}^{T} \delta^{i-1}\overline{U}$. We now state the relevant constraints.

Incentivizing Truthful Reporting Suppose that the board seeks truthful reporting at the interim date of some period t.[16] The incentive constraints that the manager truthfully

[16] Eliciting the manager's fit in t at the beginning of $t+1$ is suboptimal. It requires offering the manager the same information rent as when learning that fit in t, but without allowing for the benefit of replacing the manager earlier (see Lemma B.1 in the Appendix).

reveals her fit in t and stays if it is θ_G or leaves with a severance package if it is θ_N are

$$w_t + \theta_G \Delta w_t + \delta U_{t+1}(\theta_G, \mathbf{w}) \geq w_{s,t} + \sum_{j=t}^{T} \delta^{j-t} \overline{U} \tag{6}$$

$$w_{s,t} + \sum_{j=t}^{T} \delta^{j-t} \overline{U} \geq w_t + \theta_N \Delta w_t + \delta U_{t+1}(\theta_N, \mathbf{w}). \tag{7}$$

The fact that the manager's continuation payoff $U_{t+1}(\theta_t, \mathbf{w})$ can take on two values depending on her fit realization (θ_G or θ_N) in t means that both payoffs will play the role of state variables for characterizing the manager's contract. Furthermore, note that investing in firm-specific human capital might not be optimal after misreporting θ_N. However, the advantage of a Markov environment is that, if the manager is truthful on the equilibrium path in $t+1$, she would also be truthful off the equilibrium path in $t+1$ (i.e., after misreporting in t) after θ_{t+1} is realized.

To induce a manager to invest in firm-specific human capital in period t, the contract must further satisfy

$$U_t(\theta_{t-1}, \mathbf{w}) = \left(\begin{array}{c} e_t(\theta_{t-1})(w_t + \theta_G \Delta w_t + \delta U_{t+1}(\theta_G, \mathbf{w})) \\ +(1 - e_t(\theta_{t-1})) \left(w_{s,t} + \sum_{j=t}^{T} \delta^{j-t} \overline{U} \right) - c \end{array} \right) \geq w_{s,t} + \sum_{j=t}^{T} \delta^{j-t} \overline{U}, \tag{8}$$

where the right-hand-side of (8) captures that a manager who does not invest in firm-specific human capital does not have a good fit with certainty and, thus, is replaced at date $\tau_t = 1$ of the period. Note that if (8) is satisfied, the first incentive constraint (6) is lax.

Evaluating the Manager Based on Firm Performance Suppose, next, that the board does not offer incentives for truthful reporting and relies on the firm's cash flows to try to infer the manager's fit in some period t. In such a period, the constraint that the manager prefers investing in firm-specific human capital to not investing and forgoing the chance of being a good fit is

$$\begin{aligned} U_t(\theta_{t-1}, \mathbf{w}) &= w_t + (\theta_N + e_t(\theta_{t-1}) \Delta\theta) \Delta w_t + \delta E_{\theta_{t-1}} \left[U_{t+1}^e(\theta_t, \mathbf{w}) \right] - c \\ &\geq \sum_{j=t}^{T} \delta^{j-t} \overline{U} + w_{s,t}, \end{aligned} \tag{9}$$

where $\Delta\theta \equiv \theta_G - \theta_B$ and where, taking into account that the manager is replaced if and only if the firm's cash flows are low (which occurs with probability $1 - \theta_t$), we have defined the expected continuation payoffs

$$\begin{aligned}\mathbb{E}_{\theta_{t-1}}\left[U^e_{t+1}\left(\theta_t,\mathbf{w}\right)\right] &\equiv e_t\left(\theta_{t-1}\right)U^e_{t+1}\left(\theta_G,\mathbf{w}\right)+\left(1-e_t\left(\theta_{t-1}\right)\right)U^e_{t+1}\left(\theta_N,\mathbf{w}\right)\\ U^e_{t+1}\left(\theta_t,\mathbf{w}\right) &\equiv \theta_t U_{t+1}\left(\theta_t,\mathbf{w}\right)+\left(1-\theta_t\right)\sum_{j=t+1}^{T}\delta^{j-t-1}\overline{U}.\end{aligned}$$

Since in this case the manager prefers to stay even if her fit is θ_N, it should hold

$$w_t+\theta_N\Delta w_t+\delta U^e_{t+1}\left(\theta_N,\mathbf{w}\right)\geq\sum_{j=t}^{T}\delta^{j-t}\overline{U}+w_{s,t}. \tag{10}$$

Condition (10) will never be bind in the optimal contract, as then the board would have to increase the manager's pay to make sure she stays if her fit is θ_N. Clearly, in this case, the board would prefer to have truthful reporting in t.

The key question for the board is how the choice of whether or not to provide incentives for truthful reporting can increase its payoff by reducing the manager's rent

$$\nu_t\left(\theta_{t-1},\mathbf{w}\right):=U_t\left(\theta_{t-1},\mathbf{w}\right)-\sum_{j=t}^{T}\delta^{j-t}\overline{U}. \tag{11}$$

This rent is the "efficiency wage" a manager receives above her outside option, when she invests in firm-specific human capital. Without incentives for such investments, the board could satisfy (6) and (7), without leaving any rent to the manager by offering $w_t=\overline{U}$, $w_{s,t}=\Delta w_t=0$. However, then the incumbent would add no value to the firm, as $\theta_N<\overline{\theta}$ and $e_{t+1}\left(\theta_N\right)<e_1$. Because of this, the board always offers incentives for firm-specific human capital investment. To fully isolate the dynamics introduced by the choice of whether or not to seek truthful reporting, it is assumed that $e\left(\theta_N\right)$ is sufficiently high. This will ensure that, following a period in which the board does not seek truthful reporting, the manager invests in firm-specific human capital even if her fit in the preceding season is θ_N.[17]

3.1 Dynamics of the Manager's Contract

We can use now conditions (6)–(10) to derive the dynamics of the manager's contract for any given reporting strategy. The key state variables in any given period are the manager's continuation payoffs depending on her fit realizations, time, and the history of periods without truthful reporting.

[17] If $e_t\left(\theta_N\right)$ is low, we would need to solve additionally for whether it is optimal for the board to offer such incentives.

Proposition 1 *The board pursues one of two replacement strategies in any given period t. (i) The first strategy is not to seek truthful reporting and to replace the manager if and only if observing the low cash flow x at the end of the period ($\psi_t^1 =$ keep, $\psi_t^2 =$ keep if $x_t = x + \Delta x$, $\psi_t^2 =$ replace if $x_t = x$). Implementing this strategy in period $t < T$ goes hand-in-hand with deferring bonus payments and offering*

$$\Delta w_t = 0 \tag{12}$$

$$w_t = \max\left\{0, \sum_{j=t}^{T} \delta^{j-t}\overline{U}_j - \theta_N \Delta w_t - \delta U_{t+1}^e(\theta_N, \mathbf{w})\right\} \tag{13}$$

$$w_{s,t} = 0. \tag{14}$$

(ii) The second strategy is to provide incentives for truthful reporting (tr.rep.), in which case the manager is replaced if and only if her fit is θ_N ($\psi_t^1 =$ replace if $\widehat{\theta}_t = \theta_N$, $\psi_t^1 =$ keep if $\widehat{\theta}_t = \theta_G$, $\psi_t^2 =$ keep). Implementing this alternative requires

$$\Delta w_t = \begin{cases} \frac{c}{e_t(\theta_G)\Delta\theta} + \delta\frac{U_{t+1}(\theta_N,\mathbf{w}) - U_{t+1}(\theta_G,\mathbf{w})}{\Delta\theta} & \text{if tr.rep. in } t-1 \\ \max\left\{\begin{array}{c} \frac{c}{e_t(\theta_G)\Delta\theta} + \delta\frac{U_{t+1}(\theta_N,\mathbf{w}) - U_{t+1}(\theta_G,\mathbf{w})}{\Delta\theta}, \\ \Delta w_{t-n}^{-1}(0) \end{array}\right\} & \text{otherwise} \end{cases} \tag{15}$$

$$w_t = 0 \tag{16}$$

$$w_{s,t} = \max\left\{0, \theta_N \Delta w_t + \delta U_{t+1}(\theta_N, \mathbf{w}) - \sum_{j=t}^{T} \delta^{j-t}\overline{U}\right\}, \tag{17}$$

where $\Delta w_t = \Delta w_{t-n}^{-1}(0)$ is the minimum bonus in t, required to compensate the manager for $n \geq 1$ preceding periods with bonus deferral (i.e., periods without incentives from truthful reporting from $t-n$ to $t-1$), while satisfying (9) in these periods. The board always pursues truthful reporting in the manager's retirement period T. In the first period, $e_t(\theta_G)$ must be replaced by e_1 in (15).

Part (i) of Proposition 1 considers the case in which the board offers the manager incentives only for investment in firm-specific human capital (but not for truthful reporting). This requires offering a reward for signals indicating such an investment and a punishment for signals indicating the opposite. The signal in this case is the cash flow realization in t. Thus, the punishment is termination if this realization is low, and the reward is a bonus if the cash flow realization is high. However, reminiscent of Lazear's (1979) classical result, once a bonus is paid out, it seizes having an incentive effect. Hence, it is optimal to defer the bonus payment and make it conditional also on future success, i.e., we have $\Delta w_t = 0$.

The deferral remains in force until a period in which the manager truthfully reports her fit or until she reaches her retirement age. Clearly, offering severance pay that does not lead to truthful reporting has no benefit for the board, i.e., $w_{s,t} = 0$, and it is optimal to set $w_t = 0$, unless this makes it impossible to satisfy (10).[18]

The contract changes dramatically if the board additionally seeks truthful reporting (part (ii) of Proposition 1), as the signals about the manager's human capital investments are not the cash flows, but the manager's reports. Thus, incentivizing the manager to invest in firm-specific human capital requires that she is paid more for reporting θ_G. The problem is that the incentive constraint (7) for truthfully reporting a bad fit θ_N requires that the manager is paid severance $w_{s,t}$ that compensates her for the wage she would forgo upon dismissal. This limits the ability to incentivize investment in firm-specific human capital (condition (8)) by backloading compensation into the future. This is easily seen in the special case in which the correlation between periods is weak (i.e., $e(\theta_G) \approx e(\theta_N)$). Then, the manager's incentive to lie about her fit is especially high, as her continuation payoff beyond period t is practically the same regardless of θ_t: $U_{t+1}(\theta_G, \mathbf{w}) \approx U_{t+1}(\theta_N, \mathbf{w})$. In this case, deferring compensation can create no incentives in t, and investing in firm-specific human capital can be achieved only by promising a positive bonus $\Delta w_t > 0$ (plug in (7) into (8)). The bonus must potentially be even higher if the board has not sought truthful reporting and has deferred the manager's bonus in the preceding period(s). In particular, it must be high enough that the manager invests in firm-specific human capital even without being paid in such periods. Finally, recall that the source of the manager's rent is the need to incentivize investments in firm-specific human capital. Since the manager is not concerned with forgoing future rent in her retirement period T, incentivizing truthful reporting in that period brings no additional cost.

3.2 The Board's Choice of Truthful Reporting, Contract Horizon, and Turnover

The objective of maximizing the board's payoff (5) can be stated now as determining the optimal reporting policy for every period, subject to (12)–(17). In this problem, the two continuation payoffs, $U_{t+1}(\theta_G, \mathbf{w})$ and $U_{t+1}(\theta_N, \mathbf{w})$, time, and the history of periods with incentives for truthful reporting play the role of state descriptors, completely characterizing the dynamics of the manager's contract. Since a manager's continuation payoffs at retirement are zero ($U_{T+1}(\theta_T, \cdot) = 0$), her payoff can be derived recursively in every period for any truthful reporting policy that the board can choose from. This can be used then to calculate

[18] This is for completeness only. As noted, if (10) were binding, the board would choose truthful reporting.

the board's payoff for any such policy and to select the one that maximizes (5).

3.2.1 Illustration of Main Results With Two-Period Employment

Suppose that managers retire after two periods, i.e., $T = 2$, and assume, for illustration, that the primitives take the values from the description of Figure 3. Suppose, first, that the board seeks truthful reporting in both periods and offers $\{w_1, w_2\} = \{0, 0\}$, $\{\Delta w_1, \Delta w_2\} = \{5.1, 5.1\}$, and $\{w_{s,1}, w_{s,2}\} = \{1.7, 1.1\}$, which satisfy Proposition 1 for $\overline{U} = 1$. We can calculate the manager's payoff in $t = 1$ and $t = 2$ recursively from

$$U_t(\theta_{t-1}, \mathbf{w}) = \begin{pmatrix} e_t(\theta_{t-1})(w_t + \theta_G \Delta w_t + \delta U_{t+1}(\theta_G, \mathbf{w})) \\ +(1 - e_t(\theta_{t-1}))\left(w_{s,t} + \sum_{j=t}^{T} \delta^{j-t}\overline{U}\right) - c \end{pmatrix} \text{ if tr.rep. in } t. \tag{18}$$

This gives us $U_2(\theta_G, \mathbf{w}) = 2.1, U_2(\theta_N, \mathbf{w}) = 1.7$, and $U_1(\mathbf{w}) = 3.7$ (note that $e_t(\theta_{t-1}) = e_1$ in $t = 1$ and that $U_{T+1} = 0$). We can now verify that condition (6) is lax, while (7) and (8) are satisfied with equality. However, if the manager's outside option is lower, $\overline{U} = 0$, the incentive constraint (7) to truthfully report θ_N would not be satisfied, as then the manager would be more reluctant to lose her job. To satisfy (7), we would then have to increase the manager's severance pay to $\{w_{s,1}, w_{s,2}\}_{\overline{U}=0} = \{3.7, 2.1\}$. Two insights follow immediately. First, severance pay is needed to compensate the manager for forgoing future rent when truthfully reporting θ_N. Severance pay must be higher in $t = 1$ than in $t = 2$, as the manager needs to be compensated for forgoing rent in two periods. Second, *a manager with a higher outside option requires less severance pay to be honest* and might be preferable to the board.

Suppose now that the board does not seek truthful reporting in the first period and offers $\{w_1, w_2\} = \{0, 0\}$, $\{\Delta w_1, \Delta w_2\} = \{0, 12.4\}$, and $\{w_{s,1}, w_{s,2}\} = \{0, 4\}$. In particular, note that this contract offers no severance pay in $t = 1$ and that the first-period bonus is deferred and paid conditional on the firm also performing well in $t = 2$. By plugging in, we can now verify that conditions (6)–(10) are satisfied. Furthermore, the manager's payoff in $t = 2$ can be calculated again from (18) as $U_2(\theta_G, \mathbf{w}) = 6.4$ and $U_2(\theta_N, \mathbf{w}) = 5.6$. The payoff in $t = 1$ can be recursively calculated from

$$U_t(\theta_{t-1}, \mathbf{w}) = \begin{pmatrix} w_t + (\theta_N + e_t(\theta_{t-1})\Delta\theta)\Delta w_t \\ +\delta E_{\theta_{t-1}}\left[U_{t+1}^e(\theta_t, \mathbf{w})\right] - c \end{pmatrix} \text{ if no tr.rep. in } t. \tag{19}$$

If $\overline{U} = 1$, $U_1(\mathbf{w}) = 2.7$, which is 27% lower than with truthful reporting in both periods. Thus, there is a *trade-off between lowering the manager's rent and risking an inefficient replacement* decision by evaluating the manager based on the firm's cash flows.

$\{x, \Delta x\}$	$\overline{U}$	$V^*_{n,r}$	$V^*_{r,r}$
$\{100, 25\}$	0	2,246	2,236
	1	2,240	2,241
$\{1000, 250\}$	0	22,858	22,974
	1	22,852	22,980

Table 1: Board's expected payoff. The table compares the board's expected payoff $V^*_{r,r}$ from offering incentives for truthful reporting in both periods and from abstaining from offering such incentives in the first period, $V^*_{n,r}$, for different values of $x, \Delta x$ and $\overline{U}$.

Table 1 compares the board's residual claim (5) when sticking to each of the two strategies described above for every new hire. It illustrates some of the paper's main results. (i) *If the board seeks truthful reporting, it may prefer hiring a manager with a higher outside option*, especially if pursuing this policy in both periods (last column) (ii) *A lower outside option makes relying on performance evaluation (rather than on incentives for truthful reporting) more attractive*, especially if the firm is small ($V^*_{n,r} > V^*_{r,r}$ if $x = 100$, $\Delta x = 25$).[19] The rest of the paper generalizes and discusses the intuition for these insights. It further shows that, with longer time to retirement ($T > 2$), even large firms will regularly abstain from offering incentives for truthful reporting (Figure 3). Readers interested mainly in the implications of the analysis can skip to Implications 1–6.

3.2.2 The Cost-Benefit Mismatch of Incentivizing Truthful Reporting

The first question that should be clarified is why the board might choose not to offer incentives for truthful reporting in all periods. The first-best replacement policy would require seeking truthful reporting in every period and hiring the manager with the longest time until retirement T, as the positive correlation of fit between periods ($e(\theta_G) > e_1$) implies that a manager who stays a good fit should be kept as long as possible. However, this policy might require giving up too much information rent to the manager. Using that, following truthful reporting in $t-1$, the condition that the manager invests in firm-specific human capital in t is binding (cf. (8)), the manager's expected payoff in period t can be stated as

$$U_t(\theta_{t-1}, \mathbf{w}^r) = \left(\frac{e_t(\theta_{t-1})}{e_t(\theta_G)} - 1\right) c + w^r_{s,t} + \sum_{j=t}^{T} \delta^{j-t}\overline{U}, \tag{20}$$

where the superscript r stands for truthful reporting. Thus, offering the manager severance pay as an incentive to admit being a bad fit leads to information rent of proportionate size.

[19] The derivations are in the Appendix. If $\overline{U} = 0$, we need to change $\Delta w_2 = 10.9$ and $w_{s,2} = 4.4$ and we have $U_1(w) = 1.8$.

Suppose, now, that the board seeks truthful reporting in every period. Plugging in for the severance pay $w^r_{s,t}$ (while neglecting for a moment its zero lower bound) the manager's expected rent (11) when she is hired in period one is

$$\begin{aligned}\nu_1(\mathbf{w}^r) &= \frac{\theta_N c}{e_1\Delta\theta} + \delta\frac{\theta_G U_2(\theta_N, \mathbf{w}^r) - \theta_N U_2(\theta_G, \mathbf{w}^r)}{\Delta\theta} - \sum_{j=1}^{T}\delta^{j-1}\overline{U} \\ &= \frac{\theta_N c}{e_1\Delta\theta} - \overline{U} - \delta\frac{\theta_G}{\Delta\theta}\left(\frac{\Delta e_2}{e_2(\theta_N)}\right)c + \delta w^r_{s,2} \\ &= \frac{\theta_N c}{e_1\Delta\theta} - \overline{U} + \sum_{j=2}^{T}\delta^{j-1}\left(\left(\frac{\theta_N - \theta_G\Delta e_j}{e_j(\theta_G)\Delta\theta}\right)c - \overline{U}\right), \end{aligned} \tag{21}$$

where $\Delta e_t = e_t(\theta_G) - e_t(\theta_N)$ captures the persistence of fit between two periods.[20] This persistence implies that a bad fit today is likely to be a bad fit tomorrow, which decreases the manager's incentives to lie if her fit is θ_N and, hence, the rent that needs to be promised to her.

Proposition 2 *The board follows the first-best policy of hiring the manager with the longest time until retirement and incentivizing truthful reporting in all periods if*

$$\frac{\theta_N - \theta_G\Delta e_t}{e_t(\theta_G)\Delta\theta} \leq \overline{U} \text{ in all } t. \tag{22}$$

If the reverse inequality holds, the board may hire a manager with less time to retirement (lower T) or abstain from seeking truthful reporting in some periods. In these periods, turnover is triggered by low cash flow performance.

If condition (22) does not hold, the manager can expect an "efficiency" wage above her outside option in all periods until she is replaced. This makes her reluctant to truthfully report θ_N, especially in light of the fact that, by lying, she could stay on the job even following low cash flow realizations. Thus, the more periods she can potentially stay on the job, the higher the severance package needed to incentivize truthful reporting must be (dashed line in Figure 3). This results in a mismatch between the costs and benefits of offering incentives for truthful reporting until T. With such a policy, severance pay compensates the manager as if she would stay until retirement in T. However, the board does not realize the benefit of truthful reporting in some period t if the manager has been replaced by that point. Since the likelihood that the manager remains a good fit until retirement dwindles in T, it may be optimal not to seek truthful reporting at least in some periods (risking an inefficient retention or replacement decision) or to hire an older manager.

[20] Note that in the first period, $\Delta e_1 = 0$ and so the first term in (20) is zero.

3.2.3 Relying on Performance Measures Instead of Truthful Reporting

Suppose in all that follows that the first-best condition (22) is not satisfied. Not offering incentives for truthful reporting comes with the compelling advantage of reducing the rent that needs to be promised to the manager. This was already illustrated in Section 3.2.1 and holds generally.

Proposition 3 *Introducing a period in which the board does not offer incentives for truthful reporting in a sequence of periods with truthful reporting decreases the manager's rent not only in that period, but also in all preceding periods.*

A period in which the manager is judged based only on performance cuts through her ability to stay with the firm by lying about her fit in all periods. The prospect of lower future rent, in turn, implies that the manager is truthful about her fit even when offered lower severance pay in all preceding periods. Thus, by choosing whether to seek truthful reporting, the board faces a trade-off between making an efficient replacement decision and minimizing the manager's rent.

This trade-off is at the heart of Figure 3, which plots the optimal contract offered to the manager when the board determines the optimal sequence of truthful reporting periods that maximizes its expected payoff, as well as a contract that always offers incentives for truthful reporting. Solving for the optimal truthful reporting policy for $T > 2$ is not tractable. Numerically, this can easily be done by recursively deriving the manager's payoff from (18)–(19) in every period for any truthful reporting policy that the board can choose from, and then selecting the policy that maximizes (5).

Figure 3 illustrates that the manager's severance pay (and, thus, rent) is higher, the more periods with truthful reporting (and, thus, rent extraction) she has ahead of herself (Proposition 2). However, by introducing a period without incentives for truthful reporting, the board not only saves on the cost of offering severance pay in that period, but can also afford to offer lower severance pay in all preceding periods (Proposition 3). Figure 3 illustrates that it is optimal to rely only on performance measures to infer the manager's fit in periods six and eleven, even though her wage is only a very small fraction of the firm's value. This is because such a policy halves the manager's expected rent, while exposing the board to the relatively mild risk of making a wrong replacement decision in these periods, which the incumbent manager reaches with only 10% and 1% probability, respectively. Finally, note that the manager is promised a non-trivial bonus for achieving high cash flows in periods with truthful reporting, with bonus increases following periods without reporting to account for deferrals in such periods (Proposition 1).

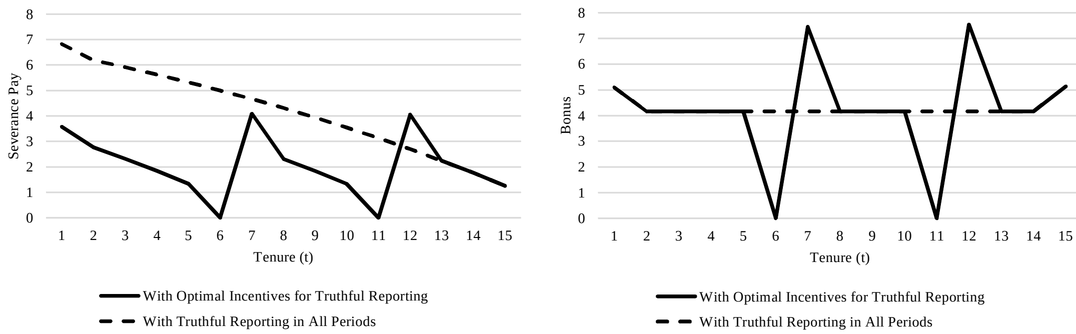

Figure 3: Optimal Incentives for Truthful Reporting vs. Truthful Reporting in All Periods. The dips in bonus and severance pay correspond to periods in which the board does not offer incentives for truthful reporting, but relies on firm performance. In terms of implementation, the dips would correspond to the end dates of renewable fixed-term contracts. The simulations are performed with $T = 15$, $e_1 = 0.55$, $e(\theta_G) = 0.65$, $e(\theta_N) = 0.45$, $c = 1$, $\theta_G = 0.7$, $\theta_N = 0.4$, $\overline{\theta} = 0.48$, $\delta = 0.95$, $x = 1000$ and $\Delta x = 250$, $\overline{U} = 1.25$. The figure illustrates that, even though the manager's pay is small relative to the firm's size, it is optimal not to elicit the manager's private information at regular intervals.

3.2.4 Implementation with Renewable Fixed-Term Contracts

A simple way to implement policies that alternate between offering and not offering incentives for truthful reporting (and, thus, severance pay) in regular intervals is with renewable fixed-term contracts (i) that stipulate severance pay for premature terminations as a multiple of the manager's bonus and remaining tenure;[21] (ii) for which severance is not paid when not renewing the manager's contract on renewal dates; and (iii) for which, absent termination, the contract continues as originally agreed upon.

This implementation is suitable, as the board can choose the end of the contract to coincide with a period in which it would be optimal not to seek truthful reporting. Specifically, the manager would have no incentives to truthfully report a deteriorating fit close to a renewal date, at which point the board could replace her without severance pay. In such cases, the board would have to judge the manager's fit based solely on her performance, and would renew the CEO's contract (according to the initial agreement) only if it is happy with that performance. Thus, both the board's and the manager's strategies would mimic those in a period without incentives for truthful reporting.

About 45% of CEO contracts used in practice are fixed-term contracts that renew automatically, unless one of the parties objects (Gillan et al., 2009). The majority of such contracts contain ex ante severance agreements (Rau and Xu, 2013; Brown, 2015). In line

[21] Termination must be triggered by the board, as the CEO cannot claim severance pay without a "good reason," such as a change of duty, diminution of pay, or relocation, (Rau and Xu, 2013). This is not in conflict with the model, as reporting a bad fit triggers such termination.

with Proposition 1, these severance agreements are usually a multiple of managers' salary and bonus and can depend on their remaining tenure. Moreover, boards pay more attention to performance, and the relation between turnover and performance is stronger close to renewal dates (Liu and Xuan, 2016; Cziraki and Groen-Xu, 201). This is in line with the prediction that the board relies on the firm's performance rather than on truthful reporting in renewal periods. Thus, the paper provides a simple intuition for the widespread use of such renewable fixed-term contracts.

3.2.5 Determinants of Contract Horizon

The next step is to highlight the wide-ranging implications of some of the factors determining the length of renewable contracts and the frequency of renewal. This requires analyzing the determinants of how often the board will evaluate the manager's fit based on the firm's performance rather than relying on severance agreements to stimulate the manager to admit to no longer being a good fit.

Lemma 1 *Take any two contracts seeking a different level of truthful reporting from a manager. (i) The contract for which the manager's rent decreases more strongly in $\overline{U}$ becomes more attractive as $\overline{U}$ increases. (ii) The contract for which the firm's expected cash flows increase more strongly in Δx becomes more attractive as Δx increases.*

Focusing, for now, on the first part of Lemma 1, Figure 4 implies that the manager's rent decreases more strongly in $\overline{U}$ when there is more truthful reporting. This is only natural, since a higher outside option makes the manager less reluctant to report that her fit is θ_N and to seek alternative employment. Hence, the rent that the board needs to promise her to reveal such a fit is lower, making this policy more attractive.

This intuition can be derived analytically for two special cases. In the first, a short-term shock affects the manager's outside option only in the first period of the employment relation. In the second, managers retire after two periods (i.e., $T = 2$). In this case, it can also be derived that incentivizing truthful reporting becomes more attractive as Δx increases, which parallels the second part of Lemma 1. This is because for larger firms and firms with higher growth prospects, there is more at stake from having the right manager in charge.

Proposition 4 *(i) Consider a short-term shock that increases the manager's outside option only in the first period. Such a shock makes seeking truthful reporting in the first period more attractive.*

(ii) Suppose that managers retire after two periods $T = 2$. Incentivizing truthful reporting becomes more attractive for the board as $\overline{U}$ and Δx increase.

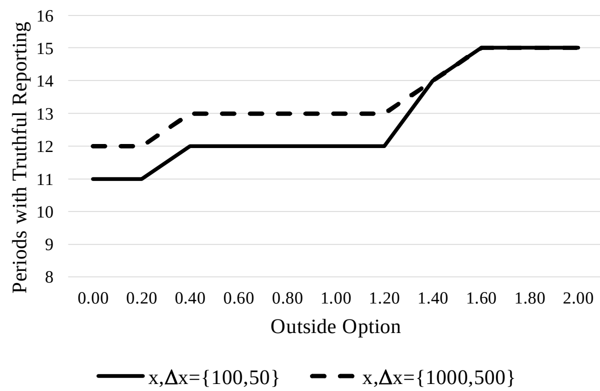

Figure 4: Comparative Statics in $\overline{U}$ and Δx. The figure plots how the number of periods in which the board incentivizes truthful reporting changes in $\overline{U}$ and Δx. The primitives are the same as in Figure 3, with $T = 15$ and $\overline{U}$ taking values from zero to two and $\{x, \Delta x\}$ taking values $\{100, 25\}$ and $\{1000, 250\}$, respectively. The figure illustrates that the board seeks more truthful reporting when the manager's outside option is higher and when the firm is larger.

Implication 1 ***Determinants of contract horizon and turnover-perfor-mance sensitivity**: (i) The length of renewable fixed-term contracts will be shorter and turnover-performance sensitivity will be higher when managers' outside options are low. (ii) Larger firms and firms with better growth options will offer contracts with longer horizons. This will go hand-in-hand with higher (average) severance pay and a higher likelihood of a timely turnover preempting underperformance.*

To the extent that managers' outside options are lower in industry downturns — e.g., because more firms are going bankrupt; fewer firms are being started; and there is more competition among the labor force for available positions — an immediate corollary is:

Implication 2 ***Turnover in downturns**: (i) Boards rely less on truthful reporting and more on performance evaluation in industry-wide downturns. This offers an alternative explanation (compared to a lack of relative performance evaluation) for the stronger turnover-performance sensitivity in downturns.*[22] *(ii) The higher reliance on performance measures to infer the manager's fit increases the risk of making a wrong retention or replacement decision, which might exacerbate downturns.*

We return to these determinants of contract horizon in Section 3.3, as they shape also other key aspects of an employment relation.

[22]Somewhat loosely speaking, one could think of this model as one in which abnormal and relative performance have already been filtered out.

3.2.6 Renegotiations and Hiring Older Managers

The preceding results show that it might be optimal for the board to abstain from offering incentives for truthful reporting in some periods in order to limit the manager's rent. However, once reaching such a period, the board could offer to renegotiate the existing contract and restructure it in a way that offers incentives for truthful reporting, as this would increase the firm's cash flows. There are several reasons why pursuing this strategy can become optimal ex post, even if it is not optimal ex ante. First, one benefit of not seeking truthful reporting in t is that it decreases the manager's rent not only in t, but also in all preceding periods. However, the latter benefit ceases to exist once both parties arrive in t.[23] Second, there is scope for renegotiations after the manager has invested in firm-specific human capital and the cost c in period t is sunk, since, at this point, incentivizing firm-specific human capital investment is no longer an objective. As is usual, though such renegotiations might be beneficial ex post, they reduce the board's expected payoff ex ante.

A commitment to avoid such renegotiations can be achieved in the present context. In particular, if the board engages in renegotiations once, all future managers would expect the same and demand only renegotiation-proof contracts from then on. Since the firm is infinitely-lived, this "trigger strategy" would prevent the board from deviating from its commitment to avoid renegotiations if δ is sufficiently high.

Still, discussing the extreme case in which the potential for renegotiations forces the board to offer a contract that incentivizes truthful reporting in all periods is also interesting. It has clear implications for the manager's age, as then the contract length coincides with her time to retirement T.[24,25] In this case, the board's problem is equivalent to that discussed in Proposition 2. Then, if the first-best condition (22) is not satisfied, the board might have to choose a manager with less time to retirement, as her rent might otherwise become too high.

To gain some intuition about the factors affecting whether to hire a younger or on older manager, it is helpful to consider again condition (22). It suggests that a higher per-period outside option $\overline{U}$ decreases the manager's rent. A more attractive outside opportunity makes the manager less reluctant to leave the firm, which reduces the severance pay she needs to be promised to truthfully report her fit. Thus, given that the cost of employing the manager

[23] As in Malcomson (2016), Definition 1 does not rule out that there might be scope for renegotiations if the manager's type has not been revealed.

[24] There are no clear predictions regarding T from the previous section except that larger firms (in the sense of higher Δx) tend to choose higher T.

[25] Indeed, renegotiations leading to increases in managers' severance pay are common in practice. Among the cases in which there was a clear conflict with the board, 42% of the time, the original contracts were renegotiated, and the departing manager received substantial severance pay (Goldman and Huang, 2015). For similar evidence from firms entering bankruptcy, see Eckbo et al. (2016).

longer is lower, while the benefit is unchanged, the board finds it optimal to offer contracts with longer horizons. Also similar to before, longer contracts are optimal if Δx is higher, as there is more at stake from holding on longer to a manager who is a good fit.

Proposition 5 *Suppose that the first-best condition (22) is not satisfied and that the board incentivizes truthful reporting in all periods. Then, the board chooses a manager with a longer time to retirement T if her per-period outside option $\overline{U}$ and the cash flow upside Δx are higher.*

Implication 3 ***CEO age***: *Suppose that the first-best condition (22) is not satisfied and that the board incentivizes truthful reporting in all periods. (i) Then, the board needs to offer younger managers higher severance pay. (ii) Furthermore, the board prefers hiring an older manager (i.e., T is lower) if $\overline{U}$ and Δx are lower. This implies that older managers are preferred in industry downturns and by firms with low growth potential.*

While the second part of Implication 3 has not been tested, the first part finds empirical support in Rau and Xu (2013).

3.3 Discussion and Extensions

3.3.1 Hiring Managers with Better Outside Options

Suppose that the pool of potential managers differs according to their success likelihood e_t and their outside options $\overline{U}$. Furthermore, assume that condition (22) is not satisfied for any $\overline{U}$ and e_t. All remaining parameters of the model remain the same.

Clearly, if all information were common knowledge, the board would prefer hiring the manager with the highest likelihood e_t of being a good fit and with the lowest outside option $\overline{U}$. However, when the manager's fit is her private information, and the board must offer incentives for truthful reporting, it might have to pay the manager an "efficiency wage" that is above her outside option. In this case, hiring a manager with a higher outside option could be beneficial since it reduces the need for generous severance pay $w_{s,t}$. This can be seen especially clearly in the extreme case in which the board stimulates truthful reporting in all periods (as in Section 3.2.6). Then, the board strictly prefers hiring a manager with a higher outside option. Thus, if the board faces the choice of selecting between a manager that has a higher likelihood e_t of being a good fit or one with a higher outside option $\overline{U}$, it might prefer the manager with the higher outside option.

Proposition 6 *(i) Take any given truthful reporting policy that the board is seeking to implement. If $\left(p\left(\mathbf{w}\right)-\delta^{T}\right)>\left(1-\delta\right)\frac{\partial}{\partial\overline{U}}U_{1}\left(\mathbf{w}\right)$, the board's payoff increases in the manager's* *outside option $\overline{U}$ for that policy.*[26] *(ii) When offering incentives for truthful reporting in all periods, the board always prefers hiring a manager with a higher outside option (as long as the manager extracts rent). (iii) Furthermore, the board might prefer a manager with a higher outside option $\overline{U}$ to one who is more likely to be a good fit (higher e_t).*

Investments in general human capital, such as taking board seats at different firms, increase CEOs' outside options. Thus, it might seem surprising that boards tolerate such behavior given that it might distract managers, and they could use it as a bargaining chip to get a higher salary. However, CEOs rarely leave the firm to become CEOs elsewhere (Fee and Hadlock, 2004), and their labor income declines, on average, by 40% following termination (Nielsen, 2017). Thus, if CEOs are paid above their outside options anyway, the more pertinent effect might be:

Implication 4 ***Investments in general human capital****: Investments in general human capital, increasing a manager's outside options, make it cheaper to offer incentives for her to be truthful about her fit. Thus, boards might tolerate such investments even if they come at the expense of investments in firm-specific human capital (lower e_t).*

3.3.2 Outside Options and Experience

Up until now, we assumed that outside options were fixed over time. However, one could imagine that, as managers stay longer with the firm, their reputation in the labor market improves and, as a result, their outside options increase. One of the main points of the present paper is that such increases do not necessarily make the manager more expensive to the firm. In fact, the exact opposite might be the case. If the manager is paid an efficiency wage above her outside option, increases in that outside option imply that it becomes easier to keep the manager honest. Thus, one could expect the board to rely more often on truthful revelation and less often on performance measures to judge the manager's fit. This insight complements Garrett and Pavan's (2012) alternative explanation that the board becomes more tolerant towards lower managerial quality over time. Further relating to the implementation of the optimal contract, we have:

Implication 5 ***Experience****: If a manager's outside options increase with her tenure: (i) The length of renewable contacts would increase with the manager's tenure. (ii)The relation between managerial turnover and firm performance will weaken with the manager's tenure.*

[26] $p(\mathbf{w})$ is defined in the Appendix. Note, however, that a higher $\overline{U}$ might lead the board to choose a different reporting policy.

Another implication of the analysis is that boards will avoid damaging departing CEOs' outside options, as this would necessitate higher severance pay. Indeed, there is evidence that severance pay is higher when firms replace CEOs with a reputation for firm mismanagement (Goldman and Huang, 2015).

Implication 6 ***Reputation****: When replacing CEOs, boards will avoid damaging their reputation, as replacing CEOs with a reputation for mismanagement requires offering higher severance pay.*

3.3.3 Relying on Both Truthful Reporting and Performance Measures

Relying on both truthful reporting and performance measures could be optimal if cash flow realizations brought additional information regarding the manager's fit relative to her signal at the interim date of a period. With more than two cash flow states, this could lead to turnover following low cash flow realizations even when the manager truthfully reports θ_G. However, it would remain true that the more periods with truthful reporting a manager can look forward to, and the lower her outside option, the stronger her incentives to lie are going to be. Thus, the properties of the solution are likely to be similar to those in Figures 3 and 4.

Reliance on both reporting and performance measures could also arise when allowing for mixed strategies. Specifically, it is conceivable that there is an equilibrium in which the manager randomizes between reporting θ_N or θ_G if her fit is θ_N, and the board subsequently randomizes between replacing and keeping the manager depending on the cash flow realization. Such stochastic replacement reduces the manager's on- and off-equilibrium continuation payoffs and, thus, the need for severance pay. However, it still remains true that having a low outside option and having the ability to stay with the firm for more periods, even following low cash flow realizations, increases the incentives to lie. Dealing with the additional complexity of mixed strategies would be an interesting, but challenging, extension, as issues related to learning (potentially over multiple periods) become central to the analysis.[27]

[27] In the presence of limited commitment, randomization generates interesting dynamics even with constant types (Hart and Tirole, 1988; Laffont and Tirole, 1990). Another level of randomization is between incentivizing and not incentivizing truthful reporting in a given period. However, this would require committing to a randomization device, as an ex ante optimal randomization is in general not optimal ex post and vice versa.

4 Conclusion

The paper analyzes optimal contract horizon and turnover in a model in which the agents' fit with the firm evolves over time, and agents are better informed about such changes. The principal can minimize the likelihood that an agent who has become a bad fit will stay with the firm, but this is not always in its best interest, as it would require generous incentive and severance packages. Thus, the principal will sometimes judge an agent's fit based on the firm's performance rather than relying on truthful reporting incentivized through generous severance pay. Such a policy might not preempt bad performance and would lead to inefficient replacement and retention decisions, but it could keep the agent's compensation from growing too large. While pervasive in various settings, a key application for which there is rich evidence is in the context of boards hiring managers.

The main predictions from the analysis, framed in the context of this application are as follows. First, it is optimal to abstain from incentives for truthful reporting in regular intervals, in which case the board judges the manager's fit based on the firm's underperformance. The resulting optimal contract can be implemented with renewable fixed-term contracts, stipulating ex ante severance pay as a multiple of the manager's wage, but allowing to costlessly replace the manager when her term expires—contracts which are widely used in practice (Gillan et al., 2009; Rau and Xu, 2013), but have not been addressed by prior theory.

The paper further offers novel insights regarding the determinants of the length of renewable contracts. A key factor is a manager's outside option. If it is low, the board will offer incentives for truthful reporting less often, and will rely more often on the firm's cash flow performance. That is, renewable contracts will be shorter. This is because in such cases, managers will be especially reluctant to lose their job. One implication of this result is that managers will have less-adequate incentives to reveal a bad fit in industry downturns, which would result in more performance-induced turnover. There is, indeed, evidence for this result, but it has hitherto been interpreted as a lack of relative performance evaluation (Jenter and Kanaan, 2015). Furthermore, the stronger reliance on noisy performance in downturns is more likely to leave firms with managers who are not a good fit, even if the board subsequently seems overly eager to replace the manager in the case of underperformance. This might exacerbate downturns. However, contract length will be longer and, thus, severance pay will be higher for larger firms and firms with better growth prospects.

The manager's age also plays a key role as a commitment device to shorter contracts, especially if there is scope for renegotiations to offer severance pay after all in periods in which the board is supposed to rely on firm performance. Hiring an older manager helps to keep the manager's pay from growing too large, as older managers would need to be offered lower severance packages.

The insight that a higher outside option makes a manager potentially cheaper to replace and employ has several other important implications for employment relations. One is that a board might appear to hire from a "select club," i.e., a manager with high outside options, even if it is unlikely that she is a better fit. Another is that boards might tolerate investments in general human capital even if they come at the expense of firm-specific human capital investments.

References

[1] Aghion, Philippe, and Jean Tirole, 1997, Real and formal authority in organizations, *Journal of Political Economy* 150(1), 1–29.

[2] Aghion, Philippe, and Matthew Jackson, 2016, Inducing leaders to take risky decisions: dismissal, tenure, and term limits, *American Economic Journal: Microeconomics* 8(3), 1–38.

[3] Almazan, Andres, and Javier Suarez, 2003, Entrenchment and severance pay in optimal governance structures, *Journal of Finance* 63(2), 519–547.

[4] Anderson, Ronald W., M. Cecilia Bustamante, Stéphane Guibaud, and Mihail Zervos, 2016, Agency, firm growth, and managerial turnover, *Journal of Finance*, forthcoming.

[5] Bernardo, Antonio, and Ivo Welch, 2001, On the evolution of overconfidence and entrepreneurs, *Journal of Economics and Management Strategy* 10(3), 301–330

[6] Bester, Helmut, and Roland Strausz, 2001, Contracting with imperfect commitment and the revelation principle: the single agent case, *Econometrica* 69(4), 1077–1098.

[7] Brown, Kareen E., 2015, Ex ante severance agreements and earnings management, *Contemporary Accounting Research* 32(3), 897–940.

[8] Crémer, Jacques, 1995, Arm's length relationships, *Quarterly Journal of Economics* 110(2), 275–295.

[9] Cziraki, Peter, and Moqi Groen-Xu, 2015, CEO career concerns and risk taking, Working Paper, LSE and University of Toronto.

[10] Eisfeldt, Andrea L., and Adriano Rampini, 2008, Managerial incentives, capital reallocation, and the business cycle, *Journal of Financial Economics* 87(1), 177–199.

[11] Eisfeldt, Andrea L., and Camelia M. Kuhnen, 2013, CEO turnover in a competitive assignment framework, *Journal of Financial Economics* 109(2), 351–372.

[12] Eckbo, Espen B., Karin S. Thorburn, and Wei Wang, 2016, How costly is corporate bankruptcy for the CEO, *Journal of Financial Economics*, forthcoming.

[13] Fee, C. Edward, and Charles J. Hadlock, 2004, Management turnover across the corporate hierarchy, *Journal of Accounting and Economics* 37(1) 3–38.

[14] Fee, C. Edward, Charles J. Hadlock, Jing Huang, and Joshua R. Pierce, 2015, Robust models of CEO turnover: new evidence on relative performance evaluation, Working Paper, Tulane University, Michigan State University, University of South Carolina, and University of Kentucky.

[15] Felli, Leonardo, and Christopher Harris, 1996, Learning, wage dynamics, and firm-specific human capital, *Journal of Political Economy* 104(4), 838–868.

[16] Garrett, Daniel, and Alessandro Pavan, 2012, Managerial turnover in a changing world, *Journal of Political Economy* 120(5), 879–925.

[17] Gervais, Simon, J. B. Heaton, and Terrance Odean, 2011, Overconfidence, compensation contracts, and capital budgeting, *Journal of Finance* 66(5), 1735–1777.

[18] Gillan, Stuart L., Jay C. Hartzell, and Robert Parrino, Explicit versus implicit contracts: evidence from CEO employment agreements, *Journal of Finance* 64(4), 1629–1655

[19] Goel, Anand M., and Anjan Thakor, 2008, Overconfidence, CEO selection, and corporate governance, *Journal of Finance* 63(6), 2737–2784.

[20] Goldman, Eitan M., and Peggy P. Huang, 2015, Contractual vs. actual separation pay following CEO turnover, *Management Science* 61(5), 1108–1120.

[21] Halac, Marina, 2012, Relational contracts and the value of relationships, *American Economic Review* 102(2), 750–779.

[22] Hart, Oliver D., and Jean Tirole, 1988, Contract renegotiation and Coasian dynamics, *Review of Economic Studies* 55(4), 509–540.

[23] He, Zhiguo, Bin Wei, Jianfeng Yu, and Feng Gao, 2017, Optimal long-term contracting with learning, *Review of Financial Studies*, forthcoming.

[24] Hermalin, Benjamin, and Michael Weisbach, 1998, Endogenously chosen boards of directors and their monitoring of the CEO, *American Economic Review* 88(1), 96–118.

[25] Hertzberg, Andrew, Jose Maria Liberti, and Daniel Paravisini, 2010, Information and incentives inside the firm: evidence from loan officer rotation, *Journal of Finance* 65(3), 795–828.

[26] Huang, Sheng, Johan Maharjan, and Anjan Thakor, 2016, Disagreement-induced CEO turnover, Working Paper, Washington University in St. Louis.

[27] Inderst, Roman, and Holger M. Mueller, 2010, CEO replacement under private information, *Review of Financial Studies* 23(8), 2935–2969.

[28] Innes, Robert D., 1990 Limited liability and incentive contracting with ex-ante choices, *Journal of Economic Theory* 52(1), 45–67.

[29] Jenter, Dirk, and Fadi Kanaan, 2015, CEO turnover and relative performance evaluation, *Journal of Finance* 70(5), 2155–2184.

[30] Jenter, Dirk, and Katharina Lewellen, 2017, Performance-induced CEO turnover, Working Paper, LSE and Darthmouth College.

[31] Jovanovic, Boyan, 1979 a, Job matching and the theory of turnover, *Journal of Political Economy* 87(5), 972–990.

[32] Jovanovic, Boyan, 1979 b, Firm-specific capital and turnover, *Journal of Political Economy* 87(6), 1246–1260.

[33] Laffont, Jean-Jacques, and Jean Tirole, 1990, Adverse selection and renegotiations in procurement, *Review of Economic Studies* 57(4), 597–925.

[34] Lazear, Edward P., 1979, Why is there mandatory retirement, *Journal of Political Economy* 87(6), 1261–1284.

[35] Lazear, Edward P., 2009, Firm-specific human capital: a skill-weights approach, *Journal of Political Economy 117(5), 914–940.*

[36] Levitt, Steven D., and Christopher M. Snyder, 1997, Is no news bad news? Information transmission and the role of "early warning" in the principal agent model, *RAND Journal of Economics* 28(4), 641–661.

[37] Liu, Ping, and Yuhai Xuan, 2016, The contract year phenomenon in the corner office: an analysis of firm behavior during CEO contract renewals, Working Paper, University of Illinois at Urbana-Champaign.

[38] Malcomson, James M., 2016, Relational incentive contracts with persistent private information, *Econometrica* 84(1), 317–346.

[39] Nielsen, Kasper M., 2017 Personal Costs of Executive Turnovers, Working Paper, Hong Kong University of Science and Technology.

[40] Prescott, Edward S., and Richard Townsend, 2006, Private information and intertemporal job assignments, *Review of Economic Studies* 73(2), 531–548.

[41] Rau, P. Raghavendra, and Jin Xu, 2013, How do ex ante severance pay contracts fit into optimal executive incentive schemes?, *Journal of Accounting Research* 51(3), 631–671.

[42] Sannikov, Yuliy, 2008, A continuous-time version of the principal-agent problem, *Review of Economic Studies* 75(3), 957–984.

[43] Stiglitz and Weiss, 1983, Incentive effects of terminations: applications to the credit and labor markets, *American Economic Review* 73(5), 912–927.

[44] Taylor, Lucian A., 2010, Why are CEOs rarely fired? Evidence from structural estimation, *Journal of Finance* 65(6), 2051–2087.

[45] Van den Steen, Eric, 2004, Rational overoptimism (and other biases), *American Economic Review* 94(4), 1141–1151

[46] Van Wesep, Edward D., 2010, Pay (be)for(e) performance: the signing bonus as an incentive device, *Review of Financial Studies* 23(10), 3812–3848.

[47] Van Wesep, Edward D., and Sean Wang, 2014, The prevention of excess managerial risk taking, *Journal of Corporate Finance* 29, 579–593.

[48] Wang, Cheng, 2011, Termination of dynamic contracts in an equilibrium labor market model, *Journal of Economic Theory* 146(1), 74–110.

[49] Wang, Cheng, and Youzhi Yang, 2015, Outside opportunities and termination, *Games and Economic Behavior* 91, 207–228.

Appendix A Omitted Proofs

Proof of Proposition 1. The proof boils down to determining which constraints bind and determine the dynamics of the manager's compensation contract. Here we only sketch the argument, and present the full proof in Appendix B. Consider, first, the case in which the board does not offer incentives for truthful reporting in period $t < T$. Since w_t does not affect the incentives to invest in firm-specific human capital, it is optimally set to zero, unless the interim participation constraint (10) becomes binding. Severance is clearly suboptimal in such a period (i.e., $w_{s,t} = 0$), as it only increases the manager's pay, without improving her effort incentives. Finally, deferring the manager's bonus to periods in which it seeks truthful reporting is a cheaper way of stimulating investment in firm-specific human capital, as a bonus paid out in t has no incentive effects in the following periods. Since the board must keep its promises, this deferral creates an additional constraint for these periods.

In a period in which the board incentivizes truthful reporting, the manager's severance pay $w_{s,t}$ is determined by (7). If this constraint were not binding, it would be optimal to decrease $w_{s,t}$, as this would reduce the manager's pay, while improving her incentives to invest in firm-specific human capital. If there is truthful reporting also in the preceding period, Δw_t is determined from the constraint that the manager invests in firm-specific human capital (8). If, instead, t follows $n \geq 1$ periods in which the board does not offer incentives for truthful reporting (from $t-n$ to $t-1$), the bonus and respective continuation payoffs in t must be high enough to satisfy (9) in the preceding n periods (for which $\Delta w_{t-n} = ... = \Delta w_{t-1} = 0$). We state this condition as $\Delta w_t \geq \Delta w_{t-n}^{-1}(0)$. Thus, Δw_t is determined by the more stringent of condition (8), $\Delta w_t \geq \Delta w_{t-n}^{-1}(0)$, and $\Delta w \geq 0$. The technical derivations bring little further insights and are, thus, relegated to Lemma B.2 in Appendix B.

It remains to argue that the board always seeks truthful reporting in the manager's retirement period T. Observe, first, that if the board seeks truthful reporting period T, we have

$$\begin{aligned}\Delta w_T &= \max\left\{0, \Delta w_{T-n}^{-1}(0), \frac{c}{e_T(\theta_G)\Delta\theta}\right\}\\ w_{s,T} &= \max\left\{0, \theta_N \Delta w_T - \overline{U}\right\}.\end{aligned}$$

Instead, if the board seeks no truthful reporting in the final period, it can no longer delay payments, and it must offer a payment that satisfies (9)–(10), while also making it optimal to set $\Delta w_t = 0$ in the immediately preceding periods without truthful reporting (in case there are such periods). Thus,

$$\begin{aligned}\Delta w_T &= \max\left\{0, \Delta w_{T-n}^{-1}(0), \frac{c}{e_T(\theta_G)\Delta\theta}\right\}\\ w_T &= \max\left\{0, \overline{U} - \theta_N \Delta w_T\right\}.\end{aligned}$$

Plugging into the manager's payoff U_T, we obtain that this payoff is identical in both cases. From (12)–(17), this implies that the manager's payoff is the same in all $t < T$ regardless of the truthful reporting policy in T. However, the board's payoff is higher with truthful reporting in T, making this policy optimal. **Q.E.D.**

Proof of Proposition 2. In what follows, Step 1 introduces some notation, and Step 2 argues to a contradiction.

Step 1. *Notation.* Let the likelihood that the manager retains her job in period t depending on whether there is truthful reporting and depending on the manager's fit from the previous period be defined as

$$\mathbf{e}_t = \begin{cases} \begin{array}{ccc} \text{if } t=1 & \text{if } 1<t\leq T & \\ \begin{pmatrix} e_1 & 0 \end{pmatrix} & \begin{pmatrix} e_t(\theta_G) & 0 \\ e_t(\theta_N) & 0 \end{pmatrix} & \text{if tr.rep.} \\ \begin{pmatrix} e_1\theta_G & (1-e_1)\theta_N \end{pmatrix} & \begin{pmatrix} e_t(\theta_G)\theta_G & (1-e_t(\theta_G))\theta_N \\ e_t(\theta_N)\theta_G & (1-e_t(\theta_N))\theta_N \end{pmatrix} & \text{otherwise} \end{array} \end{cases}.$$

The vector/matrix representation will be useful to minimize notation. Analogously, let the probability of replacement in any given period be defined as

$$\mathbf{p}_t = \begin{cases} \begin{array}{ccc} \text{if } t=1 & \text{if } 1<t\leq T & \\ 1-e_1 & \begin{pmatrix} 1-e_t(\theta_G) \\ 1-e_t(\theta_N) \end{pmatrix} & \text{if tr.rep.} \\ \begin{array}{c} e_1(1-\theta_G) \\ +(1-e_1)(1-\theta_N) \end{array} & \begin{pmatrix} e_t(\theta_G)(1-\theta_G)+(1-e_t(\theta_G))(1-\theta_N) \\ e_t(\theta_N)(1-\theta_G)+(1-e_t(\theta_N))(1-\theta_N) \end{pmatrix} & \text{otherwise} \end{array} \end{cases}.$$

We can define now the discounted likelihood of replacement (and, thus, of obtaining V^*

from hiring a new manager) over the course of the entire potential employment relation (for $T > 2$) as[28]

$$p(\mathbf{w}) := \delta \mathbf{p}_1 + \sum_{i=2}^{T-1} \delta^i \Pi_{k=1}^{i-1} \mathbf{p}_i \mathbf{e}_k + \delta^T \Pi_{k=1}^{T-1} \mathbf{e}_k \mathbf{1}, \tag{A.1}$$

which corresponds to $\mathbb{E}\left[\sum_{i=1}^{T} \delta^{i-1} \widetilde{q}_i\right] \delta$ in expression (5). Note that the replacement probability in the manager's retirement period T is $\mathbf{1} = (1; 1)$. Similarly, we can define the expected amount that the outgoing manager would be paid by the outside labor market upon her dismissal (for $T > 2$) as[29]

$$h\left(\overline{U}, \mathbf{w}\right) := \delta \mathbf{p}_1 \sum_{j=2}^{T} \delta^{j-2} \overline{U}_j + \sum_{i=2}^{T-1} \left(\delta^i \Pi_{k=1}^{i-1} \mathbf{p}_i \mathbf{e}_k \sum_{j=i+1}^{T} \delta^{j-i-1} \overline{U}_j \right), \tag{A.2}$$

which corresponds to $\sum_{i=1}^{T} \delta^{i-1} \overline{U} - \mathbb{E}\left[\sum_{i=1}^{T} \delta^{i-1} q_i \overline{U}\right]$ in expression (5). Since $\overline{U}$ is the same in all periods, we obtain

$$\begin{aligned} h\left(\overline{U}, \mathbf{w}\right) \ : \ &= \frac{\overline{U}}{1-\delta} \left(\delta \mathbf{p}_1 \left(1 - \delta^{T-1}\right) + \sum_{i=2}^{T-1} \delta^i \Pi_{k=1}^{i-1} \mathbf{e}_k \mathbf{p}_i \left(1 - \delta^{T-i}\right) \right) \\ &= \frac{\overline{U}}{1-\delta} \left(-\mathbf{p}_1 \delta^T + \underbrace{\delta \mathbf{p}_1 + \sum_{i=2}^{T-1} \delta^i \Pi_{k=1}^{i-1} \mathbf{e}_k \mathbf{p}_i}_{p(\mathbf{w}) - \delta^T \Pi_{k=1}^{T-1} \mathbf{e}_k \mathbf{1}} - \delta^T \sum_{i=2}^{T-1} \Pi_{k=1}^{i-1} \mathbf{e}_k \mathbf{p}_i \right) \\ &= \frac{\overline{U}}{1-\delta} \left(-\mathbf{p}_1 \delta^T + p(\mathbf{w}) - \delta^T \Pi_{k=1}^{T-1} \mathbf{e}_k \mathbf{1} - \delta^T \left(\mathbf{e}_1 - \Pi_{k=1}^{T-1} \mathbf{e}_k \right) \mathbf{1} \right) \\ &= \frac{\overline{U}}{1-\delta} \left(-\delta^T + p(\mathbf{w}) \right). \end{aligned} \tag{A.3}$$

Denoting further $s(\mathbf{w}) := \mathbb{E}\left[\sum_{i=1}^{T} \delta^{i-1} q_i (x_i - c)\right]$ in expression (5) and expressing $U_1(\mathbf{w})$ as $\nu_1(\mathbf{w}) + \sum_{j=1}^{T} \delta^{j-1} \overline{U}_j$ using (11), the board's equilibrium expected payoff in period one (5) can be stated as

$$V^* = -\nu_1(\mathbf{w}) - \sum_{j=1}^{T} \delta^{j-1} \overline{U}_j + s(\mathbf{w}) + h\left(\overline{U}, \mathbf{w}\right) + p(\mathbf{w}) V^*, \tag{A.4}$$

[28] For $T = 1$, we have $p(\mathbf{w}) = \delta$, and for $T = 2$, we have $p(\mathbf{w}) = \delta p_1 + \delta^2 \mathbf{e}_1 \mathbf{1}$.

[29] To be precise, in case the manager reveals that she is a bad fit, she receives $\overline{U}$ from the outside labour market in the period in which she is fired, but the firm hires a new manager for $\overline{U}$ for the remainder of the period, and the two terms cancel out in board's expected payoff. For $T = 1$, $h(\mathbf{w}) = 0$; for $T = 2$, $h(\mathbf{w}) = \delta p_1 \overline{U}$.

Using (A.3), we can simplify (A.4) to

$$V^* = \frac{s(\mathbf{w}) - v_1(\mathbf{w})}{1 - p(\mathbf{w})} - \frac{\overline{U}}{1-\delta}. \tag{A.5}$$

The functional dependence on $\mathbf{w}$ makes explicit that p, h, and s depend on the board's truthful reporting policy, since the replacement probabilities depend on this policy.

Step 2. *Optimality of Hiring an Older Manager or Abstaining from Incentives for Truthful Reporting.* We argue to a contradiction. Suppose that the board incentivizes truthful reporting in all periods. We have

$$\begin{aligned} s(\mathbf{w}) &= x + \left(\overline{\theta} + e_1\left(\theta_G - \overline{\theta}\right)\right)\Delta x - c \\ &\quad + \left(\delta e_1 + \sum_{j=2}^{T-1} \delta^j e_1 \Pi_{i=2}^{j} e_i\left(\theta_G\right)\right)\left(x + \left(\overline{\theta} + e\left(\theta_G\right)\left(\theta_G - \overline{\theta}\right)\right)\Delta x - c\right) \\ &= x + \left(\overline{\theta} + e_1\left(\theta_G - \overline{\theta}\right)\right)\Delta x - c \qquad \text{(A.6)} \\ &\quad + \left(\delta e_1 \frac{\left(1 - e\left(\theta_G\right)^{T-1}\delta^{T-1}\right)}{1 - e\left(\theta_G\right)\delta}\right)\left(x + \left(\overline{\theta} + e\left(\theta_G\right)\left(\theta_G - \overline{\theta}\right)\right)\Delta x - c\right). \end{aligned}$$

where, given the Markov structure, it is without loss to omit the subscripts of $e(\theta_G)$. Furthermore

$$\begin{aligned} 1 - p(\mathbf{w}) &= 1 - \left(\delta\left(1 - e_1\right) + \sum_{j=2}^{T-1} \delta^j e_1 e\left(\theta_G\right)^{j-2}\left(1 - e\left(\theta_G\right)\right) + \delta^T e_1 e\left(\theta_G\right)^{T-2}\right) \\ &= (1-\delta)\left(1 + \frac{\delta e_1\left(1 - e\left(\theta_G\right)^{T-1}\delta^{T-1}\right)}{1 - e\left(\theta_G\right)\delta}\right) \end{aligned} \tag{A.7}$$

Plugging in for $s(\mathbf{w})$, $p(\mathbf{w})$, as well as for $h(\mathbf{w})$ from (A.3) and $v_1(\mathbf{w})$ from (21), (A.5) becomes

$$V^* = \frac{\left(\begin{array}{c} x + \left(\overline{\theta} + e_1\left(\theta_G - \overline{\theta}\right)\right)\Delta x - c \\ + \left(\delta e_1 \frac{\left(1 - e(\theta_G)^{T-1}\delta^{T-1}\right)}{1 - e(\theta_G)\delta}\right)\left(x + \left(\overline{\theta} + e\left(\theta_G\right)\left(\theta_G - \overline{\theta}\right)\right)\Delta x - c\right) \\ - \left(\frac{\theta_N c}{e_1 \Delta\theta} - \overline{U} + \frac{\delta - \delta^T}{1-\delta}\left(\left(\frac{\theta_N - \theta_G \Delta e_t}{e_t(\theta_G)\Delta\theta}\right)c - \overline{U}\right)\right) \end{array}\right)}{(1-\delta)\left(1 + \frac{\delta e_1\left(1 - e(\theta_G)^{T-1}\delta^{T-1}\right)}{1 - e(\theta_G)\delta}\right)} - \frac{\overline{U}}{1-\delta}.$$

It is now sufficient to show that increasing $T \to \infty$, can make the board's expected payoff

V^* negative. This is the case if

$$\begin{aligned} & \left(\left(\frac{\theta_N - \theta_G \Delta e_t}{e_t(\theta_G)\Delta\theta}\right) c - \overline{U}\right) \\ > & \frac{1-\delta}{\delta}\left(\begin{array}{c} \left(x + \left(\overline{\theta} + e_1\left(\theta_G - \overline{\theta}\right)\right)\Delta x - c\right) - \frac{\theta_N c}{e_1 \Delta\theta} \\ + \frac{\delta e_1}{1-e(\theta_G)\delta}\left(x + \left(\overline{\theta} + e\left(\theta_G\right)\left(\theta_G - \overline{\theta}\right)\right)\Delta x - c - \overline{U}\right) \end{array}\right) \end{aligned} \tag{A.8}$$

The key observation now is that the RHS of (A.8) decreases towards zero as δ increases towards one (in particular, note that $\frac{1-\delta}{\delta}$ and $\frac{\delta e_1}{1-e(\theta_G)\delta}$ do not cancel out). This reflects that the manager's rent increases almost linearly in T (but for discounting) even though the likelihood that the manager stays one period longer (and, thus, that the board enjoys the benefit of offering longer employment) is only $e(\theta_G)$. By contrast, the LHS of (A.8) is independent of δ and positive if (22) is not satisfied. Thus, for any parameter constellation, there is a threshold $\widehat{\delta}$ such that for $\delta > \widehat{\delta}$, this condition is satisfied. Hence, in these cases the board will always deviate from a policy of pursuing truthful reporting in all periods and choosing T as high as possible. **Q.E.D.**

Proof of Lemma 1. Consider a contract offer $\mathbf{w}_n$ that gives the board an expected payoff of V_n^* and features n periods with truthful reporting. Compare this offer to an alternative $\mathbf{w}_{n+k}$ that gives the board V_{n+k}^* and features $n+k$ periods with truthful reporting. In what follows, it is sufficient to argue that

$$\frac{\partial}{\partial \overline{U}}\left(V_{n+k}^* - V_n^*\right) > 0 \tag{A.9}$$

$$\frac{\partial}{\partial \Delta x}\left(V_{n+k}^* - V_n^*\right) > 0. \tag{A.10}$$

The increasing differences will imply that the V_{n+k}^* offer becomes increasingly more attractive for the board as $\overline{U}$ and Δx increase. Plugging in from (A.5), (A.9) and (A.10) can be rewritten as

$$\begin{aligned} \frac{\partial}{\partial \overline{U}}\left(V_{n+k}^* - V_n^*\right) &= \frac{\partial}{\partial \overline{U}}\left(\frac{-v_1(\mathbf{w}_{n+k})}{1-p(\mathbf{w}_{n+k})} - \frac{-v_1(\mathbf{w}_n)}{1-p(\mathbf{w}_n)}\right) \\ \frac{\partial}{\partial \Delta x}\left(V_{n+k}^* - V_n^*\right) &= \frac{\partial}{\partial \Delta x}\left(\frac{s(\mathbf{w}_{n+k})}{1-p(\mathbf{w}_{n+k})} - \frac{s(\mathbf{w}_n)}{1-p(\mathbf{w}_n)}\right) \end{aligned}$$

proving the claim. **Q.E.D.**

Proof of Proposition 3. Take as a starting point the case in which the board always pursues truthful reporting, described in Proposition 2, and suppose that the board considers abstaining from offering incentives for truthful reporting in period t. From condition (9) and the fact that the manager's bonus is deferred (Proposition 1), we have that $\Delta w_t^{nr} = \frac{c}{e_t(\theta_G)\Delta\theta} + \delta\frac{U_{t+1}^e(\theta_N,\mathbf{w}) - U_{t+1}^e(\theta_G,\mathbf{w})}{\Delta\theta} = 0$. Using the first equality and that $w_t^{nr} = 0$,[30] the manager's expected payoff in period t can be stated as

$$
\begin{aligned}
U_t\left(\theta_{t-1}, \mathbf{w}^{nr}\right) &= w_t^{nr} + \left(\theta_N + e_t\left(\theta_{t-1}\right)\Delta\theta\right)\left(\frac{c}{e_t\left(\theta_G\right)\Delta\theta} + \delta\frac{U_{t+1}^e\left(\theta_N, \mathbf{w}\right) - U_{t+1}^e\left(\theta_G, \mathbf{w}\right)}{\Delta\theta}\right) \\
&\quad + \delta e\left(\theta_{t-1}\right)U_{t+1}^e\left(\theta_G, \mathbf{w}^{nr}\right) + \delta\left(1 - e\left(\theta_{t-1}\right)\right)U_{t+1}^e\left(\theta_N, \mathbf{w}^{nr}\right) - c \\
&= \delta\frac{\theta_G U_{t+1}^e\left(\theta_N, \mathbf{w}^{nr}\right) - \theta_N U_{t+1}^e\left(\theta_G, \mathbf{w}^{nr}\right)}{\Delta\theta} + \left(\frac{\left(\theta_N + e_t\left(\theta_{t-1}\right)\Delta\theta\right)}{e_t\left(\theta_G\right)\Delta\theta} - 1\right)c \\
&= \delta\frac{\theta_G\theta_N\left(U_{t+1}\left(\theta_N, \mathbf{w}^{nr}\right) - U_{t+1}\left(\theta_G, \mathbf{w}^{nr}\right)\right)}{\Delta\theta} + \sum_{j=t+1}^{T}\delta^{j-t}\overline{U}_j \\
&\quad + \left(\frac{\left(\theta_N + e_t\left(\theta_{t-1}\right)\Delta\theta\right)}{e_t\left(\theta_G\right)\Delta\theta} - 1\right)c. \qquad \text{(A.11)}
\end{aligned}
$$

where the superscript nr highlights that there is no truthful reporting in period t. Subtracting (A.11) from (20), we obtain

$$
\begin{aligned}
& \delta\frac{\theta_G U_{t+1}\left(\theta_N, \mathbf{w}^r\right) - \theta_N U_{t+1}\left(\theta_G, \mathbf{w}^r\right)}{\Delta\theta} \\
& - \left(\delta\frac{\theta_G\theta_N\left(U_{t+1}\left(\theta_N, \mathbf{w}^{nr}\right) - U_{t+1}\left(\theta_G, \mathbf{w}^{nr}\right)\right)}{\Delta\theta} + \sum_{j=t+1}^{T}\delta^{j-t}\overline{U}_j\right) \\
> & \ \delta\frac{\theta_G \nu_{t+1}\left(\theta_N, \mathbf{w}^r\right) - \theta_N \nu_{t+1}\left(\theta_G, \mathbf{w}^r\right)}{\Delta\theta} \\
= & \ \frac{\delta}{\Delta\theta}\left(-\theta_N\left(\nu_{t+1}\left(\theta_G, \mathbf{w}^r\right) - \nu_{t+1}\left(\theta_N, \mathbf{w}^r\right)\right) + \Delta\theta\nu_{t+1}\left(\theta_N, \mathbf{w}^r\right)\right) \\
= & \ \delta\left(-\frac{\theta_N\Delta e_{t+1}}{e_{t+1}\left(\theta_G\right)\Delta\theta}c + \nu_{t+1}\left(\theta_N, \mathbf{w}^r\right)\right) \\
= & \ \delta\left(\left(\frac{\theta_N - \theta_G\Delta e_{t+1}}{e_{t+1}\left(\theta_G\right)\Delta\theta}\right)c - \overline{U} + \sum_{j=t+2}^{T}\delta^{j-t-1}\left(\frac{\theta_N - \theta_G\Delta e_j}{e_j\left(\theta_G\right)\Delta\theta}c - \overline{U}\right)\right) > 0 \qquad \text{(A.12)}
\end{aligned}
$$

where the first inequality follows from $U_{t+1}\left(\theta_N, \mathbf{w}^{nr}\right) \leq U_{t+1}\left(\theta_G, \mathbf{w}^{nr}\right)$, the second and third equalities follow in analogy to (21) from

$$
\nu_{t+1}\left(\theta_t, \mathbf{w}\right) = \left(\frac{e_{t+1}\left(\theta_t\right)}{e_{t+1}\left(\theta_G\right)} - 1\right)c + \frac{\theta_N}{e_{t+1}\left(\theta_G\right)\Delta\theta}c - \overline{U} + \sum_{j=t+2}^{T}\delta^{j-t-1}\left(\frac{\theta_N - \theta_G\Delta e_j}{e_j\left(\theta_G\right)\Delta\theta}c - \overline{U}\right). \qquad \text{(A.13)}
$$

The last inequality in (A.12) follows from the assumption that (22) is not satisfied.

It remains to show that abstaining from offering incentives for truthful reporting, say in $t+1$, decreases the manager's payoff also in all preceding seasons with truthful reporting.

[30] We look at the case in which $w_t = 0$. If $w_t > 0$, the manager's rent is zero, and the result is immediate.

Plugging in for $w_{s,t}$ into the manager's expected payoff (20)), we have that

$$U_t(\theta_{t-1},\mathbf{w}) = \left(\frac{e_t(\theta_{t-1})}{e_t(\theta_G)}-1\right)c+\left(\frac{\theta_N c}{e_{t-1}(\theta_G)\Delta\theta}+\delta\frac{U_{t+1}(\theta_N,\mathbf{w})-U_{t+1}(\theta_G,\mathbf{w})}{\Delta\theta}\right) +\delta U_{t+1}(\theta_N,\mathbf{w}) \tag{A.14}$$

From (A.11), $U_{t+1}(\theta_N,\mathbf{w})-U_{t+1}(\theta_G,\mathbf{w})=\frac{\Delta e_{t+1}c}{e_{t+1}(\theta_G)}$. Hence, minimizing $U_{t+1}(\theta_N,\mathbf{w})$ minimizes $U_t(\theta_{t-1},\mathbf{w})$ and, proceeding iteratively, minimizes all payoffs until $U_1(\mathbf{w})$. More generally, Lemma B.2 shows that this is true regardless of the board's reporting strategy in periods one to t. **Q.E.D.**

Proof of Proposition 4. (i) Consider a shock that only increases the manager's outside option in period one. Clearly, the board's preference for truthful reporting when hiring a new manager after this period is unchanged by this shock. Let the board's expected payoff when hiring a new manager after period one be V^*. For period one, the board's payoff is

$$V=s(\mathbf{w})+p(\mathbf{w})V^*+h\left(\overline{U},\mathbf{w}\right)-U_1(\mathbf{w}).$$

Since the manager's outside option in period one, $\overline{U}_1$, does not enter $h\left(\overline{U},\mathbf{w}\right)$, it only affects $U_1(\mathbf{w})$ in this expression. From (20) and (A.11), this payoff depends on $\overline{U}_1$ only if the manager extracts no rent in period one, i.e., if $w_{s,1}=0$ in case of truthful reporting, and $w_1>0$ in case of no truthful reporting. In either case, a unit increase in $\overline{U}_1$ leads to a unit increase in $U_1(\mathbf{w})$ regardless of whether there is truthful reporting in that period. If the manager extracts no rent, there is no effect.

In (A.12), we have shown that the manager's rent is lower if she is not incentivized to report truthfully. Thus, there is a range of parameters for which the manager extracts rent when offered incentives for truthful reporting, but not without such incentives, in period one. For this range, an increase in $\overline{U}_1$ makes offering incentives for truthful reporting in period one more attractive (otherwise, there is no effect).

(ii) Suppose that the manager lives for at most two periods and that the first-best condition (22) is not satisfied. If employing a manager for one period only, the board always seeks truthful reporting by Proposition 1. Thus, the proposition is based on the case with $T=2$. We start by deriving the separate components of (A.5) and then plug into (A.9) and (A.10).

In the final period, the board always seeks truthful reporting by Proposition 1, implying that

$$\begin{aligned} U_2(\theta_1, \mathbf{w}) &= e_2(\theta_1)(w_2 + \theta_G \Delta w_2) + (1 - e_2(\theta_1))\left(w_{s,2} + \overline{U}\right) - c \\ &= (\theta_N + e_2(\theta_1)\Delta\theta)\Delta w_2 - c. \end{aligned}$$

where we use that $w_{s,2} = \theta_N \Delta w_2 - \overline{U}$ and $w_2 = 0$. If the board does not seek truthful reporting in the first period, the manager's bonus in the second period must be such that her bonus in period one can be set to zero, while still satisfying the manager's constraint to invest in firm-specific human capital. Specifically, for this constraint to be just satisfied, we must have that

$$\begin{aligned} 0 &= \frac{c}{e_1 \Delta\theta} + \delta \frac{U_2^e(\theta_N, \mathbf{w}) - U_2^e(\theta_G, \mathbf{w})}{\Delta\theta} \\ &= \frac{c}{e_1 \Delta\theta} + \delta \frac{\begin{array}{c}\theta_N((\theta_N + e_2(\theta_N)\Delta\theta)\Delta w_2 - c) + (1-\theta_N)\overline{U} \\ -\theta_G((\theta_N + e_2(\theta_G)\Delta\theta)\Delta w_2 - c) - (1-\theta_G)\overline{U}\end{array}}{\Delta\theta}, \end{aligned} \tag{A.15}$$

which is satisfied for $\Delta w_2 = \Delta w_1^{-1}(0) = \frac{\frac{c}{\delta e_1 \Delta\theta} + c + \overline{U}}{\delta(e_2(\theta_G)\theta_G + \theta_N(1 - e_2(\theta_N)))}$. Hence, $\Delta w_2 = \max\left\{\Delta w_1^{-1}(0), \frac{c}{e_2(\theta_G)\Delta\theta}\right\}$. If the second term is larger, $U_1(\mathbf{w}^{nr})$ is independent of $\overline{U}$. If the first term is larger, we have

$$\begin{aligned} \nu_1(\mathbf{w}^{nr}) &= U_1(\mathbf{w}^{nr}) - \sum_{j=1}^{2} \delta^{j-1}\overline{U} = \delta E_{\theta_0}\left[U_2^e(\theta_t, \mathbf{w})\right] - c - \overline{U}(1+\delta) \\ &= \delta\left[U_2^e(\theta_N, \mathbf{w}) + e_1\left(U_{t+1}^e(\theta_G, \mathbf{w}) - U_2^e(\theta_N, \mathbf{w})\right)\right] - c - \overline{U}(1+\delta) \\ &= \delta\left(\theta_N\left((e_N\theta_G + (1-e_N)\theta_N)\frac{\left(\frac{c}{\delta e_1 \Delta\theta} + c + \overline{U}\right)}{\delta(e_G\theta_G + (1-e_N)\theta_N)} - c\right) + (1-\theta_N)\overline{U}\right) \\ &\quad -\overline{U}(1+\delta) \\ &= \theta_N\left(\frac{(e_N\theta_G + (1-e_N)\theta_N)}{(e_G\theta_G + (1-e_N)\theta_N)}\left(\frac{1}{\delta e_1 \Delta\theta} + 1\right) - \delta\right)c \\ &\quad + \left(\frac{\theta_N(e_N\theta_G + (1-e_N)\theta_N)}{(\theta_G e_G + (1-e_N)\theta_N)} - 1 - \delta\theta_N\right)\overline{U}. \end{aligned}$$

where, to simplify notation, we have defined $e(\theta_G) = e_G$ and $e(\theta_N) = e_N$; the second equality follows after using that $U_2^e(\theta_N, \mathbf{w}) - U_2^e(\theta_G, \mathbf{w}) = \frac{c}{e_1\delta}$ and plugging for Δw_2 (cf. (A.15))

If the board seeks truthful reporting in the first period, then by (21) the manager's rent is simply

$$\nu_1(\mathbf{w}^r) = \frac{\theta_N}{e_1 \Delta\theta}c - \overline{U} + \delta\left(\frac{\theta_N - \theta_G \Delta e}{e_G \Delta\theta}c - \overline{U}\right),$$

By plugging $\nu_1(\mathbf{w}^r)$ and $\nu_1(\mathbf{w}^{nr})$ into (A.9), we obtain that

$$\begin{aligned}
&\frac{\partial}{\partial \overline{U}}\left(\frac{-\nu_1(\mathbf{w}^r)}{1-p(\mathbf{w}^r)}-\frac{-\nu_1(\mathbf{w}^{nr})}{1-p(\mathbf{w}^{nr})}\right) \\
=\;& \frac{1+\delta}{1-\delta(1-e_1)-\delta^2 e_1}-\frac{-\left(\frac{\theta_N(e_N\theta_G+(1-e_N)\theta_N)}{(\theta_G e_G+(1-e_N)\theta_N)}-1-\delta\theta_N\right)}{1-\delta(1-e_1\theta_G-(1-e_1)\theta_N)-\delta^2(e_1\theta_G+(1-e_1)\theta_N)} \\
=\;& \frac{(1+\delta)\delta(\mathbb{E}_1\theta-e_1)+\left(\theta_N\frac{(e_N\theta_G+(1-e_N)\theta_N)}{(\theta_G e_G+(1-e_N)\theta_N)}+\delta(1-\theta_N)\right)(1+\delta e_1)}{(1-\delta)(1+\delta e_1)(1+\delta E_1\theta)} \\
>\;& \delta\frac{(1+\delta)(\mathbb{E}_1\theta-e_1)+(1-\theta_N)(1+\delta e_1)}{(1-\delta)(1+\delta e_1)(1+\delta E_1\theta)}
\end{aligned} \tag{A.16}$$

where $\mathbb{E}_1\theta := e_1\theta_G+(1-e_1)\theta_N$. After some transformations, the last expression becomes

$$\frac{\delta(\theta_N+\theta_G e_1-2\theta_N e_1)+(\theta_G-\theta_N)e_1+1-e_1}{(1-\delta)(1+\delta e_1)(1+\delta E\theta)}>0.$$

Hence, (A.16) is positive. By Lemma 1, this proves the claim.

Approaching (A.10) similarly, we have

$$\begin{aligned}
&\frac{s(\mathbf{w}^r)}{1-p(\mathbf{w}^r)}-\frac{s(\mathbf{w}^{nr})}{1-p(\mathbf{w}^{nr})} \\
=\;& \frac{x+\left(e_1\theta_G+(1-e_1)\overline{\theta}\right)\Delta x-c+\delta e_1\left(x+\left(e_G\theta_G+(1-e_G)\overline{\theta}\right)\Delta x-c\right)}{(1-\delta)(1+\delta e_1)} \\
&-\frac{x+(e_1\theta_G+(1-e_1)\theta_N)\Delta x-c+\delta\begin{pmatrix} e_1\theta_G\left(x+\left(e_G\theta_G+(1-e_G)\overline{\theta}\right)\Delta x-c\right) \\ +(1-e_1)\theta_N\left(x+\left(e_N\theta_G+(1-e_N)\overline{\theta}\right)\Delta x-c\right)\end{pmatrix}}{(1-\delta)(1+\delta E_1\theta)}.
\end{aligned}$$

After some transformations, the terms dependent on Δx become

$$\frac{\Delta x}{(1-\delta)(1+e_1\delta)(1+\delta E_1\theta)}\begin{pmatrix}\delta\left(\theta_G-\overline{\theta}\right)((e_G-e_1)(e_1-E\theta)+\theta_N\Delta e(1-e_1)(1+\delta e_1)) \\ +\left(\overline{\theta}-\theta_N\right)(1-e_1)(1+\delta e_1)\end{pmatrix}$$

which is strictly positive even if $e_1<\mathbb{E}_1\theta$, as then

$$\begin{aligned}
&(e_G-e_1)(e_1-\mathbb{E}_1\theta)+\theta_N\Delta e(1-e_1)(1+\delta e_1) \\
>\;& \Delta e(\theta_N(1-e_1)(1+\delta e_1)-(\mathbb{E}_1\theta-e_1)) \\
>\;& \Delta e(\theta_N(1-e_1)+e_1-e_1\theta_G-(1-e_1)\theta_N)=\Delta e(e_1-e_1\theta_G)>0
\end{aligned}$$

Hence, expression (A.10) is positive, proving also the second statement. **Q.E.D.**

Proof of Proposition 5. Using (A.4) to express V^*, we have to show that

$$\frac{\partial}{\partial \overline{U}}\left(V^*\left(\mathbf{w}'\right)-V^*\left(\mathbf{w}\right)\right)=\frac{\frac{\partial}{\partial \overline{U}}h\left(\mathbf{w}'\right)\left(1-p\left(\mathbf{w}\right)\right)-\frac{\partial}{\partial \overline{U}}h\left(\mathbf{w}\right)\left(1-p\left(\mathbf{w}'\right)\right)}{\left(1-p\left(\mathbf{w}\right)\right)\left(1-p\left(\mathbf{w}'\right)\right)}>0 \tag{A.17}$$

where the offer $\mathbf{w}'$ is made to a manager with time to retirement $T'=T+1$. To obtain the equality in (A.17), we use that when condition (22) does not hold, the manager's expected payoff $U_1\left(\mathbf{w}\right)$ is independent of $\overline{U}$ (cf. (A.14)). The increasing difference in (A.17) will imply that the T'-offer becomes increasingly more attractive as $\overline{U}$ increases.

Recalling from (A.3) that $h\left(\mathbf{w}\right)=\frac{\overline{U}}{1-\delta}\left(-\delta^T+p\left(\mathbf{w}\right)\right)=\frac{\overline{U}}{1-\delta}\left(1-\delta^T+p\left(\mathbf{w}\right)-1\right)$ and plugging in from (A.7), we can express the numerator in the LHS of (A.17) as

$$\begin{aligned}
&\frac{1}{1-\delta}\left(\left(1-\delta^{T+1}+p\left(\mathbf{w}'\right)-1\right)\left(1-p\left(\mathbf{w}\right)\right)-\left(1-\delta^{T}+p\left(\mathbf{w}\right)-1\right)\left(1-p\left(\mathbf{w}'\right)\right)\right)\\
=\;&\frac{1}{1-\delta}\left(\left(1-\delta^{T+1}\right)\left(1-p\left(\mathbf{w}\right)\right)-\left(1-\delta^{T}\right)\left(1-p\left(\mathbf{w}'\right)\right)\right)\\
=\;&\left(1-\delta^{T+1}\right)\left(1+\frac{\delta e_1\left(1-\left(\delta e_G\right)^{T-1}\right)}{1-\delta e_G}\right)-\left(1-\delta^{T}\right)\left(1+\frac{\delta e_1\left(1-\left(\delta e_G\right)^{T}\right)}{1-\delta e_G}\right)
\end{aligned}$$

which after some transformations becomes

$$\begin{aligned}
&\frac{\delta^T}{1-\delta e_G}\left(\left(1-\delta\right)\left(1-\delta e_G\right)+e_1\left(\delta\left(1-\delta\right)+\left(e_G\right)^{T-1}\left(\delta^{T+1}\left(1-e_G\right)-\left(1-\delta e_G\right)\right)\right)\right)\\
\geq\;&0.
\end{aligned} \tag{A.18}$$

To see the last inequality, observe that expression (A.18) is positive if the term in brackets following e_1 is positive. If it is negative, expression (A.18) would decrease in e_1, so it would obtain its minimum value for $e_1=e_G$

$$\delta^T\left(\frac{\left(1-\delta+\left(e_G\right)^T\delta^{T+1}\left(1-e_G\right)\right)}{1-\delta e_G}-\left(e_G\right)^T\right).$$

The sign of this expression is the same as the sign of the term in brackets. The minimum of that term is zero, which is obtained for $\delta=1$.[31] Hence, for any T, $\delta\in[0,1]$, $e_G\in[0,1]$, $e_1\in[0,e_G]$, (A.18) is (weakly) positive, and it is strictly positive for $e_G<1$ and $\delta<1$,

[31] We have

$$\frac{\partial}{\partial\delta}\left(\frac{\left(1-\delta+\left(\delta e_G\right)^T\delta\left(1-e_G\right)\right)}{1-\delta e_G}-\left(e_G\right)^T\right)=\left(1-e_G\right)\frac{T\left(\delta e_G\right)^T\left(1-\delta e_G\right)+\left(\delta e_G\right)^T-1}{\left(1-\delta e_G\right)^2}$$

This term is nonpositive, as the maximum value (of zero) of the numerator is obtained for $\delta e_G=1$.

implying that we have strictly increasing differences in (A.17). By Lemma 1, this proves the claim.

Next, we argue that

$$\frac{\partial}{\partial \Delta x}\left(V^*\left(\mathbf{w}'\right)-V^*\left(\mathbf{w}\right)\right)=\frac{\frac{\partial}{\partial \Delta x}s\left(\mathbf{w}'\right)\left(1-p\left(\mathbf{w}\right)\right)-\frac{\partial}{\partial \Delta x}s\left(\mathbf{w}\right)\left(1-p\left(\mathbf{w}'\right)\right)}{\left(1-p\left(\mathbf{w}\right)\right)\left(1-p\left(\mathbf{w}'\right)\right)}>0. \tag{A.19}$$

Observe first that from (A.6), we have

$$\frac{\partial}{\partial \Delta x}s\left(\mathbf{w}\right)=\left(\overline{\theta}+e_1\left(\theta_G-\overline{\theta}\right)\right)+\delta e_1\frac{\left(1-e\left(\theta_G\right)^{T-1}\delta^{T-1}\right)}{1-e\left(\theta_G\right)\delta}\left(\overline{\theta}+e\left(\theta_G\right)\left(\theta_G-\overline{\theta}\right)\right).$$

Plugging in for $p\left(\mathbf{w}\right)$, (A.19) becomes

$$\begin{aligned}
&\left(\left(\overline{\theta}+e_1\left(\theta_G-\overline{\theta}\right)\right)+\delta e_1\frac{\left(1-e_G^T\delta^T\right)}{1-e_G\delta}\left(\overline{\theta}+e_G\left(\theta_G-\overline{\theta}\right)\right)\right)\left(1-\delta\right)\left(1+\frac{\delta e_1\left(1-e_G^{T-1}\delta^{T-1}\right)}{1-e_G\delta}\right)\\
&-\left(\left(\overline{\theta}+e_1\left(\theta_G-\overline{\theta}\right)\right)+\delta e_1\frac{\left(1-e_G^{T-1}\delta^{T-1}\right)}{1-e_G\delta}\left(\overline{\theta}+e_G\left(\theta_G-\overline{\theta}\right)\right)\right)\left(1-\delta\right)\left(1+\frac{\delta e_1\left(1-e_G^T\delta^T\right)}{1-e_G\delta}\right)\\
=&\ \delta^T e_1\left(e_G-e_1\right)e^{T-1}\left(1-\delta\right)\left(\theta_G-\overline{\theta}\right)>0
\end{aligned}$$

proving the claim. **Q.E.D.**

Proof of Proposition 6. Recall that from expressions (A.3) and (A.4), we can express

$$V^*\left(\mathbf{w}\right)=\frac{s\left(\mathbf{w}\right)+\frac{\overline{U}}{1-\delta}\left(p\left(\mathbf{w}\right)-\delta^T\right)-U_1\left(\mathbf{w}\right)}{1-p\left(\mathbf{w}\right)}.$$

Hence, $\frac{\partial}{\partial \overline{U}}V^*\left(\mathbf{w}\right)>0$ as long as $\left(p\left(\mathbf{w}\right)-\delta^T\right)>\left(1-\delta\right)\frac{\partial}{\partial \overline{U}}U_1\left(\mathbf{w}\right)$. This is trivially satisfied if the board stimulates truthful reporting in all periods and the first-best condition (22) is not satisfied. In this case, $U_1\left(\mathbf{w}\right)=\frac{\theta_N c}{e_1\Delta\theta}+\sum_{j=2}^T\delta^{j-1}\left(\frac{\theta_N-\theta_G\Delta e_j}{e_j(\theta_G)\Delta\theta}\right)c$ (cf. (21)), which is independent of $\overline{U}$. **Q.E.D.**

Appendix B For Online Publication: Supplementary Material

Lemma B.1 *If it is not optimal to elicit the manager's fit in period $t-1$, it would also not be optimal to elicit that fit in the beginning of the following period t.*

Proof of Lemma B.1. Suppose that $\mathbf{w}$ is an optimal contract that does not induce truthful reporting in period $t-1$, but induces truthful reporting in the beginning of period t. Let $\widetilde{w}_{s,t}$ be the severance pay paid to the manager in the beginning of period t for disclosing that her fit realization in $t-1$ is θ_N. Truthful reporting at the beginning of period t after no truthful reporting in period $t-1$ requires that

$$U_t(\theta_G, \mathbf{w}) \geq \sum_{j=t}^{T} \delta^{j-t}\overline{U}_j + \widetilde{w}_{s,t} = U_t(\theta_N, \mathbf{w}). \tag{B.1}$$

Multiplying all sides of (B.1) with δ and adding $w_{t-1} + \theta_G \Delta w_{t-1}$ on both sides of the inequality, we obtain

$$\begin{aligned} & w_{t-1} + \theta_G \Delta w_{t-1} + \delta U_t(\theta_G, \mathbf{w}) \\ \geq\ & w_{t-1} + \theta_G \Delta w_{t-1} + \delta \left(\sum_{j=t}^{T} \delta^{j-t}\overline{U}_j + \widetilde{w}_{s,t} \right) \\ =\ & w_{t-1} + \theta_N \Delta w_{t-1} + \delta \left(\sum_{j=t}^{T} \delta^{j-t}\overline{U}_j + \widetilde{w}_{s,t} \right) + \Delta\theta \Delta w_{t-1} \\ =\ & \sum_{j=t-1}^{T} \delta^{j-t+1}\overline{U}_j + w_{s,t-1} + \widetilde{\omega}_{t-1} + \Delta\theta \Delta w_{t-1} \end{aligned} \tag{B.2}$$

where the last equality follows from the period $t-1$ analogue of (10); $\widetilde{\omega}_{t-1}$ is defined as the difference between the left- and the right-hand-side of this analogue; and where we use that $U_t(\theta_N, \mathbf{w}) = \sum_{j=t}^{T} \delta^{j-t}\overline{U}_j + \widetilde{w}_{s,t}$ (cf. (B.1)).[32] Consider now the requirements for truth-telling in an equilibrium in which the manager reports truthfully already in period $t-1$

$$w_{t-1} + \theta_G \Delta w_{t-1} + \delta U_t(\theta_G, \mathbf{w}) \geq w_{s,t-1} + \sum_{j=t-1}^{T} \delta^{j-t+1}\overline{U}_j \geq w_{t-1} + \theta_N \Delta w_{t-1} + \delta U_t(\theta_N, \mathbf{w}).$$

[32] Note, that all subscripts are with respect to $t-1$ rather than t as in (10).

From the first and the last line of (B.2), we see that a manager with fit θ_G would would not have mimicked a manager with fit θ_N if she were asked to report that fit in period $t-1$ (cf. (6)) given a severance pay offer of $\widehat{w}_{s,t-1} := w_{s,t-1} + \widetilde{\omega}_{t-1}$. Furthermore, from the third and the fourth line of (B.2), we obtain that a manager with fit θ_N would have been indifferent to disclosing her fit also in period $t-1$ (cf. (7)) if she was compensated for doing so with $\widehat{w}_{s,t-1} = w_{s,t-1} + \widetilde{\omega}_{t-1}$. Thus, the manager's expected rent is the same regardless of whether she reports in period $t-1$ or in the beginning of period t. However, learning the manager's fit in period $t-1$ increases the board's payoff, contradicting that $\mathbf{w}$ is optimal and proving the claim. **Q.E.D.**

Binding constraints in Proposition 1. In what follows, we take the board's truthful reporting policy as given and analyze which of the conditions (7)–(10), $w_t, \Delta w_t, w_{s,t} \geq 0$ are binding when minimizing the manager's period one payoff $U_1(\mathbf{w})$. The proof proceeds by initially assuming that, to minimize $U_1(\mathbf{w})$, the board minimizes U_t in all periods t. At the end of the proof, it is shown that minimizing U_t helps to minimize U_{t-1} and, thus, preceding recursively, one minimizes the manager's expected payoff all the way to period one.

Lemma B.2 *(i) If the board does not seek truthful reporting in period $t < T$, it is optimal to set $w_{s,t} = 0$, $\Delta w_t = 0$, and w_t is determined by having the more stringent of conditions (10) and $w_t \geq 0$ be binding. (ii) If the board seeks truthful reporting in period t, $w_{s,t}$ is determined by condition (7); Δw_t is determined by condition (8) if the board seeks truthful reporting in $t-1$. Without truthful reporting from $t-n$ to $t-1$ ($n \geq 1$), Δw_t is determined by the more stringent of condition (8) and the need for Δw_t to be high enough to satisfy (9) in the preceding n periods.*

Proof of Lemma B.2. We assume initially that the board offers incentives for truthful reporting in $t-1$ (if there is such a period). This leads to four main cases depending on whether the board offers incentives for truthful reporting in period t , and $t+1$. Towards the end, we consider the case in which the board does not offer incentives for truthful reporting in $t-1$. We show the claim by induction by arguing first that it is always satisfied in period 1. We then argue that if the conditions stated in the Lemma are satisfied in t, then the $t+1$ analogue of these conditions must also be satisfied.

Truthful reporting in period t. The induction hypothesis for this case is that Δw_t, w_t, and $w_{s,t}$ are given by (15)–(17). Suppose to a contradiction that the claim is not true in the first period and that the board seeks to implement truthful reporting in that period. Recall that $e_1(\theta_{t-1}) = e_1$ in period $t = 1$. Setting $\{w_1, \Delta w_1, w_{s,1}\}$ to their minimal values maximizes the board's expected payoff, as it minimizes the manager's payoff, without affecting her incentives in the following periods. Thus, $w_{s,1}$ is determined by (7). Using this, we see that setting $w_1 = 0$ relaxes (7), while not affecting (8). Finally, Δw_1 is determined by the more stringent condition of (8) and $\Delta w_1 \geq 0$. Below we verify that the more stringent condition is (8).

Case 1: Truthful reporting in periods t ***and*** $t+1$. The manager's expected payoff when investing in firm-specific human capital in period t is

$$U_t(\theta_{t-1}, \mathbf{w}) = e_t(\theta_{t-1})(w_t + \theta_G \Delta w_t + \delta U_{t+1}(\theta_G, \mathbf{w})) + (1 - e(\theta_{t-1}))\left(w_{s,t} + \sum_{j=t}^{T} \delta^{j-t} \overline{U}_j\right) - c.$$

Using the induction hypothesis (15)–(17) to plug into w_t, Δw_t, and $w_{s,t}$, this payoff becomes

$$e_t(\theta_{t-1})\left(w_t + \frac{\theta_G c}{e_t(\theta_G)\Delta\theta} + \frac{\theta_G \delta U_{t+1}(\theta_N, \mathbf{w}) - \theta_N \delta U_{t+1}(\theta_G, \mathbf{w})}{\Delta\theta}\right)$$
$$+ (1 - e_t(\theta_{t-1})) \max\left\{\sum_{j=t}^{T} \delta^{j-t} \overline{U}_j, \frac{\theta_N c}{e_t(\theta_G)\Delta\theta} + \delta \frac{\theta_G U_{t+1}(\theta_N, \mathbf{w}) - \theta_N U_{t+1}(\theta_G, \mathbf{w})}{\Delta\theta}\right\} - c.$$

In what follows, it is shown that choosing $\{w_{s,t+1}, w_{t+1}, \Delta w_{t+1}\}$ as dictated in Proposition 1 minimizes

$$\theta_G U_{t+1}(\theta_N, \mathbf{w}) - \theta_N U_{t+1}(\theta_G, \mathbf{w}), \tag{B.3}$$

and, thus, minimizes $U_t(\theta_{t-1}, \mathbf{w})$.

The condition that the manager invests in firm-specific human capital in $t+1$ is (this is the $t+1$ analogue of (8))

$$w_{t+1} + \theta_t \Delta w_{t+1} + \delta U_{t+2}(\theta_G, \mathbf{w}) - \frac{c}{e_{t+1}(\theta_t)} \geq w_{s,t+1} + \sum_{j=t+1}^{T} \delta^{j-t-1} \overline{U}_j. \tag{B.4}$$

Truthful reporting in period $t+1$ would further require that

$$w_{s,t+1} + \sum_{j=t+1}^{T} \delta^{j-t-1} \overline{U}_j \geq w_{t+1} + \theta_N \Delta w_{t+1} + \delta U_{t+2}(\theta_N, \mathbf{w}). \tag{B.5}$$

In what follows, we show that (B.4), (B.5), and $w_{t+1} \geq 0$ will be binding.

To find the contract parameters $\{w_{s,t+1}, w_{t+1}, \Delta w_{t+1}\}$ that minimize (B.3), subject to (B.4), (B.5), and $w_{s,t+1}, w_{t+1}, \Delta w_{t+1} \geq 0$, we apply Kuhn Tucker's Theorem. Define the function

$$
\begin{aligned}
& \mathcal{L}_1(\mathbf{w}, \Lambda) \\
= \; & -\left(\theta_G \widetilde{e}_{t+1}(\theta_N) - \theta_N e_{t+1}(\theta_G)\right)\left(w_{t+1} + \theta_G \Delta w_{t+1} + \delta U_{t+2}(\theta_G, \mathbf{w})\right) \\
& -\left(\theta_G\left(1 - \widetilde{e}_{t+1}(\theta_N)\right) - \theta_N\left(1 - e_{t+1}(\theta_G)\right)\right)\left(w_{s,t+1} + \sum_{j=t+1}^{T} \delta^{j-t-1} \overline{U}_j\right) + \theta_G \widetilde{c} - \theta_N c \\
& +\lambda\left(w_{t+1} + \theta_G \Delta w_{t+1} + \delta U_{t+2}(\theta_G, \mathbf{w}) - \frac{c}{e_{t+1}(\theta_t)} - w_{s,t+1} - \sum_{j=t+1}^{T} \delta^{j-t-1} \overline{U}_j\right) \\
& +\mu\left(w_{s,t+1} + \sum_{j=t+1}^{T} \delta^{j-t-1} \overline{U}_j - w_{t+1} - \theta_N \Delta w_{t+1} - \delta U_{t+2}(\theta_N, \mathbf{w})\right) \\
& +\kappa w_{s,t+1} + \rho w_{t+1} + \chi \Delta w_{t+1}
\end{aligned}
$$

where the first two lines correspond to the negative of (B.3) (as the objective is to minimize (B.3)), and $\Lambda = \{\lambda, \mu, \kappa, \rho, \chi\}$ is the set of weakly positive Kuhn Tucker multipliers. To account for the possibility that it could be suboptimal to offer incentives to the manager to invest in firm-specific human capital if $\theta_t = \theta_N$, we have defined $\widetilde{e}_{t+1}(\theta_N) = e_{t+1}(\theta_N)$ ($\widetilde{c} = c$) if she does so and $\widetilde{e}_{t+1}(\theta_N) = 0$ ($\widetilde{c} = 0$) otherwise. Taking the first order conditions

$$
\begin{aligned}
\frac{\partial \mathcal{L}_1(\mathbf{w}, \Lambda\theta)}{\partial w_{s,t+1}} &= 0 = -\left(\theta_G\left(1 - \widetilde{e}_{t+1}(\theta_N)\right) - \theta_N\left(1 - e_{t+1}(\theta_G)\right)\right) - \lambda + \mu + \kappa \\
\frac{\partial \mathcal{L}_1(\mathbf{w}, \Lambda)}{\partial \Delta w_{t+1}} &= 0 = -\left(\theta_G \widetilde{e}_{t+1}(\theta_N) - \theta_N e_{t+1}(\theta_G)\right)\theta_G + \lambda\theta_G - \mu\theta_N + \chi \\
\frac{\partial \mathcal{L}_1(\mathbf{w}, \Lambda)}{\partial w_{t+1}} &= 0 = -\left(\theta_G \widetilde{e}_{t+1}(\theta_N) - \theta_N e_{t+1}(\theta_G)\right) + \lambda - \mu + \rho,
\end{aligned}
$$

we obtain from the second and third conditions that $\theta_G \rho = \mu \Delta\theta + \chi$. From the first and third conditions, we further have $\kappa + \rho = \Delta\theta$. Assuming now that $\Delta w_{t+1} \geq 0$ and $w_{s,t+1} \geq 0$ are not binding, i.e., $\chi = 0$ and $\kappa = 0$, we have: $\rho = \Delta\theta$, $\mu = \theta_G$, and $\lambda = \theta_G \widetilde{e}_{t+1}(\theta_N) + \theta_N\left(1 - e_{t+1}(\theta_G)\right) > 0$. Thus, $\rho, \mu, \lambda > 0$, imply that the board minimizes $w_{s,t+1}$, Δw_{t+1}, and w_{t+1}, subject to the binding constraints (B.4), (B.5), and $w_{t+1} \geq 0$, as was to be shown. Since the board knows that the manager's fit in t is θ_G, this implies that the board will choose to satisfy (B.4) for $\theta_t = \theta_G$.[33] For completeness, using that (B.5) is binding, note

[33] Note that this implies that the manager has no incentives to invest in firm-specific human capital if $\theta_t = \theta_N$, i.e., $\widetilde{e}_{t+1}(\theta_N) = 0$ and $\widetilde{c} = 0$.

that we obtain

$$\begin{aligned} w_{s,t+1} &= \theta_N \Delta w_{t+1} + \delta U_{t+2}(\theta_N, \mathbf{w}) - \sum_{j=t+1}^{T} \delta^{j-t-1} \overline{U}_j \\ \Delta w_{t+1} &= \frac{c}{e_{t+1}(\theta_G)\Delta\theta} + \frac{\delta U_{t+2}(\theta_N, \mathbf{w}) - \delta U_{t+2}(\theta_G, \mathbf{w})}{\Delta\theta}, \end{aligned}$$

which are the $t+1$ analogues of (15)–(17).

If, instead $w_{s,t+1} \geq 0$ is binding, the manager extracts no rent in period $t+1$. For this case we argue in the main text that this would also imply that she extracts no rent in period t and the board will incentivize truthful reporting in all preceding periods, making the proposed contract optimal. The same arguments will apply to Case 3 below. We verify below that $\Delta w_t, \Delta w_{t+1} > 0$.

Case 2: Truthful reporting in period t ***and no truthful reporting in period*** $t+1$**.** Similar to case 1, we can show that the board would like to minimize (B.3). The difference is that, absent truthful reporting in $t+1$, the manager's payoff in that period is

$$\begin{aligned} U_{t+1}(\theta_t, \mathbf{w}) &= w_{t+1} + (\theta_N + e_{t+1}(\theta_t)\Delta\theta)\Delta w_{t+1} - c \\ &\quad + e_{t+1}(\theta_t)\delta U^e_{t+2}(\theta_G, \mathbf{w}) + (1 - e_{t+1}(\theta_t))\delta U^e_{t+2}(\theta_N, \mathbf{w}). \end{aligned} \tag{B.6}$$

Investing in firm-specific human capital in $t+1$ requires that

$$w_{t+1} + (\theta_N + e_{t+1}(\theta_t)\Delta\theta)\Delta w_{t+1} + \delta E_{\theta_t}\left[U^e_{t+2}(\theta_{t+1}, \mathbf{w})\right] - c \geq w_{t+1} + \theta_N \Delta w_{t+1} + \delta U^e_{t+2}(\theta_N, \mathbf{w}).$$

which can be restated as

$$\Delta w_{t+1} \geq \frac{c}{e_{t+1}(\theta_t)\Delta\theta} + \delta \frac{U^e_{t+2}(\theta_N, \mathbf{w}) - U^e_{t+2}(\theta_G, \mathbf{w})}{\Delta\theta} \tag{B.7}$$

In analogy to (10), we further need to satisfy the interim participation constraint in $t+1$

$$w_{t+1} + \theta_N \Delta w_{t+1} + \delta U^e_{t+2}(\theta_N, \mathbf{w}) \geq w_{s,t+1} + \sum_{j=t+1}^{T} \delta^{j-t-1} \overline{U}_j. \tag{B.8}$$

Hence, to minimize (B.3), subject to (B.7), (B.8), and feasibility, define[34]

$$\begin{aligned}
\mathcal{L}_2 &= -\theta_G \left(w_{t+1} + \left(\theta_N + \widetilde{e}_{t+1}(\theta_N)\Delta\theta\right)\Delta w_{t+1} + \delta \widetilde{\mathbb{E}}_{\theta_N}\left[U^e_{t+2}(\theta_{t+1}, \mathbf{w})\right] - \widetilde{c} \right) \\
&+\theta_N \left(w_{t+1} + \left(\theta_N + e_{t+1}(\theta_G)\Delta\theta\right)\Delta w_{t+1} + \delta E_{\theta_G}\left[U^e_{t+2}(\theta_{t+1}, \mathbf{w})\right] - c \right) \\
&+\lambda \left(\Delta w_{t+1} - \frac{c}{e_{t+1}(\theta_t)\Delta\theta} - \delta \frac{U^e_{t+2}(\theta_N, \mathbf{w}) - U^e_{t+2}(\theta_G, \mathbf{w})}{\Delta\theta} \right) \\
&+\mu \left(w_{t+1} + \theta_N \Delta w_{t+1} + \delta U^e_{t+2}(\theta_N, \mathbf{w}) - w_{s,t+1} - \sum_{j=t+1}^{T} \delta^{j-t-1}\overline{U}_j \right) \\
&+\kappa w_{s,t+1} + \rho w_{t+1} + \chi \Delta w_{t+1}
\end{aligned}$$

giving the first-order conditions

$$\begin{aligned}
\frac{\partial \mathcal{L}_2}{\partial w_{t+1}} = 0 &= -\Delta\theta + \mu + \rho \\
\frac{\partial \mathcal{L}_2}{\partial w_{s,t+1}} = 0 &= -\mu + \kappa \\
\frac{\partial \mathcal{L}_2}{\partial \Delta w_{t+1}} = 0 &= -\left(\theta_G\left(\theta_N + \widetilde{e}_{t+1}(\theta_N)\Delta\theta\right) - \theta_N\left(\theta_N + e_{t+1}(\theta_G)\Delta\theta\right)\right) + \lambda + \theta_N \mu + \chi \\
&= -\left(\theta_N - e_{t+1}(\theta_G)\theta_N + \widetilde{e}_{t+1}(\theta_N)\theta_G\right)\Delta\theta + \lambda + \theta_N \mu + \chi
\end{aligned}$$

The first FOC implies that μ or/and ρ are positive, implying that w_{t+1} is either zero or determined by (B.8). As discussed in the main text, if $\mu > 0$, the manager extracts no rent in that period (note that if this is feasible, the board's optimal policy would be to follow the first-best rule of incentivizing truthful reporting in all periods). The second FOC implies that if $\mu \geq 0$, then $\kappa \geq 0$. Thus, it is optimal to set $w_{s,t} = 0$, as it relaxes (B.8). Finally, the first term in third FOC is negative. This implies that λ, μ, and/or χ are positive. If we don't have first-best ($\mu = 0$), this means that Δw_{t+1} is determined by the more stringent of conditions (B.7) and $\Delta w_{t+1} \geq 0$. We verify below that $\Delta w_{t+1} = \frac{c}{e_{t+1}(\theta_G)\Delta\theta} + \delta \frac{U^e_{t+2}(\theta_N, \mathbf{w}) - U^e_{t+2}(\theta_G, \mathbf{w})}{\Delta\theta} = 0$.[35]

Finally, we show that $\Delta w_t > 0$ (and so $\chi = 0$ in case 1 above) when there is truthful reporting in period t. To see this, note when the board seeks truthful reporting in t and $t+1$, $U_{t+1}(\theta_t, \mathbf{w}) = \left(\frac{e_{t+1}(\theta_t)}{e_{t+1}(\theta_G)} - 1\right) c + w^r_{s,t+1} + \sum_{j=t+1}^{T} \delta^{j-t-1}\overline{U}$ (see (20)). Plugging this into (15), we obtain

$$\Delta w_t = \left(\frac{1}{e_t(\theta_G)\Delta\theta} - \delta \frac{\Delta e_{t+1}}{e_{t+1}(\theta_G)\Delta\theta} \right) c > 0.$$

In case the board does not seek truthful reporting in $t+1$, we obtain the same expression by plugging in from the $t+1$ analogue of (A.11). **Q.E.D.**

[34] $\widetilde{E}_{\theta_N}$ is defined as E_{θ_N} but for $\widetilde{e}_{t+1}(\theta_N)$.

[35] Similar to case 1, the board knows that the manager's fit in t is θ_G and, thus, (B.7) is satisfied for $\theta_t = \theta_G$. Hence, the manager has no incentives to invest in firm-specific human capital off the equilibrium path in $t+1$.

No truthful reporting in period t**.** We continue in the steps laid out above. Suppose we have no truthful reporting in t. The aim is to minimize the manager's payoff

$$\begin{aligned} U_t(\theta_{t-1}, \mathbf{w}) &= w_t + (\theta_N + e_t(\theta_{t-1})\Delta\theta)\Delta w_t - c \\ &\quad + e_t(\theta_{t-1})\delta U^e_{t+1}(\theta_G, \mathbf{w}) + (1 - e_t(\theta_{t-1}))\delta U^e_{t+1}(\theta_N, \mathbf{w}). \end{aligned} \tag{B.9}$$

subject to (9), (10), $w_t, \Delta w_t, w_{s,t} \geq 0$. The constraint that the manager invests in firm-specific human capital (9), together with the feasibility restriction $\Delta w_t \geq 0$ can be stated as

$$\Delta w_t \geq \max\left\{0, \frac{\frac{c}{e_t(\theta_G)} + \delta\left(U^e_{t+1}(\theta_N, \mathbf{w}) - U^e_{t+1}(\theta_G, \mathbf{w})\right)}{\Delta\theta}\right\}. \tag{B.10}$$

Using that $U^e_{t+1}(\theta_t, \mathbf{w}) = \theta_t U_{t+1}(\theta_t, \mathbf{w}) + (1-\theta_t)\sum_{j=t+1}^{T}\delta^{j-t-1}\overline{U}_j$, the zero lower bound of Δw_t in (B.10) is binding if

$$\frac{\frac{c}{e_t(\theta_G)} + \delta\left(\theta_N U_{t+1}(\theta_N, \mathbf{w}) - \theta_G U_{t+1}(\theta_G, \mathbf{w}) + \Delta\theta\sum_{j=t+1}^{T}\delta^{j-t-1}\overline{U}_j\right)}{\Delta\theta} \leq 0. \tag{B.11}$$

Given the assumption of truthful reporting in $t-1$, the induction hypothesis is that $w_{s,t} = 0$, Δw_t is given by (B.10), and $w_t = 0$, unless (10) binds. Clearly, in $t = 1$, the choice of w_1, Δw_1, and $w_{s,1}$ has no effect on the payoffs in neither previous (as there are none) nor following periods. Thus, the aim is to minimize Δw_1, $w_{s,1}$ and w_1, subject to (9) and (10) proving the first induction step.

Case 3: No truthful reporting in period t ***and truthful reporting in period*** $t+1$**.** Using the induction hypothesis to plug into the manager's payoff (B.9) in period t, we have

$$\begin{aligned} &(\theta_N + e_t(\theta_{t-1})\Delta\theta)\max\left\{0, \frac{\frac{c}{e_t(\theta_{t-1})} + \delta\left(U^e_{t+1}(\theta_N, \mathbf{w}) - U^e_{t+1}(\theta_G, \mathbf{w})\right)}{\Delta\theta}\right\} - c \\ &+ e_t(\theta_{t-1})\delta U^e_{t+1}(\theta_G, \mathbf{w}) + (1 - e_t(\theta_{t-1}))\delta U^e_{t+1}(\theta_N, \mathbf{w}) \\ = &\begin{cases} \frac{\theta_G \delta U^e_{t+1}(\theta_N, \mathbf{w}) - \theta_N U^e_{t+1}(\theta_G, \mathbf{w})}{\Delta\theta} + \frac{(\theta_N + e_t(\theta_{t-1})\Delta\theta)c}{e_t(\theta_{t-1})} - c & \text{if } \Delta w_t \geq 0 \\ e_t(\theta_{t-1})\delta U^e_{t+1}(\theta_G, \mathbf{w}) + (1 - e_t(\theta_{t-1}))\delta U^e_{t+1}(\theta_N, \mathbf{w}) - c & \text{if } \Delta w_t = 0 \end{cases}. \end{aligned} \tag{B.12}$$

Clearly, if (B.11) holds (and so $\Delta w_t = 0$), the objective would be to minimize $U^e_{t+1}(\theta_t, \mathbf{w})$ and, thus, all of $\{w_{s,t+1} w_{t+1}, \Delta w_{t+1}\}$; with $w_{s,t+1}$ defined by (B.5), $w_{t+1} = 0$, and Δw_{t+1} defined by the more stringent condition of (B.4) and (B.11), which accounts for the fact that $\Delta w_t = 0$ in t. This would prove the claim. We now show that we must have, indeed,

$\Delta w_t = 0$. Suppose not. To minimize the first line of (B.12), we need to minimize

$$\begin{aligned} & \theta_G U^e_{t+1}(\theta_N, \mathbf{w}) - \theta_N U^e_{t+1}(\theta_G, \mathbf{w}) \\ = \quad & \theta_G \theta_N (U_{t+1}(\theta_N, \mathbf{w}) - U_{t+1}(\theta_G, \mathbf{w})) + \Delta\theta \sum_{j=t+1}^{T} \delta^{j-t-1} \overline{U}_j \end{aligned} \tag{B.13}$$

or, thus, equivalently minimize $U_{t+1}(\theta_N, \mathbf{w}) - U_{t+1}(\theta_G, \mathbf{w})$, subject to (B.4), (B.5), the reverse inequality in (B.11) (by contradiction assumption), and $w_{s,t+1}, w_{t+1}, \Delta w_{t+1} \geq 0$. Hence, define

$$\begin{aligned} & \mathcal{L}_3(\mathbf{w}, \Lambda) \\ = \quad & -(\widetilde{e}_{t+1}(\theta_N) - e_{t+1}(\theta_G))(w_{t+1} + \theta_G \Delta w_{t+1} + \delta U_{t+2}(\theta_G, \mathbf{w})) \\ & -((1-\widetilde{e}_{t+1}(\theta_N)) - (1-e_{t+1}(\theta_G))) \left(w_{s,t+1} + \sum_{j=t+1}^{T} \delta^{j-t-1} \overline{U}_j \right) \\ & +\lambda \left(w_{t+1} + \theta_G \Delta w_{t+1} + \delta U_{t+2}(\theta_G, \mathbf{w}) - \frac{c}{e_{t+1}(\theta_t)} - w_{s,t+1} - \sum_{j=t+1}^{T} \delta^{j-t-1} \overline{U}_j \right) \\ & +\mu \left(w_{s,t+1} + \sum_{j=t+1}^{T} \delta^{j-t-1} \overline{U}_j - w_{t+1} - \theta_N \Delta w_{t+1} - \delta U_{t+2}(\theta_N, \mathbf{w}) \right) \\ & +\kappa w_{s,t+1} + \rho w_{t+1} + \chi \Delta w_{t+1} \\ & +\sigma \left(\theta_N U_{t+1}(\theta_G, \mathbf{w}) - \theta_G U_{t+1}(\theta_G, \mathbf{w}) + \Delta\theta \sum_{j=t+1}^{T} \delta^{j-t-1} \overline{U}_j + \frac{c}{\delta e_t(\theta_G)} \right), \end{aligned}$$

where $\Lambda = \{\lambda, \mu, \kappa, \rho, \chi, \sigma\}$ is the set of weakly positive Kuhn Tucker multipliers, and $\widetilde{e}$ and $\widetilde{c}$ defined as before. Taking the first-order conditions

$$\begin{aligned} \frac{\partial \mathcal{L}_3(\mathbf{w}, \Lambda)}{\partial w_{s,t+1}} = 0 = \quad & -((1-\widetilde{e}_{t+1}(\theta_N)) - (1-e_{t+1}(\theta_G))) \\ & +\sigma(\theta_N(1-\widetilde{e}_{t+1}(\theta_N)) - \theta_G(1-e_{t+1}(\theta_G))) - \lambda + \mu + \kappa \\ \frac{\partial \mathcal{L}_3(\mathbf{w}, \Lambda)}{\partial \Delta w_{t+1}} = 0 = \quad & -(\widetilde{e}_{t+1}(\theta_N) - e_{t+1}(\theta_G))\theta_G \\ & +\sigma(\theta_N \widetilde{e}_{t+1}(\theta_N) - \theta_G e_{t+1}(\theta_G))\theta_G + \theta_G \lambda - \theta_N \mu + \chi \\ \frac{\partial \mathcal{L}_3(\mathbf{w}, \Lambda)}{\partial w_{t+1}} = 0 = \quad & -(\widetilde{e}_{t+1}(\theta_N) - e_{t+1}(\theta_G)) \\ & +\sigma(\theta_N \widetilde{e}_{t+1}(\theta_N) - \theta_G e_{t+1}(\theta_G)) + \lambda - \mu + \rho \end{aligned}$$

we obtain from the second and third condition that $\mu \Delta\theta = \theta_G \rho - \chi$ (observe that if $\rho = 0$, then we must have $\mu = \chi = 0$). From the first and third condition, we have $\sigma = \frac{\kappa+\rho}{\Delta\theta}$. To see that this implies $\sigma > 0$, suppose to the contrary that $\kappa = \rho = 0$. Since this would imply $\mu = \chi = 0$, we obtain that the RHS of the second first-order condition would be strictly positive, leading to a contradiction. Hence, we must have $\sigma > 0$, i.e., the continuation payoff is high enough that $\Delta w_t = 0$, as was to be shown. That is, the bonus in t is deferred, and Δw_{t+1} (and the continuation payoffs $U_{t+2}(\theta_{t+1}, \mathbf{w})$) must be chosen such that the more stringent of (B.11) and (B.4) is binding. If, as assumed, $e(\theta_N)$ is high enough (B.11) is more stringent than (B.4) even for $\theta_t = \theta_N$ (i.e., keeping promises regarding deferred compensation makes the constraint that the manager invests in firm specific human capital lax).

***Case 4: No truthful reporting in periods* t and $t+1$.** In analogy to Case 3, the objective is to minimize (B.12), where $U_{t+1}(\theta_t, \mathbf{w})$ is given by (B.6). We show again that $\Delta w_t \geq 0$ is binding by arguing to a contradiction. To minimize $U_{t+1}(\theta_N, \mathbf{w}) - U_{t+1}(\theta_G, \mathbf{w})$, define

$$
\begin{aligned}
& \mathcal{L}_4(\mathbf{w}, \Lambda) \\
= \quad & -\left(e_{t+1}(\theta_N) - e_{t+1}(\theta_G)\right)\Delta\theta\Delta w_{t+1} - \delta\widetilde{\mathbb{E}}_{\theta_N}\left[U^e_{t+2}(\theta_{t+1}, \mathbf{w})\right] + \delta E_{\theta_G}\left[U^e_{t+2}(\theta_{t+1}, \mathbf{w})\right] \\
& +\lambda\left(\Delta w_{t+1} - \frac{c}{e_{t+1}(\theta_t)\Delta\theta} - \delta\frac{U^e_{t+2}(\theta_N, \mathbf{w}) - U^e_{t+2}(\theta_G, \mathbf{w})}{\Delta\theta}\right) \\
& +\mu\left(w_{t+1} + \theta_N\Delta w_{t+1} + \delta U^e_{t+2}(\theta_N, \mathbf{w}) - w_{s,t+1} - \sum_{j=t+1}^{T}\delta^{j-t-1}\overline{U}_j\right) \\
& +\kappa w_{s,t+1} + \rho w_{t+1} + \chi\Delta w_{t+1} \\
& +\sigma\left(\theta_N U_{t+1}(\theta_G, \mathbf{w}) - \theta_G U_{t+1}(\theta_N, \mathbf{w}) + \Delta\theta\sum_{j=t+1}^{T}\delta^{j-t-1}\overline{U}_j + \frac{c}{\delta e(\theta_N)}\right).
\end{aligned}
$$

Taking the first order condition with respect to Δw_{t+1}, we have

$$
\begin{aligned}
\frac{\partial\mathcal{L}_4(\mathbf{w}, \Lambda)}{\partial\Delta w_{t+1}} = 0 = \quad & -\left(e_{t+1}(\theta_N) - e_{t+1}(\theta_G)\right)\Delta\theta + \lambda + \mu\theta_N + \chi \\
& +\sigma\left(\theta_N\left(\theta_N + e_{t+1}(\theta_G)\Delta\theta\right) - \theta_G\left(\theta_N + e_{t+1}(\theta_N)\Delta\theta\right)\right).
\end{aligned}
\tag{B.14}
$$

Since the first line in (B.14) is positive, it must be that $\sigma > 0$ as the term following σ can be rewritten as $-\theta_N(1 - e_{t+1}(\theta_G)) - e_{t+1}(\theta_N)\theta_G\Delta\theta$, which is strictly negative. Hence, both in Case 3 and 4, we have that $\sigma > 0$, implying that the continuation payoff must be high enough that $\Delta w_t = 0$. For completeness, note that from (B.12), we know that it is optimal to minimize w_{t+1} and $w_{s,t+1}$, as this minimizes the manager's payoff, while relaxing (B.4).

***Case: No truthful reporting in* $t-1$** Suppose that the board does not seek truthful reporting in period $t-1$ and that there is such a period. Furthermore, suppose that the board seek truthful reporting in t. The condition that the manager invests in firm-specific human capital in period $t-1$ even if $\Delta w_{t-1}=0$ is

$$
\begin{aligned}
0 &\geq \frac{c}{e_{t-1}(\theta_G)}+\delta\left(U_t^e(\theta_N,\mathbf{w})-U_t^e(\theta_G,\mathbf{w})\right)\\
&= \frac{c}{e_{t-1}(\theta_G)}+\delta\left(\begin{array}{c}\theta_N\left(\begin{array}{c}-c+(\widetilde{e}_t(\theta_N)\theta_G+(1-\widetilde{e}_t(\theta_N))\theta_N)\Delta w_t\\ +\delta(\widetilde{e}_t(\theta_N)U_{t+1}(\theta_G,\mathbf{w})+(1-\widetilde{e}_t(\theta_N))U_{t+1}(\theta_N,\mathbf{w}))\end{array}\right)\\ -\theta_G\left(\begin{array}{c}-c+(e_t(\theta_G)\theta_G+(1-e_t(\theta_G))\theta_N)\Delta w_t\\ +\delta(e_t(\theta_G)U_{t+1}(\theta_G,\mathbf{w})+(1-e_t(\theta_G))U_{t+1}(\theta_N,\mathbf{w}))\end{array}\right)\end{array}\right)\\
&\quad +\delta\Delta\theta\sum_{j=t}^{T}\delta^{j-t}\overline{U}_j
\end{aligned}
$$

Defining

$$
\Delta w_{t-1}^{-1}(0):=\frac{\begin{array}{c}\frac{c}{e_{t-1}(\theta_G)}+\delta(\Delta\theta c)+\delta\Delta\theta\sum_{j=t}^{T}\delta^{j-t}\overline{U}\\ +\delta^2\left(\begin{array}{c}(\theta_N\widetilde{e}_t(\theta_N)-\theta_G e_t(\theta_G))U_{t+1}(\theta_G,\mathbf{w})\\ -(\theta_G-\theta_N-\theta_G e_t(\theta_G)+\theta_N\widetilde{e}_t(\theta_N))U_{t+1}(\theta_N,\mathbf{w})\end{array}\right)\end{array}}{\delta\Delta\theta(\theta_N+\theta_G e_t(\theta_G)-\theta_N\widetilde{e}_t(\theta_N))},
$$

we, therefore, need that $\Delta w_t\geq\Delta w_{t-1}^{-1}(0)$. We can now proceed in analogy to Cases 1 and 2 by augmenting the induction hypothesis with

$$
\Delta w_t=\max\left\{\frac{\frac{c}{e_t(\theta_{t-1})}+\delta\left(U_{t+1}(\theta_N,\mathbf{w})-U_{t+1}(\theta_G,\mathbf{w})\right)}{\Delta\theta},\Delta w_{t-1}^{-1}(0)\right\}. \tag{B.15}
$$

It is now straightforward to show that, regardless of θ_{t-1} and whether the first or the second term is larger, the objective to minimize U_t again boils down to minimizing (B.3).

Analogously, if the board does not seek truthful reporting in t, the condition that the manager invests in firm-specific human capital in period $t-1$ even if $\Delta w_{t-1}=0$ requires that $\Delta w_t\geq\Delta w_{t-1}^{-1}(0)$, where

$$
\Delta w_{t-1}^{-1}(0):=\frac{\begin{array}{c}\frac{c}{e_{t-1}(\theta_G)}+\delta(\Delta\theta c)+\delta\Delta\theta\sum_{j=t}^{T}\delta^{j-t}\overline{U}\\ +\delta^2\left(\begin{array}{c}(\theta_N\widetilde{e}_t(\theta_N)-\theta_G e_t(\theta_G))U_{t+1}^e(\theta_G,\mathbf{w})\\ -(\theta_G-\theta_N-\theta_G e_t(\theta_G)+\theta_N\widetilde{e}_t(\theta_N))U_{t+1}^e(\theta_N,\mathbf{w})\end{array}\right)\end{array}}{\delta\Delta\theta(\theta_N+\theta_G e_t(\theta_G)-\theta_N\widetilde{e}_t(\theta_N))},
$$

We can now proceed in analogy to Cases 3 and 4 by augmenting the induction hypothesis with

$$
\Delta w_t=\max\left\{0,\frac{\frac{c}{e_t(\theta_{t-1})}+\delta\left(U_{t+1}^e(\theta_N,\mathbf{w})-U_{t+1}^e(\theta_G,\mathbf{w})\right)}{\Delta\theta},\Delta w_{t-1}^{-1}(0)\right\}. \tag{B.16}
$$

Furthermore, and in analogy to Case 3 and 4, we can show that the continuation payoff in $t+1$ is such that $\Delta w_{t-1}^{-1}(0)=0$. Hence, as stated in Proposition 1, the continuation payoff in $t+1$ must be high enough that the manager invests in firm-specific human capital in $t-1$

and t for $\Delta w_{t-1} = \Delta w_t = 0$. With more than two periods in which the board does not seek truthful reporting, we proceed analogously.[36] **Q.E.D.**

Omitted Derivations in Main Text

Lemma B.3 *A sufficient condition for (2) to hold is*

$$\begin{aligned} & \frac{e_N \theta_G}{e_N \theta_G + (1 - e_N)\theta_N} e_G + \frac{(1 - e_N)\theta_N}{e_N \theta_G + (1 - e_N)\theta_N} e_N \\ > \quad & e_1 > \frac{e_G (1 - \theta_G)}{e_G (1 - \theta_G) + (1 - e_G)(1 - \theta_N)} e_G + \frac{(1 - e_G)(1 - \theta_N)}{e_G (1 - \theta_G) + (1 - e_G)(1 - \theta_N)} e_N . \end{aligned} \tag{B.17}$$

Proof of Lemma B.3 The second inequality in (2) is most difficult to satisfy if underperformance in period t should (ex post) make it optimal to replace a manager even if her fit in $t-1$ was θ_G. The first inequality is most difficult to satisfy if it should be ex post optimal to keep the manager after realizing the high cash flow in t even if her fit in $t-1$ was θ_N. These conditions are captured by (B.17). Intuitively, they require that the board is willing to change her belief about the manager completely based on the firm's cash flows in period t. **Q.E.D.**

Calculating the Board's Expected Payoff in Section 3.2.1 In equilibrium, the board's expected payoff is V^*. Hence, we can rewrite (5) as

$$V^* = \frac{\sum_{i=1}^{2} \delta^{i-1} \overline{U} - U_1(\mathbf{w}) + \mathbb{E}\left[\sum_{i=1}^{2} \delta^{i-1} q_i \left(x_i - c - \overline{U}\right)\right]}{1 - \mathbb{E}\left[\sum_{i=1}^{2} \delta^{i-1} \widetilde{q}_i\right] \delta}.$$

If the board seeks truthful reporting in both periods, this expression becomes

$$V^* = \frac{-U_1(\mathbf{w}) + \begin{pmatrix} x + \left(\overline{\theta} + e_1\left(\theta_G - \overline{\theta}\right)\right)\Delta x - c \\ +\delta e_1 \left(x + \left(\overline{\theta} + e_2(\theta_G)\left(\theta_G - \overline{\theta}\right)\right)\Delta x - c\right) \end{pmatrix} + \delta(1 - e_1)\overline{U}}{1 - \delta(1 - e_1) - \delta^2 e_1}.$$

If instead, the board seeks truthful reporting only in the second period, we have

$$V^* = \frac{\begin{array}{c} -U_1(\mathbf{w}) + \begin{pmatrix} (x + (\theta_N + e_1(\theta_G - \theta_N))\Delta x - c) \\ +\delta e_1 \theta_G \left(x + \left(\overline{\theta} + e_2(\theta_G)\left(\theta_G - \overline{\theta}\right)\right)\Delta x - c\right) \\ +\delta(1 - e_1)\theta_N \left(x + \left(\overline{\theta} + e_2(\theta_N)\left(\theta_G - \overline{\theta}\right)\right)\Delta x - c\right) \end{pmatrix} \\ +\delta\left(e_1(1 - \theta_G) + (1 - e_1)(1 - \theta_N)\right)\overline{U} \end{array}}{1 - \left(\delta\left(e_1(1 - \theta_G) + (1 - e_1)(1 - \theta_N)\right) + \delta^2\left(e_1\theta_G + (1 - e_1)\theta_N\right)\right)}.$$

[36] Recall that for our the numerical analysis following Section 3.1, we have assumed that $e(\theta_N)$ is high enough, so that $\Delta w_{t-1}^{-1}(0)$ is always the larger term in (B.15) and (B.16) even if $\theta_{t-1} = \theta_N$.

By plugging in for $\{x, \Delta x\}$, $\overline{U}$, and $U_1(\mathbf{w})$, we obtain the values in Table 1.