A Portfolio Model of Quantitative Easing

Jens H. E. Christensen
Federal Reserve Bank of San Francisco
jens.christensen@sf.frb.org

and

Signe Krogstrup
International Monetary Fund
skrogstrup@imf.org

Abstract

This paper presents a portfolio model of asset price effects arising from central bank large-scale asset purchases, or quantitative easing (QE). Two financial frictions—segmentation of the market for central bank reserves and imperfect asset substitutability—give rise to two distinct portfolio effects. One is well known and derives from the reduced supply of the purchased assets. The other is new, runs through banks' portfolio responses to reserves expansions, and is independent of the types of assets purchased. The results imply that central bank reserve expansions can affect long-term bond prices even in the absence of long-term bond purchases.

JEL Classification: E43, E50, E52, E58, G11

Keywords: unconventional monetary policy, transmission, reserve-induced portfolio balance channel

We thank Olivier Blanchard, Bill Cline, Joseph Gagnon, John Kandrac, Arvind Krishnamurthy, Jose Lopez, and Bernd Schlusche for helpful comments and suggestions. We also thank seminar participants at the Federal Reserve Board of Governors as well as conference participants at the 2016 Bank of Canada Annual Conference, the 2017 IBEFA Summer Meeting, and the ECB Workshop on Money Markets, Monetary Policy Implementation and Central Bank Balance Sheets, including our discussants Aytek Malkhozov, Botao Wu, and Ulrike Neyer, for helpful comments. The views in this paper are solely the responsibility of the authors and do not necessarily reflect the views of the Federal Reserve Bank of San Francisco, the Federal Reserve System, the International Monetary Fund, its Management, or its Board.
This version: November 13, 2017.

1 Introduction

Since the global financial crisis of 2007-2009, a number of central banks have conducted large-scale asset purchases, frequently referred to as quantitative easing (QE), in order to provide monetary policy stimulus. Although the stated aims of such purchases have differed across countries, a common objective has been to reduce long-term interest rates, either broadly or in specific markets. In the case of the U.S. Federal Reserve, the success of its QE programs in reducing Treasury yields and mortgage rates appears to be well established; see Gagnon et al. (2011) and Krishnamurthy and Vissing-Jorgensen (2011), among many others. Similar evidence for the Bank of England's QE programs is provided in Joyce et al. (2011) and Christensen and Rudebusch (2012). More generally, it is well established that monetary policy can affect long-term interest rates when short-term policy rates are constrained at the effective lower bound (Ball et al. 2016, Swanson and Williams 2014a,b, Wright 2012).

Despite the empirical success of QE, how exactly it helps lower long-term interest rates is not well understood. Research so far has focused on two main channels.[1] One is a signaling channel, which means asset purchases send a signal to investors that lowers market expectations about future monetary policy (Christensen and Rudebusch 2012, Bauer and Rudebusch 2014). If financial market participants perceive that short-term interest rates will be lower in the future, this should translate into lower long-term interest rates today to make investors indifferent between rolling over a short-term loan and committing to a long-term loan. The other channel is a supply-induced portfolio balance channel. When a central bank purchases long-term bonds, it reduces the amount of these bonds available in the market and thereby raises their prices and lowers their returns (Gagnon et al. 2011).

When a central bank purchases assets as part of a QE program, however, it does not just reduce the supply of these assets in the market. It pays for these assets by issuing new central bank reserves. Hence, the supply of central bank reserves increases one-for-one with the reduction in the purchased assets. Such reserve expansions *per se* may play an important role in the transmission of QE to interest rates, as suggested by Bernanke and Reinhart (2004).[2] Nevertheless, the reserve expansions from QE programs have received little attention in the literature.

A notable exception is Christensen and Krogstrup (2016, henceforth CK), who examine three unique episodes in which the Swiss National Bank expanded reserves by purchasing only short-term debt securities. First, CK document that, although the supply of long-term Swiss government bonds and their closest substitutes remained unchanged, long-term yields

[1] There are other potential channels for QE to work. For example, it may affect liquidity and market functioning; see Kandrac (2014) and Christensen and Gillan (2017) for discussions and analysis in the context of U.S. QE programs. Also, it may affect the perception and pricing of risk, leading to a so-called "risk-taking channel," as discussed in Borio and Zhu (2012).

[2] Lenel (2017) argues that asset price effects from QE transactions may be a function of how the asset purchases and associated reserve creations affect the relative supplies of short- and long-term safe assets in the economy and their use in collateralized transactions.

on benchmark Swiss Confederation bonds fell following the QE announcements. Furthermore, CK show that the fall in rates cannot be explained by a lower expected path of short-term interest rates, thereby ruling out the signaling channel. Instead, they conclude that the anticipated creation of reserves was responsible for the fall in longer-term yields. Thus, there is empirical evidence that the creation of reserves from QE could matter for its transmission to long-term rates.[3,4]

The main theoretical reference for QE to give rise to portfolio effects is the model described in Vayanos and Vila (2009), which features preferred habitat behavior among investors, as originally proposed by Modigliani and Sutch (1966, 1967). Unfortunately, this model cannot account for any reserve-induced portfolio effects because it does not contain a central bank or central bank reserves. It models QE transactions as exogenous reductions in the supply of long-term bonds, thereby abstracting from the implications of the increase in central bank reserves for the balance sheets and asset demands of the private sector.[5]

This paper attempts to fill this gap by including a central bank balance sheet and modeling depository commercial banks and nonbank financial institutions separately in an otherwise simple portfolio model of the liquid asset portfolios of financial market participants, in the spirit of Tobin (1969). Our approach is to make a minimum of additional assumptions and modifications to this basic portfolio model necessary to illustrate that accounting for central bank reserves and the segmentation of their market is important for the transmission of QE to long-term yields. In the model, two financial frictions are key to our results. The first friction is standard, namely that assets, including central bank reserves, deposits, and bonds, are imperfect substitutes, and asset substitutability can differ across financial market participants. Imperfect asset substitutability is also the key friction driving traditional supply-induced portfolio balance effects in models such as Vayanos and Vila (2009). Without it, there are no portfolio balance effects. We additionally model the financial friction that central bank reserves can only be held by banks, i.e., segmentation of the market for reserves, while the assets the central bank purchases can come from banks and nonbank financial institutions alike. Both frictions are clearly empirically relevant. The requirement that asset substitutability is imperfect and can give rise to supply effects on asset prices has ample empirical support (Laubach 2009, Krishnamurthy and Vissing-Jorgensen 2012, Greenwood and Vayanos 2010, 2014, and Hamilton and Wu 2012). The nature of the asset substitutability in question has been described in different ways in the literature, e.g., as imperfect substitutabil-

[3] Another notable exception is Kandrac and Schlusche (2016), who take a step beyond the immediate portfolio balance impact of QE on bond yields and assess the effect of QE-induced reserve accumulations on bank-level lending and risk-taking activity in the U.S. Their results also suggest that the accumulation of reserves per se matters for the transmission of QE.

[4] Hancock and Passmore (2015) stress the role of the increase in the amount of bank deposits held by the private sector for the QE-induced portfolio rebalancing effects they document in the prices of U.S. mortgage-backed securities during the 2009-2013 period, but they do not quantify its effect.

[5] See Hamilton and Wu (2012) and Greenwood and Vayanos (2014) for empirical applications of the Vayanos and Vila (2009) model to the U.S. Treasury market.

ity between specific securities leading to local supply effects (D'Amico and King 2013), or as a more broad-based imperfect substitutability of duration risk in private portfolios (Gagnon et al. 2011 and Li and Wei 2013). Moreover, markets for reserves are institutionally segmented. Only certain types of financial institutions that are direct counterparties to the central bank can hold reserves. Usually, this is limited to depository banks.

Our model features the traditional portfolio effects as in Vayanos and Vila (2009) arising from the reduction in the stock of assets available to financial market participants when the central bank conducts QE—a supply-induced portfolio balance effect. Furthermore, the model shows that the reserve expansions that accompany QE asset purchases may lead to additional portfolio balance effects on asset prices more broadly. This possibility arises when QE asset purchases are executed with nonbank financial institutions. Since they do not have reserve accounts with the central bank, they cannot be paid for their assets in reserves. Instead, they receive the proceeds as deposits with their correspondent banks, and the correspondent banks—and, hence, the banking sector as a whole—see an expansion of their balance sheets with reserves on the asset side and deposits on the liability side. Importantly, banks are passive observers of these transactions that dilute the average return (or duration) of their asset portfolios. We show that if, in response to such a dilution, banks increase their demand for long-term bonds, then reserve expansions per se can produce portfolio balance effects on long-term yields.[6] In this case, the increased long-term asset demand from banks pushes up the prices of these assets further in equilibrium and hence reinforces the supply-induced portfolio balance effect. Importantly, an extended version of the model with two traded securities—a short bond and a long bond—shows that reserve- and supply-induced portfolio effects are distinct channels for the transmission of QE. It also shows that reserve-induced portfolio effects are independent of the particular assets the central bank purchases and arise due to the segmentation of the market for central bank reserves, imperfect asset substitutability, and banks' portfolio response to changes in deposit funding. Without either of these components, the reserve-induced portfolio balance channel highlighted in this paper and empirically investigated in CK shuts down.

The model implies that financial market structure, the business models of financial market intermediaries, their portfolio optimizing tools, and bank regulations may play crucial roles for the transmission of QE to long-term interest rates,[7] as these factors affect the substitutability between short- and long-term assets in the portfolio choices of banks and nonbank

[6]Haddad and Sraer (2015) demonstrate that the net interest rate exposure of U.S. banks is a significant determinant of U.S. Treasury bond excess returns. They argue that this pattern is explained by banks requiring a higher return to offset the increased interest rate risk caused by increased demand for long-term fixed-rate loans or reduced supply of deposits. This represents an example of bank behavior consistent with our assumptions.

[7]For example, Ihrig et al. (2017) find that the introduction of the leverage coverage ratio in the Basel III framework appears to have affected large banks' demand for high-quality liquid assets.

financial institutions.[8,9] As an example, we specifically consider how different types of leverage constraints may affect the transmission of QE to asset prices through banks' balance sheets.

Our analysis is related to recent theoretical work on unconventional monetary policy transmission by Farmer and Zabczyk (2016), who demonstrate how, in a general equilibrium overlapping generations model, a change in the risk composition of the central bank balance sheet can affect equilibrium asset prices. Unlike the QE analyzed in this paper, there is no expansion of the central bank balance sheet in their analysis, which focuses on central bank balance sheet compositional changes (qualitative easing rather than quantitative easing). The key friction generating the results in Farmer and Zabczyk (2016) is related to incomplete markets for financial contracts across generations, with some yet to be born. Their frictions hence work across time, while ours relate to the opportunity sets across agents. In this sense, the two analyses complement each other.

The model gives a theoretical characterization of the circumstances under which reserve-induced portfolio effects exist, and points to two potential avenues of empirically identifying them. First, QE programs that entail substantial increases in the amount of central bank reserves achieved without acquiring any long-lived securities would not affect the market supply of long-term securities. They hence would not give rise to any supply-induced portfolio balance effects on long-term interest rates. Portfolio balance effects from such programs can hence be attributed to the reserve expansion per se, after appropriately controlling for other factors as done in CK. Second, during the exit from a standard QE program, if the central bank balance sheet is normalized by letting assets mature, and if these assets are government securities rolled over into new short-term government debt (bills), then there is no change in the market supply of long-term assets coinciding with the normalization. All else equal, portfolio balance effects on long-term yields arising from such a normalization must hence be driven by reserve-induced portfolio effects, and not by standard supply effects. Future exits, if these are communicated in clear unexpected announcements, will allow us to make such assessments. Meanwhile, we note that reserve-induced effects are likely to be an empirically relevant part of the transmission of QE programs. In addition to the evidence provided by CK for the Swiss reserve expansions in August 2011, a review of recent research and data for U.S. QE programs suggests that the conditions necessary for the reserve channel to be active were met for the two most recent Federal Reserve QE programs. To fully understand the transmission of QE in the U.S., it may therefore be important to be mindful of the reserve channel and how it is influenced by bank balance sheet constraints.

Finally, the presence of reserve-induced portfolio balance effects has implications for both

[8] There is a growing literature underscoring the importance of financial intermediaries for asset prices, see Brunnermeier and Pedersen (2009), He and Krishnamurthy (2013), and Brunnermeier and Sannikov (2014), among many others.

[9] See Gertler and Karadi (2011, 2013) for analysis of a DSGE model that features financial intermediaries with endogenously determined balance sheet constraints and a central bank that can engage in QE by acting as an intermediary without constraints, but doing so less efficiently than its private-sector counterparts.

the design of future QE programs and for the exit from existing programs. One key implication is that QE may be effective at lowering long-term interest rates even in the absence of long-term bond purchases. This could matter for central banks that have to operate in markets with a limited supply of long-term securities of sufficient quality. Another implication is that the exit from QE could produce reserve-induced portfolio balance effects on long-term interest rates that result in a greater tightening of financial conditions than intended.

The remainder of the paper is structured as follows. Section 2 presents the model, while Section 3 investigates the effect of central bank asset purchases on equilibrium asset prices within the model. This section also analyzes a version of the model with two traded assets and details how constraints on bank balance sheets and leverage may influence the transmission of QE. Section 4 describes the empirical challenges of identifying reserve-induced portfolio effects, briefly reviews the U.S. experience with QE, and assesses whether the model's predictions are qualitatively consistent with U.S. data. Section 5 concludes and provides directions for future research. The appendix contains the details of the augmented version of the model with two traded assets.

2 The Model

We develop a model of a financial market with three types of agents: a central bank, nonbank financial firms, and banks. The model characterizes asset markets and abstracts from its links to the real economy. Importantly, we assume that over the horizon of the model, banks' credit portfolios do not adjust. For the same reason, we take the outstanding stock of bonds as given. The purpose of the model is to assess the short-term impact of QE transactions on asset prices before those prices affect real economic outcomes.

In the model we present here, the market for tradable securities comprises one asset. Because we have recent QE programs in mind in which long-term bonds have been acquired in exchange for reserves, we refer to this asset as a long-term liquid and safe bond with predetermined supply L and price P_L. In general, however, we can think of it as any traded asset in the economy, including short-term bonds, risky bonds, or equity. We briefly consider the case of a risky asset in Section 3.3. Having only one traded asset makes the model highly tractable and suffices to demonstrate the existence of the reserve-induced portfolio balance channel we highlight. However, this is a limitation when it comes to illustrating how central bank asset purchases of one asset can affect the prices of other assets through reserve-induced effects. We therefore consider the case with two traded assets in Section 3.2, which confirms the findings from the one-asset model and demonstrates the general nature of our findings. Importantly, we show that the reserve-induced portfolio balance channel can affect long-term bond prices even when no long-term bonds are purchased consistent with the empirical findings of CK.

In addition to holding the long-term bond, banks can hold reserves, denoted R, with the central bank and trade them with other banks. A key friction in our model—as in the real world—is that nonbank financial firms cannot hold reserves; instead they hold deposits with their correspondent banks. Another central assumption is that assets are imperfect substitutes, as is common in the portfolio balance literature.

Without any dynamic description in the model, the difference between the price of the long-term bond and its notional value of 1 can be interpreted as capturing its term premium. In our model, this term premium arises solely from imperfect substitution between bonds, reserves, and deposits and does not contain any liquidity or credit risk premiums because they are assumed away.

Unlike a deposit, which is on demand, in order to recover the money on a bond, the owner must find a buyer, which requires time and effort and involves some uncertainty about the achievable price. Investors are aware of this, and forward-looking behavior on their part provides the rationale for the existence of the term premium. Within the simplified world of the model, we think of the term premium as being positive, and without it, all agents would prefer to hold deposits.

To keep the model as simple and tractable as possible, we analyze the link between central bank asset purchases and asset prices considering a static asset market equilibrium, relying on total differentiation and comparative statics. Specifically, we study how marginal changes to the central bank asset holdings matched by similar changes in the outstanding amount of central bank reserves affect the equilibrium bond price. There are no dynamics in the model, and when we refer to changes or flows in the following, we are talking about differences between two static equilibria.

2.1 Key Behavioral Assumptions

Up front we highlight a few key behavioral assumptions that are crucial for the outcome of the model.[10]

First, the demand for bonds by firms in the nonbank financial sector, denoted L_{NB}, is assumed to be a function of the bond price and their equity E_{NB}:

$$L_{NB} = f_{NB}(P_L, E_{NB}). \tag{1}$$

Also, they are assumed to have standard preferences so that their demand is declining in the bond price

$$\frac{\partial f_{NB}(P_L, E_{NB})}{\partial P_L} < 0. \tag{2}$$

Finally, we assume that these firms do not respond in real time to short-term changes in their

[10]To keep the description in this section general, firm-specific identifiers have been dropped in the notation.

equity values by changing their demand for bonds, i.e.,

$$\frac{\partial f_{NB}(P_L, E_{NB})}{\partial E_{NB}} = 0. \tag{3}$$

This assumption allows us to abstract from interaction terms that would make the model quite intractable, but it would not be central to the mechanisms we are interested in describing.

In a similar way, to avoid detailing how banks' bond demand comes about as a solution to the optimization of their liquid safe asset portfolios, we summarize their bond demand in the following equation

$$L_B = f_B(P_L, E_B + D_B), \tag{4}$$

where banks' funding is $F_B = E_B + D_B$.[11]

As for nonbanks, we assume that banks have standard preferences for bonds. Hence, their demand is a downward sloping function of the bond price

$$\frac{\partial f_B(P_L, E_B + D_B)}{\partial P_L} < 0. \tag{5}$$

Finally, the response of banks' demand for bonds to a change in their funding satisfies the restrictions

$$0 < \frac{\partial f_B(P_L, E_B + D_B)}{\partial F_B} < 1. \tag{6}$$

This restriction is crucial in the model. If banks place all autonomous new deposit funding in reserves, i.e., $\frac{\partial f_B(P_L, E_B + D_B)}{\partial F_B} = 0$, we show in Section 3 that the reserve-induced portfolio balance channel shuts down.

Equipped with these key behavioral assumptions, we are now ready to describe the individual components of the model.

2.2 The Central Bank

The balance sheet of the central bank is given by

$$P_L L_{CB} = E_{CB} + R, \tag{7}$$

where L_{CB} is the central bank's holdings of long-term bonds, E_{CB} is the initial value of the central bank's equity, and R is the amount of outstanding reserves. Also, there are no cash balances in the model. To study QE, we assume that L_{CB} is the central bank's policy tool, which then determines the level of reserves R as a residual. Equation (7) implies that the change in the central bank's equity can be written as

$$dE_{CB} = dP_L L_{CB} + P_L dL_{CB} - dR. \tag{8}$$

[11] Note that equity and deposits are treated as equal, which is without loss of generality for our results.

In our simple setup, changes in reserves are matched by changes in the central bank's bond holdings, i.e., $dR = P_L dL_{CB}$. Consequently, changes in the central bank's equity are due solely to changes in the bond price. One practical implication of this is that a central bank engaging in QE is exposed to interest rate risk on its balance sheet; see Christensen et al. (2015) for a detailed discussion along with an empirical assessment for the U.S. Federal Reserve.

2.3 The Nonbank Financial Sector

We assume that there is a continuum of nonbank financial firms (e.g., pension funds, money market mutual funds, asset managers, hedge funds etc.) that are fully financed by a predetermined amount of equity. The representative nonbank financial firm holds some combination of bonds and bank deposits as assets. Bank deposits do not pay interest. This simplifying assumption is without much consequence in the kind of environments with near-zero interest rates that typically prevails when central banks launch and operate QE programs. The assets and liabilities of firm j must satisfy

$$P_L L^j_{NB} + D^j_{NB} = E^j_{NB}, \tag{9}$$

where L^j_{NB} is firm j's holdings of bonds, D^j_{NB} is its holdings of bank deposits, and E^j_{NB} is its initial equity value. Changes in the firm's equity are determined as a residual from the flow identity

$$dE^j_{NB} = dP_L L^j_{NB} + P_L dL^j_{NB} + dD^j_{NB}. \tag{10}$$

Firms cannot issue new debt or equity, which is another assumption justified by the short-term nature of the model. Therefore, firms can only obtain deposits by selling assets

$$dD^j_{NB} = -P_L dL^j_{NB}. \tag{11}$$

Equations (10) and (11) imply that changes in firm j's equity value derive from changes in the price of the long-term bond only

$$dE^j_{NB} = dP_L L^j_{NB}. \tag{12}$$

We make the general assumption that the firm balances its liquid portfolio between deposits and bonds, demanding positive amounts of both. This portfolio balancing arises because deposits provide liquidity without any return, while bonds generate positive returns, but with less liquidity. We do not provide micro-foundations for these assumptions, but instead note that any type of micro-foundations of the portfolio optimization behavior that conform with these minimal assumptions would produce the results we are presenting in this paper.

Thanks to the behavioral assumptions about nonbanks' demand for bonds described in

Section 2.1, it now follows that changes in firm j's demand for bonds are purely driven by changes in the bond price

$$dL^j_{NB} = \frac{\partial f_{NB}(P_L, E^j_{NB})}{\partial P_L} dP_L. \tag{13}$$

Finally, equation (11) shows that firm j's demand for deposits is a function of the change in its bond holdings. Therefore, its deposits will change with the bond price according to

$$dD^j_{NB} = -P_L \frac{\partial f_{NB}(P_L, E^j_{NB})}{\partial P_L} dP_L. \tag{14}$$

2.4 The Banking Sector

There is also a continuum of banks. Bank i's assets and liabilities must satisfy

$$R^i + P_L L^i_B = E^i_B + D^i_B, \tag{15}$$

where L^i_B is bank i's holdings of bonds and R^i is its holdings of central bank reserves. As mentioned earlier, banks' credit portfolios are assumed fixed in the short run and hence, are normalized to zero for simplicity. D^i_B is bank i's deposits from nonbank financial firms. Deposits from nonbanks are endogenously determined, and the bank cannot influence them, given that it does not create new deposits by extending credit or change deposit rates within the time horizon considered. We hence define changes in deposits by equation (14), and to keep things simple, we assume symmetry across banks and that there is an identical number of banks and nonbanks. This implies that $D^i_B = D^j_{NB}$. Finally, E^i_B denotes bank i's initial equity level. Over the horizon considered in the model, a bank cannot issue new equity or debt as this is time consuming and requires board approval, etc. Consequently, it can only actively increase its holdings of reserves by selling bonds. On the other hand, reserves can fluctuate autonomously as the bank's customers vary their deposits with the bank. Importantly, banks cannot take actions to change their deposit holdings and therefore consider them as exogenously given. To summarize, we have the following relationship for the change in bank i's reserve holdings

$$dR^i = dD^i_B - P_L dL^i_B. \tag{16}$$

In general, changes in bank equity are determined as a residual from the flow equivalent of equation (15)

$$dR^i + P_L dL^i_B + L^i_B dP_L = dE^i_B + dD^i_B. \tag{17}$$

From equation (16), which shows the change in bank i's reserves, it follows that

$$dE^i_B = L^i_B dP_L. \tag{18}$$

Thus, variation in bank i's equity is solely due to changes in the bond price.

Banks hold bonds and reserves in their liquid asset portfolios and consider them imperfect substitutes. Without loss of generality, we assume that neither reserves nor deposits pay any interest, but the long bond does because $P_L < 1$. In principle, then, an equal increase in deposits and reserves on a bank's balance sheet has no impact on its profitability in this model. Crucially, however, we assume that banks' business models are such that they respond to autonomous changes in reserves and deposit funding, with a portfolio balancing motive in mind. This assumption can be viewed as reflecting that, given asset prices, individual banks aim for a certain duration in their liquid asset portfolios. We summarize these general assumptions about the bank's liquid safe asset portfolio optimization behavior in the demand equation (4).

The bank's demand for reserves is determined as a residual from the demand for bonds given available funding

$$R_B^i = E_B^i + D_B^i - P_L f_B(P_L, E_B^i + D_B^i). \tag{19}$$

The flow equivalent of bank i's bond demand in equation (4) is given by

$$dL_B^i = \frac{\partial f_B(P_L, E_B^i + D_B^i)}{\partial P_L} dP_L + \frac{\partial f_B(P_L, E_B^i + D_B^i)}{\partial F_B}(dE_B^i + dD_B^i). \tag{20}$$

Since banks cannot respond to changes in equity valuations over the short horizon considered in the model, they are determined as a residual after other changes have taken place, and hence, they are assumed not to affect the bank's demand for bonds. Alternatively, changes in equity valuations can be interpreted as profits that are paid out to shareholders and, therefore, are not available to fund bond purchases. Either way, this implies that we can reduce equation (20) to

$$dL_B^i = \frac{\partial f_B(P_L, E_B^i + D_B^i)}{\partial P_L} dP_L + \frac{\partial f_B(P_L, E_B^i + D_B^i)}{\partial F_B} dD_B^i. \tag{21}$$

2.5 Equilibrium

Under the assumption that there is a continuum of banks identical to each other and another continuum of nonbanks also identical to each other, we can use the above equations to characterize the aggregate banking and nonbanking sectors, respectively, by dropping the i and j superscripts. Since we normalize the continuum of institutions in each category to one, we can further use the individual demand equations as characterizing aggregate sectoral demand. The total offered supply of bonds from the three types of market participants must equal their total demand for bonds, while reserves and deposits are determined as a residual.

The market equilibrium is characterized by the bond price that clears demand for bonds and makes banks' demand for reserves equal the central bank-determined supply given preferences for assets and the total stock of bonds L.

The balance sheets of the banking and nonbanking sectors and, hence, their budget constraints are linked through deposits. We can write the consolidated budget constraint as

$$P_L \left(L - L_{CB}\right) = P_L \left(L_B + L_{NB}\right), \tag{22}$$

where the flow equivalent is

$$dP_L \left(L - L_{CB}\right) + P_L \left(dL - dL_{CB}\right) = dP_L \left(L_B + L_{NB}\right) + P_L \left(dL_B + dL_{NB}\right).$$

3 The Transmission of QE to Bond Prices

In this section, we analyze the effects of central bank bond purchases in exchange for reserves on the balance sheets of banks and nonbanks in order to shed light on the transmission mechanism of such purchases to bond prices. First, we consider the economy with one traded security, as analyzed so far, before we proceed to a brief analysis of the case with two traded securities. We end the section by considering how constraints on banks' balance sheets and leverage may influence the transmission of QE. A key purpose throughout is to illustrate how the effects depend on the preferences of financial market participants.

3.1 The General Solution with One Traded Security

To arrive at the general solution with one traded security, we first derive the partial derivative of the price change of the long-term bond with respect to the central bank bond purchases. To aid intuition on how this expression relates to the asset substitutability of the two types of private-sector agents in the model, we consider two special cases. In the first case, the nonbank institutions are characterized by very low asset substitutability, and all assets are purchased from banks. In the second case, the roles are reversed and banks exhibit very low asset substitutability so that all assets are acquired from nonbanks. The resulting portfolio balance effects on asset prices differ substantially between these two cases.

The specific situation we consider is one in which the central bank increases its reserve liabilities and bond holdings in tandem without changing the total supply of bonds. Thus, the increase in central bank bond holdings must be offset by an identical decline in private-sector holdings

$$dL_{CB} > 0 \quad \text{and} \quad dL = 0 \quad \Rightarrow \quad dL_{CB} = -dL_{NB} - dL_B. \tag{23}$$

Note that these assumptions map in a direct way to the QE programs conducted by major central banks in recent years.

First, we investigate the impact of the change in bond supply and reserves on the price of bonds using the flow equations derived previously. To do so, insert the market aggregate versions of the nonbank bond demand response in equation (13) and the bank bond demand response in equation (21) into equation (23) to obtain

$$dL_{CB} = -\frac{\partial f_{NB}(P_L, E_{NB})}{\partial P_L} dP_L - \frac{\partial f_B(P_L, E_B + D_B)}{\partial P_L} dP_L - \frac{\partial f_B(P_L, E_B + D_B)}{\partial F_B} dD_B. \quad (24)$$

Next, insert the nonbank deposit response in equation (14) to arrive at

$$dL_{CB} = -\frac{\partial f_{NB}(P_L, E_{NB})}{\partial P_L} dP_L - \frac{\partial f_B(P_L, E_B + D_B)}{\partial P_L} dP_L + P_L \frac{\partial f_{NB}(P_L, E_{NB})}{\partial P_L} \frac{\partial f_B(P_L, E_B + D_B)}{\partial F_B} dP_L. \quad (25)$$

Now, the equilibrium bond price response to the central bank bond purchases can be isolated

$$\frac{dP_L}{dL_{CB}} = \frac{-1}{\frac{\partial f_{NB}(P_L,E_{NB})}{\partial P_L} + \frac{\partial f_B(P_L,E_B+D_B)}{\partial P_L} - P_L \frac{\partial f_{NB}(P_L,E_{NB})}{\partial P_L} \frac{\partial f_B(P_L,E_B+D_B)}{\partial F_B}}. \quad (26)$$

Equation (26) shows that the reaction of the equilibrium bond price to the central bank bond purchases depends on the sensitivity of market participants' demand for bonds to changes in the bond price. The first two terms in the denominator capture standard supply-induced portfolio balance effects of the central bank bond purchase on the price of bonds that arise from the reduction in the stock of bonds available to the private sector. The third term, however, captures the reserve-induced portfolio effects. Note that if the asset price sensitivity of nonbanks' demand for long-term bonds is zero, or if banks do not respond to a change in deposit funding by changing their demand for long-term bonds, the reserve-induced portfolio balance channel shuts down.

To support intuition for the two distinct portfolio balance effects in the special cases investigated below, we also derive how the quantity of deposits and, hence, the size of banks' balance sheets react to the central bank bond purchases. To see this, insert equation (26) into the market aggregate version of the nonbank deposit response in equation (14) to obtain

$$\frac{dD_B}{dL_{CB}} = -P_L \frac{\partial f_{NB}(P_L, E_{NB})}{\partial P_L} \frac{dP_L}{dL_{CB}} \quad (27)$$

or, equivalently,

$$\frac{dD_B}{dL_{CB}} = \frac{P_L \frac{\partial f_{NB}(P_L,E_{NB})}{\partial P_L}}{\frac{\partial f_{NB}(P_L,E_{NB})}{\partial P_L} + \frac{\partial f_B(P_L,E_B+D_B)}{\partial P_L} - P_L \frac{\partial f_{NB}(P_L,E_{NB})}{\partial P_L} \frac{\partial f_B(P_L,E_B+D_B)}{\partial F_B}}. \quad (28)$$

Below, we discuss in more depth the nature of reserve- and supply-induced portfolio balance effects based on these expressions.

3.1.1 Corner Solution with Bond Purchases from Banks

To better describe the standard supply-induced portfolio balance effect, we first consider the extreme case where the bond demand of the nonbank financial sector has zero sensitivity to changes in the bond price, that is, $\frac{\partial f_{NB}(P_L, E_{NB})}{\partial P_L} = 0$.

This implies that the nonbank sector holds a fixed amount of the bond that does not vary with changes in either the supply of the bond or its price. In turn, equation (28) shows that $\frac{dD_B}{dL_{CB}} = 0$. This implies that the quantity of bank deposits remains unaffected by the central bank asset purchases as banks simply sell bonds in exchange for reserves. This leaves the size of banks' aggregate balance sheet unchanged, and it leaves nonbank balance sheets unaffected in terms of both size and composition.

In this case, equation (26) shows that the asset purchases would lead to an increase in the bond price equal to

$$\frac{dP_L}{dL_{CB}} = \frac{-1}{\frac{\partial f_B(P_L, E_B + D_B)}{\partial P_L}} > 0. \tag{29}$$

This is a pure supply-induced portfolio balance effect that reflects the price increase necessary to make banks willing to substitute away from bonds and into reserves to meet the central bank bond purchases. Although the ultimate underlying cause for the effect is rooted in banks' aversion to holding more reserves at the expense of bonds, we label it a supply-induced portfolio balance effect because it can equally well be viewed as arising from the reduction in the bond supply generated by the QE bond purchases.

3.1.2 Corner Solution with Bond Purchases from Nonbanks

We now consider the alternative extreme, where banks are the ones with price-insensitive demand for bonds, that is, $\frac{\partial f_B(P_L, E_B + D_B)}{\partial P_L} = 0$. In this case, there is no bond price increase that would make banks substitute away from bonds and toward reserves. Assuming that the price sensitivity of nonbanks' demand for bonds is different from zero, the bond purchases of the central bank would have to be executed with the nonbank financial sector on the selling side. Importantly, these bond sales would result in an autonomous creation of bank deposits matched by an increase in central bank reserves on banks' balance sheets, since the nonbank financial sector cannot hold reserves. At first, the increase in deposit funding for the banks equals the total amount of bonds purchased by the central bank, i.e., $dD_B = P_L dL_{CB}$. In response to this increase in deposit funding, banks reallocate some of their new reserves toward bonds, as assumed in equation (6). In turn, this puts additional upward pressure on bond prices and gives rise to further bond sales by nonbanks to banks. This will further expand banks' balance sheets with bonds on the asset side and more deposits on the liability side. Thus, the total change in bank deposits in the new equilibrium is

$$\frac{dD_B}{dL_{CB}} = \frac{P_L}{1 - P_L \frac{\partial f_B(P_L, E_B + D_B)}{\partial F_B}} > P_L, \tag{30}$$

while the associated equilibrium bond price increase is given by

$$\frac{dP_L}{dL_{CB}} = \frac{-1}{\frac{\partial f_{NB}(P_L,E_{NB})}{\partial P_L} - P_L \frac{\partial f_{NB}(P_L,E_{NB})}{\partial P_L} \frac{\partial f_B(P_L,E_B+D_B)}{\partial F_B}} > 0, \quad (31)$$

where both inequalities follow from $0 < P_L < 1$ and $0 < \frac{\partial f_B(P_L,E_B+D_B)}{\partial F_B} < 1$.

Equation (31) shows that the effect on bond prices from central bank bond purchases with the nonbank financial sector as the counterparty comes from two sources that reinforce each other. The first is the supply-induced portfolio effect that equals the price increase needed to make the nonbank financial sector willing to give up bonds and hold deposits instead. This is captured by the first term in the denominator of equation (31). The other is the reserve-induced portfolio effect that results from the financial friction that only banks can hold reserves. Since banks now have more deposit funding, they will want to reallocate some of it towards bonds according to their preferences, as reflected in $\frac{\partial f_B(P_L,E_B+D_B)}{\partial F_B}$. However, this requires the nonbank sector to be willing to sell additional bonds, which gives rise to the additional weight $P_L \frac{\partial f_{NB}(P_L,E_{NB})}{\partial P_L}$. Importantly, because the two terms in the denominator of equation (31) have opposite signs, it follows that the reserve- and supply-induced portfolio effects reinforce each other and make the full effect greater than either in isolation.

From the two corner solutions it is clear that the initial price impact of QE asset purchases will tend to be large whenever $\frac{\partial f_{NB}(P_L,E_{NB})}{\partial P_L}$ and $\frac{\partial f_B(P_L,E_B+D_B)}{\partial P_L}$ are small, i.e., whenever bond demand is price inelastic and investor behavior could be characterized by preferred habitat. On the other hand, when bond demand is very price sensitive and the derivatives above are large for that reason, the price impact will tend to be modest. Accordingly, it will require large amounts of QE bond purchases to have a notable price impact under those circumstances.

3.2 The Case of Two Traded Securities

To further demonstrate that reserve- and supply-induced portfolio balance effects are indeed distinct and that reserve-induced effects are independent of the types of assets purchased, we develop a more general version of the model with two traded securities, a short bond S and a long bond L. The full model is described in the appendix and briefly summarized in the following.

Consider a situation in which the central bank is implementing a QE program through purchases of short bonds only, i.e., $dS_{CB} > 0$ and $dL_{CB} = 0$. This was the type of central bank asset purchases enacted by the Swiss National Bank in August 2011 and studied by CK.

Using notation similar to that introduced in Section 2, banks' demand for short and long bonds is given by

$$S_B = f_B^S(P_S, P_L, E_B + D_B) \quad \text{and} \quad L_B = f_B^L(P_S, P_L, E_B + D_B),$$

while the corresponding demand functions of the nonbank institutions are

$$S_{NB} = f^S_{NB}(P_S, P_L, E_{NB}) \quad \text{and} \quad L_{NB} = f^L_{NB}(P_S, P_L, E_{NB}).$$

Calculations provided in the appendix show that the equilibrium response of the long bond price to short bond purchases by the central bank is, in general, given by

$$\frac{dP_L}{dS_{CB}} = -\frac{\frac{\partial f^L_{NB}}{\partial P_S} + \frac{\partial f^L_B}{\partial P_S} - \frac{\partial f^L_B}{\partial F_B}(P_S \frac{\partial f^S_{NB}}{\partial P_S} + P_L \frac{\partial f^L_{NB}}{\partial P_S})}{\Delta}, \tag{32}$$

where

$$\begin{aligned}\Delta \;=\; & \Big(\frac{\partial f^L_{NB}}{\partial P_S} + \frac{\partial f^L_B}{\partial P_S} - \frac{\partial f^L_B}{\partial F_B}(P_S \frac{\partial f^S_{NB}}{\partial P_S} + P_L \frac{\partial f^L_{NB}}{\partial P_S})\Big)\Big(\frac{\partial f^S_{NB}}{\partial P_L} + \frac{\partial f^S_B}{\partial P_L}\Big) \\ - \; & \Big(\frac{\partial f^L_{NB}}{\partial P_L} + \frac{\partial f^L_B}{\partial P_L} - \frac{\partial f^L_B}{\partial F_B}(P_S \frac{\partial f^S_{NB}}{\partial P_L} + P_L \frac{\partial f^L_{NB}}{\partial P_L})\Big)\Big(\frac{\partial f^S_{NB}}{\partial P_S} + \frac{\partial f^S_B}{\partial P_S}\Big).\end{aligned}$$

This is a complex expression, and we impose a number of simplifying assumptions to make our point clear. First, we assume that the demand for short bonds by nonbank financial institutions is characterized by perfect price elasticity, i.e., $\frac{\partial f^S_{NB}}{\partial P_S} \to \infty$. Also, we assume that banks do not vary their demand for short bonds in response to changes in the short bond price, i.e., $\frac{\partial f^S_B}{\partial P_S} = 0$. Under these assumptions, the central bank purchases will be performed exclusively with nonbank entities as counterparties, which allows us to demonstrate more clearly the reserve-induced portfolio balance effect. We stress, however, that these assumptions are only sufficient, not necessary, to ensure the existence of reserve-induced effects. All we need is for the price sensitivity of the demand of nonbanks for short bonds to be strictly negative. Combined the two assumptions reduce equation (32) to

$$\frac{dP_L}{dS_{CB}} = \frac{\frac{\partial f^L_B}{\partial F_B} P_S}{-\frac{\partial f^L_B}{\partial F_B} P_S(\frac{\partial f^S_{NB}}{\partial P_L} + \frac{\partial f^S_B}{\partial P_L}) - \Big(\frac{\partial f^L_{NB}}{\partial P_L} + \frac{\partial f^L_B}{\partial P_L} - \frac{\partial f^L_B}{\partial F_B}(P_S \frac{\partial f^S_{NB}}{\partial P_L} + P_L \frac{\partial f^L_{NB}}{\partial P_L})\Big)}. \tag{33}$$

Second, we assume that all cross-price elasticities are zero, say, $\frac{\partial f^S_B}{\partial P_L} = 0$, i.e., we are in the extreme case of no substitution between the two assets. This reduces equation (33) to

$$\frac{dP_L}{dS_{CB}} = \frac{-\frac{\partial f^L_B}{\partial F_B} P_S}{\frac{\partial f^L_{NB}}{\partial P_L} + \frac{\partial f^L_B}{\partial P_L} - P_L \frac{\partial f^L_{NB}}{\partial P_L}\frac{\partial f^L_B}{\partial F_B}} > 0. \tag{34}$$

Since we assume standard preferences for banks and nonbanks, $\frac{\partial f^L_{NB}}{\partial P_L} < 0$ and $\frac{\partial f^L_B}{\partial P_L} < 0$, and we continue to assume that banks respond to changes in their funding conditions by rebalancing their portfolios, i.e., $\frac{\partial f^L_B}{\partial F_B} \in (0, 1)$, it follows that the long bond price response to QE short bond purchases in equation (34) is positive.

This shows that the price of long bonds can be positively affected when a central bank engages in QE by buying short-term bonds, as was the case in Switzerland in August 2011. This underscores that, in a general setting with multiple securities, reserve- and supply-induced portfolio balance effects are two separate transmission channels for QE asset purchases to affect long-term interest rates. Specifically, when a central bank implements a QE program by purchasing short-term bonds, there is no supply-induced portfolio effect on long bond prices. However, reserve-induced portfolio effects can continue to exist, provided the QE transactions are performed with nonbank financial institutions and run through banks' portfolio responses to the created reserves.

This effect does not come from cross-price demand elasticities as we have fixed those at zero. Instead, the effect comes from banks' increased demand for long bonds in response to the associated expansion of their balance sheets. Fixing the cross-price elasticities to zero is not necessary to obtain this result. It remains valid as long as the cross-price elasticities are smaller in absolute value than the direct own-price elasticities.

Finally, it remains the case that reserve- and supply-induced portfolio effects reinforce each other. When banks purchase long bonds from nonbanks, they pay with deposits and hence expand their balance sheets beyond what the QE purchases by themselves would imply.

3.3 The Role of Bank Balance Sheet Capacity and Regulation

The model illustrates the importance of banks' portfolio responses to deposit funding inflows in the transmission of QE to bond prices. In the real world, these responses are likely to be driven by a multitude of factors, including preferences regarding risks and returns, the presence of distortions and frictions such as limited liability, moral hazard, and leverage, and, in turn, banks' balance sheet capacity.[12] Binding regulatory bank capital and liquidity constraints are also likely to matter, and since bank regulation has changed substantially since the global financial crisis, the transmission of QE may have changed too. We have assumed general forms of banks' demand elasticities to prices and funding, allowing our model to capture different types of bank behavior. While including a full-fledged treatment of micro-founded frictions and distortions to financial intermediation is beyond the scope of this paper, we briefly illustrate how the model could be extended to discuss the role of leverage and balance sheet capacity in banks' demand for long-term bonds and their effects for the transmission of QE.

First, note that the general model presented in Section 2 is intended to describe banks' and nonbanks' holdings of safe and liquid bonds, such as government bonds, which typically have zero risk weights for regulatory purposes. A risk-weighted leverage constraint that exempts reserves and bonds will therefore not affect the outcome of the model, as substitution would take place between assets with zero risk weights.[13] The model should hence describe well the

[12] See Holmstrom and Tirole (1997) for an example. For a recent survey of the role of financial frictions in macroeconomic outcomes, see Brunnermeier et al. (2012).

[13] Note that our model is not suitable for analyzing an unweighted leverage ratio that applies to the entire

effects of QE programs carried out in assets with zero risk weights, and both reserve- and supply-induced effects should be present for central bank purchases of these types of assets.

Consider instead a QE program carried out in assets that are risky and carry positive risk weights for regulatory purposes. Suppose the risk weight on the asset is λ, while reserves have a risk weight of zero. The bank is subject to a risk-weighted leverage ratio requiring that equity to risk-weighted assets may not fall below a certain level τ:

$$\frac{E_B^i}{\lambda P_L L_B^i} \geq \tau, \tag{35}$$

where E_B^i is given by equation (15) and the change in bank equity is given by equation (18).

Without detailing the micro-foundation underlying such a leverage constraint, which among other things would require the introduction of risk, we follow Krugman (2008) and assume that frictions leading to moral hazard in bank funding and risk taking (e.g., agency problems, asymmetric information, and limited liability) imply that the bank is always operating directly on its leverage constraint. In this case, the demand for long-term bonds by the bank is directly determined by the leverage constraint imposed with equality. The leverage constraint in turn becomes the bank's demand function for a given amount of equity. This then provides us with the sensitivity of the bank's demand for long-term risky assets in response to changes in the long-term bond price ($\frac{\partial f_B^i}{\partial P_L}$) and to changes in deposit funding ($\frac{\partial f_B^i}{\partial F^i}$).

To see this, totally differentiate equation (35) (with equality imposed) and make use of equation (17) to get

$$dR^i + (P_L dL_B^i + L_B^i dP_L)(1 - \lambda\tau) = dD^i. \tag{36}$$

Furthermore, from equation (16), which shows the changes in bank i's reserves as a function of deposit inflows and long-term bond purchases, it follows that both reserves and deposits cancel out. This leaves changes in long-term bond prices as the only driver of changes in the bank's demand for long-term bonds:

$$\frac{\partial f_B^i}{\partial P_L} \equiv \frac{\partial L_B^i}{\partial P_L} = \frac{L_B^i}{P_L}\frac{1-\lambda\tau}{\lambda\tau} > 0. \tag{37}$$

Regarding the bank's bond demand function in equation (4), this result implies that its demand is independent of its funding level:

$$\frac{\partial f_B^i}{\partial F^i} = 0. \tag{38}$$

The finding in equation (37) that $\frac{\partial f_B^i}{\partial P_L} > 0$, that is, banks increase their demand as bond

balance sheet of the bank because a binding ratio would imply that the bank cannot accept new deposits on behalf of its customers, that is, it would no longer function as a bank. This is the downside to the simplicity of the model needed to make it tractable.

prices go up, is the standard financial accelerator result presented in Bernanke et al. (1999). An increase in bond prices, rather than reducing demand for assets, leads to an increase in banks' balance sheet capacity and hence an increase in banks' bond demand. This can drive bond prices up further.

Inserting equation (37) into the solution for the bond price change as a response to central bank bond purchases in equation (26), we obtain:

$$\frac{dP_L}{dL_{CB}} = \frac{-1}{\frac{\partial f_{NB}(P_L,E_{NB})}{\partial P_L} + \frac{L_B^i}{P_L}\frac{1-\lambda\tau}{\lambda\tau}}, \tag{39}$$

where we assume $\frac{\partial f_{NB}(P_L,E_{NB})}{\partial P_L} + \frac{L_B^i}{P_L}\frac{1-\lambda\tau}{\lambda\tau} < 0$.[14] When the central bank implements a QE program through purchases of risky assets and banks are operating on the limit of a risk-weighted leverage constraint, this result shows that a financial accelerator effect arising from limits on banks' balance sheet capacity works to amplify the initial supply-induced portfolio balance effect. In contrast, the reserve-induced portfolio balance effect shuts down because increases in deposits will have to be fully allocated to risk-free reserves to maintain the risk-weighted leverage ratio for a given level of risky asset prices.

These calculations show that the response of the price of the risky asset to central bank risky asset purchases now depends on the risk-weighted leverage ratio. The higher the regulatory leverage constraint is, i.e., the greater τ is, the smaller is the financial accelerator effect on asset prices arising from banks' constrained optimization of their balance sheets, and the smaller is the ultimate portfolio balance effect of asset purchases on the yields of risky assets.

While this is a highly stylized and simplified analysis of the role of balance sheet capacity and risk-weighted leverage constraints, the findings suggest that the role of financial frictions and regulation on bank behavior may be an important driver of the transmission of QE. This represents an as of yet unexplored but promising field of research for better understanding the transmission of QE.

4 Empirical Evidence of Reserve-Induced Portfolio Effects

In this section, we first discuss the circumstances under which reserve-induced portfolio balance effects can be empirically identified before we briefly assess to what extent the key condition necessary for their existence, namely that the central bank asset purchases are executed with nonbank financial institutions, appears to have been satisfied in the case of U.S. QE programs.

[14] In the case where $\frac{\partial f_{NB}(P_L,E_{NB})}{\partial P_L} + \frac{L_B^i}{P_L}\frac{1-\lambda\tau}{\lambda\tau} > 0$, the financial accelerator effect would be so large as to outdo the decline in demand from nonbanks as the bond price increases. As a result, the market demand curve would become upward sloping, which leads to implausible equilibria that we have not explored further.

4.1 Identification of Reserve-Induced Portfolio Effects

The portfolio model analyzed in the previous sections predicts that, when a central bank buys long-term bonds, both reserve- and supply-induced portfolio effects can affect bond risk premiums. The QE programs of the U.S. Federal Reserve, the Bank of England, the Bank of Japan, and the European Central Bank all included purchases of long-term securities, and there is broad consensus in the empirical literature that these programs indeed produced portfolio balance effects that lowered long-term bond risk premiums; see Ball et al. (2016) for an overview. However, because reserve- and supply-induced effects materialize simultaneously, the many event studies of the financial market reactions to the announcements of these programs cannot separately identify them and therefore merely confirm their joint occurrence, but are silent about their relative importance. To test the existence of reserve-induced portfolio balance effects empirically, our analysis points to two potential avenues.

The first approach is centered around the time of the launch of QE programs with the focus limited to QE programs that target only short-term assets because, by definition, such programs cannot produce any supply-induced portfolio effects on long-term bond prices. By analyzing the financial market reaction and the impact on long-term interest rates in response to the launch of such programs—while controlling for other confounding factors, including other QE transmission channels—it could be possible to detect and isolate reserve-induced portfolio balance effects. This is the strategy pursued by CK, who present evidence of their existence and effects on long-term bond prices around three announcements of central bank reserve expansions in Switzerland in August 2011. These expansions were exceptionally large and achieved without acquiring any long-term securities, which rules out supply-induced portfolio effects as a factor behind the documented market reaction.

The other potential window for observing reserve-induced portfolio effects arises during the exit from QE when the central bank balance sheet is being normalized. If this is achieved by merely letting assets mature, we have the reverse of the first approach, where the outstanding amount of reserves are reduced by essentially selling overnight claims back to market. As the supply of long-term assets in the economy is unchanged by definition, no supply-induced portfolio effects can materialize. However, assuming that the maturing assets are government securities, which is the empirically relevant case for all major central banks that have engaged in QE, the impact will depend on how the government obtains the revenue to pay back the principal to the central bank. If the government obtains the revenue by taxing agents in the economy, it will cause the banking sector as a whole to see a reduction in the amount of reserves matched by a reduction in the amount of deposits held by the agents paying the extra taxes. All else equal, this will lead to an increase in the duration of banks' portfolios, which creates the potential for a direct negative reserve-induced portfolio effect on long-term interest rates as banks reduce their demand for long-term bonds to offset the increased risk of their portfolios. In the more likely event that the government obtains the funds by issuing new debt, the effect would depend on the maturity of the debt issued. If the government relies on short-term bills for this financing need, it is again the case that the supply of long-term assets is left unchanged and there cannot be any supply-induced portfolio effects on long-term interest rates, only reserve-induced portfolio effects can materialize.

These considerations suggest that a careful study of the time series pattern of maturing assets held by the central bank in conjunction with an equally careful examination of new government debt issuance could offer evidence of reserve-induced portfolio effects on long-term interest rates.[15] In the case of the U.S. Federal Reserve, the amount and maturity composition of its Treasury securities holdings are publicly available information.[16] Hence, this type of analysis will be feasible when it comes time for the Fed to reduce its balance sheet. Furthermore, Lou et al. (2013) show that anticipated and repeated shocks from auctions of new securities in the U.S. Treasury market have temporary negative effects on Treasury prices. Thus, the key empirical question is whether the Treasury bond price reactions to Treasury auctions will be more negative than usual during the exit.

4.2 Empirical Support for Reserve-Induced Portfolio Effects

Rather than look for outright identification of reserve-induced portfolio effects, it is possible to verify whether the necessary conditions for their existence were empirically satisfied for key QE programs. When central bank asset purchases are conducted with a range of financial intermediaries that includes an important share of nonbank entities, commercial bank balance sheets mechanically expand, and the possibility of reserve-induced portfolio balance effects arises. The mix of counterparties to central bank asset purchases is likely to differ across countries and over time, and a fully fledged cross-country econometric analysis of counterparties and bank balance sheets is beyond the scope of the current paper. Instead, as a case study, we take a closer look at the literature and available data for the QE programs conducted by the U.S. Federal Reserve in the aftermath of the global financial crisis. The evidence suggests that the Fed's two most recent QE programs were conducted mainly with nonbanks, that bank balance sheets indeed increased in connection with those asset purchases, and that banks responded to these changes in their portfolio compositions. There is hence ample scope for reserve-induced portfolio balance effects to have played a role in the transmission of these QE programs to the prices of long-dated securities.

The question about the counterparties to the Federal Reserve's QE programs is addressed in Carpenter et al. (2015a). They analyze data on U.S. financial flows of funds and find that the Fed's purchases are mainly associated with reductions in the holdings of the targeted types of assets by nonbank entities, predominantly households, hedge funds, broker-dealers,

[15]See Greenwood and Vayanos (2014) for analysis of the maturity profile of U.S. Treasury securities.

[16]See the link: https://www.newyorkfed.org/markets/soma/sysopen_accholdings.html. See also Carpenter et al. (2015b) for analysis and projections of the Federal Reserve's balance sheet.

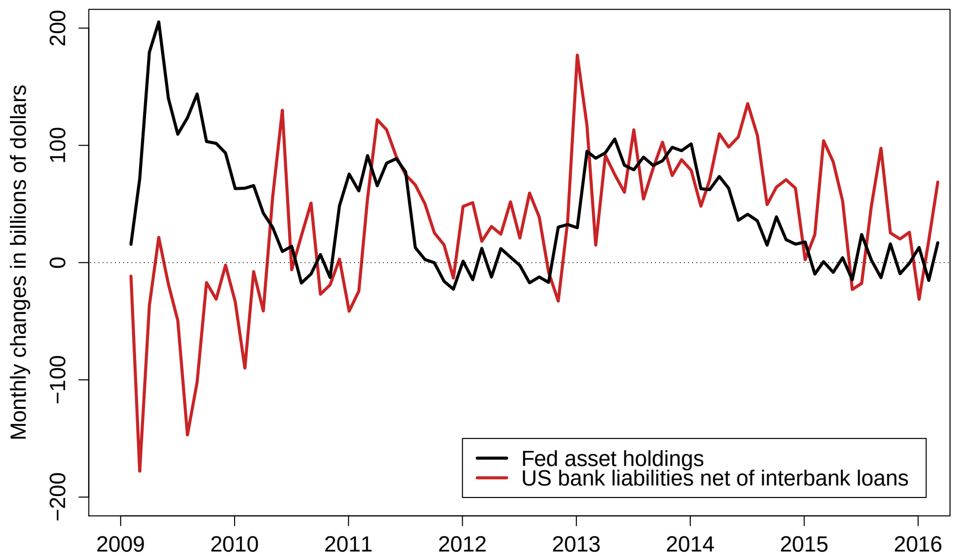

Figure 1: **Fed Asset Purchases and U.S. Bank Liabilities.**
Monthly Federal Reserve asset purchases are approximated using changes in the value of the Federal Reserves' holdings of Treasury and agency securities over the month measured in billions of dollars. Changes in U.S. bank liabilities are measured as the two-month moving average change in total bank liabilities less borrowing from U.S. banks, also measured in billions of dollars. Sources: Federal Reserve, SOMA accounts, and FRB.H8.

and insurance companies.[17]

Ennis and Wolman (2015) study the reaction of individual U.S. commercial bank balance sheets to the increase in reserves during the Federal Reserve's first and second QE programs. Their data suggest that banks played a central role as counterparties during the first QE program (QE1) in 2009, which would limit the role of reserve-induced effects at that time. Thus, banks' securities portfolios fell significantly in response to the increase in reserves during this episode, while there are few signs of changes in bank liabilities. This was a period of financial market stress that affected banks in particular, and QE1 may have helped banks deleverage in an orderly way. In contrast, during the second QE program (QE2) that operated from November 2010 to June 2011, commercial bank deposits were positively associated with increases in reserves, while commercial bank holdings of securities did not respond. Thus, bank balance sheets expanded. The transmission of QE2 is hence likely to have worked very differently from the transmission of QE1, and reserve-induced effects could have been central.

Unfortunately, the analysis of Ennis and Wolman (2015) does not extend to the Federal

[17] In the case of the U.K., Joyce et al. (2011) describe how the Bank of England's asset purchase programs were initially conducted in assets held by nonbank financial institutions with the stated intention of boosting broader monetary aggregates.

Reserve's third QE program (QE3) that operated from September 2012 to October 2014. To gain further insight, Figure 1 plots the monthly changes in the Federal Reserve's holdings of Treasury securities and mortgage-backed securities as a proxy for monthly central bank asset purchases and the (slightly smoothed) monthly changes in bank balance sheets net of interbank positions from January 2009 to February 2016. The figure confirms the findings of Ennis and Wolman (2015) for QE1 and QE2. The two series move in opposite directions in 2009 but comove strongly following the launch of QE2 in late 2010. The third wave of asset purchases during QE3 is, moreover, characterized by consistent increases in banks' balance sheets very similar to the size of the asset purchases. This association is highly statistically significant. Thus, this evidence is consistent with the findings reported in Carpenter et al. (2015a) and suggests that the conditions necessary for reserve-induced portfolio balance effects to exist were likely met for the Fed's QE2 and QE3 programs.

Finally, Kandrac and Schlusche (2016) investigate banks' reactions to increases in their reserves holdings during all three QE programs, and find that banks increased risk taking and expanded their loan portfolios. Thus, their analysis suggests that banks indeed have a portfolio response to increases in their reserves holdings, which is the other key assumption necessary in our theoretical portfolio model for reserve-induced portfolio balance effects to exist.

In summary, there is broad consensus in the literature that QE has portfolio balance effects on long-term interest rates, but the literature has focused almost entirely on QE programs that do not allow for separate identification of reserve- and supply-induced portfolio effects. Analysis of a program that does allow for such independent identification, namely the exceptional reserve expansions carried out by the Swiss National Bank in August 2011, lends strong empirical support to the existence of reserve-induced effects as documented in CK. Moreover, the conditions necessary for reserve-induced portfolio balance effects to have played a role in the financial market response to the Fed's two most recent QE programs appears to have been satisfied. The data thus broadly suggest that reserve-induced portfolio effects are likely to be a relevant transmission channel of QE to long-term interest rates that could be particularly important during the exit process when central bank balance sheets are normalized. However, more research on the strength and nature of this role is clearly called for.

5 Conclusion

In this paper, we augment a standard portfolio model to include a central bank that issues reserves in exchange for bonds and a banking sector that holds reserves and bonds financed with deposits and equity. The model contains two key frictions. First, reserves can only be held by banks. Second, central bank reserves, bank deposits, and traded securities are imperfect substitutes for each other. We use this model to study how central bank asset purchases affect the behaviors of banks and nonbank financial institutions and their implications for equilibrium asset prices.

We find that, provided a sufficient share of the central bank asset purchases are performed with nonbanks, they can give rise to two separate portfolio effects on bond prices that reinforce each other. One is due to the reduction in the available supply of bonds—a supply-induced portfolio effect. The other arises from the expansion of reserves that only banks can hold. This friction expands banks' balance sheets, dilutes the duration of their portfolios, and makes them increase their demand for bonds—a reserve-induced portfolio balance effect. In contrast, when preferences of banks and nonbanks are such that the central bank asset purchases are mainly executed with banks, only supply-induced portfolio effects materialize because there is no expansion of banks' balance sheets.

More generally, the model suggests that financial market structure, the business models of financial market intermediaries, their portfolio optimizing tools, and bank regulations may affect the transmission of QE to long-term interest rates, as these factors affect the substitutability between short- and long-term assets in the portfolios of banks and nonbank financial institutions. Regulation of financial institutions has changed dramatically since the onset of the global financial crisis and the implementation of the first QE programs. Regulatory changes may have affected the transmission of QE across time. A promising avenue for future research would be to investigate in more depth the connection between changes in bank regulation and bank balance sheet capacity and the strength of the transmission of QE to asset prices.

The analysis suggests two potential avenues to address the empirical challenge of distinguishing between reserve- and supply-induced portfolio balance effects and hence separately identify reserve-induced portfolio effects. One focuses on QE programs that entail substantial increases in the amount of central bank reserves achieved without acquiring any long-lived securities. By design, such programs would not affect the market supply of long-term securities and hence not give rise to any supply-induced portfolio balance effects on long-term interest rates. The other opportunity for detecting reserve-induced portfolio effects arises during the exit from QE when the balance sheet of the central bank is being normalized. If this is achieved by letting assets mature, and assuming that these are government securities that are rolled over into new short-term government debt (bills), we have a situation with no change in the supply of long-term assets in the economy coinciding with a reduction in banks' balance sheets. In principle, this could provide a chance to quantify reserve-induced portfolio balance effects on long-term interest rates. We leave it for future research to explore these possibilities.

The model also gives a theoretical characterization of the circumstances under which reserve-induced portfolio effects can exist. A review of recent research and data offers strong support for portfolio balance effects in general and for the existence of reserve-induced portfolio effects in particular, in the rare cases where they can be independently identified. In addition, the necessary conditions for reserve-induced effects to have been operating during the Fed's two most recent QE programs appear to have been met.

Finally, the existence of a reserve-induced portfolio balance channel may have implications for monetary policy itself. It could matter for the design of future QE programs, for the exit from such programs, and for central bank communication. Also, at the operational level, QE programs may be effective at lowering long-term bond interest rates even in the absence of long-term bond purchases. This could matter for central banks that have to operate in markets with a limited supply of long-term securities of sufficiently high credit quality. Hence, our analysis shows that more research is needed to better understand the relative importance of reserve- and supply-induced portfolio balance effects and their role for the transmission of QE.

Appendix: A Portfolio Model with Two Traded Securities

In this appendix, we present an extension of the baseline model considered in the main text with two traded securities in addition to central bank reserves and bank deposits. To keep the exposition simple, we go straight to the aggregate market equations and ignore the i and j superscripts.

The Central Bank

The central bank balance sheet is now given by

$$P_S S_{CB} + P_L L_{CB} = E_{CB} + R, \tag{40}$$

where S_{CB} is the central bank's holdings of short-term bonds. The change in the equity value of the central bank is given by

$$dE_{CB} = dP_S S_{CB} + P_S dS_{CB} + dP_L L_{CB} + P_L dL_{CB} - dR. \tag{41}$$

We now assume that S_{CB} is the central bank's policy tool and that it determines R, keeping its long-term bond holdings constant, i.e., $dL_{CB} = 0$.

The Nonbank Financial Sector

The aggregate balance sheet of the nonbank financial sector is characterized by

$$P_S S_{NB} + P_L L_{NB} + D_{NB} = E_{NB}, \tag{42}$$

where S_{NB} represents the short-term bond holdings of the nonbank financial sector.

Changes in its equity position are determined as a residual from the flow identity

$$dE_{NB} = dP_S S_{NB} + P_S dS_{NB} + dP_L L_{NB} + P_L dL_{NB} + dD_{NB}. \tag{43}$$

When it sells assets, the nonbank financial sector obtains deposits

$$P_S dS_{NB} + P_L dL_{NB} = -dD_{NB}. \tag{44}$$

By implication, changes in its equity value derive from changes in the prices of its bond holdings

$$dE_{NB} = dP_S S_{NB} + dP_L L_{NB}. \tag{45}$$

The demand for bonds by the nonbank financial sector is a function of the bond prices and equity

$$S_{NB} = f^S_{NB}(P_S, P_L, E_{NB}), \tag{46}$$

$$L_{NB} = f^L_{NB}(P_S, P_L, E_{NB}), \tag{47}$$

while the amount of demand deposits is determined as a residual from the budget constraint and is given in equation (44).

We assume standard preferences with negative own-price effects

$$\frac{\partial f^S_{NB}}{\partial P_S} < 0 \quad \text{and} \quad \frac{\partial f^L_{NB}}{\partial P_L} < 0 \tag{48}$$

and positive cross-price effects

$$\frac{\partial f^S_{NB}}{\partial P_L} > 0 \quad \text{and} \quad \frac{\partial f^L_{NB}}{\partial P_S} > 0, \tag{49}$$

so that short and long bonds are imperfect substitutes.

Furthermore, even though the equity of the nonbank financial sector is determined as a residual by the change in the bond prices as stated in equation (45), we assume that it will not respond to such equity value changes in real time by changing its demand for bonds, i.e.,

$$\frac{\partial f_{NB}^S}{\partial E_{NB}} = \frac{\partial f_{NB}^L}{\partial E_{NB}} = 0. \tag{50}$$

Combining this with equations (46) and (47), we obtain the bond demand response of the nonbank financial sector to changes in the bond prices

$$dS_{NB} = \frac{\partial f_{NB}^S}{\partial P_S} dP_S + \frac{\partial f_{NB}^S}{\partial P_L} dP_L, \tag{51}$$

$$dL_{NB} = \frac{\partial f_{NB}^L}{\partial P_S} dP_S + \frac{\partial f_{NB}^L}{\partial P_L} dP_L. \tag{52}$$

From equation (44), it then follows that the change in deposit holdings as a consequence of bond price changes is given by

$$dD_{NB} = -P_S\Big(\frac{\partial f_{NB}^S}{\partial P_S} dP_S + \frac{\partial f_{NB}^S}{\partial P_L} dP_L\Big) - P_L\Big(\frac{\partial f_{NB}^L}{\partial P_S} dP_S + \frac{\partial f_{NB}^L}{\partial P_L} dP_L\Big). \tag{53}$$

The Banking Sector

There is a continuum of banks in the economy, and their aggregate balance sheet is characterized by

$$R + P_S S_B + P_L L_B = E_B + D_B, \tag{54}$$

where S_B represents banks' holdings of short bonds, L_B is their holdings of long-term bonds, and R is their reserves, while D_B and E_B represent their deposits and equity, respectively. As in the main text, deposits are endogenously determined by the transactions of the nonbank financial sector.

Note also that we continue to assume that, over the horizon considered in the model, a bank cannot issue new equity or debt as this is time consuming and requires board approval etc. Consequently, it can only actively increase its holdings of reserves by selling bonds. On the other hand, reserves can fluctuate autonomously as the bank's customers vary their deposits with the bank. Importantly, banks cannot take actions to change their deposit holdings. Banks consider them as exogenously given. To summarize, we have the following relationship for the change in banks' reserve holdings

$$dR = dD_B - P_S dS_B - P_L dL_B. \tag{55}$$

Changes in bank equity are determined as a residual from the flow equation (54)

$$dE_B = P_S dS_B + P_S dS_B + L_B dP_L + dR - dD_B.$$

Banks hold long and short bonds and reserves in their liquid asset portfolios. While banks consider short and long bonds to be imperfect substitutes, we assume that they see short bonds and reserves as near-perfect substitutes when the yield of short bonds is near zero, i.e., near the zero lower bound.

Formally, banks' demand for short and long bonds is given by

$$S_B = f_B^S(P_S, P_L, E_B + D_B), \tag{56}$$

$$L_B = f_B^L(P_S, P_L, E_B + D_B). \tag{57}$$

Again, we assume banks have standard preferences for both types of bonds with negative own-price effects

$$\frac{\partial f_B^S}{\partial P_S} < 0 \quad \text{and} \quad \frac{\partial f_B^L}{\partial P_L} < 0 \tag{58}$$

and positive cross-price effects

$$\frac{\partial f_B^S}{\partial P_L} > 0 \quad \text{and} \quad \frac{\partial f_B^L}{\partial P_S} > 0, \tag{59}$$

so that short and long bonds are imperfect substitutes, as intended.

Finally, the demand for reserves is determined as a residual from the demand for bonds given the available initial equity funding and deposits.

As in the main text, it is key for the existence of the reserve-induced portfolio balance channel that changes in available funding lead to increased demand for short and long bonds

$$0 < \frac{\partial f_B^S}{\partial F_B} < 1 \quad \text{and} \quad 0 < \frac{\partial f_B^L}{\partial F_B} < 1. \tag{60}$$

These bond demand sensitivities to changes in deposit funding are crucial for our results. Specifically, demand for long bonds is positive, as banks that receive increased deposit funding seek to convert some of the additional liquidity into assets with positive duration.

Since banks cannot respond to changes in equity valuations over the short horizon considered in the model, changes in equity valuations are determined as a residual after other changes have taken place, and hence, they are assumed not to affect the bank's demand for bonds. Alternatively, changes in equity valuations can be interpreted as profits that are paid out to shareholders and are therefore not available to fund bond purchases. Either way, this implies that

$$\frac{\partial f_B^S(P_S, P_L, E_B + D_B)}{\partial F_B} dE_B = 0 \quad \text{and} \quad \frac{\partial f_B^L(P_S, P_L, E_B + D_B)}{\partial F_B} dE_B = 0. \tag{61}$$

This reduces the flow equivalents of banks' demand for short and long bonds in equations (56) and (57) to

$$dS_B = \frac{\partial f_B^S}{\partial P_S} dP_S + \frac{\partial f_B^S}{\partial P_L} dP_L + \frac{\partial f_B^S}{\partial F_B} dD_B, \tag{62}$$

$$dL_B = \frac{\partial f_B^L}{\partial P_S} dP_S + \frac{\partial f_B^L}{\partial P_L} dP_L + \frac{\partial f_B^L}{\partial F_B} dD_B. \tag{63}$$

The Market Equilibrium

The equilibrium in the model is characterized by a set of bond prices and a set of bond allocations across the private sector agents that ensure that the markets for short and long bonds are in equilibrium. Compared to the model with one traded security considered in the main text, we now have interaction terms due to the substitutability between short and long bonds and between bonds and reserves and deposits. This substantially complicates the derivation of the market equilibrium. Therefore, we first present the equilibrium expressions. Second, we make several assumptions we think are appropriate in order to capture the zero lower bound environment in which QE is usually performed. We then evaluate the model implications under those assumptions.

To begin, equilibrium in bond markets requires that

$$dS_{NB} + dS_B + dS_{CB} = 0, \tag{64}$$

$$dL_{NB} + dL_B + dL_{CB} = 0. \tag{65}$$

In the following, to highlight the existence of the reserve-induced portfolio balance channel, we consider the case when the central bank implements a QE program by buying short bonds only, i.e., $dS_{CB} > 0$ and $dL_{CB} = 0$. Inserting the flow demand equations of banks and nonbanks, including demand due to changes in nonbanks' demand for deposits, yields the two market clearing conditions.

First, use equations (52), (63), and (53) to determine how the demand for long bonds responds to changes in the relative asset prices

$$
\begin{aligned}
dL_{NB} + dL_B &= \frac{\partial f^L_{NB}}{\partial P_S} dP_S + \frac{\partial f^L_{NB}}{\partial P_L} dP_L + \frac{\partial f^L_B}{\partial P_S} dP_S + \frac{\partial f^L_B}{\partial P_L} dP_L + \frac{\partial f^L_B}{\partial F_B} dD_B && (66) \\
&= \frac{\partial f^L_{NB}}{\partial P_S} dP_S + \frac{\partial f^L_{NB}}{\partial P_L} dP_L + \frac{\partial f^L_B}{\partial P_S} dP_S + \frac{\partial f^L_B}{\partial P_L} dP_L && (67) \\
&\quad + \frac{\partial f^L_B}{\partial F_B} \Big(- P_S (\frac{\partial f^S_{NB}}{\partial P_S} dP_S + \frac{\partial f^S_{NB}}{\partial P_L} dP_L) - P_L (\frac{\partial f^L_{NB}}{\partial P_S} dP_S + \frac{\partial f^L_{NB}}{\partial P_L} dP_L) \Big) \\
&= 0.
\end{aligned}
$$

This can be rearranged to yield

$$
dP_S \Big(\frac{\partial f^L_{NB}}{\partial P_S} + \frac{\partial f^L_B}{\partial P_S} - \frac{\partial f^L_B}{\partial F_B} (P_S \frac{\partial f^S_{NB}}{\partial P_S} + P_L \frac{\partial f^L_{NB}}{\partial P_S}) \Big) + dP_L \Big(\frac{\partial f^L_{NB}}{\partial P_L} + \frac{\partial f^L_B}{\partial P_L} - \frac{\partial f^L_B}{\partial F_B} (P_S \frac{\partial f^S_{NB}}{\partial P_L} + P_L \frac{\partial f^L_{NB}}{\partial P_L}) \Big) = 0. \quad (68)
$$

Second, use equations (51), (62), and (53) to determine how the demand for short bonds has to balance out purchases by the central bank and the resulting relative asset price changes

$$
\begin{aligned}
dS_{NB} + dS_B &= \frac{\partial f^S_{NB}}{\partial P_S} dP_S + \frac{\partial f^S_{NB}}{\partial P_L} dP_L + \frac{\partial f^S_B}{\partial P_S} dP_S + \frac{\partial f^S_B}{\partial P_L} dP_L + \frac{\partial f^S_B}{\partial F_B} dD_B && (69) \\
&= \frac{\partial f^S_{NB}}{\partial P_S} dP_S + \frac{\partial f^S_{NB}}{\partial P_L} dP_L + \frac{\partial f^S_B}{\partial P_S} dP_S + \frac{\partial f^S_B}{\partial P_L} dP_L && (70) \\
&\quad + \frac{\partial f^S_B}{\partial F_B} \Big(- P_S (\frac{\partial f^S_{NB}}{\partial P_S} dP_S + \frac{\partial f^S_{NB}}{\partial P_L} dP_L) - P_L (\frac{\partial f^L_{NB}}{\partial P_S} dP_S + \frac{\partial f^L_{NB}}{\partial P_L} dP_L) \Big) \\
&= -dS_{CB}.
\end{aligned}
$$

This can be rearranged to yield

$$
dP_S \Big(\frac{\partial f^S_{NB}}{\partial P_S} + \frac{\partial f^S_B}{\partial P_S} - \frac{\partial f^S_B}{\partial F_B} (P_S \frac{\partial f^S_{NB}}{\partial P_S} + P_L \frac{\partial f^L_{NB}}{\partial P_S}) \Big) + dP_L \Big(\frac{\partial f^S_{NB}}{\partial P_L} + \frac{\partial f^S_B}{\partial P_L} - \frac{\partial f^S_B}{\partial F_B} (P_S \frac{\partial f^S_{NB}}{\partial P_L} + P_L \frac{\partial f^L_{NB}}{\partial P_L}) \Big) = -dS_{CB}. \quad (71)
$$

The two market equilibrium conditions give us two equations with two unknowns, namely, the bond price responses to the central bank short bond purchases ($\frac{dP_L}{dS_{CB}}$, $\frac{dP_S}{dS_{CB}}$) that we are interested in solving for. This system can be written in matrix form as

$$
\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} dP_S \\ dP_L \end{pmatrix} = \begin{pmatrix} 0 \\ -dS_{CB} \end{pmatrix}, \quad (72)
$$

where

- $\alpha = \frac{\partial f^L_{NB}}{\partial P_S} + \frac{\partial f^L_B}{\partial P_S} - \frac{\partial f^L_B}{\partial F_B} (P_S \frac{\partial f^S_{NB}}{\partial P_S} + P_L \frac{\partial f^L_{NB}}{\partial P_S})$;
- $\beta = \frac{\partial f^L_{NB}}{\partial P_L} + \frac{\partial f^L_B}{\partial P_L} - \frac{\partial f^L_B}{\partial F_B} (P_S \frac{\partial f^S_{NB}}{\partial P_L} + P_L \frac{\partial f^L_{NB}}{\partial P_L})$;
- $\gamma = \frac{\partial f^S_{NB}}{\partial P_S} + \frac{\partial f^S_B}{\partial P_S} - \frac{\partial f^S_B}{\partial F_B} (P_S \frac{\partial f^S_{NB}}{\partial P_S} + P_L \frac{\partial f^L_{NB}}{\partial P_S})$;
- $\delta = \frac{\partial f^S_{NB}}{\partial P_L} + \frac{\partial f^S_B}{\partial P_L} - \frac{\partial f^S_B}{\partial F_B} (P_S \frac{\partial f^S_{NB}}{\partial P_L} + P_L \frac{\partial f^L_{NB}}{\partial P_L})$.

Solving for the two bond price changes, we have that

$$
\begin{pmatrix} dP_S \\ dP_L \end{pmatrix} = \frac{1}{\alpha\delta - \beta\gamma} \begin{pmatrix} \delta & -\beta \\ -\gamma & \alpha \end{pmatrix} \begin{pmatrix} 0 \\ -dS_{CB} \end{pmatrix} = \begin{pmatrix} \frac{\beta dS_{CB}}{\alpha\delta - \beta\gamma} \\ \frac{-\alpha dS_{CB}}{\alpha\delta - \beta\gamma} \end{pmatrix}. \quad (73)
$$

Thus, the general result is

$$
\frac{dP_S}{dS_{CB}} = \frac{\beta}{\alpha\delta - \beta\gamma} \quad \text{and} \quad \frac{dP_L}{dS_{CB}} = -\frac{\alpha}{\alpha\delta - \beta\gamma}. \quad (74)
$$

However, these are very complicated expressions, so to help build intuition, we now focus on the case when short-term interest rates are near the zero lower bound. Consequently, we assume the short bond price to be 1, and we assume that the demand for short bonds is characterized by perfect own-price elasticity in the neighborhood of $P_S = 1$. We operationalize this by assuming that either banks, nonbanks, or both have demand for short bonds with perfect price elasticity. An immediate consequence of these assumptions is that $\frac{dP_S}{dS_{CB}} = 0$. To simplify things further, we assume that banks, faced with higher deposits, will demand more long bonds to increase the duration of their portfolios. At the same time, we assume that they will not increase their demand for assets that are similar to the reserves they obtain as their deposits increase following the QE purchases, so $\frac{\partial f_B^S}{\partial F_B} = 0$. Now, we can focus on the response of the price for the long bond

$$\frac{dP_L}{dS_{CB}} = -\frac{\frac{\partial f_{NB}^L}{\partial P_S} + \frac{\partial f_B^L}{\partial P_S} - \frac{\partial f_B^L}{\partial F_B}(P_S \frac{\partial f_{NB}^S}{\partial P_S} + P_L \frac{\partial f_{NB}^L}{\partial P_S})}{\Delta}, \tag{75}$$

where

$$\begin{aligned}
\Delta &= \alpha\delta - \beta\gamma \\
&= \Big(\frac{\partial f_{NB}^L}{\partial P_S} + \frac{\partial f_B^L}{\partial P_S} - \frac{\partial f_B^L}{\partial F_B}(P_S \frac{\partial f_{NB}^S}{\partial P_S} + P_L \frac{\partial f_{NB}^L}{\partial P_S})\Big)\Big(\frac{\partial f_{NB}^S}{\partial P_L} + \frac{\partial f_B^S}{\partial P_L}\Big) \\
&\quad -\Big(\frac{\partial f_{NB}^L}{\partial P_L} + \frac{\partial f_B^L}{\partial P_L} - \frac{\partial f_B^L}{\partial F_B}(P_S \frac{\partial f_{NB}^S}{\partial P_L} + P_L \frac{\partial f_{NB}^L}{\partial P_L})\Big)\Big(\frac{\partial f_{NB}^S}{\partial P_S} + \frac{\partial f_B^S}{\partial P_S}\Big).
\end{aligned}$$

Next, we assume that the demand for short bonds by nonbank financial institutions is characterized by perfect price elasticity, i.e., $\frac{\partial f_{NB}^S}{\partial P_S} \to \infty$. We also assume that banks do not vary their demand for short bonds in response to changes in the short bond price, i.e., $\frac{\partial f_B^S}{\partial P_S} = 0$. This gives

$$\frac{dP_L}{dS_{CB}} = \frac{\frac{\partial f_B^L}{\partial F_B} P_S}{-\frac{\partial f_B^L}{\partial F_B} P_S(\frac{\partial f_{NB}^S}{\partial P_L} + \frac{\partial f_B^S}{\partial P_L}) - \Big(\frac{\partial f_{NB}^L}{\partial P_L} + \frac{\partial f_B^L}{\partial P_L} - \frac{\partial f_B^L}{\partial F_B}(P_S \frac{\partial f_{NB}^S}{\partial P_L} + P_L \frac{\partial f_{NB}^L}{\partial P_L})\Big)}. \tag{76}$$

Assume further that all cross-price elasticities are zero, i.e., we are in the extreme case of no substitution effects between assets. This reduces the expression to

$$\frac{dP_L}{dS_{CB}} = \frac{-\frac{\partial f_B^L}{\partial F_B} P_S}{\frac{\partial f_{NB}^L}{\partial P_L} + \frac{\partial f_B^L}{\partial P_L} - P_L \frac{\partial f_B^L}{\partial F_B} \frac{\partial f_{NB}^L}{\partial P_L}} > 0. \tag{77}$$

This shows that the price of long bonds can be positively affected when the central bank engages in quantitative easing by buying short-term bonds, as was the case in Switzerland in August 2011.

Finally, we note that fixing the cross-price elasticities to zero is not necessary to obtain this result. It remains valid as long as the cross-price elasticities are smaller in absolute value than the direct own-price elasticities.

The condition for positivity makes economic sense. It says that the rate at which banks increase their purchases of long-term bonds in response to an increase in deposit funding, times the value of those bonds, should not be larger than the increase in deposits itself.

Under our zero lower bound assumptions, with nonbanks being the marginal sellers of short bonds in a central bank QE program where only short bonds are bought, all short bonds are sold by nonbanks. These will, in return, hold more deposits with their correspondent bank. The price on short bonds will only increase infinitesimally to make this shift happen, as we are at the zero lower bound. If the correspondent banks do not react to the increase in their deposits, there would be no further price changes. However, we assume that banks do react, by increasing their demand for long bonds in an effort to increase the duration of their portfolios. This generates upward pressure on long-term bond prices, or a drop in their term premiums.

If, on the other hand, banks are the sellers of short-term bonds to the central bank under a QE program at the zero lower bound, then there is no effect on asset prices. Banks effectively swap short bonds for reserves, being indifferent between the two, and see no changes to their duration as a result.

References

Ball, Laurence, Joseph Gagnon, Patrick Honohan and Signe Krogstrup, 2016, "What Else Can Central Banks Do?," *Geneva Reports on the World Economy*, No. 18.

Bauer, Michael D. and Glenn D. Rudebusch, 2014, "The Signaling Channel for Federal Reserve Bond Purchases," *International Journal of Central Banking*, Vol. 10, No. 3, 233-289.

Bernanke, Ben S., Mark Gertler and Simon Gilchrist, 1999, "The Financial Accelerator in a Quantitative Business Cycle Framework," *Handbook of Macroeconomics*, Vol. 1, 1341-1393.

Bernanke, Ben S. and Vincent Reinhart, 2004, "Conducting Monetary Policy at Very Low Short-Term Interest Rates," *American Economic Association Papers and Proceedings*, Vol. 94, No. 2, 85-90.

Borio, C. and H. Zhu, 2012, "Capital Regulation, Risk-Taking and Monetary Policy: A Missing Link in the Transmission Mechanism?," *Journal of Financial Stability*, Vol. 8, No. 4, 236-251.

Brunnermeier, Markus K. and Lasse Heje Pedersen, 2009, "Market Liquidity and Funding Liquidity," *Review of Financial Studies*, Vol. 22, No. 6, 2201-2238.

Brunnermeier, Markus K., Thomas M. Eisenbach and Yuliy Sannikov, 2012, "Macroeconomics with Financial Frictions: A Survey," *NBER Working Papers* No. 18102, National Bureau of Economic Research.

Brunnermeier, Markus K. and Yuliy Sannikov, 2014, "A Macroeconomic Model with a Financial Sector," *American Economic Review*, Vol. 104, No. 2, 379-421.

Carpenter, Seth B., Selva Demiralp, Jane E. Ihrig, and Elisabeth C. Klee, 2015a, "Analyzing Federal Reserve Asset Purchases: From Whom Does the Fed Buy?," *Journal of Banking and Finance*, Vol. 52, 230-244.

Carpenter, Seth B., Jane E. Ihrig, Elisabeth C. Klee, Daniel Quinn, and Alexander Boote, 2015b, "The Federal Reserve's Balance Sheet and Earnings: A Primer and Projections," *International Journal of Central Banking*, Vol. 11, No. 2, 237-283.

Christensen, Jens H. E. and James M. Gillan, 2017, "Does Quantitative Easing Affect Market Liquidity?," Working Paper 2013-26, Federal Reserve Bank of San Francisco.

Christensen, Jens H. E. and Signe Krogstrup, 2016, "Transmission of Quantitative Easing: The Role of Central Bank Reserves," Working Paper 2014-39, Federal Reserve Bank of San Francisco.

Christensen, Jens H. E., Jose A. Lopez, and Glenn D. Rudebusch, 2015, "A Probability-Based Stress Test of Federal Reserve Assets and Income," *Journal of Monetary Economics*, Vol. 73, 26-43.

Christensen, Jens H. E. and Glenn D. Rudebusch, 2012, "The Response of Interest Rates to U.S. and U.K. Quantitative Easing," *Economic Journal*, Vol. 122, F385-F414.

D'Amico, Stefania and Thomas B. King, 2013, "The Flow and Stock Effects of Large-Scale Treasury Purchases: Evidence on the Importance of Local Supply," *Journal of Financial Economics*, Vol. 108, No. 2, 275-564.

Ennis, Huberto M. and Alexander L. Wolman, 2015, "Excess Reserves in the United States: A View from the Cross-Section of Banks," *International Journal of Central Banking*, Vol. 11, No. 1, 251-289.

Farmer, Roger E. A. and Pawel Zabczyk, 2016, "The Theory of Unconventional Monetary Policy," NBER Working Paper 22135.

Gagnon, Joseph, Matthew Raskin, Julie Remache, and Brian Sack, 2011, "Large-Scale Asset Purchases by the Federal Reserve: Did They Work?" *International Journal of Central Banking*, Vol. 7, No. 1, 3-43.

Gertler, Mark and Peter Karadi, 2011, "A Model of Unconventional Monetary Policy," *Journal of Monetary Policy*, Vol. 58, 17-34.

Gertler, Mark and Peter Karadi, 2013, "QE 1 vs. 2 vs. 3. . . : A Framework for Analyzing Large-Scale Asset Purchases as a Monetary Policy Tool," *International Journal of Central Banking*, Vol. 9, No. 1, 5-53.

Greenwood, Robin and Dimitri Vayanos, 2010, "Price Pressure in the Government Bond Market," *American Economic Review*, Vol. 100, No. 2, 585-590.

Greenwood, Robin and Dimitri Vayanos, 2014, "Bond Supply and Excess Bond Returns," *Review of Financial Studies*, Vol. 27, No. 3, 663-713.

Haddad, Valentin and David A. Sraer, 2015, "The Banking View of Bond Risk Premia," Manuscript, UC Berkeley.

Hamilton, James D. and Jing Wu, 2012, "The Effectiveness of Alternative Monetary Policy Tools in a Zero Lower Bound Environment," *Journal of Money, Credit, and Banking*, Vol. 44, No. 1 (Supplement), pp. 3-46.

Hancock, Diana and Wayne Passmore, 2015, "How Does the Federal Reserve's Large-Scale Asset Purchases (LSAPs) Influence Mortgage-Backed Securities (MBS) Yields and U.S. Mortgage Rates," *Real Estate Economics*, Vol. 43, No. 4, 855-890.

He, Zhiguo and Arvind Krishnamurthy, 2013, "Intermediary Asset Pricing," *American Economic Review*, Vol. 103, No. 2, 732-770.

Holmstrom, Bengt and Jean Tirole, 1997, "Financial Intermediation, Loanable Funds, and The Real Sector," *The Quarterly Journal of Economics*, Vol. 112, No. 3, 663-691.

Ihrig, Jane, Edward Kim, Ashish Kumbhat, Cindy M. Vojtech, and Gretchen C. Weinbach, 2017, "How have banks been managing the composition of high-quality liquid assets?," Finance and Economics Discussion Series 2017-092. Washington: Board of Governors of the Federal Reserve System.

Joyce, Michael A. S., Ana Lasaosa, Ibrahim Stevens, and Matthew Tong, 2011, "The Financial Market Impact of Quantitative Easing in the United Kingdom," *International Journal of Central Banking*, Vol. 7, No. 3, 113-161.

Kandrac, John, 2014, "The Costs of Quantitative Easing: Liquidity and Market Functioning Effects of Federal Reserve MBS Purchases," Manuscript, Board of Governors of the Federal Reserve System.

Kandrac, John and Bernd Schlusche, 2016, "Quantitative Easing and Bank Risk Taking: Evidence from Lending," Manuscript, Board of Governors of the Federal Reserve System.

Krishnamurthy, Arvind and Annette Vissing-Jorgensen, 2011, "The Effects of Quantitative Easing on Interest Rates: Channels and Implications for Policy," *Brookings Papers on Economic Activity*, Fall 2011, 215-265.

Krishnamurthy, Arvind and Annette Vissing-Jorgensen, 2012, "The Aggregate Demand for Treasury Debt," *Journal of Political Economy*, Vol. 120, No. 2, 233-267.

Krugman, Paul, 2008, "The International Finance Multiplier," *Mimeo, Princeton University.*

Laubach, Thomas, 2009, "New Evidence on the Interest Rate Effects of Budget Deficits and Debt," *Journal of the European Economic Association*, Vol. 7, No. 4, 858-885.

Lenel, Moritz, 2017, "Safe Assets, Collateralized Lending and Monetary Policy," SIEPR Discussion Paper No. 17-010.

Li, Canlin and Min Wei, 2013, "Term Structure Modeling with Supply Factors and the Federal Reserve's Large-Scale Asset Purchase Programs," *International Journal of Central Banking*, Vol. 9, No. 1, 3-39.

Modigliani, Franco and Richard Sutch, 1966, "Innovations in Interest-Rate Policy," *American Economic Review*, Vol. 56, No. 1-2, 178-97.

Modigliani, Franco and Richard Sutch, 1966, "Debt Management and the Term Structure of Interest Rates: An Empirical Analysis of Recent Experience," *Journal of Political Economy*, Vol. 75, No. 4, Part 2, 569-589.

Swanson, Eric T. and John C. Williams, 2014a, "Measuring the Effect of the Zero Lower Bound on Medium- and Longer-Term Interest Rates," *American Economic Review*, Vol. 104, No. 10, 3154-3185.

Swanson, Eric T. and John C. Williams, 2014b, "Measuring the Effect of the Zero Lower Bound on Yields and Exchange Rates in the U.K. and Germany," *Journal of International Economics*, Vol. 92, S2-S21.

Tobin, James, 1969, "A General Equilibrium Approach to Monetary Theory," *Journal of Money, Credit and Banking*, Vol. 1, No. 1, 15-29.

Vayanos, Dimitri and Jean-Luc Vila, 2009, "A Preferred-Habitat Model of the Term Structure of Interest Rates," NBER Working Paper 15487.

Wright, Jonathan H., 2012, "What does Monetary Policy do to Long-Term Interest Rates at the Zero Lower Bound?," *Economic Journal*, Vol. 122, F447-F466.