TEACHING DELICACIES OF UTILITY AND PRODUCTION FUNCTIONS USING 3D-PRINTED PROTOTYPES:

An Innovative, Technological, Pedagogical Tool to Teach Homotheticity and Homogeneity

NC STATE UNIVERSITY

NC STATE UNIVERSITY POOLE COLLEGE OF MANAGEMENT

Seyyed Ali Zeytoon Nejad Moosavian*, szeytoo@ncsu.edu

*. Ph.D. Candidate in Economics, Department of Economics, North Carolina State University

An Intro and a Quick Look at the Literature:

- According to Naumenko and Moosavian (2016), the **spatial organization of visual models** enables the learner to process the elements as a whole all at once (Larkin and Simon, 1987).
- Additionally, they state that "the **aesthetics of visuals** play a role in learning as well. Embellished learning materials aid learning by attracting attention and reinforcing new synaptic connections" (Mangurian, 2005).
- According to Zhang (2016), "traditional and conventional approaches to teaching economics have not been friendly to students with kinesthetic preferences." By using the tools introduced in the present paper, **economics students with strong kinesthetic preferences** can also get a chance to rely on their learning preference.
- Moreover, as Nilson (2007) mentions, the **visual argument theory** hypothesizes that visuals make material more memorable, by **conveying information more efficiently** than any other tool, and as a result, they **require less working memory** and **fewer cognitive transformations** than other forms of sharing information.
- Moosavian (2016) states that "there is **enormous potential with visualization** to improve the quality of teaching and learning in economics, which has not yet been fully employed to resolve some of the issues with the teaching of economics." The present paper is indeed an attempt to take advantage of **a three-dimensional visualization method** to resolve one of these issues that has to do with understanding the nature and properties of utility and production functions.

Abstract:

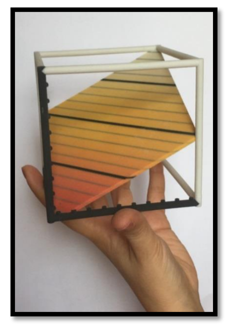
- Utility and production functions are **two major building blocks** of economics as a discipline.
- Teaching and learning the **geometric and mathematical properties** of utility and production functions (including, but not limited to, the concavity of the functions, convexity of their level curves, **homotheticity, and homogeneity**) have always been difficult in the classroom for instructors and students, while effectively teaching and completely learning these properties are of crucial importance for economics students to **thrive academically and professionally** in the discipline.
- As introduced and proposed by Zeytoon Nejad Moosavian (2017), "a novel, innovative way to teach these functions is to use the "**materialized demonstrations**" of utility and production functions, enabling students to actually "observe" what instructors usually try to describe verbally or at best graphically."
- As he explains, "this way, students can actually "**see**" and even "**touch**" the functions, and get a hands-on experience with utility and production functions. These innovative pedagogical tools can highly enhance the quality of teaching and level of learning."
- The present paper builds on Zeytoon Nejad Moosavian (2017) to introduce an innovative way to teach the mentioned properties of these functions with **a special emphasis on homogeneity and homotheticity**. Thereby, economics students can **use even the sense of touch in learning such a highly theoretical science**, i.e. economics. Advantages and applications of this approach are discussed from a pedagogical point of view in the paper. By using these tools, economics students get a chance to "see" and "touch" a set of **actual, colorfully-designed, 3D-printed prototypes and models** of multiple essential utility and production functions that have the capability to illustrate delicate geometric subtleties and desired mathematical properties of utility and production functions.

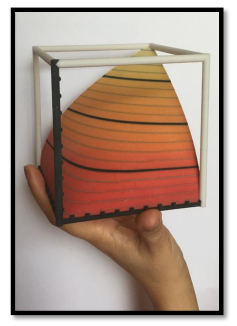
Key Words: Homotheticity, Homogeneity, 3D Utility Functions, 3D Production Functions, 3D-Printed Prototypes, Teaching of Economics, Pedagogy

JEL Classification: A10, A22, A23, C60

Prototypes for Three Main Types of Preferences:

Perfect Complements, Relative Substitutes , Perfect Substitutes

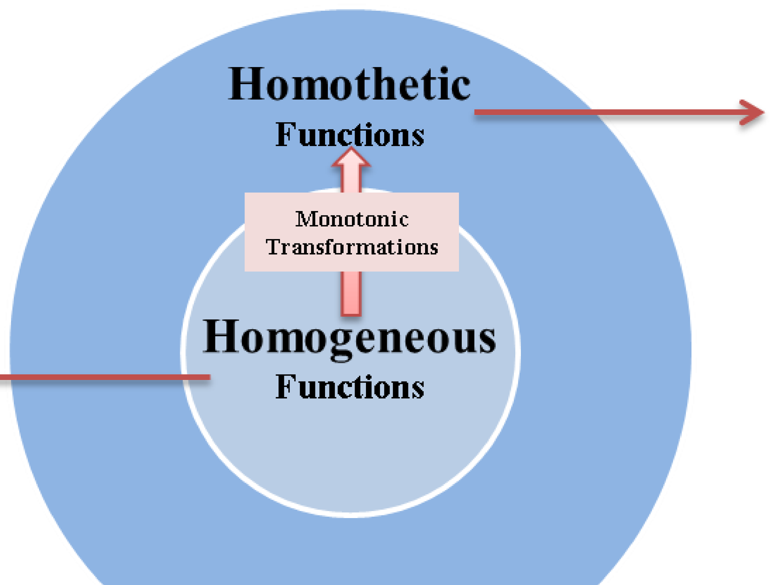




Three examples of Cobb-Douglass production functions, which are homogenous of different degrees, being less than one, one, and greater than one, from left to right, respectively

A visual summarizing the characteristics of homogeneity and homotheticity, and illustrating their relationships, connections, distinctions, similarities, examples, etc.

- The property that $f(\lambda x) = \lambda^k f(x)$
- A **cardinal** property
- Monotonic transformation of a homogeneous function
- A **stronger condition** than homotheticity
- More important for production theory and **production functions**
- Economic application: Modeling return to scale (IRTS, CRTS, DRTS)
- Examples: CD – LT – Lin production functions
- Non-examples: **Stone-Geary** production function (which is non-homothetic and non-homogenous)

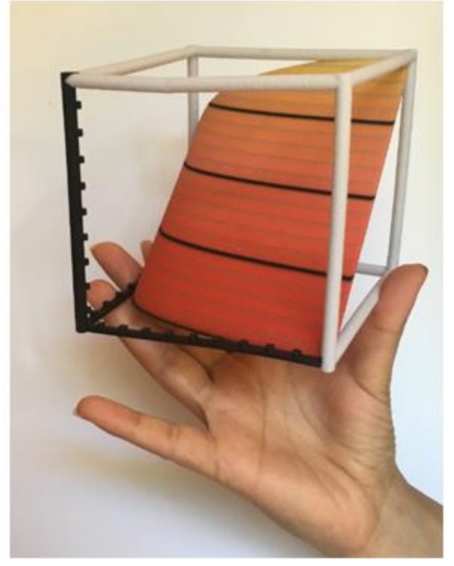


- The property that $f(x) = f(y) \Leftrightarrow f(\lambda x) = f(\lambda y)$
- An **ordinal** property
- Monotonic transformation of a homogeneous function
- A **weaker condition** than homogeneity
- More important for consumer theory and **utility functions**
- Economic application: Modeling MRS, MRTS, etc.
- Examples: CD – LT – Lin utility functions
- Non-examples: **Quasi-Linear** utility function

- An examples of a homothetic, but non-homogenous function: $f(x,y) = xy + 2$

Source: Author's own illustration

An example of a non-homogenous function (Stone-Geary function on the left) and an example of non-homothetic function (Quasi-linear function on the right)



Homogeneity

- Homogenous functions have the **property** that $f(\lambda x) = \lambda^k f(x)$ for $\forall \lambda > 0$ and for some k, which is said to be homogenous of degree k, for short HOD(k). The mathematical notion of "homogeneity" is primarily associated with the economic notion of "**return to scale.**" In economics, when a production function is HOD(k), if k>1, it is said that the function exhibits increasing return to scale, or for short **IRTS**. If k<1, then it is said that the function exhibits decreasing return to scale (**DRTS**). When k=1, it is said that the function exhibits constant return to scale (**CRTS**), which is **a popular theoretical assumption** in many economic theories and applications, such as production theory, economic growth, and growth accounting, primarily because of the **intuitive predictions** that it makes, which are close to real-world economic phenomena. Homogeneity has also interesting applications in consumer theory. Homogeneity is a **cardinal property**. Employing homogenous functions also provides economic modelers with **ease of interpretation of key economic ideas**. These functions are popular in economics because of the proper features they possess, the nice properties they exhibit, and the realistic and intuitive results they produce. Examples of homogenous production functions include Cobb–Douglas (CD), Leontief (LT), and linear production functions (Lin). A non-example of homogenous production functions is Stone-Geary production function, which in not homogenous. The figures placed above this box provide three examples of CD production functions which are homogenous of different degrees, being less than one, one, and greater than one, from left to right respectively.

Homotheticity

- Homothetic functions have the **property** that $f(x) = f(y) \Leftrightarrow f(\lambda x) = f(\lambda y)$. A homothetic function is **a monotonic transformation** of a homogeneous function. A function is monotone where ∀x,y ∈ R^n if $x \geq y \Rightarrow f(x) \geq f(y)$. Homotheticity **simplifies matters** when dealing with utility and production functions. Homotheticity of a utility function implies that the **slope of the MRS remains the same** along rays beginning at the origin (i.e. there is no impact on the MRS from changing the reference point). <u>In mathematical terminology</u>, a homothetic function of two or more variables is a function in which **the ratio of the partial derivatives depends only on the ratio of the variables**, and does not depend solely on their absolute values. <u>In economic terminology</u> in consumer theory, a utility function of two or more goods is a utility function in which **the ratio of goods demanded depends only on the ratio of their prices**. In consumer theory, there are some interesting analytic results that are brought about by homothetic utility functions. Homotheticity is an **ordinal** property. In mathematics, as mentioned above, a homothetic function is a monotonic transformation of a homogenous function; but because ordinal utility functions are defined on the basis of a monotonic transformation, the **difference** between the two concepts in consumer theory is **trivial**. The figures placed above this box provide 3D-printed prototypes of an example of a non-homogenous (Stone-Geary function on the left) and that of an example of non-homothetic function (Quasi-linear function on the right).

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