A Dynamic Theory of Mutual Fund Runs and Liquidity Management*

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Abstract

I develop a model of an open-end mutual fund that invests in illiquid assets and show that shareholder runs can occur even with a fully flexible fund NAV. The key is the fund's dynamic management of its cash buffer. Holding more cash at time t helps the fund avoid fire sales of its illiquid assets if it experiences a significant net outflow. However, the need to rebuild the cash buffer at time $t+1$ after outflows at t implies predictable sales of illiquid assets and hence a predictable decline in NAV. This generates a first-mover advantage at t, leading to shareholder runs. This mechanism differs from that underlying bank runs, which relies on fixed-NAV claims. I then study the fund's optimal dynamic cash policy in the presence of run concerns, which gives rise to the following tension. Rebuilding the cash buffer more rapidly at $t+1$ can trigger runs at t. However, lack of cash re-building makes the fund more likely to suffer another round of fire sales in the future. This tension is aggravated by a time-inconsistency problem: the fund may want to pre-commit to a less rapid cash re-building policy to avoid runs but cannot credibly convince the shareholders absent a commitment device. Therefore, despite optimal liquidity management, mutual funds are not run-free and runs can lead to higher ex-ante fire sale losses. Appropriate design of policies aiming at reducing financial stability risks of mutual funds requires taking into account the dynamic interdependence of runs and liquidity management.

KEYWORDS: open-end mutual fund, shareholder runs, flexible NAV, liquidity management.

JEL: G01, G21, G23, G32, G33, D92

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1 Introduction

There are rising concerns about the financial stability risks posed by open-end mutual funds, which promise daily liquidity to shareholders but have been increasingly holding illiquid assets such as corporate bonds.[1] Many regulators are worried about the potential for a bank-run-like scenario on mutual funds investing in illiquid assets,[2] and a number of funds, the Focused Credit Fund of Third Avenue as the most notable example, shut down redemptions in the middle of severe shareholder runs at the end of 2015.[3] However, despite the prominence of this issue, the theoretical mechanism of mutual fund runs is not well understood and the existence of runs is still in dispute. First, conventional wisdom suggests that mutual funds with a flexible end-of-day net asset value (NAV) should be immune to bank-run-like crises, which occur only with fixed-NAV claims. Second, observers also argue that careful fund liquidity management can mitigate first-mover advantages and hence prevent runs. With these two points in mind, can there really be runs on mutual funds?

In this paper, I develop a model of an open-end mutual fund that invests in illiquid assets and show that shareholder runs can occur in equilibrium even with a fully flexible NAV. My main insight is that the combination of a flexible NAV and active fund liquidity management, both of which are viewed as means to mitigate financial stability risks, can make the fund prone to shareholder runs even without any fundamental shocks to the underlying assets.

The mechanism works as follows. Holding more cash at time t helps the fund avoid fire sales of its illiquid assets if it experiences a significant net outflow. However, the need to rebuild the cash buffer at time $t+1$ after outflows at t implies predictable sales of illiquid assets and hence a predictable decline in NAV at $t+1$. This generates a first-mover advantage at t, which leads to shareholder runs in equilibrium. This logic illustrates the key trade-off in the model: cash buffers mitigate fire sales today, but the need to rebuild cash buffers tomorrow triggers runs today.

The potential for shareholder runs further gives rise to a tension regarding the fund's optimal dynamic cash management. On the one hand, rebuilding the cash buffer more rapidly at $t+1$ by aggressively selling illiquid assets can trigger runs at t for the reasons outlined above. On the other hand, rebuilding the cash buffer less rapidly makes the fund more likely to suffer another round of fire sales in the future. This tension is aggravated by a time-inconsistency problem: the fund may want to pre-commit to a less rapid cash rebuilding policy to avoid runs but cannot credibly convince the shareholders absent a commitment device. Thus, despite optimal liquidity management, mutual funds are not run-free and runs can lead to considerably higher ex-ante fire sale losses.

[1] These patterns of liquidity mismatch are significant in corporate bond mutual funds and are also pervasive in funds investing in other illiquid assets. See Appendix A.1 for facts and institutional details.

[2] The U.S. SEC voted 5-0 on Sept 22, 2015 to approve a new proposal requiring mutual funds to manage liquidity risks. Also see "Potential Emerging Threats and Vulnerabilities," Ch. 7 in the *Annual Report*, the U.S. FSOC, May 2015, "Asset Management Industry and Financial Stability," Ch. 3 in the *Global Financial Stability Report*, the IMF, April 2015, and "Old and New Risks in the Financial Landscape", Ch. 6 in the *Annual Report*, the BIS, June 2015.

[3] The Third Avenue shutdown on Dec 10, 2015 was the first case since the 1940 Act that a U.S. mutual fund shut down redemptions without getting approval from the U.S. SEC. In particular, the Focus Credit Fund was the single largest holder of many high-yield corporate bonds, the fundamental of which were still good. This suggests that the liquidity mismatch and the resulting strategic considerations among shareholders must have played an important role in the run-up to its crisis. A number of the so-called "liquid-alternative" mutual funds, operated by hedge fund managers such as Whitebox Advisors, J.P. Morgan, and Guggenheim Partners also experienced shareholder runs and were forced to close in 2015.

My theoretical predictions are consistent with new micro-level evidence. Chen, Goldstein and Jiang (2010), Feroli, Kashyap, Schoenholtz and Shin (2014), Goldstein, Jiang and Ng (2015), Shek, Shim and Shin (2015) and Wang (2015) document that current fund outflows predict a future decline in fund NAV, and the magnitude of the predictable decline in NAV is larger if the fund invests in more illiquid assets or has less cash.[4] My model provides a concrete mechanism to explain these documented patterns of run incentives and shows that they can indeed lead to runs in equilibrium. Moreover, I show that the potential for runs can in turn distort fund liquidity management, generating new testable predictions.

I formulate the ideas sketched above in a stochastic dynamic model of an open-end mutual fund with many shareholders, who may redeem their shares daily at the end-of-day flexible NAV. Section 2 lays out the model, which is built based on four realistic assumptions. First, the fund invests in both cash and many illiquid assets. Second, selling illiquid assets generates fire sale losses (Williamson, 1988, Shleifer and Vishny, 1992, 1997). Third, the fund minimizes total expected fire sale losses by managing its cash buffer over time. Finally, the fire sale prices are time-varying. Specifically, outflow-induced fire sales can create temporary price overshooting at t and subsequent reversal at $t+1$, as documented by Coval and Stafford (2007) and Duffie (2010).[5] This price pattern gives rise to a motive for fund liquidity management. After a significant outflow at t, the fund may voluntarily sell some assets to rebuild its cash buffer at $t+1$ when the selling price partially rebounds, in order to avoid potentially more severe fire sales in the future (i.e., at $t+2$) should another outflow shock comes. Other than this time-varying fire sale cost, my model does not feature other shocks or frictions at the asset market level.

I first show in Section 3 that the fund's desire to rebuild its cash buffer can induce shareholder runs, and more rapid cash rebuilding leads to more severe runs. This result is general: runs can occur in equilibrium regardless of whether the fund starts with a high cash position or a low one. However, the cost of runs, that is, the impact of runs on the risk of fire sales, is different in these two cases. The nature of strategic interactions among shareholders is also different in these two cases. Therefore, it is helpful to discuss them separately to clarify the run mechanism.

On the one hand, when the fund starts with a high cash-to-assets ratio, shareholder runs induced by fund cash rebuilding lead to higher risk of future fire sales. When a redemption shock occurs at t, the fund starting with a high cash position can satisfy the projected redemptions at both t and $t+1$ without incurring fire sales. This implies that, even if the shareholders who initially plan to redeem at $t+1$ ran at t, the fund would still have enough cash at t, and thus time-t NAV would not adjust. But since some cash is paid out at t, the fund may want to rebuild its cash buffer by voluntarily selling some illiquid assets at $t+1$. Thus, the shareholders who initially plan to redeem at $t+1$ would get a lower NAV if they waited until $t+1$, and hence may decide to run at t. Fundamentally, this occurs

[4]These papers differ in focus, but they all suggest a single point: the existence of run incentives among mutual fund shareholders. See the literature review for more detailed discussions of these papers.

[5]This assumption is consistent with many other empirical studies including Campbell, Grossman and Wang (1993), Pastor and Stambaugh (2003), Hendershott and Seasholes (2007), Mitchell, Pedersen and Pulvino (2007), Comerton-Forde, Hendershott, Jones, Moulton and Seasholes (2010), Greenwood and Thesmar (2011), Edmans, Goldstein and Jiang (2012), Jotikasthira, Lundblad, and Ramadorai (2012), Lou (2012), Nagel (2012) and Hendershott and Menkveld (2014). Duffie (2010) ascribes this pervasive pattern to the idea of "slow-moving capital," which is in turn linked to the classic work of Grossman and Miller (1988). In Appendix A.3 I provide a micro-foundation where the price pattern emerges endogenously but in the main text I assume it to highlight the run mechanism in the paper.

because cash rebuilding endogenously gives rise to strategic complementarity among shareholders. In this scenario, shareholder runs force the fund to pay more redeeming shareholders at the endogenously unchanged NAV at t, and thus the fund loses more cash.[6] Therefore, shareholder runs partially offset the fund's cash rebuilding efforts at $t+1$ and thus lead to higher risk of future fire sales at $t+2$.

On the other hand, if the fund starts with a low cash-to-assets ratio such that it cannot satisfy the projected redemptions at both t and $t+1$ without selling illiquid assets, shareholder runs further lead to higher current (i.e., time-t) fire sale losses in addition to higher risk of future (i.e., $t+2$) fire sales. Interestingly, in this scenario, a shareholder is less likely to run if more of other shareholders decide to run. This is because runs may force the fund to fire sell more of its illiquid assets at an extremely low price at t, and any shareholder who runs at t has to share that cost, that is, to get a lower NAV. This means that shareholders' run decisions can exhibit strategic substitutability. However, since the fund is already running out of cash and may voluntarily sell more assets at $t+1$ to rebuild its cash buffer, waiting may only give the shareholders an even lower NAV. Therefore, the fund's desire to rebuild its cash buffer reinforces a strong incentive for shareholders to redeem earlier despite the strategic substitutability. In this scenario, shareholder runs introduce a more severe cost by directly forcing the fund to fire sell more assets at t, a time when the fire sale price is extremely low. In addition, runs still offset the fund's cash rebuilding efforts and thus lead to higher risk of future fire sales.

Having analyzed the implications of cash rebuilding on shareholder runs for an arbitrary starting level of cash, I endogenize the dynamic cash rebuilding policy of the fund. I show in Section 4 that introducing the potential for runs gives rise to a tension absent in existing liquidity management theories. On the one hand, rebuilding the cash buffer more rapidly at $t+1$ can trigger shareholder runs at t. As described above, shareholder runs lead to higher risk of fire sales. This run concern makes a more rapid cash rebuilding policy less appealing. On the other hand, adopting a less rapid cash rebuilding policy at $t+1$ makes the fund more likely to suffer another round of future fire sales at $t+2$. Moreover, carrying less cash to $t+2$ also implies that the fund may ultimately have to rebuild its cash buffer more rapidly at time $t+3$, which can trigger future runs at $t+2$. With this tension, the fund's optimal dynamic cash rebuilding policy is significantly different from the benchmark case where there are no runs.

Moreover, I show that the potential for shareholder runs introduces a time-inconsistency problem for the fund, which aggravates the tension in choosing between a rapid or a slow cash rebuilding policy. When the cost of runs at t is relatively large, ex-ante, the fund may wish to commit itself to rebuilding its cash buffer less rapidly at $t+1$ to reduce run risks at t. However, ex-post, the fund may instead be tempted to adopt a more rapid cash rebuilding policy at $t+1$, because the time-t cost is sunk. Anticipating this, shareholders will always have strong incentives to run at t. In other words, in the absence of a commitment device, the fund cannot make credible announcement to convince shareholders not to run. I further show that, in certain circumstances, introducing a commitment device can help temper the run incentives at t and thus reduce total expected fire sale losses.

Overall, my paper provides theoretical underpinnings for understanding why open-end mutual funds may not be run-free, in contrast to what the conventional wisdom suggests. The potential for shareholder

[6] In other words, if the shareholders who initially plan to redeem at $t+1$ do not run, the fund can pay them a lower NAV when rebuilding its cash buffer and hence effectively carry more cash into $t+2$ under the same cash rebuilding policy.

runs can considerably increase fire sale losses in expectation despite optimal cash management by the fund. The dynamic interdependence of shareholder runs and fund liquidity management uncovered in this paper plays a critical role in shaping these outcomes.

Fundamentally, shareholder runs in my model are driven by a key contractual property of mutual fund NAVs: they are flexible but not forward-looking. In other words, the NAV at time t does not take into account the predictable asset sales and price impact at $t+1$. With this contractual property, fund cash rebuilding gives rise to predictable declines in NAV and thus the potential for runs.

Although they are reminiscent of bank runs in some ways, fund shareholder runs differ from classic bank runs (Diamond and Dybvig, 1983) in terms of the underlying mechanism. In my model, the first-mover advantage does not come from an exogenous fixed-NAV claim at t (like the deposit at a bank). Because NAVs in my model flexibly and endogenously adjust, a shareholder redeeming at t realizes that more early withdrawals will potentially induce more fire sales at t and thus lower the proceeds she receives. Hence, if the fund did not rebuild its cash buffer at $t+1$, the net benefit of running over waiting could be decreasing as more shareholders run. Rather, it is the fund's desire to rebuild its cash buffer at $t+1$ and the resulting predictable decline in NAV that lead to a strong first-mover advantage. This mechanism highlights a dynamic interaction between the fund and its shareholders. Such an interaction is absent in bank run models, which focus on coordination failures among depositors themselves.

The mechanism in my model is also different from that underlying market runs. Bernardo and Welch (2004) and Morris and Shin (2004) argue that if an asset market features an downward-sloping demand, investors fearing future liquidity shocks will have an incentive to front-run, fire selling the asset earlier to get a higher price. One might imagine that introducing an intermediary that helps manage liquidity shocks can alleviate such problems. Indeed, in my model, fund cash management is beneficial to shareholders because it reduces fire sale losses. However, the key tension that I document is that the fund's cash rebuilding also endogenously gives rise to predictable declines in NAV and thus run incentives. In contrast, there is no role for liquidity management in market run models. In this sense, market run models focus on asset markets themselves while my theory focuses on the role of financial intermediaries. This allows me to distinguish between risks that come from active management of financial intermediaries and those that are only a reflection of market-level frictions and would occur in the absence of intermediaries.

My model generates new policy implications, which I explore in Section 5. First, the model suggests that to introduce a flexible NAV is not a fix to money market mutual fund (MMF) runs, as also argued in Hanson, Scharfstein and Sunderam (2015). I also consider many fund-level policies, including liquidity requirements, in-kind redemptions, redemption fees and restrictions, credit lines, and swing pricing, all of which aim at mitigating financial stability risks of mutual funds. Perhaps surprisingly, these policies do not necessarily improve shareholder welfare in equilibrium because they may distort fund liquidity management, and thus lead to more fire sales. Overall, my model suggests that policies should be designed taking into account the dynamic interdependence of runs and fund liquidity management.

Section 6 explores various extensions to the baseline model, including the flow-to-performance relationship, asset price correlations, and persistent price impacts. Section 7 concludes.

Related Literature. This paper first contributes to the burgeoning literature on financial stability risks posed by open-end mutual funds.[7] Empirically, Feroli, Kashyap, Schoenholtz and Shin (2014) find that fund outflows predict future declines in NAV, suggesting the existence of run incentives for shareholders. At a more micro level, Chen, Goldstein and Jiang (2010) find that the flow-to-performance relationship is stronger for funds investing in less liquid stocks. Goldstein, Jiang and Ng (2015) echo the message by showing that corporate bond funds even exhibit a concave flow-to-performance relationship. Shek, Shim and Shin (2015) explores the underlying channel by showing that outflows are associated with future discretionary bond sales and liquidity rebuilding in an emerging market bond fund context. Wang (2015) further finds that outflows predict a stronger decline in future NAVs when the fund has less cash or invests in more illiquid bonds. My model predictions are consistent with all of these facts. Chen, Goldstein and Jiang (2010) and Morris and Shin (2014) have addressed the potential for mutual fund runs from theoretical perspectives, but their focuses and approaches are different from this paper, and they do not consider fully flexible NAV adjustment or fund liquidity management.[8]

There is a broader literature on the costs of outflows to non-trading shareholders and to future fund performance.[9] Edelen (1999), Dickson, Shoven and Sialm (2000), Alexander, Cici and Gibson (2007) and Christoffersen, Keim, and Musto (2007) find that flow-induced trades hurt fund performance, and redeeming shareholders impose externalities on non-trading shareholders through trading-related costs (including commissions, bid-ask spreads, and taxes) that are not reflected in current NAVs. Coval and Stafford (2007), Ellul, Jotikasthira and Lundblad (2011) and Manconi, Massa and Yasuda (2012) further show this by highlighting the channel of flow-induced fire sales. In addition, Chernenko and Sunderam (2015) find that even careful liquidity management of mutual funds cannot alleviate those costs. These papers do not examine the potential for shareholder runs. Especially, since runs are induced by active fund liquidity management in my model, I am able to identify a new externality that even including all the current trading-related costs in NAVs (as suggested by swing pricing) cannot internalize.

My paper also contributes to the literature of mutual fund liquidity management.[10] This literature suggests that holding cash is costly because funds have to give up other higher-yielding investment opportunities (Wermers, 2000), but cash can help them withstand redemption shocks and reduce fire

[7] The notion of open-end mutual funds does not include MMFs, insurance companies, or pension funds. Recent literature has also documented run-like dynamics associated with those financial intermediaries. For example, Schmidt, Timmermann and Wermers (2014) provide a throughout investigation of shareholder runs on MMFs, and Strahan and Tanyeri (2014) find that MMFs experiencing more outflows also reallocate their portfolio more, consistent with my prediction in a mutual fund context. More recently, Foley-Fisher, Narajabad and Verani (2015) and Da, Larrain, Sialm and Tessada (2015) provide new evidence suggesting that runs can even happen to insurance companies and pension funds, respectively.

[8] In the appendix of Chen, Goldstein and Jiang (2010), the authors build a static global game model for the purpose of hypothesis development. In that model, fund NAV is not fully flexible: shareholders are assumed to get a fixed-value claim if they run (in the spirit of Diamond and Dybvig, 1983). Their model does not consider fund liquidity management. Morris and Shin (2014) build a model of runs by funds on the asset markets, focusing on fund managers' relative performance concerns. Their model does not distinguish between open- and closed-end funds and does not consider shareholder runs.

[9] See Christoffersen, Musto and Wermers (2014) for a comprehensive review of this literature.

[10] There is also a large dynamic corporate finance literature on general liquidity management for non-financial institutions. With the premise of costly external liquidity under agency problems, this literature focuses on the role of holding internal liquidity in making investments rather than meeting redemptions. I refer readers to Bates, Kahle and Stulz (2009) for empirical evidence and Bolton, Chen and Wang (2011) for a typical modern theoretical treatment. Hugonnier and Morellec (2014), Sundaresan and Wang (2014) and Della Seta, Morellec and Zucchi (2015), among others, further push this research agenda to the bank context with a focus on the relationship between liquidity management and bank default risks. In this literature, the bank can only accumulate internal liquidity by retaining its proceeds; partial liquidation (like in my model) is not allowed. The bank in question defaults abruptly when it chooses to liquidate assets.

sales (Edelen, 1999, Christoffersen, Keim, and Musto, 2007, Coval and Stafford, 2007). Recently, Simutin (2013), Ben-Rephael (2014) and Huang (2015) investigate the determinants of cash management and the implications on fund performance for equity funds, and Chernenko and Sunderam (2015) provide more comprehensive evidence on fund cash management covering both bond and equity funds. The most relevant theory is Chordia (1996) who shows in a static model that funds hold more cash when there is uncertainty about redemptions, but funds with load and redemption fees hold less cash. My paper document a novel aspect of fund liquidity management: rebuilding cash buffers by selling illiquid assets can induce shareholder runs, which can in turn distort fund liquidity management.

Dated back to Bryant (1980) and Diamond and Dybvig (1983), there is a vast bank run literature.[11] Beyond the discussion in the introduction, I review three branches of theories that are closely related.

First, pioneered by Cooper and Ross (1998), a literature looks at how banks manage their liquidity buffers to mitigate runs.[12] By combining liquidity management and fully flexible NAV adjustment, I uncover a cost of cash rebuilding in the mutual fund context: unintended shareholder runs. In contrast, when a comparable bank rebuilds its cash buffer by selling assets, the underlying deposit value will not be affected, and thus cash rebuilding by itself would not directly trigger depositor runs.

Second, there is a growing literature about runs on non-bank but leveraged financial institutions, for instance, Liu and Mello (2011) on leveraged hedge fund runs, Martin, Skeie and von Thadden (2014) on repo runs, and Parlatore (2015) on MMF runs. These theories resemble classic bank run models in that investors still get a fixed-value claim if they run.

Third, a burgeoning literature models bank runs in dynamic contexts. Some models imbed a Diamond-Dybvig type bank run model into a dynamic growth or economic fluctuation model (Ennis and Keister, 2003, Gertler and Kiyotaki, 2014). Others model investors' dynamic run decisions directly (He and Xiong, 2012, He and Manela, 2014).[13] These dynamic models make bank run theory appealing to more contexts like debt rollovers and rumor-based runs. For tractability reasons, those models do not feature liquidity management. By developing a new framework, I can model the interaction between mutual fund shareholder runs and fund liquidity management in a tractable manner.

Finally, it is also important to distinguish my mechanism from that in the market run literature (Bernardo and Welch, 2004, Morris and Shin, 2004), as aforementioned in the introduction. The idea of market run is also present in the "cash-in-the-market" theory in Allen and Gale (1994, 2005) and the predatory treading theory in Brunnermeier and Pedersen (2005) and Carlin, Lobo and Viswanathan (2007). Based on the search model of Lagos, Rocheteau and Weill (2011), Di Maggio (2015) investigates the implications of market runs in over-the-counter (OTC) markets. My run mechanism differs from theirs by focusing on the role of financial intermediaries rather than market-level frictions themselves.

[11] See Gorton and Winton (2003) for a comprehensive review.

[12] This literature has been growing recently (Vives, 2014, Diamond and Kashyap, 2015) given the emphasis of bank liquidity requirement in Basel III.

[13] This literature has been growing rapidly. See Cheng and Milbradt (2012) and Schroth, Suarez and Taylor (2014) for recent developments built upon He and Xiong (2012). Morris (2014) gives a theoretical treatment about how such dynamic bank run models can be reconciled with static, global-game-based bank run models (Goldstein and Pauzner, 2005) and synchronization games (Abreu and Brunnermeier, 2003).

2 The Model

I first introduce the baseline model of an open-end mutual fund investing in both cash and illiquid assets. The model features the key institutional setting of open-end mutual funds, that is, possible daily shareholder redemptions at a flexible end-of-day NAV. The mechanism of this baseline model is fairly general and can be readily extended to other settings.

2.1 Setup

Time is discrete and infinite. Discount rate is normalized to 1. There is a single open-end mutual fund with many existing shareholders, who may redeem their shares on ex-ante unknown dates. The fund invests in two types of assets: 1) cash, denoted by x, which is liquid and the only consumption good, and 2) a continuum of many illiquid assets, denoted by a. These illiquid assets are assumed to have a fundamental value $R > 1$, but they are all different, which means they can have different market prices in equilibrium. The illiquid assets pay off at the end of the game (specified later) and do not generate any interim cash flows.

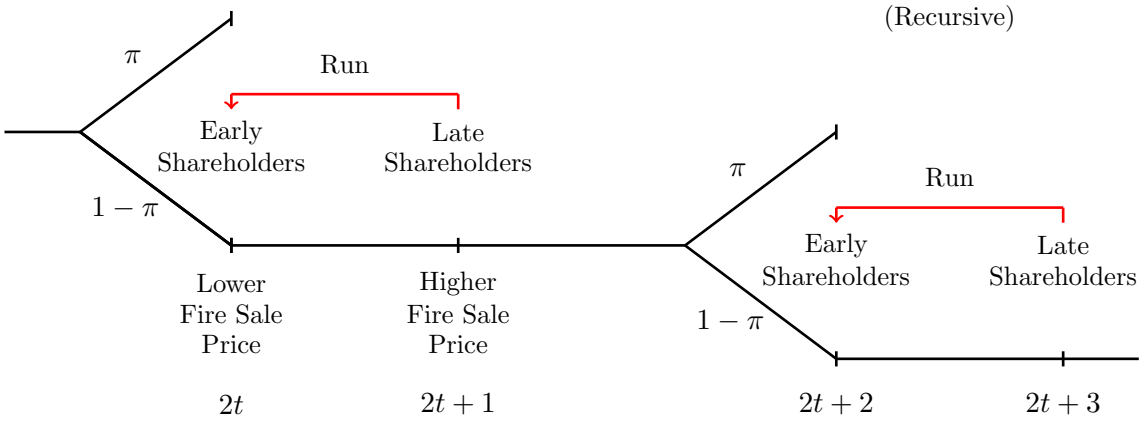


Figure 1: Timeline

Timeline. Each stage consists of two dates, an even date and an odd date.[14] I use $2t$ to denote an even date and $2t+1$ to denote an odd date. I still use t to denote a stage or a general date when the difference between even and odd dates is not important. At the beginning of any date t, the fund has x_t cash and a_t unit of the basket of illiquid assets. The fund also has n_t existing shareholders, some of whom may exit the fund by redeeming their shares in future. Redemption needs must be met in cash, so the fund may be forced to sell its illiquid assets if running short of cash, but doing this will generate fire sale losses because of the underlying illiquidity problem. Specifically, the unit fire sale price for any illiquid assets on date t is p_t, which is lower than R. To focus on redemptions, I assume that the fund has no inflows or credit lines.[15]

[14] To make distinctions between even and odd dates is a common modeling tool in the theoretical literature to capture time-varying or alternating market conditions. See Woodford (1990) and Lagos and Wright (2005) for examples.

[15] I will relax this assumption in Section 5.6. As shown then, having credit lines cannot reduce potential financial stability risks of mutual funds but may instead aggravate them in some cases.

At the beginning of each stage (i.e., right before an even date), a shock hits the economy. Specifically, with probability π the game ends; otherwise the game continues. Only if the game continues, will there be projected redemptions on the following two dates within the given stage. The random end-of-game event can be thought of as an upside event in which all the illiquid assets mature at their fundamental value, shareholders get paid off, and there will be no future redemption needs. Therefore, the shock structure parsimoniously captures the randomness of redemptions in reality, with a lower π implying that redemptions are more persistent. For these reasons, I call the shock redemption shock in what follows. Figure 1 shows the timeline with some elements to be explained shortly. In Section 2.2, I will give an intuitive interpretation of this setup and map it to real-world scenarios.

Flexible Fund NAV. The first important pillar of this model is a flexible fund NAV. The end-of-day flexible NAV will reflect all the asset sale losses during the given day. Specifically, if the fund does not sell any illiquid assets on date t, the end-of-day NAV will be

$$NAV_t = \frac{a_t R + x_t}{n_t}.$$

However, if the fund sells some assets on date t at the fire sale price p_t, the NAV will reflect that loss and hence become lower:

$$NAV_t = \frac{x_t + (a_t - a_{t+1})p_t + a_{t+1}R}{n_t}. \tag{2.1}$$

I explain the three terms in the numerator of (2.1) in order. The first term x_t is the amount of cash that the fund has initially at the beginning of date t. The second term represents the amount of cash the fund raises by selling $a_t - a_{t+1}$ unit of assets at the fire sale price p_t. The third term represents the value of the non-traded illiquid assets remained on the fund's balance sheet. In (2.1), the market prices of those different and non-traded assets will not change. This is true for illiquid assets that are different in nature, especially for those traded in OTC markets like corporate bonds. This is also consistent with the empirical evidence in Coval and Stafford (2007) that flow-induced fire sales only have temporary and local price impacts within the assets being sold. In practice, asset prices may be correlated, and mutual funds may also use matrix pricing for these non-traded assets based on the fire sale price p_t of the assets that are sold. But as I will show in Section 6.2 as an extension, asset price correlations or different accounting rules such as matrix pricing are not crucial for my model mechanism and will not change my results qualitatively. What is crucial in (2.1) is that the end-of-day NAV is flexible in the sense that it takes into account all the same-day price impact and asset sale losses, while that NAV is not forward-looking in the sense that it will not reflect any possible future price impacts and asset reallocation costs. These contractual features of fund NAV are robust regardless of the nature of different asset markets and accounting rules.

Fund Manager. The second important pillar of this model is active fund liquidity management. The fund has one fund manager. On any date t, the fund manager's objective is to minimize total expected fire sales from date t to the end of the game. The formal objective function will become clear shortly after I describe the asset market.

Since all the shareholders are ex-ante identical, having a fund manager who minimizes total expected fire sale losses implies that there is no agency friction between the fund manager and the shareholders as a whole. In this sense, the fund manager's objective parsimoniously captures the outcome of optimal contract design between investors and the asset manager (see Bhattacharya and Pfleiderer, 1985 for a classic treatment). Minimizing total fire sale losses also suggests that the fund manager's compensation is tied to the size or equivalently the assets under management (AUM) of the fund, which is common in practice.

Shareholders. Fund shareholders may redeem their shares when they have consumption needs. I define three groups of shareholders within each stage: early shareholders, late shareholders, and sleepy shareholders. Specifically, if the game continues on an even date $2t$ (with probability $1-\pi$), $\mu_E n_{2t}$ early shareholders and $\mu_L n_{2t}$ late shareholders are hit by unanticipated consumption shocks and thus must consume, where $0 < \mu_E, \mu_L < 1$ and $0 < \mu_E + \mu_L < 1$. Since consumption shocks are unanticipated, the remaining $(1 - \mu_E - \mu_L)n_{2t}$ sleepy shareholders do nothing but wait until the next stage; they do not plan ahead for future stages although they may randomly become early or late shareholders in the future.[16] Both early and late shareholders do not have any cash in advance, so they have to redeem their shares and get the endogenous and flexible end-of-day NAV.

Early shareholders must consume on date $2t$, so they always redeem their shares at the endogenous end-of-day NAV on $2t$. Late shareholders prefer to consume on date $2t+1$, but can also choose to consume on date $2t$. Formally, late shareholders' utility function is:

$$u_L(c_{2t}, c_{2t+1}) = \theta c_{2t} + c_{2t+1}\,, \tag{2.2}$$

where $0 \leqslant \theta \leqslant 1$. As late shareholders are risk neutral,[17] their consumption choice boils down to a binary problem: to redeem on date $2t$ or date $2t+1$. There is no outside storage technology, so if a late shareholder redeems on date $2t$, she gets the endogenous end-of-day NAV on $2t$ and must consume immediately; otherwise she gets the endogenous end-of-day NAV on $2t+1$ and consume then. If a late shareholder chooses to redeem and consume on date $2t$, I say that the late shareholder runs the fund. I allow late shareholders to choose mixed strategies: the run probability of late shareholder i is denoted by $\lambda_{2t}^i \in [0,1]$. As one can expect, a late shareholder's run decision will depend on the difference of NAV between the two dates, which will in turn depend on other late shareholders' run decisions and the fund manager's asset allocation decision in the given stage.

The preference parameter θ in (2.2) parsimoniously captures different types of shareholders with

[16] This is consistent with the observation that many mutual fund shareholders are mom-and-pop investors: they do not actively review their portfolios but only do so when subject to unanticipated liquidity shocks (for empirical evidence, see Agnew, Balduzzi and Sunden, 2003, Ameriks and Zeldes, 2004, Mitchell, Mottola, Utkus and Yamaguchi, 2006, Brunnermeier and Nagel, 2008, and Grinblatt and Keloharju, 2009). Institutional investors like insurance companies and pension funds also review and update their mutual fund asset portfolio infrequently. From a theoretical point of view, this helps construct a standard stochastic dynamic game with a long-run player (the fund manager) and many generations of short-run players (the shareholders). It also allows me to highlight the conflict of interests between different generations of shareholders from a dynamic perspective.

[17] In Diamond-Dybvig type bank run models, depositors are usually assumed to be risk averse, and demandable deposit emerges as the optimal contract for risk-sharing between early and late depositors. Instead, I focus on the commonly observed contractual features of open-end mutual funds rather than optimal contract design, so I assume risk-neutrality to help better document the impact of flexible NAVs on shareholders' consumption choices.

different propensities to run.[18] Intuitively, when θ is lower, late shareholders prefer late consumption more and they are less likely to run even if the NAV is lower on date $2t+1$. This setting implies that late shareholders' realized marginal utilities can be different on the two dates, a setting also commonly seen in the bank run literature (for example, Peck and Shell, 2003). My model mechanism works for any $\theta \in [0,1]$.

The presence of different types of shareholders, that is, the early, the late, and the sleepy ones, captures an important friction in my model: the conflict of interests among shareholders with different investment horizons. This conflict of interests is natural and well-documented in the empirical literature: short-term investors can impose negative impacts on a fund's long-term performance through the transaction and fire-sale costs they incur to the fund, and the fund has to manage its portfolio and liquidity carefully to mitigate such costs (for example, Edelen, 1999, Dickson, Shoven and Sialm, 2000, Alexander, Cici and Gibson, 2007, Christoffersen, Keim, and Musto, 2007, Coval and Stafford, 2007). This realistic friction plays an important role in driving the interaction between shareholder runs and fund liquidity management in my model.

Asset Market and Fire Sales. On any date, the fund manager can sell the illiquid assets to an outside investor at fire sale prices. Flow-induced fire sales are natural and pervasive (Williamson, 1988, Shleifer and Vishny, 1992, 1997), and they can create temporary price impacts (Coval and Stafford, 2007). Based on these evidence, I assume that the selling price of any unit of illiquid assets is $p_E = \delta_E R$ on date $2t$ and $p_L = \delta_L R$ on date $2t+1$, where $0 < \delta_E, \delta_L < 1$. Moreover, I assume that selling right after the shock (i.e., on date $2t$) incurs a higher price discount (and thus a lower selling price). Figure 2 illustrates a sample selling price path before the end of the game (assumed to be date 4 in this example).[19,20] Specifically, the following parameter assumption holds throughout the paper:

ASSUMPTION **1.** *The selling prices satisfy* $\delta_L > \delta_E + (1-\delta_E)(\mu_E + \mu_L)$.[21]

This selling price pattern can be micro-founded by the idea of slow-moving capital in illiquid asset markets (Duffie, 2010). When a redemption shock just hits the economy on date $2t$, because there are only a few liquidity providers available, it is very hard for the fund manager to find a good selling price. But if she can wait until the next date $2t+1$, since more liquidity providers step in, it would be easier to find a higher selling price. If the game continues on date $2t+2$, that is, when another round of redemption shock hits the economy, the selling price drops again. These transitory price impacts induced by outflows and the associated price over-shooting and reversal have been empirically

[18] There are many plausible explanations for different types of shareholders to have different values of θ. For example, Chen, Goldstein and Jiang (2010) suggest that institutional investors may have a lower θ because they often have stricter investment targets and are more likely to internalize the market impact posed by own trading activities. Alternatively, Gennaioli, Shleifer and Vishny (2015) argue that mutual funds provide trust to their shareholders. For those shareholders who value such trust, if they choose to leave the fund early, they have to give up the trust premium so can also be viewed as having a lower θ.

[19] Although my model features discrete time, the price path is intentionally depicted as a *cádlág* function that is everywhere right-continuous and has left limits everywhere to better reflect the perfectly elastic intra-day asset demand and daily price update.

[20] I also consider more persistent price impacts in Section 6.3 as an extension and show that they will only strengthen my results.

[21] Besides the fact that the fire sale cost on even dates is larger, the specific form in Assumption 1 is not crucial to my results.

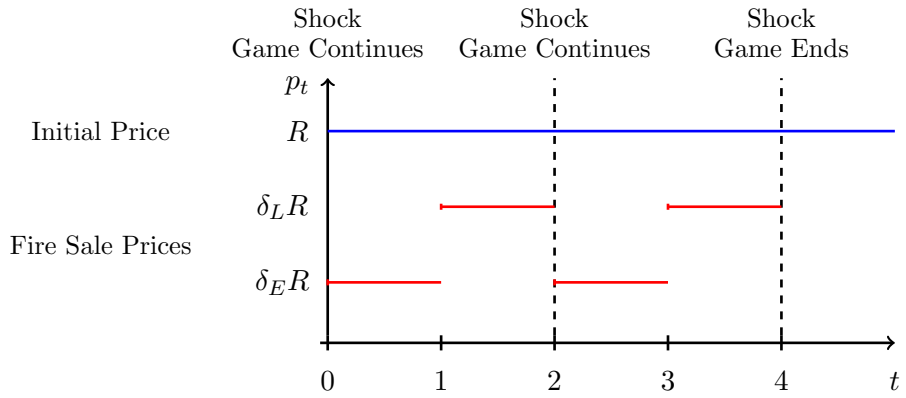


Figure 2: Sample Path of the Selling Prices

documented in various asset markets (for example, Campbell, Grossman and Wang, 1993, Pastor and Stambaugh, 2003, Coval and Stafford, 2007, Hendershott and Seasholes, 2007, Mitchell, Pedersen and Pulvino, 2007, Comerton-Forde, Hendershott, Jones, Moulton and Seasholes, 2010, Greenwood and Thesmar, 2011, Edmans, Goldstein and Jiang, 2012, Jotikasthira, Lundblad, and Ramadorai, 2012, Lou, 2012, Nagel, 2012, Hendershott and Menkveld, 2014, among others). They are also consistent with the timeline and in particular with the nature of shocks in my economy. This price pattern will be formally micro-founded in Appendix A.3, based on the idea of slow-moving capital outlined above. To focus on redemption shocks and their price impacts, my model is intentionally abstracted away from any fundamental shocks of the underlying assets.

Fund Liquidity Management. The open-end mutual fund has to meet redemption needs in cash on a daily basis.[22] Hence, the fund manager needs to manage its liquidity carefully to keep an adequate cash position. Specifically, she manages the cash position of the fund both passively and actively.

On the one hand, on any date t, if the fund does not have enough cash to meet date-t projected redemptions at the beginning-of-day NAV (i.e., NAV_{t-1}), the fund will be forced to raise cash until all redemption needs can be met. Since there are no inflows and the illiquid assets do not pay interim cash flows, the fund manager can only raise cash by selling illiquid assets passively[23] at the fire sale price p_t. However, redeeming shareholders will only get the end-of-day NAV, which will reflect asset sale losses within the given date, so that the fund manager only has to sell assets to a point at which the redemptions can be met at the end-of-day NAV. Denote the amount of illiquid assets that the fund has to sell passively by q_t, which will be endogenously determined in equilibrium.

On the other hand, in addition to selling assets passively for meeting redemptions, the fund can also manage its cash buffer actively. Specifically, the fund manager is able to voluntarily sell illiquid assets more than actual redemption needs to rebuild the cash buffer, also at the fire sale price p_t. Denote the amount of assets that the fund voluntarily sells on date t by s_t. As will become clear later, it is the time-varying fire sale price that gives rise to the motive for active cash rebuilding: the fund manager

[22] In Section 5 I discuss how this model can be extended to analyze emergency rules such as redemption restrictions and in-kind redemptions.

[23] In Section 5.6, an extension to the baseline model, I also allow the fund to have credit lines to raise cash.

may optimally choose to rebuild the fund's cash buffer on odd dates when the fire sale price is high (and thus the fire sale loss is low), in order to avoid more severe fire sales in future. In what follows, I will call s_t the fund's cash rebuilding policy on date t. Intuitively, a larger s_t means that the fund is rebuilding its cash buffer more rapidly.

Now it is natural to specify the fund manager's objective function formally. Denote by T the random date on which the game ends.[24] The fund manager chooses a sequence $\{s_\tau\}$ date by date to maximize:

$$-\mathbb{E}_t \sum_{\tau=t}^{T-1} (q_\tau + s_\tau)(R - p_\tau)\,, \tag{2.3}$$

where the expectation is taken over the random variable T. In particular, selling illiquid assets (either passively or actively) at fire sale prices will always hurt the fund NAV that redeeming shareholders are able to get. This implies that late shareholders' redemption decisions will be directly affected by the fund's cash rebuilding policies. Specifically, the late shareholders in stage t rely on the difference of NAV between date $2t$ and date $2t+1$ to make redemption decisions, so they rationally form beliefs about the fund's cash rebuilding policies $\{s_{2t}, s_{2t+1}\}$ within that stage.[25] As one can expect, the fund's cash rebuilding policies will affect shareholders' run decisions, which will in turn affect the fund's optimal cash rebuilding policies in the dynamics. This interaction is only at play when fund NAV is flexible, the key institutional setting I highlight throughout this paper.

It is important to note that, although feasible, the fund manager will never rebuild the fund's cash buffer on even dates in any generic equilibrium, that is, $s_{2t} = 0$ for any t.[26] This is intuitive because all the purpose for the fund to manage its cash buffer is to avoid extremely costly fire sales on even dates, and hence it never makes sense for the fund manager to voluntarily sell assets on even dates. As a result, the fund's cash rebuilding policy in stage t is solely determined by s_{2t+1}, the amount of illiquid assets the fund manager voluntarily sells on the odd date $2t+1$. In what follows, I consider s_{2t+1} the only choice variable of the fund manager in any stage t.

Finally, it is worth noting that my model is admittedly not intended to be a general theory of mutual fund management. In general, a fund manager may engage in other management activities (see Wermers, 2000 for a comprehensive evaluation of fund management activities such as asset-picking, style-investing, and fee-setting). Also, in addition to meeting redemption needs, a fund keeps cash for other purposes, such as making timely investments in illiquid and high-yield assets without waiting for inflows (Chordia, 1996). To focus on the interaction between fund liquidity management and shareholder runs, I assume that the purchasing price of the illiquid assets (from outside dealers) is always the fundamental value R. This is consistent with the existence of large bid-ask spreads in illiquid asset markets, and it implies that the fund will never repurchase the illiquid assets in my economy with net outflows only.

[24] By construction, T must be an even number.

[25] In the U.S., mutual funds are required to disclose their asset positions quarterly, under the scrutiny of the SEC according to 17 CFR Parts 230, 232, 239, and 274. See "Enhanced Disclosure and New Prospectus Delivery Option for Registered Open-End Management Investment Companies, Investment Company Act Release No. 28584," The SEC, Jan 13, 2009. From a theoretical point of view these requirements allow shareholders to form consistent beliefs about a fund's cash rebuilding policies.

[26] This statement will be formally proved in Appendix A.4.

2.2 Interpretation of the Setup

The settings described above represent a mutual fund crisis management scenario, during which the fund experiences persistent redemption shocks before a random recovery time. In the setup, π measures how persistent the redemptions shocks are, or in other words how likely the economy is going to recover from a bad market condition. When π is lower, the game is more likely to continue, and thus the fund is more likely to experience redemptions and potential fire sales in the next stage. As the fund manager never knows when the game will end, liquidity management indeed helps the fund minimize its total expected fire sale losses, and it matters more when π is lower. Also, I use the end-of-game event to parsimoniously capture what can possibly happen in a normal-time scenario such as inflows to the fund and dividend payouts to shareholders. In a normal-time scenario, asset prices are also more likely to be high and correctly reflecting their fundamentals, as I postulate in the setting. In this sense, my model is not intended as a general boom-bust cycle model of mutual fund management but one focusing on crisis management and the economic forces involved. From a theoretical standpoint, many dynamic crisis management theories in other contexts employ similar structures of persistent shocks followed by a random recovery time, for example, Lagos, Rocheteau and Weill (2011) on crises in OTC asset markets, He and Xiong (2012) on corporate debt runs, and He and Milbradt (2015) on maturity choices in a debt rollover crisis, among others.[27]

More importantly, the crisis management scenarios of open-end mutual funds are pervasive in reality. They can occur at both long and short time horizons, and for both fundamental and non-fundamental reasons. Since the May of 2013, the flagship Total Return fund of the Pacific Investment Management Company (PIMCO), one of the largest fund in the U.S., has experienced net outflows for more than 28 consecutive months.[28] The Prudential M&G's flagship Optimal Income fund, one of the largest bond fund in the Europe, has experienced more than 50 consecutive trading days of net outflows in mid 2015.[29] Another example features Aberdeen Asset Management, the largest listed fund manager in Europe, has experienced net outflows for 15 consecutive months as of the end of 2015.[30] This is also true at the aggregate level: between Aug 20, 2015 and Aug 26, 2015, aggregate outflows from the entire equity fund sector happened for five consecutive trading days at the total amount of 29.5 billion U.S. dollars, the largest weekly outflow on record since fund flow data began being calculated in 2002.[31] Not surprisingly, these scenarios happen to smaller funds as well. My model is designed to capture such scenarios and to explore the potential risks of shareholder runs and fire sales. Given the pervasiveness of the fund crisis management scenarios, my model is likely to have first-order implications on the potential financial stability risks of open-end mutual funds.

[27] These models all have quite distinct approaches and focuses from mine, but they share the similar structure of negative shocks followed by a random recovery time.

[28] "PIMCO Total Return Assets Drop Below 100 Billion," The *Reuters*, Sept 2, 2015.

[29] "Investors Pull 2.7 Billion from Giant M&G Bond Fund," The *Financial Times*, July 29, 2015.

[30] "Aberdeen Hit by Investor Outflows," The *Wall Street Journal*, December 1, 2015.

[31] "Running Scared: Tuesdays Stock Fund Redemptions Were Biggest Since 2007," The *Wall Street Journal*, Aug 28, 2015.

2.3 Roadmap and Solution Approach

Formally, the setup above defines an infinite-horizon stochastic game with a long-run player (the fund manager) and a sequence of short-run players (the late shareholders). I take a two-step approach to solve this game. First, in Section 3, I solve the stage game among the late shareholders who are hit by consumption shocks in stage t, given any generic cash policy of the fund within that stage. The late shareholders' equilibrium run decisions in that stage game will help determine how the asset and cash positions of the fund evolve over time. Then I turn to the fully stochastic dynamic game in Section 4, solving for the fund manager's optimal dynamic cash policies. I will offer formal equilibrium definitions accordingly in these two sections.

Both of the two steps provide new insights to the literature. The first step shows that when fund NAVs flexibly adjust, cash rebuilding by the fund can directly push shareholders to run, a point absent in classic bank run models. The second step further shows how the potential for shareholder runs distorts fund liquidity management, a point new to the mutual fund management literature.

3 Shareholder Runs

In this section, I focus on the two-date stage game, showing that the fund's desire to rebuild its cash buffer can trigger shareholder runs, and more rapid cash rebuilding leads to more severe runs. This result is very general: runs can occur in equilibrium regardless of the fund's initial cash position.

3.1 Stage-Game Equilibrium Definition and Preliminary Analysis

The two-date stage-game equilibrium is a mixed-strategy Nash equilibrium: in any stage t (consisting of dates $2t$ and $2t+1$), given the fund's initial portfolio position (a_{2t}, x_{2t}) and the late shareholders' common beliefs on the fund's cash rebuilding policy s_{2t+1}, a late shareholder's run strategy maximizes her utility given other late shareholders' strategies. Since all the late shareholders are identical, there is no loss of generality to consider symmetric equilibria when mixed strategies are allowed (Khan and Sun, 2002). Formally:

Definition **1.** *Given μ_E, μ_L, δ_E, δ_L, R, a_{2t}, x_{2t}, and s_{2t+1}, a symmetric run equilibrium of the stage-t game is defined as a run probability $\lambda_{2t}(a_{2t}, x_{2t}, s_{2t+1}) \in [0,1]$ such that*

i) given other late shareholders' run probability λ_{2t}, late investor i's optimal run probability $\lambda^i_{2t} = \lambda_{2t}$ maximizes her utility function (2.2), and

ii) all of the late shareholders have a common belief about the fund's cash rebuilding policy s_{2t+1}.[32]

By the Law of Large Numbers, in a symmetric run equilibrium, the total population of shareholders who redeem on date $2t$ is $(\mu_E + \lambda_{2t}\mu_L)n_{2t}$. Intuitively, this means when some late shareholders are going to run, there will be effectively more early shareholders and fewer late shareholders.

Before solving for the stage-game equilibrium, I first describe three cases of the stage game according to the fund's starting cash position, x_{2t}. As will become clear shortly, the consequences of runs on the risk

[32]For simplicity, when analyzing the stage game, I slightly abuse the notation s_{2t+1} to denote both the shareholders' common belief about the fund's cash rebuilding policy and the actual cash rebuilding policy itself.

of fire sales are different when the initial cash position varies. Different x_{2t} also implies different nature of strategic interactions among late shareholders. Hence, it is useful to discuss these cases separately to clarify the mechanism. These cases can be characterized by the following three cash-to-assets ratio regions.

3.1.1 Cash-to-Assets Ratio Regions

Formally, assuming no cash rebuilding and no shareholder runs as the status quo, I characterize three different cash-to-assets ratio regions of the portfolio position space $\{(a_{2t}, x_{2t})\} \subseteq \mathbb{R}^2_+$. In these different regions, the amounts of illiquid assets that the fund is forced to sell on the two adjacent even and odd dates, that is, q_{2t} and q_{2t+1}, vary. I define the cash-to-assets ratio

$$\eta_t \equiv \frac{x_t}{a_x}$$

for any date t.

LEMMA **1.** *Suppose the fund does not rebuild its cash buffer and no late shareholder is going to run, that is, $s_{2t+1} = 0$ and $\lambda_{2t} = 0$. Then there are three regions of the cash-to-assets ratio η_{2t} in the stage-t game. In these three regions, the amounts of illiquid assets that the fund has to sell passively on dates $2t$ and $2t+1$ are characterized by:*

$$\begin{cases} \textit{High Region } G_h\textit{:} & q_{2t} = 0, q_{2t+1} = 0, \quad \textit{iff} \quad \eta_{2t} \geqslant \dfrac{(\mu_E + \mu_L)R}{1 - \mu_E - \mu_L}, \\ \textit{Intermediate Region } G_m\textit{:} & q_{2t} = 0, q_{2t+1} > 0, \quad \textit{iff} \quad \dfrac{\mu_E R}{1 - \mu_E} \leqslant \eta_{2t} < \dfrac{(\mu_E + \mu_L)R}{1 - \mu_E - \mu_L}, \\ \textit{Low Region } G_l\textit{:} & q_{2t} > 0, q_{2t+1} > 0, \quad \textit{iff} \quad \eta_{2t} < \dfrac{\mu_E R}{1 - \mu_E}. \end{cases}$$

The exact values of q_{2t} and q_{2t+1} in the intermediate and low regions will be solved in Section 3.3 and Section 3.4.

The three regions of η_{2t} are intuitive. When $\eta_{2t} \in G_h$, the fund has enough cash to meet all projected redemptions on both date $2t$ and $2t+1$, and thus no forced fire sales occurs. When $\eta_{2t} \in G_m$, the fund only has enough cash to meet redemptions on date $2t$ but not on date $2t+1$, so it has to passively fire sell its illiquid asset on $2t+1$. Finally, when $\eta_{2t} \in G_l$, the fund does not even have enough cash to meet redemption needs on date $2t$, and thus has to incur forced fire sales on both dates.

Lemma 1 also implies that the stage game is scale-invariant. Neither the absolute value of (a_{2t}, x_{2t}) nor the initial population of shareholders n_{2t} plays a role in determining the three regions. This allows me to use the single variable, the cash-to-assets ratio, to characterize shareholder runs in the stage game. This property of the stage game plays an important role in making the model transparent and tractable.[33] Without loss of generality, I assume $n_{2t} = 1$ throughout this section.

[33] Existing dynamic bank run models are usually assumed to be cashless, because adding variable cash positions would introduce a second state variable that makes a dynamic model intractable. See He and Xiong (2012) and Cheng and Milbradt (2012) for discussions about this point.

Although cash rebuilding by the fund unambiguously leads to shareholder runs, Lemma 1 suggests that the detailed reasons for runs can be different in these three regions because of the different initial amounts of fire sales. In what follows, I analyze equilibrium shareholder runs and their impact on the risk of fire sales in the three regions one by one.

3.2 High Cash-to-Assets Ratio Region G_h

When the stage game is in the high cash-to-assets ratio region G_h, Lemma 1 implies that the fund is not forced to sell any illiquid assets, that is, $q_{2t} = q_{2t+1} = 0$, if no shareholder decides to run. But what would happen if some shareholders decided to run, that is, when $\lambda_{2t} > 0$? The next lemma answers this question. It shows that even if all shareholders decide to run, there will be still no forced fire sales.

LEMMA **2.** *When $\eta_{2t} \in G_h$, $q_{2t}(\lambda_{2t}) = q_{2t+1}(\lambda_{2t}) = 0$ for any given $\lambda_{2t} \in [0, 1]$.*

The intuition of Lemma 2 is clear. When some late shareholders decide to run, there will be effectively more early shareholders and fewer late shareholders, but the total population of redeeming shareholders in the given stage is not changed. Since the fund always has sufficient cash to meet all early and late redemption needs at the initial NAV, it indeed has enough cash on date $2t$ even if all of the late shareholders are going to run.

Lemma 2 has an important implication: NAV_{2t} will never change regardless of whether late shareholders run or not. In other words, in the high region, shareholders can effectively get a fixed-value claim on date $2t$ even though the NAV is flexible by nature. As long as the fund does not rebuild its cash buffer on date $2t+1$, Lemma 2 further implies that $NAV_{2t+1} = NAV_{2t}$ regardless of λ_{2t}, suggesting that there is no strategic interaction among late shareholders absent fund cash rebuilding. As a result, late shareholders will never run if the fund does not rebuild its cash buffer.

However, given the endogenously fixed NAV_{2t}, late shareholders may decide to run if the fund rebuilds its cash buffer on date $2t+1$ (i.e., $s_{2t+1} > 0$), which results in a predictable decline in NAV_{2t+1}. The following lemma characterizes late shareholders' strategic interaction when s_{2t+1} is positive.

LEMMA **3.** *When $\eta_{2t} \in G_h$, late shareholders' run decision λ_{2t} exhibits strategic complementarity if and only if $s_{2t+1} > 0$. Moreover, the strategic complementarity becomes stronger as s_{2t+1} increases. Mathematically, there are:*

$$\frac{\partial \Delta u_L(\lambda_{2t})}{\partial \lambda_{2t}} > 0 \quad \textit{and} \quad \frac{\partial^2 \Delta u_L(\lambda_{2t})}{\partial \lambda_{2t} \partial s_{2t+1}} > 0\,,$$

if and only if $s_{2t+1} > 0$, where $\Delta u_L(\lambda_{2t}) = u_L(\lambda_{2t,i} = 1; \lambda_{2t,-i} = \lambda_{2t}) - u_L(\lambda_{2t,i} = 0; \lambda_{2t,-i} = \lambda_{2t})$, while

$$\frac{\partial \Delta u_L(\lambda_{2t})}{\partial \lambda_{2t}} = 0\,,$$

when $s_{2t+1} = 0$.[34]

Lemma 3 suggests the existence of run incentives, which comes from the predictable decline in NAV when the fund rebuilds its cash buffer. Suppose the fund manager voluntarily sells $s_{2t+1} > 0$ unit of

[34]For simplicity, in what follows when I state results about strategic complementarity and substitutability I omit the mathematical definitions because they are standard.

illiquid assets on date $2t+1$ and λ_{2t} of the late shareholders decide to run. Lemma 2 first suggests that no matter what λ_{2t} is, q_{2t} is always zero and thus we have

$$NAV_{2t} = \underbrace{Ra_{2t} + x_{2t}}_{NAV_{2t-1}} . \tag{3.1}$$

However, NAV_{2t+1} becomes lower due to the fire sales of illiquid assets on date $2t+1$:

$$NAV_{2t+1} = \frac{R \overbrace{(a_{2t} - s_{2t+1})}^{\text{illiquid assets remained}} + \overbrace{x_{2t} - (\mu_E + \lambda_{2t}\mu_L)(Ra_{2t} + x_{2t})}^{\text{cash remained}} + \overbrace{\delta_L R s_{2t+1}}^{\text{cash rebuilt}}}{\underbrace{1 - (\mu_E + \lambda_{2t}\mu_L)}_{\text{shareholders remained on date } 2t+1}} \tag{3.2}$$

$$= NAV_{2t} - \frac{(1-\delta_L)Rs_{2t+1}}{1 - \mu_E - \lambda_{2t}\mu_L} . \tag{3.3}$$

The calculation in (3.2) reflects the fund manager's voluntary asset sales and the associated price impact on date $2t+1$ before the end-of-day NAV is determined. Consequently, a wedge, as shown in (3.3), emerges between the NAV of the two dates. It suggests a predictable decline in the NAV on date $2t+1$.

The predictable decline in NAV implies that the fund manager rebuilds its cash buffer at the expense of the late shareholders who initially plan to wait until date $2t+1$, giving rise to run incentives among these late shareholders. Specifically, the wedge in (3.3) is increasing in $1-\delta_L$ and s_{2t+1}, suggesting that the late shareholders who wait are hurt more if the price impact is larger or if the fund sells more. The wedge is also increasing in μ_E and λ_{2t}, suggesting that the late shareholders who wait are hurt more if more of others are running, and thus they have to bear a higher fire sale cost per share on date $2t+1$. Moreover, for given s_{2t+1} and λ_{2t}, the utility gain $\Delta u_L(\lambda_{2t})$ of running over waiting is $\theta NAV_{2t} - NAV_{2t+1}$, which is strictly increasing in λ_{2t} by (3.3). This illustrates the underlying strategic complementarity among late shareholders.

Lemma 3 suggests that both cash rebuilding and flexible NAV adjustment play crucial roles in generating the run incentives for late shareholders. If $s_{2t+1} = 0$, the stage game features no strategic interaction at all in the high region. If NAV_{2t+1} was fixed, which is the case for MMFs, rebuilding the cash buffer by selling illiquid assets would not generate a wedge of value between early and late shareholders.

I further show that the run incentives can indeed lead to shareholder runs in equilibrium. When the fund rebuilds its cash buffer rapidly enough, that is, when s_{2t+1} is large enough, late shareholders are going to run.

PROPOSITION **1.** *When $\eta_{2t} \in G_h$, late shareholders' equilibrium run behaviors are given by the following three cases:*

i) none of the late shareholders runs, that is, $\lambda_{2t} = 0$, if

$$s_{2t+1} < \underline{s}_h \equiv \frac{(1-\theta)(1-\mu_E-\mu_L)(Ra_{2t} + x_{2t})}{(1-\delta_L)R} ,$$

ii) all of the late shareholders run, that is, $\lambda_{2t} = 1$, if

$$s_{2t+1} > \overline{s}_h \equiv \frac{(1-\theta)(1-\mu_E)(Ra_{2t} + x_{2t})}{(1-\delta_L)R},$$

iii) $\lambda_{2t} \in \{0, \widetilde{\lambda}_{2t}, 1\}$, if

$$\underline{s}_h \leqslant s_{2t+1} \leqslant \overline{s}_h,$$

where $\widetilde{\lambda}_{2t}$ is the solution to

$$s_{2t+1} = \frac{(1-\theta)(1-\mu_E - \widetilde{\lambda}_{2t}\mu_L)(Ra_{2t} + x_{2t})}{(1-\delta_L)R}.$$

Moreover, there are $0 \leqslant \underline{s}_h \leqslant \overline{s}_h$.

Proposition 1 suggests that the fund's cash rebuilding indeeds leads to shareholder runs in equilibrium, and more rapid cash rebuilding can trigger more shareholders to run. The intuitions for the three cases are as follows. In Case i), when the fund sells only a few illiquid assets, NAV_{2t+1} is still high enough, the utility gain of running over waiting would be negative even if all of the late shareholders decided to run, so it turns out no one runs. In Case ii), when the fund manager voluntarily sells so many illiquid assets to a point where NAV_{2t+1} is so low and the utility gain of running would be positive even if others did not run, all of the late shareholders will run. Both Case i) and Case ii) feature a unique equilibrium. In Case iii), the utility gain of running is negative when no one runs but becomes positive when all of the late shareholders are going to run. Strategic complementarity implies that the utility gain of running is increasing when more late shareholders decide to run, so multiple equilibria emerge. Among the three possible equilibria, the partial run equilibrium $\widetilde{\lambda}_{2t}$ features a point where the utility gain of running is zero so that any late shareholder is indifferent between running or waiting.

One natural question is, would cash rebuilding (i.e., a positive s_{2t+1}) ever make sense from the fund manager's perspective? This question is sensible given that shareholder runs happen in equilibrium only if $s_{2t+1} > 0$. As I will show in Section 4, there can indeed be scenarios in which the fund manager optimally chooses to rebuild its cash buffer rapidly and bear some costs of shareholder runs, which supports the existence of run equilibria in the stage game.

The next question is: what kind of costs do shareholder runs impose on the fund? In other words, how do shareholder runs affect the fund's total risk of fire sales, given the fund manager's objective function as in (2.3)? I formally answer this question by looking at the law of motions of the fund's portfolio position (a_{2t}, x_{2t}). The law of motions is also important for the fully dynamic analysis in Section 4.

COROLLARY **1.** *When $\eta_{2t} \in G_h$, the law of motions of (a_{2t}, x_{2t}) is given by*

$$\begin{aligned} a_{2t+2} &= a_{2t} - s_{2t+1}, \text{ and,} \\ x_{2t+2} &= \underbrace{x_{2t} - (\mu_E + \mu_L)(Ra_{2t} + x_{2t})}_{\text{cash remained if no cash rebuilding}} + \underbrace{\delta_L R s_{2t+1}}_{\text{cash rebuilt}} + \underbrace{\frac{(1-\lambda_{2t})\mu_L(1-\delta_L)Rs_{2t+1}}{1-\mu_E-\lambda_{2t}\mu_L}}_{\text{cash saved due to NAV adjustment}}, \end{aligned} \tag{3.4}$$

where λ_{2t} is the equilibrium run probability induced by (a_{2t}, x_{2t}) and s_{2t+1}, as characterized in Proposition 1.

I illustrate the implications of runs on the total risk of fire sales by interpreting the two laws of motions in Corollary 1. The law of motions of a_{2t} is straightforward by Lemma 2 because $q_{2t} = q_{2t+1} = 0$ regardless of λ_{2t}. This suggests that even though cash rebuilding can trigger shareholder runs, it will not induce any forced fire sales in the current stage when $\eta_{2t} \in G_h$.

However, shareholder runs can offset the fund's cash rebuilding efforts and lead to higher risk of future fire sales. This can be seen from the law of motions of x_{2t} in (3.4). To make this clear, I organize the terms in the right hand side of (3.4) in a way to better reflect the cost of shareholder runs. The first term denotes the amount of cash remained if the fund did not rebuild its cash buffer so that the fund paid the initial NAV to all of the early and late shareholders. The second term denotes the actual amount of cash the fund can get by selling s_{2t+1} illiquid assets. Neither of these two terms depends on λ_{2t}. The third term is more interesting. It reflects the fact that the fund can give the late shareholders less cash when it rebuilds its cash buffer on date $2t+1$. Specifically, when s_{2t+1} is positive, NAV_{2t+1} becomes lower as shown in (3.3). Thus, more cash remains on the fund's balance sheet than that indicated by the first term in (3.4). But the third term is strictly decreasing in λ_{2t}, suggesting that this benefit of cash saving to the fund becomes smaller when more late shareholders are running.[35] As a result, when more shareholders run in equilibrium, the fund loses more cash in the given stage, carries less cash to future stages under the same cash rebuilding policy s_{2t+1}, and thus faces higher risk of future fire sales.

With the intuition outlined above, it is convenient to combine the last two terms in (3.4) and define

$$\widehat{p}_L(\lambda_{2t}) \equiv \left[\delta_L + \frac{(1-\lambda_{2t})\mu_L(1-\delta_L)}{1-\mu_E-\lambda_{2t}\mu_L}\right] R \tag{3.5}$$

as the effective selling price on the odd dates $2t+1$. It is decreasing in λ_{2t}, meaning that the effective selling price on odd dates is lower when more shareholders are going to run. Intuitively, the fund prefers a higher effective selling price on odd dates because that helps reduce future risk of fire sales. I will frequently refer to this definition in the dynamic analysis in Section 4.

3.3 Low Cash-to-Assets Ratio Region G_l

Now I turn to the low cash-to-assets ratio region G_l. In this region, the fund's starting cash position is so low that it cannot even meet the redemption needs of the early shareholders. Thus, it is forced to fire sell its illiquid assets on both dates $2t$ and $2t+1$. If late shareholders decide to run, the fund has to

[35] There are two effects captured by the numerator and denominator of the third term in (3.4), respectively. On the one hand, as more late shareholders run on date $2t$, the fund has to pay the (endogenously) fixed NAV_{2t} to more shareholders. This is reflected in the numerator in the sense that only $1-\lambda_{2t}\mu_L$ late shareholders are left to bear the asset sale cost on date $2t+1$, so that the benefit of cash saving due to flexible NAV adjustment becomes lower to the fund. On the other hand, as reflected by the denominator as well as the wedge term in (3.3), NAV_{2t+1} becomes lower as more shareholders are going to run, suggesting that the fund can give less to each of those late shareholders who wait. In equilibrium, the former effect dominates, suggesting that shareholder runs impose an unambiguous negative effect on the net amount of cash the fund can get by selling s_{2t+1} illiquid assets.

passively sell even more. The following lemma formally characterizes how late shareholder runs affect the fund's forced fire sales on the two dates.

LEMMA **4.** *When $\eta_{2t} \in G_l$, there are*

$$q_{2t}(\lambda_{2t}) = \frac{\overbrace{(\mu_E + \lambda_{2t}\mu_L)(Ra_{2t} + x_{2t}) - x_{2t}}^{\text{cash gap}}}{\underbrace{[\delta_E + (1-\delta_E)(\mu_E + \lambda_{2t}\mu_L)]\, R}_{\text{effective selling price with NAV adjustment}}}, \text{ and,} \tag{3.6}$$

$$q_{2t+1}(\lambda_{2t}) = \frac{\overbrace{(1-\lambda_{2t})\mu_L \cdot \frac{R(a_{2t} - q_{2t})}{1 - \mu_E - \lambda_{2t}\mu_L}}^{\text{cash gap}}}{\underbrace{\left[\delta_L + \frac{(1-\lambda_{2t})\mu_L(1-\delta_L)}{1 - \mu_E - \lambda_{2t}\mu_L}\right] R}_{\text{effective selling price with NAV adjustment}}}, \tag{3.7}$$

where q_{2t} is increasing in λ_{2t}, q_{2t+1} is decreasing in λ_{2t}, and $q_{2t} + q_{2t+1}$ is increasing in λ_{2t}.

I first interpret the intuition behind the expressions of $q_{2t}(\lambda_{2t})$ and $q_{2t+1}(\lambda_{2t})$. In determining the amounts of forced fire sales, one needs to know 1) the amount of cash that the fund is forced to raise (i.e., the "cash gap"), and 2) the price at which the fund can sell its assets. Specifically, at the beginning of each date, the cash gap is defined as the difference between the fund's initial cash position and the amount of cash needed to meet projected redemptions at the beginning-of-day NAV, as shown in the numerators of (3.6) and (3.7). However, the fund does not really have to raise that much cash in equilibrium, because the NAV goes down as the fund sells its assets, and redeeming shareholders are only entitled to the end-of-day NAV, which reflects those asset sale costs. Effectively, this is equivalent to a counterfactual in which the fund still sells assets to close the initial cash gap but at a higher effective selling price as the denominators of (3.6) and (3.7) indicate.

Thus, like (3.5), I also formally define the notion of effective selling price on the even dates $2t$:

$$\widehat{p}_E(\lambda_{2t}) \equiv [\delta_E + (1-\delta_E)(\mu_E + \lambda_{2t}\mu_L)]\, R\,. \tag{3.8}$$

Lemma 4 shows how shareholder runs affect the amounts of forced fire sales on the two dates, respectively. When more late shareholders decide to run, the fund has to meet more redemptions on date $2t$ while fewer redemptions on date $2t+1$. Hence, it is forced to fire sell more assets on date $2t$ while fewer assets on date $2t+1$.[36]

More importantly, Lemma 4 also illustrates that runs unambiguously lead to higher total amounts of forced fire sales in the given stage, as shown in the monotonicity of $q_{2t} + q_{2t+1}$ in λ_{2t}. This is because

[36] Formally, for q_{2t}, there are two effects: the cash gap is larger when λ_{2t} is larger, but the effective selling price is also higher. The cash gap effect dominates so that q_{2t} is increasing in λ_{2t}. For q_{2t+1}, there are three effects. When λ_{2t} gets larger, fewer late shareholders choose to wait, and fewer illiquid assets remain as well (since q_{2t} becomes larger). Both lead to a smaller cash gap. However, the effective selling price is lower as well. Again the cash gap effect dominates so that q_{2t} is increasing in λ_{2t}.

the effective selling price on date $2t$ is always lower than that on date $2t+1$,[37] which means that more early redemptions have to be met by selling assets at a lower effective selling price while fewer late redemptions will be met by selling assets at a higher effective selling price. Hence, the increase of q_{2t} will dominate the decrease of q_{2t+1} when more shareholders are going to run. As a result, shareholder runs, if occurring in equilibrium, lead to unambiguously more severe current-stage fire sales.

Lemma 4 implies that both NAV_{2t} and NAV_{2t+1} will be lower when shareholder runs occur. There are

$$\begin{aligned} NAV_{2t}(\lambda_{2t}) &= R\overbrace{(a_{2t}-q_{2t}(\lambda_{2t}))}^{\text{illiquid assets remained}} + \overbrace{x_{2t}}^{\text{cash existed}} + \overbrace{\delta_E R q_{2t}(\lambda_{2t})}^{\text{cash raised}} \\ &= \underbrace{Ra_{2t}+x_{2t}}_{NAV_{2t-1}} - (1-\delta_E)Rq_{2t}(\lambda_{2t}), \end{aligned} \tag{3.9}$$

and

$$NAV_{2t+1}(\lambda_{2t}) = \frac{\overbrace{R(a_{2t}-q_{2t}(\lambda_{2t})-q_{2t+1}(\lambda_{2t})-s_{2t+1})}^{\text{illiquid assets remained}} + \overbrace{\delta_L R(q_{2t+1}(\lambda_{2t})+s_{2t+1})}^{\text{cash raised and rebuilt}}}{\underbrace{1-\mu_E-\lambda_{2t}\mu_L}_{\text{shareholders remained on date } 2t+1}}, \tag{3.10}$$

where $q_{2t}(\lambda_{2t})$ and $q_{2t+1}(\lambda_{2t})$ are given in (3.6) and (3.7).

Like in the high region, a predictable decline in NAV_{2t+1} emerges because of the fund's forced fire sales and active cash rebuilding on date $2t+1$, as shown in (3.10). However, different from the high region, $NAV_{2t}(\lambda_{2t})$ is no longer fixed but decreasing in λ_{2t}, as shown in (3.9). Intuitively, because the fund NAV flexibly adjusts, shareholders who choose to run have to bear all the fire sale costs incurred on date $2t$. This feature changes the nature of the stage game.

LEMMA **5.** *When $\eta_{2t} \in G_l$, late shareholders' run decision λ_{2t} exhibits strategic substitutability for any λ_{2t} satisfying $\theta NAV_{2t}(\lambda_{2t}) \geqslant NAV_{2t+1}(\lambda_{2t})$ and any feasible s_{2t+1}.*[38] *However, when s_{2t+1} increases, the strategic substitutability becomes weaker and $\theta NAV_{2t}(\lambda_{2t}) - NAV_{2t+1}(\lambda_{2t})$ becomes larger, reinforcing a stronger incentive to redeem earlier.*

The intuition behind Lemma 5 is as follows. On the one hand, the predictable decline in NAV_{2t+1} can give rise to an incentive to redeem earlier (i.e., to run on date $2t$). On the other hand, a late shareholder who decides to run has to accept a lower NAV_{2t} when more of other late shareholders decide to run. In particular, as more shareholders are going to run, the difference between $NAV_{2t}(\lambda_{2t})$ and $NAV_{2t+1}(\lambda_{2t})$ becomes smaller, implying strategic substitutability among late shareholders. Intuitively, a shareholder redeeming at t realizes that more early withdrawals will potentially induce more fire sales at t and thus

[37] More formally, by the monotonicity of the effective selling prices $\widehat{p}_L(\lambda_{2t})$ and $\widehat{p}_E(\lambda_{2t})$, there is

$$\widehat{p}_E(\lambda_{2t}) \leqslant \widehat{p}_E(1) < \widehat{p}_L(1) \leqslant \widehat{p}_L(\lambda_{2t})$$

for any $\lambda_{2t} \in [0,1]$. In other words, the potential for runs may change the effective selling prices on the two adjacent dates in the given stage, but the effective selling price on $2t$ is still lower than that on $2t+1$ regardless of shareholder runs.

[38] In equilibrium λ_{2t} will be an endogenous function of s_{2t+1}. But in showing the strategic interaction among late shareholders of the stage game, λ_{2t} should be treated as an independent variable. This also applies to the analysis of the intermediate region.

lower the proceeds she receives, and thus the expected utility gain of running over waiting would be decreasing as more shareholders run.

However, when the fund rebuilds its cash buffer, the strategic substitutability may not reduce the run incentive (when θ is large enough). Concretely, when the fund voluntarily sells a sufficiently large amount of assets to rebuild its cash buffer, the resulting large predictable decline in NAV_{2t+1} reinforces a sufficiently strong run incentive. As a result, a late shareholder may still decide to run even if all of the other late shareholders have already run, despite the strategic substitutability.

Consequently, the next proposition fully characterizes late shareholders' equilibrium run behaviors in the low region.

PROPOSITION **2.** *When $\eta_{2t} \in G_l$, late shareholders' equilibrium run behaviors are given by the following three cases:*

i) none of the late shareholders runs, that is, $\lambda_{2t} = 0$, if

$$s_{2t+1} < \underline{s}_l \equiv \frac{Ra_{2t} - \theta(1-\mu_E)(Ra_{2t}+x_{2t}) - (1-\theta(1-\delta_E)(1-\mu_E))Rq_{2t}(0)}{(1-\delta_L)R} - q_{2t+1}(0)\,,$$

ii) all of the late shareholders run, that is, $\lambda_{2t} = 1$, if

$$s_{2t+1} > \overline{s}_l \equiv \frac{Ra_{2t} - \theta(1-\mu_E-\mu_L)(Ra_{2t}+x_{2t}) - (1-\theta(1-\delta_E)(1-\mu_E-\mu_L))Rq_{2t}(1)}{(1-\delta_L)R}\,,$$

iii) some of the late shareholders run, that is, $\lambda_{2t} = \widetilde{\lambda}_{2t}$, if

$$\underline{s}_l \leqslant s_{2t+1} \leqslant \overline{s}_l\,,$$

where $\widetilde{\lambda}_{2t}$ is the solution to

$$s_{2t+1} = \frac{Ra_{2t} - \theta(1-\mu_E-\widetilde{\lambda}_{2t}\mu_l)(Ra_{2t}+x_{2t}) - (1-\theta(1-\delta_E)(1-\mu_E-\widetilde{\lambda}_{2t}\mu_l))Rq_{2t}(\widetilde{\lambda}_{2t})}{(1-\delta_L)R} - q_{2t+1}(\widetilde{\lambda}_{2t})\,.$$

All of the $q_{2t}(\lambda_{2t})$ and $q_{2t+1}(\lambda_{2t})$ are given in Lemma 4. Moreover, there are $\overline{s}_l \geqslant 0$ and $\overline{s}_l > \underline{s}_l$.

Like Proposition 1, Proposition 2 also suggests that the fund's cash rebuilding leads to shareholder runs in equilibrium, and more rapid cash rebuilding can trigger more shareholders to run, despite the strategic substitutability. In Case i), when the fund does not rebuild its cash buffer or only sells a few illiquid assets, NAV_{2t+1} can be still higher regardless of shareholders' redemption decisions, so that late shareholders will not run. In Case ii), when the fund voluntarily sells so many illiquid assets, NAV_{2t+1} is so low that the utility gain of running over waiting is positive even if all of the late shareholders have already run. In other words, fund cash rebuilding can reinforce a strong run incentive despite the strategic substitutability. In addition, all of the late shareholders do not run unless the fund rebuilds its cash buffer (since $\overline{s}_l \geqslant 0$), suggesting that only cash rebuilding by the fund can push all the shareholders to run in this mutual fund context.[39] In Case iii), the utility gain of running over waiting is positive when no shareholder runs but becomes negative when all of the late shareholders are going to run. In

[39]This is not true for a comparable bank with fixed-value deposits, in which all shareholders can run in equilibrium even if the bank does not do anything by itself.

this case, there exists some run equilibrium in which the utility gain of running over waiting is zero, so that late shareholders are indifferent between running or waiting.

Again, I show how shareholder runs affect the risk of forced fire sales, by looking at the law of motions of the fund portfolio position (a_{2t}, x_{2t}).

COROLLARY **2.** *When $\eta_{2t} \in G_l$, the law of motions of (a_{2t}, x_{2t}) is given by*

$$a_{2t+2} = a_{2t} - \overbrace{(q_{2t}(\lambda_{2t}) + q_{2t+1}(\lambda_{2t}))}^{\text{forced fire sales}} - s_{2t+1}\,, \text{ and}, \tag{3.11}$$

$$x_{2t+2} = \underbrace{\delta_L R s_{2t+1}}_{\text{cash rebuilt}} + \underbrace{\frac{(1-\lambda_{2t})\mu_L(1-\delta_L)Rs_{2t+1}}{1-\mu_E-\lambda_{2t}\mu_L}}_{\text{cash saved due to NAV adjustment}} = \widehat{p}_L(\lambda_{2t})s_{2t+1}\,, \tag{3.12}$$

where λ_{2t} is the equilibrium run probability induced by (a_{2t}, x_{2t}) and s_{2t+1}, as characterized in Proposition 2.

The two laws of motions here are different from those in Corollary 1, but can be unified under the same intuition. The law of motions of a_{2t} in (3.11) is different because of the newly introduced forced fire sales terms: shareholder runs result in more forced fire sales in the given stage. The law of motions of x_{2t} in (3.12) loses the first term in (3.4). This is natural because by construction no cash would remain in the low region if the fund did not rebuild its cash buffer. Perhaps surprisingly, the two terms in (3.12) are exactly the same as the last two terms in (3.4) despite the more complicated forced asset sales and NAV updating in the low region. But this is still intuitive: all the proceeds from forced fire sales accrue to the redeeming shareholders, and thus they will not affect the amount of cash that the fund can carry into future stages. This suggests that, in rebuilding its cash buffer the fund is still selling at the effective selling price $\widehat{p}_L(\lambda_{2t})$.

Importantly, Corollary 2 implies two different costs of shareholder runs. First, shareholder runs force the fund to fire sell more illiquid assets in the current stage (remember Lemma 4 shows that $q_{2t} + q_{2t+1}$ is increasing in λ_{2t}). Second, like that in the high region, shareholder runs lead to a lower effective selling price on date $2t+1$ when the fund rebuilds its cash buffer. This means that runs partially offset the fund's efforts of cash rebuilding and thus lead to higher risk of future fire sales. The second cost is also present in the high region as shown in (3.4).

Compared to the analysis for the high region in Section 3.2, Proposition 2 and Corollary 2 suggest that starting with a low cash position makes a fund financially more fragile. Being in the low cash-to-assets ratio region makes the fund more prone to forced fire sales initially. Even worse, because the fund is running out of cash, it is likely to rebuild its cash more rapidly (as formally shown in Section 4), leading to more severe runs. In particular, runs in the low region are more detrimental to the fund: they not only increase current-stage forced fire sales but also lead to higher risk of future fire sales.

3.4 Intermediate Cash-to-Assets Ratio Region G_m

I then analyze the intermediate cash-to-assets ratio region G_m. In this region, the fund's starting cash position is moderate in the sense that it can meet all of the early shareholders' redemption needs but

then falls short for late shareholders' redemption requests.

In the intermediate region, we still have the universal results of shareholder runs as those in the high and low regions. Specifically, the fund's cash rebuilding can lead to runs in equilibrium, and more rapid cash rebuilding implies more severe runs. Also, there are two costs of runs as those in the low region: more current-stage forced fire sales and higher risk of future fire sales.

However, the underlying strategic interaction among shareholders becomes more involved in the intermediate region. When only a few late shareholders decide to run, the fund will not be forced to fire sell its illiquid assets on date $2t$, and thus NAV_{2t} will be endogenously fixed. However, when many late shareholders decide to run, the fund will be forced to sell its assets on date $2t$, and thus both NAV_{2t} and NAV_{2t+1} vary. In this sense, the stage game in the intermediate region can be viewed as a hybrid of one game in the high region and another one in the low region, which can switch from strategic complementarity to substitutability as more shareholders decide to run. However, it is still the fund's cash rebuilding and the resulting predictable decline in NAV_{2t+1} that reinforce a strong run incentive. Given the results in Sections 3.2 and 3.3, I defer the full investigation of the intermediate region to Appendix A.2. The formal results are stated there as Lemma 10, Proposition 13, and Corollary 4.

3.5 The Differences from Bank Runs and Market Runs

Before analyzing the fully dynamic model of fund cash management, it is useful to contrast the shareholder run mechanism to those underlying classic bank runs (Diamond and Dybvig, 1983) and market runs (Bernardo and Welch, 2004, Morris and Shin, 2004). This helps highlight the contribution of this paper to the existing run literature.

First, fund shareholder runs differ from classic bank runs in terms of the underlying mechanism. In my model, the first-mover advantage does not come from an exogenous fixed-NAV claim on date $2t$ like the deposit at a bank. Instead, it is the fund's desire to rebuild its cash buffer on date $2t+1$ and the resulting predictable decline in NAV_{2t+1} that lead to a strong first-mover advantage and thus the potential for shareholder runs. In contrast, when a comparable bank rebuilds its cash buffer by selling assets, the underlying deposit value will not change,[40] and thus bank cash rebuilding by itself cannot directly generate depositor runs.

To illustrate the differences between fund shareholder runs and classic bank runs more in depth, it is again helpful to separate the cases with a high and a low starting cash position.

When the fund starts with a high cash position, late shareholders are going to run only if the fund voluntarily sells a sufficient amount of assets to rebuild its cash buffer, which generates a large enough predictable decline in NAV. On the one hand, although the late shareholders who run are expecting to get a fixed NAV on date $2t$, NAV_{2t} is fully endogenous and flexible by nature. This is in contrast to bank run models in which a fixed-value claim is either exogenously assumed or derived as the optimal contract in an outer risk-sharing problem. On the other hand, in a typical two-date bank run model, strategic complementarity arises because run-induced asset liquidation on the early date hurts what a

[40] To be precise, banks are always financed by some equity, so that the asset sale costs go to the equity holders. This is consistent with the reality in the sense that deposit banks are always subject to some capital requirements.

waiting shareholder can get on the late date; no asset liquidation happens on the late date. But in my model, no fire sales ever occur on date $2t$ when the fund is in the high cash-to-assets region. Instead, runs hurt shareholders who wait because fewer of them are left on date $2t+1$ and thus each has to bear a higher asset sale cost per share when the fund rebuilds its cash buffer. As will be clear in the dynamic analysis, when more shareholders run, the fund loses more cash on date $2t$ and thus may want to rebuild it cash buffer more rapidly, creating an negative externality on the late shareholders who wait.

In contrast, when the fund starts with a low cash position, the stage game can even feature strategic substitutability, but the fund's cash rebuilding reinforces a strong run incentive. In classic two-date bank run models, the game exhibits strategic substitutability only if the bank goes bankruptcy on the early date and an equal sharing rule is applied among depositors. In other words, strategic substitutability emerges only because more depositors are going to share the fixed liquidation value on the early date, and thus everyone gets less. Such strategic substitutability is absent as long as the bank is still solvent. However, in my mutual fund model with a fully flexible NAV, the fund never goes bankruptcy. Strategic substitutability instead emerges from the fact that the fund may be forced to fire sell more illiquid assets on date $2t$ and thus NAV_{2t} becomes lower. Indeed, some fund managers refer to this strategic substitutability to deny the existence of fund shareholder runs. But as I have already showed, the fund's cash rebuilding and the resulting predictable decline in NAV can lead to shareholder runs in equilibrium despite the strategic substitutability.

Regardless of the starting cash position, the shareholder run mechanism further highlights a dynamic interaction between the fund and its shareholders, which is absent in bank run models that focus on coordination failures among depositors themselves. Specifically, a shareholder's run decision not only depends on her belief about other shareholders' run decisions, but also depends on her belief about the fund's cash rebuilding policy,[41] which in turn depends on all the future generations of shareholders' run decisions in the fully dynamic model.

Observationally, fund shareholder runs also tend to be "slow-moving," in the sense that their costs are reflected in higher fire sale losses over time rather than an abrupt bankruptcy as that in bank run models.[42] This nature suggests that the negative impacts of shareholder runs on fund performance can be gradual but long-lasting.

The run mechanism in my model is also different from that underlying market runs. The notion of market runs is formally proposed by Bernardo and Welch (2004) and Morris and Shin (2004), which independently argue that if an asset market features an exogenous downward-sloping demand curve, investors fearing future liquidity shocks will have an incentive to front-run, selling the asset earlier to get a higher selling price. This run mechanism leads to massive fire sales in equilibrium. In my model, shareholders get access to the underlying assets only through the fund, and fund cash management is indeed beneficial to shareholders because it helps reduce total expected fire sale losses. However, the key tension that I document is that the fund's dynamic cash rebuilding also endogenously gives rise to

[41] In this sense, the shareholder run mechanism within the stage game more closely resemble the idea of fundamental runs as argued by Chari and Jagannathan (1988), Jacklin and Bhattacharya (1988) and Allen and Gale (1998).

[42] In my model, as long as the entire population of shareholders are not going to run (i.e., $\mu_E + \mu_L < 1$), the fund will stay solvent. This is not true for a bank, which can be forced to go bankruptcy even if only a fraction of depositors run.

a predictable decline in NAV and thus a new kind of run incentives. No cash management implies no runs in my model. In contrast, there is no role for cash management in market run models. In this sense, market run models focus on the asset market itself while my theory focuses on the role of financial intermediaries. This allows me to distinguish between risks that come from the active management of financial intermediaries and those that are only a reflection of the underlying asset market frictions.

4 Fund Liquidity Management in the Presence of Runs

In this section, I turn to the fully dynamic game and endogenize the fund's optimal dynamic cash rebuilding policy. I show that the potential for runs gives rise to a new tension: rebuilding the cash buffer more rapidly can trigger runs, while rebuilding it less rapidly puts the fund at higher risk of future fire sales as well as future runs. I then show that a time-inconsistency problem further complicates this tension, leading to severe fire sales in expectation despite optimal cash management by the fund.

4.1 Dynamic Equilibrium Definition and Preliminary Analysis

The dynamic equilibrium is a Markov perfect equilibrium[43]: in any stage t (consisting of dates $2t$ and $2t+1$), as long as the game continues, both the fund manager and the late shareholders' strategies are functions of the state variables a_{2t} and x_{2t}, the fund's starting portfolio position, and the strategy profile is subgame perfect. Formally, I have:

Definition **2.** *Given μ_E, μ_L, δ_E, δ_L, and R, a Markov perfect equilibrium is defined as a combination of the fund manager's optimal cash rebuilding policy function $s^*_{2t+1}(a_{2t}, x_{2t})$ and the late shareholders' run decision $\lambda_{2t}(a_{2t}, x_{2t}, s_{2t+1})$ such that*

i) given any state (a_{2t}, x_{2t}) and any generic common belief of the cash rebuilding policy $s_{2t+1}(a_{2t}, x_{2t})$, the late investors' run decision $\lambda_{2t}(a_{2t}, x_{2t}, s_{2t+1}) \in [0,1]$ constructs a symmetric run equilibrium as defined in Definition 1, which also determines q_{2t} and q_{2t+1} in any stage,

ii) the fund manager's optimal cash rebuilding policy function $s^*_{2t+1}(a_{2t}, x_{2t}, \lambda_{2t})$ solves the following Bellman equation:

$$V(a_{2t}, x_{2t}) = -(1-\delta_E)Rq_{2t} - (1-\delta_L)Rq_{2t+1} + \max_{s^*_{2t+1}}[-(1-\delta_L)Rs_{2t+1} + (1-\pi)V(a_{2t+2}, x_{2t+2})], \tag{4.1}$$

iii) the state variables (a_{2t}, x_{2t}) are govern by the endogenous laws of motions as described in Corollaries 1, 2 and 4, according to the respective cash-to-assets ratio regions.

I use a guess-and-verify approach to solve for the equilibrium.[44] Specifically, I will first characterize some important properties of the value function $V(a_{2t}, x_{2t})$. With the help of these properties, I solve

[43] As shown in Bhaskar, Morris and Mailath (2013), an equilibrium is purifiable if some close-by behavior is consistent with equilibrium when agents' payoffs in each stage are perturbed additively and independently, and for infinite stochastic games with at most one long-run player all purifiable equilibria are Markov. My model can be viewed as a special case of the general class of games described in Bhaskar, Morris and Mailath (2013). As the fund manager is the only long-run player in my model, to restrict attention to Markov equilibria involves no loss of generality in the sense of finding all purifiable equilibria.

[44] Formally, the usual first-order approach does not apply here, because the value function is only piecewise-differentiable.

for the equilibrium for different parameter values of θ, which governs the late shareholders' propensity to run.

The stage game may admit multiple equilibria in some circumstances, and thus an equilibrium selection mechanism is needed. Since equilibrium selection is not crucial to my main point about the dynamic interdependence between shareholder runs and fund liquidity management, I assume that late shareholders will coordinate to the worst equilibrium whenever multiple equilibria occur.[45] This equilibrium selection mechanism can be motivated by that the fund manager may be ambiguity averse to the potential for shareholder runs, so that she wants to find the most conservative cash rebuilding policies. Alternative equilibrium selection mechanisms such as selecting the best equilibrium or the static global game approach (Goldstein and Pauzner, 2005) will not qualitatively change my results.

The following proposition establishes the existence and some important analytical properties of the value function.

PROPOSITION **3.** *A value function $V(a_{2t}, x_{2t})$ exists under the Markov strategies proposed in Definition 2. In particular, $V(a_{2t}, x_{2t})$ is homogeneous of degree one (HD1) in (a_{2t}, x_{2t}).*

The fact that $V(a_{2t}, x_{2t})$ is HD1 in (a_{2t}, x_{2t}) is important. It implies that the dynamic game is also scale-invariant, and thus the cash-to-assets ratio η_{2t} becomes the single effective state variable of the fully dynamic model. This property will make the analysis of the fund's dynamic cash rebuilding policies more transparent.

Finally, before analyzing the fully dynamic equilibrium in general, I analyze how different values of π, the probability at which the game ends, shape the fund's optimal cash rebuilding policy. Intuitively, when the shock is less persistent (i.e., π is large), the future risk of fire sales is small, and thus cash buffers become less valuable. Therefore, when π is sufficiently large, it makes little sense for the fund to rebuild its cash buffer ex-ante, because doing do only induces current sales of assets for sure but generates little future benefit. As a result, the model admits a type of equilibria in which the fund finds it optimal not to rebuild its cash buffer at all. In these equilibria, ex-post, the fund may be forced to fire sell many illiquid assets in future, because it may not have enough cash buffer when the game continues. This is consistent with the view that if agents underestimate the probability of bad shocks they are likely to suffer huge losses ex-post (Gennaioli, Shleifer and Vishny, 2012, 2013).

LEMMA **6.** *When π is sufficiently large, the equilibrium features $s^*_{2t+1}(a_{2t}, x_{2t}) = 0$ for any starting portfolio position (a_{2t}, x_{2t}).*

To better illustrate the key trade-off involved in the dynamic model, in the following analysis, I will consider an arbitrarily small (but still positive) π. Intuitively, this means that the redemption shocks are sufficiently persistent, consistent with the crisis management scenarios discussed in Section 2.2. This will introduce significant future risk of fire sales and thus give rise to a significant trade-off between current runs and future fire sales.[46]

[45] See Postlewaite and Vives (1987), Allen and Gale (1998), Cooper and Ross (1998), among other more recent papers, for a similar treatment; some of those papers assume shareholders to coordinate to the best equilibrium to justify the existence of banks.

[46] It should be noted that those equilibria characterized by Lemma 6 are still intuitive and consistent with the model

4.2 The No-Run and Extreme-Run Scenarios: $\theta = 0$ and $\theta = 1$

First, I consider two extreme scenarios, which are sufficient to illustrate the key trade-off underlying the fund's optimal cash management. One is the scenario of $\theta = 0$, in which there are completely no runs. The other is the scenario of $\theta = 1$, in which late shareholders are indifferent between early and late consumptions so that they have the strongest propensity to run.

I show that, in both scenarios, the fund optimally rebuilds its cash buffer when its cash position falls below some threshold. But the threshold and the optimal amount of cash rebuilding are different in these cases, reflecting different trade-offs between current runs and future fire sales.

I start by defining some new notations to streamline the presentation. First, the dynamic game is scale-invariant according to Proposition 3, so it is convenient to define

$$\sigma_{2t+1} \equiv \frac{s_{2t+1}}{a_{2t}},$$

the fraction of illiquid assets that the fund voluntarily sell on odd dates $2t+1$ (relative to the beginning-of-stage asset position a_{2t}), to denote the cash rebuilding policy. Moreover, Corollaries 1, 2, and 4 suggest that η_{2t+2} is uniquely determined by (a_{2t}, x_{2t}) and σ_{2t+1}.[47] Given (a_{2t}, x_{2t}) as the state variables and η_{2t} as the only effective state variable, it is also convenient to use η_{2t+2} to denote the fund's cash rebuilding policy.

I also recap the definitions of different cash-to-assets ratio regions. In particular, I further divide the high region G_h into three different sub-regions: the high-low region G_{hl}, the high-intermediate region G_{hm}, and the high-high region G_{hh}:

Region	Cash-to-Assets Ratio
G_l	$\eta_{2t} < \dfrac{\mu_E R}{1-\mu_E}$
G_m	$\dfrac{\mu_E R}{1-\mu_E} \leqslant \eta_{2t} < \dfrac{(\mu_E+\mu_L)R}{1-\mu_E-\mu_L}$
G_{hl}	$\eta_{2t} \geqslant \dfrac{(\mu_E+\mu_L)R}{1-\mu_E-\mu_L}$ and $\eta_{2t+2} < \dfrac{\mu_E R}{1-\mu_E}$ if $\sigma_{2t+1} = 0$
G_{hm}	$\eta_{2t} \geqslant \dfrac{(\mu_E+\mu_L)R}{1-\mu_E-\mu_L}$ and $\dfrac{\mu_E R}{1-\mu_E} \leqslant \eta_{2t+2} < \dfrac{(\mu_E+\mu_L)R}{1-\mu_E-\mu_L}$ if $\sigma_{2t+1} = 0$
G_{hh}	$\eta_{2t} \geqslant \dfrac{(\mu_E+\mu_L)R}{1-\mu_E-\mu_L}$ and $\eta_{2t+2} \geqslant \dfrac{(\mu_E+\mu_L)R}{1-\mu_E-\mu_L}$ if $\sigma_{2t+1} = 0$

The three sub-regions of the high region G_h are defined from a dynamic perspective, and they will be useful in describing the optimal dynamic cash rebuilding policy. When the fund starts from G_h and does not rebuild its cash buffer, by definition, after meeting redemptions in the given stage the fund still has a non-negative cash position in the next stage. If the fund ends up into the low region G_l in the next stage, I say that the fund starts from the high-low region G_{hl}. If instead the fund ends up into the

settings. They are just less relevant to the main point of this paper: the dynamic interdependence of shareholder runs and fund liquidity management in a crisis management scenario.

[47] Keep in mind that the fixed stage-game equilibrium selection mechanism is used when needed.

intermediate region G_m in the next stage, I say that the fund starts from the high-intermediate region G_{hm}. The high-high region G_{hh} is defined in the same manner. Clearly, there is

$$G_h = G_{hl} \cup G_{hm} \cup G_{hh} .$$

Now I characterize the fully dynamic equilibria when $\theta = 0$ and $\theta = 1$. First, I analyze the behavior of shareholder runs in these two scenarios, given any generic and feasible cash rebuilding policy of the fund. These results are useful not only because they illustrate whether and when shareholders will run in equilibrium, but also because they can help pin down all the possible off-equilibrium paths.

LEMMA **7.** *When $\theta = 0$, none of the late shareholders run in stage t, that is, $\lambda_{2t}(a_{2t}, x_{2t}) = 0$ for any (a_{2t}, x_{2t}) and any cash rebuilding policy σ_{2t+1}.*

Lemma 7 is straightforward. If a shareholder gets nothing when running, they will never run. In other words, there will be completely no runs in this scenario.

LEMMA **8.** *When $\theta = 1$, all of the late shareholders run in stage t, that is, $\lambda_{2t}(a_{2t}, x_{2t}) = 1$ for any (a_{2t}, x_{2t}) and any positive and feasible cash rebuilding policy $\sigma_{2t+1} > 0$.*

Lemma 8 shows that when the shareholders' propensity to run is the highest (i.e., $\theta = 1$), all of the late shareholders decide to run even if the fund only sells a small amount of assets to rebuild its cash buffer, regardless of the fund's initial cash position. In intuitive terms, in this case shareholders are extremely sensitive to the fund's cash rebuilding. This is because when $\theta = 1$ the late shareholders simply compare between NAV_{2t} and NAV_{2t+1} to decide whether to run. As long as the fund rebuilds its cash buffer, there will be a predictable decline in NAV_{2t+1} regardless of either the fund's initial cash position or other shareholders' run behavior, and thus all of the late shareholders are going to run.

Now I turn to the fund's equilibrium cash rebuilding policy. With the help of Lemma 7, the following proposition first characterizes the optimal cash rebuilding policy when $\theta = 0$.

PROPOSITION **4.** *When $\theta = 0$, the equilibrium cash rebuilding policy of the fund is characterized by:*[48]

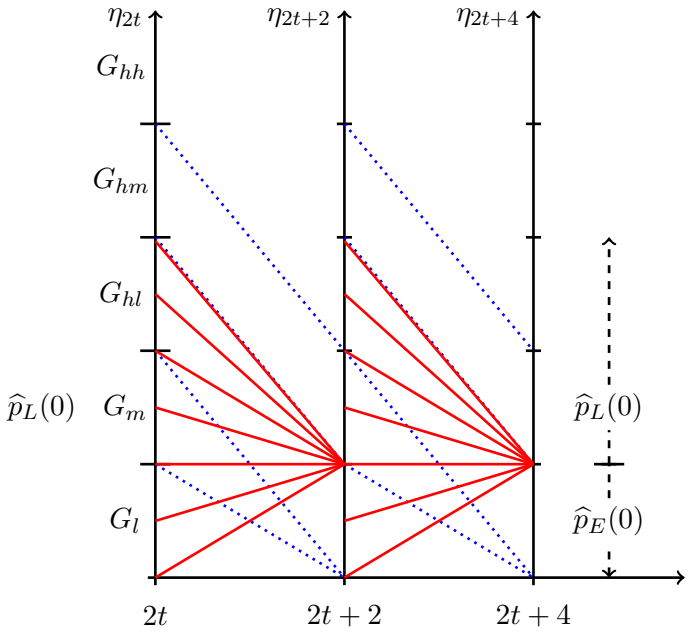
i) if $\eta_{2t} \in G_l \cup G_m \cup G_{hl}$, the fund chooses $\sigma^*_{2t+1} > 0$ such that

$$\eta^*_{2t+2} = \frac{\mu_E R}{1 - \mu_E} , \text{ and},$$

ii) if $\eta_{2t} \in G_{hm} \cup G_{hh}$, the fund does not rebuild its cash buffer, that is, $\sigma^*_{2t+1} = 0$.

The fund's optimal dynamic cash rebuilding policy when $\theta = 0$, as characterized in Proposition 4, is illustrated in Figure 3. In this figure, the horizontal axis denotes date, while the vertical axis denotes the cash-to-assets ratio. The blue dotted line depicts the evolution of the cash-to-assets ratio if the fund does not rebuild its cash buffer at all. The red line depicts the evolution of the cash-to-assets ratio when the fund follows the optimal dynamic cash rebuilding policy in equilibrium. Because of the

[48] Keep in mind that the dynamic equilibrium requires sequential optimality. In other words, the fund's cash rebuilding policy is optimal in a stage only when in the next stage the fund also follows its optimal cash rebuilding policy, which is again conditional on the fund's optimal cash rebuilding policy in the following stage, and so on.



The horizontal axis denotes date. The vertical axis denotes the cash-to-assets ratio (the effective state variable) by different regions. The blue dotted line depicts the evolution of the cash-to-assets ratio if the fund does not rebuild its cash buffer at all. The red line depicts the equilibrium evolution of the cash-to-assets ratio when the fund follows the optimal dynamic cash rebuilding policy. From the perspective of any stage t (including dates $2t$ and $2t+1$), the effective selling price $\widehat{p}_L(\cdot)$ on the left side is that at which the fund can rebuilds its cash buffer in the current stage, while the effective selling prices $\widehat{p}_E(\cdot)$ and $\widehat{p}_L(\cdot)$ on the right side are those at which the fund can raise cash in the next stage in different cash-to-assets regions. See the main text for more explanations.

Figure 3: Equilibrium Cash Rebuilding Policy When $\theta = 0$

scale-invariance and the resulting stationarity of the dynamic game, the equilibrium cash rebuilding policy (conditional on the effective state variable, the cash-to-assets ratio η_{2t}) always follows the same pattern in different stages as long as the game continues.

Proposition 4 says that as long as its initial cash position falls below the high-intermediate region G_{hm}, the fund optimally rebuilds its cash buffer until the next-stage cash-to-assets ratio η_{2t+2} reaches the cutoff between the low region G_l and the intermediate region G_m. But if the fund starts with a higher initial cash position, it will not rebuild its cash buffer. In either case, the fund has a strictly positive cash target for the next stage.

Since there are no runs in equilibrium (by Lemma 7), the main insight behind Proposition 4 is a trade-off between current-stage active asset sales (under a policy of more rapid cash rebuilding) and future-stage forced fire sales (under a policy of no or less rapid cash rebuilding). Intuitively, because the fund manager cares about total expected fire sale losses, it is worthwhile for her to voluntarily sell more assets at the current stage (on date $2t+1$), if the cash buffer rebuilt can help avoid more severe fire sales in the next stage (on dates $2t+2$).

Importantly, due to flexible NAV adjustment, what matter for the dynamic trade-off of current and future amounts of fire sales are not the physical but the effective selling prices as defined in (3.5) and (3.8). To see this, on the one hand, suppose there is a cash gap $\Delta x_{2t+2} > 0$ on date $2t+2$. In other words, the difference between the fund's initial cash position x_{2t+2} and the amount of cash needed to

meet projected redemptions on date $2t+2$ at the beginning-of-day NAV (i.e., NAV_{2t+1}) is Δx_{2t+2}. As a result, there will be forced fire sales on date $2t+2$. But since redeeming shareholders on date $2t+2$ will only get a lower end-of-day NAV, the fund manager can effectively sell at the effective selling price on date $2t+2$ to meet the initial cash gap Δx_{2t+2}. On the other hand, the fund manager can choose to actively sell more assets on date $2t+1$, also at the corresponding effective selling price, to rebuild Δx_{2t+2} unit of cash buffer in advance on date $2t+1$, carry it to date $2t+2$, and thus avoid forced fire sales on date $2t+2$. Moreover, the physical selling price on date $2t+1$ is always higher than that on date $2t+2$. Hence, when the effective selling price on date $2t+1$ is higher than that on date $2t+2$, the fund can sell a smaller amount of assets at a higher physical price on date $2t+1$ in order to avoid higher fire sale losses on date $2t+2$. Hence, a more rapid cash rebuilding policy is better for the purpose of minimizing total fire sale losses, and thus it is optimal for the fund to do so.

I discuss different cash-to-asset ratio regions to illustrate this trade-off in more depth.

First, the fund always rebuilds its cash buffer when starting from the low, the intermediate, or the high-low cash-to-assets ratio region (i.e., $\eta_{2t} \in G_l \cup G_m \cup G_{hl}$). The reason is the following. If the fund did not rebuild its cash buffer, it would end up in the low region in the next stage (i.e., $\eta_{2t+2} \in G_l$). Since the fund will be forced to fire sell its assets then (as the game continues with a high probability $1-\pi$), the fund manager may want to rebuild its cash buffer on date $2t+1$ to avoid fire sales on $2t+2$. Specifically, because late shareholders never run (by Lemma 7), the effective selling price to rebuild cash buffers actively on date $2t+1$ is $\widehat{p}_L(0)$, while the effective selling price to raise cash passively on date $2t+2$ is $\widehat{p}_E(0)$. As $\widehat{p}_L(0) > \widehat{p}_E(0)$, the fund manager always finds it optimal to rebuild its cash buffer on date $2t+1$.

Given that the fund rebuilds its cash buffer, what is the optimal amount of active asset sales? In equilibrium, the fund manager will rebuild the cash buffer up to a point where η_{2t+2} just hits the cut-off between the low and the intermediate region. This is because, on the one hand, a lower cash target still implies forced fire sales on date $2t+2$ at a lower effective selling price $\widehat{p}_E(0)$ and thus is not optimal. On the other hand, any more cash rebuilding on date $2t+1$ means the fund will still have a strictly positive cash buffer on date $2t+3$ after outflows on date $2t+2$. This is also not optimal because that cash buffer is excessive from the perspective of date $2t+1$. In other words, even if asset sales occur on date $2t+3$, the fund manager will be able to sell at the higher effective selling price $\widehat{p}_L(0)$ then. Since the game only has a less than one probability to continue, selling at the same effective price $\widehat{p}_L(0)$ on date $2t+1$ to build that excessive cash buffer is not profitable.

Then, it is straightforward to understand the equilibrium cash rebuilding policy in the high-intermediate and the high-high regions (i.e., $\eta_{2t} \in G_{hm} \cup G_{hh}$) with the intuition outlined above. Here, even if the fund does not rebuild its cash buffer, it will end up at least in the intermediate region, where the fund can raise cash at the effective selling price $\widehat{p}_L(0)$ when needed. As a result, any cash buffer rebuilt on date $2t+1$ (by selling assets also at the effective selling price $\widehat{p}_L(0)$) is excessive, and thus the fund finds $\sigma^*_{2t+1} = 0$ to be optimal.

Next, I characterize the optimal cash rebuilding policy when $\theta = 1$ and contrast it to that in the scenario of $\theta = 0$. This illustrates how the potential for runs distorts a fund's dynamic liquidity

management.

PROPOSITION **5.** *When $\theta = 1$, the equilibrium cash rebuilding policy of the fund is characterized by:*

i) if $\eta_{2t} \in G_l \cup G_m \cup G_{hl} \cup G_{hm}$, the fund chooses $\sigma^*_{2t+1} > 0$ such that

$$\eta^*_{2t+2} = \frac{(\mu_E + \mu_L)R}{1 - \mu_E - \mu_L}, \text{ and,}$$

ii) if $\eta_{2t} \in G_{hh}$, the fund does not rebuild its cash buffer, that is, $\sigma^*_{2t+1} = 0$.

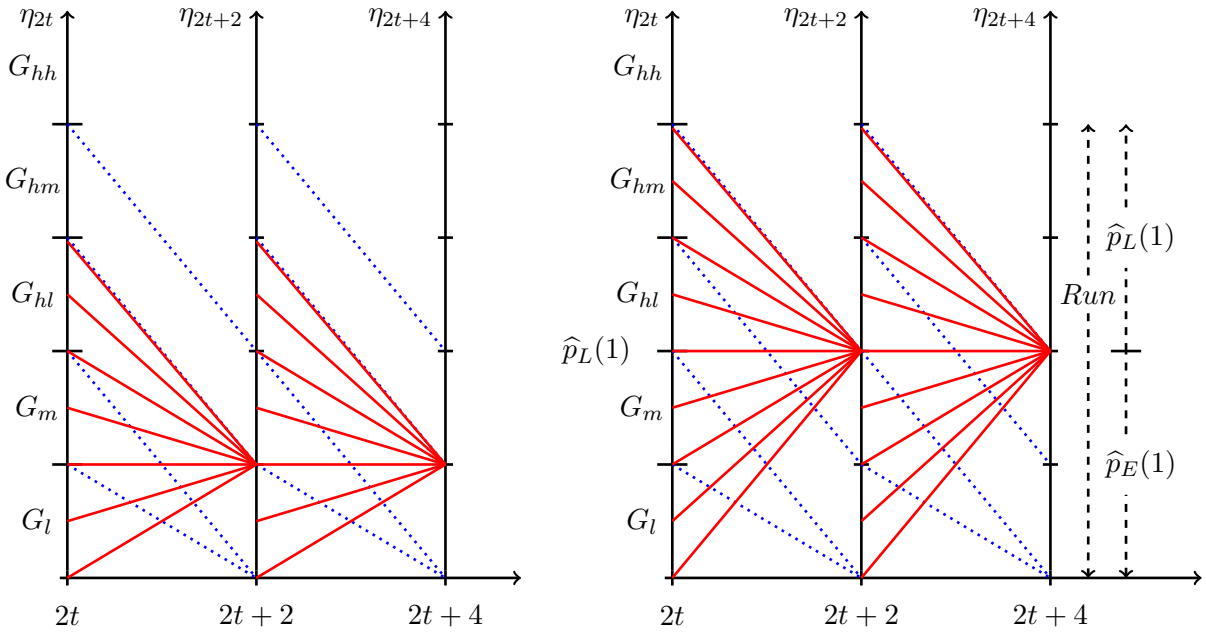


Figure 4: Equilibrium Cash Rebuilding Policies When $\theta = 0$ (Left) and $\theta = 1$ (Right)

The right panel of Figure 4 illustrates the equilibrium cash rebuilding policy when $\theta = 1$. To recap and better show the difference, I illustrate the equilibrium cash rebuilding policy when $\theta = 0$ on the left. As one can see, the equilibrium when $\theta = 1$ differs significantly from that when $\theta = 0$ because of the interdependence between shareholder runs and fund liquidity management.

Proposition 5 says that the fund starts to rebuild its cash buffer at a higher starting cash position, and it also rebuilds the cash buffer more rapidly, compared to the scenario without runs. Specifically, as long as the fund's initial cash position falls below the high-high region G_{hh}, it rebuilds its cash buffer until the next-stage cash-to-assets ratio η_{2t+2} reaches the cutoff between the intermediate region G_m and the high region G_h.

Although Proposition 5 still features the trade-off between current- and future-stage fire sales, this trade-off becomes more subtle in the presence of runs. By Corollaries 1, 2 and 4, runs in equilibrium result in less effective cash rebuilding (i.e., a lower effective selling price) on odd dates and more forced fire sales on even dates. Thus, when current-stage run risks are relatively high, the fund wants to choose a less rapid cash rebuilding policy. In contrast, when future-stage risk of fire sales is relatively high, in particular, when future-stage runs lead to more severe future-stage fire sales, the fund prefers a more rapid cash rebuilding policy.

Again, I discuss different cash-to-asset ratio regions to illustrate how shareholder runs interact with fund cash rebuilding in equilibrium.

First, suppose the fund starts from the low or the intermediate region (i.e., $\eta_{2t} \in G_l \cup G_m$). By Lemma 8, all of the late shareholders are going to run on date $2t$ (because $NAV_{2t} > NAV_{2t+1}$) in this case, which implies a lower effective selling price (for cash rebuilding) $\widehat{p}_L(1)$ on date $2t+1$. However, if the fund did not rebuild its cash buffer, it would end up in the low region in the next stage, where the fund would have to fire sell at an effective price $\widehat{p}_E(1)$. Because $\widehat{p}_L(1) > \widehat{p}_E(1)$, the risk of future fire sales is relatively larger. Hence, the fund still finds it optimal to rebuild its cash buffer on date $2t+1$ to avoid more costly fire sales on date $2t+2$, despite the runs on date $2t$.

However, different from the scenario when $\theta = 0$, when $\theta = 1$ the fund does not stop rebuilding its cash buffer even if the next-stage cash-to-assets ratio hits the cutoff between the low and the intermediate region. The reason is as follows. If the fund ended up in the intermediate region in the next stage (i.e., $\eta_{2t+2} \in G_m$), again by Lemma 8, all of the late shareholders in the next stage will run on date $2t+2$ too. Thus, the fund would be forced to fire sell its assets on date $2t+2$ at the effective selling price $\widehat{p}_E(1)$ even if starting in the intermediate region then. Fundamentally, future-stage runs lead to higher risk of future fire sales. As a result, the fund will keep rebuilding its cash buffer even when $\eta_{2t+2} \in G_m$.

In equilibrium, the fund manager will rebuild the cash buffer up to a point where η_{2t+2} hits the cutoff between the intermediate and the high region, which is higher than the counterpart when $\theta = 0$ (as shown in Proposition 5). As analyzed above, a lower cash target implies forced fire sales on date $2t+2$ at a lower effective selling price $\widehat{p}_E(1)$ and thus is not optimal. Also, a higher cash target becomes excessive despite runs in the next stage. Specifically, a higher cash target implies that the fund would end up in the high region in the next stage (i.e., $\eta_{2t+2} \in G_h$), where runs only lead to a lower effective selling price $\widehat{p}_L(1)$ on date $2t+3$. Since the game only has a less than one probability to continue, selling at the same effective price $\widehat{p}_L(1)$ on date $2t+1$ to build that excessive cash buffer is not profitable.

Second, suppose the fund starts from the high-low or the high-intermediate region (i.e., $\eta_{2t} \in G_{hl} \cup G_{hm}$). If the fund did not rebuild its cash buffer, it would end up in the low or the intermediate region (i.e., $\eta_{2t+2} \in G_l \cup G_m$), where there would be expected forced fire sales in the next stage. Since the risk of next-stage fire sales is still relatively higher, the fund still finds it optimal to suffer current-stage runs (i.e., to incur a lower effective selling price) and choose a more rapid cash rebuilding policy. For the same reason as discussed above, the fund still optimally rebuilds its cash buffer to the cutoff between the intermediate and the high region, and any more cash buffer would be excessive.

Lastly, I consider the high-high region (i.e., $\eta_{2t} \in G_{hh}$). Without cash rebuilding, the fund would end up in the high-low or high-intermediate region in the next stage (i.e., $\eta_{2t} \in G_{hl} \cup G_{hm}$) as long as the starting cash position is not sufficiently high.[49] As discussed above, all the next-stage late shareholders would decide to run on date $2t+2$ in those two regions, because the fund would rebuild its cash buffer on date $2t+3$. Would it make sense for the fund to rebuild its cash buffer on date $2t+1$ to prevent these runs on date $2t+2$? The answer is no in this case. This is because in the future stage the fund will be at least in the high region, where runs will not result in forced fire sales on date $2t+2$. Although

[49] If η_{2t} is sufficiently high, there can be $\eta_{2t+2} \in G_{hh}$ with $\sigma_{2t+1} = 0$. I can further divide region G_{hh} into three separate regions to make the argument more precise. But doing this adds little to provide new insights.

future-stage runs on date $2t+2$ still lead to a lower effective selling price $\widehat{p}_L(1)$ for cash rebuilding on date $2t+3$, the effective selling price on date $2t+1$ would also be $\widehat{p}_L(1)$ if the fund chose to rebuild the cash buffer and triggered runs on date $2t$. Since the game only has a less than one probability to continue, the risk of future-stage fire sales becomes relatively smaller while current-stage costs of runs become relatively larger. As a result, the fund optimally chooses not to rebuild its cash buffer in the high-high region.

Overall, compared to the scenario without runs (i.e., $\theta = 0$), Proposition 5 and the intuition above suggest that the trade-off in fund cash rebuilding becomes more complicated in the presence of runs. When the starting cash position is lower, future risk of fire sales (in particular, future-stage forced fire sales induced by future runs) is relatively higher, and thus the fund optimally chooses a more rapid cash rebuilding policy. On the contrary, when the starting cash position is higher, current-stage costs of runs are relatively higher, and thus the fund optimally chooses a less rapid cash rebuilding policy or does not rebuild the cash buffer at all.

Finally, it is helpful to contrast the equilibrium cash rebuilding policies here to those in the classic (s, S)-type inventory problem.[50] In the (s, S)-type inventory problems, a firm holds inventory because it helps the firm to meet future demand more easily. However, to accumulate inventory is also costly. It may incur additional adjustment costs, which are often assumed to be exogenous or following a quadratic form. When there are no runs, my fund liquidity management framework resembles a (s, S)-type inventory problem with a zero fixed cost and different variable costs on different dates. However, when the potential for runs is introduced, it features a novel cash inventory problem in which the cost structure is endogenously determined by shareholders' run decisions, which are in turn endogenously determined by the fund's cash rebuilding policy itself.

4.3 The General Scenarios

I proceed to characterize the fully dynamic equilibria in the general scenarios when $\theta \in (0, 1)$. In these general scenarios, the equilibrium cash policies and the resulting run behaviors become more involved. This is because when the shareholders have a moderate propensity to run, they become less sensitive to the fund's cash rebuilding than they would in the $\theta = 1$ scenario, while a sufficiently rapid cash rebuilding policy can still push them to run. This in turn shapes the fund's optimal cash rebuilding policy in equilibrium.

However, despite the complexity of the general scenarios, all the equilibrium results can be still unified under the same trade-off between current-stage runs and future-stage fire sales as discussed in Section 4.2. The formal result is stated in Proposition 6.

PROPOSITION **6.** *When $\theta \in (0, 1)$, there exist two endogenously determined thresholds $0 < \underline{\theta} < \overline{\theta} < 1$, such that*

i) if $\theta \in (0, \underline{\theta}]$, the equilibrium cash rebuilding policy is characterized by Proposition 4, that is, the

[50] See Stokey and Lucas (1989) for a textbook treatment of the classic (s, S)-type inventory problems and Strebulaev and Whited (2012) for a review on the modern applications of (s, S)-type problems to dynamic corporate liquidity management. None of the existing (s, S)-type problems considers a mutual fund context as I do.

cash rebuilding policy follows that in the scenario of $\theta = 0$,

ii) if $\theta \in (\underline{\theta}, \overline{\theta})$, *the equilibrium cash policies are characterized by*

a) if $\eta_{2t} \in G_l \cup G_m \cup G_{hl} \cup \underline{G_{hm}}$, *the fund chooses* $\sigma^*_{2t+1} > 0$ *such that*

$$\eta^*_{2t+2} = \eta(\check{\lambda}) \equiv \frac{(\mu_E + \check{\lambda}\mu_L)R}{1 - \mu_E - \check{\lambda}\mu_L},$$

where $\check{\lambda}$ *is given by*

$$\begin{cases} \lambda^*_{2t} > 0 & \textit{iff} \quad \eta_{2t} < \eta(\check{\lambda}), \\ \lambda^*_{2t} = 0 & \textit{iff} \quad \eta_{2t} \geqslant \eta(\check{\lambda}), \end{cases}$$

in which λ^*_{2t} *denotes the equilibrium run behaviors under the optimal cash rebuilding policy* σ^*_{2t+1}, *and*

$$\underline{G_{hm}} \equiv \left\{ \eta_{2t} | \eta_{2t} \geqslant \frac{(\mu_E + \mu_L)R}{1 - \mu_E - \mu_L} \textit{ and } \frac{\mu_E R}{1 - \mu_E} \leqslant \eta_{2t+2} < \frac{(\mu_E + \check{\lambda}\mu_L)R}{1 - \mu_E - \check{\lambda}\mu_L} \textit{ for } \sigma_{2t+1} = 0 \right\},$$

b) if $\eta_{2t} \in \overline{G_{hm}} \cup G_{hh}$, *then* $\sigma^*_{2t+1} = 0$, *where* $\overline{G_{hm}} = G_{hm}/\underline{G_{hm}}$,

iii) if $\theta \in [\overline{\theta}, 1)$, *the equilibrium cash rebuilding policy is characterized by Proposition 5, that is, the cash rebuilding policy follows that in the scenario of* $\theta = 1$.

Figure 5 illustrates the optimal cash rebuilding policies when θ varies. As suggested by Proposition 6, when θ is close to 0, the equilibrium is the same as that when $\theta = 0$, while when θ approaches 1 the equilibrium is the same as that when $\theta = 1$. As θ increases, shareholder runs spread to more cash-to-assets regions, and the fund also chooses a more rapid cash rebuilding policy in equilibrium to better avoid future-stage fire sales induced by future-stage runs. Figure 5 illustrates the scenarios with a moderate value of θ, in which the equilibrium is different from the two extreme scenarios when $\theta = 0$ and $\theta = 1$.

4.4 The Time-Inconsistency Problem

I illustrate the time-inconsistency problem associated with fund cash rebuilding by asking the following question. From Propositions 4, 5, and 6, it can be seen that for any θ and in any equilibrium path, the fund never allows its target of next-stage cash-to-assets ratio below the intermediate region G_m regardless of the starting cash position. Why? In other words, can there be any circumstances in which the fund finds it optimal to adopt a less rapid cash rebuilding policy such that the next stage game falls into the low region (i.e., $\eta^*_{2t+2} \in G_l$)?

This question is very valid in views of the trade-off between current runs and future fire sales. Especially, as suggested by Corollaries 2 and 4, more shareholder runs result in more severe current-stage forced fire sales when the fund starts from the low or the intermediate region. Why does not the fund choose a less rapid cash rebuilding policy to prevent current-stage runs and thus reduce those forced fire sale losses?

Proposition 7 gives an answer. It suggests that, in the absence of a commitment device, such a less rapid cash rebuilding policy as mentioned above will never appear in any equilibrium path. But it may

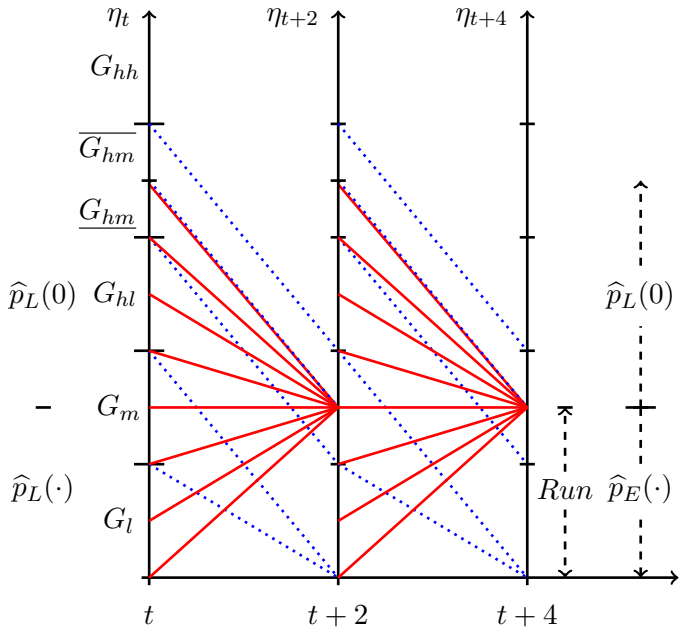


Figure 5: Equilibrium Cash Rebuilding Policy When $\theta \in (\underline{\theta}, \overline{\theta})$

indeed be optimal if the fund can credibly announce and commit to such a policy on date $2t$. Figure 6 illustrates this problem.

PROPOSITION **7.** *A cash rebuilding policy involving*

$$\eta^*_{2t+2} < \frac{\mu_E R}{1-\mu_E}$$

cannot happen in any equilibrium path unless the fund is able to credibly commit to such a policy.

The intuition behind Proposition 7 is a time-inconsistency problem, which aggravates the tension in choosing between a rapid or a slow cash rebuilding policy. Starting from the low region or the intermediate region, the fund indeed has a relatively large current-stage cost of shareholder runs because they lead to severe current-stage forced fire sales. Thus, on date $2t$, the fund may wish to commit itself to rebuilding its cash buffer less rapidly on date $2t+1$ to reduce such run risks on date $2t$. However, on date $2t+1$, because all the date-$2t$ costs of runs are sunk, the fund may instead be tempted to adopt a more rapid cash rebuilding policy on date $2t+1$. Importantly, what matters for shareholders' run decisions on date $2t$ are their beliefs about the fund's cash rebuilding policy on date $2t+1$. In equilibrium, they can always anticipate the fund manager's date-$2t+1$ temptation to rebuild the cash buffer more rapidly, and thus will always have strong incentives to run on date $2t$. Mathematically, the intuition outlined above can be also seen from the dynamic equilibrium definition (Definition 2) and in particular from the Bellman equation (4.1) in the non-commitment benchmark.

Proposition 7 suggests a fundamental difficulty in reducing fund shareholder runs in practice, in which a commitment device can be hard to implement. In the mutual fund context, shareholders decide to run not only because they expect other shareholders to run at the same time, but more importantly because they expect the fund to rebuild its cash buffer in the future, which gives rise to the predictable

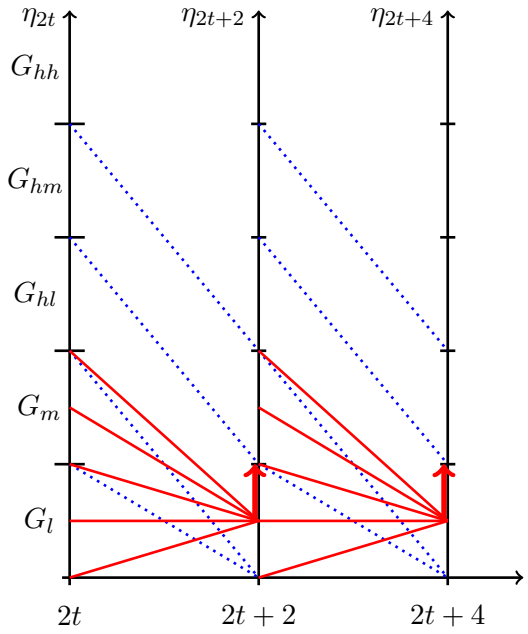


Figure 6: The Time-Inconsistency Problem

NAV and thus the run incentives. As will be shown in Section 5, policies that are effective in preventing bank runs may fail in preventing fund shareholder runs, because they are not designed taking into account the dynamic interdependence of shareholder runs and fund liquidity management.

4.5 Expected Total Fire Sale Losses

Finally, I show in Proposition 8 that the potential for shareholder runs can lead to unambiguously higher total fire sale losses ex-ante, regardless of the fund's initial portfolio position. This occurs in a world where both the fund manager and the shareholders are rational, and the fund's cash rebuilding policy is optimal. It suggests that the potential financial stability risks induced by mutual fund shareholder runs can be significant and thus should not be overlooked.

PROPOSITION **8.** *When θ increases, the ex-ante total fire sale losses become higher for any positive starting portfolio position (a_{2t}, x_{2t}).*

As suggested by Proposition 7, the lack of a commitment device contributes to the occurrence of run problems despite optimal liquidity management by the fund. I show in Proposition 9 that introducing a commitment device can indeed help reduce total fire sale losses in expectation by tempering shareholders' run incentives.

PROPOSITION **9.** *When the fund can pre-commit to a cash policy s_{2t+1} on date $2t$, the ex-ante total fire sale losses become lower for any positive (a_{2t}, x_{2t}) and any $\theta > 0$.*

Intuitively, introducing a commitment device helps reduce total fire sales through two ways. On the one hand, as suggested by Proposition 7, since the fund is able to pre-commit to a less rapid cash rebuilding policy, it can directly reduce current-stage forced fire sales by reducing shareholder runs.

On the other hand, from a dynamic perspective, the risk of future-stage fire sales also becomes lower thanks to less severe future runs, and thus the fund is also more comfortable in choosing a less rapid cash rebuilding policy by selling assets less aggressively in the current stage.

5 Policy Implications

Many regulators and practitioners have proposed fund-level policies, aiming at mitigating potential financial stability risks of open-end mutual funds. By documenting the dynamic interdependence between shareholder runs and fund liquidity management, my model delivers new policy implications. Rather than making definitive policy prescriptions, I emphasize how the mechanism in this model adds new dimensions to the current policy debates.

5.1 MMF Reforms

First of all, my model contributes to the recent debate about MMF reforms. A major proposal of reforming the MMFs is to adopt floating NAV accounting, which will effectively make MMFs like regular open-end mutual funds.[51] However, as discussed by Hanson, Scharfstein and Sunderam (2015), flexible NAV adjustment may not be a fix to run problems on MMFs. My formal model also suggests the same view. In particular, MMFs adopting a flexible NAV are no longer prone to an abrupt "breaking-the-buck," but will be prone to a new type of shareholder runs as I have shown. This is in particular relevant in bad times when funds' cash positions are low while redemption shocks are large and persistent.

In the following, I turn to several other fund-level policies that are specific to regular open-end mutual funds. I show that, perhaps surprisingly, some of them are less effective than commonly thought in mitigating potential financial stability risks of mutual funds. The key insight is again the dynamic dependence between runs and liquidity management.

5.2 Liquidity Requirements

I first discuss the proposal of imposing liquidity requirements on open-end mutual funds. Although no formal liquidity requirements have been proposed to mutual funds, many have been imposed on banks and MMFs by the U.S. SEC[52] and the International Regulatory Framework for Banks (Basel III)[53] since the past financial crisis. Recently, the U.S. SEC voted 5-0 on Sept 22, 2015 to approve a new proposal requiring mutual funds to better manage their liquidity risks. Also, the IMF and the BIS have both proposed potential stress tests for mutual funds in 2015,[54] which resemble fund liquidity

[51] This proposal has been introduced by the SEC in 2013 in amendments to its 2a-7 rules and then formally adopted in 2014. It applies to institutional prime money market funds. See Hanson, Scharfstein and Sunderam (2015) for a comprehensive discussion.

[52] In 2010, the U.S. SEC changed the 2a-7 rules to institute overnight and weekly liquidity thresholds for MMFs. Specifically, the U.S. SEC mandated that any MMF holds at least 10% of their assets in securities maturing within one business day and at least 30% of assets in securities maturing within one week.

[53] The Basel III accord of 2010 and 2011 require banks to hold a certain amount of high-quality liquid assets to meet expected outflows. Given typical projections of outflows, the threshold for retail banks is set to be 3% of stable retail deposits.

[54] "IMF Seeks Stress Tests for Asset Managers," The *Financial Times*, April 8, 2015, and "Asset Managers' Push into Bonds Prompts Regulatory Scrutiny," The *Financial Times*, June 2, 2015.

requirements. These developments suggest that formal liquidity requirements for mutual funds are likely to be introduced in the future, given the increasing concerns about potential financial stability risks.

My model offers a new building block to help assess the effectiveness of liquidity requirements for mutual funds. From a bank run perspective, Vives (2014) and Diamond and Kashyap (2015) are among the first attempts to evaluate the effectiveness of these new liquidity requirements in mitigating bank runs. Interestingly, as I have already shown, in a bank context, selling illiquid assets to meet liquidity requirements would not induce runs per se as long as the bank is solvent, but doing so can indeed lead to shareholder runs in a mutual fund context. This suggests that designing any appropriate mutual fund liquidity requirements needs to take account of the dynamic interdependence of shareholder runs and fund liquidity management.

Specifically, the results in Proposition 6 suggest state-contingent and fund-specific optimal cash targets for mutual funds. On the one hand, when persistent redemption shocks are likely to occur in the future, mutual fund managers should be warned about the possibility of future shareholder runs. Hence, a more stringent liquidity requirement is preferred. This ensures that when future redemption shocks realize, mutual funds will have higher cash-to-assets ratios and thus can better avoid severe fire sales and runs.

On the other hand, when persistent redemption shocks have already hit the economy as my model describes, the optimal liquidity requirement design should be more fund-specific. For funds whose shareholders have a lower propensity to run (i.e., θ is lower), a more stringent liquidity requirement is likely to help the fund better avoid future fire sales. However, for funds whose shareholders have a higher propensity to run (i.e., θ is higher), a less stringent liquidity requirement as suggested by Proposition 7 is likely to be appropriate, which can better mitigate the fund's time-inconsistency problem and thus reduce shareholder runs.

Of course, the design of any implementable fund liquidity requirements calls for a more comprehensive assessment of other relevant factors. At the bottom line, the results of my model suggest that the one-size-fits-all liquidity requirements as those currently designed for banks and MMFs are unlikely to be appropriate for mutual funds.[55] Instead, the dynamic interdependence of shareholder runs and fund liquidity management should be better taken into account.

5.3 Redemption Fees

The second policy proposal is to increase or eliminate the cap on redemption fees. Open-end mutual funds can charge their shareholders redemption fees when they redeem their shares. Currently, the SEC requires mutual fund redemption fees to be lower than 2%.[56] Therefore, some observers argue that to increase or eliminate the cap, at least in crisis times, is likely to mitigate potential financial stability risks of mutual funds.

My model suggests that higher redemption fees may help reduce shareholder runs. Suppose $1-\kappa$ of the redemption proceeds are collected as redemption fees, where $\kappa \in (0,1)$. Thus, any shareholder

[55] The recent work of Diamond and Kashyap (2015) suggests that those one-size-fits-all liquidity requirement rules may not be optimal for banks as well, but for different reasons.

[56] This is according to Rule 22c-2 of the Investment Company Act of 1940.

who redeems on date t only gets κNAV_t.[57] Also, redemption fees are paid back directly to the fund, implying that the fund can save $(1-\kappa)NAV_t$ cash per share redeemed. To better contrast to the baseline model without redemption fees, I consider $\theta = 1$, that is, when the shareholders' propensity to run is the highest. The following proposition shows that the introduction of redemption fees can lead to less shareholder runs in equilibrium.

PROPOSITION **10.** *For any given starting portfolio position (a_{2t}, x_{2t}), any feasible cash rebuilding policy s_{2t+1}, and any proportional redemption fee $1-\kappa > 0$, there is $\lambda_{2t}^{\kappa} \leqslant \lambda_{2t}$, where λ_{2t}^{κ} is the equilibrium run probability in the game with the redemption fee while λ_{2t} is that in the game without redemption fees, all other things being equal.*

When the stage game starts from the high cash-to-assets region, redemption fees have a stronger effect. In contrast to the baseline model where any cash rebuilding (i.e., any $s_{2t+1} > 0$) leads to shareholder runs when $\theta = 1$, with the redemption fee there can be completely no runs in equilibrium when s_{2t+1} is small, that is, when the fund only sells a few assets to rebuild its cash buffer.

COROLLARY **3.** *For any given starting portfolio position (a_{2t}, x_{2t}) satisfying $\eta_{2t} \in G_h$ and any proportional redemption fee $1-\kappa > 0$, there exists a strictly positive $\overline{s} > 0$ such that $\lambda_{2t}^{\kappa} = 0$ constructs the unique equilibrium when $s_{2t+1} \leqslant \overline{s}$.*

Proposition 10 and Corollary 3 suggest that redemption fees can directly reduce shareholders' run incentives. Intuitively, with redemption fees, redeeming shareholders effectively get a value lower than the prevailing NAV, implying a wealth transfer from redeeming shareholders to staying ones. Moreover, in any stage, redemption fees allow the fund to save more cash proportionally, making it easier to meet redemption needs without incurring fire sales.

However, as suggested by Proposition 10, redemption fees do not directly alter the dynamic interdependence between runs and fund liquidity management. They cannot solve the time-inconsistency problem associated with the fund's dynamic cash rebuilding policy either. Only when the fund imposes a 100% redemption fee, or equivalently, chooses to be closed-ending, can shareholder runs be completely prevented. But closed-end mutual funds cannot provide liquidity service to their shareholders (Stein, 2005), and thus it will be obviously not optimal to push all the mutual funds to be closed-ending just for preventing runs.

Redemption fees may also be less effective in practice for the following reasons. First, in my model, redemption fees are introduced ex-ante. However, if redemption fees are first introduced on an odd date $2t+1$ but are expected on the previous date $2t$, the late shareholders will have higher incentives to run to avoid the fees. This represents a real-world concern that imposing higher redemption fees by itself can lead to one-time market turmoil. Other unmodeled but plausible reasons include negative effects on future fund share sales and on the reputation of fund managers. In practice, many funds still stick to zero or lower than 2% redemption fees even when market volatility or outflows are high (Nanda,

[57] According to Rule 22c-2, the U.S. SEC prohibits discriminative redemption fees solely conditional on shareholder identities as those would effectively create classes of shareholder seniority. This implies that, in my model, the fund cannot intentionally impose different redemption fees on early and late shareholders.

Narayanan and Warther, 2000), suggesting that to increase or eliminate the 2% cap on redemption fees may only have limited effects.

5.4 In-Kind Redemptions

By nature, open-end mutual funds can satisfy redemption requests by delivering a portion of the underlying basket of assets invested, including cash, which is known as "in-kind redemptions." Many practitioners argue that the option to elect to in-kind redemptions can largely mitigate any financial stability risks of mutual funds, at least during crisis times. Are in-kind redemptions really a relief?

My model suggests that in-kind redemptions can be very effective in preventing fund shareholder runs within a fund, but perhaps surprisingly, they do not necessarily help reduce total fire sale losses or improve total shareholder welfare. The following proposition offers a sufficient condition for the episodes in which the negative effects of in-kind redemptions dominate.

PROPOSITION **11.** *Electing to in-kind redemptions completely prevents shareholder runs, that is,* $\lambda(a_{2t}, x_{2t}) = 0$ *for any* (a_{2t}, x_{2t}). *However, when* θ, μ_L *are sufficiently small and* δ_L *is sufficiently larger than* δ_E, *in-kind redemptions lead to higher total fire sale loss ex-ante than a counterfactual in which the fund sticks to cash redemptions, all other things being equal.*

The intuition behind Proposition 11 relies on three progressive reasons. First, adopting in-kind redemptions completely eliminates any run incentives. This is because late shareholders always get the same basket of assets regardless of the time they redeem, and they would have to sell the illiquid assets at a lower price p_E for consumptions if they ran, so they prefer not to run. Second, since the fund manager only cares about total fire sale losses at the fund level, liquidity management becomes irrelevant. In other words, the fund will never rebuild its cash buffer, and the initial cash-to-assets ratio η_0 will never change. Third, early shareholders have to fire sell the illiquid assets they get at the extremely low price p_E for consumptions. These fire sale losses could have been avoided if the fund manager actively managed its cash buffer. If these fire sale losses are significant, shareholders will become worse-off than the counterfactual with cash redemptions.

At a fundamental level, Proposition 11 suggests that in-kind redemptions are not a free lunch even during crisis times, because shareholders who ask their fund to elect to in-kind redemptions effectively give up any benefit they could get from active liquidity management by the fund. This point again highlights the dynamic interdependence between shareholder runs and fund liquidity management. In addition, given that in-kind redemptions are obviously costly during normal times since they discourage the sales of shares, the overall benefit of adopting in-kind redemptions can be even more ambiguous. Moreover, in reality, in-kind redemptions can be hard to implement; they are seldom used for both legal and practical reasons.[58]

Furthermore, the analysis about in-kind redemptions here sheds new light on the potential financial stability risks of exchange-traded funds (ETFs). Observers may argue that ETFs should be immune

[58]Rule 18f-1 of the Investment Company Act of 1940 only enables mutual funds to limit in-kind redemptions, which implies that in-kind redemptions will not be effected unless specific approval is first obtained from the SEC. This rule is intended to facilitate mutual fund share sales in jurisdictions where in-cash redemptions are required.

to shareholder runs because they are not directly subject to outflows from shareholders. Specifically, unlike open-end mutual funds, ETF sponsors can issue and redeem shares only with market-making firms known as authorized participants (APs). In other words, APs are the only market player who can directly redeem ETF shares by trading with fund sponsors. In particular, transactions between an ETF sponsor and an AP are typically settled in-kind, where the AP delivers or receives a basket of assets almost identical to the ETF's holdings,[59] known as the creation/redemption basket. Therefore, by the logic underlying Proposition 11, there will be no direct shareholder or AP runs on ETFs, and ETFs hold less cash buffer than their mutual-fund counterparts do.[60] However, when APs redeem the shares they have bought (presumably at a discount to fund NAV) and get the underlying assets, they are less likely to hold them on their balance sheets for a long time. Rather, they typically sell the underlying assets immediately, trying to lock in the arbitrage profits. This mechanism, known as the AP arbitrage mechanism of ETFs, plays an important role to keep ETF prices as close as possible to fund NAVs. But as suggested by Proposition 11, this may potentially lead to more fire sales (by APs) of the underlying illiquid assets and potentially larger price impacts in the underlying markets. Overall, this analysis suggests that ETFs may also generate previously overlooked financial stability risks, the mechanism of which can be unified under the dynamic interaction between shareholder runs and fund liquidity management.[61]

5.5 Redemption Restrictions

A similar emergency rule is redemption restrictions, which give a fund the right to suspend redemptions in given periods as permitted by regulators, for example, the U.S. SEC.[62] Can redemption restrictions prevent shareholder runs?

In my framework, I model redemption restrictions by assuming that the fund is able to deny any individual shareholder's redemption request on any date with probability $1-\zeta$, $\zeta \in (0,1)$. To better contrast to the baseline model without redemption fees, I also consider $\theta = 1$ when the shareholders' propensity to run is the highest. The following proposition characterizes the nature of this game with redemption restrictions.

PROPOSITION **12.** *For any given starting portfolio position (a_{2t}, x_{2t}) and any redemption restriction $1-\zeta > 0$, there is $\lambda_{2t}^{\zeta} \leqslant \lambda_{2t}$, where λ_{2t}^{ζ} is the equilibrium run probability in the game with the redemption restriction while λ_{2t} is that in the game without redemption restrictions, all other things being equal.*

Proposition 12 suggests that redemption restrictions can help reduce shareholder runs. Interestingly, introducing redemption restrictions closely resembles the introduction of redemptions fees as analyzed

[59] There are a few exceptions where cash transactions may be required. This is because some ETF holdings, such as some specific emerging market assets, are subject to legal restrictions that prevent in-kind transactions.

[60] The fact that ETFs on average hold less liquidity buffer than open-end mutual funds do is documented by new empirical evidence (Ben-David, Franzoni and Moussawi, 2014). This allows ETFs to better track the underlying benchmark while keeping a minimal tracking error.

[61] In a complementary paper, Pan and Zeng (2015) explores this AP arbitrage mechanism of ETFs in greater detail and documents the financial stability risks that arise from this mechanism.

[62] According to Rule 22e of the Investment Company Act, an open-end mutual fund is generally prohibited from suspending the right or redemption or postponing the payment of redemption proceeds for more than seven days. However, the SEC has the right to deem emergency periods during which a fund is able to suspend redemptions.

in Proposition 10 and Corollary 3. The intuition for Proposition 12 is clear, because by the Law of Large Numbers, only ζ of the redeeming shareholders can get cash out of the fund. Therefore, there will be effectively fewer redemptions. But like the introduction of redemption fees, the introduction of redemption restrictions cannot fully prevent shareholder runs or solve the time-inconsistency problem associated with fund liquidity management.

In addition, my model suggests that redemption restrictions can be indeed hard to implement in reality, given that they have to be introduced at the discretion of regulators. Due to the "slow-moving" nature of shareholder runs, no abrupt bankruptcy or events like "breaking-the-buck" by MMFs can be observed. Even in scenarios where run-induced fire sales are extremely severe, as suggested by Proposition 6, the fund can still keep a positive cash-to-assets ratio in each stage, making it hard for the regulatory authority to deem such periods emergent. In practice, unsurprisingly, the U.S. SEC has seldom deemed a period to be an emergency to allow open-end mutual funds to use redemption restrictions. One and the only recent example for the SEC to permit redemption restrictions happened in 2008 for the Reserve Primary Fund,[63] which was an MMF.

5.6 Credit Lines

Although my baseline model focuses on a crisis management scenario during which there are no net inflows, mutual funds may turn to pre-established credit lines to raise cash. For example, in 2015 BlackRock increased the amount that its mutual funds can collectively borrow to meet redemptions to $2.1 billion, a historically high level.[64] Can fund credit lines prevent shareholder runs?

My model suggests that using credit lines may temporarily mitigate the negative effects of current-stage shareholder runs, but can induce more severe fire sales and runs in the future. Specifically, in stage t, suppose the fund uses pre-established credit lines (rather than selling assets) when it is in the low or the intermediate cash-to-assets region. Thus, the fund does not have to fire sell any illiquid assets in meeting redemptions on dates $2t$ and $2t+1$. As a result, the NAV will not change within stage t, that is, $NAV_{2t+1} = NAV_{2t} = NAV_{2t-1}$, and thus $(\lambda_E + \lambda_L)n_{2t}$ shareholders leave the fund with such an intact NAV. However, in the next stage (if the game continues) the fund will have no cash to start (i.e., $\eta_{2t+2} = 0$). If the credit lines have a sufficiently long maturity, the fund does not have to pay back the debts immediately, but have to face more severe fire sales unless it can borrow more. What is worse, if the fund is required to pay back its debts first on date $2t+2$, it will have to fire sell even more illiquid assets or simply default. Intuitively, using credit lines makes a fund' life easier temporarily, but the fund also forgoes the option value of cash rebuilding, which is higher when the redemption shocks are more persistent (i.e., π is smaller). This idea resembles that outlined for in-kind redemptions in Section 5.4, suggesting that it will be naive to shut down a fund's active liquidity management when attempting to prevent runs.

Moreover, credit lines may expose a fund back to the risk of debt runs as suggested by He and Xiong (2012) and by other bank run models. This is in particular relevant when the redemption shocks are

[63] See Investment Company Act Release No. 28,487.

[64] "BlackRock Leads Funds Raising Credit Lines Amid Review," The *Bloomberg Business*, January 21, 2015.

persistent so that the fund has to repeatedly turn to credit lines established with multiple creditors or to rollover existing debts. In a crisis management scenario, creditor banks may also be subject to aggregate risks, making credit lines riskier and less reliable than cash buffers (Acharya, Almeida and Campello, 2013). Thus, having credit lines is hardly a relief for open-end mutual funds.

5.7 Swing Pricing

Some observers argue that swing pricing, which allows current NAVs to reflect commissions to asset brokers and dealers, bid-ask spreads, taxes, and other trading-related charges, can reduce the negative externalities imposed by redeeming shareholders on non-trading ones. A growing number of open-end mutual funds has been adopting swing pricing, while as of September 2015 the use of swing pricing is still voluntary and not required by the U.S. SEC.[65] Will full swing pricing prevent shareholder runs?

My model suggests that the answer is no. In fact, swing pricing, in its currently observed form, has already been incorporated into my baseline model, This is because flow-induced fire sales are the only type of trading-related costs in the model, and current NAVs have already taken them into account. However, my model suggests that, since they still do not incorporate future asset sale costs, they are not able to mitigate the risk of runs induced by active fund liquidity management. In this sense, my theory identifies a form of negative externality that even introducing swing pricing cannot internalize.

Rather than swing pricing in its current form, my model suggests that forward-looking NAVs may help reduce shareholder runs. This is equivalent to requiring shareholders to contract on future NAVs directly. However, from the perspective of market incompleteness, shareholders cannot fully contract on future NAVs because mutual funds promise to provide daily liquidity service to their shareholders. In other words, if shareholders instead contracted on future NAVs and they had common and rational beliefs on future NAVs, they would effectively go back to "separate accounts," or equivalently direct holdings of the underlying assets by the shareholders, and there will be no liquidity service provided by the funds. In this sense, there is no point to have a mutual fund in the first place. As a result, forward-looking NAV rules may be hard to implement in reality, and runs can be viewed as a cost that shareholders have to bear to have mutual funds engage in liquidity transformation.[66] To investigate optimal mechanism design for liquidity provision in a mutual fund context (like Green and Lin, 2003 and Peck and Shell, 2003 in a bank context) is beyond the scope of this paper, and thus I leave it for future research.

[65] "Fund Investors May Pay Fees for Withdrawals Amid Turmoil," The *Bloomberg Business*, September 11, 2015.

[66] One can go further along this line to ask why mutual funds provide liquidity. Gorton and Pennacchi (1990) argue that banks and bank-like financial intermediaries are the best candidate to provide liquidity because debt value is the least sensitive to asset-side value fluctuations. But as suggested by Stein (2005), in competing for funding, mutual funds are also eager to provide liquidity (i.e., to adopt open-ending) to shareholders through the means of equity. To make such liquidity appealing to shareholders, mutual funds invest in higher-yielding but illiquid assets with the help of deliberate cash management, effectively engaging themselves in liquidity transformation. On the one hand, this logic suggests that mutual funds are not a simple pass-through; they are not as "plain-vanilla" as practitioners usually argue. On the other hand, more importantly, as fund shareholders get the promised liquidity at the flexible NAV, their liquidity is more sensitive to asset-side fluctuations. This nature makes mutual funds immune to the usual notion of debt runs that stem from fixed claims, but prone to a new notion of equity runs that result from asset-side value adjustment.

6 Extensions

My baseline model parsimoniously captures the novel interdependence between shareholder runs and fund liquidity management. The underlying mechanism is fairly general and robust to other aspects which may either aggravate or mitigate mutual fund financial stability risks. Here I explore several extensions, one at a time, with a focus on their interactions with the main mechanism of the baseline model.

6.1 Flow-to-Performance Relationship

The baseline model assumes random redemption shocks, but the realized population of redeeming shareholders in each stage is taken as exogenous. One may argue that future fund flows are likely to be positively correlated with past returns, known as the flow-to-performance relationship. Earlier research finds that future flows mostly respond to past good performance (Ippolito, 1992, Sirri and Tufano, 1998), but recent evidence suggests that they also respond to bad performance in particular when the underlying assets are illiquid (for example, Spiegel and Zhang, 2013, Goldstein, Jiang and Ng, 2015). Does such flow-to-performance relationship interact with fund shareholder runs?

My model suggests that, in the presence of shareholder runs, introducing the flow-to-performance relationship implies higher total expected fire sale losses despite optimal cash management by the fund. To see this, I can incorporate the flow-to-performance relationship into my baseline model. For any stage, I define the fund return as

$$r_{2t+1} = \frac{NAV_{2t+1}}{NAV_{2t}},$$

which is positive but no greater than one in my baseline model.[67] I then assume that in any stage t the populations of early and late shareholders are $\gamma_{2t}\lambda_E n_{2t}$ and $\gamma_{2t}\lambda_L n_{2t}$ for the even date $2t$ and the odd date $2t+1$, respectively, where $\gamma_{2t}(r_{2t-1}) \geqslant 1$ for $t > 1$ is a decreasing function of r_{2t-1} satisfying $\gamma_{2t}(1) = 1$, and $\gamma_0 = 1$. This implies that, if current fund return is lower, there will be more shareholders redeeming in the next stage if the game continues, capturing the flow-to-performance relationship.

In this extended setting, the flow-to-performance relationship does not directly alter shareholders' run incentives in any stage game, but will complicate the tension in choosing between a rapid or slow cash rebuilding policy by the fund manager. This can be seen from Proposition 5. Suppose the fund starts from the joint region $G_l \cup G_m \cup G_{hl} \cup G_{hm}$ where it is optimal to sell some illiquid assets to rebuild the cash buffer (i.e., $\sigma^*_{2t+1} > 0$). When the flow-to-performance relationship is introduced, σ^*_{2t+1} suggested by Proposition 5 is no longer optimal. To see this, notice that $\sigma^*_{2t+1} > 0$ implies $r_{2t+1} < 1$ and then $\gamma_{2t+2} > 1$. As a result, the fund either has to increase σ_{2t+1} to prevent more severe future fire sales due to a larger population of redeeming shareholders in the next stage, or to decrease σ_{2t+1} to sustain a higher current fund return but suffer higher risk of future fire sales. Either way, the fund incurs higher risk of shareholder runs and higher total expected fire sale losses as well.

At a fundamental level, the extended setting suggests a new amplification mechanism to explain fund performance persistence in bad times. The flow-to-performance relationship first implies that

[67] But it can be larger than one in the model with redemption fees or redemption restrictions.

it is harder for the fund to manage its cash buffer. Due to the interdependence of shareholder runs and fund liquidity management, this further suggests more severe runs and fire sales, leading to worse performance. Only those funds with a sufficiently high cash-to-assets ratio are likely to withstand these hard times without incurring shareholder runs and fire sales.

6.2 Asset Price Correlations

In the baseline model, flow-induced fire sales will not impact the market prices of the non-traded assets that still remain on the fund's balance sheet. This is realistic given that mutual funds invest in many different illiquid assets, and flow-induced fire sales only have local and temporary price impacts (Coval and Stafford, 2007). One may argue that asset prices can be correlated with each other, and the fund manager may want to use alternative accounting rules such as matrix pricing for these non-traded assets. Will those differences have any qualitative effects on shareholder runs?

My model suggests no. To see why, I assume that asset prices are perfectly correlated (while still keeping the realistic assumption that the price impacts induced by fire sales are temporary; they only last for one stage). This can be effectively viewed as a world with only a single illiquid asset. In this alternative setting, if the fund sells some assets on date t at the fire sale price p_t, the end-of-day flexible NAV will be:

$$NAV_t = \frac{x_t + (a_t - a_{t+1})p_t + a_{t+1}p_t}{n_t}.^{68} \tag{6.1}$$

Clearly, the only difference between (6.1) and the baseline model NAV (2.1) is the last term in the numerator, which reflects the fact that the market prices of non-traded assets are also updated to p_t, the temporary fire-sale price, in this extended setting. In both (2.1) and (6.1), the NAV is flexible in the sense that it takes into account all the same-day price impact and asset sale losses, while it is not forward-looking in the sense that it will not reflect any possible future price impacts and asset reallocation costs. By similar analysis as that in Section 3, as long as these contractual features of fund NAV are present, fund cash rebuilding still gives rise to a predictable decline in NAV and thus the run incentives. As a result, introducing asset price correlations or alternative accounting rules at the fund level would not change my results qualitatively.[69]

6.3 Persistent Price Impacts

Although the price impacts induced by fire sales tend to be temporary, in some asset classes there can be more persistent or even long-term price impacts. For example, in illiquid asset markets where dealers actively manage their inventory, a higher inventory level implies a higher price concession to compensate for their inventory risks. Would persistent price impacts change shareholders' run behaviors?

To answer this question, I can incorporate persistent price impacts into the baseline model. I illustrate that there can be more severe shareholder runs and fire sales in equilibrium in this extended

[68] To be more precise, in this case the fire sale price p_t will be assumed to be a decreasing function of the amount of fire sales within the given date, and the slope of this intra-day downward-sloping demand curve is steeper on an even date than that on an odd date.

[69] Depending on how correlated asset prices are, the run incentives can be quantitatively different in equilibrium. This quantitative difference is not crucial for the key mechanism of this model.

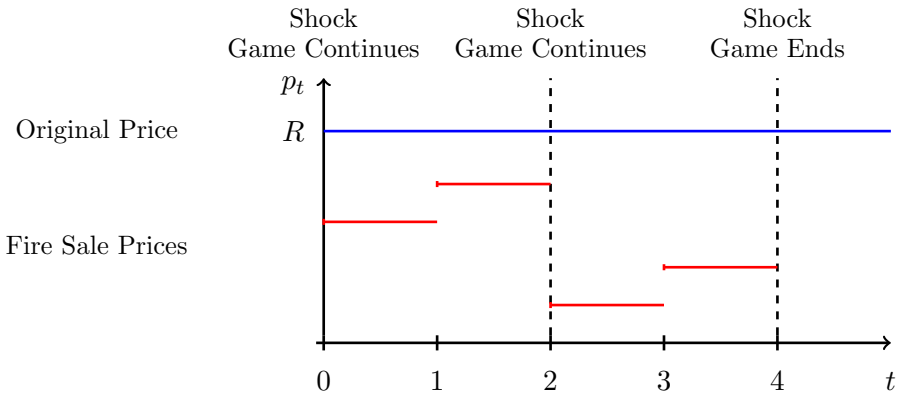


Figure 7: Price Path with Persistent Price Impacts

setting, despite optimal cash management by the fund.

Specifically, I assume that in stage t, the fire sale price on the even date $2t$ is $\beta^t \delta_E R$, while that on the odd date $2t+1$ is $\beta^t \delta_L R$, where $\beta \in (0,1]$. When $\beta = 1$, this goes back to the baseline model. Figure 7 illustrates a sample path of selling prices with persistent price impacts. There are two important ingredients of this price pattern. First, in the short run (i.e., within each stage), it still features temporary price overshooting and subsequent reversal. This is consistent with the baseline model.[70] Second, in the long run (i.e., across different stages) but before the game ends, the selling prices for the illiquid assets become lower over time.

Like the analysis in Section 6.1, persistent price impacts do not directly change shareholders' run incentives, but will significantly change the fund's dynamic optimal cash rebuilding policy, which in turn alters shareholders' equilibrium run behaviors. Again, this can be seen from Proposition 6. In the baseline model, the fund rebuilds its cash buffer on the odd date $2t+1$ to prevent forced fire sales on the next even date $2t+2$. The fund is not worried about potential asset sales on the next odd date $2t+3$ because of the same asset selling price there. However, in the extended setting with persistent price impacts, the asset selling price on the next odd date $2t+3$ will become lower, which gives rise to an incentive for the fund to sell more assets on date $2t+1$. Therefore, by the results in Propositions 1, 2 and 13, more rapid cash rebuilding unambiguously leads to more severe run problems and potentially higher total fire sale losses in expectation.

This extension has two implications. On the one hand, it suggests that shareholder runs and the dynamic interdependence between runs and fund liquidity management do not rely on the existence of persistent price impacts described here. In other words, there can be shareholder runs in equilibrium without persistent price impacts. On the other hand, the existence of persistent price impacts can make the run problems more severe. Given that persistence price impacts can indeed exist in some illiquid asset markets, this extension suggests that the concerns about shareholder runs can be indeed very relevant.

[70] Note that, this extended setting still keeps the structure of one shock per two dates, and thus the micro-foundation in Appendix A.3 is still valid.

7 Conclusion

In this paper, I build a model of an open-end mutual fund with a flexible NAV, and show that shareholder runs can occur in equilibrium despite optimal liquidity management by the fund. With a flexible NAV, fund cash rebuilding by selling illiquid assets implies a predictable decline in NAV and thus a first-mover advantage, leading to runs. The presence of shareholder runs further complicates the fund's efforts in liquidity management, leading to higher total fire sale losses in expectation. Hence, appropriate design of policies aiming for mitigating financial stability risks of mutual funds should take into account the dynamic interdependence of shareholder runs and fund liquidity management.

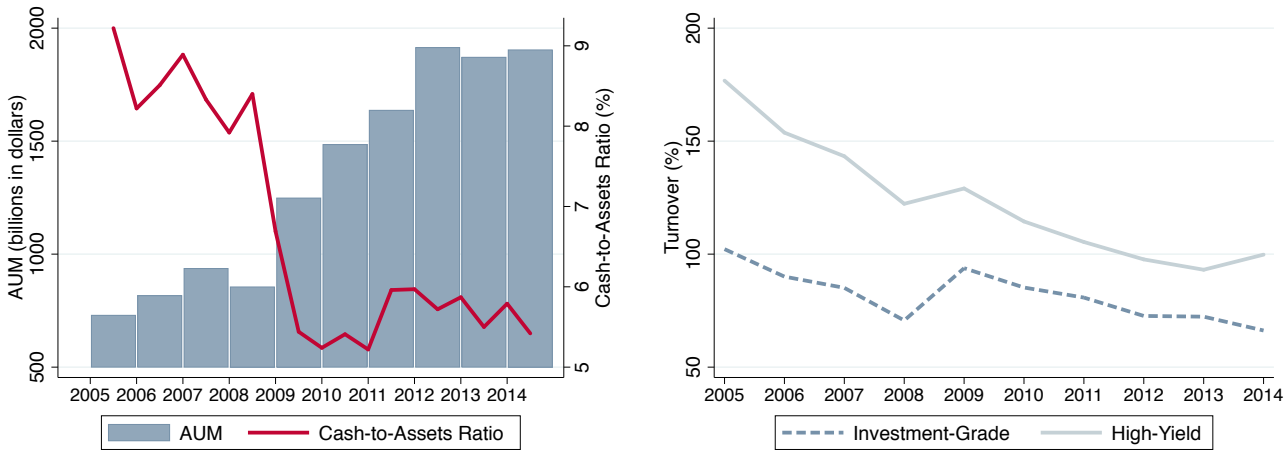
At a fundamental level, shareholder runs are driven by a key contractual property of mutual fund NAVs: they are flexible but not forward-looking. Specifically, the NAV at t does not take into account the predictable asset sales and price impact at $t+1$. This contractual property comes from a form of market incompleteness that shareholders cannot fully contract on future NAVs. This property implies that cash rebuilding can give rise to predictable declines in NAV and thus the potential for runs.

Finally, my model sheds new light on potential systemic risks posed by mutual funds. As mutual fund runs can lead to more fire sales, the underlying asset markets may become even more illiquid. As suggested by Stein (2014) and formally shown in He and Milbradt (2014), this effect can cause more corporate bond defaults and impose considerate risks on real economic activities. This channel becomes increasingly relevant given the bank-bond substitution after the crisis (Becker and Ivashina, 2014, Crouzet, 2015) as well as the increasing "reaching-for-yield" behavior in corporate bond markets (Becker and Ivashina, 2015). To be clear, I do not claim that mutual fund runs cause more systemic risks. The systemic implications of mutual fund runs depend not only on the contagion from secondary-market fire sales to primary-market investment losses, but also on how non-bank financial intermediaries interact with other bank-like financial institutions. A thorough investigation covering all these issues is beyond the scope of this paper, but the results here can naturally serve as a building block for future research on these issues.

A Appendix

A.1 Institutional Background

In this appendix, I depict the current trends of U.S. corporate bond mutual funds. These trends show that these funds are growing rapidly in size, investing in more illiquid assets, and holding less cash relative to their assets. The settings in my model are consistent with these trends.



The left panel plots total assets under management (data from the Investment Company Institute) and weighted-average cash-to-assets ratios (data from the SEC form N-SAR) of corporate bond funds. Cash equivalents such as repurchase agreements and other short-term debt securities are included in the SEC cash holdings data. The right panel plots annual turnovers of the underlying corporate bond markets (data from Barclays Research).

Figure 8: Trends in Corporate Bond Mutual Funds

It is worth noting that, although the trends illustrated here suggest increasing financial stability risks of mutual funds and potentially large fire sale losses when runs occur, the mechanism of this paper is general and does not rely on the trends.

A.2 The Analysis of the Intermediate Cash-to-Assets Ratio Region G_m

In this appendix, I provide a complete equilibrium analysis of the stage game when the fund starts at the intermediate cash-to-assets ratio region G_m.

Again, I first characterize how shareholder runs affect the fund's forced fire sales on dates $2t$ and $2t+1$, respectively. For convenience, I define

$$\widehat{\lambda}_{2t} \equiv \frac{x_{2t} - \mu_E(Ra_{2t} + x_{2t})}{\mu_L(Ra_{2t} + x_{2t})}.$$

By construction, within the intermediate region, there is always $\widehat{\lambda}_{2t} \in [0, 1)$. The economic meaning of $\widehat{\lambda}_{2t}$ will become clear shortly.

LEMMA **9.** *When $\eta_{2t} \in G_m$, there are:*

i) if $\lambda_{2t} \in [0, \widehat{\lambda}_{2t}]$, *then*

$$\begin{aligned} q_{2t}(\lambda_{2t}) &= 0\,, \\ q_{2t+1}(\lambda_{2t}) &= \frac{(\mu_E + \mu_L)(Ra_{2t} + x_{2t}) - x_{2t}}{\widehat{p}_L(\lambda_{2t})}\,, \end{aligned} \tag{A.1}$$

where q_{2t+1} *is increasing in* λ_{2t}, *and,*

ii) if $\lambda_{2t} \in (\widehat{\lambda}_{2t}, 1]$, *then*

$$q_{2t}(\lambda_{2t}) = \frac{(\mu_E + \lambda_{2t}\mu_L)(Ra_{2t} + x_{2t}) - x_{2t}}{\widehat{p}_E(\lambda_{2t})}\,, \tag{A.2}$$

$$q_{2t+1}(\lambda_{2t}) = \frac{(1-\lambda_{2t})\mu_L \cdot \dfrac{R(a_{2t} - q_{2t})}{1 - \mu_E - \lambda_{2t}\mu_L}}{\widehat{p}_L(\lambda_{2t})}\,, \tag{A.3}$$

where q_{2t} *is increasing in* λ_{2t} *but* q_{2t+1} *is decreasing in* λ_{2t}.

Moreover, $q_{2t} + q_{2t+1}$ *is increasing in* λ_{2t} *for all* $\lambda_{2t} \in [0, 1]$.

I first discuss the intuition behind the results when $\lambda_{2t} \leqslant \widehat{\lambda}_{2t}$. In this case, the fund has enough cash to satisfy all the $\mu_E + \lambda_{2t}\mu_L$ redeeming shareholders on date $2t$ at the initial NAV. Thus, no illiquid assets are forced to sell on date $2t$, that is, $q_{2t}(\lambda_{2t}) = 0$. However, in the intermediate region the fund does not have enough cash to satisfy all the late shareholders on the odd date. Specifically, the cash gap at the beginning of date $2t+1$ is indicated by the numerator of (A.1). Following the same intuition of Lemma 4, the fund manager will close the gap by selling at the effective price $\widehat{p}_L(\lambda_{2t})$. As λ_{2t} increases, the effective selling price $\widehat{p}_L(\lambda_{2t})$ becomes lower, suggesting that the fund will be forced to sell more on date $2t+1$, that is, q_{2t+1} becomes larger.

The situation becomes different when $\lambda_{2t} > \widehat{\lambda}_{2t}$. Compared to Lemma 4, the two conditions (A.2) and (A.3) are exactly the same as conditions (3.6) and (3.7) there. This is because when $\lambda_{2t} > \widehat{\lambda}_{2t}$ the cash position x_{2t} becomes inadequate to satisfy the $\mu_E + \lambda_{2t}\mu_L$ redeeming shareholders on date $2t$ at the initial NAV, so that the stage game effectively jumps into the low cash-to-assets ratio region. The monotonicity of q_{2t}, q_{2t+1}, and $(q_{2t} + q_{2t+1})$ all follows the same intuition there.

It is worth noting that, regardless of whether $\lambda_{2t} \leqslant \widehat{\lambda}_{2t}$ or $\lambda_{2t} > \widehat{\lambda}_{2t}$, more late shareholder runs always lead to unambiguously higher forced fire sales within the entire stage (including both date $2t$ and $2t+1$).

Similarly, I can characterize the NAVs in the intermediate region. When $\lambda_{2t} \in [0, \widehat{\lambda}_{2t}]$, by Lemma 9 there is

$$NAV_{2t}(\lambda_{2t}) = Ra_{2t} + x_{2t}\,, \tag{A.4}$$

and

$$NAV_{2t+1}(\lambda_{2t}) = \frac{\overbrace{R\left(a_{2t}-q_{2t+1}(\lambda_{2t})-s_{2t+1}\right)}^{\text{illiquid assets remained}}+\overbrace{x_{2t}-(\mu_E+\lambda_{2t}\mu_L)(Ra_{2t}+x_{2t})}^{\text{cash remained}}+\overbrace{\delta_L R(q_{2t+1}(\lambda_{2t})+s_{2t+1})}^{\text{cash raised and rebuilt}}}{\underbrace{1-(\mu_E+\lambda_{2t}\mu_L)}_{\text{shareholders remained on date } 2t+1}}$$

$$= NAV_{2t} - \frac{(1-\delta_L)R\left(q_{2t+1}(\lambda_{2t})+s_{2t+1}\right)}{1-\mu_E-\lambda_{2t}\mu_L}. \tag{A.5}$$

where $q_{2t+1}(\lambda_{2t})$ is given in (A.1). Clearly, the NAV on date $2t$ as in (A.4) is also constant and the same as (3.1) in the high region. The NAV on date $2t+1$ as in (A.5) also features the same expression as (3.3) in the high region. These suggest that shareholders' strategic interaction in this sub-region is the same as that in the high region.

When $\lambda_{2t} \in (\widehat{\lambda}_{2t}, 1]$, by Lemma 9 there are

$$NAV_{2t}(\lambda_{2t}) = Ra_{2t} + x_{2t} - (1-\delta_E)Rq_{2t}(\lambda_{2t}), \tag{A.6}$$

and

$$NAV_{2t+1}(\lambda_{2t}) = \frac{R\left(a_{2t}-q_{2t}(\lambda_{2t})-q_{2t+1}(\lambda_{2t})-s_{2t+1}\right)+\delta_L R\left(q_{2t+1}(\lambda_{2t})+s_{2t+1}\right)}{1-\mu_E-\lambda_{2t}\mu_L}, \tag{A.7}$$

where $q_{2t}(\lambda_{2t})$ and $q_{2t+1}(\lambda_{2t})$ are given in (A.2) and (A.3). Note that, the NAVs as in (A.6) and (A.7) are exactly the same as (3.9) and (3.10) in the low region, suggesting that shareholders' strategic interaction in this sub-region is the same as that in the low region.

Formally, the following lemma tells us that the stage game in the intermediate region indeed features a switch from strategic complementarity to substitutability.

LEMMA **10.** *When $\eta_{2t} \in G_m$, there are:*

i) if $\lambda_{2t} \in [0, \widehat{\lambda}_{2t}]$, late shareholders' run decision λ_{2t} exhibits strategic complementarity for any feasible $s_{2t+1} \in [0, a_{2t} - q_{2t+1}(\lambda_{2t})]$, and the strategic complementarity becomes stronger as s_{2t+1} increases, and,

ii) if $\lambda_{2t} \in (\widehat{\lambda}_{2t}, 1]$, late shareholders' run decision λ_{2t} exhibits strategic substitutability for any λ_{2t} satisfying $\theta NAV_{2t}(\lambda_{2t}) \geqslant NAV_{2t+1}(\lambda_{2t})$ and any feasible $s_{2t+1} \in [0, a_{2t} - q_{2t}(\lambda_{2t}) - q_{2t+1}(\lambda_{2t})]$, and the strategic substitutability becomes weaker as s_{2t+1} increases.

Not surprisingly, Lemma 10 can be understood in view of Lemma 3 (in the analysis for the high region) and Lemma 5 (in the analysis for the low region). In the first sub-region $[0, \widehat{\lambda}_{2t}]$, shareholders who run can get the endogenously fixed NAV on date $2t$ at the expense of shareholders who wait. More running shareholders or a more rapid cash rebuilding policy implies a larger magnitude of predictable decline in the NAV on date $2t+1$, leading to a stronger strategic complementarity. In the second sub-region, however, running shareholders have to accept an endogenously lower NAV themselves because the fund is forced to sell its illiquid assets on date $2t$, when the fire sale price is extremely low. The resulting higher fire sale losses suggest that more shareholder runs make the other shareholders who plan

to wait less likely to run. But again, more rapid cash rebuilding still gives rise to a larger magnitude of predictable decline in the NAV on date $2t+1$ and thus reinforces the run incentive.

Because of the switch of strategic interaction, shareholders' equilibrium run behaviors exhibit a richer pattern. Despite the complicated equilibrium construction in the intermediate region, it still indicates that fund cash rebuilding leads to runs and more rapid cash rebuilding triggers more severe runs in equilibrium.

PROPOSITION **13.** *When $\eta_{2t} \in G_m$, late shareholders' equilibrium run behaviors are given by the following five cases:*

i) none of the late shareholders runs, that is, $\lambda_{2t} = 0$, if

$$s_{2t+1} < \underline{s}_m \equiv \frac{Ra_{2t} - \theta(1-\mu_E - \widehat{\lambda}_{2t}\mu_L)(Ra_{2t}+x_{2t})}{(1-\delta_L)R} - q_{2t+1}(\widehat{\lambda}_{2t})\,,$$

ii) if

$$s_{2t+1} > \overline{s}_m \equiv \frac{Ra_{2t} - \theta(1-\mu_E)(Ra_{2t}+x_{2t})}{(1-\delta_L)R} - q_{2t+1}(0)\,,\ \textit{then},$$

a) all of the late shareholders run, that is, $\lambda_{2t} = 1$, if

$$s_{2t+1} > \overline{\overline{s}}_m \equiv \frac{Ra_{2t} - \theta(1-\mu_E-\mu_l)(Ra_{2t}+x_{2t}) - (1-\theta(1-\delta_E)(1-\mu_E-\mu_l))Rq_{2t}(1)}{(1-\delta_L)R}\,,$$

b) some of the late shareholder runs, that is, $\lambda_{2t} = \widetilde{\widetilde{\lambda}}_{2t} \in [\widehat{\lambda}_{2t}, 1)$, if

$$s_{2t+1} \leqslant \overline{\overline{s}}_m\,,$$

where $\widetilde{\widetilde{\lambda}}_{2t}$ is the solution to

$$s_{2t+1} = \frac{Ra_{2t} - \theta(1-\mu_E - \widetilde{\widetilde{\lambda}}_{2t}\mu_l)(Ra_{2t}+x_{2t}) - (1-\theta(1-\delta_E)(1-\mu_E-\widetilde{\widetilde{\lambda}}_{2t}\mu_l))Rq_{2t}(\widetilde{\widetilde{\lambda}}_{2t})}{(1-\delta_L)R} - q_{2t+1}(\widetilde{\widetilde{\lambda}}_{2t})\,,$$

iii) if $\underline{s}_m \leqslant s_{2t+1} \leqslant \overline{s}_m$, then,

c) $\lambda_{2t} \in \{0, \widetilde{\lambda}_{2t}, 1\}$, if

$$s_{2t+1} > \overline{\overline{s}}_m\,,$$

where $\widetilde{\lambda}_{2t}$ is the solution to

$$s_{2t+1} = \frac{(1-\theta)(1-\mu_E - \widetilde{\lambda}_{2t}\mu_L)(Ra_{2t}+x_{2t})}{(1-\delta_L)R} - q_{2t+1}(\widetilde{\lambda}_{2t})\,,$$

d) $\lambda_{2t} \in \{0, \widetilde{\lambda}_{2t}, \widetilde{\widetilde{\lambda}}_{2t}\}$, if

$$s_{2t+1} \leqslant \overline{\overline{s}}_m\,,$$

where $\widetilde{\lambda}_{2t}$ is given in Case c) and $\widetilde{\widetilde{\lambda}}_{2t}$ is given in Case b).
All of the $q_{2t}(\lambda_{2t})$ and $q_{2t+1}(\lambda_{2t})$ are given in Lemma 9. Moreover, $\overline{s}_m \geqslant 0$ and $\overline{s}_m > \underline{s}_m$.

The intuition behind Proposition 13 is clear in view of Propositions 1 and 2. By Lemma 10, the

stage game in the intermediate region starts with strategic complementarity when only a small fraction of late shareholders decides to run. Hence, it is the strategic complementarity in the first sub-region $[0, \widehat{\lambda}_{2t}]$ that determines whether any late shareholder will run at all. As in Case i), when $\widehat{\lambda}_{2t}$ of the late shareholders decide to run, if the utility gain of running over waiting is still not positive, none of the late shareholders will ever run. In Case ii), the utility gain of running over waiting is already positive even if no one runs, so that at least $\widehat{\lambda}_{2t}$ of the late shareholders will run due to the strategic complementarity in the sub-region $[0, \widehat{\lambda}_{2t}]$. However, as the stage game switches to the second sub-region $(\widehat{\lambda}_{2t}, 1]$, there can be strategic substitutability. In sub-case a), the fund uses a rapid cash rebuilding policy so that all of the late shareholders run despite the strategic substitutability, while in sub-case b) the substitutability is strong so that $\lambda_{2t} = \widetilde{\widetilde{\lambda}}_{2t} \in [\widehat{\lambda}_{2t}, 1)$ of the late shareholders are going to run. Finally, in Case iii), the strategic complementarity in the first sub-region $[0, \widehat{\lambda}_{2t}]$ is moderate. When this happens, the worst equilibrium will be determined by the magnitude of strategic substitutability in the sub-region $(\widehat{\lambda}_{2t}, 1]$, as shown in Case c) and Case d).

Interestingly, Proposition 2 suggests that the risk of shareholder runs can be higher in the intermediate region than that in the low region (with the same set of cash rebuilding policy s_{2t+1} and other model parameters). This is because the run threshold $\underline{s}_m$ in Proposition 13 can be smaller than $\underline{s}_l$ in Proposition 2. This predication may be surprising given that the fund has more cash in the intermediate region. But it is intuitive in view of the strategic complementarity in the intermediate region. Concretely, when the fund starts from either the low or the intermediate region, there are forced fire sales on date $2t+1$. But only when starting from the intermediate region, can some of the running shareholders get an endogenously fixed NAV on date $2t$. As a result, shareholders may be more willing to run to get the higher NAV on date $2t$ when the fund starts from the intermediate region, compared to the case when if the fund starts from the low region where they would always have to accept a lower NAV on date $2t$ if they run.

As usual, I show how shareholder runs increase the risk of forced fire sales by exploring the laws of motions in the intermediate region.

COROLLARY **4.** *When $\eta_{2t} \in G_m$, the law of motions of (a_{2t}, x_{2t}) is given by*

$$a_{2t+2} = a_{2t} - (q_{2t}(\lambda_{2t}) + q_{2t+1}(\lambda_{2t})) - s_{2t+1}\,, \text{ and}, \tag{A.8}$$

$$x_{2t+2} = \widehat{p}_L(\lambda_{2t}) s_{2t+1}\,, \tag{A.9}$$

where λ_{2t} is the equilibrium run probability induced by (a_{2t}, x_{2t}) and s_{2t+1}, as characterized in Proposition 13.

Similarly, Corollary 4 is easier to understand in view of Corollary 2; the two laws of motions (A.8) and (A.9) share the same mathematical expressions with (3.11) and (3.12) in the low region.[71] They again suggest two different costs of shareholder runs: more forced fire sales in the current stage and higher risk of future-stage fire sales.

[71] However, it is worth noting that the equilibrium determination and the endogenous functions of $q_{2t}(\lambda_{2t})$ and $q_{2t+1}(\lambda_{2t})$ are still different between the two regions. So that the intermediate region still features a different law of motions of (a_{2t}, x_{2t}).

A.3 A Micro-foundation for the Pattern of Selling Prices

In this appendix, I show that the pattern of selling prices in the baseline model can emerge endogenously by modeling the slow-moving of liquidity providers in the spirit of Grossman and Miller (1988) and Duffie (2010). It shows that the reduced-form assumption can be rationalized as the outcome of a full-fledged equilibrium model with both liquidity demanders and providers. To make the idea more transparent, I set the micro-foundation in continuous time.[72] I follow the building blocks in Duffie, Garleanu and Pedersen (2005, 2007), Weill (2007) and Lagos, Rocheteau and Weill (2011) to model the gradual entry of liquidity providers and focus on the equilibrium price implications.

Time is continuous and infinite. A probability space $(\Omega, \mathcal{F}, P)$ is fixed with an information filtration $\{\mathcal{F}_t, t \geqslant 0\}$ satisfying the usual measurability conditions (Sun, 2006). There is a common discount rate $r > 0$. There is a continuum of 1 of risk-neutral, infinitely lived, and competitive investors. There is a centralized market with many different assets. The total supply of all assets is $S \in [0, 1)$. Investors can hold at most one unit of assets and cannot short sell the assets. There is also a riskfree saving account with return r, which can be interpreted as cash equivalents. Under usual non-arbitrage conditions, this implies that the fundamental value of the assets is $1/r$.

There are two types of investors: liquidity providers and liquidity demanders. Liquidity providers enjoy a high utility flow per time by holding one unit of assets, which is normalized to 1, while liquidity demanders enjoy a low utility flow $\delta \in (0, 1)$.

At the beginning, the economy is hit by an unanticipated liquidity shock that makes all investors liquidity demanders. However, as time goes by, they will randomly and pairwise independently switch to liquidity providers.[73] Specifically, the times at which investors switch to liquidity providers are i.i.d. exponentially distributed with a parameter α. Denote the endogenous population of liquidity providers by $\rho(t)$. By the exact law of larger numbers (Sun, 2006, Theorem 2.16), there is

$$\rho(t) = 1 - \exp(-\alpha t)\,. \tag{A.10}$$

Intuitively, this implies that there is no liquidity provider available right at the shock time (i.e., $t = 0$), while there will be more and more liquidity providers stepping into the market after the shock.

In this simple framework, the following proposition shows the pattern of asset selling price over time:

PROPOSITION **14.** *The asset selling price at time t is characterized by*

$$p(t) = \frac{\delta + (1 - \delta)\exp(-r(t_S - t))}{r}\,,$$

where t_S satisfies $\rho(t_S) = S$ and $\rho(\cdot)$ is given by (A.10).

Intuitively, the selling price drops discontinuously at $t = 0$ from the fundamental value, but rebounds gradually over time (as more liquidity providers become available) until it gets back to the fundamental

[72] This baseline model is set in discrete time to highlight the discrete nature of daily redemptions and the end-of-day NAV. But in the micro-foundation, the discrete nature is no longer important. As a result, setting a continuous-time model incurs no loss of generality but makes the derivation mathematically more convenient.

[73] This dynamic process is in the spirit of Grossman and Miller (1988), in which liquidity providers only enter the market one period after the initial liquidity shock.

value at time t_S. When the next shock comes, this process repeats itself, giving rise to the price pattern in the baseline model.

It is instructive to provide the proof here to help build intuition. First of all, I show that there is a time at which the selling positions can be completely absorbed by liquidity providers so that the price goes back to the fundamental. Specifically, condition (A.10) shows that more liquidity providers step into the market as time goes by after the shock. Denote the endogenous time by which liquidity providers can absorb all the asset supply by t_S, which implies that $\rho(t_S) = S$. Since $\rho(t_S)$ is monotone, this uniquely determines t_S. This corresponds to the baseline model that if the game ends (i.e., there are no future shocks), the asset selling price will ultimately reflect the fundamental value.

Then I show that, between the shock time 0 and the full recovery time t_S (before the next possible shock), the asset selling price first drops and then rebounds gradually, as that in the baseline model. Note that, at any time t between 0 and t_S, there are no enough liquidity providers in the market, so that the marginal investor is a liquidity demander who has a low valuation of the assets. Since this liquidity demander is infinitely lived, the Hamilton-Jacobi-Bellman equation leads to:

$$rp(t)dt = \delta dt + p(t) \tag{A.11}$$

This condition has an intuitive interpretation. At any time t between 0 and t_S, the left hand side of (A.11) denotes the return of selling the unit of assets at t and investing the proceeds in cash equivalents in the time interval $[t, t + dt)$, while the right hand side denotes the valuation flow by holding one unit of assets in the time interval $[t, t + dt)$ plus the proceeds from selling it after that. In any equilibrium path, the liquidity demander should be indifferent between these two options of selling earlier or later. Therefore, solving the differential equation implied by (A.11) with the boundary conditions yields the equilibrium selling price. This concludes the proof.

Fundamentally, this micro-foundation follows the spirit of Grossman and Miller (1988) and Duffie (2010), but differs in an important way. Specifically, liquidity providers in their models share risks with liquidity demanders, while in both my baseline model and the micro-foundation, all the investors are risk neutral.[74] However, similar selling price pattern emerges. This is because liquidity providers in my model have higher valuation of the underlying assets, which more resembles the notion of natural buyers in Shleifer and Vishny (1992, 1997) and thus is closer to the fire sale interpretation in the baseline model. Like that in Grossman and Miller (1988) and Duffie (2010), liquidity providers step into the market only gradually after the shock, implying that only a few liquidity providers are present in the market right after the shock. Hence, investors who want to sell the assets right after the shock have to incur an extremely low fire sale price. As time goes by (but before the next possible shock comes), more liquidity providers with high valuation of the underlying assets step into the market, implying that it becomes increasingly easier for the liquidity demanders to find a better selling price.

This micro-foundation has other nice properties, which are also consistent with other ingredients of the baseline model. First, different from Grossman and Miller (1988), all investors in the micro-

[74] This assumption of risk neutrality also appears in other search-based models (see Duffie, Garleanu and Pedersen, 2005, Weill, 2007, Lagos, Rocheteau and Weill, 2011, among many others).

foundation are infinitely lived and perfectly forward-looking. This implies that the resulting selling price pattern does not come from any myopia of the investors. Second, as suggested by Duffie, Garleanu and Pedersen (2005, 2007) and many follow-up models, the setting of low and high valuations parsimoniously captures many potential benefits and costs of holding the assets and the resulting trading motives, which are consistent with the baseline model. When a mutual fund experiences a large redemption, holding the illiquid assets would incur more costs and thus imply a lower valuation.

A.4 Proofs

In this appendix, I provide proofs for all the results in the main text.

Proof of Lemma 1. First, in the high cash-to-assets ratio region, the fund needs to sell no illiquid assets on either date $2t$ or $2t+1$. Since no fire sale losses are incurred in this region, both early and late shareholders are able to get the same NAV as that at the beginning of date $2t$, that is,

$$NAV_{2t} = NAV_{2t+1} = \frac{Ra_{2t} + x_{2t}}{n_{2t}}.$$

Moreover, the initial cash position should be large enough to meet the redemption needs of all shareholders on dates $2t$ and $2t+1$ at such a constant NAV:

$$x_{2t} \geqslant (\mu_E + \mu_L) n_{2t} \cdot \frac{Ra_{2t} + x_{2t}}{n_{2t}},$$

yielding

$$\eta_{2t} \geqslant \frac{(\mu_E + \mu_L)R}{1 - \mu_E - \mu_L}, \tag{A.12}$$

the criterion for the high region.

Then, in the intermediate region, as no fire sale is incurred on date $2t$, the initial cash position is high enough to meet the redemption needs of early shareholders at the initial NAV but insufficient to meet late shareholders' redemption needs:

$$\mu_E n_{2t} \cdot \frac{Ra_{2t} + x_{2t}}{n_{2t}} \leqslant x_{2t} < (\mu_E + \mu_L) n_{2t} \cdot \frac{Ra_{2t} + x_{2t}}{n_{2t}},$$

which leads to

$$\frac{\mu_E R}{1 - \mu_E} \leqslant \eta_{2t} < \frac{(\mu_E + \mu_L)R}{1 - \mu_E - \mu_L}, \tag{A.13}$$

the criterion for the intermediate region.

Finally, in the low region, the cash position is even inadequate to meet early shareholders' redemption needs at the initial NAV. This means

$$x_{2t} < \mu_E n_{2t} \cdot \frac{Ra_{2t} + x_{2t}}{n_{2t}},$$

which yields

$$\eta_{2t} < \frac{\mu_E R}{1 - \mu_E}, \tag{A.14}$$

the criterion for the intermediate region.

It is straightforward to check that (A.12), (A.13), and (A.14) are also sufficient conditions. This concludes the proof. □

Proof of Lemma 2. Suppose $\lambda_{2t}\mu_L$ late shareholders decide to run. This situation is equivalent to a counterfactual in which there are initially $\mu'_E = \mu_E + \lambda_{2t}\mu_L$ early shareholders and $\mu'_L = (1-\lambda_{2t})\mu_L$ late shareholders but no late shareholder runs. Since $\mu'_E + \mu'_L = \mu_E + \mu_L$, by Lemma 1, $q_{2t} = q_{2t+1} = 0$ is true in the counterfactual situation and so is true in the original situation with $\lambda_{2t}\mu_L$ late shareholders running. This concludes the proof. □

Proof of Lemma 3. By Lemma 2 and the definition of $\Delta u_L(\lambda_{2t})$:

$$\Delta u_L(\lambda_{2t}) = \theta NAV_{2t} - NAV_{2t+1} = (\theta - 1)(Ra_{2t} + x_{2t}) + \frac{(1-\delta_L)Rs_{2t+1}}{1 - \mu_E - \lambda_{2t}\mu_L}\,.$$

Taking derivatives yields:

$$\frac{\partial \Delta u_L(\lambda_{2t})}{\partial \lambda_{2t}} = \frac{(1-\delta_L)\mu_L R s_{2t+1}}{(1 - \mu_E - \lambda_{2t}\mu_L)^2} > 0\,,$$

which takes value 0 when $s_{2t+1} = 0$, and

$$\frac{\partial^2 \Delta u_L(\lambda_{2t})}{\partial \lambda_{2t} \partial s_{2t+1}} = \frac{(1-\delta_L)\mu_L R}{(1 - \mu_E - \lambda_{2t}\mu_L)^2} > 0\,.$$

This concludes the proof. □

Proof of Proposition 1. By Lemma 3, the stage game exhibits strategic complementarity when $s_{2t+1} > 0$. Also notice that any shareholder runs only if $\theta NAV_{2t} \geqslant NAV_{2t+1}$. Thus, in Case i), none of the late shareholders runs if

$$\theta NAV_{2t} < NAV_{2t+1}(1)\,, \tag{A.15}$$

in which $NAV_{2t+1}(\lambda_{2t})$ is a function of λ_{2t}. Solving inequality (A.15) leads to

$$s_{2t+1} < \frac{(1-\theta)(1 - \mu_E - \mu_L)(Ra_{2t} + x_{2t})}{(1-\delta_L)R} \equiv \underline{s}_h\,.$$

Alternatively, in Case ii), all of the late shareholders run if

$$\theta NAV_{2t} > NAV_{2t+1}(0)\,, \tag{A.16}$$

the solution of which is

$$s_{2t+1} > \frac{(1-\theta)(1 - \mu_E)(Ra_{2t} + x_{2t})}{(1-\delta_L)R} \equiv \overline{s}_h\,.$$

Finally, in Case iii), if neither (A.15) nor (A.16) holds, there exists a $\widetilde{\lambda}_{2t} \in [0, 1]$ that solves

$$\theta NAV_{2t} = NAV_{2t+1}(\widetilde{\lambda}_{2t})\,.$$

Note that, $\widetilde{\lambda}_{2t}$ constructs an equilibrium because by definition $\Delta u_L(\widetilde{\lambda}_{2t}) = 0$ and thus no shareholder would have an incentive to deviate from it. In addition, in this case, again by Lemma 3, there are $\theta NAV_{2t} \geqslant NAV_{2t+1}(1)$ and $\theta NAV_{2t} \leqslant NAV_{2t+1}(0)$, which means $\lambda_{2t} = 1$ and $\lambda_{2t} = 0$ are also two equilibria when (A.15) and (A.16) are both violated. This concludes the proof. □

Proof of Corollary 1. By Lemma 2, $q_{2t}(\lambda_{2t}) = q_{2t+1}(\lambda_{2t}) = 0$ for any arbitrary $\lambda_{2t} \in [0, 1]$. Thus, the evolution of the asset position directly follows:

$$a_{2t+2} = a_{2t} - q_{2t} - q_{2t+1} - s_{2t+1} = a_{2t} - s_{2t+1}\,.$$

For the evolution of the cash position, the fund pays all the redeeming shareholders by cash at the respective end-of-day NAVs on date $2t$ and $2t+1$, and rebuilds its cash buffer on date $2t+1$. Note that there will be no cash raised by forced fire sales. Thus:

$$\begin{aligned} x_{2t+2} &= x_{2t} - (\mu_E + \lambda_{2t}\mu_L)NAV_{2t} - (1-\lambda_{2t})\mu_L NAV_{2t+1} + p_L s_{2t+1} \\ &= x_{2t} - (\mu_E + \mu_L)(Ra_{2t} + x_{2t}) + \delta_L R s_{2t+1} + \frac{(1-\lambda_{2t})\mu_L(1-\delta_L)Rs_{2t+1}}{1-\mu_E-\lambda_{2t}\mu_L}\,. \end{aligned}$$

This concludes the proof. □

Proof of Lemma 4. Recall that, when forced fire sales occur, the fund sells up to a point at which it can satisfy the redemptions at the end-of-day NAV, which will take into account the losses from forced fire sales. On the one hand, on date $2t$ the fund starts with a cash position x_{2t}. Hence, on date $2t$, q_{2t} solves

$$x_{2t} + p_E q_{2t} = (\mu_E + \lambda_{2t}\mu_L)[(a_{2t} - q_{2t})R + x_{2t} + p_E q_{2t}]\,,$$

yielding

$$q_{2t}(\lambda_{2t}) = \frac{(\mu_E + \lambda_{2t}\mu_L)(Ra_{2t} + x_{2t}) - x_{2t}}{[\delta_E + (1-\delta_E)(\mu_E + \lambda_{2t}\mu_L)]\,R}\,. \tag{A.17}$$

On the other hand, on date $2t+1$, by construction, the fund has no cash at all at the beginning. Hence, q_{2t+1} solves

$$p_L q_{2t+1} = (1-\lambda_{2t})\mu_L \frac{(a_{2t} - q_{2t} - q_{2t+1})R + p_L q_{2t+1}}{1-\mu_E-\lambda_{2t}\mu_L}\,,$$

yielding

$$q_{2t+1} = \frac{(1-\lambda_{2t})\mu_L(a_{2t} - q_{2t})}{(1-\mu_E-\lambda_{2t}\mu_L)\delta_L + (1-\lambda_{2t})\mu_L(1-\delta_L)}\,. \tag{A.18}$$

Plugging (A.17) into (A.18) leads to

$$q_{2t+1}(\lambda_{2t}) = \frac{(1-\lambda_{2t})\mu_L \cdot \dfrac{R(a_{2t} - q_{2t})}{1-\mu_E-\lambda_{2t}\mu_L}}{\left[\delta_L + \dfrac{(1-\lambda_{2t})\mu_L(1-\delta_L)}{1-\mu_E-\lambda_{2t}\mu_L}\right]R}\,. \tag{A.19}$$

For the monotonicity of $q_{2t}(\lambda_{2t})$, taking derivative of (A.17) leads to

$$\frac{\partial q_{2t}(\lambda_{2t})}{\lambda_{2t}} = \frac{\mu_L(\delta_E R a_{2t} + x_{2t})}{[\mu_E(1-\delta_E) + (1-\lambda_{2t}\mu_L)\delta_E + \lambda_{2t}\mu_L]^2} > 0\,,$$

implying that $q_{2t}(\lambda_{2t})$ is increasing in λ_{2t}. Similar procedures based on (A.17) and (A.19) show that $q_{2t+1}(\lambda_{2t})$ is decreasing in λ_{2t} while $q_{2t}(\lambda_{2t})+q_{2t+1}(\lambda_{2t})$ is increasing in λ_{2t}. This concludes the proof. □

Proof of Lemma 5. By Lemma 4 and the definition of $\Delta u_L(\lambda_{2t})$:

$$\begin{aligned}\Delta u_L(\lambda_{2t}) &= \theta NAV_{2t} - NAV_{2t+1} \\ &= \theta[(Ra_{2t}+x_{2t}) - (1-\delta_E)Rq_{2t}] - \frac{(Ra_{2t}+x_{2t}) - Rq_{2t} - (1-\delta_L)R(q_{2t+1}+s_{2t+1})}{1-\mu_E-\lambda_{2t}\mu_L}\,, \quad \text{(A.20)}\end{aligned}$$

in which q_{2t} and q_{2t+1} are functions of λ_{2t} by Lemma 4. It is straightforward that $\Delta u_L(\lambda_{2t})$ is larger when s_{2t+1} increases. To focus on the value of λ_{2t} that satisfies $\theta NAV_{2t} \geqslant NAV_{2t+1}$, there is no loss of generality to consider $\theta = 1$, and the analysis for a general θ naturally follows by considering subsets of λ_{2t}. Now plug (A.17) and (A.18) into (A.20) and then take derivative with respect to λ_{2t}. After rearrangement, this yields:

$$\frac{\partial \Delta u_L(\lambda_{2t})}{\partial \lambda_{2t}} = -\frac{(1-\delta_L)\mu_L\left(x_{2t}C_1 - \dfrac{R\left(s_{2t+1}C_2 - a_{2t}\delta_E(1-\mu_E-\lambda_{2t}\mu_L)^2C_1\right)}{(1-\mu_E-\lambda_{2t}\mu_L)^2}\right)}{((1-\lambda_{2t})\mu_L + \delta_L(1-\mu_E-\mu_L))^2(\mu_E+\lambda_{2t}\mu_L+\delta_E(1-\mu_E-\lambda_{2t}\mu_L))^2}\,, \quad \text{(A.21)}$$

where

$$\begin{aligned}C_1 &= (1-\delta_E)(1-\lambda_{2t})^2\mu_L^2 + \delta_L(1-\mu_E-\mu_L)(\mu_E+\mu_L) + \delta_E\delta_L(1-\mu_E-\mu_L)^2 > 0\,, \\ C_2 &= ((1-\lambda_{2t})\mu_L + \delta_L(1-\mu_E-\mu_L))^2\,(\mu_E+\lambda_{2t}\mu_L+\delta_E(1-\mu_E-\lambda_{2t}\mu_L))^2 > 0\,.\end{aligned}$$

Consider

$$C = x_{2t}C_1 - \frac{R\left(s_{2t+1}C_2 - a_{2t}\delta_E(1-\mu_E-\lambda_{2t}\mu_L)^2C_1\right)}{(1-\mu_E-\lambda_{2t}\mu_L)^2}$$

for any $0 \leqslant x_{2t} < \mu_E n_{2t}(Ra_{2t}+x_{2t})$ and any $0 \leqslant s_{2t+1} \leqslant a_{2t} - q_{2t}(\lambda_{2t}) - q_{2t+1}(\lambda_{2t})$. Since $C_1 > 0$, there is $x_{2t}C_1 > 0$ and thus

$$C > \frac{R\left(a_{2t}\left(\delta_E(1-\mu_E-\lambda_{2t}\mu_L)^2C_1 - C_2\right)\right)}{(1-\mu_E-\lambda_{2t}\mu_L)^2}\,. \quad \text{(A.22)}$$

Notice that C_1 and C_2 are only functions of λ_{2t}, μ_E, μ_L, δ_E, δ_L, and are independent of a_{2t} and x_{2t}. By construction,

$$\delta_E(1-\mu_E-\lambda_{2t}\mu_L)^2C_1 - C_2 \geqslant 0$$

for any $\lambda_{2t} \in [0,1]$.

As a result, since $a_{2t} > 0$ and $R > 0$, inequality (A.22) implies that $C > 0$. Plugging back to (A.21) finally yields

$$\frac{\partial \Delta u_L(\lambda_{2t})}{\partial \lambda_{2t}} < 0\,,$$

implying strategic substitutability.

It is straightforward that $\Delta u_L(\lambda_{2t})$ is larger when s_{2t+1} increases. Also, by definition, C is decreasing in s_{2t+1} when $0 \leqslant s_{2t+1} \leqslant a_{2t} - q_{2t}(\lambda_{2t}) - q_{2t+1}(\lambda_{2t})$. By (A.21) and the derivation above, the strategic substitutability becomes weaker when s_{2t+1} increases. This concludes the proof. □

Proof of Proposition 2. Notice that any shareholder runs only if $\theta NAV_{2t} \geqslant NAV_{2t+1}$. Also by Lemma 5, the stage game exhibits strategic substitutability whenever an incentive to redeem earlier exists. Thus, in Case i), none of the late shareholders runs if

$$\theta NAV_{2t}(0) < NAV_{2t+1}(0)\,, \tag{A.23}$$

which implies that $\theta NAV_{2t}(\lambda_{2t}) < NAV_{2t+1}(\lambda_{2t})$ for any λ_{2t} by using the expressions in Lemma 4. Thus, solving inequality (A.23) leads to

$$s_{2t+1} < \frac{Ra_{2t} - \theta(1-\mu_E)(Ra_{2t}+x_{2t}) - (1-\theta(1-\delta_E)(1-\mu_E))Rq_{2t}(0)}{(1-\delta_L)R} - q_{2t+1}(0) \equiv \underline{s}_l\,.$$

Alternatively, in Case ii), all of the late shareholders run if

$$\theta NAV_{2t}(1) > NAV_{2t+1}(1)\,, \tag{A.24}$$

which implies that $\theta NAV_{2t}(\lambda_{2t}) > NAV_{2t+1}(\lambda_{2t})$ for any λ_{2t} despite the underlying strategic substitutability suggested by Lemma 5. Solving inequality (A.24) leads to

$$s_{2t+1} > \frac{Ra_{2t} - \theta(1-\mu_E-\mu_L)(Ra_{2t}+x_{2t}) - (1-\theta(1-\delta_E)(1-\mu_E-\mu_L))Rq_{2t}(1)}{(1-\delta_L)R} \equiv \overline{s}_l\,.$$

Using the expressions in Lemma 4, plugging $q_{2t}(0)$, $q_{2t+1}(0)$ and $q_{2t}(1)$ into the definition of $\underline{s}_l$ and $\overline{s}_l$ directly yields $\overline{s}_l \geqslant 0$ and $\overline{s}_l > \underline{s}_l$.

Finally, in Case iii), there exists some $\widetilde{\lambda}_{2t} \in [0,1]$ that solves

$$\theta NAV_{2t}(\widetilde{\lambda}_{2t}) = NAV_{2t+1}(\widetilde{\lambda}_{2t})\,,$$

where $\widetilde{\lambda}_{2t}$ constructs an equilibrium because by definition $\Delta u_L(\widetilde{\lambda}_{2t}) = 0$ and thus no shareholder would have an incentive to deviate from it. This leads to

$$s_{2t+1} = \frac{Ra_{2t} - \theta(1-\mu_E-\widetilde{\lambda}_{2t}\mu_l)(Ra_{2t}+x_{2t}) - (1-\theta(1-\delta_E)(1-\mu_E-\widetilde{\lambda}_{2t}\mu_l))Rq_{2t}(\widetilde{\lambda}_{2t})}{(1-\delta_L)R} - q_{2t+1}(\widetilde{\lambda}_{2t})\,.$$

This concludes the proof. □

Proof of Corollary 2. By Lemma 4, $q_{2t}(\lambda_{2t}) > 0$ for any arbitrary $\lambda_{2t} \in [0,1]$ and $q_{2t+1}(\lambda_{2t}) > 0$ for any arbitrary $\lambda_{2t} \in [0,1)$. Thus, the evolution of the asset position directly follows:

$$a_{2t+2} = a_{2t} - q_{2t}(\lambda_{2t}) - q_{2t+1}(\lambda_{2t}) - s_{2t+1}\,.$$

For the evolution of the cash position, notice that all the proceeds from forced fire sales $q_{2t}(\lambda_{2t})$ and $q_{2t}(\lambda_{2t+1})$ go to the redeeming shareholders. Also, by definition of the low cash-to-assets region, the fund starts with no cash on date $2t+1$. Thus:

$$\begin{aligned} x_{2t+2} &= p_L(q_{2t+1}(\lambda_{2t}) + s_{2t+1}) - (1-\lambda_{2t})\mu_L NAV_{2t+1} \\ &= \delta_L R s_{2t+1} + \frac{(1-\lambda_{2t})\mu_L(1-\delta_L)Rs_{2t+1}}{1-\mu_E-\lambda_{2t}\mu_L}. \end{aligned}$$

This concludes the proof. □

Proof of Proposition 3. The existence of a Markov equilibrium of the fully dynamic game follows a special case of Theorem 2 and Corollary 6 in Khan and Sun (2002). The key is to find a measurable selection of Nash equilibria in each stage game determined by the state variables (a_{2t}, x_{2t}). The Arsenin-Kunugui Theorem (see Kechris, 1995 for a textbook treatment) guarantees that any usual equilibrium selection mechanism such as selecting the best, the worst or the one based on the global game approach is measurable.

Under any Markov strategy profile, by definition, the strategies of both the fund manager and all the shareholders are functions of the two state variables (a_{2t}, x_{2t}), and their strategies are mutually best responses as well. In other words, strategies played in the past stages influence current-stage strategies only through the two state variables. For convenience, in what follows I call a stage game (a_{2t}, x_{2t}) when the fund starts from the portfolio position (a_{2t}, x_{2t}) on date $2t$.

Consider any arbitrary $\phi \in (0,1)$. Define $a'_{2t} = \phi a_{2t}$, $x'_{2t} = \phi x_{2t}$, and $s'_{2t+1} = \phi a_{2t+1}$. By Lemma 1, game (a_{2t}, x_{2t}) and game (a'_{2t}, x'_{2t}) start from the same cash-to-assets ratio region. By Propositions 1, 2, and 13, if λ_{2t} constructs a run equilibrium in game (a_{2t}, x_{2t}) under the cash rebuilding policy s_{2t+1}, it must also construct a run equilibrium in game (a'_{2t}, x'_{2t}) under the cash rebuilding policy s'_{2t+1}. Hence, by Lemmas 2, 4, and 9, the equilibrium amounts of forced fire sales in game (a'_{2t}, x'_{2t}) must be $q'_{2t} = \phi q_{2t}$, $q'_{2t+1} = \phi q_{2t+1}$, where q_{2t} and q_{2t+1} are the equilibrium amounts of forced fire sales in game (a_{2t}, x_{2t}).

Then consider the dynamics. Fix a consistent equilibrium selection mechanism if multiple equilibria occur. Let (a_{2t+2}, x_{2t+2}) be the next stage game when game (a_{2t}, x_{2t}) is played under the cash rebuilding policy s_{2t+1}. By Corollaries 1, 2 and 4, the next stage game must be (a'_{2t+2}, x'_{2t+2}), where $a'_{2t+2} = \phi a_{2t+2}$ and $x'_{2t+2} = \phi x_{2t+2}$, if the current stage game (a'_{2t}, x'_{2t}) is played under the cash rebuilding policy s'_{2t+1}. Therefore, if $s_{2t+1}(a_{2t}, x_{2t})$ is the optimal cash rebuilding policy in stage t for game (a_{2t}, x_{2t}), $s'_{2t+1}(a'_{2t}, x'_{2t}) = \phi s_{2t+1}(a_{2t}, x_{2t})$ must be the optimal cash rebuilding policy in stage t for game (a'_{2t}, x'_{2t}). Hence, $V(a'_{2t}, x'_{2t}) = \phi V(a_{2t}, x_{2t})$ is indeed the value function for the dynamic game with a starting position (a'_{2t}, x'_{2t}).

Finally, it is straightforward to see that $V(0,0) = 0$. This concludes the proof. □

Proof of Lemma 6. If $a_{2t} = 0$, it is trivial that $s^*_{2t+1}(a_{2t}, x_{2t}) = 0$. So it is only worth considering a strictly positive a_{2t}.

On the one hand, consider a perturbation $\varepsilon > 0$ of cash rebuilding around $s^*_{2t+1}(a_{2t}, x_{2t}) = 0$. On

date $2t+1$ (in stage t), regardless of the starting portfolio position (a_{2t}, x_{2t}), the effective selling price on $2t+1$ is at most $\widehat{p}_L(0) > 0$. Thus, the fire sale loss in stage t is at least

$$\frac{\varepsilon(1-\delta_L)R}{\widehat{p}_L(0)} > 0\,.$$

On other other hand, consider an initial cash gap ε on date $2t+2$ (in stage $t+1$). Regardless of the starting portfolio position (a_{2t+2}, x_{2t+2}), the effective selling price on $2t+2$ is at least $\widehat{p}_E(0) > 0$, and the physical selling price in stage $t+1$ is at least $\delta_E R$. Thus, the expected fire sale loss in stage $t+1$ due to this cash gap is at most

$$\frac{\varepsilon(1-\delta_E)R}{\widehat{p}_E(0)} > 0\,.$$

Therefore, for any π satisfying

$$\pi > 1 - \frac{(1-\delta_L)\widehat{p}_E(0)}{(1-\delta_E)\widehat{p}_L(0)} \in (0,1)\,,$$

it is optimal to choose $s^*_{2t+1}(a_{2t}, x_{2t}) = 0$. This concludes the proof. □

Proof of Lemma 7. This directly follows late shareholders' utility function. □

Proof of Lemma 8. First consider the case of $\eta_{2t} \in G_h$. By Proposition 1, $\theta = 1$ implies that $\overline{s}_h = 0$. Again by Proposition 1, there is $\lambda_{2t} = 1$ for any $s_{2t+1} > 0$ regardless of (a_{2t}, x_{2t}).

Then consider the case of $\eta_{2t} \in G_l$. Similarly, by Proposition 2, $\theta = 1$ implies that $\overline{s}_l = 0$. Again by Proposition 2, there is $\lambda_{2t} = 1$ for any $s_{2t+1} > 0$ regardless of (a_{2t}, x_{2t}).

Finally, consider the case of $\eta_{2t} \in G_m$. By Proposition 13, $\theta = 1$ implies that $\overline{s}_m = \overline{\overline{s}}_m = 0$. Again by Proposition 13, there is $\lambda_{2t} = 1$ for any $s_{2t+1} > 0$ regardless of (a_{2t}, x_{2t}). This concludes the proof. □

Proof of Proposition 4. I consider two cases according to the starting cash-to-assets ratio on date $2t$.

Case 1. $\eta_{2t} \in G_l \cup G_m \cup G_{hl}$. First, consider a perturbation $-\varepsilon < 0$ of cash rebuilding around σ^*_{2t+1} that satisfies $\eta^*_{2t+2} = \mu_E R/(1-\mu_E)$. On date $2t+1$ (in stage t), since there are no runs (by Lemma 7), the effective selling price on $2t+1$ is $\widehat{p}_L(0)$. Thus, the fire sale loss saved in stage t is

$$\frac{\varepsilon(1-\delta_L)R}{\widehat{p}_L(0)} > 0\,.$$

Now consider the same cash gap ε on date $2t+2$ (under the perturbed cash rebuilding policy $(\sigma^*_{2t+1}, -\varepsilon)$. This implies that $\eta_{2t+2} \in G_l$. Since there are no runs, the fund has to fire sell its assets on date $2t+2$ at the effective selling price $\widehat{p}_E(0) > 0$. Hence, the expected increase of fire sale loss in stage $t+1$ due to this cash gap ε is

$$\frac{\varepsilon(1-\delta_E)R}{\widehat{p}_E(0)} > 0\,.$$

Since $\delta_E < \delta_L$ and $\widehat{p}_E(0) < \widehat{p}_L(0)$, there is

$$\frac{\varepsilon(1-\delta_L)R}{\widehat{p}_L(0)} < (1-\pi)\frac{\varepsilon(1-\delta_E)R}{\widehat{p}_E(0)}$$

for a sufficiently small but positive π, implying that the perturbation $-\varepsilon$ is not profitable.

Next, consider another perturbation $\varepsilon > 0$ of cash rebuilding around σ^*_{2t+1} that satisfies $\eta^*_{2t+2} = \mu_E R/(1-\mu_E)$. On date $2t+1$ (in stage t), similarly, the fire sale loss increased in stage t is

$$\frac{\varepsilon(1-\delta_L)R}{\widehat{p}_L(0)} > 0\,.$$

Under the perturbed cash rebuilding policy $(\sigma^*_{2t+1}, \varepsilon)$ the fund gets ε more cash in stage $t+1$. This implies that $\eta_{2t+2} \in G_m$. Since there are no runs, the fund does not have to fire sell its assets on date $2t+2$. Rather, the marginal cash saves the fund's active asset sales on date $2t+3$ at the effective selling price $\widehat{p}_L(0) > 0$. Hence, the expected fire sale saved in stage $t+1$ due to this marginal cash ε is also

$$\frac{\varepsilon(1-\delta_L)R}{\widehat{p}_L(0)} > 0\,.$$

Since $\pi \in (0,1)$, this perturbation ε is also not profitable. This verifies the optimality of $\eta^*_{2t+2} = \mu_E R/(1-\mu_E)$ when $\eta_{2t} \in G_l \cup G_m \cup G_{hl}$.

Case 2. $\eta_{2t} \in G_{hm} \cup G_{hh}$. Consider a perturbation $\varepsilon > 0$ of cash rebuilding around $\sigma^*_{2t+1} = 0$. On date $2t+1$ (in stage t), similarly, the fire sale loss increased in stage t is

$$\frac{\varepsilon(1-\delta_L)R}{\widehat{p}_L(0)} > 0\,.$$

Similarly, under the perturbed cash rebuilding policy $(\sigma^*_{2t+1}, \varepsilon)$ the fund gets ε more cash in stage $t+1$. The expected fire sale loss saved in stage $t+1$ due to this marginal cash ε is also

$$\frac{\varepsilon(1-\delta_L)R}{\widehat{p}_L(0)} > 0\,.$$

Since $\pi \in (0,1)$, this perturbation ε is again not profitable. This verifies the optimality of $\sigma^*_{2t+1} = 0$ when $\eta_{2t} \in G_{hm} \cup G_{hh}$. This finally concludes the proof. □

Proof of Proposition 5. I consider two cases according to the starting cash-to-assets ratio on date $2t$.

Case 1. $\eta_{2t} \in G_l \cup G_m \cup G_{hl} \cup G_{hm}$. First, consider a perturbation $-\varepsilon < 0$ of cash rebuilding around σ^*_{2t+1} that satisfies $\eta^*_{2t+2} = (\mu_E + \mu_L)R/(1-\mu_E-\mu_L)$. On date $2t+1$ (in stage t), since $\lambda_{2t} = 1$ (by Lemma 8), the effective selling price on $2t+1$ is $\widehat{p}_L(1)$. Thus, the fire sale loss saved in stage t is

$$\frac{\varepsilon(1-\delta_L)R}{\widehat{p}_L(1)} > 0\,.$$

Now consider the same cash gap ε on date $2t+2$ (under the perturbed cash rebuilding policy $(\sigma^*_{2t+1}, -\varepsilon)$. This implies that $\eta_{2t+2} \in G_l \cup G_m$. Since $\lambda_{2t+2} = 1$, by Lemmas 4 and 9 the fund always has to fire sell its assets on date $2t+2$ at the effective selling price $\widehat{p}_E(1) > 0$, even if $\eta_{2t+2} \in G_m$.

Hence, the expected increase of fire sale loss in stage $t+1$ due to this cash gap ε is

$$\frac{\varepsilon(1-\delta_E)R}{\widehat{p}_E(1)} > 0\,.$$

Since $\delta_E < \delta_L$ and $\widehat{p}_E(1) < \widehat{p}_L(1)$, there is

$$\frac{\varepsilon(1-\delta_L)R}{\widehat{p}_L(1)} < (1-\pi)\frac{\varepsilon(1-\delta_E)R}{\widehat{p}_E(1)}$$

for a sufficiently small but positive π. Hence, the perturbation $-\varepsilon$ is not profitable.

Next, consider another perturbation $\varepsilon > 0$ of cash rebuilding around σ^*_{2t+1} that satisfies $\eta^*_{2t+2} = (\mu_E+\mu_L)R/(1-\mu_E-\mu_L)$. On date $2t+1$ (in stage t), similarly, the fire sale loss increased in stage t is

$$\frac{\varepsilon(1-\delta_L)R}{\widehat{p}_L(1)} > 0\,.$$

Under the perturbed cash rebuilding policy $(\sigma^*_{2t+1}, \varepsilon)$ the fund gets ε more cash in stage $t+1$. This implies that $\eta_{2t+2} \in G_h$. Hence, by Lemma 2, regardless of runs the fund does not have to fire sell its assets on date $2t+2$. Rather, the marginal cash saves the fund's active asset sales on date $2t+3$ at the effective selling price $\widehat{p}_L(1) > 0$. Hence, the expected fire sale saved in stage $t+1$ due to this marginal cash ε is also

$$\frac{\varepsilon(1-\delta_L)R}{\widehat{p}_L(1)} > 0\,.$$

Since $\pi \in (0,1)$, this perturbation ε is also not profitable. This verifies the optimality of $\eta^*_{2t+2} = (\mu_E+\mu_L)R/(1-\mu_E-\mu_L)$ when $\eta_{2t} \in G_l \cup G_m \cup G_{hl} \cup G_{hm}$.

Case 2. $\eta_{2t} \in G_{hh}$. Consider a perturbation $\varepsilon > 0$ of cash rebuilding around $\sigma^*_{2t+1} = 0$. On date $2t+1$ (in stage t), similarly, the fire sale loss increased in stage t is

$$\frac{\varepsilon(1-\delta_L)R}{\widehat{p}_L(1)} > 0\,.$$

Similarly, under the perturbed cash rebuilding policy $(\sigma^*_{2t+1}, \varepsilon)$ the fund gets ε more cash in stage $t+1$. The expected fire sale loss saved in stage $t+1$ due to this marginal cash ε is also

$$\frac{\varepsilon(1-\delta_L)R}{\widehat{p}_L(1)} > 0\,.$$

Since $\pi \in (0,1)$, this perturbation ε is again not profitable. This verifies the optimality of $\sigma^*_{2t+1} = 0$ when $\eta_{2t} \in G_{hh}$. This finally concludes the proof. □

Proof of Proposition 6. This proof proceeds in three steps. First, I show that when θ is sufficiently small, the equilibrium is the same as that characterized by Proposition 4. Second, I characterize the equilibrium when θ takes an intermediate value. Lastly, I show that when θ is sufficiently large, the equilibrium is the same as that characterized by Proposition 5.

Step 1. Recall that when $\theta = 0$, the equilibrium cash rebuilding policy is characterized by Proposition 4. By Propositions 1, 2, and 13, $\underline{s}_h$, $\underline{s}_l$, and $\underline{s}_m$ are all continuous in θ. Hence, there

exists a $\underline{\theta} > 0$ (explicit expression will be calculated in the next step) such that when $\theta \in (0, \underline{\theta}]$, none of the late shareholders chooses to run in any region if the fund still follows the cash rebuilding policy as described in Proposition 4. In addition, the proof of Proposition 4 only relies on the fact that there are no shareholder runs. This confirms that the cash rebuilding policy as described in Proposition 4 is still optimal when $\theta \in (0, \underline{\theta}]$, which in turn confirms the late shareholders' run decision $\lambda_{2t} = 0$.

Step 2. By the definition of $\underline{\theta}$, when $\theta > \underline{\theta}$ there exists a non-zero-measure set G_{run} in which at least some of the late shareholders will run given the cash rebuilding policy described in Proposition 4. I first show that G_{run} takes the form of

$$G_{run} = G_l \cup \underline{G_m}\,, \tag{A.25}$$

where $\underline{G_m} \subseteq G_m$ is connected and

$$\inf \underline{G_m} = \frac{\mu_E R}{1-\mu_E}\,.$$

To see this, first recall the definition of $\underline{s_l}$:

$$\underline{s_l} = \frac{Ra_{2t} - \theta(1-\mu_E)(Ra_{2t}+x_{2t}) - (1-\theta(1-\delta_E)(1-\mu_E))Rq_{2t}(0)}{(1-\delta_L)R} - q_{2t+1}(0)\,.$$

Note that, for every pair of (a_{2t}, x_{2t}) and η_{2t+2}, there is an implied s_{2t+1}. Using that as the threshold $\underline{s_l}$ and solving for θ backward yields that, under the cash rebuilding policy $\eta_{2t+2} = \mu_E R/(1-\mu_E)$, when

$$\theta > \frac{\delta_L}{\mu_E + \mu_L + \delta_L(1-\mu_E-\mu_L)} \equiv \underline{\theta} \in (0,1)$$

there must be $\lambda_{2t} > 0$ for $\eta_{2t} \in G_l$.

Similarly, consider the definitions of $\underline{s_m}$ and $\underline{s_h}$. Also under the cash rebuilding policy $\eta_{2t+2} = \mu_E R/(1-\mu_E)$, solving for the threshold θ backward yields that, when

$$\theta > \frac{\delta_L}{\mu_E + \mu_L + \delta_L(1-\mu_E-\mu_L)} = \underline{\theta}$$

there must be $\lambda_{2t} > \widehat{\lambda}_{2t}$ for $\eta_{2t} \in G_m$, while when

$$\theta > \frac{\delta_L + \mu_L - \delta_L\mu_L}{\mu_E + \mu_L + \delta_L(1-\mu_E-\mu_L)} \equiv \overline{\overline{\theta}} \in (0,1)$$

there must be $\lambda_{2t} > 0$ for $\eta_{2t} \in G_h$.

Notice that

$$\underline{\theta} < \overline{\overline{\theta}}\,.$$

Thus, under the cash rebuilding policy $\eta_{2t+2} = \mu_E R/(1-\mu_E)$, when $\theta \in (\underline{\theta}, \overline{\overline{\theta}})$, there is $\lambda_{2t} = 0$ when $\eta_{2t} \in G_h$. This confirms the claim in (A.25).

Now define

$$\eta(\check{\lambda}) \equiv \frac{(\mu_E + \check{\lambda}\mu_L)R}{1-\mu_E-\check{\lambda}\mu_L} \in G_m\,.$$

For any $\check{\lambda} \in (0,1)$, consider the following cash rebuilding policy:

$$\eta_{2t+2} = \eta(\check{\lambda})\,.$$

Since

$$\frac{\mu_E R}{1-\mu_E} < \eta(\check{\lambda}) < \frac{(\mu_E+\mu_L)R}{1-\mu_E-\mu_L}\,,$$

there exists a $\check{\theta} \in (\underline{\theta}, \overline{\overline{\theta}})$ such that when $\theta = \check{\theta}$, there is

$$\begin{cases} \lambda_{2t} > 0 & \text{iff} \quad \eta_{2t} < \eta(\check{\lambda})\,, \\ \lambda_{2t} = 0 & \text{iff} \quad \eta_{2t} \geqslant \eta(\check{\lambda})\,. \end{cases}$$

Thus, it is natural to define that

$$\underline{G_m} \equiv \left\{ \eta_{2t} | \frac{\mu_E R}{1-\mu_E} \leqslant \eta_{2t} < \eta(\check{\lambda}) \right\},$$

and

$$\underline{G_{hm}} \equiv \left\{ \eta_{2t} | \eta_{2t} \geqslant \frac{(\mu_E+\mu_L)R}{1-\mu_E-\mu_L} \text{ and } \frac{\mu_E R}{1-\mu_E} \leqslant \eta_{2t+2} < \eta(\check{\lambda}) \text{ for } \sigma_{2t+1} = 0 \right\}.$$

Now I confirm that $\eta^*_{2t+2} = \eta(\check{\lambda})$ is the optimal cash rebuilding policy when $\theta = \check{\theta}$ and $\eta_{2t} \in \underline{G_m}$. First, consider a perturbation $-\varepsilon < 0$ of cash rebuilding around σ^*_{2t+1} that satisfies $\eta^*_{2t+2} = \eta(\check{\lambda})$. On date $2t+1$ (in stage t), since $\lambda_{2t} = 1$ (by Lemma 8), the effective selling price on $2t+1$ is $\widehat{p}_L(\check{\lambda})$. Thus, the fire sale loss saved in stage t is

$$\frac{\varepsilon(1-\delta_L)R}{\widehat{p}_L(\check{\lambda})} > 0\,.$$

Now consider the same cash gap ε on date $2t+2$ (under the perturbed cash rebuilding policy $(\sigma^*_{2t+1}, -\varepsilon)$. This implies that $\eta_{2t+2} \in \underline{G_{hm}}$. Since $\lambda_{2t+2} \geqslant \check{\lambda}$, the fund always has to fire sell its assets on date $2t+2$ at most at the effective selling price $\widehat{p}_E(\check{\lambda}) > 0$. Hence, the expected increase of fire sale loss in stage $t+1$ due to this cash gap ε is at least

$$\frac{\varepsilon(1-\delta_E)R}{\widehat{p}_E(\check{\lambda})} > 0\,.$$

Since $\widehat{p}_E(\check{\lambda}) < \widehat{p}_L(\check{\lambda})$, there is

$$\frac{\varepsilon(1-\delta_L)R}{\widehat{p}_L(\check{\lambda})} < (1-\pi)\frac{\varepsilon(1-\delta_E)R}{\widehat{p}_E(\check{\lambda})}$$

for a sufficiently small but positive π. Hence, the perturbation $-\varepsilon$ is not profitable.

Next, consider another perturbation $\varepsilon > 0$ of cash rebuilding around σ^*_{2t+1} that satisfies $\eta^*_{2t+2} = \eta(\check{\lambda})$. On date $2t+1$ (in stage t), similarly, the fire sale loss increased in stage t is

$$\frac{\varepsilon(1-\delta_L)R}{\widehat{p}_L(\check{\lambda})} > 0\,.$$

Under the perturbed cash rebuilding policy $(\sigma^*_{2t+1}, \varepsilon)$ the fund gets ε more cash in stage $t+1$. This implies that $\eta_{2t+2} \in G_h$. Since there will be no runs on date $2t+2$, the marginal cash saves the fund's active asset sales on date $2t+3$ at the effective selling price $\widehat{p}_L(1) > 0$. Hence, the expected fire sale saved in stage $t+1$ due to this marginal cash ε is

$$\frac{\varepsilon(1-\delta_L)R}{\widehat{p}_L(1)} > 0\,.$$

Since $\widehat{p}_L(\check{\lambda}) < \widehat{p}_L(1)$ (and also $\pi \in (0,1)$), this perturbation ε is also not profitable. This verifies the optimality of $\eta^*_{2t+2} = (\mu_E + \mu_L)R/(1-\mu_E-\mu_L)$ when $\eta_{2t} \in \underline{G_m}$. This analysis can be readily extended to other subset of $G_l \cup G_m \cup G_{hl} \cup \underline{G_{hm}}$ as well as $\overline{G_{hm}} \cup G_{hh}$ following the same argument.

Finally, define

$$\overline{\theta} \equiv \check{\theta}(\check{\lambda} = 1) \in (\underline{\theta}, \overline{\overline{\theta}})\,.$$

By construction, when $\theta = \overline{\theta}$, there are $\lambda_{2t} > 0$ for $\eta_{2t} \in G_l \cup G_m$ while $\lambda_{2t} = 0$ for $\eta_{2t} \in G_h$ under the corresponding optimal cash rebuilding policy $\eta^*_{2t+2} = \eta(1)$.

Step 3. This step shows that when $\theta > \overline{\theta}$ there can not be equilibria other than that descried by Proposition 5. In this step, I use Figure 9 to help illustrate the idea. I first show that, when $\theta > \overline{\theta}$, there must be $G_{run} = G_l \cup G_m \cup G_h$. Note that, by Step 2, there must be $G_l \cup G_m \subseteq G_{run}$ when $\theta > \overline{\theta}$, and thus it suffices to show that it cannot be that

$$\sup G_{run} < \sup G_h\,.$$

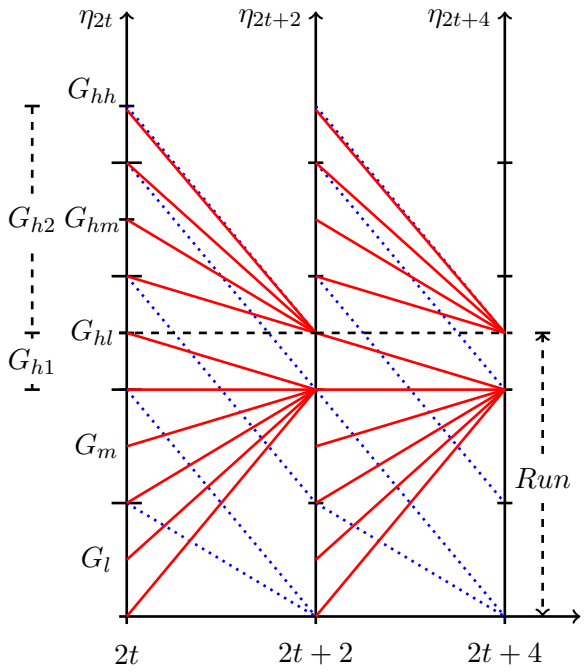


Figure 9: Hypothetical Equilibrium Cash Rebuilding Policy When $\theta \in [\overline{\theta}, 1)$

I prove this by contradiction. First suppose $\sup G_{run} \in G_{hl} \cup G_{hm}$. Define

$$G_{h1} \equiv G_{run}/(G_l \cup G_m)\,.$$

By the argument in the proof of Proposition 5, when $\eta_{2t} \in G_{run}$ the equilibrium cash rebuilding policy still features

$$\eta^*_{2t+2} = \frac{(\mu_E + \mu_L)R}{1 - \mu_E - \mu_L}\,. \tag{A.26}$$

However, because $\sup G_{run} \in G_{hl} \cup G_{hm}$, one can now find another non-zero-measure connected set $G_{h2} \subseteq G_{hl} \cup G_{hm}$ that satisfies

$$\inf G_{h2} = \sup G_{h1}\,,$$

in which shareholders will not run under the cash rebuilding policy (A.26).

By construction, the optimal cash rebuilding policy when $\eta_{2t} \in G_{h2}$ should be

$$\eta^*_{2t+2} \geqslant \sup G_{h1} > \frac{(\mu_E + \mu_L)R}{1 - \mu_E - \mu_L}\,. \tag{A.27}$$

To see this, consider a perturbation $-\varepsilon$ of cash rebuilding around this cash rebuilding policy. On date $2t+1$ (in stage t), since there are no runs when $\eta_{2t} \in G_{h2}$, the effective selling price on $2t+1$ is $\widehat{p}_L(0)$. Thus, the fire sale loss saved in stage t is

$$\frac{\varepsilon(1-\delta_L)R}{\widehat{p}_L(0)} > 0\,.$$

Now consider the same cash gap ε on date $2t+2$ (under the perturbed cash rebuilding policy $(\sigma^*_{2t+1}, -\varepsilon)$. This implies that $\eta_{2t+2} \in G_{h1}$. Because of shareholder runs on date $2t+2$, the fund will sell its assets at the effective selling price $\widehat{p}_L(1) > 0$. Hence, the expected increase of fire sale loss in stage $t+1$ due to this cash gap ε is

$$\frac{\varepsilon(1-\delta_L)R}{\widehat{p}_L(1)} > 0\,.$$

Since $\widehat{p}_L(1) < \widehat{p}_L(0)$, there is

$$\frac{\varepsilon(1-\delta_L)R}{\widehat{p}_L(0)} < (1-\pi)\frac{\varepsilon(1-\delta_L)R}{\widehat{p}_L(1)}$$

for a sufficiently small but positive π, implying that the perturbation $-\varepsilon$ is not profitable.

However, under the new, more rapid cash rebuilding policy (A.27), by the definition of $\underline{s}_h$, there must be a subset of G_{h2} in which late shareholders are going to run. This violates the definition of G_{h2}: a contradiction.

Now instead suppose $\inf G_{hh} \leqslant \sup G_{run} < \sup G_h$. Again by the monotonicity and continuity of $\underline{s}_h$ in a_{2t} and x_{2t}, there is no loss of generality to assume that $\sup G_{run} = \inf G_{hh} + \epsilon$, where $\epsilon > 0$ is arbitrarily small. Similarly, the optimal cash rebuilding policy when $\eta_{2t} = \inf G_{hh} + \epsilon$ should be

$$\eta^*_{2t+2} = \inf G_{hh} > \frac{(\mu_E + \mu_L)R}{1 - \mu_E - \mu_L}\,. \tag{A.28}$$

However, when $\eta_{2t} = \sup G_{hm} = \inf G_{hh}$, the optimal cash rebuilding policy already leads to runs. By the definition of $\underline{s}_h$, there must be shareholder runs when $\eta_{2t} = \inf G_{hh} + \epsilon$ under the cash rebuilding policy (A.28). This is again a contraction. As a result, there must be $G_{run} = G_l \cup G_m \cup G_h$.

Finally, by Proposition 5, the optimal cash rebuilding policy must be the same as described there because the pattern of shareholder runs is the same as described by Lemma 8. This ultimately concludes the proof. □

Proof of Proposition 7. It directly follows Propositions 4, 5, and 6 that for any θ, the proposed cash rebuilding policy

$$\eta^*_{2t+2} < \frac{\mu_E R}{1-\mu_E}$$

is not optimal when there is no commitment device. Here I provide a sufficient condition that it can be optimal if a commitment device is introduced.

Consider $\underline{\theta}$ as defined in Proposition 6. By definition, when $\theta = \underline{\theta} + \epsilon$, where $\epsilon > 0$ is arbitrarily small, and $\eta_{2t+2} = \mu_E R/(1-\mu_E)$, there is $\lambda_{2t} > 1$ for any $\eta_{2t} \in G_l$. Consider a perturbation $-\varepsilon < 0$ of cash rebuilding around $\eta_{2t+2} = \mu_E R/(1-\mu_E)$ when $\eta_{2t} = \mu_E R/(1-\mu_E)$, where the perturbation is chosen such that there is $\lambda_{2t} = 0$ for any $\eta_{2t} \in G_l$. On the one hand, since $\lambda_{2t+2} = 0$, the cash gap resulted from this perturbation on date $2t+2$ leads to the following expected increase of fire sale loss in stage $t+1$:

$$\frac{\varepsilon(1-\delta_E)R}{\widehat{p}_E(0)}\,.$$

On the other hand, when a commitment device is introduced, the determination of η_{2t+2} on $2t$ directly affects q_{2t} and q_{2t+1} through λ_{2t}. Thus, under the proposed perturbation $-\varepsilon < 0$, there are no runs on date $2t$, and thus the fire sale loss saved in stage t is

$$\Delta q_{2t}(1-\delta_E)R + \left(\frac{\varepsilon}{\widehat{p}_L(\lambda_{2t})} + \Delta q_{2t+1}\right)(1-\delta_L)R\,,$$

where λ_{2t} solves

$$\frac{\varepsilon}{\widehat{p}_L(\lambda_{2t})} = \frac{Ra_{2t} - \theta(1-\mu_E-\lambda_{2t}\mu_l)(Ra_{2t}+x_{2t}) - (1-\theta(1-\delta_E)(1-\mu_E-\lambda_{2t}\mu_l))Rq_{2t}(\lambda_{2t})}{(1-\delta_L)R} - q_{2t+1}(\lambda_{2t})\,,$$

and $\Delta q_{2t} = q_{2t}(\lambda_{2t}) - q_{2t}(0)$ and $\Delta q_{2t+1} = q_{2t+1}(0) - q_{2t+1}(\lambda_{2t})$.

Note that $\Delta q_{2t}(1-\delta_E)R + \Delta q_{2t+1}(1-\delta_L)R > 0$. Thus, if

$$\Delta q_{2t}(1-\delta_E) + \Delta q_{2t+1}(1-\delta_L) > (1-\pi)\frac{\varepsilon(1-\delta_E)}{\widehat{p}_E(0)} - \frac{\varepsilon(1-\delta_L)}{\widehat{p}_L(\lambda_{2t})}$$

satisfies, it is optimal for the fund to choose a less rapid cash rebuilding policy $\eta_{2t+2} < \mu_E R/(1-\mu_E)$. This concludes the proof. □

Proof of Proposition 8. This directly follows Propositions 4, 5, and 6. Under a Markov strategy profile, because the equilibrium is stationary, it suffices to show that a higher θ leads to higher total fire sale losses within stage t for any given positive (a_{2t}, x_{2t}). I then consider two cases.

Case 1. $\eta_{2t} \in G_l \cup G_m$. By Propositions 4, 5, and 6, the optimal cash rebuilding policy is always $\eta^*_{2t+2}(\theta) > 0$ where $\eta^*_{2t+2}(\theta)$ is increasing in θ. By definition, $\eta_{2t+2} = 0$ if $s_{2t+1} = 0$. Thus, the total fire sale losses in stage t is given by

$$L_t(\theta) = (1-\delta_E)Rq_{2t} + (1-\delta_L)Rq_{2t+1} + \frac{\eta^*_{2t+2}}{\widehat{p}_L + \eta^*_{2t+2}}(1-\delta_L)R(a_{2t} - q_{2t} - q_{2t+1})\,,$$

where q_{2t}, q_{2t+1}, and $\widehat{p}$ are both functions of λ_0 and in turn functions of θ. Propositions 4, 5, and 6 also imply that λ_{2t} is increasing in θ for any given positive (a_{2t}, x_{2t}). Hence, it follows Lemmas 4 and 9 that $L(\theta)$ is increasing in θ.

Case 2. $\eta_{2t} \in G_h$. By Lemma 2, $q_{2t} = q_{2t+1} = 0$ regardless of λ_{2t} or θ. Thus, the total fire sale losses in stage t is given by

$$L_t(\theta) = (1-\delta_L)Rs^*_{2t+1}\,.$$

Define $\underline{\eta_{2t+2}}$ as the target cash-to-assets ratio if $s_{2t+1} = 0$ and $\eta_{2t} \in G_h$. By Propositions 4, 5, and 6, the difference $\eta^*_{2t+2}(\theta) - \underline{\eta_{2t+2}}$ is increasing in θ. Moreover, $\widehat{p}_L$ is decreasing in λ_{2t} and thus decreasing in θ. This implies that s^*_{2t+1} is increasing in θ and so is $L(\theta)$ in this case. This finally concludes the proof. □

Proof of Proposition 9. Recall that, the Bellman equation for the non-commitment case is:

$$V(a_{2t}, x_{2t}) = -(1-\delta_E)Rq_{2t} - (1-\delta_L)Rq_{2t+1} + \max_{s^*_{2t+1}}[-(1-\delta_L)Rs_{2t+1} + (1-\pi)V(a_{2t+2}, x_{2t+2})]\,. \tag{A.29}$$

When a commitment device is introduced, the Bellman equation instead becomes:

$$V(a_{2t}, x_{2t}) = \max_{s^*_{2t+1}}[-(1-\delta_E)Rq_{2t} - (1-\delta_L)R(q_{2t+1} + s_{2t+1}) + (1-\pi)V(a_{2t+2}, x_{2t+2})]\,. \tag{A.30}$$

Also, the fund manager's objective function, which is to minimize the total expected fire sale losses, can be re-written as

$$\max_{\{s^*_{2\tau+1}\}_{\tau=t}^{\infty}} \mathbb{E}_t \sum_{\tau=t}^{T-1}[-(1-\delta_E)Rq_{2t} - (1-\delta_L)R(q_{2t+1} + s_{2t+1})]\,,$$

where the expectation is taken over the random variable T,[75] which is in turn govern by probability π. By the Principle of Optimality, the solution to (A.30) maximizes the fund manager's objective function, while the solution to (A.29) is feasible for the sequential problem associated with (A.30). This concludes the proof. □

Proof of Proposition 10 and Corollary 3. First, according to the starting cash-to-assets ratio η_{2t}, I still divide the stage game into three different regions. Without loss of generality, I consider $n_{2t} = 1$ as in the baseline model. Suppose the fund does not rebuild its cash buffer and no late shareholder is going to run, that is, $s_{2t+1} = 0$ and $\lambda_{2t} = 0$. Then there are three regions of the cash-to-assets ratio

[75] To be precise, the random variable T here denotes the stage (rather than the date) before which the game ends.

η_{2t} in the stage-t game. In these three regions, the amounts of illiquid assets that the fund has to sell passively on dates t and $t+1$ are characterized by:

$$\begin{cases} \text{High Region } G_h^\kappa: & q_{2t}^\kappa = 0, q_{2t+1}^\kappa = 0\,, \\ \text{Intermediate Region } G_m^\kappa: & q_{2t}^\kappa = 0, q_{2t+1}^\kappa > 0\,, \\ \text{Low Region } G_l^\kappa: & q_{2t}^\kappa > 0, q_{2t+1}^\kappa > 0\,. \end{cases}$$

I use the superscript κ to indicate the existence of the redemption fees. Note that, if a starting position (a_{2t}, x_{2t}) falls into a region G_j, $j \in \{h, m, l\}$, it does not necessarily falls into the same region G_j^κ when redemption fees are introduced. But by construction, there is

$$G_h^\kappa \cup G_m^\kappa \cup G_l^\kappa = G_h \cup G_m \cup G_l\,,$$

and

$$G_j^\kappa \cap G_k^\kappa = \varnothing, j \neq k\,.$$

Thus it suffices to consider the three regions G_h^κ, G_m^κ, and G_l^κ separately. Here I provide a complete analysis of the high region G_h^κ and the derivation for the other two regions directly follows.

In the high region G_h^κ, when $q_{2t}^\kappa = 0$ and $\lambda_{2t} = 0$, there is

$$NAV_{2t}^\kappa = Ra_{2t} + x_{2t}\,,$$

and

$$NAV_{2t+1}^\kappa = \frac{1-\kappa\mu_E}{1-\mu_E}(Ra_{2t} + x_{2t})\,.$$

Thus, $q_{2t}^\kappa = 0$ and $q_{2t+1}^\kappa = 0$ imply

$$\eta_{2t} \geqslant \frac{\left(\kappa\mu_E + \kappa\mu_L \dfrac{1-\kappa\mu_E}{1-\mu_E}\right) R}{1 - \kappa\mu_E - \kappa\mu_L \dfrac{1-\kappa\mu_E}{1-\mu_E}}\,.$$

This suggests that $G_h \subseteq G_h^\kappa$. This also suggests that Lemma 2 still holds. That means, for any λ_{2t}:

$$NAV_{2t}^\kappa(\lambda_{2t}) = Ra_{2t} + x_{2t}\,.$$

Meanwhile, when shareholder runs and cash rebuilding are introduced, there is

$$NAV_{2t+1}^\kappa(\lambda_{2t}) = \frac{R(a_{2t} - s_{2t+1}) + x_{2t} - \kappa(\mu_E + \lambda_{2t}\mu_L)(Ra_{2t} + x_{2t}) + \delta_L R s_{2t+1}}{1 - \mu_E - \lambda_{2t}\mu_L}\,.$$

Therefore, when $\theta = 1$, late shareholders' run incentives are governed by

$$\Delta NAV^{\kappa}(\lambda_{2t}) = \frac{\delta_L R s_{2t+1}}{1-\mu_E-\lambda_{2t}\mu_L} - \frac{(1-\kappa)(\mu_E+\lambda_{2t}\mu_L)(Ra_{2t}+x_{2t})}{1-\mu_E-\lambda_{2t}\mu_L}\,. \tag{A.31}$$

Clearly, when there are no redemption fees, that is, when $\kappa = 1$, this goes back to wedge (3.3) in the baseline model. For any $\kappa \in (0,1)$ and any $\lambda_{2t} \in [0,1]$, the second term in (A.31) is strictly positive. This directly implies that for any feasible s_{2t+1}, there is $\lambda_{2t}^{\kappa} \leqslant \lambda_{2t}$, where λ_{2t}^{κ} is the equilibrium run probability in the game with the redemption fee while λ_{2t} is that in the game without redemption fees, leading to the results in Proposition 10.

Also, for any (a_{2t}, x_{2t}) and any $\kappa \in (0,1)$, define

$$\overline{s} = \inf_{\lambda_{2t}\in[0,1]} \inf_{s_{2t+1}} \{s_{2t+1} | \Delta NAV^{\kappa}(s_{2t+1};\lambda_{2t}) \geqslant 0\}\,.$$

By construction, there is $\overline{s} > 0$. Then the result follows because $\Delta NAV^{\kappa}(\lambda_{2t})$ is strictly increasing in s_{2t+1}. This leads to the results in Corollary 3 and finally concludes the proof. □

Proof of Proposition 11. Under in-kind redemptions, any shareholder who redeems on date t will get a_t/n_t unit of assets and x_t/n_t unit of cash. Since there will be no forced fire sales at the fund level, the fund will no longer manage its cash buffer. This implies $\eta_t = \eta_0$ for any date t, where η_0 is the initial cash-to-assets ratio.

Consider any late shareholder on any odd date $2t+1$. If she redeems and consumes on date $2t+1$, she gets $\delta_L R a_0/n_0 + x_0/n_0$, while if she redeemed and consumed on date $2t$, she would get $\delta_E R a_0/n_0 + x_0/n_0$. Since $\delta_L > \delta_E$, no late shareholder will ever run in an equilibrium.

Now I consider total fire sale losses when $\theta = 0$. There is no loss of generality to consider $\eta_{2t} = \mu_E R/(1-\mu_E)$, which is the steady-state cash-to-assets ratio in the baseline model. Again due to the scale-invariance of the dynamic game, it suffices to consider an arbitrary state t. In the baseline model, by Proposition 4, the fire sale losses in stage t under the optimal cash rebuilding policy are:

$$\begin{aligned} L_t &= (1-\delta_L)R(q_{2t+1}+s^*_{2t+1}) \\ &= \frac{(1-\delta_L)R\mu_E\left(a_{2t} - \frac{\mu_L(Ra_{2t}+x_{2t})}{\left(\delta_L + \frac{\mu_L(1-\delta_L)}{1-\mu_E}\right)}\right)}{(1-\mu_E)\delta_L + \mu_L(1-\delta_L)+\mu_E} + \frac{(1-\delta_L)R\mu_L(Ra_{2t}+x_{2t})}{\left(\delta_L + \frac{\mu_L(1-\delta_L)}{1-\mu_E}\right)}\,, \end{aligned} \tag{A.32}$$

while the fire sale losses in stage t under in-kind redemptions are

$$L_t^{in-kind} = (1-\delta_E)R\mu_E a_{2t} + (1-\delta_L)R\mu_L a_{2t}\,. \tag{A.33}$$

Note that, when $\mu_L = 0$, (A.32) reduces to

$$L_t = \frac{(1-\delta_L)R\mu_E a_{2t}}{(1-\mu_E)\delta_L + \mu_E}\,,$$

while (A.33) reduces to

$$L_t^{in-kind} = (1 - \delta_E) R \mu_E a_{2t} \,.$$

Clearly, when δ_L is sufficiently larger than δ_E such that

$$1 - \delta_E > \frac{1 - \delta_L}{(1 - \mu_E)\delta_L + \mu_E} \,,$$

there is $L_t^{in-kind} > L_t$. Since fire sale losses are continuous in μ_L, and by Proposition 6 they are also continuous in θ, this concludes the proof. □

Proof of Proposition 12. This directly follows the proof of Proposition 10. The only difference is that under redemption fees any individual redeeming shareholder gets κNAV_t on date t while she gets NAV_t (if not being denied) under redemption restrictions. In expectation, any shareholder gets ζNAV_t when she redeems her shares under redemption restrictions. As such, this proof is identical to that of Proposition 10. □

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