Pricing of Idiosyncratic Equity and Variance Risks *

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Abstract

This paper decomposes the risk premia of individual stocks into contributions from systematic and idiosyncratic risks. I introduce an affine jump-diffusion model, which accounts for both the factor structure of asset returns and that of the variance of idiosyncratic returns. The estimation is performed on a time series of returns and option prices from 2006 to 2012. I find that investors not only require compensation for the systematic movements in returns and variance, but also for non hedgeable idiosyncratic risks. For the stocks of the Dow Jones, these risks account for an average of 50% and 80% of the equity and variance risk premia, respectively. I provide a categorization of sectors based on the risk profile of their Exchange Traded Funds and highlight the high prices of idiosyncratic risks in the Energy, Financial and Consumer Discretionary sectors. Other sectors are found to be appealing alternatives for investors who are not willing to be exposed to non diversifiable risks.

JEL Classification: C38, C51, G12, G13.

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1 Introduction

It is well known that individual stocks carry a positive equity risk premium and a negative variance risk premium. The former constitutes the compensation that investors require to accept bearing the risk of asset price fluctuations. The latter represents the amount that they are willing to pay to be hedged against fluctuations in price variance. In a perfect capital market, these premia should only reflect the exposure of stocks to systematic risk. Standard asset pricing theories indeed predict that idiosyncratic risks can be diversified, and should therefore not be priced. However, the recent literature pointed to the possibility that these risks be in fact priced, which may come from violations of the assumption of perfect capital market. Structural or behavioral constraints may indeed prevent investors from holding a perfectly diversified portfolio.[1]

This paper re-examines the pricing of equity and variance risks using an integrated panel of large stocks and option data. I estimate a parametric jump-diffusion model of stock dynamics, which enables me to decompose equity and variance risk premia into systematic and idiosyncratic components, each of which can be further decomposed into parts stemming from diffusion and jump risks. Up to recently, the pricing of idiosyncratic risks had only been studied based on the sole analysis of stock returns, hence without making use of the information content of options. As a consequence, the resulting estimated measures of idiosyncratic risks were usually low frequency, typically monthly or quarterly. Including options allows me to obtain a higher frequency measure of idiosyncratic variance, but most importantly to use their information content on the aversion of investors towards different types of risks and investment horizons, and on the pricing of these various risks. I have been made aware of two papers which were written in parallel with this one, namely Boloorforoosh (2014) and Bégin, Dorion, and Gauthier (2015). These papers find results which are consistent with mine.

[1] Reasons that may prevent investors from holding a well diversified portfolio include transaction costs, taxes, selling restrictions on employee compensation plans, private information, behavioural biases in portfolio choice (Benartzi (2001), Benartzi. and Thaler (2001), Huberman (2001)) as well as preferences on the moments of returns' distribution (Langlois (2013)). Studies highlight the lack of diversification in private investors' portfolios, see, e.g., Barber and Odean (2000), Polkovnichenko (2005), Goetzmann and Kumar (2008). Theories that assume under-diversification predict that there be a positive relation between idiosyncratic risk and expected returns, see Levy (1978), Merton (1987) and Malkiel and Xu (2001). Surprisingly, Ang, Hodrick, Xing, and Zhang (2006), Brockman and Yan (2008), Ang, Hodrick, Xing, and Zhang (2009), Jiang, Xu, and Yao (2009) and Guo and Savickas (2010) find an anomalous negative relation between past idiosyncratic variance and expected returns. Bali and Cakici (2008), Huang, Liu, Rhee, and Zhang (2010) and Han and Lesmond (2011) however dispute the robustness of this relation. Fama and MacBeth (1973), Bali, Cakici, Yan, and Zhang (2005) and Fink, Fink, and He (2012) find that idiosyncratic volatility is not a priced risk factor. In contrast, Spiegel and Wang (2005) and Fu (2009) find a positive relation between expected idiosyncratic variance and expected returns.

However, they do not account for the factor structure of idiosyncratic variances, which is found to be a crucial feature of the model. I further explore the link to sectors and market frictions, which has, up to my knowledge, not been done so far.

Over the period from 2006 until 2012, and for the stocks of the Dow Jones Industrial Average Index, I find that there is idiosyncratic risk priced in individual assets, with magnitude increasing in the variance of idiosyncratic returns. The idiosyncratic component of the equity premium accounts for 50% of the total premium on average. Whereas the total equity risk premium has an upward sloping term structure, which changes sign in times of high market volatility, its idiosyncratic component is found to have a decreasing term structure on average, suggesting that idiosyncratic equity risk is mainly priced in short-term investments. Interestingly, the contribution of tail risk to the idiosyncratic equity risk premium amounts to 34% on average. The role of priced idiosyncratic risk is even stronger in the variance risk premium, and represents on average 80% of the total premium. However, the idiosyncratic component of variance risk premia exhibits a term structure similar to that of the total premia, and becomes more negative as the horizon increases. Idiosyncratic variance risk is therefore persistently priced, over short- and long-term horizons. The idiosyncratic component of the variance risk premium can be further decomposed into a part which is due to the common movements in variance and another part due to the residual idiosyncratic variance. I find that the first term carries a positive risk premium, with an upward sloping term structure. In contrast, the second term carries a dominant negative risk premium, which decreases with the time-to-maturity of the investment. I relate the results obtained to arguments based on demand pressure and constraints borne by financial intermediaries.

These results are obtained using a parametric approach to model the different components impacting the evolution of asset returns. I introduce an jump-diffusion model that is able to reproduce the factor structure of returns. The model is an extension of those of Collin-Dufresne, Goldstein, and Yang (2012) and Christoffersen, Fournier, and Jacobs (2013). My framework assumes that individual stock returns' dynamics consist of a systematic component which is function of the movements of a market factor, and an idiosyncratic component that solely depends on the firm's characteristics. As risk premia strongly depend on higher order moments of the underlying returns, using jumps in the returns and variance process accommodates for the rich stylized facts of the empirical and risk-neutral distributions of the underlying stocks and market factor. Each stock can have a different exposure to the diffusive movements and jumps of the market factor. The model enables disentangling and analyzing their respective impact. The variances of idiosyncratic components of returns are referred to as idiosyncratic variances. I account for the results of Schürhoff and Ziegler (2011) and Herskovic, Kelly, Lustig, and van Nieuwerburgh (2015), who surprisingly observe comovements of idiosyncratic variances. This suggests a strong factor structure of idiosyncratic variances, which Herskovic, Kelly, Lustig, and van Nieuwerburgh (2015) link to the risk faced by households. To represent this phenomenon, I allow idiosyncratic variances to be linearly related to the market variance. The remaining component, referred to as the residual idiosyncratic variance of an asset, is specific to the firm and independent of the market variance. The model belongs to the affine class of Duffie, Pan, and Singleton (2000). It is thus tractable and enables estimation to a cross-section of stocks' returns and option prices over a relatively long time-series of data.

Based on particle filtering techniques, the estimation reconciles the vast information present in the data in order to quantify each component of the asset returns' dynamics, and better understand which risks are priced. High-frequency returns and deep out-of-the-money options with short times-to-maturity permit identifying the jumps occurrences and the moments of their magnitude. Options with longer maturities allow me to make inferences on the properties of the term structure of risk premia. A two-step estimation allows for separating the impact of the market factor from the idiosyncratic movements of stocks. The estimation procedure is based on the Auxiliary Particle Filter of Pitt and Shephard (1999) which, combined with a Maximum Likelihood estimation, yields time-consistent parameters as well as estimates of the distributions of unobservable processes and their jumps over time. The estimation period is from 2006 to 2009. Because it covers the financial crisis, it contains valuable information on jumps, as these typically do not occur frequently and are hard to detect empirically. The subsequent period, from 2010 to 2012, is used to assess the model's out-of-sample performance.

I show that the model is able to capture stylized facts that have been non-parametrically highlighted in the literature. In particular, estimation results confirm the relationship between the beta parameters and the risk-neutral variance of returns. Furthermore, an analysis of pricing errors confirms that the model reasonably fits option prices. I test the performance of the model to reproduce variance swap rates. I synthesize swaps from available options and regress the resulting rates on model-implied rates for all stocks considered. Cross-sectional (Fama-MacBeth) and panel regressions indicate that the model accurately represents the synthesized swaps. I further assess its reliability by evaluating how well it forecasts future variance swap rates. I compare the predictions it delivers to martingale predictions, whose best guess of the future rate is the current rate. Panel regressions reveal that the model significantly outperforms the martingale predictions for all future times considered.

Once the reliability of the model is shown, I perform a thorough analysis of the different risk sources in the variances of stock returns. Based on the estimated trajectories of the variance processes, I find that the idiosyncratic variance amounts to 63% of the total variance on average. The total variance is mainly governed by its idiosyncratic component in times of market calm. The market impact becomes substantial during turmoil periods. Idiosyncratic variances exhibit a strong factor structure that is similar to the one of the total variance, the first component of a Principal Component Analysis accounting for 81% of the variation. This justifies the decomposition of the dynamics of idiosyncratic variance into common movements and residual idiosyncratic movements.

Knowledge of idiosyncratic variance is important because it can guide portfolio allocation. I point out that an investment over the period 2006-2012 in a dynamically rebalanced portfolio of stocks which have an idiosyncratic variance level in the highest quartile yields a Sharpe ratio of 3.41, against -2.00 when one buys the stocks whose idiosyncratic variance is in the lowest quartile. This result is robust to different measures of idiosyncratic variance and confirms that idiosyncratic variance contains information on future returns.

I perform the same analysis on a different test asset, considering sectors instead of single stocks. An investigation of sector Exchange Traded Funds reveals that three sectors contain particularly high levels of non hedgeable risk. The idiosyncratic component of the equity and variance risk premia of the Financial, Energy and Consumer Discretionary sectors reaches 20 to 30% and includes substantial tail risk. The jump part of the idiosyncratic equity risk premia represents 40% and 43%, respectively, of the total one-month equity risk premium for the Energy and Financial sectors. The Industrials, Health Care and Materials sectors contain medium priced idiosyncratic equity and variance risk (below 5% during calm times and 15% during the crisis). The Technology and Consumer Staples sectors only contain negligible priced equity and variance risk (below 5% in calm times and 10% during the crisis). A reason for this is that they have lower idiosyncratic variance than these components, due to a diversification effect. Therefore, they provide an interesting investment alternative for agents who are not willing to be exposed to non diversifiable return risk.

Many of the papers investigating the equity and variance risk premia rely on non-parametric approaches. The main advantage of these methods is that their results are model independent. However they must overcome the fact that risk premia are not observable using approximations and proxies. For example, the expectation of the future integrated variance under the empirical measure is often approximated by the realized variance over the past days or weeks. The model I propose aims to represent the stylized facts highlighted empirically, and enables estimating risk premia and their components analytically.

The paper which is closest to mine is that of Schürhoff and Ziegler (2011), who estimate the systematic and idiosyncratic variance risk premia of single stocks using a very innovative approach based on synthesized variance swaps. They assume that the idiosyncratic variance risk premium is solely driven by the common movements in the variance process. The main difference between their paper and mine comes from the methodology. Whereas they use variance swaps with one-month time-to-maturity to summarize the information contents of options, I use all the available options without transformation. This provides me with more accurate information on the term structure of variance and on jumps which, as I show, account for a substantial part of the variance risk premia. Furthermore, they estimate quarterly premia, whereas my approach delivers daily estimates, and can therefore be used for tasks which require being more responsive to swift changes in the market.

The remainder of this paper is structured as follows. Section 2 presents a preliminary data analysis. The model description and its properties are given in Section 3. Section 4 details the estimation procedure. Section 5 investigates the behaviour of idiosyncratic variance for individual stocks and sectors. Finally, Section 6 examines the estimated equity and variance risk premia and presents inferences made on the pricing of idiosyncratic risks.

2 Preliminary data analysis

2.1 Data

The single stocks examined in this paper are the components of the Dow Jones Industrial Average index. Visa was excluded from the sample as it completed its initial public offering in March 2008. For every stock in the sample, the dataset includes daily dividend-adjusted prices obtained from CSRP, five-minute trading prices from the TAQ database as well option prices from OptionMetrics. The sampling period of high-frequency prices has been chosen to avoid adverse effects of microstructure noise while keeping a frequency which is high enough to make inference on the properties of jumps. All data span the time period from the 1st of March 2006 until the end of 2012. I eliminate options with missing bid/ask prices or zero bid prices, as well as options which have a negative bid-ask spread. I further remove options with zero open interest and those that violate the arbitrage conditions: $C(t, T, K) < S_t$ and $C(t, T, K) > S_t - Ke^{-r(T-t)}$, where $C(t, T, K)$ is a price at time t of a call option with maturity T and strike K, S_t is the price process of its underlying and r is the risk-free interest-rate. To account for the American feature of options, I follow Broadie, Chernov, and Johannes (2007) and calculate American volatilities implied from a binomial tree for all mid prices, and then treat the options as European, with an implied volatility equal to the one found. The error due to this adjustment is negligible for short-maturity options which have an early exercise premium close to zero, and increases with the maturity of options. To avoid beginning- and end-of-week effects, I follow standard practice and only keep Wednesday option prices. Table 1 summarizes the available stocks, provide their acronyms and gives the minimum and maximum moneyness of options on each stock. Maturities range from one week to one year.

The second set of test data I use consists of SPDR Exchange traded funds (ETF). They were introduced in 1998 and track the performance of the Global Industry Classification Standard sectors of the S&P 500. Every fund is a portfolio of the components of the index in the corresponding sector. American options started trading on the funds and realized a fast-growing success. A summary of the data available on sector ETFs is provided in Table 2. All options are of American type. The dataset is treated similarly to the options on single stocks.

The S&P 500 combines the market capitalizations of the 500 largest US stocks, and therefore is a natural proxy for the market factor in the US market. It is complemented by the VIX index, which represents the market expectation of the future volatility of S&P 500 returns over the next thirty days, and is often considered as a fear gauge of the markets. Options on both indices are liquidly traded. Options on the S&P 500 have a maturity that ranges from four days to a year, and a moneyness $m = K/S$ between 0.39 and 1.37. Options on the VIX have a maturity between five days and one year, and a moneyness between 0.23 and 5.46.

2.2 Factor structure of implied volatilities

In addition to providing useful information on the individual stock returns' distributions, I show in this section that options also contain information on the factor structure of stocks.

I analyze the factor structure of implied volatility (IV) surfaces using Functional Principal Component Analysis (FPCA). While standard PCA decomposes the covariance matrix of a set of time-series to identify the main vectors of variation, FPCA performs a similar task with time-series of surfaces.[2] The detailed procedure is described in Appendix A.

I construct for every Wednesday in the sample a smooth estimator of the implied volatility surface $IV_t(m, \tau)$ over a grid of moneynesses $m \in [m_{min}, m_{max}]$ and times-to-maturity $\tau \in [\tau_{min}, \tau_{max}]$ using available data, and a non-parametric Nadaraya-Watson estimator as detailed in Härdle (1992) and Aït-Sahalia and Lo (1998). The outputs of the FPCA are a set of eigenmodes that represent the main directions of variation of the surface, and principal component processes that are the projection of the surface at every point in time onto the eigenmodes.

The eigenmodes and principal component processes that result from the FPCA of the S&P 500 index are represented in Appendix A, in Figures 13 and 14. The first eigenmode represents the variation coming from the level of the IV surface, i.e. the upwards and downwards shifts of the entire surface. This mode is not completely flat, there is a slight upward tilt towards short maturities and smaller moneynesses and a slight downward tilt towards longer maturities and larger moneynesses. Furthermore, the mode is slightly decreasing as maturity and moneyness increase. This reflects the fact that deep out-of-the-money (OTM) put IVs tend to move more and deep OTM call IVs less than other IVs. The second mode is a skew mode. OTM put IVs move in opposite directions from OTM call IVs. The mode changes sign at the ATM level. Overall magnitudes tend to decrease as the time-to-maturity increases. The third mode is a term structure mode. Short-term IVs move in the opposite direction from long-term IVs. The magnitude tends to decrease as the moneyness increases. Finally, the fourth mode is a convexity mode. Deep OTM put and call IVs tend to fluctuate more than ATM option IVs. These findings are consistent with those of Cont and da Fonsecca (2002).

[2] Similar analyses have been applied to indices by Skiadopoulos, Hodges, and Clewlow (2000), Cont and da Fonsecca (2002), Cont, da Fonsecca, and Durrleman (2002), Daglish, Hull, and Suo (2007). Furthermore, Christoffersen, Fournier, and Jacobs (2013) decompose the IV surface of single stocks into an affine function of moneyness and time-to-maturity. Standard PCA is then applied to the time-series of loadings. This procedure is a sub-case of the one I apply, in which the first direction of variation of the IV is affine in the moneyness and the second is proportional to the time-to-maturity. I allow eigenmodes to be polynomial functions of both moneyness and maturity.

They find that the third mode is the convexity mode, instead of the term structure mode. But this is likely due to the fact that they use a shorter time-series of data.

The same analysis is conducted on IV surfaces of single stocks. The resulting modes as well as the percentages of variance they explain are described in Table 3. The first three eigenmodes are usually the same as those found for the S&P 500. In some cases the term structure mode explains more variation than the skewness mode. Overall, the first mode explains a large percentage of the variation of the surfaces, on average 97.5%. The corresponding principal component process is highly correlated with the one of the S&P 500 surface decomposition, with a correlation of 91.6% on average. The second and third eigenmodes are also significantly correlated with the corresponding eigenmodes of the S&P 500 decomposition, with respective average coefficients of 51.6% and 42.6%.

Functional PCAs therefore indicate a strong factor structure of the main driver of IV surfaces, i.e., the level of the surface. The other drivers, which roughly represent the skewness and term structure of the surfaces, also exhibit a large positive correlation with the corresponding market principal components. However, I interpret the fact that this correlation is always substantially lower than 1 as an indication of firm specific variation in IV. These results are consistent with those of Christoffersen, Fournier, and Jacobs (2013).

2.3 Factor structure of high-frequency returns

High-frequency data contain valuable information on the fine structure of data. In particular, diverse measures of variation of log-price increments have been introduced that enable disentangling the contribution of the diffusive and jump components of the dynamics of returns.[3] In this section, I use high-frequency data to analyze the potential factor structure of some of these measures.[4]

For each stock I calculate the daily Realized Variance (RV), which captures the total variation of high-frequency returns over a day, and can be decomposed into the Continuous Variation (CV), which represents the variation due to continuous movements of the returns, and the Jump Variation (JV) which captures discontinuous movements of returns. For definitions and convergence properties, see,

[3] See for example Andersen, Bollerslev, Diebold, and Labys (2003), Barndorff-Nielsen and Shephard (2004) and Barndorff-Nielsen and Shephard (2006)

[4] Luciani and Veredas (2012) and Barigozzi, Brownlees, Gallo, and Veredas (2014) introduce a dynamic factor model for realized volatilities.

e.g., Bollerslev, Todorov, and Li (2013). I apply a PCA to these measures.[5] The same analysis is performed after taking their n-day moving averages, with $n \in [3\ 180]$.

The percentage of variance explained by the first principal component as a function of n is displayed in Figure 1. Using a larger n leads to the first component explaining a larger percentage of the variance of all measures, with that percentage being very close to 100% when the 6-month moving average is considered, except for the jump variation measure. Along the same line, a smaller number of factors are needed to explain the total amount of variation when n increases. I interpret this result as an indication that the factor structure explains the overall tendencies of variation measures, whereas the daily fluctations are mainly caused by firm-specific events. The much smaller percentage of variation of the jump measure explained by the first components indicates that most jumps are idiosyncratic, and that the factor structure of the diffusive movements is much stronger than the one of jumps. These results are in line with Bollerslev, Todorov, and Li (2013), who find that the number of filtered idiosyncratic jumps in stocks that belong to the S&P 500 exceeds the number of systematic jumps for all the stocks contained in their sample.

I propose to represent the factor structure of asset returns using a parametric modeling approach. Most studies using high-frequency data or option prices use non-parametric or semi-parametric approaches, which present more flexibility in analyzing the data and do not face the challenge of estimation. However, with such approaches it is only possible to draw inferences, e.g., they cannot be used for out-of-sample exercises, which are important parts of this paper.

3 Model

3.1 Model setup

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ be a filtered probability space satisfying the usual assumptions, where $\mathbb{P}$ denotes the empirical measure. I denote by $M = (M_t)_{t\geq 0}$ the value process of the market factor. Its dynamics under $\mathbb{P}$ follow an extension of the Heston model with jumps. They are specified as follows:

[5] A log-transformation is applied to the RV.

$$\frac{dM_t}{M_{t^-}} = (r_t - \lambda^M(v_{t^-}^M, m_t)\mathbb{E}^{\mathbb{P}}[e^{Z_t^M} - 1] + \gamma_t^M(v_{t^-}^M, m_t))dt + \sqrt{v_{t^-}^M}dW_t^{M(\mathbb{P})} + (e^{Z_t^M} - 1)dN_t^{M(\mathbb{P})}. \quad (1)$$

The risk-free rate r_t is assumed to be a deterministic function of time. The function $\gamma_t^M(v_{t^-}^M, m_t)$ represents the market equity risk premium (ERP). It represents the instantaneous return that the investor receives in addition to the risk-free rate for bearing the risk in the market factor. It depends on the specification of the pricing kernel. Price returns are driven by a Brownian motion $W_t^{M(\mathbb{P})}$ and a Poisson process $N_t^{M(\mathbb{P})}$. The variance of the diffusive component of returns is driven by two factors:

$$dv_t^M = \kappa_v^{M(\mathbb{P})}\left(\frac{\kappa_v^M}{\kappa_v^{M(\mathbb{P})}}m_t - v_{t^-}^m\right)dt + \sigma_v^M\sqrt{v_{t^-}^M}dB_t^{M(\mathbb{P})} + y_t^{M(\mathbb{P})}dN_t^{M(\mathbb{P})} \quad (2)$$

$$dm_t = \kappa_m^{M(\mathbb{P})}(\theta^{M(\mathbb{P})} - m_t)dt + \sigma_m^M\sqrt{m_t}d\bar{B}_t^{M(\mathbb{P})}. \quad (3)$$

The first variance process $(v_t^M)_{t\geq 0}$ mean-reverts towards a stochastic central tendency $(m_t)_{t\geq 0}$, up to a scaling factor. Its mean-reversion speed is controlled by $\kappa_v^{M(\mathbb{P})}$. The parameter κ_v^M corresponds to the speed of mean-reversion of the variance under an equivalent martingale measure $\mathbb{Q}$. The stochastic central tendency mean-reverts itself towards a fixed long-run mean $\theta^{M(\mathbb{P})}$, with speed $\kappa_m^{M(\mathbb{P})}$. Jumps in the returns process occur simulateneously with jumps in the first variance factor v_t^M, with an intensity that is affine in the two variance processes: $\lambda^M(v_{t^-}^M, m_t) = \lambda_0^M + \lambda_1^M v_{t^-}^M + \lambda_2^M m_t$.[6] The sizes of jumps in the returns are assumed to be independent and identically distributed following a normal distribution. Jumps in the variance are assumed to be exponentially distributed.[7] This implies that the variance can only jump upwards and prevents it from becoming negative. The leverage effect that impacts the market factor is represented by the correlation coefficient ρ^M defined such that $d\langle W^{M(\mathbb{P})}, B^{M(\mathbb{P})}\rangle_t = \rho^M dt$.

The model for individual stocks is a continuous-time extension of the CAPM.[8] I separate the impact of

[6] Based on a non-parametric study, Bollerslev and Todorov (2011) argue in favor of time-varying jump intensities and a tendency for fatter tails in period of high overall volatility.

[7] Amengual and Xiu (2014) show that it is more realistic to also include negative jumps in the variance. The assumption of exponential jumps aims to ensure that the variance remains positive.

[8] The model is related to that of Collin-Dufresne, Goldstein, and Yang (2012), but additionally features idiosyncratic jumps and stochastic volatility. The framework which is closest to mine is the one of Christoffersen, Fournier, and Jacobs (2013), who calibrate a diffusive model to a cross-section of stock returns and option prices. There are some major differences however between their work and mine. First, they assume that the idiosyncratic variance is independent from

the market factor into two contributions. On one side the diffusive random variation of single stocks' returns is dependent on the Brownian motion that drives market returns through the coefficient β^j_{diff}. On the other side discontinuous movements in the market returns can also translate into jumps in single stocks' returns, though the coefficient β^j_{jump}. A jump of size Z^M_t in the market log level triggers a jump of $\beta_{jump} Z^M_t$ in the individual log stock price.[9] This allows disentangling the effects of both market drivers on single stocks' returns. Under $\mathbb{P}$, the dynamics of firm j's dividend-adjusted stock prices S^j_t are given by:

$$\frac{dS^j_t}{S^j_{t^-}} = (r_t + \gamma^j_t(v^j_{t^-}, v^M_{t^-}, m_t))dt + dR^{sys}_t + dR^{idio}_t, \tag{4}$$

where the function $\gamma^j_t(v^j_{t^-}, v^M_{t^-}, m_t)$ represents stock's k equity risk premium. It is the instantaneous return that the investor receives in addition to the risk-free rate in exchange for bearing the risk in the stock. dR^{sys}_t denotes the increment of the systematic martingale return, and dR^{idio}_t is the increment of the idiosyncratic martingale return:

$$dR^{sys}_t = \overbrace{\beta^j_{diff}\sqrt{v^M_{t^-}}dW^{M(\mathbb{P})}_t}^{\text{Systematic diffusive returns}} + \overbrace{(e^{\beta^j_{jump}Z^M_t} - 1)dN^{M(\mathbb{P})}_t - \lambda^M(v^M_{t^-}, m_t)\mathbb{E}^{\mathbb{P}}[e^{\beta^j_{jump}Z^M_t} - 1]dt}^{\text{Systematic discontinuous returns}}, \tag{5}$$

$$dR^{idio}_t = \overbrace{\sqrt{v^j_{t^-}}dW^{j(\mathbb{P})}_t}^{\text{Idiosyncratic diffusive returns}} + \overbrace{(e^{Z^j_t} - 1)dN^{j(\mathbb{P})}_t - \lambda^j(v^j_{t^-})\mathbb{E}^{\mathbb{P}}[e^{Z^j_t} - 1]dt}^{\text{Idiosyncratic discontinuous returns}}. \tag{6}$$

The variance of the idiosyncratic diffusive returns, later referred to as idiosyncratic variance, has the

the market variance. An examination of the idiosyncratic variances of stocks suggests that a richer factor structure would be needed, as further advocated by Serban, Lehoczky, and Seppi (2008), Schürhoff and Ziegler (2011) and Herskovic, Kelly, Lustig, and van Nieuwerburgh (2015). My model accounts for this and decomposes idiosyncratic variance movements into a common and a residual idiosyncratic component. Second, they assume that idiosyncratic variance risk is not priced. In other words, the variance risk premium of single stocks is assumed to be zero. Although it has been shown to be smaller in magnitude than the variance risk premium of the index, there is empirical evidence for a non-zero variance risk premium in individual stocks. Third, they estimate the trajectory of the unobservable spot variance of the index and of single stocks using a least-square algorithm, i.e., its dynamics are not ensured to be consistent with the specified model. The use of Bayesian methods allows me to have an estimation which guarantees that the filtered trajectory of the spot variance is consistent with the model specification. Last, their model does not include jumps, which have been shown to be important, especially during the financial crisis, to fit option prices, see, e.g., Bakshi, Cao, and Zhong (2012).

[9] Based on a model selection exercise, Bakshi, Cao, and Zhong (2012) propose to use a model with jumps in the returns and in the volatility of individual stocks.

following dynamics:

$$dv_t^j = \underbrace{\beta_v^j dv_t^M}_{\text{Common idiosyncratic variance movements}} \underbrace{+ \kappa_v^{j(\mathbb{P})} \left(\frac{\kappa_v^j}{\kappa_v^{j(\mathbb{P})}} \theta^j - v_{t^-}^j \right) dt + \sigma_v^j \sqrt{v_{t^-}^j} dB_t^{j(\mathbb{P})} + y_t^j dN_t^{j(\mathbb{P})}}_{\text{Residual idiosyncratic variance movements}}. \tag{7}$$

The leverage effect for individual stocks is accounted for: $d\langle W^{j(\mathbb{P})}, B^{j(\mathbb{P})} \rangle_t = \rho^j dt$. In line with the preliminary data analysis, I assume that idiosyncratic returns and variance can jump. As for the market factor, jumps occur in both processes simultaneously. They are driven by the Poisson process N_t^j, whose intensity is affine in the idiosyncratic variance: $\lambda^j(v_{t^-}^j) = \lambda_0^j + \lambda_1^j v_{t^-}^j$. The sizes of jumps in the returns are assumed to be normally distributed with mean $\mu^{j(\mathbb{P})}$ and variance $(\sigma^{j(\mathbb{P})})^2$. Jumps in the variance follow an exponential distribution with mean $\nu^{j(\mathbb{P})}$.

The model allows for two new features. On the one hand, it disentangles the respective impacts of the diffusive and jump components of the market factor on individual stocks. Such differentiation is justified by the notion of crash aversion introduced by Bates (2008), who argues that the investors treat jump and volatility risks differently. For example, an investor may want to hedge tail risk, in which case he will find stocks with a larger β_{jump}^j more attractive. Both volatility and jump risks are priced in the cross-section of returns, as has been shown by Cremers, Halling, and Weinbaum (2013).

On the other hand, my model allows the idiosyncratic variance of individual stocks to be correlated with the market variance. Schürhoff and Ziegler (2011) and Herskovic, Kelly, Lustig, and van Nieuwerburgh (2015) show that the idiosyncratic variance of individual stocks can be decomposed into a residual idiosyncratic component that is firm-specific and a common idiosyncratic component, which Herskovic, Kelly, Lustig, and van Nieuwerburgh (2015) identify as being linked to household income risk. To represent this factor structure of idiosyncratic variance, I divide their change over an infinitesimal period of time into two parts. The first one, $\beta_v^j dv_t^M$, measures the correlation of a common factor, which is chosen to be the market variance, with the idiosyncratic variance. The coefficient β_v^j serves as scaling factor, and quantifies the exposure of stock j's idiosyncratic variance to the market variance. The residual component of the idioyncratic variance is driven by a firm-specific Brownian motion $B_t^{j(\mathbb{P})}$ and a Poisson process $N_t^{j(\mathbb{P})}$. Its drivers are assumed to be independent from the drivers of the market variance. Such representation preserves the affine structure of the model.

The model can be easily extended to multiple factors driving underlying stocks' returns. Several factors have been proposed in the asset pricing literature. The size and value factors of the popular Fama-French three-factor model account for the extra risk present in small companies' stocks and value stocks. However, adding factors in the proposed setup requires additional parametric assumptions on their dynamics, and involves further model and estimation error. Finally, traded derivatives' prices would be needed to estimate the risk premia of these factors. Such generalization would therefore be challenging to apply in practice.

One limitation of my model is that the beta coefficients are assumed to be constant in order to preserve the affine structure of the setup. This assumption is, however, justified by the time-series of data considered. Adrian and Franzoni (2009) show that the beta in its traditional sense varies over time when looking at a long time-series of data, typically including several decades. The advantage of such exercise is to dispose of a sufficient amount of data points. However, various issues such as the non-stationarity of the data may arise in this case. As I work with options, the amount of available data is already very large and is sufficient to draw inferences for the recent years covering the financial crisis, while keeping the betas constant over this time horizon.

3.2 Risk premia specification

Due to incompleteness of the market, the risk-neutral measure is not unique. Following standard practice, I parameterize the pricing kernel L_t defined in equation (8), which is then calibrated to the market. The corresponding risk-neutral measure is denoted by $\mathbb{Q}$. L_t can be decomposed into two components which respectively contain the market prices of risk of the diffusive and jump processes that play a role in the dynamics of assets returns:

$$L_t = e^{-rt} \left.\frac{d\mathbb{Q}}{d\mathbb{P}}\right|_{\mathcal{F}_t} = e^{-rt} L_t^{diff} L_t^{jump}. \tag{8}$$

The diffusive component of the pricing kernel is equal to

$$L_t^{diff} = \exp\left(\int_0^t \mathbf{\Lambda}_\mathbf{s}' d\mathbf{W}_\mathbf{s} - \frac{1}{2}\int_0^t \mathbf{\Lambda}_\mathbf{s}'\mathbf{\Lambda}_\mathbf{s} ds\right)$$

where $\mathbf{W_t}$ is the vector of diffusive random processes that drive asset returns at time t and $\mathbf{\Lambda_t}$ is the vector of premia attached to every element of $\mathbf{W_t}$, assumed to be proportional to the volatility levels:

$$\begin{aligned}\mathbf{W_t'} &= \left(\; W_t^{M(\mathbb{P})}, \quad B_t^{M(\mathbb{P})}, \quad \bar{B}_t^{M(\mathbb{P})}, \quad W_t^{j(\mathbb{P})}, \quad B_t^{j(\mathbb{P})} \; \right) \\ \mathbf{\Lambda_t'} &= \left(\; \eta^M \sqrt{v_t^M}, \quad \tfrac{\kappa_v^M - \kappa_v^{M(\mathbb{P})}}{\sigma_v^M} \sqrt{v_t^M}, \quad \tfrac{\kappa_m^M - \kappa_m^{M(\mathbb{P})}}{\sigma_m^M} \sqrt{m_t^M}, \quad \eta^j \sqrt{v_t^j}, \quad \tfrac{\kappa_v^j - \kappa_v^{j(\mathbb{P})}}{\sigma_v^j} \sqrt{v_t^j} \; \right). \end{aligned} \tag{9}$$

The parameters without superscript ($\mathbb{P}$) in equation (9) either correspond to the $\mathbb{Q}$-counterpart of the $\mathbb{P}$-parameters, or are invariant under both measures. This specification allows for non-zero equity and variance risk premia in the market factor and in individual stock returns. The affine structure of the model is preserved under $\mathbb{Q}$, so that options can be efficiently priced using Fourier methods.

The jump component L_t^{jump} accounts for different mean and variance of jumps under $\mathbb{P}$ and $\mathbb{Q}$. For the sake of identification, the intensity of jumps is assumed to be equal under both measures.[10]

I make the assumption that the leverage and the beta coefficients are the same under the risk-neutral and objective measures. The latter assumption is consistent with Serban, Lehoczky, and Seppi (2008), who find estimates which are statistically close under both measures.

Under this specification of the pricing kernel, the market equity risk premium (ERP) function $\gamma_t^M(v_{t^-}^M, m_t)$ present in equation (1) is equal to the sum of a diffusive contribution that is proportional to the market variance level, and a jump contribution that reflects the different jump size distributions under the measures $\mathbb{P}$ and $\mathbb{Q}$:

$$\gamma_t^M(v_{t^-}^M, m_t) = \eta^M v_{t^-}^M + \lambda^M(v_{t^-}^M, m_t) \left(\mathbb{E}^{\mathbb{P}}[e^{Z_t^M} - 1] - \mathbb{E}^{\mathbb{Q}}[e^{Z_t^M} - 1] \right).$$

[10]The jump component of the pricing kernel is defined as:

$$\begin{aligned} L_t^{jump} = & \prod_{n=1}^{N_t^{M(\mathbb{P})}} \frac{\phi^{M(\mathbb{Q})}(Z_n^M)}{\phi^{M(\mathbb{P})}(Z_n^M)} \exp\left(\int_0^t \left\{ \int_{\mathbb{R}} \lambda^M(v_s^M, m_s) \left(\phi^{M(\mathbb{P})}(z) - \phi^{M(\mathbb{Q})}(z) \right) dz \right\} ds \right) \\ & \prod_{j \in \{1,\dots,J\}} \prod_{n=1}^{N_t^{j(\mathbb{P})}} \frac{\phi^{j(\mathbb{Q})}(Z_n^j)}{\phi^{j(\mathbb{P})}(Z_n^j)} \exp\left(\int_0^t \left\{ \int_{\mathbb{R}} \lambda^j(v_s^j) \left(\phi^{j(\mathbb{P})}(z) - \phi^{j(\mathbb{Q})}(z) \right) dz \right\} ds \right) \end{aligned}$$

where J is the number of individual stocks considered. The functions $\phi^M(Z_n^M)$ and $\phi^j(Z_n^j)$ refer to the normal density function, with parameters equal to the mean and variance of jump sizes in the market factor and single stock j, under the specified measure.

Similarly, the single stock's ERP function $\gamma_t^j(v_{t-}^j, v_{t-}^M, m_t)$, which is present in equation (4), can be decomposed into a diffusive and a jump contribution. Each of them is itself the sum of an idiosyncratic component and a market component, whose impacts are quantified by β_{diff}^j and β_{jump}^j:

$$\begin{aligned}\gamma_t^j(v_{t-}^j, v_{t-}^M, m_t) =& \eta^j v_{t-}^j + \beta_{diff}^j \eta^M v_{t-}^M \\ &+ \lambda^j(v_{t-}^j)\left(\mathbb{E}^{\mathbb{P}}[e^{Z_t^j} - 1] - \mathbb{E}^{\mathbb{Q}}[e^{Z_t^j} - 1]\right) + \lambda^M(v_{t-}^M, m_t)\left(\mathbb{E}^{\mathbb{P}}[e^{\beta_{jump} Z_t^M} - 1] - \mathbb{E}^{\mathbb{Q}}[e^{\beta_{jump} Z_t^M} - 1]\right).\end{aligned}$$

Under this specification of the change of measure, the dynamics of the market factor and individual returns are given in Appendix C.

3.3 Properties of the model

Due to the affine structure of the model, option prices are available in closed-form and involve the characteristic functions of returns, which are known up to the resolution of Ordinary Differential Equations (ODEs). Assuming the existence of derivatives on the market factor, the characteristic function of market returns is given in Bardgett, Gourier, and Leippold (2014). The one of single stocks' returns can be derived similarly.

Proposition 3.1. *The Laplace transform of the single stocks' log returns $\tilde{S}_t^j = \log S_t^j$ is exponential affine in the factor processes:*

$$\begin{aligned}\Psi_{\tilde{S}_T}(t, \tilde{s}_t, v_t^j, v_t^M, m_t; \omega) :=& \mathbb{E}^{\mathbb{Q}}\left[e^{\omega \tilde{S}_T} | \mathcal{F}_t\right] \\ =& e^{\alpha(T-t)\cdot \tilde{S}_t + \beta(T-t)\cdot v_t^j + \gamma(T-t)\cdot v_t^M + \chi(T-t)\cdot m_t + \eta(T-t)}\end{aligned}$$

where the coefficients α, β, γ, χ and η are functions defined on $[0, T]$ by ODEs presented in Appendix D and $\omega \in \mathbb{C}$ is chosen so that the conditional expectations are well defined.

The availability and tractability of the Laplace transforms are desirable features, as they allow using Fourier methods to calculate option prices in a fast and efficient manner.

Another important property of the model is that risk premia are available in closed-form. The integrated ERP (IERP) is defined as follows[11]:

$$IERP^j(t,T) = \frac{1}{T-t}\left(\mathbb{E}_t^{\mathbb{P}}\left[\log\frac{S_T^j}{S_t^j}\right] - \mathbb{E}_t^{\mathbb{Q}}\left[\log\frac{S_T^j}{S_t^j}\right]\right). \tag{10}$$

It represents the difference between the expected return made by an investor who buys stock j at time t and keeps it until time T, under the empirical and risk-neutral measures. It is composed of two parts. On the one hand, the diffusive part depends on the market price of idiosyncratic risk η^j and on the price of risk of the market factor, η^M. On the other hand, the jump component depends on premia attached to the mean and variance of the jump sizes.

The integrated variance risk premium (IVRP) reflects the amount that an investor would be willing to pay to be hedged against the fluctuations in an asset's variance. It is defined as follows:

$$IVRP^j(t,T) = \frac{1}{T-t}\left(\mathbb{E}_t^{\mathbb{P}}\left[QV_{[t,T]}^j\right] - \mathbb{E}_t^{\mathbb{Q}}\left[QV_{[t,T]}^j\right]\right) \tag{11}$$

where $QV_{[t,T]}$ denotes the quadratic variation of the log price process, i.e., the sum of the integrated variance of returns and the squared jumps in the time interval $[t,T]$.

As a result of the affine property of the model, the IERP and IVRP are available in closed-form and can be expressed as linear functions of the latent variance processes. Details on the calculation of the IERP and IVRP are available in Appendix E.

Finally, the model is consistent with empirical facts highlighted in the literature on the relationship between the beta and risk-neutral moments of the conditional distribution of log-returns. Duan and Wei (2009) find that firms with higher betas tend to have a high level of risk-neutral variance. In my model, the risk-neutral variance rate can be decomposed into a diffusion part $v_{t-}^j + (\beta_{diff}^j)^2 v_{t-}^M$ that increases with β_{diff}^j, and a jump part $\left[(\sigma^j)^2 + (\mu^j)^2\right]\lambda^j(v_{t-}^j) + (\beta_{jump}^j)^2\left[(\sigma^M)^2 + (\mu^M)^2\right]\lambda^M(v_{t-}^M, m_t)$ that increases with β_{jump}^j. Appendix F shows that the total risk-neutral variance of stock j over $[t,T]$ also increases in both betas.

[11] This definition is equivalent to the usual definition $IERP^j(t,T) = \frac{1}{T-t}\left(\mathbb{E}_t^{\mathbb{P}}\left[\frac{S_T^j - S_t^j}{S_t^j}\right] - \mathbb{E}_t^{\mathbb{Q}}\left[\frac{S_T^j - S_t^j}{S_t^j}\right]\right)$ up to the convexity adjustment $O\left(\mathbb{E}_t^{\mathbb{P}}\left[\left(\frac{S_T^j - S_t^j}{S_t^j}\right)^2\right] - \mathbb{E}_t^{\mathbb{Q}}\left[\left(\frac{S_T^j - S_t^j}{S_t^j}\right)^2\right]\right)$. In the following, the notation $\mathbb{E}_t[.]$ is used in place of $\mathbb{E}[.|\mathcal{F}_t]$.

Furthermore, Dennis and Mayhew (2002) and Duan and Wei (2009) find that higher beta firms have more negatively skewed risk-neutral distributions. In my model and as detailed in Appendix F, the skewness rate can be decomposed into a market effect and an idiosyncratic component. The market component increases in absolute value proportionally to $(\beta_{diff})^3$, and is negative. The idiosyncratic component is closer to zero. Therefore my model reflects the findings of Dennis and Mayhew (2002) and Duan and Wei (2009).

4 Estimation

I estimate the model using the data described in Section 2.1. The estimation period ranges from 2006 until to 2009, and therefore covers the financial crisis. This period is subsequently referred to as the in-sample period. The model performance is examined on the subsequent time period (out-of-sample period) spanning 2010 up to the end of 2012. Table 4 reports the amount of options included in the in- and out-of-sample periods for every stock. As the number of traded options tends to increase over time, for many stocks it is higher in the out-of-sample period than in the in-sample period, which is a challenge for the model.

4.1 Estimation method

I estimate the model in two steps. The first step consists in estimating the parameters of the market equations (1) - (3). I reconcile the dynamics of the S&P 500 and VIX indices following the estimation procedure outlined in Bardgett, Gourier, and Leippold (2014). The affine characteristics of the model allows me to use the Fourier Cosine method introduced by Fang and Oosterlee (2008) to calculate the prices of VIX options. My procedure is based on the Auxiliary Particle Filter of Pitt and Shephard (1999) which, combined with a Maximum Likelihood estimation, yields time-consistent parameters as well as estimates of the distributions of unobservable processes over time and their jumps.[12] Given the parameters and latent processes driving the market factor [13], the second step uses a similar algorithm to estimate the parameters and filter the processes that drive every individual

[12]This procedure allows obtaining at each point in time an estimate for the density of the latent factors, consistent with the specified stochastic differential equation (SDE). In that sense it provides a significant improvement over the typically used least-square estimation, see e.g, Christoffersen, Fournier, and Jacobs (2013), where the estimation treats the unobservable factors as parameters.

[13]At every time t, I use the empirical mean of the filtered density of these factors as point estimate.

stock. In this second step, all estimations can be run in parallel, making the procedure particularly efficient. I cast the problem in a state-space form, which consists of a transition equation and three measurement equations. The transition equation is an Euler discretization of the SDEs satisfied by the variance process of the individual stock prices under $\mathbb{P}$, on a uniform time grid of $M+1$ points $t \in \{t_0 = 0, t_1 = \Delta t, ..., t_M = M\Delta t\}, M \in \mathbb{N}^\star$ so that $M\Delta t$ corresponds to the end of the period considered. For $t \in \{0, ..., M-1\}$:

$$\Delta v_t^j = \beta_v^j \Delta v_t^M + \kappa_v^{j(\mathbb{P})} \left(\frac{\kappa_v^j}{\kappa_v^{j(\mathbb{P})}} \theta^j - v_t^j \right) \Delta t + \sigma_v^j \sqrt{v_t^j} \Delta B_t^{j(\mathbb{P})} + y_t^{j(\mathbb{P})} \Delta N_t^{j(\mathbb{P})}.$$

The measurements consist of log-returns, option prices and daily values of the RV. The first measurement equation is obtained using the Euler discretization of the dynamics of log-returns. The second matches model-implied option prices to market data. The error is defined as the relative difference between both. The measurement equation is a vector equation, whose dimension is equal to the number of options that are traded at time t. The error term is assumed to follow a normal distribution centered at 0, with a heteroskedastic variance that is exponential affine in the bid-ask spread, the log-moneyness of the option and its time-to-maturity. The last measurement equation expresses the logarithm of the variance rate as an affine function of the logarithm of the realized variance, following Wu (2011). The innovations are assumed to be normally distributed centered at zero, with variance proportional to v_t^j to account for the fact that the error made when estimating the variance tends to increase with the variance level.

The parameters driving the jump distribution under the empirical measure were found hard to estimate precisely. Therefore, they were estimated in a preliminary investigation of high-frequency returns and fixed throughout the particle filtering exercise. The algorithm used to estimate them is outlined in Appendix B. Estimated intra-day jumps were aggregated on a daily basis, so that the empirical mean and standard deviation of daily jumps could be calculated.

The outputs of the estimation are the parameters for which the likelihood of the measurements reaches a global maximum, and the filtered trajectories of the latent processes: the idiosyncratic variances, daily probability of jump occurence and the jump size.

4.2 Estimation results

The estimation performance is evaluated by examining root mean-square errors (RMSEs) and relative errors, as well as the evolution of sequential likelihoods in- and out-of-sample. RMSEs are reported in Table 5. For the sake of conciseness, the other performance measures are not reported but are available upon request. There is a slight loss of quality in the estimation of deep OTM options. This is not surprising as the tails of the risk-neutral distribution are typically hard to estimate. However, given the large amount of prices to fit, the model performs well.

Parameter estimates as well as their standard errors are displayed in Table 6. The mean size of return jumps under $\mathbb{P}$ is very close to zero, which is in line with Yan (2011). Under the risk-neutral measure, it tends to be more negative, consistent with a positive jump equity risk premium. Furthermore, the risk-neutral speed of mean reversion of the variance process is always significantly lower than its empirical counterpart, which pulls the variance risk premium towards negative values.

The beta coefficients are reported in Table 7. Stocks are grouped by industry. The diffusive beta β^j_{diff} varies between 0.450 for Procter & Gamble and 1.221 for Goldman Sachs. Companies specialized in financial services have the highest values of β^j_{diff}. As those firms were at the core of the financial crisis, this result was expected. Companies specialized in consumer staples, such as The Coca-Cola Company (KO), tend to have a relatively low exposure to diffusive market movements.

The exposure of a stock to the market is directly linked to its risk-neutral variance, as Figure 2 illustrates. As a benchmark measure of the risk-neutral variance, I use the two-month at-the-money (ATM) IV.[14] Its evolution is displayed for three stocks on the left panel of the figure, over the time period March 2006 - December 2012. The estimated diffusive betas of these stocks are respectively: 0.497 for JNJ, 0.670 for IBM and 1.221 for GS. The graph clearly indicates that the overall level of IV increases with β^j_{diff}. In the absence of jumps, my model specification implies that the risk-neutral variance is proportional to the square of β^j_{diff}. The right graph of Figure 2 represents the median of the two-month ATM IV versus the estimated value of β^j_{diff} squared across stocks. As expected, the graph indicates a monotonic relationship between the two quantities. I do not find any evidence that the risk-neutral variance is increasing in the jump exposure coefficient β^j_{jump}, which is justified by the fact that the variance is mainly driven by its continuous part.

[14] The ATM IV has been interpolated on the basis of available market data, using the biharmonic spline interpolation method of T.Sandwell (1987).

The coefficients β_v^j and β_{jump}^j are found more difficult to estimate and have larger standard errors. β_v^j is between 0.2 and 0.3 for most stocks, which confirms the factor structure of variance. The estimation of β_{jump}^j does not provide convincing evidence in favor of a strong factor structure of jumps, which is in line with the data analysis conducted in Section 2.

4.3 Out-of-sample performance: test on variance swaps

As a consequence of the affine structure of the model, variance swap (VS) rates are available in closed form. The rate with term τ for company j is given by:

$$\mathrm{VS}_{t,\tau}^{j,\mathrm{model}} = \mathbb{E}_t^{\mathbb{Q}}\left[\int_t^{t+\tau} \mathrm{VarRate}^j(u)du\right] \tag{12}$$

where the variance rate is given by:

$$\begin{aligned}\mathrm{VarRate}^j(t) =& v_{t^-}^j + (\beta_{diff}^j)^2 v_{t^-}^M \\ &+ \left[(\sigma^j)^2 + (\mu^j)^2\right] \lambda^j(v_{t^-}^j) + (\beta_{jump}^j)^2 \left[(\sigma^M)^2 + (\mu^M)^2\right] \lambda^M(v_{t^-}^M, m_t).\end{aligned} \tag{13}$$

4.3.1 Out-of-sample pricing

To evaluate the out-of-sample performance of the model, I compare synthesized VS rates with maturity one month to model-implied rates. Every week I synthesize VS using a portfolio of OTM options as described in Carr and Wu (2009). The resulting VS rates are denoted by $\mathrm{VS}^{j,\mathrm{data}}$. The replication procedure assumes that the underlying stock does not jump. If this assumption is not satisfied, the error is dependent on the third moment of jumps. I perform a linear regression with the synthesized one-month VS rate on the left hand-side and its model-implied counterpart on the right hand-side. The error term includes different potential sources of discrepancies between the two quantities. First, the replication involves interpolation and extrapolation procedures that take as input a finite number of options with different maturities and moneynesses. The amount of available options varies from week to week, and yields as output the implied volatilities of options defined on a grid of moneynesses, with maturity one month. The quality of the replication depends on the number of options with maturity close to one month that are available on a specific date, and on the range of their moneynesses. Second, there is inherent noise in the data due, for example, to the bid-ask spread, that is further reflected in the error term. Finally, the model is calibrated to a large amount of datapoints, and the estimation is clearly subject to model specification error as well as estimation error. The intercept should be zero if there is no systematic bias in the estimation. The slope parameter reflects the ability of the model to capture the amplitude of the movements of VS rates, it should be one if the model perfectly replicates the magnitude of the data on average. Non-systematic errors will be reflected in the error term.

I consider two types of regressions in Table 8: repeated cross-sectional regressions (Fama-MacBeth) and a pooled panel regression. For the cross-sectional regressions, I report the time-series mean of the coefficient estimates and the R^2. The standard errors are calculated using the Newey-West procedure to account for autocorrelation, with eight lags. Cluster-robust standard errors are used in the panel regression following the method of Cameron, Gelbach, and Miller (2011), to account for cross-sectional correlation and time-series auto- and cross-correlations.

Both cross-sectional and panel regressions yield estimates of the intercept that are close to zero, which indicates that the model does not present a bias in its representation of one-month VS rates. Estimates of the slope parameter are both slightly smaller than 1. Therefore the model tends to underestimate the magnitude of VS rates. The hypothesis that the model correctly prices VS, i.e., that the intercept is zero and the slope coefficient one, has a p-value of 96% when the regression is run every week, and 87% when it is run for the whole panel at once. The average R^2 from the cross-sectional regressions is 82%, it goes down to 70% in the panel regression. In summary, the model fits suynthesized VS rates remarkably well.

4.3.2 Forecasting

I further evaluate the out-of-sample performance of the model in forecasting VS rates. The model-implied conditional expectation of a time-u VS rate is $\mathbb{E}_t^{\mathbb{P}}[\mathrm{VS}_u]$. I regress the ex ante VS rate at time u on this expectation, using cross-sectional and panel regressions. I further challenge the model by comparing the predictions it delivers to a prediction based on the current VS rate at time t. The latter method is referred to a as martingale prediction, it assumes that the VS rates follow a random walk, in which case the best guess of future rate that it gives is the rate of today. Given the high persistence in VS rates dynamics, the martingale prediction is a rather challenging benchmark. If the forecast is accurate, the intercept of the regressions should be zero and the slope parameter one. I test this hypothesis and compare the p-values delivered by both prediction methods, for different future times u.

Table 9 reports the results of the regressions for three values of $u - t$: one week, one month and one month and a half. The R^2 of the regression using the model prediction is comparable to the one using the martingale prediction, for all values of $u - t$. As expected, the R^2 decreases with $u - t$ and is higher for the cross-sectional regressions than for the panel regressions. The p-value of the null hypothesis is close to 1 for $u - t$ equal to one week with the model prediction. The martingale prediction, in turn, yields a p-value of only 0.10 in the panel regression. For $u-t$ equal to a month and six weeks. the p-values of the cross-sectional regressions remain high, around 0.8, for both the model and the martingale prediction. The panel regressions, in turn, highlight the better performance of the model predictions, with a p-value of 0.19 against 0.05 for $u - t$ equal to one month and of 0.24 against 0.03 for $u - t$ equal to six weeks. To summarize, the model accurately predicts VS rates and outperforms a martingale model.

5 Idiosyncratic variance

5.1 Decomposition of the variance rate

A major advantage of using a parametric approach for the variance of returns is that daily estimates of it components are filtered as part of the model estimation. In particular, the trajectory of the idiosyncratic variance is estimated on a daily basis, in contrast to most of the asset pricing literature which relies on quarterly estimates. These estimates are usually calculated as the standard deviations of the innovations of a factor model.

The total variance swap rate at time t, given by equation (13), decomposes into a diffusive part $v^j_{t-} + (\beta^j_{diff})^2 v^M_t$, which is measure-independent, and a jump part $[(\sigma^j)^2 + (\mu^j)^2]\lambda^j(v^j_{t-}) + (\beta^j_{jump})^2[(\sigma^M)^2 + (\mu^M)^2]\lambda^M(v^M_{t-}, m_t)$, which is different under $\mathbb{P}$ and $\mathbb{Q}$ because of the jump risk premia. Under $\mathbb{P}$, the diffusive variance represents on average 86% of the total variance rate, and 85% under $\mathbb{Q}$.

Figure 3 displays the decomposition of the diffusive and jump parts of the variance rate into an idiosyncratic component and a component that comes from the effect of the market factor. Every quantity is displayed on average over the stocks of the Dow Jones. The idiosyncratic variance accounts for 63% of the total variance on average. The impact of the market factor is negligible until the beginning of 2008, because the market volatility is very low. The variance rate is then almost solely driven by the idiosyncratic variance of the firm, which is typically much higher than the market variance due to the averaging effect in the market portfolio. In 2008, the market variance peaks and causes a strong increase in the total variance. The magnitude of that increase is scaled by the β^j_{diff} parameter. For a typical stock, the influence of the market factor is of the same order of magnitude as the idiosyncratic variance during the market variance peak, causing the total variance of the stock to be multiplied by a factor of approximately 2. The ratio of idiosyncratic variance over total variance then reverts back up, to reach a level around 80%, before dropping again at the end of 2011, to reach a level below 40%. The decomposition of the jump part of variance rates follows a similar pattern.

The preliminary data analysis highlighted a strong common component in single stocks' variances. This phenomenon is also apparent for their idiosyncratic components. Applying PCA to the total instantaneous variance rate and its idiosyncratic component, I find that the first principal component explains more than 92% of the variation of the total variance rate, and the first three components more than 97%. The numbers are only slightly lower for the idiosyncratic variance, as the first component explains 81% of its variation and the three first components more than 92%. These results are generally in line with Herskovic, Kelly, Lustig, and van Nieuwerburgh (2015)[15].

The analysis of Herskovic, Kelly, Lustig, and van Nieuwerburgh (2015) further suggests that idiosyncratic variance varies with the size of the company and the industry it belongs to.[16] To analyze the influence of the company's size on its idiosyncratic variance, I sort the considered firms by their market capitalization as of the end of 2012 and divide them into five buckets. Into each of these buckets, I calculate the average total diffusive and jump variance rates across time. Taking the variance rate of the bucket of firms with the smallest market capitalizations as a reference process, I regress the time-series of the average variance rates in the other buckets on the reference time-series. Table 10 summarizes the results of the regressions. They indicate that firms with the largest market

[15]Herskovic, Kelly, Lustig, and van Nieuwerburgh (2015) use a completely different methodology to obtain these results. They calibrate a factor model to returns and compute the idiosyncratic variance as the variance of the residuals.

[16]Berrada and Hugonnier (2013) suggest that idiosyncratic volatility is generated by aggregated forecast error, in an incomplete information setting. It seems intuitive that forecast error be larger for companies of smaller size.

capitalization have a smaller variance rate on average. Exceptions are companies that suffered more from the financial crisis, such as GE, which presents higher variance rates than the other companies in the bucket.

I conduct the same analysis on the idiosyncratic variances and find the same pattern, as shown in Table 11. As parameter estimates do not seem to indicate that companies of smaller size tend to have a larger exposure to the market factor, I conclude that the idiosyncratic variance is responsible for the fact that the total variance rate decreases when the size of the company increases. This ordering does not hold anymore for the residual idiosyncratic variance, which pinpoints the common movements in idiosyncratic variances as the determinant of this phenomenon.

To examine the extent to which the idiosyncratic variance of a stock depends on the industry it belongs to, I repeat the estimation exercise using data which are industry related, based on data on sector ETFs. The model defined in equations (4)-(7) is fitted to ETF returns and option prices, using the estimation methodology described in Section 4.1.

Figure 4 represents the evolution of the idiosyncratic variance v^j, as estimated by the particle filter, for four sectors and the stocks of the Dow Jones that belong to them.[17] The variance levels are compared to the trajectory of the market factor. Until the end of 2007, most sectors have idiosyncratic variances which are comparable to that of the market factor, between 0 and 3%, and lower than those of the considered stocks. The variance of a portfolio is composed of a weighted average of the individual variances, where the weights are equal to the square of the weights that form the portfolio, and a term that corresponds to the covariances. If the correlation between the stocks in the portfolio is low, the covariance term is close to zero. Then it is common that the portfolio's variance be smaller than the individual variances, thus exhibiting diversification benefits. The Energy sector distinguishes itself from others because it has a higher idiosyncratic variance, oscillating around 6%. Because the variance tends to increase when the market capitalization of the stock decreases, it is reasonable to assume that most stocks of the S&P 500 that have not been represented because they do not belong to the Dow Jones, have larger levels of idiosyncratic variance than the stocks considered which are in the same sector. The two stocks considered in the Energy sector, Exxon Mobil Corp (XOM) and Chevron Corp (CVX), represent around 30% of the holdings of the ETF. Therefore, the large idiosyncratic variance of the Energy sector in 2006 and 2007 may be explained

[17] Only four sectors are displayed, but all of them were studied. The remaining graphs are available upon request.

by a larger variance of the smaller stocks in the portfolios. Until August 2007, the variance of the Financial sector overlaps with the market variance. However, when the crisis of the quant-strategy hedge funds starts, the Financial market becomes more volatile than the market, which will remain the case during the rest of the period considered. In March 2008, the acquisition of Bear Sterns by JPMorgan triggers an increase in the variance of the Financial sector, which reaches 15%. Such an early increase is specific to that sector and does not occur in any other sector. During the market variance peak, all sectors exhibit increases in idiosyncratic variance. However, the Consumer Staples, Industrials and Health Care sectors are only marginally affected. They reach a level of 10 to 15%. The idiosyncratic variance of the Materials and Consumer Discretionary sectors peaks to 25-30%. For the Energy and Technology sectors, the maximum is 35 to 40%. The Financial sector stands out with a maximum idiosyncratic variance that is close to the highest variance of the market factor, around 60% shortly after the bankrupty of Lehman Brothers and the failure of the TARP to pass Congress. The variance of the Financial sector then goes down, simultaneously with the market variance, to around 25% (at this point the market variance is about 15%). A striking peculiarity of this sector is that it goes up again early in 2009 and reaches a peak of almost 50% following the distress of Bank of America. This second peak is much more pronounced than the one of the market and specific to the Financial sector. Not surprisingly, stocks such as General Electric (GE), which belongs to the Industrials sector, have a variance which is similar to that of the Financial sector. Indeed, GE used to have a consumer lending division, and was therefore particularly exposed to the risks specific to the financial industry.

Figure 5 illustrates the decomposition of the evolution of idiosyncratic variance into residual idiosyncratic increments and market-induced increments, on average over stocks of the Dow Jones. The idiosyncratic movements control the evolution of the variance in the beginning of the time-series, and more generally in times of low volatility periods. The ratio of residual idiosyncratic volatility over total idiosyncratic volatility is indeed close to 1. In contrast, this ratio drops in high volatility times. On average, it drops to less than 50% during the peak at the end of 2008, then reverts back to a level close to 1 in 2010, and drops again to close to a low level at the end of 2011. For some stocks in the financial sector, whose volatility peak does not replicate that of the market, the residual idiosyncratic variance remains in control of the total idiosyncratic variance. This is the case for GE (General Electric), JPMorgan Chase and Goldman Sachs. For these stocks, the volatility peak lasts longer than the one of the market, and therefore the idiosyncratic variance takes over the entire peak.

5.2 Relation to returns

According to standard asset pricing theory, idiosyncratic volatility should not be priced in the cross-section of returns, as investors should be able to diversify this risk away. However, in the last decade many papers have found a relation between idiosyncratic volatility and expected returns. There has been a long debate on the sign of this relation. Explanations have been proposed to explain why investors may hold under-diversified porfolio, resulting in idiosyncratic risk being priced in the cross-section of returns. Fink, Fink, and He (2012) review the different methods for calculating idiosyncratic volatility, and argue that measures of idiosyncratic variance which are calculated from the information set available at time t do not have any predictive power.

All the studies previously mentioned calculate the idiosyncratic variance by estimating a Fama-French factor model (sometimes with additional factors) and taking the standard deviation of the innovations. My approach is fundamentally different in two ways. From a technical standpoint, calculating the standard deviation of residuals requires setting windows of time over which the empirical moment is calculated, typically a month. The resulting measure of idiosyncratic variance is constrained to low frequency, and heavily depends on the chosen window. In contrast, my estimate of idiosyncratic variance is daily, and is therefore more responsive to economic changes, which is important during the crisis period. From a contents perspective, the information contained in my estimate differs from the one contained in the traditional measure, because it is estimated using option prices. As a result, the resulting idiosyncratic variance contains forward-looking information that reflects the perception of market participants. This fact is particularly important, as recent papers document the predictive power of some option-derived variables on future returns, e.g., Bali and Hovakimian (2009), Cremers and Weinbaum (2010), Xing, Zhang, and Zhao (2010), An, Ang, Bali, and Cakici (2014). To visualize the effect of the difference in construction of the idiosyncratic variance, I estimate a three-Factor Fama French model using the daily values of the factors available on the website of Kenneth R. French, and calculate the idiosyncratic variance using monthly windows. The resulting time-series are referred to as FF idiosyncratic variance. I regress the trajectory filtered from the estimation of my model on the FF idiosyncratic variance for each stock of the Dow Jones. To make the time-series comparable, I only keep the last value of filtered idiosyncratic variance each month. I obtain, on average, an intercept of 0.03, a slope coefficient of 0.84 and an R^2 value of 0.51. When I average the filtered values of the idiosyncratic variance over each month instead of only keeping the last value, the R^2 slightly increases and equals 0.53. Therefore the filtered variance tends to be larger than the FF idiosyncratic variance. This is consistent with the use of options, which typically imply a larger variance than the one effectively observed in returns, see, e.g., Johannes, Polson, and Stroud (2009).

To analyze the relationship between idiosyncratic volatility and returns, I form four weekly-rebalanced portfolios, rebalanced every week according to the stocks' filtered idiosyncratic variances. The values of idiosyncratic variances are sorted, and each portfolio contains the stocks in one quartile. Portfolio 1 contains the seven stocks which have the smallest idiosyncratic variance level, portfolio 4 contains those that have the largest variance levels. Stocks are given equal weights. Figure 6 illustrates the evolution of the wealth for the four portfolios. I assume an initial level of wealth of 100. Table 12 reports the main statistics of the portfolios. An investor who buys a portfolio of stocks with a higher idiosyncratic variance tends to receive higher returns, and a larger Sharpe ratio. The portfolio that contains the first quartile of stocks has a negative weekly Sharpe ratio of -2.00 whereas the portfolio that contains the last quartile of stocks has a Sharpe ratio of 3.41. The results remain unchanged if one sorts portfolios according to the idiosyncratic variance rate (which also includes the idiosyncratic jumps in returns), the ratio of idiosyncratic variance over total variance or the level of residual idiosyncratic variance. They are also robust to a different weighting mechanism, where the position in each stock is proportional to the level of its variance. These results suggest that idiosyncratic variance and, in particular its residual idiosyncratic component, may contain useful information on future returns.

6 Pricing of idiosyncratic risks

This section examines whether idiosyncratic risk is priced in the cross-section of assets by evaluating the contribution of idiosyncratic equity and variance risks to the equity and variance risk premia. The risk premia are calculated following equations (10) and (11). They are available in closed-form at every time t conditionally on the values of the latent variance factors. These values are estimated following the procedure outlined in Section 4.1.

6.1 Equity Risk Premium

The typical behaviour of the Integrated Equity Risk Premium (IERP) is illustrated in Figure 7. It is positive for all stocks and of smaller magnitude than the risk premium of the market. Its term structure is similar to the one of the index and behaves pro-cyclically, in line with the findings of Van Binsbergen, Koijen, Hueskes, and Vrugt (2013). Its magnitude increases when the term increases during market calm, indicating that investors require a larger compensation when they invest on a longer-term horizon. In contrast, during market turmoil, the IERP becomes larger for smaller terms, as investors are unsure of how markets will behave in the near future and require compensation for that. The decomposition into an idiosyncratic and market component for a short-term horizon investment varies from stock to stock. For some stocks such as Boeing, idiosyncratic risk is not priced in the IERP, which suggests that investors think this risk can be diversified away. For other stocks, the idiosyncratic component of the IERP constitutes an important part of the total IERP. On average, the idiosyncratic IERP represents 50% of the total IERP of single stocks. Figure 7 also compares the term structure of the total IERP with the one of its idiosyncratic component (Panels A and C). The difference between the average value of the six-month IERP and the one of the instantaneous IERP is displayed, on average over the stocks of the Dow Jones. Whereas the slope of the term structure of the total IERP is positive in times of market calm, disregarding whether the stock has a small or a large variance, in the case of the idiosyncratic IERP, the slope is always negative. It is more negative for stocks with larger idiosyncratic variance. The sign of the difference between the IERP of a short-term versus long-term investment remains the same throughout the time series. The term structure of the idiosyncratic IERP is therefore not found to be pro-cyclical. Looking at tail risk provides an explanation for this finding. Indeed, the part of the idiosyncratic IERP due to jumps account to 34% of the idiosyncratic IERP on average. Therefore, idiosyncratic tail risk is an important priced risk, which mainly affects short-term risk premia.

Figure 8 represents the one-month IERPs of four sectors and compares them to the IERP of the Dow Jones components which belong to them. The sector IERP is usually smaller than the single stocks' IERPs, except in the Financial sector. The idiosyncratic IERP represents between 13 and 58% of the total IERP for all sectors except the Financial sector. The latter sector is characterized by a particularly large portion of idiosyncratic variance, which represents 75% of the total variance. The Consumer Discretionary (XLY) and Energy (XLE) sectors also exhibit sizable IERPs, 58%

and 54% of which is idiosyncratic. Going one step further and separating the jump and diffusion contributions of the idiosyncratic IERP reveals that jumps only play a non negligible role for the Energy and Financial sectors, where they represent 40% and 43% of the total IERP.

Investing in ETFs on the Technology, Consumer Staples or Health Care sectors is a way for investors to be exposed to less diversifiable risk than what they would otherwise have to bear if investing in single stocks. Speculators who, in turn, would like to be exposed to idiosyncratic risk, and particularly to tail risk, can invest in the Energy or Financials ETFs. Kelly and Lustig (2011) show that government guarantees removed part of the tail risk of the financial sector. My results show that there is still substantial tail risk that remains priced.

6.2 Variance Risk Premium

The typical behavior of the Integrated Variance Risk Premium (IVRP) is represented in Figure 9. It is negative for all stocks, which means that investors are willing to pay to get protection against the variance fluctuations in most stocks. The amplitude of the IVRP varies very much from stock to stock. The pattern is similar across stocks, i.e., the IVRP is small in absolute value in times of market calm, and peaks downward during market turmoil. The term structure is similar to the one of the market factor, i.e., the VRP becomes more negative when the term increases. This indicates that investors are willing to pay larger amounts to be covered against fluctuations in stock volatility, when they invest for longer terms. Such term structure can be benefited from in a trading strategy, as detailed in Filipović, Gourier, and Mancini (2013). Furthermore, the term structure of the idiosyncratic IVRP is very similar to the term structure of the total IVRP. Note that my results do not contradict those of Driessen, Maenhout, and Vilkov (2009) who find no evidence of a negative risk premium on individual variance risk. Indeed, they are considering relatively short terms, from 14 to 60 days, and have option data from 1996 until 2003, i.e., prior to the volatility peak.

A substantial part of the IVRP consists of its idiosyncratic component, for both short and long terms. The idiosyncratic IVRP represents 80% of the total IVRP on average, this number varying from 13% to 96% depending on the stock. Therefore, the compensation that investors require to bear systematic risk only explains 20% of the total IVRP, on average.

Han and Zhou (2013) construct the variance risk premium from implied volatilities and high frequency data, and show that stocks' expected returns increase with the premium. Using portfolio sorts similarly to what was done in part 5.2, I investigate whether the idiosyncratic IVRP contains information on future returns. Figure 12 displays the evolution of the wealth trajectory of four portfolios that are weekly rebalanced. The first portfolio contains the stocks which have the idiosyncratic IVRP that is in the first quartile (closest to zero). The last portfolio contain the stocks with the most negative idiosyncratic IVRP. The six-month IVRP is used on Panel A, the one-month IVRP on Panel B. Table 13 reports the main statistics of the two portfolios. Portfolio 4 has a Sharpe ratio of 4.06, against 0.90 for portfolio 1. The results suggest that the idiosyncratic component of the IVRP contains information on the future returns. They remain qualitatively unchanged when one chooses to weigh the stocks differently, or to use different terms of IVRP.

Figure 10 represents the six-month IVRPs of sectors and compares them to the IVRP of the Dow Jones components which belong to them. Because the magnitude of the IVRP increases with the term, the six-month term is chosen for illustration purposes. Results are qualitatively similar for other terms. The analysis of the graphs suggests that sectors belong to one of three groups. The Financial sector is the only component of the first group, as its IVRP is by far the one that has the largest magnitude. Its minimum level goes below -20% during the variance peak. No other sector has an IVRP that goes beyond -15%. The IVRP of the Financial sector crosses this threshold when Lehman Brothers collapses in September 2008, and remains below until mid-2009. The second group includes the Energy, Materials and Consumer Discretionary sectors. The total IVRP of these three sectors decreases almost linearly with time from mid 2007 until September 2008, when they are around -6%. The variance peak pulls them down to about -12%, but they quickly recover and get back to a level oscillating around 5%. Finally, the last group includes the remaining sectors, which do not have substantial priced variance risk.

The part of the total IVRP that corresponds to idiosyncratic risk is very high: between 59 and 95%. In particular, in the two riskiest categories of sectors, it ranges from 83 to 95%. Therefore, in those sectors, only 5 to 17% of the IVRP represents compensation for systematic market risk. The three categories of sectors exhibit different behaviours. The IVRP of the Financial sector reaches a minimum value which goes down to -21%. Long-term investments contain much more priced risk than short-term investments, with a minimum IVRP of around -10% for a one-month term. For the sectors in the second group, the minimum IVRP stabilizes around 12% towards the four-month term. The sectors in the third group have a minimum IVRP between 0 and 6% for all terms. The jump component does not play a large role in the variance risk premia, it represents 5 to 18% of the total premium.

If I go one step further and decompose the continuous part of the idiosyncratic IVRP into a component that is due to the common idiosyncratic variance, controlled by β_v^j, and another due to the residual idiosyncratic variance, I obtain the results displayed on Figure 11. Interestingly, the common idiosyncratic variance carries a risk premium that tends to be close to zero in times of market calm, and significantly positive during market turmoil. This result is in line with the results of Schürhoff and Ziegler (2011). They justify the sign of the part of the idiosyncratic IVRP due to common movements in idiosyncratic variances by the compensation required by financial intermediaries for bearing the risk of variance co-movements.

However, I find that a large part of the continuous idiosyncratic IVRP is unexplained by the common movements of idiosyncratic variance. This part is negative, due to the risk premium carried by the residual idiosyncratic variance.

6.3 Discussion

To explain the large amount of priced idiosyncratic risk in the Financial, Energy and Consumer Discretionary sectors, I borrow from the literature on financial intermediation. Garleanu, Pedersen, and Poteshman (2009) investigate the effect of demand pressure on option prices and introduce a demand-based option pricing model. An increase of public demand for options forces dealers to be able to increase their supply, thereby exposing them to more inventory risk. This is especially the case for OTM put options, which are difficult to hedge. Such a phenomenon would imply that demand be positively correlated with option prices and the magnitude of risk premia. Bollen and Whaley (2004) indeed show that net buying pressures are positively related to changes in implied volatilities. When market makers cannot fully hedge their positions, they require higher compensation which increases the magnitude of the VRP. In a related paper, Pan and Poteshman (2006) show that option trading volume contains information on future stock returns. I examine the relationship between demand and VRP by regressing the idiosyncratic VRP on the total Open Interest (OI) for sectors. I obtain a significantly negative slope coefficient, which indicates that the magnitude of the idiosyncratic VRP is positively correlated with OI, see Table 14. This finding holds in both cross-sectional and panel regressions. The R^2 is respectively 41% and 31%. The slope coefficient becomes more negative when considering the OI of deep OTM puts, with moneyness smaller than 0.75. This suggests that there is a link between priced idiosyncratic risk and demand for options, in particular for deep OTM options.

In the sector of Energy, the priced risk may be due to inelastic demand and limits to arbitrage. During calm market times, contractors invest in long-term projects, i.e., building an oil platform. If a crash occurs, catching market participants by surprise, these investments lead to an excess of supply leading to inventory risk. In the context of electricity markets, Bessembinder and Lemmon (2002) show that in the absence of storage, hedging demand affects both spot and futures prices. Acharya, Lochstoer, and Ramadorai (2013) look at oil and gas markets and show that this is still the case, even when storage is possible, when speculators are capital constrained and producers are hedging constrained. Such limits to arbitrage would likely have a stronger effect during financial distress, and distort the resulting risk premia. They may therefore provide an explanation for the priced idiosyncratic risk found. In the sector of Consumer Discretionaries, the substantial priced risk during market turmoil can be explained by the pro-cyclicality of demand. Indeed, this sector contains, among others, luxuries and media companies. Aït-Sahalia, Parker, and Yogo (2004) use the demand for luxuries as a measure of consumption. In times of expansion, people invest in luxuries and in media products (television sets for example). In contrast, in times of recession, these are the products that they will cut from their spendings. The link with the Financial sector is trivial, as this sector absorbes all the shocks borne by financial intermediaries.

The link between OI and VRP must however be interpreted with caution, keeping in mind that OI measures the option transactions among all market participants, including transactions that do not involve financial intermediaries. These results should therefore be complemented by a study of data on the parties involved in each transaction, in order to investigate the link with demand and the risk-bearing capacities of financial intermediaries. The relation between inventory risk, market dealers' wealth and the VRP of the S&P 500 is further analyzed by Fournier (2014).

Chen, Joslin, and Ni (2014) study the relationship between public demand and the expensiveness of deep OTM put options on the S&P 500 and find a significant negative relationship. In turn, they find empirical evidence in favor of an alternative theory, which predicts that a shock in intermediation introduces more constraints on financial intermediaries, pushing the prices of options up and the demand down. Along the same line, Barras and Malkhozov (2014) find that option-implied VRP not only reflect investors' risk attitude, but also include information on the risk-bearing capacity of financial intermediaries.

This line of reasoning could provide an explanation for the large priced idiosyncratic risk of the Financial, Energy and Consumer Discretionary sectors. A thorough investigation of the link between the risk-bearing capacity of financial intermediaries and risk premia for single stocks and sectors is left for further research.

7 Conclusion

This paper revisits the question: does idiosyncratic risk matter? It introduces a flexible parametric framework for the study of the risks embedded in individual stocks, which allows me to disentangle the contributions of the different sources of risk. Equity risk is thus decomposed into two components: the first one reflects the impact of a market factor on single stocks' returns and the second one is idiosyncratic. Similarly, the variances of idiosyncratic returns are shown to also follow a factor structure. They are therefore decomposed into a systematic and an idiosyncratic component. Both types of shocks contain a diffusive and a jump part, to reflect the fact that investors may exhibit different levels of risk aversion towards small and large price movements. The estimation approach combines the information that underlying levels and options contain on the distributions of returns and on the pricing kernel. The resulting analysis of the price of risk of each component reveals that idiosyncratic equity and variance risks matter. They are indeed found to be substantial parts of the equity and variance risk premia. Furthermore, I find that the idiosyncratic component of the equity risk premium contains non negligible tail risk.

I compare the risk present in individual stocks to the one of the sectors they belong to. This analysis is based on the information contained in sector ETFs and derivatives on them. I show that most sector ETFs provide an interesting investment alternative for agents who are not willing to be exposed to non diversifiable risks, as their idiosyncratic risk premia are close to zero. The three exceptions are the Financial, Energy and Consumer Discretionary sectors, which provide significant exposure to idiosyncratic equity and variance risks, and include substantial tail risk. I propose explanations for this result which are based on demand arguments and linked to the risk-bearing capacities of financial intermediaries. Increased demand, intermediation shocks or trading constraints may lead to inventory risk, thereby increasing the exposure of financial intermediaries to unhedgeable risks, which may impact option prices and the associated risk premia. A thorough study of these effects on the risk premia of single stocks and sectors is left for future research.

References

Acharya, V. V., L. A. Lochstoer, and T. Ramadorai, 2013, "Limits to arbitrage and hedging: Evidence from commodity markets," *Journal of Financial Economics*, 109(2), 441–465.

Adrian, T., and F. Franzoni, 2009, "Learning about beta: Time-varying factor loadings, expected returns, and the conditional CAPM," *Journal of Empirical Finance*, 16(4), 537–556.

Aït-Sahalia, Y., and A. Lo, 1998, "Nonparametric Estimation of State-Price-Densities Implicit in Financial Asset Prices," *Journal of Finance*, 53(2), 499–547.

Aït-Sahalia, Y., J. Parker, and M. Yogo, 2004, "Luxury Consumption and the Equity Premium," *Journal of Finance*, 59, 2959–3004.

Amengual, D., and D. Xiu, 2014, "Resolution of Policy Uncertainty and Sudden Declines in Volatility," Chicago Booth Paper No. 13-78.

An, B.-J., A. Ang, T. G. Bali, and N. Cakici, 2014, "The Joint Cross Section of Stocks and Options," *Journal of Finance*, 69(5), 2279–2337.

Andersen, T., T. Bollerslev, F. Diebold, and P. Labys, 2003, "Modeling and Forecasting Realized Volatility," *Econometrica*, 71, 579–625.

Ang, A., R. J. Hodrick, Y. Xing, and X. Zhang, 2006, "The Cross-Section of Volatility and Expected Returns," *Journal of Finance*, 61(1), 259–299.

——— , 2009, "High idiosyncratic volatility and low returns: International and further U.S. evidence," *Journal of Financial Economics*, 91(1), 1–23.

Bakshi, G., C. Cao, and Z. Zhong, 2012, "Assessing Models of Individual Equity Option Prices," Working Paper.

Bali, T., and N. Cakici, 2008, "Idiosyncratic volatility and the cross-section of expected returns," *Journal of Financial and Quantitative Analysis*, 43, 29–58.

Bali, T. G., N. Cakici, X. Yan, and Z. Zhang, 2005, "Does Idiosyncratic Risk Really Matter?," *Journal of Finance*, 60(2), 905–929.

Bali, T. G., and A. Hovakimian, 2009, "Volatility Spreads and Expected Stock Returns," *Management Science*, 55, 1797–1812.

Barber, B. M., and T. Odean, 2000, "Trading is hazardous to your wealt: The common stock investment performance of individual investors," *Journal of Finance*, 55, 773–806.

Bardgett, C., E. Gourier, and M. Leippold, 2014, "Inferring volatility dynamics and risk premia from the S&P 500 and VIX markets," Swiss Finance Institute Research Paper No. 870.

Barigozzi, M., C. Brownlees, G. M. Gallo, and D. Veredas, 2014, "Disentangling Systematic and Idiosyncratic Dynamics in Panels of Volatility Measures," *Journal of Econometrics*, 182(2), 364–384.

Barndorff-Nielsen, O. E., and N. Shephard, 2004, "Power and Bipower Variation with Stochastic Volatility and Jumps," *Journal of Financial Econometrics*, 2(1), 1–37.

——— , 2006, "Econometrics of Testing for Jumps in Financial Economics using Bipower Variation," *Journal of Financial Econometrics*, 4, 1–30.

Barras, L., and A. Malkhozov, 2014, "When Variance Risk Has Two Prices: Evidence from the Equity and Option Markets," Working Paper.

Bates, D. S., 2008, "The market for crash risk," *Journal of Economic Dynamics and Control*, 32(7), 2291–2321.

Bégin, J.-F., C. Dorion, and G. Gauthier, 2015, "The Pricing of Idiosyncratic Risk in Option Markets," .

Benartzi, S., 2001, "Excessive extrapolation and the allocation of 401(k) accounts to company stock," *Journal of Finance*, 56, 1747–1764.

Benartzi., S., and R. H. Thaler, 2001, "Naive diversification strategies in defined contribution saving plan," *American Economic Review*, 91, 79–98.

Berrada, T., and J. Hugonnier, 2013, "Incomplete information, idiosyncratic volatility and stock returns," *Journal of Banking & Finance*, 37(2), 448–462.

Bessembinder, H., and M. Lemmon, 2002, "Equilibrium Pricing and Optimal Hedging in Electricity Forward Markets," *Journal of Finance*, 62(3), 1347–1382.

Bollen, N., and R. Whaley, 2004, "Does net buying pressure affect the shape of implied volatility functions?," *Journal of Finance*, 59, 711–753.

Bollerslev, T., and V. Todorov, 2011, "Tails, Fears and Risk Premia," *Journal of Finance*, 66(6), 2165–2211.

Bollerslev, T., V. Todorov, and S. Z. Li, 2013, "Jump Tails, Extreme Dependencies and the Distribution of Stock Returns," *Journal of Econometrics*, 172(2), 307–324.

Boloorforoosh, A., 2014, "Is Idiosyncratic Volatility Risk Priced? Evidence from the Physical and Risk-Neutral Distributions," Concordia University; National Bank of Canada.

Broadie, M., M. Chernov, and M. Johannes, 2007, "Model Specification and Risk Premia: Evidence from Futures Options," *Journal of Finance*, 62(3), 1453–1490.

Brockman, P., and X. S. Yan, 2008, "The Time-Series Behavior and Pricing of Idiosyncratic Volatility: Evidence from 1926 to 1962," Working Paper. University of Missouri at Columbia.

Cameron, A. C., J. B. Gelbach, and D. L. Miller, 2011, "Robust Inference With Multiway Clustering," *Journal of Business and Economic Statistics*, 29(2), 238–249.

Carr, P., and L. Wu, 2009, "Variance risk premiums," *Review of Financial Studies*, 22(3), 1311–1341.

Chen, H., S. Joslin, and S. X. Ni, 2014, "Demand for Crash Insurance, Intermediary Constraints, and Stock Return Predictability," AFA 2013 San Diego Meetings Paper.

Christoffersen, P., M. Fournier, and K. Jacobs, 2013, "The Factor Structure in Equity Option Prices," Working Paper.

Collin-Dufresne, P., R. S. Goldstein, and F. Yang, 2012, "On the Relative Pricing of Long-Maturity Index Options and Collateralized Debt Obligations," *Journal of Finance*, 67(6), 1983–2014.

Cont, R., and J. da Fonsecca, 2002, "Dynamics of implied volatility surfaces," *Quantitative Finance*, 2, 45–60.

Cont, R., J. da Fonsecca, and V. Durrleman, 2002, "Stochastic Models of Implied Volatility Surfaces," *Economic Notes*, 31(2), 361–377.

Cremers, M., M. Halling, and D. Weinbaum, 2013, "Aggregate Jump and Volatility Risk in the Cross Section of Stock Returns," working paper, Johnson School Research Paper Series No. 13-2010.

Cremers, M., and D. Weinbaum, 2010, "Deviations from Put-Call Parity and Stock Return Predictability," *Journal of Financial Quantitative Analysis*, 45, 335–367.

Daglish, T., J. Hull, and W. Suo, 2007, "Volatility surfaces: theory, rules of thumb, and empirical evidence," *Quantitative Finance*, 7, 507–524.

Dennis, P., and S. Mayhew, 2002, "Risk-Neutral Skewness: Evidence from Stock Options," *Journal of Financial and Quantitative Analysis*, 37(2), 471–493.

Driessen, J., P. J. Maenhout, and G. Vilkov, 2009, "The price of correlation risk: evidence from equity options," *Journal of Finance*, 64(3), 1377–1406.

Duan, J., and J. Wei, 2009, "Systematic Risk and the Price Structure of Individual Equity Options," *Review of Financial Studies*, 22, 1981–2006.

Duffie, D., J. Pan, and K. J. Singleton, 2000, "Transform Analysis and Asset Pricing for Affine Jump-Diffusions," *Econometrica*, 68(6), 1343–1376.

Fama, E., and J. MacBeth, 1973, "Risk, returns, and equilibrium: Empirical tests," *Journal of Political Economy*, 81, 607–636.

Fang, F., and C. Oosterlee, 2008, "A novel pricing method for European options based on Fourier-cosine series expansions," *SIAM Journal on Scientific Computing*, 31(2), 826–848.

Filipović, D., E. Gourier, and L. Mancini, 2013, "Quadratic Variance Swap Models - Theory and Evidence," Swiss Finance Institute Research Paper No. 13-06.

Fink, J., K. Fink, and H. He, 2012, "Expected Idiosyncratic Volatility Measures and Expected Returns," *Financial Management*, 41(3), 519–553.

Fournier, M., 2014, "Inventory Risk, Market-Maker Wealth, and the Variance Risk Premium," Working Paper.

Fu, F., 2009, "Idiosyncratic Risk and the Cross-Section of Expected Stock Returns," *Journal of Financial Economics*, 91(1), 24–37.

Garleanu, N., L. Pedersen, and A. Poteshman, 2009, "Demand-Based Option Pricing," *Review of Financial Studies*, 22(10), 4259–4299.

Goetzmann, W. N., and A. Kumar, 2008, "Equity Portfolio Diversification," *Review of Finance*, 12(3), 433–463.

Guo, H., and R. Savickas, 2010, "Relation between time-series an cross-sectional effects of idiosyncratic variance on stock returns," *Journal of Banking and Finance*, 34(7), 1637–1649.

Han, B., and Y. Zhou, 2013, "Variance Risk Premium and Cross-Section of Stock Returns," Working Paper.

Han, Y., and D. Lesmond, 2011, "Liquidity biases and the pricing of cross-sectional idiosyncratic volatility," *Review of Financial Studies*, 24, 1590–1629.

Härdle, W., 1992, *Applied Nonparametric Regression*. Cambridge University Press, Part of Econometric Society Monographs.

Herskovic, B., B. Kelly, H. Lustig, and S. van Nieuwerburgh, 2015, "The Common Factor in Idiosyncratic Volatility: Quantitative Asset Pricing Implications," *Journal of Financial Economics*, Forthcoming.

Huang, W., Q. Liu, S. Rhee, and L. Zhang, 2010, "Return reversals, idiosyncratic risk, and expected returns," *Review of Financial Studies*, 23, 147–168.

Huberman, G., 2001, "Familiarity breeds investment," *Review of Financial Studies*, 14, 659–680.

Jiang, G. J., D. Xu, and T. Yao, 2009, "The Information Content of Idiosyncratic Volatility," *Journal of Financial and Quantitative Analysis*, 44(1), 1–28.

Johannes, M. S., N. G. Polson, and J. R. Stroud, 2009, "Optimal Filtering of Jump Diffusions: Extracting Latent States from Asset Prices," *Review of Financial Studies*, 22(7), 2759–2799.

Kelly, B., and H. Lustig, 2011, "Too-Systemic-To-Fail: What Option Markets Imply About Sector-wide Government Guarantees," NBER Working Paper No. 17149.

Langlois, H., 2013, "Asset Pricing with Return Asymmetries: Theory and Tests," Working Paper. HEC Paris.

Levy, H., 1978, "Equilibrium in an imperfect market: a constraint on the number of securities in the portfolio," *American Economic Review*, 68, 643–658.

Luciani, M., and D. Veredas, 2012, "Estimating and Forecasting Large Panels of Volatilities with Approximate Dynamic Factor Models," ECARES Working Paper 2011/28.

Malkiel, B., and Y. Xu, 2001, "Idiosyncratic risk and security returns," Working paper. University of Dallas.

Mancini, C., 2009, "Non-Parametric Threshold Estimation for Models with Stochastic Diffusion Coefficient and Jumps," *Scandinavian Journal of Statistics*, 36, 270–296.

Merton, R., 1987, "A simple model of capital market equilibrium with incomplete information," *Journal of Finance*, 42, 483–510.

Pan, J., and A. Poteshman, 2006, "The information in option volume for future stocks prices," *Review of Financial Studies*, 19, 871–908.

Phoon, K. K., S. P. Huang, and S. T. Quek, 2002, "Implementation of Karhunen-Loeve expansion for simulation using a wavelet Garlekin scheme," *Probabilistic Engineering Mechanics*, 17, 293–303.

Pitt, M. K., and N. Shephard, 1999, "Filtering via Simulation: Auxiliary Particle Filters," *Journal of the American Statistical Association*, 94(446), 590–599.

Polkovnichenko, V., 2005, "Household Portfolio Diversification: A Case for Rank-Dependent Preferences," *Review of Financial Studies*, 18(4), 1467–1502.

Schürhoff, N., and A. Ziegler, 2011, "Variance Risk, Financial Intermediation, and the Cross-Section of Expected Option Returns," CEPR Discussion Paper Series No. 8268.

Serban, M., J. Lehoczky, and D. Seppi, 2008, "Cross-Sectional Stock Option Pricing and Factor Models of Returns," Working Paper.

Skiadopoulos, G., S. Hodges, and L. Clewlow, 2000, "The Dynamics of the S&P 500 Implied Volatility Surface," *Review of Derivatives Research*, 3(3), 263–282.

Spiegel, M., and X. Wang, 2005, "Cross-sectional variation in stock returns: Liquidity and idiosyncratic risk," Yale ICF Working Paper No. 05-13.

T.Sandwell, D., 1987, "Biharmonic spline interpolation of GEOS-3 and SEASAT altimeter data," *Geophysical Research Letters*, 2, 139–142.

Van Binsbergen, J., R. S. Koijen, W. Hueskes, and E. Vrugt, 2013, "Equity Yields," *Journal of Financial Economics*, 110(3), 503–519.

Wu, L., 2011, "Variance Dynamics: Joint Evidence from Options and High-Frequency Returns," *Journal of Econometrics*, 160(1), 280–287.

Xing, Y., X. Zhang, and R. Zhao, 2010, "What Does the Individual Option Volatility Smirk Tell Us About Future Equity Returns," *Journal of Financial and Quantitative Analysis*, 45, 641–662.

Yan, S., 2011, "Jump risk, stock returns and slope of implied volatility smile," *Journal of Financial Economics*, 99(1), 216–233.

Figures and Tables

Figure 1: *Panel A:* These graphs report the variance explained by the first principal component in the PCA of high-frequency measures of returns' variation of individual stocks'. The stocks considered are the elements of the Dow Jones, except Visa. The measures considered are the Realized Variance (RV), the Bipower Variation (BV) and the Continuous Variation (CV) on Panel A, and the Jump Variation on Panel B. They are computed based on 5-minute tick data from March 3, 2006 until December 26, 2012.

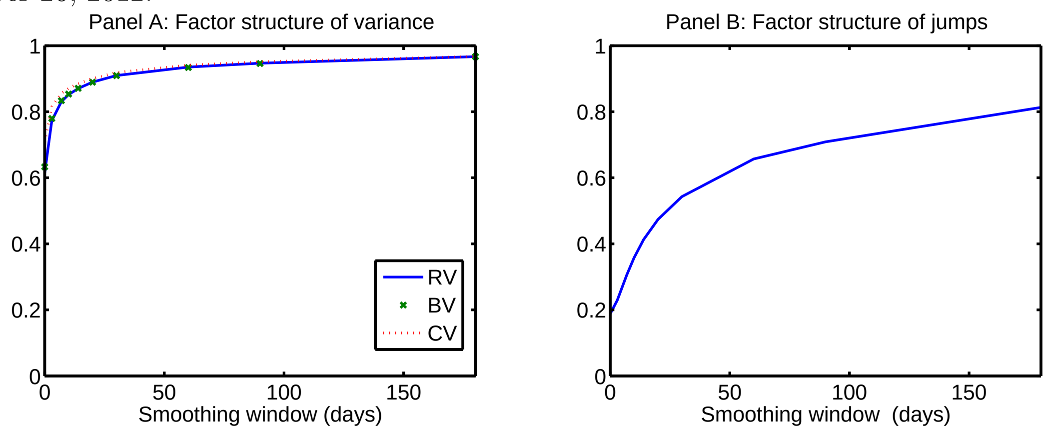


Figure 2: *Left panel:* This graph represents the evolution of the two-month at-the-money (ATM) level of the implied volatility (IV) of single stock options from January 4, 2006 until December 26, 2012. *Right panel:* This graph represents the median level of interpolated two-month ATM IV from March 3, 2006 until December 26, 2012, versus the point estimate of the parameter $(\beta^j_{diff})^2$, which measures the impact of diffusive market movements on the returns of stock j.

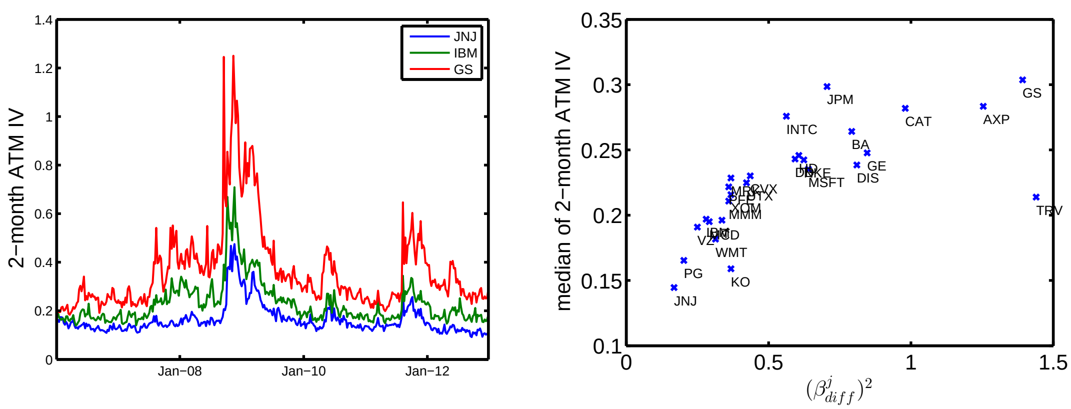


Figure 3: This figure represents the decomposition of the variance rate, where each quantity is calculated on average over the stocks of the Dow Jones. *Panel A* represents the decomposition of the evolution of the diffusive component of the variance rate (Total var. rate) into an idiosyncratic component (Idio. part) and a component that results from the impact of the market factor (Market part). *Panel B* represents the ratio of total diffusive variance that is composed of idiosyncratic variance. *Panel C* represents the decomposition of the evolution of the jump component of the variance rate (Total var. rate) into an idiosyncratic component (Idio. part) and a component that results from the impact of the market factor (Market part). Finally, *Panel D* represents the ratio of total discontinuous variance that is composed of idiosyncratic variance. The estimation was performed on the period from March 3, 2006 until end of 2009. The grey area represents the out-of-sample period, from January 2010 until December 26, 2012.

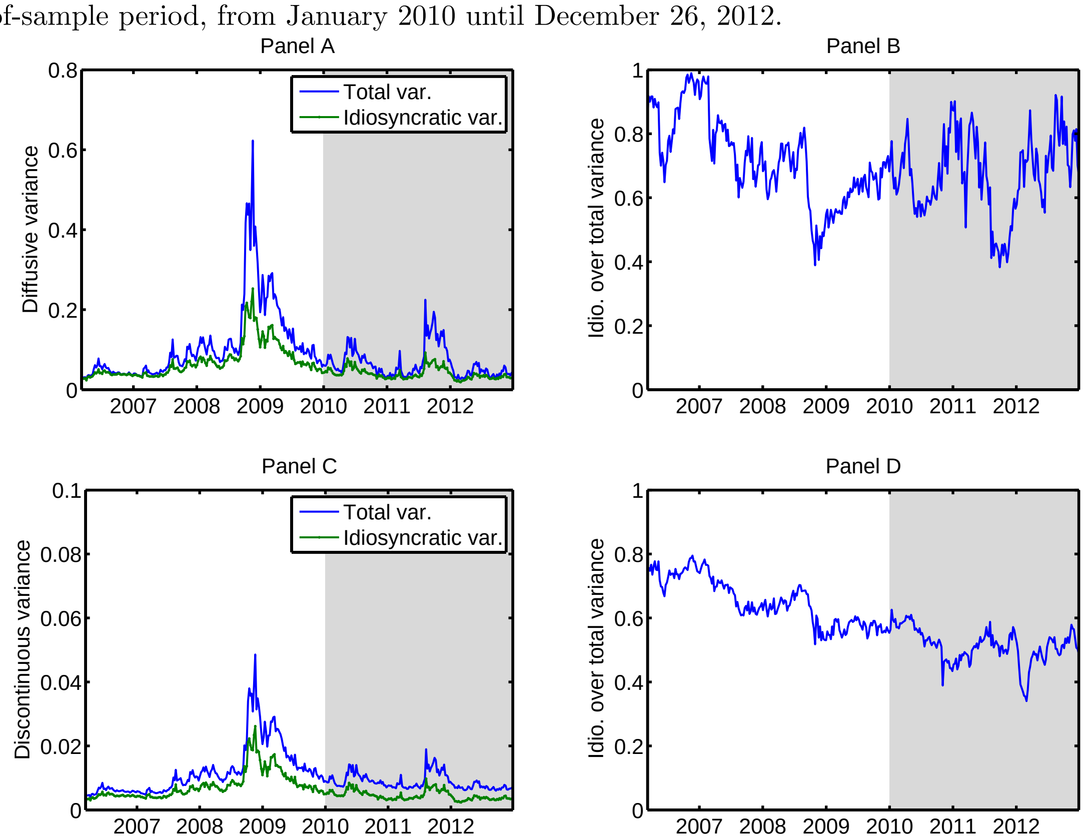


Figure 4: These graphs represent the total idiosyncratic variance of four sectors (Energy: $\beta_v^j = 0.59$, Consumer Staples: $\beta_v^j = 0.25$, Industrials: $\beta_v^j = 0.10$ and Financials: $\beta_v^j = 0.40$) versus the variance of the Dow Jones stocks that are part of these sectors, from March 3, 2006 until end of 2012.

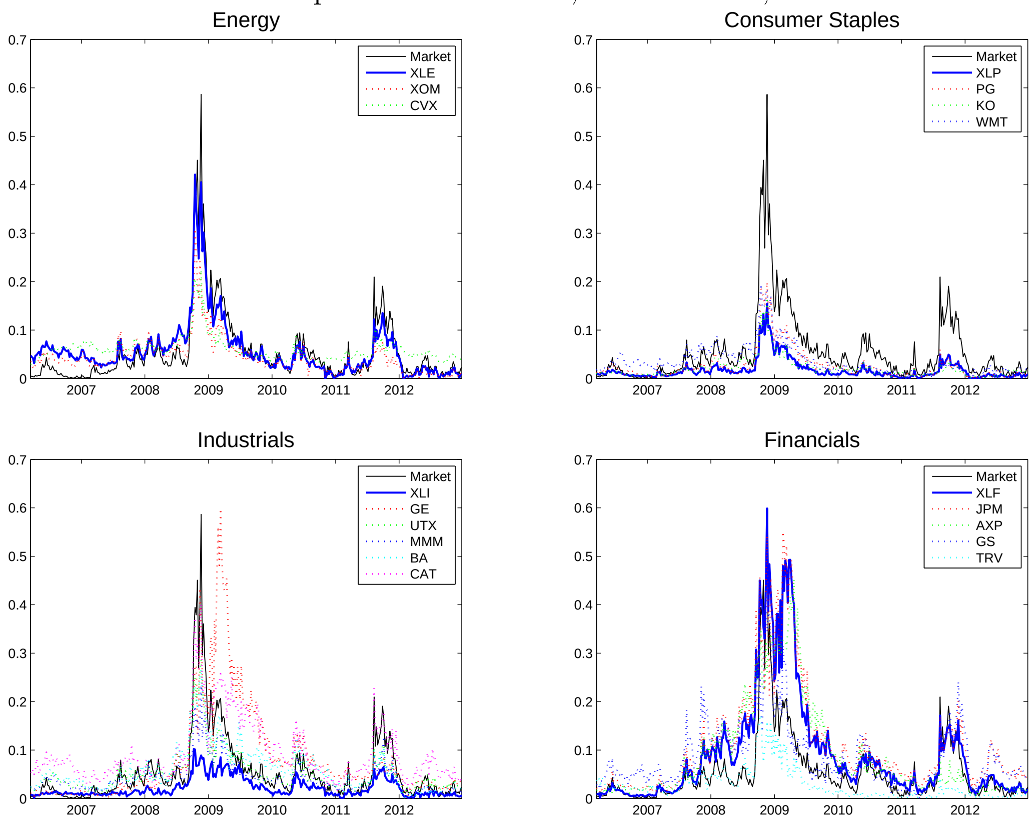


Figure 5: *Panel A* represents the decomposition of the evolution of the idiosyncratic variance, calculated on average over the stocks of the Dow Jones, into residual idiosyncratic movements and common movements. *Panel B* represents the ratio of total idiosyncratic variance that common idiosyncratic variance movements account for. The estimation was performed on the period from March 3, 2006 until end of 2009. The grey area represents the out-of-sample period, from January 2010 until December 26, 2012.

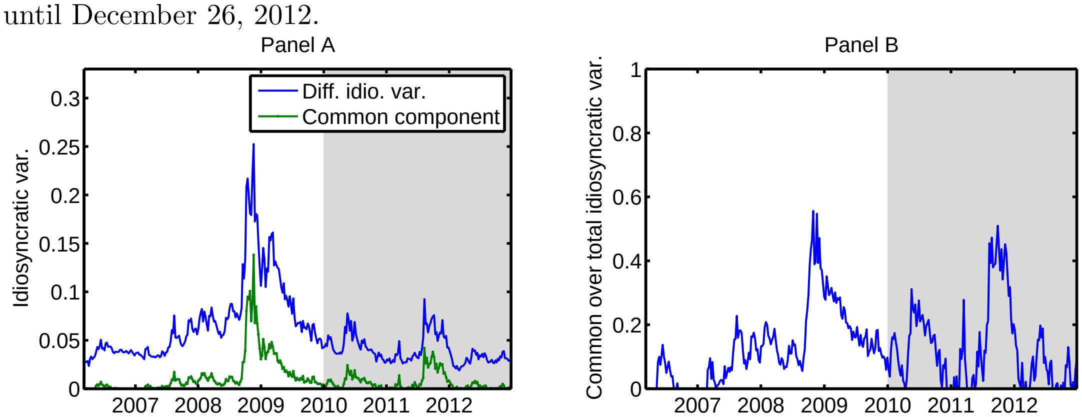


Figure 6: These graphs represent the evolution of the wealth process of four portfolios that are weekly rebalanced depending on the level of idiosyncratic variance (*Panel A*) and of residual idiosyncratic variance (*Panel B*) of each stock. Portfolio 1 contains every week a combination of the 7 stocks with the smallest idiosyncratic variance. Portfolio 4 contains the 7 stocks with the largest idiosyncratic variance. Stocks are equally weighted. The initial wealth level is 100. The time period covers from March 3, 2006 until the end of 2012.

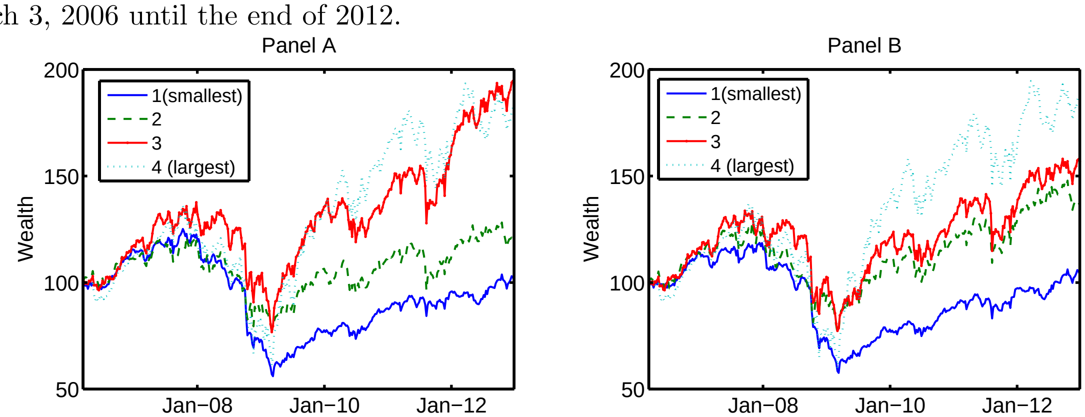


Figure 7: These graphs represent the term structure and decomposition of the IERP, calculated on average over the stocks of the Dow Jones, from March 3, 2006 until December 26, 2012. *Panel A* displays the instantaneous (Δt) versus the six-month (6m) IERP. *Panel B* displays the decomposition of the 1-month IERP into systematic and idiosyncratic components. *Panel C* displays the instantaneous versus the 6-month idiosyncratic component of the IERP. Finally, *Panel D* displays the decomposition of the idiosyncratic IERP into jump and diffusive components.

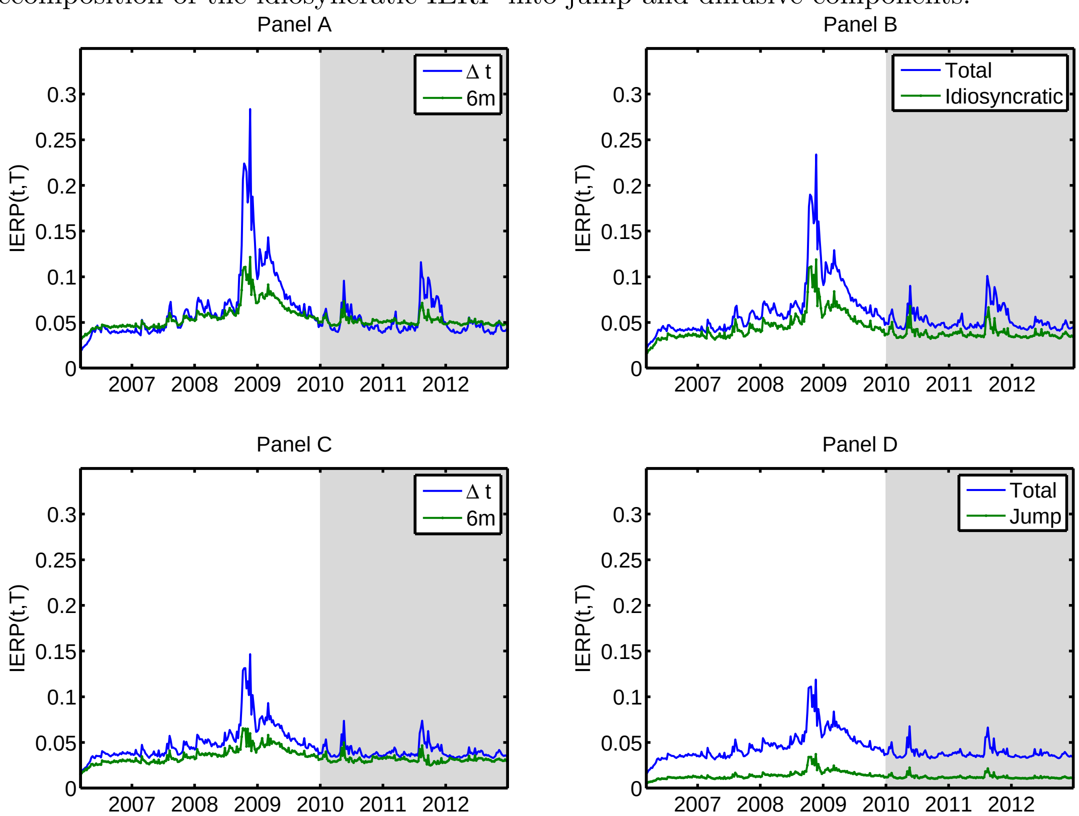


Figure 8: These graphs represent the one-month total IERP of four sectors (Energy, Consumer Staples and Financials) versus the IERP of the components of the Dow Jones that are part of these sectors. The time period covers from March 3, 2006 until the end of 2012.

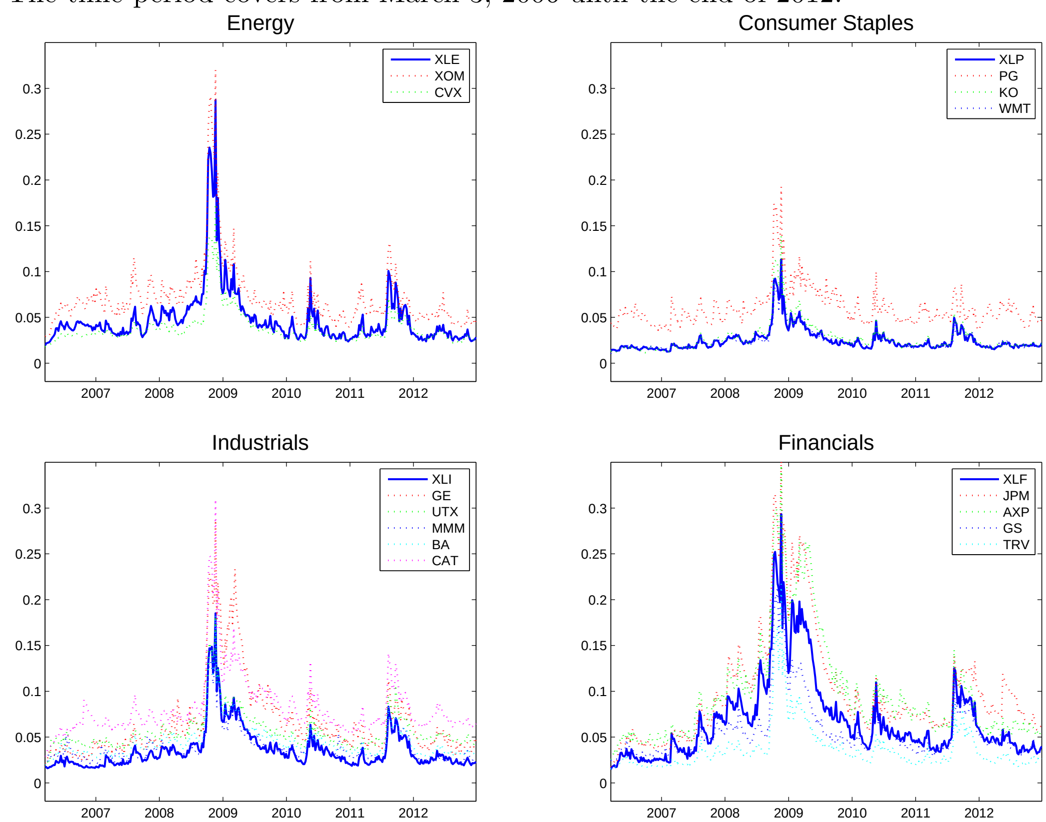


Figure 9: These graphs represent the term structure and decomposition of the IVRP, calculated on average over the stocks of the Dow Jones, from March 3, 2006 until December 26, 2012. *Panel A* displays the instantaneous (Δt) versus the six-month (6m) IVRP. *Panel B* displays the decomposition of the 1-month IVRP into systematic and idiosyncratic components. *Panel C* displays the instantaneous versus the 6-month idiosyncratic component of the IVRP. Finally, *Panel D* displays the decomposition of the idiosyncratic IVRP into jump and diffusive components.

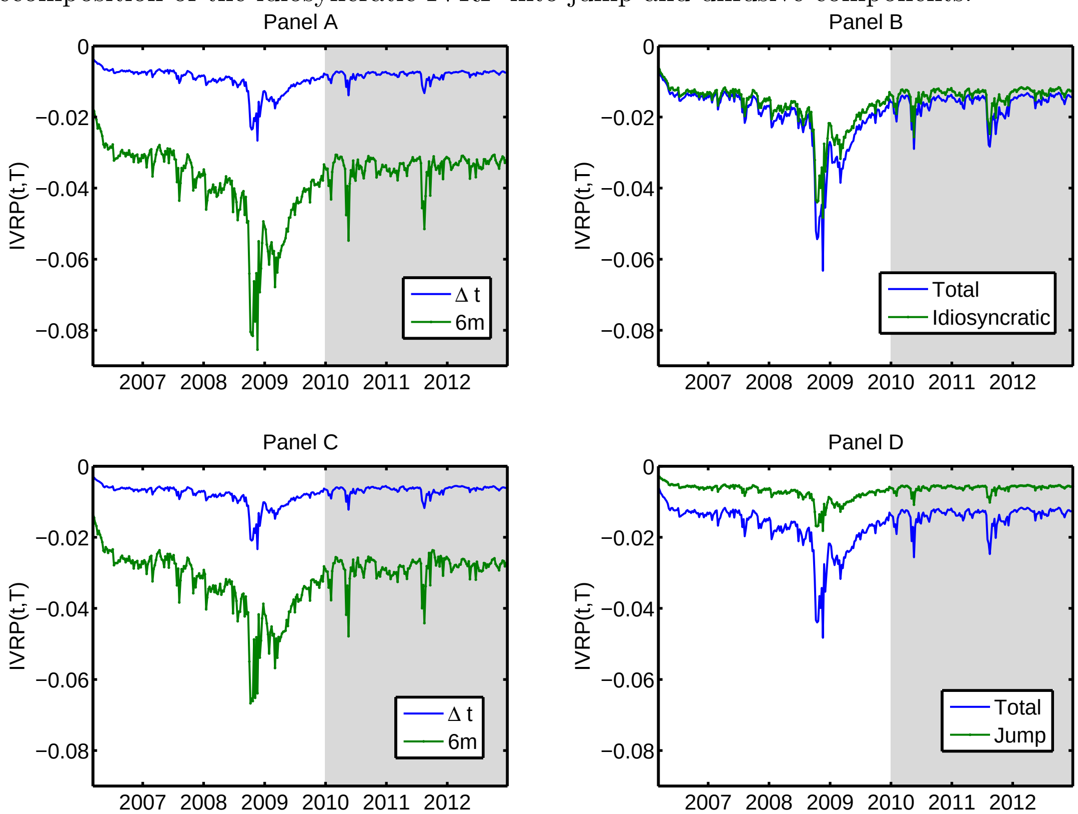


Figure 10: These graphs represent the six-month total IVRP of four sectors (Energy, Consumer Staples, Industrials and Financials) versus the IVRP the components of the Dow Jones that are part of these sectors. The time period covers from March 3, 2006 until the end of 2012.

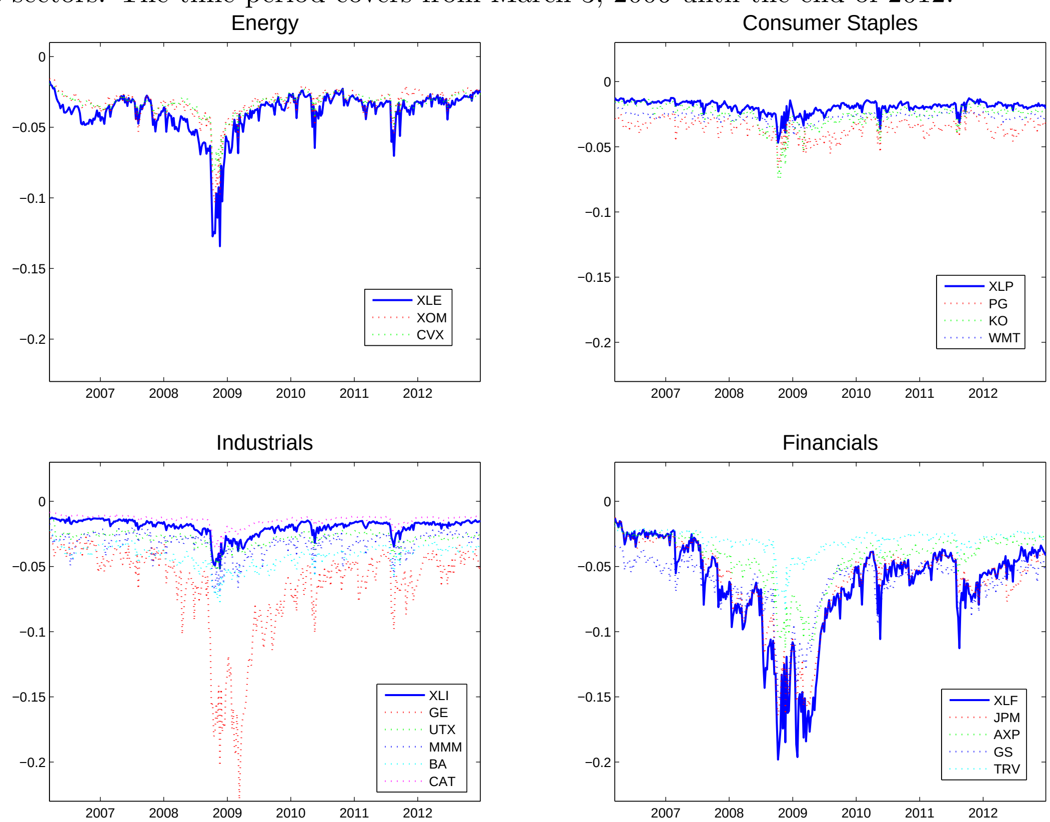


Figure 11: These graphs represent the part of the six-month idiosyncratic IVRP of four sectors (Energy, Consumer Staples, Industrials and Financials) due to common movements in idiosyncratic variances, versus the same quantity for the components of the Dow Jones which are part of these sectors. The time period covers from March 3, 2006 until the end of 2012.

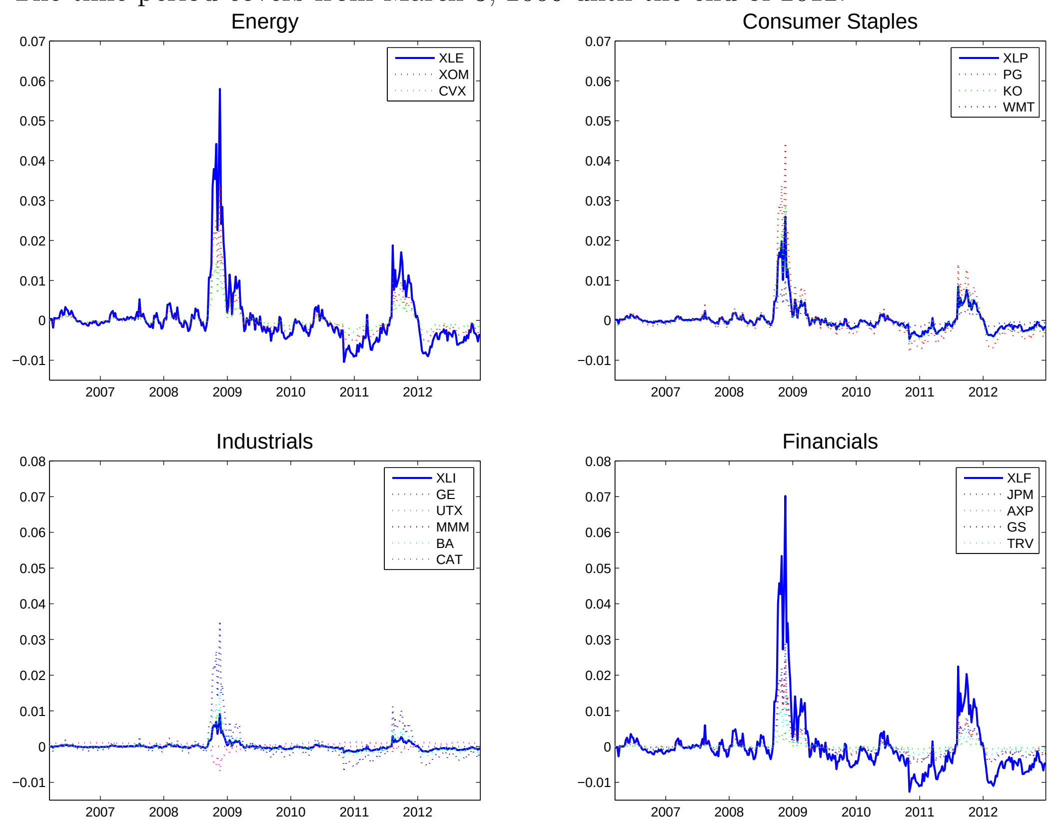


Figure 12: These graphs represent the evolution of the wealth process of four portfolios that are weekly rebalanced depending on the level of 6-month (Panel A) and 1-month (Panel B) idiosyncratic variance risk premium of each stock. Portfolio 1 contains every week a combination of the 7 stocks with the smallest idiosyncratic IVRP in absolute value. Portfolio 4 contains the 7 stocks with the largest idiosyncratic variance in absolute value. Stocks are equally weighted. The initial wealth level is 100. The time period covers from March 3, 2006 until the end of 2012.

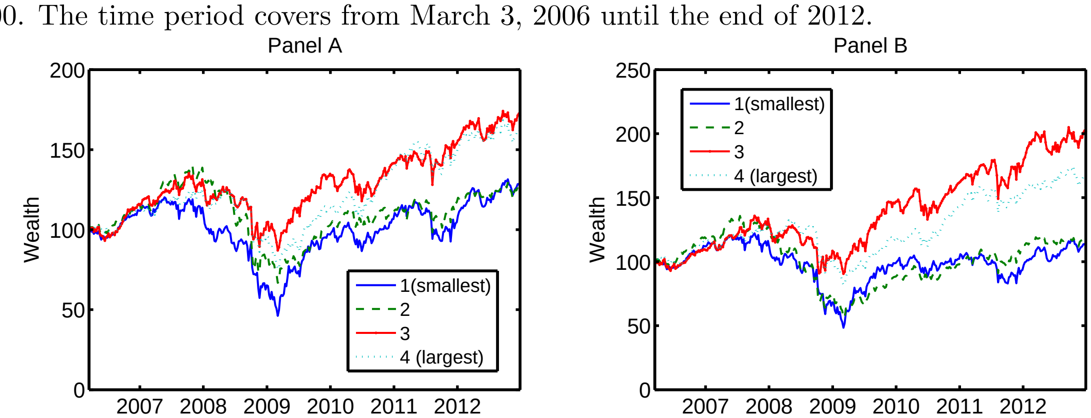


Table 1: This table lists the stocks used in the paper and gives their acronyms. For each stock, the two last colums provide the minimum and maximum moneynesses of the available options over the period March 2006 - December 2012. The moneyness at time t of an option with strike K and underlying price S_t is defined as $m_t = K/S_t$.

Company	Acronym	$\min m_t$	$\max m_t$
American Express Co	AXP	0.378	3.260
Boeing Co	BA	0.349	2.839
Caterpillar Inc	CAT	0.338	3.516
JPMorgan Chase and Co	JPM	0.372	3.328
Chevron Corp	CVX	0.305	1.965
AT&T Inc	T	0.380	2.395
Cisco Systems Inc	CSCO	0.630	0.843
The Coca-Cola Co	KO	0.316	1.687
Walt Disney Co	DIS	0.325	2.623
E I du Pont de Nemours and Co	DD	0.343	2.725
Exxon Mobil Corp	XOM	0.334	2.162
General Electric Co	GE	0.350	3.704
Goldman Sachs Group Inc	GS	0.363	4.024
Home Depot Inc	HD	0.349	3.030
Intel Corp	INTC	0.345	3.394
International Business Machines Corp	IBM	0.314	1.922
Johnson & Johnson	JNJ	0.319	1.688
McDonald's Corp	MCD	0.314	1.618
Merck & Co Inc	MRK	0.335	2.358
Microsoft Corp	MSFT	0.337	2.947
3M Co	MMM	0.355	2.121
Nike Inc	NKE	0.349	2.300
Pfizer Inc	PFE	0.351	2.676
Procter & Gamble Co	PG	0.319	1.803
Travelers Companies Inc	TRV	0.386	2.459
United Technologies Corp	UTX	0.323	1.927
Verizon Communications Inc	VZ	0.335	2.004
Wal-Mart Stores Inc	WMT	0.322	2.154
UnitedHealth Group Inc	UNH	0.344	2.877

Table 2: This table lists the number of options included in the in-sample (e.g., estimation period) and out-of-sample exercises for each sector ETF. The following sectors are examined: Energy (XLE), Utilities (XLU), Technology (XLK), Materials (XLB), Consumer Staples (XLP), Consumer Discretionary (XLY), Industrials (XLI), Health Care (XLV) and Financials (XLF). The in-sample period covers from March 6, 2006 until end of 2009. The out-of-sample period ranges from the beginning of 2010 until the end of 2012.

	XLE	XLU	XLK	XLB	XLP	XLY	XLI	XLV	XLF
In-sample period									
All options	48568	7341	5609	10959	6411	8251	8323	5611	10606
$m \leq 0.7$	3519	149	98	631	347	365	328	66	956
$0.7 < m \leq 0.95$	16933	2928	2176	4252	2345	3158	3092	2005	3783
$0.95 < m \leq 1.05$	9162	2546	1725	2690	1447	2399	2535	2319	1865
$1.05 < m \leq 1.2$	9669	1488	1284	2202	1495	1758	1746	1082	1954
$m > 1.2$	9285	234	326	1184	777	574	622	139	2048
$\tau \leq 2m$	10511	1887	1186	2833	1980	1932	1822	1314	2949
$2m < \tau \leq 6m$	20987	3588	2665	5171	2634	3864	3918	2744	4615
$\tau > 6m$	17070	1866	1758	2955	1797	2455	2583	1553	3042
Out-of-sample period									
All options	45988	7698	7869	11617	11033	11017	11666	8100	19939
$m \leq 0.7$	5038	875	1130	1401	1721	1428	1842	905	2575
$0.7 < m \leq 0.95$	18475	3928	3306	4678	4709	4804	4664	3947	6596
$0.95 < m \leq 1.05$	8508	2000	1801	2352	2105	2540	2347	2237	3437
$1.05 < m \leq 1.2$	9344	860	1357	2371	1692	1913	2131	980	4262
$m > 1.2$	4630	35	275	815	806	332	682	31	3069
$\tau \leq 2m$	10311	2003	1807	3261	3592	2835	2846	2013	4881
$2m < \tau \leq 6m$	19570	3675	3585	5525	4612	5195	5368	3844	9053
$\tau > 6m$	16107	2020	2477	2831	2829	2987	3452	2243	6005

Table 3: This table reports the results of the Functional Principal Analysis (FPCA) of the IV surfaces of single stocks. The stocks considered are the constituents of the Dow Jones. The second column describes what the top three eigenmodes (EM) of the decomposition represent. The next three columns provide the percentage of variance explained by each of them. The last three columns give the correlation between each principal component (PC) process of the stocks' FPCA and the corresponding principal component process of the S&P 500 FPCA. All numbers are in percentages.

	Top 3 eigenmodes	Var. explained by EM no. k			Correl. of PC no. j with PC of S&P 500 IV		
		$k=1$	$k=2$	$k=3$	$j=1$	$j=2$	$j=3$
AXP	level, skewness, TS	99.6	0.2	0.2	91.6	49.8	40.3
BA	level, TS, skewness	98.8	0.6	0.5	95.4	65.6	43.6
CAT	level, skewness, convexity	91.8	6.3	1.0	92.0	24.7	5.1
CVX	level, skewness, TS	96.8	2.5	0.4	94.5	72.0	56.5
DD	level, skewness, TS	98.8	0.7	0.5	97.3	60.9	43.7
DIS	level, skewness, TS	98.7	0.6	0.5	96.7	44.9	32.9
GE	level, TS, skewness	99.7	0.2	0.1	89.6	46.1	32.3
GS	level, skewness, TS	99.1	0.5	0.3	88.2	34.2	33.8
JPM	level, TS, skewness	99.6	0.2	0.1	91.4	37.1	40.5
KO	level, skewness, TS	98.8	0.7	0.4	95.3	53.7	57.8
T	level, skewness, TS	94.8	3.5	1.1	87.0	57.8	52.9
XOM	level, skewness, TS	86.8	9.4	2.3	85.1	45.2	24.2
HD	level, skewness, TS	98.6	0.9	0.4	88.6	55.6	47.8
INTC	level, skewness, TS	97.6	1.5	0.6	88.0	56.0	33.6
IBM	level, skewness, TS	98.0	1.1	0.7	94.0	59.7	39.7
JNJ	level, skewness, TS	97.7	1.6	0.5	95.4	52.2	42.5
MCD	level, skewness, TS	96.0	2.7	0.8	81.2	55.1	26.4
MRK	level, skewness, TS	98.1	1.0	0.6	87.7	60.8	44.7
MSFT	level, TS, skewness	98.9	0.7	0.3	94.1	48.3	59.1
MMM	level, skewness, TS	97.7	1.4	0.6	97.1	63.3	46.2
NKE	level, TS, skewness	98.5	0.7	0.6	95.4	55.0	57.0
PFE	level, skewness, TS	98.7	0.6	0.5	93.8	44.8	54.3
PG	level, skewness, TS	97.7	1.4	0.7	94.7	51.0	51.5
TRV	level, TS, skewness	99.6	0.2	0.1	90.2	47.8	41.2
UTX	level, skewness, TS	98.4	0.9	0.5	97.8	63.2	53.0
VZ	level, TS, skewness	97.2	1.9	0.6	85.7	29.6	47.7
WMT	level, skewness, TS	97.5	1.7	0.5	85.3	58.5	48.4
CSCO	level, TS, skewness	96.5	2.1	1.0	92.3	29.8	46.3
UNH	level, skewness, TS	99.1	0.6	0.1	85.8	51.5	27.1

Table 4: This table reports the number of options included in the in-sample (e.g., estimation period) and out-of-sample exercises for each stock in the Dow Jones. The in-sample period starts in March 2006, until the end of 2009. The out-of-sample period ranges from the beginning of 2010 until the end of 2012.

	AXP	BA	CAT	CVX	CSCO	DD	DIS	GE	GS	JPM	KO	T	XOM	HD	INTC
In-sample period															
All options	7969	9702	12038	8070	6337	6411	5357	7899	16351	10606	6129	7009	8655	7843	6550
$m \leq 0.7$	531	760	1044	730	657	347	360	456	1958	956	322	622	829	639	570
$0.7 < m \leq 0.95$	2810	3873	4311	2801	2249	2345	1931	2412	5491	3783	2443	2437	3059	2531	2003
$0.95 < m \leq 1.05$	1480	1668	2008	1409	1035	1447	1078	1368	2228	1865	1667	1331	1432	1282	973
$1.05 < m \leq 1.2$	1563	1840	2160	1731	1151	1495	1187	1614	2705	1954	1156	1601	1633	1646	1305
$m > 1.2$	1585	1561	2519	1399	1245	777	801	2049	3969	2048	541	1018	1702	1745	1699
$\tau \leq 2m$	2493	2639	3556	1963	1970	1980	1500	2204	4903	2949	1571	2287	2469	1997	2002
$2m < \tau \leq 6m$	3197	4259	5360	3389	2410	2634	2222	3452	6182	4615	2636	2760	3378	3429	2585
$\tau > 6m$	2279	2804	3122	2718	1957	1797	1635	2243	5266	3042	1922	1962	2808	2417	1963
Out-of-sample period															
All options	10564	11583	13499	10745	8739	11033	11213	10518	14722	19939	8625	9262	10121	11834	11458
$m \leq 0.7$	1405	1885	2147	2141	1182	1721	1712	1542	2099	2575	2028	1381	2237	1940	1370
$0.7 < m \leq 0.95$	4160	3942	4494	4233	3314	4709	4420	3644	4327	6596	3552	3995	3824	4687	4061
$0.95 < m \leq 1.05$	2158	1993	1892	1540	1343	2105	2166	1602	1808	3437	1767	1787	1691	2163	1816
$1.05 < m \leq 1.2$	1959	2114	2286	1593	1563	1692	2079	2034	2560	4262	1041	1737	1515	2164	2501
$m > 1.2$	882	1649	2680	1238	1337	806	836	1703	3928	3069	237	362	854	880	1710
$\tau \leq 2m$	3731	3095	3606	2726	2247	3592	3696	2281	4288	4881	2239	2406	2895	3423	2814
$2m < \tau \leq 6m$	4213	4965	5712	4545	3902	4612	4683	4866	5633	9053	3758	4272	3943	5374	5307
$\tau > 6m$	2620	3523	4181	3474	2590	2829	2834	3371	4801	6005	2628	2584	3283	3037	3337

	IBM	JNJ	MCD	MRK	MSFT	MMM	NKE	PFE	PG	TRV	UNH	UTX	VZ	WMT
In-sample period														
All options	9335	3742	7858	7089	7870	5889	6129	5605	5772	3991	8184	6274	6500	8544
$m \leq 0.7$	1107	260	695	609	705	361	621	451	450	369	648	584	600	638
$0.7 < m \leq 0.95$	3337	1566	3187	2685	2501	2153	2159	1824	2411	1386	2663	2215	2293	3194
$0.95 < m \leq 1.05$	1624	966	1663	1429	1359	1168	1123	986	1405	713	1336	1138	1280	1762
$1.05 < m \leq 1.2$	1854	642	1625	1360	1682	1284	1242	1134	1049	811	1605	1304	1466	1925
$m > 1.2$	1413	308	688	1006	1623	923	984	1210	457	712	1932	1033	861	1028
$\tau \leq 2m$	2649	1018	2025	2055	2522	1596	1660	1333	1707	1011	2072	1474	1922	2291
$2m < \tau \leq 6m$	3731	1406	3417	2890	3086	2242	2404	2422	2244	1593	3601	2693	2552	3596
$\tau > 6m$	2955	1318	2416	2144	2262	2051	2065	1850	1821	1387	2511	2107	2026	2657
Out-of-sample period														
All options	13770	7054	10178	10302	13156	8554	9765	9790	7501	3726	11307	10129	13925	8194
$m \leq 0.7$	2827	1448	2004	1647	1710	1556	1619	1426	1746	553	1478	1963	2422	1903
$0.7 < m \leq 0.95$	4913	3061	4075	4007	4593	3036	3728	3848	3187	1562	4509	3575	5776	3425
$0.95 < m \leq 1.05$	2124	1441	2018	2214	2225	1697	1719	1739	1508	823	2127	1784	2691	1543
$1.05 < m \leq 1.2$	2342	817	1443	1942	2892	1533	1482	1886	860	588	2109	1850	2493	980
$m > 1.2$	1564	287	638	492	1736	732	1217	891	200	200	1084	957	543	343
$\tau \leq 2m$	4078	2056	2767	3383	3230	2511	2765	2135	2211	944	3517	2570	3652	2171
$2m < \tau \leq 6m$	5233	2848	4350	4312	6050	3335	3926	4558	3063	1482	4956	4511	6541	3590
$\tau > 6m$	4459	2150	3061	2607	3876	2708	3074	3097	2227	1300	2834	3048	3732	2433

Table 5: This table reports the root mean-square errors (RMSEs) which result from the estimation for each stock's options, in the in- and out-of-sample periods. The in-sample period starts in March 2006, until the end of 2009. The out-of-sample period ranges from the beginning of 2010 until the end of 2012.

	AXP	BA	CAT	CVX	CSCO	DD	DIS	GE	GS	JPM	KO	T	XOM	HD	INTC
In-sample period															
All options	0.386	0.243	0.305	0.599	0.110	0.213	0.128	0.179	1.418	0.430	0.209	0.213	0.485	0.189	0.089
$m \leq 0.7$	0.418	0.255	0.338	0.543	0.083	0.267	0.062	0.239	1.463	0.453	0.214	0.169	0.482	0.251	0.080
$0.7 < m \leq 0.95$	0.353	0.201	0.327	0.615	0.099	0.221	0.077	0.216	1.639	0.349	0.224	0.219	0.555	0.217	0.085
$0.95 < m \leq 1.05$	0.387	0.293	0.359	0.660	0.129	0.227	0.169	0.188	1.616	0.488	0.201	0.255	0.530	0.201	0.112
$1.05 < m \leq 1.2$	0.429	0.311	0.309	0.650	0.134	0.198	0.180	0.170	1.361	0.542	0.207	0.226	0.482	0.171	0.110
$m > 1.2$	0.384	0.173	0.175	0.442	0.102	0.157	0.088	0.100	0.888	0.375	0.150	0.124	0.269	0.106	0.059
$\tau \leq 2m$	0.198	0.146	0.148	0.322	0.063	0.075	0.077	0.113	0.816	0.223	0.096	0.084	0.221	0.078	0.047
$2m < \tau \leq 6m$	0.352	0.228	0.295	0.530	0.096	0.186	0.116	0.162	1.308	0.352	0.194	0.180	0.410	0.162	0.088
$\tau > 6m$	0.550	0.325	0.429	0.799	0.155	0.325	0.175	0.246	1.902	0.639	0.282	0.330	0.693	0.271	0.118
Out-of-sample period															
All options	0.264	0.274	0.476	0.483	0.100	0.251	0.125	0.095	0.710	0.224	0.233	0.137	0.791	0.150	0.080
$m \leq 0.7$	0.245	0.274	0.403	0.411	0.049	0.213	0.096	0.107	0.611	0.229	0.113	0.097	0.520	0.163	0.069
$0.7 < m \leq 0.95$	0.267	0.282	0.540	0.602	0.097	0.265	0.120	0.119	0.809	0.221	0.197	0.162	1.167	0.160	0.073
$0.95 < m \leq 1.05$	0.301	0.319	0.577	0.523	0.147	0.289	0.169	0.099	1.006	0.283	0.344	0.147	0.478	0.163	0.100
$1.05 < m \leq 1.2$	0.255	0.281	0.476	0.315	0.114	0.239	0.100	0.063	0.732	0.234	0.290	0.096	0.287	0.124	0.088
$m > 1.2$	0.193	0.167	0.310	0.170	0.059	0.138	0.124	0.035	0.388	0.109	0.162	0.059	0.134	0.077	0.065
$\tau \leq 2m$	0.097	0.122	0.148	0.184	0.058	0.082	0.064	0.042	0.336	0.090	0.089	0.041	0.432	0.062	0.040
$2m < \tau \leq 6m$	0.232	0.230	0.440	0.455	0.078	0.213	0.120	0.065	0.577	0.179	0.189	0.085	1.014	0.133	0.051
$\tau > 6m$	0.426	0.400	0.671	0.653	0.148	0.404	0.180	0.144	1.026	0.335	0.346	0.232	0.730	0.229	0.128

	IBM	JNJ	MCD	MRK	MSFT	MMM	NKE	PFE	PG	TRV	UNH	UTX	VZ	WMT
In-sample period														
All options	0.500	0.215	0.375	0.204	0.168	0.350	0.265	0.074	0.331	1.042	0.281	0.288	0.211	0.332
$m \leq 0.7$	0.516	0.245	0.369	0.187	0.129	0.398	0.318	0.078	0.264	0.834	0.257	0.278	0.212	0.344
$0.7 < m \leq 0.95$	0.593	0.240	0.403	0.232	0.188	0.340	0.236	0.087	0.277	0.989	0.289	0.230	0.190	0.305
$0.95 < m \leq 1.05$	0.529	0.219	0.410	0.220	0.201	0.397	0.279	0.093	0.389	1.047	0.289	0.374	0.240	0.392
$1.05 < m \leq 1.2$	0.416	0.174	0.341	0.179	0.172	0.400	0.315	0.060	0.419	1.211	0.240	0.365	0.248	0.365
$m > 1.2$	0.258	0.078	0.183	0.125	0.105	0.173	0.190	0.031	0.216	1.026	0.304	0.160	0.135	0.197
$\tau \leq 2m$	0.251	0.125	0.138	0.104	0.083	0.161	0.127	0.044	0.151	0.442	0.128	0.141	0.105	0.132
$2m < \tau \leq 6m$	0.427	0.193	0.299	0.200	0.161	0.308	0.220	0.056	0.306	0.857	0.271	0.239	0.174	0.257
$\tau > 6m$	0.710	0.283	0.560	0.270	0.236	0.478	0.372	0.104	0.460	1.462	0.373	0.401	0.306	0.499
Out-of-sample period														
All options	0.520	0.199	0.230	0.168	0.092	0.385	0.302	0.077	0.196	0.301	0.215	0.330	0.149	0.150
$m \leq 0.7$	0.379	0.136	0.131	0.129	0.078	0.310	0.278	0.061	0.089	0.224	0.197	0.304	0.108	0.090
$0.7 < m \leq 0.95$	0.542	0.202	0.232	0.196	0.096	0.486	0.324	0.085	0.184	0.311	0.192	0.375	0.181	0.140
$0.95 < m \leq 1.05$	0.690	0.251	0.295	0.187	0.115	0.417	0.375	0.089	0.279	0.352	0.246	0.391	0.155	0.226
$1.05 < m \leq 1.2$	0.526	0.210	0.251	0.124	0.091	0.255	0.267	0.073	0.229	0.294	0.256	0.261	0.103	0.142
$m > 1.2$	0.377	0.072	0.169	0.071	0.056	0.100	0.148	0.035	0.117	0.169	0.173	0.144	0.065	0.083
$\tau \leq 2m$	0.214	0.085	0.092	0.061	0.042	0.131	0.172	0.034	0.080	0.095	0.099	0.121	0.053	0.085
$2m < \tau \leq 6m$	0.439	0.172	0.186	0.147	0.064	0.310	0.269	0.049	0.162	0.244	0.182	0.313	0.105	0.135
$\tau > 6m$	0.753	0.290	0.345	0.266	0.145	0.578	0.412	0.120	0.294	0.431	0.338	0.451	0.248	0.205

Table 6: This table reports the point estimates of the parameters which govern single stock returns' dynamics. Below each estimate, the corresponding standard error is given in italic. The parameter $\theta^{j(\mathbb{P})}$ satisfies $\theta^{j(\mathbb{P})}\kappa_v^{j(\mathbb{P})} = \theta^j\kappa_v^j$. The moments of returns' jumps are estimated from high-frequency data. The estimation period starts in March 2006, until the end of 2009.

	$\kappa_v^{j(\mathbb{P})}$	$\theta^{j(\mathbb{P})}$	$y^{j(\mathbb{P})}$	$\mu^{j(\mathbb{P})}$	$\sigma^{j(\mathbb{P})}$	κ_v^j	θ^j	y^j	μ^j	σ^j	λ_0^j	λ_1^j	σ_v^j	ρ^j	η^j
AXP	5.900	0.001	0.040	0.001	0.017	2.900	0.002	0.080	-0.010	0.110	0.080	4.700	0.300	-0.550	0.850
	1.491		*0.067*			*1.183*	*0.016*	*0.042*	*0.049*	*0.063*	*0.042*	*2.194*	*0.113*	*0.105*	*0.208*
BA	7.000	0.008	0.030	0.000	0.012	2.900	0.020	0.120	-0.040	0.090	0.120	6.300	0.300	-0.600	0.000
	1.666		*0.078*			*1.390*	*0.027*	*0.064*	*0.048*	*0.060*	*0.064*	*1.804*	*0.103*	*0.062*	*0.341*
CAT	3.700	0.028	0.070	0.000	0.013	4.500	0.023	0.040	-0.060	0.130	0.040	5.500	0.200	-0.550	0.250
	2.280		*0.042*			*1.328*	*0.024*	*0.052*	*0.036*	*0.102*	*0.052*	*1.767*	*0.091*	*0.092*	*0.162*
CVX	4.600	0.006	0.050	0.000	0.011	2.500	0.011	0.040	-0.020	0.130	0.040	5.200	0.200	-0.600	0.250
	1.840		*0.060*			*1.543*	*0.023*	*0.033*	*0.039*	*0.037*	*0.033*	*2.427*	*0.115*	*0.050*	*0.189*
DD	5.700	0.002	0.150	-0.001	0.012	5.500	0.002	0.060	-0.070	0.060	0.060	6.700	0.250	-0.600	0.585
	1.214		*0.060*			*1.630*	*0.014*	*0.018*	*0.017*	*0.048*	*0.018*	*1.721*	*0.055*	*0.029*	*0.159*
DIS	6.100	0.002	0.040	0.000	0.014	1.200	0.008	0.060	-0.040	0.090	0.060	5.900	0.450	-0.450	0.750
	1.460		*0.066*			*1.561*	*0.030*	*0.028*	*0.058*	*0.058*	*0.028*	*1.677*	*0.186*	*0.126*	*0.348*
GE	8.500	0.001	0.030	0.001	0.013	3.500	0.003	0.020	-0.020	0.150	0.020	7.600	0.500	-0.500	0.401
	2.837		*0.065*			*1.186*	*0.016*	*0.029*	*0.021*	*0.051*	*0.029*	*1.806*	*0.194*	*0.137*	*0.255*
GS	7.900	0.001	0.070	0.001	0.018	2.400	0.003	0.050	-0.030	0.150	0.050	5.400	0.390	-0.540	0.100
	1.922		*0.024*			*1.504*	*0.022*	*0.010*	*0.013*	*0.017*	*0.010*	*1.876*	*0.057*	*0.020*	*0.152*
JPM	4.600	0.001	0.110	0.001	0.016	2.400	0.002	0.040	-0.010	0.090	0.040	9.500	0.300	-0.600	0.536
	1.572		*0.063*			*1.314*	*0.016*	*0.026*	*0.037*	*0.058*	*0.026*	*2.840*	*0.075*	*0.047*	*0.213*
KO	7.389	0.001	0.027	0.000	0.009	1.649	0.005	0.030	-0.020	0.100	0.030	7.389	0.202	-0.589	0.300
	1.696		*0.013*			*1.124*	*0.011*	*0.015*	*0.042*	*0.053*	*0.015*	*1.877*	*0.105*	*0.116*	*0.164*
T	6.300	0.002	0.090	0.001	0.011	2.600	0.006	0.040	-0.020	0.110	0.040	9.300	0.330	-0.600	0.390
	1.194		*0.047*			*1.626*	*0.014*	*0.028*	*0.037*	*0.063*	*0.028*	*3.443*	*0.061*	*0.059*	*0.176*
XOM	6.360	0.003	0.074	0.000	0.010	2.460	0.007	0.033	-0.060	0.080	0.033	6.050	0.273	-0.666	0.789
	1.243		*0.027*			*1.939*	*0.017*	*0.034*	*0.033*	*0.040*	*0.034*	*1.749*	*0.070*	*0.062*	*0.168*
HD	5.474	0.002	0.067	0.000	0.013	2.460	0.004	0.055	-0.050	0.060	0.055	9.400	0.273	-0.589	0.850
	1.129		*0.028*			*1.410*	*0.022*	*0.019*	*0.017*	*0.023*	*0.019*	*2.650*	*0.070*	*0.022*	*0.133*
INTC	6.500	0.011	0.100	0.000	0.017	2.400	0.029	0.060	-0.060	0.190	0.060	4.900	0.300	-0.550	-0.050
	1.825		*0.033*			*0.840*	*0.028*	*0.029*	*0.051*	*0.060*	*0.029*	*2.407*	*0.118*	*0.096*	*0.243*
IBM	4.700	0.001	0.150	0.001	0.011	2.300	0.003	0.100	-0.010	0.070	0.100	7.500	0.300	-0.500	0.578
	1.163		*0.054*			*2.097*	*0.027*	*0.045*	*0.031*	*0.055*	*0.045*	*4.369*	*0.106*	*0.041*	*0.249*
JNJ	8.200	0.000	0.110	0.000	0.007	2.300	0.000	0.020	-0.020	0.130	0.020	5.500	0.350	-0.650	0.650
	2.341		*0.048*			*1.211*	*0.013*	*0.038*	*0.043*	*0.036*	*0.038*	*2.473*	*0.127*	*0.097*	*0.117*
MCD	6.700	0.011	0.070	0.000	0.010	4.500	0.017	0.040	-0.020	0.080	0.040	6.500	0.400	-0.600	0.600
	0.964		*0.032*			*1.622*	*0.016*	*0.055*	*0.037*	*0.100*	*0.055*	*1.682*	*0.131*	*0.059*	*0.154*
MRK	6.686	0.002	0.037	-0.001	0.013	1.649	0.006	0.033	-0.040	0.130	0.033	9.974	0.223	-0.589	0.199
	1.163		*0.054*			*2.097*	*0.027*	*0.045*	*0.031*	*0.055*	*0.045*	*4.369*	*0.106*	*0.041*	*0.249*
MSFT	6.100	0.008	0.070	0.001	0.014	3.500	0.014	0.020	-0.030	0.090	0.020	8.100	0.200	-0.500	0.900
	1.351		*0.043*			*1.147*	*0.016*	*0.047*	*0.030*	*0.087*	*0.047*	*2.204*	*0.108*	*0.101*	*0.136*
MMM	8.200	0.000	0.050	0.001	0.011	1.900	0.002	0.080	-0.020	0.090	0.080	4.400	0.300	-0.550	0.511
	1.679		*0.021*			*0.456*	*0.022*	*0.037*	*0.027*	*0.026*	*0.037*	*2.371*	*0.061*	*0.039*	*0.123*
NKE	7.029	0.002	0.052	0.000	0.012	2.117	0.006	0.041	-0.060	0.050	0.041	5.474	0.273	-0.547	0.500
	5.285		*2.979*			*1.446*	*5.059*	*3.132*	*0.034*	*2.138*	*3.132*	*3.499*	*1.812*	*2.682*	*0.163*
PFE	6.680	0.001	0.030	0.001	0.016	1.640	0.005	0.070	-0.080	0.150	0.070	9.770	0.310	-0.590	-0.100
	4.934		*2.978*			*0.995*	*5.072*	*3.172*	*0.036*	*2.119*	*3.172*	*7.787*	*1.824*	*2.732*	*0.225*
PG	9.000	0.004	0.100	0.000	0.009	3.300	0.011	0.040	-0.020	0.090	0.040	7.900	0.400	-0.550	0.900
	2.757		*0.030*			*0.595*	*0.010*	*0.040*	*0.045*	*0.049*	*0.040*	*2.112*	*0.118*	*0.051*	*0.101*
TRV	9.000	0.001	0.030	0.000	0.016	4.000	0.002	0.160	-0.010	0.090	0.160	3.500	0.250	-0.550	0.150
	0.900		*0.015*			*0.128*	*0.017*	*0.030*	*0.040*	*0.030*	*0.030*	*0.782*	*0.105*	*0.049*	*0.108*
UTX	5.200	0.010	0.100	0.001	0.011	3.200	0.017	0.040	0.000	0.100	0.040	6.500	0.350	-0.500	0.600
	1.481		*0.045*			*0.997*	*0.017*	*0.053*	*0.044*	*0.106*	*0.053*	*2.131*	*0.106*	*0.100*	*0.184*
VZ	5.700	0.010	0.100	0.000	0.010	3.200	0.017	0.040	0.020	0.120	0.040	6.500	0.350	-0.500	0.400
	1.267		*0.049*			*0.792*	*0.012*	*0.036*	*0.028*	*0.122*	*0.036*	*1.631*	*0.100*	*0.124*	*0.255*
WMT	5.700	0.006	0.100	-0.020	0.190	3.600	0.010	0.040	0.020	0.070	0.040	6.500	0.350	-0.500	0.400
	1.540		*0.040*			*1.170*	*0.022*	*0.038*	*0.081*	*0.081*	*0.038*	*1.955*	*0.121*	*0.130*	*0.219*
CSCO	7.600	0.013	0.130	-0.010	0.050	4.900	0.020	0.060	-0.060	0.150	0.060	3.700	0.250	-0.550	0.900
	1.858		*0.035*			*1.999*	*0.032*	*0.024*	*0.027*	*0.056*	*0.024*	*2.530*	*0.100*	*0.052*	*0.318*
UNH	4.700	0.015	0.140	0.010	0.110	2.400	0.030	0.020	-0.100	0.130	0.020	4.600	0.250	-0.550	0.200
	1.912		*0.039*			*1.428*	*0.021*	*0.053*	*0.054*	*0.036*	*0.053*	*3.698*	*0.078*	*0.077*	*0.122*

Table 7: This table reports the point estimates and standard errors (in italic) of the beta coefficients. The estimation period starts in March 2006, until the end of 2009.

		β^j_{diff}		β^j_{jump}		β^j_v	
		Est.	Std. err.	Est.	Std. err.	Est.	Std. err.
Consumer Staples	KO	0.607	*0.130*	0.607	*0.129*	0.150	*0.375*
	PG	0.450	*0.189*	9.000	*0.408*	0.300	*0.209*
	WMT	0.560	*0.144*	0.650	*0.283*	0.300	*0.412*
Health Care	JNJ	0.410	*0.094*	8.200	*0.229*	0.200	*0.486*
	MRK	0.607	*0.110*	6.686	*0.164*	0.301	*0.446*
	UNH	0.810	*0.258*	1.250	*0.228*	0.000	*0.618*
	PFE	0.600	*0.878*	6.680	*2.264*	0.490	*0.000*
Energy	CVX	0.660	*0.086*	1.100	*0.221*	0.200	*0.520*
	XOM	0.607	*0.119*	1.105	*0.192*	0.259	*0.438*
Technology	IBM	0.530	*0.110*	4.700	*0.446*	0.150	*0.164*
	MSFT	0.800	*0.149*	6.100	*0.545*	0.250	*0.246*
	VZ	0.500	*0.215*	5.700	*0.411*	0.150	*0.216*
	T	0.580	*0.143*	0.950	*0.093*	0.250	*0.093*
	CSCO	1.140	*0.268*	1.000	*0.001*	0.000	*0.463*
	INTC	0.750	*0.103*	6.500	*0.215*	0.250	*0.628*
Industrials	MMM	0.600	*0.162*	8.200	*0.480*	0.200	*0.222*
	UTX	0.650	*0.209*	0.002	*0.382*	0.150	*0.250*
	GE	0.920	*0.163*	0.400	*0.593*	0.000	*0.310*
	BA	0.890	*0.163*	1.450	*0.738*	0.150	*0.283*
	CAT	0.990	*0.150*	0.650	*0.407*	0.450	*0.278*
Materials	DD	0.770	*0.080*	0.300	*0.225*	0.550	*0.690*
Consumer Discretionary	MCD	0.540	*0.210*	0.600	*0.232*	0.150	*0.409*
	HD	0.779	*0.073*	5.474	*0.109*	0.273	*0.349*
	NKE	0.790	*1.053*	7.029	*1.942*	0.157	*1.314*
	DIS	0.900	*0.216*	1.100	*0.244*	0.100	*0.234*
Financials	AXP	1.120	*0.384*	0.550	*0.301*	0.050	*0.171*
	JPM	0.840	*0.219*	0.950	*0.228*	0.250	*0.439*
	TRV	1.200	*0.160*	0.950	*0.000*	0.150	*0.113*
	GS	1.180	*0.199*	0.600	*0.107*	0.200	*0.404*

Table 8: This table reports the results of a regression comparing synthesized VS rates following the procedure described in Carr and Wu (2009) with model-implied VS rates, following equation (12). The sampling frequency is weekly. The first column is based on repeated cross-sectional regressions (Fama-MacBeth). The reported coefficients are averages of the weekly coefficient estimates. Standard errors are calculated using the method of Newey-West with 8 lags. The standard errors of the intercept coefficients are multiplied by 100. The second column reports the results of a pooled panel regression. The last line reports the p-value when testing a hypothesis test with null $\mathcal{H}_0$: the model perfectly represents VS rates, i.e., the intercept is zero and the slope is one.

	Cross-sectional	Panel
Intercept $\times 100$	0.98	-0.27
	(0.20)	*(0.27)*
Slope	0.88	0.96
	(0.02)	*(0.03)*
R^2	0.82	0.70
N	357	9639
$\mathcal{H}_0$ p-value [%]	0.82	0.79

Table 9: This table reports the results of a regression comparing ex ante synthesized VS rates at time $t + \tau$, where VS rates are computed following the procedure described in Carr and Wu (2009), with model-implied predictions at time t, given by $\mathbb{E}^{\mathbb{P}}[\mathrm{VS}_{t+\tau}]$. The model predictions are compared to predictions given by the martingale method, whose prediction of future VS rate is the current VS rate. Cross-sectional regressions (Fama-MacBeth) as well as pooled panel regressions are conducted, for three values of τ: one week (1w), one month (1m) and 6 weeks (5w). For cross-sectional regressions, the reported coefficients are averages of the weekly coefficient estimates. Standard errors are calculated using the method of Newey-West with 8 lags. The standard errors of the intercept coefficients are multiplied by 100. The last line reports the p-value when testing a hypothesis test with null $\mathcal{H}_0$: the prediction perfectly matches future VS rates, i.e., the intercept is zero and the slope is one.

	$\tau = 1w$				$\tau = 1m$				$\tau = 6w$			
	Model		Martingale		Model		Martingale		Model		Martingale	
	Cr. sec.	Panel	Cr. sec.	Panel	Cr. sec.	Panel	Cr. sec.	Panel	Cr. sec.	Panel	Cr. sec.	Panel
Intercept ×100	1.03	0.34	0.58	2.38	1.59	-1.21	1.27	3.06	2.04	-0.23	1.51	3.50
	(0.21)	*(0.32)*	*(0.16)*	*(1.27)*	*(0.30)*	*(0.57)*	*(0.28)*	*(1.19)*	*(0.39)*	*(0.74)*	*(0.35)*	*(1.13)*
Slope	0.86	0.99	0.95	0.71	0.94	1.44	0.87	0.63	0.99	1.42	0.85	0.58
	(0.02)	*(0.05)*	*(0.01)*	*(0.15)*	*(0.06)*	*(0.10)*	*(0.03)*	*(0.14)*	*(0.08)*	*(0.15)*	*(0.05)*	*(0.13)*
R^2	0.78	0.63	0.85	0.51	0.68	0.42	0.67	0.40	0.63	0.26	0.62	0.34
N	356	10324	356	10324	353	10237	353	10237	351	10179	351	10179
$\mathcal{H}_0$ p-value [%]	0.93	0.96	0.97	0.10	0.84	0.19	0.84	0.05	0.80	0.24	0.79	0.03

Table 10: This table reports the results of a regression comparing the average total diffusive variance rate of the bucket of firms with the smallest market capitalizations at the end of 2012 with the average variance rates in other size buckets. The notation 'k/1' is used to refer to the regression of the average diffusive variance rate in bucket k on the rate in bucket 1. Bucket 1 contains the firms with the smallest market capitalizations, bucket 5 those with the largest. In the last column, the variance of GE was removed from the calculation of the average. Standard errors are given in parentheses and are in percentages.

	2/1	3/1	4/1	5/1	5/1 (w/o GE)
Intercept×100	0.30	0.34	-0.94	0.05	0.25
	(0.09)	*(0.09)*	*(0.11)*	*(0.08)*	*(0.09)*
Slope	0.87	0.78	0.59	0.66	0.55
	(0.56)	*(0.60)*	*(0.65)*	*(0.50)*	*(0.56)*
R^2	0.98	0.98	0.97	0.98	0.97

Table 11: This table reports the results of a regression comparing the average idiosyncratic diffusive variance rate of the bucket of firms with the smallest market capitalizations at the end of 2012 with the average variance rates in other size buckets. The notation 'k/1' is used to refer to the regression of the average diffusive variance rate in bucket k on the rate in bucket 1. Bucket 1 contains the firms with the smallest market capitalizations, bucket 5 those with the largest. In the last column, the variance of GE was removed from the calculation of the average. Standard errors are given in parentheses and are in percentages.

	2/1	3/1	4/1	5/1	5/1 (w/o GE)
Intercept×100	0.34	0.18	-0.66	0.09	0.24
	(0.11)	*(0.11)*	*(0.14)*	*(0.09)*	*(0.10)*
Slope	0.93	0.86	0.57	0.74	0.59
	(1.35)	*(1.32)*	*(1.49)*	*(1.14)*	*(1.25)*
R^2	0.93	0.92	0.85	0.98	0.86

Table 12: This table reports the statistics of the returns of four portfolios that are weekly rebalanced depending on the level of idiosyncratic variance (Panel A) and residual idiosyncratic variance (Panel B) of each stock, from the beginning of March 2006 until the end of December 2012. Portfolio 1 contains every week a combination of the 7 stocks with the smallest idiosyncratic variance. Portfolio 4 contains the 7 stocks with the largest idiosyncratic variance. The initial wealth level is 100. The average return, its standard deviation (Std. dev.) and its Sharpe ratio are multiplied by 100. All quantities are quoted on a weekly basis.

	Average return	Std. dev.	Sharpe ratio	Skewness	Kurtosis
Panel A: Sort by idiosyncratic variance					
1	-0.00	1.99	-2.00	-1.31	12.23
2	0.05	2.42	0.85	-0.68	6.29
3	0.18	2.70	5.51	-0.92	8.21
4	0.17	3.79	3.41	-0.57	8.29
Panel B: Sort by residual idiosyncratic variance					
1	-0.00	1.93	-2.01	-0.94	9.70
2	0.10	2.40	2.85	-0.48	5.95
3	0.14	2.78	4.00	-0.93	10.48
4	0.15	3.83	3.06	-0.69	8.06

Table 13: This table reports the statistics of the returns of four portfolios that are weekly rebalanced depending on the level of idiosyncratic 6-month (Panel A) and 1-month (Panel B) integrated variance risk premium (IVRP) of each stock, from the beginning of March 2006 until the end of December 2012. Portfolio 1 contains every week a combination of the 7 stocks with the smallest (i.e., closest to zero) idiosyncratic IVRP. Portfolio 4 contains the 7 stocks with the largest idiosyncratic IVRP. The initial wealth level is 100. The average return, its standard deviation (Std. dev.) and its Sharpe ratio are multiplied by 100. All quantities are quoted on a weekly basis.

	Average return	Std. dev.	Sharpe ratio	Skewness	Kurtosis
	Panel A: Sort by 6-month idiosyncratic IVRP				
1	0.06	3.45	0.90	-0.53	7.54
2	0.06	2.84	0.92	-0.59	9.65
3	0.15	2.44	4.70	-1.14	9.77
4	0.13	2.39	4.06	-0.74	6.99
	Panel B: Sort by 1-month idiosyncratic IVRP				
1	0.03	9.44	0.89	-0.45	7.28
2	0.04	2.84	0.92	-1.11	10.25
3	0.19	2.62	4.70	-0.55	7.31
4	0.14	2.20	4.06	-1.00	8.53

Table 14: This table reports the results of a regression comparing the idiosyncratic IVRP of sectors with total Open Interest (OI) and OI of OTM puts (moneyness ≤ 0.75). OI values are scaled by 10^6. The sampling frequency is weekly. The first column is based on repeated cross-sectional regressions (Fama-MacBeth). The reported coefficients are averages of the weekly coefficient estimates. Standard errors are calculated using the method of Newey-West with 8 lags. The coefficients and standard errors are multiplied by 100. The second column reports the results of a pooled panel regression.

	Total OI		OI of OTM puts	
	Cross-sectional	Panel	Cross-sectional	Panel
Intercept	-1.78	-0.00	-2.05	-0.00
	(0.02)	*(0.00)*	*(0.03)*	*(0.00)*
Slope	-1.67	-1.88	-26.14	-2.29
	(0.06)	*(0.42)*	*(2.38)*	*(0.46)*
R^2	0.41	0.31	0.37	0.16
N	355	2840		

Technical Appendix

A Functional Principal Component Analysis

A.1 Procedure

I first construct for every Wednesday in my sample a smooth estimator of the implied volatility surface $IV_t(m, \tau)$ over a grid of moneynesses $m \in [m_{min}, m_{max}]$ and times-to-maturity $\tau \in [\tau_{min}, \tau_{max}]$, using a non-parametric Nadaraya-Watson estimator as detailed in Härdle (1992) and Aït-Sahalia and Lo (1998). The boundaries of the grid are chosen such that enough data points are available over time to accurately intrapolate the IV surface. Following Cont and da Fonsecca (2002), I calculate the weekly variations $\Delta X_t(m, \tau) = \ln IV_t(m, \tau) - \ln IV_{t-1}(m, \tau)$ of the logarithm of the implied volatility at every grid point and consider the random field $U_t(m, \tau) = \Delta X_t(m, \tau)$. I assume stationarity and apply a Karhunen-Loève (K-L) decomposition to $U_t(m, \tau)$, which is a generalization of the classic PCA to higher dimensional random fields. Intuitively, the K-L expansion provides a second-moment characterization of random surface in terms of orthogonal functions f_k and uncorrelated random variables U_k as follows:

$$U(m, \tau) = \sum_{k=1}^{N_{KL}} U_k f_k(m, \tau).$$

The f_k are the eigenfunctions of the covariance function $K(m, \tau, m', \tau')$ for (m, τ) and $(m', \tau') \in A = [m_{min}, m_{max}] \times [\tau_{min}, \tau_{max}]$, defined as follows:

$$K(m, \tau, m', \tau') = cov(U(m, \tau), U(m', \tau')).$$

The covariance function is numerically computed from the time-series of smooth implied volatility surfaces. Its eigenfunctions result from a spectral decomposition of the function, done by solving the homogeneous Fredholm integral equation of the second kind:

$$\int_A K(x_1, x_2) f_k(x_1) dx_1 = \lambda_k f_k(x_2),$$

where the λ_k are the eigenvalues of the covariance function, x_1 and $x_2 \in A$. A difficulty of solving the Fredholm equation lies in the availability of eigenfunctions. To reduce the dimensionality of the problem I use as in Cont and da Fonsecca (2002) a Garlekin procedure[18], which expresses the eigenfunctions as a linear combination of basis functions h_j:

$$f_k(m,\tau) = \sum_j a_{ij} h_j(m,\tau) + \epsilon. \tag{14}$$

The coefficients a_{ij} define a matrix A. Let scalar product of two surfaces u and $v : A \to \mathbb{R}$ is defined as:

$$\langle u, v \rangle = \int_A u(x)v(x)dx.$$

The error term ϵ is set to be orthogonal to each basis function h_j so that one ultimately has to solve for a finite number of equations. The symmetric matrices B and C are defined as:

$$B_{ij} = \langle h_i, h_j \rangle,$$

$$C_{ij} = \int_A dm d\tau \int_A dm' d\tau' h_i(m,\tau) K(m,\tau,m',\tau') h_j(m',\tau')$$

In practice, each coefficient is calculated numerically using Simpson's rule of integration. The final problem is summarized in the following matrix eigenvalue problem, where one has to solve for D and A, D being a diagonal matrix with the eigenvalues of the covariance function λ_k on its main diagonal: $CA = DBA$.

The eigenfunctions are recovered from equation (14) and the principal components processes $x_k(t)$ obtained by projecting the random surface U_t on the eigenfunctions f_k:

$$x_k(t) = \langle X_t - X_0, f_k \rangle.$$

[18] Alternative methods involving, e.g., wavelets have been proposed to deal with potentially non-smooth surfaces and processes that exhibit non-stationarity or autocorrelation. See for example Phoon, Huang, and Quek (2002)

A.2 Results for the S&P 500

Figure 13: Eigenmodes resulting from the FPCA of weekly S&P 500 options' implied volatilities from March 1st, 2006 until December 31, 2012. The first eigenmode explains 98%, and the sum of the first four eigenmode explain 99.88% of the variation of the IV surface over time.

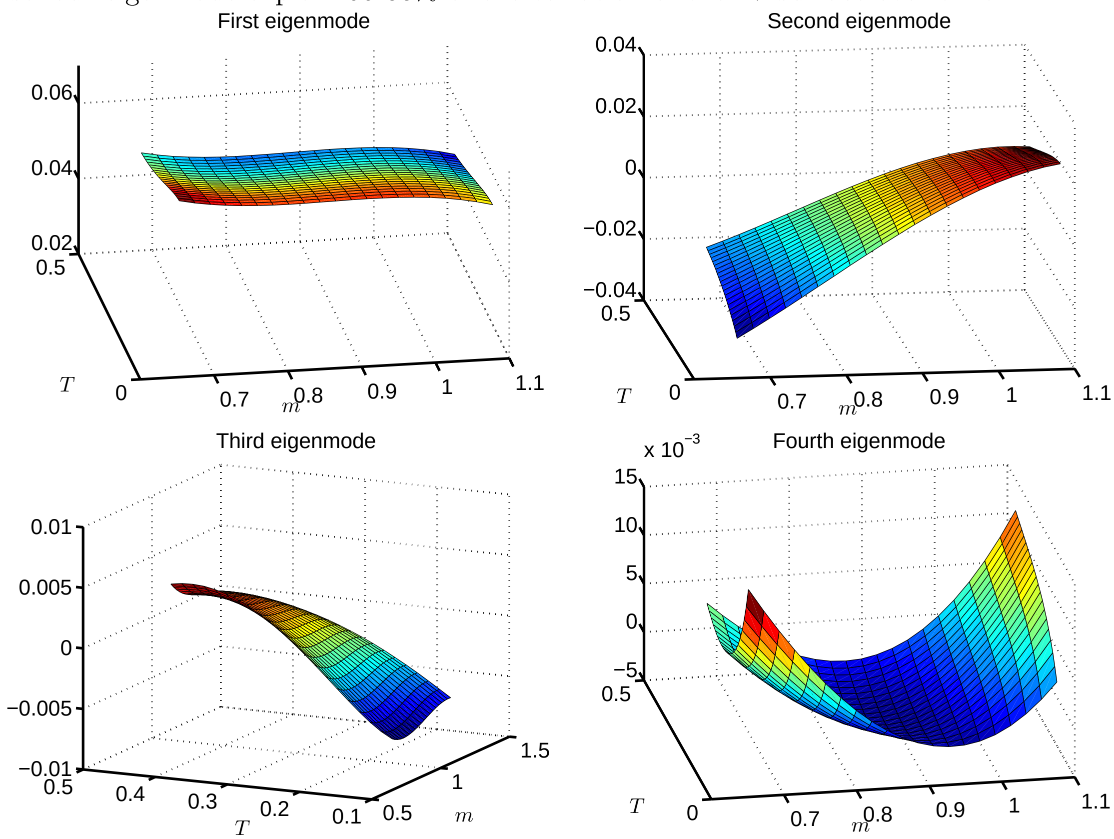
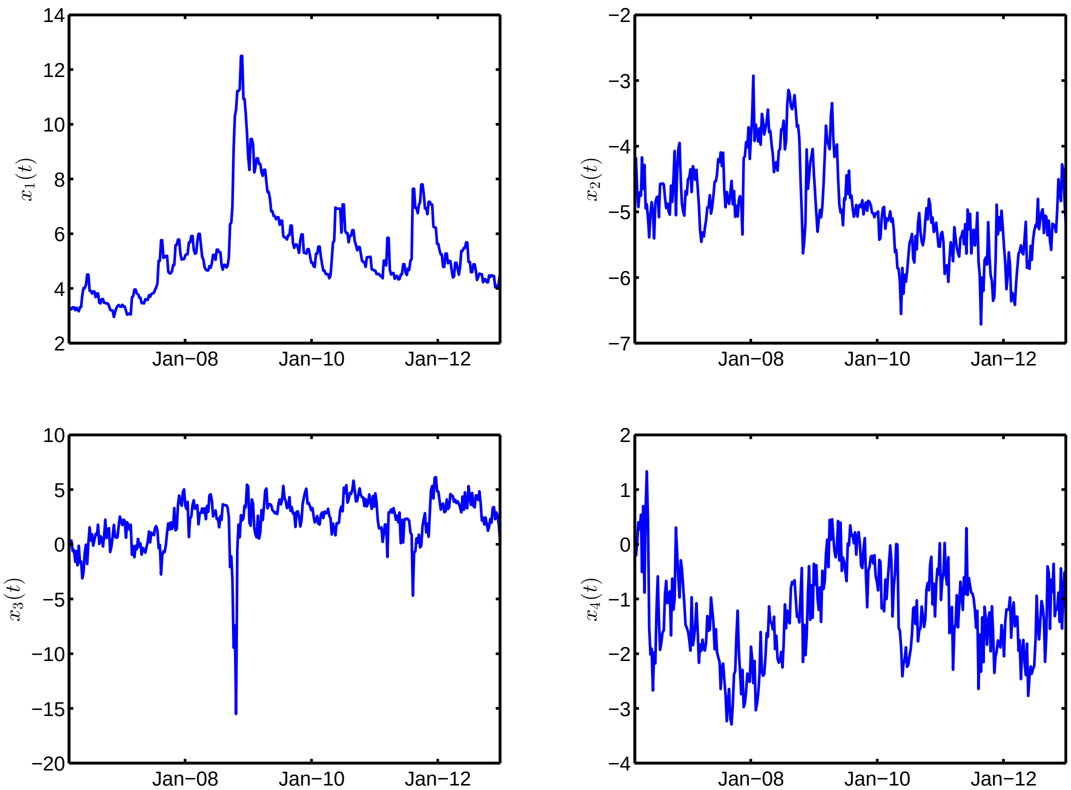


Figure 14: Principal component processes resulting from the FPCA of weekly S&P 500 options' implied volatilities from March 1st, 2006 until December 31, 2012.



B High-frequency data analysis

For each stock I denote by $\Delta_i^n p = p_{\frac{i}{n}} - p_{\frac{i-1}{n}}$ the variation of log-price increments over time grid $\left[\frac{i-1}{n}, \frac{i}{n}\right]$ every day $t = 1, ..., T$. The following measures of variation of log price returns are calculated based on 5-minute tick data. These measures have been introduced for generic semimartingale processes, see Andersen, Bollerslev, Diebold, and Labys (2003), Bollerslev, Todorov, and Li (2013), Mancini (2009), and used extensively since.

The **realized variance** RV_t is defined as the sum of intra-day squared returns, it converges in probability towards the quadratic variation of the underlying return process:

$$RV_t = \sum_{i=tn+1}^{tn+n} |\Delta_i^n p|^2 \rightarrow^{\mathbb{P}} QV_{[t,t+1]},$$

The realized variance can be decomposed into the **continuous variation** CV_t that captures the "small" movements of the returns, which are likely to be generated by a continuous process, and the **jump variation** JV_t, which captures the larger movements, better represented by a jump process:

$$CV_t = \sum_{i=tn+1}^{tn+n} |\Delta_i^n p|^2 \mathbf{1}_{\{|\Delta_i^n p| \leq \alpha_i n^{-\bar{\omega}}\}}.$$

$$JV_t = RV_t - CV_t \rightarrow^{\mathbb{P}} \int_t^{t+1} \int_{\mathbb{R}} x^2 \mu(ds, dx).$$

$\mu(.,.)$ refers to the jump measure of the underlying returns. The threshold α_i is set following Bollerslev and Todorov (2011) and Bollerslev, Todorov, and Li (2013). The estimated mean and standard deviation of jump sizes under the empirical measure are set to the empirical moments of aggregated intra-day returns that exceed the level α_i.

C Change of measure

The market dynamics are as follows under a risk-neutral measure $\mathbb{Q}$, where the parameters without superscript are either $\mathbb{Q}$-parameters or parameters that are invariant under both measures:

$$\begin{aligned}
\frac{dM_t}{M_{t^-}} &= (r - \lambda^M(v_{t^-}^M, m_t)\mathbb{E}^{\mathbb{Q}}[e^{Z_t^M} - 1])dt + \sqrt{v_{t^-}^M} dW_t^M + (e^{Z_t^M} - 1)dN_t^M \\
dv_t^M &= \kappa_v^M(m_t - v_{t^-}^M)dt + \sigma_v^M \sqrt{v_{t^-}^M} dB_t^M + y_t^M dN_t^M \\
dm_t &= \kappa_m^M(\theta^M - m_t)dt + \sigma_m^M \sqrt{m_t} d\bar{B}_t^M
\end{aligned}$$

with $\mathbb{E}^{\mathbb{Q}}[e^{Z_t^M} - 1] = e^{\mu^M + \frac{1}{2}(\sigma^M)^2} - 1$ and

$$\begin{aligned}
dW_t^{M(\mathbb{P})} &= dW_t^{M(\mathbb{Q})} - \eta^M \sqrt{v_{t^-}^M} dt \\
dB_t^{M(\mathbb{P})} &= dB_t^{M(\mathbb{Q})} + \sqrt{v_{t^-}^M} \frac{\kappa_v^{M(\mathbb{P})} - \kappa_v^{M(\mathbb{Q})}}{\sigma_v^M} dt \\
d\bar{B}_t^{M(\mathbb{P})} &= d\bar{B}_t^{M(\mathbb{Q})} + \sqrt{m_t} \frac{\kappa_m^{M(\mathbb{P})} - \kappa_m^{M(\mathbb{Q})}}{\sigma_m^M} dt \\
\theta^{M(\mathbb{P})} &= \frac{\kappa_m^{M(\mathbb{Q})} \theta^{M(\mathbb{Q})}}{\kappa_m^{M(\mathbb{P})}}
\end{aligned}$$

Individual stocks have the following dynamics:

$$\begin{aligned}
\frac{dS_t^j}{S_{t^-}^j} =& rdt + \beta_{diff}^j \sqrt{v_{t^-}^M} dW_t^M + \sqrt{v_{t^-}^j} dW_t^j + (e^{Z_t^j} - 1)dN_t^j - \lambda^j(v_{t^-}^j)\mathbb{E}^{\mathbb{Q}}[e^{Z_t^j} - 1]dt \\
&+ (e^{\beta_{jump}^j Z_t^M} - 1)dN_t^M - \lambda^M(v_{t^-}^M, m_t)\mathbb{E}^{\mathbb{Q}}[e^{\beta_{jump}^j Z_t^M} - 1]dt \\
dv_t^j =& \beta_v^j \, dv_t^M + \kappa_v^j(\theta^j - v_{t^-}^j)dt + \sigma_v^j \sqrt{v_{t^-}^j} dB_t^j + y_t^j dN_t^j.
\end{aligned}$$

with $\mathbb{E}^{\mathbb{Q}}[e^{Z_t^j} - 1] = e^{\mu^j + \frac{1}{2}(\sigma^j)^2} - 1$ and

$$\begin{aligned}
dW_t^{j(\mathbb{P})} &= dW_t^j - \eta^j \sqrt{v_{t^-}^j} dt \\
dB_t^{j(\mathbb{P})} &= dB_t^j + \sqrt{v_{t^-}^j} \frac{\kappa_v^{j(\mathbb{P})} - \kappa_v^j}{\sigma_v^j} dt.
\end{aligned}$$

D Option pricing

Assume that the characteristic functions of the process $\tilde{S}$ takes the following exponential form:

$$
\begin{aligned}
\Psi_{\tilde{S}_T}(t, \tilde{s}_t, v_t^j, v_t^M, m_t; \omega) &:= \mathbb{E}^{\mathbb{Q}}\left[e^{\omega \tilde{S}_T} | \mathcal{F}_t\right] \\
&= e^{\alpha(T-t)\cdot \tilde{S}_t + \beta(T-t)\cdot v_t^j + \gamma(T-t)\cdot v_t^M + \chi(T-t)\cdot m_t + \eta(T-t)},
\end{aligned}
$$

. with $\omega \in \mathbb{C}$. Let us define the joint Laplace transforms of jumps in returns and variances:

$$
\begin{aligned}
\theta_Z(\phi^j, \phi^M) &= \mathbb{E}^{\mathbb{Q}}[e^{\phi^j Z^j + \phi^M Z^M}] = e^{\phi^j \mu^j + \frac{1}{2}(\phi^j \sigma^j)^2}\ e^{\phi^M \mu^M + \frac{1}{2}(\phi^M \sigma^M)^2} \\
\theta_y(\psi^j, \psi^M) &= \mathbb{E}^{\mathbb{Q}}[e^{\psi^j y^j + \psi^M y^M}] = \left(\frac{1}{1-\psi^j y^j}\right)\left(\frac{1}{1-\psi^M y^M}\right).
\end{aligned}
$$

Coefficients are functions of the time-to-maturity and satisfy a system of ODEs, $\forall t \in (0, T]$. The first equation reduces to $\alpha(T-t) = \alpha(0) = \omega$. The remaining ODEs are as follows:

$$
\begin{aligned}
\beta'(T-t) = \;& \omega\left[-\lambda_1^j(\theta_Z(1,0)-1) - \frac{1}{2}\right] - \beta(T-t)\kappa_v^j + \frac{1}{2}\omega^2 + \frac{1}{2}\left(\beta(T-t)\sigma_v^j\right)^2 + \omega\beta(T-t)\sigma_v^j\rho^j + \\
& \lambda_1^j\left[\theta_Z(\omega,0)\theta_y(\beta(T-t),0) - 1\right] \\
\gamma'(T-t) = \;& \omega\left[-\lambda_1^M(\theta_Z(0,\beta^j_{jump})-1) - \frac{1}{2}(\beta^j_{diff})^2\right] - \beta(T-t)\beta_v^j\kappa_v^M - \gamma(T-t)\kappa_v^M + \frac{1}{2}\left(\omega\beta^j_{diff}\right)^2 + \\
& \frac{1}{2}\left(\beta(T-t)\beta_v^j\sigma_v^M\right)^2 + \frac{1}{2}\left(\gamma(T-t)\sigma_v^M\right)^2 + \omega\beta(T-t)\beta^j_{diff}\beta_v^j\sigma_v^M\rho^M + \omega\gamma(T-t)\beta^j_{diff}\sigma_v^M\rho^M + \\
& \beta(T-t)\gamma(T-t)\beta_v^j(\sigma_v^M)^2 + \lambda_1^M\left[\theta_Z(0,\omega\beta^j_{jump})\theta_y(0,\beta(T-t)\beta_v^j + \gamma(T-t)) - 1\right] \\
\chi'(T-t) = \;& -\omega\lambda_2^M\left[\theta_Z(0,\beta^j_{jump}) - 1\right] + \kappa_v^M\left[\beta(T-t)\beta_v^j + \gamma(T-t)\right] - \chi(T-t)\kappa_m + \frac{1}{2}\left(\chi(T-t)\sigma_m\right)^2 + \\
& \lambda_2^M\left[\theta_Z(0,\omega\beta^j_{jump})\theta_y(0,\beta(T-t)\beta_v^j + \gamma(T-t)) - 1\right] \\
\eta'(T-t) = \;& \omega\left[r - \lambda_0^j(\theta_Z(1,0)-1) - \lambda_0^M(\theta_Z(0,\beta^j_{jump}) - 1)\right] + \beta(T-t)\kappa_v^j\theta^j + \chi(T-t)\kappa_m\theta^M + \\
& \lambda_0^j\left[\theta_Z(\omega,0)\theta_y(\beta(T-t),0) - 1\right] + \lambda_0^M\left[\theta_Z(0,\omega\beta^j_{jump})\theta_y(0,\beta(T-t)\beta_v^j + \gamma(T-t)) - 1\right].
\end{aligned}
$$

Boundary conditions are given by $\alpha(0) = \omega$, $\beta(0) = \gamma(0) = \chi(0) = \eta(0) = 0$.

E Integrated Risk Premia

Under the definition given in equation (10), the IERP can be decomposed into an idiosyncratic component $IERP^{Idio}(t,T)$ and a market component $IERP^{Market}(t,T)$, with:

$$
\begin{aligned}
IERP^{Idio}(t,T) =& \eta^j \mathbb{E}_t^{\mathbb{P}}\left[\int_t^T v_s^j ds\right] - \mathbb{E}^{\mathbb{Q}}[e^{Z^j}-1]\left(\mathbb{E}^{\mathbb{P}}\left[\int_t^T \lambda^j(v_s^j)ds\right] - \mathbb{E}^{\mathbb{Q}}\left[\int_t^T \lambda^j(v_s^j)ds\right]\right) \\
& - \frac{1}{2}\left(\mathbb{E}^{\mathbb{P}}\left[\int_t^T v_s^j ds\right] - \mathbb{E}^{\mathbb{Q}}\left[\int_t^T v_s^j ds\right]\right) + \mu^{j(\mathbb{P})}\mathbb{E}^{\mathbb{P}}\left[\int_t^T \lambda^j(v_s^j)ds\right] - \mu^j \mathbb{E}^{\mathbb{Q}}\left[\int_t^T \lambda^j(v_s^j)ds\right].
\end{aligned}
$$

$$
\begin{aligned}
IERP^{Market}(t,T) =& \beta_{diff}^j \eta^M \mathbb{E}_t^{\mathbb{P}}\left[\int_t^T v_s^M ds\right] \\
& - \mathbb{E}^{\mathbb{Q}}[e^{\beta_{jump}^j Z^M}-1]\left(\mathbb{E}^{\mathbb{P}}\left[\int_t^T \lambda^M(v_s^M, m_s)ds\right] - \mathbb{E}^{\mathbb{Q}}\left[\int_t^T \lambda^M(v_s^M, m_s)ds\right]\right) \\
& - \frac{1}{2}(\beta_{diff}^j)^2\left(\mathbb{E}^{\mathbb{P}}\left[\int_t^T v_s^M ds\right] - \mathbb{E}^{\mathbb{Q}}\left[\int_t^T v_s^M ds\right]\right) \\
& + \beta_{jump}^j\left(\mu^{M(\mathbb{P})}\mathbb{E}^{\mathbb{P}}\left[\int_t^T \lambda^M(v_s^M, m_s)ds\right] - \mu^M \mathbb{E}^{\mathbb{Q}}\left[\int_t^T \lambda^M(v_s^j, m_s)ds\right]\right).
\end{aligned}
$$

The calculation of the IERP therefore amounts to computing the expectation of the integrated variances under the empirical and risk-neutral measures. Due to the affine property of the model, these expectations can be written as linear functions of v_t^j, v_t^M and m_t. Coefficients are derived by solving the ODE satisfied by $f(t,T) = \mathbb{E}[v_T]$, under $\mathbb{P}$ and $\mathbb{Q}$, for $v_T = v_T^j$ and $v_T = v_T^M$. The ODE is obtained directly from the stochastic differential equation satisfied by the variance processes. For conciseness, coefficients are not provided but available upon request.

Similarly, the IVRP can be decomposed into an idiosyncratic component $IVRP^{Idio}(t,T)$ and a market component $IVRP^{Market}(t,T)$, such that

$$
\begin{aligned}
IVRP^{Idio}(t,T) =& \frac{1}{T-t}\left(\mathbb{E}_t^{\mathbb{P}}\left[\int_t^T v_s^j ds\right] - \mathbb{E}_t^{\mathbb{Q}}\left[\int_t^T v_s^j ds\right]\right) \\
&+ \frac{1}{T-t}\left(\mathbb{E}_t^{\mathbb{P}}\left[\sum_{n=N_t^j+1}^{N_T^j} (Z_n^j)^2\right] - \mathbb{E}_t^{\mathbb{Q}}\left[\sum_{n=N_t^j+1}^{N_T^j} (Z_n^j)^2\right]\right)
\end{aligned}
$$

and

$$
\begin{aligned}
IVRP^{Market}(t,T) =& \frac{(\beta_{diff}^j)^2}{T-t}\left(\mathbb{E}_t^{\mathbb{P}}\left[\int_t^T v_s^M ds\right] - \mathbb{E}_t^{\mathbb{Q}}\left[\int_t^T v_s^M ds\right]\right) \\
&+ \frac{(\beta_{jump}^j)^2}{T-t}\left(\mathbb{E}_t^{\mathbb{P}}\left[\sum_{n=N_t^j+1}^{N_T^j} (Z_n^M)^2\right] - \mathbb{E}_t^{\mathbb{Q}}\left[\sum_{n=N_t^j+1}^{N_T^j} (Z_n^M)^2\right]\right).
\end{aligned}
$$

The expectation of the squared jumps is obtained using the independence between the jump sizes and the Poisson process driving their occurrence time. For example in the case of the individual stock:

$$
\mathbb{E}_t\left[\sum_{n=N_t^j+1}^{N_T^j} (Z_n^j)^2\right] = [(\mu^j)^2 + (\sigma^j)^2]\int_t^T \mathbb{E}[\lambda^j(v_s^j)]ds.
$$

F Model properties

In this section I examine the relationship between the betas and the risk-neutral moments of the returns. Let $t = t_0 < t_1 < ... < t_n = T$ be a uniform time grid over a given time period $[0, T]$, with $\Delta t = t_i - t_{i-1},\ i > 0$. The total risk-neutral variance of stock j over $[t, T]$ is given by[19]:

$$
\begin{aligned}
\text{TotalVar}^j(t,T) &= \mathbb{E}_t\left[\left(\int_t^T d\log S_u^j - \mathbb{E}_t[d\log S_u^j]\right)^2\right] \\
&= \int_t^T \text{VarRate}^j(u)du + o\left(\int_t^T \text{VarRate}^j(u)du\right)
\end{aligned}
$$

where VarRate is the instantaneous variance rate defined in equation (13) and the rest is a function of the characteristics of the jump distributions. As the variance swap rate, the total variance is

[19]All expectations are taken under $\mathbb{Q}$.

therefore increasing in the betas.

I define the risk-neutral skewness rate of stock j as follows:

$$\begin{aligned}
\text{SkewRate}^j(t) &= \lim_{\Delta t \to 0} \frac{1}{\Delta t^{1/2}} \frac{\mathbb{E}_t\left[\left(\Delta \log S_t^j - \mathbb{E}_t[\Delta \log S_t^j]\right)^3\right]}{\mathbb{E}_t\left[\left(\Delta \log S_t^j - \mathbb{E}_t[\Delta \log S_t^j]\right)^2\right]^{3/2}} \\
&= \text{SkewRate}^M(t)(\beta_{jump}^j)^3 \left(\frac{\text{VarRate}^M(t)}{\text{VarRate}^j(t)}\right)^{3/2} + \text{SkewRate}^{Idio(t)} \left(\frac{\text{VarRate}^{Idio}(t)}{\text{VarRate}^j(t)}\right)^{3/2}
\end{aligned}$$

where $\text{VarRate}^M(t)$ and $\text{SkewRate}^M(t)$ are the spot variance and skewness of the market factor, and $\text{VarRate}^{Idio}(t)$ and $\text{SkewRate}^{Idio}(t)$ are those of stock j obtained by setting $\beta_{jump}^j = \beta_{diff}^j = 0$. As μ^M is typically negative, the market skewness is negative, and pulls the total skewness towards negative values. The idiosyncratic skewness is for typical parameter values close to zero. Note that the instantaneous skewness rate is only non zero because of the jump term. In a diffusion setup, as the third moment of Brownian motion increments is zero, there is no instantaneous skewness. Following Christoffersen, Fournier, and Jacobs (2013), the total skewness of stock j over the time interval $[t, T]$ is defined as:

$$\text{TotalSkew}^j(t) = \frac{\mathbb{E}_t\left[\left(\int_t^T d\log S_u^j - \mathbb{E}_t[d\log S_u^j]\right)^3\right]}{\mathbb{E}_t\left[\left(\int_t^T d\log S_u^j - \mathbb{E}_t[d\log S_u^j]\right)^2\right]^{3/2}}$$

The total skew can be decomposed similarly to the instantaneous skew rate into an idiosyncratic skew and a market skew, whose diffusive and jump parts are respectively proportional to $(\beta_{diff})^3$ and $(\beta_{jump})^3$. A larger $(\beta_{diff})^3$ pushes the skew to be more negative. The impact of $(\beta_{jump})^3$ is, however, not as clear as for the spot skew, because of an additional cross-term that is close to zero for small idiosyncratic jumps.