Bank Information Sharing and Liquidity Risk*

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January 2016

Abstract

This paper proposes a novel rationale for the existence of bank information sharing schemes. We suggest that banks can voluntarily disclose borrowers' credit history in order to maintain asset market liquidity. By entering an information sharing scheme, banks will face less adverse selection when selling their loans in secondary markets. This reduces the cost of asset liquidation in case of liquidity shocks. The benefit, however, has to be weighed against higher competition and lower profitability in prime loan markets. Information sharing can arise endogenously as banks tradeoff between asset liquidity and rent extraction. Different from the previous literature, we allow for borrower's non-verifiable credit history, and show that banks still have incentives to truthfully disclose such information in competitive credit markets.

JEL Classification: G21.

Keywords: Information Sharing, Funding Liquidity Risk, Market Liquidity, Adverse Selection in Secondary Market.

*We would like to thank Thorsten Beck, Christoph Bertsch, Sudipto Dasgupta, Xavier Freixas, Vasso Ioannidou, Lei Mao, Marco Pagano, Kasper Roszbach, Lucy White and seminar participants at Riksbank, University of Gothenburg, University of Lancaster, University of Warwick for useful comments. The usual disclaimer applies.

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1 Introduction

One of the rationales for the existence of banks is their roles in liquidity transformation. Borrowing short-term and lending long-term, banks face funding liquidity risk which is an innate characteristic of financial intermediation (Diamond and Dybvig, 1983). This paper argues that such funding risk can be at the root of the existence of information sharing agreements among banks. The need of information sharing arises because banks in need of liquidity have to sell their assets in secondary markets. Information asymmetry in such markets can make the cost of asset liquidation particularly high (i.e., fire-sales). In order to mitigate adverse selection problems, banks could find it convenient to share information about the quality of assets that they hold. This reduces the cost of asset liquidation when liquidity needs materialize. Information sharing allows banks to reduce adverse selection in secondary loan markets, which in turn reduces the damage of asset fire sales in case of liquidity needs.[1]

The benefit of information sharing, however, has to be traded off with its potential cost. Letting other banks know the credit worthiness of its own borrowers, an incumbent bank sacrifices its market power. Likely its competitors will forcefully compete for the good borrowers. The intensified competition will reduce the incumbent bank's profitability. We develop a simple model to analyze this trade-off.

We consider an economy made of two banks, one borrower, and many asset buyers. One of the banks is a relationship bank that has a long standing lending relationship with the borrower. It knows both the credit worthiness (i.e., the type) and the credit history (i.e., the repayments) of the borrower's. While the information on borrower credit worthiness cannot be communicated, the credit history can be shared. The second bank is a distant bank, and it has no lending relationship with the borrower so it does not have any information about the borrower's credit worthiness or history. This bank can however compete for the borrower by offering competitive loan rates. The borrower can be risky or safe. While both types have projects of positive NPV, the safe borrower surely brings the project to maturity while the risky one does so only with a certain probability. The distant bank can lose from lending if it cannot price the loan correctly.

The relationship bank is subject to liquidity risk, which we model as a possibility of

[1] A similar argument can be made for collateralized borrowing and securitization, where the reduced adverse selection will lead to lower haircut and higher prices for securitized assets.

an (idiosyncratic) bank run. When liquidity need arises, the relationship bank can sell in a secondary market the loan it has granted to the borrower. Since the quality of the loan is unknown to third parties, the secondary market for asset is characterized by adverse selection. Even if the relationship bank holds a safe loan, to sell that at a discount can incur the risk of bankruptcy. Sharing information ex-ante is beneficial because it reduces the adverse selection problem and boosts the liquidation value. The relationship bank trades off higher asset liquidity against rent extraction, and it will voluntarily share information when the benefit outweighs the cost.

The analysis unfolds in three steps. First, we provide an existence result, pinning down the conditions under which information sharing can save the relationship bank from illiquidity. This happens when the participation in an information sharing scheme actually boosts the asset price in the secondary market. This result is not trivial to obtain because information sharing has two countervailing effects on the asset price. On the one hand, observing a good credit history, the asset buyers are willing to pay more for the bank's loan on sale. As the quality of the loan (i.e., the borrower's type) is more likely to be high quality. The adverse selection on the high quality borrower reduces as a result of information sharing. On the other hand, the distant bank competes more aggressively for this loan exactly for the same reason. This drives down the loan rate charged by the relationship bank on the high quality borrower. Since the loan is less profitable, its price in the secondary market decreases. We show that the first effect always dominates.

Second, we look at the equilibrium and characterize the conditions when the relationship bank actually chooses to share information. These conditions coincide with the existence conditions if the relationship bank's probability of becoming illiquid (bank run) is sufficiently high. Indeed, the benefit of information sharing is high and the relationship bank finds it optimal to share information whenever is feasible. Otherwise, when the probability of a run is low, the parameter constellation in which the relationship bank chooses to share information is smaller than the one in which information sharing saves the relationship bank from illiquidity. This occurs because the reduction in expected profits due to more intense competition overcomes the expected benefit of the higher asset's liquidation value.

Lastly, we relax the common assumption in the existing literature that the shared credit history is verifiable. The relationship bank can lie about the borrower's credit history when it shares this information. There are both theoretical and practical reasons to think that such assumption is quite restrictive. From a theoretical point of view, a natural way to sustain truth telling would be to employ a dynamic setting where banks have some reputation at stake. This would induce them to say the truth. We use instead a static game to show that truth telling under information sharing can be indeed a perfect Bayesian equilibrium. From a practical point of view, the verifiability assumption can be rationalized in certain contexts, but it maybe be quite unrealistic in other circumstances. For example, Giannetti et al. (2015) show that banks manipulate the credit ratings of their borrowers in the Argentinian credit registry. On a more casual level, information manipulation can take place in the form of 'zombie' lending, like it occurred in Japan with the ever-greening phenomenon or in Spain where banks kept on lending to real estate firms likely to be in distress after housing market crash. We allow for the possibility that banks can manipulate credit reporting and overstate past loan performance. We show under which conditions the relationship bank has an incentive to truthfully disclose the information on the borrower's credit history. It turns out that it exists a narrower parameter constellation than the one in which information sharing is chosen in equilibrium under the assumption of verifiable credit history. In particular, banks have the incentive to truthfully communicate borrower' credit history when credit market is competitive. In fact, one necessary condition for information sharing to be sustained as a truth-telling equilibrium is that the relationship bank can increase the loan rate charged on borrower with bad credit history.

The conjecture that information sharing is driven by market liquidity is novel and complementary to existing rationales. Previous literature has mostly rationalized the presence of information sharing by focusing on the loan market. Sharing information can either reduce adverse selection (Pagano and Jappelli, 1993) or mitigate moral hazard (Padilla and Pagano, 1997 and 2000). In their seminal paper, Pagano and Jappelli (1993) rationalize the existence of information sharing as a mechanism to have more accurate information about borrowers that change location and therefore the bank from which they borrow. Sharing ex-ante information about borrowers reduces their riskiness and increases banks' expected profits. This beneficial role is traded off against the cost of losing the information advantage over the competitors. We see information sharing as stemming also from frictions on the secondary market for asset sale instead only on the prime loan market. The two explanations are in principle not mutually exclusive but complementary.

Another strand of the literature argues that information sharing allows the incumbent bank to extract more monopolistic rent. When competition for borrowers occurs in two periods, inviting the competitor to enter in the second period by sharing information actually dampens the competition in the first period (Bouckaert and Degryse, 2004; Gehrig and Stenbacka 2007). Sharing information about the past defaulted borrowers deters the entry of competitor, which allows the incumbent bank to capture those unlucky but still good borrowers (Bouckaert and Degryse, 2004). This mechanism is also present in our model, and it is related to our analysis with unverifiable credit history. The incumbent (relationship) bank has an incentive to report the true credit history if it can charge higher loan rates to a good borrower with bad credit history.

Finally, a couple of papers link information sharing to other banking activities. For example, information sharing can complement collateral requirement since the bank is able to charge high collateral requirement only after the high risk borrowers are identified via information sharing (Karapetyan and Stacescu 2014b). Information sharing can also complement information acquisition. After hard information is communicated, collecting soft information to boost profit becomes a more urgent task for the bank (Karapetyan and Stacescu 2014a). In those papers, the goal is not to provide a rationale of why banks voluntary choose to share information but how information sharing affects other dimensions of bank lending decisions.

Our novel theoretical exposition also opens road for future empirical research. The model generates complementary empirical implications that information sharing will facilitate banks' liquidity management and loan securitization. The model also suggests that information sharing system can be more easily established, and can work more effectively, in countries with competitive banking sector, and in credit market segments where competition is strong. These empirical predictions would complement the existing empirical literature which has mostly focused on the impact of information sharing on bank risks and firms' access to bank financing.[2]

The remainder of this paper is organized as follows. In the next section we present the model. In Section 3 we show under which conditions information sharing arises endogenously when borrower's credit history is verifiable. Section 4 shows when information

[2] Doblas-Madrid and Minetti (2013) finds that information sharing reduces contract delinquencies. Houston, et al. (2010) finds that information sharing is correlated with lower bank insolvency risk and likelihood of financial crisis. Brown et al. (2009) find that information sharing improves credit availability and lower cost of credit to firms in transition countries.

sharing is still chosen in equilibrium when credit history is not verifiable. Section 5 analyzes welfare and policy implication. Section 6 concludes.

2 Model Setup

The economy consists of banks, a relationship bank and a distant bank, one borrower and many depositors and asset buyers. All agents are risk neutral. The gross return of the risk-free asset is indicated as r_0.

For simplicity, we assume that a bank has one loan on its balance sheet. The loan requires 1 unit of initial funding, and its returns depend on the type of the borrower. The borrower can be either safe (H-type) or risky (L-type). The ex-ante probability of the safe type $\Pr(H)$ is equal to α, and for the risky type $\Pr(L)$ is equal to $1-\alpha$. A safe borrower always generates a payoff $R > r_0$, and a risky borrower has a payoff that depends on an aggregate state $s = \{G, B\}$. In the good state G, the payoff is the same as a safe borrower R, but in the bad state B the payoff is 0. The ex-ante probabilities of the two states are $\Pr(G) = \pi$ and $\Pr(B) = 1 - \pi$, respectively. Throughout the paper, we assume no credit rationing. Even a risky loan has a positive NPV, that is, $\pi R > r_0$.[3]

The relationship bank has an ongoing lending relationship with a borrower. It privately observes both the credit worthiness (i.e., the type) and the payment history of the relationship borrower. The distant bank, on the other hand, has no lending relationship with the borrower and observes no information about the borrower's type. It does not know the credit history either, unless the relationship bank shares such information. The distant bank can compete for the borrower by offering lower loan rates, but to initiate the new lending relationship it bears a fixed cost c. Such cost instead represents a sunk cost for the relationship bank.[4]

We make a distinction between soft and hard information. While borrower's credit

[3] One potential interpretation is to consider the H-type being prime mortgage borrowers, and L-type being subprime borrowers. While both can pay back their loans in a housing boom, the subprime borrowers will default once housing price drops. However, the probability of a housing market boom is sufficiently large that it is still profitable to lend to both types.

[4] One possible interpretation of the fixed cost c can be the fixed cost paid by the bank to establish new branches, hire and train new staffs, etc. Alternatively it can represents the borrower's switching cost that is paid by the bank.

worthiness (type) is assumed to be soft information and cannot be communicated to the others, credit history is assumed to be hard information and can be shared with third parties. We model information sharing as a unilateral decision of the relationship bank. If the bank chooses to share the credit history of its borrower, it makes announcement about whether the borrower had defaulted or not. We label a credit history with previous defaults as D, and a credit history without defaults as $\overline{D}$. A safe borrower has a credit history $\overline{D}$ with probability 1, and a risky borrower has a credit history $\overline{D}$ with probability π and a credit history D with probability $1-\pi$.[5]

The relationship bank and the distant bank compete for the borrower by offering loan rates. The banks are financed solely by deposits. We abstract from risk-shifting induced by limited liability, and assume that there is perfect market discipline so that deposit rates are determined based on bank's risk. Depositors are assumed to have the same information about the borrower as the distant bank. In a competitive deposit market, the depositors demand to earn the risk-free rate r_0 in expectation.

To capture funding liquidity risk, we assume the probability that the relationship bank faces a run equals to $\Pr(\text{run}) = \rho$. In such a case all depositors withdraw their funds. Otherwise, we have no bank run with probability $\Pr(\text{no run}) = 1-\rho$. When a run happens, the relationship bank needs to raise liquidity to meet the depositors' withdrawals. We assume that physical liquidation of the bank's loan is not feasible, and only financial liquidation—a loan sale to asset buyers—is possible. We also assume that the loan is indivisible and the bank has to sell it as a whole. The state $s = \{G, B\}$ realizes after the loan competition, and it becomes public information. Asset buyers observe the true state, but are uninformed of the credit worthiness of the relationship borrower's. They can nevertheless condition their bids on the borrower's credit history if the relationship bank shares the information. We assume that the secondary asset market is competitive, and risk neutral asset buyers only require to break even in expectation.

Notice that bank asset can be on sale for two reason: either due to funding liquidity need, in which case H-type loans can be on sale, or due to strategic sale for arbitrage reason, in which case only L-type loans will be sold. The possibility of strategic asset

[5] It is equivalent to assume there was a first round of lending before the current model. If the borrower is safe, it generated a non default credit history. If the borrower is risky, its payoff depended on the state when the first round of lending occurred. If the state was good, the risky borrower did not default as well, instead if the state was bad, the risky borrower had a credit history of D.

sale leads to adverse selection in the secondary asset market. Therefore, H-type loans are underpriced during asset sale and even a solvent relationship bank owning an H-type loan can fail due to illiquidity. In case of a bank failure, we assume that bankruptcy costs result in zero salvage value. Such liquidity risk and costly liquidation gives the relationship bank the incentive to disclose the credit history of its borrower, in the hope that such information sharing can boost asset market liquidity.

The sequence of events is summarized in Figure 1.

[Put Figure 1 here]

The timing captures the fact that information sharing is a long-term decision (commitment), while competition in the loan market and the liquidity risk faced by the bank are shorter-term concerns.

At $t = 0$ the relationship bank inherits a lending relationship and decides to participate in the information sharing scheme or not. At $t = 1$, the borrower's type and credit history realizes. The relationship bank privately observes these information and announces the borrower's credit history if it chose to participate in information sharing scheme in the previous stage. At $t = 2$, the two banks compete in loan rates for the opportunity to lend to the borrower again. The winning bank is financed by competitive depositors. At $t = 2.5$, the aggregate state realizes and is publicly observed. The relationship bank's liquidity risk realizes and is only privately known. The relationship bank raises liquidity by selling its loan on the secondary asset market. Finally, at $t = 3$ the loan pays off.

3 Verifiable Credit History

We solve the decentralized solution by backward induction. Therefore we proceed as follows: i) determine the prices at which loans are traded in the secondary asset market; ii) compute the deposit rates at which depositors supply their fund to the bank; iii) determine the loan rates at which the bank offers credit to the borrower; iv) decide if the relationship bank wants to share or not the information on the borrower's credit history.

Depending on whether banks share information or not, the game has different information structures. Without information sharing, asset prices, loan rates and deposit rates cannot be conditional on the borrower's credit history. On the contrary, such variables will depend on credit history if information is shared. Through this section we follow the literature and assume that credit history, once shared, is perfectly verifiable. In Section 4 we allow for the possibility that the relationship bank can manipulate the credit history and overstate past loan performance.

3.1 Asset Prices

We determine at which price loans are traded in the secondary market taking as given loan rates and deposit rates. We indicate with P_i^s the asset price in state $s = \{G, B\}$ and with information-sharing regime $i = \{N, S\}$, where N is no information sharing in place, and S refers to the presence of information sharing. Like all other agents, asset buyers can perfectly observe state s, but they cannot observe whether the loan sale is for liquidity reason or for arbitrage. Accordingly, the pricing of loans is state-contingent and takes into account the relationship bank's strategic behaviors.

Without information sharing, if the aggregate state is good, the borrower will generate the same payoff, and therefore $P_N^G = R_N$ independently of the borrower's type. That is, asset buyers are competitive so they bid until zero profit. If the state is bad, the L-type borrower will generate a zero payoff. Asset buyers cannot update their prior beliefs since the relationship bank does not share any information on borrower's credit history. For any positive price, L-type loan will be on sale even if the relationship bank faces no bank run. Due to the presence of L-type loan, H-type loan will be sold at a discount. Consequently, it is sold by the relationship bank only if there is urgent liquidity needs to meet the depositors' withdrawals. The market is characterized by adverse selection. The price P_N^B is determined by the following break-even condition of asset buyers

$$\Pr(L)(0 - P_N^B) + \Pr(H)\Pr(\text{run})(R_N - P_N^B) = 0,$$

which implies

$$P_N^B = \frac{\alpha\rho}{(1-\alpha) + \alpha\rho} R_N. \tag{1}$$

It follows immediately that the H-type loan is underpriced (lower than the fundamental value R_N) because of adverse selection in the secondary asset market.

With information sharing, asset prices can be conditional on the borrower's credit history D and $\overline{D}$ too. If the state is good no loan will default, the prices equal to the face value of loans. We have

$$P_S^G(D) = R_S(D)$$

and

$$P_S^G(\overline{D}) = R_S(\overline{D}),$$

where $R_S(D)$ and $R_S(\overline{D})$ denote the loan rates for a borrower with and without default history, respectively. Notice that asset prices are different because the loans rate are different, conditional on the information released. When the state is bad, asset buyers can update their beliefs accordingly. When the relationship bank announce a previous default then the borrower is perceived as a L-type for sure, therefore posterior beliefs are $\Pr(H \mid D) = 0$ and $\Pr(L \mid D) = 1$. Since a L-type loan defaults in state B with certainty, we have $P_S^B(D) = 0$. When the announced credit history is $\overline{D}$ (no default), then posterior beliefs, according to Bayesian rule, are

$$\Pr(H \mid \overline{D}) = \frac{\Pr(H, \overline{D})}{\Pr(\overline{D})} = \frac{\alpha}{\alpha + (1-\alpha)\pi} > \alpha$$

and

$$\Pr(L \mid \overline{D}) = \frac{\Pr(L, \overline{D})}{\Pr(\overline{D})} = \frac{(1-\alpha)\pi}{\alpha + (1-\alpha)\pi} < 1 - \alpha.$$

Intuitively, asset buyers uses the credit history as a noisy signal of the loan quality. A loan with a good credit history $\overline{D}$ is more likely to be of H-type, thus $\Pr(H \mid \overline{D}) > \alpha$.

Given the posterior beliefs, asset buyers anticipate that the relationship bank always sells L-type loan and withholds the H-type loan to maturity if no bank run occurs, therefore the price $P_S^B(\overline{D})$ they are willing to pay is given by the following break even condition

$$\Pr(L \mid \overline{D})[0 - P_S^B(\overline{D})] + \Pr(H \mid \overline{D}) \Pr(\text{run})[R_s(\overline{D}) - P_S^B(\overline{D})] = 0,$$

which implies

$$P_S^B(\overline{D}) = \frac{\alpha\rho}{(1-\alpha)\pi + \alpha\rho} R_s(\overline{D}). \tag{2}$$

Comparing (1) with (2), conditional on $\overline{D}$-history, the perceived chance that a loan is H-type is higher under information sharing. This is because a L-type borrower with bad credit history D can no longer be pooled with a H-type in asset sales. Information sharing therefore mitigates the adverse selection problem. However, we cannot yet draw a final conclusion on the relationship between the asset prices until we determine the equilibrium loan rates R_N and $R_s(\overline{D})$.

3.2 Deposit Rates

We assume that deposits are fairly priced for the risk and that depositors have the same information on the credit worthiness of loan applicants as the distant bank. Consequently, the pricing of deposit rates can be conditional on the riskiness of bank's loan as well as the past credit information of the loan applicants if the relationship bank shared this piece of information. We determine equilibrium deposit rates r_i, with $i = \{N, S\}$, taking as given the loan rates.

On the equilibrium path, it will be the relationship bank that finances the loan. We first discuss the deposit rates charged to the relationship bank, i.e. the deposit rates on equilibrium path. Besides the fundamental asset risk, the liquidity risk faced by the relationship bank is endogenized in pricing the deposit rates. A *necessary* condition for a candidate deposit rate to be an equilibrium one is that the depositors break even by earning zero expected payoff under this rate. The break-even condition is only necessary because we have to check the depositors do not have a profitable deviation by charging a lower rate than the break-even one. Since deposits can be either risky or safe, a break-even deposit rate can be so high that the relationship bank cannot survive a run. In this case, lowering the deposit rate can save the relationship bank and it can guarantee to the depositors a positive payoff.

Consider the situation where relationship bank does not participate in the information sharing program, and denote r_N as the equilibrium deposit rate. When the loan opportunity is risky, define $\widehat{r}_N$ as the break-even rate for risky deposit, we have

$$[\Pr(G) + \Pr(H)\Pr(B)\Pr(\text{no run})]\widehat{r}_N = r_0,$$

which implies

$$\widehat{r}_N = \frac{r_0}{\pi + \alpha(1-\pi)(1-\rho)} > r_0.$$

Notice that deposit rate is charged before the realization of the state s and of the (possible) bank run. Facing a bank run, the relationship bank will be bankrupt. We implicitly assume that the parameter values are such that $P_N^B < \widehat{r_N}$ in the case of risky deposits. Recall that there is zero salvage value when bankruptcy occurs, then a candidate equilibrium rate is $\widehat{r}_N$ in case of risky deposits. On the other hand, if the parameter values are such that $P_N^B > \widehat{r_N}$, the relationship bank will survive a bank run. The deposits are safe, then a candidate equilibrium deposit rate is simply r_0 in case of safe deposits.

The following Lemma characterizes the equilibrium deposit rates in case information sharing is not in place.

Lemma 1 *Assume there is no information sharing, then deposit rates are as follows: (i) If* $P_N^B \geq r_0$ *then* $r_N = r_0$*; (ii) If* $P_N^B < r_0$ *then* $r_N = \widehat{r}_N$*.*

The proof is in the Appendix. The intuition is that when the price of the asset to liquidate is greater than or equal to the risk-free rate, then deposits are not risky and depositors can be remunerated with the risk-free rate. Otherwise, if the price of the asset is less than the risk-free rate, bankruptcy occurs and deposits become risky. Depositors anticipate this possibility, and they have to be remunerated with the interest $\widehat{r}_N$ higher than the risk-free rate.

We now characterize deposit rates when the relationship bank adopts the information sharing regime. The deposit rates are now conditional on the credit history of the borrower. If the borrower has a credit history with default (i.e., a D-history) then depositors knows the borrower is surely L-type and $P_S^B(D) = 0$. Therefore depositors are paid only if the state is G. This leads depositors to ask a deposit rate $r_S(D)$ that satisfies the break-even condition $\Pr(G) r_S(D) = r_0$. Accordingly we have

$$r_S(D) = \frac{r_0}{\pi} > r_0. \tag{3}$$

When the borrower has a $\overline{D}$-history (i.e., no default) the analysis is similar to the no information sharing, and the candidate equilibrium deposit rates depend on parameter values. Defining the break-even deposit rate for risky deposits by $\widehat{r}_S(\overline{D})$, we have

$$[\Pr(G) + \Pr(B) \Pr(H \mid \overline{D}) \Pr(\text{no run})] \widehat{r}_S(\overline{D}) = r_0$$

that implies

$$\widehat{r}_S(\overline{D}) = \frac{\alpha + (1-\alpha)\pi}{\alpha + (1-\alpha)\pi^2 - (1-\pi)\alpha\rho} r_0 > r_0.$$

Again, if the parameter values are such that $P_S^B(\overline{D}) > r_0$, a candidate equilibrium deposit rate is $\widehat{r}_S(\overline{D})$. Instead, if the parameter values are such that $P_S^B(\overline{D}) > r_0$, a candidate equilibrium deposit rate is simply the risk-free rate r_0. The following Lemma characterizes the equilibrium deposit rates when the no default history $\overline{D}$ is reported.

Lemma 2 *Assume information sharing is in place and the borrower has a* $\overline{D}$*-history, then deposits rates are as follows: (i) If* $P_S^B(\overline{D}) \geq r_0$ *then* $r_S(\overline{D}) = r_0$*; (ii) If* $P_S^B(\overline{D}) < r_0$ *then* $r_S(\overline{D}) = \widehat{r}_S(\overline{D})$*.*

The proof is provided in the Appendix, and the intuition is similar to Lemma 1. When the price of the asset in the secondary market is sufficiently high, the equilibrium deposit rate is equal to the risk-free rate. Otherwise, deposits are risky and consequently the equilibrium deposit rate is higher than the risk-free rate.

We now compute the break-even deposit rates r_i^E with $i = \{N, S\}$ charged to the distant bank. These deposit rates are off-equilibrium rates since it is the relationship bank that finances the loan in equilibrium. Remember that the distant bank only faces the fundamental asset risk.[6] Without information sharing, the deposit rate r_N^E is determined by depositors' break-even condition as follows

$$\Pr(H) r_N^E + \Pr(L) \Pr(G) r_N^E = r_0,$$

which implies

$$r_N^E = \frac{r_0}{\alpha + (1-\alpha)\pi} > r_0. \tag{4}$$

Under the information sharing regime, the deposit rate $r_S^E(\overline{D})$ charged when the borrower has no previous default is determined by depositors' break even condition

$$\Pr(H \mid \overline{D}) r_S^E(\overline{D}) + \Pr(L \mid \overline{D}) \Pr(G) r_S^E(\overline{D}) = r_0,$$

which implies

$$r_S^E(\overline{D}) = \frac{\alpha + (1-\alpha)\pi}{\alpha + (1-\alpha)\pi^2} r_0 > r_0. \tag{5}$$

Finally, the deposit rate $r_S^E(D)$ charged when the borrower has a default history is given by the depositors' break even condition $\Pr(G) r_S^E(D) = r_0$, which implies $r_S^E(D) = r_0/\pi$.

3.3 Loans Rates

We assume the credit market is contestable, then the loan rates charged to the borrower are determined by the break-even condition of the distant bank that tries to enter the loan market. We call R_i^E the loan rate offered by the distant (entrant) bank to the borrower under information-sharing regime $i = \{N, S\}$. As noticed, we assume that the distant bank does not face liquidity risk but only fundamental asset risk.

[6] While the relationship bank faces the liquidity risk, that the distant bank does not face, the relationship bank has an extra tool (information sharing decision) to manage that risk. Our set up is symmetric in this respect.

Without information sharing, the distant bank holds the prior belief on the borrower's type. The break-even condition for the distant bank is

$$\Pr(H)(R_N^E - r_N^E) + \Pr(L)\Pr(G)(R_N^E - r_N^E) = c,$$

where c is the fix entry cost and r_N^E is the deposit rate paid by the distant bank to its depositors determined in (4). Combining the two expressions, we get

$$R_N^E = \frac{c + r_0}{Pr(H) + Pr(L)Pr(G)} = \frac{c + r_0}{\alpha + (1-\alpha)\pi}.$$

With information sharing in place, loan rates are contingent on credit history. If the distant bank observes a previous default, then the borrower is surely an L-type. The distant bank's break-even condition is

$$\Pr(G)[R_S^E(D) - r_S^E(D)] = c,$$

where $r_S^E(D) = r_0/\pi$. Combining these two expressions, we get

$$R_S^E(D) = \frac{c + r_0}{\pi}.$$

When the credit history of the borrower is $\overline{D}$, the distant bank updates its belief and its break-even condition is

$$\Pr(H \mid \overline{D})[R_S^E(\overline{D}) - r_S^E(\overline{D})] + \Pr(L \mid \overline{D})\Pr(G)[R_S^E(\overline{D}) - r_S^E(\overline{D})] = c,$$

where $r_S^E(\overline{D})$ is given by (5). Combining the two expressions, we get

$$R_S^E(\overline{D}) = \frac{c + r_0}{Pr(H|\overline{D}) + Pr(L|\overline{D})Pr(G)} = \frac{\alpha + (1-\alpha)\pi}{\alpha + (1-\alpha)\pi^2}(c + r_0).$$

A simple comparison of the loan rates makes it possible to rank them as follows.

Lemma 3 *The ranking of the loan rates charged by the distant bank is* $R_S^E(\overline{D}) < R_N^E < R_S^E(D)$.

When information sharing is in place, and the borrower has the no-default history $\overline{D}$, the distant bank offers the lowest loan rate since it is more likely that the borrower is H-type. On the contrary, if the credit history presents defaults the distant bank charges the highest loan rate since the borrower is surely an L-type. Without information sharing, the distant bank offers an average loan rate (reflecting the prior belief about borrower's type).

The equilibrium loan rate also depends on the contestability of the loan market. Suppose $R_i^E > R$, then the payoff R from the project (loan) is too low and entry into such loan market is never profitable for the distant bank. Then the relationship bank can charge the monopolistic loan rate taking the entire payoff from the project. Suppose, otherwise, $R_i^E \leq R$. In this case the payoff R is high enough to induce the distant bank to enter the loan market. The relationship bank in this case can only undercut the loan rate to R_i^E. The equilibrium loan rate is determined by the break-even loan rate charged by the distant bank. Let us indicate the equilibrium loan rate as R_i^* under information-sharing regime $i = \{N, S\}$. The following lemma characterizes the equilibrium loan rates.

Lemma 4 *In equilibrium the loan is financed by the relationship bank. The equilibrium loan rates depend on the relationship between the distant bank's break-even loan rates and the project's return R. We have the following four cases:*

- *Case 0: If* $R \in R_0 = (c + r_0, R_S^E(\overline{D})]$ *then* $R_S^*(\overline{D}) = R_N^* = R_S^*(D) = R$
- *Case 1: If* $R \in R_1 = (R_S^E(\overline{D}), R_N^E]$ *then* $R_S^*(\overline{D}) = R_S^E(\overline{D})$ *and* $R_N^* = R_S^*(D) = R$
- *Case 2: If* $R \in R_2 = (R_N^E, R_S^E(D)]$ *then* $R_S^*(\overline{D}) = R_S^E(\overline{D})$*,* $R_N^* = R_N^E$ *and* $R_S^*(D) = R$
- *Case 3: If* $R \in R_3 = (R_S^E(D), \infty)$ *then* $R_S^*(\overline{D}) = R_S^E(\overline{D})$*,* $R_N^* = R_N^E$ *and* $R_S^*(D) = R_S^E(D)$*.*

Where R_j, with $j = \{0, 1, 2, 3\}$, denotes the set of payoffs of the project's return R for each case j. Consider Case 0, the payoff R is so low that distant bank does find convenient to enter the loan market. In this case, the loan market is least contestable, and the relationship bank charges the monopolistic loan rate R irrespective of the borrower's credit history. The higher R, and the more contestable the loan market becomes. In Case 3, the loan market is the most contestable since R is so high that the distant bank competes for a loan even when the borrower shows the defaulted D-history. The four cases are mutually exclusive, as it is clear from Figure 2 that represent them graphically.

[Put Figure 2 here]

Recall expressions (1) and (2), and the fact that the perceived loan quality is higher for a $\overline{D}$-loan with information sharing than for a loan with unknown credit history. The benefit of information sharing is to mitigate the adverse selection. However, we noticed that there is also a second effect that goes through the equilibrium loan rates R_N^* and $R_s^*(\overline{D})$. As $R_S^*(\overline{D}) \leq R_N^*$, it seems that information sharing *may* result in $P_S^B(\overline{D}) < P_N^B$ as it decreases loan rate from R_N^* to $R_S^*(\overline{D})$. We establish in Proposition 1 in the next section that the effect of reduced adverse selection is of the first order importance, and it is always true that $P_S^B(\overline{D}) > P_N^B$.

3.4 The Benefit of Information Sharing

We now show that in each of the cases $j = \{0, 1, 2, 3\}$, corresponding to different degree of loan market contestability, there exists a set of parameter values that guarantees the existence of a region where information sharing is indeed beneficial to the relationship bank. To be more specific, we show that there exists a parameter region where the relationship bank owning an H-type loan will survive from bank run when sharing information but will fail otherwise in the bad state. To understand intuitively when this can be the case, recall the analysis in Section 3.1 about the asset prices in the secondary loan market.

When the state is bad (B) only an H-type loan generates positive payoff, and there is adverse selection in the secondary market. If asset buyers do not know the exact type of a loan, it results in the underpricing of an H-type loan. Relationship bank may fail from a bank run even if it hold a safe H-type loan. Sharing information on credit history could therefore boost the asset price in the secondary market by mitigating the adverse selection. However, the distant bank also competes more fiercely with the relationship bank in the prime loan market for a borrower with good credit history. Accordingly, the relationship bank's profitability of financing an H-type of loan decreases. This in turn negatively affects the asset price in the secondary market.

The following result establishes the existence of a set of parameter values that guarantees that information sharing indeed promotes market liquidity in the bad state. Under such parameter values, the positive effect of mitigating adverse selection dominates the negative effect of lower profitability. We have

Proposition 1 *In the bad state, the equilibrium asset price is $P_S^B(\overline{D}) > P_N^B$. Whenever $P_S^B(\overline{D}) > r_0 > P_N^B$ information sharing can save the relationship bank from illiquidity.*

The proof is in the Appendix. The result can be easily verified with Case 0, where equilibrium loan rates are equal to R regardless of the information sharing regime. Indeed with $R_S^*(\overline{D}) = R_N^* = R$, the comparison between expression (1) and (2) is straightforward and we have $P_S^B(\overline{D}) > P_N^B$.

The result also hold for all other cases because of the presence of adverse selection both in the prime loan market and in the secondary asset market. We discuss Case 2 to give some the intuition. The best way to examine the relationship between $P_S^B(\overline{D})$ and P_N^B is to consider their ratio, which can be decomposed into a product of two elements

$$\frac{P_N^B}{P_S^B(\overline{D})} = \underbrace{\left(\frac{\Pr(L,\overline{D}) + \Pr(H)\Pr(\text{run})}{\Pr(L) + \Pr(H)\Pr(\text{run})}\right)}_{(1)}\underbrace{\left(\frac{\Pr(H) + \Pr(L,\overline{D})\Pr(G)}{\Pr(H) + \Pr(L)\Pr(G)}\frac{1}{\Pr(\overline{D})}\right)}_{(2)}.$$

Part (1) represents an increase in asset quality in the secondary market due to information sharing. It is a ratio of the expected average asset quality of $\overline{D}$-type loan under information sharing and the average asset quality under no information sharing regime in the secondary market. This ratio has a upper bound because of adverse selection in the secondary market

$$\frac{\Pr(L,\overline{D}) + \Pr(H)\Pr(\text{run})}{\Pr(L) + \Pr(H)\Pr(\text{run})} \leq \Pr(\overline{D}).$$

When the probability of a run increases, it becomes less likely that assets are on sale for strategic reason. As a result, the adverse selection in the secondary market decreases, and the gap in asset qualities under the two information regimes diminishes. However, it reaches a limit when $\Pr(\text{run}) \to 1$. Indeed, the adverse selection in the secondary market completely disappear when $\Pr(\text{run}) = 1$, and Part (1) reaches its upper bound $\Pr(\overline{D})$.[7]

Part (2) represents the extra rent that the relationship bank can extract from a $\overline{D}$-type borrower by not sharing information, and this rent diminishes when the adverse selection is mitigated in the prime loan market. Suppose a L-type borrower always generates a default credit history D in the previous lending relationship, the adverse selection would disappear in the prime loan market. Since under this assumption, the non default credit history (default credit history) must be generated by a H-type (L-type) borrower. With

[7] Without information sharing, the average loan quality $\Pr(L) + \Pr(H)\Pr(\text{run})$ tends to 1, loan of any type will be sold for liquidity when $\Pr(\text{run}) = 1$. Similarly, with information sharing, any $\overline{D}$-type loan has to be sold for liquidity when $\Pr(\text{run}) = 1$, the average loan quality $\Pr(L,\overline{D}) + \Pr(H)\Pr(\text{run})$ tends to $\Pr(\overline{D})$.

$\Pr(L, \overline{D}) \to \Pr(L)$, Part (2) reaches its upper bound $1/\Pr(\overline{D})$[8]

$$1 < \frac{\Pr(H) + \Pr(L, \overline{D})\Pr(G)}{\Pr(H) + \Pr(L)\Pr(G)} \frac{1}{\Pr(\overline{D})} \leq \frac{1}{\Pr(\overline{D})}.$$

The stronger the adverse selection in the prime loan market is, or the bigger the gap between $\Pr(L, \overline{D})$ and $\Pr(L)$ is, and the smaller Part (2) becomes. When adverse selection is mitigated for $\overline{D}$-type loan, the relationship bank extracts less profitability from financing $\overline{D}$-type of loan because the distant bank undercuts more for this type of loan.

Since both Part (1) and Part (2) are bounded from above, and the upper bounds are $\Pr(\overline{D})$ and $1/\Pr(\overline{D})$ respectively, we can conclude that $P_N^B < P_S^B(\overline{D})$ always holds. The benefit of information sharing from the increase in average asset quality dominates the losses of information sharing from the reduction in rent extraction on the $\overline{D}$-type of borrower. Once this result is established, there must exist a set of parameters where the risk-free rate r_0 lies between the two prices and information sharing can save the bank from illiquidity. We will focus on those cases throughout the paper.

A corollary of Proposition 1 regards the equilibrium deposit rates that make information sharing valuable. We have

Corollary 1 *If $r_N = \widehat{r}_N$ and $r_S(\overline{D}) = r_0$ then information sharing can save the relationship bank from illiquidity.*

The intuition is as follows. For information sharing to be valuable, it must be able to prevent bank illiquidity. On the one hand, without information sharing, the relationship bank must face liquidity risk and it fails because of the run when the state is bad, even if it holds the safe H-type loan. This implies that the equilibrium deposit rate without information sharing r_N has to be risky. On the other hand, with information sharing, the relationship bank must never fail because of the run when the state is bad, even if it lends to the L-type borrower (in that case it would sell the asset for arbitrage which is the source of adverse selection). This implies that the equilibrium deposit rate with information sharing $r_S(\overline{D})$ has to be equal to the risk-free rate r_0.

Information sharing can endogenously emerge only inside the set of parameters specified in Proposition 1. Under this parameters restriction, the equilibrium deposit rates are those specified in Corollary 1. All other combinations of parameter values would not allow

[8]This is true because $\Pr(L, \overline{D}) \leq \Pr(L)$.

information sharing to be an equilibrium outcome. For example, assume $r_N = r_S(\overline{D}) = r_0$, then the relationship bank does not face any liquidity risk, therefore it will always survive with and without information sharing. Given that information sharing does not reduce liquidity risk, but it only intensify competition on the loan rates, the relationship bank will not choose to share its information on borrower's credit history. Similarly, assume $r_N = \widehat{r}_N$ and $r_S(\overline{D}) = \widehat{r}_S(\overline{D})$. The relationship bank faces liquidity risk and it would fail in case of a run both with and without information sharing. The bank again does not gain anything to disclose its information on the borrower. Finally, consider the case $r_N = r_0$ and $r_S(\overline{D}) = \widehat{r}_S(\overline{D})$. The relationship bank would fail in case of a run with information sharing and it survives without information sharing. The choice about sharing information is again clear. Notice however that the last case cannot exist since the parameter restrictions generate an empty set.

Given the result in Proposition 1, we define the set F_j with $j = \{0, 1, 2, 3\}$ such that the condition $P_S^B(\overline{D}) > r_0 > P_N^B$ holds. This is the set of parameters in each case j such that the relationship bank with $\overline{D}$-history loan survives from bank run in bad state when sharing information and fails because illiquidity in the bad state without information sharing. Recall that R_j, with $j = \{0, 1, 2, 3\}$, gives the set of payoffs R that defines different levels of contestability in the prime loan market. We define the intersection set $\Psi_j = R_j \bigcap F_j$ with $j = \{0, 1, 2, 3\}$. We have:

- $\Psi_0 = R_0 \bigcap F_0$ with $F_0 = \{R | \frac{(1-\alpha)\pi+\alpha\rho}{\alpha\rho} r_0 < R < \frac{(1-\alpha)+\alpha\rho}{\alpha\rho} r_0\}$.
- $\Psi_1 = R_1 \bigcap F_1$ with $F_1 = \{R | R < \frac{\alpha\rho+(1-\alpha)}{\alpha} r_0 \quad \text{and} \quad c + r_0 > \frac{\alpha\rho+(1-\alpha)\pi}{\alpha\rho} \frac{\alpha+(1-\alpha)\pi^2}{\alpha+(1-\alpha)\pi} r_0\}$.
- $\Psi_2 = R_2 \bigcap F_2$ with $F_2 = \{R | \frac{(1-\alpha)\pi+\alpha\rho}{\alpha\rho} \frac{\alpha+(1-\alpha)\pi^2}{\alpha+(1-\alpha)\pi} r_0 < c + r_0 < \frac{(1-\alpha)+\alpha\rho}{\alpha\rho} [\alpha+(1-\alpha)\pi] r_0\}$.
- $\Psi_3 = R_3 \bigcap F_3$ with $F_3 = F_2$.

Notice that the prices P_N^B and $P_S^B(\overline{D})$ are the same under Case 2 and Case 3. This is because the prime loan market is more contestable under these two cases. The distant bank competes with the relationship bank for a loan without knowing the credit history as well as for a loan with good credit history. Therefore we have $F_3 = F_2$. Figure 3 presents Cases 0, 1, 2 and 3 each with its respective shaded area in which the condition in Proposition 1 holds. In each of the four cases the relevant area exists, and we indicate this area as Ψ_j with $j = \{0, 1, 2, 3\}$. The non-shaded areas in Figure 3 correspond to the set of parameters in which information sharing is not beneficial in saving the relationship bank from illiquidity and then it cannot emerge in equilibrium.[9] We therefore do not further consider in our analysis such parameter values.

[Put Figure 3 here]

3.5 Equilibrium Information Sharing

We are now in a position to determine when information sharing emerges as an equilibrium of our game. We focus on the regions Ψ_j with $j = \{0, 1, 2, 3\}$. At $t = 0$, the relationship bank decides whether to choose the information sharing regime or the no information sharing regime by comparing the expected profits in those two regimes. Let us call the relationship bank's expected profits at $t = 0$ with V_i, where like before $i = \{N, S\}$.

The relationship bank's expected profits under no information sharing regime is

$$V_N = [\Pr(G) + \Pr(B)\Pr(H)\Pr(\text{no run})](R_N^* - r_N).$$

In the good state, the relationship bank will always survive irrespective of the type of its loan. However, in the bad state the relationship bank holding an H-type loan will survive only if there is no bank run.[10] Without information sharing scheme, the relationship bank cannot charge discriminative prices conditional on the borrower's type. Otherwise, it will reveal the borrower's type to the distant bank. Recall that the equilibrium deposit rate r_N under no information sharing regime is risky. That is, $r_N = \widehat{r}_N$ which is determined by $[\Pr(G) + \Pr(B)\Pr(H)\Pr(\text{no run})]\widehat{r}_N = r_0$. Therefore, we obtain

$$V_N = [\alpha + (1-\alpha)\pi^2]R_N^* + (1-\alpha)(1-\pi)\pi R_N^* - \alpha(1-\pi)\rho R_N^* - r_0.$$

When the relationship bank participates in the information sharing regime, its expected profits V_S are

$$V_S = \Pr(\overline{D})[\Pr(H|\overline{D})V_S^H(\overline{D}) + \Pr(L|\overline{D})V_S^L(\overline{D})] + \Pr(D)V_S^L(D), \qquad (6)$$

[9] To guarantee that the area where information sharing is beneficial exists in all four cases, we impose a further parameter restricion ($\frac{(1-\alpha)\pi+\alpha\rho}{\alpha\rho} > \frac{1}{\pi}$). The analysis of the relevant areas would be the same without such restriction.

[10] Recall that we focus on the case where the relationship bank with an H-type loan will survive from bank run when sharing information but will fail otherwise.

where $V_S^H(\overline{D})$ and $V_S^L(\overline{D})$ are the expected profits of financing an H-type and an L-type borrower, respectively, when they generate the non default credit history $\overline{D}$. While $V_S^L(D)$ is the expected profit of financing an L-type borrower with default credit history D. Notice that when a loan has a credit history $\overline{D}$, with posterior probability $\Pr(H|\overline{D})$ it is an H-type loan. Moreover, $\Pr(D) = \Pr(L)\Pr(B) = (1-\alpha)(1-\pi)$ and $\Pr(\overline{D}) = 1 - \Pr(D) = \alpha + (1-\alpha)\pi$.

The expected profit of financing an H-type borrower with credit history $\overline{D}$ is

$$V_S^H(\overline{D}) = [\Pr(G) + \Pr(B)\Pr(\text{no run})]R_S^*(\overline{D}) + \Pr(B)\Pr(\text{run})P_S^B(\overline{D}) - r_0.$$

Notice that, given that we focus on the case in which information sharing saves the relationship bank from illiquidity, we have $r_S(\overline{D}) = r_0$. Moreover, the relationship bank will withhold H-type loan to maturity if no bank run occurs because $P_S^B(\overline{D}) = \frac{\alpha\rho}{(1-\alpha)\pi+\alpha\rho}R_S^*(\overline{D}) < R_S^*(\overline{D})$. Similarly, the expected profit of financing a L-type borrower with credit history $\overline{D}$ is given by

$$V_S^L(\overline{D}) = \Pr(G)R_S^*(\overline{D}) + \Pr(B)P_S^B(\overline{D}) - r_0.$$

When the relationship bank holds an L-type loan, in the bad state B the bank will sell it on the secondary market even without facing a run. Finally, a borrower that generates a default credit history D must be an L-type borrower. The equilibrium deposit rate is risky, that is $r_S(D) = r_0/\pi$. The expected profit of financing such a loan is

$$V_S^L(D) = \Pr(G)[R_S^*(D) - r_0/\pi] = \Pr(G)R_S^*(D) - r_0.$$

Insert the expressions of $V_S^H(\overline{D})$, $V_S^L(\overline{D})$ and $V_S^L(D)$ into equation (6), and we get after rearranging

$$V_S = [\alpha + (1-\alpha)\pi^2]R_S^*(\overline{D}) + (1-\alpha)(1-\pi)\pi R_S^*(D) - r_0.$$

Information sharing is preferred by the relationship bank if and only if $V_S - V_N > 0$. The difference between the expected profits in the two regimes can be rewritten as

$$V_S - V_N = \underbrace{[\alpha + (1-\alpha)\pi^2](R_S^*(\overline{D}) - R_N^*)}_{(1)} + \underbrace{(1-\alpha)(1-\pi)\pi(R_S^*(D) - R_N^*)}_{(2)} + \underbrace{\alpha(1-\pi)\rho R_N^*}_{(3)}.$$

The interpretation of the three terms is quite intuitive. Term (1) represents the *competition* effect, and it has a negative consequence on the adoption of the information sharing regime since $R^*_S(\overline{D}) \leq R^*_N$. Sharing information about the credit history encourages the distant bank to compete for the borrower with good credit history, i.e. $\overline{D}$-history. The expected profits of the relationship bank is reduced due to this effect because the entrant bank undercuts the loan rate when $\overline{D}$-history is observed. Term (2) is understood as the *capturing* effect, and it has positive impact on sharing information since $R^*_S(D) \geq R^*_N$. Sharing information about the borrower with bad credit history, i.e. D-history, deters the entry of distant bank. Thus the relationship bank can discriminate the borrower with D-history by charging higher loan rate. The expected profits of the relationship bank increases due to this effect. Finally, Term (3) denotes the *liquidity* effect, which is always positive. Sharing credit information of a borrower with good credit history reduces the adverse selection in the secondary credit market. In the bad state of nature, the relationship bank will be saved from potential bank run. This effect increases the expected profits of the relationship bank by avoiding costly asset liquidation.

The overall effect crucially depends if the capturing effect together with the liquidity effect dominate the competition effect. In that case the relationship bank chooses information sharing regime to maximize its expected profits. Denote with φ_j where $j = \{0, 1, 2, 3\}$ the set of parameters in which $V_S > V_N$ holds, then we have

Proposition 2 *The relationship bank chooses voluntarily to share information on* $\varphi_j = \Psi_j$ *with* $j = \{0, 3\}$ *and on* $\varphi_j \subseteq \Psi_j$ *with* $j = \{1, 2\}$*. Moreover, if* $\rho > (1-\alpha)(1-\pi)$ *then information sharing is chosen on* $\varphi_j = \Psi_j$ $\forall j$*.*

The proof is in the Appendix. The intuition is the following. In Cases 0 and 3 the set of parameters φ_j in which the relationship bank decide to share information coincides with the set Ψ_j in which information sharing saves the relationship bank from illiquidity. The reason is that there is no cost for the relationship bank to share information in both cases. In Case 0 because the distant bank never compete for the borrower, and in Case 3 because the distant bank always compete for the borrower. This is not true in Cases 1 and 2. In those two cases, the competition effect could overcome the sum of the capturing and the liquidity effects and the relationship bank would find it profitable to not sharing information. This reduces the set of parameters φ_j in which sharing information is actually chosen versus the set of parameters Ψ_j in which is actually beneficial. However, when the probability of bank run is sufficiently high, the benefit from sharing information becomes sufficiently high that the relationship bank find it convenient to share information whenever is beneficial to do so also in Cases 1 and 2 .

[Put Figure 4 here]

Figure 4 shows Cases 0, 1, 2 and 3 corresponding to different degree of loan market contestability. In each graph, the double-shaded area corresponds to the set of parameters φ_j. Clearly the double-shaded areas in Cases 0 and 3 correspond to the shaded areas in Figure 3. When ρ is low, the double-shaded areas in the graphs of Cases 1 and 2 are smaller than the corresponding areas in Figure 3 (the red line is the boundary of the double-shaded area in which the relationship bank voluntarily chooses to share information). When ρ is sufficiently high Figure 3 and 4 coincide.

4 Unverifiable Credit History

In this section we relax the assumption of verifiable credit history. If the reported borrower's credit history is not verifiable, the relationship bank that chooses to share such information may have an incentive to misreport the borrower's credit history after observing it. In particular, the relationship bank may have an incentive to overstate the borrower's credit history, that is to report a default credit history D as a non default credit history $\overline{D}$.[11] We have the following

Proposition 3 *The relationship bank truthfully discloses the borrower's credit history only if it leads to an increase in the loan rate for borrowers who have a default history D. This does not occur on φ_j with $j = \{0, 1\}$, and it does occur on φ_2, for sufficiently low ρ, and always on φ_3.*

The proof in the Appendix. In order to sustain truthfully reporting the credit history as an equilibrium, a necessary condition is that the relationship bank must suffer a loss when deviating from the equilibrium strategy. Consider the case in which the relationship

[11] We assume misreporting $\overline{D}$ as D to be impossible, that is the relationship bank cannot claim non-defaulted borrower as defaulted. This is because borrowers have means and incentive to correct it or act against it (e.g., FCA in US). Moreover, according to the documentations in www.doingbusiness.com, borrowers can access their own credit record. A false report about defaulting can result in a legal dispute.

bank lends to an L-type of borrower, which generated a default credit history D. If the relationship bank truthfully reveals the credit history, it is able to charge the loan rate $R_S^*(D)$. Yet, the relationship bank will not survive if the state is bad (i.e., with probability $1-\pi$), because the asset buyers know that a loan with a credit history D is L-type and will generate zero payoff in state B. If the relationship bank lies about the credit history, the asset buyers as well as the distant bank will perceive the borrower to be more likely an H-type. Accordingly, the loan rate charged by the relationship bank is $R_S^*(\overline{D})$, which could be lower than $R_S^*(D)$ due to the intensified competition. However, cheating gives the relationship bank more resilience against the future liquidity shock since it can sell the loan in the secondary market at the price $P_S^B(\overline{D}) > r_0$ when the state is bad. The relationship bank always survives when cheating. Thus, the relationship bank trades off the benefit of market liquidity (surviving in state B) versus the loss in profitability (potential decrease in loan rate from $R_S^*(D)$ to $R_S^*(\overline{D})$) when deciding to tell the truth about the reported credit history. Notice that a pre-requisite for the relationship bank to manipulate the reported credit history is that it must choose the information sharing regime in the first place. Thus, we focus our discussion on the intuition in each case, on the parameter sets φ_j, with $j = \{0, 1, 2, 3\}$, defined in Section 3.5.

Consider Case 0. We have $R_S^*(D) = R_S^*(\overline{D}) = R$ therefore the relationship bank always has incentive to misreport the true D-history as $\overline{D}$-history in the parameters space φ_0. The loan market is least contestable and we have $R_S^*(D) = R_S^*(\overline{D}) = R$. Assuming truthfully reporting, ex-ante participating in information sharing is more profitable for the relationship bank in the parameter set φ_0. However, when the relationship bank observes a credit history D ex-post, it will incur no loss in profit to misreport the credit history as $\overline{D}$ because $R_S^*(D) = R_S^*(\overline{D})$. Consequently, the relationship bank will always misreport the true D-history as $\overline{D}$-history in the parameters set φ_0. Truthfully reporting the credit history can never be an equilibrium in Case 0.

Since in the other cases we have $R_S^*(D) > R_S^*(\overline{D})$, there is hope for the relationship bank to report the true credit history. However, as noticed, this is only a necessary condition. Even if ex-post the relationship bank has an incentive to tell the truth, it is possible that ex-ante it is not willing to share information. The parameters that guarantee the ex-post truth telling have to be consistent with those that induce ex-ante information sharing.

Consider Case 1. On the one hand, assuming truthfully reporting, the relationship bank ex-ante prefers to participate in information sharing scheme when R is low. This is because its expected profit without sharing information is increasing in R ($R_N^* = R$), while the expected profit with information sharing is increasing in R only if the relationship bank lend to an L-type borrower. On the other hand, in order to make the relationship bank report the true credit history ex-post, R must be high. This is because the deviating penalty increases with R, that is $R_S^*(D) = R$ while $R_S^*(\overline{D})$ is an internal solution thus it does not depend on R. It turns out that the ex-ante and ex-post conditions on R determine an empty set and therefore truthfully reporting can not be sustained as an equilibrium in the parameter space φ_1.

Consider Case 2. On the one hand, assuming truthfully reporting, the relationship bank ex-ante prefers to participate information sharing scheme when R is high. This is because the loan market becomes more contestable, the expected profit without information sharing does not depend on R any more (R_N^* becomes an internal solution), while the expected profit with information sharing is increasing in R (with L-type borrower, the loan rate is $R_S^*(D) = R$). On the other hand, in order to make the relationship bank report the true credit history ex-post, the return R must be high since, as in Case 1, $R_S^*(D) = R$ and $R_S^*(\overline{D})$ is an internal solution. It turns out that the ex-ante condition on R is more restrictive than the ex-post condition only if ρ is lower than a critical value $\widehat{\rho}$ (i.e., the value in which the relationship bank is indifferent between reporting the true credit history D and the false credit history $\overline{D}$). Under this condition, whenever the relationship bank finds ex-ante optimal to share information it also will report ex-post the true credit history, and truthful reporting can be sustained as an equilibrium in the parameter space φ_2.

Finally, consider Case 3. Assuming truthfully reporting, the relationship bank ex-ante always prefer information sharing (irrespective of R). Moreover, the prime loan market is most contestable, $R_S^*(D) = \frac{c+r_0}{\pi} > R_S^*(\overline{D})$. It turns out that the relationship bank earns a strictly negative profit by ex-post misreporting D history with $\overline{D}$. This is because, $R_S^*(D)$ is substantially higher than $R_S^*(\overline{D})$, so the relationship bank's expected loss in profit overcomes its expected gain from market liquidity by misreporting the credit history. As a result, truthful reporting is sustained as an equilibrium in the parameter space φ_3.

To sum up, by truthfully reporting the credit history, the relationship bank can discriminate the L-type borrower by charging higher loan rate. When misreport the credit history, the relationship bank has to charge a lower loan rate but benefits from the higher market liquidity to survive potential runs. If the market is less contestable, the profit from the discriminative loan pricing is bounded above by the loan's return R. Thus, in Case 0 and 1, the benefit from the higher market liquidity to save the bank from run in state B, dominates the loss in profit. The relationship bank will lie in those Cases. However, in Case 2 and 3, the return R is sufficiently large and the profit from discriminative loan pricing tends to dominate the benefit from market liquidity. Truthfully reporting the credit history can be sustained as equilibrium in those two Cases.

Figure 5 shows Cases 0, 1, 2 and 3 each with its respective dark-blue area corresponding to the set of parameters in which truth-telling is an equilibrium. In Cases 0 and 1 such area is empty since truth-telling is not possible under these Cases. In Case 2 we show a situation where truth-telling can be sustained in a subset of φ_2, which occurs when $\rho < \min[\widehat{\rho}, (1-\alpha)(1-\pi)]$. In case 3, since truth-telling is always sustained in the entire region φ_3, the dark-blue area coincide with the area in Figure 4.

[Put Figure 5 here]

5 Welfare and Policy Implication

We first notice what is the socially efficient level of information sharing. Suppose a benevolent social planner knows borrower's type, then the planner would always invest (all positive NPV projects). Moreover, there are two sources of frictions: i) information power of the relationship bank over the borrower; ii) adverse selection in the secondary market for loan sale. Since both frictions are reduced by information sharing, from a social perspective maximum information sharing is preferred. Indeed, the planner does not care about friction i), but reducing friction ii) is better for everybody.

From a private perspective, relationship bank values information sharing since it reduces the adverse selection problem in the secondary asset market enhancing asset market liquidity. But it also reduces market power vis a vis the borrower. This can generates a private level of information sharing that is less than the efficient one.

This is seen comparing the shaded areas in Figure 3 and the double-shaded areas in Figure 4. In Cases 0 and 3 the two areas coincide so there is no inefficient choice. However in Cases 1 and 2 the relationship bank chooses a level of information sharing that is less than what would be (socially) optimal. In this Cases sharing information is costly, and the private cost of the relationship bank is higher than the social cost.

The endogenous arise of private registries is rational from the bank's point of view, but can be inefficiently low in some circumstances. A public registry can increase welfare in Cases 1 and 2, without harming in Cases 0 and 3.

6 Conclusion

This paper formally analyzes the conjecture according to which banks' decision to share information about the credit history of their borrowers is driven by the needs for market liquidity. To meet urgent liquidity needs, banks have to make loan sale in the secondary market. However, the information friction in loan markets makes this sale costly and good loans can be priced below their fundamental value. This concern became very evident during the financial crisis started in the summer of 2007. Several potentially solvent banks risk to fail because they could not raise enough short term liquidity.

This basic observation implies that banks could find convenient to share information on their loans in order to reduce the information asymmetry about their quality in case they have to sell them in the secondary market. Information sharing can be a solution to reduce the cost of urgent liquidity needs so to make banks more resilient to funding risk. Clearly, sharing information makes banks to lose the rent they extract if credit information were not communicated. Banks may be no longer able to lock in their loan applicants because competing banks also know about the quality of those loans. Eventually, the benefit of a greater secondary market liquidity has to be traded off with the loss in information rent. We show that it possible to rationalize information sharing as such device. We show under which conditions information sharing is feasible, and when is actually chosen by the banks in equilibrium.

We also show that our rationale for information sharing is robust to truth telling. A common assumption in the literature is that when banks communicate the credit information, they share it truthfully. We allow banks to manipulate the information they release by reporting bad loans as good ones. The reason is for the banks to increase the liquidation value in the secondary market. We show that when banks lose too much in information rent from good borrowers with bad credit history, then information sharing is a truth telling device.

Coherently with previous theoretical model of information sharing, the existing empirical literature has mostly focused on the impact of information sharing on bank risks and firms' access to bank financing. Our theoretical contribution generates new empirical implications. In particular, information sharing should facilitate banks liquidity management and loan securitization. The model also suggests that information sharing can be more easily established, and work more effectively, in countries with competitive banking sector, and in credit market segments where competition is strong.

Appendix: Proofs

Proof of Lemma 1. Recall that the depositors' break even rates are r_0 when deposits are safe and $\widehat{r}_N$ ($> r_0$) when deposits are risky. Depositors are competitive so they bid against each other in determining the equilibrium deposit rate r_N to finance the bank. Depositors take the asset price P_N^B and the break even rates as given. Under the assumption of perfect competition, a necessary condition for the equilibrium deposit rate is that it has to guarantee zero expected profits to depositors.

We prove statement (i) by contradiction. Let us consider three cases.

Case (a), the parameters are such that $P_N^B > \widehat{r}_N$. Assume the equilibrium deposit rate is $r_N > P_N^B > \widehat{r}_N$. If this rate were indeed the equilibrium rate, then the deposits were risky because the asset price P_N^B is not enough to repay r_N in equilibrium. Their break even rate is $\widehat{r}_N$. However, the depositors could make positive profit if this were the case, since $r_N > \widehat{r}_N$. A deposit rate higher than P_N^B cannot be an equilibrium. Assume the equilibrium deposit rate is $P_N^B \geq r_N > \widehat{r}_N$. If this were the case, the deposits are safe, depositors' break even rate is r_0. But if this rate were the equilibrium rate, again the depositors could make positive profit since $\widehat{r}_N > r_0$. The equilibrium deposit rate can not be higher than $\widehat{r}_N$. Assume the equilibrium rate is $\widehat{r}_N \geq r_N > r_0$. Deposits are again safe, and depositors can make positive profit since r_N is larger than r_0. Lastly, assume the equilibrium rate is $r_0 > r_N$. If this were the case, the depositors make negative profit. As a result, the only candidate equilibrium deposit rate is $r_N = r_0$. Under this rate, the deposits are safe and depositors make zero expected profit. Each depositor does not have incentive to undercut below $r_N = r_0$. Thus $r_N = r_0$ is the unique equilibrium deposit rate.

Case (b), the parameters are such that $P_N^B = \widehat{r}_N$. Assume the equilibrium deposit rate is $r_N > P_N^B = \widehat{r}_N$, then the deposits are risky. But if r_N were the equilibrium rate, the depositors would earn positive profit because $r_N > \widehat{r}_N$. Assume the equilibrium rate is $P_N^B = \widehat{r}_N \geq r_N > r_0$, then the deposits are again safe but depositors would earn positive profit since $r_N > r_0$. If $r_0 > r_N$ depositors make negative profit. Thus, the unique equilibrium deposit rate is again $r_N = r_0$, under which the depositors have no incentive to undercut.

Case (c), the parameters are such that $r_0 \leq P_N^B < \widehat{r}_N$. Assume $r_N > \widehat{r}_N > P_N^B$, then deposits are risky. The rate r_N is making depositors earn positive profit. Assume $r_N = \widehat{r}_N > P_N^B$, then deposits are risky and depositors earn zero profit. But if this rate were the equilibrium rate, then the depositors can offer an alternative rate as $r_N = P_N^B - \epsilon$. Under this new rate $r_N < P_N^B$, the deposits become safe and the depositors can instead make positive profit as $r_N = P_N^B - \epsilon \geq r_0$. There exists a profitable deviation. Assume $\widehat{r}_N > r_N > P_N^B$, the deposits are risky and the depositors will never finance the bank as they make negative profit. Assume $\widehat{r}_N > P_N^B \geq r_N > r_0$, the deposits are risk-free but the depositors could make positive profit. Lastly, assume $r_0 > r_N$, the depositors again make negative profit. We have the unique equilibrium deposit rate is $r_N = r_0$. Under this rate, the deposits are safe and the depositors make zero profit. The depositors have no incentive to undercut further otherwise they make negative profit.

In sum, the unique equilibrium deposit rate is $r_N = r_0$, and deposits are safe.

To prove statement (ii), notice that the only case to consider is $\widehat{r}_N > r_0 > P_N^B$. Assume $r_N > \widehat{r}_N > r_0 > P_N^B$, the deposits are risky yet under this rate the depositors could make positive profit. Assume $\widehat{r}_N > r_N > r_0 > P_N^B$ or $\widehat{r}_N > r_0 \geq r_N > P_N^B$, the deposits are also risky but the depositors make negative profit. Assume $\widehat{r}_N > r_0 > P_N^B \geq r_N$, the deposits are safe but the depositors make negative profit. Lastly, assume $r_N = \widehat{r}_N > r_0 > P_N^B$, then the deposits are risky but make zero expected profit. They have no incentive to undercut further since otherwise they will make negative profit. Thus, the unique equilibrium deposit rate is $r_N = \widehat{r}_N$ and deposits are risky. ■

Proof of Lemma 2. The logic of the proof is similar to the one provided in Lemma 1, with the only difference that we focus on the loan with a past non-defaulted history $\overline{D}$. The depositors' break even rates are r_0 when their deposits are safe and $\widehat{r_S}(\overline{D})$ when the deposits are risky. Depositors are competitive, so they bid against each other in determining the equilibrium deposit rate $r_S(\overline{D})$.

We prove statement (i) by contradiction, and we consider three cases.

Case (a), the parameters are such that $P_S^B(\overline{D}) > \widehat{r_S}(\overline{D})$. Assume $r_S(\overline{D}) > P_S^B(\overline{D}) > \widehat{r_S}(\overline{D})$, we have risky deposits but positive profit. Assume $P_S^B(\overline{D}) > r_S(\overline{D}) > \widehat{r_S}(\overline{D}) > r_0$ and $\widehat{r_S}(\overline{D}) > r_S(\overline{D}) > r_0$, we have safe deposits but positive profit. Assume $r_0 > r_S(\overline{D})$, we have negative profit. The unique equilibrium rate is $r_S(\overline{D}) = r_0$.

Case (b), the parameters are such that $P_S^B(\overline{D}) = \widehat{r_S}(\overline{D}) > r_0$. Assume $r_S(\overline{D}) > P_S^B(\overline{D}) = \widehat{r_S}(\overline{D})$, we have risky deposits but positive profit. Assume $\widehat{r_S}(\overline{D}) > r_S(\overline{D}) > r_0$, we again have safe deposits but positive profit. Assume $r_0 > r_S(\overline{D})$, we have negative profit. The unique equilibrium rate is $r_S(\overline{D}) = r_0$.

Case (c), the parameters are such that $P_S^B(\overline{D}) < \widehat{r_S}(\overline{D})$. Assume $r_S(\overline{D}) > \widehat{r_S}(\overline{D}) > P_S^B(\overline{D})$, we have the deposits are risky but depositors are making positive profit. Assume $r_S(\overline{D}) = \widehat{r_S}(\overline{D}) > P_S^B(\overline{D})$, the deposits are again risky and the depositors earn zero profit. But the depositors can undercut to offer $r_S(\overline{D}) = P_S^B(\overline{D}) - \epsilon$ to make the deposits safe and earn positive profit. Assume $\widehat{r_S}(\overline{D}) > r_S(\overline{D}) > P_S^B(\overline{D})$, the deposits are risky and the depositors make negative profit. Assume $\widehat{r_S}(\overline{D}) > P_S^B(\overline{D}) \geq r_S(\overline{D}) > r_0$, the deposits are risk-free but the depositors could make positive profit. Last, assume $r_0 > r_S(\overline{D})$ and depositors get negative profit. We have the unique equilibrium deposit rate is $r_S(\overline{D}) = r_0$. Under this rate, the deposits for the bank with a loan of past history $\overline{D}$ are safe, the depositors make zero expected profit. The depositors have no incentive to undercut otherwise they make negative profit.

To prove statement (ii), notice that we have to consider the case in which $\widehat{r_S}(\overline{D}) > r_0 > P_S^B(\overline{D})$. Assume $r_S(\overline{D}) > \widehat{r_S}(\overline{D}) > r_0 > P_S^B(\overline{D})$, then deposits are risky yet the depositors make positive profit. Assume $\widehat{r_S}(\overline{D}) > r_S(\overline{D}) > r_0 > P_S^B(\overline{D})$, then deposits are risky and the depositors make negative profit. Assume $r_S(\overline{D}) = \widehat{r_S}(\overline{D}) > r_0 > P_S^B(\overline{D})$, then deposits are risky but the depositors make zero expected profit. They have no incentive to undercut as well since otherwise they will make negative profit. Thus, the unique equilibrium deposit rate is $r_S(\overline{D}) = \widehat{r_S}(\overline{D})$ and we have risky deposits for a bank with a loan of past history $\overline{D}$. ■

Proof of Proposition 1. Recall expressions (1) and (2) that determines equilibrium asset prices in the secondary market. They are

$$P_N^B = \frac{\alpha\rho}{(1-\alpha)+\alpha\rho} R_N^*$$

and

$$P_S^B(\overline{D}) = \frac{\alpha\rho}{(1-\alpha)\pi+\alpha\rho} R_S^*(\overline{D}),$$

where R_N^* and $R_S^*(\overline{D})$ are the equilibrium loan rates under no information sharing and information sharing regime, respectively. Notice that the average loan quality in the secondary market without information sharing ($\frac{\alpha\rho}{(1-\alpha)+\alpha\rho}$) is lower than the average loan quality with information sharing ($\frac{\alpha\rho}{(1-\alpha)\pi+\alpha\rho}$).

Consider Case 0. The distant bank does not compete for any loan even if the relationship bank shared the credit history of the borrower. The relationship bank extracts the entire payoff of the loan irrespective of the information sharing regime, that is $R_S^*(\overline{D}) = R_N^* = R$.

Information sharing solely brings in the benefit from boosting asset liquidity for loan with $\overline{D}$ history. Consequently, $P_S^B(\overline{D}) > P_N^B$.

Consider Case 2 (for the easy of exposition it is convenient to analyze this case first). Distant bank competes both under information sharing (and the borrower has no default history $\overline{D}$) and when there is no information sharing. The equilibrium loan rates are therefore

$$R_N^* = \frac{c+r_0}{\alpha+(1-\alpha)\pi} > \frac{\alpha+(1-\alpha)\pi}{\alpha+(1-\alpha)\pi^2}(c+r_0) = R_S^*(\overline{D}).$$

We want to show that

$$P_N^B = \frac{\alpha\rho}{(1-\alpha)+\alpha\rho}\frac{c+r_0}{\alpha+(1-\alpha)\pi} < \frac{\alpha\rho}{(1-\alpha)\pi+\alpha\rho}\frac{\alpha+(1-\alpha)\pi}{\alpha+(1-\alpha)\pi^2}(c+r_0) = P_S^B(\overline{D}),$$

which can be rewritten as

$$\frac{(1-\alpha)\pi+\alpha\rho}{(1-\alpha)+\alpha\rho}\frac{\alpha+(1-\alpha)\pi^2}{[\alpha+(1-\alpha)\pi]^2} < 1.$$

To show that the last inequality holds, we notice that the ratio $\frac{(1-\alpha)\pi+\alpha\rho}{(1-\alpha)+\alpha\rho}$ is increasing in ρ, so its maximum value is reached when $\rho = 1$ and it equal to $(1-\alpha)\pi + \alpha$ $(= Pr(\overline{D}))$. Therefore, the maximum value of the LHS of the last inequality can written as

$$[(1-\alpha)\pi+\alpha]\frac{\alpha+(1-\alpha)\pi^2}{[\alpha+(1-\alpha)\pi]^2} = \frac{\alpha+(1-\alpha)\pi^2}{\alpha+(1-\alpha)\pi},$$

which is smaller than 1 since $\pi \in (0,1)$. Thus, $P_S^B(\overline{D}) > P_N^B$.

Consider Case 1. The distant bank only competes for the loan with past non-defaulted history $\overline{D}$. The equilibrium loan rate $R_S^*(\overline{D})$ is determined by the distant bank. Without information sharing, the relationship bank can discriminate the borrower by charging $R_N^* = R > R_S^*(\overline{D})$. The competition effect is clearly smaller than under Case 2. Since $P_S^B(\overline{D}) > P_N^B$ always holds in Case 2, then it necessarily holds also in Case 1.

Consider Case 3. The distant bank competes no matter the past history of the borrower. The relevant equilibrium loan rates R_N^* and $R_S^*(\overline{D})$ do not change with respect Case 2. The relationship between the prices $P_S^B(\overline{D})$ and P_N^B is the same as the one analyzed in Case 2. Thus, $P_S^B(\overline{D}) > P_N^B$.

Since we have that is all cases $P_N^B < P_S^B(\overline{D})$, by continuity when r_0 is located in between these two prices the relationship bank survives from illiquidity under information sharing regime and fails under no information sharing regime. ■

Proof of Proposition 2. For each Case $j = \{0, 1, 2, 3\}$ we consider the parameter set Ψ_j defined in Proposition 1.

Consider Case 0. We have: $V_S = [\alpha + (1-\alpha)\pi]R - r_0$ and $V_N = [\alpha - \alpha(1-\pi)\rho + (1-\alpha)\pi]R - r_0$. Then $V_S > V_N$ for the entire region Ψ_0. Thus $\varphi_0 = \Psi_0$.

Consider Case 1. We have: $V_S = [\alpha + (1-\alpha)\pi](c + r_0) + (1-\alpha)(1-\pi)\pi R - r_0$ and $V_N = [\alpha - \alpha(1-\pi)\rho + (1-\alpha)\pi]R - r_0$. Therefore,

$$V_S - V_N = [\alpha + (1-\alpha)\pi](c + r_0) - [(1-\alpha)\pi^2 + \alpha - \alpha(1-\pi)\rho]R.$$

Notice that $(1-\alpha)\pi^2 + \alpha - \alpha(1-\pi)\rho > 0$. We have that $V_S - V_N > 0$ if and only if

$$R < \frac{\alpha + (1-\alpha)\pi}{\alpha - \alpha(1-\pi)\rho + (1-\alpha)\pi^2}(c + r_0) \equiv R_1.$$

We define the region φ_1 as follows

$$\varphi_1 = \Psi_1 \bigcap \{R | R < R_1\} \subseteq \Psi_1.$$

If R_1 is greater than the upper bound R_N^E of R defining Case 1 then information sharing is preferred for the entire region Ψ_1. That is, if

$$R_1 = \frac{\alpha + (1-\alpha)\pi}{\alpha - \alpha(1-\pi)\rho + (1-\alpha)\pi^2}(c + r_0) > \frac{\alpha + (1-\alpha)\pi}{\alpha + (1-\alpha)\pi^2}(c + r_0) = R_N^E$$

the set φ_1 coincides with Ψ_1. We can simplify the last inequality as

$$\rho > (1-\alpha)(1-\pi).$$

Otherwise, when $\rho < (1-\alpha)(1-\pi)$, we have $\varphi_1 \subset \Psi_1$. Indeed, notice that R_1 is increasing in ρ. When $\rho \to 0$, we have $R_1 \to \frac{\alpha+(1-\alpha)\pi}{\alpha+(1-\alpha)\pi^2}(c + r_0) = R_S^E(\overline{D})$. Recall the definition of region Ψ_1, we always have such $\varphi_1 = \Psi_1 \bigcap \{R | R < R_1\}$ non-empty for any value of $\rho \in (0, 1)$ and $\varphi_1 \subset \Psi_1$ when $\rho < (1-\alpha)(1-\pi)$.

Consider Case 2. We have $V_S = [\alpha + (1-\alpha)\pi](c + r_0) + (1-\alpha)(1-\pi)\pi R - r_0$ and $V_N = [\alpha - \alpha(1-\pi)\rho + (1-\alpha)\pi]\frac{c+r_0}{\alpha+(1-\alpha)\pi} - r_0$. Therefore,

$$V_S - V_N = [\alpha + (1-\alpha)\pi](c + r_0) + (1-\alpha)(1-\pi)\pi R - [1 - \frac{\alpha(1-\pi)\rho}{\alpha + (1-\alpha)\pi}](c + r_0).$$

We have $V_S - V_N > 0$ if and only if

$$R > [1 - \frac{\alpha\rho}{(1-\alpha)[\alpha + (1-\alpha)\pi]}]\frac{c + r_0}{\pi} \equiv R_2.$$

We define the set φ_2 as follows

$$\varphi_2 = \Psi_2 \bigcap \{R | R > R_2\} \subseteq \Psi_2.$$

If R_2 is lower than the lower bound of R defining Case 2 then information sharing is preferred for the entire region Ψ_2. That is, if

$$[1 - \frac{\alpha\rho}{(1-\alpha)[\alpha + (1-\alpha)\pi]}]\frac{c + r_0}{\pi} < \frac{c + r_0}{\alpha + (1-\alpha)\pi}$$

the set $\varphi_2 = \Psi_2$. We can simplify the last inequality again as

$$\rho > (1-\alpha)(1-\pi).$$

Otherwise, when $\rho < (1-\alpha)(1-\pi)$ we have $\varphi_2 \subset \Psi_2$. Indeed, also R_2 is decreasing in ρ. When $\rho \to 0$, we have $R_2 \to \frac{c+r_0}{\pi} = R_S^E(D)$. Recall the definition of region Ψ_2, we always have such φ_2 non-empty for all $\rho \in (0, 1)$ and $\varphi_2 \subset \Psi_2$ when $\rho < (1-\alpha)(1-\pi)$.

Consider Case 3. We have $V_S = c$ and $V_N = c - \alpha(1-\pi)\rho\frac{c+r_0}{\alpha+(1-\alpha)\pi}$, therefore

$$V_S - V_N = \alpha(1-\pi)\rho\frac{c + r_0}{\alpha + (1-\alpha)\pi} > 0.$$

In this case we have $\varphi_3 = \Psi_3$ and information sharing is preferred by the relationship bank.

■

Proof of Proposition 3. Suppose the distant bank, depositors and asset buyers all hold the belief that the relationship bank will tell the truth about the credit history of the borrower. We analyze the profitable deviation of the relationship bank to announce truthfully a defaulted D-history under such belief. We focus our discussion on the parameter set φ_j with $j = \{0, 1, 2, 3\}$ defined in Proposition 2.

Consider Case 0. We first compute the relationship bank's expected profit at $t = 1$ of truthfully reporting a loan with default credit history D. Recalling that $R_S^*(D) = R$ in this case, we have

$$V_S(D) = \pi R_S^*(D) - r_0 = \pi R - r_0. \tag{7}$$

The expected profit of misreporting the borrower's true credit history (i.e., reporting the false $\overline{D}$-history) is

$$V_S(D, \overline{D}) = \Pr(G)R_S^*(\overline{D}) + \Pr(B)P_S^B(\overline{D}) - r_0 = \pi R + (1-\pi)\frac{\alpha\rho}{\alpha\rho + (1-\alpha)\pi}R - r_0.$$

Notice the relationship bank does not fail by misreporting the credit history. Clearly we have $V_S(D) - V_S(D, \overline{D}) < 0$. The relationship bank finds it profitable to misreport the borrower's credit history. The benefit from the deviation $(1-\pi)\frac{\alpha\rho}{\alpha\rho+(1-\alpha)\pi}R$ is the expected liquidation loss in case of bank run. Under this case, the belief of outsiders can not be rationalized, and truthful information sharing can not be sustained as a Perfect Bayesian Equilibrium in the set of parameter φ_0.

Consider Case 1. Like in Case 0, the relevant equilibrium loan rate is $R_S^*(D) = R$. Then reporting the true default history gives the same expected profit as in (7). The expected profit of misreporting the true credit history with the false $\overline{D}$-history can be expressed as

$$\begin{aligned} V_S(D,\overline{D}) &= \Pr(G)R_S^*(\overline{D}) + \Pr(B)P_S^B(\overline{D}) - r_0 \\ &= \pi\frac{\alpha+(1-\alpha)\pi}{\alpha+(1-\alpha)\pi^2}(c+r_0) + (1-\pi)\frac{\alpha\rho}{\alpha\rho+(1-\alpha)\pi}\frac{\alpha+(1-\alpha)\pi}{\alpha+(1-\alpha)\pi^2}(c+r_0) - r_0, \end{aligned}$$

since $R_S^*(\overline{D}) = \frac{\alpha+(1-\alpha)\pi}{\alpha+(1-\alpha)\pi^2}(c+r_0)$ in this Case. Then we have

$$V_S(D,\overline{D}) = \frac{\alpha\rho+(1-\alpha)\pi^2}{\alpha\rho+(1-\alpha)\pi}\frac{\alpha+(1-\alpha)\pi}{\alpha+(1-\alpha)\pi^2}(c+r_0) - r_0. \tag{8}$$

Then the ex-post incentive compatibility constraint to tell the truth is

$$V_S(D) - V_S(D,\overline{D}) = \pi R - [\pi + (1-\pi)\frac{\alpha\rho}{(1-\alpha)\pi+\alpha\rho}]\frac{\alpha+(1-\alpha)\pi}{\alpha+(1-\alpha)\pi^2}(c+r_0) > 0,$$

which can be simplified as

$$R > \left[\frac{\alpha\rho+(1-\alpha)\pi^2}{\alpha\rho+(1-\alpha)\pi}\frac{\alpha+(1-\alpha)\pi}{\alpha+(1-\alpha)\pi^2}\right]\frac{c+r_0}{\pi} \equiv \underline{R}. \tag{9}$$

Information sharing is ex-ante chosen in Case 1 when (recall the definition of R_1 in the proof of Proposition 2)

$$R < \frac{\alpha+(1-\alpha)\pi}{\alpha-\alpha(1-\alpha)\rho+(1-\alpha)\pi^2}(c+r_0) \equiv R_1.$$

It can be shown that $R_1 - \underline{R} = -\alpha^2(1-\rho)\rho(1-\pi) < 0$. Consequently, there exists no R such that the relationship bank will ex-ante participate in information sharing scheme and ex-post report the true default credit history of a borrower. The belief of outsiders can not be rationalized and truthful information sharing can not be sustained as a Perfect Bayesian Equilibrium in the set of parameter φ_1.

Consider Case 2. We again have $R_S^*(D) = R$. Reporting the true default history gives the same expected profit as in (7). The expected profit of misreporting the true credit history is the same as in expression (8), since $R_S^*(\overline{D}) = \frac{\alpha+(1-\alpha)\pi}{\alpha+(1-\alpha)\pi^2}(c+r_0)$ also in this Case.

Therefore the condition on R to ensure ex-post the relationship bank tells the truth is the same as in (9). Information sharing is ex-ante chosen in Case 2 when (recall the definition of R_2 in the proof of Proposition 2)

$$R > \left[1 - \frac{\alpha\rho}{(1-\alpha)[\alpha + (1-\alpha)\pi]}\right] \frac{c + r_0}{\pi} \equiv R_2.$$

Information sharing can be sustained as a Perfect Bayesian Equilibrium only if both the inequality $R > R_2$ and the condition (9) are satisfied. In particular, we find a region of parameters in which whenever is ex-ante optimal for the relationship bank to share information is also ex-post convenient for it to tell the true credit history. This implies to impose the following restriction

$$1 - \frac{\alpha\rho}{(1-\alpha)[\alpha + (1-\alpha)\pi]} > \frac{\alpha\rho + (1-\alpha)\pi^2}{\alpha\rho + (1-\alpha)\pi} \frac{\alpha + (1-\alpha)\pi}{\alpha + (1-\alpha)\pi^2}. \tag{10}$$

Note that the expression (10) can be rewritten as

$$1 - \frac{\alpha\rho}{(1-\alpha)[\alpha + (1-\alpha)\pi]} - \frac{\alpha\rho + (1-\alpha)\pi^2}{\alpha\rho + (1-\alpha)\pi} \frac{\alpha + (1-\alpha)\pi}{\alpha + (1-\alpha)\pi^2} = 0.$$

We define a function $F(\rho) = 1 - \frac{\alpha\rho}{(1-\alpha)[\alpha+(1-\alpha)\pi]} - \frac{\alpha\rho+(1-\alpha)\pi^2}{\alpha\rho+(1-\alpha)\pi} \frac{\alpha+(1-\alpha)\pi}{\alpha+(1-\alpha)\pi^2}$. It can be checked that

$$F'(\rho) = -\frac{\alpha}{(1-\alpha)[\alpha + (1-\alpha)\pi]} - \frac{\alpha(1-\alpha)\pi(1-\pi))}{[\alpha\rho + (1-\alpha)\pi]^2} \frac{\alpha + (1-\alpha)\pi}{\alpha + (1-\alpha)\pi^2} < 0.$$

Moreover, we can take the limits

$$\begin{aligned}
\lim_{\rho\to 0} F(\rho) &= 1 - \frac{\alpha\pi + (1-\alpha)\pi^2}{\alpha + (1-\alpha)\pi^2} > 0 \\
\lim_{\rho\to 1} F(\rho) &= -\frac{\alpha}{(1-\alpha)[\alpha + (1-\alpha)\pi]} < 0.
\end{aligned}$$

Thus, there exists a unique $\hat{\rho}$ such that $F(\hat{\rho}) = 0$. Whenever $0 < \rho < \hat{\rho}$, we have $F(\rho) > 0$ and expression (10) holds. Then truth telling can be sustained as a Perfect Bayesian Equilibrium in the set of parameter φ_2. Recall that we established in Proposition 2 that φ_2 is non-empty for all $\rho \in (0, 1)$.

Consider Case 3. In this Case we have $R_S^*(D) = (c + r_0)/\pi$ since the distant bank competes also for the defaulted borrower. Reporting the true default history gives an expected profit equal to

$$V_S(D) = \pi R_S^*(D) - r_0 = c.$$

The expected profit of misreporting the credit history is the same as in (8), and since

$$\frac{\alpha\rho + (1-\alpha)\pi^2}{\alpha\rho + (1-\alpha)\pi}\frac{\alpha + (1-\alpha)\pi}{\alpha + (1-\alpha)\pi^2} < 1,$$

we have $V_S(D, \overline{D}) - V_S(D) < 0$. The belief of outsiders can be rationalized, and truthful information sharing can be sustained as a Perfect Bayesian Equilibrium in the set of parameter φ_3. ■

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Figure 1: Time line of the model

t = 0	t = 1	t = 2	t = 2.5	t = 3
1. The relationship bank inherits a lending relationship from history. 2. The bank decides whether to share borrower's credit history or not.	1. Borrower credit worthiness (type) and credit history realize. 2. The information is privately observed by the relationship bank. 3. The relationship bank announces the borrower's credit history if it chooses to share such information.	1. The relationship bank and the distant bank compete for the borrower by offering loan rates. 2. The winner is financed by fairly priced deposits.	1. State s realizes and is publicly observed. 2. The relationship bank's liquidity risk is realized, and is privately observed by the bank. 3. A secondary loan market opens; and the relationship bank can sell its loan to asset buyers.	The bank loan pays off.

Figure 2: Equilibrium loan rates: Interior and corner solutions

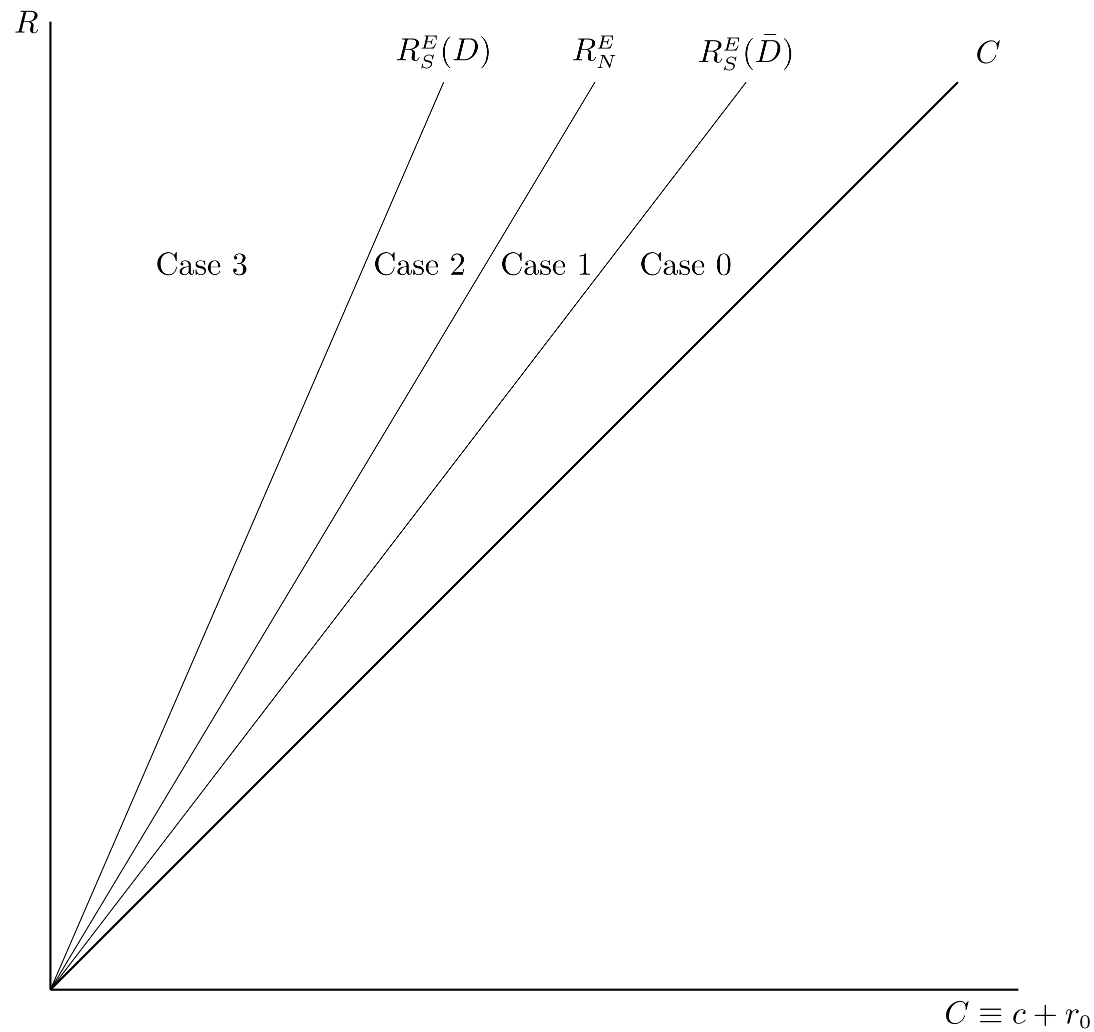

Figure 3: Regions where information sharing can save the relationship bank from illiquidity

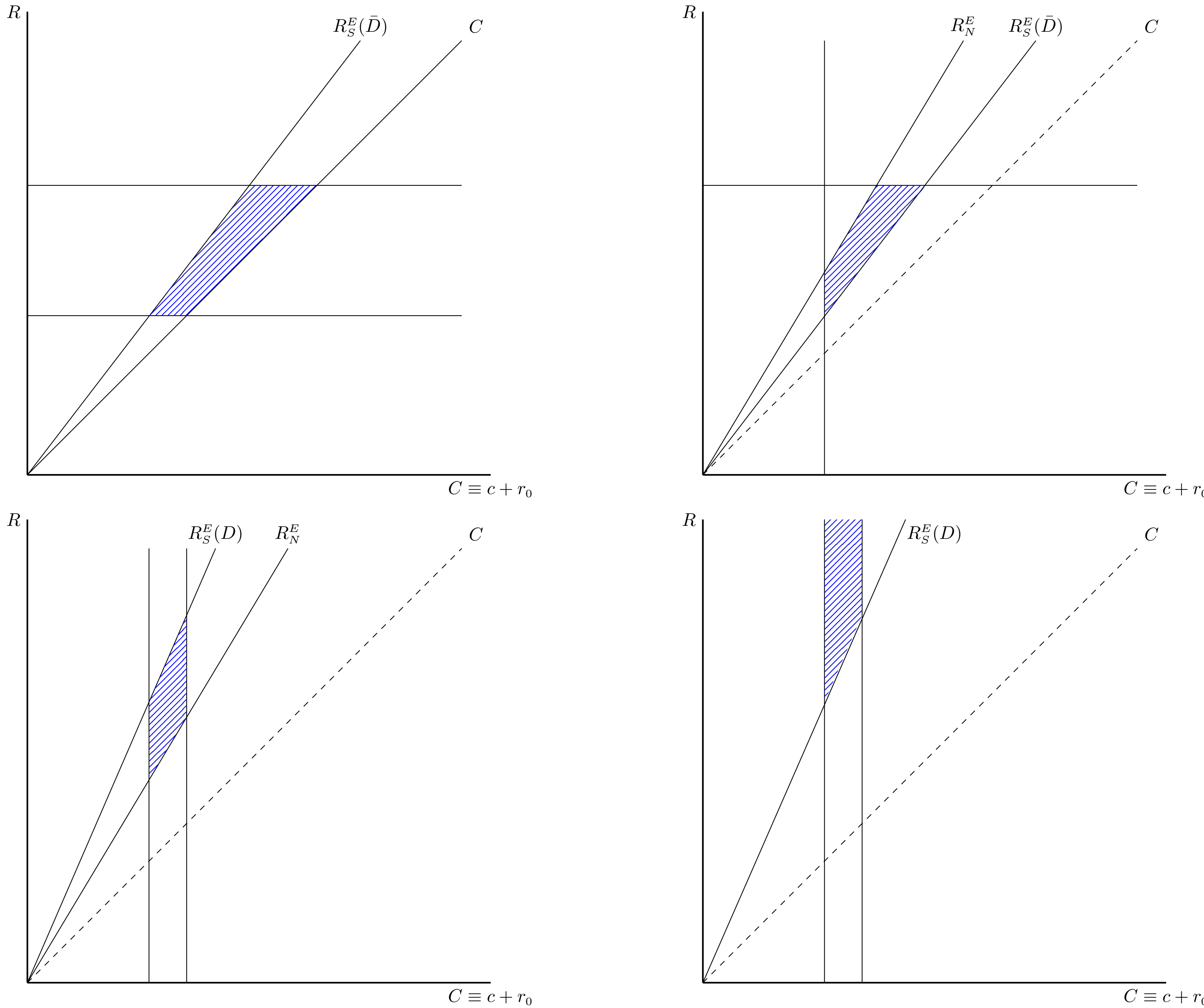

Figure 4: Regions where information sharing leads to greater value for the relationship bank(for $\rho < (1-\alpha)(1-\pi)$)

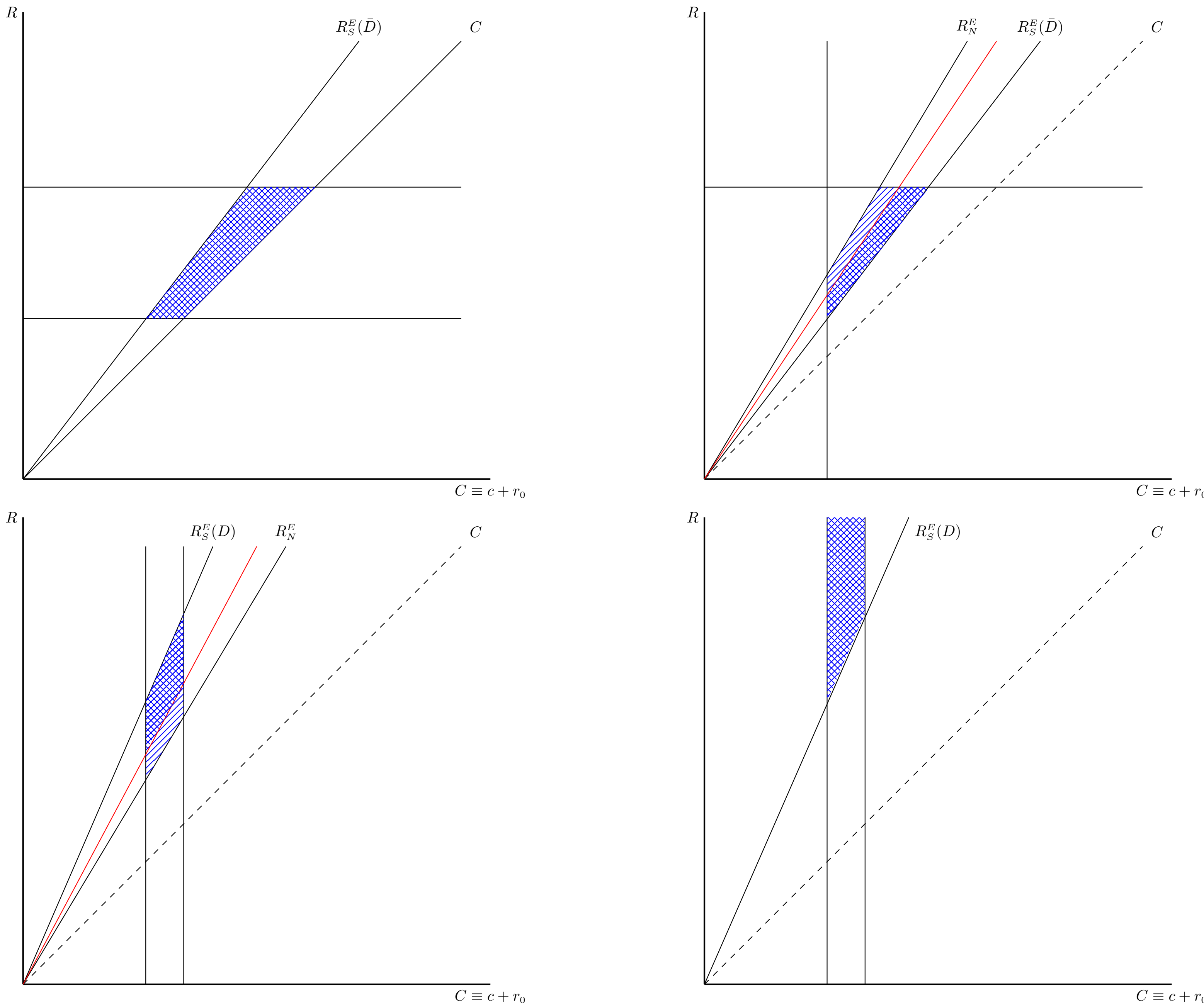

Figure 5: Regions where truthful information sharing can be sustained in a perfect Bayesian equilibrium (for $\rho < \min\{\hat{\rho}, (1-\alpha)(1-\pi)\}$)