Helping the Poor with Selfish Intentions: Trade, Foreign Aid and Growth

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Abstract

Departing from explanations that involve either altruistic or political motivations of donors, we link a donor's willingness to give aid to economic incentives and returns. Since targeted aid might increase the returns from trade and FDI linkages for donors, there exists a non-altruistic basis for helping poor countries. We explore this idea by developing a two-country dynamic strategic interaction model of foreign aid and trade. We endogenize aid-tying process by incorporating it into the donor's optimization problem. We analyze the dynamics of aid composition and show how different components of foreign aid, such as education, investment, commodity and general budget support are optimally supplied over time. We show even under donor-ideal conditions aid might fail to generate growth for the recipient.

1 Introduction

Foreign aid might have several indirect benefits for the donor country when recipient and donor are linked by trade and direct investment. For example, by increasing the return on investment, aid might increase the income growth rate of the recipient country, which can build its capacity to repay debt and and increase its demand for donor country's goods and services. In this scenario, it might be beneficial for the donor to provide aid geared towards increasing overall productivity and returns from direct investments and an altruistic motive might not be necessary for the donor.

We believe this aspect of foreign aid is not fully explored by previous work, which mostly motivates the existence foreign aid by ascribing altruistic preferences to the donor. While altruism might be a rational approach in justifying aid as a means for improving donor's welfare, it leaves questions on how the amount and the composition of aid are determined by the donor, what portion of aid should be tied by the donor and especially why the donor should give tied aid in the first place, not fully answered[1]

In this paper, we incorporate trade and direct investment links between a donor and a recipient in a two-country strategic and dynamic interaction to highlight economic returns of foreign aid both for the recipient and the donor. The model has two novelties: *(i)* it takes into account the optimal allocations of both the recipient and the donor country agents when they are connected through trade and direct investment links, *(ii)* it pays attention to the optimal dynamic composition of foreign aid as an outcome of a strategic interaction rather than only to its amount in ad hoc fashion.

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[1]As a technical consequence, the deep parameters of these general equilibrium models are more difficult to forecast. Some studies establish a correlation between the political will in a donor country and its foreign aid behavior. For instance, Fleck and Kilby (2006) establish a political link between political will and the composition of aid. They demonstrate that when extending foreign aid, liberal U.S. Congresses tend to emphasize developmental and human rights goals whereas conservative U.S. Congresses tend to emphasize advancing U.S. commercial interests in the recipient countries. Such studies, however, seldom address the interaction between the donor and the recipient or elaborate on how to design such aid extension schemes. Other studies motivate foreign aid by assuming that donors are inequality averse (Azam and Laffont, 2003).

We set up the interaction between the donor and the recipient governments as a simple repeated stage game, where the donor acts similarly to a Stackelberg leader in choosing the optimal policy response given the recipients strategy space. Specifically, we construct a setup where the donor government determines the amount and the composition of aid optimally by taking the best response decisions of the recipient government as well as domestic households and firms as given. Initially, the recipient government has no means of committing to any action other than that of a Stackelberg follower, once it observes the donors actions, it only runs the domestic fiscal policy and decides on how much to invest in infrastructure[2].

Such a situation might arise, for example, when the donor and the recipient form a close political union in which the donor acts as a central authority that exerts power in the unions harmonized fiscal framework. In this ideal environment, we find that aid is capable of helping the recipient economy to overcome its structural handicaps such as low level of infrastructure but it is not necessarily able to increase its growth rates permanently. On the other hand, even if aid is given in accordance with purely selfish interests of the donor it leads to a higher welfare and higher growth rates for the donor country and therefore are provided in positive amounts.

A prominent feature of the model is that by abstracting away from altruistic preferences the model enables us to break down foreign aid into its dynamic components and solve for their optimal combination from donor's point of view. How to compose aid disbursements over a time span is one of the most contentious areas in the optimal provision of aid debate. Disregarding heterogeneity of aid components assumes that each aid sector contributes equally to growth and welfare. Such an omission may constitute a pitfall in measuring aid's impact on the recipient economy growth rate by suppressing the magnitudes of estimated coefficients.

To gain a first insight on how different components of aid have behaved over the last

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[2]We use the term infrastructure interchangeably with public capital

century we provide a look at the OECD classification, which distinguishes between eleven types of aid[3]. Figure 1 shows the change of sectors' shares in total aid flows using OECD definitions. There have been substantial changes in the composition of total aid since 1960's. Specifically, production aid and commodity aid have decreased after 1980's whereas total infrastructure aid, which consists of both social and economic infrastructure aid, has relatively increased since 1960's and holds the highest share in total aid as of 2009. In this article, we focus on social and economic infrastructure aid, aid given to production sector, as well as commodity aid and debt relief together which make up the bulk of total foreign aid. This classification makes this study also comparable to previous recent studies such as Chatterjee et al (2003), Chatterjee and Turnovsky (2005) and Scholl(2009) in which infrastructure aid and debt relief play prominent roles.

Our results indicate that aid is supplied in positive amounts at all times even when donor's motives are completely non-altruistic. We find infrastructure aid is supplied in relatively high amounts whereas debt relief is insignificant when compared to other forms of aid such as education and manufactured commodity aid. Specifically, a donor-preferred aid package consists of a high level of infrastructure aid coupled with manufactured commodity aid, education aid, debt relief and budget aid which are respectively decreasing in significance. We find no role for basic commodity aid under our scenario. We further analyze the effects of a foreign aid composite on a variety of macroeconomic outcomes. We compare our results both to autarky and to a situation where there are trade and investment links but no aid transfers. With the introduction of a composite aid transfer, we find increased growth rates are possible but not necessarily sustainable. On the other hand, welfare is improved for both the donor and the recipient. In terms of compositional dynamics, we find infrastructure aid is increasing but commodity aid is decreasing in growth transition which

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[3]Social Infrastructure and Services, Economic Infrastructure and Services, Production Sectors, Multi-sector / Cross-Cutting,Commodity Aid / General Programme Assistance, Action Relating to Debt, Humanitarian Aid, Administrative Costs of Donors, Support to NGOs, Refugees in Donor Countries, Unallocated / Unspecified.
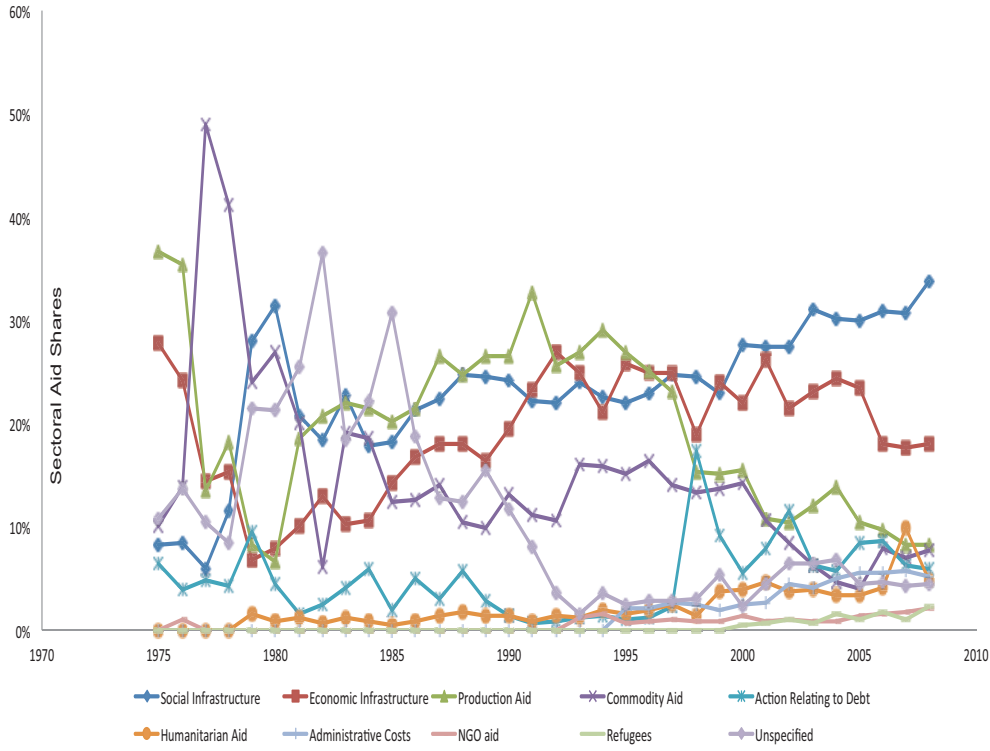
Figure 1: Composition of World Foreign Aid. Source: OECD Trade Data

is roughly in agreement with Figure 1.

Similar to previous studies we find optimal tax rates are decreasing for the recipient. However, we also find that the recipient savings rates are lower, which derives our first result that growth rates are not sustainable under certain scenarios. While the share of productive government spending such as infrastructure investment is lower with the simultaneous introduction of foreign aid, trade and foreign direct investment, its rate is decreasing over time. In contrast, donor's infrastructure investment as well as its rate are higher when compared to autarky.

Donor implements higher taxes to finance aid and investment expenditures. The recipient's debt to output ratio is lower at the new steady state, but the recipient's contribution, to it, is also lower. This occurs through several channels. By increasing returns on education aid helps the recipient to bridge the labor productivity gap between itself and the donor, and

improvements in labor productivity increases returns to investment in intermediate goods sector. By increasing global demand for the more productive sector in the donor country, aid causes a more efficient allocation of resources in the donor country. In this sense aid can be thought of an indirect subsidy to donor country intermediate goods sector. By increasing returns to investment, it increases the speed of further specialization that trade forces. By channeling donor resources to where the productivity gains are higher a composite aid creates a positive externality for intermediate goods producers and on investment returns from low productivity goods.

This paper is organized as follows. In Section 2, we motivate our paper with a brief overview of related literature. Section 3 presents the strategic interaction between the donor and the recipient. In this section, we also introduce the economic agents and their optimization problems and describe the equilibrium. In section 4, we provide simulation results for our model and discuss findings. Section 5 concludes.

2   Literature Review

Despite the voluminous empirical literature on the subject, no consensus has been established on foreign aid's contribution to recipient countries' economic growth and long run welfare[4]. Some of the discrepancy in these findings can be attributed to the variety of empirical specifications employed in these studies[5], but they might also be due to a "lack of clear guidance by theory" (Easterly, 2003).

Regardless of such discrepancies, there is virtually an agreement on foreign aid falling short of expectations in terms of improving growth and welfare of the recipient country.

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[4]Boone(1996) finds no impact of aid on growth whereas Hansen and Tarp(2001)maintains that aid increases the growth rate. A positive impact has also been found by Burnside and Dollar(2000), Lensink and White (2001) and Economides et al (2008). Dalgaard and Hansen (2001) and Easterly et al (2004) condition this positive correlation on good policy. See Hansen and Tarp(2001) for a summary of the results found by empirical literature and Easterly(2009) for a more recent discussion

[5]Burnside and Dollar(2000) use an interaction term between aid and policy, Lensink and White(2001) use aid squared as the regressor.

Common explanations for this failure include usage of aid for unintended purposes by the recipient country such as wasteful government consumption or corrupt activities, and uneven distribution of aid which exacerbates social conflicts that hurt growth (Feyzioglu et al., 1998; Burnside and Dollar, 2000). Established institutions, enacted policies and climate in the recipient country also play a significant role in determining the effectiveness of aid in helping growth and welfare of the recipient country. More classical explanations in growth framework emphasize the hypothesis of diminishing returns to aid and limited absorptive capacity of the recipient[6].

Studies on foreign aid can be classified according to the emphasis put on each end of the transfer. Roughly, these studies can be grouped under three categories: *(i) recipient-oriented , (ii) donor-oriented*, and *(iii) interaction-oriented* Recipient-oriented studies tend to introduce foreign aid as a transfer from the donor, and focus on this transfer's role in the shaping of the policies of the recipient state. The amount of this transfer is determined either exogenously or by endogenous rules, such as giving aid as a simple fraction of the recipient's gross domestic product. These studies condition the positive correlation between aid and growth on the quality the recipient's economic policies (Collier and Dollar, 2002) or its political institutions in general (Collier and Dollar 2002, Burnside and Dollar 2004). Svensson (2000) specifically maintains that democratic recipients utilize aid more effectively to achieve economic growth by allocating aid towards productive sectors. Wintrobe(1998) and de Mesquita and Smith(2007) posit that leaders who are subject to larger selectorates tend to invest aid in growth inducing projects. Wright (2010) echoes this argument by demonstrating that personalist rule in the recipient state decreases the effectiveness of aid on the recipient's level of income. Our study may fall under both the donor- and strategic-oriented categories as we attempt to introduce the donors perspective in a dynamic and

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[6]Early works by Keynes(1929) , Pigou(1932) and Samuelson (1952) argue that in a world without distortions international transfers are beneficial for the recipient but not necessarily for the donor. Turunen and Woodland (1998) argue Pareto improvement can be achieved through multilateral transfers, when distortions are present.

strategic interaction setup.

In an early attempt to model the effect donor preferences have on the allocation of aid, Dudley and Montmarquette (1976) interpret the impact of aid as a public good consumed by the donor country residents. In their study, the optimal amount of aid is determined by the first order condition which equates the marginal utilities of the consumable good to that of impact of foreign aid. They posit three channels for this impact to be created: political, economic and humanitarian. Dudley and Montmarquette further argue that higher levels of political and economic interdependence between the two parties increase efficacy of aid. Their study as well as later studies by Alesina and Dollar (2000) and Alesina and Weder (2002) have lent reasonable empirical evidence to these arguments. Other studies focus on the how donors *design* aid schemes. Chatterjee and Turnovsky (2007), for instance, analyze the impact of transfers tied to investments in public infrastructure. They find that tied transfers has a higher impact than pure transfers but the nature of the impact depends on the initial stock of public capital.

A related paper by Chatterjee and Turnovsky (2005) deal with an international transfer problem where the degree to which transfers are tied to infrastructure investments and borrowing costs are determinants of the long-run effectiveness of aid. In this paper, we endogenize all such transfers made by the donor country in an environment ideal from a donors perspective. In our model, there are direct investment as well as trade linksbetween the donor and recipient and the decision on aid composition is endogenized by giving relevant roles to different forms aid in the optimal choices of both governments.

While treating donor or recipient behavior as exogenous render valuable insights to the behavior of the other party, the donors and recipients perhaps strategically interact in their allocation decisions. Accounting for such strategic interaction has often lent itself to game-theoretic models. An important strand within this group of models have focused on the effect institutional environments of the participants have on their policy decisions. Specifically,

these studies examined the political conflict arising between the donor state (or a multilateral agency) and the recipient state regarding policy reforms the donor wants implemented and the recipient reluctant to undertake. For example, Scholl(2009) analyzes optimal aid policy in a neoclassical growth framework, when there are incentive compatibility problems, that arise from this conflict. In such a situation, optimal self-enforcing conditional aid increases growth rate. Such a contract can be written on conditions that can be sustained with a threat of permanent aid cutoff at any point in time. Azam and Laffont 2003, Mac Donald and Hoddinott 2004)

In an overlapping generations framework, Dalgaard (2008) employs an endogenous aid allocation rule that depends on the recipient's economic performance as well as the donors' political interests and their aversion to risk. Different donor policies result in different "trajectories for GDP growth" in the recipient countries. Similar Dalgaard (2008) we let donors' policy choices to influence aid outcomes, but instead of assigning exogenous parameters to donor policies we emphasize the nature of economic returns for the donor in its strategic interaction with the recipient and thus endogenize such policy responses.

Our paper differs from the aforementioned literature in several more ways. First we let the donor act similar to a Stackelberg leader, in that at each stage it makes its decision after observing the optimal allocation choices of the recipient. Secondly, we decompose aid into several dynamic components each of which has a different role in shaping the economic exchange between the donor and the recipient and determining equilibrium outcomes. In our model, all realized transfers are optimal from the donors point of view. Finally, we allow for trade and investment links between the donor and the recipient to account for nested interests between the parties.

Two points for further inquiry stand out from this brief literature survey. First, scholars have seldom endogenized foreign aid and decomposed it into its constituent parts. However as some practitioners argue, and as we demonstrate below, specific combinations of foreign

aid can stimulate the recipient's economy in varying ways. Furthermore, the few studies that decompose foreign aid as a stimulant of economic growth, do not employ a strategic interaction framework that also feature trade and investment links between the parties. Our model contributes to the existing debate by simultaneously decomposing foreign aid into its constituent parts and examining these parts' effects on economic growth over time in a strategic framework.

3 The Analytical Framework

3.1 Main Assumptions, A Summary of the Model and Description of the Strategic Interaction

In this section, we present our main assumptions, describe the nature and timing of the strategic interaction among agents and present a summary of our solution methodology.

Assume there are two countries: the donor and the recipient, denoted by subscripts D and R, respectively. The donor country is characterized by its developed economy and high-skilled labor force whereas the recipient country is characterized by its developing small economy and low-skilled labor force. In each country, there are three types of agents: firms, households and the government. Production by firms takes place in two sectors: a final goods sector and an intermediate goods sector. The donor and recipient countries are connected via trade in final goods, via direct investment in intermediate goods sector and via various forms aid flows from donor government to recipient government and households. We assume there are two goods produced exclusively in each country. For quick reference, we arbitrarily name the good produced in the donor country as the "manufactured good" and the one produced in the recipient country as the "basic commodity." To construct a foreign direct investment link between the two countries, we assume that the aggregate capital in each country is the sum of domestic and foreign capital supplied by the households.

In both countries, household dynasties live forever, earn wage and rental income, pay taxes and consume a bundle of basic commodity and manufactured goods. In addition, the recipient country households receive aid in form of manufactured goods and basic commodities from the donor country government. Both governments collect taxes, invest in infrastructure and decide on debt policy. The recipient country receives general budget support and debt relief from donor country government. The donor country government gives donations in form of manufactured goods, basic commodities, infrastructure and education aid, debt relief and general budget support. We assume that there are no informational asymmetries among agents. The governments are fully informed of the firms' and households' strategies and determine their optimal allocation strategies in light of these best response functions. In each period, governments take action against each other in a turn-based fashion following a "within-period" sequential game. We will describe the nature and timing of the interaction later.

We solve the allocation problem in each period by backward induction. Once the firms' and households' problems are introduced, the competitive equilibrium is derived as a set of best response functions whose arguments reflect the governments' decision vector and competitive prices. The structure of the model implicitly assumes governments are completely benevolent and the policy environment, institutions and markets are all perfect, so there are no frictions on growth. There are also no elite groups, ruling classes or any other distortionary force that would compete over the aid transfers.

The decision problems of the firms, the households and the governments are described in Sections 3.3 through 3.5. We characterize the world competitive equilibrium resulting from firm and household actions in both countries in Section 3.6. We describe timing of events, actions and payoff functions of each government in Section 3.7.

3.2   Composition of Aid

We assume in each period t the donor country transfers the total amount A_t to the recipient country. This amount, is distributed to different sectors of the recipient economy. The following equation holds in each period:

$$A_t = a_{It} + a_{Gt} + a_{Dt} + L_t^R \left(a_{Et} + a_{ct} + a_{mt} \right) \tag{1}$$

Where A_t is the total amount of aid given, a_{It} is the aid invested in economic infrastructure, a_{Gt} is the general budget support, a_{Dt} is debt relief, L_t^R is the population of the recipient country, a_{Et} is per capita aid spent on education, a_{ct} is per capita basic commodity aid and a_{mt} is per capita manufactured good aid.

3.3   Production

In each country, the final goods (FG) sector, operating under perfect competition, produces consumables and the intermediate capital goods (ICG) sector, operating under perfect monopoly, produces a variety of capital inputs for final goods production.

3.3.1   Final Good Firms

We assume there are two types of final good firms, each producing either a manufactured good or a basic commodity exclusively. Manufactured good producing firms are located only in the donor country whereas the basic commodity producing firms are located in the recipient country.

Both the manufactured good and the basic commodity are produced by using labor, intermediate goods and economic infrastructure. Final good production in each country is subject to the following specifications.

$$Y_t^R = \left(I_t^R + a_{It}\right)^{\varphi^R} \left(Z_t^R L_t^R\right)^{1-\alpha^R} \sum_{j \in J^R} (q_{jt})^{1-\alpha^R} \left(X_{jt}^R\right)^{\alpha^R} \tag{2}$$

$$Y_t^D = \left(I_t^D\right)^{\varphi^D} \left(Z_t^D L_t^D\right)^{1-\alpha^D} \sum_{j \in J^D} (q_{jt})^{1-\alpha^D} \left(X_{jt}^D\right)^{\alpha^D} \tag{3}$$

Here Y_t is the level of final, output I_t denotes the level of infrastructure expenditures of the government, Z_t denotes the labor productivity, L_t denotes the size of the labor force, X_t denotes the amount of capital, q_j is the quality of intermediate good and the set of intermediate capital good qualities are given by J^D and J^R. The infrastructure in the recipient country is financed partly by the government and partly by foreign aid. This specification is adapted from Aghion and Howitt (2009) by adding infrastructure investments and labor productivity. It allows foreign direct investment to be identified by the location of firm origin. Firms maximize profit by choosing the amount of labor and capital. In both countries firms solve the following problem.

$$\max_{\{L_t^R, X_t^R\}} \quad p_t^c Y_t^R - w_t^R L_t^R - p_t^R X_t^R \tag{4}$$

$$\max_{\{L_t^D, X_t^D\}} \quad p_t^m Y_t^D - w_t^D L_t^D - p_t^D X_t^D \tag{5}$$

Prices w_t^R, w_t^D, p_t^R and p_t^D are found by taking the usual first order conditions. A part of the total production, α, is paid to the producers of intermediate capital goods. The remaining $1 - \alpha$ is paid to the labor. Labor is supplied inelastically and evolves exogenously characterized by the equations (6) and (7).

$$L_{t+1}^R = \left(1 + n^R\right) L_t^R \tag{6}$$

$$L_{t+1}^D = \left(1 + n^D\right) L_t^D \tag{7}$$

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where n^R and n^D denote the population growth rate in recipient and donor countries, respectively.

3.3.2 Intermediate Goods Producers

There are J number of intermediate good (IG) producing firms which use capital to produce output. These firms operate on an international scale, but are each owned completely by the consumers of either country. Specifically, if $j \in J^R$ the firm is owned by the consumers in the recipient country and $j \in J^D$ the firm is owned by the consumers in the donor country. There are no other possibilities, therefore $J^R \cup J^D = J$. All profits flow to the owners. The maximization problems of IG producing firms are, thus, given as:

$$if \; j \in J^R : \quad \max_{X_{jt}^D, X_{jt}^R} \quad \left(p_{jt}^D X_{jt}^D - r_t^R f_{jt}^R \right) + \left(p_{jt}^R X_{jt}^R - r_t^R k_{jt}^R \right) \tag{8}$$

$$if \; j \in J^D : \quad \max_{X_{jt}^D, X_{jt}^R} \quad \left(p_{jt}^D x_{jt}^D - r_t^D k_{jt}^D \right) + \left(p_{jt}^R X_{jt}^R - r_t^D f_{jt}^D \right) \tag{9}$$

where X_{jt}^i is amount of IG supplied to country i by firm j, f_{jt}^i is the capital amount transferred to country i by firm j and k_{jt}^i is the capital amount left in country i by firm j for $i = R, D$. In this setup, production of IG can be carried out in each country by using capital borrowed from home country. Substituting the p_{jt} values derived from solving the final good production problem, amounts of IG production can be found as:

$$X_{jt}^R = \left(\frac{\left(\alpha^R \right)^2 p_t^c \left(I_t^R + a_{It} \right)^{\varphi^R}}{r_{kt}^R} \right)^{\frac{1}{1 - \alpha^R}} \left(Z_t^R L_t^R \right) (q_{jt}) \tag{10}$$

$$X_{jt}^D = \left(\frac{\left(\alpha^D \right)^2 p_t^m \left(I_t^D \right)^{\varphi^D}}{r_t^D} \right)^{\frac{1}{1 - \alpha^D}} \left(Z_t^D L_t^D \right) (q_{jt}) \tag{11}$$

These production levels set the price levels at:

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$$p_{jt}^R = \frac{r_{kt}^R}{\alpha^R} \tag{12}$$

$$p_{jt}^D = \frac{r_{kt}^D}{\alpha^D} \tag{13}$$

Assuming equilibrium in the capital markets:

$$L_t^R k_t^R + L_t^D f_t^D = \sum_{j=1}^J X_{jt}^R = \left(\frac{\left(\alpha^R\right)^2 p_t^c \left(I_t^R + a_{It}\right)^{\varphi^R}}{r_{kt}^R} \right)^{\frac{1}{1-\alpha^R}} \left(Z_t^R L_t^R\right) (\bar{q}_t) \tag{14}$$

$$L_t^D k_t^D + L_t^R f_t^R = \sum_{j=1}^J X_{jt}^D = \left(\frac{\left(\alpha^D\right)^2 p_t^m \left(I_t^D\right)^{\varphi^D}}{r_{kt}^D} \right)^{\frac{1}{1-\alpha^D}} \left(Z_t^D L_t^D\right) (\bar{q}_t) \tag{15}$$

where $\bar{q}_t$ is a total quality measure. Then r values can be found as:

$$r_{kt}^R = \left(\alpha^R\right)^2 p_t^c \left(I_t^R + a_{It}\right)^{\varphi^R} \left(\frac{\left(Z_t^R L_t^R\right) (\bar{q}_t)}{L_t^R k_t^R + L_t^D f_t^D} \right)^{1-\alpha^R} \tag{16}$$

$$r_{kt}^D = \left(\alpha^D\right)^2 p_t^m \left(I_t^D\right)^{\varphi^D} \left(\frac{\left(Z_t^D L_t^D\right) (\bar{q}_t)}{L_t^D k_t^D + L_t^R f_t^R} \right)^{1-\alpha^D} \tag{17}$$

It is possible to determine how much capital is allocated among sectors by using previous results.

$$x_{jt}^R = \left(\frac{q_{jt}}{\bar{q}_t} \right) \left(L_t^R k_t^R + L_t^D f_t^D \right) \tag{18}$$

$$x_{jt}^D = \left(\frac{q_{jt}}{\bar{q}_t} \right) \left(L_t^D k_t^D + L_t^R f_t^R \right) \tag{19}$$

Using all the results, we can now write the per capita incomes as functions of the gross domestic products of respective countries:

$$\tilde{w}_t^R = \frac{1}{L_t^R}\left[(1-\alpha^R) + (\alpha^R)^2\left(\frac{\bar{q}_t^R}{\bar{q}_t}\right) + \frac{\bar{q}_t^R}{\bar{q}_t}\alpha^R(1-\alpha^R) + (\alpha^R)^2\left(\frac{1}{L_t^R k_t^R + L_t^D f_t^D}\right)\right]p_c\left[Y_t^R\right]$$
$$+ \frac{1}{L_t^R}\left[(\alpha^D)^2\left(\frac{\bar{q}_t^R}{\bar{q}_t}\right) + \frac{\bar{q}_t^R}{\bar{q}_t}\alpha^D(1-\alpha^D)\right]p_m\left[Y_t^D\right] \tag{20}$$

$$\tilde{w}_t^D = \frac{1}{L_t^D}\left[(1-\alpha^D) + (\alpha^D)^2\left(\frac{\bar{q}_t^D}{\bar{q}_t}\right) + \alpha^D(1-\alpha^D)\frac{\bar{q}_t^D}{\bar{q}_t}\right]p_t^m\left[Y_t^D\right]$$
$$+ \frac{1}{L_t^D}\left[(\alpha^R)^2\left(\frac{\bar{q}_t^D}{\bar{q}_t}\right) + \alpha^R(1-\alpha^R)\frac{\bar{q}_t^D}{\bar{q}_t} + (\alpha^R)^2\left(\frac{1}{L_t^R k_t^R + L_t^D f_t^D}\right)\right]p_t^c\left[Y_t^R\right] \tag{21}$$

Substituting (2) in (21) leads to the following proposition.

Proposition 1. *Foreign Aid indirectly increases income and welfare for the consumers in the donor country.* This result follows from the fact that intermediate goods production occur in both countries and an increase in infrastructure aid increases the income of the recipient and results in a higher revenue stream for donor ICG producers in both countries through both increased demand for their exports and through increased productive capacity in the recipient country.

3.4 Households

There are household dynasties in both countries, living forever and maximizing the sum of their discounted time-separable utility functions subject to their budget constraints, given the competitive prices and the governments' policy vectors. In households' problems, β denotes the discount factor, m and c denote the amount of manufactured good and basic commodity consumed, respectively, $\tilde{\omega}$ denotes the per capita income (sum of labor and capital income), and δ denotes the depreciation rate. Furthermore, n denotes the population growth rate, τ denotes the per capita lump sum taxes. Let i denote the households country of residence then x^{iD} and x^{iR} denote the amount of per capita capital rented to the production sector in the donor and recipient country by country $i = R, D$.

The problem of the recipient country households is defined by the equations (22), (23), (24) and (25).

$$\max_{\left\{m_t^R, c_t^R, x_{t+1}^{RR}, x_{t+1}^{RD}\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \left(\beta^R\right)^t \left(m_t^R\right)^{\sigma^R} \left(c_t^R\right)^{1-\sigma^R} \tag{22}$$

subject to

$$\frac{p_t^m}{p_t^c}\left(m_t^R - a_{mt}\right) + \left(c_t^R - a_{ct}\right)$$
$$= \frac{\tilde{\omega}_t^R}{p_t^c} + (1-\delta)\left(x_t^{RR} + \frac{p_t^m}{p_t^c}x_t^{RD}\right) - \left(1+n^R\right)\left(x_{t+1}^{RR} + \frac{p_t^m}{p_t^c}x_{t+1}^{RD}\right) - \tau_t^R \tag{23}$$

Additionally there are forced choice constraints to guarantee the commodity aid supplied is at most the amount demanded for each consumable.

$$c_t^R \geq a_{ct} \tag{24}$$

$$m_t^R \geq a_{mt} \tag{25}$$

The first order conditions are derived from the Lagrangian problem where λ_t^R is the multiplier on the budget constraint (23), μ_t^{R1} is the multiplier on the forced choice constraint (24) and μ_t^{R2} is the multiplier on the forced choice constraint (25).

$$m_t^R : \sigma^R \left(\beta^R\right)^t \left(m_t^R\right)^{\sigma^R - 1} \left(c_t^R\right)^{1-\sigma^R} - \lambda_t^R \frac{p_t^m}{p_t^c} + \mu_t^{R1} = 0 \tag{26}$$

$$c_t^R : \left(1 - \sigma^R\right) \left(\beta^R\right)^R \left(m_t^R\right)^{\sigma^R} \left(c_t^R\right)^{-\sigma^R} - \lambda_t^R + \mu_t^{R2} = 0 \tag{27}$$

$$x_{jt+1}^{RR} : -\lambda_t^R \left(1 + n^R\right) + \lambda_{t+1}^R \left(\frac{\partial \tilde{\omega}_{t+1}^P}{\partial x_j^{RR}} + 1 - \delta\right) = 0 \tag{28}$$

$$x_{jt+1}^{RD} : -\lambda_t^R \frac{p_t^m}{p_t^c}\left(1 + n^R\right) + \lambda_{t+1}^R \frac{p_t^m}{p_t^c}\left(\frac{\partial \tilde{\omega}_{t+1}^P}{\partial x_j^{RD}} + 1 - \delta\right) = 0 \tag{29}$$

Substituting the intratemporal Euler conditions into the budgets gives the demands as a

function of capital allocations and competitive prices. For simplicity, assume the demand for consumables in the Recipient country always exceeds the commodity aids, therefore setting $\mu_t^{R1} = \mu_t^{R2} = 0$.

$$c_t^R = \left(1 - \sigma^R\right) \left[\begin{array}{l} \frac{\tilde{\omega}_t^R}{p_t^c} + (1 - \delta) \left(\sum_{j \in JRR} x_{jt}^R + \frac{p_t^m}{p_t^c} \sum_{j \in JRD} x_{jt}^D \right) \\ - \left(1 + n^R\right) \left(\sum_{j \in JRR} x_{jt+1}^R + \frac{p_t^m}{p_t^c} \sum_{j \in JRD} x_{jt+1}^D \right) + \frac{p_t^m}{p_t^c} a_{mt} + a_{ct} - \tau_t^R \end{array} \right] \tag{30}$$

$$m_t^R = \sigma^R \frac{p_t^c}{p_t^m} \left[\begin{array}{l} \frac{\tilde{\omega}_t^R}{p_t^c} + (1 - \delta) \left(\sum_{j \in JRR} x_{jt}^R + \frac{p_t^m}{p_t^c} \sum_{j \in JRD} x_{jt}^D \right) \\ - \left(1 + n^R\right) \left(\sum_{j \in JRR} x_{jt+1}^R + \frac{p_t^m}{p_t^c} \sum_{j \in JRD} x_{jt+1}^D \right) + \frac{p_t^m}{p_t^c} a_{mt} + a_{ct} - \tau_t^R \end{array} \right] \tag{31}$$

from which Proposition 2 follows.

Proposition 2. *Foreign Aid indirectly increases demand for tradable consumables in the recipient country.*

The problem of the donor country households is defined by the equations (32) and (33).

$$\max_{\{m_t^D, c_t^D, x_{t+1}^{DR}, x_{t+1}^{DD}\}} \sum_{t=0}^{\infty} \left(\beta^D\right)^t \left(m_t^D\right)^{\sigma^D} \left(c_t^D\right)^{1 - \sigma^D} \tag{32}$$

subject to

$$m_t^D + \frac{p_t^c}{p_t^m} c_t^R \\ = \frac{\tilde{\omega}_t^D}{p_t^m} + (1 - \delta) \left(x_t^{DD} + \frac{p_t^c}{p_t^m} x_t^{DR} \right) - \left(1 + n^D\right) \left(x_{t+1}^{DD} + \frac{p_t^c}{p_t^m} x_{t+1}^{DR} \right) - \tau_t^D \tag{33}$$

The first order conditions are derived again from the Lagrangian problem where λ_t^D is the multiplier on the budget constraint (33).

$$m_t^D : \left(1 - \sigma^D\right) \left(\beta^D\right)^t \left(m_t^R\right)^{-\sigma^D} \left(c_t^D\right)^{\sigma^D} - \lambda_t^D = 0 \tag{34}$$

$$c_t^D : \sigma^D \left(\beta^D\right)^t \left(m_t^D\right)^{1 - \sigma^D} \left(c_t^R\right)^{\sigma^D - 1} - \lambda_t^D \frac{p_t^c}{p_t^m} = 0 \tag{35}$$

$$x_{jt+1}^{DR} : \quad -\lambda_t^D \left(1 + n^D\right) + \lambda_{t+1}^D \left(\frac{\partial \tilde{\omega}_t^D}{\partial x_j^{DR}} + 1 - \delta\right) = 0 \tag{36}$$

$$x_{jt+1}^{DD} : \quad -\lambda_t^D \frac{p_t^c}{p_t^m} \left(1 + n^D\right) + \lambda_{t+1}^D \frac{p_{t+1}^c}{p_{t+1}^m} \left(\frac{\partial \tilde{\omega}_t^D}{\partial x_{jt}^{DD}} + 1 - \delta\right) = 0 \tag{37}$$

3.5   Labor Productivity

We assume social infrastructure investments such as education and health care contribute towards closing the labor productivity gap between the two countries. The evolution of labor productivity levels throughout time is determined by the equations (38) and (39).

$$Z_t^R = \left(1 + z^R\right) Z_{t-1}^R + \left(\frac{Z_{t-1}^D - Z_{t-1}^R}{Z_{t-1}^R}\right)^\chi \left(1 - e^{-\gamma a_{Et}}\right) \tag{38}$$

$$Z_t^D = \left(1 + z^D\right) Z_{t-1}^D \tag{39}$$

The term $\left(\frac{Z_{t-1}^D - Z_{t-1}^R}{Z_{t-1}^R}\right)^\chi$ determines the degree of relative catch-up effect between the two productivity levels. The speed of convergence depends on the education aid through the term $1 - e^{-\gamma a_{Et}}$. γ represents the effectiveness of education aid and χ determines the speed of convergence. When $\gamma = 0$ the labor productivity in the recipient country evolves exogenously. The right hand side of the plus sign in equation (38) goes to zero as the productivity gap closes or education aid goes to zero. In the benchmark case, we let $\gamma = 0$ and compare the results to the model with aid which we obtain by calibrating both χ and γ.

3.6   Governments

3.6.1   Payoff functions

The recipient country government has three revenue sources: taxes collected form its own citizens, borrowings and general budget support from the donor country government. It can

invest these funds as infrastructure or pay some of its debt. It maximizes the welfare of its citizens by determining the per capita lump-sum taxes, debt and contribution made to the infrastructure subject to the budget constraint (41).

$$\max_{\left\{I_t^R, D_{t+1}^R, \tau_t^R\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \left(\beta^R\right)^t u^R \left(\hat{c}_t^R, \hat{m}_t^R\right) \tag{40}$$

subject to

$$I_t^R + (1 + i_t) \left(D_t^R - a_{Dt}\right) = L_t^R \tau_t^R + D_{t+1}^R + a_{Gt} + a_{It} \tag{41}$$

The donor country government has two revenue sources: taxes collected from its own citizens and debt repayments received from the poor country government. It can invest these funds as infrastructure or lend some money or supply foreign aid to the recipient country government.

$$\max_{\left\{I_t^D, a_{ct}, a_{Dt}, a_{Et}, a_{It}, a_{Gt}, a_{mt}, \tau_t^D\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \left(\beta^D\right)^t u^D \left(\hat{c}_t^D, \hat{m}_t^D\right) \tag{42}$$

subject to

$$I_t^D + D_{t+1}^R + L_t^R a_{Et} + a_{It} + a_{Gt} + L_t^R a_{ct} + L_t^R a_{mt} = L_t^D \tau_t^D + (1 + i_t) \left(D_t^R - a_{Dt}\right) \tag{43}$$

3.7 Competitive Equilibrium

We define the world competitive equilibrium in each period as follows:

i) *Consumers maximize their lifetime utility by choosing the amounts of basic commodity, manufactured good, and foreign and domestic investment for the next period, given prices.*

ii) *Intermediate good producing firm j maximizes its profits by choosing how much intermediate good j will be produced in respective countries. The level of production also equals to its demand for ordinary capital.*

iii) *Final goods producing firms maximize their profits by choosing the amount of labor*

and intermediate good used, given prices.

iv) *There exists a $(2J + 6) \times 1$ price vector consisting of equilibrium prices of the intermediate goods, basic commodity, manufactured good, labor and capital in both countries such that:*

v) *World demand for the basic commodity equals world supply, i.e. poor country's gross domestic product equals per capita demands for basic commodity multiplied by the populations.*

$$Y_t^R = L_t^D c_t^D + L_t^R c_t^R$$

vi) *Each government maximizes the welfare of its citizens as given in section 3.6.1*

vii) *World demand for the manufactured good equals world supply, i.e. rich country's gross domestic product equals per capita demands for manufactured good multiplied by the populations.*

$$Y_t^D = L_t^D m_t^D + L_t^R m_t^R$$

viii) *Aggregate ordinary capital demand of intermediate good production in each country equals the sum of domestic and foreign capital inflows.*

$$\sum_{j=1}^{J} x_{jt}^D = K_t^D = L_t^D k_t^D + L_t^R f_t^R$$

$$\sum_{j=1}^{J} x_{jt}^R = K_t^R = L_t^R k_t^R + L_t^D f_t^D$$

ix) *The dynamic resource constraint is given by*

$$K_{t+1}^D + K_{t+1}^R = Y_t^D + Y_t^R + (1 - \delta)\left(K_t^D + K_t^D\right) - L_t^D \left(c_t^D + m_t^R + \tau_t^D\right) - L_t^R \left(c_t^R + m_t^R + \tau_t^R\right)$$

$$(44)$$

x) *World resource constraint holds, i.e. sum of total production and depreciated capital*

stock equals the sum of consumption, investment and taxes.

$$\hat{Y}_t^R + \hat{Y}_t^D + (1 - \delta) \left(L_t^R \left(\hat{x}_t^{RR} + \hat{x}_t^{RD} \right) + L_t^D \left(\hat{x}_t^{DD} + \hat{x}_t^{DR} \right) \right)$$
$$= L_t^D \left(\hat{c}_t^D + \hat{m}_t^D + \left(1 + n^D\right) \left(\hat{x}_{t+1}^{DD} + \hat{x}_{t+1}^{DR} \right) \right) + L_t^R \left(\hat{c}_t^R + \hat{m}_t^R + \left(1 + n^R\right) \left(\hat{x}_{t+1}^{RR} + \hat{x}_{t+1}^{RD} \right) \right) + \tau_t^R + \tau_t^D$$

Competitive equilibrium consists of quantities that solve the households' and firms' problem and a competitive price vector for each period. The quantities are determined by response functions, that have competitive prices and government decisions, G, as arguments. The competitive equilibrium is denoted by the following response functions.

$$\hat{c}_t^R = c^R\left(G; \hat{p}, t\right), \hat{c}_t^D = c^D\left(G; \hat{p}, t\right),$$

$$\hat{m}_t^R = m^R\left(G; \hat{p}, t\right), \hat{m}_t^D = m^D\left(G; \hat{p}, t\right),$$

$$\hat{x}_{t+1}^{RR} = x^{RR}\left(G; \hat{p}, t\right), \hat{x}_{jt+1}^{DD} = x^{DD}\left(G; \hat{p}, t\right)$$

$$\hat{x}_{t+1}^{RD} = x^{RD}\left(G; \hat{p}, t\right), \hat{x}_{t+1}^{DR} = x^{DR}\left(G; \hat{p}, t\right)$$

In the competitive equilibrium, the world total output is distributed according to (45) and (46).

$$\tilde{\omega}_t^R = \pi_t^{RR}\hat{Y}_t^P + \pi_t^{RD}\hat{Y}_t^D \tag{45}$$

$$\tilde{\omega}_t^D = \pi_t^{DD}\hat{Y}_t^D + \pi_t^{DR}\hat{Y}_t^R \tag{46}$$

where π_t^{ij} represent country i's share of claims in the gross domestic product of country j.

3.7.1   Timing of Events and Actions

Each period consists of two stages. At the beginning of each period, firms and households in each country maximize their objective functions given their governments' policy parameters.

At the second stage, the recipient country's government solves a Ramsey problem by choosing infrastructure investments and tax level to maximize social welfare of its own citizens, taking competitive equilibrium at home and the foreign government's action parameters as given. At the final stage, the donor country's government acts as the leader in a sequential game versus the recipient country's government. Specifically, it solves a Ramsey problem by determining infrastructure investments, tax levels and the amount and composition of foreign aid to maximize social welfare of its own citizens, given the competitive equilibrium at home and foreign government's decision rules that are based on its own actions.
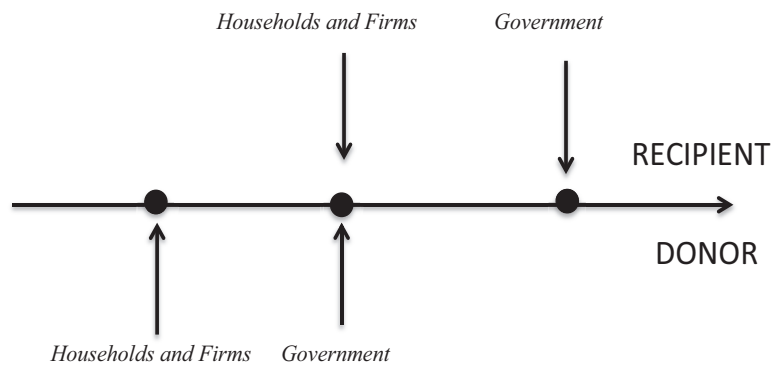
Figure 2: Timeline of Optimizing Actions

Governments are fully informed about the firms' and households' best response functions that determine the competitive equilibrium and determine their policies with the strategic objective of maximizing the welfare of their own citizens given others' actions. Specifically, they play a repeated game, where the donor government acts as a leader and the recipient government acts as a follower. Each period in the game consists of three stages. At the beginning of each period, donor households and firms maximize their objective functions as described in sections 3.3.2-4. At the second stage both governments take the set of competitive equilibrium decision rules by firms and households in each country as given and individually solve Ramsey problems. In addition, the recipient government takes the composition of aid determined by the donor government also given. At the second stage,

the donor government acts as the leader in a sequential game versus the recipient govern-ment. Specifically, it solves a Ramsey problem by determining tax levels, $\left\{\tau_t^D\right\}_{t=0}^{\infty}$, debt, $\left\{D_{t+1}^D\right\}_{t=0}^{\infty}$, infrastructure investments, $\left\{I_t^D\right\}_{t=0}^{\infty}$, and the amount and composition of foreign aid, $\left\{a_{ct}, a_{Dt}, a_{Et}, a_{It}, a_{Gt}, a_{mt}\right\}_{t=0}^{\infty}$ to maximize social welfare of its own citizens, given the competitive equilibrium at home and foreign government's decision rules that are based on its own contingent actions. At the last stage the recipient government chooses per capita lump-sum taxes, $\left\{\tau_t^R\right\}_{t=0}^{\infty}$, debt, $\left\{D_{t+1}^R\right\}_{t=0}^{\infty}$, and infrastructure investments, $\left\{I_t^R\right\}_{t=0}^{\infty}$ to maximize welfare. In our setup, there are some further constraints to sustain a Stackelberg equilibrium. We assume donor must knows ex-ante that the recipient observes his action. The recipient has no means of committing to a future non-Stackelberg follower action and the donor has full information on this.

4 Numerical Analysis

4.1 Algorithm and Parameters

Since further solutions cannot be carried out analytically, we resort to numerical methods to illustrate the results obtained from the model [7]. Our two-tiered simulation algorithm is as follows:

i) *Employ a grid-search method to locate the steady states.*

ii) *Compute transition paths using decision rules of agents.*

iii) *Use backwards induction based on Figure 2 starting from the last step to solve for the optimal reaction functions of each government numerically at previous stage, alternating between donor and the recipient .*

iv) *Restart at i) until convergence is achieved.*

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[7]A supplement describing the full set of steady state equilibria, the solution of the model as well as the accompanying Matlab code are available from authors

v) *Use optimal decision rules of agents, who respond to government fiscal and aid policy adjustments at the previous stage, by adjusting consumption and investment at the current stage as well as government reaction functions at the current stage to solve for optimal levels of consumption, investment, capital and fiscal and aid policies.*

Firstly, we calibrate the parameters of the system under autarky and without aid. We match our data to observed import shares which act also as preference parameters due to the assumption that donor and recipient specialize in a specific type of good. Import shares are found by calculating the fraction of total import flows to countries that receive greater than 3% of their GDP as aid[8]. This exercise pins down the import shares for the recipient at 0.3 and for the donor at 0.7. The knowledge-transfer rate, γ and the catch-up effect, χ are found by calibrating the model to these values. Table 1 contains the parameters used to calibrate the benchmark economy as well as calibrated parameters. The rest of the parameters are standard for developing countries and are taken from World Bank studies. To understand the effects of aid, we first simulate a benchmark model with trade and investment links but without any catch up effect, i.e. $\gamma = 0$ and compute its steady state values. Next, we simulate the model with positive aid flows and a positive catchup effect as well as with positive aid flows and a zero catch up effect. In this case, $\gamma = 0$ describes effectively the situation when all aid flows except education is non-negative and education aid is exactly zero in equilibrium.

4.2 Simulation Results

4.2.1 Growth and Welfare Comparisons

In Table 2, we report the steady state properties of the model and compare results, that are obtained by setting the effect of education aid to zero, to results obtained from the calibrated model. The first case ($\gamma = 0$) refers to a situation where individual country labor

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[8]For this exercise we utilize **?** data

Table 1: The benchmark economy

Parameters	
Import Shares:	$\sigma^R = .3, \sigma^D = .7$
Production parameters:	$\varphi^R = .2, \varphi^D = .2, \alpha^R = .3, \alpha^D = .3$
Depreciation rate	$\delta = .0635$
Discount rates	$\beta^R = .95, \beta^D = .95$
Population growth	$n^R = .02, n^D = .01$
Labor productivity growth rate	$z^R = .01, z^D = .01$
Knowledge transfer parameter	$\gamma = 10.1$

Table 2: Steady State Properties

Catch-up parameter	$\gamma = 0$	$\gamma = 10.1$
Growth rates		
Benchmark	$\bar{g}_R = 0.0221, \bar{g}_D = 0.0161$	
With Aid	$\bar{g}_R = 0.0155, \bar{g}_D = 0.0232$	$\bar{g}_R = 0.0244, \bar{g}_D = 0.0226$
Tax rates		
Benchmark	$\bar{t}_R = \%23.16, \bar{t}_D = \%15.29$	
With Aid	$\bar{t}_R = \%26.66, \bar{t}_D = \%17.32$	$\bar{t}_R = \%14.06, \bar{t}_D = \%29.59$
Capital-output ratio		
Benchmark	$(\frac{\bar{K}}{Y})_R = 0.29, (\frac{\bar{K}}{Y})_D = 0.59$	
With Aid	$(\frac{\bar{K}}{Y})_R = 0.41, (\frac{\bar{K}}{Y})_D = 0.68$	$(\frac{\bar{K}}{Y})_R = 0.39, (\frac{\bar{K}}{Y})_D = 0.61$
Recipient debt-output ratio		
Benchmark	$(\frac{\bar{D}}{Y}) = 0.45$	
With Aid	$(\frac{\bar{D}}{Y}) = 0.69$	$(\frac{\bar{D}}{Y}) = 0.61$
Aid Components		
Benchmark	$\bar{a}_I = \bar{a}_G = \bar{a}_D = \bar{a}_E = \bar{a}_c = \bar{a}_m = 0$	
With Aid	$\bar{a}_I = 0.86, \bar{a}_G = 0.04, \bar{a}_D = 0.01$	$\bar{a}_I = 0.74, \bar{a}_G = 0.02, \bar{a}_D = 0.03$
	$\bar{a}_E = 0, \bar{a}_m = 0.09, \bar{a}_c = 0$	$\bar{a}_E = 0.06, \bar{a}_m = 0.15, \bar{a}_c = 0$

productivities evolve exogenously, whereas the second case ($\gamma = 10.1$) represents a situation where recipients' productivity level is positively affected by education aid. In addition, we also report steady state values under two different scenarios. The benchmark scenario refers to the situation when there are trade and investment links but no aid flows. Here, both governments run only fiscal policies and decide on infrastructure investments. The scenario with aid refers to the situation when there are positive aid, investment and trade flows between countries. In this scenario, the donor government decides also on aid composition.

When labor productivities evolve exogenously, we find that foreign aid reduces steady state growth rate of the recipient by 0.66 percentage points, while it increases the donor's growth rate by 0.007 [9]. When there are positive productivity returns to education aid, foreign aid increases recipient's growth rate by 0.23 and donor's growth rate by 0.006. In the first case, recipient tax rates increase by 3.50 percentage points and donor tax rates increase by 2.03. With the introduction of education aid, donor tax rates jump by 14.30 percentage points, whereas recipient tax rates drop by 9.10. Steady state physical capital-output ratios are higher for the recipient and the donor in both cases. The recipient's capital-output ratio drops from 0.41 to 0.39 and the donor's capital-output ratio drops from 0.68 to 0.61 when we allow for education aid. The steady state debt-output ratios of the recipient mimics its tax rates. With the introduction of budget support and debt relief, the steady state debt-output ratio of the recipient reaches to 0.69. After the introduction of education aid, however, it drops to 0.61. Finally, infrastructure aid constitutes the biggest part of an optimally combined aid package irrespective of whether there are positive returns to education aid. The introduction of education aid leads to simultaneous increases in debt relief and manufactured commodity aid but it decreases infrastructure aid and budget support.

Figure 2 shows growth rate comparisons between the benchmark model and two models

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[9]Note that, in this case education aid is optimally zero.

with aid along the transition path. It also shows welfare comparisons between the benchmark model and the model with education aid. When the optimal aid package does not contain education aid, recipient growth rate increases temporarily fourfold, but drops below the benchmark level during transition. Inclusion of education aid increases the growth rate even more, by about six times initially, but the growth rate steadily drops until just above the benchmark level. For the donor, transitions are markedly different. Whether the aid package includes education aid or not growth rates are permanently higher than the benchmark level. Recipient welfare is improved with aid whereas donor's welfare first drops but then eventually reaches above its equilibrium path that it would follow without aid.

4.2.2 Savings Rates, Tax Rates, Infrastructure Investments and Debt Accumulation

Figure 3 shows the evolution of savings, tax and investment rates as well as debt to output ratio both at the benchmark steady state and after the introduction of aid flows in both countries. We find that, in transition, recipient households' savings rate first increases and then drops below the benchmark level. Savings rate in the donor country first drops significantly but recovers to 28%, roughly 4% below its benchmark level. Recipient country's equilibrium tax rate initially jumps by 1.2% with aid flows but eventually drops 9% below the benchmark level whereas donor's tax rate initially jumps by 3.2% and continues to increase towards steady state level of 29.59%.

Since savings rates are related to consumption smoothing behavior in both countries, they respond to both movements in tax rates as well as movements in labor productivity that are in turn partly induced by infrastructure and education aid flows. Once the aid flows start, both countries increase tax rates which cause the recipient households to save significantly less but the effect is dampened over time due to benefits from increased aid, trade and investment flows. The donor households, on the other hand, increase their savings
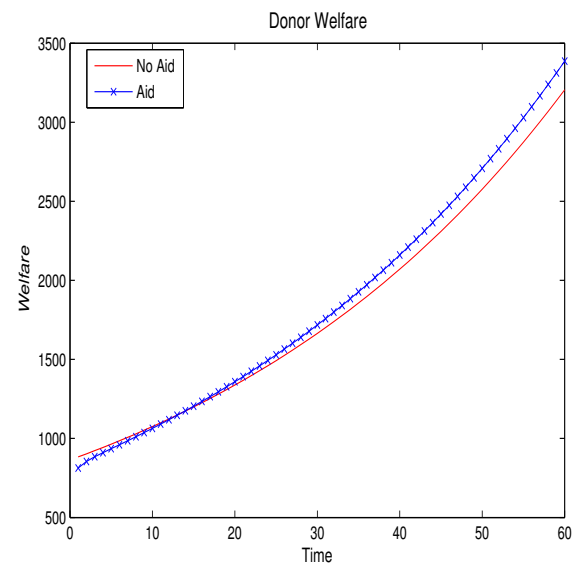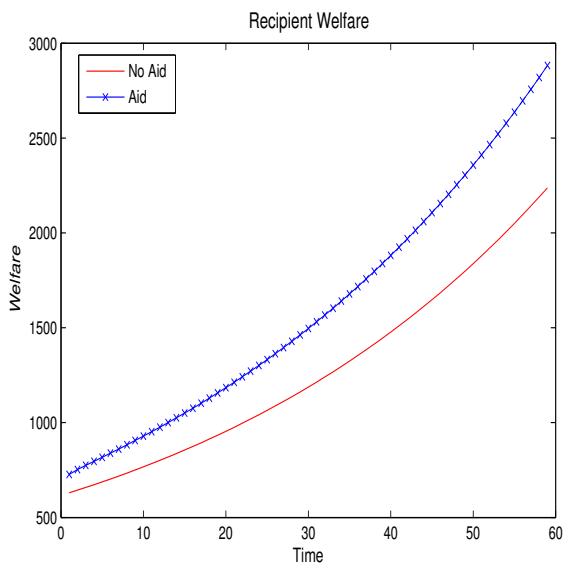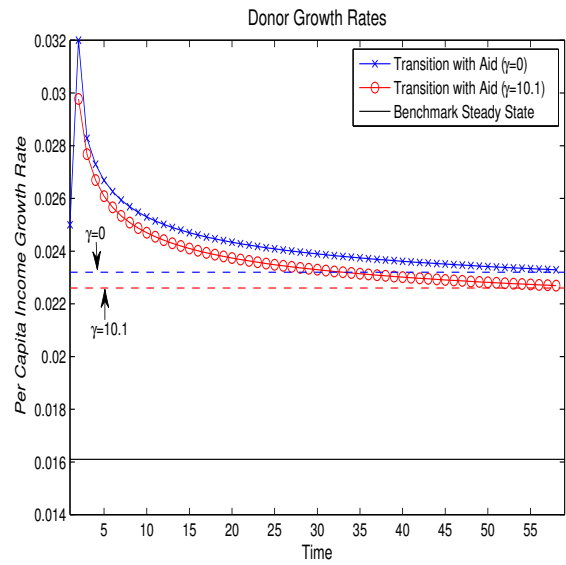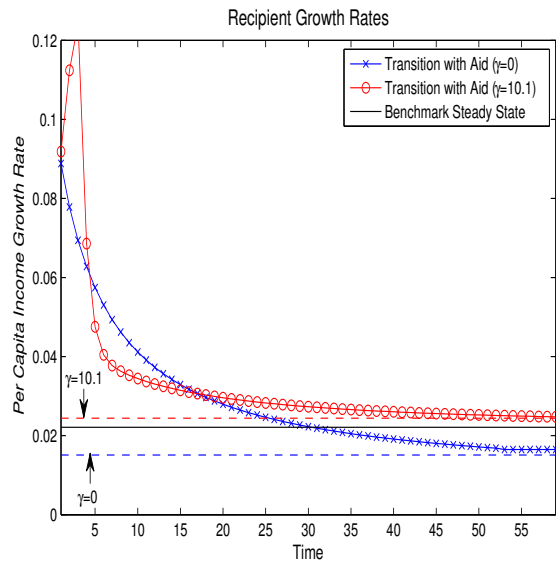
Figure 3: Growth and Welfare

rate which, however, also drops below its benchmark level toward steady state.

In our setup, how the infrastructure investment and the debt of the recipient country is financed by each party can be determined optimally by the governments of two countries. In figure 3, we report the infrastructure investment rates, defined as the the total investment expenditures as a share of total output, in the recipient country and the percentage contribution of each government to finance the debt of the recipient country. The investment expenditures by the donor country in the recipient country increases to 22.0% of the recipient output once we introduce aid flows. Recipient investment rates almost double with the introduction of aid but they eventually drop to 22.4% at the steady state. This is hardly surprising , as infrastructure aid turns out to be the biggest component of the optimal aid package, a result we introduce in the next section. Once aid flows start the debt-output ratio of the recipient country increases steadily to 0.61. While this ratio is below the level that is attained when there is no education aid it is still significantly higher than the benchmark level. The recipients contribution drops to 70.1% after aid flows and further drops to 66.1% whereas the donor finances 33.9% of the recipient debt at the steady state.

4.2.3 Composition of Aid and Dynamics of Aid Components

One of the main arguments of this paper is that the optimal aid shares are not constant and evolve over time. As we presented in Figure 1, actual evidence points to an increasing infrastructure aid, relatively low levels of commodity aid and budget aid including debt relief. Figure 4 shows the transition of aid shares toward steady state. We find that all types of aid except basic commodity aid are supplied optimally in positive amounts at all times. Broadly consistent with the actual evidence we also find that infrastructure aid makes up the bulk of total aid. Manufactured commodity aid has a larger share than all other types combined except infrastructure. Inclusion of education aid increases optimal levels of infrastructure aid because it increases returns to infrastructure investments and leads to
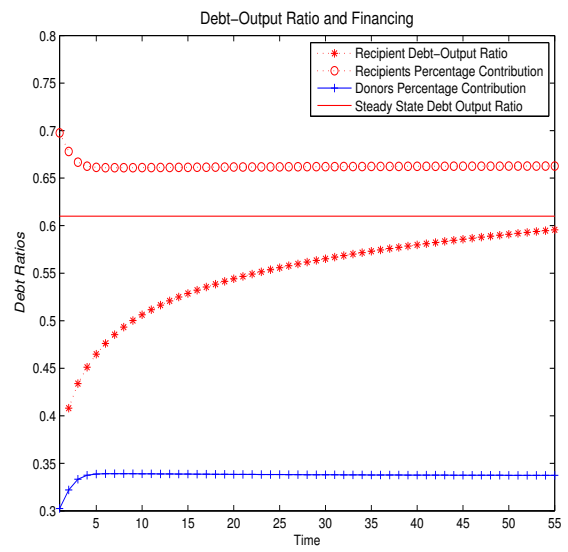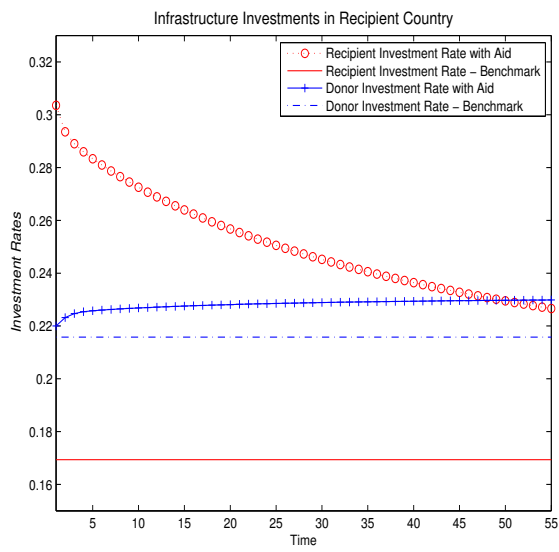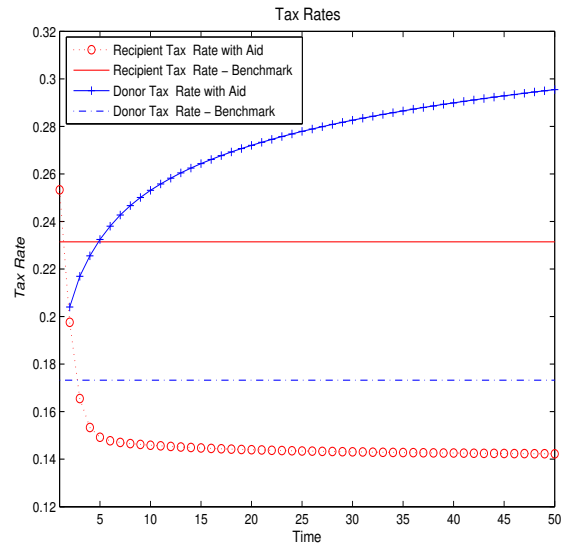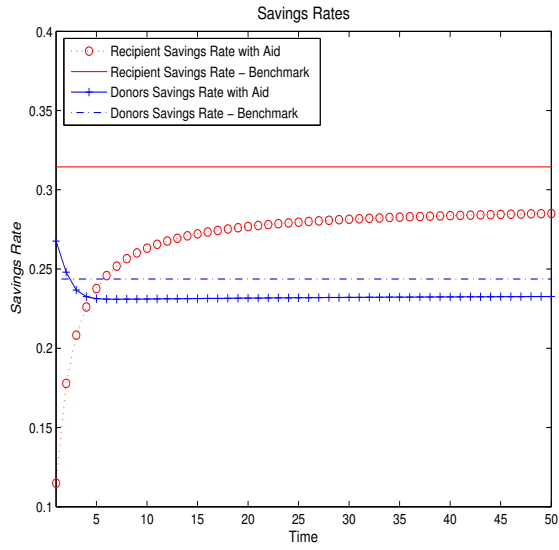
Figure 4: Savings, Taxes and Financing

efficiency gains. Our results suggest, when combined, all types of aid except commodity aid increase recipients growth. When combined, three categories of aid, education aid, capital aid and infrastructure aid in combination can even generate substantial growth levels that are sustainable. The aid process generates a convergence in labor productivity levels, which eventually increases the overall productivity and increases returns to infrastructure investments.
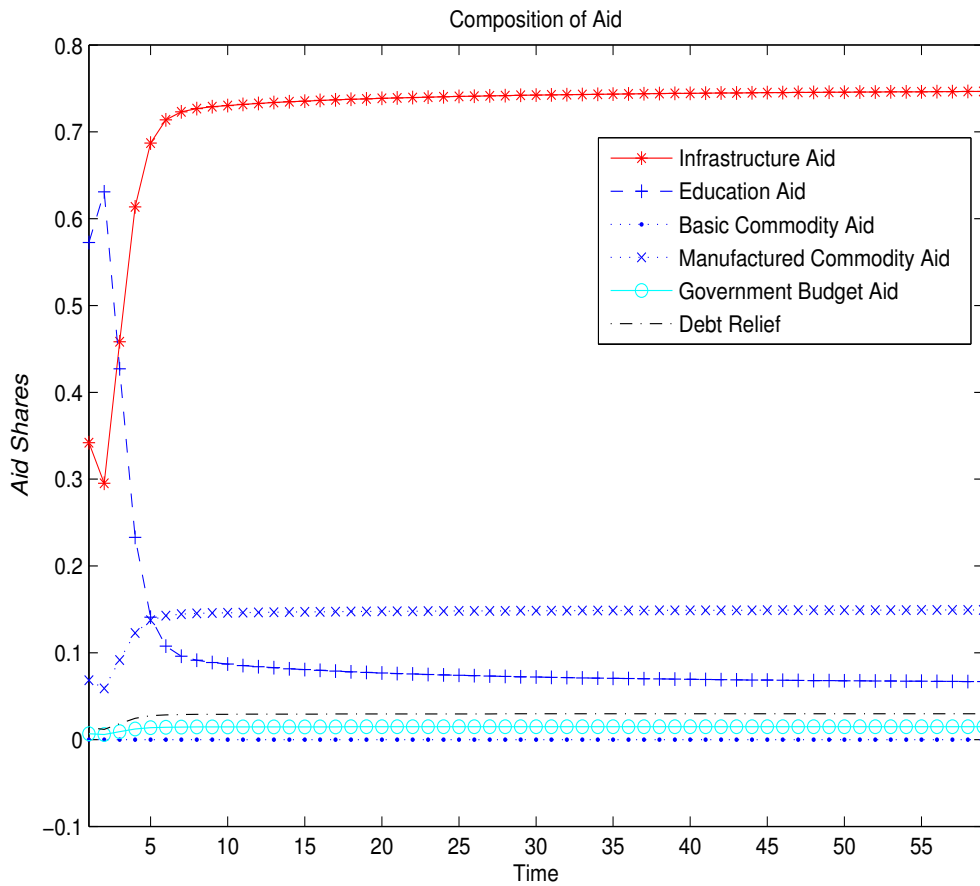

Figure 5: Evolution of Aid Components

4.2.4 Sensitivity Analysis

We run a sensitivity analysis where the recipient discounts the future at a higher rate than the donor. Table 3 summarizes the results obtained from this exercise. In our setup, a higher discount rate leads to lower growth, higher tax rates and a lower capital-output ratio for the recipient. A higher discount rate also increases the magnitude of the donors response to debt relief and budget aid. It causes the recipient country to increase taxes by 1% whereas the recipient debt output ratio increases by 8%. Our findings are in line with **?** who points out that higher discount rate for the recipient can account for political economy features that cause overspending. The degree of externality of public capital is an important source of dynamic adjustment in our model. We run a further sensitivity analysis where we increase the externality of public capital in the recipient country to 0.3 from 0.2. When the externality of public capital in the recipient country is increased to 0.3, growth rates increase considerably. With a higher public capital externality, private capital accumulation speeds up and welfare improves significantly. The donor responds by increasing infrastructure and education aid. Our simulations highlight the complementarity between foreign aid and the externality of public capital. The results suggest that aid policies where the recipient government is required to maintain its commitment to public capital yields the highest level effects to capital accumulation, debt and output.

5 Conclusion

Modeling an aid transaction, with no altruism on part of the donor, is difficult, because aid has no direct returns for the donor when not tied. Provided that aid is neither a pure gift, nor a pure medium of exchange, a standard growth setup does not yield the usual comparable first order conditions. There is an apparent rate of return dominance problem: if aid has less direct returns when it is not tied, then why should donors be giving it? To

Table 3: Sensitivity of Aid Effectiveness: Discount Rates

Discount Rates	$\beta_R = 0.95,\ \beta_D = 0.95$	$\beta_R = 0.90,\ \beta_D = 0.95$
Growth rates	$\bar{g}_R = 0.0244,\ \bar{g}_D = 0.0226$	$\bar{g}_R = 0.0192,\ \bar{g}_D = 0.0211$
Tax rates	$\bar{t}_R = \%14.06,\ \bar{t}_D = \%29.59$	$\bar{t}_R = \%15.69,\ \bar{t}_D = \%29.06$
Capital-output ratio	$(\frac{\bar{K}}{Y})_R = 0.39,\ (\frac{\bar{K}}{Y})_D = 0.61$	$(\frac{\bar{K}}{Y})_R = 0.44,\ (\frac{\bar{K}}{Y})_D = 0.57$
Recipient debt-output ratio	$(\frac{\bar{D}}{Y}) = 0.65$	$(\frac{\bar{D}}{Y}) = 0.73$
Aid Components	$\bar{a}_I = 0.74, \bar{a}_G = 0.02, \bar{a}_D = 0.03$ $\bar{a}_E = 0.06, \bar{a}_m = 0.15, \bar{a}_c = 0$	$\bar{a}_I = 0.60, \bar{a}_G = 0.09, \bar{a}_D = 0.14$ $\bar{a}_E = 0.08, \bar{a}_m = 0.09, \bar{a}_c = 0$

Table 4: Sensitivity of Aid Effectiveness: Externality of Public Capital

Degree of Externality	$\varphi^R = .1, \varphi^D = .1$	$\varphi^R = .15, \varphi^D = .1$
Growth rates	$\bar{g}_R = 0.0244,\ \bar{g}_D = 0.0226$	$\bar{g}_R = 0.0301,\ \bar{g}_D = 0.0244$
Tax rates	$\bar{t}_R = \%14.06,\ \bar{t}_D = \%29.59$	$\bar{t}_R = \%15.87,\ \bar{t}_D = \%28.55$
Capital-output ratio	$(\frac{\bar{K}}{Y})_R = 0.39,\ (\frac{\bar{K}}{Y})_D = 0.61$	$(\frac{\bar{K}}{Y})_R = 0.52,\ (\frac{\bar{K}}{Y})_D = 0.60$
Recipient debt-output ratio	$(\frac{\bar{D}}{Y}) = 0.65$	$(\frac{\bar{D}}{Y}) = 0.44$
Aid Components	$\bar{a}_I = 0.74, \bar{a}_G = 0.02, \bar{a}_D = 0.03$ $\bar{a}_E = 0.06, \bar{a}_m = 0.15, \bar{a}_c = 0$	$\bar{a}_I = 0.77, \bar{a}_G = 0.01, \bar{a}_D = 0.001$ $\bar{a}_E = 0.07, \bar{a}_m = 0.14, \bar{a}_c = 0$

tackle this issue, we assume an ideal situation for the donor whose aid is spent in accordance with its interests. In our setup, donor maximizes the welfare of its citizens who enjoy returns from bilateral trade and direct investment that are at least partially affected by the amount and the composition of aid. This setup endogenizes aid-tying process by incorporating it into the donor's optimization problem instead of tying aid to recipient policies in an ad-hoc manner. This setup is not far from reality. It is applicable to countries or establishments that are harmonizing their economic policies via tax-laws and at the same time delegating more power to a central authority. In this sense, European Union is a case in point.

This study investigates the effects of aid flows on both the recipient and the donor country. We focus on the determination of optimal aid composition from a donors point of view and show that under ideal conditions. To our best knowledge, this study is the first attempt to incorporate trade and direct investment links between a donor and a recipient in a strategic setup. It is also first attempt in which the donor dynamically optimizes the composition of aid to maximize its own welfare. To keep the model tractable, we introduce a simple setup where we do not allow strategic deviations from a dynamic contract which might or might not be enforceable by credible threats.

The model we construct ignores altruistic political motivations of the donor or the recipient in order to understand the effects of economic returns of the aid process. In a world, where the donor has trade and investment links with the recipient, there are economic returns to giving certain forms of aid. These returns flow mainly through two channels. Firstly, increased production in the recipient economy yields higher returns to foreign investments made by the donor. Secondly, a higher income for the donor and increased foreign aid together imply a higher income for the recipient economy, which translates to increased demand for donor exports. Not all forms of aid contributes to increasing growth. The model predicts no economic role for humanitarian or commodity aid as well as any kind of untied transfer made to the recipient government such as general budget support.In our

setup, the governments of two countries can optimally determine the share of infrastructure investments in the recipient economy, that are to be financed by each party.

Our study suggests, when combined, all except one form of aid contribute to growth. When combined, three categories of aid, education aid, capital aid and infrastructure aid can generate even substantial growth. It is assumed that there is an initial gap between the donor and the recipient countries' labor productivity levels. It is also assumed that this gap can be closed by making the necessary investments in education, which can be financed by education aid. This process generates a convergence in labor productivity levels, which eventually increases the overall productivity and increases returns to infrastructure investments. This is one of the key propositions of the model constructed in this paper. Based on our results, assumptions on regarding how education aid affects labor productivity in a dynamic setting in the recipient country turns out to be crucial in determining aid outcomes. If we allow labor productivities evolve exogenously, foreign aid does not lead to higher growth rates that are sustainable in the long run.

Trade and investment links between a donor and a recipient opens up a new framework, in which returns to aid for the donor and the recipient can be analyzed simultaneously. This model can be extended to allow the recipient country to have a dynamic contract with the donor while maintaining such links. Although there has been an impetus in recent research to analyze such dynamic contracts, a more realistic analysis of the economic exchange between the donor and the recipient, who are interconnected in more than one way, is still missing.

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