Generalized Stability of Monetary Unions under Regime Switching in Monetary and Fiscal policies*

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Abstract

Earlier studies on the stability of monetary unions show that an inflation-targeting central bank imposes strict budgetary requirements on fiscal policy to obtain a unique stable equilibrium. Failure of only one fiscal authority to meet these requirements already results in non-existence of equilibrium. Nevertheless, it might prove useful to temporarily depart from such requirements in order to absorb country-specific shocks. We show that such departures are feasible if fiscal authorities commit to switch to more sustainable fiscal regimes in the future. Fiscal bailouts and debt devaluation may also broaden the range of policy stances under which monetary unions are stable.

JEL Classification: E62, E63

Keywords: Markov switching, monetary union, equilibrium stability and uniqueness, monetary-fiscal interactions

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1 Introduction

The stability of monetary unions has become a topic of considerable interest in recent years. Much of the literature on the stability and uniqueness of rational expectations equilibria, and the policies required to deliver them, has assumed policy rules to be constant throughout time. However, given the sizable response of policymakers to the recent global recession, the empirical relevancy of this assumption is cast into doubt. Moreover, without autonomous control over monetary instruments, member states of a monetary union are required to adjust their fiscal stance from time to time to absorb country-specific shocks, whilst ensuring a sustainable path for sovereign debt. A more realistic characterization of policy behavior, therefore, is one that allows policymakers to alter their stance in response to changing economic conditions. In this paper, we study the implications of such regime-switching behavior for equilibrium stability in a monetary union.

Using a simple general equilibrium model for an endowment economy, we show that equilibrium stability depends, not only on the policy stance within a particular regime, but also on the *frequency* with which the economy switches between regimes. Our model economy consists of two countries who form a monetary union. Monetary policy is controlled by a supranational central bank, while fiscal policy is conducted nationally (and independently) in each country. Our main contribution is that we allow policymakers to alter their stance over time according to an exogenous Markov process. The model is solved using the 'forward solution method' developed by Cho (2016). This method assumes that all agents are aware of the Markov-switching nature of the economy and know the corresponding transition matrix. The ability to alter the policy stance gives rise to different policy *regimes*, i.e. different

combinations of monetary and fiscal policies.

Our point of departure is a regime in which monetary policy aims to anchor inflation expectations by actively targeting union-wide inflation through appropriate adjustments in the nominal interest rate. Fiscal policy is conducted differently across the two countries. In one country, taxes respond endogenously to changes in government debt in such a way that long-term debt sustainability is ensured. In contrast, taxes are kept constant in the other country. As shown in Leeper (1991), this constellation of policies, if held fixed, is unable to deliver a stable equilibrium. In particular, for stability to be achieved in the fixed-regime case, taxes ought to offset changes in government debt, such that the growth rate of debt is below the real interest rate. Yet, if taxes are held constant, national debt can grow without bounds. This result has been shown by Bergin (2000) to carry over to monetary unions, in which stability can be obtained only if *all* member states maintain a sufficient feedback between debt and taxes (provided the central bank actively targets inflation). However, the case in which a member state of a monetary union ignores (at least temporarily) the accumulation of its debt is particularly interesting, given the strong reliance on expansionary fiscal policy to ward-off adverse country-specific shocks. Yet, despite its policy relevance, current macroeconomic theory has little to say about this regime as it is not feasible in fixed-regime models. This paper aims to fill this gap by focusing on the feasibility of this regime in a regime-switching setup. In particular, we consider three alternative regimes to which the economy can move.

In the first alternative regime, all member states target their public debt through appropriate adjustments in the tax rate, while monetary policy remains dedicated to stabilize inflation (expectations). We show that, if the frequency with which the economy moves to this alternative 'Ricardian' regime is

sufficiently high, a stable equilibrium can be obtained, even if one member state departs from its debt target in the initial regime. Furthermore, the weaker is the feedback between taxes and debt in the initial regime, the more vigorous must be the response of taxes to debt growth in the alternative regime. Hence, an intertemporal trade-off arises between weak fiscal consolidation today and aggressive consolidation in the future. The *slope* of this trade-off is determined by the fraction of time the economy resides in the initial regime.

In the second alternative regime, the central bank temporarily abandons its inflation target. It does so to allow inflation to reduce the *real* value of national debt to a level consistent with the government's intertemporal budget constraint. Again, if occurring sufficiently often, switching between the initial regime and this alternative 'debt devaluation' regime can deliver a stable and unique equilibrium. Notice that this result is a generalization of the Fiscal Theory of the Price Level (see e.g. Leeper, 1991, Sims, 1994, and Woodford, 1998): when the fiscal response to debt is insufficiently strong, a weak monetary response to inflation is a necessary condition for stability, yet unlike the fixed-regime case it is not a *sufficient* condition. Whether the condition is sufficient under regime-switching now also depends on the fraction of time spent in the initial regime. Although switching to either the first or second alternative regime can deliver stable solutions, the latter necessarily requires greater bouts of inflation in order to force down the debt burden in real terms. In fact, we show that a deficit-financed tax cut raises inflation under the possibility of moving to the second alternative regime, whereas inflation is entirely unresponsive if debt stabilization is achieved through fiscal measures only (due to Ricardian equivalence), as is the case under the first alternative regime.

In the third alternative regime, we allow the partner state to provide a fiscal bailout and thereby assume (part of) the debt burden of the other country.

As in the first two cases, this alternative 'bailout' regime allows one of the member states to temporarily disregard its debt obligations without necessarily jeopardizing the stability of the monetary union. In this case, however, the onus of stabilizing debt now falls upon the bailout donor and is increasing in the frequency with which the economy moves to the bailout regime. Therefore, this alternative regime unavoidably entails a transfer of wealth between countries required to ensure equilibrium stability.

Our results on the implications of switching between various policy regimes offer new insights into the requirements for the smooth functioning of monetary unions. For instance, when facing country-specific disturbances, our results suggest that member states can (temporarily) divert their fiscal tools away from debt stabilization and gear them towards the stabilization of economic activity and inflation, without threatening the stability of the monetary union. Whether such actions are feasible hinges on the credibility of the alternative future policy regime in which long-run fiscal solvency is once again a main priority. In turn, credible regime switching requires an appropriate institutional setup that allows for fiscal accommodation at the national level during severe economic crises, whilst preserving fiscal sustainability for all member states (see Corsetti et al., 2016).

This paper is related to a growing literature on the study of monetary-fiscal interactions, that dates back to Sargent and Wallace (1981). Leeper (1991) was one of the first to demonstrate the tight relation between the monetary and fiscal policy requirements for equilibrium stability and determinacy. Bergin (2000) shows that these requirements also apply to monetary unions and that price stability depends, not only on the monetary stance taken by the common central bank, but also on the fiscal stance of each individual member state.

Although insightful, these results are based on regime-invariant models and

thereby imply a characterization of monetary and fiscal policy too restrictive to be realistic or even practical. In fact, there is a large body of empirical work that shows that policy stances vary across time (Favero and Monacelli, 2005; Davig and Leeper, 2006; Gonzalez-Astudillo, 2013; Chen et al., 2015; Aldama and Creel, 2016; Bianchi and Ilut, 2017; Chang and Kwak, 2017). Much of this literature has focused on the relationship between changes in policy regimes and inflation dynamics in post-war US history. Bianchi (2012a, 2012b) and Bianchi and Ilut (2017), for instance, show that the appointment of Federal Reserve Chairman Volcker in 1979 marked a shift from passive to active monetary policy that led to more stable inflation during the 1980s as compared to the high inflation era of the 1960s and 1970s. Gonzalez-Astudillo (2013) and Chang and Kwak (2017) show that such shifts in US monetary policy have been synchronous with changes in the fiscal policy stance, while Favero and Monacelli (2005) and Davig and Leeper (2006) also find regime switching in both fiscal and monetary policy, yet without exhibiting synchronization between the two.

The broad empirical support for regime switching in fiscal and monetary policies has inspired researchers to allow for time variation in the parameters governing the fiscal and monetary response to policy targets in dynamic macroeconomic models. For instance, Davig and Leeper (2007), using a Fisherian model that abstracts from fiscal policy, show that regime-switching possibilities relax the determinacy requirements for monetary policy. In that case, a 'long-run Taylor principle' arises that states that determinacy does not require monetary policy to be active at all times, yet only that it is sufficiently active often enough in the future. Canzoneri et al. (2001) note that a similar idea applies to fiscal policy, and that a stable equilibrium requires Ricardian policies to prevail sufficiently often, but not necessarily always. Expanding on that

notion and using a standard New Keynesian model, Ascari et al. (2017) show that regime-switching possibilities may broaden the set of fiscal and monetary policies that deliver a stable and unique equilibrium, along similar lines as Davig and Leeper (2007). We build on this strand of literature by extending the analysis to a monetary union with switching in fiscal and monetary policies and cross-border fiscal transfers to investigate the conditions that allow for the realistic scenario in which (some of the) national fiscal authorities temporarily abandon their debt targets.

The rest of the paper is organized as follows. Section 2 describes the model and the different policy regimes over which the economy may switch. In this section, we also describe the equilibrium properties in the absence of regime switching. The solution method and equilibrium properties under regime switching are discussed in Section 3. Then, in Section 4, we calibrate the model parameters and examine the implications of regime-switching possibilities for equilibrium stability and uniqueness. In Section 5, we discuss the effects of transitory tax cuts under regime switching. Finally, Section 6 concludes.

2 A regime-switching model for a monetary union

In this section, we introduce a simple model of a monetary union that is made up of two endowment economies. The endowments both countries receive are perfectly substitutable and tradable. This means that the law of one price holds within the monetary union.

In the first part of this section, we focus on the supra-national monetary

policy rule implemented by the monetary authority. Next, we consider the fiscal policy rules used by the national governments of the two member states. We then consider how these policy decisions influence the households' optimal savings decisions. Our main contribution is to allow for the policies followed by the central bank and the national governments to vary over time. In the final part of this section, we introduce the set of policy regimes, and possible transitions between them, that we consider.

2.1 Monetary policy rule

The member countries of the monetary union that we study are subject to one single central bank which sets the gross nominal risk-free interest rate, R_t, in order to stabilize union-wide gross inflation, $\pi_t \equiv P_t/P_{t-1}$, where P_t is the aggregate price level in the union. In particular, we assume that the central bank targets a gross inflation rate of $\overline{\pi}$ and does so by following a feedback rule of the form

$$\frac{R_t}{R} = \left(\frac{\pi_t}{\overline{\pi}}\right)^{\phi_{\pi,s_t}}. \tag{1}$$

Here, the coefficient $\phi_{\pi,s_t} \geq 0$ determines the degree to which the central bank responds to deviations from the inflation target. This parameter varies across policy regimes, which are indexed by s_t. When $\phi_{\pi,s_t} > 1$ monetary policy is *active* and the central bank raises the real interest rate when inflation increases. Monetary policy is referred to as *passive* if this is not the case. Because the Taylor-rule coefficient varies across policy regimes, monetary policy does not always have to be active or passive, yet can switch between these two stances, as in Davig and Leeper (2007).

2.2 Fiscal policy and bailouts

The two countries, which we index by $j \in \{1, 2\}$, that make up the monetary union each pursue their own fiscal policy. In each period, the governments levy a lump-sum tax, $\tau_{j,t}$, and issue one-period nominal government bonds. We denote the nominal value of the one-period sovereign debt of country j in time t by $B_{j,t}$. The gross nominal return on this debt is given by $R_{j,t}$. These taxes and bonds are used to finance public consumption, $g_{j,t}$, and payment of the principal and interest on previous period's debt. To keep the model tractable and to avoid having to solve portfolio choice decisions, we assume that all government debt is domestically held by the household sector of the same country.[1]

We allow for a regime in which the debt liabilities of country 1 are (partly) taken on by country 2 in the form of a *bailout*. This is, effectively, a union-wide redistributive fiscal policy from country 2 to country 1. We formalize it as follows. In case of a bailout, country 2 takes over a fraction $\gamma_{s_t} \in (0, 1]$ of country 1's debt. If there is no bailout, $\gamma_{s_t} = 0$. In case of bailout, the bailout size is determined by country 1's outstanding public debt:

$$\gamma_{s_t} = \left(\frac{b_{1,t-1}}{b_1} \right)^{\phi_{\gamma,s_t}} - 1, \tag{2}$$

where $\phi_{\gamma,s_t} \geq 0$ denotes the bailout elasticity, $b_{1,t} \equiv B_{1,t}/P_t$ real government debt and b_1 is the steady-state value of government debt in country 1.[2] Note that, in steady state, $\gamma = 0$ for any regime s_t. The real costs of the bailout for

[1] One can think of this assumption as capturing the exposure of the domestic banking system to sovereign debt of the home country.

[2] In principle, one can add an intercept and scaling parameter to this equation as well. However, adding such parameters does not qualitatively change the results we present in our analysis. Therefore, we use this much more parsimonious representation.

country 2 are given by $\gamma_{s_t} R_{1,t-1} B_{1,t-1}/P_t$. The probability of such a bailout is a function of the transition probabilities between the different regimes. We discuss these transition probabilities in the fourth part of this section.

The asymmetry between country 1, which potentially is the recipient of a bailout, and country 2, which finances the potential bailout of country 1, is reflected in the government budget constraints. The public flow budget constraint of country 1 can be written as

$$B_{1,t} = (1 - \gamma_{s_t}) R_{1,t-1} B_{1,t-1} - P_t (\tau_{1,t} - g_{1,t}). \tag{3}$$

Here, the bailout size, γ_{s_t}, reduces the (expected) repayments on the previous period's debt obligations. In addition, the path of sovereign debt outstanding is determined by the real primary surplus, $\tau_{1,t} - g_{1,t}$ and the price level, P_t.

Since we assume that country 2 always fulfills its debt obligations, its budget constraint does not include a bailout term. It does, however, contain the cost of bailing out country 1. Country 2's public flow budget constraint reads

$$B_{2,t} = R_{2,t-1} B_{2,t-1} - P_t (\tau_{2,t} - g_{2,t} - \Theta_{s_t}), \tag{4}$$

where $\Theta_{s_t} \equiv \gamma_{s_t} R_{1,t-1} B_{1,t-1}/P_t$.

We assume that the government of country j sets the path of taxes according to a fiscal rule that targets a real debt level of $\bar{b}_j$. To achieve this goal, both governments set their taxes in proportion to deviations of the stock of outstanding government debt from $\bar{b}_j$, i.e.

$$\tau_{j,t} = \phi_{b_j,s_t} \left(b_{j,t-1} - \bar{b}_j \right) + z_{\tau_j,t}. \tag{5}$$

Here, $z_{\tau_j,t}$ is a tax shock that represents random, but possibly persistent, devi-

ations from this policy rule. The coefficient $\phi_{b_j,s_t} \geq 0$, which varies by policy regime s_t, characterizes the responsiveness of a country's fiscal policy with respect to debt its level.

Throughout, we use the terminology introduced by Leeper (1991) and call fiscal policy *passive* when $\phi_{b_j,s_t} > (1-\beta)/\beta$, where $\beta \in (0,1)$ denotes the household's discount factor. This is the case in which the government prevents public debt from growing faster than the real interest rate. Fiscal policy is called *active* if this is not the case. The distinction between passive and active fiscal policy is important because if public debt persistently grows faster than the real interest rate, the expected future present discounted value of government debt will be infinite.

Of course, the growth of public debt in country 1 is potentially mitigated when it gets bailed out by country 2. To which degree bailouts stem the growth of public debt depends on the frequency with which they occur and the magnitude of the size of the bailouts, γ_{s_t}. Moreover, the growth rate of public debt in country 2 potentially increases in the frequency with which it bails out country 1 and in the magnitude of these bailouts.

2.3 The household sector

Each country's household sector consists of a representative household. Part of the income of households in each period in both countries is a constant endowment y_j. The rest of the household sectors' income is made up of the interest and principle payments they receive on the one-period government bonds that they invested in during the previous period, i.e. $R_{j,t-1}B_{j,t-1}$. Households spend this income on three different things. First of all, they buy consumption goods, $c_{j,t}$, at the price P_t. Second, they invest in government bonds, $B_{j,t}$. Finally,

they pay taxes, $P_t\tau_{j,t}$. Combining these sources of income and expenditure yields the following flow budget constraint of the household sector:

$$P_t c_{j,t} + B_{j,t} + P_t\tau_{j,t} = R_{j,t-1}B_{j,t-1} + P_t y_j. \tag{6}$$

We assume that the representative households in each country have the same log-utility preferences. Households in country j choose their paths of consumption, $c_{j,t}$, and savings, $B_{j,t}$, to maximize expected life-time utility, given by

$$E_t \sum_{k=0}^{\infty} \beta^k \log c_{j,t+k}. \tag{7}$$

They do so subject to the budget constraint given by (6). This yields the following Euler equation, that is the intertemporal optimality condition for the households in country j:

$$\frac{1}{c_{j,t}} = \beta R_{j,t} E_t \left[\frac{1}{\pi_{t+1}} \frac{1}{c_{j,t+1}} \right]. \tag{8}$$

In addition, the household's path of consumption and savings satisfies the transversality condition that the expected present discounted value of savings infinitely far in the future is zero. Assuming the no-arbitrage condition holds, the interest rates on public debt must be equal to the risk-free rate set by the central bank, i.e. $R_{j,t} = R_t$.

For simplicity, we assume that real government consumption remains constant over time in both countries, i.e. $g_{j,t} = g_j$ for all t. Perfect substitutability and tradability of the endowments, y_1 and y_2, across countries then imply the following goods market clearing condition for the monetary union,

$$c_{1,t} + c_{2,t} + g_1 + g_2 = y_1 + y_2. \tag{9}$$

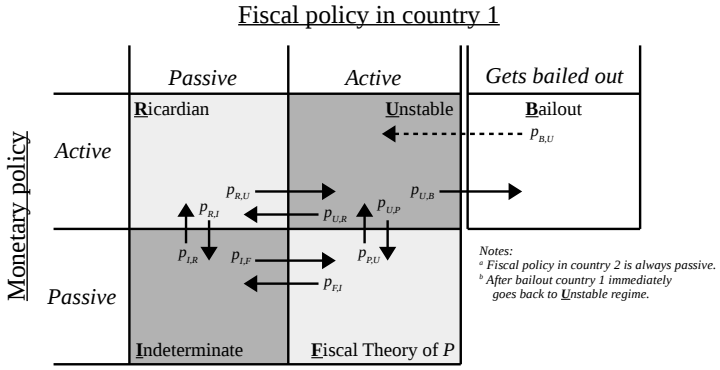

Figure 1: Policy regimes, names, and transition probabilities

which in turn implies household consumption in the monetary union is constant.[3]

2.4 Policy regimes and regime switches

Thus far, we have discussed the different monetary and fiscal rules that the central bank and the governments can follow. Here we focus on the likelihood of transitions between these policy rules. In particular, we consider the transitions between different policy *regimes*.

Figure 1 depicts the grid with the five different policy regimes that we consider. We index the regimes by their first letter. The regimes can be split up into three groups. The first group consists of the light-grey shaded regimes in Figure 1. These are the regimes under which, in the absence of regime changes, there is a well-defined rational expectations equilibrium. They are:

- **Ricardian**. This is the regime under which the policy stances by the

[3]Because aggregate endowments in the union are constant, there is no stimulative role of either fiscal or monetary policy at the monetary union level. All stimulus, in terms of consumption, in one country comes at the cost of consumption in the other.

central bank and the government of country 1 result in a stable and unique rational expectations equilibrium. This is the case when monetary policy is active, and follows the Taylor principle, and fiscal policy in country 1 is passive or, following the terminology used in Woodford (2001), *Ricardian* . That is, $\phi_{\pi,s_t} > 1$ and $\phi_{b_1,s_t} > (1-\beta)/\beta$. This is the type of equilibrium that is commonly studied in New-Keynesian models.

- **Fiscal Theory of the Price Level**. This is the regime under which fiscal policy in country 1 does not assure long-run fiscal stability for any path of inflation. In that case, the price level jumps to affect real debt holdings by the government such that the present discounted value of future real government debt is finite (see Leeper, 1991, Sims, 1994, and Woodford, 1998). In this regime, monetary policy is passive, $\phi_{\pi,s_t} \leq 1$, and fiscal policy in country 1 is active, $\phi_{b_1,s_t} \leq (1-\beta)/\beta$.

The second group is made up of two regimes under which, if they were permanent, there is no well-defined rational expectations equilibrium. These are the dark-grey shaded regimes in Figure 1. They are:

- **Indeterminate**. This is the regime in which the fiscal authority in country 1 runs a passive policy, $\phi_{b_1,s_t} > (1-\beta)/\beta$, that assures long-run fiscal stability no matter what the path of inflation. However, the central bank runs a passive monetary policy, $\phi_{\pi,s_t} \leq 1$, that does not pin down a unique path of inflation expectations. As a consequence, for any level of real government debt by countries 1 and 2, inflation expectations are not uniquely determined and there are multiple equilibrium paths consistent with rational expectations.

- **Unstable**. Under this regime, the central bank pursues an active mone-

tary policy, $\phi_{\pi,s_t} > 1$, that aims to uniquely pin down inflation expectations. However, the government of country 1 runs an active fiscal policy, $\phi_{b_1,s_t} \leq (1-\beta)/\beta$, which results in a non-zero expected long-run present discounted value of the country's real debt level for the path of inflation implied by monetary policy. As Bergin (2000) shows, in this case there does not exist a stable rational expectations equilibrium.

The third group is made up of the **Bailout** regime in which real debt holdings of country 1 are reduced through a bailout by country 2.

The focus of this paper is on the **U**nstable regime, which we consider as the benchmark starting point throughout the rest of the analysis. Regime **U** is particularly interesting within the context of monetary union, as member states, having relinquished their control over monetary policy, must rely on accommodative fiscal policy to offset country-specific disturbances. This may require fiscal authorities to, occasionally, loosen their debt target or, equivalently, adopt an active fiscal policy. Such actions may arise when the common central bank pursues an active monetary policy, implying a move towards regime **U**.

Unfortunately, the inexistence of a rational expectations equilibrium in the **U**nstable regime means that current economic theory has little to say about monetary unions where the central bank sticks to the Taylor principle while fiscal authorities' decisions are not consistent with long-run solvency. The only thing we learn from current theory is that, if the policy stances in this regime were *permanent*, then the governments with the explosive debt path would not have access to sovereign debt markets.

However, if the policy stances are not permanent, then the economy might cycle between regimes in which the debt of country 1 grows precipitously and

cycles where its growth slows down or even reverses. If the economy is not in the unstable regime too frequently and excessive debt growth in the periods in the unstable regime are offset by slowdowns in growth or reductions of real debt levels in other regimes then, even though the economy might sometimes transition through the unstable regime, the rational expectations equilibrium could still be stable and determinate.

In order to formalize this intuition, we have to be explicit about how the economy moves through the different policy regimes. Following other studies that use Markov-Switching Rational Expectations models, we assume that the transition probabilities between policy regimes only depend on the policy regime that the economy is in and that they do not depend on other equilibrium variables that determine the state of the economy we consider.[4] We assume that these transition probabilities are constant over time. Throughout, $p_{s_{t-1}s_t}$ denotes the probability of being in regime s_t in period t conditional on being in regime s_{t-1} in the previous period.

For tractability purposes, we consider regime switching between the unstable regime and only *one* of the other alternative regimes. Hence, the transition probability matrix $\mathcal{P}$ is a 2x2 matrix governing the transition from and to the unstable regime and one of the other alternative regimes:

$$\mathcal{P} = \begin{bmatrix} p_{UU} & p_{Us_t} \\ p_{s_{t-1}U} & p_{s_{t-1}s_t} \end{bmatrix}, \qquad s_t \in \{R, F, I, B\}, \tag{10}$$

with $p_{UU} + p_{Us_t} = p_{s_{t-1}U} + p_{s_{t-1}s_t} = 1$. Of course, expanding the number

[4]For example, the probability of a bailout depends on whether the economy is in the Unstable regime, yet does not depend on the real debt levels of countries 1 and 2. Current state-of-the-art solution techniques for rational expectations models with regime switching are not able to handle such state-dependence of the regime transition probabilities. This is why we abstract from it.

of transition possibilities would make the model more realistic, yet would also obscure the channels that explain how switching to an alternative regime helps deliver equilibrium stability and determinacy once the economy cycles through the **U**nstable regime.

When considering switching between the **U**nstable and **B**ailout regimes, we impose the condition that, after a bailout, the government of country 1 resumes the active fiscal policy stance it took before, i.e. $p_{BU} = 1$. Also, we ignore switching between the **U**nstable and **I**ndeterminate regimes, since neither regime is favorable in terms of equilibrium stability.

3 Equilibrium conditions and properties

Throughout the rest of this paper we investigate the stability, determinacy, and dynamic properties of the equilibrium of the Markov Switching Rational Expectations (MSRE) model that we introduced in the previous section. In this section, we discuss the solution method we use and the associated equilibrium conditions that we check for stability and determinacy. In addition, we describe the additional equilibrium properties that we focus on in the numerical results we present in the subsequent sections.

3.1 Solution method and stability conditions

Analyses of conventional Linear Rational Expectations (LRE) models without regime switching generally rely on the eigenvalue conditions described in Blanchard and Kahn (1980) to establish stability and determinacy of the equilibrium. However, as Farmer et al. (2009) point out, these conditions are necessary but not sufficient in the context of models with regime changes. For

this reason, we instead use the solution method introduced by Cho (2016).[5,6] Cho's method is a generalization of the application of the method of undetermined coefficients in LRE models developed by Cho and McCallum (2015) to MSRE models. It allows for both investigating the necessary and sufficient conditions for stability and determinacy, as well as the dynamics of the equilibrium through impulse response functions.

The main insight of the method is that the solution of the MSRE model can be written in a specific matrix form. The elements of these matrices are the undetermined coefficients that need to be solved for to obtain the solution. Unfortunately, the system of matrix equations can not be solved directly. It turns out, though, that a solution can be obtained by iterating over the equations until convergence. This is what Cho and Moreno (2011) call the "Forward Method".

Necessary and sufficient conditions for determinacy, indeterminacy, and instability can then be derived in terms of the solution matrices. In our numerical analysis, it is these conditions that we check to see whether our model yields a mean-square stable Rational Expectations Equilibrium and to establish whether this equilibrium is either determinate or indeterminate.

3.2 Dynamic response to shocks in the unstable regime

Because the solution method yields a complete representation of the solution of the MSRE model, it also allows for the calculation of impulse response

[5]In particular, Cho (2016) derives these necessary and sufficient conditions for the case where stability, determinacy, and indeterminacy in the MSRE model are defined in terms of mean-square stable solutions. See also Costa et al. (2005) and Farmer et al. (2009).

[6]The mathematical details of how we linearize our model and write it in the representation used by Cho (2016), as well as the main results from Cho's paper that we utilize, can be found in the Online Appendix.

functions, which differ depending on the policy regime in place when shocks occur. We use these impulse response functions to document the response of the economy to a fiscal shock for parameter combinations for which the model with regime switching yields a determinate equilibrium in which there is a positive probability of the economy ending up passing through the **U**nstable policy regime. In particular, we consider the response of the economy in case the fiscal shock occurs when the economy is in the **U** regime.

The reason we focus on this case is because, in the absence of regime switching, an equilibrium path does not even exist. Thus, we focus on the dynamics of a monetary union that starts off in a situation in which the central bank pursues an active monetary policy and in which some of its members, as captured here by country 1, pursue a fiscal policy that, if unaltered, does not assure long-run fiscal sustainability. This is a very relevant real-life scenario, in particular for countries that belong to a monetary union, for which current theory has little more to offer than the observation that the monetary union is not stable in that case.

We focus on how the stability and determinacy properties of the equilibrium change in case of regime switching in the next section. We document the dynamic response of the economy to a fiscal shock in Section 5.

4 Generalized stability conditions

In this section, we illustrate that episodes of combined active monetary policy and active fiscal policy do not necessarily lead to instability of a monetary union. In such a case, the stability of the union in our model can potentially be established by offsetting such episodes through three different channels. The first is through periods of *fiscal austerity* in which the government of country 1

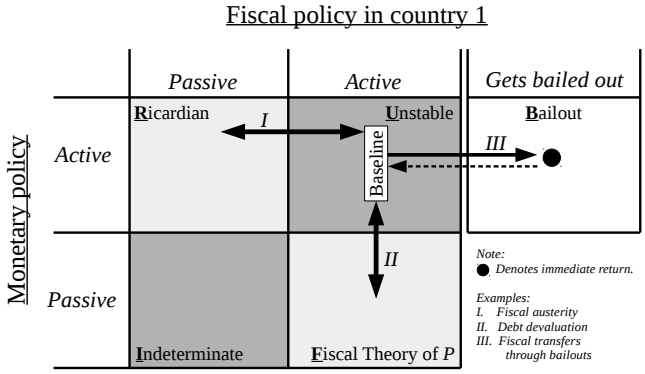

Figure 2: Three examples of generalized stability conditions

temporarily pursues a passive fiscal policy. The second channel is through *debt devaluation* when the central bank sometimes abandons its active monetary policy and allows the path of inflation to adjust to reduce the real sovereign debt burden of country 1. The last potential channel is through *fiscal transfers from country 2 to country 1 in the form of bailouts*.

We illustrate our main point for each of these channels using three examples. Each of the examples isolates one of the three respective channels. The most important thing they have in common is that they all involve the economy being in the **U**nstable regime periodically. In fact, our baseline case is the one in which the economy is *always* in the **U**nstable regime and there, thus, does not exist a rational expectations equilibrium. Figure 2 depicts the three examples we consider in the regime-grid introduced in Figure 1. The three examples, numbered I through III in the figure, correspond to the respective channels discussed above. The baseline regime is highlighted as the origin of the arrows for each of the examples.

Beside the periodic episodes in the **U**nstable regime, the examples also have a set of benchmark parameter values in common. Keeping these parame-

Table 1: Benchmark parameter values

	Description	**Value**	**Interpretation**
β	Discount factor	0.99	4 percent annual real interest rate
ρ_τ	Tax-smoothing parameter	0.9	High persistence of tax shocks
b_j	Steady-state debt ratio	2.4	60 percent annualized debt ratio
y_j	Output levels	0.5	Monetary union of "equals"
g_j	Steady-state public spending ratio	0.2	Long-run OECD average
ϕ_{π,s_t}	Monetary policy stance ($s_t \neq F$)	1.5	Ensures active monetary policy
ϕ_{b_2}	Fiscal policy stance country 2	0.02	Ensures passive fiscal policy

ters fixed enhances the comparability across examples and allows us to isolate the importance of the parameters we change for the stability conditions of the monetary union in our model. The benchmark parameter values across examples are listed in Table 1 and are based on a quarterly frequency for t. Lines 4 and 5 of the table, together with the goods market clearing condition (9), imply that consumption makes up 60 percent of output.

4.1 Fiscal austerity (I)

The first example that we consider is the one in which periods of *fiscal austerity* in country 1 potentially offset the explosive growth of real government debt under the active fiscal policy stance the government of country 1 pursues when in the Unstable regime that is our baseline. Such episodes of *fiscal austerity* are periods during which the government of country 1 implements a passive fiscal policy rule focused on stabilizing its real debt level to sustainable levels. We use this example to point out three main insights. The first is that, for a given frequency of switching between fiscal policy regimes in country 1, the more explosive real debt levels are in the baseline **U** regime, the more austere fiscal policy needs to be in the other regime to assure stability of real debt levels of country 1. Fiscal authorities therefore face an intertemporal trade-off

between loose fiscal policy today and budgetary tightening in the future that is required to guarantee a stable equilibrium.

To illustrate this, we consider variation in $\phi_{b_1,R}$ versus $\phi_{b_1,U}$ for a given pair of transition probabilities, p_{UR} and p_{RU}. By (10), the choice of the pair of transition probabilities also pins down $p_{UU} = 1 - p_{UR}$ and $p_{RR} = 1 - p_{RU}$. Solving for the steady state of the two-state Markov process implied by these transition probabilities yields that the fraction of time the economy spends in the **U**nstable regime, f_U, in this case equals

$$f_U = \frac{1}{1 + \frac{p_{UR}}{p_{RU}}}.$$

So, what really matters for the stability of the economy in this case is the *ratio* of the transition probabilities. We report the results of our experiment in terms of how the stability of the monetary union depends on f_U.[7] We consider which combinations of $(\phi_{b_1,U}, \phi_{b_1,R})$, where $\phi_{b_1,U} \leq (1-\beta)/\beta < \phi_{b_1,R}$, result in a stable equilibrium for a given fraction of time the economy spends in the **U**nstable regime.

The theoretical benchmark, based on Bergin (2000), is the case in which $f_U = 1$ and the unstable regime is an absorbing state. In that case the monetary union is unstable no matter what the fiscal policy stances $(\phi_{b_1,U}, \phi_{b_1,R})$. Figure 3 shows the combinations of $(\phi_{b_1,U}, \phi_{b_1,R})$ that result in a stable equilibrium for different levels of $f_U < 1$. In particular, it plots the boundary of the stable parameter set for different values of f_U. For each of these boundaries, the points on the lower-left side are the policy stances under which there is no stable equilibrium while those on the upper-right are part of the stable policy

[7] Since f_U only depends on the ratio p_{UR}/p_{RU}, we normalize p_{UR} to equal 0.05 such that the expected duration of an episode in the **U**nstable regime is twenty quarters (5 years).

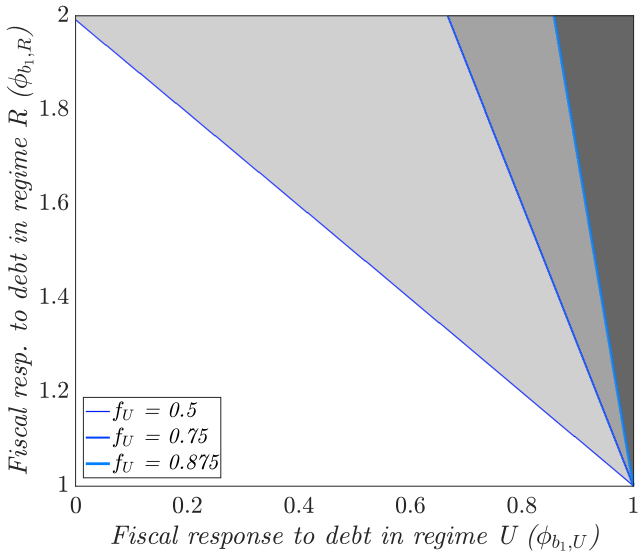

Figure 3: Stable parameter sets under Fiscal Austerity experiment

Notes: The figure shows the stable parameter sets in $(\phi_{b_1,U}, \phi_{b_1,R})$-space for three different frequencies of occurrence of the **U**nstable regime, f_U. The areas to the lower-left of the boundaries of the sets for the different f_U reflect parameter combinations where there is no stable equilibrium. The gray-shaded areas depict parameter combinations that result in a stable equilibrium.

parameter space for the given level of f_U that the boundary corresponds with.

As this figure shows, the more active fiscal policy is in the unstable regime, i.e. the lower $\phi_{b_1,U}$, the more austere fiscal policy needs to be when it is passive, i.e. the higher $\phi_{b_1,R}$, to assure stability of the monetary union. This can be seen from the fact that the boundary of the stable parameter set is downward sloping in the Figure for all the values $f_U < 1$.

The second main insight is that there is another trade-off between the degree and frequency of austerity. That is, for a given active fiscal policy in country 1 in the baseline regime, the more frequently the government of country 1 temporarily takes a passive fiscal policy stance the less severe the degree of austerity during such periods has to be to assure long-run fiscal solvency.

This can also be seen from the figure when one considers the evolution of the stable parameter set as a function of f_U for a given $\phi_{b_1,U}$. Namely, the more time the economy spends in the unstable regime, i.e. the higher f_U, the more passive fiscal policy needs to be in periods of austerity to assure stability. That is, the higher $\phi_{b_1,R}$. Thus, in addition to the intertemporal trade-off between fiscal stances across current and future regimes, there is also a direct trade-off between the frequency and severity of austerity measures needed to stabilize the monetary union.

The final point we make with this example is that if monetary policy does not have any real effects on economic outcomes in country 1 and is always active, then stability is purely a fiscal policy issue. That is, the fiscal policy stances that result in stability of the monetary union do not depend on the monetary policy rule, i.e. on $\phi_{\pi,s_t} > 1$. In terms of Figure 3, as long as $\phi_{\pi,R} > 1$ and $\phi_{\pi,U} > 1$, then the boundaries of the stable parameter sets plotted in the figure are invariant to the actual values of $\phi_{\pi,R}$ and $\phi_{\pi,U}$.

The reason for this is that the dynamics of the evolution of inflation expectations under active monetary policy in this example do not depend on fiscal policy. In particular, this evolution is given by

$$E_{t|s_t}\left[\pi_{t+1}\right] = E_{t|s_t}\left[\phi_{\pi,s_t}\right]\pi_t \text{ for } s_t \in \{U,R\}\,. \tag{11}$$

Thus, because $\phi_{\pi,R} > 1$ and $\phi_{\pi,U} > 1$,

$$E_{t|s_t}\left[\phi_{\pi,s_t}\right] > 1 \text{ for } s_t \in \{R,U\}\,.$$

The path of inflation expectations is therefore always an unstable node and inflation expectations are anchored, no matter what the fiscal policy stances,

$\phi_{b_1,R}$ and $\phi_{b_1,U}$, and the frequency of occurrence of regimes, f_U. Conditional on these anchored inflation expectations, stability then is a matter of fiscal policy assuring a zero expected long-run discounted value of real debt burdens. This long-run stability of the real debt burden depends on the weighted average of the fiscal policy stances in country 1 under the **R** and **U** regimes, where the weight of the latter equals the frequency with which the economy is in the **U**nstable regime, i.e. f_U. This is what drives the trade offs in the first two points of this example and the shape of the stable parameter sets in Figure 3.

4.2 Debt devaluation (II)

Where in the previous example stability and determinacy of the equilibrium was the result of the fiscal authority in country 1 frequently stepping on the break during periods of austerity to stem real debt growth, in this example we consider the case in which stability and determinacy are achieved through the reduction of the real value of country 1's nominal debt obligations as a consequence of increased inflation. Thus, this example is a case of debt devaluation. In the context of the interaction of fiscal and monetary policy that we consider here, of course, this case is more often referred to as the Fiscal Theory of the Price Level. So, in this example, we consider the stability and determinacy of the equilibrium in this economy when it switches between the baseline **U**nstable regime, in which both the central bank and the government of country 1 take an active policy stance, and the **F**iscal Theory of the Price Level regime, in which the central bank abandons its active monetary policy stance. The stable and determinate equilibrium outcomes in this case involve jumps in the price level, such that the level of inflation depends on the deviation of country 1's real debt level from its target, $\bar{b}_1$.

This experiment is thus about the interaction of the monetary policy and fiscal policy responses in the **U**nstable and the **F**iscal-Theory-of-the-Price-Level regimes. These policies, in principle, involve four parameters, namely $\phi_{\pi,U}$, $\phi_{\pi,F}$, $\phi_{b_1,U}$, and $\phi_{b_1,F}$. In our example we reduce this to two parameters. This is because we keep $\phi_{\pi,U} = 1.5$ at its benchmark value. Moreover, we assume that the stance of fiscal policy does not vary across the **U**nstable and the **F**iscal Theory of the Price Level regimes, such that $\phi_{b_1,U} = \phi_{b_1,F}$. Given these two restrictions, we can consider the stability of equilibrium in $(\phi_{b_1,U}, \phi_{\pi,F})$-space. Just like in the previous example, we consider these stability sets for different frequencies with which the economy is in the unstable regime, i.e. f_U. In this case, this frequency is given by $f_U = 1/\left(1 + p_{UF}/p_{FU}\right)$. The resulting stability sets for the parameters are plotted in Figure 4.

The most striking feature of the figure is that the boundaries of the stable parameter sets are horizontal. This indicates that, conditional on persistent active fiscal policy in country 1, the only thing that matters for stability of the equilibrium is whether the central bank pursues a passive enough monetary policy. This monetary passiveness allows inflation to adjust to stabilize country 1's real debt levels, no matter how active that country's fiscal policy stance.

The reason for this is that, as we illustrated in (11), whether inflation expectations are anchored or not does not depend on the fiscal policy stances of countries 1 and 2. So, whether inflation expectations are able to adjust to stabilize real debt levels only depends on the stance of monetary policy across the regimes and the frequency with which the regimes occur. Again, this is the result of the assumption that monetary policy does not have any real effects and thus does not affect real debt growth in country 1 in that way.

The second feature of Figure 4 is that the more frequently the economy is in the **U**nstable regime, in which fiscal policy is active, the more passive monetary

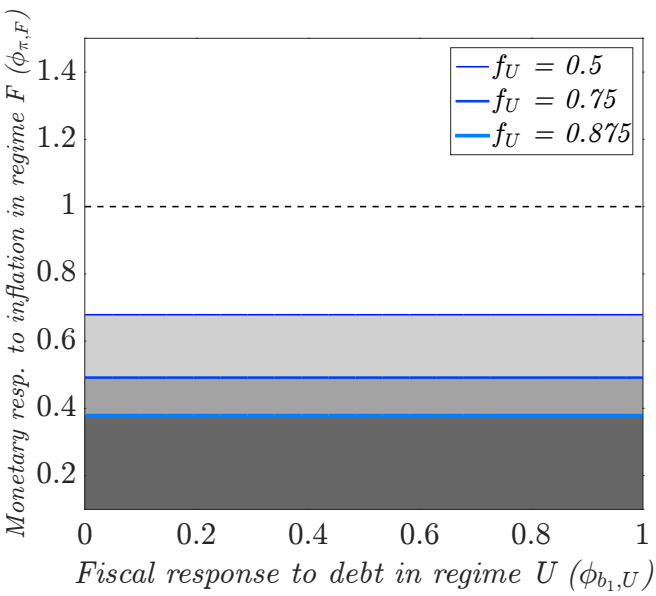

Figure 4: Stable parameter sets under Debt Devaluation experiment

Notes: The figure shows the stable parameter sets in $(\phi_{b_1,U}, \phi_{\pi,F})$-space for three different frequencies of occurrence of the **U**nstable regime, f_U. The areas above the boundaries of the sets for the different f_U reflect parameter combinations where there is no stable equilibrium. The gray-shaded areas depict parameter combinations that result in a stable equilibrium.

policy needs to be in the **F** regime. Because the economy does not reside in the **F** regime forever, the adjustment of the price level required to bring down the real debt burden and satisfy the government's intertemporal budget constraint gets 'interrupted' by active monetary policy in the **U** regime. The higher is f_U, the more frequent these interruptions and so the more passive monetary policy must be to guarantee stability. In this figure, we keep the monetary stance in the **U** regime fixed at $\phi_{\pi,U} = 1.5$. However, if we let $\phi_{\pi,U}$ vary, then we would also find that the more active monetary policy is in the **U** regime, the more passive it has to be when the central bank temporarily abandons the Taylor principle.

4.3 Fiscal transfers (III)

This example illustrates the third channel through which stability of the monetary union can be achieved in case the fiscal authority of part of the union tends to pursue an active policy rule that results in a potentially explosive real debt burden. This channel involves fiscal transfers in the form of bailouts from country 2 to country 1.

Similar to the first example of fiscal austerity, what emerges from this example are two main trade-offs. The first is that, conditional on a certain frequency of bailouts, the size of the bailout necessary for stability of the monetary union is increasing in how active fiscal policy of country 1 is in the **U**nstable regime (decreasing in $\phi_{b_1,U}$).

We illustrate this in Figure 5. This figure shows the stable parameter sets for the combinations of the fiscal policy parameter in the **U**nstable regime, $\phi_{b_1,U}$, and the elasticity of the fraction of debt of country 1 that is transferred to country 2 in each bailout, i.e. $\phi_{\gamma,B}$. The former parameter is plotted on the horizontal axis while the latter, which is our proxy for the size of the bailouts, is on the vertical axis. The figure contains the sets of stable parameter combinations for different frequencies of bailout, p_{UB}. For comparison purposes we translate this bailout probability again into the fraction of time the economy is in the **U**nstable regime, in this case, $f_U = 1/\left(1 + p_{UB}/p_{BU}\right)$.

What you can see from the figure is that, for a given frequency of bailouts, i.e. for a given f_U, a more active fiscal policy of country 1 in the **U**nstable regime, i.e. the lower $\phi_{b_1,U}$, the larger the size of the bailouts, as captured by $\phi_{\gamma,B}$, that is needed to assure stability of the debt in country 1 and of the monetary union. This is reflected in the boundaries of the stable parameter sets being downward sloping. By assuming a fraction of country 1's debt when

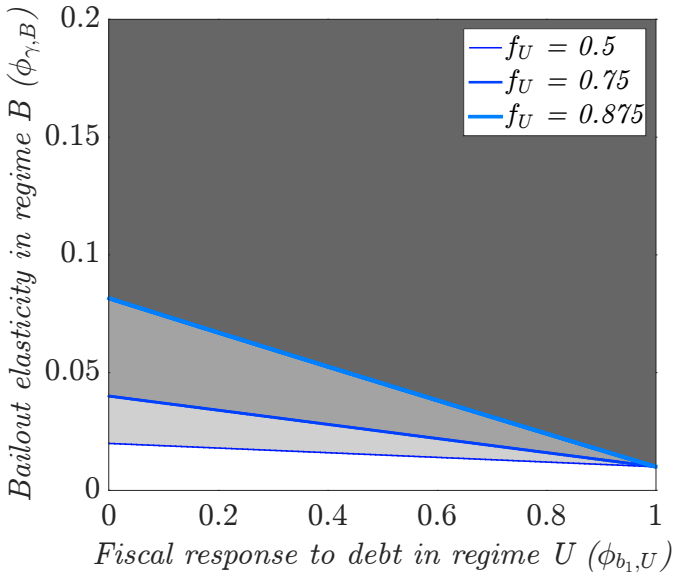

Figure 5: Stable parameter sets under Fiscal Transfers experiment

Notes: The figure shows the stable parameter sets in $(\phi_{b_1,U}, \phi_{\gamma,B})$-space for three different frequencies of occurrence of the **U**nstable regime, f_U. The areas below the boundaries of the sets for the different f_U reflect parameter combinations where there is no stable equilibrium. The gray-shaded areas depict parameter combinations that result in a stable equilibrium.

the economy slips into the **B**ailout regime, the fiscal authority of country 2 slows down the growth rate of debt by enough to prevent the intertemporal budget constraint from being violated. The latter is made possible by the assumption that fiscal policy in country 2 is always passive. Also, because taxes are lump-sum in our model, there is no limit to tax revenue in country 2 due to the existence of a Laffer curve. This means that the relative sizes of countries 1 and 2 do not affect the stable parameter sets in Figure 5. The only limitation on the relative size of country 1 compared to country 2 is that the required bailouts don't violate the monetary union's budget constraint. If they do, then the bailouts implied by our parameterization are simply infeasible.

The flip side of the trade-off above is that, conditional on the size of the

bailouts, the necessary frequency of bailouts, p_{UB}, is increasing in the rate of growth of real debt in country 1, i.e. decreasing in $\phi_{b_1,U}$. This can also be seen from Figure 5 in that for a given level of $\phi_{\gamma,B}$ on the vertical axis, the minimum frequency of time the economy spends in the **U**nstable regime, f_U, necessary for stability is decreasing in the fiscal policy parameter $\phi_{b_1,U}$.

Just like in the first example, because our stylized model does not include real effects of monetary policy, as long as the central bank always pursues an active monetary policy, i.e. $\phi_{\pi,U} > 1$ and $\phi_{\pi,B} > 1$, then the set of fiscal policy parameters that lead to stability and determinacy of equilibrium in the model does *not* depend on the monetary policy parameter. Thus, as long as this is the case, the shape of the stability sets plotted in Figure 5 does not change.

4.4 Generalizations and implications

Our model and examples are necessarily stylized. They do make an important point, however. Namely, the conclusion of traditional macroeconomic models of fiscal and monetary policies that a monetary union in which both the monetary authority and (part of) the fiscal authority pursue an active policy stance is unstable (Bergin, 2000) is predicated on the assumption that these active policy stances are permanent. If these policy stances are not, then a monetary union that goes through periods in which both monetary and (part of) fiscal policy are active can still be stable and equilibrium in that case determinate.

In both the *Fiscal Austerity* as well as the *Fiscal Transfers* examples, we found that if the central bank commits to permanently satisfying the Taylor Principal and the classical dichotomy holds, then stability of a monetary union is solely dependent on fiscal policy choices. We deliberately chose these examples to illustrate that under active monetary policy that has limited real

effects, especially on countries that accumulate an explosive real debt burden, the stability of a monetary union mainly depends on fiscal policy choices. To assure stability, such choices necessarily need to include mechanisms to alleviate the debt burden of countries that tend to pursue active fiscal policies (country 1 in our case). Our results indicate that there are really only two options for such a fiscal relieve valve. Either have the taxpayers in country 1 pay off the real debt burden their government accumulates by implementing frequent periods of *fiscal austerity* in country 1. Or have the taxpayers in the rest of the monetary union pay for the real debt burden of country 1 through *fiscal transfers*, in our model in the form of bailouts.[8]

As far as the interaction of monetary and fiscal policy choices is concerned, our results can be interpreted as a generalization of those in Leeper (1991) to monetary unions with Markov regime switching in policies. Leeper shows that, for a stable monetary-fiscal-policy equilibrium to exist, one of the policy stances needs to be active while the other is passive. Moreover, what makes fiscal policy active or passive only depends on the fiscal policy parameters and not on the monetary policy rule. The reverse is true for monetary policy. As we showed in our *Fiscal Austerity*, *Fiscal Transfers*, and *Debt Devaluation* examples, our model shares this property with Leeper (1991). The only difference is that 'active' and 'passive' are not defined in terms of permanent policy stances. Instead they are defined in terms of a weighted average of the parameters across policy regimes, where the weights are determined by the regime transition probabilities. This means that, even in a monetary union

[8]In a similar spirit, Corsetti et al. (2016) propose an institutional setup for the European Monetary Union that allows active fiscal policies be pursued during times of severe economic crises through enhanced risk sharing among member states. In their proposal, the latter is achieved through the issuance of a 'non-defaultable' Eurobond by a 'euro area fund'. Although not the same as a fiscal bailout, mechanically such a proposal delivers the same type of relieve valve as the cross-border fiscal transfers we consider.

that we consider, monetary and fiscal policy actions need to be coordinated in the sense that one of them is, on average, active and the other passive. However, the degree to which one is active does not affect the conditions under which the other one is passive. Thus, in many ways, our results are to Bergin (2000) what Davig and Leeper (2007) are to the Taylor principle.

What is important is that, when one realizes that a monetary union is not necessarily unstable when it faces episodes in which both the central bank and (part of) its fiscal authority pursue active policies, it is possible to analyze the economy's response to shocks and policy decisions even though it is in the **U**nstable regime when they occur. In the next section, we present such impulse responses.

5 Fiscal shocks in the unstable regime

In this section, we examine the effects of expansionary fiscal policy in country 1 on the monetary union. In particular, we simulate an exogenous tax cut of 1% (i.e. a fiscal policy shock) from steady state in country 1 that occurs when the economy is in the **U**nstable regime. We then study the effects of the tax cut on union-wide inflation and the debt positions of the two member states. As in the examples in the previous section, we allow the economy to switch between the **U**nstable regime and one other regime. Corresponding to the three examples in the previous section, these other regimes are the **R**icardian, **F**iscal Theory of the Price Level, and the **B**ailout regimes. The impulse responses, presented in Figure 6, are *expected* outcomes, where expectations are taken over the path of possible future policy regimes conditional on starting in the **U**nstable one. Each row of the three panels in Figure 6 corresponds to the respective case, I-III, from the previous section.

Table 2: Regime-specific parameters used for impulse response functions

Regime (s_t)	**Fiscal policy** (ϕ_{b_1,s_t})	**Monetary policy** (ϕ_{π,s_t})	**Bailout** (ϕ_{γ,s_t})	**Transition probabilities** $\left(p_{s_{t-1}s_t}\right)$
U	0	1.5	0	$p_{UU} = 1 - \frac{1-f_U}{f_U} p_{\mathcal{R}U}$
R	0.07	1.5	0	$p_{RU} = 0.25$
F	0	0.5	0	$p_{FU} = 0.25$
B	0	1.5	0.15	$p_{BU} = 1$

Notes: Parameters characterizing the policy regimes considered, as well as the associated transition probabilities, with $\mathcal{R} = \{R, F, B\}$.

The impulse responses are plotted for the benchmark parameters listed in Table 1. In addition, we chose specific values of the policy parameters, ϕ_{b_1,s_t}, ϕ_{π,s_t}, and ϕ_{γ,s_t}, in the different regimes that clearly illustrate the qualitative properties of the IRFs. The values of these policy parameters are listed in Table 2.

The final column of this table contains specific transition probabilities for the three examples of impulse responses we consider. The particular shape of the IRFs depends on these parameters because they determine the expected path of future policy regimes conditional on currently being in the **U**nstable one. The qualitative properties of the IRFs that is our focus in this section, however, do not depend on these parameters and are best captured by the unconditional probability of being in the **U**nstable regime, f_U, that we focused on before. This is why we label the IRFs in terms of this unconditional probability.

As we discussed in the previous section, stability of the equilibrium in the Fiscal Austerity (I) and Bailout (III) scenarios required inflation expectations to be anchored. Such anchored inflation expectations mean that inflation does not budge in response to a fiscal policy shock. This can be seen in the first and third rows of column 1 of Figure 6. They show that inflation remains at

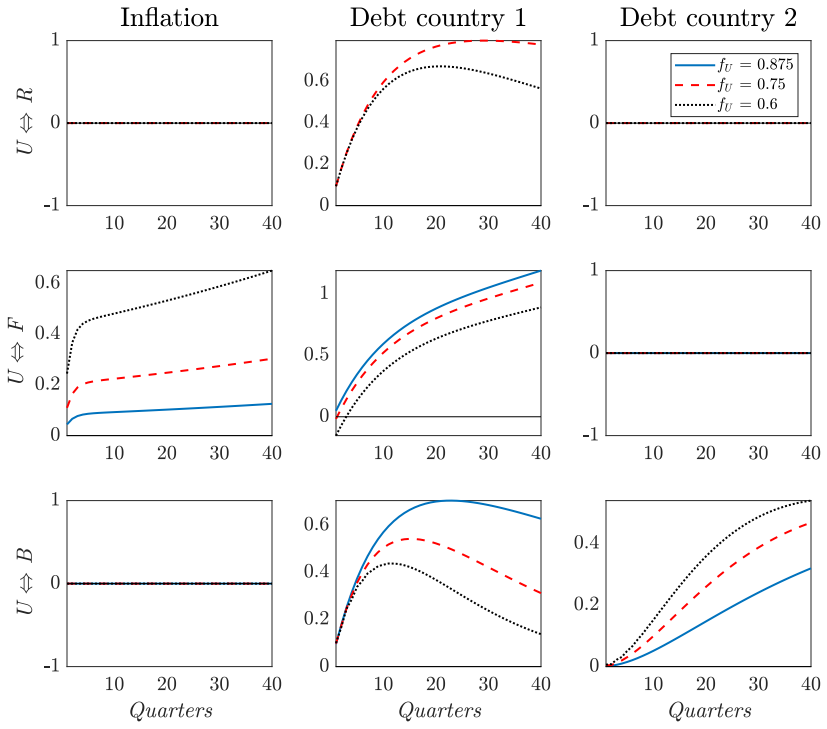

Figure 6: Responses to a temporary tax cut in country 1

Notes: The figure shows the responses of inflation and government debt in country 1 and 2, following an exogenous reduction in taxes in country 1 of 1 percent while in steady state in the **U**nstable regime. The axis-labels on the left-hand side show between **U** and which other regime the economy switches. IRFs are plotted as log-deviations from steady state. f_U denotes the frequency with which the economy is in regime **U**. The impulse responses are plotted conditional on the shock occurring in the **U**nstable regime.

its target level in response to the tax cut in country 1 in the Fiscal Austerity (I) and Bailout (III) examples.

In the Fiscal Austerity (I) example, the tax cut also does not affect the public finances of country 2. It just results in country 1 running up excess (above target) government debt. This debt growth is restrained by passive fiscal policy once the economy is in regime **R**. Thus, as you can see from the second panel in the first row of the figure, the longer the episodes in the **U**nstable regime last, i.e. the higher f_U, the more pronounced the run up of country 1's debt in response to the tax cut. In fact, in the case where the economy is in the **U**nstable regime 87.5 percent of the time, the austerity measures are not enough to offset country 1's debt growth and the equilibrium is unstable. This is why there is no IRF with the solid (blue) line plotted in the top row of Figure 6.

In the Bailout (III) example, the public finances of country 2 are affected by the tax cut in country 1. This is because country 2 takes over some of country 1's debt obligations to stem the explosive debt growth in country 1 and assure stability of the monetary union. As can be seen from the middle and right panels in the bottom row of Figure 6, the more frequent the bailouts, i.e. the less time the economy is in the unstable regime and the lower is f_U, the more the tax cut in country 1 affects real debt balances in country 2.

Finally, the three panels in the middle row of Figure 6 show the response of inflation and government debt to a tax cut in country 1 in the **U**nstable regime when stability of the monetary union is accomplished through debt devaluation. This is the case where Markov regime switching has the most profound impact on the impulse response functions.

In the absence of policy regime switches, when the economy is always in the **F**iscal Theory of the Price Level regime, a tax cut in country 1 would have

no effect on real debt holdings and would be immediately offset by a jump in the price level, as in the one-country case discussed in Kocherlakota and Phelan (1999). The middle row of Figure 6 reveals that this is not the case when the economy switches between the **U** and **F** regimes. In that case, the expected response is a joint run up in the debt level of country 1 as well as an increase in inflation that stems country 1's excessive debt growth. The more the economy is in the unstable regime, i.e. the higher f_U, the more pronounced the debt cycle in country 1 after the tax cut and the less expected inflation increases as a result of the tax cut.

Of course, the examples in this section are deliberately stylized to isolate the impact of Markov Regime Switching on the impulse responses in our model. They are more realistic in one dimension than those of existing models of fiscal and monetary policy in a monetary union. Namely, they consider the real-life relevant case of an unexpected fiscal policy move by a fiscal authority in a monetary union that pursues an active policy while the central bank adheres to the Taylor principle.

In addition to the direct takeaways from the examples, there is one more thing one should realize from our analysis. In order to understand the effect of fiscal policy shocks in a monetary union with episodes of jointly active fiscal and monetary policy, one has to take a stand on what types of shifts in policy regimes will occur in order to assure stability of the monetary union as well as how frequently such shifts occur.

6 Conclusion

Current macroeconomic models of joint monetary and fiscal policies in monetary unions have little to say about the case in which both the central bank

as well as (part of) the fiscal authority take active policy stances. In fact, Bergin (2000) showed that, if these stances are permanent, then no stable rational expectations equilibrium even exists. This result is true no matter how small a part of the fiscal authority pursues this active policy. This is rather unsatisfactory, because the reality is that in almost all monetary systems of the world, where the central bank sticks to the Taylor principle, there is at least some fiscal authority that pursues an active fiscal policy.[9]

In this paper, we illustrated that the result in Bergin (2000) hinges on the assumption of the permanence of the policy regime. If one allows for the fiscal and monetary authorities to switch their policy stances between active and passive, as well as allow for potential fiscal transfers within the monetary union, then the monetary union can be stable in spite of exhibiting episodes of jointly active monetary and fiscal policies. In a sense, our paper is to Bergin (2000) what Davig and Leeper (2007) is to the Taylor principle.

We explored three potential ways to alleviate the explosive debt growth that occurs under active fiscal policies. The first involved frequent episodes of fiscal austerity where the fiscal authority with the active policy reigns in debt growth by pursuing passive policy aimed at reducing government debt levels. The second involved episodes where the central bank abandons the Taylor principle. This results in a run up of inflation that reduces the value of the explosive real government debt holdings and in stability of the monetary union. The third involved bailouts in which the part of the fiscal authority that pursues a passive policy takes on some of the growing debt burden of the active fiscal policy authority. Such fiscal redistributions can also stem government debt growth and stabilize the equilibrium.

[9]For example, Greece before 2011 in the Euro Area and Puerto Rico in the U.S. Dollar system.

The main theoretical insight of our analysis is that allowing for policy regime switches enables one to think about economic dynamics in the empirically relevant case of a central bank following the Taylor principle and part of the fiscal authority of the monetary union pursuing fiscal policies that, if permanent, would violate transversality conditions. Once one realizes this, it also becomes apparent that such policy switches are inevitable in monetary unions where fiscal standards can not always be credibly enforced. In such monetary unions it is imperative that periods of austerity and fiscal transfers, as well as episodes of unconventional monetary policy that ignores the Taylor principle, are not just one-off events. They are part of the policy fabric and institutions necessary to assure stability of the monetary union.

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