Market Power and Price Informativeness*

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Abstract

The asset ownership structure in financial markets worldwide has been changing rapidly over the last few decades. Institutional investors, both active and passive, own a larger fraction of assets and the distribution of the ownership is more concentrated. We develop a general equilibrium portfolio-choice model with endogenous information acquisition and market power. We show that, in the cross section, an increase in active (passive) institutional ownership increases (decreases) price informativeness, and an increase in concentration of ownership leads to lower informativeness. In contrast to the cross-sectional results, the policy experiments of changing ownership structure indicate a non-monotonic relationship between the levels of ownership and price informativeness. Further, we show that increasing the passive share of the market prompts active investors to adopt learning strategies that exacerbate the reduction in aggregate price informativeness. We conclude that any policy targeting ownership structure should factor in its effects on welfare through price informativeness.

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1 Introduction

In the last few decades, equity ownership of large asset management companies has drastically increased, as has the concentration of those holdings. From 1980 to 2015, the average institutional ownership of U.S. stocks increased from 25% to 60%, and ownership levels among the top-10 largest asset management companies almost doubled, from 18% to 35%. At the same time, the share of funds investing passively has jumped as well, rising from about 11% before the financial crisis, to about 25% at the end of 2016.

These striking changes in market structure attracted keen interest from various market participants, including asset managers, regulators, media, and individual investors. Some critics point to potential impact of the changes for financial stability; others emphasize consequences for market prices, and therefore for efficient allocation of capital in the economy. This paper takes one important step towards understanding these consequences—by examining the implications of various market structures for price informativeness.

While some theoretical and empirical work examining the effects of market power on price informativeness exists, we extend this discussion to a micro-founded general equilibrium framework along three main dimensions: (i) the overall size of institutional investors; (ii) the relative size of institutional investors; and (iii) the relative ability of investors to collect information. In particular, we highlight the tradeoff between size (price impact) and information acquisition in financial markets.

Our model features an economy with many assets and many large investors (herein referred to as oligopolists), who each have the ability to internalize the effect of their trades on equilibrium prices. The model also features a continuum of smaller, fringe investors, each with a smaller capacity to process information. All agents can learn about the future paths of prices, and then oligopolists engage in a Cournot game in the asset market. Inputs into the model are the size of the agents, based on assets under management, and agents' learning capacities. The presence of large investors is a novel component of portfolio choice models with endogenous information acquisition. The combination of endogenous learning and trading in a multi-asset model is a key deviation from the theoretical literature on market power, where most papers focus either on exogenous information or a single asset market.

We identify three key channels that affect price informativeness. The first one is the degree to which prices track fundamentals, which can be viewed as *average ownership-weighted information*. This effect increases with agents' ability to learn, and is positively correlated with price informativeness. The second one is the sensitivity of the oligopolists' quantities to information, or what we call the *information passthrough to quantities*. This sensitivity is increasing in the size of the oligopoly sector, because the larger an oligopolist is, the more she worries about price impact. Relatedly, passthrough increases when the agent is less risk averse, or when the volatility in the market is lower. Passthrough is also positively correlated with price informativeness, as increases in passthrough reduce price volatility. Third and the final one is the effect of concentration, which in our model takes the form of the Herfindahl-Hirschman Index, weighted by agents' learning decisions—we call this the *learning HHI* or *LHHI*. This channel arises due to the noise introduced by large players' trades, and is independent of any learning by the fringe. It is, therefore, unambiguously negatively related to price informativeness. The overall effect on price informativeness is thus a result of a tension between the first two channels and the third one.

In the cross section of assets, we find that (i) higher levels of institutional (oligopolistic) ownership of an asset correspond to higher degrees of price informativeness, and (ii) higher levels of institutional concentration in ownership of an asset correspond to lower levels of price informativeness. The more agents learn about an asset, the more they participate in that market, and therefore, the higher the degree of this asset's price informativeness. On the other hand, if very few agents learn about a particular asset, concentration in that market is much higher, and the asset's price informativeness is lower.

The above results focus mainly on the effects of learning and volatility on price informativeness, but for a fixed market structure. From a policy perspective, it may be important to analyze how changing market structure can affect price informativeness. The key to understanding the results of a changing structure on price informativeness is to understand the effects of market power on a single investor. When investors are atomistic they will specialize in their information collection, choosing only one asset to learn about. As they increase in size, they continue to specialize for a time, but they also internalize the effect of their learning on prices. When the magnitude of that effect gets large enough, agents no longer want to specialize and so *when agents cross a certain threshold in size, they diversify their learning.* This is a novel finding, and one that is central to the results that follow.

We alter market structure (the distribution of sizes) along three dimensions (total size, concentration, and active/passive mix). We find that price informativeness has a *non-monotonic, hump shape in the size of the oligopoly sector.* The intuition is that as oligopolists increase in size, they not only diversify their learning, but also reduce their participation in the asset market. In the limit, when the oligopolists are the market, they will not trade at all, as their trades affect prices perfectly. Second, we find that increases in the concentration of oligopoly sizes unambiguously reduce price informativeness. A concentrated distribution of oligopolists means that comparatively larger parts of the market are controlled by agents who are very concerned with their price impact, and so they trade more cautiously.

Finally, we find that an increase in the size of a passive investor, at the expense of an active investor unambiguously reduces price informativeness. This result, while not surprising, sheds light on the information effect of passive traders' size on active traders' behaviors. There are two aspects to this effect. First, assets in the market get diverted from active to passive investors, resulting in less informed funds being present for trade. Second, as the active investors lose market power, they specialize more in their learning. Specialization leads to a decrease in aggregate price informativeness, but because of agents' preference for learning about high-volatility assets over low-volatility assets, the decrease is not necessarily uniform across all assets.

To further emphasize the role of the endogenous information acquisition in markets, we contrast the results of our benchmark model with those of a model in which the attention allocation is exogenous, as in Kyle (1985). We show that the benchmark model predicts a very different relation of ownership and concentration with price informativeness. In particular, we show that the policy prescriptions about the optimal level of institutional ownership coming from a model with exogenous information choice can be biased upwards or downwards relative to the fully endogenous model, depending on the exogenous information structure one assumes. We conclude that modeling endogenous information choices is crucial when making normative statements about the size and structure of the institutional asset management sector.

In the last part of the paper, we contrast our model with the data from the U.S. market. We find strong empirical support for some of the main theoretical predictions.

In Section 2, we present a set of motivating facts from the U.S. data on institutional ownership and its concentration. Section 3 presents the theoretical framework, the equilibrium concept, and derives basic theoretical tradeoffs between the ownership structure and price informativeness. In Section 4, we derive numerical solutions for the more general settings and discuss various policy experiments. Section 5 provides empirical results corroborating some of the model's predictions. Section 6 concludes. Any omitted proofs and derivations are in the Appendix.

1.1 Related Literature

Our paper spans several research themes. First, our general equilibrium model is anchored in the literature on the endogenous information choice, in the spirit of Sims (1998, 2003). More closely related to our application are the models of costly information of Van Nieuwerburgh and Veldkamp (2009, 2010), Mondria (2010), Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016), and Kacperczyk, Nosal, and Stevens (2017). Ours is the first theoretical study to introduce market power into a model with endogenous information acquisition. This novel aspect allows us to study strategic responses of oligopolistic traders in terms of their demand and information choices.

The literature on informed trading with market power dates back to Kyle (1985) whose setup is one strategic trader, and Holden and Subrahmanyam (1992), which extends the model of Kyle into an oligopolistic framework. Lambert, Ostrovsky, and Panov (2016) extend the Kyle's model to study the relation between the number of strategic traders and information content of prices.[1] In all these studies, information is an exogenous process, which is a key dimension along which our model works. Also, they do not examine the role of concentration of ownership among strategic traders, which is the main focus of our study. Kyle, Ou-Yang, and Wei (2011) allows for endogenous information acquisition but their mechanism depends on differences in risk aversion. Also, they focus on the contracting features of delegation and allow for only one risky asset. In turn, our framework operates through heterogeneity in information capacity and multi-asset economy.

We also contribute to the literature on information production and asset prices. Bond, Edmans, and Goldstein (2012) survey the literature on information production in financial markets, emphasizing the differences between new information produced in markets (revelatory price efficiency: RPE) and what is already known and merely reflected in prices (forecasting price efficiency: FPE). Our focus is solely on RPE and is largely dictated by the modeling framework we use.[2] Stein (2009) develops a model of market efficiency and sophisticated (arbitrage) capital in the presence of capital constraints. Garleanu and Pedersen (2015) examine the role of search frictions in asset

[1] Models in which traders condition on others' decisions also include Foster and Viswanathan (1996) and Back, Cao, and Willard (2000).

[2] Theoretical work on asset prices and real efficiency also includes Dow and Gorton (1997), Subrahmanyam and Titman (1999), Kurlat and Veldkamp (2015), and Edmans, Goldstein, and Jiang (2015).

management for price efficiency. On an empirical front, Chen, Goldstein, and Jiang (2007) and Bakke and Whited (2010) find that the relation between stock prices and investment is stronger for firms with more informative stock prices, whereas Baker, Stein, and Wurgler (2003) find that it is stronger for firms that issue equity more often. None of the above studies investigates the role of market power and endogenous information acquisition. The exception is Bai, Philippon, and Savov (2016) who show empirically that price informativeness is greater for stocks with greater institutional ownership. We confirm their findings for the range of the ownership values. However, we show that beyond certain levels (not observed in their data) ownership may in fact reduce price informativeness. Separately, we also investigate the role of ownership concentration and provide a micro-founded general equilibrium model that allows us to study the underlying economic mechanism in more depth. In a contemporaneous work, Farboodi, Matrey, and Veldkamp (2017) examine differences in price informativeness between companies included and not included in the S&P 500 index. They show that the indexed companies exhibit larger efficiency, which they attribute to composition effect of these companies, being older and larger. Their focus, however, is not on market power and changes in market structure.

Finally, we add to a growing empirical literature that studies the impact of market structure in asset management on various economic outcomes. Following the diseconomies of scale argument of Chen et al. (2004), Pástor, Stambaugh, and Taylor (2015) show significant diseconomies of scale at the industry level. Using a merger between BlackRock and BGI as a shock to market power, Massa, Schumacher, and Yan (2016) study the asset allocation responses of their competitors. They find that competitors scale down positions which overlap with those held by the merged entity. More broadly, He and Huang (2014) and Azar, Schmalz, and Tecu (2016) study consequences of common asset ownership by large blockholders for product market competition and prices. Our work complements these studies by studying, theoretically and empirically, the effect of ownership structure on price informativeness.

2 Motivating Facts

In this section, we present the three empirical facts that motivate our study. First, we show that institutional stock ownership has increased widely over the last thirty-five years. Second, we show that the ownership structure is skewed towards the largest owners. Third, we show that, in the recent times, the ownership mix has shifted from active to passive investors.

The growth in institutional ownership has been previously documented in several studies, including Gompers and Metrick (2001). The evidence on concentration is much more sparse. Similarly, evidence on passive investments has been large explored, theoretically, from the agency perspective (e.g., Basak and Pavlova (2013)). However, except for the recent paper by Bai, Philippon, and Savov (2016) which emphasizes the first fact, no other study has exploited the implications of these facts for longer-horizon price informativeness.[3]

Our data on institutional stock ownership come from Thomson Reuters and span the period 1980–2015. Even though the formal requirements to report holdings allow smaller companies not to report, the representation of institutions in the data is more than 98% in value-weighted terms. We calculate the stock-level institutional ownership by taking the ratio of the number of stocks held by financial institutions at the end of a given year to the number of shares outstanding at the same time. Next, we aggregate the measures across stocks by taking a simple average across all stocks in our sample. Using equal weighting, rather than value weighting, gives a conservative metric of the trends in the data. Subsequently, we calculate a similar measure, but only taking into account the holdings of the top-10 largest holders for a given stock. We present the time-series dynamics of the two quantities in Figure 1.

Both series indicate a clear pattern underlying the recent policy discussions: In-

[3]A parallel microstructure literature (Boehmer and Kelley (2009)) examines empirically the relation between institutional ownership and price efficiency due to trading intensity. Efficiency there is measured using variance ratios and pricing errors. The conclusions from this literature are akin to those reported in our paper.

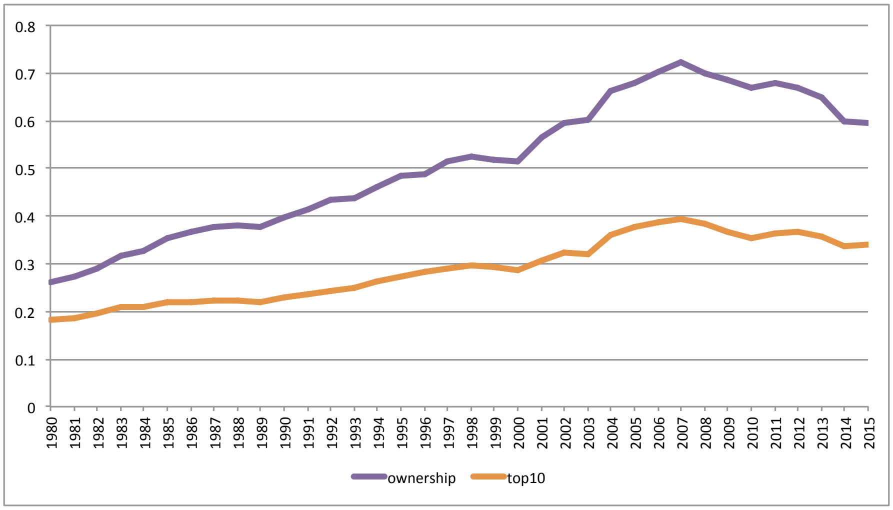


Figure 1: Institutional Ownership: Unconditional and Top-10 Holders

stitutional ownership has grown and the increase is mostly fueled by the growing concentration of ownership. The magnitudes of the growth are economically large: Over the period of over 35 years, each ownership statistic has more than doubled. While we focus here on the average trends in the data, even stronger effects can be observed in the cross section of stocks with different characteristics.

In our model, a more natural way to measure concentration is the Herfindahl-Hirshman Index (HHI), defined as the sum of squared shares of all institutional owners of a given stock. However, the problem with using a raw index value is its mechanical correlation with the number of investors in the data. To the extent that the number of institutions has been growing steadily over the same period the unadjusted index would reflect two effects going in opposite direction. To filter out this mechanical sorting, we take out the predicted component in the HHI accounted by the number of investors. We plot the filtered series in Figure 2.

The results indicate that the concentration levels have been generally going up over time. This pattern has been particularly visible since the early 1990s. The magnitude of the growth is economically large and the large values of concentration, especially in the last few years, reflect the concerns policy makers have voiced with

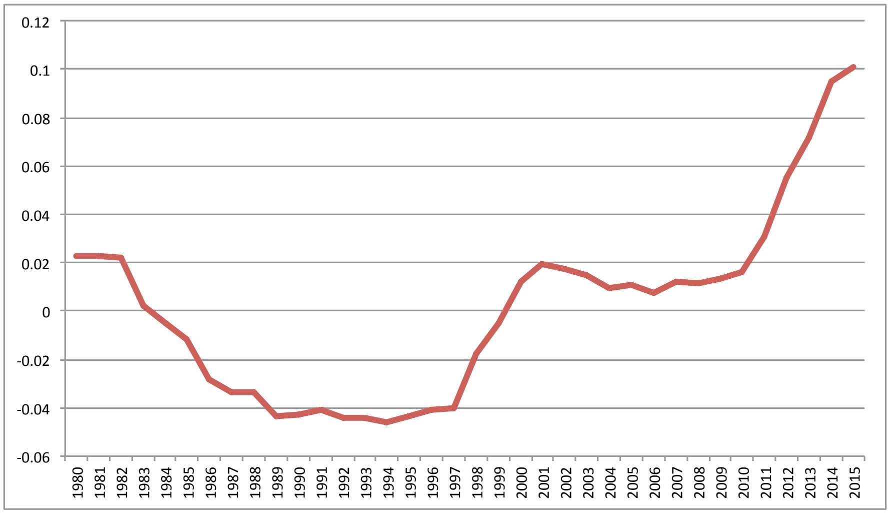


Figure 2: Adjusted Ownership Concentration

regard to this phenomenon.

To illustrate the effects on ownership mix we define active investors as those engaged in information acquisition process and passive investors as those who strictly invest in pre-defined index portfolios. The latter group includes both index mutual funds and ETFs. Because identifying passive funds in the institutional investors data is not trivial we borrow the evidence from the Investment Company Institute (ICI) Fact Book. We show the time-series evolution of the percentage of passively managed equity mutual funds in the U.S. in Figure 3.

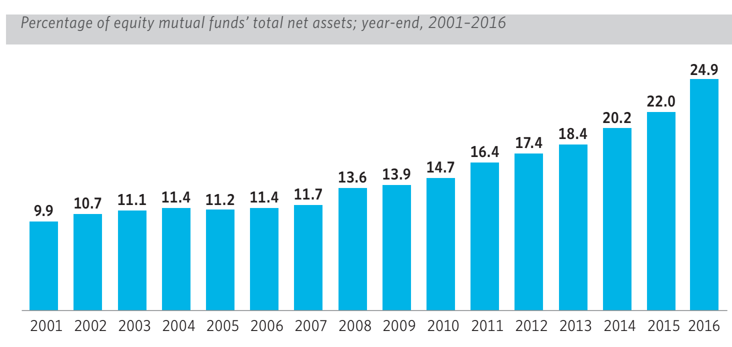


Figure 3: Passive Equity Fund Share

The results indicate a significant increase in passive ownership in the period of

2001-2016. While passive funds accounted for less than 10% of total equity fund market in the U.S., this share has increased to almost 25% by 2016. In the paper, we take this trend as given and merely focus on its consequences for stock price informativeness.

To conclude, we note that while the motivating facts we present relate to institutional investors, the model we present next is a general theory of asset allocation and information acquisition by investors with market power. We believe institutional investors are natural candidates for this type of investors.

3 Model

This section presents a noisy rational expectations portfolio choice model in which investors are constrained in their capacity to process information about assets payoffs. The setup departs from the information choice model of Kacperczyk et al. (2017) by introducing market power for some investors. Also, we solve for price informativeness of the aggregate economy and individual assets differentiated by their volatility.

3.1 Setup

The model features a finite continuum of traders, divided into $l+1$ many segments, represented by λ. The traders in the first segment, λ_0, are atomistic – these traders act as a **competitive fringe**, in that they are able to pay attention to innovations in asset prices, but do not have any market power. They are indexed by h. Measures $\{\lambda_1, \lambda_2, ..., \lambda_l\}$ of investors act as **oligopolists**, indexed by j. Each measure collects information and trades, as the fringe does, but the oligopolists collect and trade as a unit, and therefore they have market power in information, *and* market power as traders.

Every member of the fringe, and every oligopolist observe signals about innovations in asset prices. The vector of signals for the oligopolist for asset j is

$s_j = (s_{j1}...s_{jl})$. The vector of signals for the fringe for asset j is indexed by h. Investors of both types maximize mean-variance utility function, with common risk aversion ρ.

The market has one risk-free asset, with a price normalized to one, and a net payout of r, and $n > 1$ risky assets, indexed by i, with prices p_i and independent payoffs $z_i = \bar{z} + \varepsilon_i$, where $\varepsilon_i \sim N(0, \sigma_i^2)$. The risk-free asset has unlimited supply, and each risky asset has a stochastic supply with mean $\bar{x}$ and variance $\{\sigma_{xi}\}$. We can think of these as noisy supply shocks.

Agents make portfolio decisions and can choose to obtain information about the price innovations for some or all of the risky assets. The capacity to process information for the oligopolists is denoted $\{K_j\}$, while the capacity of each member of the fringe is constant at K_h. We place no restrictions on the values of K_j and K_h other than they must be finite. Investors do not learn from prices. Oligopolists and members of the fringe can use their capacities to receive informative signals about the payoff of the asset and reduce that variance accordingly. We model signal choice using entropy reduction as in Sims (2003).

We denote an agent's posterior variance as $\hat{\sigma}^2$. For simplicity, we also define $\alpha_{ji} \equiv \frac{\sigma_i^2}{\hat{\sigma}_{ji}^2} \geq 1$. We conjecture and later verify the following price structure:

$$p_i = a_i + b_i \varepsilon_i - c_i \nu_i - \sum_{j=1}^{n} d_{ji} \zeta_{ji} \tag{1}$$

where ε_i and ν_i are the innovations in the payoff and noisy supply shocks, respectively. The first term corresponds to the base price, and the second one to the innovation. The innovation is not typically revealed completely in prices, because agents cannot perfectly observe it. The third term corresponds to noise or liquidity shocks, while the fourth one is defined as follows: First, define δ_{ji} as the data loss of oligopolist j: $\delta_{ji} \equiv z_i - s_{ji}$, then define $\zeta_{ji} \equiv \delta_{ji} - \frac{1}{\alpha_{ji}} \varepsilon_i$ to be the portion of the dataloss that is uncorrelated with the price innovation. Then $p_i \sim \mathcal{N}\left(a_i, \sigma_{pi}^2\right)$ where σ_{pi} can be expressed as:

$$\sigma_{pi}^2 \quad = \quad b_i^2\sigma_i^2 + c_i^2\sigma_{xi}^2 + \sum_{j=1}^{n} d_{ji}^2 \left(1 - \frac{1}{\alpha_{ji}}\right) \widehat{\sigma}_{ji}^2 \tag{2}$$

See the Appendix 7 for the derivations. Before solving the oligopolists's problem, we first turn to the problem faced by the competitive fringe.

3.1.1 Competitive fringe

Portfolio problem The portfolio problem of the fringe is as follows. Given posterior beliefs and equilibrium prices, each competitive investor h solves the following problem:

$$U_h = \max_{\{q_{hi}\}_{i=1}^n} E_h(W_h) - \frac{\rho}{2} V_h(W_h) \quad s.t. \quad W_j = r\left(W_{0h} - \sum_{i=1}^{n} q_{hi}p_i\right) + \sum_{i=1}^{n} q_{hi}z_i \tag{3}$$

where E_h and V_h are the perceived mean and variance of investor h conditional on her information set, and W_{0h} is initial wealth. Then, optimal portfolio holdings are:

$$q_{hi} = \frac{\hat{\mu}_{hi} - rp_i}{\rho\hat{\sigma}_{hi}^2} \tag{4}$$

where $\widehat{\mu}_{hi}$ and $\widehat{\sigma}_{hi}^2$ are the mean and variance of investor h's posterior beliefs about payoff z_i.

Given this 'second-stage' problem, the fringe agents have a 'first-stage' information choice problem. Each member of the fringe can choose to receive signals s_{hi} on each asset payoff ϵ_i. The vector of signals is subject to an information capacity constraint, based off Shannon (1948)'s mutual information measure: $I(z; s_h) \leq K_h$. Since K_h is finite, this expression constrains the ability of fringe members to reduce the uncertainty of signals.

Information problem Each member of the fringe faces the following information problem:

$$\max_{\{\widehat{\sigma}_{hi}^2\}_{i=1}^n} U_{0h} \equiv \frac{1}{2\rho}\sum_{i=1}^{n}\frac{E_{0h}\left(\widehat{\mu}_{hi}-rp_i\right)^2}{\widehat{\sigma}_{hi}^2} \tag{5}$$

subject to the relative entropy constraint

$$\prod_{i=1}^{n}\frac{\sigma_i^2}{\widehat{\sigma}_{hi}^2} \leq e^{2K_h}. \tag{6}$$

The information problem can also be written as:

$$U_{0h} = \sum_{i=1}^{n} G_i \frac{\sigma_i^2}{\widehat{\sigma}_{hi}^2}, \tag{7}$$

We obtain a corner solution: each investor h learns about one asset $l_h \in \arg\max\{G_i\}$. The gain to the competitive investors from learning about asset i is:

$$G_i \equiv \frac{\left(\overline{z}-ra_i\right)^2}{\sigma_i^2} + \left(1-rb_i\right)^2 + r^2 c_i^2 \frac{\sigma_{xi}^2}{\sigma_i^2} + r^2\left(\sum_{j=1}^{n} d_{ji}^2\left(1-\frac{1}{\alpha_{ji}}\right)\widehat{\sigma}_{ji}^2\right) - \frac{\widehat{\sigma}_{hi}^2}{\sigma_i^2}(1-2rb_i)$$

Derivation in Appendix 7. The gain from learning about a particular asset is the same across all competitive investors. However, this gain is a function of the learning that the monopolist does in that asset (namely, it is a function of the oligopolist's posterior variance, $\widehat{\sigma}_{ji}^2$). The gains to learning about an asset's payoff are that the fringe traders can take advantage of deviations in the price from their perception of the asset's value. The first term of G_i corresponds to the gains of trade from the fundamental; the second one to the gains of trade from deviations in the innovation; the third one to the gains of trade from noise traders; and the fourth one to alterations in price due to data-loss by the oligopolists.

3.1.2 Oligopolist

Portfolio problem Oligopolists have a similar trading problem to the fringe and the quantity demanded by each oligopolist is:

$$q_{ji} = \frac{\widehat{\mu}_{ji} - rp_i\left(q_{ji}\right)}{\rho\widehat{\sigma}_{ji}^2 + r\frac{dp_i(q_{ji})}{dq_{ji}}}, \tag{8}$$

The derivative in the denominator reflects the fact that oligopolists have market power. Each oligopolist internalizes the fact that their asset purchase decisions affect the equilibrium price. Using market clearing, we can solve for this derivative to get:

$$\frac{dp_i(q_{ji})}{dq_{ji}} = \frac{\lambda_j \rho \sigma_i^2}{\lambda_0 r(1+\Phi_{hi})} > 0, \tag{9}$$

where

$$\Phi_{hi} \equiv m_{hi}\left(e^{2K_h} - 1\right), \tag{10}$$

and m_{hi} is the mass of competitive investors learning about asset i. Hence, how sensitive the price is to an oligopolist's demand depends (inversely) on what fraction of the competitive fringe is learning about that asset, and how much.

The oligopolist's demand becomes:

$$q_{ji} = \frac{\widehat{\mu}_{ji} - rp_i}{\rho\left(\widehat{\sigma}_{ji}^2 + \widehat{\lambda}_{ji}\sigma_i^2\right)} \tag{11}$$

where $\hat{\lambda}_{ji} = \frac{\lambda_j}{\lambda_0(1+\Phi_{hi})}$ - essentially a ratio of the effective shares of the oligopolists to the fringe. Given the expression for quantity, demanded we can then calculate indirect utility:

$$U_j = \frac{1}{2\rho}\sum_{i=1}^{n}\left(\widehat{\mu}_{ji} - rp_i\right)^2\left[\frac{\widehat{\sigma}_{ji}^2 + 2\widehat{\lambda}_{ji}\sigma_i^2}{\left(\widehat{\sigma}_{ji}^2 + \widehat{\lambda}_{ji}\sigma_i^2\right)^2}\right], \tag{12}$$

Derivation in Appendix 7. As with the fringe, oligopolists' expected utilities depend positively on the deviations of their personal estimates from the equilibrium price (larger deviations mean larger quantities demanded). The smaller the oligopolists' posterior variance the larger their utility. The larger the oligopolist's market power (or conversely the smaller the fringe, or the less informed the fringe), the larger is the oligopolist's price impact, and therefore the smaller her utility.

Information problem The oligopolist's information problem is to solve

$$\max_{\left\{\widehat{\sigma}_{ji}^{2}\right\}_{i=1}^{n}} U_{0j} \quad s.t. \quad \prod_{i=1}^{n} \frac{\sigma_i^2}{\widehat{\sigma}_{ji}^2} \leq e^{2K_j}, \tag{13}$$

We can also write the constraint as

$$\prod_{i=1}^{n} \alpha_{ji} \leq e^{2K_j} \Leftrightarrow \sum_{i=1}^{n} \ln\alpha_{ji} \leq 2K_j, \tag{14}$$

with

$$\ln\alpha_{ji} \geq 0. \tag{15}$$

The Lagrangean is [dropping $1/2\rho$]

$$\mathcal{L} = \sum_{i=1}^{n} \left[u_i\left(\alpha_{ji}\right) - \mu\ln\alpha_{ji} + \eta_i \ln\alpha_{ji}\right] + n\gamma 2K_j, \tag{16}$$

The optimality conditions are

$$u_i'\left(\alpha_{ji}\right) - \frac{\mu}{\alpha_{ji}} + \frac{\eta_i}{\alpha_{ji}} = 0, \quad \forall i = 1, ..., n. \tag{17}$$

The capacity constraint is always binding, so $\sum_{i=1}^{n} \ln\alpha_{ji} = 2K_j$ and $\mu > 0$. Let L denote the set of assets that are learned about by the oligopolist. We have that

$$\alpha_{jl} > 1 \quad \text{and} \quad \eta_l = 0 \quad \text{and} \quad \mu = \alpha_{jl} u_l'\left(\alpha_{jl}\right) \quad \forall l \in L \tag{18}$$

and

$$\sum_{l=L} \ln\alpha_{jl} = 2K_j. \tag{19}$$

For assets $i \notin L$,

$$\alpha_{jl} = 1 \quad \text{and} \quad \eta_l = \mu - u_i'(1) \geq 0 \quad \Leftrightarrow \quad \alpha_{ji} u_i'(\alpha_{ji}) \geq u_i'(1) \quad \forall l \in L. \tag{20}$$

These conditions yield the oligopolist's allocation of attention across assets, $\{\alpha_{ji}\}$, as a function of the equilibrium price coefficients, a_i, b_i, c_i, d_i, and the share of competitive investors learning about each asset, m_{hi}. Given the choice of the oligopolist of the set $\{\alpha_{ji}\}$, variance of the posterior belief of the monopolist is σ_i^2/α_{ji} and the mean is just the signal s_{ji}. The signal is distributed, conditional on the realizations $z_i = \bar{z} + \varepsilon_i$, as

$$E(s_{ji}|z_i) = \bar{z} + \left(1 - \frac{\hat{\sigma}_{ji}^2}{\sigma_i^2}\right)\varepsilon_i = \bar{z} + \left(1 - \frac{1}{\alpha_{ji}}\right)\varepsilon_i,$$

$$Var(s_{ji}|z_i) = \sigma_i^2 \left(1 - \frac{\hat{\sigma}_{ji}^2}{\sigma_i^2}\right)\frac{\hat{\sigma}_{ji}^2}{\sigma_i^2} = \left(1 - \frac{1}{\alpha_{ji}}\right)\frac{1}{\alpha_{ji}}\sigma_i^2.$$

3.2 Equilibrium

We can now solve for the coefficients of the equation (1) posited earlier. Doing so yields expressions for a_i, b_i, c_i, d_{ki}, and d_{ji} (derivation in Appendix 7):

$$a_i = \frac{\bar{z}}{r} - \frac{\bar{x}}{r}\frac{N_i \rho \sigma_i^2}{\lambda_0(1+\Phi_{hi})} \tag{21}$$

$$b_i = N_i \left(\sum_{j=1}^{n} \frac{M_{ji}(\alpha_{ji}-1)}{r\alpha_{ji}} + \frac{\Phi_{hi}}{r(1+\Phi_{hi})}\right) \tag{22}$$

$$c_i = \frac{N_i \rho \sigma_i^2}{r\lambda_0(1+\Phi_{hi})} \tag{23}$$

$$d_{ji} = \frac{N_i M_{ji}}{r} \tag{24}$$

where $M_{ji} \equiv \frac{\hat{\lambda}_{ji}\sigma_i^2}{\left(\widehat{\sigma}_{ji}^2+\widehat{\lambda}_{ji}\sigma_i^2\right)}$ and $N_i \equiv \frac{1}{1+\sum_{j=1}^{n} M_{ji}}$. The fundamental component of the price, a_i, unsurprisingly depends positively on $\bar{z}$. An increase in supply will also decrease a_i, as will increased risk aversion and fundamental volatility. As the fringe's size or attentional capacity increase, their demand increases, and thus prices increase. As the oligopolists' size increases, or as their attention to asset i increases, demand goes up, M_{ji} increases, and N_i decreases, again driving up the price.

b_i depends almost exclusively on the information choices of the fringe and oligopolists. If the fringe cannot pay attention, then Φ drops to zero, as does the second term of the expression. If the oligopolists cannot pay attention, each α_{ji} goes to zero. b_i is increasing in Φ_{hi} and α_{ji}, because increased attention increases investors' predictive power of the innovation, and therefore their information will be better reflected in prices.

The same reasons that demand fluctuates in a_i apply to c_i, as c_i corresponds to the random component, while a_i corresponds to the mean component. We are able to show the existence of an equilibrium.

Proposition 1. *An equilibrium in learning exists.*

3.3 Comparative statics

We first solve a special case of the model, to generate much of the intuition for the mechanisms at play. Specifically, let us assume that there is only one institutional investor, a *monopolist*, for whom $K_j > 0$, and that the fringe cannot learn.

Then the first order conditions for the monopolist are:

$$\begin{aligned} \mu &= \frac{\alpha_i}{(1+2\hat{\lambda}\alpha_i)^2} X_i \\ \prod_{i=1}^{n} \alpha_i &= e^{2K_j} \end{aligned}$$

where $X_i = \frac{1}{2\rho}\left(\left(\frac{\rho}{\lambda_0}\right)^2(\bar{x}^2+\sigma_{ix}^2)\sigma_i^2+1+2\hat{\lambda}\right)$. For any two assets i and k that the monopolist learns about:

$$\frac{\alpha_k}{(1+2\hat{\lambda}\alpha_k)^2}X_k = \frac{\alpha_i}{(1+2\hat{\lambda}\alpha_i)^2}X_i \tag{25}$$

Lemma 1. *The monopolist wants to learn about high-volatility assets first.*

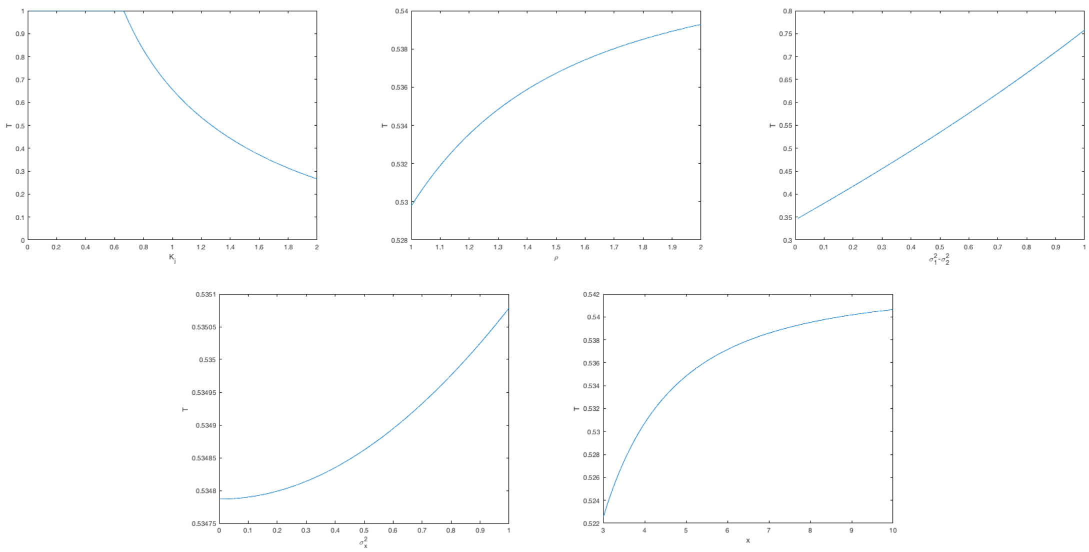
The returns to learning are increasing in the volatility of the asset. One way to interpret this finding is that there is more to learn about, when payoffs are volatile. This leads naturally to our first main proposition:

Proposition 2. *For sufficiently low levels of market power, the monopolist will learn only about the most volatile asset.*

If the monopolists were infinitesimally small, they would behave like a member of the fringe and specialize. What Proposition 2 shows is that specialization lasts until a critical level of size is reached. At that level, the marginal benefit of information is low enough that the monopolists will learn about multiple assets. The threshold level of specialization is characterized by the value T, such that:

$$\frac{e^{2K_j}}{(1+2\frac{T}{\lambda_0}e^{2K_j})^2}\left(\left(\frac{\rho}{\lambda_0}\right)^2(\bar{x}^2+\sigma_x^2)\sigma_1^2+1+2\frac{T}{\lambda_0}\right)-\frac{1}{(1+2\frac{T}{\lambda_0})^2}\left(\left(\frac{\rho}{\lambda_0}\right)^2(\bar{x}^2+\sigma_x^2)\sigma_2^2+1+2\frac{T}{\lambda_0}\right)=0 \tag{26}$$

where σ_1 is the most volatile asset, and σ_2 is the next most volatile asset. The first addend represents the marginal benefit of information once the monopolist has learnt *only* about the most volatile asset, while the second represents the marginal benefit of not learning at all about the second asset. When the two addends are equal, it means that the monopolist is satisfied learning exclusively about only one asset, but that even a slight increase in the monopolists size would lead to diversification. Because $\sigma_1 > \sigma 2$, the expression is decreasing in T. Therefore, we can see that the monopolist will specialize for longer as σ_1 increases, because the marginal benefit of learning about the most volatile asset is higher. As ρ, $\bar{x}$, and σ_x^2 increase, the marginal benefit of learning about any asset increases, so the monopolist will specialize for longer. Further the threshold decreases in K_j, because price impact is larger the more informed the monopolist is. The plots below show the relationships between the model's parameters and the threshold level past which the monopolist diversifies.



Proposition 3. *Higher levels of market power for the monopolist increases the price informativeness of low-volatility assets, and reduces the price informativeness of high-volatility assets.*

α therefore only moves monotonically with volatility, and faces distributional changes in response to movements in all other parameters. The expression for price informativeness is:

$$\frac{Cov(p_i, z_i)}{\sigma_{pi}} = \frac{\lambda_1(\alpha_{ji}-1)\sigma_i}{\sqrt{\left[\lambda_1^2(\alpha_{ji}-1)^2 + (1+\hat{\lambda}_{ji}\alpha_{ji})^2\rho^2\sigma_i^2\sigma_{xi}^2 + \lambda_1^2\right]}} \tag{27}$$

$$= \frac{(\alpha_{ji}-1)\sigma_i^2}{\sqrt{\left[(\alpha_{ji}-1)^2\sigma_i^2 + \frac{\sigma_{xi}^2\alpha_{ji}^2}{W^2} + \sigma_i^2\right]}} \tag{28}$$

Where $W = \frac{\partial \lambda_1 q_{ji}(\hat{\mu}_{ji})}{\partial \hat{\mu}_{ji}} = \frac{\lambda_1 \alpha_{ji}}{\rho \sigma_i^2 (1+\hat{\lambda}_{ji}\alpha_{ji})}$. Thus, price informativeness only depends on λ through W. W represents the sensitivity of asset quantity decisions to the signal received about the shock - and only affects price informativeness through the terms related to the noise traders c_i.

3.3.1 A Marginal Cost of Information

Under the assumption that the monopolist has a capacity for information, that capacity must be used completely. However, if we instead assume that the monopolist has a marginal cost of information, we can examine how much attention she chooses to pay overall as a function of the model's parameters. The first order condition of such a problem (again assuming no learning by the fringe) would be:

$$\mu = \frac{\alpha_i}{2\rho(1+2\hat{\lambda}\alpha_i)^2}\left(\left(\frac{\rho}{\lambda_0}\right)^2(\bar{x}^2+\sigma_{ix}^2)\sigma_i^2+1+2\hat{\lambda}\right)$$

where μ is the marginal cost to the monopolist of increasing the total amount of attention paid. The FOC will only hold, of course, if there is an interior solution. Since there is no upper bound on how much attention can be paid, the condition for an interior solution for asset i is that:

$$\mu < \frac{1}{2\rho(1+2\hat{\lambda})^2}\left(\left(\frac{\rho}{\lambda_0}\right)^2(\bar{x}^2+\sigma_{ix}^2)\sigma_i^2+1+2\hat{\lambda}\right)$$

Conditional on there being an internal solution, the total amount of attention paid to asset i is increasing in ρ, $\bar{x}$, σ_{ix}^2, σ_i^2, and decreasing in λ.

3.4 Price Informativeness

Now, let us turn to the general form of the model (still assuming a fringe that cannot learn). Price informativeness in the model is given by the covariance of the price with the fundamental shock, normalized by the standard deviation of the price.

Alternatively, this can be seen as the correlation of the price with the fundamental, multiplied by the asset's variance. This definition is taken from Bai, Philippon, and Savov (2016).

$$PI = \frac{b_i \sigma_i}{\sqrt{b_i^2 + c_i^2 \sigma_{xi}^2 / \sigma_i^2 + \sum_j d_{ji}^2 \frac{\alpha_{ji}-1}{\alpha_{ji}^2}}},$$

where a_i, b_i, c_i and d_{ji} are the coefficients of the equilibrium price function. In the expression for PI, b_i parameterizes the covariance of the price with the shock z_i; the second term in the denominator captures noise in the price coming from the noise trader demand shock, and the third term in the denominator captures the noise in the price coming from the noise in the oligopolists' private signals. Clearly, the lower are the noise terms relative to the signal term b_i, the higher is price informativeness. Plugging in terms, we get:

We can use the equilibrium expressions for the price coefficients to express PI as

$$PI = \frac{\sigma_i \sum_j \omega_{ji} \frac{\alpha_{ji}-1}{\alpha_{ji}}}{\sqrt{\left(\sum_j \omega_{ji} \frac{\alpha_{ji}-1}{\alpha_{ji}}\right)^2 + \frac{1}{(\sum_j W_{ji})^2} \frac{\sigma_{xi}^2}{\sigma_i^2} + \sum_j \omega_{ji}^2 \frac{\alpha_{ji}-1}{\alpha_{ji}^2}}}, \tag{29}$$

where ω_{ji} is the average share held by oligopolist j of asset i, given by

$$\omega_{ji} \equiv \frac{Q_{ji}}{\sum_k Q_{ki}},$$

and Q_{ji} is the average quantity held of asset i by oligopolist j, and W_{ji} is the responsiveness of the quantity traded of asset i by oligopolist j to the private signal of oligopolist j, i.e.

$$W_{ji} = \frac{\partial \lambda_j q_{ji}}{\partial \hat{\mu}_{ji}} = \frac{\lambda_j \alpha_{ji}}{\rho \sigma_i^2 (1 + \hat{\lambda}_{ji} \alpha_{ji})}.$$

We term this the *passthrough of information to quantities*. Intuitively, PI is increasing in the term $\sum_j \omega_{ji} \frac{\alpha_{ji}-1}{\alpha_{ji}}$, which is the ownership share-weighted average of the reduction in uncertainty[4] about asset is payoffs due to learning by oligopolist j. PI is also increasing in the information passthrough to quantities. The more information affects trading, the more it shows up in prices.

Finally, PI is decreasing in the term $\sum_j \omega_{ji}^2 \frac{\alpha_{ji}-1}{\alpha_{ji}^2}$, which is given by the weighted sum of the noise in private signals, with weights given by the square ownership shares of each oligopolist. We term this the *Learning HHI* or LHHI. To see that this expression is related to ownership concentration, notice that in a symmetric case of $\alpha_{ji} = \alpha_i$, it simplifies to $\frac{\alpha_i - 1}{\alpha_i^2} HHI_i$, where HHI_i is the Herfindahl index for asset i. Therefore, if the noise in oligopolists' signals is equally volatile, high concentration hurts PI through this channel.

The expression in (30) highlights the importance of modeling the choice of information for price informativeness. For an exogenously fixed learning structure (i.e. fixed $\{\alpha_{ji}\}_{j=1,..,l,i=1,...,n}$, putting high weight on the highest α oligopolist always increases the numerator of PI and hence is beneficial. However, working through the third term in the denominator, high ownership concentration could be detrimental (e.g. for equal αs), or beneficial (e.g. for very unequal distribution of α). Hence, the information structure one assumes in an exogenous information model will dictate the conclusion on the benefits of ownership concentration.

3.5 Passive and active large investors

In this section, we consider the role of passive and active institutional investors and their interaction in determining price informativeness of asset i. In order to make the analysis tractable and allow for clear exposition, we focus on a case where investors 1 through k are active, and investors $k+1$ through l are passive. In this

[4]Note that $(\alpha_{ji} - 1)/\alpha_{ji} = (\sigma_i^2 - \hat{\sigma}_{ji}^2)/\sigma_i^2$

case, price informativeness is:

$$PI = \frac{\sigma_i \sum_{j\le k} \omega_{ji} \frac{\alpha_{ji}-1}{\alpha_{ji}}}{\sqrt{\left(\sum_{j\le k} \omega_{ji} \frac{\alpha_{ji}-1}{\alpha_{ji}}\right)^2 + \frac{1}{(\sum_{j\le k} W_{ji})^2 + \sum_{j>k} W_{ji}^2} \frac{\sigma_{xi}^2}{\sigma_i^2} + \sum_{j\le k} \omega_{ji}^2 \frac{\alpha_{ji}-1}{\alpha_{ji}^2}}}, \tag{30}$$

The presence of passive investors does not contribute to price informativeness via increased correlation of prices with the fundamental, nor does it detract from price informativeness due to increased noise in signal acquisition. However, the term associated with noise trading decreases. Obviously, for the passive investors there is no 'information passthrough'—the W term is expressing how sensitive the agent would be should her information set change. The change in the noise term reflects the fact that a large player will trade less than a fringe set of the same size. The lower levels of trade reduce the contribution of noise traders to the volatility of the price.

Price informativeness is therefore lower than it would be should the passive traders be instead active. On the other hand, it is higher than it would be if all passive traders were fringe traders instead. The first part of that claim is unsurprising, but the impact of increasing the size of passive players at the expense of active sector must have an effect on the learning decisions of the active players. We find that the effects exacerbate the reduction in price informativeness, which we summarize in the following proposition.

Proposition 4. *If the passive sector increases at the expense of the active sector, active investors specialize in their learning, further reducing Price Informativeness.*

4 Numerical Analysis

In this section, we provide a set of quantitative results from the solution to the equilibrium of the model.[5] We select parameter values for the return distribution

[5]This involves solving a fixed point of the best responses of the oligopolists to each other's learning and trading policies.

$\bar{z}$ and $\{\sigma_i\}_{i=1}^n$, the liquidity distribution $\bar{x}$ and $\{\sigma_{xi}\}_{i=1}^n$, the risk-free return r, risk aversion ρ, learning capacities K_h and $\{K_j\}_{j=1}^l$, and fringe and oligopolist sizes λ_0 and $\{\lambda_j\}_{j=1}^l$. The simulation generates equilibrium levels of price informativeness, oligopoly holdings, and oligopoly concentration for each asset.

In our simulations, we choose the parameters with two goals in mind: they have to be in an empirically relevant region of the parameter space and the solution needs to involve some degree of learning. Specifically, we consider parameters such that the benchmark model exhibits: (i) learning about all assets, (ii) aggregate institutional holding share of between 60 and 70% (which corresponds to the information in Figure 1), (iii) market excess real return of around 7% (which corresponds to the average over 1980-2015). For the results reported below, we set the number of assets to $n = 10$ and the number of oligopolists to $l = 6$. We report parameter values in Table 1.

Table 1: Parameter values

Parameter	Symbol	Value
Mean payoff, supply	$\bar{z}_i$, $\bar{x}_i$	10, 5 for all i
Number of assets	n, l	$10, 6$
Risk-free rate	r	2.5%
Vol. of noise shocks	σ_{xi}	0.41 for all i
Vol. of asset payoffs	σ_i	$\in [1, 1.5]$, linear distribution
Risk aversion	ρ	1.3
Information capacities	K_h, $\{K_j\}$	$0, 4.5$, constant
Investor masses	$\lambda_0, \lambda_l/\lambda_1$	$0.45, 4$ λ_js linearly distributed

4.1 Cross-sectional patterns

We begin by analyzing the cross section of equilibrium output variables across assets for the benchmark parameter values in Table 1. Figure 4 presents the relation between equilibrium price informativeness per asset (on the y-axis) and equilibrium oligopoly holdings per asset (on the x axis). The intuition for this result is based on our analysis in Section **??**—agents want to learn about high-volatility assets first, because those are the most rewarding. Therefore, price informativeness is increasing in underlying volatility, and so are total oligopoly holdings.

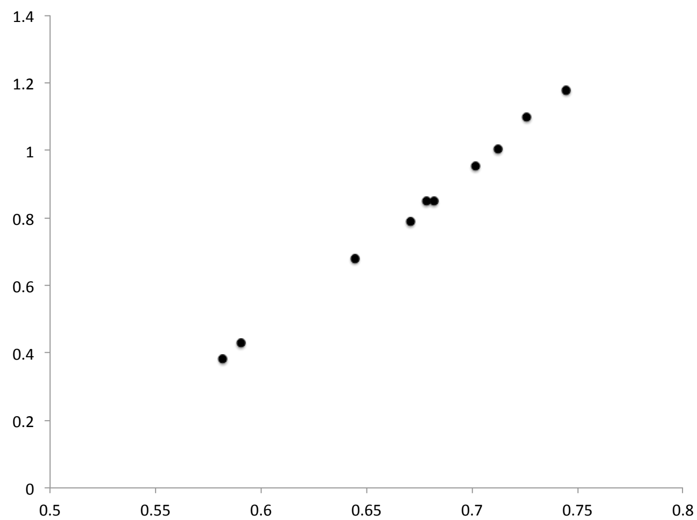


Figure 4: Price informativeness and institutional ownership

Figure 5 presents the relation between equilibrium price informativeness and equilibrium oligopoly concentration. The larger an oligopolist's presence in a particular asset's market, the more likely she is to internalize the price effect of her trade. As such, she would like to be less informed than she would be if she had a small presence. As a result, concentration in a particular asset is associated with lower levels of price informativeness.

In Figure 6, we present the above cross-sectional relations for the part of price informativeness due to only the correlation of prices and shocks. That is the part of the information measure that is endogenous to the information choices of agents, and does not come from pure cross-sectional dispersion of the exogenous shocks (see, equation (27)). As the figure indicates, the positive relation with institutional holdings and the negative relation with concentration hold for the correlation part of price informativeness, consistently with the empirical patterns documented before.

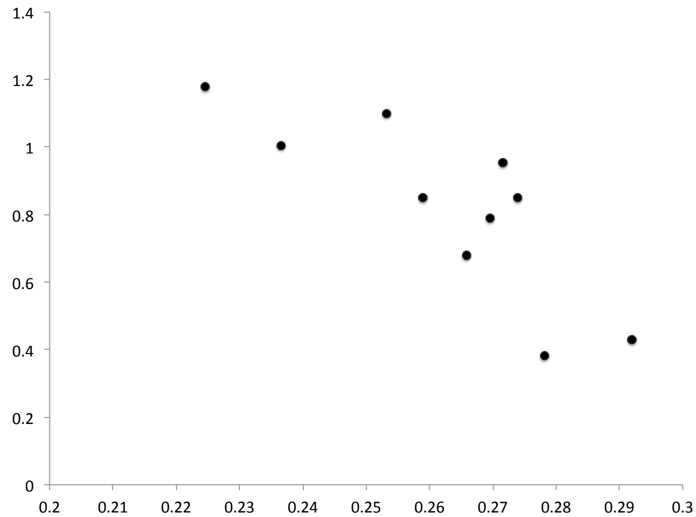


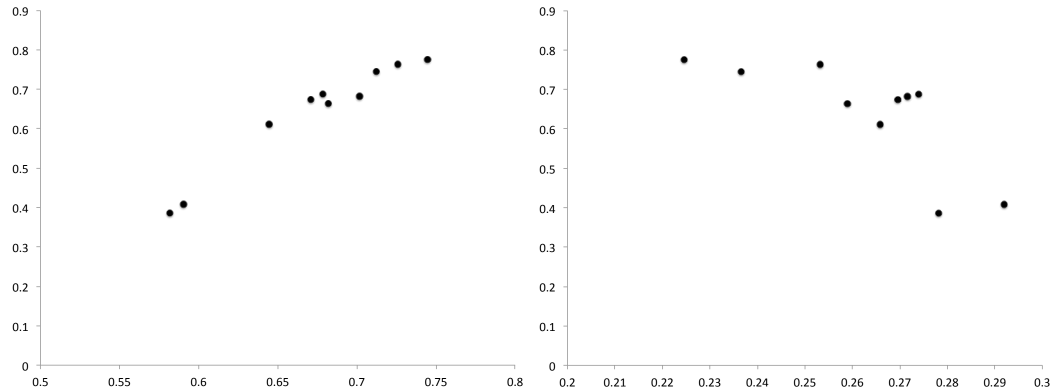
Figure 5: Price informativeness and concentration

4.2 Policy experiments

The different signs of the relation suggest an interesting interaction between high ownership and high concentration for the overall effect on price informativeness. We now move on to analyze the effect of policy on the aggregate price informativeness. While in Figures 4 and 5, each point corresponded to one asset, in the following exercise, each point corresponds to one iteration of a full financial market (with several assets). The experiments are useful as a way to isolate the relative effects of institutional concentration and holdings on price informativeness.

The size of the oligopoly In our first experiment, we look at how average price informativeness across assets changes in response to different levels of λ_0. Holding the relative distribution of λ_j fixed, we look at simulations of the model for varying λ_0 from 0.05 to 0.95. The type of policy being tested here could be thought of as a limit on entry, or a limit on a per-agent size in a given market, which would then affect the *composition* of ownership in the market, keeping the total mass of investors constant.

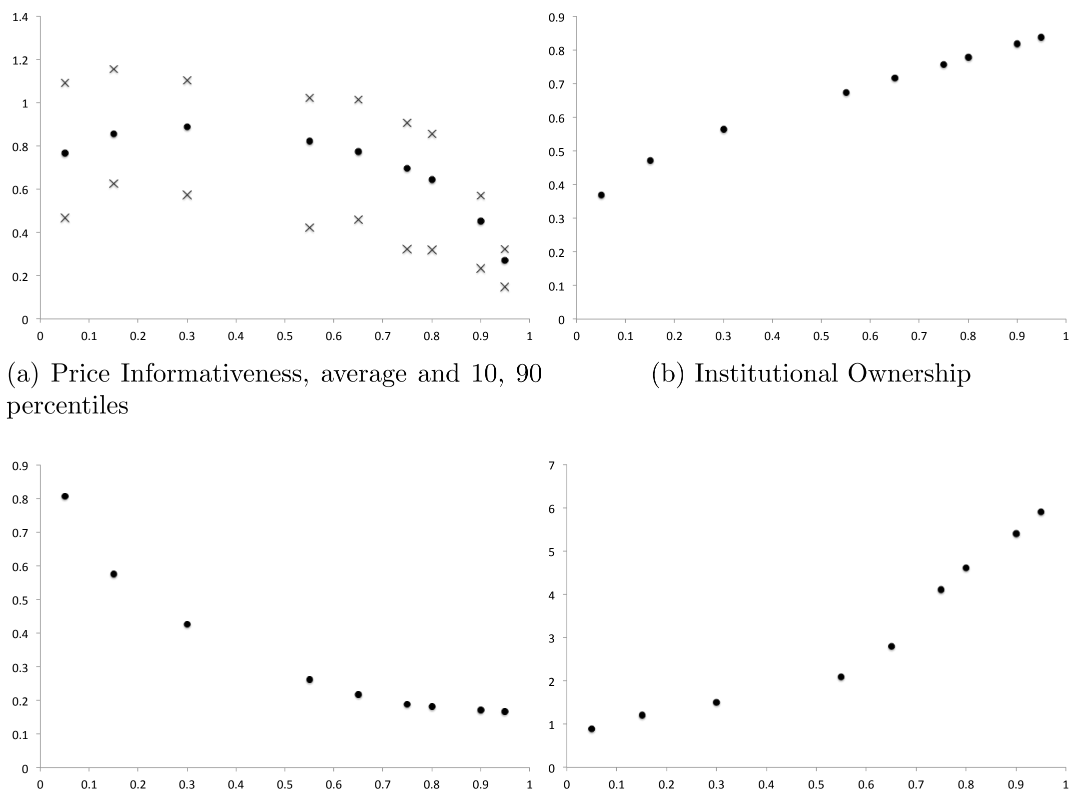
Figure 7 shows the relation between the size of the institutional sector (param-



(a) Relative to institutional ownership. (b) Relative to concentration.

Figure 6: Price informativeness: correlation only

eterized by $1 - \lambda_0$) and endogenous variables of interest. The price informativeness in Panel (a) shows a hump-shaped relation, on average and also for each asset (as indicated by interquantile 10-90 range) with the parameterized size, and hence also with the actual realized ownership which is monotonically increasing (Panel (b)). The model results point to an interior solution to optimal institutional sector size. This result can be explained by a tradeoff between more efficient (i.e. diversified) learning due to larger size of the institutional sector, and an inefficiency due to the endogenous restriction of size of trades (quantities) due to the price impact considerations of the large investors. When initially the size of the institutional sector is small, the oligopolists' price impact considerations are not very important in their quantity decisions, which means that increasing their size will mean more diversified learning without adverse impact of how quantities react to individual signals (and hence show up in price). As the size of the institutional sector increases further, learning about each asset becomes more diversified: additional oligopolists start learning (Panel (d)) and trading any given asset, which results in a large drop in concentration (Panel (c)), and increased ownership (Panel (b)). The increased diversification in learning means more efficient price informativeness while still relatively small size means that the negative quantity effects have not kicked in. Above a certain size of the institutional sector, the information choice is fully diversified and does not change by much further, but the size considerations are very significant and result in *decreasing* the size of trades as the size of the sector goes up – too much of the information is revealed in prices as quantity reacts to private signals. These effects give rise to a hump-shaped relation in the model between price informativeness and both institutional ownership and the concentration measure of that ownership. We present these results in Figure 8.



(a) Price Informativeness, average and 10, 90 percentiles

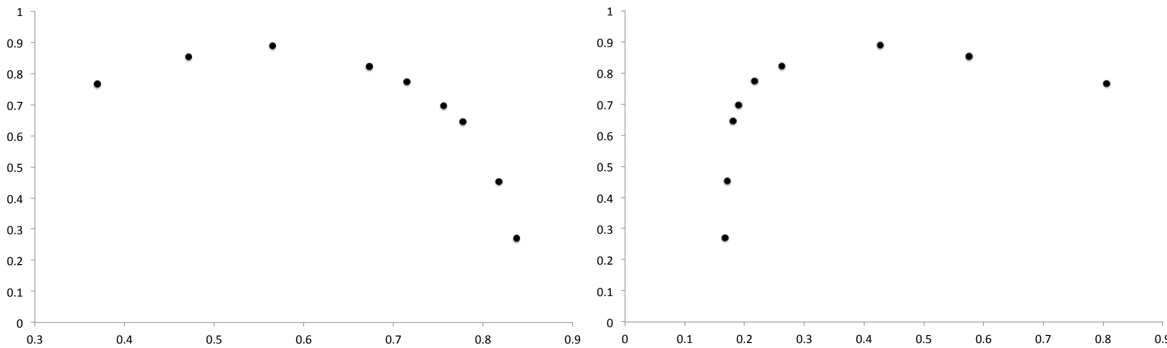
(b) Institutional Ownership

(c) Concentration

(d) Average Num. of Active Traders per Asset

Figure 7: Response of model to changing size of institutional sector $(1 - \lambda_0)$.

The concentration of the oligopoly Now, we consider the effects of a policy that affects the concentration of the actively trading oligopolists. Holding λ_0 constant, we vary the size distribution of $\{\lambda_j\}$ in order to measure an impact on the concentration



(a) Relative to institutional ownership

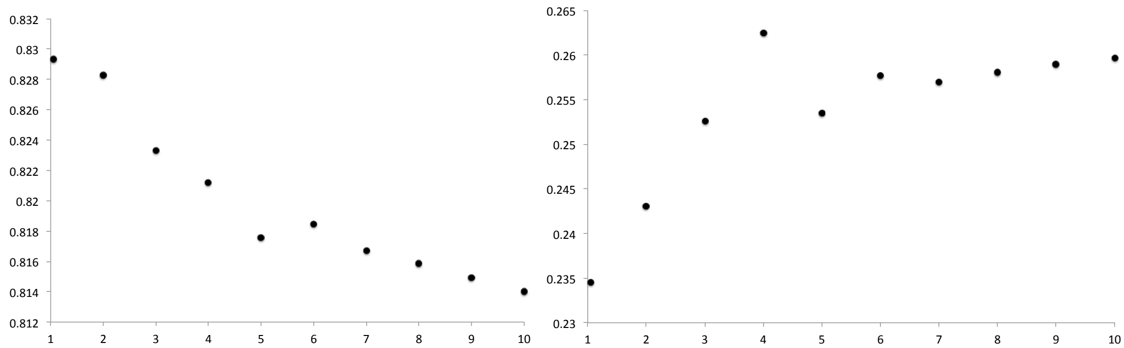
(b) Relative to concentration.

Figure 8: Price informativeness in the model when changing $1 - \lambda_0$.

measure. Specifically, we vary λ_l/λ_1 from 1.05 to 10, with intermediate λ_js growing linearly from λ_1 to λ_l. In doing so, the sum of all λ_js is kept equal to $1 - \lambda_0$ to isolate the effect of concentration on endogenous variables. Figure 9 presents the results for price informativeness and concentration.[6] The results with respect to concentration are roughly monotonic: Holding ownership relatively constant, a decrease in concentration increases the price informativeness in the aggregate. This is in line with the intuition from the previous exercise. If ownership is relatively stable, then there is no change in *average* market power across these markets. However, changing the size distribution of the oligopolists towards a more unequal one increases the concentration of ownership and hence increases market power of some of the oligopolists, distorting their quantity choices more. That leads to a negative relation between concentration and price informativeness, while keeping the ownership stable.

Passive investors We further explore the predictions of the model by considering the role played by passive investors—those who have market power, but no capacity for informational investment. The growing importance of such investors (e.g., BlackRock, Vanguard) has been an important element of the asset management landscape in the last few decades. To this end, we consider two versions of the model. In one,

[6]Institutional ownership in this case varies only by 1.4% of the mean—by design—and hence we do not show its graph explicitly.



(a) Price Informativeness (b) Concentration

Figure 9: Average price informativeness and concentration relative to dispersion λ_l/λ_1

the smallest half of the institutional investors has zero information capacity (*small passive*); in the other, the largest half of the institutional investors has zero capacity (*big passive*). In Figure 10, we repeat the first experiment, and plot *average* price informativeness across different institutional ownership levels (via λ_0 variation). Not surprisingly, informativeness is always higher in the small passive case. In the model, size is an impediment to learning efficiently, as it makes more information get revealed via actual trades. Hence, having small informed agents implies their allocation of learning capacity is more efficient, which is reflected in aggregate informativeness.

4.3 The role of endogenous learning choice

In this section, we present a comparison of the model with endogenous learning choice (our benchmark) to a model in which the information structure is given and set the same as in the benchmark model. The model with a fixed information structure is similar in spirit to that presented in Kyle (1985), in that the effect of market power in the absence of endogenous reoptimization of information choices depends entirely on how the quantities adjust.

Figure 11 presents the interaction of institutional ownership and price informativeness, where the different points are generated by varying λ_0. The black dots represent the benchmark case in which we allow both quantities and information

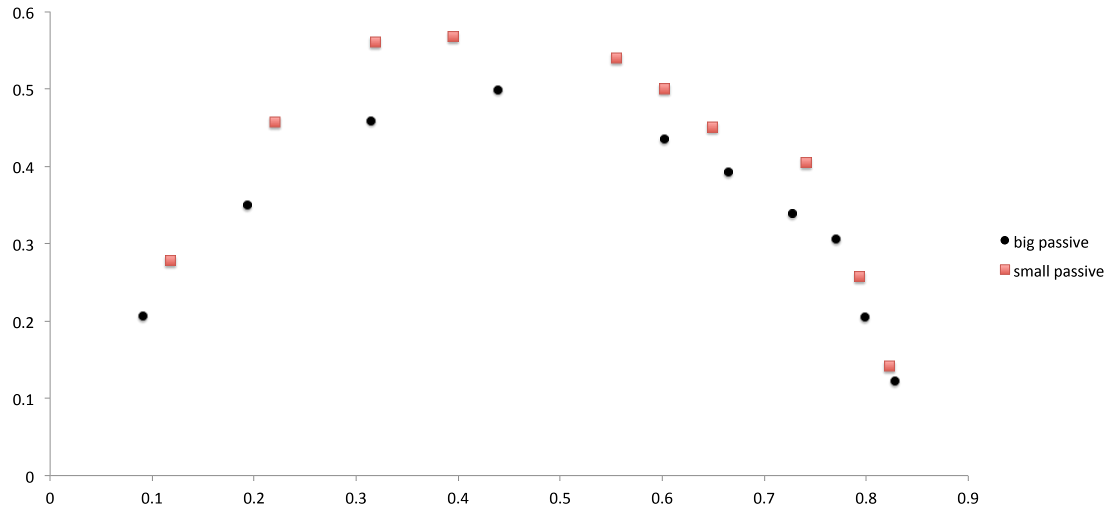
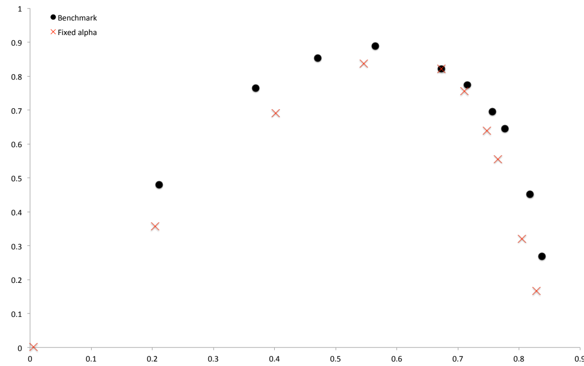


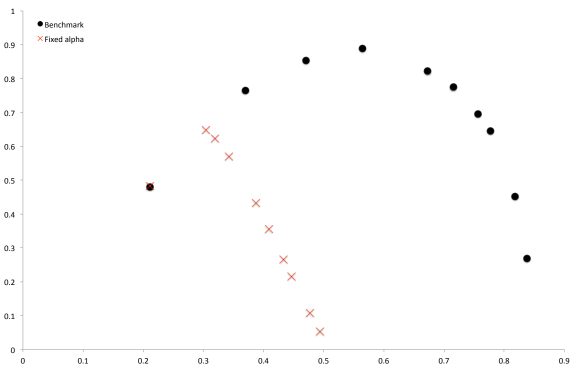
Figure 10: Aggregate price informativeness and institutional ownership: The case of big or small passive investors

choices to adjust in response to changing λ_0. The red crosses correspond to a case with a fixed learning structure. For the fixed learning cases, the information choice is either fixed at the benchmark value (i.e., $\lambda_0 = 0.45$, Panel (a)), or at values such that information structures are optimal at $\lambda_0 = 0.999$ (small oligopolists, Panel (b)) and $\lambda_0 = 0.05$ (large oligopolists, Panel (c)). In all the fixed-information cases, the level of price informativeness is below that of the benchmark model for which capacity choice adjusts optimally. The gains in price informativeness from optimal learning can be quite large. For example, for the benchmark specification of fixed alphas, price informativeness is reduced by up to 40%. More important, fixing the learning choices leads to very different conclusions about the optimal size of the institutional sector. Depending on what values of learning one exactly fixes, the optimal size lies either below or above the actual optimum derived when all the choices are endogenous. This finding underscores the importance of modeling the information choice margin when making normative statements about the size of the institutional sector.

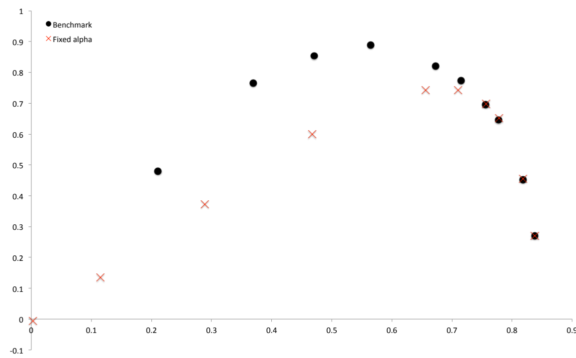
Next, we evaluate the 'concentration of oligopoly' exercise of Section 4.2, in which we hold λ_0 fixed but vary λ_l/λ_1. Figure 12 presents the relation between concentration of ownership and price informativeness for the benchmark model with endogenous



(a) Fixing information at benchmark market structure



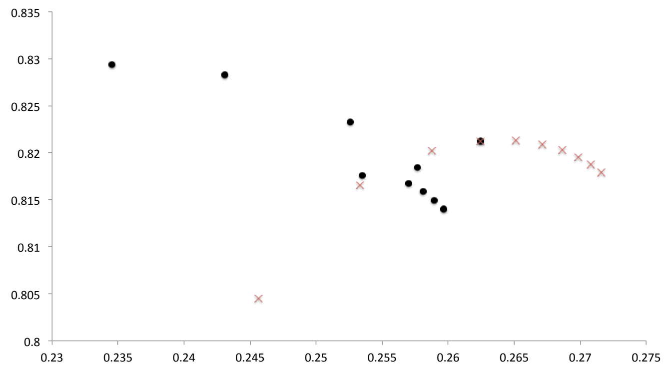
(b) Fixing information when $\lambda_0 = 0.999$



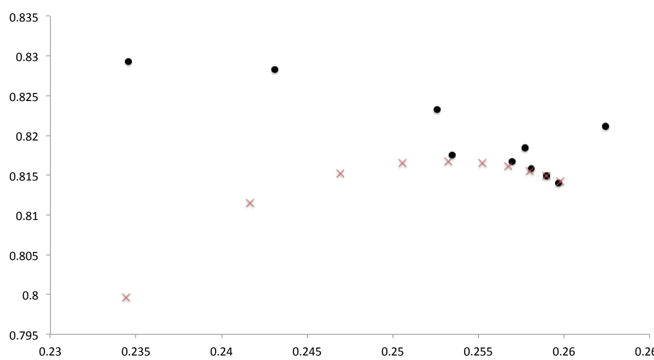
(c) Fixing information when $\lambda_0 = 0.05$

Figure 11: Aggregate price informativeness and institutional ownership with varying λ_0

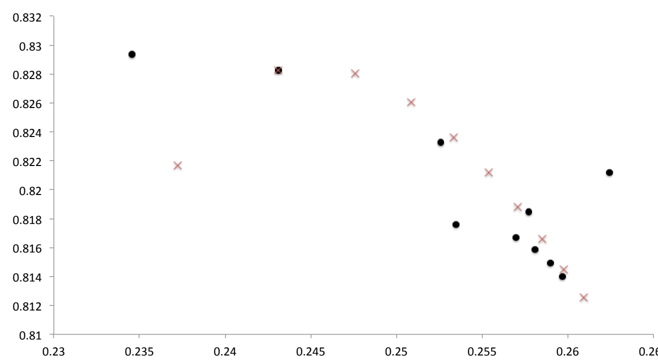
information choice (black dots), as well as three cases of fixing the information choice at the benchmark values (i.e., for $\lambda_l/\lambda_1 = 4$, Panel (a)), as well as at values that are optimal at two extremes of the size distribution of the oligopolists, $\lambda_l/\lambda_1 = 9$ (Panel (b)) and $\lambda_l/\lambda_1 = 2$ (Panel (c)). For all the three cases, the exogenous and endogenous information models give remarkably different predictions in terms of the relation of concentration and price informativeness. In particular, for the benchmark model, lower concentration always increases price informativeness. In contrast, models with fixed information structure exhibit a hump-shaped relation between concentration and price informativeness. Similar to the previous exercise, the two models give very different recommendations regarding the level of concentration that maximizes price informativeness. The exogenous information models optimally imply an intermediate level of concentration, while at the same time the fully endogenous model prescribes a concentration level that is at the lower bound of the potential values.



(a) Fixed alpha at benchmark value



(b) Fixed alpha for $\lambda_l/\lambda_1 = 9$



(c) Fixed alpha for $\lambda_l/\lambda_1 = 2$

Figure 12: Aggregate price informativeness and concentration of institutional ownership with varying λ_l/λ_1

Overall, we conclude that the predictions resulting from a model with endogenous learning choices are not a simple extension of the model where information choices are fixed. The differences are not only quantitatively important but also qualitatively relevant from the perspective of policy making.

5 Empirical Results

In this section, we provide an empirical verification of the main predictions coming from our model. In particular, we focus on the relations between ownership levels and its concentration, and price informativeness. Our goal is a more modest qualitative assessment of comparative statics rather than an attempt to match the quantities from the model. The specifics of our empirical methodology follow closely those in Bai, Philippon, and Savov (2016).

We begin by constructing an empirical measure of price informativeness. The measure captures the covariance between the price and fundamental information and is formally defined as:

$$PI_{t,h} = b_{t,h} * \sigma_t(log(M/A)), \tag{31}$$

where M denotes the market value of equity, A denotes the book value of assets, h is the investment horizon, and $b_{t,h}$ is a coefficient obtained from the following regression:

$$\frac{E_{i,t+h}}{A_{i,t}} = a_{t,h} + b_{t,h}log(\frac{M_{i,t}}{A_{i,t}}) + c_{t,h}log(\frac{E_{i,t}}{A_{i,t}}) + d_{t,h}SIC_{i,t} + \epsilon_{i,t,h}, \tag{32}$$

E is the value of earnings (EBIT) and SIC is an indicator variable for each one-digit SIC code. In our empirical tests, we set the horizon period h to be equal to one year.

To estimate the measure, we consider all companies in Compustat with valid financial information. Our data are recorded at an annual frequency and cover the period of 1980-2015. Following the observation in Bai et al. (2016) who argue that large firms have the most stable characteristics we also considered a subset of only large firms and also a set that excludes financial companies. The results remain qualitatively similar in all those cases and are available upon request. The PI measure is defined for each time period using a given cross-section of firms. In our tests, we sort companies into portfolio based on various characteristics of interest and thus the respective PI measures correspond to a particular portfolio.

Specifically, each year, we sort companies into deciles according to their ownership levels. For each decile portfolio, we calculate the equal-weighted PI measure. Next, we aggregate information for each individual portfolio using the time-series dimension. We present the results in column (2) of Table 2.

Table 2: Decile Sorts and Price Informativeness

Decile	Ownership	Concentration	Residualized Concentration
1	-0.0214	0.0127	0.0075
2	-0.0147	0.0105	0.0039
3	-0.0083	0.0065	0.0020
4	-0.0061	0.0018	0.0006
5	-0.0020	-0.0011	-0.0012
6	0.0010	-0.0076	-0.0023
7	0.0068	-0.0089	-0.0050
8	0.0073	-0.0117	-0.0063
9	0.0093	-0.0161	-0.0085
10	0.0115	-0.0185	-0.0126

We observe that price informativeness is monotonically increasing with ownership levels. Companies with lowest ownership have the least informative prices and companies with highest ownership have the most informative prices. This result is consistent with predictions of our model and also confirms the empirical results in Bai et al. (2016).

Next, we perform a similar sort, this time based on measure of concentration. Our measure of concentration is the Herfindahl-Hirshman Index (HHI), defined in Section 2. We present the results from the sort in column (3). Consistent with our predictions, we observe that price informativeness is decreasing in the level of concentration. Companies with the highest levels of ownership concentration have the least informative prices. The opposite is true for companies which have the most dispersed ownership.

Since the HHI mechanically depends on the number of institutional investors, the issue arises whether the results we identify are a function of pure concentration or result from the relation between number of institutions and price informativeness. Given that ownership is positively related to PI, this would suggest the pure concentration effect should be in fact even larger than the one we identify. To evaluate this claim, we obtain residuals from the regression of the index on the number of participants and use it as a sorting variable in our exercise. We present the results in column (4).

The PI measure we use in our analysis nests two economic effects: the effect on the correlation structure between prices and fundamentals and the effect of volatility of volatility of prices. While the correlation is directly related to information story, volatility in the data could change for reasons other than information. To assess whether our analysis is not driven by any non-information component, we consider the correlation between prices and fundamentals as an alternative information measure. We perform similar three sorts and before and present our results in Table 3.

In column (2), we present the results for ownership-based sorts. Again, we find a very strong monotonic relation between the levels of ownership and correlation between prices and fundamentals. The magnitude of the differences between the top and the bottom decile is economically very large and equals approximately 0.2. Similarly, in column (3), we report the results for the sorts based on levels of ownership concentration and document a similar negative relation between concentration and the correlation measure. The most concentrated portfolio has a correlation value which is lower by approximately 0.23 than the correlation of the least concentrated portfolio. Finally, in column (4), we report the results for the sorts based on residualized concentration measure and find very similar results. Overall, our results suggest that

Table 3: Decile Sorts and Correlation between Price and Fundamentals

Decile	Ownership	Concentration	Residualized Concentration
1	-0.0769	0.1626	0.0679
2	-0.0533	0.1147	0.0386
3	-0.0312	0.0629	0.0253
4	-0.0257	0.0246	0.0068
5	-0.0026	0.0004	-0.0004
6	0.0146	-0.0327	-0.0108
7	0.0525	-0.0360	-0.0203
8	0.0682	-0.0445	-0.0232
9	0.1004	-0.0585	-0.0314
10	0.1270	-0.0692	-0.0433

the variation in PI that we document in the paper is unlikely to be driven only by non-information forces. Moreover, our results strongly confirm the theoretical predictions coming from our model. Since we do not attempt the model and the data on quantities, we cannot determine whether the information effect is the sole driver of the variation in price informativeness.

6 Concluding Remarks

The last few decades have observed important changes in institutional equity ownership structure, with significant consequences for financial stability and social welfare. These trends have triggered an active response from financial regulators and finance industry. While several participants in the debate have raised important reasons for or against regulatory changes, the ultimate verdict is difficult to reach in the absence of a well-specified economic model. This paper aims to take one step in this direction by offering a general equilibrium model in which asymmetric information, market power, and heterogeneity of assets play an important role. We think this setting is a good way to characterize the world of equity ownership. Our goal is to rank various equilibria by comparing their average price informativeness.

We show that for the level of ownership equal to the currently observed levels in the U.S. (roughly 60%), to average price efficiency is positively related to the levels of institutional ownership and negatively related to its concentration. This cross-sectional result is strongly supported by the data. Further, we show that the average price informativeness across assets is maximized for the values of ownership and concentration that are strictly within the range of admissible outcomes. This result suggests an interesting role for policy makers to enforce optimal structure of equity ownership.

Our model can be flexibly applied to settings with rich cross-section of assets, differences in information asymmetry across agents, and differences in market power. Hence, it can generate interesting policy implications at the aggregate and cross-sectional dimensions. It can also be a good tool to evaluate asset pricing implications in the presence of market power and information asymmetry. We leave these questions for future research. At the same time, while our research can inform the debate for the role of institutional owners for price informativeness and learning in the economy, it naturally abstracts from other important dimensions relevant for policy makers, such as investment costs or flows of funds in and out of the sector.

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7 Appendix: Proofs

7.1 Model

7.1.1 Derivation of Proposition 4

Proof. In order to apply Kakutani's Fixed Point Theorem, we need to define a few terms. Agents select α_i First, define $A_i(\{\alpha_{-j}\})$ to be the best response correspondence for oligopolist j. Next define $\alpha = \{\alpha_1, \alpha_2, ..., \alpha_L\}$, and let $\aleph$ define the set of all possible α. Then the best response correspondence can be defined as $A : \aleph \Rightarrow \aleph$ such that for all $\alpha \in \aleph$, we have that $A(\alpha) = [A_j(\alpha_{-j})]_{j \in L}$. This best response function takes into account the associated demand schedule for every oligopolist, as well as the learning and demand decisions for the fringe. Now we need to check whether there is a fixed point to A.

- $\aleph$ is compact and convex. Each α_j must satisfy the capacity constraint. Therefore each α_j is convex, closed, and bounded, and therefore compact. Therefore $\aleph$ is as well.

- A is non-empty. This is trivially true if an interior solution exists. If an interior solution does not exist, then the solutions are corners, so A is always non-empty.

- A is convex-valued. A is convex iff A_i are all convex. The oligopolist's objective function is weakly more concave than the fringe's due to size. We show here that the second derivative is negative.

$$\frac{2\rho\mu\left(\lambda_0+\lambda_j\alpha_{ji}\right)^4}{\alpha_{ji}N_i^2} = X_i(\lambda_0+2\lambda\alpha)\left(\lambda_0^2-\lambda_j^2\alpha_{ji}^2-2N\lambda_0\lambda_j\alpha_{ji}+\alpha_{ji}\right)+Y_i\left[2\lambda_j-2\lambda_j(N_i\lambda_0+\lambda_0+\lambda_j\alpha_{ji})(\lambda_0+2\lambda_j\alpha_{ji})\right]$$

$$\underbrace{u' - P(x)u^2 - Q(x)u - R(x)}_{=0,\text{ since } u \text{ is a particular solution.}}$$

$$\frac{2\rho\mu\left(\lambda_0+\lambda_j\alpha_{ji}\right)^2}{\alpha N_i} = \left[2N_i'(\lambda_0+2\lambda_j\alpha_{ji}) - \frac{2\lambda_j N_i\lambda_j\alpha_{ji}}{(\lambda_0+\lambda_j\alpha_{ji})}\right]$$

$$\left[\frac{\alpha_{ji}\rho^2\sigma_i^2}{\lambda_0}(\bar{x}^2+\sigma_{ix}^2) + \left(\frac{1-N_iM_{ji}}{N_i}\right)^2\lambda_0(\alpha_{ji}-1)\right]$$

$$+\left[N_i(\lambda_0+2\lambda_j\alpha_{ji})\right]\left[\frac{\rho^2\sigma_i^2}{\lambda_0}(\bar{x}^2+\sigma_{ix}^2) + \left(\frac{1-N_iM_{ji}}{N_i}\right)^2\lambda_0\right]$$

$$\frac{2\rho\mu\left(\lambda_0+\lambda_j\alpha_{ji}\right)^2}{\alpha N_i} = \frac{\rho^2\sigma_i^2}{\lambda_0}(\bar{x}^2+\sigma_{ix}^2)\left[(N_i+2\alpha_{ji}N_i')(\lambda_0+2\lambda_j\alpha_{ji}) - \frac{2\lambda_jN_i\lambda_j\alpha_{ji}^2}{(\lambda_0+\lambda_j\alpha_{ji})}\right]$$

$$+\left(\frac{1-N_iM_{ji}}{N_i}\right)^2\lambda_0\left[(N_i+2(\alpha_{ji}-1)N_i')(\lambda_0+2\lambda_j\alpha_{ji}) - \frac{2\lambda_jN_i\lambda_j\alpha_{ji}(\alpha_{ji}-1)}{(\lambda_0+\lambda_j\alpha_{ji})}\right]$$

$$\frac{2\rho\mu\left(\lambda_0+\lambda_j\alpha_{ji}\right)^3}{\alpha N_i^2} = \frac{\rho^2\sigma_i^2}{\lambda_0}(\bar{x}^2+\sigma_{ix}^2)\left[((\lambda_0+\lambda_j\alpha_{ji})-2\alpha_{ji}N_i\frac{\lambda_j\lambda_0}{(\lambda_j\alpha_{ji}+\lambda_0)})(\lambda_0+2\lambda_j\alpha_{ji})-2\lambda_j^2\alpha_{ji}^2\right]$$

$$+\left(\frac{1-N_iM_{ji}}{N_i}\right)^2\lambda_0\left[((\lambda_0+\lambda_j\alpha_{ji})-2(\alpha_{ji}-1)N_i\frac{\lambda_j\lambda_0}{(\lambda_j\alpha_{ji}+\lambda_0)})(\lambda_0+2\lambda_j\alpha_{ji})-2\lambda_j^2\alpha_{ji}(\alpha_{ji}-1)\right]$$

$$\frac{2\rho\mu\left(\lambda_0+\lambda_j\alpha_{ji}\right)^3}{\alpha N_i^2} = \left(\frac{\rho^2\sigma_i^2}{\lambda_0}(\bar{x}^2+\sigma_{ix}^2)+\left(\frac{1-N_iM_{ji}}{N_i}\right)^2\lambda_0\right)\left[((\lambda_0+\lambda_j\alpha_{ji})-2\alpha_{ji}N_i\frac{\lambda_j\lambda_0}{(\lambda_j\alpha_{ji}+\lambda_0)})(\lambda_0+2\lambda_j\alpha_{j}\right.$$

$$\left.+\left(\frac{1-N_iM_{ji}}{N_i}\right)^2\lambda_0\left[(2N_i\frac{\lambda_j\lambda_0}{(\lambda_j\alpha_{ji}+\lambda_0)})(\lambda_0+2\lambda_j\alpha_{ji})+2\lambda_j^2\alpha_{ji}\right]\right.$$

$$\frac{\partial^2 U}{\partial\alpha_{ji}^2}\propto-\left[\frac{(N_i\lambda_0+\lambda_0+\lambda_j\alpha_{ji})(\lambda_0+2\lambda_j\alpha_{ji})-(\lambda_0+\lambda_j\alpha_{ji})^2}{(\lambda_0+\lambda_j\alpha_{ji})^4}\right](4X_iN_iN_i'\lambda_j\alpha_{ji}+2X_iN_i^2\lambda_j-4Y_iN_iN_i'\lambda_j)$$

$$-\frac{(2X_iN_i^2\lambda_j\alpha_{ji}-2Y_iN_i^2\lambda_j)}{(\lambda_0+\lambda_j\alpha_{ji})^8}\left[(\lambda_0+\lambda_j\alpha_{ji})^4\left((N_i'\lambda_0+\lambda_j)(\lambda_0+2\lambda_j\alpha_{ji})+2\lambda_j(N_i\lambda_0+\lambda_0+\lambda_j\alpha_{ji})-2\lambda_j(\lambda_0+\lambda_j\right.\right.$$

$$\left.-4\lambda_j\left(\lambda_0+\lambda_j\alpha_{ji}\right)^3\left((N_i\lambda_0+\lambda_0+\lambda_j\alpha_{ji})(\lambda_0+2\lambda_j\alpha_{ji})-(\lambda_0+\lambda_j\alpha_{ji})^2\right)\right]$$

$$+\left[\frac{(\lambda_0+\lambda_j\alpha_{ji})^2\left(2N_iN_i'(\lambda_0+2\lambda_j\alpha_{ji})+2N_i^2\lambda_j\right)-2\lambda_jN_i^2(\lambda_0+2\lambda_j\alpha_{ji})(\lambda_0+\lambda_j\alpha_{ji})}{(\lambda_0+\lambda_j\alpha_{ji})^4}\right]X_i$$

$$\frac{\partial^2 U}{\partial\alpha_{ji}^2}\propto-\underbrace{\left[\frac{(N_i\lambda_0+\lambda_0+\lambda_j\alpha_{ji})(\lambda_0+2\lambda_j\alpha_{ji})-(\lambda_0+\lambda_j\alpha_{ji})^2}{(\lambda_0+\lambda_j\alpha_{ji})^4}\right]}_{\frac{A-1}{(\lambda_0+\lambda_j\alpha_{ji})^2}>0}\underbrace{2N_i^2\lambda_j\left(-2N_i\frac{\lambda_j\lambda_0}{(\lambda_j\alpha_{ji}+\lambda_0)^2}(\alpha_{ji}X_i-Y_i)+X_i\right)}_{>0}$$

$$-\underbrace{\frac{(2X_iN_i^2\lambda_j\alpha_{ji}-2Y_iN_i^2\lambda_j)}{(\lambda_0+\lambda_j\alpha_{ji})^8}}_{>0}\left[(\lambda_0+\lambda_j\alpha_{ji})^4\left((N_i'\lambda_0+\lambda_j)(\lambda_0+2\lambda_j\alpha_{ji})+2\lambda_jN_i\lambda_0\right)\right.$$

$$\left.-\underbrace{4\lambda_j\left(\lambda_0+\lambda_j\alpha_{ji}\right)^3\left((N_i\lambda_0+\lambda_0+\lambda_j\alpha_{ji})(\lambda_0+2\lambda_j\alpha_{ji})-(\lambda_0+\lambda_j\alpha_{ji})^2\right)}_{>0}\right]$$

$$+\underbrace{\left[\frac{(\lambda_0+\lambda_j\alpha_{ji})^2\left(2N_iN_i'(\lambda_0+2\lambda_j\alpha_{ji})+2N_i^2\lambda_j\right)-2\lambda_jN_i^2(\lambda_0+2\lambda_j\alpha_{ji})(\lambda_0+\lambda_j\alpha_{ji})}{(\lambda_0+\lambda_j\alpha_{ji})^4}\right]X_i}_{<0}$$

$$\frac{\partial^2 U}{\partial \alpha_{ji}^2} \propto -\left[\frac{A-1}{(\lambda_0+\lambda_j\alpha_{ji})^4}\right] 2N_i^2\lambda_j\left(-2N_i\lambda_j\lambda_0(X_i\alpha_{ji}-Y_i)+X_i(\lambda_j\alpha_{ji}+\lambda_0)^2\right)$$

$$-\frac{2N_i^2\lambda_j(X_i\alpha_{ji}-Y_i)}{(\lambda_0+\lambda_j\alpha_{ji})^4}\left[\big((N_i'\lambda_0+\lambda_j)(\lambda_0+2\lambda_j\alpha_{ji})+2\lambda_jN_i\lambda_0\big)-4\lambda_j\Big((A-1)\,(\lambda_0+\lambda_j\alpha_{ji})\Big)\right]$$

$$+\left[\frac{(\lambda_0+\lambda_j\alpha_{ji})^2\left(2N_iN_i'(\lambda_0+2\lambda_j\alpha_{ji})+2N_i^2\lambda_j\right)-2\lambda_jN_i^2(\lambda_0+2\lambda_j\alpha_{ji})(\lambda_0+\lambda_j\alpha_{ji})}{(\lambda_0+\lambda_j\alpha_{ji})^4}\right]X_i$$

$$\frac{\partial^2 U}{\partial \alpha_{ji}^2} \propto -(A-1)2N_i^2\lambda_j\left(-2N_i\lambda_j\lambda_0(X_i\alpha_{ji}-Y_i)+X_i(\lambda_j\alpha_{ji}+\lambda_0)^2\right)$$

$$-(2N_i^2\lambda_j(X_i\alpha_{ji}-Y_i))\left[\big((N_i'\lambda_0+\lambda_j)(\lambda_0+2\lambda_j\alpha_{ji})+2\lambda_jN_i\lambda_0\big)-4\lambda_j\Big((A-1)\,(\lambda_0+\lambda_j\alpha_{ji})\Big)\right]$$

$$+\Big[(\lambda_0+\lambda_j\alpha_{ji})^2\left(2N_iN_i'(\lambda_0+2\lambda_j\alpha_{ji})+2N_i^2\lambda_j\right)-2\lambda_jN_i^2(\lambda_0+2\lambda_j\alpha_{ji})(\lambda_0+\lambda_j\alpha_{ji})\Big]X_i$$

$$\frac{\partial^2 U}{\partial \alpha_{ji}^2} \propto -2N_i^2\lambda_j\Big(\big((A-1)\,(-2N_i\lambda_j\lambda_0-4\lambda_j(\lambda_0+\lambda_j\alpha_{ji}))+\big((N_i'\lambda_0+\lambda_j)(\lambda_0+2\lambda_j\alpha_{ji})+2\lambda_jN_i\lambda_0\big)\big)\,(X_i\alpha_{ji}-Y_i$$

$$+X_i(\lambda_j\alpha_{ji}+\lambda_0)^2(A-1)\Big)$$

$$+\Big[(\lambda_0+\lambda_j\alpha_{ji})^2\left(2N_iN_i'(\lambda_0+2\lambda_j\alpha_{ji})+2N_i^2\lambda_j\right)-2\lambda_jN_i^2(\lambda_0+2\lambda_j\alpha_{ji})(\lambda_0+\lambda_j\alpha_{ji})\Big]X_i$$

$$\frac{\partial^2 U}{\partial \alpha_{ji}^2} \propto \left((2AN_i\lambda_j\lambda_0+(A-1)4\lambda_j(\lambda_0+\lambda_j\alpha_{ji}))+\left(\left(\frac{2N_i^2\lambda_j\lambda_0}{(\lambda_j\alpha_{ji}+\lambda_0)^2}\lambda_0-\lambda_j\right)(\lambda_0+2\lambda_j\alpha_{ji})\right)\right)(X_i\alpha_{ji}-Y_i)$$

$$-X_i(\lambda_j\alpha_{ji}+\lambda_0)^2(A-1)-\big[2N_i\lambda_0(\lambda_0+2\lambda_j\alpha_{ji})-(\lambda_0+\lambda_j\alpha_{ji})^2+(\lambda_0+\lambda_j\alpha_{ji})(\lambda_0+2\lambda_j\alpha_{ji})\big]X_i$$

$$\frac{\partial^2 U}{\partial \alpha_{ji}^2} \propto -\left((2AN_i+4A-5)\lambda_j\lambda_0+(4A-6)\lambda_j^2\alpha_{ji}+\frac{2N_i\lambda_j\alpha_{ji}\lambda_0}{(\lambda_j\alpha_{ji}+\lambda_0)^2}\lambda_0(\lambda_0+2\lambda_j\alpha_{ji})\right)Y_i$$

$$-X_i\left[(\lambda_j\alpha_{ji}+\lambda_0)^2(A-1)+\big[2N_i\lambda_0(\lambda_0+2\lambda_j\alpha_{ji})-(\lambda_0+\lambda_j\alpha_{ji})^2+(\lambda_0+\lambda_j\alpha_{ji})(\lambda_0+2\lambda_j\alpha_{ji})\big]\right.$$

$$-\left((2AN_i + 4A - 5)\lambda_j\alpha_{ji}\lambda_0 + (4A-6)\lambda_j^2\alpha_{ji}^2 + \frac{2N_i\lambda_j\alpha_{ji}\lambda_0}{(\lambda_j\alpha_{ji}+\lambda_0)^2}\lambda_0(\lambda_0 + 2\lambda_j\alpha_{ji})\right)\Bigg]$$

$$\frac{\partial^2 U}{\partial \alpha_{ji}^2} \propto -\left((2AN_i + 4A - 5)\lambda_j\lambda_0 + (4A-6)\lambda_j^2\alpha_{ji} + \frac{2N_i\lambda_j\alpha_{ji}\lambda_0}{(\lambda_j\alpha_{ji}+\lambda_0)^2}\lambda_0(\lambda_0 + 2\lambda_j\alpha_{ji})\right) Y_i$$

$$-\frac{X_i}{\lambda_0^2 + \lambda_j^2\alpha_{ji}^2 + 2\lambda_0\lambda_j\alpha_{ji}}\Bigg[(\lambda_j\alpha_{ji}+\lambda_0)^2((N_i+1)\lambda_0^2+2\lambda_j^2\alpha_{ji}^2+\lambda_0\lambda_j\alpha_{ji}(2N_i+3)-(\lambda_j\alpha_{ji}+\lambda_0)^2)+2N_i\lambda_0(\lambda_0+2\lambda_j\alpha_{ji})($$

$$-(\lambda_j\alpha_{ji}+\lambda_0)^4 + (\lambda_0 + \lambda_j\alpha_{ji})^3(\lambda_0 + 2\lambda_j\alpha_{ji}) - 2N_i\lambda_j\alpha_{ji}\lambda_0\lambda_0(\lambda_0+2\lambda_j\alpha_{ji})$$

$$-\left((2N_i+4)((N_i+1)\lambda_0^2 + 2\lambda_j^2\alpha_{ji}^2 + \lambda_0\lambda_j\alpha_{ji}(2N_i+3)) - 5(\lambda_j\alpha_{ji}+\lambda_0)^2\right)\lambda_j\lambda_0\alpha_{ji}$$

$$-(4((N_i+1)\lambda_0^2 + 2\lambda_j^2\alpha_{ji}^2 + \lambda_0\lambda_j\alpha_{ji}(2N_i+3)) - 6(\lambda_j\alpha_{ji}+\lambda_0)^2)\lambda_j^2\alpha_{ji}^2\Bigg]$$

$$\frac{\partial^2 U}{\partial \alpha_{ji}^2} \propto -\left(\left((2AN_i+4A-5)\lambda_j\lambda_0+(4A-6)\lambda_j^2\alpha_{ji}\right)(\lambda_j^2\alpha_{ji}^2+\lambda_0^2+2\lambda_0\lambda_j\alpha_{ji})+2N_i\lambda_j\alpha_{ji}\lambda_0^2(\lambda_0+2\lambda_j\alpha_{ji})\right)Y_i$$

$$-X_i\left[3N_i\lambda_0^4 + \left((3+2N_i-2N_i^2)\lambda_0^2 + (9-2N_i)\lambda_j^2\alpha_{ji}^2 + (5N_i+15)\lambda_0\lambda_j\alpha_{ji}\right)\lambda_j\lambda_0\alpha_{ji}\right]$$

$$\begin{aligned}
2AN_i+4A-5 &= (2N_i+4)((N_i+1)\lambda_0^2+2\lambda_j^2\alpha_{ji}^2+\lambda_0\lambda_j\alpha_{ji}(2N_i+3))-5(\lambda_0^2+\lambda_j^2\alpha_{ji}^2+2\lambda_0\lambda_j\alpha_{ji}) \\
&= \lambda_0^2(2N_i^2+6N_i-1) + \lambda_j^2\alpha_{ji}^2(4N_i+3) + \lambda_0\lambda_j\alpha_{ji}(4N_i^2+14N_i+2) \\
4A-6 &= 4((N_i+1)\lambda_0^2 + 2\lambda_j^2\alpha_{ji}^2 + \lambda_0\lambda_j\alpha_{ji}(2N_i+3)) - 6(\lambda_0^2 + \lambda_j^2\alpha_{ji}^2 + 2\lambda_0\lambda_j\alpha_{ji}) \\
&= \lambda_0^2(4N_i-2) + 2\lambda_j^2\alpha_{ji}^2 + \lambda_0\lambda_j\alpha_{ji}(8N_i)
\end{aligned}$$

Therefore" $\frac{\partial^2 U}{\partial \alpha_{ji}^2} < 0$.

- A has a closed graph. The first order conditions of the oligopolist are continuous, so this is trivial. (see above).

$\square$

7.1.2 Derivation of Equation 2

$$\begin{aligned}
\sigma_{pi}^2 &= b_i^2\sigma_i^2 + c_i^2\sigma_{xi}^2 + \sum_{j=1}^{n}\left(\widehat{\sigma}_{ji}^2 + \frac{\widehat{\sigma}_{ji}^4}{\sigma_i^4}\sigma_i^2 - \frac{2}{\alpha_{ji}}Cov(\epsilon_i,\delta_{ji}) - 2b_id_{ji}Cov\left(\varepsilon_i,\zeta_{ji}\right) - 2c_id_{ji}Cov\left(\nu_i,\delta_{ji}\right)\right) \\
&\quad + \sum_{j=1}^{n}\sum_{k\neq j} 2d_{ji}d_{ki}Cov(\zeta_{ji},\zeta_{ki}) \\
&= b_i^2\sigma_i^2 + c_i^2\sigma_{xi}^2 + \sum_{j=1}^{n}\left(d_{ji}^2\left(\widehat{\sigma}_{ji}^2 + \frac{\widehat{\sigma}_{ji}^4}{\sigma_i^2} - \frac{2\widehat{\sigma}_{ji}^2}{\sigma_i^2}\widehat{\sigma}_{ji}^2\right) - 2b_id_{ji}\left(\widehat{\sigma}_{ji}^2 - \frac{\sigma_i^2}{\alpha_{ji}}\right)\right) \\
&\quad + \sum_{j=1}^{n}\sum_{j\neq k} 2d_{ji}d_{ki}Cov\left(\delta_{ji} - \frac{1}{\alpha_{ji}}\varepsilon_i, \delta_{ki} - \frac{1}{\alpha_{ki}}\varepsilon_i\right) \\
&= b_i^2\sigma_i^2 + c_i^2\sigma_{xi}^2 + \sum_{j=1}^{n} d_{ji}^2\left(1 - \frac{1}{\alpha_{ji}}\right)\widehat{\sigma}_{ji}^2 + \\
&\quad \sum_{j=1}^{n}\sum_{k\neq j} 2d_{ji}d_{ki}\left(\frac{1}{\alpha_{ji}\alpha_{ki}}\sigma_i^2 - \widehat{\sigma}_{ki}^2 - \widehat{\sigma}_{ji}^2 + \sigma_i^2 - \frac{1}{\alpha_{ki}}\widehat{\sigma}_{ki}^2 - \frac{1}{\alpha_{ji}}\widehat{\sigma}_{ji}^2\right) \\
&= b_i^2\sigma_i^2 + c_i^2\sigma_{xi}^2 + \sum_{j=1}^{n} d_{ji}^2\left(1 - \frac{1}{\alpha_{ji}}\right)\widehat{\sigma}_{ji}^2 + \\
&\quad \sum_{j=1}^{n}\sum_{k\neq j} 2d_{ji}d_{ki}\left(\frac{1+\alpha_{ji}\alpha_{ki}}{\alpha_{ji}\alpha_{ki}}\sigma_i^2 - \frac{1+\alpha_{ki}}{\alpha_{ki}}\widehat{\sigma}_{ki}^2 - \frac{1+\alpha_{ji}}{\alpha_{ji}}\widehat{\sigma}_{ji}^2\right)
\end{aligned}$$

7.1.3 Derivation of Equation 8

The objective is $U_{0h} = \frac{1}{2\rho}\sum_{i=1}^{n}\frac{E_{0h}(\widehat{\mu}_{hi} - rp_i)^2}{\widehat{\sigma}_{hi}^2} = \frac{1}{2\rho}\sum_{i=1}^{n}\frac{\widehat{R}_i^2 + \widehat{V}_{hi}}{\widehat{\sigma}_{hi}^2}$, where

$\widehat{R}_i \equiv E_{0h}\left(\widehat{\mu}_{hi} - rp_i\right) = \overline{z} - r\overline{p}_i = \overline{z} - ra_i$, $\widehat{V}_{hi} \equiv V_{0h}\left(\widehat{\mu}_{hi} - rp_i\right) = Var\left(\widehat{\mu}_{hi}\right) + r^2\sigma_{pi}^2 - 2rCov\left(\widehat{\mu}_{hi}, p_i\right)$.

$Var\left(\widehat{\mu}_{hi}\right) = \sigma_i^2 - \widehat{\sigma}_{hi}^2$. $\sigma_{pi}^2 = b_i^2\sigma_i^2 + c_i^2\sigma_{xi}^2 + \sum_{j=1}^{n} d_{ji}^2\left(1 - \frac{1}{\alpha_{ji}}\right)\widehat{\sigma}_{ji}^2 + \sum_{j=1}^{n}\sum_{k\neq j} 2d_{ji}d_{ki}\left(\frac{1+\alpha_{ji}\alpha_{ki}}{\alpha_{ji}\alpha_{ki}}\sigma_i^2 - \frac{1+\alpha_{ki}}{\alpha_{ki}}\widehat{\sigma}_{ki}^2 - \frac{1+\alpha_{ji}}{\alpha_{ji}}\widehat{\sigma}_{ji}^2\right)$.

Posterior beliefs and prices are conditionally independent given payoffs.

$$\begin{aligned}
Cov\left(\widehat{\mu}_{hi}, p_i\right) &= \frac{1}{\sigma_i^2} Cov\left(\widehat{\mu}_{hi}, z_i\right) Cov\left(z_i, p_i\right) \\
&= \frac{1}{\sigma_i^2}\left(\sigma_i^2 - \widehat{\sigma}_{hi}^2\right)\left(Cov(\varepsilon_i, b_i\varepsilon_i) - \sum_{j=1}^{n} Cov(\varepsilon_i, d_{ji}\zeta_{ji})\right) \\
&= \frac{1}{\sigma_i^2}\left(\sigma_i^2 - \widehat{\sigma}_{hi}^2\right)\left(b_i\sigma_i^2 - \sum_{j=1}^{n} d_{ji} Cov\left(\varepsilon_i, \delta_{ji} - \frac{1}{\alpha_{ji}}\varepsilon_i\right)\right) \\
&= \frac{1}{\sigma_i^2}\left(\sigma_i^2 - \widehat{\sigma}_{hi}^2\right)\left(\left(b_i + \sum_{j=1}^{n} \frac{d_{ji}}{\alpha_{ji}}\right)\sigma_i^2 - \sum_{j=1}^{n} d_{ji}\widehat{\sigma}_{ji}^2\right) \\
&= \left(\sigma_i^2 - \widehat{\sigma}_{hi}^2\right) b_i
\end{aligned}$$

Hence

$$\begin{aligned}
\widehat{V}_{hi} &= \sigma_i^2 - \widehat{\sigma}_{hi}^2 + r^2\left(b_i^2\sigma_i^2 + c_i^2\sigma_{xi}^2 + \sum_{j=1}^{n} d_{ji}^2\left(1 - \frac{1}{\alpha_{ji}}\right)\widehat{\sigma}_{ji}^2 + \right. \\
&\left. \sum_{j=1}^{n}\sum_{k\neq j} 2d_{ji}d_{ki}\left(\frac{1+\alpha_{ji}\alpha_{ki}}{\alpha_{ji}\alpha_{ki}}\sigma_i^2 - \frac{1+\alpha_{ki}}{\alpha_{ki}}\widehat{\sigma}_{ki}^2 - \frac{1+\alpha_{ji}}{\alpha_{ji}}\widehat{\sigma}_{ji}^2\right)\right) - 2r\left(\sigma_i^2 - \widehat{\sigma}_{hi}^2\right) b_i
\end{aligned}$$

Expected utility becomes Hence $U_{0h} = \frac{1}{2\rho}\sum_{i=1}^{n} G_i \frac{\sigma_i^2}{\widehat{\sigma}_{hi}^2} - \frac{1}{2\rho}\sum_{i=1}^{n}(1 - 2rb_i)$, where

$$G_i \equiv G^{KNS} + r^2\left(\sum_{j=1}^{n} d_{ji}^2\left(1 - \frac{1}{\alpha_{ji}}\right)\widehat{\sigma}_{ji}^2 + \sum_{j=1}^{n}\sum_{k\neq j} 2d_{ji}d_{ki}\left(\frac{1+\alpha_{ji}\alpha_{ki}}{\alpha_{ji}\alpha_{ki}}\sigma_i^2 - \frac{1+\alpha_{ki}}{\alpha_{ki}}\widehat{\sigma}_{ki}^2 - \frac{1+\alpha_{ji}}{\alpha_{ji}}\widehat{\sigma}_{ji}^2\right)\right).$$

Note: $G_i^{KNS} \equiv \frac{\widehat{R}_i^2}{\sigma_i^2} + (1 - rb_i)^2 + r^2 c_i^2 \frac{\sigma_{xi}^2}{\sigma_i^2}$.

7.1.4 Derivation of Equation 12

Market clearing for each asset i is

$$\begin{aligned}
x_i &= \sum_{j=1}^{n} \lambda_j q_{ji} + \int_{H_i} q_{hi} dh \\
&= \sum_{j=1}^{n} \lambda_j q_{ji} + \int_{H_i} \frac{\widehat{\mu}_{hi} - rp_i}{\rho \widehat{\sigma}_{hi}^2} dh \\
&= \sum_{j=1}^{n} \lambda_j q_{ji} + \frac{\lambda_0}{\rho \sigma_i^2} \left[e^{2K_h} \int_{H_i} \widehat{\mu}_{hi} dh - m_{hi} e^{2K_h} rp_i + (1 - m_{hi})(\overline{z} - rp_i) \right]
\end{aligned}$$

where H_i is the mass of competitive investors learning about asset i, of measure m_{hi}.

Using $E[s_{hi}|z_i] = \begin{cases} \overline{z} + \left(1 - e^{-2K_h}\right) \varepsilon_i & \text{if } i = l_h \\ \overline{z} & \text{if } i \neq l_h, \end{cases}$

$\int_{H_i} \widehat{\mu}_{hi} dh = m_{hi} \left[\overline{z} + \left(1 - e^{-2K_h}\right) \varepsilon_i \right].$

Then market clearing becomes

$x_i = \sum_{j=1}^{n} \lambda_j q_{ji} + \frac{\lambda_0}{\rho \sigma_i^2} \left[\left(1 - m_{hi} + e^{2K_h} m_{hi}\right) \overline{z} + \left(e^{2K_h} - 1\right) \varepsilon_i m_{hi} - \left(1 - m_{hi} + e^{2K_h} m_{hi}\right) rp_i \right]$

Defining $\Phi_{hi} \equiv m_{hi}(e^{2K_h} - 1)$,

$x_i = \sum_{j=1}^{n} \lambda_j q_{ji} + + \frac{\lambda_0}{\rho \sigma_i^2} [\bar{z}(1 + \Phi_{hi}) + \Phi_{hi} \varepsilon_i - rp_i(1 + \Phi_i)]$
which becomes
$\frac{\rho \sigma_i^2}{\lambda_0} x_i = \frac{\rho \sigma_i^2}{\lambda_0} \sum_{j=1}^{n} \lambda_j q_{ji} + \bar{z}(1 + \Phi_{hi}) + \Phi_{hi} \varepsilon_i - rp_i(1 + \Phi_i)$
and then
$rp_i = \frac{\rho \sigma_i^2}{\lambda_0 (1 + \Phi_{hi})} \sum_{j=1}^{n} \lambda_j q_{ji} + \bar{z} + \frac{\Phi_{hi}}{1 + \Phi_{hi}} \varepsilon_i - \frac{\rho \sigma_i^2}{\lambda_0 (1 + \Phi_{hi})} x_i$
Hence,
$\frac{dp_i(q_{ji})}{dq_{ji}} = \frac{\lambda_j \rho \sigma_i^2}{\lambda_0 r (1 + \Phi_{hi})} > 0$
Let $\hat{\lambda}_{ji} \equiv \frac{\lambda_j}{\lambda_0 (1 + \Phi_{hi})}$.
Then $q_{ji} = \frac{\widehat{\mu}_{ji} - rp_i}{\rho \left(\widehat{\sigma}_{ji}^2 + \widehat{\lambda}_{ji} \sigma_i^2\right)}$, and similarly for k.
Plugging in the expression for q_{ji}:
$rp_i = \sum_{j=1}^{n} \hat{\lambda}_{ji} \rho \sigma_i^2 \frac{\widehat{\mu}_{ji} - rp_i}{\rho \left(\widehat{\sigma}_{ji}^2 + \widehat{\lambda}_{ji} \sigma_i^2\right)} + \bar{z} + \frac{\Phi_{hi}}{1 + \Phi_{hi}} \varepsilon_i - \frac{\rho \sigma_i^2}{\lambda_0 (1 + \Phi_{hi})} x_i$
which becomes
$rp_i \left(1 + \sum_{j=1}^{n} \frac{\hat{\lambda}_{ji} \rho \sigma_i^2}{\rho \left(\widehat{\sigma}_{ji}^2 + \widehat{\lambda}_{ji} \sigma_i^2\right)} \right) = \sum_{j=1}^{n} \frac{\hat{\lambda}_{ji} \rho \sigma_i^2}{\rho \left(\widehat{\sigma}_{ji}^2 + \widehat{\lambda}_{ji} \sigma_i^2\right)} \widehat{\mu}_{ji} + \bar{z} + \frac{\Phi_{hi}}{1 + \Phi_{hi}} \varepsilon_i - \frac{\rho \sigma_i^2}{\lambda_0 (1 + \Phi_{hi})} x_i$
dividing through gives

$rp_i = \frac{\sum_{j=1}^n \frac{\hat{\lambda}_{ji}\rho\sigma_i^2}{\rho\left(\widehat{\sigma}_{ji}^2+\widehat{\lambda}_{ji}\sigma_i^2\right)}\widehat{\mu}_{ji}+\bar{z}+\frac{\Phi_{hi}}{1+\Phi_{hi}}\varepsilon_i-\frac{\rho\sigma_i^2}{\lambda_0(1+\Phi_{hi})}x_i}{\left(1+\sum_{j=1}^n \frac{\hat{\lambda}_{ji}\rho\sigma_i^2}{\rho\left(\widehat{\sigma}_{ji}^2+\widehat{\lambda}_{ji}\sigma_i^2\right)}\right)}$

The indirect utility function $U_j = \sum_{i=1}^n q_{ji}\left(\widehat{\mu}_{ji} - rp_i\right) - \frac{\rho}{2}\sum_{i=1}^n q_{ji}^2\widehat{\sigma}_{ji}^2$ becomes

$U_j = \sum_{i=1}^n \left[q_{ji}^2\rho\left(\widehat{\sigma}_{ji}^2 + \widehat{\lambda}_{ji}\sigma_i^2\right) - \frac{\rho}{2}q_{ji}^2\widehat{\sigma}_{ji}^2\right]$

$U_j = \sum_{i=1}^n \left[\rho q_{ji}^2\left(\widehat{\sigma}_{ji}^2 + \widehat{\lambda}_{ji}\sigma_i^2 - \frac{1}{2}\widehat{\sigma}_{ji}^2\right)\right]$

$U_j = \sum_{i=1}^n \left\{\frac{(\widehat{\mu}_{ji}-rp_i)^2}{\rho\left(\widehat{\sigma}_{ji}^2+\widehat{\lambda}_{ji}\sigma_i^2\right)^2}\left(\frac{1}{2}\widehat{\sigma}_{ji}^2 + \widehat{\lambda}_{ji}\sigma_i^2\right)\right\}$

$U_j = \frac{1}{2\rho}\sum_{i=1}^n \left\{\left(\widehat{\mu}_{ji} - rp_i\right)^2\left[\frac{\widehat{\sigma}_{ji}^2+2\widehat{\lambda}_i\sigma_i^2}{\left(\widehat{\sigma}_{ji}^2+\widehat{\lambda}_i\sigma_i^2\right)^2}\right]\right\}.$

More detailed expression for U: We can rewrite $E_{0j}(\widehat{\mu}_{ji} - rp_i)^2$ as $\widehat{R}_i^2 + \widehat{V}_{ji}$, where $\widehat{R}_i$ and $\widehat{V}_{ji}$ denote the ex-ante mean and variance of expected excess returns, which means: $\widehat{R}_i \equiv E_{0j}\left(\widehat{\mu}_{ji} - rp_i\right) = \frac{\frac{\rho\sigma_i^2}{\lambda_0(1+\Phi_{hi})}}{\left(1+\sum_{j=1}^n \frac{\hat{\lambda}_{ji}\rho\sigma_i^2}{\rho\left(\widehat{\sigma}_{ji}^2+\widehat{\lambda}_{ji}\sigma_i^2\right)}\right)}\bar{x}$ Define $M_{ji} \equiv \frac{\hat{\lambda}_{ji}\sigma_i^2}{\left(\widehat{\sigma}_{ji}^2+\widehat{\lambda}_{ji}\sigma_i^2\right)}$ $\quad N_i \equiv \frac{1}{1+\sum_{j=1}^n M_{ji}}$

$\widehat{V}_{ji} \equiv V_{0j}\left(\widehat{\mu}_{ji} - rp_i\right)$
$= V_{0j}\left(\widehat{\mu}_{ji} - N_i\sum_{k=1}^n M_{ki}\widehat{\mu}_{ki} - N_i\bar{z} - N_i\frac{\Phi_{hi}}{1+\Phi_{hi}}\varepsilon_i + N_i\frac{\rho\sigma_i^2}{\lambda_0(1+\Phi_{hi})}x_i\right)$
$= V_{0j}\left(N_i\left(\widehat{\mu}_{ji} + \sum_{k=1}^n M_{ki}(\widehat{\mu}_{ji} - \widehat{\mu}_{ki})\right) - N_i\frac{\Phi_{hi}}{1+\Phi_{hi}}\varepsilon_i + N_i\frac{\rho\sigma_i^2}{\lambda_0(1+\Phi_{hi})}x_i\right)$
$= N_i^2 V_{0j}\left(\widehat{\mu}_{ji} + \sum_{k\neq j}^n M_{ki}\widehat{\mu}_{ji} - \frac{\Phi_{hi}}{1+\Phi_{hi}}\varepsilon_i + \frac{\rho\sigma_i^2}{\lambda_0(1+\Phi_{hi})}x_i\right)$
$= N_i^2\left(1+\sum_{k\neq j}^n M_{ki}\right)^2(\sigma_i^2 - \widehat{\sigma}_{ji}^2) + \left(\frac{N_i\Phi_{hi}}{1+\Phi_{hi}}\right)^2\sigma_i^2 + \left(\frac{N_i\rho\sigma_i^2}{\lambda_0(1+\Phi_{hi})}\right)^2\sigma_{xi}^2 - 2\left(\frac{N_i^2\Phi_{hi}\left(1+\sum_{k\neq j}^n M_{ki}\right)}{1+\Phi_{hi}}\right)(\sigma_i^2 - \widehat{\sigma}_{ji}^2)$

$$
\begin{aligned}
U_{0j} \quad = \quad & \frac{1}{2\rho}\sum_i \frac{N_i^2(\widehat{\sigma}_{ji}^2 + 2\widehat{\lambda}_i\sigma_i^2)}{\left(\hat{\sigma}_{ji}^2 + \hat{\lambda}\sigma_i^2\right)^2}\Bigg[\left(\frac{\rho\sigma_i^2}{\lambda_0(1+\Phi_{hi})}\right)^2(\bar{x}^2 + \sigma_{ix}^2) + \left(\frac{\Phi_{hi}}{1+\Phi_{hi}}\right)^2\sigma_i^2 \\
& + \left(\frac{(1+\Phi_{hi})\left(1+\sum_{k\neq j}^n M_{ki}\right) - 2\Phi_{hi}}{1+\Phi_{hi}}\right)(\sigma_i^2 - \hat{\sigma}_i^2)\left(1 + \sum_{k\neq j}^n M_{ki}\right)\Bigg]
\end{aligned}
$$

$$
\begin{aligned}
& = \frac{1}{2\rho}\sum_i \frac{N_i^2(1+2\hat{\lambda}\alpha_{ji})}{(1+\hat{\lambda}\alpha_{ji})^2}\Bigg[\left(\frac{\Phi_{hi}}{1+\Phi_{hi}}\right)^2\alpha_{ji} + \left(\frac{\rho\sigma_i^2}{\lambda_0(1+\Phi_{hi})}\right)^2(\bar{x}^2 + \sigma_{ix}^2)\frac{\alpha_{ji}}{\sigma_i^2} \\
& + \left(\frac{(1+\Phi_{hi})\left(1+\sum_{k\neq j}^n M_{ki}\right) - 2\Phi_{hi}}{1+\Phi_{hi}}\right)\left(1 + \sum_{k\neq j}^n M_{ki}\right)(\alpha_{ji} - 1)\Bigg]
\end{aligned}
$$

7.1.5 Derivation of Equations 21

The market clearing condition is

$$rp_i = \frac{\sum_{j=1}^{n} \frac{\hat{\lambda}_{ji}\rho\sigma_i^2}{\rho\left(\widehat{\sigma}_{ji}^2+\widehat{\lambda}_{ji}\sigma_i^2\right)}\widehat{\mu}_{ji} + \bar{z} + \frac{\Phi_{hi}}{1+\Phi_{hi}}\varepsilon_i - \frac{\rho\sigma_i^2}{\lambda_0(1+\Phi_{hi})}x_i}{\left(1 + \sum_{j=1}^{n} \frac{\hat{\lambda}_{ji}\rho\sigma_i^2}{\rho\left(\widehat{\sigma}_{ji}^2+\widehat{\lambda}_{ji}\sigma_i^2\right)}\right)} \tag{33}$$

From here we identify the price coefficients as a function of the monopolist learning and the competitive fringe learning. Now, conditionally on z_i, we have

$$\hat{\mu}_{ji} = s_{ji}$$

and s_{ji} is normally distributed with mean $\bar{z} + (1 - \frac{1}{\alpha_{ji}})\varepsilon_i$ and variance $(1 - \frac{1}{\alpha_{ji}})\frac{1}{\alpha_{ji}}\sigma_i^2$. What we want is to express the posterior mean in terms of delta as in $z_i = s_i + \delta_i$. Given that,

$$\delta_{ji} = z_i - s_{ji} = -\frac{1}{\alpha_{ji}}\varepsilon_i + noise$$

$$rp_i = N_i \sum_{j=1}^{n} M_{ji}\left(\bar{z} + \left(1 - \frac{1}{\alpha_{ji}}\right)\epsilon_i - \zeta_{ji}\right) + N_i\left[\bar{z} + \frac{\Phi_{hi}}{1+\Phi_{hi}}\varepsilon_i - \frac{\rho\sigma_i^2}{\lambda_0(1+\Phi_{hi})}x_i\right] \tag{34}$$

$$\begin{aligned} rp_i &= \bar{z} - \bar{x}\frac{N_i\rho\sigma_i^2}{\lambda_0(1+\Phi_{hi})} + \epsilon_i N_i\left(\sum_{j=1}^{n}\frac{M_{ji}(\alpha_{ji}-1)}{\alpha_{ji}} + \frac{\Phi_{hi}}{1+\Phi_{hi}}\right) \\ &\quad -\frac{N_i\rho\sigma_i^2}{\lambda_0(1+\Phi_{hi})}\nu_i - N_i\sum_{j=1}^{n} M_{ji}\zeta_{ji} \end{aligned}$$

If only the monopolist can learn, then $\Phi_{hi} = 0$. First we can write: $M_{ji} = \frac{\hat{\lambda}_{ji}\alpha_{ji}}{1+\hat{\lambda}_{ji}\alpha_{ji}}, N_i = \frac{1+\hat{\lambda}_{ji}\alpha_{ji}}{1+2\hat{\lambda}_{ji}\alpha_{ji}}$. Then we need to solve the monopolist's information problem:

$$\begin{aligned} 0 &= \frac{\partial}{\partial\alpha_{ji}}\frac{1}{2\rho}\sum_i \frac{1}{1+2\hat{\lambda}\alpha_{ji}}\left[\left(\frac{\rho\sigma_i^2}{\lambda_0}\right)^2(\bar{x}^2+\sigma_{ix}^2)\frac{\alpha_{ji}}{\sigma_i^2} + (\alpha_{ji}-1)\right] \\ 0 &= \frac{1}{1+2\hat{\lambda}_{ji}\alpha_{ji}}\left[\left(\frac{\rho\sigma_i^2}{\lambda_0}\right)^2(\bar{x}^2+\sigma_{ix}^2)\frac{1}{\sigma_i^2} + 1\right] - \\ &\quad \frac{2\hat{\lambda}_{ji}}{(1+2\hat{\lambda}_{ji}\alpha_{ji})^2}\left[\left(\frac{\rho\sigma_i^2}{\lambda_0}\right)^2(\bar{x}^2+\sigma_{ix}^2)\frac{\alpha_{ji}}{\sigma_i^2} + (\alpha_{ji}-1)\right] + 2\rho\frac{\eta_i-\mu}{\alpha_{ji}} \end{aligned}$$

$$
\begin{aligned}
\frac{2\rho\mu(1+2\hat{\lambda}_{ji}\alpha_{ji})}{\alpha_{ji}} &= \left[\left(\frac{\rho\sigma_i^2}{\lambda_0}\right)^2(\bar{x}^2+\sigma_{ix}^2)\frac{1}{\sigma_i^2}+1\right]-\frac{2\hat{\lambda}_{ji}}{1+2\hat{\lambda}_{ji}\alpha_{ji}}\left[\left(\frac{\rho\sigma_i^2}{\lambda_0}\right)^2(\bar{x}^2+\sigma_{ix}^2)\frac{\alpha_{ji}}{\sigma_i^2}+(\alpha_{ji}-1)\right] \\
&= \left[\left(\frac{\rho\sigma_i^2}{\lambda_0}\right)^2(\bar{x}^2+\sigma_{ix}^2)\frac{1}{\sigma_i^2}+1\right]\left(1-\frac{2\hat{\lambda}_{ji}\alpha_{ji}}{1+2\hat{\lambda}_{ji}\alpha_{ji}}\right)+\frac{2\hat{\lambda}_{ji}}{1+2\hat{\lambda}_{ji}\alpha_{ji}} \\
\frac{2\rho\mu(1+2\hat{\lambda}_{ji}\alpha_{ji})^2}{\alpha_{ji}} &= \left[\left(\frac{\rho}{\lambda_0}\right)^2(\bar{x}^2+\sigma_{ix}^2)\sigma_i^2+1\right]+2\hat{\lambda}_{ji}
\end{aligned}
$$

$$
\begin{aligned}
\mu &= \frac{\alpha_{ji}}{(1+2\hat{\lambda}_{ji}\alpha_{ji})^2}X_i \\
X_i &= \frac{1}{2\rho}\left(\left[\left(\frac{\rho}{\lambda_0}\right)^2(\bar{x}^2+\sigma_{ix}^2)\sigma_i^2+1\right]+2\hat{\lambda}_{ji}\right) \\
\frac{(1+2\hat{\lambda}_{ji}\alpha_{ji})^2}{(1+2\hat{\lambda}_{jk}\alpha_{jk})^2} &= \frac{\alpha_{ji}X_i}{\alpha_{jk}X_k} \\
\prod_{k=1}^{n}\frac{(1+2\hat{\lambda}_{ji}\alpha_{ji})^2}{(1+2\hat{\lambda}_{jk}\alpha_{jk})^2} &= \prod_{k=1}^{n}\frac{\alpha_{ji}X_i}{\alpha_{jk}X_k} \\
\frac{(1+2\hat{\lambda}_{ji}\alpha_{ji})^{2n}}{\prod_{k=1}^{n}(1+2\hat{\lambda}_{jk}\alpha_{jk})^2} &= \frac{(\alpha_{ji}X_i)^n}{e^{2K_h}\prod_{k=1}^{n}X_k} \\
M_{ji} &= \frac{\lambda_1\alpha_{ji}}{\lambda_0+\lambda_1\alpha_{ji}} \\
N_i &= \frac{\lambda_0+\lambda_1\alpha_{ji}}{\lambda_0+2\lambda_1\alpha_{ji}} \\
a_i &= \frac{\bar{z}}{r}-\frac{\bar{x}}{r}\frac{(\lambda_0+\lambda_1\alpha_{ji})\rho\sigma_i^2}{\lambda_0(\lambda_0+2\lambda_1\alpha_{ji})} \\
b_i &= \frac{\lambda_1(\alpha_{ji}-1)}{r(\lambda_0+2\lambda_1\alpha_{ji})} \\
c_i &= \frac{(\lambda_0+\lambda_1\alpha_{ji})\rho\sigma_i^2}{r\lambda_0(\lambda_0+2\lambda_1\alpha_{ji})} \\
d_{ji} &= \frac{\lambda_1\alpha_{ji}}{r(\lambda_0+2\lambda_1\alpha_{ji})} \\
\sigma_{pi}^2 &= b_i^2\sigma_i^2+c_i^2\sigma_{xi}^2+d_{ji}^2\left(1-\frac{1}{\alpha_{ji}}\right)\widehat{\sigma}_{ji}^2 \\
&= \frac{1}{r^2(\lambda_0+2\lambda_1\alpha_{ji})^2}\left[\lambda_1^2(\alpha_{ji}-1)^2\sigma_i^2+(\lambda_0+\lambda_1\alpha_{ji})^2\rho^2\sigma_i^4\sigma_{xi}^2\lambda_0^{-2}+\lambda_1^2\sigma_i^2\right]
\end{aligned}
$$

$$
\begin{aligned}
\frac{Cov(p_i, z_i)}{\sigma_{pi}} &= \frac{Cov(a_i + b_i\epsilon_i - c_i\nu_i - d_i\zeta_{ji}, z_i)}{\sigma_{pi}} \\
&= \frac{\lambda_1(\alpha_{ji}-1)\sigma_i^2}{\sqrt{\left[\lambda_1^2(\alpha_{ji}-1)^2\sigma_i^2 + (\lambda_0+\lambda_1\alpha_{ji})^2\rho^2\sigma_i^4\sigma_{xi}^2\lambda_0^{-2} + \lambda_1^2\sigma_i^2\right]}} \\
&= \frac{\lambda_1(\alpha_{ji}-1)\sigma_i}{\sqrt{\left[\lambda_1^2(\alpha_{ji}-1)^2 + (1+\hat{\lambda}_{ji}\alpha_{ji})^2\rho^2\sigma_i^2\sigma_{xi}^2 + \lambda_1^2\right]}}
\end{aligned}
$$

Proof of Proposition 3

Proof. Define $f(\lambda, \alpha_j) = \frac{\alpha_j}{(1+2\hat{\lambda}\alpha_j)^2}X_j$. Then $f_\lambda = -\frac{2\alpha_j(2\alpha_j\hat{\lambda}+2\alpha(X_j-2\hat{\lambda})-1)}{(2\hat{\lambda}\alpha_j+1)^3}$ which is decreasing in α_j. Since $\alpha_j'(\sigma_j) > 0$. Therefore assuming wlog that $\sigma_j > \sigma_i$, we get that $\alpha_j > \alpha_i$, and therefore $f_\lambda(i) > f_\lambda(j)$. Similarly $f_\alpha = -\frac{1-2\alpha_j\hat{\lambda}}{(1+2\alpha\hat{\lambda})^3}X$ is decreasing in α_j. Therefore in order to satisfy the first order conditions $\alpha_j(\lambda) < 0$ and $\alpha_i(\lambda) > 0$. □

Proof of Proposition **??**

Proof.

$$
\begin{aligned}
PI_i'(\lambda_1) &= \frac{\partial PI_i}{\partial \lambda_1} \\
&= \frac{(\alpha-1)\sigma_i^3\rho^2\sigma_{xi}^2(\hat{\lambda}+1)}{\left[\lambda_1^2(\alpha_{ji}-1)^2 + (1+\hat{\lambda}_{ji}\alpha_{ji})^2\rho^2\sigma_i^2\sigma_{xi}^2 + \lambda_1^2\right]^{1.5}} \\
&> 0
\end{aligned}
$$

□

If both the monopolist and the fringe can learn, we need to solve the monopolist's information problem when $\Phi_{hi} > 0$. First we can write: $M_{ji} = \frac{\hat{\lambda}_{ji}\alpha_{ji}}{1+\hat{\lambda}_{ji}\alpha_{ji}}, N_i = \frac{1+\hat{\lambda}_{ji}\alpha_{ji}}{1+2\hat{\lambda}_{ji}\alpha_{ji}}$

$$
\begin{aligned}
0 = \; & \frac{\partial}{\partial \alpha_{ji}}\frac{1}{2\rho}\sum_i \frac{1}{1+2\hat{\lambda}\alpha_{ji}}\Bigg[\left(\frac{\Phi_{hi}}{1+\Phi_{hi}}\right)^2\alpha_{ji} + \left(\frac{\rho\sigma_i^2}{\lambda_0(1+\Phi_{hi})}\right)^2(\bar{x}^2+\sigma_{ix}^2)\frac{\alpha_{ji}}{\sigma_i^2} \\
& + \left(\frac{(1+\Phi_{hi})\left(1+\sum_{k\neq j}^n M_{ki}\right) - 2\Phi_{hi}}{1+\Phi_{hi}}\right)\left(1+\sum_{k\neq j}^n M_{ki}\right)(\alpha_{ji}-1)\Bigg]
\end{aligned}
$$

$$
\begin{aligned}
\frac{2\rho\mu}{\alpha_{ji}} &= \frac{-2\hat{\lambda}_{ji}}{(1+2\hat{\lambda}_{ji}\alpha_{ji})^2}\left[\left(\frac{\Phi_{hi}}{1+\Phi_{hi}}\right)^2\alpha_{ji}+\left(\frac{\rho\sigma_i^2}{\lambda_0(1+\Phi_{hi})}\right)^2(\bar{x}\Phi_{hi}^2+\sigma_{ix}^2)\frac{\alpha_{ji}}{\sigma_i^2}+\left(\frac{1-\Phi_{hi}}{1+\Phi_{hi}}\right)(\alpha_{ji}-1)\right] \\
&\quad +\frac{1}{1+2\hat{\lambda}_{ji}\alpha_{ji}}\left[\left(\frac{\Phi_{hi}}{1+\Phi_{hi}}\right)^2+\left(\frac{\rho\sigma_i^2}{\lambda_0(1+\Phi_{hi})}\right)^2(\bar{x}^2+\sigma_{ix}^2)\frac{1}{\sigma_i^2}+\left(\frac{1-\Phi_{hi}}{1+\Phi_{hi}}\right)\right] \\
\frac{2\rho\mu}{\alpha_{ji}} &= \frac{1}{(1+2\hat{\lambda}_{ji}\alpha_{ji})^2}\left[\left(\frac{\Phi_{hi}}{1+\Phi_{hi}}\right)^2+\left(\frac{\rho\sigma_i^2}{\lambda_0(1+\Phi_{hi})}\right)^2(\bar{x}^2+\sigma_{ix}^2)\frac{1}{\sigma_i^2}+\left(\frac{1-\Phi_{hi}}{1+\Phi_{hi}}\right)(1+2\hat{\lambda}_{ji})\right] \\
Y_i &= \frac{1}{2\rho}\left[1+2\hat{\lambda}_{ji}+\left(\frac{\rho\sigma_i^2}{\lambda_0}\right)^2(\bar{x}^2+\sigma_{ix}^2)\frac{1}{\sigma_i^2}\right] \\
\mu &= \frac{\alpha_{ji}Y_i+\frac{\hat{\lambda}_{ji}\Phi_{hi}^2}{\rho}}{(1+2\hat{\lambda}_{ji}\alpha_{ji})^2(1+\Phi_{hi})^2} \\
\lambda_1 &> 0 \\
M_{ji} &= \frac{\lambda_1\alpha_{ji}}{\lambda_0(1+\Phi_{hi})+\lambda_1\alpha_{ji}} \\
N_i &= \frac{\lambda_0(1+\Phi_{hi})+\lambda_1\alpha_{ji}}{\lambda_0(1+\Phi_{hi})+2\lambda_1\alpha_{ji}} \\
a_i &= \frac{\bar{z}}{r}-\frac{\bar{x}}{r}\frac{(\lambda_0(1+\Phi_{hi})+\lambda_1\alpha_{ji})\rho\sigma_i^2}{\lambda_0(1+\Phi_{hi})(\lambda_0(1+\Phi_{hi})+2\lambda_1\alpha_{ji})} \\
b_i &= \frac{\lambda_1(\alpha_{ji}-1)}{r(\lambda_0(1+\Phi_{hi})+2\lambda_1\alpha_{ji})}+\frac{\lambda_0(1+\Phi_{hi})+\lambda_1\alpha_{ji}}{\lambda_0(1+\Phi_{hi})+2\lambda_1\alpha_{ji}}\frac{\Phi_{hi}}{r(1+\Phi_{hi})} \\
c_i &= \frac{(\lambda_0(1+\Phi_{hi})+\lambda_1\alpha_{ji})\rho\sigma_i^2}{\lambda_0(1+\Phi_{hi})(\lambda_0(1+\Phi_{hi})+2\lambda_1\alpha_{ji})r} \\
d_{ji} &= \frac{\lambda_1\alpha_{ji}}{r(\lambda_0(1+\Phi_{hi})+2\lambda_1\alpha_{ji})} \\
\sigma_{pi}^2 &= b_i^2\sigma_i^2+c_i^2\sigma_{xi}^2+d_{ji}^2\left(1-\frac{1}{\alpha_{ji}}\right)\widehat{\sigma}_{ji}^2
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{r^2(\lambda_0(1+\Phi_{hi})^2+2\lambda_1\alpha_{ji})^2}\left[\lambda_1^2(\alpha_{ji}-1)^2\sigma_i^2+\frac{(\lambda_0(1+\Phi_{hi})+\lambda_1\alpha_{ji})^2}{(1+\Phi_{hi})^2}+\right. \\
&\quad \left.(\lambda_0(1+\Phi_{hi})+\lambda_1\alpha_{ji})^2\rho^2\sigma_i^4\sigma_{xi}^2\lambda_0^{-2}+\lambda_1^2\sigma_i^2\right] \\
\frac{Cov(p_i,z_i)}{\sigma_{pi}} &= \frac{Cov(a_i+b_i\epsilon_i-c_i\nu_i-d_i\zeta_{ji},z_i)}{\sigma_{pi}} \\
&= \frac{(\lambda_1(\alpha_{ji}-1)+\lambda_0(1+\Phi_{hi})+\lambda_1\alpha_{ji}\frac{\Phi_{hi}}{(1+\Phi_{hi})})\sigma_i}{\sqrt{\left[\lambda_1^2(\alpha_{ji}-1)^2+\frac{(\lambda_0(1+\Phi_{hi})+\lambda_1\alpha_{ji})^2}{(1+\Phi_{hi})^2}+(\lambda_0(1+\Phi_{hi})+\lambda_1\alpha_{ji})^2\rho^2\sigma_i^2\sigma_{xi}^2\lambda_0^{-2}+\lambda_1^2\right]}}
\end{aligned}
$$

Proof of Proposition **??**

Proof.

$$U_{0j} = \frac{1}{2\rho}\sum_i \frac{N_i^2(\widehat{\sigma}_{ji}^2 + 2\widehat{\lambda}_i\sigma_i^2)}{\left(\hat{\sigma}_{ji}^2 + \hat{\lambda}\sigma_i^2\right)^2}\left[\left(\frac{\rho\sigma_i^2}{\lambda_0(1+\Phi_{hi})}\right)^2(\bar{x}^2+\sigma_{ix}^2) + \left(\frac{\Phi_{hi}}{1+\Phi_{hi}}\right)^2\sigma_i^2 + \right.$$

$$\left.\left(\frac{(1+\Phi_{hi})\left(1+\sum_{k\neq j}^n M_{ki}\right) - 2\Phi_{hi}}{1+\Phi_{hi}}\right)(\sigma_i^2 - \hat{\sigma}_i^2)\left(1+\sum_{k\neq j}^n M_{ki}\right)\right]$$

$$U_{0ji} = \frac{1}{2\rho}\frac{N_i^2(\widehat{\sigma}_{ji}^2 + 2\widehat{\lambda}_i\sigma_i^2)}{\left(\hat{\sigma}_{ji}^2 + \hat{\lambda}\sigma_i^2\right)^2}\left[\left(\frac{\rho\sigma_i^2}{\lambda_0}\right)^2(\bar{x}^2+\sigma_{ix}^2) + \left(1+\sum_{k\neq j}^n M_{ki}\right)^2(\sigma_i^2-\hat{\sigma}_i^2)\right]$$

$$U_{0ji} = \frac{1}{2\rho}\frac{N_i^2(\lambda_0 + 2\lambda_j\alpha_{ji})}{(\lambda_0+\lambda_j\alpha_{ji})^2}\left[\frac{\alpha_{ji}\rho^2\sigma_i^2}{\lambda_0}(\bar{x}^2+\sigma_{ix}^2) + \left(1+\sum_{k\neq j}^n M_{ki}\right)^2\lambda_0(\alpha_{ji}-1)\right]$$

$$\frac{\mu}{\alpha} = \frac{1}{2\rho}\left[\frac{(2N_iN_i'(\lambda_0+2\lambda_j\alpha_{ji}) + 2N_i^2\lambda_j)}{(\lambda_0+\lambda_j\alpha_{ji})^2} - \frac{2\lambda_jN_i^2(\lambda_0+2\lambda_j\alpha_{ji})}{(\lambda_0+\lambda_j\alpha_{ji})^3}\right]$$

$$\left[\frac{\alpha_{ji}\rho^2\sigma_i^2}{\lambda_0}(\bar{x}^2+\sigma_{ix}^2) + \left(1+\sum_{k\neq j}^n M_{ki}\right)^2\lambda_0(\alpha_{ji}-1)\right]$$

$$+\left[\frac{1}{2\rho}\frac{N_i^2(\lambda_0+2\lambda_j\alpha_{ji})}{(\lambda_0+\lambda_j\alpha_{ji})^2}\right]\left[\frac{\rho^2\sigma_i^2}{\lambda_0}(\bar{x}^2+\sigma_{ix}^2) + \left(1+\sum_{k\neq j}^n M_{ki}\right)^2\lambda_0\right]$$

$$N_i' = \frac{-1}{\left(1+\sum_j M_{ji}\right)^2}M_{ji}' = -N_i^2M_{ji}' = -N_i^2\frac{\lambda_j\lambda_0}{(\lambda_j\alpha_{ji}+\lambda_0)^2}$$

$$\frac{\mu}{\alpha_{ji}} = \frac{1}{2\rho}\left[\frac{(-2N_i^3\lambda_j\lambda_0(\lambda_0+2\lambda_j\alpha_{ji}) + 2N_i^2\lambda_j\left(\lambda_0+\lambda_j\alpha_{ji}\right)^2)}{(\lambda_0+\lambda_j\alpha_{ji})^4} - \frac{2\lambda_jN_i^2(\lambda_0+2\lambda_j\alpha_{ji})}{(\lambda_0+\lambda_j\alpha_{ji})^3}\right]$$

$$\underbrace{\left[\frac{\alpha_{ji}\rho^2\sigma_i^2}{\lambda_0}(\bar{x}^2+\sigma_{ix}^2) + \left(1+\sum_{k\neq j}^n M_{ki}\right)^2\lambda_0(\alpha_{ji}-1)\right]}_{\text{positively linear in } \alpha_{ji}}$$

$$+\left[\frac{1}{2\rho}\frac{N_i^2(\lambda_0+2\lambda_j\alpha_{ji})}{(\lambda_0+\lambda_j\alpha_{ji})^2}\right]\underbrace{\left[\frac{\rho^2\sigma_i^2}{\lambda_0}(\bar{x}^2+\sigma_{ix}^2)+\left(1+\sum_{k\neq j}^{n}M_{ki}\right)^2\lambda_0\right]}_{\text{does not depend on }\alpha_{ji}}$$

$$\frac{2\rho\mu}{\alpha_{ji}}=-2N_i^2\lambda_j\left[\frac{(N_i\lambda_0+\lambda_0+\lambda_j\alpha_{ji})(\lambda_0+2\lambda_j\alpha_{ji})-(\lambda_0+\lambda_j\alpha_{ji})^2}{(\lambda_0+\lambda_j\alpha_{ji})^4}\right](X_i\alpha_{ji}-Y_i)+\left[\frac{N_i^2(\lambda_0+2\lambda_j\alpha_{ji})}{(\lambda_0+\lambda_j\alpha_{ji})^2}\right]X_i$$

$$\frac{2\rho\mu\,(\lambda_0+\lambda_j\alpha_{ji})^2}{\alpha_{ji}N_i^2(\lambda_0+2\lambda_j\alpha_{ji})}=2\lambda_j\left[\frac{(\lambda_0+\lambda_j\alpha_{ji})^2-(N_i\lambda_0+\lambda_0+\lambda_j\alpha_{ji})(\lambda_0+2\lambda_j\alpha_{ji})}{(\lambda_0+2\lambda_j\alpha_{ji})\,(\lambda_0+\lambda_j\alpha_{ji})^2}\right](X_i\alpha_{ji}-Y_i)+X_i$$

$$\frac{2\rho\mu\,(\lambda_0+\lambda_j\alpha_{ji})^4}{\alpha_{ji}N_i^2}=2\lambda_j\left[(N_i\lambda_0+\lambda_0+\lambda_j\alpha_{ji})(\lambda_0+2\lambda_j\alpha_{ji})-(\lambda_0+\lambda_j\alpha_{ji})^2\right]Y_i$$

$$+\left[(\lambda_0+4\lambda_j\alpha_{ji})\,(\lambda_0+\lambda_j\alpha_{ji})^2-2\lambda_j\alpha_{ji}(N_i\lambda_0+\lambda_0+\lambda_j\alpha_{ji})(\lambda_0+2\lambda_j\alpha_{ji})\right]X_i$$

$$2\rho\mu=\frac{2N_i^2\lambda_j\alpha_{ji}\left[(N_i\lambda_0+\lambda_0+\lambda_j\alpha_{ji})(\lambda_0+2\lambda_j\alpha_{ji})-(\lambda_0+\lambda_j\alpha_{ji})^2\right]}{(\lambda_0+\lambda_j\alpha_{ji})^4}Y_i$$

$$+\frac{\left[(\lambda_0+4\lambda_j\alpha_{ji})\,(\lambda_0+\lambda_j\alpha_{ji})^2-2\lambda_j\alpha_{ji}(N_i\lambda_0+\lambda_0+\lambda_j\alpha_{ji})(\lambda_0+2\lambda_j\alpha_{ji})\right]\alpha_{ji}N_i^2}{(\lambda_0+\lambda_j\alpha_{ji})^4}X_i$$

If λ_j decreases for any reason, α_{ji} must increase to compensate (holding μ constant), which means that the marginal benefit of information increases as size decreases - that is, agents will specialize more in learning as their size decreases. Therefore, if the passive sector increases at the expense of the active sector, which ever active oligopolists are decreased in size will specialize more in their learning - thus reducing aggregate price informativeness. □