Early Childhood Development, Earnings Inequality and Social Mobility in an Education Signaling Model*

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Abstract

The growing income inequality has been a big concern for economists and policy makers around the world. Many factors are responsible for the observed burgeoning income inequality, such as capital outflow, relocation of jobs, declining labor union, i.e., declining bargaining power of the labor, poor regulation of financial institutions, corruption, and all-encompassing globalization. Incomes of the bottom 99 percent population in a society comes mainly from earnings, and much of the earnings inequality results from the inequality of skill formation. The children of poor socioeconomic status stays behind skill accusations as compared to their rich counterpart. In modern technology-rich economies, providing high quality education to the talented individuals and matching their jobs with the highly productive technical sector is crucial for economic growth, earnings inequality and social mobility. Because education is used as a signal for a worker's unobserved endowment of talents, its acuisition by various social groups distorts productive efficiency, lowers social mobility and increases earnings inequality. This paper provides a signaling equilibrium framework to study these issues.

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1 Introduction

The growing income inequality has been a big concern for economists and policy makers around the world. Many factors are responsible for the observed burgeoning income inequality, such as capital outflow, relocation of jobs, declining labor union, i.e., declining bargaining power of the labor, poor regulation of financial institutions, corruption, and all-encompassing globalization (Bourguignon, 2015; Piketty, 2014; Stiglitz, 2015). In most economies, incomes of the bottom 99 percent come mainly from earnings and much of the inequality in earnings results from the inequality of skill formations (Autor, 2014). A growing consensus reached among researchers is that children of poor SES (Socioeconomic Status) are not prepared for college because they were not prepared for school to begin with. The most effective intervention for the children of poor SES should be introduced at the preschool stage so that these children are prepared for school and college (see, for instance, Cunha and Heckman, 2009; Currie, 2011; Currie and Almond, 2011; Duncan et al., 2010; Garca et al., 2016; Heckman et al., 2010; Heckman and Raut, 2016; Keane and Wolpin, 1997, 2001).

Children of poor SES perform poorly in skill acquisition and in the labor market. Many children of poor SES do not complete high school and many of them perform poorly in schools. Gaps in test scores between rich and poor children are substantial, and unequal schooling does little to widen this gap. In spite of its positive effects on test scores and earnings, the effects of improved school quality on school dropout rates is marginal.

Most earlier research in the last century focussed on cognitive skills as the main determinant of socioeconomic behaviors, school performances and labor market outcomes. One line of influential but controversial research argues that poor parents have poor cognitive abilities and that is why they are poor; children of poor SES inherit poor cognitive abilities from their parents; thus very little can be done to improve the cognitive skills of the disadvantaged children, and hence their school performance and labor market outcomes, see Herrnstein and Murray (1994) and other references in Plomin and Deary (2015). This view has been refuted using more appropriate data and statistical techniques. More recent research in neuroscience, psychology, economics and experimental game theory emphasize that it is the interplay of emotions and cognition that determine most social and economic behaviors (see, for instance, Camerer et al., 2005; Kahneman, 2013; Winter, 2014; Zak, 2004).

In psychology literature, especially in the work of Bowlby (Bowlby (1982)), it has been argued that affect (emotion) dysregulation which begins to form immediately after birth, especially during the first two years of age, from low quality interaction of the primary care-taker (generally the mother) with the baby can have long lasting effects on emotional development of the child in later ages. NETWORK (2004) carried a longitudinal study and found evidence for such affect dysregulation mechanisms. The emotional dysregulation also conditions cognitive developments of children. More recent neurological research on this phenomena confirms this, see for instance, Schore (2005).

Using the fMRI images of brain areas a number of neurological studies found that poverty has significant negative effects on development a child's certain areas of brain that are responsible for cognition, executive functions and emotions. For instance, a large scale neurological study by Noble et al. (2015) found that family income significantly affects children's brain size, particularly in the surface area of the cerebral cortex that does most of the cognitive processing. See also their earlier study, Noble et al. (2012) and the commentary in Balter (2015). Hair et al. (2015) conducted a large longitudinal neurological studies on children starting at an early age, and followed them up into their school years. They measured their scores on cognitive and academic achievements, and development of brain tissue, including gray matter of the total brain, frontal lobe, temporal lobe, and hippocampus. They found significant negative effects of poverty on developments of these areas and on their academic achievements.

In modern technology-rich economies, providing high quality education to the talented children and matching their jobs with the highly productive technical sector is crucial for economic growth, social mobility and earnings inequality. Individuals know their own abilities but the employers do not observe them. Employers use education as a predictor of a worker's level of unobserved cognitive abilities. Because education acts as an imperfect predictor of one's cognitive abilities, and children of poor SES have disadvantages of the type mentioned above in acquiring education, the individual investment in education in the economy distorts productive efficiency, lowers social mobility and increases earnings inequality. The paper will address these issues in a signaling model, adapting the asymmetric information frameworks of Stiglitz (1975) and Spence (1974).

The rest of the paper is organized as follows: Section 2 describes the basic model of human capital acquisition in a signaling equilibrium framework. Section 3 studies the properties of signaling equilibria specializing to the lognormal case and shows that use of education as a signal accentuates earnings inequality and reduces social mobility. Section 4 specializes the model to two schooling levels and two levels of unobserved cognitive ability and then studies the nature of equilibrium earnings inequality, social mobility and growth when children of poor SES are adversely affected in obtaining education.

2 The Basic Model

The economy consists of an overlapping generations of agents and of risk neutral competitive producers. In each period there is a continuum of adult population, who live for one period. At the end of the period, he dies and a new adult child is born to each parent. Denote by τ an individual's cognitive ability which affects his productivity at workplace and learning in school. I assume for simplicity that τ is one dimensional, and it takes a $\mathcal{T} = (0, \infty)$. In the set $\mathcal{T}$, a higher number denotes a greater level of cognitive ability. An individual's productivity depends on his schooling level and his level of cognitive ability. The cost of schooling depends on his own schooling level, level of cognitive ability and his family background, denoted here with his parent's schooling level s_{t-1}. The level of schooling can be used to signal one's productivity level. An individual chooses a schooling level s_t which which together with his cognitive ability determine his productivity level, *productivity function* $e(s, \tau)$. Possible education levels are assumed to be from the set $\mathcal{S} = (0, \infty)$, a higher number representing a higher education level.[1] I assume that the cognitive level $\tau_t \in \mathcal{T}$ of a child born to a parent of cognitive ability τ_{t-1} and schooling level s_{t-1}follows probability distribution independent of $\tau_{t-1}$and s_{t-1}, which is characterized by the pdf $g(\tau_t)$[2]

I consider only human capital investment in education, other important forms of human capital investment such as health and nutrition are not considered here. Attainment of an education level by an individual is a more complex decision making process than assumed here. Generally, parents make the initial investments such as pre-school investments and investments up to college or so, until the child reaches enough maturity to make his own schooling decision. Family background can have great influence on educational attainment in several other ways. For instance, suppose that the quality of pre-school investment of parents' time at home affect children's motivation and persistence to continue schooling. Then, of course, more highly educated parents can provide better learning environment for their children at home. Similarly, more highly educated parents with their better knowledge base of child care, or simply because of their higher incomes can provide better pre-natal and post-natal care, and health care for proper cognitive and affective developments of

[1] The general practice in the human capital literature is, however, to treat S as continuous variable, more realistically it is a discrete set.

[2] There is a long controversy over the issue of whether children's innate ability is genetically inherited from parent's innate ability. The scientific consensus is that the correlation between parent's innate ability and a child's innate ability is somewhere between 0.3 to 0.7. I assumed it to be zero, for simplification. There are other controversies regarding talent, ability and intelligence. Some believe that one is born with a fixed level of intelligence, and training and environment has no effect on intelligence. Others do not agree with it, and believe that ability, intelligence and talent could be improved to some extent with better environment and training. Some believe that intelligence or innate ability is fixed when one is born, and less intelligent people can learn and do complex things that we face in our everyday life, in school curricula, and in modern jobs, except that they might take longer, and thus less productive; this is the view we take in this paper.

their children.[3]

An individual in period t of ability τ_t and family background $s_{t-1}$is denoted by the index $\xi_t = (\tau_t, s_{t-1})$ .The distributions of individuals in period t is characterized by the pdf $f_{\xi_t}(\tau_t, s_{t-1}) = g(\tau_t).\pi_{t-1}(s_{t-1})$, where $\pi_{t-1}(.)$ is the pdf of the schooling distribution of the parents in period t. The effect of the above types of family ground is assumed to affect his cost of obtaining a level of education s_t. Denote this cost function for agent (τ_t, s_{t-1}) by $\theta_t(s_t, \tau_t, s_{t-1})$[4]

Assume that all individuals have identical linear[5] utility function $u(c_t)$, where c_t is the consumption of an adult of period t. An adult of period t with cognitive ability $\tau_t \in \mathcal{T}$ and parental educational background s_{t-1}, takes the wage function $w_t(s_t)$ of period t as given and decides his education level $s_t \in \mathcal{S}$ by solving the following problem:

$$\sigma_t(\tau_t, s_{t-1}) = \arg\max_{s_t \in \mathcal{S}} u(w_t(s_t) - \theta(s_t, \tau_t, s_{t-1})) \tag{1}$$

For regular cases, there is a unique optimal solution s_t for each agent $\xi_t = (\tau_t, s_{t-1})$. Notice that in this framework, all individuals with cognitive ability τ_t and family background s_{t-1}behave identically. Denote the optimal solution of the choice problem in Eq. (1) for agent (τ_t, s_{t-1}) by $\sigma_t(\tau_t, s_{t-1})$.

I assume that the production sector is competitive; the producer is risk neutral; there is no affirmative action in hiring, i.e., workers with the same level of schooling are treated the same way, no matter what their family backgrounds are. In each period $t \geq 1$, a producer announces a wage schedule $w_t(s_t)$ for hiring purposes. He observes the education level s_t of a worker but not his innate productive ability level τ_t. The employer holds a subjective belief about his productivity level $e(s_t, \tau_t)$ given his education level s_t. This belief is represented by a condition density function $q_t(e_t|s_t), e \in \mathcal{E}, s_t \in \mathcal{S}$. Perfect competition, and expected profit maximization imply that $w_t(s_t) = \int e_t \, q_t(e_t|s_t) \, de_t$ in equilibrium.

The economy begins at time $t = 1$ with an adult population whose parents' education level is distributed as $\pi_0(s)$. Given π_0, the transition probability density function $p_t(s_t|s_{t-1})$ determines the dynamics of the schooling distributions $\pi_t, t \geq 1$.

The signalling equilibrium is recursively defined over time as follows: At the beginning of time period t, the population density function $\pi_{t-1}(s_{t-1})$ is known. A competitive producer knows these and he knows the distribution of τ_t in the population, but he does not observe an individual agent's

[3]There are other ways education of parents can influence the educational achievement of their children, for instance, by providing role models.

[4]The assumption that $\theta_t(s_t, \tau_t, s_{t-1})$ varies with τ_t is necessary for education to act as a signal for talent, see Stiglitz (1975), Spence (1973) or Kreps (1990) for a justification.

[5]Thus we abstract away from bearings on our results from risk sharing between employers and workers.

cognitive ability level τ_t. The employer holds a subjective belief $q(e_t|s_t)$ and announces an earning function $w_t(s_t) = \int e_t \; q(e_t|s_t) \; de_t$ for hiring purpose. Given $w_t(s_t)$, each worker (τ_t, s_{t-1}) decides his optimal education level $\sigma_t \left(\tau_t, s_{t-1}\right)$ as in Eq.(1). Given the probability distributions over family backgrounds, $\pi_{t-1}\left(s_{t-1}\right)$ and innate ability $g(\tau_t)$, the optimal schooling decision variable $\sigma_t\left(\tau_t, s_{t-1}\right)$ induces a joint probability distribution of (s_t, τ_t) in period t. Denote the joint pdf of the (s_t, τ_t) by $f_{s_t,\tau_t}\left(.,.\right)$. This joint distribution of (s_t, τ_t) induces a conditional distribution of τ_t given s_t,denoted by $f_{\tau_t|s_t}\left(.\right)$. This conditional distribution together with the productivity function $e_t = e\left(s_t, \tau_t\right)$ produces the observed distribution of productivity levels $\hat{q}_t(e|s_t)$ for each level of s_t. We have a signalling equilibrium, when the anticipated distribution coincides with the above observed distribution, i.e., $q_t\left(e_t|s_t\right) = \hat{q}_t\left(e_t|s_t\right)$ for all education levels that are chosen by some agent in the population.

Notice that optimal schooling choices $s_t = \sigma\left(\tau_t, s_{t-1}\right)$ determines the transition probability measure $P_t\left(s_t \in A|s_{t-1}\right)$ of an individual born in the family background s_{t-1} moves to a family background $s_t \in A$ as follows:

$$P_t(s_t \in A|s_{t-1}) = \int \mathcal{I}_A\left(\sigma\left(\tau_t, s_{t-1}\right)\right)\; g(\tau_t)\; d\tau_t \tag{2}$$

The transition probabilities for earnings between period $t-1$ and period t can be defined similarly. The transition probability distribution $P_t\left(s_t \in A|s_{t-1}\right)$ determines π_t, the distribution of s_t in each period t as follows

$$\pi_t\left(s_t \in A\right) = \int P_t(s_t \in A|s_{t-1})\, d\pi_{t-1}\left(s_{t-1}\right). \tag{3}$$

The economy moves to the next period with known π_t and the above process starts all over again.

Definition 1 *Initial distribution π^0 of social groups in $\mathcal{S}$, is given. A* ***signaling equilibrium*** *is a squence of probability distributions $\left\{q_t\left(e_t|s_t\right)\right\}_1^\infty$and a sequence of optimal schooling decision rules $\left\{\sigma_t\left(\tau_t, s_{t-1}\right)\right\}_1^\infty$ defined in (1) such that at each period $t \geq 1$,*

1. *The induced wage schedule $w_t\left(s\right) = \int e_t q_t\left(e_t|s_t\right)\; de_t$ is a smooth concave function.*

2. *Given $w_t\left(s\right)$, the funciton $\sigma_t\left(\tau_t, s_{t-1}\right)$ solves schooling decision problem (1) of each agent $\left(\tau_t, s_{t-1}\right)$.*

3. *The induced conditional distribution $\hat{q}_t\left(e_t|s_t\right)$ of e_t given the optimal solution $s_t = \sigma_t\left(\tau_t, s_{t-1}\right)$ obtained by using Bayes rule coincides with the anticpated conditional distribution $q_t\left(e_t|s_t\right)$ for all s_t.*

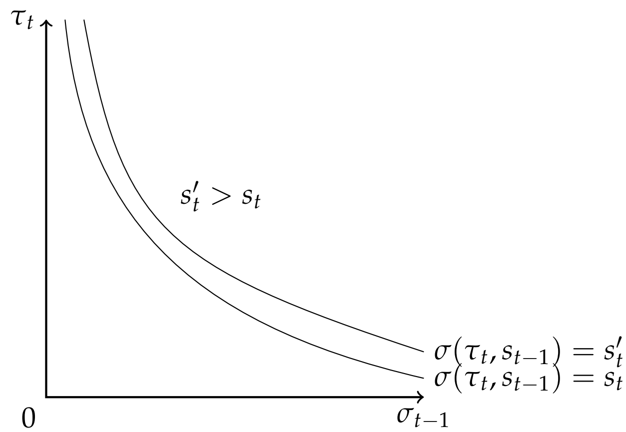

Figure 1: Set of individuals $(\tau_t, st-1)$ for whom s_t is the optimal schooling level.

Assume that

$$\theta(s_t, \tau_t, s_{t-1}) = \theta_1(s_t).\theta_2(\tau_t, s_{t-1}) \tag{4}$$

where $\theta_1(s_t)$ is a monotonically increasing smooth function of s_t, and $\theta_2(\tau_t, s_{t-1})$ is a smooth function decreasing in each variable τ_t and s_{t-1}. The distributions of agents in period t is characterized by the pdf $f_{\xi_t}(\tau_t, s_{t-1}) = g(\tau_t).\pi_{t-1}(s_{t-1})$ .Assume that there is no affirmative action in hiring, i.e., workers with the same level of schooling are treated the same way, no matter what their family backgrounds are. The first order condition of the schooling choice problem in Eq. (1) is given by

$$\frac{w_t'(s_t)}{\theta_1'(s_t)} = \theta_2(\tau_t, s_{t-1}) \tag{5}$$

The left hand side of Eq. (5) is a strictly monotonic function of s_t, and hence we can solve s_t as a function of agent characteristics (τ_t, s_{t-1}), which we denote by $s_t = \sigma_t(\tau_t, s_{t-1})$.

Note that for each τ_t, s_t, we can solve s_{t-1} as a function of (s_t, τ_t), which we denote by $s_{t-1} = s_t^{*-1}\left(\tau_t, w_t'(s_t)/\theta_1'(s_t)\right)$. Let the bivariate random variable $X_t \equiv (s_t, \tau_t)$ be the optimal schooling level s_t and the cognitive productivity level τ_t of the child $\xi_t = (\tau_t, s_{t-1})$. From the probability distribution of $\xi_t = (\tau_t, s_{t-1})$, we derive the joint probability distribution $f_{(s_t,\tau_t)}(s_t, \tau_t)$ of s_t, τ_t using the transformation $s_{t-1} = s_t^{*-1}\left(\tau_t, w_t'(s_t)/\theta_1'(s_t)\right)$, $\tau_t = \tau_t$. Note that the Jacobian of the transformation is given by

$$\begin{aligned} \frac{\partial(\tau_t, s_{t-1})}{\partial(s_t, \tau_t)} &= \det\begin{pmatrix} \frac{\partial \tau_t}{\partial s_t} & \frac{\partial \tau_t}{\partial \tau_t} \\ \frac{\partial s_{t-1}}{\partial s_t} & \frac{\partial s_{t-1}}{\partial \tau_t} \end{pmatrix} \\ &= -\frac{w_t''(s_t)\,\theta_1'(s_t) - w_t'(s_t)\,\theta_1''(s_t)}{\left[\theta_1'(s_t)\right]^2 \partial\theta_2\left(\tau_t, s_t^{*-1}\left(\tau_t, w_t'(s_t)/\theta_1'(s_t)\right)\right)/\partial s_{t-1}} \end{aligned}$$

Hence the joint pdf of s_t, τ_t is given by

$$f_{(s_t,\tau_t)}(s_t,\tau_t) = g(\tau_t).\pi_{t-1}\left(s_t^{*-1}\left(\tau_t, \frac{w_t'(s_t)}{\theta_1'(s_t)}\right)\right)\left|\frac{\partial(\tau_t, s_{t-1})}{\partial(s_t,\tau_t)}\right|$$

A period-t signaling equilibrium is a wage schedule $w_t(s_t)$ such that

$$\begin{aligned} w_t(s_t) &= \int_T e(s_t,\tau_t) f_{\tau_t|s_t}(\tau_t)\, d\tau_t = \frac{\int_T e(s_t,\tau_t) f_{X_t}(s_t,\tau_t)\, d\tau_t}{\int_T f_{X_t}(s_t,\tau_t)\, d\tau_t} \\ &= \frac{\int_T e(s_t,\tau_t) g(\tau_t)\pi_{t-1}\left(s_t^{*-1}\left(\tau_t, \frac{w_t'(s_t)}{\theta_1'(s_t)}\right)\right)\left[\frac{\partial\theta_2}{\partial s_{t-1}}\left(\tau_t, s_t^{*-1}\left(\tau_t, \frac{w_t'(s_t)}{\theta_1'(s_t)}\right)\right)\right]^{-1} d\tau_t}{\int_T g(\tau_t)\pi_{t-1}\left(s_t^{*-1}\left(\tau_t, \frac{w_t'(s_t)}{\theta_1'(s_t)}\right)\right)\left[\frac{\partial\theta_2}{\partial s_{t-1}}\left(\tau_t, s_t^{*-1}\left(\tau_t, \frac{w_t'(s_t)}{\theta_1'(s_t)}\right)\right)\right]^{-1} d\tau_t} \\ &= \Psi\left(w_t'(s_t), s_t\right) \text{ say} \end{aligned}$$

The right hand side of the above equation is the observed or realized average productivities of the signal class s_t, which in other words, is the conditional expectation of $e(s_t,\tau_t)$ with respect to the observed empirical conditional distribution of τ_t given s_t.The above is a first order non-linear differential equation which under general conditions have smooth solution $w_t(s_t)$, which is unique when we provide an initial condition. We take the initial condition $w(0) = 0$, i.e., the labor with no education has zero productivity.

I do not examine conditions under which there exists a signalling equilibrium. Instead I specialize to lognormal specification of the distributions of productivity level τ_t and the family background level $s_{t-1}$and explicitly compute the signalling equilibrium and study the properties of equilibrium earnings inequality and social mobility.

3 Log-Normal Economy

I examine how inequality in earnings and schooling changes over time with the specification of lognormal distributions for cognitive ability τ and the initial distribution of population over the observed schooling levels of parents.

The notation $X \sim \Lambda(\mu, \sigma^2)$ means the random variable X is log-normally distribution with parameters μ, and σ^2,i.e. $\ln X$ is normally distributed with mean μ and variance σ^2. Assume that

$$\begin{aligned} s_{t-1} &\sim \Lambda\left(\mu_{s_{t-1}}, \sigma^2_{s_{t-1}}\right) \\ \tau_t &\sim \Lambda\left(\mu_\tau, \sigma^2_\tau\right) \\ e(s_t,\tau_t) &= s_t^\rho.\tau_t,\ \rho > 0. \end{aligned} \tag{6}$$

For simplicity and without loss of much generality, I assume that $\theta_1(s_t) = s_t$.I consider two cases below. First I consider the case in which cost of education depends only on τ and not on family background s_{t-1}. In this case, from the observable optimal schooling level s_t, the unobserved cognitive ability level τ_t or the productivity level $e(s_t, \tau_t)$ of the worker could be predicted perfectly. Then I consider the case in which cost of education depends on both τ_t and the family background s_{t-1}. In this case the observed optimal education level can predict the unobserved cognitive skill level imperfectly. I then compare how the distribution of earnings and education levels become more unequal due to signaling role of education.

3.1 Signaling cost does not depend on family background

Assume that cost of education does not depend on the family background of the child, i.e. family background does not have effect on child development. Let it be more specific as $\theta_2(\tau_t, s_{t-1}) = \tau_t^{-\alpha}$ where, $\alpha > 0$. The pdf of τ_t is assumed to be lognormal as follows:

$$f(\tau_t) = \frac{1}{(2\pi\sigma_\tau)^{1/2}\tau} \exp\left\{-\frac{1}{2}\left[\frac{(\ln\tau - \mu_\tau)^2}{\sigma_\tau^2}\right]\right\}$$

From the first order condition of the agent's schooling choice problem Eq. (5), we have $w_t'(s_t) = \tau_t^{-\alpha}$. Note that given schooling level s_t, one can perfectly predict his ability level τ_t as I mentioned earlier. Denote this forecasting rule by $\tau_t = (w_t'(s_t))^{-1/\alpha}$. This prediction of τ_t given the optimal schooling level s_t is equivalent to the degenerate pdf $f_{\tau_t|s_t}(\tau_t|s_t) = 1$ if $\tau_t = (w_t'(s_t))^{-1/\alpha}$and $f_{\tau_t|s_t}(\tau_t|s_t) = 0$ otherwise. The equilibrium is attained if $w_t(s_t) = \int e(s_t, \tau_t) f_{\tau_t|s_t}(\tau_t|s_t)\, d\tau_t$,which in our case simplifies to the following first order differential equation,

$$\frac{dw_t(s_t)}{ds_t} = \left[\frac{s_t^\rho}{w_t(s)}\right]^\alpha$$

The above first order non-linear differential equation is in the separation-of-variables form and can be solved explicitly. The general solution of this differential equation is given by

$$w_t(s_t) = \left[c + \frac{\alpha+1}{\alpha\rho+1} s_t^{\alpha\rho+1}\right]^{\frac{1}{1+\alpha}}, \text{ where } c \text{ is a constant of integration.}$$

Each value of c will give a signalling equilibrium and there are continuum of them. Using the initial condition $w(0) = 0$, the equilibrium earnings function becomes,

$$w_t(s_t) = \left[\frac{\alpha+1}{\alpha\rho+1}\right]^{\frac{1}{1+\alpha}} s_t^{\frac{\alpha\rho+1}{\alpha+1}}. \tag{7}$$

To compute the equilibrium distribution of schooling levels and the wages, note that $\tau_t = (w_t'(s_t))^{-1/\alpha}$. Substituting the value of $w_t'(s)$ from the above, and taking natural log, i.e. ln on both sides, we see that $\ln(s_t) = \frac{1}{1-\rho}\ln\frac{\alpha\rho+1}{1+\alpha} + \frac{\alpha+1}{1-\rho}\ln\tau$. Thus, the equilibrium schooling distribution in period t follow the following lognormal distribution,

$$s_t \sim \Lambda\left(\frac{1}{1-\rho}\ln\frac{\alpha\rho+1}{1+\alpha} + \frac{\alpha+1}{1-\rho}\mu_\tau, \left[\frac{1+1/\alpha}{1-\rho}\right]^2\alpha^2\sigma_\tau^2\right) \tag{8}$$

and the equilibrium wage distribution in period t is also a lognormal,

$$w_t \sim \Lambda\left(\frac{\rho}{1-\rho}\ln\frac{\alpha\rho+1}{\alpha+1} + \frac{\alpha\rho+1}{1-\rho}\mu_\tau, \left[\frac{\alpha\rho+1}{\alpha+1}\cdot\frac{1+\rho/\alpha}{1-\rho}\right]^2\alpha^2\sigma_\tau^2\right). \tag{9}$$

One can compute the transition probability density function for schooling levels of two generations $f_{s_t|s_{t-1}}(s_t|s_{t-1})$and the transition probability density function for earnings $f_{w_t|w_{t-1}}(w_t|w_{t-1})$ and compute a measure of schooling mobility and earnings mobility.

The Gini-coefficient for schooling inequality in period t is

$$\begin{aligned} G_{s_t} &= 2\Phi\left(\frac{1}{\sqrt{2}}\frac{1+1/\alpha}{1-\rho}\sqrt{\alpha^2\sigma_\tau^2}\right) - 1 \\ G_{w_t} &= 2\Phi\left(\frac{1}{\sqrt{2}}\frac{\alpha\rho+1}{\alpha+1}\cdot\frac{1+1/\alpha}{1-\rho}\sqrt{\alpha^2\sigma_\tau^2}\right) - 1 \end{aligned} \tag{10}$$

where Φ is the erf function. The Gini coefficient of earning distribution is smaller than the Gini coefficient of schooling distribution. How these compare when schooling cost depends on the family background, and thus education is an imperfect signal of cognitive skill.

3.2 Signal cost depends on ability and family background

I now assume that schooling cost depends on family background, which is incorporated by assuming that $\theta_2(\tau_t, s_{t-1}) = \tau_t^{-\alpha}\cdot s_{t-1}^{-\gamma}$, where, $\alpha, \gamma > 0$.I assume that family background of is lognormally distributed as $s_{t-1}w_t \sim \Lambda\left(\mu_{s_t-1}, \sigma_{s_{t-1}}^2\right)$.The rest of the specifications are as in the previous subsection.

The joint pdf of (τ_t, s_{t-1}) is given by

$$f_{(\tau_t,s_{t-1})}(\tau_t, s_{t-1}) = \frac{1}{2\pi\sigma_\tau\sigma_{s_{t-1}}\tau_t s_{t-1}}\exp\left\{-\frac{1}{2}\left[\frac{(\ln\tau_t - \mu_\tau)^2}{\sigma_\tau^2} + \frac{\left(\ln s_{t-1} - \mu_{s_{t-1}}\right)^2}{\sigma_{s_{t-1}}^2}\right]\right\}$$

The first order condition of the agent is

$$w_t'(s_t) = \theta_2(\tau_t, s_{t-1}) = \tau_t^{-\alpha}\cdot s_{t-1}^{-\gamma}$$

Assume as before $\theta_t'(s_t) = 1$. Notice that the above implicitly defines a transformation $(\tau_t, s_{t-1}) \longmapsto (\tau_t, s_t)$ and the Jacobian of this transformation is given by

$$\begin{aligned}
\frac{\partial(\tau_t, s_{t-1})}{\partial(\tau_t, s_t)} &= \det \begin{bmatrix} 1 & 0 \\ \frac{\partial s_{t-1}}{\partial \tau_t} & -\frac{1}{\gamma}\left[w_t'(s_t)\right]^{-\frac{\gamma+1}{\gamma}} \cdot \tau_t^{-\frac{\alpha}{\gamma}} \cdot w_t''(s_t) \end{bmatrix} \\
&= -\frac{1}{\gamma}\left[w_t'(s_t)\right]^{-\frac{\gamma+1}{\gamma}} \cdot \tau_t^{-\frac{\alpha}{\gamma}} \cdot w_t''(s_t)
\end{aligned}$$

Thus the joint pdf of (s_t, τ_t) is given by

$$f_{(s_t,\tau_t)}(s_t, \tau_t) = \frac{w_t''(s_t)}{2\pi\sigma_\tau\sigma_{s_{t-1}}\tau_t w_t'(s_t)} e^{-\frac{1}{2}\left[\frac{1}{\sigma_\tau^2}(\ln\tau_t - \mu_\tau)^2 + \frac{1}{\sigma_{s_{t-1}}^2}\left(\frac{\alpha}{\gamma}\ln\tau_t + \frac{1}{\gamma}\ln w_t'(s_t) + \mu_{s_{t-1}}\right)^2\right]}$$

The bracketed term in the above exponential can be rewritten as

$$\begin{aligned}
[\cdot] &= \frac{1}{\sigma_\tau^2}(\ln\tau_t - \mu_\tau)^2 + \frac{1}{\sigma_{s_{t-1}}^2}\left(\frac{\alpha}{\gamma}(\ln\tau_t - \mu_\tau) + \left[\frac{1}{\gamma}\ln w_t'(s_t) + \left(\mu_{s_{t-1}} + \frac{\alpha}{\gamma}\mu_\tau\right)\right]\right)^2 \\
&= \left(\frac{1}{\sigma_\tau^2} + \frac{\alpha^2}{\gamma^2\sigma_{s_{t-1}}^2}\right)(\ln\tau_t - \mu_\tau)^2 + 2\frac{\alpha}{\gamma^2\sigma_{s_{t-1}}^2}(\ln\tau_t - \mu_\tau)\cdot\left[\ln w_t'(s_t) + \left(\gamma\mu_{s_{t-1}} + \alpha\mu_\tau\right)\right] \\
&\quad + \text{a term involving } w_t'(s_t) \text{ but not } \tau_t \\
&= \frac{\left[\ln\tau_t - \left([1-\alpha\beta^*]\mu_\tau - \gamma\beta^*\mu_{s_{t-1}} - \beta^*\ln w_t'(s_t)\right)\right]^2}{\sigma^{*2}} \\
&\quad + \text{a term involving } w_t'(s_t) \text{ but not } \tau_t
\end{aligned}$$

where

$$\beta^* = \frac{\alpha\sigma_\tau^2}{\gamma^2\sigma_{s_{t-1}}^2 + \alpha^2\sigma_\tau^2}, \quad \text{and} \quad \sigma^{*2} = \frac{\gamma^2\sigma_\tau^2\sigma_{s_{t-1}}^2}{\gamma^2\sigma_{s_{t-1}}^2 + \alpha^2\sigma_\tau^2}$$

Hence the conditional pdf of $\tau_t|s_t$ is given by

$$f_{\tau_t|s_t}(\tau_t) = \frac{1}{\sqrt{2\pi}\sigma^*\tau_t}\exp\left\{-\frac{1}{2}\frac{\left[\ln\tau_t - \left([1-\alpha\beta^*]\mu_\tau - \gamma\beta^*\mu_{s_{t-1}} - \beta^*\ln w_t'(s_t)\right)\right]^2}{\sigma^{*2}}\right\}$$

which is a log-normal distribution.

In this case, we have

$$\begin{aligned}
w_t(s_t) &= \int e(s_t, \tau_t) f_{\tau_t|s_t}(\tau_t)\, d\tau_t \\
&= s_t^\rho . \exp\left\{\left([1-\alpha\beta^*]\mu_\tau - \gamma\beta^*\mu_{s_{t-1}} - \beta^*\ln w_t'(s_t)\right) + \sigma^{*2}/2\right\} \\
&= s_t^\rho \exp\left\{d - \beta^*\ln w_t'(s_t)\right\}, \text{ where } d = [1-\alpha\beta^*]\mu_\tau - \gamma\beta^*\mu_{s_{t-1}} + \sigma^{*2}/2
\end{aligned}$$

from which we have

$$w_t'(s_t) = \left[\frac{s_t^{\rho}\widetilde{\mu}}{w_t(s_t)}\right]^{1/\beta^*}, \text{ where } \tilde{\mu} = \exp(d) = \exp\left([1-\alpha\beta^*]\mu_\tau - \gamma\beta^*\mu_{s_{t-1}} + \sigma^{*2}/2\right)$$

A general solution of this differential equation is given by

$$w_t(s_t) = \left[c + \frac{1+\beta^*}{\rho+\beta^*}\widetilde{\mu}^{1/\beta^*} s_t^{(\rho+\beta^*)/\beta^*}\right]^{\beta^*/(1+\beta^*)}$$

where, c is a constant of integration. The above is a one parameter family, each c represents a signaling equilibrium with an associated self-fulfilling employer expectations regarding the relationship between education level and productivity level.[6] Using the same initial condition $w_t(0) = 0$ as in the previous subsection, we have $c = 0$. Thus, equilibrium wage function is given by,

$$w_t(s_t) = \widetilde{\mu}^{1/(1+\beta^*)}\left[\frac{1+\beta^*}{\rho+\beta^*}\right]^{\beta^*/(1+\beta^*)} s_t^{(\rho+\beta^*)/(1+\beta^*)} \tag{11}$$

We want to find the equilibrium income distribution, i.e., the distribution of s_t, and invariant distribution for (s_t, τ_t) and the long-run growth rate.

To find the equilibrium distribution of s_t, let us denote by $z = w_t'(s_t)$. Notice that $z = \tau_t^{-\alpha} \cdot s_{t-1}^{-\gamma}$. Thus we know that $z \sim \Lambda\left(-\alpha\mu_\tau - \gamma\mu_{s_{t-1}}, \alpha^2\sigma_\tau^2 + \gamma^2\sigma_{s_{t-1}}^2\right)$. Under the assumption that $c = 0$, we have

$$w_t'(s_t) = K \cdot s_t^{(\rho-1)/(1+\beta^*)}, \text{ where } K = \left(\tilde{\mu}\frac{\rho+\beta^*}{1+\beta^*}\right)^{1/(1+\beta^*)} \cdot \widetilde{\mu}^{1/(1+\beta^*)}$$

Hence, we have $\ln s_t = \frac{\ln\left(\tilde{\mu}\frac{\rho+\beta^*}{1+\beta^*}\right)}{1-\rho} + \frac{(1+\beta^*)}{1-\rho}(\alpha\ln\tau + \gamma\ln s_{t-1})$. Hence we have that

$$s_t \sim \Lambda\left(\left[\frac{\ln\left(\tilde{\mu}\frac{\rho+\beta^*}{1+\beta^*}\right)}{1-\rho} + \frac{(1+\beta^*)}{1-\rho}\left(\alpha\mu_\tau + \gamma\mu_{s_{t-1}}\right)\right], \frac{(1+\beta^*)^2}{(\rho-1)^2}\cdot\left[\alpha^2\sigma_\tau^2 + \gamma^2\sigma_{s_{t-1}}^2\right]\right) \tag{12}$$

and

$$w_t s \sim \Lambda\left(\mu_w, \left[\frac{\rho+\beta^*}{1+\beta^*}\cdot\frac{(1+\beta^*)}{1-\rho}\right]^2\left[\alpha^2\sigma_\tau^2 + \gamma^2\sigma_{s_{t-1}}^2\right]\right) \tag{13}$$

For this economy, the Gini-coefficient for schooling inequality in period t is

$$\begin{aligned} G_{s_t}' &= 2\Phi\left(\frac{1}{\sqrt{2}}\frac{1+\beta^*}{1-\rho}\sqrt{\alpha^2\sigma_\tau^2 + \gamma^2\sigma_{s_{t-1}}^2}\right) - 1. \\ G_{w_t}' &= 2\Phi\left(\frac{1}{\sqrt{2}}\frac{\rho+\beta^*}{1+\beta^*}\cdot\frac{1+\beta^*}{1-\rho}\sqrt{\alpha^2\sigma_\tau^2 + \gamma^2\sigma_{s_{t-1}}^2}\right) - 1. \end{aligned} \tag{14}$$

[6] We should check what happens to net income for each agent $\xi_t = (\tau_t, s_{t-1})$ as c changes, and check to see if $c = 0$, gives the highest net income.

Comparing Gini coefficients for schooling level, $G'_{s_t} < G_{s_t}$ and $G'_{w_t} < G_{w_t}$ Compared to the previous case, the Gini-coefficient has two sources of variation variance of τ_t and σ_{t-1}. However, it is clear from Figure 1) that the signaling equilibrium in the second case has pooling of individuals of varying cognitive skills from various family backgrounds who chose same schooling level and thus earned the same wage. This will make the schooling inequality and earnings inequality smaller in the second case.

It is possible that when other labor market mechanisms such as quits layoffs which would break the pooling of individuals to smaller and finer sub classes and thus the inequality will be further increased and social mobility will also improve. We see in the lognormal case that when labor market mechanisms that help to lower the pooling groups in the equilibrium increases inequality and increases social mobility. I show those in the finite case in the next section.

4 Finite number of ability and schooling types

To gain further insights about the nature of the equilibrium dynamics of earnings inequality, social mobility and growth, I consider the following simple economy for much of this paper. Let $\mathcal{T} = \{1,2\}$, $S = \{1,2\}$. Assume that the number of talented workers with high education level create social productive knowledge which generate growth in earnings.

$$e(s,\tau) = \begin{cases} e_1 & \text{if } s = 1, \forall \tau \in \mathcal{T} \\ e_2 & \text{if } s = 2, \tau = 1 \\ e_3 & \text{if } s = 2, \tau = 2 \end{cases} \tag{15}$$

An interpretation of the above is that the workers with education level 1 are unskilled workers and the talent of the unskilled workers do not affect their productivity; however, higher educated talented workers have higher productivity than higher educated not-so-talented workers.

Does there exist any signaling equilibrium, and if there exists one, are there many equilibria? Is there an equal opportunity separating equilibrium? Does any of these equilibria attain maximal growth and social mobility? The answers to these questions depend on the productivity technology $e(\tau,s)$ and the cost function, $\theta(s_t,\tau_t,s_{t-1})$. I assume that the cost function $\theta(s_t,\tau_t.s_{t-1})$ satisfies the following:

$$\left.\begin{array}{c} \theta(1,\tau_t,s_{t-1}) = 0 \; \forall \tau_t, s_{t-1}, \text{ and} \\ \theta(2,2,2) < \theta(2,1,2) < (e_2 - e_1) + p\,(e_3 - e_2) < \theta(2,2,1) < \theta(2,1,1) \end{array}\right] \tag{16}$$

Signaling equilibrium 1: Suppose the employers in period t hold the following subjective probability distribution $q_t\,(e|s)$)of productivity level e given his schooling level s, which in matrix form is given by

$$[q_t\,(e|s)]_{\substack{e=e_1,e_2,e_3\\ s=1,2}} = \begin{bmatrix} 1 & 0 \\ 0 & 1-p \\ 0 & p \end{bmatrix}$$

Given the above expectations, the employer announces the following wage schedule:

$$w_t(s_t) = \begin{cases} 1 & \text{if } s_t = 1 \\ e_2.(1-p) + e_3.p & \text{if } s_t = 2 \end{cases} \quad \text{for all } t \geq 0$$

Given the above wage schedule, one can easily verify that the equilibrium schooling decisions $\sigma_t(\tau_t, s_{t-1})$ of an agent of talent type τ_t from the family background s_{t-1} is as follows:

$$\sigma_t(\tau_t, s_{t-1}) = \begin{cases} 1 & \forall \tau_t \in \mathcal{T} \; if \; s_{t-1} = 1 \\ 2 & \forall \tau_t \in \mathcal{T} \; if \; s_{t-1} = 2 \end{cases} \quad \text{for all } t \geq 0$$

It can be easily checked that given the above optimum solution, the observed conditional probability distribution of e given s_t will coincide with the anticipated one. Note that the the transition matrix associated with $\sigma_t(.)$ is the following:

$$P_t = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \forall t \geq 0$$

Thus in this economy there is no intergenerational mobility. Furthermore, the economy is in steady-state from the beginning. Thus, the number of highly talented highly educated workers $R_t = p \cdot \pi_0^2$, and hence the productivity growth rate is given by $\gamma(p\pi_0^2)$ which is strictly less than $\gamma(p)$, the maximum attainable productivity growth rate for the economy when all talented individuals from all socio-occupational groups obtain higher education.

This equilibrium is not equal opportunity separating, nor maximal growth separating type. In this equilibrium, all talent types of the children from each type of family backgrounds are pooled.

Could there be any other equilibrium for the above economy? For a certain subclass of the above economies, there is another equilibrium, which is growth enhancing separating and is Pareto superior to the above equilibrium. To see this, consider the following:

Signaling equilibrium 2: let $v_t \equiv \frac{p}{p\pi_{t-1}^1 + \pi_{t-1}^2}$. Note that $v_t > p \; \forall t \geq 1$. At $t = 1$, v_1 is known. Let us suppose that apart from the assumption (16), the cost function also satisfies the condition:

$$\theta(2,2,1) < (e_2 - e_1) + v_1\,(e_3 - e_2) < \theta(2,1,1)$$

Suppose the employer holds the following subjective probability distribution for the productivity type E_t given S_t:

$$\overline{q}_t\,(e|s) = \begin{bmatrix} 1 & 0 \\ 0 & 1-v_t \\ 0 & v_t \end{bmatrix} \quad \text{for all } t \geq 1 \tag{17}$$

According to (4), given above expectations, the employer announces the following wage schedule:

$$\overline{w}(s_t) = \begin{cases} 1 & \text{if } s_t = 1 \\ e_2.(1 - v_t) + e_3.v_t & \text{if } s_t = 2 \end{cases}$$

Given the above wage schedule, the original $\sigma_t(\tau_t, s_{t-1})$ will be optimal for all (τ_t, s_{t-1}) except for $\tau_t = 2$, $s_{t-1} = 1$, who will choose $s_t = 2$. It can be easily checked that for this optimal solution, the observed conditional probability distribution of e_t given s_t will coincide with the anticipated one in equation (**??**). Note that the transition matrix associated with this new optimal schooling decision $\overline{s}_t^*(.)$ is as follows:

$$\overline{P}_t = \begin{pmatrix} 1-p & p \\ 0 & 1 \end{pmatrix}$$

Thus in this economy there is intergenerational mobility. The proportion of population with higher education will go on increasing and the proportion of the population with lower education will go on decreasing. This process, however, cannot go on for ever, since in that case $v_t \to p$, as $t \to \infty$, which will mean that there will be some finite $t_0 > 1$ such that $v_{t_0} > \theta(2, 2, 1)$ for the first time and then on the equilibrium will switch on to the previous one with no mobility. Note, however that the new steady-state equilibrium growth rate will be $\gamma\left(\pi_{t_0}^2 \cdot p\right)$ since $\pi_{t_0}^2 > \pi_0^2$. Furthermore, the short-run growth rate up to period t_0, is higher in the second equilibrium than in the first type; and the second equilibrium is Pareto superior to the first.

Furthermore, notice that there will be a positive wage growth during all periods $t \leq t_0$, and after t_0, the source of growth is only from factor productivity growth.

Thus, in this economy there may exist multiple equilibria; which one will actually realize depends on the expectations of the employers. The question is then, how the employer's expectations are formed? We need a theory of expectations formation of the producers to select an equilibrium, and we do not pursue this theory here.

Also note that the first signaling equilibrium will be in stationary state from time $t = 1$, will produce no social mobility in any periods. The second signaling equilibrium will produce upward mobility from social class $s = 1$ to $s = 2$ up to time $t = t_0$ according to the transition matrix $\overline{P}_t$, and during this period, there will be a positive wage growth due to upward mobility; after period t_0, however, the process will revert to the mobility pattern of the first signalling equilibrium. Two equilibria, however, will produce two different long-run income distributions.

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