Macroeconomic Risk and Idiosyncratic Risk-Taking*

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Abstract

We develop and estimate a dynamic model of risk-shifting over the business cycle. First, equity holders with Epstein-Zin preferences increase their taking of idiosyncratic risk substantially more than the standard model in repeated games, because they perceive the arrival probability of bad states higher than the actual probability and prefer an early resolution of macroeconomic uncertainty. Second, sudden switches to bad states and large shocks in the bad states induce the countercyclical and "synchronized" idiosyncratic risk. Third, combined with high market risk premium in the bad states, the clustered risk-taking generates the countercyclical idiosyncratic volatility discount on equity returns.

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1 Introduction

Macroeconomic risk affects a firm's financing, default and risk-taking policies and its asset prices. In a structural model that features macroeconomic uncertainty and investors' preference for an early resolution of the uncertainty, we examine how macroeconomic risk impacts corporate risk-taking policy and its asset pricing implications. Our model generates two novel results. First, firms increase their taking of idiosyncratic risk simultaneously in recessions rather than in expansions, causing a countercyclical and "synchronized" risk as documented in Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2015). Second, combined with the countercyclical market risk premium, the clustering of idiosyncratic risk-taking decreases equity returns much more in recessions than in booms, resulting in a countercyclical idiosyncratic volatility discount (Ang, Hodrick, Xing, and Zhang, 2006).[1]

In a traditional risk-shifting model, Jensen and Meckling (1976) argue that, because equity holders are not obligated to pay back debt holders from their own pockets at bankruptcy, they have limited downside risk but can receive unlimited upside profits. Once debt is in place, equity holders of a distressed firm strategically increase risk in an effort to save the firm. By doing so, equity holders effectively shift this excessively increased risk to debt holders in this one-time game. Leland (1998) extends it to repeated games by allowing the once-distressed firm to return to debt markets and refinance its debt upward if it has managed to survive. He concludes the agency cost is less severe in repeated games, because equity holders face an implicit cost – too much risk-taking might cause their firm to go bankrupt earlier than otherwise, therefore causing the loss of future tax benefits.

In our structural model, the corporate risk-taking decision is made by equity holders with Epstein and Zin (1989) (EZ) preferences, who prefer an early resolution of uncertainty. We show that, within this framework, the EZ-type equity holders have stronger incentives to increase their taking of idiosyncratic risk in the presence of macroeconomic uncertainty risk. The macroeconomic uncertainty risk we emphasize is the aggregate state-switching risk stemming from large shocks,

[1] The so-called "idiosyncratic volatility discount" is based on the finding by Ang et al. (2006) that firms with a high realized idiosyncratic stock volatility receive stock returns lower than those with high volatility by 1.06% per month in both domestic and international stock markets. Huang (2009) finds a similar result that firms with high *cash flow* volatility earn abnormally lower stock returns than their counterparts with low volatility, by 1.35% per month.

different from (Brownian) risk stemming from small shocks within one miwenaggregate state. The intuition is as follows. First, because of a countercyclical market risk premium in bad times, the risk-adjusted firm value perceived by risk-neutral equity holders in recessions is much lower than that in expansions. Similar to the argument by Almeida and Philippon (2007) that the risk-adjusted distress cost is much higher than that without risk adjustment, the risk-adjusted firm continuation value in our model is much lower than that without risk adjustment. Therefore, small, negative shocks induce equity holders to take on more idiosyncratic risk earlier in recessions than in expansions. Second, when a large negative marketwide shock shifts the aggregate economy from the good state to the bad state, many firms choose to default because the large shock can easily decrease their cash flow levels to a low bound, inducing them to increase risk simultaneously and causing the synchronized idiosyncratic risk (Herskovic et al., 2015).[2] Lastly, and more important, in good times, the probability of the economy sliding into a recession, as perceived by the risk-neutral agent, is greater than the actual probability, even if the recessions are transitory and the economy is in expansion most of the time. Therefore, those agents who are concerned about the arrival of bad times, increase their taking of idiosyncratic risk earlier than otherwise, because they prefer an early resolution of the macroeconomic uncertainty.

Our paper emphasizes the taking of idiosyncratic risk instead of total risk, compared to Leland (1998), because of the countercyclical market risk and risk premium. The intuition is straightforward: Equity holders of a distressed firm want to increase idiosyncratic risk in the hope that the "idiosyncratic investments" might lead the firm to a different direction than the sliding economy, if the aggregate economy is deteriorating. Consistent with the above reasoning, our model predicts that, because of the countercyclical market risk premium and investors' preference for an early resolution of uncertainty, risk-neutral equity holders have greater incentives to take on more idiosyncratic risk, particularly in recessions. This prediction helps explain recent empirical findings of countercyclical idiosyncratic risk-taking. Bloom (2009) reports the percentage increase in various dispersion measures of cash flow shocks in recessions relative to expansions to be between 0.23 and 0.67. Herskovic et al. (2015) find that the average volatilities of idiosyncratic cash flow and stock return residuals are high in recessions, a manifestation of idiosyncratic risk clustering. Recently,

[2] In contrast, a large positive shock in good times does not hit the firm's low bound of risk-taking. Therefore, it is less likely for the risk-taking clustering to be observed in expansions.

Bartram, Brown, and Stulz (2016) have confirmed that both idiosyncratic cash flow volatility and return volatility increase with market risk. While Herskovic et al. (2015) show that customer-supplier net formation creates a common factor in firm-level idiosyncratic risk in a network model, their model does not explicitly link the idiosyncratic risk-taking behavior with the business cycle, but we do.

The corporate risk-taking strategy has significant implications for asset prices. We start with a single-state baseline model and derive a closed-form solution for equity returns to deliver the intuition. Intuitively, levered equity is a long position on a call option, and its value increases with idiosyncratic volatility. Therefore, equity holders of a distressed firm, who are aware of this advantage, increase their idiosyncratic volatility, particularly in recessions. Doing so allows them to capture upside profits and become less sensitive to declining cash flows and asset values. Therefore, firms with high idiosyncratic volatility have low systematic risk exposure, and receive lower equity returns than those with lower idiosyncratic volatility.

By extending the single-state model to a two-state model, we further demonstrate the macroeconomic uncertainty amplifies the idiosyncratic volatility discount effect from the single-state economy. First, the countercyclical clustering of idiosyncratic risk-taking causes the procyclical sensitivity of equity to the systematic cash flows risk. That is, the greater taking of idiosyncratic risk causes the lower systematic risk exposure in bad aggregate states. Second, because the expected market risk premium is high in the bad states, the equity risk exposure is negatively correlated with this expected market premium. The negative covariance between the equity risk exposure and the expected market risk premium further reduces the expected equity returns for firms with high idiosyncratic volatility, resulting in a countercyclical idiosyncratic volatility discount.

To take the model to the data, we use simulated method of moments to estimate the fully-fledged two-state model and show its quantitative implications for asset prices. First, in our model parameter sensitivity analysis, we find that the timing of risk-shifting is much more sensitive to the macroeconomic state-switching risk, such as the aggregate state-switching probability and the state-switching risk premium, relative to the market volatility and the market risk premium. Second, our calibrations demonstrate that our model is able to generate a sizable idiosyncratic risk discount on equity returns, particularly in recessions. The countercyclical idiosyncratic volatility discount is robust to various model specifications, estimation methods and parameter changes. The data strongly support for this prediction. When we use the idiosyncratic cash flow volatility to form portfolios, firms with low volatility earn higher returns than those with high volatility, by 8.227% per year. This difference is 15.042% in recessions and 7.499% in expansions, respectively. When we use the idiosyncratic equity return volatility to form portfolios, firms with low return volatility earn higher equity returns on average than those with high equity return volatility, by 11.051% for the whole sample, 15.716% in recessions and 9.929% in expansions, respectively.

Our work is related to three strands of the literature. The first strand is the emerging literature that examines the impacts of macroeconomic risk on corporate financing and investment decisions as well as credit risk. Hackbarth, Miao, and Morellec (2006) were the first to introduce macroeconomic dynamics to dynamic capital structure/credit risk models (i.e., Leland (1994); Fischer, Heinkel, and Zechner (1989); Goldstein, Ju, and Leland (2001)). Bhamra, Kuehn, and Strebulaev (2010a,b) introduce consumption-based asset pricing to this framework, and study the dynamics of aggregate leverage and the equity risk premium. Similarly, Chen (2010) seeks to explain two related empirical puzzles, i.e., observed low financial leverage and high credit spreads. Along these lines, scholars examine the effects of macroeconomic uncertainty on credit risk. For example, Koijen, Lustig, and Van Nieuwerburgh (2010) show that bond factors from different business cycle horizons are priced in the cross-section of stock returns, Kuehn and Schmid (2014) examine the importance of investment options in modeling credit risk, and Gomes and Schmid (2016) develop a general equilibrium model and connect the prices of stocks and bonds with endogenous leverage and aggregate volatility. However, they do not examine the corporate endogenous risk-taking behavior over the business cycle.[3]

The second strand concerns the theoretical literature on the well known risk-shifting problem between equity and debt holders. Since Jensen and Meckling (1976), the risk-shifting behavior of corporations has been well studied, theoretically. The well known examples are Leland (1998) and Ericsson (2000). Recent theoretical works introduce various costs of taking excess risk. We follow Hennessy and Tserlukevich (2008) and model a value-destroying cost, the cost being proportional to the excess taking of risk.[4] Additionally, we introduce and estimate the upfront cost of increasing

[3]Chen, Collin-Dufresne, and Goldstein (2009), Arnold, Wagner, and Westermann (2013), and Chen, Cui, He, and Milbradt (2014) examine credit spreads. Ai and Kiku (2013) study the value premium with the presence of the time-varying macroeconomic risk.

[4]Panageas (2010) introduces the bailout into the risk-shifting problem. The implicit cost of increasing risk is the loss of the opportunity to be bailed out, as potential bailouters will be reluctant to save a high-risk firm.

idiosyncratic risk and the cost of reversing the increased risk. In our structural estimation, we find that the value-destroying cost is the most important determinant, compared with the two upfront risk adjustment costs. Second, between the two upfront costs, the cost of reversing the increased risk plays a more important role than the cost of increasing the risk, implying that the future cost in repeated games is a crucial part of understanding the firm's risk-shifting behavior.

Our paper belongs to a third strand of literature that connects agency conflicts with asset valuations.[5] Davydenko and Strebulaev (2007) demonstrate that strategic default decisions by equity holders have an adverse effect on bond prices. McQuade (2016) examines the implications of the strategic default option, in addition to the growth option, in the presence of stochastic volatility, for credit spreads and stock returns. Favara, Schroth, and Valta (2011) and Hackbarth, Haselmann, and Schoenherr (2015) study the effect of equity holders' bargaining power at bankruptcy on stock returns. By studying another agency conflict, we demonstrate that the negative association between idiosyncratic volatility and the future stock return is driven by strategic risk-shifting behavior, particularly in recessions.

The remainder of the paper proceeds as follows. We present the baseline model in Section 2 and the fully-fledged model with macroeconomic uncertainty risk in Section 3. Section 4 discusses the asset pricing implications of the risk-taking. Section 5 describes the data and simulated method of moments. Section 6 presents estimation results and Section 7 examines the implications of the estimated model for corporate risk-shifting and equity returns. Section 8 concludes the paper.

2 Baseline Model

Building on Leland (1998), we develop a baseline risk-shifting model that allows equity holders to increase their firm's idiosyncratic risk when the firm's condition is deteriorating.[6] If their firm

[5] A recent body of literature examines how the changes in operating cash flow risk affect stock returns. Galai and Masulis (1976) show the negative impact of asset growth volatility on stock returns. Along the same lines, Johnson (2004) introduces uncertainty of asset growth volatility into Merton (1974). Additionally, Babenko, Boguth, and Tserlukevich (2016) and Bhamra and Shim (2013) link idiosyncratic cash flow volatility with equity risk via investment options. Our paper differs both because the risk-taking decision is endogenous in our model and because we consider the interaction between macroeconomic risk and idiosyncratic risk. Other papers that study the cross-section of stock returns in a dynamic model include Berk, Green, and Naik (1999), Carlson, Fisher, and Giammarino (2004), Gomes, Kogan, and Zhang (2003), Zhang (2005) and Gomes and Schmid (2010). However, those papers do not consider agency conflicts between equity and debt holders.

[6] We do not study conflicts between managers and stock/debt holders. Albuquerue and Wang (2008) examine the impacts of corporate governance on stock valuation and show that countries with weaker investor protection have more incentives to overinvest, lower Tobin's q, and larger risk premia. Carlson and Lazrak (2010) show that

survives, it comes back to the debt market to restructure its debt upward in repeated games. We follow Hennessy and Tserlukevich (2008) and extend Leland (1998) by introducing a value-destroying cost. Additionally, we introduce two risk adjustment costs.

2.1 Setup

The baseline model is partial equilibrium with a pricing kernel, m_t, following

$$\frac{dm_t}{m_t} = -rdt - \theta dZ_t, \tag{1}$$

where r is the constant risk-free rate, θ is the market price of risk, and Z_t is a standard Brownian motion.

The economy consists of a large number of firms. A representative firm that possesses assets produces instantaneous cash flow X_t, when it is solvent. The cash flow X_t is governed by the following stochastic differentiation equation:

$$\frac{dX_t}{X_t} = \hat{\mu}_{v_t} dt + \sigma^m d\hat{W}_t^m + \sigma_{v_t}^{i,X} d\hat{W}_t^i, \tag{2}$$

where $\hat{\mu}_{v_t}$ is the expected growth rate, σ^m is the systematic volatility, $\sigma_{v_t}^{i,X}$ is the idiosyncratic volatility, and $\hat{W}_t^m$ and $\hat{W}_t^i$ are standard Brownian motions. We assume that the systematic risk of the production asset is constant, but its idiosyncratic risk $\sigma_{v_t}^{i,X}$ has two levels, high-risk and low-risk, i.e., $v_t = \{H, L\}$. The total volatility of the cash flow growth rate is $\sigma_{v_t} = \sqrt{(\sigma^m)^2 + (\sigma_{v_t}^{i,X})^2}$. We use $\hat{}$ to denote the physical measure, and define $\hat{\mu}_{v_t} = \mu_{v_t} + \zeta$, where μ_{v_t} is the risk-neutral counterpart of $\hat{\mu}_{v_t}$, and ζ is the constant risk premium, i.e., $\zeta = \theta\sigma^m$.

According to Gordon's growth model, the asset value under the risk-neutral measure Q is as follows:

$$A_{t,v_t} \equiv A(X_t, v_t) = \mathbb{E}^Q \left[\int_t^\infty X_\tau e^{-r\tau} d\tau \right] = \frac{X_t}{r - \mu_{v_t}}. \tag{3}$$

managerial stock compensation induces risk-shifting behavior that helps explain the rates of credit default swaps (CDS) and leverage choices.

Because A_{t,v_t} is linear in X_t, it follows that

$$\frac{dA_{t,v_t}}{A_{t,v_t}} = \hat{\mu}_{v_t} dt + \sigma^m d\hat{W}_t^m + \sigma_{v_t}^{i,X} d\hat{W}_t^i. \tag{4}$$

Hence, the assets A_{t,v_t} and their generated cash flows, X_t, share the same parameters of growth and volatility.

2.1.1 Time line

To illustrate the time line of this dynamic model, Figure 1 plots four possible paths a firm could take in one refinancing cycle. At time 0, the firm enters the market and finances its low-risk investments with a mix of equity and debt. The installed assets produce cash flows, X_t, that are characterized by a physical growth rate, $\hat{\mu}_L$, and a total volatility parameter, σ_L. The firm uses the generated cash flows to pay taxes to the government and dividends to equity holders. The effective tax rate is τ. The dividend received by equity holders is the entire cash flow X_t, net of coupon payments c to debt holders and tax payments, i.e., $d_t = (1-\tau)(X_t - c)$.

In observing its dynamic cash flows, the firm makes financing, risk-taking and default decisions. Path 1 shows that, when its cash flow level reaches an upper threshold X_u, the firm decides to issue more debt to take advantage of tax benefits. Following Goldstein et al. (2001), we assume that the firm calls back its outstanding debt at par and issues a greater amount of debt to take advantage of tax benefits. In contrast, if the cash flow level, X_t, declines to a low threshold X_r along Path 2, equity holders thus choose to take on investments that produce cash flows with a low expected growth rate $\hat{\mu}_H$ and high volatility σ_H, hoping that a cash flow windfall due to the increased σ_H might save the firm. The low $\hat{\mu}_H$ is caused by the value-destroying cost, which will be introduced in the next subsection.

After taking more risk, there are two possibilities. The first possibility is shown in Path 3. The increased idiosyncratic risk may quickly release the firm from financial distress, and eventually leads to a subsequent debt restructuring at the *same* restructure threshold X_u. In order to enjoy the same cost of debt, i.e., coupon payment, the firm has incentives to adjust its level of idiosyncratic risk back to its previous level immediately before the refinancing at X_u, as if it had never increased the risk earlier. The second possibility is that the new risky projects may cause a more severe cash flow shortfall, as shown in Path 4. When cash flows cannot cover the coupon payments, the firm may issue new equity to cover the shortfall. If the firm continues to deteriorate, equity holders will no longer be willing to inject more capital, and will decide to go bankrupt at X_d, which is probably earlier than the firm would have gone bankrupt if it had not increased the idiosyncratic risk. Bankruptcy leads to immediate liquidation, in which equity holders receive nothing.

2.1.2 Costly Risk Adjustment

In this baseline model, equity holders choose the optimal timing X_r to increases their taking of idiosyncratic risk. This excess taking of risk, ϵ, is reversible with a cost when the firm returns to the debt markets to refinance. The change in the idiosyncratic risk, $\sigma_L^{i,X}$, to $\sigma_H^{i,X}$ is $\epsilon \geq 0$.[7]

The idiosyncratic risk-taking is costly. Compared to the static model of Leland (1994), the benefits of tax shelter are greater because this dynamic model allows the firm to repeatedly refinance debt upward at X_u and enjoy greater tax benefits as long as it manages to survive after the risk-shifting. However, if equity holders increase the level of idiosyncratic volatility too early, they might lose their opportunity to wait for the firm to bounce back, because the excessive risk-taking exacerbates the looming default. The undesirable earlier default causes the equity holders to lose the greater tax benefits. This is an implicit cost in repeated games.

There are three explicit costs as well. The first is the value-destroying cost. We follow Hennessy and Tserlukevich (2008) and assume that the investment with excess taking of idiosyncratic volatility is value-destroying. That is, this investment is an inferior project that decreases the expected growth rate by $\eta\epsilon^2$, where $\eta \geq 0$ is a cost proportional to excess taking of idiosyncratic risk, ϵ. This value-destroying cost causes a reduction in the growth rate as follows:

$$\hat{\mu}_H = \hat{\mu}_L - \eta\epsilon^2. \tag{5}$$

Because this reduction in the growth rate decreases the cash flow by $\eta\epsilon^2 X_t$, it is a flow cost. For example, it is more costly for sinking firms to attract talented workers for their "idiosyncratic"

[7]An asset is more idiosyncratic if it cannot easily be redeployed by other firms for common operations, if the firm has to resell it. For example, R&D investment is generally regarded as less redeployable than other assets (Titman, 1984). Practically, a firm can invest more in R&D projects to increase its idiosyncratic risk-taking. R&D expenses for more idiosyncratic (or unique) projects are generally greater than those for less idiosyncratic projects, because they demand more research inputs.

projects. If those firms were to eventually go bankrupt, their workers would have difficulty finding new jobs, given that their specific skills might not be applicable to other jobs (Titman, 1984). To attract and keep the talented workers, the distressed firms have to pay a high wage, which increases their operating cost and decreases the cash flow X_t.

The other two risk adjustment costs are upfront costs. They occur to the firm whenever it increases or decreases its level of idiosyncratic risk. We assume a proportional cost of increasing the risk at the threshold X_r, i.e, $\xi^+\epsilon^2 A(X_r)(1-\tau)$, where ξ^+ is the proportional cost of searching for and purchasing new risky investments, and $A(.)$ denotes the firm value at X_r. If the post-shifting firm survives and is able to refinance its debt upward, it faces a proportional risk adjustment cost $\xi^-\epsilon^2 A(X_u)(1-\tau)$, where ξ^- is the proportional cost of undoing the previous investment and adjusting the risk down. This adjustment cost can be thought of as the cost related to selling the installed high-risk investment, or setting up a risk management office to diversify away the excess risk. Otherwise, the firm will have to pay a higher coupon because of the increased excess taking of idiosyncratic risk. Although we do not explicitly model the reputation cost due to the risk-shifting behavior, the reputation cost can be considered part of this risk adjustment cost.

Although all three different costs deter the firm from undertaking excess risk, the value-destroying cost is different from the upfront adjustment costs, because the flow cost η reduces the firm's continuation value and induces an early bankruptcy and the two other upfront cost does not. While the upfront cost of increasing the risk, ξ^+, *explicitly* reduce a firm's risk-taking incentives, the future cost of reversing the increased risk, ξ^-, *implicitly* deters a precautionary firm from taking on excess risk if it anticipates to return to the debt markets in the future.

2.2 Optimal Policies

Since the dynamic model is solved by backward induction, we first show how to determine optimal default policy X_d. Then, we present the optimal risk-taking policy X_r. Lastly, we present the optimal refinancing policy X_u that maximizes the firm value.

Default Policy

If the risk-taking action does not save the firm, equity holders choose the optimal bankruptcy threshold X_d to maximize their own equity value $E(X_t, v_t)$ as follows:[8]

$$\lim_{X_t \downarrow X_d} E'(X_t, H) = 0, \tag{6}$$

where $E'(X_t, v_t)$ denotes the first-order partial derivative of the equity value function $E(X_t, v_t)$ with respect to X_t. Equation (6) is the smooth-pasting condition that allows equity holders to choose the optimal bankruptcy threshold by considering a tradeoff between the costs of keeping the firm alive and future tax shelter benefits (Leland, 1994).

Risk-Taking Policy

When the firm's condition is deteriorating, equity holders choose the optimal timing X_r to increase the level of idiosyncratic risk from $\sigma_L^{i,X}$ to $\sigma_H^{i,X}$ by ϵ. We use the following smooth-pasting condition to determine the optimal risk-shifting threshold X_r:

$$\lim_{X_t \downarrow X_r} E(X_t, L)' = \lim_{X_t \uparrow X_r} E(X_t, H)' - \xi^+ \epsilon^2 A'(X_t)(1-\tau), \tag{7}$$

where $E(X_t, v_t)'$ denotes the first-order partial derivative of the equity value function $E(X_t, v_t)$ with respect to X_t, and $A'(X_t)$ is the partial derivative of the asset function $A(X_t)$.

Refinancing Policies

At time 0, immediately after the debt is in place, the firm chooses the optimal timing of debt refinancing, X_u, *ex ante* to maximize the present value of the firm. The firm value is the sum of the equity value and the par value of debt, net of a proportional flotation cost ϕ, as follows:

$$\arg\max_{X_u} \quad E(X_0, L) + (1-\phi) D(X_0, L), \tag{8}$$

subject to equations (6) and (7).

Overall, as shown in Figure 1, the upper bound X_u and the lower bound X_d characterize the length of each refinancing cycle. The optimal X_r between X_u and X_d determines the relative weight of $\sigma_L^{i,X}$ and $\sigma_H^{i,X}$ within each refinancing cycle. The higher the threshold X_r, the greater the expected level of $\sigma^{i,X}$. Moreover, according to the dynamic paths of the underlying cash flow, we

[8] Davydenko (2008) documents that the majority of *negative*-net-worth firms do not default for at least a year, and that equity holders of distressed firms renegotiate with debt holders and violate bond covenants.

assume that the order of the optimal thresholds within each refinancing cycle is

$$X_d < X_r < X_0 < X_u. \tag{9}$$

This order can be easily satisfied given a reasonable set of parameter values in the literature.

2.3 Scaling Property of Optimal Policies

Because of the log-normal distribution of cash flow X_t and the proportional cost of debt, Goldstein et al. (2001) and Leland (1998) show that the dynamic model has a scaling property and can be reduced to a static problem for each refinancing cycle.

The scaling property states that, within each refinancing cycle, the optimal default, refinancing and risk-shifting thresholds, and the values of debt and equity, are all homogeneous of degree one in the cash flow X_t. The intuition is as follows. Given the same parameters of a firm's cash flow process, at two adjacent refinancing points, the firm faces identical problems in the above three optimal thresholds, except that the cash flow levels are different. In other words, if the cash flow level has doubled, it is optimal to double the default, risk-shifting and refinancing boundaries. However, the reversible increment in idiosyncratic volatility, ϵ, has to be the same across different refinancing cycles to ensure that the scaling property holds across any two consecutive refinancing cycles.

Our extended model keeps this scaling property because the two upfront adjustment costs and the value-destroying cost are proportional to assets. This property is particularly useful when we simulate the model to estimate the costs of risk-taking, because we do not have to solve for the optimal policies whenever the firms refinance their debt and increase their equity size repeatedly. This is the reason why we assume *proportional* adjustment costs. We take further advantage of this property when we expand our model to the two-state model.

3 Fully-Fledged Model with Macroeconomic Uncertainty Risk

We extend our baseline, single-state model by endogenizing a firm's financing, default and risk-taking decisions over the business cycle. We follow Bhamra et al. (2010b) and Chen (2010), and introduce the Epstein-Zin (EZ) preferences (Epstein and Zin, 1989) and time-varying macroeconomic risk into the single-state model. In this extended framework, a representative firm operates in two economic states, s_t, and is able to alter its idiosyncratic risk level, v_t, in response to both firm-level small shocks and state-level large shocks. In a nutshell, we have two *exogenous* aggregate states of the economy and two *endogenous* firm levels of idiosyncratic volatility.

3.1 Setup

Considering an economy with business-cycle fluctuations, and without loss of generality, we assume the economy has two aggregate states, i.e., $s_t = \{G, B\}$ for good (G) and bad (B) states, respectively. In addition to the standard Brownian risk in the baseline model, the pricing kernel includes the macroeconomic uncertainty risk as follows:

$$\frac{dm_t}{m_t} = -r_{s_t}dt - \theta_{s_t}d\hat{W}_t^m + \sum (\kappa_{s_t} - 1)d\hat{M}_t^{s_t}. \tag{10}$$

where r_{s_t} is the risk-free rate, θ_{s_t} is the market price of risk of small shocks in the state s_t, $\hat{W}_t^m$ is a standard Brownian motion, $\hat{M}_t^{s_t}$ is a compensated Poisson process with an intensity of $\hat{\lambda}_{s_t}$ that determines the aggregate switching between the good and bad states, and κ_{s_t} is the aggregate state-switching risk premium that determines the market price of large shocks in the aggregate economy, i.e., $\kappa_B = 1/\kappa_G$.

As noted by Bhamra et al. (2010b) and Chen (2010), the switching risk premium, κ_{s_t}, depends on investors' preference for the resolution of the aggregate economic uncertainty, and $\kappa_G > 1$ given preferences for an early resolution of uncertainty (Epstein and Zin, 1989).[9] The risk-neutral measure adjusts for such a preference by raising the probability that the economy will enter into a bad state. For example, the risk-neutral switching intensity from the good state to the bad state $\lambda_G = \hat{\lambda}_G\kappa_G > \hat{\lambda}_G$ when $\kappa_G > 1$. This preference parameter κ_{s_t} plays a crucial role in our model of

[9] Bhamra et al. (2010b) assume that the representative agent has the continuous-time analog of EZ preferences of stochastic differential utility type (Duffie and Epstein, 1992). The utility index U_t over a consumption process C_s solves

$$U_t = \mathbb{E}^P\left[\int_t^\infty \frac{\rho}{1-\delta}\frac{C_s^{1-\delta} - ((1-\iota)U_s)^{\frac{1-\delta}{1-\iota}}}{((1-\iota)U_s)^{\frac{1-\delta}{1-\iota}} - 1}ds|\mathcal{F}_t\right],$$

in which ρ is the rate of time preference, ι the coefficient of relative risk aversion, and $\psi = \frac{1}{\delta}$ the elasticity of inter-temporal substitution for deterministic consumption paths. Incorporating the separability of time and state preferences and assuming $\psi > 1$, the representative agent has a preference for early resolution of uncertainty.

risk-shifting.

When the firm is solvent, it produces instantaneous cash flows X_t governed by the following stochastic process:

$$\frac{dX_t}{X_t} = \hat{\mu}_{s_t,v_t} dt + \sigma^m_{s_t} d\hat{W}^m_t + \sigma^{i,X}_{s_t,v_t} d\hat{W}^i_t, \tag{11}$$

where $\hat{\mu}_{s_t,v_t}$ is the expected growth rate in the state of s_t and the volatility regime of v_t, $\sigma^m_{s_t}$ is the systematic volatility, $\sigma^{i,X}_{s_t,v_t}$ is the idiosyncratic volatility of cash flows, and $\hat{W}^m_t$ and $\hat{W}^i_t$ are standard Brownian motions. The total volatility of cash flows $\sigma_{s_t,v_t} = \sqrt{(\sigma^m_{s_t})^2 + (\sigma^{i,X}_{s_t,v_t})^2}$. Moreover, $\hat{\mu}_{s_t,v_t} = \mu_{s_t,v_t} + \zeta_{s_t}$, where μ_{s_t,v_t} is the risk-neutral counterpart of $\hat{\mu}_{s_t,v_t}$ and $\zeta_{s_t} = \sigma^m_{s_t}\theta_{s_t}$ is the risk premium in the state s_t. Same as in the baseline model, the value-destroying cost η causes a reduction in the growth rate $\hat{\mu}_{s_t,v_t}$, which differs within each aggregate state, s_t, as follows:

$$\hat{\mu}_{s_t,H} = \hat{\mu}_{s_t,L} - \eta\epsilon^2_{s_t}. \tag{12}$$

For simplicity, we assume η is constant across the two economic states.

Simply put, $\sigma^m_{s_t}$ and θ_{s_t} are constant within each state but vary across the two states. $\hat{\mu}_{s_t,v_t}$ and $\sigma^{i,X}_{s_t,v_t}$ vary across the two states $s_t \in (B, G)$ and two volatility levels $v_t \in (H, L)$.

3.2 Time Line

The time line is largely similar to that for the baseline model. We mainly highlight the differences in notation here. At time 0, the firm finances its investments with a mix of equity and debt in the initial state s_0. The initial debt and its associated coupon payment $c(s_0)$ affect the firm's decisions on when to increase its debt holding, when to increases its operational riskiness, and when to go bankrupt in the future state s_t. Following Chen (2010), we allow a countercyclical liquidation cost α_{s_t}, i.e. $\alpha_B > \alpha_G$, because fire sales in bad times are more costly.

When cash flow increases to a high threshold $X_u(s_t; s_0)$ in the aggregate state s_t, the firm first calls back its outstanding debt and then issues more debt with a new coupon payment $c(s_t)$. When cash flow X_t declines to a low threshold $X_r(s_t; s_0)$ in either state s_t, equity holders who anticipate a continuing deterioration choose to take on more high-idiosyncratic-risk investments that produce cash flows with a low growth rate $\hat{\mu}_{s_t,H}$ but a high volatility level $\sigma_{s_t,H}$. Immediately before the firm refinances its debt at the same threshold $X_u(s_t; s_0)$, equity holders adjust their idiosyncratic risk from $\sigma^{i,X}_{s_t,H}$ back to the original low level of $\sigma^{i,X}_{s_t,L}$. In contrast, if the firm's performance is still deteriorating, its equity holders are no longer willing to inject more capital, and decide to go bankrupt at $X_b(s_t; s_0)$.

3.3 Optimal Policies

In this fully-fledged model, equity holders with EZ preferences chooses optimal timing of financing, risk-taking and default to maximize its equity holders' value in the presence of macroeconomic uncertainty. Because the firm operates in the fluctuating aggregate economy, it makes decisions in the initial state s_0 by anticipating the economy switching into another state s_t. Suppose the firm enters the economy in good times, $s_0 = G$. It has to account for situations in which the economy may slide into bad times, i.e., recessions. It is worth noting again that the risk-neutral probability of the economy switching from good times to bad, as perceived by risk-neutral equity holders, is higher than the actual switching probability.

Default Policy

After they has already increased their firm's idiosyncratic risk to a high level, $v_t = H$, in either aggregate state s_t, equity holders choose their optimal bankruptcy thresholds $X_d(s_t; s_0)$ by making a tradeoff between the costs of keeping the firm alive and the future tax benefits (Leland, 1994). We have the following smooth-pasting conditions to determine the optimal $X_d(s_t; s_0)$:

$$\lim_{X_t \downarrow X_d(B;s_0)} E^{'}(X_t, B, H; s_0) = 0, \tag{13}$$

$$\lim_{X_t \downarrow X_d(G;s_0)} E^{'}(X_t, G, H; s_0) = 0, \tag{14}$$

where $E(X_t, s_t, v_t; s_0)$ is the equity value function of the firm with a level of idiosyncratic risk, v_t, in the aggregate state s_t, conditional on the initial state s_0, and $E'(X_t, s_t, v_t; s_0)$ denotes its first-order partial derivative with respect to X_t.

Risk-shifting Policy

We use the following smooth-pasting conditions to determine the optimal risk-shifting threshold

$X_r(s_t; s_0)$:

$$\lim_{X_t \downarrow X_r(B;s_0)} E^{'}(X_t, B, H; s_0) = \lim_{X_t \downarrow X_r(B;s_0)} E^{'}(X_t, B, L; s_0) - \xi^{+}\epsilon_B^2 A^{'}(X_t, B, H)(1-\tau), \tag{15}$$

$$\lim_{X_t \downarrow X_r(G;s_0)} E^{'}(X_t, G, H; s_0) = \lim_{X_t \downarrow X_r(G;s_0)} E^{'}(X_t, G, L; s_0) - \xi^{+}\epsilon_G^2 A^{'}(X_t, G, H)(1-\tau). \tag{16}$$

where $A^{'}(X_t, s_t, v_t)$ is the partial derivative of the asset value function $A(X_t, s_t, v_t)$.

Refinancing Policy

At time 0, the firm is born in an initial aggregate economic state, s_0, and has a low level of idiosyncratic volatility (i.e., $v_t = L$). After the debt is in place, equity holders choose the optimal timing of debt refinancing $\mathbf{X_u}(s_0)$ so as to maximize the present value of the firm, where the vector $\mathbf{X_u}(s_0) = \{X_u(B; s_0), X_u(G; s_0)\}$, as follows:

$$\max_{\mathbf{X_u}(s_0)} \quad E(X_0, s_0, L; s_0) + (1 - \phi_{s_t}) D(X_0, s_0, L; s_0), \tag{17}$$

$$\text{subject to equations (13) to (16),}$$

where $D(X_0, s_t, L; s_0)$ denotes the debt value function of a firm with the level of idiosyncratic volatility v_t in the aggregate state s_t, conditional on the initial state s_0.

In total, we have six optimal thresholds to be determined, given an initial state s_0. We impose the following order of the optimal thresholds:

$$X_d(G; s_0) < X_d(B; s_0) < X_r(G; s_0) < X_r(B; s_0) < X_0 < X_u(G; s_0) < X_u(B; s_0). \tag{18}$$

It is intuitive that the firm file for bankruptcy earlier in the bad state than in the good state, i.e., $X_d(G; s_0) < X_d(B; s_0)$. Similarly, the firm might take corrective action earlier in the bad state, i.e., $X_r(G; s_0) < X_r(B; s_0)$. We also impose that the firm refinances debt earlier in the good state than in the bad state, i.e., $X_u(G; s_0) < X_u(B; s_0)$. This order can easily be satisfied under a set of reasonable parameter values drawn from the literature.

3.4 Scaling Property across Two States

The scaling property described in Section 2.3 only holds within the same state. Chen (2010) and Bhamra et al. (2010b) extend this scaling property to different states. Across two initial states, s_0, due to the homogeneity, the optimal thresholds in both states, s_t, are proportional to the coupons issued in the initial states as follows:

$$\frac{X_d(s_t;G)}{X_d(s_t;B)} = \frac{X_r(s_t;G)}{X_r(s_t;B)} = \frac{X_u(s_t;G)}{X_u(s_t;B)} = \frac{c(G)}{c(B)}. \tag{19}$$

We assume that the firm starts in the good state, i.e., $s_0 = G$, and solve for a set of policies. With the cross-state scaling property, we can obtain the policies for firms that start in the bad state $s_0 = B$. Combining the within-the-state scaling property and the across-the-state scaling property, we can determine all the optimal policies over the business cycle.

4 Asset Pricing Implications

The risk-shifting behavior affects equity risk and expected returns. To demonstrate the implications of risk-shifting for asset prices, we first simplify the baseline model and use a closed-form solution to demonstrate how the strategically increased idiosyncratic volatility reduces the exposure of equity to systematic cash flow risk. Then, in the fully-fledged model with macroeconomic risk, we show how the interaction between the reduction in the risk exposure and the countercyclical market risk premium further reduces the expected equity returns.

4.1 Simplified Baseline Model

In this simplified version, the firm has no option to reverse its risk-taking or refinance its debt. It has an option to increase idiosyncratic risk and an option to go bankrupt. Because the firm does not return to the debt market, the option to increase risk is a one-time game in this simplified case. This simplified version has a semi-closed-form solution for the equity and equity return, which allows us to flesh out how equity holders strategically take full advantage of the American put option to go bankrupt, which in turn reduces the sensitivity of equity to cash flow.

The following proposition states the expected excess return in this simplified model.[10]

Proposition 1 *Before bankruptcy, $X_t > X_d$, the expected excess return of equity for a firm with a level v_t of idiosyncratic volatility is as follows:*

$$r^e_{t,v_t} = \mathbb{E}_t[r^E_{t,v_t}] - rdt = \gamma_{t,v_t}\zeta dt. \tag{20}$$

After the firm increases its taking of idiosyncratic risk to $v_t = H$ at the optimal threshold X_r, the sensitivity of its stock value to the underlying assets or cash flows, $\gamma_{t,H}$, is

$$\gamma_{t,H} = \frac{\partial E_{t,H}/E_{t,H}}{\partial X_t/X_t} = \frac{\partial E_{t,H}/E_{t,H}}{\partial A_{t,H}/A_{t,H}}, \tag{21}$$

$$= 1 + \underbrace{\frac{c/r(1-\tau)}{E_{t,H}}}_{\textit{Leverage}} \underbrace{-(1-\omega_{1,H})\frac{(c/r - A_{d,H})}{E_{t,H}}\left(\frac{X_t}{X_d}\right)^{\omega_{1,H}}(1-\tau)}_{\textit{American Put Option of Delaying Bankruptcy (+)}} \tag{22}$$

where equity value $E_{t,H}$ is given by

$$E_{t,H} = \left[\underbrace{\left(A_{t,H} - \frac{c}{r}\right)}_{\textit{Equity-in-Place}} + \underbrace{\left(\frac{c}{r} - A_{d,H}\right)\left(\frac{X_t}{X_d}\right)^{\omega_{1,H}}}_{\textit{Option of Delaying Bankruptcy}}\right](1-\tau), \tag{23}$$

the optimal default threshold X_d is

$$X_d = \frac{c}{r}\frac{(r-\mu_H)\omega_{1,H}}{(\omega_{1,H}-1)}, \tag{24}$$

and the optimal risk-shifting threshold X_r is

$$X_r = \left[\frac{(c/r - A_{d,H})(\omega_{1,H} - \omega_{1,L})}{X_d^{\omega_{1,H}}\left(\frac{1}{r-\mu_L} - \frac{1-\eta\epsilon^2}{r-\mu_H}\right)(1-\omega_{1,L})}\right]^{\frac{1}{1-\omega_{1,H}}}. \tag{25}$$

Proof: See the Appendix C.2.

Equation (20) shows that the expected excess equity return, r^e_{t,v_t}, is the product of the *systematic* risk premium, ζ, and the sensitivity of stocks to underlying assets, γ_{t,v_t}. The time-varying element

[10]We derived the closed-form solution for the firm prior to the risk-shifting as well, which is available upon request.

for the expected excess stock return is then γ_{t,v_t} in equation (21). We denote by γ_{t,v_t} the "equity-cash flow sensitivity"or the "equity-asset sensitivity"because, strictly speaking, it measures the percentage changes in the equity value in response to one percentage change in cash flows or asset values.

After the firm increases its idiosyncratic volatility, the sensitivity is $\gamma_{t,H}$, which consists of three components, as shown in equation (22). The first is the baseline sensitivity, which is normalized to one. The second is related to financial leverage, as c/r can be regarded as risk-free equivalent debt. Not surprisingly, the equity-cash flow sensitivity is positively associated with the financial leverage. Because the coupon c is fixed after debt is in place, the increased excess risk ϵ increases $E_{t,H}$, thereby reducing the financial leverage and the equity-cash flow sensitivity.

The last component, the option of delaying bankruptcy, decreases the equity-cash flow sensitivity. This option, which is essentially an American put option, protects equity holders from downside risk. Given limited liability, equity holders choose to go bankrupt only when the asset value $A_{d,H}$ falls below the risk-free equivalent debt c/r.[11] Hence, $c/r - A_{d,H} > 0$.

The equity value after risk-shifting shown in equation (23) has two components, equity-in-place and the option of delaying bankruptcy. The optimal default threshold in equation (24) increases with the coupon payment c. Similarly, the optimal risk-shifting threshold in equation (25) shows that the risk-shifting threshold increases with the risk-free equivalent debt c/r, which is consistent with our intuition that greater financial leverage induces firms to increase their risk-taking earlier.

The greater the cash flow volatility, the more opportunities equity holders have to receive a cash flow windfall. Therefore, equity holders of a firm with high idiosyncratic cash flow volatility have more incentives to delay bankruptcy. The delayed bankruptcy implies a smaller asset value $A_{d,H}$, i.e., $\partial A_{d,H}/\partial \sigma_H^{i,X} < 0$. Everything else being equal, the payoff of the put option, $c/r - A_{d,H}$, increases with $\sigma_H^{i,X}$. Therefore, the increase in the value of the American put option due to strategically increased volatility, ϵ, decreases the equity-cash flow sensitivity.

In a contemporaneous work, McQuade (2016) shows that the endogenous default option is important for distressed firms to hedge against the uncertainty. Thus, this option lowers the equity holders' risk exposure and equity returns. The endogenously increased volatility in our model can

[11] Empirically, Davydenko (2008) documents that the majority of *negative*-net-worth firms do not default for at least a year, and that the mean (median) of the market value of assets at default is only 66% (61.6%) of the face value of debt. This finding shows the importance of the option to delay bankruptcy.

further amplify the negative effect from the endogenous default option on equity risk and returns, because it increases the value of this option. We share Tim McQuade's view of the importance of the strategic default option in lowering equity risk and returns, and additionally, emphasize the option to endogenously increase the idiosyncratic cash flow volatility.

To increase the idiosyncratic risk, the firm has to make capital investments to alter its risk profile. In the investment/production based asset pricing framework, Liu, Whited, and Zhang (2009) show that firms with more investments receive low stock returns. This negative investment effect on future stock returns is similar to the negative effect of idiosyncratic volatility on future stock returns. We complement the investment literature by focusing on the "idiosyncratic" investments and provide an explicit theoretical link between the idiosyncratic volatility and stock returns via the closed-form solution in equations (20) to (22).

In short, the strategically increased idiosyncratic cash flow growth volatility, $\sigma_H^{i,X}$, lowers the equity-cash flow sensitivity, $\gamma_{t,H}$, and therefore the expected excess stock returns, r^e_{t,v_t}, for firms with a high level of idiosyncratic volatility.

4.2 Fully-Fledged Model with Macroeconomic Risk

We proceed to show the expected excess stock return in the fully-fledged model that features the macroeconomic uncertainty risk. In this two-state economy, a firm endogenously chooses the timing of increasing its taking of idiosyncratic risk in response to the exogenous switches of the economy between aggregate good and bad states.

The following proposition shows that the expected excess stock return differs across the two states $s_t \in (G, B)$ and the two levels of idiosyncratic volatility, $v_t \in (H, L)$.

Proposition 2 *After entering the market at the initial state s_0, the firm then operates in the two aggregate states s_t. The conditional expected excess return of equity with a level v_t of idiosyncratic volatility is*

$$r^e_{s_t,v_t} = \mathbb{E}_t[r^E_{s_t,v_t}] - rdt = \zeta_{s_t}\gamma_{s_t,v_t}dt + \psi_{s_t,v_t}(1-\kappa_{s_t})\hat{\lambda}_{s_t}dt \tag{26}$$

where $\gamma_{s_t,v_t} = \frac{X_t \partial E_{s_t,v_t,s_0}}{E_{s_t,v_t,s_0}\partial X_t}$, which measures the sensitivity of equity to the cash flow X_t, $\psi_{s_t,v_t} = (\frac{E_{s_t^+,v_t,s_0}}{E_{s_t,v_t,s_0}} - 1)$, which measures the percentage change in equity value in response to the changes in the aggregate economy from the state s_t to the other state s^+, and $E_{s_t,v_t,s_0} \equiv E(X_t, s_t, v_t; s_0)$.

Proof: See the Appendix C.3.

Equation (26) has two components, the risk premium due to small market shocks within one aggregate state and the risk premium due to large switches across the two aggregate states, s_t. In the first component, different from the constant price of risk in equation (20) in the baseline model, the price of risk $\zeta_{s_t} = \theta_{s_t}\sigma^m_{s_t}$ is countercyclical, because it is well known that the market price of risk $\theta_B > \theta_G$ and the market volatility $\sigma^m_B > \sigma^m_G$ (see e.g., Bhamra et al. (2010b) and Chen et al. (2009)). Therefore, everything else being equal, given the same cross-sectional spread in the idiosyncratic volatility, the countercyclical market premium causes a larger spread in equity returns in recessions than in expansions.

Moreover, the equity-cash flow sensitivity, γ_{s_t,v_t}, in the first component is procyclical at the portfolio level or at the aggregate level.[12] That is, γ_{s_t,v_t} is low in the bad aggregate state, $s_t = B$. Because a large, negative market shock could push the level of cash flows substantially to the low threshold X_r, and cause many firms to increase the level of idiosyncratic volatility to $v_t = H$, this increased level of idiosyncratic volatility lowers the sensitivity according to equation (22). That is, everything else being equal, $\gamma_{B,v_t} < \gamma_{G,v_t}$, because $\sigma^{i,X}_{B,v_t} > \sigma^{i,X}_{G,v_t}$ at the portfolio level. Therefore, the sensitivity γ_{s_t,v_t} and the market risk premium ζ_{s_t} covary negatively. For example, for a portfolio of firms with high idiosyncratic volatility, $v_t = H$, this negative covariance further reduces the the unconditional expectation of the first component of equation (26) as follows:[13]

$$\mathbb{E}[\gamma_{s_t,H}\zeta_{s_t}dt] = \overline{\gamma_{s_t,H}}\,\overline{\zeta_{s_t}}dt + \mathbf{cov}(\gamma_{s_t,H}, \zeta_{s_t})dt, \tag{27}$$

where $\overline{\gamma_{s_t,H}}$ is the expected sensitivity, $\overline{\zeta_{s_t}}$ is the expected risk premium, and $\mathbf{cov}(\gamma_{s_t,H}, \zeta_{s_t})$is the covariance between $\gamma_{s_t,H}$ and ζ_{s_t} across the two states. Therefore, $\mathbf{cov}(\gamma_{s_t,H}, \zeta_{s_t}) < 0$, causing a further reduction in the unconditional expected equity return for the portfolio of high volatility firms.

The second component, $\psi_{s_t,v_t}(1-\kappa_{s_t})\hat{\lambda}_{s_t}dt$, captures macroeconomic uncertainty risk. The price

[12] We emphasize the portfolio level or the aggregate level because not necessarily every single firm increases its risk-taking in recessions.

[13] This is in the same spirit as Jagannathan and Wang (1996). They argue that the covariance between the market beta and the expected market risk premium plays an important role in the conditional CAPM.

of the uncertainty risk, $\hat{\lambda}_{s_t}(1-\kappa_{s_t})$, is countercyclical, because the EZ-type equity holders prefer an early resolution of macroeconomic state-switching uncertainty. According to Bhamra et al. (2010b) among others, the preference for an early resolution implies $\kappa_G > 1$. That is, when the economy is in the good state, $s_t = G$, investors like this good state and are willing to charge (pay) a negative (positive) risk premium for staying in the good state. In contrast, when the economy is in the bad state, investors do not like this bad state, and demand a positive risk premium for staying this state. In other words, the state-switching premium is negative in good times $(1-\kappa_G \leq 0)$, but positive in bad times.

Our model has two risk factors, one related to small Brownian shocks and another related to large state-switching shocks. The endogenously changed level of idiosyncratic volatility does not enter the two factors.[14] Instead, it changes the equity holders' exposure to the two risk factors. In the model of Herskovic et al. (2015), common idiosyncratic volatility (CIV) is a systematic risk factor because of the not fully diversifiable labor income. The CIV changes over time and the shocks in CIV carry a negative market price of risk. Therefore, stocks with a high loading on this CIV risk receive lower equity returns. Simply put, our model is different from theirs because idiosyncratic volatility in our model affects the equity return via the *exposure* of equity holders to risk factors.

In summary, we first derive the closed-form solution to demonstrate that the high idiosyncratic volatility causes a low equity-cash flow sensitivity and equity returns in the simplified one-state baseline model. Then, by extending the one-state model to the two state one with countercyclical market risk premium, we show that the negative covariance between the equity-cash flow sensitivity and the expected risk premium results in a further reduction in the bad states for the firms with a high level of idiosyncratic volatility.

5 Data and Estimation Method

We perform structural estimation for the fully-fledged model. We describe the model inputs and estimation method, and the intuition behind the structural estimation.

[14]Note that the value-destroying cost, $\eta\epsilon^2_{s_t}$, does not enter the pricing kernel as well, although it decreases the equity value for the firms with a high level of idiosyncratic volatility.

5.1 Data

We obtain accounting information from quarterly Compustat industrial data. For availability reasons, our sample period is from January 1975 to December 2014. We restrict the sample to firm-quarter observations with non-missing values for operating income and total assets, with positive total assets. We include common stocks listed on the NYSE, AMEX, and NASDAQ with CRSP share code 10 or 11. We exclude firms from the financial and utility sectors.

Debt is the sum of current liabilities (Compustat item DLCQ) and long-term debt (item DLTTQ). If the debt is missing, we set it to zero. Quasi-market leverage (QML) is the ratio of the book value of debt to the sum of debt and equity (PRCCQ*CSHOQ). We use the growth of assets (item ATQ) to proxy for cash flow growth, because in our model the asset growth and cash flow growth share the same parameters, as in equation (4). We follow the standard empirical procedure and use 20 quarter cash flow residuals to calculate the rolling standard deviation, which we use to proxy for the idiosyncratic cash flow volatility $\sigma_t^{i,X}$. The cash flow residuals are obtained from a regression of cash flow growth rates of the past 20 quarters on the simple average of cash flow growth rates among all the firms.

Following Strebulaev and Whited (2012), we remove the heterogeneity of financial leverage and idiosyncratic cash flow volatility by demeaning the time-series mean of the variables, and adding the sample mean of each variable, because our model is for a representative firm and we do not allow heterogeneity of parameter values in the model simulation.

5.2 Estimation Method

Following Bloom (2009), we use simulated method of moments (SMM) to estimate the model. We aim to estimate the value-destroying parameter η, the risk adjustments ξ^+ and ξ^-, and the increments of ϵ_{s_t}. The vector of the parameters to be estimated, $\mathbf{b}$, is as follows:

$$\mathbf{b} = [\eta \quad \xi^+ \quad \xi^- \quad \epsilon_{s_t}]. \tag{28}$$

To keep the estimation parsimonious, we assume the increases in idiosyncratic cash flow volatility are the same in the bad and good states, i.e., $\epsilon_B = \epsilon_G$. We relax this restriction in our robustness tests.

Let M denote the $K \times 1$ vector of data moments. Given a parameter vector $\mathbf{b}$, for each simulation $s = 1, \cdots, S$, we simulate a time series of length T and compute a vector of moments from the simulated data, $\tilde{M}_s(\mathbf{b})$, that serves as an analog to the data moments, M. The method of moments estimator for the parameters is defined as

$$\hat{\mathbf{b}} = \min_{\mathbf{b}} \quad J = \left(M - \frac{1}{S}\sum_{s=1}^{S} \tilde{M}_s(\mathbf{b})\right)' W \left(M - \frac{1}{S}\sum_{s=1}^{S} \tilde{M}_s(\mathbf{b})\right) \tag{29}$$

where W is a positive semidefinite weighting matrix. We discuss in detail about the covariance matrix of data and estimates, model simulation and model-generated variables in Appendix D.

5.3 Identification Moments

Identification in structural estimation means choosing moments whose predicted values move with the model's parameter values and choosing enough moments that there are unique parameter values that ensure the model fits the data as closely as possible. Therefore, the key to identification is to choose moments M that are informative about the parameters. Because all the parameters to be estimated are related to the process of idiosyncratic cash flow volatility, our strategy is to match the empirical distribution of the cash flow volatility process.

For the three cost parameters, because η and ξ^+ determine the timing of increasing excess risk, and ξ^- determines the timing of decreasing risk, they largely determine the upper and lower bounds of idiosyncratic volatility across all the firms, each quarter. They can be pinned down by the variance of $\sigma_t^{i,X}$, the interquartile time series of $\sigma_t^{i,X}$, and the first-order autocorrelation of this interquartile. Given the initial low level of idiosyncratic cash flow volatility, the increment ϵ_{s_t} can be determined by the time series mean of $\sigma_t^{i,X}$ from the data.

More importantly, to ensure the idiosyncratic volatility changes are driven by the risk-shifting mechanism, we match the averaged quasi-market financial leverage QML_{t-1} and the sensitivity of $\sigma_t^{i,X}$ to QML_{t-1}. The sensitivity is estimated by regressing the sample mean of $\sigma_t^{i,X}$ on an intercept and the sample mean of QML_{t-1}.

It is worth noting that we do not use the cross-sectional equity returns or the idiosyncratic equity return volatility as matching moments, in order to mitigate the concern that the parameters of the idiosyncratic volatility process are directly implied by equity returns. In addition to the above targeted moments, we also examine whether our model can match other data characteristics, including the average cash flow rates, interest coverage, equity market Sharpe ratio, equity market return volatility, firm-level volatility of equity returns, and default probability.

Taken together, the identification moments vector, M, includes six moments for four unknown parameters, such as the averaged idiosyncratic cash flow volatility $\sigma_t^{i,X}$, the variance of $\sigma_t^{i,X}$, the average of the interquartile time series of $\sigma_t^{i,X}$, the first-order autocorrelation of this interquartile time series, quasi-market leverage QML_{t-1}, and the sensitivity of $\sigma_t^{i,X}$ to QML_{t-1}.

6 Model Estimation Results

We start with the preliminary analysis for the baseline model, and then present the results of structural estimation for the fully-fledged model. With the estimated model, we examine the sensitivity of optimal policies and model-generated moments to estimated parameters and macroeconomic variables.

6.1 Analysis for the Baseline Model

We perform comparative statics and sensitivity analysis for the costs of risk-taking and for market risk variables. To compare the relative importance across all the variables, we investigate the sensitivity of the optimal policies to the three costs and two macroeconomic risk variables.

We set the parameter values for the widely used parameters, and list them in Panel A of Table 1. For the adjustment costs, we set the value-destroying cost η to 0.03, the cost of increasing risk ξ^+ to 0.05, and the cost of decreasing risk ξ^- to 0.05. The increment in idiosyncratic volatility, ϵ, is set to 0.15, so that $\sigma_{s_t,H}^{i,X} = 0.25$ $(\sigma_{s_t,L}^{i,X} + \epsilon_{s_t})$. Given those predetermined values, we solve the baseline model and find that the risk-shifting threshold X_r is 0.149, which is far below the coupon c of 0.4 and slightly above the default threshold X_d of 0.025. The very low threshold X_r indicates that the firm has little incentive to shift risk before the bankruptcy.

Panel B displays the results from comparative statics analysis. First, consistent with our intuition, all the three costs have a negative impact on the risk-shifting threshold. Second, out of all the three costs, the value-destroying flow cost η plays the most important role in deterring firms from taking on excess idiosyncratic risk, as evidenced by the largest decrease in X_r from 0.149 to

0.105. This is consistent with our early discussion that the flow cost η reduces the firm's continuation value and the opportunity to survive, which in turn reduces the likelihood to return to the markets to refinance debt upward for greater tax benefits in repeated games. Third, the increases in the market risk and the market price of risk induce a substantial increase in X_r, motivating us to incorporate the state-varying macroeconomic risk in the full model.

Compared with the comparative statics analysis, the sensitivity analysis allows us to compare the relative importance across different variables, because the sensitivity is measured by the percentage changes in the policies due to a one-percentage change in the variable. We present the results in Panel C. While the results show the same directional impacts as those from the comparative statics, two observations across the parameters are worth noting. First, the market risk variables have much greater impacts in absolute value on the the risk-shifting threshold than the three risk-taking costs in determining the risk-shifting threshold X_r. For example, the sensitivity to the market volatility σ^m is 6.555, which is the highest in Panel C, followed by the sensitivity to the market risk premium 4.753. Second, among the three costs, the greatest impact comes from the value-destroying cost η, with a negative sensitivity of –0.355.

In short, the market risk variables are far more important than the three cost variables when firms make risk-taking decisions. Among the three explicit costs, the value-destroying cost is the most significant determinant.

6.2 Structural Estimation for the Full Model

Having seen the relative importance of the parameters in the baseline model, we proceed to estimate our model via the simulated method of moments. The predetermined parameter values are listed in Panel A Table 2 and their justifications can be found in Appendix Subsection D.4.

As shown in Panel B, all the estimates are statistically significant, except for the cost of increasing idiosyncratic risk, ξ^+. The estimate of the value-destroying cost η is 0.031, with a t-statistic of 31.198, which is economically and statistically significant. The estimated cost of decreasing risk, ξ^-, is 0.015 (t-stat = 8.825), greater than the cost of increasing risk. The difference between the estimated ξ^- and ξ^+ captures the capital loss incurred in reversing investments with high idiosyncratic risk. As for the model fitness, the p-value of χ^2 is 0.932, indicating the model cannot be rejected. Panel C presents the optimal policies for initial state $s_0 = G$, given the predetermined and estimated parameters. The optimal policies for the initial state $s_0 = B$ can be obtained via the cross-state scaling property in equation (19). Compared with the optimal policies for the benchmark values shown in Table 1, the optimal risk-shifting threshold $X_r(G; s_0 = G)$ is 0.577, much greater than the 0.149 for the benchmark parameter value in Panel B of Table 1.

Table 3 reports the targeted and untargeted moments. The model-generated moments are averaged across 100 simulated economies. The targeted moments in Panel A are mainly used to identify the time series and cross-sectional dynamics of idiosyncratic risk. A first glimpse shows that all the targeted moments are well matched, with the largest deviation for the first-order autocorrelation of the interquartile of $\sigma_t^{i,X}$. The average level and interquartile of the idiosyncratic cash flow volatility, $\sigma_t^{i,X}$, are 0.127 and 0.055, respectively. The average quasi-market leverage, QML_t, from the model is 0.238, and the sensitivity of $\sigma_t^{i,X}$ to QML_{t-1} is 0.203. The differences between the model-generated moments and the data moments are very small, and the t-statistics in the last column suggest that, for all the moments except for the QML_t, the differences are not statistically different from zero.

In Panel B, we report the other moments that we do not choose to match in our estimation. From the model-generated samples, the average asset growth rate is 0.074, the interest coverage is 2.481, and the standard deviation of the market return is 0.150. All of them are close to the data. The average equity Sharpe ratio from our model is 0.309, close to the 0.329 generated in Gomes and Schmid (2016), and 25% of the 100 simulated economies has a Sharpe ratio above 0.458. Both the firm-level volatility of quarterly equity returns and default probability are relatively lower than those in the data. The firm-level volatility is 0.315, slightly lower than the 0.339 reported by Kuehn and Schmid (2014).

Overall, our model delivers a reasonable job in matching both targeted and untargeted moments.

6.3 Importance of the Value-Destroying Cost and the Risk Adjustment Costs

In this section, we examine the sensitivity of the optimal policies and the targeted moments to the estimated parameters. Because the optimal risk-taking policy determines the targeted moments of the idiosyncratic volatility process, the sensitivity analysis allows us to evaluate whether our targeted moments are sensitive to the estimated parameters. High sensitivities indicate the identification moments are well selected and the model is well identified.

The value-destroying cost and the two adjustment costs affect the optimal timing of risk-taking and -reversing. Three observations are worth noting from Panel A of Table 4. First, the value-destroying cost η significantly deters the distressed firms from increasing idiosyncratic risk, as evidenced by the negative sensitivity, –0.634 for $X_r(B;G)$ and –0.637 for $X_r(G;G)$, which is consistent with the finding in Table 1 for the baseline model. Second, the two adjustment costs have different impacts on the timing of risk-shifting. While the cost ξ^+ of increasing risk has a trivial effect on the risk-taking policies $X_r(s_t;G)$ and other policies, the future cost ξ^- of reversing the increased risk has a relatively stronger negative impact. These two observations are also in line with the findings in Table 1. Third, the increase in idiosyncratic volatility ϵ_{s_t} ($\epsilon_B = \epsilon_G$) decreases the risk-shifting threshold, because the cost of excess risk-taking is proportional to ϵ_{s_t}. The increase in idiosyncratic risk also causes a delay in default and refinancing, because the endogenously increased idiosyncratic risk further boosts the values of the options to default and to refinance debt upward.

The optimal timings of risk-taking and -reversing consequently determine the distribution and moments of the idiosyncratic cash flow volatility in the simulated samples. Panel B shows that, in general, the targeted moments are more sensitive to the value-destroying costs η and the increment ϵ_{s_t}, but less sensitive to the two adjustment costs. The sensitivity of $\sigma_t^{i,X}$ to QML_{t-1} is informative about all the estimated parameters, except for the cost of increasing idiosyncratic risk.

Simply put, the sensitivity analyses demonstrate that our model performs well in the parametrization and estimation of the risk-shifting mechanism. The flow value-destroying cost dominates the other two upfront adjustment costs when a firm makes risk-shifting decisions. Among the two adjustment costs, the future upfront cost of reversing risk plays a relatively more important role to the precautionary firms than the current upfront cost of increasing risk in repeated games.

6.4 Importance of Macroeconomic Risk

We have shown that a greater market risk and price of market risk induce a higher optimal risk-taking threshold in Table 1. We proceed to examine how the aggregate state-switching risk affects the firms' risk-taking decisions.

The sensitivity analysis allows us to compare the relative effects of macroeconomic parameters on the optimal policies. In Panel A of Table 5, three observations are worth noting. First, compared with those in Panel C of Table 1, the risk-shifting and default policies are much less sensitive to both the price of risk θ_{s_t} and market volatility $\sigma^m_{s_t}$, because the aggregate state-switching risk dominates their effects. Second, among all the state-switching parameters, κ_G has the most significant impact for all the policies in both states, s_t. This is consistent with our early discussion that equity holders, who prefer an early resolution of the state-switching risk, have more incentives to exercise their option of risk-taking when the increased κ_G induces a high risk-adjusted probability $\lambda_G = \kappa_G \hat{\lambda_G}$. Third, the probability of leaving the current bad state, $\hat{\lambda_B}$, and the probability of leaving the current good state, $\hat{\lambda_G}$, have opposite effects on the timing of risk-shifting and default. An increase in $\hat{\lambda_B}$ causes a delay in both risk-shifting and default. Because firms could benefit from the economy switching into a good state, they have incentives to wait for such a switch, instead of increasing the risk or defaulting too early. In contrast, an increase in $\hat{\lambda_G}$ pushes the firm to take more risk or go bankrupt early, because the increased likelihood of a switch into the bad state makes the aforementioned waiting less meaningful.

The effect of the state-switching risk on the optimal policy consequently shows up in the model-generated moments. The last three columns of Panel B show the absolute value of each of the sensitivities to be above one. Because κ_G increases the risk-shifting threshold, it increases the mean and variance of $\sigma_t^{i,X}$, the mean and the first-order autocorrelation of the interquartile of $\sigma_t^{i,X}$, and the sensitivity of $\sigma_t^{i,X}$ to leverage. Similarly, $\hat{\lambda_B}$ and $\hat{\lambda_G}$ have strong opposing effects on the mean and variance of $\sigma_t^{i,X}$, the mean of the interquartile of $\sigma_t^{i,X}$, and the sensitivity of $\sigma_t^{i,X}$ to financial leverage.

More important, when comparing the sensitivities with those in Table 4, we find that all the optimal policies and resulting model moments are more sensitive to the macroeconomic state-switching risk variables than they are to the three cost variables. This confirms the importance of the macroeconomic uncertainty risk in shaping a firm's risk-taking decisions in our study.

7 Model Implications

In this section, we use the estimated model to demonstrate the risk-shifting behavior over the business cycle, and its implications for the cross-sectional equity returns.

7.1 Visual Inspection: Idiosyncratic Risk-Taking and Default Events over the Business Cycle

To gain preliminary insights, we visually inspect a firm's risk-taking and default behavior over the business cycle for a typical economy. The economy consists of 1,000 firms.

We first plot one sample path of the simple average of financial leverage over time in Panel A of Figure 2. Gray areas in the figure correspond to periods when the economy is in the bad states. Consistent with Korajczyk and Levy (2003) and Bhamra et al. (2010a), we find that financial leverage is strongly countercyclical. That is, financial leverage is high in the bad states but low in the good states. The countercyclical financial leverage drives the countercyclical taking of idiosyncratic risk. As shown in Panel B, both risk-shifting and default events increase during the bad states, and the probability of risk-taking is substantially higher than that of default events. For example, the risk-shifting probability increases dramatically to about 25% immediately after year 40 when the economy is sliding into a bad state. The reason for this spike in the risk-shifting probability is intuitive: The risk-shifting threshold increases for all firms and force many troubled firms into taking immediate corrective action to save themselves. Moreover, the longer the economy stays in the bad states, the more firms become distressed and eventually take on additional risk. After taking on the excess idiosyncratic risk, these firms keep their high risk profiles until they manage to return to the markets and refinance their debt upward.

The high likelihood of risk-taking in the bad states consequently results in a high level of idiosyncratic cash flow risk. As shown in Figure 3, both the average idiosyncratic cash flow risk in Panel A and the interquartile of the idiosyncratic risk in Panel B increase dramatically during recessions, particularly after year 40. The large spread in the idiosyncratic volatility in the bad states, proxied by the interquartile, is likely to cause a large cross-sectional spread in equity returns, which we will show in Section 7.3. However, the increases in the idiosyncratic risk in the bad states appear to lag behind the actual occurrence of risk-shifting events in Panel B of Figure 2. This lag is likely due to measurement errors in calculating the rolling idiosyncratic volatility, when we follow the standard empirical procedure and use cash flow residual shocks of the past 20 quarters to calculate the idiosyncratic volatility.[15]

[15] For example, for the firms that have increased the level of idiosyncratic volatility for one year, the volatility calculated from this procedure is *underestimated* because the large weight is given for the past 4 years for the 5-

It is worth noting that, even if we assume the same initial level $\sigma^{i,X}_{s_t,L}$ and the same increment ϵ_{s_t} for both aggregate states in our estimation, our model is able to generate the countercyclical levels and interquartiles of idiosyncratic volatility over the business cycle. The risk-taking behavior depends on the realized cash flow shocks relative to the risk-shifting threshold across different firms. When a large negative marketwide shock brings the firm-level cash flows to a low level in bad times, a big fraction of firms choose to increase their taking of idiosyncratic volatility simultaneously, thereby generating countercyclical idiosyncratic volatility in the simulated economy, which is consistent with the finding of the synchronized high idiosyncratic risk in recessions by Herskovic et al. (2015).[16]

In short, we provide preliminary visual evidence that financial leverage drives the idiosyncratic risk-taking, particularly in the bad states. We also find that the clustering of the taking of idiosyncratic risk can be attributed to large and sudden shocks that cause the aggregate economy to switch from the good state to the bad state.

7.2 Idiosyncratic Risk-Taking and Default Events

Having visually identified the clustering of risk-shifting and default events in recessions for one simulated sample, we now seek statistical evidence by running standard probit regressions on 100 simulated economies. After dropping the first 100 years of observations, each economy contains 40 years of records on annual risk-taking and default events for 1,000 firms. For each simulated economy, a pooled probit regression is run for each of the following three horizons: one, two and five years. We regress the probability of risk-taking on an intercept, interest coverage X_t/c, a current state indicator $\mathbf{1_{s_t=B}}$, and their interaction.[17]

Table 6 reports the results. Panel A shows that, when T = 1, the interest coverage is negatively associated with the the future likelihood of risk-taking, with a coefficient of –0.423 (t-stat = –9.539), consistent with our intuition that the firms that have difficulties covering their coupon payments are more likely to shift risk. The second column shows that, when firms are in a bad state, $\mathbf{1_{s_t=B}}$, they have low cash flows (and low interest coverage) and are likely to increase their risk as well. The

year rolling window. In contrast, for the firms that have just reversed its increased risk for one year, the volatility calculated from this rolling procedure is *overestimated* relative to the true volatility level. The measure error occurs both in the real data and in the model-generated data.

[16]We obtain a stronger countercyclical pattern when we allow the different ϵ_{s_t} in the bad and good states.

[17]Instead of using the financial leverage we have already employed in the simulated method of moments, we use the interest coverage to complement those results because the interest coverage X_t/c is calculated in a way roughly similar to the risk-shifting threshold scaled by the coupon, X_r/c.

third column shows that their interaction terms are not statistically and economically significant. When the horizon T increases to two years, their individual impacts become smaller in the forth and fifth columns. In the sixth column where we include the interaction term, the coefficient of the aggregate state indicator $\mathbf{1_{s_t=B}}$ drops to –0.067 and becomes statistically insignificant. Therefore, the transitory bad state has no long-lasting predicting power. However, equity holders in a good state with EZ preferences perceive the risk-neutral arrival probability of the bad state greater than the actual probability and shift the risk earlier than necessary.

Panel B exhibits the results predicting default events. For the prediction horizon of $T = 1$, the coefficient of interest coverage in the first column is –16.062 (t = –4.919) and the *pseudo*-R^2 is 0.675. Both are much stronger than their counterparts in Panel A in predicting the risk-shifting behavior, indicating that the interest coverage has a strong predicting power for the defaults. Even when the horizon increases to $T = 5$, interest coverage still weakly determines a firm's default decision. In contrast, the current state of the economy has much weaker predicting power.

In short, we find that the interest coverage has strong and persist predicting power for both risk-taking and default events. The transitory bad state has no long-lasting predicting power.

7.3 Implications for the Cross-Section of Expected Equity Returns

In this section, we use the estimated full model to examine the quantitative implications of the risk-shifting behavior for equity returns. Additionally, we use the standard portfolio approach to investigate this idiosyncratic volatility discount over the business cycle.[18] We include firms in the recession subsample if the last quarter before the portfolio construction was in an NBER recession. It is worth noting again that the portfolio returns are not targeted moments in our structural estimation and that the changes in idiosyncratic volatility are entirely driven by the risk shifting mechanism in the simulated data.

7.3.1 Sorts on Idiosyncratic Cash Flow Volatility

Table 7 shows the portfolio performance using the real data and model-generated data. We report the average value-weighted excess returns and the CAPM alphas of decile portfolios. At the beginning of each quarter, we sort firms into deciles based on the idiosyncratic volatility of

[18] Our results from Fama-MacBeth regressions are very similar and available upon request.

cash flow residuals, $\sigma_{t-1}^{i,X}$, of the last quarter. Then, we form decile portfolios, compute value-weighted portfolio returns over the next quarter, and rebalance the portfolios each quarter. LMH is the hedge portfolio that is long on the low-idiosyncratic-volatility portfolio and short on the high-idiosyncratic-volatility portfolio.

Our empirical results in Panel A largely resemble the findings of Huang (2009).[19] Firms with lower cash flow volatility receive higher equity returns than those with higher cash flow volatility, on average, by 8.227% (t-statistic = 2.042). The LMH portfolio earns 15.042% in recessions, more than double the average of 7.499% in expansions, implying the idiosyncratic volatility discount is countercyclical. This observation is also consistent with the empirical finding by Avramov, Chordia, Jostova, and Philipov (2013) that the idiosyncratic volatility discount is more pronounced in distressed firms. While their finding relates to firm-level distress, ours relates to aggregate bad states.

Following the empirical procedure, we use the model-generated idiosyncratic cash flow volatility to form the portfolios based on the volatility of the last quarter. Two observations in Panel B are as follows: First, as shown in the first row, the average excess return of the decile portfolios decreases from 5.389% to –3.661%. The LMH portfolio earns 9.050% per year, and its CAPM alpha is 8.047% (t-statistic = 3.695). This suggests the CAPM fails in our model. This is not surprising, because our simulated economy has two state variables (as in equation (10)) instead of one single market variable. Second, when splitting the sample into the subsamples of recessions and expansions, we find the LMH portfolio earns 14.623% in recessions, more than double the 6.814% in expansions. All the values are close to those from the empirical data, even though our SMM estimation does not include cross-sectional equity returns as targeted moments. This supports our prediction of the countercyclical idiosyncratic volatility discount in Proposition 2.

[19] We follow Huang (2009) and calculate the idiosyncratic cash flow volatility. The cash flow is operating incomes scaled by sales (the sum of Compustat item IBQ and XintQ divided by the sales of the last quarter). The cash flow residuals are obtained by regressing the firm-level cash flow growth on the simple average of cash flow growth across all the firms.

7.3.2 Sorts on Idiosyncratic Equity Return Volatility

Next, we investigate whether the high equity return volatility causes low future equity returns.[20] Sightly different from $\sigma_{t-1}^{i,X}$, the idiosyncratic equity return volatility $\sigma_{t-1}^{i,E}$ is calculated using the stocks returns of the past 24 months from CRSP. We form the decile portfolio at the beginning of each month, compute value-weighted excess returns over the next month, and rebalance the portfolios each month.

Table 8 reports the results of portfolio performance. As shown in Panel A, the idiosyncratic volatility discount is significant as well when we form the portfolios on the idiosyncratic return volatility of the past month. Firms with high equity return volatility earns a lower equity return than those with low return volatility, on average, by 7.809% per year for the whole sample, 11.317% in recessions, and 6.622% in expansions. Panel B shows that the model-generated equity returns exhibit the same patterns as those in the data, and with similar magnitudes. The average excess return declines from 5.579% to –1.416% with the idiosyncratic return volatility. The LMH portfolio earns 6.996% with a t-statistic of 2.299. More importantly, the idiosyncratic volatility discount is countercyclical. That is, the discount in recessions is 9.804% (t-statistic = 1.448), greater than 5.723% (t-statistic = 1.925) in expansions.

In summary, consistent with the qualitative prediction in Proposition 2, our calibration using the estimated model demonstrates quantitatively that the negative association between idiosyncratic volatility and the equity returns is much stronger in recessions than expansions, for both idiosyncratic cash flow volatility and equity return volatility. Hence, the clustering of the strategic risk-taking recessions could help us understand the countercyclical idiosyncratic volatility discount.

8 Concluding Remarks

While corporate financing and investment decisions have been well studied in the presence of macroeconomic uncertainty risk, there has been no study on the interaction between macroeco-

[20] Empirical evidence regarding on the relation between idiosyncratic return volatility and stock returns is mixed. While Fu (2009) presents evidence that the conditional *expected* EGARCH idiosyncratic volatility is positively related to stock return, Ang et al. (2006) find a negative relation between the *realized* idiosyncratic volatility and the stock return of the next month. However, a recent paper by Guo, Kassa, and Ferguson (2014) raised a concern about the potential look-ahead bias in Fu's work. Specifically, they state "A spurious positive relation between exponential generalized autoregressive conditional heteroskedasticity (EGARCH) estimates of expected month t idiosyncratic volatility and month t stock returns arises when the month t return is included in estimation of model parameters."

nomic risk and corporate risk-taking policies, to our best knowledge. We are the first to introduce macroeconomic uncertainty into the standard risk-shifting model (Leland, 1998) and endogenize a firm's financing, default and risk-taking decisions over the business cycle.

We estimate the model via the SMM and obtain three novel results. First, equity holders with EZ preferences in our model increase their taking of idiosyncratic risk substantially more than in Leland (1998) in repeated games, because they perceive the probability of the arrival of bad times as higher than the actual probability and prefer an early resolution of macroeconomic uncertainty. Second, because of large, negative shocks in bad states and sudden switches from good to bad states, our model generates the clustering of idiosyncratic risk-taking, which provides a novel explanation for recent empirical findings of countercyclical and "synchronized" idiosyncratic volatility (Bloom, 2009, 2016; Herskovic, Kelly, Lustig, and Nieuwerburgh, 2015). Lastly, combined with the countercyclical market risk premium, the clustered idiosyncratic risk-taking generates the countercyclical idiosyncratic volatility discount on equity returns. Our calibration using the estimated model shows that our model is able to generate quantitative implications for the idiosyncratic volatility discount over the business cycle.

While our paper assumes that managers act on behalf of equity holders and ignores another layer of the agency conflict between the managers and equity holders, it will be fruitful to extend our work in this direction in our future research agenda.

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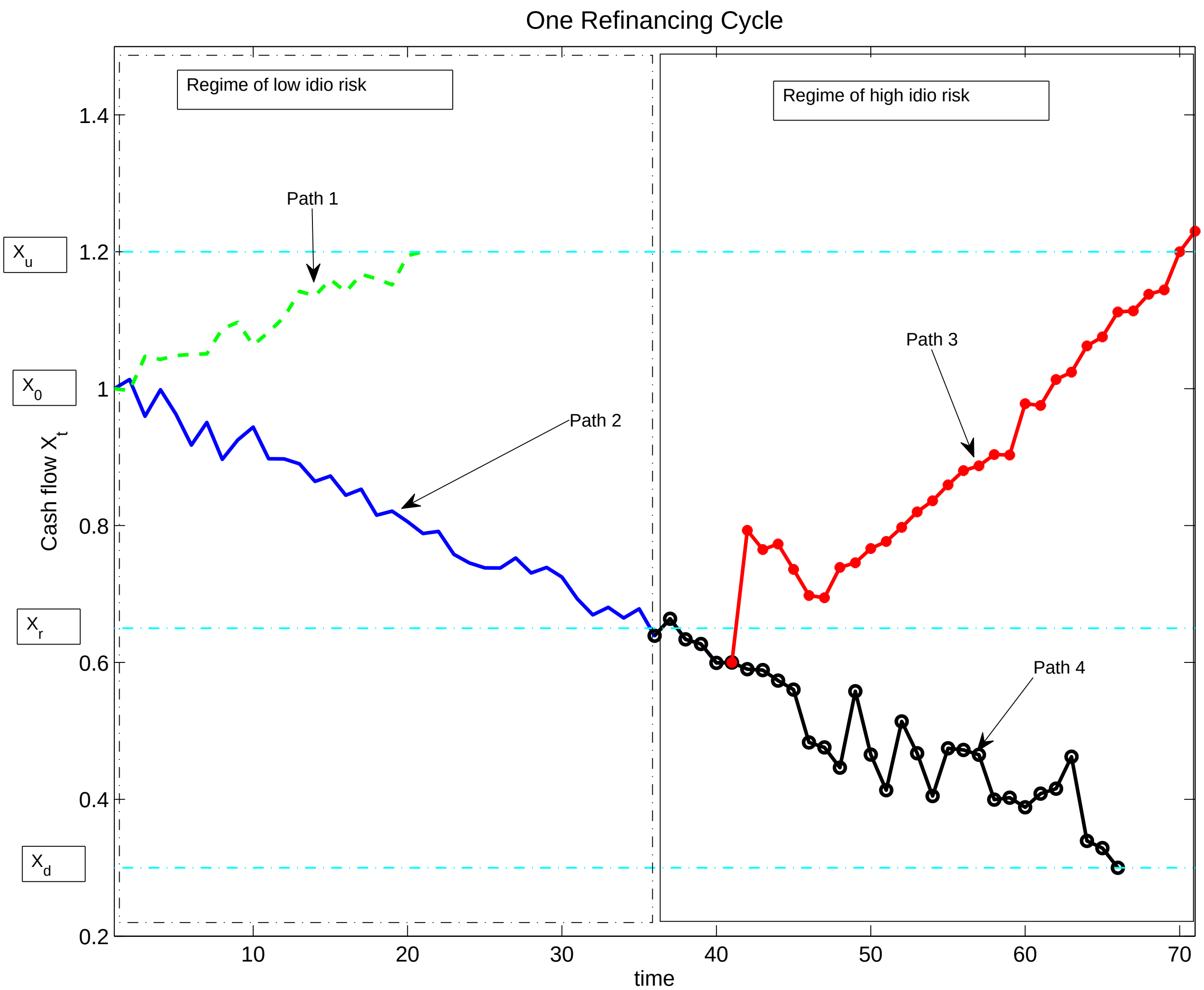
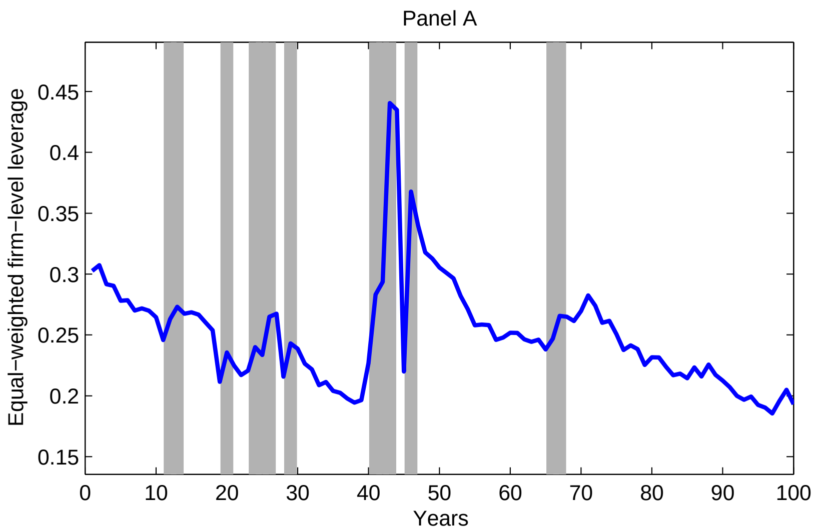


Figure 1: **Dynamic Paths**
This figure plots four possible paths that a firm could take within one refinancing cycle, which can be repeated infinitely. In observing its dynamic asset value, the firm makes financing, risk-taking and default decisions. Path 1 shows that, when its cash flows reach an upper threshold X_u, the firm decides to issue more debt to take advantage of tax benefits. In contrast, if the cash flows X_t decline to a low threshold X_r along Path 2, equity holders thus choose to make high-risk investments with high idiosyncratic risk $\sigma_H^{i,X}$ that produce cash flows with a low expected growth rate $\hat{\mu}_H$ and high total volatility σ_H, hoping that a cash flow windfall due to the increased σ_H might save the firm. After taking more risk, there are two possibilities. The first possibility is shown in Path 3. The increased idiosyncratic risk may quickly release the firm from financial distress. This eventually leads to a subsequent debt restructuring at the *same* restructure threshold X_u. In order to enjoy the same cost of debt, i.e., coupon payment, the firm has incentives to adjust its level of idiosyncratic risk back to its previous level immediately before the refinancing at X_u, as if it had never increased it earlier. The second possibility is that the new risky projects may cause a more severe cash flow shortfall, as shown in Path 4. If the firm continues to deteriorate, equity holders will no longer be willing to inject more capital, and decide to go bankrupt at X_d. Bankruptcy leads to immediate liquidation.



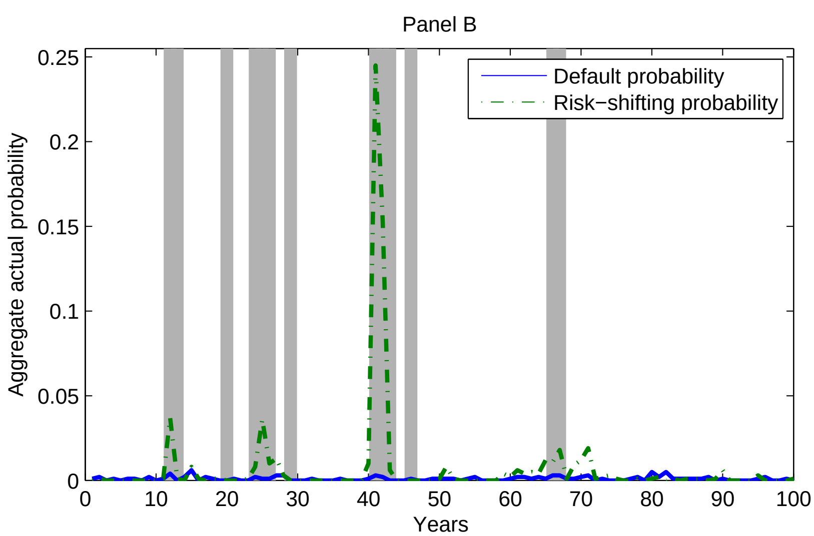


Figure 2: **Financial Leverage, and Probability of Default and Risk-Shifting**
This figure plots the time series of the cross-sectional average of firm-level leverage in Panel A, and the time series of default probability (solid line) and risk-shifting probability (dotted line) in Panel B, for a typical economy. The economy consists of 1,000 firms. Firm-level financial leverage is the ratio of debt to the market value of assets for an individual firm. Both risk-shifting and default probabilities are cumulative over one year. Gray areas show times when the economy is in the bad state, i.e., $s_t = B$.

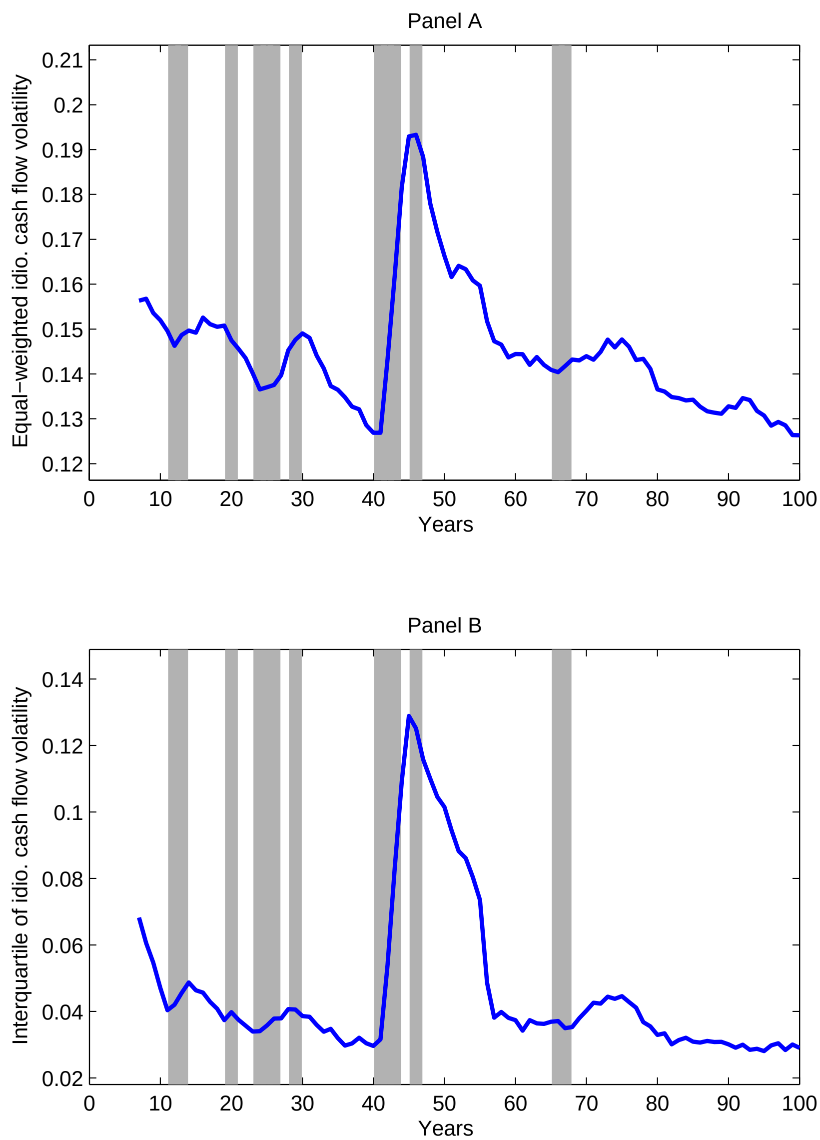


Figure 3: **Idiosyncratic Volatility of Cash Flows**
This figure plots the equal-weighted average (Panel A) and the interquartile (Panel B) of the idiosyncratic volatility of cash flow growth across 1,000 firms against years, for a typical economy. Gray areas show times when the economy is in the bad state, i.e., $s_t = B$.

Table 1: **Comparative Statics Analysis for the Baseline Model**

This table presents the comparative statics analysis for the parameter values for the baseline model. In Panel A, we list the benchmark values of the parameters according to the literature, except for the value-destroying cost and the two risk adjustment costs. In Panel B, we perform comparative statics analysis by varying the values of the market price of risk θ, the systematic volatility σ^m, the value-destroying cost η, the cost of increasing risk ξ^+, and the cost of decreasing risk ξ^-. In Panel C, we report the sensitivity analysis for the same set of parameters.

Panel A. Parameters	
	Value
Nominal Interest rate, r	0.040
Market price of risk, θ	0.200
Systematic volatility, σ^m	0.100
Debt issue cost, ϕ	0.010
Liquidation cost, α	0.150
Effective tax rate, τ	0.200
Coupon, c	0.400
Physical growth rate, $\hat{\mu}_L$	0.055
Low cash flow idio volatility, $\sigma_L^{i,X}$	0.100
Increment in idio volatility, ϵ	0.150
Value destroying cost, η	0.030
Cost of increasing risk, ξ^+	0.050
Cost of decreasing risk, ξ^-	0.050

Panel B. Comparative Statics						
	Benchmark values	η =0.05	ξ^- =0.1	ξ^+ =0.1	θ =0.25	σ^m =0.15
X_r	0.149	0.105	0.146	0.143	0.507	0.798
X_d	0.025	0.025	0.025	0.025	0.048	0.065
X_u	1.403	1.402	1.403	1.403	1.431	1.369

Panel C. Sensitivity of Optimal Policies to Parameters					
	η	ξ^-	ξ^+	θ	σ^m
X_r	−0.355	−0.001	−0.075	4.753	6.555
X_d	0.117	−0.000	0.023	3.725	3.009
X_u	−0.000	−0.000	−0.000	−0.070	−0.065

Table 2: **Parameter Estimation for the Full Model** This table presents the estimation results from the simulated method of moments. We list the predetermined parameters from the existing literature in Panel A. We report the estimates of the value-destroying parameter η, the risk adjustment costs ξ^+ and ξ^-, and the increments in ϵ_{s_t}, with their t-statistics in parentheses in Panel B. The p-value of the χ^2 statistic is also reported. Panel C presents the optimal policies given the predetermined and estimated parameter values.

Panel A. Parameters from the Literature		
	s_t = B	s_t = G
Probability of leaving current state s_t, $\hat{\lambda}_{s_t}$	0.500	0.100
Aggregate state-switching risk premium, κ_{s_t}	0.500	2.000
Nominal interest rate, r_{s_t}	0.040	0.040
Market price of risk, θ_{s_t}	0.220	0.170
Systematic volatility, $\sigma^m_{s_t}$	0.120	0.100
Debt issue cost, ϕ_{s_t}	0.010	0.010
Liquidation cost, α_{s_t}	0.150	0.100
Effective tax rate, τ_{s_t}	0.200	0.200
Coupon, c_{s_t}	0.380	0.400
Physical growth rate, $\hat{\mu}_{s_t,L}$	−0.010	0.080
Low cash flow idio volatility, $\sigma^{i,X}_{s_t,L}$	0.100	0.100

Panel B. Parameter Estimated from SMM		
	Estimates	t-statistic
Value destroying cost, η	0.031	31.198
Cost of increasing risk, ξ^+	0.015	0.023
Cost of decreasing risk, ξ^-	0.022	8.825
Increment in idio volatility, $\epsilon_B = \epsilon_G$	0.155	21.496
χ^2	0.138	
$p - value$	0.933	

Panel C. Optimal Policies						
	$X_r(B;G)$	$X_r(G;G)$	$X_d(B;G)$	$X_d(G;G)$	$X_u(B;G)$	$X_u(G;G)$
Estimates	0.644	0.577	0.073	0.061	3.496	1.363

Table 3: **Moments of Generated Samples from the Full Model**
This table reports the targeted moments, used in the simulated method of moments, in Panel A, and untargeted moments in Panel B, from 100 model-generated samples. Each sample contains the quarterly observations for 1,000 firms over 40 years (the first 100 years of observations have been discarded). The statistics are averaged across all the samples. In Panel A, the targeted moments include averaged idiosyncratic cash flow volatility $\sigma_t^{i,X}$, averaged quasi-financial leverage, the average of the interquartile time series of $\sigma_t^{i,X}$, the variance of $\sigma_t^{i,X}$, the first-order autocorrelation (AC(1)) of the interquartile time series of $\sigma_t^{i,X}$, and the sensitivity of $\sigma_t^{i,X}$ to quasi-market leverage (QML_{t-1}). We estimate the sensitivity of $\sigma_t^{i,X}$ by regressing the sample mean of $\sigma_t^{i,X}$ on an intercept and the sample mean of quasi-market leverage. In Panel B, we report the untargeted moments, including average firm-level cash flow growth rate, interest coverage, Sharpe ratio, unconditional (uncond.) standard deviation (std) of market returns, firm-level unconditional standard deviation of equity returns, and default probability. 'Data' shows the moments from the data, 'Model' the average of the moments across all samples generated from the full model, and 'M–D' the difference between those two columns. We also report the 5th, 25th, 50th, 75th, and 95th percentiles of the moments from the 100 samples.

Panel A. Targeted Moments

	D(ata)	M(odel)	5th	25th	50th	75th	95th	M–D	t-stat(M–D)
Ave idiosyncratic volatility $\sigma_t^{i,X}$	0.127	0.127	0.107	0.115	0.125	0.135	0.161	−0.001	−0.056
Ave. quasi-market leverage (QML_t)	0.238	0.246	0.179	0.209	0.240	0.276	0.355	0.008	2.527
Interquartile of $\sigma_t^{i,X}$	0.055	0.054	0.025	0.029	0.052	0.060	0.126	−0.001	−0.093
Variance of $\sigma_t^{i,X}$	0.016	0.017	0.011	0.013	0.016	0.019	0.026	0.000	0.003
AC(1) of Interquartile of $\sigma_t^{i,X}$	0.941	0.964	0.780	0.958	0.971	0.987	0.996	0.024	0.652
Sensitivity of $\sigma_t^{i,X}$ on QML_{t-1}	0.203	0.203	0.074	0.151	0.201	0.283	0.328	0.000	0.003

Panel B. Untargeted Moments

	D(ata)/Literature	M(odel)	5th	25th	50th	75th	95th	M–D
Ave. firm-level cash flow growth	0.077	0.074	0.042	0.058	0.072	0.089	0.108	−0.004
Interest coverage	2.410	2.481	2.115	2.387	2.534	2.617	2.681	0.071
Equity market Sharpe ratio	0.329	0.309	−0.043	0.184	0.301	0.458	0.569	−0.020
Uncond. std of equity market returns	0.172	0.150	0.115	0.134	0.151	0.162	0.181	−0.022
Firm-level uncond. std of equity returns	0.339	0.315	0.258	0.274	0.309	0.333	0.418	−0.024
Default probability (%)	0.553	0.465	0.020	0.150	0.213	0.700	1.405	−0.088

Table 4: **Sensitivities of Optimal Policies and Model Moments to Estimated Parameters**
This table presents the sensitivities of the optimal policies (in Panel A) and targeted model moments (in Panel B) to the estimated parameters. The sensitivity is measured as the percentage change in the optimal policies or model moments in response to a one-percentage change in the estimated parameters. The estimated parameters are the value-destroying parameter η, the risk adjustment costs ξ^+ and ξ^-, and the increments in ϵ_{s_t}. In Panel A, the optimal policies include the refinancing threshold, $X_u(s_t; s_0 = G)$, the default threshold, $X_d(s_t; s_0 = G)$, and the risk-shifting threshold, $X_r(s_t; s_0 = G)$, for both states, $s_t = G, B$. In Panel B, the moments include averaged idiosyncratic cash flow volatility $\sigma_t^{i,X}$, averaged quasi-market leverage (QML_{t-1}), the time series average of the interquartile of $\sigma_t^{i,X}$, the variance of $\sigma_t^{i,X}$, the first-order autocorrelation (AC(1)) of the interquartile time series of $\sigma_t^{i,X}$, and the sensitivity of $\sigma_t^{i,X}$ to QML_{t-1}.

Panel A. Sensitivity of Optimal Policies to Estimated Parameters

	η	ξ^+	ξ^-	$\epsilon_B = \epsilon_G$
$X_r(B;G)$	-0.634	-0.004	-0.026	-0.276
$X_r(G;G)$	-0.637	-0.000	-0.025	-0.277
$X_d(B;G)$	0.013	-0.000	0.000	-0.424
$X_d(G;G)$	0.013	-0.000	0.000	-0.438
$X_u(B;G)$	-0.233	0.036	0.075	0.756
$X_u(G;G)$	0.077	0.001	-0.000	0.082

Panel B. Sensitivity of Targeted Moments to Estimated Parameters

	η	ξ^+	ξ^-	$\epsilon_B = \epsilon_G$
Ave $\sigma_t^{i,X}$	-0.565	-0.000	-0.061	-0.169
Ave. quasi-market leverage	-0.567	0.001	-0.094	-0.401
Interquartile of $\sigma_t^{i,X}$	-1.975	0.000	-0.080	-1.542
Variance of $\sigma_t^{i,X}$	-1.186	-0.000	-0.107	-0.428
AC(1) of interquartile of $\sigma_t^{i,X}$	0.043	-0.000	0.017	0.606
Sensitivity of $\sigma_t^{i,X}$ on QML_{t-1}	-1.129	-0.003	0.384	2.318

Table 5: **Sensitivities of Optimal Policies and Model Moments to Macroeconomic Variables**

This table presents the sensitivities of the optimal policies (in Panel A) and targeted model moments (in Panel B) to macroeconomic parameters. The sensitivity is measured as the percentage change in the optimal policies or model moments in response to a one-percentage change in the macroeconomic variables. The macroeconomic risk parameters include the market volatility, $\sigma^m_{s_t}$, the market price of risk, θ_{s_t}, the macroeconomic risk-switching premium κ_G, the actual probability of leaving the current state B, $\hat{\lambda}_B$, and the actual probability of leaving the current state G, $\hat{\lambda}_G$. In Panel A, the optimal policies include the refinancing threshold, $X_u(s_t; s_0 = G)$, the default threshold, $X_d(s_t; s_0 = G)$, and the risk-shifting threshold, $X_r(s_t; s_0 = G)$, for both states, $s_t = G, B$. In Panel B, the moments include averaged idiosyncratic cash flow volatility $\sigma^{i,X}_t$, averaged quasi-market leverage QML_{t-1}, the time series average of the interquartile of $\sigma^{i,X}_t$, the variance of $\sigma^{i,X}_t$, the first-order autocorrelation (AC(1)) of the interquartile time series of $\sigma^{i,X}_t$, and the sensitivity of $\sigma^{i,X}_t$ to QML_{t-1}.

Panel A. Sensitivity of Optimal Policies to Macro Parameters							
	σ^m_B	θ_B	σ^m_G	θ_G	$\kappa_G = 1/\kappa_B$	$\hat{\lambda}_B$	$\hat{\lambda}_G$
$X_r(B; G)$	0.715	0.770	1.033	1.132	4.168	−1.816	2.354
$X_r(G; G)$	0.715	0.771	1.072	1.175	4.263	−1.827	2.438
$X_d(B; G)$	0.414	0.522	0.593	0.641	2.579	−1.183	1.397
$X_d(G; G)$	0.453	0.488	0.578	0.681	2.552	−1.086	1.466
$X_u(B; G)$	−0.362	0.041	−0.089	−0.114	0.143	−0.390	−0.183
$X_u(G; G)$	−0.014	−0.026	−0.023	−0.068	−0.233	0.086	−0.147

Panel B. Sensitivity of Targeted Moments to Macro Parameters							
	σ^m_B	θ_B	σ^m_G	θ_G	$\kappa_G = 1/\kappa_B$	$\hat{\lambda}_B$	$\hat{\lambda}_G$
Ave $\sigma^{i,X}_t$	0.831	0.905	1.502	1.173	2.947	−1.166	1.394
Ave. QML_t	1.128	1.335	2.162	1.593	4.560	−1.537	2.096
Interquartile of $\sigma^{i,X}_t$	2.184	2.838	5.441	4.084	12.720	−6.230	3.985
Variance of $\sigma^{i,X}_t$	1.736	1.891	3.088	2.423	6.046	−2.469	2.856
AC(1) of Interquartile of $\sigma^{i,X}_t$	0.892	1.173	1.569	1.331	3.233	1.096	2.598
Sensitivity of $\sigma^{i,X}_t$ on QML_{t-1}	4.878	1.529	6.413	7.199	15.952	−3.576	8.408

Table 6: **Regressions of Idiosyncratic Risk-Taking and Default Events**
This table reports the results of probit regressions used to predict the idiosyncratic risk-taking and the default over different horizons. We simulate 100 artificial panels of data at a quarterly frequency for a period of 140 years for 1,000 firms, and discard the first 100 years of observations. We use records on annual defaults for 1,000 firms. For each data set, a pooled probit regression is run for each of the following three horizons: one, two, and five years. The dependent variable equals 1 if the firm defaults within the specified horizon. Independent variables are interest coverage (X_t/c) and the indicator variable $\mathbf{1}_{s_t=B}$ if the current state of the economy $s_t = B$. The t-statistics are given in parentheses. Log-like is the log-likelihood ratio, and pseudo-R^2 is the McFadden pseudo-R^2. The coefficients and t-statistics are averaged across all the panels.

Panel A. Prediction Regression of Risk-Shifting									
		T = 1			T = 2			T = 5	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Intercept	−1.577	−2.810	−1.847	−1.900	−2.647	−1.984	−2.683	−2.392	−2.656
(t)	(−18.098)	(−56.598)	(−14.256)	(−13.621)	(−44.597)	(−11.135)	(−9.167)	(−32.656)	(−7.877)
X_t/c	−0.423		−0.435	−0.274		−0.281	0.064		0.097
(t)	(−9.539)		(−6.795)	(−3.853)		(−3.316)	(0.767)		(0.675)
$\mathbf{1}_{s_t=B}$		0.596	0.511		0.214	−0.067		−0.586	−0.632
(t)		(9.640)	(2.905)		(3.917)	(0.705)		(−1.056)	(−0.312)
$X_t/c\mathbf{1}_{s_t=B}$			0.046			0.115			0.012
(t)			(0.674)			(0.561)			(0.014)
Log-like	106.506	196.841	292.401	20.631	51.171	72.836	4.878	22.342	29.907
pseudo-R^2	0.071	0.088	0.156	0.035	0.049	0.082	0.013	0.037	0.054

Panel B. Prediction Regression of Default									
Intercept	2.999	−3.235	3.083	0.830	−3.003	1.100	−0.948	−3.043	−0.831
(t)	(3.354)	(−36.687)	(2.108)	(0.668)	(−30.682)	(0.850)	(−2.415)	(−18.848)	(−1.868)
X_t/c	−16.062		−16.930	−7.803		−8.736	−2.675		−2.867
(t)	(−4.919)		(−3.882)	(−3.439)		(−3.064)	(−2.131)		(−1.937)
$\mathbf{1}_{s_t=B}$		−0.113	−2.471		−0.403	−3.292		−0.496	−1.978
(t)		(1.218)	(0.085)		(0.151)	(−0.052)		(0.012)	(0.044)
$X_t/c\mathbf{1}_{s_t=B}$			6.460			5.036			0.126
(t)			(0.218)			(0.169)			(0.009)
Log-like	227.727	4.637	78.473	94.986	−155.910	99.774	25.462	−34.928	29.235
pseudo-R^2	0.675	0.012	−1.883	0.559	−1.469	0.579	0.392	−0.556	0.429

Table 7: **Sort on Idiosyncratic Cash Flow Growth Volatility over the Business Cycle**
This table reports the averages of the value-weighted excess returns of decile portfolios from 100 model-generated samples. We simulate 100 artificial panels of data at a quarterly frequency for a period of 140 years for 1,000 firms and discard the first 100 years of observations. At the beginning of a quarter, firms are sorted into deciles based on the idiosyncratic volatility of cash flow growth over the last 20 quarters. Idiosyncratic volatility of cash flow growth is the standard deviation of residuals, which are obtained from a regression of cash flow growth rates of the past 20 quarters on the simple average of cash flow growth rates all the firms. The average equity returns are computed over the next quarter and the portfolios are rebalanced each quarter. We report annualized value-weighted returns for the whole sample as well as subsamples of recessions (Recess) and expansions (Expans). Firms are included in the recession subsample if the last quarter before the portfolio construction belonged to a recession. The excess returns r^e, and CAPM alphas, α^{CAPM}, are annualized and reported in percentages. The t-statistics in parentheses are calculated based on the heteroskedasticity-consistent standard errors of Newey and West (1987).

Panel A. Data

		L(ow)	2	3	4	5	6	7	8	9	H(igh)	LMH
Whole Sample	r^e (%)	8.564	8.982	8.960	7.619	8.207	6.468	5.714	5.171	4.350	0.338	8.227
	(t)	(4.065)	(3.891)	(3.519)	(2.786)	(2.912)	(2.358)	(1.703)	(1.436)	(1.055)	(0.065)	(2.042)
	α^{CAPM} (%)	2.306	2.031	1.548	−0.129	0.394	−1.657	−2.959	−4.285	−5.801	−11.303	13.609
	(t)	(2.840)	(2.121)	(1.959)	(−0.146)	(0.384)	(−1.517)	(−2.056)	(−2.778)	(−2.379)	(−3.580)	(3.882)
Recess Sample	r^e (%)	14.459	13.248	13.211	12.571	13.941	8.136	6.914	8.406	6.341	−0.583	15.042
	(t)	(1.551)	(1.227)	(1.133)	(0.963)	(1.115)	(0.666)	(0.457)	(0.547)	(0.437)	(−0.031)	(1.244)
	α^{CAPM} (%)	6.121	3.358	2.889	1.092	2.827	−3.581	−5.250	−5.065	−6.768	−16.830	22.951
	(t)	(2.295)	(1.531)	(1.155)	(0.424)	(1.228)	(−0.800)	(−0.879)	(−1.298)	(−1.062)	(−2.295)	(2.541)
Expans Sample	r^e (%)	7.798	8.283	8.507	7.042	7.181	6.302	5.513	5.140	3.963	0.299	7.499
	(t)	(3.670)	(3.559)	(3.289)	(2.665)	(2.522)	(2.294)	(1.665)	(1.398)	(0.886)	(0.054)	(1.687)
	α^{CAPM} (%)	1.746	1.749	1.469	−0.235	−0.184	−1.406	−2.790	−3.852	−5.895	−10.700	12.446
	(t)	(2.013)	(1.568)	(1.634)	(−0.242)	(−0.158)	(−1.322)	(−1.988)	(−2.233)	(−2.185)	(−3.011)	(3.163)

Panel B. Model

		L(ow)	2	3	4	5	6	7	8	9	H(igh)	LMH
Whole Sample	r^e (%)	5.389	5.138	5.030	5.021	4.975	4.640	4.295	3.076	0.626	−3.662	9.050
	(t)	(2.561)	(2.451)	(2.383)	(2.383)	(2.347)	(2.208)	(2.071)	(1.602)	(0.618)	(−1.051)	(3.724)
	α^{CAPM} (%)	0.860	0.581	0.454	0.456	0.422	0.103	−0.215	−1.284	−3.369	−7.187	8.047
	(t)	(1.326)	(0.895)	(0.718)	(0.788)	(0.741)	(0.471)	(0.304)	(−0.542)	(−1.786)	(−3.646)	(3.695)
Recess Sample	r^e (%)	11.214	11.151	10.558	10.108	10.283	9.075	8.654	6.104	2.578	−3.408	14.623
	(t)	(1.847)	(1.713)	(1.572)	(1.594)	(1.520)	(1.351)	(1.341)	(0.665)	(0.082)	(−0.792)	(2.252)
	α^{CAPM} (%)	2.069	2.037	0.993	0.782	0.867	−0.346	−0.798	−2.953	−5.539	−8.882	10.951
	(t)	(1.490)	(1.418)	(0.518)	(0.647)	(1.081)	(0.407)	(0.039)	(−1.254)	(−1.435)	(−2.396)	(2.344)
Expans Sample	r^e (%)	3.933	3.746	3.695	3.787	3.753	3.547	3.136	2.191	0.275	−2.881	6.814
	(t)	(1.906)	(1.828)	(1.797)	(1.802)	(1.820)	(1.726)	(1.543)	(1.172)	(0.426)	(−0.751)	(3.549)
	α^{CAPM} (%)	0.656	0.424	0.349	0.469	0.427	0.180	−0.266	−1.218	−3.260	−6.615	7.270
	(t)	(1.144)	(0.706)	(0.550)	(0.784)	(0.809)	(0.457)	(0.171)	(−0.469)	(−1.628)	(−3.736)	(3.956)

Table 8: **Sort on Idiosyncratic Equity Return Volatility over the Business Cycle**
This table reports the averages of the value-weighted excess returns of decile portfolios from 100 model-generated samples. We simulate 100 artificial panels of data at a monthly frequency for a period of 140 years for 1,000 firms and discard the first 100 years of observations. At the beginning of a month, firms are sorted into deciles based on the idiosyncratic volatility of equity returns of the last 24 months. Idiosyncratic volatility of equity returns is the standard deviation of residuals, which are obtained from a regression of market equity returns. The average equity returns are computed over the next month and the portfolios are rebalanced each month. We report annualized value-weighted returns for the whole sample as well as subsamples of recessions (Recess) and expansions (Expans). Firms are included in the recession subsample if the last quarter before the portfolio construction belonged to a recession. The excess returns r^e, and CAPM alphas, α^{CAPM}, are annualized and reported in percentages. The t-statistics in parentheses are calculated based on the heteroskedasticity-consistent standard errors of Newey and West (1987).

Panel A. Data

	L(ow)	2	3	4	5	6	7	8	9	H(igh)	LMH
Whole Sample r^e (%)	6.186	6.995	7.305	8.129	6.912	7.572	7.322	4.764	2.833	−1.623	7.809
(t)	(3.262)	(3.128)	(2.769)	(2.780)	(2.161)	(2.168)	(1.879)	(1.100)	(0.626)	(−0.320)	(1.867)
α^{CAPM} (%)	1.385	1.045	0.531	0.777	−0.755	−0.793	−1.294	−4.558	−6.510	−11.481	12.866
(t)	(2.167)	(1.689)	(0.632)	(0.743)	(−0.536)	(−0.492)	(−0.632)	(−1.945)	(−2.560)	(−3.721)	(3.663)
Recess Sample r^e (%)	6.036	10.912	11.300	8.370	6.670	10.232	9.226	5.623	2.311	−5.281	11.317
(t)	(0.730)	(1.075)	(1.003)	(0.676)	(0.493)	(0.708)	(0.608)	(0.347)	(0.140)	(−0.317)	(1.079)
α^{CAPM} (%)	1.260	4.803	4.615	1.238	−0.360	2.271	1.421	−3.186	−6.119	−14.129	15.389
(t)	(0.691)	(2.724)	(1.910)	(0.426)	(−0.081)	(0.441)	(0.281)	(−0.587)	(−0.940)	(−2.250)	(2.068)
Expans Sample r^e (%)	6.206	6.351	6.689	7.882	7.074	7.324	7.323	4.868	3.175	−0.416	6.622
(t)	(3.420)	(3.028)	(2.633)	(2.797)	(2.268)	(2.114)	(1.854)	(1.086)	(0.684)	(−0.076)	(1.397)
α^{CAPM} (%)	1.416	0.468	−0.128	0.439	−0.864	−1.230	−1.609	−4.715	−6.499	−10.607	12.024
(t)	(1.974)	(0.699)	(−0.144)	(0.390)	(−0.569)	(−0.722)	(−0.696)	(−1.799)	(−2.352)	(−2.983)	(2.968)

Panel B. Model

	L(ow)	2	3	4	5	6	7	8	9	H(igh)	LMH
Whole Sample r^e (%)	5.579	5.433	5.187	5.197	4.937	5.095	4.969	4.205	2.577	−1.416	6.996
(t)	(2.611)	(2.596)	(2.483)	(2.494)	(2.280)	(2.454)	(2.406)	(2.016)	(1.380)	(−0.043)	(2.299)
α^{CAPM} (%)	1.222	1.060	0.818	0.776	0.536	0.735	0.625	−0.150	−1.593	−5.424	6.647
(t)	(1.574)	(1.561)	(1.300)	(1.206)	(0.731)	(1.136)	(1.370)	(0.325)	(−0.467)	(−2.060)	(2.317)
Recess Sample r^e (%)	10.677	10.518	9.782	10.687	8.888	9.738	8.968	7.851	5.317	0.872	9.804
(t)	(1.789)	(1.747)	(1.556)	(1.630)	(1.231)	(1.632)	(1.445)	(1.219)	(0.761)	(0.262)	(1.448)
α^{CAPM} (%)	2.333	2.140	1.469	1.925	0.427	1.938	0.879	−0.591	−3.380	−7.088	9.421
(t)	(1.395)	(1.106)	(1.021)	(1.596)	(0.113)	(1.377)	(0.937)	(−0.060)	(−0.302)	(−2.494)	(1.856)
Expans Sample r^e (%)	4.329	4.251	4.099	4.010	4.000	4.010	4.037	3.372	2.101	−1.394	5.723
(t)	(2.072)	(2.056)	(1.998)	(1.886)	(1.900)	(1.926)	(1.921)	(1.600)	(1.121)	(−0.087)	(1.925)
α^{CAPM} (%)	0.971	0.901	0.710	0.591	0.586	0.611	0.658	−0.129	−1.450	−5.408	6.379
(t)	(1.465)	(1.413)	(1.261)	(1.022)	(0.864)	(1.036)	(1.504)	(0.489)	(−0.343)	(−1.888)	(2.191)

Online Appendix (Not for Publication)

A Baseline Model

For the baseline model, we present the value functions for equity and debt and then the boundary conditions to solve for the value functions.

A.1 Asset Valuations

Under the risk-neutral measure, the Bellman equation describes the valuation of any claim $J(X_t, v_t)$ on operating cash flows X_t in state, s, as follows:

$$J(X_t, v_t) = CF_t dt + e^{-rdt}\mathbb{E}^Q(J(X_t + dX_t, v_t)), \tag{A1}$$

where CF_t denotes the cash flows accruing to claim holders. Standard dynamic programming suggests that $J(X_t, v_t) \equiv J_{t,v_t}$ must satisfy the ordinary differential equation

$$\mu_{v_t} X J'_{t,v_t} + \frac{\sigma^2_{v_t}}{2} X^2 J''_{t,v_t} - rJ_{t,v_t} + CF_t = 0, \tag{A2}$$

where $J(X_t, v_t)$, J'_{t,v_t} and J''_{t,v_t} denote the first and second-order derivatives of J_{t,v_t} with respect to X_t, respectively.

Because the cash flows generated by the assets is $CF_t = X_t$, the value of assets-in-place, A_{t,v_t}, under the risk-neutral measure Q, is

$$A_{t,v_t} \equiv A(X_t, v_t) = \frac{X_t}{r - \mu_{v_t}}. \tag{A3}$$

The cash flow accruing to equity holders is $CF_t = (X_t - c)(1 - \tau)$. Hence, the value function of equity is

$$E(X_t, v_t) = (1-\tau)\left(\frac{X_t}{r - \mu_{v_t}} - \frac{c}{r}\right) + e_{v_t,1} X_t^{\omega_{v_t,1}} + e_{v_t,2} X_t^{\omega_{v_t,2}} \tag{A4}$$

$$= (1-\tau)\left(A_{t,v_t} - \frac{c}{r}\right) + e_{v_t,1} X_t^{\omega_{v_t,1}} + e_{v_t,2} X_t^{\omega_{v_t,2}}, \tag{A5}$$

where $\omega_{v_t,1} < 0$ and $\omega_{v_t,2} > 1$ are the two roots of the characteristic equation

$$\frac{1}{2}\sigma^2_{v_t}\omega_{v_t}(\omega_{v_t} - 1) + \mu_{v_t}\omega_{v_t} - r = 0. \tag{A6}$$

The cash flow accruing to debt holders is $CF_t = c$. Hence, the value function of debt at the level of v_t is

$$D(X_t, v_t) = \frac{c}{r} + d_{v_t,1} X_t^{\omega_{v_t,1}} + d_{v_t,2} X_t^{\omega_{v_t,2}}. \tag{A7}$$

A.2 Boundary Conditions

Equity Boundary Conditions To solve for $e_{L,1}$, $e_{L,2}$, $e_{H,1}$ and $e_{H,2}$, we will use the following boundary conditions as follows:

$$\lim_{X_t \uparrow X_d} E(X_t, H) = 0; \tag{A8}$$

$$\lim_{X_t \uparrow X_u} E(X_t, L) = \lim_{X_t \downarrow X_u} \frac{X_t}{X_0} \left(E(X_0, L) + P(1-\phi)\right) - P; \tag{A9}$$

$$\lim_{X_t \uparrow X_u} E(X_t, H) = \lim_{X_t \downarrow X_u} \frac{X_t}{X_0} \left(E(X_0, L) + P(1-\phi)\right) - P - \xi^- \epsilon^2 A(X_t)(1-\tau); \tag{A10}$$

$$\lim_{X_t \uparrow X_r} E(X_t, L) = \lim_{X_t \downarrow X_r} E(X_t, H) - \xi^+ \epsilon^2 A(X_t)(1-\tau). \tag{A11}$$

Equation (A8) states that equity holders receive nothing at bankruptcy, if increasing idiosyncratic volatility to the high level, i.e., $v_t = H$, does not save the firm. Equation (A9) states that the equity value $E(X_u, L)$ increases by a scaling factor $\frac{X_u}{X_0}$ at the refinancing threshold X_u, after retiring the existing debt-in-place at par P and issuing more debt $\frac{X_u}{X_0}P(1-\phi)$. Equation (A10) is similar to Equation (A9), but with an addition adjustment cost $\xi^- \epsilon^2 A(X_t)(1-\tau)$ to reverse the risk level. The value-matching condition of (A11) states that, by paying the cost $\xi^+ \epsilon^2 A(X_t)(1-\tau)$, equity holders increase the idiosyncratic volatility and therefore the equity value from $E(X_r, L)$ to $E(X_r, H)$.

Debt Boundary Conditions To solve for four coefficients, $d_{L,1}$, $d_{L,2}$, $d_{H,1}$ and $d_{H,2}$, we use the following boundary conditions, (A12) to (A15).

$$\lim_{X_t \uparrow X_d} D(X_t, H) = \lim_{X_t \downarrow X_d} (1-\alpha) A(X_t, H)(1-\tau); \tag{A12}$$

$$\lim_{X_t \uparrow X_u} D(X_t, L) = P; \tag{A13}$$

$$\lim_{X_t \uparrow X_u} D(X_t, H) = P; \tag{A14}$$

$$\lim_{X_t \uparrow X_r} D(X_t, L) = \lim_{X_t \downarrow X_r} D(X_t, H). \tag{A15}$$

Equation (A12) shows that debt holders take over the assets and receive the residual value of assets $V(X_d, H)(1-\tau)$ after the liquidation cost α. Equation (A13) states that debt holders receive the par value of debt P when equity holders retire the existing debt if they never increase risk before. Equation (A14) states that debt holders receive the par value of debt P after equity holders reverse their taking of idiosyncratic risk. The value-matching condition of (A15) is to ensure no arbitrage opportunity at X_r.

B Full Model with Macroeconomic Risk

For the fully fledged model, we present the value functions for equity and debt, and then the boundary conditions to solve for the value functions.

B.1 Asset Valuations

We first provide the general valuation framework, and then present the value function for equity and debt.

B.1.1 General Valuations

Extending the one-state case in equation (A2), we have that any claim $J_{s_t,v_t} \equiv J(t, s_t, v_t)$ that pay CF_{s_t,v_t} contingent on cash flows X_t satisfies

$$(r_B + \lambda_B)J_{B,v_t} = CF_{B,v_t} + \mu_{B,v_t}X_tJ'_{B,v_t} + \frac{1}{2}\sigma^2_{B,v_t}X_tJ''_{B,v_t} + \lambda_BJ_{G,v_t}, \tag{B1}$$

$$(r_G + \lambda_G)J_{G,v_t} = CF_{G,v_t} + \mu_{G,v_t}X_tJ'_{G,v_t} + \frac{1}{2}\sigma^2_{G,v_t}X_tJ''_{G,v_t} + \lambda_GJ_{B,v_t}. \tag{B2}$$

In the matrix form,

$$\left[\begin{pmatrix} r_B + \lambda_B & -\lambda_B \\ -\lambda_G & r_G + \lambda_G \end{pmatrix} - \begin{pmatrix} \mu_{B,v_t} & 0 \\ 0 & \mu_{G,v_t} \end{pmatrix} X_t \tfrac{\partial}{\partial X_t} - \tfrac{1}{2}\begin{pmatrix} \sigma^2_{B,v_t} & 0 \\ 0 & \sigma^2_{G,v_t} \end{pmatrix} X_t^2 \tfrac{\partial^2}{\partial X_t^2}\right] \begin{pmatrix} J_{B,v_t} \\ J_{G,v_t} \end{pmatrix} = \begin{bmatrix} CF_{B,v_t} \\ CF_{G,v_t} \end{bmatrix} \tag{B3}$$

For each initial state, s_0, there are a total of five cash flow regions for both equity and debt. The cash flow regions are divided as follows:

$$\mathbb{R}_1 = X_{G,d} \leq X_t < X_{B,d}; \tag{B4}$$

$$\mathbb{R}_2 = X_{B,d} \leq X_t < X_{B,r}; \tag{B5}$$

$$\mathbb{R}_3 = X_{B,r} \leq X_t < X_{G,r}; \tag{B6}$$

$$\mathbb{R}_4 = X_{G,r} \leq X_t < X_{G,u}; \tag{B7}$$

$$\mathbb{R}_5 = X_{G,u} \leq X_t < X_{B,u}. \tag{B8}$$

For regions of $\mathbb{R}_1$ and $\mathbb{R}_2$, firms have high cash flow risk, $v_t = H$ in both states $s_t = G, B$, as they have already increased their risk in both states. For region of $\mathbb{R}_3$, the firms have a low level of idiosyncratic risk in the good state, but has shifted to a high level of risk in the bad state. For regions of $\mathbb{R}_4$ and $\mathbb{R}_5$, firms have a low level of idiosyncratic risk, $v_t = L$ in both states. We will successively characterize the values of equity and debt for each region.

B.1.2 Equity Value Functions

$\mathbb{R}_1 = \mathbf{X_{G,d} \leq X_t < X_{B,d}}$

After switching to the high-risk regime, $v_t = H$, the firm has already gone bankrupt in the bad state, but not yet in the good state. Because equity holders receive nothing at bankruptcy, $E(X_t, B, H; s_0) = 0$, in the bad state. They still receive the residuals after interest and taxes before bankruptcy in the good state. In addition, a sudden switch of the economy from the good state to the bad state will cause the firm to go bankrupt immediately.

The value function of $E(X_t, G, H; s_0)$ satisfies the following ODE:

$$(r_G+\lambda_G)E(X_t, G, H; s_0) = (1-\tau)(X_t-c(s_0))+\mu_{G,H}X_tE^{'}(X_t, G, H; s_0)+\frac{1}{2}\sigma^2_{G,H}X_t^2E^{''}(X_t, G, H; s_0) \tag{B9}$$

Assume that the function form of the equity value is

$$E(X_t, G, H; s_0) = (A(X_t, G, H) - C_G(s_0))(1-\tau) + \sum_{i=1}^{2} a^E_{G,H,i}(s_0)X_t^{\psi_{H,i}}, \tag{B10}$$

where $\psi_{H,i}$ is the roots of

$$\frac{1}{2}\sigma^2_{G,H}\psi_H(\psi_H - 1) + \mu_{G,H}\psi_H - r_G - \lambda_G = 0. \tag{B11}$$

We can easily verify that the particular parts of the above function form are, respectively,

$$A(X_t, G, H) = \frac{X_t}{r_G + \lambda_G - \mu_{G,H}}, \tag{B12}$$

and

$$C_G(s_0) = \frac{c(s_0)}{r_G + \lambda_G}. \tag{B13}$$

It is evident that the unleveled asset value $A(X_t, G, H)$ is decreasing with the probability of leaving the good state for the bad state, λ_G, in line with our intuition. While $A(X_t, G, H)$ is independent of initial state s_0, $C_G(s_0)$ is dependent on the initial state where the firm enters market and issue debt.

$\mathbb{R}_2 = \mathbf{X_{B,d}} \leq \mathbf{X_t} < \mathbf{X_{B,r}}$

In this region, the firm has taken on high risk investments, i.e., $v_t = H$, but have no gone bankrupt in both states. Equity holders receive $(1-\tau)(X_t - c(s_0))$ in both states so that $E(X_t, B, H; s_0)$ and $E(X_t, G, H; s_0)$ satisfy the following system of ODEs:

$$\begin{aligned}(r_B + \lambda_B)E(X_t, B, H; s_0) &= (1-\tau)(X_t - c(s_0)) + \mu_{B,H}X_tE^{'}(X_t, B, H; s_0)\\ &+ \frac{1}{2}\sigma^2_{B,H}X_t^2E^{''}(X_t, B, H; s_0) + \lambda_BE(X_t, G, H; s_0) \qquad \text{(B14)}\\ (r_G + \lambda_G)E(X_t, G, H; s_0) &= (1-\tau)(X_t - c(s_0)) + \mu_{G,H}X_tE^{'}(X_t, G, H; s_0)\\ &+ \frac{1}{2}\sigma^2_{G,H}X_t^2E^{''}(X_t, G, H; s_0) + \lambda_GE(X_t, B, H; s_0). \qquad \text{(B15)}\end{aligned}$$

Assume the functional form of the solution in state s_t is

$$E(X_t, s_t, H; s_0) = (A(X_t, s_t, H) - C_{s_t}(s_0))(1-\tau) + \sum_{i=1}^{4} e^E_{s_t,H,i}X_t^{\omega_{H,i}}. \tag{B16}$$

Plugging (B16) into the ODEs (B14) and (B15), we obtain the solutions to the particular parts as follows:

$$\begin{bmatrix} A(X_t, B, H) \\ A(X_t, G, H) \end{bmatrix} = \begin{bmatrix} r_B - \mu_{B,H} + \lambda_B & -\lambda_B \\ -\lambda_G & r_G - \mu_{G,H} + \lambda_G \end{bmatrix}^{-1} \begin{bmatrix} X_t \\ X_t \end{bmatrix} \tag{B17}$$

and

$$\begin{bmatrix} C_B(s_0) \\ C_G(s_0) \end{bmatrix} = \begin{bmatrix} r_B + \lambda_B & -\lambda_B \\ -\lambda_G & r_G + \lambda_G \end{bmatrix}^{-1} \begin{bmatrix} c(s_0) \\ c(s_0) \end{bmatrix} \tag{B18}$$

For the homogenous part of the solution, we verify that, for each pair of $e^E_{B,H,i}(s_0)X_t^{\omega_{H,i}}$ and $e^E_{G,H,i}(s_0)X_t^{\omega_{H,i}}$, we have

$$\left[\begin{pmatrix} r_{B,H} + \lambda_B & -\lambda_B \\ -\lambda_G & r_{G,H} + \lambda_G \end{pmatrix} - \begin{pmatrix} \mu_{B,H} & 0 \\ 0 & \mu_{G,H} \end{pmatrix} \omega_{H,i} - \tfrac{1}{2} \begin{pmatrix} \sigma^2_{B,H} & 0 \\ 0 & \sigma^2_{G,H} \end{pmatrix} \omega_{H,i}(\omega_{H,i} - 1)\right] \begin{pmatrix} e^E_{B,H,i}(s_0) \\ e^E_{G,H,i}(s_0) \end{pmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{B19}$$

Moreover, $e^E_{B,H,i}(s_0) = g_{H,i}e^E_{G,H,i}(s_0)$, where

$$g_{H,i} = \frac{1}{\lambda_G}(\frac{1}{2}\sigma^2_{G,H}\omega_{H,i}(\omega_{H,i} - 1) + \mu_{G,H}\omega_{H,i} - r_G - \lambda_G), \tag{B20}$$

and $\omega_{H,i}$ is one of two positive roots and two negative roots of the following function

$$(\frac{1}{2}\sigma^2_{B,H}\omega_H(\omega_H - 1) + \mu_{B,H}\omega_H - r_B - \lambda_B)(\frac{1}{2}\sigma^2_{G,H}\omega_H(\omega_H - 1) + \mu_{G,H}\omega_H - r_G - \lambda_G) = \lambda_B\lambda_G. \tag{B21}$$

$\mathbb{R}_3 = \mathbf{X_{G,r}} \leq \mathbf{X_t} < \mathbf{X_{B,r}}$

The firm has shifted to a high level of idiosyncratic risk in the bad state, but has not done so in the good state and has a low level of cash flow risk. A sudden switch from the bad state to the good state could lead the firm to increase its risk immediately and increase $E(X_t, G, L; s_0)$ to $E(X_t, B, H; s_0)$.

The following ODE describes the value of equity in this region:

$$\begin{aligned}(r_G + \lambda_G)E(X_t, G, L; s_0) &= (1 - \tau)(X_t - c(s_0)) + \mu_{G,L}X_tE'(X_t, G, L; s_0) \\ &+ \frac{1}{2}\sigma^2_{G,L}X_t^2E''(X_t, G, L; s_0) + \lambda_GE(X_t, B, H; s_0).\end{aligned} \tag{B22}$$

Assume the functional form of the solution is as follows:

$$E(X_t, G, L; s_0) = (A(X_t, G, L) - C_G(s_0))(1 - \tau) + \sum_{i=1}^{2} e^E_{G,L,i}(s_0)X_t^{\omega_{M,i}}, \tag{B23}$$

where $\omega_{M,i}$ is the positive and negative roots of the following function

$$\frac{1}{2}\sigma^2_{G,L}\omega_M(\omega_M - 1) + \mu_{G,L}\omega_M - r_G - \lambda_G. \tag{B24}$$

As before, plugging (B23) into ODEs (B22), we can easily verify that the particular parts of the above function form are, respectively,

$$A(X_t, G, L) = \frac{X_t}{r_G + \lambda_G - \mu_{G,L}} \tag{B25}$$

and

$$C_G(s_0) = \frac{c(s_0)}{r_G + \lambda_G}. \tag{B26}$$

$\mathbb{R}_4 = \mathbf{X_{B,r}} \leq \mathbf{X_t} < \mathbf{X_{G,u}}$

The firm has not risk-shifted and has a low risk profile in both states in this region. Hence, equity value functions $E(X_t, G, L; s_0)$ and $E(X_t, B, L; s_0)$ satisfy the following system of ODEs

$$\begin{aligned}(r_G + \lambda_G)E(X_t, G, L; s_0) &= (1-\tau)(X_t - c(s_0)) + \mu_{G,L} X_t E^{'}(X_t, G, L; s_0)\\ &\quad + \frac{1}{2}\sigma_{G,L}^2 X_t^2 E^{''}(X_t, G, L; s_0) + \lambda_G E(X_t, B, L; s_0), \end{aligned}\tag{B27}$$

$$\begin{aligned}(r_G + \lambda_B)E(X_t, B, L; s_0) &= (1-\tau)(X_t - c(s_0)) + \mu_{B,L} X_t E^{'}(X_t, B, L; s_0)\\ &\quad + \frac{1}{2}\sigma_{B,L}^2 X_t^2 E^{''}(X_t, B, L; s_0) + \lambda_B E(X_t, G, L; s_0). \end{aligned}\tag{B28}$$

Assume the functional form of the value function is

$$E(X_t, s_t, L; s_0) = (A(X_t, s_t, L) - C_{s_t}(s_0))(1-\tau) + \sum_{i=1}^{4} e_{s_t,L,i}^E(s_0) X_t^{\omega_{L,i}}. \tag{B29}$$

Plugging (B23) and (B29) into ODEs (B27) and (B28), we obtain its particular solutions $A(X_t, s_t, L)$ and $C_{s_t}(s_0)$ in the matrix form are as follows:

$$\begin{bmatrix} A(X_t, B, L) \\ A(X_t, G, L) \end{bmatrix} = \begin{bmatrix} r_B - \mu_{B,L} + \lambda_B & -\lambda_B \\ -\lambda_G & r_G - \mu_{G,L} + \lambda_G \end{bmatrix}^{-1} \begin{bmatrix} X_t \\ X_t \end{bmatrix}, \tag{B30}$$

and

$$\begin{bmatrix} C_B(s_0) \\ C_G(s_0) \end{bmatrix} = \begin{bmatrix} r_B + \lambda_B & -\lambda_B \\ -\lambda_G & r_G + \lambda_G \end{bmatrix}^{-1} \begin{bmatrix} c(s_0) \\ c(s_0) \end{bmatrix}. \tag{B31}$$

We can verify for each item $e_{B,L,i}^E(s_0)X_t^{\omega_{L,i}}$ and $e_{G,L,i}^E(s_0)X_t^{\omega_{L,i}}$ of the homogenous solution is

$$\left[\begin{pmatrix} r_{B,L} + \lambda_B & -\lambda_B \\ -\lambda_G & r_{G,L} + \lambda_G \end{pmatrix} - \begin{pmatrix} \mu_{B,L} & 0 \\ 0 & \mu_{G,L} \end{pmatrix} \omega_{L,i} - \tfrac{1}{2} \begin{pmatrix} \sigma_{B,L}^2 & 0 \\ 0 & \sigma_{G,L}^2 \end{pmatrix} \omega_{L,i}(\omega_{L,i} - 1)\right] \begin{pmatrix} e_{B,L,i}^E(s_0) \\ e_{G,L,i}^E(s_0) \end{pmatrix} = \begin{bmatrix} 0 \\ 0. \end{bmatrix} \tag{B32}$$

Additionally, $e_{B,L,i}^E(s_0) = g_{L,i} e_{G,L,i}^E(s_0)$, where

$$g_{L,i} = \frac{1}{\lambda_G}(\frac{1}{2}\sigma_{G,L}^2 \omega_{L,i}(\omega_{L,i} - 1) + \mu_{G,L}\omega_{L,i} - r_G - \lambda_G), \tag{B33}$$

and ω_L is two positive roots and two negative roots of the following function

$$(\frac{1}{2}\sigma^2_{B,L}\omega_L(\omega_L-1)+\mu_{B,L}\omega_L-r_B-\lambda_B)(\frac{1}{2}\sigma^2_{G,L}\omega_L(\omega_L-1)+\mu_{G,L}\omega_L-r_G-\lambda_G)=\lambda_B\lambda_G. \quad \text{(B34)}$$

$\mathbb{R}_5 = \mathbf{X_{G,u} \leq X_t < X_{B,u}}$

In this region, the firm in the good state has already refinanced their debt upward, but not yet in the bad state. By retiring the existing debt at par $D(X_0, s_0, L; s_0)$ and issuing new debt $D(X_{G,u}, G, L; G)$ at a fraction cost ϕ, equity holders increase their own wealth to $E(X_t, G, L; s_0) = (1-\phi)\, D(X_t, G, L; G) + E(X_t, G, L; G) - D(X_0, s_0, L; s_0)$. By scaling property, we have the following equity value at the refinancing threshold $X_{G,u}$:

$$E(X_{G,u}, G, L; s_0) = \frac{X_{G,u}}{X_0}\left((1-\phi)\, D(X_0, G, L; G) + E(X_0, G, L; G)\right) - D(X_0, s_0, L; s_0).$$

In contrast, equity holders in the bad state have not refinanced their debt yet. However, an exogenous switch from the bad state to the good state induces equity holders to refinance their debt immediately. Hence, the equity value function in the bad state, $E(X_t, B, L; s_0)$, satisfies the following ODE

$$\begin{aligned}(r_B+\lambda_B)E(X_t, B, L; s_0) &= (1-\tau)(X_t - c(s_0)) + \mu_{B,L}X_tE^{'}_{B,L} + \frac{1}{2}\sigma^2_{B,L}X_tE^{''}_{B,L} \\ &\quad + \lambda_B\left(\frac{X_t}{X_0}\left((1-\phi)\, D(X_0, G, L; G) + E(X_0, G, L; G)\right) - D(X_0, s_0, L; s_0)\right).\end{aligned} \quad \text{(B35)}$$

Its solution is

$$E(X_t, B, L; s_0) = A(X_t, B, L) - C_B(s_0) + \sum_{i=1}^{2} a^E_{B,L,i}(s_0)X_t^{\psi_{L,i}} \quad \text{(B36)}$$

where $\psi_{L,i}$ is the negative and positive roots of

$$\frac{1}{2}\sigma^2_{B,L}\psi_L(\psi_L-1)+\mu_{B,L}\psi_L-r_B-\lambda_B=0. \quad \text{(B37)}$$

We can verify the particular parts of the value function are as follows:

$$A(X_t, B, L) = \frac{X_t(1-\tau)+\lambda_B\frac{X_t}{X_0}\left((1-\phi)\, D(X_0, G, L; G) + E(X_0, G, L; G)\right)}{r_B+\lambda_B-\mu_{B,,L}}, \quad \text{(B38)}$$

and

$$C_B(s_0) = \frac{c(s_0)(1-\tau)+\lambda_B D(X_0, s_0, L; s_0)}{r_B+\lambda_B}. \quad \text{(B39)}$$

In total, we have 14 unknown coefficients for equity value function for an initial state s_0.

B.1.3 Debt Value Functions

$\mathbb{R}_1 = \mathbf{X_{G,d}} \leq \mathbf{X_t} < \mathbf{X_{B,d}}$

In this region, the firm has gone bankrupt in the bad state. Debt holders take over the assets and receive the residual value after the liquidation cost, i.e., $D(X_t, B, H; s_0) = (1-\alpha_B)A(X_{B,d}, B, H)(1-\tau)$. In the good state, debt holders still receive the fixed coupon $c(s_0)$ before bankruptcy. Hence, its value function $D(X_t, G, H; s_0)$ satisfies the following ODE:

$$\begin{aligned}(r_G + \lambda_G)D(X_t, G, H; s_0) &= c(s_0) + \mu_{G,H} X_t D'(X_t, G, H; s_0) \\ &+ \frac{1}{2}\sigma_{G,H}^2 X_t^2 D''(X_t, G, H; s_0) + \lambda_G (1-\alpha_B) A(X_{B,d}, B, H)(1-\tau)\end{aligned} \tag{B40}$$

The solution of the debt value function is

$$D(X_t, G, H; s_0) = C_G(s_0) + \sum_{i=1}^{2} a_{G,H,i}^D(s_0) X_t^{\psi_{H,i}} + a_d \lambda_G (1-\alpha_B) A(X_{B,d}, B, H)(1-\tau), \tag{B41}$$

where $C_G(s_0)$ is defined in equation (B13), $\psi_{H,i}$ in (B11),

$$a_d = \frac{1}{r_G + \lambda_G - \mu_{G,H}}, \tag{B42}$$

and

$$A(X_t, B, H) = \frac{X_t}{r_B + \lambda_B - \mu_{B,H}}. \tag{B43}$$

$\mathbb{R}_2 = \mathbf{X_{B,d}} \leq \mathbf{X_t} < \mathbf{X_{G,r}}$

In this region, the firm has a high risk profile and its debt holders receive a stream of fixed coupon $c(s_0)$ in both states. $D(X_t, B, H; s_0)$ and $D(X_t, G, H; s_0)$ satisfy the following system of ODEs:

$$\begin{aligned}(r_B + \lambda_B)D(X_t, B, H; s_0) &= c(s_0) + \mu_{B,H} X_t D'(X_t, B, H; s_0) \\ &+ \frac{1}{2}\sigma_{B,H}^2 X_t^2 D''(X_t, B, H; s_0) + \lambda_B D(X_t, G, H; s_0)\end{aligned} \tag{B44}$$

$$\begin{aligned}(r_G + \lambda_G)D(X_t, G, H; s_0) &= c(s_0) + \mu_{G,H} X_t D'(X_t, G, H; s_0) \\ &+ \frac{1}{2}\sigma_{G,H}^2 X_t^2 D''(X_t, G, H; s_0) + \lambda_G D(X_t, B, H; s_0),\end{aligned} \tag{B45}$$

The debt value function in state s_t is

$$D(X_t, s_t, H; s_0) = C_{s_t}(s_0) + \sum_{i=1}^{4} e_{s_t,H,i}^D X_t^{\omega_{H,i}} \tag{B46}$$

where $C_{s_t}(s_0)$ is shown in (B18) and $\omega_{H,i}$ in (B21). Similar to the equity value function in the same region, $e_{B,H,i}^D(s_0) = g_{H,i} e_{G,H,i}^D(s_0)$, where $g_{H,i}$ is in equation (B20).

$\mathbb{R}_3 = \mathbf{X_{G,r}} \leq \mathbf{X_t} < \mathbf{X_{B,r}}$

Debt holders have the fixed coupon, $c(s_0)$, dependent on the initial state s_0 where debt was issued.

A sudden switch from the bad state to the good state could lead the firm to increase its risk immediately and change $D(X_t, G, L; s_0)$ to $D(X_t, B, H; s_0)$. The value function of debt $D(X_t, G, L; s_0)$ satisfies the following ODE:

$$(r_G + \lambda_G) D(X_t, G, L; s_0) = c(s_0) + \mu_{G,L} X_t D^{'}(X_t, G, L; s_0) + \frac{1}{2}\sigma^2_{G,L} X_t^2 D^{''}(X_t, G, L; s_0) + \lambda_B D(X_t, B, H; s_0) \quad \text{(B47)}$$

The value functions of debt is as follows:

$$D(X_t, G, L; s_0) = C_G(s_0) + \sum_{i=1}^{2} e^D_{G,L,i}(s_0) X_t^{\omega_{M,i}}, \quad \text{(B48)}$$

where $C_{s_t}(s_0)$ is shown in (B26) and $\omega_{M,i}$ in (B24).

$\mathbb{R}_4 = \mathbf{X_{B,r}} \leq \mathbf{X_t} < \mathbf{X_{G,u}}$

The firm has not risk-shifted in both states and possesses a low level of idiosyncratic risk. Debt value functions $D(X_t, G, L; s_0)$ and $D(X_t, B, L; s_0)$ satisfy the following system of ODEs:

$$(r_G + \lambda_G) D(X_t, G, L; s_0) = c(s_0) + \mu_{G,L} X_t D^{'}(X_t, G, L; s_0) + \frac{1}{2}\sigma^2_{G,L} X_t^2 D^{''}(X_t, G, L; s_0) + \lambda_G D(X_t, B, L; s_0), \quad \text{(B49)}$$

$$(r_G + \lambda_B) D(X_t, B, L; s_0) = c(s_0) + \mu_{B,L} X_t D^{'}(X_t, B, L; s_0) + \frac{1}{2}\sigma^2_{B,L} X_t^2 D^{''}(X_t, B, L; s_0) + \lambda_B D(X_t, G, L; s_0). \quad \text{(B50)}$$

And the solution function in both states is

$$D(X_t, s_t, L; s_0) = C_{s_t}(s_0) + \sum_{i=1}^{4} e^D_{s_t,L,i}(s_0) X_t^{\omega_{L,i}}. \quad \text{(B51)}$$

where $C_{s_t}(s_0)$ is shown in (B31) and $\omega_{L,i}$ in (B34). Similar to the equity value function in the same region, $e^D_{B,L,i}(s_0) = g_{L,i} e^D_{G,L,i}(s_0)$, where $g_{L,i}$ is in equation (B33).

$\mathbb{R}_5 = \mathbf{X_{G,u}} \leq \mathbf{X_t} < \mathbf{X_{B,u}}$

Because the firm refinances earlier in the good state than in the bad state, debt holders have already redeemed the par value, $D(X_t, G, L; s_0) = D(X_0, s_0, L; s_0)$, in the good state. Because debt holders have not received the payment at par in the bad state, the debt value function, $D(X_t, B, L; s_0)$, satisfies the following ODE:

$$(r_B + \lambda_B) D(X_t, B, L; s_0) = c(s_0) + \mu_{B,L} X_t D^{'}(X_t, B, L; s_0) + \frac{1}{2}\sigma^2_{B,L} X_t^2 D^{''}(X_t, B, L; s_0) + \lambda_B D(X_0, s_0, L; s_0) \quad \text{(B52)}$$

Its solution is

$$D(X_t, B, L; s_0) = C_B(s_0) + \sum_{i=1}^{2} a^D_{B,L,i}(s_0) X_t^{\psi_{L,i}} \quad \text{(B53)}$$

where $\psi_{L,i}$ is in (B37) and

$$C_B(s_0) = \frac{c(s_0) + \lambda_B D(X_0, s_0, L; s_0)}{r_B + \lambda_B}. \tag{B54}$$

In total, we have 14 unknown coefficients for debt value function for an initial state s_0.

B.2 Boundary Conditions

B.2.1 Equity Boundary Conditions

When the firm has a high level of idiosyncratic volatility $v_t = H$, we have the following conditions for the two aggregate states, s_t:

$$\lim_{X_t \downarrow X_d(B;s_0)} E(X_t, B, H; s_0) = 0, \tag{B55}$$

$$\lim_{X_t \downarrow X_d(G;s_0)} E(X_t, G, H; s_0) = 0, \tag{B56}$$

$$\lim_{X_t \uparrow X_d(B;s_0)} E(X_t, G, H; s_0) = \lim_{X_t \downarrow X_d(B;s_0)} E(X_t, G, H; s_0), \tag{B57}$$

$$\lim_{X_t \uparrow X_d(B;s_0)} E^{'}(X_t, G, H; s_0) = \lim_{X_t \downarrow X_d(B;s_0)} E^{'}(X_t, G, H; s_0). \tag{B58}$$

Equations (B55) and (B56) state that equity holders receive nothing at bankruptcy at both aggregate state, $s_t = B, G$. Equations (B57) and (B58) are to ensure that the equity value function $E(X_t, G, H; s_0)$ be continuous and smooth at $X_d(B; s_0)$.

Before the firm goes bankrupt or restructures its debt, it switches between the high- and low levels of idiosyncratic volatility. We impose the following conditions for equity value functions :

$$\lim_{X_t \downarrow X_r(B;s_0)} E(X_t, B, L; s_0) = \lim_{X_t \uparrow X_r(B;s_0)} E(X_t, B, H; s_0) - \xi^+ \epsilon_B^2 A(X_t, B, H)(1-\tau), \tag{B59}$$

$$\lim_{X_t \downarrow X_r(G;s_0)} E(X_t, G, L; s_0) = \lim_{X_t \uparrow X_r(G;s_0)} E(X_t, G, H; s_0) - \xi^+ \epsilon_G^2 A(X_t, G, H)(1-\tau), \tag{B60}$$

$$\lim_{X_t \downarrow X_r(B;s_0)} E(X_t, G, L; s_0) = \lim_{X_t \uparrow X_r(B;s_0)} E(X_t, G, L; s_0), \tag{B61}$$

$$\lim_{X_t \downarrow X_r(B;s_0)} E^{'}(X_t, G, L; s_0) = \lim_{X_t \uparrow X_r(B;s_0)} E^{'}(X_t, G, L; s_0). \tag{B62}$$

Equations (B59) and (B60) are value matching conditions, which ensure no arbitrage at the risk-shifting threshold $X_r(s_t; s_0)$ for the same state, s_t. While equations (B61) and (B62) are to ensure that $E(X_t, G, L; s_0)$ is continuous and smooth at $X_r(B; s_0)$.

When the firm is currently in a low level of idiosyncratic volatility, i.e., $v_t = L$ in both aggregate states, it restructures its debt upward. We impose the boundary conditions as follows:

$$\lim_{X_t \uparrow X_u(B;s_0)} E(X_t, B, L; s_0) = \lim_{X_t \downarrow X_u(B;s_0)} \frac{X_t}{X_0}[(1-\phi)D(X_0, B, L; B) + E(X_0, B, L; B)] - D(X_0, B, L; s_0), \tag{B63}$$

$$\lim_{X_t \uparrow X_u(G;s_0)} E(X_t, G, L; s_0) = \lim_{X_t \downarrow X_u(G;s_0)} \frac{X_t}{X_0}[(1-\phi)D(X_0, G, L; G) + E(X_0, G, L; G)] - D(X_0, G, L; s_0), \tag{B64}$$

$$\begin{aligned}\lim_{X_t \uparrow X_u(B;s_0)} E(X_t, B, H; s_0) = & \lim_{X_t \downarrow X_u(B;s_0)} \frac{X_t}{X_0}[(1-\phi)D(X_0, B, L; B) + E(X_0, B, L; B)] \\ & - D(X_0, B, L; s_0) - \xi^{-}\epsilon_B^2 A(X_t, B, L)(1-\tau),\end{aligned} \tag{B65}$$

$$\begin{aligned}\lim_{X_t \uparrow X_u(G;s_0)} E(X_t, G, H; s_0) = & \lim_{X_t \downarrow X_u(G;s_0)} \frac{X_t}{X_0}[(1-\phi)D(X_0, G, L; G) + E(X_0, G, L; G)] \\ & - D(X_0, G, L; s_0) - \xi^{-}\epsilon_G^2 A(X_t, G, L)(1-\tau),\end{aligned} \tag{B66}$$

$$\lim_{X_t \uparrow X_u(G;s_0)} E(X_t, B, L; s_0) = \lim_{X_t \downarrow X_u(G;s_0)} E(X_t, B, L; s_0), \tag{B67}$$

$$\lim_{X_t \uparrow X_u(G;s_0)} E^{'}(X_t, B, L; s_0) = \lim_{X_t \downarrow X_u(G;s_0)} E^{'}(X_t, B, L; s_0). \tag{B68}$$

Equations (B63) and (B64) are value matching conditions for firms that never increase idiosyncratic risk. They states that, at the restructuring threshold, $X_u(s_t; s_0)$, equity holders retire debt at par $D(X_0, s_0, L; s_0)$, which was issued at the initial state s_0, and issue more debt $D(X_t, s_t, L; s_0)$ at the current aggregate state $s_t = B, G$ if the firm has a low level of volatility, i.e., $v_t = L$. *The scaling property applies only within the same aggregate state, s_t.* That is, if the firm starts at the initial state $s_0 = B$ but refinance at $s_t = G$, we scale up the firm value to $X_t/X_0[(1-\phi)D(X_0, G, L; G) + E(X_0, G, L; G)]$ as if it starts at $s_0 = G$.

Slightly different from equations (B63) and (B64), equations (B65) and (B66) apply to the firms that increased the idiosyncratic volatility before. They have to pay a cost $\xi^{-}\epsilon_G^2 A(X_t, s_t, L)(1-\tau)$ of adjust the volatility back to the low level immediately before they need to refinance at the state s_t. Equations (B67) and (B68) are to ensure that equity value function $E(X_t, B, L; s_0)$ is continuous and smooth at $X_u(G; s_0)$.

B.2.2 Debt Boundary Conditions

When the firm has a high level of idiosyncratic volatility, $v_t = H$, we have the following conditions for debt value functions:

$$\lim_{X_t \downarrow X_b(B;s_0)} D(X_t, B, H; s_0) = (1-\alpha) A(X_b, B, H), \tag{B69}$$

$$\lim_{X_t \downarrow X_b(G;s_0)} D(X_t, G, H; s_0) = (1-\alpha) A(X_b, G, H), \tag{B70}$$

$$\lim_{X_t \uparrow X_d(B;s_0)} D(X_t, G, H; s_0) = \lim_{X_t \downarrow X_d(B;s_0)} D(X_t, G, H; s_0), \tag{B71}$$

$$\lim_{X_t \uparrow X_d(B;s_0)} D^{'}(X_t, G, H; s_0) = \lim_{X_t \downarrow X_d(B;s_0)} D^{'}(X_t, G, H; s_0). \tag{B72}$$

Equations (B69) to (B70) states that debt holders receive the asset value after liquidation cost α in both states $s_t = B, G$. Equations (B71) and (B72) are to ensure that the debt value function $D(X_t, G, H; s_0)$ be continuous and smooth at $X_d(B; s_0)$.

We impose the following conditions for debt value functions before the firm goes bankrupt or restructures its debt:

$$\lim_{X_t \uparrow X_r(B;s_0)} D(X_t, B, H; s_0) = \lim_{X_t \downarrow X_r(B;s_0)} D(X_t, B, L; s_0), \tag{B73}$$

$$\lim_{X_t \uparrow X_r(G;s_0)} D(X_t, G, H; s_0) = \lim_{X_t \downarrow X_r(G;s_0)} D(X_t, G, L; s_0), \tag{B74}$$

$$\lim_{X_t \uparrow X_r(B;s_0)} D(X_t, G, L; s_0) = \lim_{X_t \downarrow X_r(B;s_0)} D(X_t, G, L; s_0), \tag{B75}$$

$$\lim_{X_t \uparrow X_r(B;s_0)} D^{'}(X_t, G, L; s_0) = \lim_{X_t \downarrow X_r(B;s_0)} D^{'}(X_t, G, L; s_0). \tag{B76}$$

The interpretations for equations (B73) to (B76) for debt value functions are similar to those for equations (B59) to (B62) for equity value functions, except that debt holders do not have to pay the adjust costs.

When the firm is is in the low level of idiosyncratic volatility, $v_t = L$, and it restructures its debt upward, we have the following conditions:

$$\lim_{X_t \uparrow X_u(B;s_0)} D(X_t, B, L; s_0) = P(X_0; s_0), \tag{B77}$$

$$\lim_{X_t \uparrow X_u(G;s_0)} D(X_t, G, L; s_0) = P(X_0; s_0), \tag{B78}$$

$$\lim_{X_t \uparrow X_u(B;s_0)} D(X_t, B, H; s_0) = P(X_0; s_0), \tag{B79}$$

$$\lim_{X_t \uparrow X_u(G;s_0)} D(X_t, G, H; s_0) = P(X_0; s_0), \tag{B80}$$

$$\lim_{X_t \uparrow X_u(G;s_0)} D(X_t, B, L; s_0) = \lim_{X_t \downarrow X_u(G;s_0)} D(X_t, B, L; s_0), \tag{B81}$$

$$\lim_{X_t \uparrow X_u(G;s_0)} D^{'}(X_t, B, L; s_0) = \lim_{X_t \downarrow X_u(G;s_0)} D^{'}(X_t, B, L; s_0). \tag{B82}$$

Equations (B77) and (B78) are value matching conditions which indicate that debt holder receive par value at the debt refinancing threshold $X_u(s_t; s_0)$ at both states, respectively, for a firm that equity holders did not increase the idiosyncratic volatility. Regardless of which the current aggregate state s_t is, debt holders receive the par value $P(X_0; s_0)$ determined at the initial state s_0 where debt is issued. Slightly different from Equations (B77) and (B78), Equations (B79) and (B80) state that equity holders increased the idiosyncratic volatility before, but have not adjusted the volatility back to the low (L) level until they need to refinance. Equations (B81) and (B82) are to ensure that debt value function $D(X_t, B, L; s_0)$ is continuous and smooth at $X_u(G; s_0)$.

C Equity Returns

In this section, we provide the proofs for equity returns in propositions 1 and 2.

C.1 Proof of Proposition 1

Proposition 1 is for the simplified baseline model, in which the firm does not have the option to reverse its risk-taking and the option to refinance its debt upward. Instead, the firm has only an option to increase idiosyncratic risk and an option to go into bankruptcy.

C.2 Proof of Proposition 1

To prove the Proposition 1, we start with the general formula for the equity return and then use the boundary conditions. Ito's lemma implies that the equity value $E(X_t, v_t) \equiv E_{t,v_t}$ satisfies

$$\frac{dE_{t,v_t}}{E_{t,v_t}} = \frac{1}{E_{t,v_t}} \left(\frac{\partial E_{t,v_t}}{\partial t} + \hat{\mu}_{v_t} X_t \frac{\partial E_{t,v_t}}{\partial X_t} + \frac{\sigma_{v_t}^2}{2} X_t^2 \frac{\partial^2 E_{t,v_t}}{\partial X_t^2} \right) dt + (\sigma^m d\hat{W}_t^m + \sigma_{v_t}^{i,X} d\hat{W}_t^i) \frac{X_t}{E_{t,v_t}} \frac{\partial E_{t,v_t}}{\partial X_t}. \tag{C1}$$

The standard asset pricing argument gives

$$\mathbb{E}\left[\frac{dE_{t,v_t} + D_t dt}{E_{t,v_t}}\right] - rdt = -\mathbb{E}\left(\frac{dE_{t,v_t}}{E_{t,v_t}}, \frac{dm_t}{m_t}\right) = \frac{X_t}{E_{t,v_t}} \frac{\partial E_{t,v_t}}{\partial X_t} \sigma^m \theta dt. \tag{C2}$$

Denoting $(dE_{t,v_t} + D_t dt)/E_{t,v_t}$ by r_{t,v_t}^E and $(X_t \partial E_{t,v_t})/(E_{t,v_t} \partial X_t)$ by γ_{t,v_t}, we have the excess equity return

$$r_{t,v_t}^e(X_t) = \mathbb{E}_t[r_{t,v_t}^E] - rdt = \gamma_{t,v_t} \sigma^m \theta dt = \gamma_{t,v_t} \zeta dt. \tag{C3}$$

The sensitivity of the equity to the underlying cash flows γ_{t,v_t} can be obtained by differentiating

(A5), and is as follows:

$$\begin{aligned}
\gamma_{t,v_t} =& \frac{X_t \partial E_{t,v_t}}{E_{t,v_t} \partial X_t} = \frac{A_{t,v_t} \partial E_{t,v_t}}{E_{t,v_t} \partial A_{t,v_t}} \\
=& \frac{1}{E_{t,v_t}} (X_t(1-\tau) + e_{v_t,1}\omega_{v_t,1} X_t^{\omega_{v_t,1}} + e_{v_t,2}\omega_{v_t,2} X_t^{\omega_{v_t,2}}) \\
=& \frac{1}{E_{t,v_t}} \left(E_{t,v_t} + \frac{c(1-\tau)}{r} - e_{v_t,1} X_t^{\omega_{v_t,1}} + e_{v_t,1}\omega_{v_t,1} X_t^{\omega_{v_t,1}} - e_{v_t,2} X_t^{\omega_{v_t,2}} + e_{v_t,2}\omega_{v_t,2} X_t^{\omega_{v_t,2}} \right) \\
=& 1 + \frac{c(1-\tau)}{rE_{t,v_t}} + \frac{(\omega_{v_t,1}-1)}{E_{t,v_t}} e_{v_t,1} X_t^{\omega_{v_t,1}} + \frac{(\omega_{v_t,2}-1)}{E_{t,v_t}} e_{v_t,2} X_t^{\omega_{v_t,2}}
\end{aligned} \tag{C4}$$

With the general formula for the expected excess return, we are ready to obtain the equity value function in equation (23) and the elasticity equation (22) by determining the coefficient $e_{v_t,1}$ and $e_{v_t,2}$ according to the boundary conditions.

After the risk-shifting, the firm does not have the upward refinancing opportunity in this simplified case. Therefore, the no-bubble condition implies $e_{H,2} = 0$, and the value-matching condition of equation (A8) gives $e_{H,1} = -(A_{d,H} - c/r)(1-\tau)/X_d^{\omega_{H,1}}$. Simply substituting $e_{H,1}$ and $e_{H,2}$ into equations (C4), we obtain the elasticity in equation (22). Plugging $e_{H,1}$ and $e_{H,2}$ into (A5), we obtain the equity value function (23).

Given the equity value function in equation (23), we use the condition of equation (6) and easily obtain the optimal default threshold X_d as in equation (24). However, to obtain the optimal risk-shifting threshold X_r, we need to know the equity value function of the firm prior to the risk-shifting before we can apply the smooth pasting condition of equation (7).

Similarly, the firm does not have the upward refinancing opportunity in this simplified case before it risk-shifts. Hence, $e_{L,2} = 0$ because of the no-bubble condition, and $e_{L,1}$ can be obtained by the value-matching condition of equation (A11) as follows:

$$\begin{aligned}
\left(A_{r,L} - \frac{c}{r}\right)(1-\tau) + e_{L,1} X_r^{\omega_{L,1}} =& \left(A_{r,H} - \frac{c}{r}\right)(1-\tau) + \left(\frac{c}{r} - A_{d,H}\right)\left(\frac{X_r}{X_d}\right)^{\omega_{H,1}} (1-\tau) \\
& - \eta\epsilon^2 A_{r,H}(1-\tau).
\end{aligned} \tag{C5}$$

Hence,

$$e_{L,1} = \frac{(1-\tau)}{X_r^{\omega_{L,1}}} \left[(A_{r,H}(1-\eta\epsilon^2) - A_{r,L}) + \left(\frac{c}{r} - A_{d,H}\right)\left(\frac{X_r}{X_d}\right)^{\omega_{H,1}} \right]. \tag{C6}$$

Substituting $e_{L,1}$ and $e_{L,2}$ into equation (A5), we obtain the equity value before the risk-shifting

$$E_{L,t} = \left[\left(A(X_t, L) - \frac{c}{r}\right) + (A_{r,H}(1-\eta\epsilon^2) - A_{r,L})\left(\frac{X_t}{X_r}\right)^{\omega_{L,1}} + (\frac{c}{r} - A_{d,H})\left(\frac{X_r}{X_d}\right)^{\omega_{H,1}}\left(\frac{X_t}{X_r}\right)^{\omega_{L,1}} \right](1-\tau). \tag{C7}$$

Using smooth-pasting condition in equation (7) for equations (23) and (C7), we obtain the optimal risk-shifting threshold X_r in equation (25) after some algebraic manipulation.

C.3 Proof of Proposition 2

To prove Proposition 2, we apply Ito's Lemma to equation (11) and obtain

$$\frac{dE_{s_t,v_t}}{E_{s_t,v_t}} = \frac{\partial E_{s_t,v_t}}{\partial X_t E_{s_t,v_t}} dX_t + \frac{1}{2}\frac{\partial^2 E_{s_t,v_t}}{\partial X_t^2}\frac{(dX_t)^2}{E_{s_t,v_t}} + \frac{E_{s_t^+,v_t} - E_{s_t,v_t}}{E_{s_t,v_t}}(\hat{\lambda}_{s_t}dt + dM_{s_t,t}) \tag{C8}$$

$$= \frac{X_t \partial E_{s_t,v_t}}{E_{s_t,v_t}\partial X_t}\left(\hat{\mu}_{s_t}dt + \sigma^m_{s_t}d\hat{W}^m_t + \sigma^{i,X}_{s_t,v_t}d\hat{W}^i_t\right) + \frac{1}{2}\frac{X_t^2\partial^2 E_{s_t,v_t}}{E_{s_t,v_t}\partial X_t^2}\sigma^2_{s_t,v_t}dt + \frac{E_{s_t^+,v_t} - E_{s_t,v_t}}{E_{s_t,v_t}}(dM_{s_t,t} + \hat{\lambda}_{s_t}dt) \tag{C9}$$

$$= \frac{1}{E_{s_t,v_t}}\left(\frac{X_t\partial E_{s_t,v_t}}{\partial X_t}\hat{\mu}_{s_t} + \frac{X_t^2\sigma^2_{s_t,v_t}}{2}\frac{\partial^2 E_{s_t,v_t}}{\partial X_t^2} + \frac{E_{s_t^+,v_t} - E_{s_t,v_t}}{E_{s_t,v_t}}\hat{\lambda}_{s_t}\right)dt + \frac{E_{s_t^+,v_t} - E_{s_t,v_t}}{E_{s_t,v_t}}dM_{s_t,t}$$

$$+ \frac{X_t}{E_{s_t,v_t}}\frac{\partial E_{s_t,v_t}}{\partial X_t}\left(\sigma^m_{s_t}d\hat{W}^m_t + \sigma^{i,X}_{s_t,v_t}d\hat{W}^i_t\right) \tag{C10}$$

For two aggregate states, $s_t \in (G,B)$, and two levels of idiosyncratic risk, $v_t \in (H,L)$, the excess equity return is given by

$$r^e_{s_t,v_t}(X_t) = \mathbb{E}_t[r^E_{s_t,v_t}] - rdt \tag{C11}$$

$$= -\mathbb{E}\left[\frac{dm_{s_t,t}}{m_{s_t,t}}, \frac{dE_{s_t,v_t}}{E_{s_t,v_t}}\right] = \left[(\frac{X_t\partial E_{s_t,v_t}}{E_{s_t,v_t}\partial X_t})(\sigma^m_{s_t}\theta_{s_t}) - (\frac{E_{s_t^+,v_t}}{E_{s_t,v_t}} - 1)(\kappa_{s_t} - 1)\hat{\lambda}_{s_t}\right]dt. \tag{C12}$$

Let $\frac{X_t\partial E_{s_t,v_t,s_0}}{E_{s_t,v_t,s_0}\partial X_t} = \gamma_{s_t,v_t}$ and $(\frac{E_{s_t^+,v_t,s_0}}{E_{s_t,v_t,s_0}} - 1) = \psi_{s_t,v_t}$, we have

$$r^e_{s_t,v_t} = \gamma_{s_t,v_t}\zeta_{s_t}dt + \psi_{s_t,v_t}(1-\kappa_{s_t})\hat{\lambda}_{s_t}dt. \tag{C13}$$

∎

D Simulated Method of Moments

We discuss the covariance matrix of data and estimates, model simulation and the definitions of model-generated variables.

D.1 Covariance Matrix

The $\hat{\mathbf{b}}$ is asymptotically normally distributed as follows:

$$\sqrt{T}(\hat{\mathbf{b}} - \mathbf{b}) \rightarrow \mathcal{N}[0, (1+\frac{1}{S})(H'WH)^{-1}(H'W\Sigma_0 WH)(H'WH)^{-1}] \tag{C1}$$

where S denotes the number of simulations of length T, and

$$H = \mathbb{E}\Big[\frac{\partial \tilde{M}_s(\mathbf{b})}{\partial \mathbf{b}}\Big]. \tag{C2}$$

Σ_0 is the variance matrix calculated from the data but not from simulated data.

The weighting matrix W determines the weight of each element of the vector M in the objective function J. If $W = \Sigma_0{}^{-1}$, the estimator is optimal or efficient in the sense that the variance is as small as possible. Duffie and Singleton (1993) show that, under the appropriate conditions,

$$\sqrt{T}(\hat{\mathbf{b}} - \mathbf{b}) \rightarrow \mathcal{N}[0, (1 + 1/S)(H'\Sigma_0^{-1}H)^{-1}]. \tag{C3}$$

While the inverse of the variance matrix, $\Sigma_0{}^{-1}$, provides efficient estimates, the identity weight matrix gives a better economic weight for each moment(Cochrane, 1991, 1996). We use the identity weight matrix in our main analysis and conduct a robustness test using the inverse of Σ_0. As pointed out by Strebulaev and Whited (2012), the identity weight matrix gives the same weight to each of the selected moments. For example, the variance is usually small relative to the mean. Hence, its change has a relatively small effect on the objective function, J. To ensure our estimation is less dependent on the weighting matrix, we scale up the time-varying idiosyncratic volatility of cash flows, $\sigma_t^{i,X}$, by 10 to ensure that each moment carries enough weight in the objective function.

We follow Taylor (2010) and Wang (2017) and construct the covariance matrix Σ_0 of data moments using the seemingly unrelated regressions approach. Specifically, we express moments as the coefficients from a system of regression equations, each of which takes the form

$$Y_i = Z_i\beta_i + e_i, \tag{C4}$$

in which Z, Y, and e are vectors and the subscript i indicates the equation. Y_i is $T_i \times 1$ and β_i is $N_i \times 1$. The covariance, Σ_0, between moments estimators β_i and β_j is

$$\Sigma_0 = (Z_i'Z_i)^{-1}Z_i'\Sigma_{i,j}Z_j(Z_j'Z_j)^{-1} \tag{C5}$$

where $\Sigma_{i,j} = \mathbf{cov}(e_i, e_j)$. We also adjust the covariance matrix using Newey and West (1987) with 4 lags.

D.2 Model Simulation

To generate the model-implied moments, we simulate the model and generate 100 artificial panels of data at the quarterly frequency for a period of 140 years. The first 100 years of observations are discarded to reduce the dependence on initial values. In each panel there are 1,000 identical firms *ex ante* with the same initial parameters. At time 0, firms with an initial low level of idiosyncratic risk $v_t = L$ enter the market in the initial state $s_0 = G$. In observing the dynamics of their cash flows X_t, they take action whenever X_t crosses the refinancing threshold X_u, or falls below the risk-shifting threshold X_r. Following Strebulaev (2007), we assume that, when a firm terminates at the bankruptcy threshold X_d, debt holders take over the firm and a new firm emerges immediately. Cash flow parameters, including growth rate and volatility, are reset to the initial level after the takeover by debt holders.

Whenever a refinancing threshold is reached, all the optimal policies are scaled up. First, given the predetermined and estimated parameters, we solve for the three pairs of optimal policies on risk-shifting, $X_r(s_t; s_0)$, default, $X_d(s_t; s_0)$, and debt-refinancing, $X_u(s_t; s_0)$ for the initial state $s_0 = G$. Then, we apply the scaling property across two initial states in equation (19) to obtain three pairs of policies for $s_0 = B$. With these optimal policies, we calculate the equity E_t and debt values D_t along the path of cash flow X_t according to the value functions in different five regions in the online Appendix B.

D.3 Definitions of Variables

Using the simulated time series of each firm, we calculate the variables of interest in the same fashion as we do in standard empirical tests.

Interest Coverage In the same way as we calculate the interest coverage for the data, we calculate its theoretical counterpart as $X_t/c(s_0)$.

Financial Leverage Following Strebulaev (2007), we calculate the quasi-market leverage (QML) in the same fashion as we construct financial leverage from the data. Assuming the market value of debt is not observable, as in the Compustat data, we use the debt value at the issue date.

Equity Returns We calculate the equity value E_t along the path X_t. The semi-closed solution of E_t for different regions of X_t can be found in Section B.1. Accordingly, the return on equity for an individual firm i is

$$r_t^E = \frac{E_t + (X_t - c)(1-\tau)\Delta t - ACF_t}{E_{t-\Delta t}} - 1 \tag{C6}$$

where ACF_t denotes additional cash flows to equity holders when they pay the adjustment cost to adjust the level of idiosyncratic risk or pay the flotation cost to issue new debt.

Idiosyncratic Volatility To mimic the empirical procedure, we use the rolling standard deviation of five years (20 quarters) of asset growth shocks to proxy for the idiosyncratic cash flow volatility $\sigma_t^{i,X}$ and equity return volatility $\sigma_t^{i,E}$. To calculate $\sigma_t^{i,X}$, we first regress the percentage returns of individual firms dX_t/X_t on the market cash flow percentage return X_t^M, which is the simple average of X_t across all firms. Then, we calculate the standard deviation of the cash flow growth residuals to proxy for the idiosyncratic volatility. Similarly, to calculate idiosyncratic equity return volatility $\sigma_t^{i,E}$, we first use the individual equity returns from equation (C6) to construct the equity market returns, and then regress the individual equity returns of each firm on the time series of the equity market return. Finally, we use the standard deviation of the 20 residuals to calculate the idiosyncratic equity return volatility for each firm.

D.4 Predetermined Parameters

We set the parameters that are widely used in the literature to predetermined values. The parameter values are listed in Panel A of Table 2, and are largely based on the literature (Bhamra et al. (2010b), Bhamra et al. (2010a), Chen et al. (2009) and Chen et al. (2014)). Starting with the macroeconomic variables, we set the risk- free rate $r_G = r_B = 4\%$ in both aggregate states to abstract away from any term structure effects. Following Chen et al. (2014), the transition intensities of the Markov chain are chosen to match the average duration of National Bureau of Economic Research (NBER)-dated expansions and recessions, i.e., $\hat{\lambda}_G = 0.5$ and $\hat{\lambda}_B$ =0.1, which gives average durations of 10 years for expansions and 2 years for recessions over the business cycle. We set the state-switching risk premium $\kappa_G = 1/\kappa_B = 2$, which implies the risk-neutral probability of switching from the good state to the bad is two times as high as the actual probability. The rest of the macroeconomic parameter values are set as standard. We set the state-dependent systematic volatility σ^m, and liquidation cost α_{s_t} to be countercyclical. The effective tax rate τ is set to 0.2 and debt issuance cost ϕ_{s_t} to 0.01 in both states.

For the firm-level parameters, we obtain the parameter values of the expected growth rate and volatility of cash flows from the data for their initial values when the firms start with a low level of idiosyncratic volatility. That is, the growth rate $\hat{\mu}_{s_t,L} = 0.08$ and –0.01 for the good and bad state, respectively. We set $\sigma^{i,X}_{s_t,L}$ to 0.1. The starting level of cash flow $X_0 = 1$. We also set the coupon rate to 0.4 and 0.38 for the good state and the bad state, which is consistent with the empirical finding of interest coverage of 2.5.

E Robustness Tests

The negative association between the idiosyncratic volatility and future equity returns generated from the model might depend on the model specification and predetermined parameter values. We conduct robustness tests to ensure our results hold under different specifications, a different weighting matrix, and different predetermined parameter values. For the parameters, we mainly examine the aggregate state-switching risk premium κ_{s_t} and the switching rate $\hat{\lambda}_{s_t}$, because our results on the sensitivity of optimal policies in Tables 4 and 5 show they are the two variables with the greatest influence over the risk-shifting policies.

E.1 Parameter Estimation

To ensure our model does not rely on a particular model specification, we first perform a different specification by allowing the firm to increase idiosyncratic volatility by ϵ_{s_t}, which differs between the good and bad states. We denote their simple average $m = (\epsilon_B + \epsilon_G)/2$, and their spread $s = (\epsilon_B - \epsilon_G)/2$. If the spread $s = 0$, then the specification boils down to our original specification. As shown in Panel A of Table A1, the estimates of the three costs are close to those in Panel B of Table 2. The simple average $m = (\epsilon_B + \epsilon_G)/2$ is 0.162, slightly above the estimates of ϵ_B and ϵ_G in the original specification. The spread, $s = (\epsilon_B - \epsilon_G)/2$, is 0.049 with a t-statistic of 1.739, which is not statistically significant. Because the original specification used to produce the main results is a special case of this specification, i.e., $s = 0$, we also perform the D-test to compare the two nested specifications. The D-test of the two nested models gives a p-value of 0.352, suggesting the difference between the two models is small. Therefore, we choose the specification $\epsilon_B = \epsilon_G$ as our main specification, to keep the model parsimonious.

We also estimate the model using the inverse of the variance matrix of data moment as the weighting matrix. The estimates of all the cost parameters are largely similar to those in our original specification, with the largest deviation in the increment in volatility. That is, the estimate of ϵ_{s_t} becomes 0.139, slightly lower than the 0.155 seen in Table 2 for which an identity weighting matrix is used.

Lastly, we experiment with different macroeconomic state-switching risk premiums κ_G and the state-switching probability $\hat{\lambda}_G$. In Panel C, where we increase κ_G from 2 to 2.1, the estimate of the value-destroying cost η increases from 0.031 to 0.041. The model cannot be rejected because the p-value is 0.594. In Panel D, where we increase the actual probability $\hat{\lambda}_G$ from 0.1 to 0.11, the increment ϵ_{s_t} further decreases to 0.135 and the p-value drops substantially to 0.025.

E.2 Idiosyncratic Volatility Discount on Equity Returns

The changes in the model specifications and parameters impact cross-sectional equity returns. Using the estimated parameters from Table A1, we simulate the model and report the model-generated excess equity returns r_t^e for the whole sample in Table A2.[21]

In Panel A, where we allows different increases in the idiosyncratic risk in the different aggregate states (i.e., $s = (\epsilon_B - \epsilon_G)/2 = 0.049$), the LMH portfolio sorted on the idiosyncratic cash flow volatility $\sigma_t^{i,X}$ earns 15.007% per year, much greater than the 9.050% seen in Panel B of Table 7, while the LMH portfolio sorted on the idiosyncratic equity return volatility $\sigma_t^{i,E}$ earns 9.284% per year, slighter greater than the 9.161% seen in Panel B of Table 8. These two increases are not surprising because the spread allows a greater cross sectional variation in idiosyncratic volatility. Moreover, the changes in the equity returns of LMH portfolio in Panel B, where we use a different weighting matrix, are very close to those in Panel A.

In Panel C, where we increase κ_G to 2.1, the average excess return increases to 17.475% and 10.140%, respectively, for the LMH portfolio sorted on the idiosyncratic cash flow volatility and the equity return volatility. This is consistent with our previous result on the sensitivity of the risk-shifting threshold to κ_G in Panel A of Table 5. When the risk-adjusted probability of entering a bad state from the good state (i.e., $\lambda_G = \kappa_G \hat{\lambda}_G$) increases, they increases the risk-shifting threshold X_r. The greater the increase in the idiosyncratic risk, the lower the stock returns for high volatility firms, and the greater the LMH portfolio return.

Lastly, Panel D shows that the returns of the LMH portfolio sorted on $\sigma_t^{i,X}$ and $\sigma_t^{i,E}$ are even greater than those in Panel C. The reasoning is as follows. Everything else being equal, the high

[21]The results for the recession and expansion subsamples are similar to those in Tables 7 and 8. They are available upon request.

actual switching probability means a high risk-neutral switching probability, which in turn induces a high risk-taking incentive as well. Consequently, the increased idiosyncratic volatility lowers the equity holders' exposure to cash flow risk and results in a low equity return.

In short, we demonstrate that our quantitative results regarding the risk-shifting mechanism for equity returns are robust to different specifications, weighting matrix, and parameter values.

Table A1: **Robustness Tests for Model Estimation**

This table presents the parameter values estimated from the simulated method of moments under a different specification, different weighting matrix, and different pre-determined parameter values. The t-statistics of the estimated parameters are reported in parentheses, and the p-value of the χ^2 statistic is also reported.

Panel E. τ =0.175

	η	ξ^+	ξ^-	$\epsilon_B = \epsilon_G$	p-value
Estimates	0.029	0.016	0.023	0.155	0.943
(t)	(40.913)	(4.302)	(2.009)	(103.358)	

Panel F. α_B =0.1

	η	ξ^+	ξ^-	$\epsilon_B = \epsilon_G$	p-value
Estimates	0.031	0.016	0.023	0.145	0.874
(t)	(44.493)	(0.032)	(0.750)	(6.733)	

Panel G. r_{s_t} =0.045

	η	ξ^+	ξ^-	$\epsilon_B = \epsilon_G$	p-value
Estimates	0.048	0.017	0.017	0.143	0.602
(t)	(7.893)	(0.019)	(0.543)	(2.317)	

Panel H. ϕ_B =0.015

	η	ξ^+	ξ^-	$\epsilon_B = \epsilon_G$	p-value
Estimates	0.031	0.016	0.022	0.146	0.909
(t)	(62.954)	(0.005)	(2.351)	(215.256)	

Panel I. σ_G^m =0.12

	η	ξ^+	ξ^-	$\epsilon_B = \epsilon_G$	p-value
Estimates	0.044	0.012	0.019	0.180	0.371
(t)	(46.451)	(0.889)	(0.752)	(7.152)	

Panel J. θ_G =0.2

	η	ξ^+	ξ^-	$\epsilon_B = \epsilon_G$	p-value
Estimates	0.040	0.016	0.023	0.125	0.056
(t)	(59.542)	(0.499)	(2.411)	(5.861)	

Table A2: **Robustness Tests for Cross-Sectional Equity Returns**

This table presents value-weighted excess equity returns, given the parameter values estimated from the simulated method of moments in Table A1 under a different specification, different weighting matrix, and different pre-determined parameter values. The portfolios are sorted on the idiosyncratic volatility of cash flow growth $\sigma_{t-1}^{i,X}$ and equity returns, $\sigma_{t-1}^{i,E}$, respectively. The procedures to form portfolio and calculate equity returns are the same as in that in Tables 7 and 8. The t-statistics are reported in parentheses.

Panel E. τ =0.175											
r_t^e sorted on $\sigma_t^{i,X}$	5.667	5.360	5.274	5.075	5.146	4.822	4.430	3.123	−1.333	−9.820	15.488
(t)	(2.690)	(2.537)	(2.524)	(2.422)	(2.433)	(2.313)	(2.179)	(1.689)	(0.313)	(−2.205)	(3.957)
r_t^e sorted on $\sigma_t^{i,E}$	6.715	6.559	6.633	6.787	6.478	6.546	6.292	5.523	2.776	−2.643	9.358
(t)	(2.546)	(2.482)	(2.522)	(2.570)	(2.457)	(2.482)	(2.376)	(2.100)	(1.165)	(−0.439)	(2.855)
Panel F. α_B =0.1											
r_t^e sorted on $\sigma_t^{i,X}$	5.616	5.160	5.214	5.143	5.021	4.867	4.262	2.142	−2.161	−10.555	16.171
(t)	(2.690)	(2.466)	(2.489)	(2.440)	(2.378)	(2.361)	(2.079)	(1.321)	(−0.027)	(−2.826)	(4.406)
r_t^e sorted on $\sigma_t^{i,E}$	6.714	6.691	6.760	6.701	6.652	6.559	6.040	5.186	2.634	−2.405	9.119
(t)	(2.552)	(2.524)	(2.551)	(2.552)	(2.517)	(2.488)	(2.265)	(2.001)	(1.110)	(−0.485)	(2.879)
Panel G. r_{s_t} =0.045											
r_t^e sorted on $\sigma_t^{i,X}$	5.535	5.123	5.049	5.010	4.963	5.015	4.196	2.242	−1.944	−11.470	17.006
(t)	(2.608)	(2.429)	(2.404)	(2.353)	(2.321)	(2.385)	(1.960)	(1.283)	(0.068)	(−2.793)	(4.416)
r_t^e sorted on $\sigma_t^{i,E}$	6.797	6.584	6.771	6.598	6.639	6.500	6.279	4.740	2.504	−2.932	9.729
(t)	(2.516)	(2.456)	(2.519)	(2.458)	(2.520)	(2.396)	(2.341)	(1.769)	(1.043)	(−0.566)	(2.868)
Panel H. ϕ_B =0.015											
r_t^e sorted on $\sigma_t^{i,X}$	5.640	5.249	5.238	5.061	5.161	4.973	4.457	2.606	−1.785	−10.678	16.318
(t)	(2.674)	(2.506)	(2.489)	(2.421)	(2.435)	(2.370)	(2.181)	(1.505)	(0.095)	(−2.652)	(4.295)
r_t^e sorted on $\sigma_t^{i,E}$	6.840	6.684	6.723	6.795	6.477	6.646	6.165	5.115	3.127	−2.554	9.394
(t)	(2.591)	(2.522)	(2.535)	(2.563)	(2.445)	(2.535)	(2.349)	(1.970)	(1.235)	(−0.493)	(3.021)
Panel I. σ_G^m =0.12											
r_t^e sorted on $\sigma_t^{i,X}$	6.033	5.866	5.708	5.561	5.736	5.649	4.993	3.581	−0.365	−9.884	15.917
(t)	(2.556)	(2.459)	(2.424)	(2.370)	(2.400)	(2.396)	(2.144)	(1.660)	(0.526)	(−2.166)	(4.061)
r_t^e sorted on $\sigma_t^{i,E}$	7.155	7.247	7.264	7.363	7.568	6.983	7.009	6.675	4.068	−0.790	7.945
(t)	(2.529)	(2.538)	(2.572)	(2.569)	(2.638)	(2.475)	(2.444)	(2.347)	(1.462)	(0.041)	(2.406)
Panel J. θ_G =0.2											
r_t^e sorted on $\sigma_t^{i,X}$	5.641	5.237	5.395	4.973	5.049	4.459	3.120	0.021	−3.338	−12.632	18.273
(t)	(2.729)	(2.502)	(2.574)	(2.418)	(2.340)	(2.142)	(1.682)	(0.701)	(−0.570)	(−3.248)	(4.796)
r_t^e sorted on $\sigma_t^{i,E}$	7.259	7.379	7.386	7.184	7.167	7.140	6.150	4.958	2.683	−4.288	11.546
(t)	(2.777)	(2.843)	(2.852)	(2.778)	(2.772)	(2.754)	(2.333)	(1.855)	(1.036)	(−0.934)	(3.188)