Commodity Financialization: Risk Sharing and Price Discovery in Commodity Futures Markets

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Abstract

We theoretically study how commodity financialization affects trading behavior, prices and welfare through affecting risk sharing and price discovery in futures markets. In our model, the general equilibrium feature makes financial traders either provide or demand liquidity in the futures market, depending on the information environment. Consistent with recent evidence, commodity financialization reduces the futures price bias through broadening risk sharing and injecting information into the market. Each financial trader loses and final-end commodity consumers benefit in the process of commodity financialization. Commercial hedgers can either lose or win, and their welfare improves with commodity financialization only when the number of active financial traders takes intermediate values.

Keywords: Financialization of commodities, risk sharing, price discovery, liquidity providers/demanders, futures price bias, welfare

JEL Classifications: D82, G14

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1 Introduction

Historically, futures market was introduced for commodity producers (such as farmers) and demanders (such as manufacturers) to share later spot price risks and control costs. Over the past decade, particularly after the year of 2004, commodity futures have become a popular asset to financial investors, such as hedge funds and commodity index traders, and this process has been referred to as the financialization of commodity markets (Basak and Pavlova, 2014; Cheng and Xiong, 2014). There is an emerging empirical literature that documents important facts about the trading behavior of financial traders in futures markets and the resulting pricing implications.

The fundamental role of futures market is to facilitate risk sharing and price discovery. In this paper, we study how commodity financialization affects these functions of futures markets and explore the resulting implications for trading behavior, prices of futures and commodity goods, and welfare of the relevant interest groups. Specifically, we aim to address the following questions: How does the trading of financial traders in the futures market affect the behavior of later spot price? Are financial traders liquidity providers or liquidity demanders in the futures market? How does the presence of more financial traders affect the futures price biases (i.e., the deviations of the future price from the expected later spot price)? How does it affect the risk-sharing and price discovery roles of futures market? Who benefit and who lose in the process of commodity financialization? Why and when? To answer these questions, some formal modeling is needed and our paper takes up this task.

We develop an asymmetric information model with one commodity good (labeled as "wheat") and two periods ($t = 0$ and 1). At date 1, there is a wheat spot market where a group of consumers who derive utility from wheat consumption trade against a group of commodity suppliers (labeled as "growers"). The resulting spot price is determined by two shocks, a demand shock $\tilde{\theta}$ built in consumers' preference and a supply shock $\tilde{c}$ built in growers' production technology. Information is symmetric in the date-1 spot market. At date 0, the growers and K financial traders trade contracts on the future delivery of the commodity. Commodity financialization is parameterized by the number K of financial traders active in the futures market. Both growers and financial traders are risk averse, and growers in the futures market represent commercial hedgers in reality. The futures market features asymmetric information: growers as wheat producers are informed of the supply side shock $\tilde{c}$, while financial traders have diverse private signals about the demand shock $\tilde{\theta}$. This information structure enables us to capture the idea that financial traders can bring new information to the market, and that the more of this type of traders, the more information they bring in aggregate. Growers also make their real production decisions at date 0, through which we establish the real effect of futures market on the later spot market.

We develop two sets of results, one on positive variables and the other on agents' welfare. The positive implications concern the trading behavior of financial traders and the futures and spot prices of the commodity. First, we find that the general equilibrium feature can endogenously make financial traders either provide or demand liquidity in the futures market, depending on the information environment. Specifically, when financial traders have very accurate private information about the demand shock $\tilde{\theta}$, their speculative trading motive is strong, and at this moment they tend to demand liquidity. By contrast, when the supply shock $\tilde{c}$ is very volatile, growers' hedging motive is strong, and so they tend to demand liquidity, which in turn makes financial traders provide liquidity in equilibrium on the other side. In addition, we find that financial traders may change faces in the process of financialization. For instance, financial traders may switch from liquidity providers to demanders as there are more of them in the futures market. These results help to reconcile the mixed empirical evidence that financial traders both provide and demand liquidity in commodity futures market.[1]

Second, we identify a supply channel through which the futures price affects the later spot price (see equation (10) in Lemma 1). In growers' date-0 decision problem, because both the futures contract and their production technology expose them to the same source of risks, growers can effectively treat the futures price as their wheat selling price in making production decisions. As a result, when the presence of more financial traders in futures

[1]For instance, Moskowitz, Ooi and Pedersen (2012) argue that their finding is consistent with that fiancancial traders as speculators provide liquidity. By contrast, according to Kang, Rouwenhorst and Tang (2014), it is hedgers who provide liquidity to speculator in commodity futures market. Cheng, Kirilenko and Xiong (2014) document that financial traders can either provide or demand liquidity at different time periods, and in particular, during the recent crisis, financial traders reduced their commodity futures positions instead of facilitating the hedging needs of hedgers.

market pushes up the futures price, growers will supply more commodities in the later spot market, which will drive down the spot price. This observation can be useful for understanding the recent decline in the crude oil price. Back to 2007-2008, the futures price was very high, toping \$140 per barrel, due to the huge inflow of financial traders. This increased futures price may have caused oil producers to develop more of those oil fields with relatively high marginal costs. After a few years, these oil fields have been fully developed and the new resulting oil wells start to operate, which leads to the oil glut nowadays, causing the price decline.

Third, we show that the futures market can either feature a normal backwardation (i.e., a downward bias in futures price relative to the expected value of the later spot price) or a contango (i.e., an upward bias in future price). When the average production cost is relatively low, a normal backwardation ensues, and otherwise, a contango follows. This is again related to the supply channel we discussed above: when the average production cost is low, growers tend to produce more wheat and therefore short more futures to hedge, thereby depressing the futures price. We then demonstrate that commodity financialization reduces the futures price bias through two effects. The direct effect is that more financial traders in the futures market can share the risk off-loaded by growers. The indirect effect is that more financial traders bring more new information to the market, which, through the futures price, reduces the risks faced by all market participants. Consistent with our finding, Hamilton and Wu (2014) have recently documented that the risk premium in crude oil futures on average decreased and became more volatile since 2005.

Our normative analysis yields insights on how commodity financialization affects the welfare of consumers, financial traders, and particularly, growers (which correspond to commercial hedgers in the futures market in reality). First, we show that as more financial traders come to the futures market, the welfare of consumers increases, while the welfare of each individual financial trader decreases. Intuitively, the presence of more financial traders pushes the futures price up, which induces growers to supply more wheat at a lower price in the spot market, thereby benefiting wheat consumers. Each financial trader loses in the process of commodity financialization simply because the presence of more financial traders reduces the futures price bias, which wipes potential trading opportunities that can be explored by each financial trader as a speculator.

Second and more interestingly, we find that the welfare of growers improves with commodity financialization only when the number of financial traders active in the futures market takes intermediate values. In other words, growers' welfare first decreases, then increases, and finally decreases again with the number of financial traders. This result is in contrast to the traditional view that adding more traders should benefit hedgers through broader risk sharing. Our result arises from the interaction of two competing forces. The positive force is that more financial traders push up the futures price, and since the futures price is the effective wheat selling price of growers, their profit from wheat production improves. The negative force comes from the fact that more financial traders also reduces the trading gains of growers in the futures market. This negative force is particularly strong when the number of financial traders is sufficiently small or sufficiently large. When there are not many traders in the market, the effect of adding a new trader to share the risk is relatively large. When the number of financial traders is sufficiently large and when financial traders have private information, adding a new financial trader will cause all the existing traders to read better information from the price, and through this price discovery channel, the trading gain of each market participant can decrease a lot due to an effect similar to Hirshleifer (1971).

Our paper is broadly related to two strands of literature. The first is the recent growing literature on financialization of commodities. So far, this literature is largely empirical and it documents the trading behavior of financial traders in futures markets and their pricing impact.[2] The theoretical research is relatively scarce. Basak and Pavlova (2014) and Baker (2014) construct dynamic equilibrium models to study asset price dynamics. While their analyses offer important insights, their models feature symmetric information, which is therefore not suitable for our goal of analyzing how financialization affects price discovery in futures markets. In addition, we consider some dimensions that their models do not (such as welfare implications) and their models consider some dimensions that we do not (such as return dynamics), and therefore, our analysis complements theirs.

Recently, Sockin and Xiong (2014) and Goldstein, Li and Yang (2014) also develop asymmetric information models to explore the implications of the financialization of commodity

[2] See Irwin and Sanders (2011) and Cheng and Xiong (2014) for excellent surveys.

futures markets. Our paper and those studies complement each other in many important dimensions. Sockin and Xiong (2014) focus on information asymmetry in the spot market, and their main theory insight is that a high spot price may further spur the commodity demand through an informational channel, which may therefore fuel a short-term bubble in spot prices. In contrast, our analysis focuses on information asymmetry in futures market and the real effect on spot prices is through a supply channel in our setting. Goldstein, Li and Yang (2014) emphasize that in the futures market, because financial institutions are limited to trade in the futures contracts for speculation purposes, while commodities producers trade the futures contracts mostly for hedging, these two groups of traders they may respond to the *same* information in opposite directions. This can lead to a reduction in price informativeness and an increase in the futures risk premium. By contrast, in our setting, financial institutions and commodities producers observe *different* information, and commodity financialization always reduces the futures price biases. Moreover, we have provided both positive and normative analyses, while Sockin and Xiong (2014) and Goldstein, Li and Yang (2014) mainly focus on pricing implications.

Our paper is also related to the classic literature on futures market.[3] This literature has developed theories of "hedging pressure" (Keynes, 1930; Hirshleifer, 1988, 1990) or "storage" (Kaldor, 1939; Working, 1949) to explain futures prices. Notably, the literature has also developed asymmetric information models on futures market (e.g., Grossman, 1977; Danthine, 1978; Bray, 1981; Stein, 1987). However, because commodity financialization is just a recent phenomenon, these models have focused on different research questions, for instance, on whether the futures market is viable (Grossman, 1977), on whether the futures price is fully revealing (Danthine, 1978; Bray, 1981), and on whether speculative trading can reduce welfare (Stein, 1987).

Among these models, Stein (1987) is closest in terms of research topics, but his analysis concerns the comparison between an economy with futures and an economy without, and he shows that introducing a new asset can harm welfare by generating price volatility. However, he does not explore the general implications of financialization for risk sharing and price discovery, such as the effect on the trading behavior of financial traders. Also, his model

[3] See Section 1.1 of Acharya, Lochstoer and Ramadorai (2013) for a brief literature review on this literature.

features an endowment economy, while our setup has commodity production, which is crucial for many of our results. In terms of model setup, our model is closer to Danthine (1978). His main finding is that the futures price provides a sufficient statistics used by rational traders in formulating their probability distributions. The futures price is not fully revealing in our setup, because we have introduced two sources of uncertainty, similar to Bray (1981) and Goldstein and Yang (2014). In addition, our analysis has focused on topics emphasized by the recent empirical literature on financialization of commodities.

2 The Model

We consider an asymmetric information model similar to Grossman (1977), Danthine (1978) and Stein (1987) to study the implications of commodity financialization. There are two consumption goods—a commodity good (call it wheat) and a numeraire good (call it money)—and two tradable assets, a futures contract on the commodity and a risk-free asset (with the net risk-free rate normalized at zero). Time lasts two periods: $t = 0$ and 1. At date 0, $K \in \mathbb{N}$ financial investors, such as hedge funds or commodity index traders, trade futures contracts against $J \in \mathbb{N}$ commodity producers (call them growers). Here we use parameter K to capture financialization of commodities, that is, the process of commodity financialization corresponds to an increase in K.[4] At date 0, growers also make their investment on the commodity production, which in turn determines the commodity supply at the date-1 spot market. At date 1, $J \in \mathbb{N}$ consumers purchase commodity from the spot market, and all agents consume and exit the economy.[5] The timeline of the economy is summarized in Figure 1. We next describe in detail the behavior and information structure of each type of agents.

[4] Alternatively, we can follow Hirshleifer (1988) and assume that it costs $\varphi > 0$ for a financial trader to participate in the futures market to endogenize the number K of financial traders active in the futures market. In this augmented setting, an increase in K can be interpreted as a decrease in φ.

[5] For simplicity, we have assumed that the number of consumers and the number of growers are the same. This assumption is just a normalization and it is without loss of generality.

2.1 Consumers

There are $J > 0$ identical consumers. At date 1, a representative consumer derives utility from the two consumption goods according to the following Marshallian quasi-linear utility function:

$$U_C(y, m) = -\frac{1}{2}y^2 + \tilde{\theta}y + m, \tag{1}$$

where y is the wheat consumption, m is the left-over money, and $\tilde{\theta}$ denotes a preference shock which follows a normal distribution with a mean of $\bar{\theta} \in \mathbb{R}$ and a precision (the inverse of variance) of $\tau_\theta > 0$ (i.e., $\tilde{\theta} \sim N\left(\bar{\theta}, 1/\tau_\theta\right)$).[6] Consumers know their preference shock $\tilde{\theta}$ and the spot price $\tilde{v}$ when they make their consumption choice, which differs from Sockin and Xiong (2014) whose analysis emphasizes the information inference problem in commodity demanders' decisions.

Preference shock $\tilde{\theta}$ can also be interpreted as a technology shock. For instance, consumers do not directly consume wheat, and they have to transform wheat into bread according to a concave technology, $B = -\frac{1}{2}y^2 + \tilde{\theta}y$, where B is the bread output and y is the wheat input. Then, preference (1) follows directly. We are agonistic about both interpretations and the key is that $\tilde{\theta}$ represents demand shocks in the commodity spot market. We also follow Stein (1987) and assume that consumers do not trade in the futures market back at date 0.

The behavior of consumers generates the commodity demand in the spot market. Their effective role in the model is to provide a parsimonious device that determines the wheat spot price. Specifically, let $\tilde{v}$ denote the spot price of wheat and normalize each consumer's initial endowment at 0. Then, a representative consumer's problem is

$$\max_y \left(-\frac{1}{2}y^2 + \tilde{\theta}y - \tilde{v}y\right),$$

which yields the following wheat demand function of each consumer:

$$y = \tilde{\theta} - \tilde{v}. \tag{2}$$

Accordingly, the aggregate wheat demand is $J \times y = J\left(\tilde{\theta} - \tilde{v}\right)$. Note that the spot price $\tilde{v}$ is also a random variable, whose distribution will be endogenously derived in subsequent sections.

[6] Throughout the paper, we use a tilde ($\tilde{}$) to signify a random variable, where a bar denotes its mean and τ denotes its precision. That is, for a random variable $\tilde{z}$, we have $\bar{z} \equiv E(\tilde{z})$ and $\tau_z = \frac{1}{Var(\tilde{z})}$.

2.2 Growers

There are $J > 0$ identical growers who only consume the numeraire good. We make growers risk averse to introduce their hedging motives. Specifically, a representative grower has a constant absolute risk aversion (CARA) utility with a risk aversion coefficient of $\alpha > 0$; that is, his utility function is $U_G(W) = -e^{-\alpha W}$, where W is his final wealth at the end of date 1. Growers make decisions at date 0 and these decisions are twofold. First, they decide how much wheat to produce, which will in turn determine the wheat supply at the date-1 spot market. Second, they decide how many futures to invest in the futures market in order to both hedge their wheat production and to speculate on their private information that is introduced shortly.

Consider a representative grower. If he decides to produce x units of wheat, he needs to pay a production cost[7]

$$C(x) = \tilde{c}x + \frac{1}{2}x^2, \tag{3}$$

where the cost shock $\tilde{c} \sim N(\bar{c}, 1/\tau_c)$ (with $\tau_c > 0$) is independent of the demand shock $\tilde{\theta}$. The cost function (3) is convex in x. The cost shock $\tilde{c}$ is growers' private information. We introduce this shock for two reasons. First, it is empirically relevant that commodity producers indeed have valuable private information when they trade futures. Second, later on, we will allow financial traders to possess private information on $\tilde{\theta}$, and by doing so, we will have a structure that financial traders and growers possess different information, so that the interactions between these two dimensional information can endogenously make financial traders either provide or demand liquidity in equilibrium.

The representative grower also participates in the futures market. The payoff on the futures contract is the wheat spot price $\tilde{v}$ at date 1. Each unit of futures contract is traded at an endogenous price $\tilde{p}$. This price is observable to all market participants and it contains valuable information. So, the grower's information set is $\mathcal{I}_G \equiv \{\tilde{c}, \tilde{p}\}$. His decision is to

[7] The cost function $C(x)$ can be alternatively interpreted as an inventory cost. For instance, suppose that the date-0 wheat spot price is v_0 and carrying an inventory of x units of wheat incurs a cost of $\tilde{c}x + \frac{1}{2}x^2$. Then the total cost of storing x units of wheat is $C(x) = (\tilde{c} + v_0)x + \frac{1}{2}x^2$, which is essentially equation (3) with a renormalization of the mean of $\tilde{c}$. However, this interpretation is made in a partial equilibrium setting since the date-0 spot price v_0 is exogenous. We can fully endogenize this spot price as well at the expense of introducing one extra source of uncertainty, because otherwise the prices of futures and current spot price will jointly fully reveal the shocks (see Grossman, 1977).

choose wheat production x and futures investment d_G (and investment in the risk-free asset) to maximize

$$E\left(-e^{-\alpha \tilde{W}_G}\middle| \tilde{c}, \tilde{p}\right) \tag{4}$$

subject to

$$\tilde{W}_G = \tilde{v}x - \tilde{c}x - \frac{1}{2}x^2 + (\tilde{v} - \tilde{p})\, d_G, \tag{5}$$

where we have normalized grower's initial endowment as 0.

Remark 1 *To better connect our setup to previous models, we have followed the literature (e.g., Danthine, 1978) and interpreted growers as commodity suppliers. In effect, a more precise interpretation of growers should be commercial hedgers, because as become clear later, their futures demand contains a hedging component (see equation (12)). In this sense, growers can be either commodity providers or commodity demanders. Specifically, if $x < 0$, then in equation (5) the term $\tilde{v}x - \tilde{c}x - \frac{1}{2}x^2$ can be interpreted as the quasi-linear utility from consuming $-x$ units of wheat, and so growers are wheat demanders. We can also extend the model to have multiple growers who receive different cost shocks, so that in equilibrium some growers supply wheat while others demand wheat. We abstract from this extension for simplicity. Relatedly, the essential role of consumers in our model is to provide a residual wheat demand function to determine the spot price $\tilde{v}$. To see this, note that growers' information set does not include the later spot price $\tilde{v}$ and thus their decisions on wheat production x does not depend on $\tilde{v}$ (see equation (9)). Therefore, we use consumers' wheat demand to bring $\tilde{v}$ into the market clearing condition of the spot market. In this sense, consumers may be better interpreted as those parties in reality who deal with physical commodities but do not participate in futures market, such as some airline companies.*

2.3 Financial traders

There are $K \geq 0$ financial traders who derive utility only from their consumption of the numeraire good at date 1. They have a CARA utility with a risk aversion coefficient of $\gamma > 0$. Financial traders trade futures only for speculation, not for hedging any real production of commodities. To capture the fact that financial traders can bring new information to the market, we assume that they have private information about the demand shock $\tilde{\theta}$ in the later spot market (recall that growers have private information about the supply shock $\tilde{c}$). In addition, we assume that financial traders have different private information to implement the notion that as more financial traders come to the market, they bring more information in aggregate. Specifically, at date 0, before trading a financial trader k receives a private signal

$$\tilde{s}_k = \tilde{\theta} + \tilde{\varepsilon}_k \text{ with } \tilde{\varepsilon}_k \sim N(0, 1/\tau_\varepsilon) \text{ and } \tau_\varepsilon \geq 0, \tag{6}$$

where $\tilde{\varepsilon}_k$ is independent of each other and of other random variables. Parameter τ_ε controls the quality of their private signals. We allow the possibility of $\tau_\varepsilon = 0$ which corresponds to the case that financial traders do not have private information at all.

Financial traders also observe the future price $\tilde{p}$, and thus financial trader k has an information set $\mathcal{I}_{F,k} \equiv \{\tilde{s}_k, \tilde{p}\}$. His decision problem is to choose date-0 financial investments in futures and the risk-free asset to maximize the conditional expected utility. Specifically, let $d_{F,k}$ be the futures demand of financial trader k, and his decision problem is

$$\max_{d_{F,k}} E\left[-e^{-\gamma(\tilde{v}-\tilde{p})d_{F,k}} \middle| \tilde{s}_k, \tilde{p} \right], \tag{7}$$

where we have also normalized the initial endowment of financial traders to be zero. For simplicity, we have assumed that financial traders do not trade other assets such as stocks. As long as the payoffs of other assets are independent of futures payoff, our results will not change even if more assets are introduced into the economy. However, if the payoffs are correlated across assets, then financial traders may trade futures also for hedging motives, which will complicate the analysis significantly. We do not expect this modification changes the main results of the paper.

3 Equilibrium Characterization

An equilibrium in our economy consists of two subequilibria: the spot market equilibrium at date 1 and the futures market equilibrium at date 0. Intuitively, at date 1, consumers maximize their preference, yielding the wheat demand function, which in turn clears the wheat supply provided by growers at the prevailing spot price $\tilde{v}$. Because the wheat demand depends on the demand shock $\tilde{\theta}$ and the wheat supply depends on cost shock $\tilde{c}$ and futures price $\tilde{p}$, the spot price $\tilde{v}$ is expected to be a function of $\left(\tilde{\theta}, \tilde{c}, \tilde{p}\right)$. At date 0, there is a competitive rational expectations equilibrium (REE) in the futures market. Given growers have private information $\tilde{c}$ and financial traders have private information $\{\tilde{s}_k\}_{k=1}^K$, the futures price $\tilde{p}$ is expected to depend on $(\tilde{c}, \tilde{s}_1, ..., \tilde{s}_K)$; that is, there is a futures price function $p(\tilde{c}, \tilde{s}_1, ..., \tilde{s}_K)$. Growers and financial traders extract information from observing $\tilde{p}$ as well as their own private information to maximize their utility at the prevailing price.

Formally, an equilibrium is defined as follows.

Definition 1 *An equilibrium consists of a spot price function,* $v\left(\tilde{\theta}, \tilde{c}, \tilde{p}\right) : \mathbb{R}^3 \to \mathbb{R}$, *a futures price function,* $p(\tilde{c}, \tilde{s}_1, ..., \tilde{s}_K) : \mathbb{R}^{K+1} \to \mathbb{R}$, *a commodity production policy,* $x(\tilde{c}, \tilde{p}) : \mathbb{R}^2 \to \mathbb{R}$, *a trading strategy of growers,* $d_G(\tilde{c}, \tilde{p}) : \mathbb{R}^2 \to \mathbb{R}$, *and a trading strategy of financial traders,* $d_F(\tilde{s}_k, \tilde{p}) : \mathbb{R}^2 \to \mathbb{R}$, *such that:*
(a) at date 1, spot market clears, $J \times \left[\tilde{\theta} - v\left(\tilde{\theta}, \tilde{c}, \tilde{p}\right)\right] = J \times x(\tilde{c}, \tilde{p}) \Leftrightarrow \tilde{\theta} - v\left(\tilde{\theta}, \tilde{c}, \tilde{p}\right) = x(\tilde{c}, \tilde{p})$;
(b) at date 0, given $\tilde{v}$ *is defined by* $v\left(\tilde{\theta}, \tilde{c}, \tilde{p}\right)$, *(b1)* $x(\tilde{c}, \tilde{p})$ *and* $d_G(\tilde{c}, \tilde{p})$ *solve for growers' problem given by (4) and (5); (b2)* $d_F(\tilde{s}_k, \tilde{p})$ *solves financial traders' problem (7); and (b3) the futures market clears,* $J \times d_G(\tilde{c}, \tilde{p}) + \sum_{k=1}^{K} d_F(\tilde{s}_k, \tilde{p}) = 0$.

We next construct an equilibrium in which the price functions $v\left(\tilde{\theta}, \tilde{c}, \tilde{p}\right)$ and $p(\tilde{c}, \tilde{s}_1, ..., \tilde{s}_K)$ are linear. As standard in the literature, we solve the equilibrium backward from date 1.

3.1 Date-1 spot market equilibrium

The wheat demand is given by equation (2), $y = \tilde{\theta} - \tilde{v}$. The wheat supply is determined by growers' date-0 investment decisions. Solving growers' problem (given by (4) and (5)) yields the following first-order conditions:

$$x + d_G = \frac{E(\tilde{v}|\mathcal{I}_G) - \tilde{p}}{\alpha Var(\tilde{v}|\mathcal{I}_G)}, \tag{8}$$

$$x = \tilde{p} - \tilde{c}. \tag{9}$$

The above expressions are similar to those in Danthine (1978). The intuition is as follows. Given both real investment x and financial investment d_G expose a grower to the same risk source $\tilde{v}$, his overall exposure to this risk is given by the standard demand function of a CARA-investor, as expressed in (8). Expression (9) says that since the grower can sell his wheat at the futures price $\tilde{p}$ (after controlling the total risk given by (8)), he essentially treats $\tilde{p}$ as the wheat selling price when making real production decisions. Aggregating (9) across all growers delivers the aggregate wheat supply at the spot market: $J \times x = J(\tilde{p} - \tilde{c})$. By the market clearing condition $J \times y = J \times x \Leftrightarrow y = x$ and equations (2) and (9), we can solve the spot price $\tilde{v}$, which is given by the following lemma.

Lemma 1 *The date-1 spot price $\tilde{v}$ is given by*

$$\tilde{v} = \tilde{\theta} + \tilde{c} - \tilde{p}. \tag{10}$$

3.2 Date-0 futures market equilibrium

We conjecture the following futures price function:

$$\tilde{p} = p_0 + p_s \tilde{S} + p_c \tilde{c} \text{ with } \tilde{S} \equiv \frac{1}{K} \sum\nolimits_{k=1}^{K} \tilde{s}_k, \tag{11}$$

where p_0, p_c and p_s are undetermined coefficients. Next, we compute the demand function of futures market participants and use the market clearing condition to construct such a linear REE price function.

By (8) and (9), growers' demand for futures is

$$d_G(\tilde{c}, \tilde{p}) = \underbrace{\frac{E(\tilde{v}|\mathcal{I}_G) - \tilde{p}}{\alpha Var(\tilde{v}|\mathcal{I}_G)}}_{\text{speculation}} - \underbrace{(\tilde{p} - \tilde{c})}_{\text{hedging}}. \tag{12}$$

Growers trade futures for two reasons. First, they hedge their real commodity production of $x = \tilde{p} - \tilde{c}$. Second, because they also have private information on their production cost $\tilde{c}$, they speculate on this private information. Given that growers' information set is $\mathcal{I}_G \equiv \{\tilde{c}, \tilde{p}\}$, they can use the price function (11) to back out signal $\tilde{S}$ (provided $p_s \neq 0$), which provides information about the demand component $\tilde{\theta}$ in the later spot price $\tilde{v}$. Thus, growers' conditional forecast is

$$E(\tilde{v}|\mathcal{I}_G) = E\left(\tilde{\theta}\middle|\tilde{S}\right) + \tilde{c} - \tilde{p} = \frac{\tau_\theta \bar{\theta} + K\tau_\varepsilon \tilde{S}}{\tau_\theta + K\tau_\varepsilon} + \tilde{c} - \tilde{p}, \tag{13}$$

$$Var(\tilde{v}|\mathcal{I}_G) = Var\left(\tilde{\theta}\middle|\tilde{S}\right) = \frac{1}{\tau_\theta + K\tau_\varepsilon}. \tag{14}$$

Solving financial trader k's problem in (7), we can compute his futures demand as follows:

$$d_F(\tilde{s}_k, \tilde{p}) = \frac{E(\tilde{v}|\mathcal{I}_{F,k}) - \tilde{p}}{\gamma Var(\tilde{v}|\mathcal{I}_{F,k})}. \tag{15}$$

Clearly, financial trader k trades only for speculating on information $\mathcal{I}_{F,k} \equiv \{\tilde{s}_k, \tilde{p}\}$. Provided $p_c \neq 0$ (which is true in equilibrium), the price $\tilde{p}$ is equivalent to the following signal in predicting $\tilde{v}$:

$$\tilde{s}_p \equiv \frac{\tilde{p} - p_0}{p_c} = \rho \tilde{S} + \tilde{c} \text{ with } \rho \equiv \frac{p_s}{p_c}. \tag{16}$$

Using Bayes' law, we can compute his forecast as follows:

$$Var\left(\tilde{v}|\tilde{s}_k,\tilde{p}\right) = \frac{(K-1)\left(\tau_c+\tau_\theta+K\tau_\varepsilon\right)\rho^2-2K\tau_\varepsilon\left(K-1\right)\rho+K^2\tau_\varepsilon}{\tau_c\left(K-1\right)\left(\tau_\theta+K\tau_\varepsilon\right)\rho^2+K^2\tau_\varepsilon\left(\tau_\theta+\tau_\varepsilon\right)}, \tag{17}$$

$$E\left(\tilde{v}|\tilde{s}_k,\tilde{p}\right) = \bar{\theta}+\bar{c}+\beta_{F,p}\left(\tilde{s}_p-\rho\bar{\theta}-\bar{c}\right)+\beta_{F,s}\left(\tilde{s}_k-\bar{\theta}\right)-\tilde{p}, \tag{18}$$

where

$$\beta_{F,p} = \frac{K\tau_\varepsilon\left(\tau_c\left(K-1\right)\rho+K\left(\tau_\theta+\tau_\varepsilon\right)\right)}{\tau_c\left(K-1\right)\left(\tau_\theta+K\tau_\varepsilon\right)\rho^2+K^2\tau_\varepsilon\left(\tau_\theta+\tau_\varepsilon\right)}, \tag{19}$$

$$\text{and } \beta_{F,s} = \frac{K\tau_\varepsilon\left(-\left(\tau_\theta+K\tau_\varepsilon\right)\rho+K\tau_\varepsilon\right)}{\tau_c\left(K-1\right)\left(\tau_\theta+K\tau_\varepsilon\right)\rho^2+K^2\tau_\varepsilon\left(\tau_\theta+\tau_\varepsilon\right)}. \tag{20}$$

Using the market clearing condition,

$$J\times d_G\left(\tilde{c},\tilde{p}\right)+\sum\nolimits_{k=1}^{K} d_F\left(\tilde{s}_k,\tilde{p}\right)=0, \tag{21}$$

and the expressions of the demand functions, and comparing coefficients, we can establish the following proposition regarding the existence and uniqueness of REE in the date-0 futures market.

Proposition 1 *There exists a linear REE where the futures price $\tilde{p}$ and spot price $\tilde{v}$ are given respectively by*

$$\tilde{p} = p_0+p_s\tilde{S}+p_c\tilde{c},$$
$$\tilde{v} = \tilde{\theta}+\tilde{c}-\tilde{p},$$

where $\tilde{S}\equiv\frac{1}{K}\sum\nolimits_{k=1}^{K}\tilde{s}_k$ and the coefficients p_0, $p_s\geq 0$ and $p_c>0$ are given in the appendix. The equilibrium is characterized by $\rho\equiv\frac{p_s}{p_c}\in\left[\frac{K\tau_\varepsilon}{\tau_\theta+K\tau_\varepsilon+\alpha},\frac{K\tau_\varepsilon}{\tau_\theta+K\tau_\varepsilon}\right]$, with ρ being determined by

$$\rho=\frac{J\frac{K\tau_\varepsilon}{\alpha}+\frac{K}{\gamma}\frac{K\tau_\varepsilon\left[-\left(\tau_\theta+K\tau_\varepsilon\right)\rho+K\tau_\varepsilon\right]}{(K-1)(\tau_c+\tau_\theta+K\tau_\varepsilon)\rho^2-2K\tau_\varepsilon(K-1)\rho+K^2\tau_\varepsilon}}{J\left(\frac{\tau_\theta+K\tau_\varepsilon}{\alpha}+1\right)}. \tag{22}$$

Moreover, if growers' risk aversion coefficient α is sufficiently small, then the equilibrium is unique within the linear price function class.

4 Trading and Pricing Implications

4.1 Liquidity providers and demanders

The empirical literature on commodity financialization finds that financial traders can either provide liquidity or demand liquidity in the futures market (see Cheng and Xiong, 2014; Cheng, Kirilenko and Xiong, 2014; Kang, Rouwenhorst and Tang, 2014). Our model is useful for understanding this phenomenon. In our analysis, we follow Vayanos and Wang

(2012) and define liquidity demanders and providers as follows.

Definition 2 *For any type $t \in \{G, F\}$, if $Cov\left(d_t\left(\mathcal{I}_t\right), \tilde{p}\right) > 0$, then type-t traders are liquidity demanders; that is, they buy the asset when the price goes up and sell the asset when the price goes down. Otherwise, if $Cov\left(d_t\left(\mathcal{I}_t\right), \tilde{p}\right) < 0$, then type-t traders are liquidity providers.*

Intuitively, if a trader demands liquidity, then it is very likely that he initiates the trade. Thus, if he wants to buy the asset, he has to offer a price high enough to attract the other side to engage in the trade. Similarly, if he wants to sell the asset, then he has to sell it at a sufficiently low price to convince the other side to buy in the asset. As a result, a liquidity demander's equilibrium order flow tends to be positively correlated with the equilibrium asset price. By the same token, a liquidity suppler's order flow tends to be negatively correlated with the equilibrium price. Given that financial traders trade against growers in our economy, it must be the case that when financial traders provide liquidity, growers demand liquidity, and vice versa.

In our model, the general equilibrium feature endogenously makes financial traders admit either the role of liquidity demanders or of suppliers, depending on the information environment. Loosely speaking, financial traders tend to demand liquidity when the precision τ_ε of their private information is high and/or when the precision τ_c of the cost shock is high, and they tend to supply liquidity when the opposite is true. The intuition is as follows. First, financial traders speculate on their private information $\tilde{s}_k$, and so if their information is very precise (i.e., if τ_ε is very high), they will trade aggressively, and as a result, their order flows will more likely move prices in the same direction. That is, financial traders demand liquidity when τ_ε is high. Second, growers trade futures both for hedging and for speculation, where the strength of their trading motives is captured by the variance $\frac{1}{\tau_c}$ of the cost shock $\tilde{c}$ in their production technology. Thus, when $\frac{1}{\tau_c}$ is high, or when τ_c is small, growers' trading incentives are strong, and it is more likely for them to demand liquidity, which implies that in equilibrium financial traders have to provide liquidity on the other side. As a consequence, financial traders tends to provide liquidity when τ_c is small. Formally, we have the following proposition.

Proposition 2 *Financial traders tend to supply liquidity when τ_ε or τ_c are small, and they tend to demand liquidity when τ_c is large. That is, for a given $K < \infty$, we have $Cov\left(d_F\left(\tilde{p}, \tilde{s}_k\right), \tilde{p}\right) < 0$ for small values of τ_ε or τ_c, and $Cov\left(d_F\left(\tilde{p}, \tilde{s}_k\right), \tilde{p}\right) > 0$ for large values of τ_c.*

Figure 2 provides an illustration for Proposition 2 under the parameter configuration $\tau_\theta = \alpha = \gamma = 1$, $J = K = 20$, $\bar{\theta} = 5$ and $\bar{c} = 1$. We use "+" to indicate the region of $(\tau_\varepsilon, \tau_c)$ for which financial traders demand liquidity in equilibrium, while the blank region indicates the values of of $(\tau_\varepsilon, \tau_c)$ for which financial traders supply liquidity. Indeed, we find that financial traders tend to demand liquidity when either τ_ε or τ_c is high, and they tend to provide liquidity when the opposite is true. Thus, our analysis shows that financial traders can either demand or provide liquidity depending on the information environment, even when financial traders always behave as speculators in futures market (that is, their demand function (15) does not have a hedging component). This is in sharp contrast to the literature which typically relies on financial traders' hedging need, say due to portfolio concerns, to make them become liquidity demanders (e.g., Cheng, Kirilenko and Xiong, 2014; Kang, Rouwenhorst and Tang, 2014).

Figure 3 conducts a different exercise. Here, we fix the values of $(\tau_\varepsilon, \tau_c)$ and examine how the number K of financial traders affects their liquidity provision/demanding behavior. The general patterns depend on the comparison between τ_c and τ_θ. Specifically, in Panel A, we choose $\tau_c = 2$ and $\tau_\theta = 1$, while in Panel B, we choose $\tau_c = 0.5$ and $\tau_\theta = 1$. In both panels, the other parameters are fixed at $\tau_\varepsilon = 0.1$, $\alpha = \gamma = 1$, $J = 20$, $\bar{\theta} = 5$ and $\bar{c} = 1$. We find that in Panel A, financial traders start to be liquidity providers as K is small, and then they become liquidity demanders when K becomes large. In contrast, in Panel B, financial traders keep providing liquidity independent of the values of K. Note that Panel A of Figure 3 suggests that as more financial traders come to the futures market, it is possible that financial traders may switch from providing liquidity to demanding liquidity.

The result of Figure 3 can be understood as follows. Given that the qualitative difference between Panels A and B happens when K is large, we consider the limiting case of $K \to \infty$. In the limit, the average signal $\tilde{S} \equiv \frac{1}{K}\sum_{k=1}^{K} \tilde{s}_k \to \tilde{\theta}$, and thus growers perfectly know $\tilde{\theta}$ and $\tilde{c}$. As a result, their trade must force the futures price $\tilde{p}$ to fully reveal $\tilde{v}$, that is, $\tilde{p} = \tilde{v} = \frac{\tilde{\theta}+\tilde{c}}{2}$, where the second equality follows from equation (10). Since the price is close to be fully revealing, the speculation component in $d_G(\tilde{c}, \tilde{p})$ is close to $d_F(\tilde{s}_k, \tilde{p})$ except for the different risk aversion coefficients. Thus, the market clearing condition implies

$$\sum\nolimits_{k=1}^{K} d_F(\tilde{s}_k, \tilde{p}) \propto \tilde{p} - \tilde{c} = \frac{\tilde{\theta} + \tilde{c}}{2} - \tilde{c} = \frac{\tilde{\theta} - \tilde{c}}{2}.$$

As a consequence, we have

$$Cov\left(\sum\nolimits_{k=1}^{K} d_F(\tilde{s}_k, \tilde{p}), \tilde{p}\right) \propto Cov\left(\tilde{\theta} - \tilde{c}, \tilde{\theta} + \tilde{c}\right) = \frac{1}{\tau_\theta} - \frac{1}{\tau_c},$$

and thus, $Cov\left(\sum\nolimits_{k=1}^{K} d_F(\tilde{s}_k, \tilde{p}), \tilde{p}\right) > 0$ if and only if $\tau_c > \tau_\theta$.

4.2 Futures price biases

The literature has looked at "futures price bias," that is, the deviation of the futures price from the expectation of the later spot price, $E(\tilde{v} - \tilde{p})$. A downward bias in the futures price is termed "normal backwardation," while an upward bias in the futures price is termed "contango." A major branch of literature on futures pricing has attributed bias to hedging pressures of commodity producers. Hamilton and Wu (2014) document that the futures price bias in crude oil futures on average decreased since 2005. Our model sheds light on how commodity financialization affects the average futures price, the average spot price and the resulting futures price bias.

In the appendix, we show that the futures price bias $E(\tilde{v} - \tilde{p})$ is given as follows:

$$E(\tilde{v} - \tilde{p}) = \frac{\bar{\theta} - \bar{c}}{2A + 1}, \tag{23}$$

where

$$A \equiv \frac{1}{\alpha Var\left(\tilde{\theta} \middle| \tilde{S}\right)} + \frac{K}{J} \frac{1}{\gamma Var(\tilde{v}|\tilde{s}_k, \tilde{p})}. \tag{24}$$

Parameter A is a normalized capacity of the market to absorb risks. To see this, note that by the demand functions (12) and (15) and the market clearing condition (21), the futures price can be understood as determined by J CARA-investors with risk aversion α and K CARA-investors with risk aversion γ, while the effective supply is $J(\tilde{p} - \tilde{c})$, which is the aggregate hedging pressure from growers. Thus, in (24), the conditional variances $Var\left(\tilde{\theta} \middle| \tilde{S}\right)$ and $Var(\tilde{v}|\tilde{s}_k, \tilde{p})$ are the payoff risks faced by growers and financial traders, respectively. We then adjust the payoff risks by their respective risk aversions to capture the effect of preferences. The second term is also adjusted by a ratio of $\frac{K}{J}$ to capture the fact that there are K financial traders while the total futures supply is proportional to the number J of growers.

By equations (23) and (24), we have $E(\tilde{v}-\tilde{p})>0$ if and only if $\bar{c}<\bar{\theta}$. This result is intuitive. When the average cost shock $\bar{c}$ is low, growers tend to produce more wheat and thus they will short more futures to hedge their wheat production, thereby depressing the futures price. But this result is non-trivial, because $\tilde{c}$ affects both the futures price $\tilde{p}$ and the later spot price $\tilde{v}$ in the same direction (see Proposition 1). The key observation is that $\tilde{c}$ affects $\tilde{p}$ more than $\tilde{v}$. Fama and French (1987) used 21 commodities to test the futures risk premium hypothesis, and indeed, they found that some markets feature "normal backwardation," while others feature "contango." According to our theory, this difference can be explained by the relative sizes of the average supply shock and the average demand shock.

Increasing the number K of financial traders tends to decrease futures price bias, that is, $\frac{\partial|E(\tilde{v}-\tilde{p})|}{\partial K}<0$, which is consistent with the empirical evidence provided by Hamilton and Wu (2014). This is because the market's aggregate risk bearing capacity A may increase with K through two channels. First, the newly added financial traders directly share the risk that is loaded off from the hedging needs of growers. Second, since financial traders can bring more information into the market, other existing market participants also learn more information about the futures payoff from reading the futures price, which effectively reduces the payoff risks faced by the market.

Similarly, we can show

$$E(\tilde{p})=\bar{c}+\frac{A}{2A+1}\left(\bar{\theta}-\bar{c}\right) \text{ and } E(\tilde{v})=\bar{\theta}-\frac{A}{2A+1}\left(\bar{\theta}-\bar{c}\right), \tag{25}$$

and thus, parameter K also affects the average futures price $E(\tilde{p})$ and the average spot price $E(\tilde{v})$ through its effect on the capacity A of the market to absorb risks. When $\bar{\theta}>\bar{c}$, growers need to hedge a lot of their wheat production so that the effective futures supply is high, the average futures price $E(\tilde{p})$ increases with the risk absorption capacity A. Due to the increased average futures price, the average spot price $E(\tilde{v})=\bar{\theta}+\bar{c}-E(\tilde{p})$ decreases with A. When $\bar{\theta}<\bar{c}$, the opposite is true.

Summarizing the above discussions yields the following proposition.

Proposition 3 *(a) There is a downward bias (i.e., normal backwardation) in futures price* *relative to the expected value of the later spot price if and only if* $\bar{c} < \bar{\theta}$*. That is,* $E(\tilde{v} - \tilde{p}) > 0$ *if and only if* $\bar{c} < \bar{\theta}$*.*

(b) Suppose the risk aversion α *of growers is sufficiently small.*

(b1) Commodity financialization decreases futures price bias. That is, $\frac{\partial |E(\tilde{v}-\tilde{p})|}{\partial K} < 0$.

(b2) If $\bar{c} < \bar{\theta}$*, then commodity financialization increases average futures price and decreases average spot price. If* $\bar{c} > \bar{\theta}$*, then the opposite is true. That is, if* $\bar{c} < \bar{\theta}$*, then* $\frac{\partial E(\tilde{p})}{\partial K} > 0$ *and* $\frac{\partial E(\tilde{v})}{\partial K} < 0$*, and if* $\bar{c} > \bar{\theta}$*, then* $\frac{\partial E(\tilde{p})}{\partial K} < 0$ *and* $\frac{\partial E(\tilde{v})}{\partial K} > 0$.

Figure 4 graphically illustrates Proposition 3. In the top two panels, we set $\bar{\theta} = 5$ and $\bar{c} = 1$, while in the bottom two panels, we set $\bar{\theta} = 1$ and $\bar{c} = 5$. In all panels, the other parameters are: $\tau_\varepsilon = 0.1, \tau_c = \tau_\theta = \alpha = \gamma = 1$ and $J = 20$. Consistent with Proposition 3, we observe that in Panel A1, there is a downward futures price bias and it declines with the number of the number K of financial traders. In Panel B1, there is an upward bias, and its absolute value also declines with K. In addition, in Panel A2 where $\bar{\theta} > \bar{c}$, the average spot price $E(\tilde{v})$ decreases with K, while the average futures price $E(\tilde{p})$ increases with K, while in Panel B2 where $\bar{\theta} < \bar{c}$, $E(\tilde{v})$ increases with K and $E(\tilde{p})$ decreases with K.

Panel B1 offers a possible explanation for the recent behavior of the crude oil market. Since June 2014, the crude oil price has kept declining from more than \$110 per barrel down to about \$50 per barrel, a more than 50% plunge. Many observers believe that this falling price is predominantly a supply effect.[8] As we know, it takes time to search and develop oil fields. It is plausible that back to 2007-2008, commodity financialization has pushed oil futures price way too high, with its peak close to \$140 per barrel, and oil producers may have started to develop too many oil fields in which they would not invest otherwise. A few years later, these oil fields are fully developed and the new resulting oil wells generate excess oil supply, which is responsible for the price decline in the current oil market.

[8]For instance, in speaking to *Wall Street Journal*, Christine Lagarde, managing director of the International Monetary Fund, says: "What we do first is try to analyze whether it's a supply or demand effect. And in the present circumstances, it's predominantly supply. It's 80% supply, 20% demand." ("How the IMF's Christine Lagarde Sees the World's Economic Hot Spots," *Wall Street Journal*, 2014 December 9).

5 Welfare Implications

5.1 Welfare computations

We use the date-0 ex ante expected utility to represent the welfare of each group of agents. For consumers, we insert their date-1 wheat demand function (2) into their preference expression (1) and use the equilibrium spot price (10) to obtain consumers' date-1 indirect utility as follows:

$$\tilde{U}_{C,1} = \frac{1}{2}\left(\tilde{p} - \tilde{c}\right)^2. \tag{26}$$

Recall that the wheat supply is given by $x = \tilde{p} - \tilde{c}$ in equation (9), and thus consumers' date-1 indirect utility is quadratic in the equilibrium wheat supply. This makes sense, given that consumers' preference is quasi-linear and they have to consume wheat in equilibrium. Taking expectation over $\tilde{U}_{C,1}$ yields consumers' date-0 expected utility (or certainty equivalent given that their preference is linear in money consumption and thus they are risk neutral) as follows:

$$CE_{C,0} \equiv E\left(\tilde{U}_{C,1}\right) = \frac{1}{2}\left[E\left(\tilde{p} - \tilde{c}\right)\right]^2 + \frac{1}{2}Var\left(\tilde{p} - \tilde{c}\right). \tag{27}$$

For financial traders, we can compute their indirect utility after trading as

$$\widetilde{CE}_{F,1} = \frac{\left[E\left(\tilde{v} - \tilde{p}|\mathcal{I}_{F,k}\right)\right]^2}{2\gamma Var\left(\tilde{v}|\mathcal{I}_{F,k}\right)}, \tag{28}$$

which is essentially the trading gains captured by financial traders in the futures market conditional on the realizations of the futures price $\tilde{p}$ and the private signal $\tilde{s}_k$. Then, an argument similar to Grossman and Stiglitz (1980) shows that the date-0 certainty equivalent before the realizations of $\tilde{p}$ and $\tilde{s}_k$ is

$$\begin{aligned} CE_{F,0} &\equiv -\frac{1}{\gamma}\log\left[E\left(e^{-\gamma\widetilde{CE}_{F,1}}\right)\right] \\ &= \frac{\left[E\left(\tilde{v} - \tilde{p}\right)\right]^2}{2\gamma Var\left(\tilde{v} - \tilde{p}\right)} + \frac{1}{2\gamma}\log\left[\frac{Var\left(\tilde{v} - \tilde{p}\right)}{Var\left(\tilde{v}|\mathcal{I}_{F,k}\right)}\right]. \end{aligned} \tag{29}$$

This expression is intuitive. The first term is the certainty equivalent that a trader can obtain without making demands dependent on the equilibrium price or any private information, where the numerator $\left[E\left(\tilde{v} - \tilde{p}\right)\right]^2$ captures the potential gains due to the deviations between futures price and later payoff and the denominator $2\gamma Var\left(\tilde{v} - \tilde{p}\right)$ captures the risk in trading. This term represents the trading gains due to the market making role of financial traders. The second term in (29) represents the additional benefit from trading with superior information (i.e., the benefit associated with the speculation role of financial traders), where $Var\left(\tilde{v} - \tilde{p}\right)$

is the benchmark risk when there is no information, while $Var(\tilde{v}|\mathcal{I}_{F,k})$ is the reduced risk due to the additional information in futures price $\tilde{p}$ and the private signal $\tilde{s}_k$.

Similarly, we can compute the indirect utility of growers after trading and production as follows:

$$\widetilde{CE}_{G,1} = \frac{[E(\tilde{v}-\tilde{p}|\mathcal{I}_G)]^2}{2\alpha Var(\tilde{v}|\mathcal{I}_G)} + \frac{1}{2}(\tilde{p}-\tilde{c})^2, \tag{30}$$

where the first term captures the trading gains from participating in the futures market, while the second term is the expected profit from producing wheat, with the expectation being taken after the realizations of the futures price $\tilde{p}$ and the private information $\tilde{c}$. We can also compute the date-0 ex ante certainty equivalent before the realizations of $\tilde{p}$ and $\tilde{c}$ as follows:

$$\begin{aligned} CE_{G,0} &\equiv -\frac{1}{\alpha}\log\left[E\left(e^{-\alpha\widetilde{CE}_{G,1}}\right)\right] \\ &= \frac{1}{\alpha}\left(Q_0 - \frac{1}{2}Q_1'(I+2\Sigma Q_2)^{-1}\Sigma Q_1\right) + \frac{1}{2\alpha}\log|I+2\Sigma Q_2|, \end{aligned} \tag{31}$$

where I is the 2×2 identity matrix, and

$$\begin{aligned} Q_2 &= \begin{bmatrix} \frac{\tau_\theta + K\tau_\varepsilon}{2} & 0 \\ 0 & \frac{\alpha}{2} \end{bmatrix}, Q_1 = \begin{bmatrix} (\tau_\theta + K\tau_\varepsilon)E(\tilde{v}-\tilde{p}) \\ \alpha E(\tilde{p}-\tilde{c}) \end{bmatrix}, \\ \text{and } Q_0 &= \frac{(\tau_\theta + K\tau_\varepsilon)[E(\tilde{v}-\tilde{p})]^2}{2} + \frac{\alpha}{2}[E(\tilde{p}-\tilde{c})]^2. \end{aligned}$$

Again, in (31), the first term captures the certainty equivalent that a grower can obtain without making decisions based on private information $\tilde{c}$ and the futures price $\tilde{p}$, while the second term is the extra benefit from speculating on superior information.

5.2 Symmetric information economies: The role of risk sharing

We now set $\tau_\varepsilon = 0$, so that the presence of more financial traders in the futures market simply adds more market participants without adding new information to the market. In this case, financial traders can infer the private information $\tilde{c}$ owned by growers, so that the futures market features symmetric information. Since no information on $\tilde{\theta}$ is brought into the market, the variations in futures price $\tilde{p}$ are fully driven by the variations in the cost $\tilde{c}$ of the wheat production technology. In this limiting economy, we can compute the welfare expressions analytically, which are given in the appendix.

Consumers' welfare increases with the number K of financial traders. Intuitively, when more financial traders participate in the futures market, they help to share the risk and push up the futures price $\tilde{p}$. Since the futures price is the effective wheat selling price from the eyes of growers, they produce more wheat in equilibrium. As a result, consumers can consume more wheat at a cheaper price in the date-1 spot market, and their welfare improves.

The welfare of each financial trader decreases with the number K of financial traders in the futures market for two reasons. First, as more financial traders participate in the market, they bring down the futures price bias $[E(\tilde{v}-\tilde{p})]^2$ by Proposition 3, and thus, the first term in (29), which corresponds to the trading gains without information, will decrease. Second, the second term in (29) also decreases with K, because more financial traders, who have the same information as growers, also bring the price $\tilde{p}$ closer to its payoff $\tilde{v}$, which effectively reduces $Var(\tilde{v}-\tilde{p})$.

The effect of commodity financialization on the welfare of growers is ambiguous: increasing K will benefit growers if and only if there are a sufficiently large number of financial traders. This is in contrast to our conventional wisdom that expanding the traders base would benefit growers (as hedgers) through more risk sharing in the market. We can demonstrate the intuition most clearly by examining equation (30), which is the indirect utility $\widetilde{CE}_{G,1}$ post trading and production. We also set $\tau_c = \infty$ so that $\tilde{c} = \bar{c}$. By so doing, we essentially remove the randomness in the futures price $\tilde{p}$, and thus $\widetilde{CE}_{G,1} = CE_{G,0}$. We can show that increasing K affects the two terms of (30), $\frac{[E(\tilde{v}-\tilde{p}|\mathcal{I}_G)]^2}{2\alpha Var(\tilde{v}|\mathcal{I}_G)}$ and $\frac{1}{2}(\tilde{p}-\tilde{c})^2$, in opposite directions: it increases $\frac{1}{2}(\tilde{p}-\tilde{c})^2$ but decreases $\frac{[E(\tilde{v}-\tilde{p}|\mathcal{I}_G)]^2}{2\alpha Var(\tilde{v}|\mathcal{I}_G)}$. First, when there are more financial traders, they can share the risk loaded off by growers, thereby increasing the futures price $\tilde{p}$ and the profit $\frac{1}{2}(\tilde{p}-\tilde{c})^2$ made by growers. Second, the presence of more financial traders also lowers the futures price bias, which therefore reduces the trading gains $\frac{[E(\tilde{v}-\tilde{p}|\mathcal{I}_G)]^2}{2\alpha Var(\tilde{v}|\mathcal{I}_G)}$. This second negative effect dominates when there are not many financial traders in the market and so the futures bias is initially large.

Interestingly, we can also show that the dominance of the negative effect on growers' welfare only arises in our production economy where growers make real investment decisions. In contrast, in an endowment economy where growers are given with an exogenous amount of commodities, increasing the number of financial traders always benefits growers. That is, the negative effect is stronger in our production economy. This is because when the futures price $\tilde{p}$ increases due to the added financial traders, growers also supply more wheat in the later spot price, which therefore also endogenously changes the payoff on the futures contract, making the decrease in the futures price bias particularly severe.

We summarize the above discussions in the following proposition.

Proposition 4 *Suppose* $\tau_\varepsilon = 0$.
(a) Increasing the number K *of financial traders benefits consumers and harms financial traders. That is,* $\frac{\partial CE_{C,0}}{\partial K} > 0$ *and* $\frac{\partial CE_{F,0}}{\partial K} < 0$.
(b) If, in addition $\tau_c = \infty$ *and* $\bar{c} \neq \bar{\theta}$, *growers' welfare improves with financialization if and only if* $\frac{K}{\gamma} > \frac{J}{\alpha}$. *That is, if* $\tau_c = \infty$ *and* $\bar{c} \neq \bar{\theta}$, *then* $\frac{\partial CE_{G,0}}{\partial K} > 0$ *if and only if* $\frac{K}{\gamma} > \frac{J}{\alpha}$.

5.3 Asymmetric information economies: The role of price discovery

Now we allow $\tau_\varepsilon > 0$, so that the futures market participation of financial traders also brings new information about the demand shock $\tilde{\theta}$ into the market. We find that this new feature mainly changes the implication for growers' welfare. Because of the complexity of the welfare expressions, it is difficult to establish analytical results. We therefore use a numerical example to illustrate our analysis. We have tried numerous parameter configurations and found that the pattern we identified is quite robust.

In Figure 5, we choose parameter values similar to those in previous figures. That is, $\tau_\varepsilon = 0.1$, $\tau_c = \tau_\theta = \alpha = \gamma = 1$, $J = 20$, $\bar{\theta} = 5$ and $\bar{c} = 1$. The interesting observation concerns the welfare of growers in Panel A. Recall that in Proposition 4, when $\tau_\varepsilon = 0$, growers' welfare $CE_{G,0}$ first decreases and then increases with K. Now when $\tau_\varepsilon > 0$, we find that $CE_{G,0}$ first decreases, then increases, but finally decreases again with K. This suggests that the negative welfare effect on the trading gains is particularly strong when either K is small or K is large. The intuition for the case of small K is still the same as before, that is, when there are not many financial traders in the market, the futures bias can be large, leaving a large room for it to decline. Now, when $\tau_\varepsilon > 0$, recall that financial traders bring information to the market. Thus, when there are many financial traders who infer information from prices, a newly added financial trader will cause all these traders to reform their forecast, and after aggregating their trading, the price can reveal a lot more information. This price discovery process will hurt the growers' trading gains through a channel similar to the Hirshleifer effect (1970). That is, more financial traders bring more information about the payoff on the futures payoff, thereby destroying the potential trading gains that can be captured by market participants.

Panels B and C of Figure 5 suggest that commodity financialization still benefits consumers and harms financial traders. The aforementioned price discovery effect also adversely affects financial traders. However, since when $\tau_{\varepsilon} = 0$, the welfare of each financial trader has already declined with K, the extra negative effect due to price discovery only strengthens this pattern and will not change the direction. Given that the welfare $CE_{F,0}$ of each financial trader declines with K, if there is a market participation cost $\varphi > 0$ for financial traders to participate in the futures market, then the number K of financial traders active in the market can be uniquely pinned down by setting $CE_{F,0} = \varphi$. This echoes the argument made in our previous footnote 4. Also note that here it is the welfare of each individual financial trader that decreases with K. As a group, financial traders' aggregate welfare $K \times CE_{F,0}$ actually first increases and then decreases with K.

6 Conclusion

In the past decade, there is a sharp increase in the inflow of financial investors into commodity futures markets, which is labelled as the financialization of commodities. In this paper, we develop a model to study the implications of this phenomenon for trading behavior, asset prices and welfare through the lens of risk sharing and price discovery. We find that in the futures market, financial traders, as speculators, can either provide liquidity to or demand liquidity from other market participants such as commercial hedgers, depending on the information environment. Commodity financialization helps to reduce futures price bias, not only because financial traders help to share risk, but also because they bring new information to the market, which reduces the risk faced by all market participants. Commodity financialization generally harms financial traders and benefits the final end consumers. Unlike the conventional wisdom that argues that commercial hedgers benefit from the presence of more market participants, commercial hedgers can actually lose in the process of commodity financialization, because more financial traders active in the futures market also reduces the trading gains of commercial hedgers through bringing down the futures price bias.

Appendix

Proof of Proposition 1

In order to get equation (22) determining ρ, we first plug all the conditional moments into the demand functions of each type of traders, then plug the expressed demand function into the market clearing condition, to write the equilibrium price $\tilde{p}$ as functions of $\left(\tilde{c}, \tilde{S}\right)$, and finally compare compare coefficients. Specifically, in the aggregate order flow, information about $\tilde{c}$ is brought by growers, and $\tilde{S}$ is brought by growers and financial traders. We compute the coefficient on $\tilde{c}$ in the aggregate order flow as $J\left[\frac{1}{\alpha Var\left(\tilde{\theta}\middle|\tilde{S}\right)}+1\right]$ and the coefficient on $\tilde{S}$ is $J\frac{K\tau_\varepsilon}{\alpha}+K\frac{\beta_{F,s}}{\gamma Var(\tilde{v}|\tilde{s}_k,\tilde{p})}$. Thus, we have:

$$\rho=\frac{p_s}{p_c}=\frac{J\frac{K\tau_\varepsilon}{\alpha}+K\frac{\beta_{F,s}}{\gamma Var(\tilde{v}|\tilde{s}_k,\tilde{p})}}{J\left[\frac{1}{\alpha Var\left(\tilde{\theta}\middle|\tilde{S}\right)}+1\right]}. \tag{A1}$$

Plugging the expressions of $\beta_{F,s}$ and $Var\left(\tilde{v}|\tilde{s}_k,\tilde{p}\right)$ into the above expression yields (22).

Examining equation (22), we find that when $\rho=0$, the RHS is positive, and that when $\rho\to\infty$, the RHS is finite. Thus, by the intermediate value theorem, there exists a solution $\rho\in(0,\infty)$ to equation (22). In effect, we can further narrow down the range of ρ as $\left[\frac{K\tau_\varepsilon}{\tau_\theta+K\tau_\varepsilon+\alpha},\frac{K\tau_\varepsilon}{\tau_\theta+K\tau_\varepsilon}\right]$. To see this, suppose $\rho>\frac{K\tau_\varepsilon}{\tau_\theta+K\tau_\varepsilon}$, so that $\beta_{F,s}<0$. Then, by equation (22), we must have $\rho\leq\frac{J\frac{K\tau_\varepsilon}{\alpha}}{J\left(\frac{1}{\alpha Var(\tilde{\theta}|\tilde{S})}+1\right)}=\frac{\frac{K\tau_\varepsilon}{\alpha}}{\frac{\tau_\theta+K\tau_\varepsilon}{\alpha}+1}=\frac{K\tau_\varepsilon}{\tau_\theta+K\tau_\varepsilon+\alpha}$. A contradiction. Thus, we must have $\rho\leq\frac{K\tau_\varepsilon}{\tau_\theta+K\tau_\varepsilon}$ and $\beta_{F,s}\geq 0$. Accordingly, by equation (22), we have $\rho\geq\frac{J\frac{K\tau_\varepsilon}{\alpha}}{J\left(\frac{1}{\alpha Var(\tilde{\theta}|\tilde{S})}+1\right)}=\frac{K\tau_\varepsilon}{\tau_\theta+K\tau_\varepsilon+\alpha}$.

Next, we establish the uniqueness of the equilibrium when α is sufficiently small. Note that only $\frac{\beta_{F,s}}{Var(\tilde{v}|\tilde{s}_k,\tilde{p})}$ depends on ρ in the RHS of (A1). By the expressions of $\beta_{F,s}$ and

$Var\left(\tilde{v}|\tilde{s}_k,\tilde{p}\right)$, we have

$$\begin{aligned}
&\frac{\partial}{\partial\rho}\frac{\beta_{F,s}}{Var\left(\tilde{v}|\tilde{s}_k,\tilde{p}\right)}\\
=\ &\frac{\partial}{\partial\rho}\frac{K\tau_\varepsilon\left(-\left(\tau_\theta+K\tau_\varepsilon\right)\rho+K\tau_\varepsilon\right)}{\left(K-1\right)\left(\tau_c+\tau_\theta+K\tau_\varepsilon\right)\rho^2-2K\tau_\varepsilon\left(K-1\right)\rho+K^2\tau_\varepsilon}\\
=\ &K\tau_\varepsilon\frac{\left[\begin{array}{c}-\left(\tau_\theta+K\tau_\varepsilon\right)\left(\left(K-1\right)\left(\tau_c+\tau_\theta+K\tau_\varepsilon\right)\rho^2-2K\tau_\varepsilon\left(K-1\right)\rho+K^2\tau_\varepsilon\right)\\-\left(-\left(\tau_\theta+K\tau_\varepsilon\right)\rho+K\tau_\varepsilon\right)\left(2\left(K-1\right)\left(\tau_c+\tau_\theta+K\tau_\varepsilon\right)\rho-2K\tau_\varepsilon\left(K-1\right)\right)\end{array}\right]}{\left(\left(K-1\right)\left(\tau_c+\tau_\theta+K\tau_\varepsilon\right)\rho^2-2K\tau_\varepsilon\left(K-1\right)\rho+K^2\tau_\varepsilon\right)^2}.
\end{aligned}$$

The numerator of the above expression is quadratic and it is downward sloping for $\rho\leq\frac{K\tau_\varepsilon}{\tau_\theta+K\tau_\varepsilon}$. In addition, this quadratic numerator is negative at $\rho=\frac{K\tau_\varepsilon}{\tau_\theta+K\tau_\varepsilon}$. Note that in equilibrium, $\rho\in\left(\frac{K\tau_\varepsilon}{\tau_\theta+K\tau_\varepsilon+\alpha},\frac{K\tau_\varepsilon}{\tau_\theta+K\tau_\varepsilon}\right)$, and thus, when α is sufficiently small, $\frac{\partial}{\partial\rho}\frac{\beta_{F,s}}{Var(\tilde{v}|\tilde{s}_k,\tilde{p})}<0$ for all $\rho\in\left[\frac{K\tau_\varepsilon}{\tau_\theta+K\tau_\varepsilon+\alpha},\frac{K\tau_\varepsilon}{\tau_\theta+K\tau_\varepsilon}\right]$. As a result, the RHS of (A1) is downward sloping in ρ, while its RHS is upward sloping. Therefore, uniqueness is established.

Finally, using the market clearing condition, we can compute the expressions of p_0,p_s and p_c as follows:

$$\begin{aligned}
p_0 &= \frac{J\frac{\tau_\theta\bar{\theta}}{\alpha}+K\frac{\left(1-\rho\beta_{F,p}-\beta_{F,s}\right)\bar{\theta}+\left(1-\beta_{F,p}\right)\bar{c}}{\gamma Var(\tilde{v}|\tilde{s}_k,\tilde{p})}}{J\left(\frac{2(\tau_\theta+K\tau_\varepsilon)}{\alpha}+1\right)+\frac{2K}{\gamma Var(\tilde{v}|\tilde{s}_k,\tilde{p})}},\\
p_s &= \frac{J\frac{K\tau_\varepsilon}{\alpha}+\frac{K\left(\beta_{F,p}\rho+\beta_{F,s}\right)}{\gamma Var(\tilde{v}|\tilde{s}_k,\tilde{p})}}{J\left(\frac{2(\tau_\theta+K\tau_\varepsilon)}{\alpha}+1\right)+\frac{2K}{\gamma Var(\tilde{v}|\tilde{s}_k,\tilde{p})}},\\
p_c &= \frac{J\left(\frac{\tau_\theta+K\tau_\varepsilon}{\alpha}+1\right)+\frac{K\beta_{F,p}}{\gamma Var(\tilde{v}|\tilde{s}_k,\tilde{p})}}{J\left(\frac{2(\tau_\theta+K\tau_\varepsilon)}{\alpha}+1\right)+\frac{2K}{\gamma Var(\tilde{v}|\tilde{s}_k,\tilde{p})}}.
\end{aligned}$$

Note that $p_s\geq0$ and $p_c>0$ because $J>0$, $K\geq0$ and both $\beta_{F,p}$ and $\beta_{F,s}$ are non-negative.

Proof of Proposition 2

We prove $Corr\left(d_F,\tilde{p}\right)<0$ for small values of τ_ε or τ_c by considering two limiting economies. First, for any given $\tau_c\in[0,\infty)$, when $\tau_\varepsilon\to0$, we have $Corr\left(d_F,\tilde{p}\right)\to-1$. To see this, by setting $\tau_\varepsilon=0$, we can use Proposition 1 to show

$$\tilde{p}=\frac{\tau_\theta\left(\frac{J}{\alpha}+\frac{K}{\gamma}\right)}{2\tau_\theta\left(\frac{J}{\alpha}+\frac{K}{\gamma}\right)+J}\bar{\theta}+\frac{\tau_\theta\left(\frac{J}{\alpha}+\frac{K}{\gamma}\right)+J}{2\tau_\theta\left(\frac{J}{\alpha}+\frac{K}{\gamma}\right)+J}\tilde{c},$$

$$d_F\left(\tilde{s}_k,\tilde{p}\right)=\frac{E\left(\tilde{v}-\tilde{p}|\tilde{s}_k,\tilde{p}\right)}{\gamma Var\left(\tilde{v}|\tilde{s}_k,\tilde{p}\right)}\propto\tilde{p}-\tilde{c}=\frac{\tau_\theta\left(\frac{J}{\alpha}+\frac{K}{\gamma}\right)}{2\tau_\theta\left(\frac{J}{\alpha}+\frac{K}{\gamma}\right)+J}\left(\bar{\theta}-\tilde{c}\right).$$

Thus, as long as $Var(\tilde{c}) > 0$, we have $Corr(\tilde{p} - \tilde{c}, \tilde{p}) = -1$. Second, for any given $\tau_\varepsilon > 0$, when $\tau_c \to 0$, we also have $Corr(d_F, \tilde{p}) \to -1$. Note that when $\tau_c \to 0$, we have $Var(\tilde{c}) \to \infty$, and thus the variations in $d_F(\tilde{s}_k, \tilde{p})$ and $\tilde{p}$ are primarily driven by variations in $\tilde{c}$. Again, by Proposition 1, we can show that as long as $\tau_\varepsilon > 0$, the coefficient p_c on $\tilde{c}$ in the price $\tilde{p}$ is given by

$$p_c = \frac{J\left(\frac{\tau_\theta + K\tau_\varepsilon}{\alpha} + 1\right) + \frac{K}{\gamma Var(\tilde{v}|\tilde{s}_k, \tilde{p})}}{J\left(\frac{2(\tau_\theta + K\tau_\varepsilon)}{\alpha} + 1\right) + \frac{2K}{\gamma Var(\tilde{v}|\tilde{s}_k, \tilde{p})}} \in (0,1)$$

and the coefficient of $\tilde{c}$ in $d_F(\tilde{s}_k, \tilde{p})$, it is proportional to

$$1 - 2p_c - \frac{J}{J\left(\frac{2(\tau_\theta + K\tau_\varepsilon)}{\alpha} + 1\right) + \frac{2K}{\gamma Var(\tilde{v}|\tilde{s}_k, \tilde{p})}} < 0.$$

Thus, $Corr(d_F, \tilde{p}) \to -1$ as $\tau_c \to 0$ for a given $\tau_\varepsilon > 0$.

We prove $Corr(d_F, \tilde{p}) > 0$ for large values of τ_c by considering a limiting economy of $\tau_c \to \infty$. Setting $\tau_c \to \infty$ in Proposition 1, we can show

$$\tilde{p} = \frac{\left(\frac{J}{\alpha} + \frac{K}{\gamma}\right)\tau_\theta\bar{\theta} + \left(\frac{J}{\alpha} + \frac{K}{\gamma}\right)(\tau_\theta + K\tau_\varepsilon)\bar{c} + J\bar{c} + \left(\frac{J}{\alpha} + \frac{K}{\gamma}\right)K\tau_\varepsilon\tilde{S}}{J + 2\left(\frac{J}{\alpha} + \frac{K}{\gamma}\right)(\tau_\theta + K\tau_\varepsilon)},$$

and $d_F \propto \tilde{p} - \bar{c}$. As a result, we have $Corr(d_F, \tilde{p}) = 1$.

Proof of Proposition 3

By the demand functions (12) and (15) and the market clearing condition (21), we can show

$$\bar{v} - \bar{p} = \frac{J(\bar{p} - \bar{c})}{\frac{J}{\alpha Var(\tilde{\theta}|\tilde{S})} + \frac{K}{\gamma Var(\tilde{v}|\tilde{s}_k, \tilde{p})}} = \frac{\bar{p} - \bar{c}}{A}, \tag{A2}$$

where A is given by (24) and $\bar{v} = E(\tilde{v})$ and $\bar{p} = E(\tilde{p})$. Then, we use the expression of $\tilde{v}$ in (10) to obtain

$$\bar{v} = \bar{\theta} + \bar{c} - \bar{p}. \tag{A3}$$

From equations (A2) and (A3), we can solve

$$\begin{aligned} \bar{p} &= \bar{c} + \frac{A}{2A+1}\left(\bar{\theta} - \bar{c}\right), \\ \bar{v} &= \bar{\theta} - \frac{A}{2A+1}\left(\bar{\theta} - \bar{c}\right), \\ \bar{v} - \bar{p} &= \frac{\bar{\theta} - \bar{c}}{2A+1}. \end{aligned}$$

Clearly, $\bar{v} - \bar{p} > 0$ if and only if $\bar{\theta} > \bar{c}$.

Also, K affects $\bar{p}$ and $\bar{v}$ through affecting A. Given the expressions of $Var\left(\tilde{\theta}\middle|\tilde{S}\right)$ and

$Var\left(\tilde{v}|\tilde{s}_k,\tilde{p}\right)$ in (14) and (17), we can express A as follows:

$$A=\frac{\tau_\theta+K\tau_\varepsilon}{\alpha}+\frac{K}{J}\frac{\tau_c\left(K-1\right)\left(\tau_\theta+K\tau_\varepsilon\right)\rho^2+K^2\tau_\varepsilon\left(\tau_\theta+\tau_\varepsilon\right)}{\gamma\left[\left(K-1\right)\left(\tau_c+\tau_\theta+K\tau_\varepsilon\right)\rho^2-2K\tau_\varepsilon\left(K-1\right)\rho+K^2\tau_\varepsilon\right]}.$$

Suppose $\tau_\varepsilon>0$. Then, when α is small, the derivative $\frac{\partial A}{\partial K}$ is mainly driven by the first term, which increases with K. That is, $\frac{\partial A}{\partial K}\approx\frac{\tau_\varepsilon}{\alpha}>0$. This effect occurs through the information effect on growers' trading. If $\tau_\varepsilon=0$, then,

$$A=\frac{\tau_\theta}{\alpha}+\frac{K}{J}\frac{\tau_c\tau_\theta}{\gamma\left(\tau_c+\tau_\theta\right)},$$

and still, increasing K will increase A directly. This effect occurs through the effect of expanding the traders population. Therefore, independent of the value of τ_ε, A increases with K for small levels of α. All the other results follow immediately.

Proof of Proposition 4

By Proposition 1, we can show that when $\tau_\varepsilon=0$, we have

$$\begin{aligned}\tilde{p} &= \frac{\tau_\theta\left(\frac{J}{\alpha}+\frac{K}{\gamma}\right)}{2\tau_\theta\left(\frac{J}{\alpha}+\frac{K}{\gamma}\right)+J}\bar{\theta}+\frac{\tau_\theta\left(\frac{J}{\alpha}+\frac{K}{\gamma}\right)+J}{2\tau_\theta\left(\frac{J}{\alpha}+\frac{K}{\gamma}\right)+J}\tilde{c},\\ \tilde{p}-\tilde{c} &= \frac{\tau_\theta\left(\frac{J}{\alpha}+\frac{K}{\gamma}\right)}{2\tau_\theta\left(\frac{J}{\alpha}+\frac{K}{\gamma}\right)+J}\left(\bar{\theta}-\tilde{c}\right).\end{aligned}$$

Thus, we can compute

$$Var\left(\tilde{p}-\tilde{c}\right) = \left[\frac{\tau_\theta\left(\frac{J}{\alpha}+\frac{K}{\gamma}\right)}{2\tau_\theta\left(\frac{J}{\alpha}+\frac{K}{\gamma}\right)+J}\right]^2\frac{1}{\tau_c}, \tag{A4}$$

$$\left[E\left(\tilde{p}-\tilde{c}\right)\right]^2 = \left[\frac{\tau_\theta\left(\frac{J}{\alpha}+\frac{K}{\gamma}\right)}{2\tau_\theta\left(\frac{J}{\alpha}+\frac{K}{\gamma}\right)+J}\right]^2\left(\bar{\theta}-\bar{c}\right)^2. \tag{A5}$$

It is clear that both $\left[E\left(\tilde{p}-\tilde{c}\right)\right]^2$ and $Var\left(\tilde{p}-\tilde{c}\right)$ increase with K.

Given the expression of $CE_{C,0}$ in (27), we know that $\frac{\partial CE_{C,0}}{\partial K}>0$.

We can also show that

$$\widetilde{CE}_{F,1} = \frac{1}{2\gamma\tau_\theta}(\tilde{p}-\tilde{c})^2\left(\frac{J}{\frac{J}{\alpha}+\frac{K}{\gamma}}\right)^2 \Rightarrow$$

$$CE_{F,0} = \frac{1}{2\gamma}\log\left[1+2\left(\frac{J\tau_\theta}{\sqrt{2\tau_\theta}}\frac{1}{2\tau_\theta\left(\frac{J}{\alpha}+\frac{K}{\gamma}\right)+J}\right)^2\frac{1}{\tau_c}\right]$$

$$+\frac{1}{\gamma}\frac{\left(\frac{J\tau_\theta}{\sqrt{2\tau_\theta}}\frac{1}{2\tau_\theta\left(\frac{J}{\alpha}+\frac{K}{\gamma}\right)+J}\right)^2\left(\bar{\theta}-\bar{c}\right)^2}{1+2\left(\frac{J\tau_\theta}{\sqrt{2\tau_\theta}}\frac{1}{2\tau_\theta\left(\frac{J}{\alpha}+\frac{K}{\gamma}\right)+J}\right)^2\frac{1}{\tau_c}}.$$

As K increases, both the first term and the second term of $CE_{F,0}$ decrease. Thus, $CE_{F,0}$ decreases with K.

For growers, we can show

$$\widetilde{CE}_{G,1} = \frac{1}{2}(\tilde{p}-\tilde{c})^2\left[\frac{1}{\alpha\tau_\theta}\left(\frac{J}{\frac{J}{\alpha}+\frac{K}{\gamma}}\right)^2+1\right] \Rightarrow$$

$$CE_{G,0} = \frac{1}{2\alpha}\log\left[1+\alpha\left[\frac{1}{\alpha\tau_\theta}\left(\frac{J}{\frac{J}{\alpha}+\frac{K}{\gamma}}\right)^2+1\right]Var\left(\tilde{p}-\tilde{c}\right)\right]$$

$$+\frac{1}{2}\frac{\left[\frac{1}{\alpha\tau_\theta}\left(\frac{J}{\frac{J}{\alpha}+\frac{K}{\gamma}}\right)^2+1\right]}{1+\alpha\left[\frac{1}{\alpha\tau_\theta}\left(\frac{J}{\frac{J}{\alpha}+\frac{K}{\gamma}}\right)^2+1\right]Var\left(\tilde{p}-\tilde{c}\right)}\left[E\left(\tilde{p}-\tilde{c}\right)\right]^2.$$

Now suppose $\tau_c = \infty$ and $\bar{\theta} \neq \bar{c}$. Then, the first term in $CE_{G,0}$ vanishes by (A4), and then combined with (A5), direct computation shows $\frac{\partial CE_{G,0}}{\partial K} > 0 \Leftrightarrow \frac{K}{\gamma} > \frac{J}{\alpha}$.

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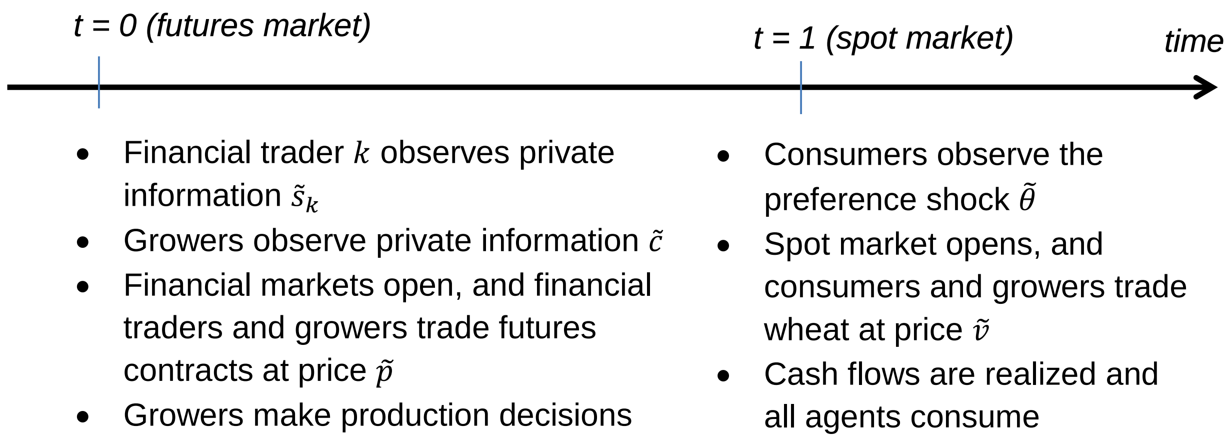
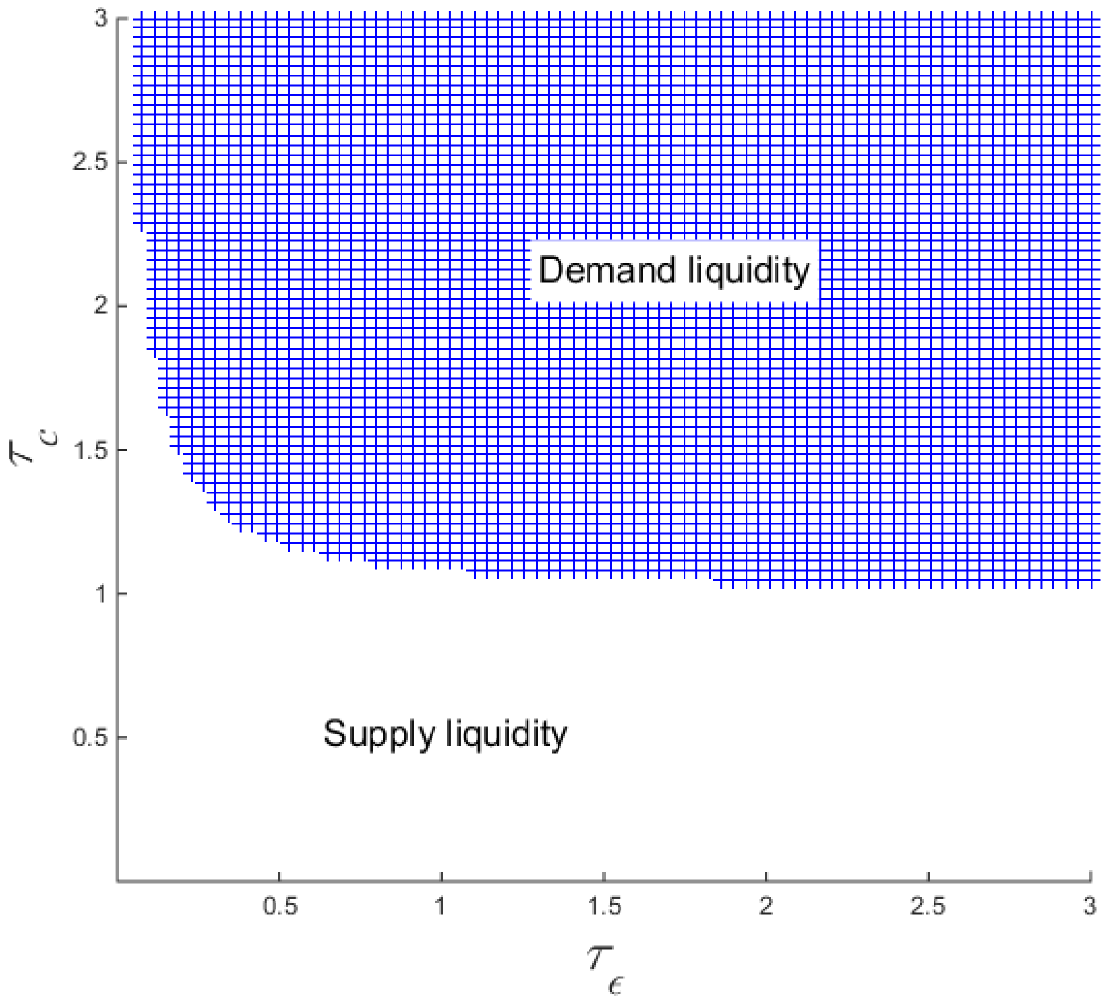
Figure 1: Timeline



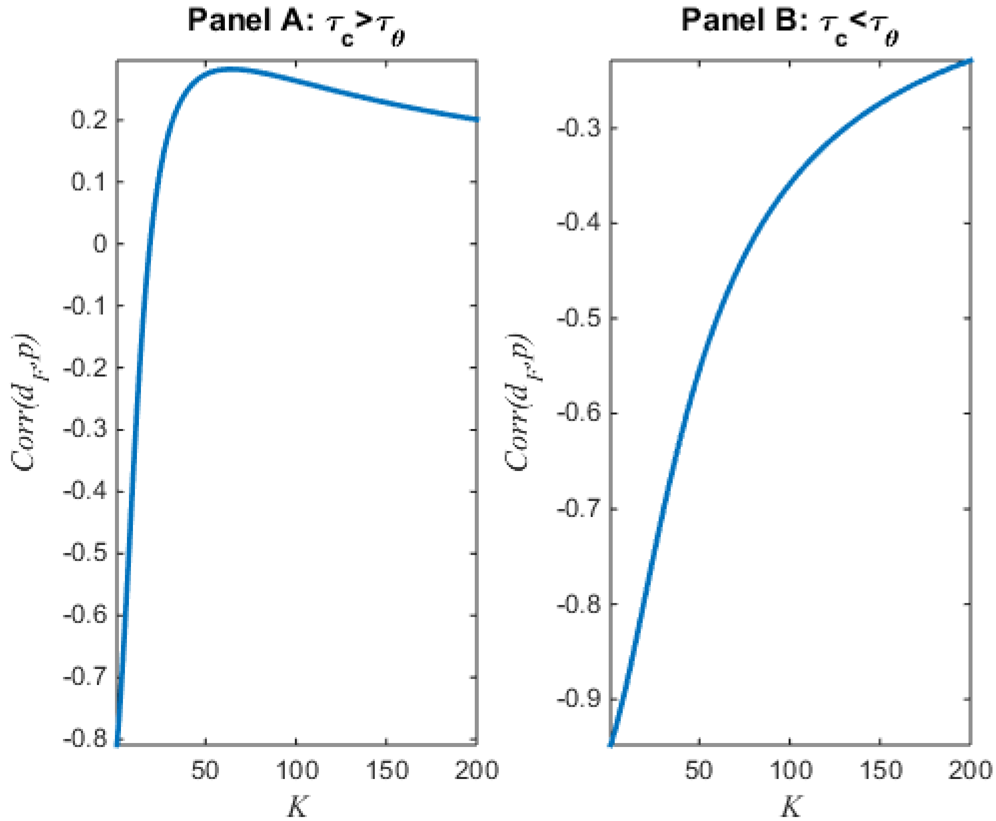
Note: This figure plots the order of events in the economy.

Figure 2: The Region of Financial Traders as Liquidity Suppliers/Demanders



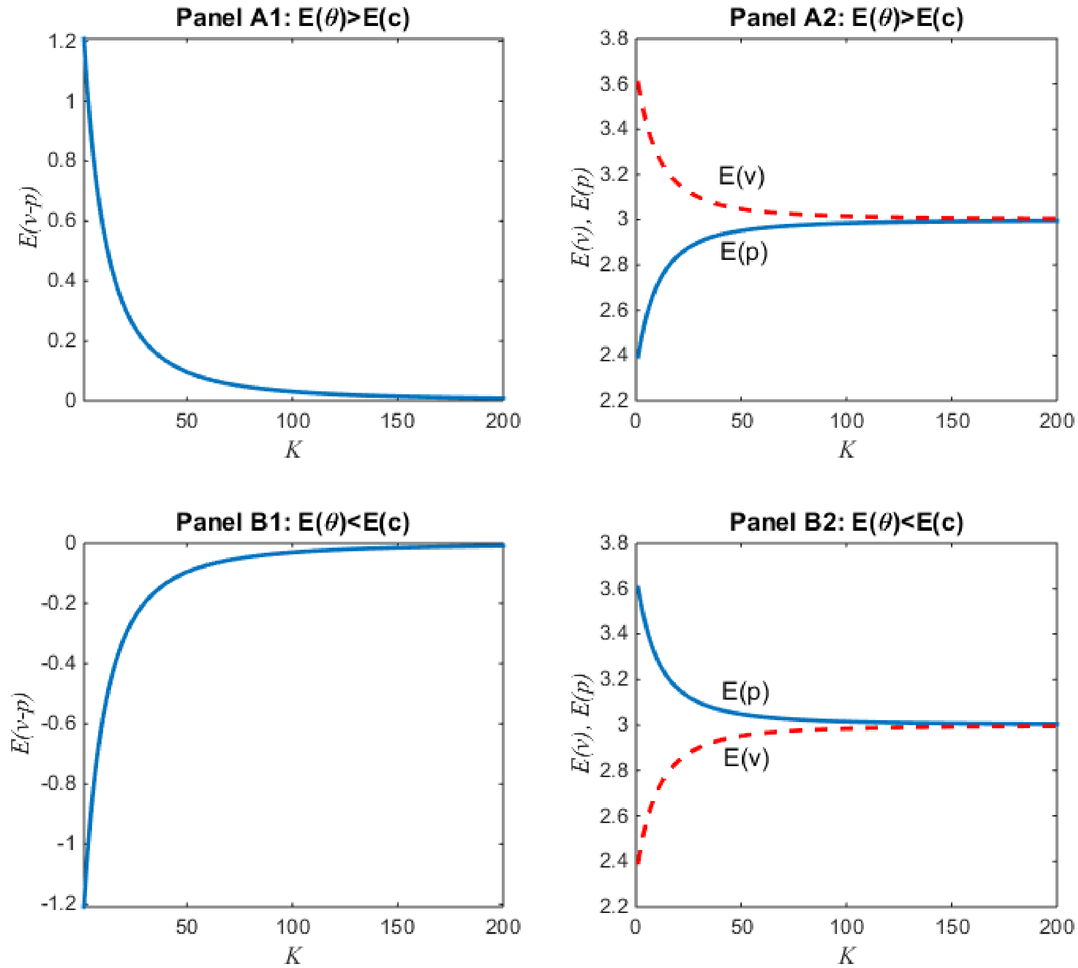
Note: The symbol "+" indicates the region for which financial traders demand liquidity in equilibrium. Financial traders demand liquidity if $Cov(d_F, \tilde{p}) > 0$, and they supply liquidity if $Cov(d_F, \tilde{p}) < 0$. The parameter values are: $\tau_\theta = \alpha = \gamma = 1, J = K = 20, \bar{\theta} = 5$ and $\bar{c} = 1$.

Figure 3: Trading Implications of Commodity Financialization



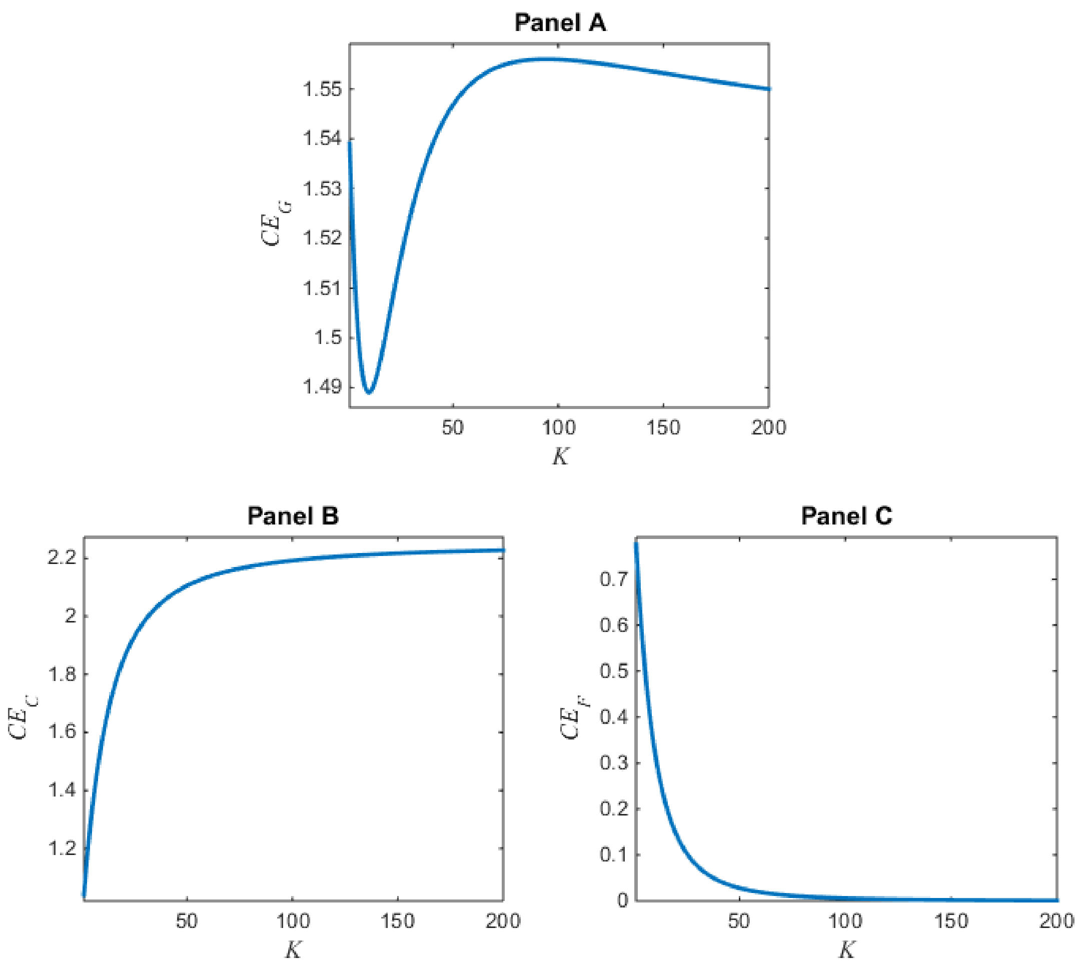
Note: This figure plots $Corr(d_F, \tilde{p})$ against the number K of financial traders. In Panel A, $\tau_c = 2$ and $\tau_\theta = 1$. In Panel B, $\tau_c = 0.5$ and $\tau_\theta = 1$. In both panels, the other parameters are: $\tau_\varepsilon = 0.1$, $\alpha = \gamma = 1$, $J = 20$, $\bar{\theta} = 5$ and $\bar{c} = 1$.

Figure 4: Pricing Implications of Commodity Financialization



Note: This figure plots the futures price bias $E(\tilde{v}-\tilde{p})$, the average spot price $E(\tilde{v})$, and the average futures price $E(\tilde{p})$ against the number K of financial traders. In Panels A1 and A2, $\bar{\theta}=5$ and $\bar{c}=1$. In Panels B1 and B2, $\bar{\theta}=1$ and $\bar{c}=5$. In all four panels, the other parameters are: $\tau_\varepsilon=0.1, \tau_c=\tau_\theta=1, \alpha=\gamma=1$ and $J=20$.

Figure 5: Welfare Implications of Commodity Financialization



This figure plots how the number K of financial traders affects the welfare of growers (Panel A), the welfare of consumers (Panel B), and the welfare of financial traders (Panel C). The other parameters are: $\tau_\varepsilon = 0.1$, $\tau_c = \tau_\theta = 1$, $\alpha = \gamma = 1$, $J = 20$, $\bar{\theta} = 5$ and $\bar{c} = 1$.