

THE HECKSCHER-OHLIN-SAMUELSON MODEL AND THE CAMBRIDGE CAPITAL CONTROVERSIES

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1. Introduction

- The purpose of this paper is to review the development of the HOS model in relation to the Cambridge capital controversies.
- We show that the modern dynamic HOS model, where capital is reproducible, still has such a structure that it circumvents the issues pointed out in the capital controversies.
- We pay much attention to the validity of the **factor price equalisation theorem (FPET)**.
 - ✓ Capital as a primary factor of production: Samuelson (1953), Gale and Nikaido (1965), Samuelson (1966a), Nikaido (1972), Kuga (1972), Mas-Colell (1979a,b), Blackorby et al. (1993).
 - ✓ Capital as a bundle of reproducible commodities: we present a numerical example based on the Leontief production model with alternative techniques in order to show the possibility that the FPET does not hold even in absence of capital intensity reversal.

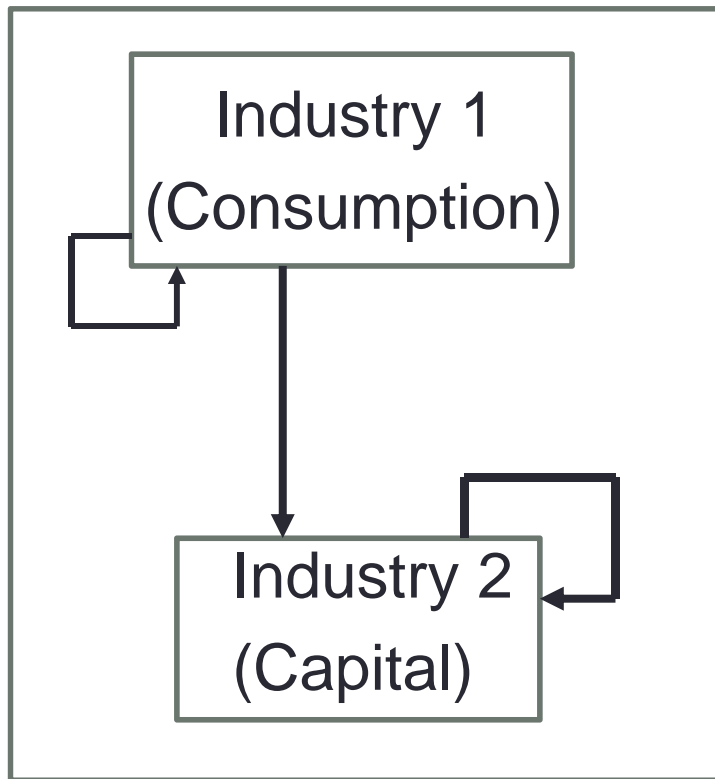
2. FPET under Capital as a Primary Factor

- Samuelson (1953), Gale and Nikaido (1965), etc.
 - ✓ using the Jacobian matrix of cost functions to obtain the sufficient condition for the FPET.
- Samuelson (1966a), Nikaido (1972), Mas-Colell (1979a, b), etc.
 - ✓ The factor intensity is defined by using the cost share of factor i to the total cost.
- Kuga (1972), Blackorby et al. (1993)
 - ✓ The necessary and sufficient condition for the FPET is characterised that factor prices are solely dependent on commodity prices determined by free trade and are entirely independent of factor endowments.

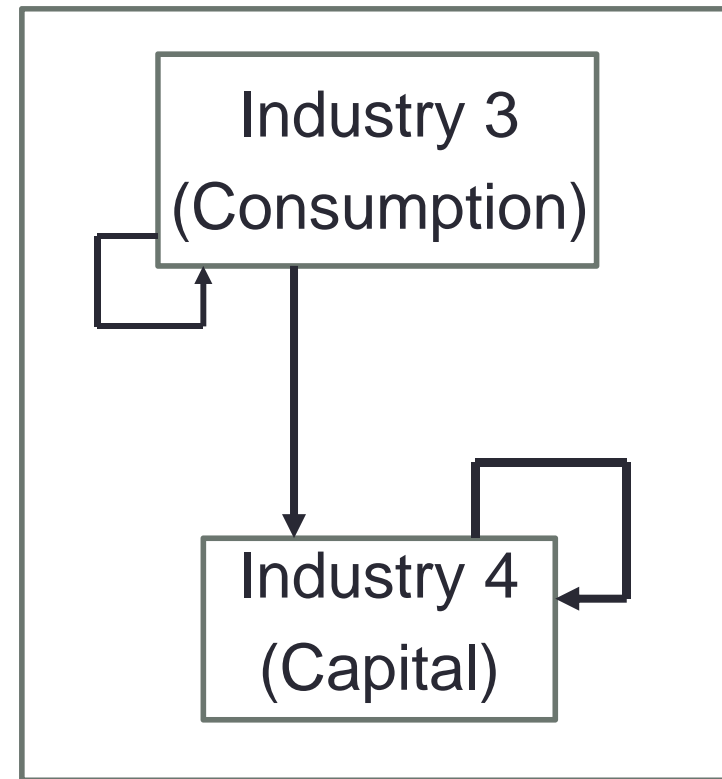
3. Capital as a Bundle of Reproducible Commodities

- The outcome of the Cambridge capital controversies makes us doubt the validity of the FPET (Metcalfe and Steedman, 1972, 1973; Mainwaring 1984).
 - ✓ In the Leontief production model with two commodities, no capital intensity reversal is the necessary and sufficient condition for factor price equalisation.
 - ✓ The technical change which reverses capital intensity is not peculiar, given the Leontief model with alternative techniques.
 - ✓ Our two-integrated-sector example shows the invalidity of the FPET in the absence of capital intensity reversal, when capital consists of reproducible commodities.

The numerical example



Sector 1



Sector 2

Available Techniques

- Industry 1

$$\begin{aligned}(a_{11}^{\alpha}, a_{21}^{\alpha}, l_1^{\alpha}) &= (0.38, 0.63, 0.06), \\ (a_{11}^{\beta}, a_{21}^{\beta}, l_1^{\beta}) &= (0.4188, 0.424, 0.265), \\ (a_{11}^{\gamma}, a_{21}^{\gamma}, l_1^{\gamma}) &= (0.52, 0.01, 0.65).\end{aligned}$$

- Industry 2

$$(a_{12}, a_{22}, l_2) = (0.08, 0, 1).$$

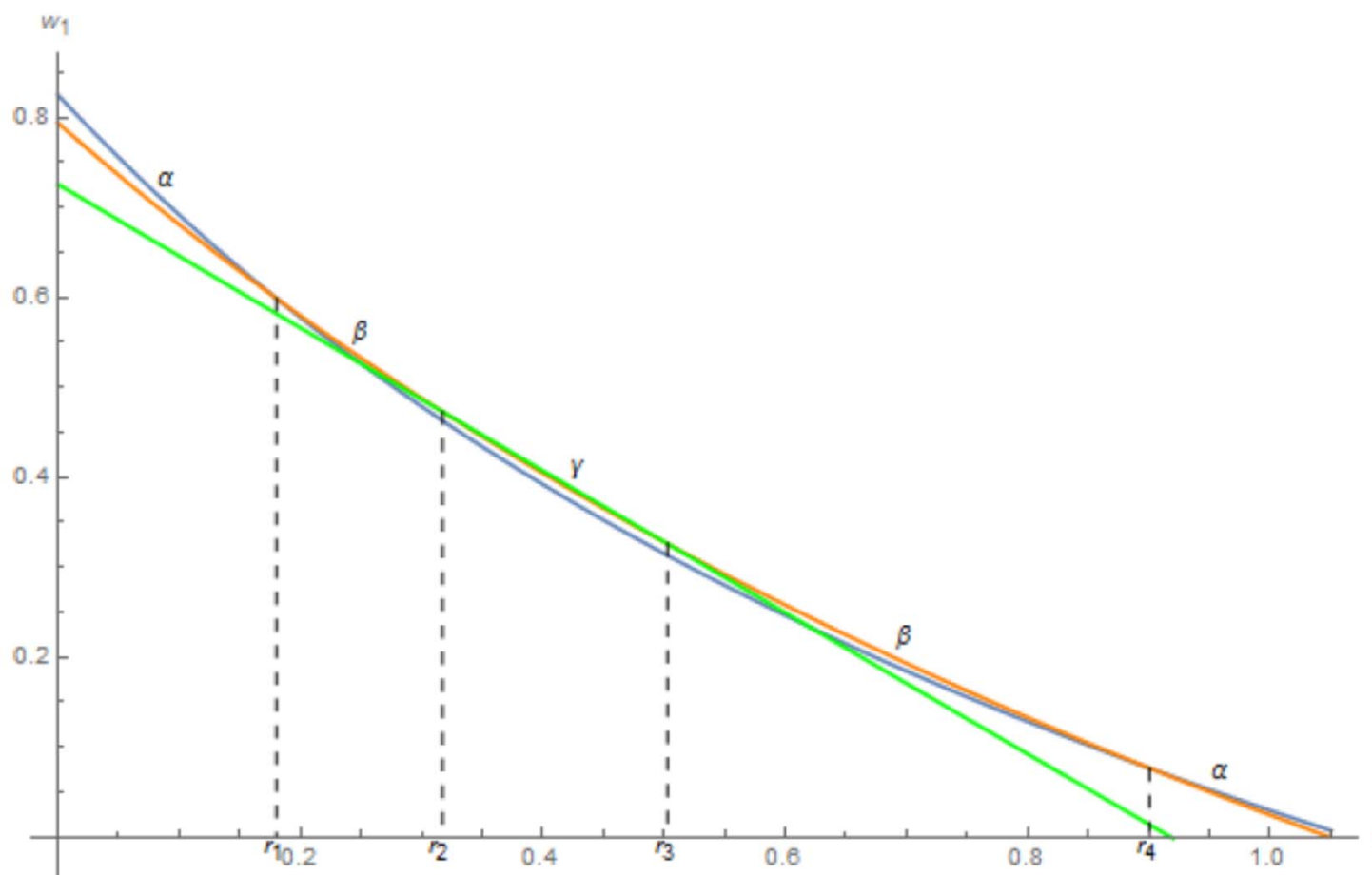
- Industry 3

$$\begin{aligned}(a_{33}^{\delta}, a_{43}^{\delta}, l_3^{\delta}) &= (0.2, 0.485, 0.03), \\ (a_{33}^{\varepsilon}, a_{43}^{\varepsilon}, l_3^{\varepsilon}) &= (0.3, 0.41, 0.02).\end{aligned}$$

- Industry 4

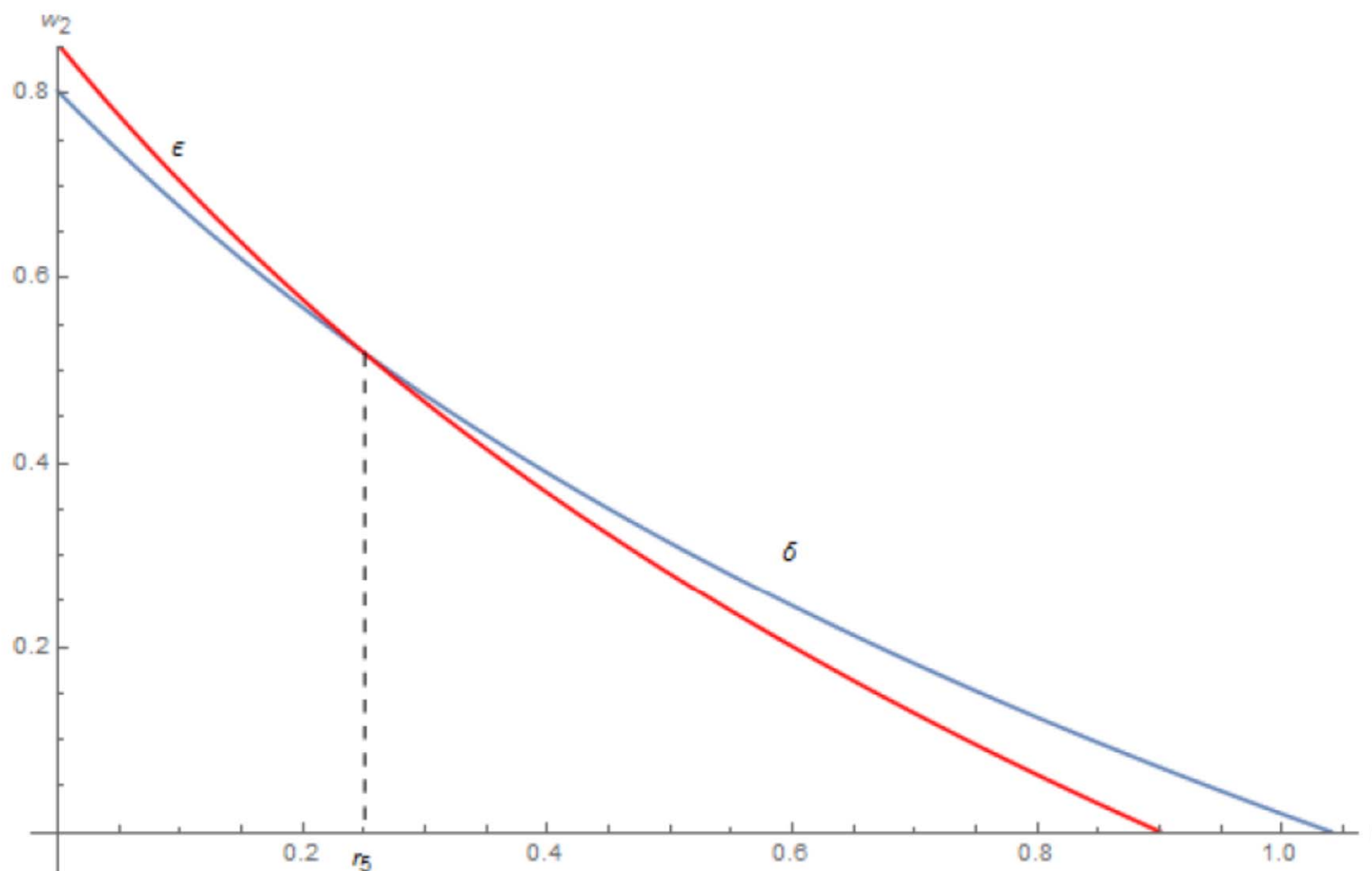
$$(a_{34}, a_{44}, l_4) = (0.29, 0, 1.61).$$

The Wage-profit Curve of Sector 1



Both capital reversing and reswitching occur in Sector 1.

The Wage-profit Curve of Sector 2



Neither capital reversing nor reswitching occurs.

Summary of the Results

r	w_1	w_2	$\frac{p_2}{p_1}$	k_1	k_2
0	0.826	0.854	0.966	1.451	1.565
0.1	0.692	0.705	0.981	1.336	1.36
0.2	0.58	0.577	1.004	1.073	1.392
0.3	0.488	0.481	1.015	1.021	1.125
0.4	0.407	0.396	1.0295	0.797	1.075
0.5	0.328	0.319	1.0277	0.796	1.015
0.6	0.258	0.250	1.030	0.894	0.967
0.7	0.193	0.187	1.0320	0.859	0.924
0.8	0.133	0.128	1.0322	0.827	0.884
0.9	0.076	0.074	1.031	0.798	0.846
1.0	0.029	0.023	1.238	0.796	0.978
1.04	0.011	0.004	2.723	0.783	2.118

The table shows that:

- No capital intensity reversal occurs; sector 2 is always more capital intensive than sector 1:
- The relative price is not a monotonic function of the rate of profit, which implies that factor prices are not necessarily equalised even though no capital intensity reversal occurs.
- The capital as a bundle of reproducible commodities jeopardises the validity of the FPET since the monotonicity of the rate of profit and relative price is not ensured.

4. The HOS model after the Controversies

- Burmeister (1978):

$$[\mathbf{p}, \mathbf{s}] = \mathbf{w}\mathbf{e} + (1 + r)\mathbf{p}\mathbf{A},$$

$\mathbf{p} \in \mathbb{R}^n$: capital good prices, $\mathbf{s} \in \mathbb{R}^m$: consumption good prices, $\mathbf{w} \in \mathbb{R}^h$: primary factor prices, \mathbf{e} : primary factor coefficient matrix of $h \times (n + m)$ order, \mathbf{A} : capital coefficient matrix of $n \times (n + m)$ order.

- The consumption goods are the ‘non-basic’ in this model.

- Burmeister showed that the FPET holds if either condition given below is satisfied:
 - ◆ **SSS-I Condition:** All the diagonal elements of \mathbf{A}^{-1} are positive and all the non-diagonal elements are negative.
 - ◆ **SSS-II Condition:** All the diagonal elements of \mathbf{A}^{-1} are negative and all the non-diagonal elements are positive.
- The Simplified Burmeister model 1: There exist one consumption good, one capital good, one primary factor (labour).

$$[p, s] = [(1 + r)p, w]\bar{\mathbf{A}},$$

$$\text{where } \bar{\mathbf{A}} \equiv \begin{bmatrix} a_{11} & a_{12} \\ l_1 & l_2 \end{bmatrix}.$$

Although capital is reproducible in the above model, it is a *de-facto* one-good model with respect to the determination of factor price. The monotonicity of capital price and the rate of profit is always sustained.

- The Simplified Burmeister Model 2: there exist commodities that can be used as both capital and consumption goods, both of which are the basic goods.

$$\bar{\mathbf{A}} = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ l_1 & l_2 & 0 \end{bmatrix}$$

$\bar{\mathbf{A}}$ is singular. Therefore, the theorems derived from Burmeister (1978) cannot apply to this case.

- Burmeister model is constructed so as to circumvent the issues pointed out in the capital controversies, although the modern dynamic HOS models have the same construction as Burmeister's.

5. Concluding

- If capital consists of a bundle of reproducible commodities, the FPET is not valid even if no capital intensity reversal occurs.
- It suggests that it is necessary to construct a basic theories of international trade that do not rely on factor price equalisation and to treat capital as a bundle of reproducible inputs.