Risk-Taking Dynamics and Financial Stability*
An Evolutionary Perspective

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December 2017

We study how selection effects in the financial sector drive the dynamics of aggregate risk-taking and lead to novel effects of financial policy interventions. When financial market participants differ in their risk-taking, good shock realizations increase the capital of high risk-takers more than that of low risk-takers. This raises the fraction of wealth controlled by high risk-takers and, under incomplete markets, increases aggregate risk-taking. The opposite conclusions apply for bad shocks. As a result, aggregate risk-taking is pro-cyclical – in a sense, booms sow the seeds of the next crisis. Public policy interventions work primarily via selection effects, i.e. by affecting the composition of the financial sector, in contrast to the static restriction on choice sets that is the focus of most conventional economic frameworks. For example, bailouts have deleterious effects not because they affect incentives but because they interfere with the natural capitalist selection process. Interventions to stabilize aggregate risk-taking, such as growth limits or stress tests, bring the economy closer to the first-best, increasing expected growth and reducing aggregate volatility. Non-financial policies such as monetary policy or housing policy may also have significant selection effects on the composition of the financial sector.

JEL Codes: E14, E44, G18

Keywords: Risk-taking, evolutionary dynamics, financial stability

*The authors would like to thank Toni Ahnert, Jarda Borovicka, Matthieu Darracq-Paries, Martin Oehmke, Anatoli Segura, Jeremy Stein, Joe Stiglitz and Michael Woodford as well as participants at seminars at Columbia University and UNC Chapel Hill and the 2016 ASSA Meetings as well as the 2017 Bank of Spain-CEMFI Financial Stability Conference, the 2017 Bank of Canada Annual Research Conference, and the First ECB/IMF Macroprudential Policy Conference for helpful comments and discussions.

1 Introduction

Almost a decade after the Great Financial Crisis of 2008/09, the world economy has yet to recover from the damage created by excessive risk-taking in the financial sector. An important part of the economics profession has spent the time since trying to understand how phenomena such as exuberance, distorted incentives and market imperfections combined to trigger the crisis and – with even greater urgency – what lessons we can derive to make our financial system more stable for the future. These phenomena are typically studied by focusing on a representative financial institution, since they affect the financial sector as a whole.

Our paper, by contrast, focuses on how the composition of the financial sector drives aggregate outcomes. If most of the net worth in the financial sector is controlled by high risk-takers, the sector as a whole becomes riskier, and vice versa. The dynamics of how net worth is distributed across heterogeneous agents thus determine aggregate risk-taking in the economy. During booms, i.e. when the economy experiences a number of high aggregate shocks, high risk-takers earn higher returns and their wealth grows at a faster pace. During busts, i.e. when the economy experiences low aggregate shocks, high risk-takers incur larger losses; therefore the relative wealth of low risk-takers increases.

Changes in the composition of the financial sector typically play a large role in booms and busts, both in advanced and emerging economies. For example, in the US mortgage market, both the boom of the first half of the 2000s and the subsequent bust were driven in large part by risky players such as Countrywide Financial and Washington Mutual: Countrywide grew to become the largest US mortgage lender, capturing more than 20% of the market and originating loans amounting to 3.5% of US GDP in 2006; in January 2008, it was rescued in an emergency take-over by Bank of America. Its spin-off Indymac followed a similar trajectory and was taken into conservatorship by the FDIC in July 2008. Washington Mutual followed an aggressive expansion strategy in the early 2000s and grew to be the largest savings and loan association and the sixth-largest bank in the US, only to end in the largest bank failure in US history in September 2008. AIG became the largest player in the market for credit default swaps by aggressively selling credit insurance against close to half a trillion dollar of securities; in September 2008, it experienced a run and received the largest government bailout in US history. Conversely, institutions that followed a safer strategy experienced the opposite dynamics: JP Morgan Chase, for example, underperformed its peers in the first half of the 2000s but came to be the largest US financial institution in the aftermath of the financial crisis.

Similarly, in emerging economies, credit booms are commonly driven by a small set of financial institutions that specialize in channeling dollar credit into the domestic economy that comes with low interest rates but high currency risk. These institutions grow fast during boom times but often collapse when the tide reverses.

We study these phenomena in a framework of heterogeneous financial market participants that are subject to aggregate shocks but differ in the set of investment opportunities to which they have access. Some institutions engage in a relatively low-risk business model – they can be interpreted e.g. as conservative savings banks. Other institutions have a business model that produces higher risk but higher potential rewards, e.g. investment banking or subprime lending. An important condition for compositional changes to matter in such an environment is that risk-sharing in the economy is imperfect – otherwise each institution would only invest into the market portfolio. We assume that risk-sharing is limited because of agency problems that require that each institution to have a sufficiently large equity stake in its own business.

The first result of our paper is to show that the described compositional effects produces financial market dynamics that generate pro-cyclicality – good times sow the seeds of the next financial crisis. A series of high shocks increases the fraction of net worth controlled by high risk-takers – in the language of evolutionary theory, the financial sector adapts to a benign economic environment and becomes riskier. This leads to greater risk-taking in aggregate and makes the economy more vulnerable to low shocks. These results can also be interpreted as a formalization of the financial instability hypothesis postulated by Minsky (1986). Through the lens of evolutionary theory, pro-cyclicality is the result of temporary mal-adaptation – of a financial sector that has adapted to a benign risk environment and is unprepared for bad returns. The converse happens in response to a series of low shocks.

Secondly, we show that it is socially desirable to lean against the fluctuations in net worth and in aggregate risk-taking in the laissez-faire economy, i.e. to stabilize aggregate risk-taking at an intermediate level. This replicates the allocation that would prevail if risk markets were complete: it maximizes the expected growth rate of aggregate capital and results in a more stable financial system, mitigating the described pro-cyclical dynamics. It also reflects a broader theme in evolutionary theory, that preserving diversity enhances the robustness of a population and allows it to better deal with aggregate shocks.

Our third insight is to identify a novel channel through which public policy interventions shape risk-taking dynamics: public policy affects the dynamic composition of the financial sector, not only the static choice set of agents in the period it is conducted, as in traditional models of financial sector policy. A one-time intervention influences the distribution of net worth of agents going forward, which can have long-lasting dynamic effects on aggregate risk-taking. For example, financial regulation that limits risk-taking during booms reduces the profits of high risk-takers and slows down the reallocation of net worth towards them, which limits the fall-out during the next bust. There is no rationale for policies such as limits on credit growth on specific sub-sectors in traditional economic models, but in our setting, regulators are responding to the justified risk of mal-adaptation of the financial sector.

Fourth, we show how the nature of idiosyncratic shocks to risk types affects aggregate dynamics. We assume that idiosyncratic shocks are described by a transition matrix that captures the probabilities with which risk types change. In evolutionary terms, such idiosyncratic shocks correspond to mutation in risk types. In the financial sector, by contrast, the idiosyncratic dynamics of risk types can be interpreted as arising from three conceptually distinct phenomena: (i) idiosyncratic shocks to the set of investment opportunties of bankers, (ii) changes in the set of financial institutions that are operative or (iii) reallocations of funds by external investors. All these phenomena are also affected by the regulatory environment, for example by how much the environment encourages dynamism and experimentation in the financial sector. Symmetric idiosyncratic shocks are generally desirable because they introduce a form of mean reversion in risk types that brings the economy closer to the optimal capital allocation. Idiosyncratic shocks that are state-dependent, i.e. correlated with the aggregate shock, introduce the potential for momentum or contrarian dynamics of risk types. Momentum-based dynamics in risk types generally exacerbate the financial instability dynamics; contrarian reallocations generally lead to mean reversion and reduce volatility. In fact, if the economy starts out at the optimal capital allocation, the right magnitude of contrarian reallocation will preserve the optimal capital ratio at all times.

Finally, we analyze the spillovers of the described financial sector dynamics on the real economy. We assume that the financial sector intermediates capital to the real economy and creates jobs for households. The fate of the household sector is thus intricately linked to levels of wealth and risk-taking in the financial sector: during boom times, the household sector benefits from ample capital investment. During busts, losses in the financial system spill over into the real economy. Our earlier lessons on boom-bust dynamics and on how desirable it is to hold capital shares constant carry over to this extension: Households collectively prefer a stable supply of capital. Their welfare is maximized when the fraction of net worth allocated to the different risk types is held constant. Furthermore, spillovers to the real economy may justify rescue packages and bailouts when the financial sector is under-capitalized. We show that bailouts interfere in a major way with the natural selection process of capitalist economies. Traditional economic theories emphasizes the incentive effects of bailouts. By contrast, in our evolutionary setting, the adverse effects of bailouts come from selection not incentives: a bailout allows high risk-takers to continue to operate at the expense of low risk-takers. In the language of evolutionary theory, it allows mal-adapted agents to continue to operate at the expense of better-adapted agents.

Literature Our work is related to a large strand of literature that studies the importance of net worth in the financial sector for the real economy. When financial markets are imperfect, Jensen and Meckling (1976) and Stiglitz and Weiss (1981) emphasize that net worth matters for economic activity. Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) emphasize furthermore that the distribution of net worth between more and less productive agents drives aggregate economic activity. In the aftermath of the Great Financial Crisis, a flourishing literature including Gertler and Karadi (2011), He and Krishnamurthy (2013) and Brunnermeier and Sannikov (2014), among others, have emphasized that low net worth in the financial sector reduces financial intermediation to the rest of the economy and depresses economic activity. Our work focuses, in addition, on how dynamic changes in the composition of net worth within the financial sector drive aggregate volatility.

A closely related strand of literature aims to understand incentives for risk-taking in the financial sector and uses the resulting insights to motivate financial regulation. Geanakoplos and Polemarchakis (1986) as well as Greenwald and Stiglitz (1986) show that financial market imperfections commonly give rise lead to pecuniary externalities that call for policy intervention. This is of particular importance for risk-taking when there are fire sales, as studied e.g. by Lorenzoni (2008), Korinek (2011) and Davila (2014). Our paper is closely related to this literature in that it also exhibits incomplete markets that give rise to pecuniary externalities. However, it focuses on how compositional effects within the financial sector give rise to risk-taking dynamics that are inefficient in both directions – during booms and busts. Alternative explanations for excessive risk-taking in the financial sector inlcude Farhi and Werning (2016) and Korinek and Simsek (2016) who focus on aggregate demand externalities. Admati et al. (2010) propose moral hazard as the leading explanation for risk-taking in the financial sector. A growing branch of literature also analyzes how deviations from perfect rationality may have led to the observed risk-taking behavior. For an example in the context of the financial crisis see Barberis (2013). Daniel and Hirshleifer (2015) survey the implications of overconfidence in financial markets.

A third strand of literature to which our work is related analyzes the effects of hetereogeneity among financial market participants. A common form of heterogeneity in this literature is that financial market participants differ in their beliefes. Friedman (1953) hypohtesized that the market always selects the investors with the most accurate beliefs, but Blume and Easley (2006) demonstrated that this no longer holds when financilal markets are incomplete. In a similar vein, Fostel and Geanakoplos (2008) and Geanakoplos (2009) describe a leverage cycle that is driven by wealth reallocations between optimists and pessimists. Burnside et al. (ming) analyze how booms and busts may arise from social dynamics when agents have heterogeneous beliefs about long-run fundamentals and when beliefs are subject to contagion dynamics. Both papers share with our paper that there is heterogeneity among financial market participants—they assume that different agents arrive at different actions because of heterogeneous beliefs; in our work, by contrast, agents have identical beliefs but differ in their technology. The main benefit is that this enables us to conduct a detailed welfare analysis and arrive at a number of interesting policy implications. As in the literature on heterogeneous firms (see e.g. Hopenhayn, 1992), heterogeneous agents in our setup do not have access to complete risk markets.

Finally, a number of papers study how evolutionary dynamics shape the preferences and by implication the behavior of economic agents. See e.g. Robson and Samuelson (2011) for a survey and Brennan and Lo (2011) for an application to financial markets. Our work is similar in methodology but focuses on the implications of the dynamics of net worth among heterogeneous actors in financial markets for aggregate risk-taking.

2 Baseline Model

Population We consider a population of financial market participants called "bankers" who live in infinite discrete time $t = 0, 1, 2, ...$ and who are of different types $i \in \mathfrak{I} = \{1, ...N\}$. The types differ along three dimensions from each other: in their beliefs, patience, and investment opportunities. More specifially, bankers of type i value consumption according to the utility function

$$U_i = \mathbb{E}_i \left[\sum_t (\beta_i)^t \log c_{it} \right]$$

where $\mathbb{E}_i [\cdot]$ is an expectations operator that captures subjective beliefs and β_i is their discount rate.

Each type i consists of a continuum of agents in the unit interval $z \in [0, 1]$ who are endowed with total initial capital $k_{i0} = \int_0^1 k_{i0}(z)dz$, which we assume positive $k_{i0} > 0$ for all types. Throughout our analysis, it is sufficient to keep track of the total capital $k_{it} = \int_0^1 k_{it}(z)dz$ managed by bankers of each type i – how this capital is distributed across individual bankers of type i is irrelevant since they all behave in the same way. In the following, we will call k_{it} the "type i capital."

In a given time period, type i bankers have access to a set $\mathfrak{S}_i$ of investment opportunities. An investment strategy $S \in \mathfrak{S}_i$ delivers a stochastic one-period return $\tilde{R}_S$ that is distributed according to the function $F_S(\tilde{R}_S)$ which satisfies $F_S(0) = 0$. Investment returns depends on an aggregate state of nature $\omega_t \in \Omega$ that is, for simplicity, independent across time periods.

The optimization problem of type i bankers is

$$\max_{c_{it}, S_{it} \in \mathfrak{S}_i, k_{it+1}} \mathbb{E}_i \left[\sum_t (\beta_i)^t \log c_{it} \right] \qquad \text{s.t.} \qquad c_{it} + k_{it+1} = \tilde{R}_{S_t} k_{it}$$

Remark (Heterogeneity) The assumption that bankers differ in beliefs, patience and investment opportunities allows us to capture that there is a considerable degree of heterogeneity in the financial sector – otherwise, the financial sector would behave like a representative agent. It is well-documented that bankers differ in beliefs and patience. Furthermore, it is also clear that bankers follow different business strategies that provide access to different types of investment opportunities. For example, some bankers specialize in lending whereas others specialize in trading; some are better at evaluating safe investments whereas others are specialists in risky opportunities. Such heterogeneity in technologies is amply documented across firms of all types (see e.g. Bernard et al., 2003). Adrian and Shin (2010) provide empirical evidence of heterogeneity in the return characteristics of different firms in the financial sector.

In all models of firm heterogeneity, there is furthermore an assumption that firms cannot perfectly insure their idiosyncratic shocks (see Hopenhayn, 1992). If they could, then heterogeneity would not matter and we could focus on the behavior of a single representative firm. In our baseline model we assume that risk markets are completely absent. In Section 5.1 we will consider the case that individual bankers can share up to a fraction $1-\phi$ of their business risk. This assumption deserves further discussion since economists generally believe that the financial sector is extremely efficient at allocating and sharing risk. However, the risk that is shared efficiently is *portfolio risk* and is distinct from the *business risk* of individual bankers: the portfolio risk of the financial assets that a financial institution holds on its balance sheet (e.g. mortgages, business loans, or even equities) can be shared relatively easily via re-trading, syndication, securitization, or credit default swaps. By contrast, the risk inherent in the franchise of a financial institution is much more difficult to share: for example, it is difficult for an investment bank to insure against the risk of primary markets drying up which deprives them of much of their business, or for a mortgage lender to insure against the risk of mortgage markets drying up, which inhibits their main business activity. This is the incompleteness in risk markets that we consider here. In the following we spell out two examples for different sets of investment strategies to make our setup more tangible.

Example 1 (Choice of Leverage)**.** One of the classic decision variables of bankers is to choose the leverage at which they are operating. Assume that type i bankers have access to a risky return $\tilde{Q}_i$ with minimum realization $Q_i^{\min}$, for example from lending to their natural type i constituency, as well as the risk-free return r, which represents the risk-free world interest rate. Then the set of investment strategies and the corresponding returns can be described as a function of the leverage choice x_i such that[1]

$$\mathfrak{S}_i = \left\{ S_i\left(x_i\right) : x_i < \frac{r}{r - Q_i^{\min}} \right\} \text{ where } \tilde{R}_i\left(x_i\right) = x_i \tilde{Q}_i + \left(1 - x_i\right) r$$

Example 2 (Diversification)**.** Another typical decision problem for bankers is how much to diversify their risk exposure in financial markets. We capture this by assuming that each banker type i has access to a risky investment opportunity with return $\tilde{Q}_i$ that stems from its specific sector of activity, for example mortgage lending, or business lending, or securities investments. We assume that a type i banker has to invest at least a fraction ϕ of its capital in its own sector at return $\tilde{Q}_i$ to guarantee

[1]Given log-utility, bankers would otherwise always avoid bankruptcy in our setup, as captureed by the constraint on x_i. The example could also be extended to allow for bankruptcy protection that provides a minimal subsistence return $\tilde{R}_{it}$ to bankers that is positive but close to zero.

proper effort, but it can diversify the remaining fraction $1-\phi$ in the returns $\tilde{Q}_j$ of the remaining banker types $\mathfrak{I}\backslash\{\mathfrak{i}\}$. Then the set of investment strategies and the corresponding returns can be parameterized as a function of the portfolio weights $\{x_{ij}\}$ such that

$$\mathfrak{S}_i = \left\{ S_i\left(\{x_{ij}\}\right) : x_{ii} \geq \phi, \sum_j x_{ij} = 1 \right\} \text{ where } \tilde{R}_i\left(\{x_{ij}\}\right) = \sum x_{ij}\tilde{Q}_j$$

Naturally, the examples can be combined with each other and/or with additional decision variables of bankers.

The following lemma characterizes the optimal behavior of a given type i banker.

Lemma 1 (Optimal Strategy)**.** *In the decentralized equilibrium, type i bankers follow a fixed investment strategy $S_i \in \mathfrak{S}_\mathfrak{i}$ each period that maximizes the geometric mean return*

$$S_i = \arg\max_{S\in\mathfrak{S}_i} E_i\left[\log \tilde{R}_S\right] \tag{1}$$

They earn a return $\tilde{R}_{it} = \tilde{R}_{S_it}$ each period t, consume a constant fraction $(1-\beta_i)$ of their wealth, and accumulate capital according to the law-of-motion

$$k_{it+1} = \beta_i \tilde{R}_{it} k_{it} \tag{2}$$

(ii) If the frontier of investment strategies is described by a continuously differentiable parameter x_i for type i bankers, then the optimal interior portfolio choice is described by

$$E_i\left[\frac{\tilde{R}'\left(x_i\right)}{\tilde{R}\left(x_i\right)}\right] = 0 \forall i, t \tag{3}$$

Proof. (i) Given the log-utility and i.i.d. nature of shocks, the terms $E_i\left[\log \tilde{R}_S\right]$ enters the optimization problem of bankers additively. Statement (1) follows immediately. Log-utility furthermore implies the law-of-motion (2).

(ii) The optimality condition represents the first-order condition to the problem $\max_{x_i} E_i\left[\log \tilde{R}\left(x_i\right)\right]$. □

The optimal strategy for each type i maximizes the geometric mean return, i.e. the growth rate of its capital. This criterion follows in a straightforward manner from the utility function of bankers. It is well-known in the literature on optimal investment strategies, in which it is frequently referred to as the "Kelly Criterion" after Kelly

(1956) or the capital growth criterion since it maximizes the average growth rate of the bankers' portfolio. Given the law-of-motion for capital, the log of type i capital $\log k_{it}$ follows a martingale with drift $E[\ln \beta_i \tilde{R}_{S_i}]$.

The second part of the lemma applies if the portfolio choice of bankers can be described by a continuous parameter x_i, as in our examples above. It captures that the optimal value of x_i is then such that the excess return from varying x_i is zero at the optimum, given the pricing kernel of bankers, which satisfies $u'(c_{it}) = 1/[(1-\beta_i) k_{it}] \simeq 1/\left[\tilde{R}(x_i) k_{it-1}\right] \simeq 1/\tilde{R}(x_i)$ where k_{it-1} drops out of the optimality condition since it is given at the time of the portfolio choice for period t.

Vector Notation For compactness of notation, let us denote by $\tilde{R}_i = \tilde{R}_{S_i}$ the random variable that describes the return process of the optimal strategy S^i chosen by type i bankers, and by $\tilde{R}_{it}$ the period t realization of that random variable. Furthermore, we collect in the diagonal matrix $\tilde{R}_t = diag[\tilde{R}_{1t}, \tilde{R}_{2t}, ...\tilde{R}_{Nt}]$ the stochastic returns of the strategies chosen by all bankers. Then the vector of capital positions $k_t = (k_{1t}, k_{2t}, ...k_{nt})'$ follows the law-of-motion

$$k_{t+1} = \tilde{R}_t k_t \tag{4}$$

We denote the aggregate capital stock in the economy by the capital letter $K_t = \sum_{i \in \mathfrak{I}} k_{it} = \iota_N k_t$ where $\iota_N = (1, ...1)_N$ is a row vector of ones. Given a vector k_t, the aggregate capital stock in the following period will be

$$K_{t+1} = \sum \tilde{R}_{it} k_{it} = \iota_N \tilde{R}_t k_t$$

Only bankers who earn the maximum geometric mean return in the economy will survive over time. Conversely, those who earn a geometric mean return below the maximum will be excluded over time by natural selection. We denote the maximum geometric mean return across all types of bankers $i \in \mathfrak{I}$ by

$$\ln \bar{R} = \max_{i \in \mathfrak{I}} E\left[\ln \tilde{R}_{S^i}\right]$$

Then we find:

Lemma 2. (Exclusion of Inferior Strategies) *Bankers who earn a geometric mean return below the maximum $E[\ln R_{S^j}] < \ln \bar{R}$ will see the fraction of their capital* *in the economy converge to zero,*

$$\lim_{T\to\infty} k_{jT}/K_T = 0 \quad a.s.$$

Proof. The proof follows from the weak law of large numbers applied to the logged variables. $\square$

Order by Increasing Riskiness We assume w.l.o.g. that the set of bankers $\mathfrak{I}$ is ordered by increasing variance of the investment strategy $Var(\tilde{R}_i)$, i.e. if $i > j$ then $Var(\tilde{R}_i) > Var(\tilde{R}_j)$. Accordingly, we call i the risk type of bankers. This proves useful since the bankers that survive over time will not significantly differ in their geometric mean return according to Lemma 2 – types with geometric mean return that is significantly below $\bar{R}$ will be rapidly excluded from the population – but may differ greatly in riskiness. This makes the riskiness of each type its main distinguishing feature.

Let us define an ordinal measure of the riskiness of different capital allocations:

Definition 1. (Riskiness of Capital Allocation) For two capital allocations k_t and k'_t with aggregate capital K_t and K'_t respectively, we call allocation k_t riskier than k'_t if $\sum_{0\le i\le n} k_{it}/K_t \le \sum_{0\le i\le n} k'_{it}/K'_t \forall n \in \{1,...N\}$ with strict inequality for some n.

Intuitively, an allocation k_t is riskier than another allocation k'_t if for any risk level n, there is a smaller fraction of capital allocated to strategies safer than n under allocation k_t than under allocation k'_t.

Let us also define a measure of the volatility of the aggregate capital stock:

Definition 2. (Volatility) The n-period-ahead volatility of the aggregate capital stock is

$$V_{t+n} = \frac{Std\,(K_{t+n})}{K_t}$$

Our measure V_{t+n} consists of the average standard deviation of returns of the different investment strategies weighted by the fraction k_{it}/K_t of each risk type. If $n = 1$, we will simply speak of the period-ahead volatility.

2.1 A Two-by-Two Example

Let us now consider an economy in which there are two states of nature in every period (low or high) and two types of bankers (safe and risky). Following the logic of Lemma 2, we limit our attention to the case in which the two types of bankers have each access to a single investment opportunity that earns the same geometric mean return $\bar{R}$ but with different riskiness. Technically speaking, the logs of the different types of capital are martingales with equal drift $\log \bar{R}$ but increasing variance. This implies that the two types exhibit zero selective difference in the long run and, in expectation, all survive with a positive share of capital.

In the short run, however, different types of bankers have different exposure to the aggregate shock. In each period, selection favors those types that are better adapted to the realized shock: in response to a high aggregate shock, risky types earn higher returns than safe types, and the relative fraction of risky capital in the economy rises. Conversely, in response to a negative shock, risky types suffers greater losses than safe types, and the relative fraction of risky capital declines.

The aggregate state that is realized in each period is "low" with probability p and "high" with probability $1-p$. When there are only two states of nature, all random variables with geometric mean return $\bar{R}$ are of the form

$$\tilde{R}_s = \begin{cases} \bar{R}(1+s)^{-\frac{1}{p}} & \text{in the low state } L \text{ (with prob. } p) \\ \bar{R}(1+s)^{\frac{1}{1-p}} & \text{in the high state } H \text{ (with prob. } 1-p) \end{cases}$$

for some return dispersion $s \geq 0$. For the resulting family of random variables, although the geometric mean return is the same, the simple average return $E[\tilde{R}_s]$ and the variance $Var(\tilde{R}_s)$ are increasing functions of s. Intuitively, higher variance is compensated by higher return so that the expected log utility of the different options is the same. In the described class of random variables, each risk type $i \in \mathfrak{I}$ is thus fully described by a return dispersion s_i. Since we ordered risk types by increasing riskiness, the return dispersion s_i is increasing in i. Furthermore, Condition **??** is satisfied, i.e. all higher moments are an increasing function of i.

Volatility and Pro-cyclicality We now use the described setup to analyze the dynamics of aggregate risk-taking and capital in the economy. A few immediate results follow:

Proposition 1. (Volatility) *For any horizon n, the n-period ahead volatility of the aggregate capital stock V_{t+n} increases the riskier the period t capital allocation k_t of the banking sector.*

(Pro-Cyclicality) *Risk-taking in the economy is pro-cyclical, i.e. starting from an initial allocation of capital k_t, the more positive shocks the economy has experienced,* *(i) the greater the absolute and relative loss of aggregate capital from a negative shock (and the greater the gain from a positive shock) in the following period,*

(ii) the greater the n-period ahead volatility V_{t+n} of the aggregate capital stock for any horizon n.

Proof. We observe that the n-period-ahead volatility of the aggregate capital stock is given by

$$V_{t+n} = \frac{Std\left(K_{t+n}\right)}{K_t} = \frac{\sum_{i\in\mathfrak{I}} Std\left(\tilde{R}_{it}^{n}\right) k_{it}}{K_t} = \frac{\iota_N Std\left(\tilde{R}_{t}^{n}\right) k_t}{K_t} \tag{5}$$

where $Std(\tilde{R}_t^n)$ is taken element-by-element. The additivity of standard deviations in the second equality follows since all returns are perfectly correlated with the aggregate shock. The variance of the n-th power of $\tilde{R}_s$ is

$$Var\left(\tilde{R}_s^n\right) = p\left(1-p\right)\bar{R}^{2n}\left[\left(1+s\right)^{-\frac{n}{p}} - \left(1+s\right)^{\frac{n}{1-p}}\right]^2$$

and is strictly increasing in s. This observation together with equation (5) and the definition of riskiness implies the result on volatility.

For the results on pro-cyclicality, observe that a positive shock in a given period t increases K_{t+1} and renders the capital distribution k_{t+1} riskier than k_t. The riskier the capital allocation, the greater the relative loss from a low shock and, since K_{t+1} is higher, the greater the absolute loss from a low shock, proving point (i). Furthermore, equation (5) implies that the n-period-ahead volatility is greater. □

Financial Instability Hypothesis It is straightforward to interpret the financial instability and pro-cyclicality as articulated e.g. by Minsky (1986) through the lens of our framework: Minsky's the observation was that "booms sow the seeds of the next crisis;" in our framework, booms reallocate capital into the hands of riskier bankers who invest the economy's capital stock in riskier investments. When the next adverse shock hits the economy, the economy is highly exposed to aggregate risk and there is a large correction.

In evolutionary terms, the phenomenon can be interpreted as an instance of temporary maladaptation – after a series of good shocks, the risk profile of the capital stock has adapted to a safer environment than what is appropriate as high risk types own an ever larger fraction of the aggregate capital stock. When a negative shock hits, it turns out that the sector is maladapted to a low return environment.

Simulation 1 (Volatility and Pro-Cyclicality) Let us illustrate the pro-cyclicality in a simple example: assume $N = 2$ risk types $i = 1, 2$ with $s_i = ir$ for $r = 5\%$ and with initial capital distributed equally so $k_{i0} = \frac{1}{N}$. We may call the capital invested under two types risky capital and safe capital. We assume that the probability of the low state is $\pi = 10\%$. For simplicity we set $\bar{R} = 1$ so there is no trend growth in our illustration. Figure 1 shows a typical path of the two types of capital k_{it} as well as the aggregate capital position of the economy K_t and the relative fraction $\kappa_t = k_{2t}/K_t$ of the risky type.

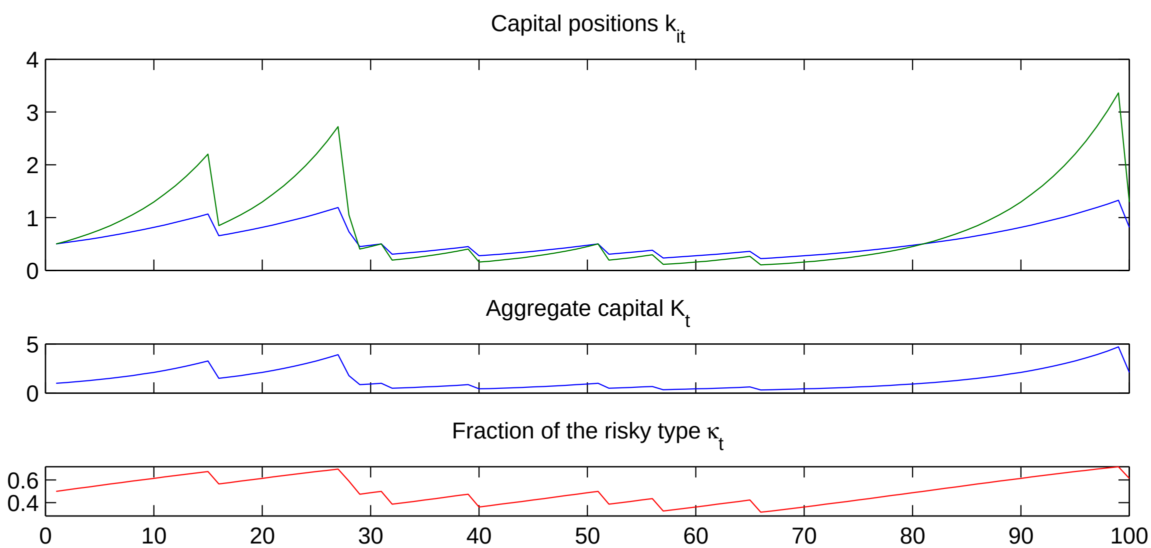

Figure 1: Simulation of Risk-Taking Dynamics

The results of Proposition 1 can be seen clearly: at first, a series of good shocks favors riskier bankers and causes aggregate capital to quadruple – at the height of the boom, the capital of risky bankers has increased more than five-fold and makes up 70% of the total, as displayed in the bottom panel, whereas the capital of safe bankers declines to 30% of the total. As the economy experiences a number of negative shocks around period 30, the fortunes reverse – risky bankers loose 96% of their capital, contributing heavily to an 89% decline in the aggregate capital stock. Towards the end of the simulation period, the risky type catches up again due to a series of positive shocks.

2.2 First-Best Capital Allocation

This raises the question of what the optimal allocation of capital among the different types of bankers would be in order to maximize the growth of the aggregate capital stock. As a benchmark, we start by characterizing the first best. This corresponds to a situation in which the planner can freely allocate capital to different types of bankers, instruct each type which investment strategy to choose, and then optimally redistribute the returns according to a set of welfare weights $\theta_i \geq 0$ that satisfy $\sum_i \theta_i = 1$. The planner's optimization problem can then be expressed as

$$\max_{c_{it},k_{it},S_{it}} \sum_t \beta^t \sum_i E\left[\log c_{it}\right] \quad \text{where} \quad \sum c_{it} + k_{it} = \sum \tilde{R}\left(S_{it}\right) k_{it-1} \forall t \tag{6}$$

Proposition 2. (First Best) *(i) The first best features fixed capital shares $\kappa_i^* = k_{it}/K_t \forall t$ allocated to the different types of bankers and fixed investment strategies S_i^* for each type that solve the static optimization problem*

$$\max_{\kappa_i \in [0,1], S_i \in \mathcal{S}_i} E\left[\log \sum_i \kappa_i \tilde{R}\left(S_i\right)\right] \tag{7}$$

(ii) For types $i \in \mathfrak{I}$ for which the portfolio choice κ_i is interior, the planner equalizes risk-adjusted returns,

$$E\left[\lambda^* R\,\tilde{(S_i)}\right] = c \tag{8}$$

where the planner's pricing kernel $\lambda^* \simeq 1/\sum \kappa_i^* \tilde{R}\left(S_i^*\right)$ is time-invariant.

(iii) If returns are continuously differentiable in a strategy parameter S_i for type i, then the planner's optimality condition is

$$E\left[\lambda^* \tilde{R}'\left(S_i\right)\right] = 0 \tag{9}$$

(iv) The planner's allocation leads to faster growth, i.e. denoting by K_t and K_t^* the path of aggregate capital under the planner's optimum and the decentralized equilibrium, we find $\lim_{T\to\infty} K_T/K_T^* = 0$ a.s.

Proof. For part (i), defining the capital shares $\kappa_{it} = k_{it}/K_t$, the argument of the planner's dynamic optimization problem (6) can be re-written as

$$
\begin{aligned}
E\left[\ln K_T\right] &= E\left[\ln \sum_i \tilde{R}\left(S_{it}\right) \kappa_{iT-1}\right] + E\left[\ln K_{T-1}\right] = \\
&= \sum_{t=0}^{T-1} E\left[\ln \sum_i \tilde{R}\left(S_{it}\right) \kappa_{it}\right] + \ln K_0
\end{aligned}
$$

The terms $E[\log \sum_i \tilde{R}\left(S_{it}\right) \kappa_{it}]$ enter the dynamic optimization problem additively without any interactions. Therefore setting the fixed portfolio weights $\kappa_{it} = \kappa_i^*$ and strategies $S_{it} = S_i^*$ that solve the static problem (7) in each period also maximizes the dynamic optimization problem. If the objective is continuously differentiable and the solution is interior, the optimality conditions (9) are the first-order conditions to the static problem.

Parts (ii) and (iii) are standard first-order optimality conditions to the static optimization problem. The pricing kernel λ^* is proportional to marginal utility, which satisfies $u'\left(c_{it}\right) = 1/c_{it} = \left(1-\beta\right)\theta_i K_{it} \simeq \sum_i \kappa^* \tilde{R}\left(S_i^*\right)$.

For part (iv), denote by κ_{it} the capital shares in the decentralized equilibrium and observe that $E[\sum_i \log \kappa_{it} \tilde{R}_{it}] \leq E[\sum_i \log \kappa_i^* \tilde{R}\left(S_i^*\right)] \forall t$ with strict inequality except when $\kappa_{it} = \kappa_i^*$ and $S_{it} = S_i^*$ since the starred values maximize the objective. The result then follows from the weak law of large numbers. □

Part (i) of the proposition captures that the first-best is described by a standard static portfolio allocation problem every period. This makes it desirable to choose a fixed strategy for each type of banker and to keep the weights attached to each investment strategy constant over time. This contrasts markedly with the booms and busts under the free market allocation described in Proposition 1. The difference arises because the planner can reassign the economy's capital across to different investment opportunities every period to optimize the risk/return trade-off of the aggregate capital stock. By contrast, individual bankers in the laissez faire equilibrium can only invest in their own techologies. As a result, part (ii) of the proposition shows that the planner's allocation will, in the long run, always outperform the decentralized boom-and-bust dynamics.

Simulation 2 (First Best) We include an illustration of the optimal capital allocation in our earlier Simulation 1 of decentralized capital dynamics. For $N = 2$ risk types, we denote by $\kappa = k_1/K$ the capital share in strategy 1 and express the portfolio allocation problem of the planner as

$$\max_{\kappa\in[0,1]} E\left[u\left(\tilde{R}\right)\right] \quad \text{where} \quad \tilde{R} = \kappa\tilde{R}_1 + (1-\kappa)\,\tilde{R}_2$$

If the optimum is interior, then it equates the risk-adjusted returns of the two investment opportunities,

$$E\left[u'\left(\tilde{R}\right)\left(\tilde{R}_1 - \tilde{R}_2\right)\right] = 0$$

which can be solved for

$$\kappa^* = \frac{pR_2^H}{R_2^H - R_1^H} + \frac{(1-p)\,R_2^L}{R_2^L - R_1^L} \tag{10}$$

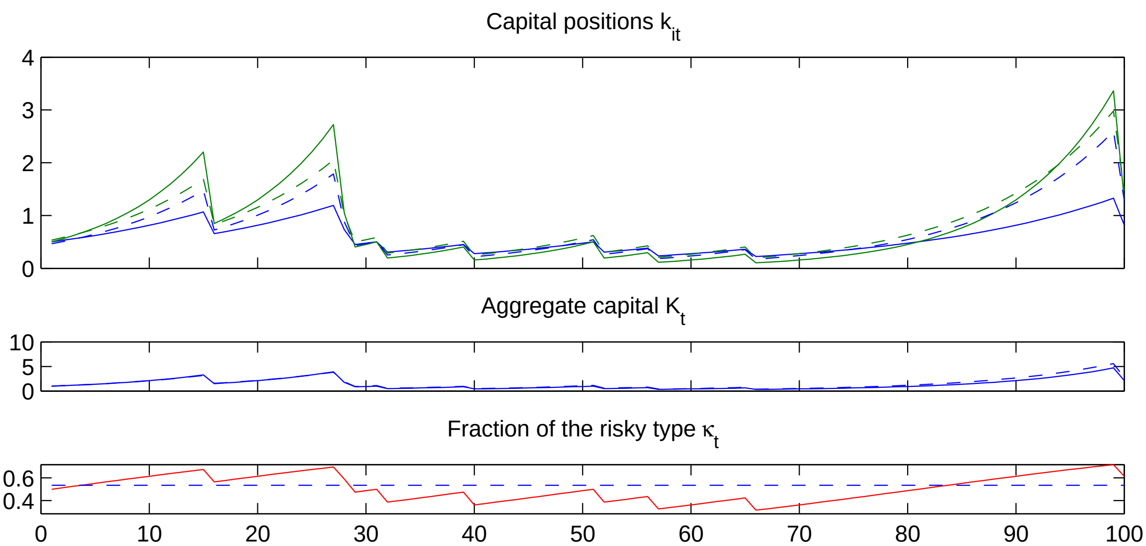

Figure 2: Decentralized risk-taking dynamics versus optimal capital allocation

Given our earlier parameters, we find that the optimal share of capital allocated to investment opportunity 1 is $\kappa^* = 46.6\%$. Figure 2 compares the decentralized dynamics from Simulation 1 (in solid lines) to the socially optimal capital allocation (dashed lines), using the same realizations of the stochastic process as in the original figure. The aggregate capital stock grows faster under the socially optimal allocation. Furthermore, it leads to constant volatility whereas the volatility in the decentralized allocation fluctuates greatly.

2.3 Constrained Optimal Investment Strategies

Next we consider the planner's constrained optimal choice of investment strategies if she has to respect the internal capital accumulation decisions of bankers given by (4), but can instruct each type to choose a particular investment strategy $\hat{S}_{it} \in \mathcal{S}_i \forall i, t$ and can reallocate payouts. This corresponds to a private ownership economy in which the risk-taking strategies of bankers are regulated and dividend payments are subject to taxation and transfers. Given the ability to redistribute dividend payouts, the planner's optimization problem in such an economy is to maximize the log of the sum of dividends each period, i.e.

$$\max_{k_{it}, S_{it}, K_t} \sum_t \beta^t E\left[\log \sum (1-\beta)\, k_{it}\right] \quad \text{where} \quad k_{it} = \tilde{R}_{S_{it-1}} k_{it-1} \forall t \tag{11}$$

Proposition 3. (Constrained Optimal Investment Strategies) *(i) Given capital shares $\kappa_{it} = k_{it}/K_t$ in a given period, the constrained planner instructs each type i to invest in the strategy*

$$S_{it} = \arg\max_{S_{it} \in \mathcal{S}_i} E\left[\log \sum_i \tilde{R}_{S_{it}} \kappa_{it}\right] \tag{12}$$

If the frontier of investment strategies is continuous in a parameter x_{it} for each $i \in \mathcal{I}$, then the planner's optimality condition is

$$E\left[\frac{\partial \tilde{R}_{S_{it}}/\partial x_{it}}{\tilde{R}_t}\right] = 0 \forall i, t \tag{13}$$

where $\tilde{R}_t = \sum \tilde{R}_{S_{it}} \kappa_{it}$ is the state-contingent return on the aggregate portfolio at date t.

(ii) The constrained optimal strategies can be implemented by imposing a tax on the portfolio decision of bankers of

(iii) The planner's allocation leads to faster growth, i.e. denoting by K_t and $\hat{K}_t$ the path of aggregate capital in the decentralized equilibrium, the constrained optimum, and the first best, we find $\lim_{T\to\infty} K_T/\hat{K}_T = 0$ a.s. and $\lim_{T\to\infty} \hat{K}_T/K_T^* = 0$ a.s.

Proof. The proof is analogous to the proof of Proposition 2. □

The proposition describes how a constrained planner would optimally regulate the investment choices of bankers if she cannot interfere in their law-of-motion of capital accumulation. It provides several notable insights:

First, the optimal investment strategies of different banker types are time-varying as their capital shares κ_{it} in the economy vary.

Secondly, comparing optimality conditions (9) and (13), we can see that the

3 Spillovers to the Real Economy

Our baseline model assumed that investment opportunities did not require any inputs other than capital and delivered constant returns to scale. This section extends the baseline by assuming that production requires not only capital but also complementary factors such as labor and land, which cannot easily be reproduced. As the capital stock in the economy grows, the limited supply of these complementary factors reduces the returns to capital. Conversely, declines in the aggregate capital stock imply that complementary factors are more abundant and capital earns higher returns. The resulting decreasing returns to capital lead to selection dynamics that imply that the aggregate capital stock in the economy is bounded and fluctuates around an ergodic steady state, no matter how productive (or unproductive) the economy's investment opportunities are.

A second important insight from this section is that the net worth of the financial sector is an important driver of output and labor income in the real economy. The boom and bust dynamics in the financial sector that we analyzed before generate spillovers to the real economy. This implies that the policies to smooth booms and busts in the financial sector also stabilize the real economy – financial policy is a key element of macroeconomic stabilization policy.

Let us introduce a unit mass of households $h \in [0,1]$ who each supply one unit of labor every period. We assume that households themselves do not have access to financial markets and live hand-to-mouth – they derive period utility $u(w_t) = \ln w_t$ from their wage income.[2] Bankers invest capital in their desired investment opportunity, and after the returns $\tilde{R}_t$ are realized, they lend all capital $K'_t = \sum_i \tilde{R}_i k_i$ to competitive firms in the real economy who have access to a Cobb-Douglas production technology $y = Ak^{\alpha}\ell^{1-\alpha}$ that combines capital and labor to produce output. Capital fully depreciates, and the output from production is consumed or invested as future capital. Given this setup, we find:

[2] This is a reasonable characterization for a majority of households worldwide. In the US, for example, 76% of households are living paycheck-to-paycheck as defined by having liquid savings of less than three months worth of income (CNN Money, 2013). Only 18% participated in markets for aggregate risk as defined by holding liquid equity investments (see Kennickel, 2013).

Lemma 3. *(Bank Capital and Wages) Wages and the return on capital in the economy are given by*

$$\begin{aligned} w_t &= w\left(K_t'\right) = (1-\alpha)\, A(K_t')^{\alpha} \\ r_t &= r\left(K_t'\right) = \alpha A(K_t')^{\alpha-1} \end{aligned} \tag{14}$$

Proof. The result follows since firms competitively maximize profit and markets for capital and labor have to clear at the available quantities $\ell = 1$ and $K_t^{'}$. □

The lemma shows that both wages and the return to capital depend crucially on the capital of the banking sector. Higher capital leads to greater wages but decreasing returns on capital. Both wages and the return on capital are taken as given by individual agents – the effects of aggregate capital in the banking sector on the two variables thus represent pecuniary externalities.

The law-of-motion for net worth of sector i bankers is now

$$k_{it+1} = r_t \tilde{R}_{it} k_{it}$$

The optimal behavior of bankers as well as that of a social planner in our extended setting is unchanged from the baseline model:

Lemma 4. *(i) The optimal strategy of type i bankers continues to be given by the maximum geometric mean criterion described in Lemma 1.*
(ii) The optimal strategy of a social planner who maximizes either banker welfare, worker welfare, or aggregate welfare is given by the constant capital shares described in Proposition 2.

Proof. (i) The optimization problem of a type i banker is described by the period-by-period problem

$$\max_{S\in\mathfrak{S}^i} E\left[\ln r_t \tilde{R}_S\right] = E\left[\ln r_t\right] + \max_{S\in\mathfrak{S}^i} E\left[\ln \tilde{R}_S\right]$$

Since individual bankers take r_t as given and the term enters the optimization problem additively, the solution to the problem is the same as in Lemma 1.

(ii) The optimal strategy of a planner who maximizes banker welfare $U = E[\ln K_T]$ is given by the period-by-period problem

$$\max_{\kappa_i\in[0,1]} E\left[\ln r_t K_t'\right] = \max_{\kappa_i\in[0,1]} E\left[\ln \alpha A (K_t')^{\alpha}\right] = \max_{\kappa_i\in[0,1]} \alpha E\left[\ln \sum_i \tilde{R}_i \kappa_i\right] + E\left[\ln \alpha A\right]$$

The solution to this problem is given by Proposition 2.

The optimal strategy of a planner who maximizes worker welfare $U^w = \sum \beta^t \ln w_t$ is given by

$$\arg\max_{\kappa_{it}\in[0,1]} \sum_t \beta^t E\left[\ln (1-\alpha) A \left(K_t'\right)^\alpha\right] = \arg\max_{\kappa_{it}\in[0,1]} \sum_t \beta^t E\left[\ln K_t'\right] \quad \text{where} \quad K_t' = \sum_i \tilde{R}_i \kappa_i$$

where we drop constant additive and multiplicative terms. For $t = 0$, the solution is given by the optimal capital shares described in Proposition 2. To maximize the term $E[\ln K_t']$ in the sum above for any given t, we observe that

$$\begin{aligned} \max_{\kappa_{it}\in[0,1]} E\left[\ln K_t'\right] &= \max_{\kappa_{it}\in[0,1]} E\left[\ln K_t \sum_i \tilde{R}_{it}\kappa_{it}\right] = \max_{\kappa_{it}\in[0,1]} E\left[\ln \alpha A (K_{t-1}')^\alpha \sum_i \tilde{R}_i \kappa_i\right] \\ &= \alpha E\left[\ln K_{t-1}'\right] + E\left[\ln \alpha A\right] + \max_{\kappa_{it}\in[0,1]} E\left[\ln \sum_i \tilde{R}_i \kappa_i\right] \end{aligned} \tag{15}$$

where the second step employs the relationship $K_t = r_{t-1}K_{t-1}' = \alpha A (K_{t-1}')^\alpha$. Following the logic of induction, if the strategy of Proposition 2 solves the maximization problem for all terms up to period $t-1$, then equation (15) shows that the same strategy also solves the maximization problem in period t.

A planner who maximizes aggregate welfare maximizes a weighted sum of banker and worker welfare $U + \gamma U^w$. The proof follows along the same lines as for bankers and workers separately. □

Let us now investigate the effects of the strategies of decentralized bankers and of the social planner on the dynamics in the real economy. We find the following:

Proposition 4. (Smoothing Spillovers to the Real Economy) *The social planner's allocation exhibits (i) a smaller range of fluctuations for capital, output and wages and (ii) higher geometric mean levels for the three variables compared to the decentralized equilibrium.*

Proof. In the decentralized equilibrium, the lower bound $\underline{K}$ (upper bound $\bar{K}$) on capital is reached asymptotically if all capital is held by the type with the lowest (highest) possible shock realization $\underline{\tilde{R}}$ (or $\bar{\tilde{R}}$) and a large number of the lowest (highest) shocks materialize. The capital level then coverges towards a level defined by the fixed point $\underline{K} = r(\underline{K})\underline{\tilde{R}}\underline{K}$ (or $\bar{K} = r(\bar{K})\bar{\tilde{R}}\bar{K}$) or, equivalently,

$$\underline{K} = \left[\alpha A \left(\underline{\tilde{R}}\right)^\alpha\right]^{\frac{1}{1-\alpha}} \quad \text{and} \quad \bar{K} = \left[\alpha A \left(\bar{\tilde{R}}\right)^\alpha\right]^{\frac{1}{1-\alpha}}$$

In the planner's allocation, capital is always allocated in constant fractions, resulting in stochastic returns $\tilde{R}^*$ that satisfy $\underline{\tilde{R}}^* = \min\{\tilde{R}^*\} > \underline{\tilde{R}}$ and $\overline{\tilde{R}}^* = \max\{\tilde{R}^*\} < \overline{\tilde{R}}$ and corresponding bounds on capital that satisfy $\underline{K} < \underline{K}^* < \bar{K}^* < \bar{K}$. Output and wages are strictly monotonic transformations of the aggregate capital stock, so this proves point (i). Next observe that the planner's strategy in any period t amounts to maximizing the $E[\log K_t']$. Given the Cobb-Douglas production function, this also maximizes the geometric mean of output and wages, proving point (ii). $\square$

Simulation 4 (Decreasing Returns and Spillovers to the Real Economy) We simulate the described dynamics building on the parameterization and shock process in Simulation 1 and setting the capital share $\alpha = 1/3$ and productivity A such that $\alpha A = 1$. The result is depicted in Figure 3. Compared to our earlier Figures 1 and 2, the decreasing returns introduce two novel considerations: first, as illustrated in the top panel, growth in the capital of one type comes at the expense of the other type, i.e. due to the decreasing returns, the sector that experiences the relatively lower return shock $\tilde{R}_{it}$ shrinks (even if the return shock $\tilde{R}_{it}$ is positive). For example, during the initial boom, the capital of safe bankers declines both in relative and absolute terms. Secondly, there is strong mean reversion – as a result, aggregate capital never exceeds an upper threshold $\bar{K}$ that satisfies $\alpha A(R_2^H \bar{K})^\alpha = \bar{K}$ or a lower threshold $\underline{K}$ that satisfies $\alpha A(R_2^L \underline{K})^\alpha = \underline{K}$, as illustrated in the second panel of the figure (the two thresholds are indicated by dotted lines).

The third panel shows the dynamics of the wage w_t (top line) and the rental rate of capital r_t (bottom line), which follow the dynamics of the aggregate capital stock. Finally, the bottom panel shows that the relative fraction of capital held by the risky versus the safe type is unchanged from our baseline simulation.

3.1 Bailouts

The welfare of workers in the real economy depends critically on a well-capitalized financial sector since wages are a function $w(K_t')$. When the financial sector is under-capitalized, workers may thus find it collectively desirable to provide transfers (or "bailouts") to the bankers. In particular, we observe the following:

Lemma 5. (Bailout Threshold *If $K_t' < \hat{K}$ then the welfare of workers increases if they provide a transfer of $T(K_t') = \hat{K} - K_t'$ to bankers where the threshold $\hat{K}$ is given by $w'(\hat{K}) = 1$ or, equivalently,*

$$\hat{K} = [\alpha (1 - \alpha) A]^{\frac{1}{1-\alpha}}$$

Figure 3: Simulation with Decreasing Returns and Spillovers to the Real Economy

Proof. As long as $K_t' < \hat{K}$, we find $w'(K_t') > 1$, i.e. a marginal unit of additional capital transferred to the banking sector raises wages by more than a marginal unit and raises the consumption of workers by $w'(K_t') - 1 > 0$. Their period t consumption thus increases to $c_t = w(\hat{K}) - (\hat{K} - K_t') > w(K_t')$. $\square$

The lemma thus provides a simple theory of endogenous bailouts that are voluntarily provided from workers to bankers. The role of bailouts is simply to mitigate the financial market imperfection that makes a shortage of capital in the banking sector so costly.

One of the most hotly debated questions after the recent financial crisis was how bailouts may introduce distortions into the financial sector. A unifying theme in the related literature was that bailouts distort incentives. By contrast, we identify a novel channel through which bailouts introduce distortions into the financial sector – borrowing from the language of evolutionary theory, bailouts interfere with the natural selection mechanism in capitalist economies. To isolate this novel mechanism from the traditional argument about distorted incentives, let us assume that the set of investment opportunities $\mathfrak{S}^i$ of each risk type is a singleton, i.e. the optimal strategy S^i of each risk type with returns $\tilde{R}_i$ is pre-determined.

The effects of bailouts generally depend on the manner in which they are allocated to individual bankers. We generally believe that the least distortive manner of providing bailouts is if they are given in a lump-sum fashion. This implies in particular that they are provided independently of any endogenous variables that are affected by the choices of the banker. In the following, we make the following assumption:

Assumption 1. (Uniform lump-sum transfers) *Bailouts are provided as uniform lump-sum transfers across bankers, i.e. each type i banker receives an exogenous transfer*

$$T_t^i = \frac{T(K_t')}{N}$$

We then observe the following effects:

Proposition 5. (Interfering with the Capitalist Natural Selection Process) *(i) Bailouts allow for the survival of high risk types with inferior geometric mean return that would go extinct in the decentralized equilibrium.*

(ii) They may instead cause safer risk types with superior geometric mean return to go extinct.

(iii) In the extreme, only high risk types to which a planner would assign zero weight will survive.

Proof. Let us define the (multiplicative) bailout return factor that is earned by each type as $R_{it}^B = 1 + T_t^i / k'_{it} = 1 + T(K'_t)/(Nk'_{it})$ which is decreasing in k'_{it}, i.e. for a given bailout $T(K'_t) > 0$, risk types with higher k'_{it} have a lower bailout return factor. In the presence of bailouts, the long-run survival of each risk type depends on the expected geometric mean return $E[\log \tilde{R}_i + \log R_{it}^B]$. (The return r_t affects all types equally and can thus be left out of the comparison.)

Critically, bailouts are given when aggregate capital K'_t is low, which occurs after low shock realizations. However, after low shock realizations, high risk types have on average lost more than low risk types so $k'_{it} < k'_{jt}$ for $i > j$. This implies that high risk types experience a higher bailout return factor, which raises their geometric mean return compared to low risk types and allows for their long-run survival, proving part (i). If the comparison of expected geometric mean returns results in a strict inequality in favor of a high risk type, lower risk types with superior geometric mean return may go extinct, proving part (ii). □

Intuitively, bailouts that are distributed uniformly to all risk types benefit risky bankers most since these suffer from the largest capital shortfalls precisely in those states of nature in which bailouts are provided. This interferes with the capitalist natural selection process and allows for the survival of inefficient risk types that would otherwise become extinct.[3]

The channel through which bailouts affect aggregate risk-taking is in marked contrast to much of the existing literature on the topic: we find that bailouts increase risk-taking via their effects on the capitalist natural selection process, whereas much of the existing literature emphasizes how bailouts adversely affect the *incentives* of individual bankers. The existing view on the adverse incentive effects of bailouts has been put in question in recent years since there is little evidence to support the hypothesis that bankers knowingly exposed their firms to existential risk (see e.g. Cheng et al, 2014). Our setup explains how bailouts can have deleterious effects on risk-taking even though the incentives of bankers are unaffected.

[3]The Proof of Proposition 5 also hints at how bailouts would have to be provided to be neutral for the selection process – they would have to deliver equal R_{it}^B to all types. In practice, this is unfortunately very difficult to implement since it would imply smaller bailouts to those who need them most in the sense that they have recently suffered the highest losses.

4 Capital Reallocations

So far our analysis has assumed that the net worth of heterogeneous firms follows purely the dynamics that result from internal accumulation of earnings. In practice, there are a number of mechanisms that are wide-spread in financial markets and by which capital is reallocated among risk types in a predictable manner. This section extends our analysis to study such dynamics. The mechanisms for capital reallocations include the following phenomena:

First, they may capture *idiosyncratic shocks to the type* of bankers. For a given bank, such shocks may arise from stochastic changes in management, from changes in the set of insiders who have decisionmaking power, e.g. a shift of managerial power from traditional lending to the trading desk, or changes in the information set of decisionmakers. Secondly, they may also capture *changes in the set of financial institutions* that are operative. For example, financial institutions may be subject to mergers and take-overs, or they may exit the market and be replaced by new bankers of different types. Third, capital reallocations may capture *public policy actions* whereby a policymaker imposes taxes and subsidies or equivalent measures on bankers that redistribute among types. Fourth, in a somewhat broader interpretation of our setup, the law of motion may capture *reallocations of funds by external investors* that are not modeled in further detail.

Building on our baseline setup, all such reallocations are described by a Markov process with transition matrix $M = (m_{ij})$, where element m_{ij} captures the probability that a banker of risk type i turns into risk type j in a given time period. An equivalent interpretation is that the element m_{ij} in the transition matrix captures the probability that a dollar of risk type i moves under risk type j in a given time period. The diagonal elements m_{ii} capture the probability that a banker remains of type i. We assume that the matrix M satisfies the standard properties of a transition matrix and is irreducible. The resulting law of motion of the vector of capital positions is

$$k_{t+1} = M\tilde{R}_t k_t$$

First-Best Capital Reallocation If we ask what transition matrix M would maximize long-run capital growth in the economy without imposing any restrictions, we find a familiar result:

Proposition 6. ***(First-Best Reallocations)*** *The optimal transition matrix is time-*

invariant and has identical columns

$$M^* = \begin{pmatrix} \kappa_1^* & \cdots & \kappa_1^* \\ \cdots & \cdots & \cdots \\ \kappa_N^* & \cdots & \kappa_N^* \end{pmatrix} \tag{16}$$

where $(\kappa_1^*, ..., \kappa_N^*)$ *are the optimal fractions of aggregate capital to be invested in opportunities* 1 *to* N *that we characterized in Proposition 2.*

Proof. The matrix M^* implements the first-best allocation characterized in Proposition 2 for any initial vector of capital holdings k_t. □

This hints at what kind of dynamics policy should aim to encourage: since it is desirable for capital to be allocated in constant proportions, policy should aim to encourage dynamics that undo the inefficient boom-bust dynamics that arise in the decentralized equilibrium.

However, it is unlikely that the four phenomena described above will give rise to these optimal dynamics in practice. Let us thus investigate two alternative scenarios that capture risk type dynamics that are more likely to be encountered in the real world: random symmetric capital reallocation and state-dependent capital reallocations that capture momentum and contrarian dynamics of risk types. We limit our attention to two risk types $\mathfrak{I} = \{1, 2\}$ to obtain simple analytic results.

4.1 Symmetric Capital Reallocations

Symmetric shocks can be interpreted in the four ways listed in the beginning of the section as long as the capital reallocations are independent from the realization of aggregate shocks, i.e. as long as idiosyncratic shocks to banker types, changes in the set of financial institutions who are active, and the reallocations driven by policy or by external investors are determined by factors unrelated to the aggregate shock process.

For two risk types, symmetric capital reallocations are described by a transition probability $\mu \in (0, 1]$ and a transition matrix

$$M^{sym} = \begin{pmatrix} 1-\mu & \mu \\ \mu & 1-\mu \end{pmatrix}$$

This matrix is no longer able to implement the optimal capital allocation that we described above. However, under random reallocation, we can make two interesting observations:

Proposition 7. ***(Symmetric Reallocations, Two Risk Types)*** *(i) Introducing a small transition probability $\mu > 0$ is desirable if $\kappa_t < \min\{\kappa^*, 1/2\}$ or $\kappa_t > \max\{\kappa^*, 1/2\}$ and undesirable for $\kappa_t \in (\kappa^*, 1/2)$;*

(ii) The optimal transition probability μ is a function $\mu(\kappa_t)$ of the relative capital allocation κ_t at time t; for $\kappa_t \neq 1/2$ it is given by

$$\mu^*(\kappa_t) = \min\{\max\{0, \mu(\kappa_t)\}, 1\} \quad \textit{where} \quad \mu(\kappa_t) = \frac{\kappa_t - \kappa^*}{1 - 2\kappa_t} \tag{17}$$

For $\kappa_t = 1/2$, any μ is optimal since random symmetric reallocation does not affect the allocation of capital and is irrelevant.

Proof. We drop the time subscript and observe that for a given capital allocation κ, the transition probability μ that maximizes geometric mean growth in the following period solves

$$\max_{\mu\in[0,1]} E\left[\ln \tilde{K}\right] \quad \text{where} \quad \tilde{K} = [(1-\mu)\kappa + \mu(1-\kappa)]\tilde{R}_1 + [\mu\kappa + (1-\mu)(1-\kappa)]\tilde{R}_2$$

The optimality condition to this problem is

$$\frac{dE\left[\ln \tilde{K}\right]}{d\mu} = \frac{p(1-2\kappa)\left(R_1^L - R_2^L\right)}{K^L} + \frac{(1-p)(1-2\kappa)\left(R_1^H - R_2^H\right)}{K^H} = 0$$

Evaluating the derivative at $\mu = 0$, we obtain

$$\left.\frac{dE\left[\ln \tilde{K}\right]}{d\mu}\right|_{\mu=0} = \Theta(1-2\kappa)(\kappa - \kappa^*) \quad \text{where} \quad \Theta < 0$$

This immediately delivers point (i).

For point (ii), the optimality condition is satisfied for any μ if $\kappa = 1/2$. Otherwise, substituting for K^L and K^H delivers an optimality condition for μ that depends on κ and that be solved for the expression $\mu(\kappa_t)$ in (17). If $\mu(\kappa_t)$ is outside the unit interval, then the corner of the interval $[0, 1]$ that is closest to $\mu(\kappa_t)$ represents the constrained optimum transition probability, since the objective function is monotonic in μ in the relevant range. This is captured by the min-max expression in equation (17). □

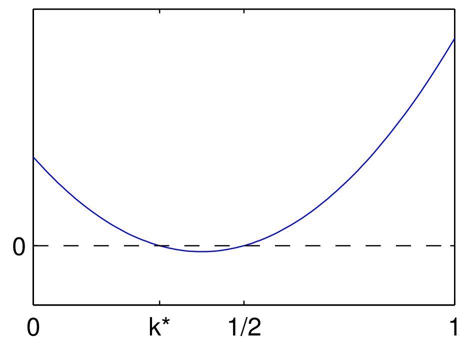

Figure 4: Benefit of random symmetric reallocation $dE[\ln \tilde{K}]/d\mu$ at $\mu = 0$

To further illustrate the results of the Proposition, the effect of introducing a small probability of symmetric reallocation is depicted in Figure 4. The figure evaluates $dE[\ln \tilde{K}]/d\mu$ at $\mu = 0$ over the domain $\kappa \in [0, 1]$ for the case $\kappa^* < 1/2$. If the capital allocation is close to the corners of the interval $[0, 1]$ then it is far from the efficient value κ^* and symmetric reallocation is desirable. By contrast, if the capital allocation is in the interval $(\kappa^*, 1/2)$, symmetric reallocation moves the capital allocation away from κ^* towards $1/2$, which is undesirable. Intuitively, symmetric reallocation leads to mean regression in risk types that pushes the capital shares towards $1/2$. This is desirable if it brings the capital shares closer to the optimum κ^*.

Simulation 3
(Effects of Symmetric Capital Reallocations) We incorporating symmetric capital reallocation dynamics into the example of Simulation 1. Figure 5 compares the wealth dynamics without capital reallocations (solid lines) to those with capital reallocations (dashed lines) for the two risk types for $\mu = 5\%$. It can be seen that the dynamics with symmetric capital reallocations are generally less extreme (top panel) and exhibit smaller variation in relative capital shares (bottom panel). In the simulation, the aggregate capital stock also ends up growing at a slightly higher rate since the allocation of capital to the risky type is, on average, closer to its optimum κ^*.

4.2 State-Dependent Capital Reallocations

In financial markets, capital reallocations are frequently correlated with the aggregate state of nature. For example, aggregate shocks may lead to systematic changes in management that affect the set of investment opportunities of banks, or to mergers, take-overs, exit decisions that systematically affect the set of financial institutions that

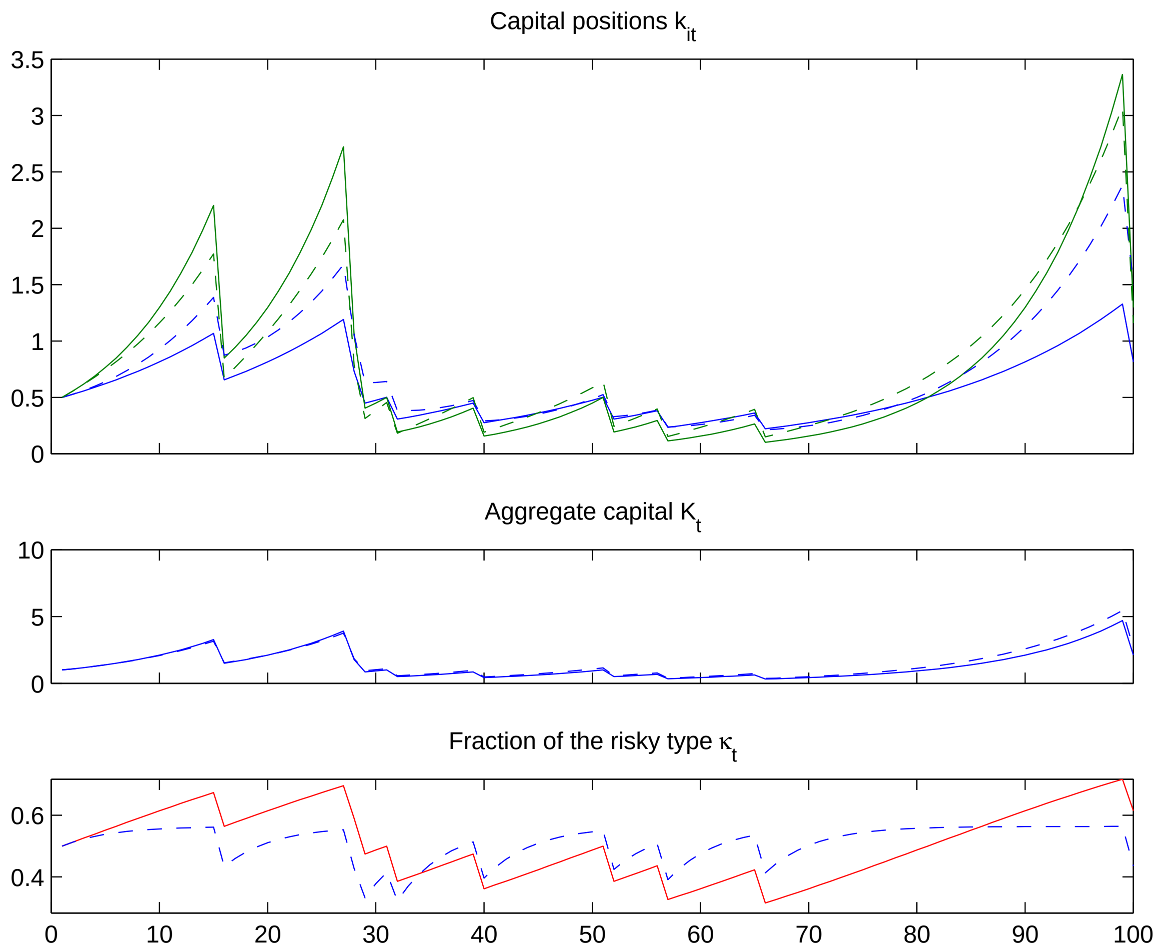

Figure 5: Simulation of Risk-Taking Dynamics with Random Reallocation

continues operation, or to systematic reallocations of funds by external investors who either chase momentum or act as contrarians. We capture such effects by assuming a stochastic transition matrix $\tilde{M}_t$ that is a function of the aggregate state of nature. The law of motion for the vector of capital positions is then

$$k_{t+1} = \tilde{M}_t \tilde{R}_t k_t$$

By imposing further structure on the transition matrices, state-dependent capital reallocation allows us to capture the following interesting phenomena that are of particular interest in financial markets. Let us denote by M^L and M^H the transition matrices in the low and high state.

Definition 3. (Momentum) We call a state-dependent capital reallocation process momentum-based if the matrix M^L is upper-triangular and M^H is lower-triangular.

(Contrarian) Conversely, we call it contrarian if the matrix M^L is lower-triangular and M^H is upper-triangular.

A momentum-based reallocation process implies that capital is, on average, reallocated from risk types that have just performed relatively poorly to risk types that have just performed relatively well. In practice, momentum-based reallocations of capital can occur because traders or managers who have performed well are promoted whereas those who under-perform are replaced, because firm exit and corporate take-overs are often concentrated on underperforming firms, or because funds are moved by external investors who chase momentum. A contrarian shock process implies the opposite: capital is reallocated from well-performing strategies to recently under-performing strategies. This type of shock process is somewhat rarer in financial markets.

Two Risk Types We introduce momentum-based and contrarian reallocation processes in our example with two risk types by considering the two matrices

$$M^+ = \begin{pmatrix} 1-\nu^+ & 0 \\ \nu^+ & 1 \end{pmatrix} \quad \text{and} \quad M^- = \begin{pmatrix} 1 & \nu^- \\ 0 & 1-\nu^- \end{pmatrix} \tag{18}$$

The transition matrix M^+ moves a fraction $\nu^+ \in (0, 1]$ of capital from risk type 1 to type 2 and therefore increases the average riskiness of the economy. The matrix M^- reallocates a fraction $\nu^- \in (0, 1]$ from type 2 to type 1 and decreases the aggregate riskiness of the economy. A momentum-based reallocation shock process implies that

$M^L = M^-$ and $M^H = M^+$; conversely, a contrarian shock process implies that $M^L = M^+$ and $M^H = M^-$.

Optimal Capital Allocation There is one particular configuration of triangular state-contingent transition matrices that preserves optimal capital allocations:

Proposition 8. ***(Optimal Contrarian Capital Reallocation)*** *Starting from an optimal capital allocation $k_t = (\kappa^*, 1-\kappa^*)K_t$, the contrarian reallocation matrices M^+ and M^- with*

$$\nu^+ = (1-\kappa^*)\left(1 - \frac{R_2^L}{R_1^L}\right) \quad \textit{and} \quad \nu^- = \kappa^*\left(1 - \frac{R_1^H}{R_2^H}\right)$$

preserve the optimal capital ratio $(\kappa^*, 1-\kappa^*)$ at all times.

Proof. Starting from a capital allocation k_t, the capital positions after a low shock realization is

$$\begin{aligned} k_{t+1} = M^+ R^L k_t &= \begin{pmatrix} 1-\nu^+ & 0 \\ \nu^+ & 1 \end{pmatrix} \begin{pmatrix} R_1^L & 0 \\ 0 & R_2^L \end{pmatrix} \begin{pmatrix} \kappa^* \\ 1-\kappa^* \end{pmatrix} K_t \\ &= \left[\kappa^* R_1^L + (1-\kappa^*) R_2^L\right] \begin{pmatrix} \kappa^* \\ 1-\kappa^* \end{pmatrix} K_t \end{aligned}$$

which is an optimal allocation. A similar result can be verified for $M^- R^H k_t$. □

The role of the transition matrices M^+ and M^- defined in the Proposition is to precisely undo the differential capital growth of the two sectors: a low shock realization increases the fraction of capital allocated to the low risk strategy, but the transition matrix M^+ undoes the increase by reallocating capital from the low-risk strategy to the high-risk strategy. Conversely, a high shock realization leads to disproportionate growth of the high risk strategy, but the transition matrix M^- restores the optimal ratio $(\kappa^*, 1-\kappa^*)$.

Momentum and Contrarian Capital Reallocation For allocations that differ from the optimal ratio κ^*, it is useful to denote the capital ratio after the period t shock is realized but before reallocation has taken place by prime variables,

$$k'_t = \tilde{R}_t k_t \quad \text{and} \quad \kappa'_t = k'_{t1}/(k'_{t1} + k'_{t2})$$

This allows us to establish the following result:

Lemma 6. *The optimal triangular reallocation matrix for given κ_t' satisfies*

$$\begin{array}{llll} \textit{if } \kappa_t' \in [0,\kappa^*) & \textit{then } M = M^- & \textit{with } \nu^- = \frac{\kappa^*-\kappa_t'}{1-\kappa_t'} \\ \textit{if } \kappa_t' = \kappa^* & \textit{then } M = I_N & \\ \textit{if } \kappa_t' \in (\kappa^*,1) & \textit{then } M = M^+ & \textit{with } \nu^+ = \frac{\kappa_t'-\kappa^*}{\kappa_t'} \end{array}$$

Proof. The proof for the first case follows readily from observing that

$$\begin{pmatrix} 1 & \frac{\kappa^*-\kappa_t'}{1-\kappa_t'} \\ 0 & 1-\frac{\kappa^*-\kappa_t'}{1-\kappa_t'} \end{pmatrix} \begin{pmatrix} \kappa_t' \\ 1-\kappa_t' \end{pmatrix} = \begin{pmatrix} \kappa^* \\ 1-\kappa^* \end{pmatrix}$$

The proof of the other two cases is analogous. □

In the first case of the lemma, κ_t' is suboptimally low and it is desirable to increase it, making it optimal to apply the reallocation matrix M^+ with the given value of ν^+. The third case reflects the opposite situation. Naturally, at $\kappa_t' = \kappa^*$, no reallocation is indicated.

Next let us observe that there are two thresholds $\kappa^L < \kappa^* < \kappa^H$ such that $\kappa_t' < \kappa^*$ for $S_t = L$ and $\kappa_t' > \kappa^*$ for $S_t = H$ for any $\kappa_t \in \left[\kappa^L, \kappa^H\right]$. In other words, if $\kappa_t < \kappa^L$, then the capital ratio is sufficiently far below the optimal value κ^* that it will still be below κ^* if the next shock is L; if $\kappa_t > \kappa^H$, then the ratio is sufficiently far above the optimal value κ^* that it will still be above if the next shock is H. This observation has the following straightforward implication for the desirability of momentum-based vs. contrarian capital reallocation processes:

Proposition 9. (State-Dependent Reallocation and Volatility) *(i) For $\kappa_t \in \left[\kappa^L, \kappa^H\right]$, the optimal capital reallocation process is contrarian. For $\kappa_t < \kappa^L$, it is always desirable to apply the transition matrix M^+; conversely, for $\kappa_t > \kappa^H$ it is always desirable to apply the transition matrix M^-, with the optimal transition rates ν^+ and ν^- given by Lemma 6.*

(ii) For $\kappa_t \in \left[\kappa^L, \kappa^H\right]$, contrarian reallocation reduces period-ahead volatility V_{t+1} whereas momentum-based reallocation increases period-ahead volatility.

Proof. The proofs follow directly from the lemma and the discussion above. □

The proposition captures that it is generally desirable to have contrarian reallocation for intermediate capital ratios $\kappa_t \in \left[\kappa^L, \kappa^H\right]$, and that this reduces volatiltiy. By contrast, momentum-based reallocation generally increases volatility.

However, these results no longer apply if the capital ratio has veered towards one of the two corners of the unit interval as a result of consecutive shocks that go in the same direction – in that case, it becomes desirable to move back towards κ^* no matter what the prior shock.

The general desirability of contrarian forces for intermediate capital ratios poses a dilemma, since many of the drivers of idiosyncratic shocks that we described have an inherent tendency to introduce momentum. Sometimes, the best that policy can do is to aim to stem against the momentum-based shocks inherent in the financial system.

Simulation 3 (Effects of Momentum-Based Capital Reallocation) Given the prevalence of forces that lead to momentum-based reallocation in financial markets in practice, we illustrate their effects in a variant of our earlier Simulation 1 that includes the state-dependent transition matrices $M^H = M^+$ and $M^L = M^-$ as defined in equation (18) where we set $\nu^+ = 5\%$ and ν^- such that $(1-\nu^-)^\pi = (1-\nu^+)^{(1-\pi)}$. Intuitively, momentum reinforces the tendency of our economy to exhibit boom-bust patterns since the reallocations favor recent high performers and penalize recent poor performers.

Panel 1 of Figure 6 shows that fluctuations of the high and low risk types are accentuated by momentum-based reallocation, increasing risk-taking in booms and reducing it in busts. As a result, panel 2 shows that aggregate capital is more volatile than it would be in the absence of momentum-based reallocation. In the given example, a long series of positive shocks raises the capital stock that is allocated to high risk types to close to 100% towards the end of the simulation period as indicated in panel 3. However, the momentum-based reallocation also implies that a fraction ν^- of capital is returned to the low risk type after a bad shock strikes at the end of the simulation period, which prevents the low type from going extinct in the long run.

5 Extensions

5.1 Improvements in Risk-Sharing / Financial Development

This section relaxes our ealier assumption on incomplete risk markets by considering the case that bankers can invest up to a fraction ϕ of their net worth into the investment opportunities of other bankers. The microfoundation is that bankers need to invest a minimum fraction $1-\phi$ of their net worth in their own set of investment

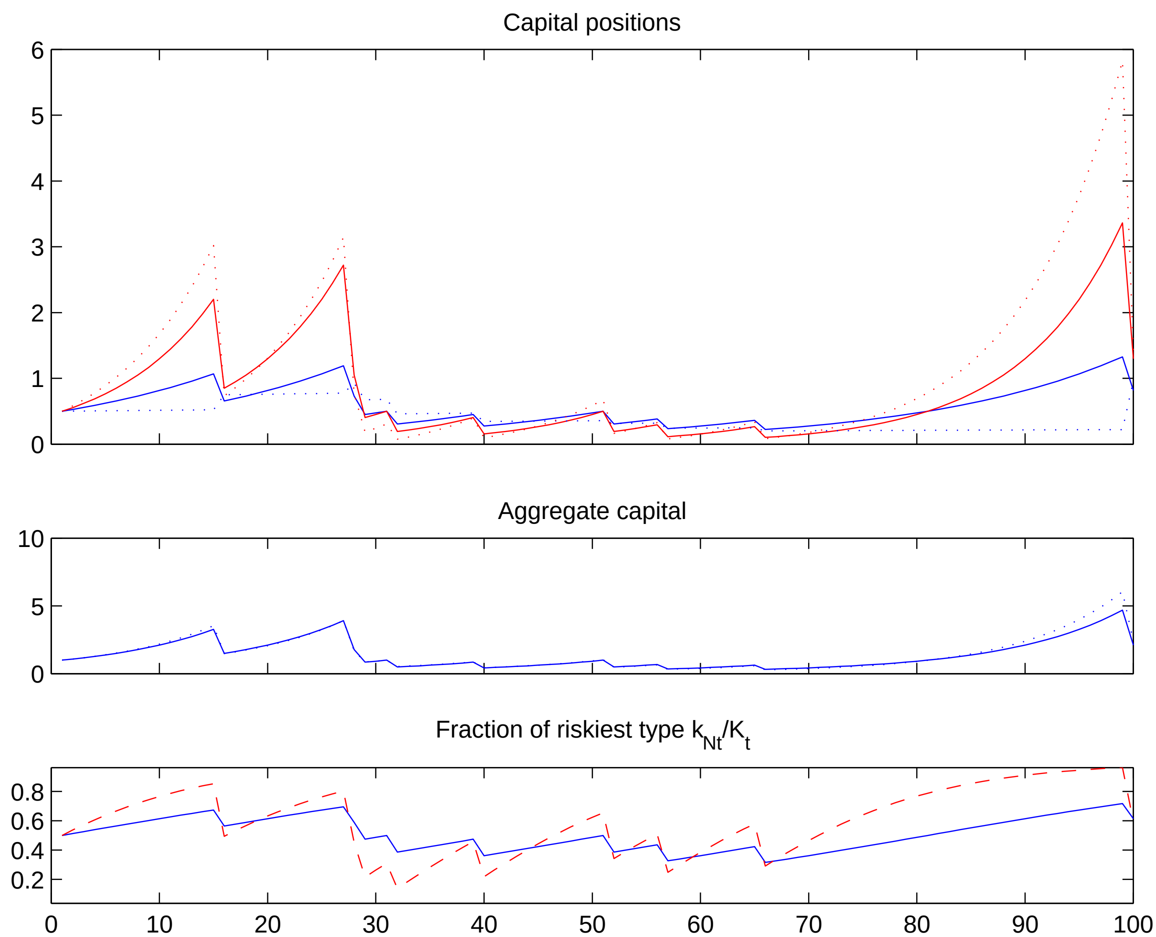

Figure 6: Simulation of Risk-Taking Dynamics with Momentum-Based Idiosyncratic Shocks

opportunities to guarantee proper effort. The case $\phi = 0$ nest our baseline model. Increases in ϕ beyond zero capture a form of financial development.

We find the following results for this extension:

Proposition 10 (Financial Development)**.** *(i) As long as $\phi < 1 - \kappa_i \forall i$, all bankers diversify a fraction ϕ of their net worth by investing in the investment opportunities of other risk types. Their geometric mean return increases, but they do not achieve the optimal risk allocation. The economy continues to experience volatility and procyclicality.*

(ii) For $\min\{1-\kappa_i\} \leq \phi < \max\{1-\kappa_i\}$, those risk types i for whom $\phi \geq 1-\kappa_i$ can achieve the first-best allocation. Their geometric mean return exceeds that of all other risk types, and the other risk types for whom $\phi < 1-\kappa_i$ will go extinct. In the long run, the economy achieves the first-best.

(iii) For $\phi \geq \max\{1-\kappa_i\}$, all risk types will invest in the optimal capital allocation and the economy immediately achieves the first-best.

Proof. See appendix. □

Intuitively, financial development allows bankers to improve risk-sharing and overcome the incompleteness in risk markets. If the fraction κ_i that should optimally be allocated to a given type is high enough, $\kappa_i \geq 1-\phi$, then bankers of this type simply keep κ_i in their own investment opportunities and allocate the remaining $1-\kappa_i$ to other risk types. This allows them to implement the optimal risk allocation in the economy.

6 Policy Interventions

Our evolutionary framework creates a novel role for public policy interventions that is quite distinct from the way policy is traditionally evaluated – policy affects the economy through dynamic changes in the composition of the financial sector rather than by constraining the static choice set of agents. Risk-taking dynamics are driven primarily by such compositional effects, i.e. by changes in the relative wealth of different types of bankers. In the simple framework described so far, the first welfare theorem holds – private agents engage in optimizing behavior and do not affect each other – so there is no role for policy intervention to improve welfare. However, it is useful to analyze the effects of different policy interventions in our baseline model to provide lessons for more general versions of the model, for example the ones in which the financial sector creates spillovers to the real economy.

We start by considering restrictions on the set $\mathfrak{S}^*$ of investment strategies, for example in the form of limits on risk-taking. Formally, this corresponds to imposing a ceiling $\bar{V}$ on the volatility of investment opportunities of bankers. We denote the remaining set of investment opportunities by $\mathfrak{S}^* = \{S \in \mathfrak{S}^i : Std(R_S) \leq \bar{V}\}$ and assume that it is non-empty.

We can then decompose the effects of limits on risk-taking in the following manner:

Corollary 1. (Limits on risk-taking) *Imposing a ceiling $\bar{V}$ on risk-taking for a given period t leads to*

(i) the static effect *of reducing period t risk-taking, which lowers one period-ahead volatility to*

$$V_{t+1} = k_t' \cdot \min\left\{\bar{V}, \ln \tilde{R}, \right\}$$

(ii) the dynamic effect *of changing the composition of capital in the following period,*

(ii.a) which lowers future volatilities V_{t+2}, V_{t+3} etc. if the period t shock is high;

(ii.b) which raises future volatilities V_{t+2}, V_{t+3} etc. if the period t shock is low.

Proof. The proof of part (i) is immediate. For the proof of part (ii.a), observe that a high shock increases the riskiness of the capital stock distribution. It follows from Proposition 1 that future volatilities are higher. For part (ii.b), a low shock reduces the riskiness of the distribution of the capital stock, and the opposite conclusions apply. □

The static effect in part (i) of the corollary corresponds to the usual model of financial regulation as restricting the choice set of economic agents. By contrast, the effects in part (ii) of the corollary represent dynamic effects that are not present in traditional models of homogenous agents and that are introduced by dynamic changes in the composition of the financial sector, i.e. in how capital is allocated across different risk types. As illustrated by the corollary, these dynamic effects of regulation have a long-lasting impact on the volatility of the sector.

Furthermore, as described in points (ii.a) and (ii.b), the impact of the dynamic effects on future volatility is counter-cyclical – they reduce volatility following high shocks and increase it following low shocks. In other words, they counteract the natural pro-cyclical tendencies of the financial system that are described in Proposition 1.

In practice, such changes in the composition of the financial sector play a major role during booms and busts in the financial sector. For example, most of the financial institutions that went bust during the Financial Crisis of 2008/09, such as Countrywide, Washington Mutual, etc. were among the fastest-growing players in the financial sector – in large part due ot their aggressive risk-taking practices during the upswing.

Given the importance of the dynamic effects in Corollary 1, it is natural that regulators in some jurisdictions have directly imposed limits on the growth of financial institutions. Limits on asset growth are often viewed as an archaic intstrument since they are difficult to motivate in standard models of regulation. In an evolutionary framework, by contrast, their role is straightforward: they ensure that the financial sector does not come to be dominated by the riskiest types. This has a stabilizing effect on the economy, again without reducing the utility of bankers.

Let us return to our example with two types of bankers only. Formally, we assume a restriction on the capital growth of bankers to a factor G over any n time periods, or $\prod_{k=t-n+1}^{t} R_{st} < G$. This implications of such regulation are as follows:

Corollary 2. (Limits on Asset Growth) *A restriction to grow at most by a factor G over n time periods,*

(i) will never affect low risk types with s s.t. $\left(R_s^H\right)^n \leq G$,

(ii) but will restrict the risk-taking of high risk types s if their cumulative return over the previous m periods satisfies

$$\prod_{k=t-m+1}^{t} R_{sk} > G/(\bar{R}^{n-m-1} R_s^H) \quad \textit{for some} \quad m < n, \tag{19}$$

(iii) and lowers the volatility of the capital stock V_{t+1} if condition (19) is satisfied for any risk type.

Proof. Part (i) holds since a banker satisfying the condition will meet the growth restriction even after n positive aggregate shocks. For part (ii), observe that if condition (19) holds, then the return of type s over the preceding m periods is sufficiently high that another high return realization of strategy s would violate the growth limit, even if the banker's funds are parked in the risk-free strategy for the remaining $n - m - 1$ periods. In that case, type s bankers are forced to switch to a safer strategy $s' < s$, which reduces the one-period ahead volatility of the capital stock V_{t+1}, proving point (iii). □

Aside from the two policy measures discussed in Corollaries 1 and 2 that directly affect the set of permissible investment strategies of bankers, the risk composition of the financial sector is affected by many other public policy measures. For example, it has been argued that low interest rates may favor high-risk investment strategies and shift bankers in that direction, with long-lasting effects on the composition of the financial sector.

Capital Reallocations: Policy can also be aimed at affecting the capital reallocation process. Examples include:

- *Competition policy that favors take-overs and management changes* accelerates momentum-based selection and thus tends to increase aggregate financial sector volatility. This effect is absent in traditional economic models.

  – conservative, boring banks are desirable

- Arranged take-overs, e.g. during financial crises

The commonality of the described policy interventions is that they all work primarily by affecting the dynamic composition of the financial sector, not the static incentives. This effect has not been systematically studied in the existing literature.

7 Conclusion

In short, our paper follows the evolutionary dynamics of heterogeneous bankers in a traditional economic model based on individual optimization in order to leverage the benefits of both approaches and develop more robust models of financial markets and financial policies. We find that an explicit focus on these dynamics delivers a number of novel insights about both positive dynamics in financial markets and the effects of public policy interventions.

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