

Currency Manipulation*

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VERY PRELIMINARY AND INCOMPLETE

Abstract

We propose a novel, risk-based, transmission mechanism for the effects of currency manipulation: Policies that systematically induce a country's currency to appreciate in bad times lower its risk premium in international markets. As a result, these policies lower the country's risk-free interest rate and increase domestic capital accumulation and wages. Currency manipulations by large countries also have external effects on foreign interest rates and capital accumulation. Applying this logic to policies that lower the variance of the bilateral exchange rate relative to some target country ("currency pegs"), we find that a small economy pegging its currency to a large economy increases domestic capital accumulation and wages. The size of this effect increases with the size of the target country, offering a potential explanation why the vast majority of currency pegs in the data are to the US dollar, the currency of the largest economy in the world. A large economy (such as China) pegging to a larger economy (such as the US) diverts capital accumulation from the target country to itself, increasing domestic wages while decreasing wages in the target country.

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1 Introduction

Differences in interest rates across developed economies are large and persistent, where some countries have lower interest rates than others for decades rather than years. These long-lasting differences in interest rates correlate with differences in capital-output ratios across countries and account for the majority of excess returns on the carry trade, a trading strategy where international investors borrow in low interest rate currencies, such as the Japanese Yen, and lend in high interest rate currencies, such as the New Zealand dollar (Lustig, Roussanov, and Verdelhan (2011); Hassan and Mano (2015)).

A growing literature studying these “unconditional” differences in currency returns argues that they may be attributable to heterogeneity in the stochastic properties of exchange rates: currencies with low interest rates pay lower returns because they tend to appreciate in bad times and depreciate in good times, providing a hedge to international investors and making them a safer investment (Lustig and Verdelhan (2007); Menkhoff, Sarno, Schmeling, and Schrimpf (2013)). This literature has explored various potential drivers of heterogeneity of the stochastic properties of countries’ exchange rates, ranging from differences in country size (Hassan (2013); Martin (2012)), and financial development (Maggiori (2013)) to differential resilience to disaster risk (Farhi and Gabaix (2015)). The common theme across these papers is that whatever makes countries different from each other results in differential sensitivities of their exchange rates to various shocks, such that some currencies tend to appreciate systematically in states of the world when the shadow price of traded goods is high. Currencies with this property then pay lower expected returns and have lower risk-free interest rates.

In this paper, we argue that this risk-based view of currency returns provides a novel way of thinking about the effects of currency manipulation: interventions in currency markets that change the stochastic properties of exchange rates should change the expected returns on currencies and other assets. In particular, policies that induce a country’s currency to appreciate in bad times should lower domestic interest rates, lower the cost of capital for the production of non-traded goods, and, as a result, increase capital accumulation. Moreover, if these interventions are large enough, that is, if the country manipulating its exchange rate is large relative to the world, its policies will affect the

interest rates and capital accumulation in other countries, potentially diverting capital accumulation from other countries to itself. Policies that change the variances and covariances of exchange rates should thus, via their effect on interest rates and asset returns, affect the allocation of capital across countries.

After making this argument in its most general form, we illustrate the implications of this view with an application to currency pegs. Table 1 shows that 88% of countries representing 47% of world GDP manipulate their exchange rates by pegging their currency relative to some target country (Reinhart and Rogoff (2004)). Such currency pegs specify a target currency (two thirds of them the US dollar) and set an upper bound for the volatility of the real or nominal exchange rate relative to that target country. A “hard” peg may set this volatility to zero while a “soft” peg may officially or unofficially specify a band of allowable fluctuations around some mean. The common feature of all of these policies is that they manipulate the variances and covariances of exchange rates by changing the states of the world and the extent to which they appreciate and depreciate, without necessarily manipulating the level of the exchange rate.

Table 1: 2010 Exchange Rate Arrangements based on Reinhart and Rogoff (2004), Reinhart and Rogoff (2011))

	% of Countries	% of GDP
Panel A	<i>Exchange rate arrangement</i>	
Floating	3%	34%
Pegged	88%	47%
soft	47%	32%
hard	41%	15%
Currency union	9%	19%
Panel B	<i>Target currencies of pegs</i>	
Dollar	67%	80%
Euro	27%	19%

Notes: Classification of exchange rate regimes as of 2010 according to Reinhart and Rogoff (2004), Reinhart and Rogoff (2011). All data are available on Carmen Reinhart’s website at www.carmenreinhart.com/data/browse-by-topic/topics/11.

We analyze the effects of such pegs on interest rates, capital accumulation, and wages within a generic model of exchange rate determination: households consume a bundle

of a freely traded good and a country-specific nontraded good. The nontraded good is produced using capital and labor as inputs. In equilibrium, the real exchange rate fluctuates in response to country-specific (supply) shocks to the productivity in production of nontraded goods across countries, (demand) shocks to preferences, and (monetary) shocks to the inflation rates of national currencies.

As a stand-in for the various potential sources of heterogeneity in the stochastic properties of countries' exchange rates suggested by the literature outlined above, we add heterogeneity in country size to this canonical setup as in (Hassan (2013)). That is, we assume that all shocks are common within countries and some countries account for a larger share of world GDP than others. This heterogeneity in country size generates differences in the stochastic properties of countries' exchange rates, where the currencies of larger countries tend to appreciate in “bad” times: households react to supply, demand, and monetary shocks by shipping traded goods across countries in an effort to share risk across borders. However, the shocks that affect larger countries are harder to diversify internationally. For example, when a country has a low per capita output of nontraded goods, its nontraded good becomes relatively more expensive and its real exchange rate appreciates. To compensate for the shortfall of nontraded goods the country imports additional traded goods from the rest of the world. However, a low output of nontraded goods in a large country simultaneously triggers a rise in the world market price of traded goods, while a low output of nontraded goods in a small country does not. As a consequence, the currencies of large countries tend to appreciate when the world market price of traded goods is high, offering a hedge to world-wide consumption risk. Because of these hedging properties, the currencies of large countries pay lower expected returns, have lower risk-free interest rates. The lower interest rates in turn lower the cost of capital in these countries, prompting them to install higher capital-output ratios and pay higher wages in equilibrium.

Within this economic environment we study the positive effects of a class of policies that lower the variance of one “pegging” country's real exchange rate relative to a “target” country's currency, while leaving the mean of the real exchange rate unaffected. Throughout we focus on real pegs, that is, policies manipulating the real rather than the nominal exchange rate, although all of our main results generalize to nominal pegs. We

also focus almost exclusively on the positive predictions of our model, mainly because these predictions are invariant to whether real exchange rates move as the result of supply, demand, or monetary shocks and thus appear highly robust. By contrast, the welfare effects of currency pegs depend on many details of the model, such as the degree of market completeness and the relative importance of monetary shocks.

To sustain the peg, the pegging country's government alters the state-contingent plan of shipments of traded goods to and from the country. In particular, when the target country appreciates, it matches this appreciation by reducing traded goods consumption and thus raising domestic marginal utility. Similarly, when the pegging country suffers a shock that increases domestic marginal utility that would ordinarily result in an appreciation, it imports additional traded goods to lower domestic marginal utility. In our model, the pegging country's government implements these policies using a set of state-contingent taxes on Arrow-Debreu securities and a lump sum transfer financed using an independent source of wealth ("currency reserves"). More generally, we might imagine such policies being implemented by offering to exchange foreign currencies for domestic currency at a pre-determined rate or other kinds of interventions in currency markets.

We first consider the case in which the pegging country is small and thus only affects its own price of consumption. A small country that imposes a hard peg on a larger country inherits the stochastic properties of the large country's exchange rate: the pegged exchange rate now tends to appreciate when the marginal utility of traded consumption is high in world markets, making it a better hedge against consumption risk and lowering its risk-free interest rate and the return on its currency. Similarly, investments in its capital stock now become more valuable, increasing its capital-output ratio and raising wages within the country.

To sustain the peg, the pegging country ships additional traded goods to the rest of the world when the target country appreciates. If the target country is large, these states again tend to be states when the shadow price of traded goods is high. As a result, pegging to a larger target country generates an insurance premium, making it cheaper to peg to larger countries. If households are sufficiently risk averse and the target country is sufficiently large, this insurance premium may be so large that the currency peg generates positive revenues, that is, it accumulates reserves rather than depleting them.

This revenue-generating effect of currency pegs to larger countries however diminishes when the pegging country itself becomes larger. The reason is that the peg exaggerates the spikes in the pegging country's own demand for traded goods, increasing its price impact: in states of the world in which the pegging country has high marginal utility and would ordinarily appreciate relative to the target country it must import even more traded goods than it would have in the absence of the peg to prevent appreciation. When the pegging country is large enough to affect the equilibrium shadow price of traded goods, the peg thus induces an unfavorable change in the state-contingent prices of traded goods. The larger the pegging country, the more reserves are required to maintain the peg.

Our model also allows us to solve for the effects of the peg on the target country: a country that becomes the target of a peg imposed by a country that is large enough to affect world prices (or the target of multiple pegs imposed by a non-zero measure of small countries) experiences a rise in its risk-free interest rate, a decrease in its capital-output ratio and a decrease in wages. The reason is that, to sustain its peg, the pegging country supplies additional traded goods to the world market whenever the target country appreciates. This activity dampens the impact of the shocks affecting the target country on the shadow price of traded goods, reducing their spill-over to the world market. The lower this impact the lower is the co-movement between the shadow price of traded goods and the target country's exchange rate. The currency of a large country that is the target of a peg thus becomes a less attractive hedge for international investors, raising its risk-free interest rate.

In various robustness checks we show that this broad set of conclusions arises regardless of whether variation in exchange rates are driven primarily by supply, demand, or monetary shocks, regardless of whether the peg is real or nominal, and regardless of whether financial markets are complete or segmented within countries.

We also examine the welfare effects of currency pegs for a special case of our model where markets are complete and exchange rates vary exclusively as a result of supply shocks. In this simpler model, currency pegs are never welfare increasing for the pegging country because any gains in revenues from the peg or from the increase in capital accumulation are outweighed by the adverse effect of an increase in the volatility of consumption. Conversely, becoming the target of a peg reduces one's variance of consumption, resulting

in a net welfare increase, despite the detrimental effects on the target country's capital stock. However, these welfare results depend strongly on the details of the model.

Taken together, we believe our results provide a novel way of thinking about currency manipulation in a world in which risk-premia affect the level of interest rates. First, by manipulating exchange rates policymakers may be able to manipulate the allocation of capital across countries. Second, although currency pegs do not appear optimal under standard welfare measures, our model shows that policymakers might have a motive to peg if their objective is to increase wages, increase capital accumulation, or raise revenue. For example, we might think of political reasons why policymakers might have an interest in raising wages or of externalities that may make it optimal to increase capital accumulation. Third, whatever the motive for pegging, pegs to larger countries appear to be cheaper to implement and more impactful on all dimensions, offering a potential explanation for the fact that almost all pegs in the data are imposed on the euro and the dollar. Fourth, our model speaks to the external effects of pegs on the target country, providing a meaningful notion of what it means to be at the center of the world monetary system: countries that peg to a common target divert capital accumulation from the target while dampening the effects of shocks emanating from the target on the world economy.¹

This latter point also offers an interesting perspective on the large public debate on the Chinese exchange rate regime: U.S. policymakers have often voiced concern that China may be undervaluing its exchange rate and that this undervaluation may be bad for U.S. workers and good for Chinese workers. The official Chinese response to these allegations has been that China is merely reducing the volatility of the dollar - RMB exchange rate but not systematically distorting its level. The implication of our analysis is that even if this assertion is accurate, the mere fact that China is pegging to the dollar may divert capital accumulation from the U.S. to China, a policy that is bad for U.S. workers.

A large literature studies the effects of monetary stabilization and exchange rate pegs in the presence of nominal frictions.² Most closely related are Kollmann (2002) and

¹In this sense, our paper also relates to a growing literature that argues for a special role of the US dollar in world financial markets. See for example Gourinchas and Rey (2007), Lustig et al. (2011), Maggiori (2013), and Miranda-Agrippino and Rey (2015).

²One strand of the literature analyzes optimal monetary policy in small open economies with fixed exchange rates (Parrado and Velasco (2002), Kollmann (2002), Gali and Monacelli (2005)) while another

Bergin and Corsetti (2015) where currency pegs affect markups and the level of capital accumulation through their effects on nominal rigidities. We add to this literature in three ways. First, we study a novel effect of currency pegs on risk premia that operates even in a frictionless economy where money is neutral. Second, we are able to study how the effects of currency pegs vary with the choice of the target country. Third, we are able to study the external effect of the currency peg on the target country.

More broadly, our paper also relates to a large literature on capital controls.³ Similar to the work by Costinot, Lorenzoni, and Werning (2014), who argue that capital controls may be thought of as a manipulation of intertemporal prices, we show that currency pegs, and other policies altering the stochastic properties of exchange rates, may be thought of as manipulation of state-contingent prices. The key difference between the two concepts is that capital controls affect allocations through market power and rents, while currency manipulation affects allocations through risk premia even when the country manipulating its exchange rate is small. In addition, our work shows that, in contrast to capital controls, currency pegs cannot be rationalized as optimal policies within a frictionless neoclassical model.

Finally, as mentioned above, our paper relates to a growing empirical literature that argues that “unconditional” differences in currency returns may be attributable to heterogeneity in the stochastic properties of exchange rates.⁴ The theoretical side of this literature has explored various potential drivers of heterogeneity of the stochastic properties of countries’ exchange rates.⁵ We add to this literature by showing that this class of model implies that exchange rate manipulation may transmit itself through its effect on currency risk premia.

deals with the choice of the exchange rate regime in the presence of nominal rigidities (Helpman and Razin (1987), Schmitt-Grohe Uribe (2011), Bacchetta and van Wincoop (2000), Devereux and Engel (2003), Corsetti, Dedola, and Leduc (2010), Bergin and Corsetti (2015)).

³See for example Korinek (2013) and Bianchi (2011).

⁴See for example Lustig and Verdelhan (2007), Campbell, Serfaty-De Medeiros, and Viceira (2010), Menkhoff, Sarno, Schmeling, and Schrimpf (2012), Lustig, Roussanov, and Verdelhan (2011), and David, Henriksen, and Simonovska (2014), Verdelhan (2015). Also see the evidence in some of the aforementioned papers, including Hassan (2013), Tran (2013), and Ready, Roussanov, and Ward (2013) and Richmond (2015).

⁵ These include differences in country size (Hassan (2013); Martin (2012); Govillot, Rey, and Gourinchas (2010), the size and volatility of shocks affecting the nontraded sector (Tran (2013)), financial development (Maggiore (2013)), factor endowments (Ready, Roussanov, and Ward (2013); Thomas 2015), trade centrality (Richmond (2015)), and resilience to disaster risk (Farhi and Gabaix (2015)).

2 Reduced Form Model of Exchange Rates

We begin by deriving the main insights of our analysis in their most general form. Consider a class of models in which the utility of a representative household in each country n depends on its consumption of a final good consisting of a country-specific nontraded good and a freely traded good. In this class of models, we may write the price of the final good in country n in reduced form as

$$p^n = a\lambda_T - bx^n, \tag{1}$$

where p^n is the log of the number of traded goods required to purchase one unit of the final good in country n , λ_T is the log shadow price traded goods in the world market, b is a constant greater than zero, and $x \sim N(0, \sigma_x^2)$ is a normally distributed shock to the log price of consumption in country n . We may think of this shock interchangeably as the effect of a country-specific supply, demand, or monetary shock, a stand-in for any factor that affects the price of consumption in one country more than in others. The higher x , the lower is the price of domestic consumption.

If households can share risk in world markets by shipping traded goods between countries, these country-specific shocks will also be reflected in the equilibrium shadow price of traded goods in the world. Thus, if many countries have adverse shocks, the shadow price of traded goods will be high in the world and vice versa. In the model, we derive below this relationship is linear with

$$\lambda_T = - \sum_n w^n x^n, \tag{2}$$

where the weights $w^n > 0$ may differ across countries.

The real exchange rate between two countries f and h is the relative price between their respective final goods. We can write the log real exchange rate as

$$s^{f,h} = p^f - p^h.$$

The risk-based view of differences in currency returns applies some elementary asset pric-

ing to this expression. Using the Euler equation of an international investor, one can show that the log expected return to borrowing in country h and lending in country f is

$$r^f + E s^{f,h} - r^h = cov(\lambda_T, p^h - p^f) = b(w^h - w^f) \sigma_x^2 \quad (3)$$

where r^n is the risk-free interest rate in country n .⁶ This statement means that a currency that tends to appreciate when the shadow price of traded goods is high pays a lower expected return and, if $E s^{f,h} = 0$, has a lower risk-free interest rate. Currencies that appreciate in bad times thus provide a hedge against world-wide consumption risk and must pay lower returns in equilibrium. These “systemic” currencies are the currencies of countries that have a relatively large w^n , that is, the currencies of countries whose shocks spill over to world markets more than the shocks of other countries.

This line of argument (equations (1)-(3)) is the main ingredient of any risk-based model of unconditional differences in interest rates across countries, where different approaches model differences in w^n as the result of heterogeneity in country size, the volatility of shocks to the non-traded sector, trade specialization, financial development, factor endowments, etc.

We make a simple point relative to this literature: if there is merit to this risk-based view of currency returns, policies that alter the covariance between a country’s exchange rate and the shadow price of traded goods can alter interest rates, currency returns, and the allocation of capital across countries. In particular, a country that adopts policies to increase the price of domestic consumption in states of the world where λ_T is high can lower its risk-free interest rate relative to all other countries in the world. As an example, consider a “pegging” country (p) that levies a state-contingent tax on domestic consumption of traded goods that is proportional to the realization of x in some target country t , such that

$$p^p = a\lambda_T - bx^p - cx^t.$$

Note that this state-contingent tax is zero on average across states ($E(x^t) = 0$), such that it affects only the stochastic properties but not the level of country p ’s exchange rate. If the target country’s shock affects the world price of traded goods, that is if $w^t > 0$, this

⁶See Appendix A for a formal proof.

policy increases the covariance between p^p and λ_T and, as a result, lowers country p 's interest rate relative to all other countries in the world. The larger w^t , the larger is this effect. In this sense, currency manipulations that hone in on the shocks affecting the most systemic countries in the world are most impactful.

In addition, and this will become clear when we move to our fully specified model, the state-contingent tax also impacts the country p 's state-contingent plan of shipping traded goods to and from the world. Specifically, a tax that increases the price of consumption when x^t is low induces shipments of traded goods from country p to the rest of the world in those states. If the country manipulating its exchange rate is itself large in the sense that its actions affect the equilibrium shadow price of traded goods, its policy thus reduces the the target country's weight w^t in (2). That is, it dampens the extent to which the target country's shock spills over to other countries. As a consequence, the covariance between p^t and λ_T falls, increasing the interest rate in the target country. A state-contingent tax of the form above thus *raises* the interest rate in the target country while lowering it in the country manipulating its exchange rate.

If interest rates play a role in allocating capital across countries (as is the case in our fully specified model), manipulations of the stochastic properties of exchange rates can thus divert capital accumulation from the target country of the manipulation to the country conducting the manipulation and, more broadly, alter the equilibrium allocation of capital across countries.

The remainder of this paper fleshes out this simple argument in the context of a canonical model of exchange rate determination.

3 A Model of Currency Pegs

In this section, we set up our fully specified model in which the allocation of capital across countries and the stochastic properties of real exchange rates are jointly determined as a function of supply and demand shocks. The model generalizes the framework in Hassan (2013) and Hassan, Mertens, and Zhang (2015) by allowing for currency manipulation. The model nests the canonical real business cycle model of exchange rate determination (Backus and Smith, 1993) augmented with preference shifters as in Pavlova and Rigobon

(2007), as well as a simplified version of the incomplete-markets model with monetary shocks by Alvarez, Atkeson, and Kehoe (2002). The purpose of including multiple types of shocks is simply to show the generality of our argument, all results go through with just one type of shocks. Within this canonical model of exchange rate determination, one country, labelled the “pegging” country, deviates from the competitive equilibrium by imposing a hard or soft peg on its real or nominal exchange rate with respect to a “target” country.

3.1 Setup

There are two discrete time periods, $t = 1, 2$. There exists a unit measure of households $i \in [0, 1]$, partitioned into three subsets Θ^n of measure θ^n . Each subset represents the constituent households of a country. We label these countries $n = p, t, o$ for the “pegging”, “target”, and “outside” country, respectively. Households make an investment decision in the first period. All consumption occurs in the second period.

Households exhibit constant relative risk aversion according to

$$U(i) = \frac{1}{1 - \gamma} \mathbb{E} [(\exp(\chi^n) C_2(i))^{1-\gamma}] \quad (4)$$

where $C_2(i)$ is the consumption index for household i , χ^n is a common shock to the preferences of households in country n ,

$$\chi^n \sim N\left(-\frac{1}{2}\sigma_\chi^2, \sigma_\chi^2\right)$$

and $\gamma > 0$ is the coefficient of relative risk aversion. The consumption index is defined as

$$C_2(i) = C_{T,2}(i)^\tau C_{N,2}(i)^{1-\tau} \quad (5)$$

where $C_{N,2}$ is consumption of the country-specific nontraded good, $C_{T,2}$ is consumption of the traded good, and $\tau \in (0, 1)$.

At the start of the first period, each household receives a deterministic endowment of traded goods ($Y_{T,1}^n$) and one unit of capital. Traded goods can be stored for consumption

in the second period and are freely shipped internationally. Capital goods can be freely shipped only in the first period when they are invested for use in the production of nontraded goods in the second period.

In each country, there exists a firm that produces nontraded goods using a Cobb-Douglas production technology employing capital and labor. Firms purchase capital in international markets in the first period. Each household owns one share of the firm in its country and inelastically supplies one unit of labor to it. The per capita output of nontraded goods is

$$Y_{N,2}^n = \exp(\eta^n) (K_N^n)^\nu \quad (6)$$

where $0 < \nu < 1$ is the capital share in production, K_N^n is the *per capita* stock of capital in country n and η^n is a country-specific productivity shock to the production of nontraded goods realized at the start of the second period,

$$\eta^n \sim N\left(-\frac{1}{2}\sigma_N^2, \sigma_N^2\right).$$

Throughout we use the traded consumption good as the numéraire, such that all prices and returns are accounted for in the same units.

In the first period, a fixed proportion ϕ of households within each country trade a complete set of state-contingent securities in international markets. Label these households as “active”. The remaining $1 - \phi$ fraction of households within each country are excluded from trading state-contingent securities. Label these households as “inactive”. Inactive households cede the claims to their endowments, their wages, and firm profits to active households in return for a nominal bond. Each active household thus receives a fraction $\frac{1}{\phi}$ of per capita second period wages and firm profits.

The nominal bond given to inactive households pays off one unit of the country’s nominal consumer price index. We write this payment to inactive households as $P_2^n e^{-\mu^n}$, where μ^n is a country-specific inflation shock to the price of one unit of the traded good in terms of the currency of country n ,

$$\mu^n \sim N\left(-\frac{1}{2}\tilde{\sigma}, \tilde{\sigma}\right)$$

To simplify notation, let ω represent the realization of productivity, preference and inflation shocks and let $g(\omega)$ be the associated multivariate density. All households take prices as given. Active households maximize their utility (4) subject to the constraint

$$\begin{aligned} & \int Q(\omega) \left(P_2^n(\omega) C_2^n(\omega) + \frac{1-\phi}{\phi} P_2^n(\omega) e^{-\mu^n} \right) d\omega \\ & \leq \frac{1}{\phi} (Y_{T,1}^n + q_1) + \int Q(\omega) \frac{1}{\phi} (w_2^n(\omega) + \pi_2^n(\omega)) d\omega + \kappa^n \end{aligned} \quad (7)$$

where $Q(\omega)$ is the price of a security that pays one unit of the tradable good if state ω occurs, P_2^n denotes the number of traded goods required to buy one unit of the country-specific consumption index in country n , and $\frac{1-\phi}{\phi}$ is the number of inactive households per active household in each country. $w_2^n(\omega)$ is the wage paid to each unit of labor and $\pi_2^n(\omega)$ is the per capita share of profits, where again we use the traded good as the numeraire. q_1 is the world-market price of a unit of capital in the first period. To simplify the derivation, we also assume that active households receive a country-specific transfer, κ^n , before trading begins such that the decentralized problem coincides with the Social Planner's allocation with unit Pareto weights.

Inactive households also maximize (4), but subject to the constraint

$$P_2^n \hat{C}_2(i) \leq P_2^n(\omega) e^{-\mu^n}, \quad (8)$$

where we use hats to denote the consumption of inactive households.

Firms take prices as given and purchase an optimal quantity of capital in the first period to maximize expected discounted profits. Firms produce non-traded goods and pay out wages and profits in the second period to maximize

$$\max_{K_N^n} \int Q(\omega) \pi_2(\omega) d\omega = \int Q(\omega) (P_{N,2}^n(\omega) Y_{N,2}^n(\omega) - w_2^n(\omega)) d\omega - q_1 K_N^n. \quad (9)$$

3.2 Currency Pegs

The pegging country's government has the ability to levy a state-contingent consumption tax and has access to an independent supply of traded goods (currency reserves) that

it can use to finance its taxation scheme. The government's objective is to decrease fluctuations of its country's log real exchange rate with the target country by a fraction $\zeta \in (0, 1]$ relative to the freely-floating regime without distorting the conditional mean of the log real exchange rate. As a result, it chooses a taxation scheme such that

$$\text{var}(s^{t,p}) = (1 - \zeta)^2 \text{var}(s^{t,p*}) \quad (\text{P1})$$

and

$$\mathbb{E}[s^{t,p}|K^n] = \mathbb{E}_1[s^{t,p*}|K_N^n]. \quad (\text{P2})$$

where asterisks denote values under a free floating exchange rate regime and we refer to $\zeta = 1$ as a "hard" peg.

We also consider pegs of the nominal exchange rate that decrease the variance of the log nominal exchange rate between the pegging and target countries, $\text{var}(\tilde{s}^{t,p}) = (1 - \zeta)^2 \text{var}(\tilde{s}^{t,p*})$, while keeping the conditional mean of the log nominal exchange rate unchanged, $\mathbb{E}[\tilde{s}^{t,p}|K_N^n] = \mathbb{E}[\tilde{s}^{t,p*}|K_N^n]$.

The government achieves this policy through a combination of a state contingent tax on consumption goods delivered in the country, $Z(\omega)$ and a lump sum transfer, \bar{Z} . Formally, households in the pegging country face the following budget constraint

$$\begin{aligned} & \int Z(\omega)Q(\omega) \left(P_2^p(\omega)C_2^p(\omega) + \frac{1-\phi}{\phi}P_2^p(\omega)e^{-\mu^n} \right) d\omega \\ & \leq \frac{1}{\phi}(Y_{T,1}^p + q_1) + \int Q(\omega)\frac{1}{\phi}(w_2^p(\omega) + \pi_2^p(\omega)) d\omega + \kappa^p + \bar{Z}. \end{aligned} \quad (10)$$

3.3 Equilibrium

The market clearing conditions for traded, nontraded, and capital goods are

$$\int_i C_{T,2}(i, \omega) di = \sum_n \theta^n Y_{T,1}^n(\omega), \quad (11)$$

$$\int_{i \in \theta^n} C_{N,2}(i, \omega) di = \theta^n Y_{N,2}^n(\omega), \quad (12)$$

and

$$\sum_n \theta^n K_N^n = \sum_n \theta^n = 1 \quad (13)$$

The economy is at an equilibrium when all households maximize utility subject to their budget constraints, firms maximize profits, and goods markets clear.

3.4 Solving the Model

Although financial markets are incomplete, the model's solution remains tractable because a subset of the population in each country (active households) have access to complete financial markets. As a result, we can solve for the equilibrium allocation by solving the Social Planner's problem, subject to the equilibrium behavior of firms, inactive households, and households in the pegging country. Appendix B shows the solution to the inactive household's problem and the corresponding adjustments to the Social Planner's constraints (the procedure is identical to that in Alvarez, Atkeson, and Kehoe (2002)). Their behavior is relevant only for understanding how monetary shocks affect the equilibrium, which we discuss in detail below.

The Social Planner chooses the allocation of traded and nontraded goods to maximize the utility of active households in the target and outside countries subject to the consumption of inactive households, the consumption of the pegging country's active households, the firms' investment decisions, and the resource constraints in the economy.⁷ Because all active households within a given country are identical, we can write their consumption bundle as $(C_{T,2}^n, C_{N,2}^n)$ and henceforth drop the household index i .

The first-order conditions with respect to $C_{T,2}^n$ equate the shadow price of tradable consumption across active households in the target and outside countries

$$\tau (\chi^n)^{1-\gamma} (C_2^n(\omega))^{-\gamma} (C_{T,2}^n(\omega))^{-1} = \Lambda_{T,2}(\omega) \quad (14)$$

and the first-order conditions with respect to $C_{N,2}^n$ define the shadow prices of nontradable

⁷See Appendix E for a formal setup and proof.

goods within each country

$$(1 - \tau) (\chi^n)^{1-\gamma} (C_2^m(\omega))^{-\gamma} (C_{N,2}^n(\omega))^{-1} = \Lambda_{N,2}^n(\omega). \quad (15)$$

In addition, it is useful to keep track of the (redundant) first-order condition with respect to the consumption index C_2^n , because it pins down the marginal utility of consumption of active households in each country

$$(\chi^n)^{1-\gamma} (C_2^m(\omega))^{-\gamma} = \Lambda_2^n(\omega) \quad (16)$$

By definition, the real exchange rate between two countries h and f equals the ratio of these shadow prices,

$$S^{f,h}(\omega) = \Lambda_2^f(\omega) / \Lambda_2^h(\omega)$$

Because markets are competitive, all prices must coincide with ratios of shadow prices from the Social Planner's problem. In particular, we can solve for the state-contingent price of an Arrow-Debreu security paying one unit of the traded good in state ω in the target or outside country as

$$Q(\omega) = \frac{\Lambda_{T,2}(\omega)}{\Lambda_{T,1}} g(\omega), \quad (17)$$

where $\Lambda_{T,1} = \mathbb{E} [\Lambda_{T,2}(\omega)]$ can be interpreted as the shadow price of a tradable good in the first period, and prior to the realization of shocks. Thus, all active households outside of the pegging country price assets using the ratio of marginal utilities from tradable consumption as the unique stochastic discount factor.

Solving the problem of active households in the pegging country (see Appendix C) yields the first-order conditions for the consumption of traded goods

$$\tau (\chi^p)^{1-\gamma} (C_2^p(\omega))^{-\gamma} (C_{T,2}^p(\omega))^{-1} g(\omega) = Z(\omega) \Lambda_{T,2}(\omega) \quad (18)$$

and nontraded goods

$$(1 - \tau) (\chi^p)^{1-\gamma} (C_2^p(\omega))^{-\gamma} (C_{N,2}^p(\omega))^{-1} g(\omega) = Z(\omega) \Lambda_{N,2}^p, \quad (19)$$

where the state-contingent tax implementing the peg now appears simply as a consumption wedge.

Finally, firms in each country decide how much capital is purchased from households in international markets. We take first-order conditions of the firm's problem given by equation (9), use equations (17) and the fact that competitive markets imply $P_{N,2}^n(\omega) = \Lambda_{N,2}^n(\omega)/\Lambda_{T,2}(\omega)$ and simplify to derive the following Euler equation

$$K_N^n = \frac{\nu}{\Lambda_{T,1}q_1} \mathbb{E} [\Lambda_{N,2}^n Y_{N,2}^n], \quad (20)$$

which defines the level of capital accumulation in country n as a function of first-period prices and the stochastic properties of $\Lambda_{N,2}^n$ and $Y_{N,2}^n$.

Importantly, this Euler equation holds in all countries, even the pegging country. To see this, note that the pegging government's intervention alters the firm's problem in the pegging country in two offsetting ways: although the price impact of the tax alters the domestic price of each Arrow-Debreu securities, and thus the state-contingent valuation of output ($Q(\omega)Z(\omega)$), the firm's profit is paid in the form of units of consumption to its shareholders and is thus itself subject to the tax, delivering $\pi_2^n(\omega)/Z(\omega)$ to the shareholder. The two effects cancel such that (9) holds in all countries. In other words, implementing a currency peg requires a consumption wedge but does not require distorting capital accumulation, such that both domestic and international investors agree on the valuation of the firm.

Combining the six first order conditions (18), (19), (14), (15), with three Euler Equations for capital investment (20) and the five resource constraints (11), (12), (13) yields a system of 14 equations. These 14 equations implicitly define the following 14 endogenous variables: $\{C_{N,2}^n, C_{T,2}^n, K_N^n, \Lambda_{N,2}^n\}_{n \in \{p,t,o\}}, \Lambda_{T,2}$ and q_1 .

4 Results

To study the model in closed form, we log-linearize the model around the deterministic solution — the point at which the variances of all shocks are zero ($\sigma_{\chi,n}, \sigma_{N,n}, \tilde{\sigma} = 0$) and all firms have a capital stock that is fixed at the deterministic steady state level. That is, we

will study the *incentives* to accumulate different levels of capital across countries while holding the capital stock fixed, thus ignoring the feedback effect of differential capital accumulation on the size of risk premia. Doing so allows us to simplify the exposition of the solution. The appendix shows equivalent expressions where we allow capital to adjust endogenously. Although these expressions are slightly more complicated they generate identical qualitative results ⁸.

We begin by characterizing the state contingent taxes that the pegging country can implement to impose a real or nominal exchange rate peg. Throughout, lowercase variables refer to natural logs.

Lemma 4.1

A tax on all assets paying off consumption goods in the pegging country p of the form

$$z(\omega) = \zeta \frac{1 - \tau}{\tau(\tau + \phi(1 - \tau))} (y_N^p - y_N^t) + \zeta \frac{(1 - \tau)(1 - \phi)}{\tau(\tau + \phi(1 - \tau))} (\mu^p - \mu^t) + \zeta \frac{\gamma - 1}{\tau + \phi(1 - \tau)} (\chi^t - \chi^p)$$

implements a real exchange rate peg of strength ζ .

A tax on all assets paying off consumption goods in the pegging country p of the form

$$\tilde{z}(\omega) = z(\omega) + \zeta \frac{\gamma\tau + \phi(1 - \tau)}{\tau + \phi(1 - \tau)} (\mu^p - \mu^t)$$

implements a nominal exchange rate peg of strength ζ .

The cost of the peg, κ_{Cost}^p is the sum of the change in the cost to purchase state-contingent claims to tradable consumption, the change in the cost of purchasing capital for the firm and the change in the income from selling capital to firms.

$$\kappa_{Cost}^p = \int Q(\omega) C_T^p d\omega - \int Q^*(\omega) C_T^{p*} d\omega$$

Proof. See Appendix D. ■

⁸See Appendix F for the log-linear system of equations.

4.1 Real Business Cycle Model

We first analyze the case where all households are active, $\phi = 1$, and the variance of the preference shock is zero ($\sigma_{\chi,n} = 0$). In this case, all endogenous variables are determined exclusively by shocks to the production of nontraded goods and our model coincides with the canonical real business cycle model of exchange rate determination (Backus and Smith, 1993).

4.1.1 The freely floating regime

In the absence of currency manipulation ($\zeta = 0$), equilibrium consumption of tradable goods in an arbitrary country n (recall that lowercase variables denote logs) is given by

$$c_T^{n*} = \frac{(1 - \tau)(\gamma - 1)}{(1 - \tau) + \gamma\tau} (\bar{y}_N - y_N^n),$$

where $\bar{y}_N = \sum_n \theta^n y_N^n$ is the average log per capita output of nontraded goods across countries. The expression shows that households use the traded good to insure against risk in their nontradable sectors. If $\gamma > 1$, households receive additional tradables whenever they have a lower than average output of nontradable goods and vice versa.

This risk-sharing behavior generates a shadow price of tradable goods of the form given in (2),

$$\lambda_T^* = -(\gamma - 1)(1 - \tau) \sum_n \theta^n y_N^n, \quad (21)$$

where each country's weight is proportional to its size: shocks to the productivity of larger countries affect a larger measure of households and thus tend to spill over to the rest of the world in the form of higher shadow prices of traded goods. When $\gamma > 1$, the shadow price of traded goods falls with the average output of nontraded goods across countries. Thus, λ_T tends to be low in “good” states of the world when countries experience positive productivity shocks in their nontradable sectors.

The real exchange rate between two arbitrary countries f and h is

$$s^{f,h*} = \lambda^{f*} - \lambda^{h*} = \frac{\gamma(1 - \tau)}{(1 - \tau) + \gamma\tau} (y_N^h - y_N^f)$$

The country with the lower per-capita output of nontradable goods appreciates because its final consumption bundle is expensive relative to that in other countries.

Inspecting λ_T^* and $s^{f,h*}$ shows that larger countries tend to appreciate when the shadow price of tradable goods is high: whenever a country suffers a low productivity shock, the relative price of its nontradable goods rises and its real exchange rate appreciates. For a given percentage decline in productivity, this appreciation occurs independently of how large the country is (note that $s^{f,h*}$ is independent of θ). However, a shock to a larger country has a larger impact on the rest of the world. For example, in states of the world in which the US (the largest economy in the world) draws a low productivity shock, it imports a large share of the world's tradable goods, raising the shadow price of tradable goods for every country. As a result, the US dollar tends to appreciate when λ_T is high, producing a positive covariance between the US real exchange rate and λ_T . It immediately follows from the first equality in (3) that larger countries have a lower risk-free rate.

$$r^{f*} + \mathbb{E}s^{f,h*} - r^{h*} = cov(\lambda_T^*, p^{h*} - p^{f*}) = \frac{(\gamma - 1)\gamma(1 - \tau)^2}{1 + (\gamma - 1)\tau} (\theta^h - \theta^f) \sigma_N^2$$

To see that these differences in interest rates across countries translate into differential incentives to accumulate capital, rearrange (20) to obtain a form similar to (3): take logs of both sides of the equation, substitute $\lambda_{N,2}^n = p_{N,2}^n + y_{N,2}^n$, and take differences across countries to obtain

$$k_N^{f*} - k_N^{h*} = \frac{1}{2}var(p_N^f + y_N^f) - \frac{1}{2}var(p_N^h + y_N^h) + cov(p_N^f + y_N^f - p_N^h - y_N^h, \lambda_T) \quad (22)$$

where we can interpret $p_N^f + y_N^f$ as the value of nontradable output in terms of traded goods, or as the payoff of a unit of stock in the non-tradable sector of country f . Ignoring the two variance terms on the right hand side for the moment, this expression suggests that countries whose output increases in value when λ_T is high should accumulate more capital per capita. The solution of the model yields

$$p_N^{f*} + y_N^{f*} = \frac{(1 - \tau)(\gamma - 1)}{1 + (\gamma - 1)\tau} (\bar{y}_N - y_N^f).$$

It shows that differences in the payoff of stocks behave in the same way as exchange rates:

when country f suffers a low productivity shock, its currency appreciates and the value of its firm's output in terms of traded goods increases. If country f is large, the same adverse productivity shock also raises λ_T , inducing a positive covariance between λ_T and the value of the firm's output.

Larger countries thus not only have lower interest rates but also have incentives to accumulate higher capital-output ratios. Solving for the variances and covariances in (22) yields

$$k_N^{f*} - k_N^{h*} = \frac{(\gamma - 1)^3(1 - \tau)^2\tau}{1 + (\gamma - 1)\tau} (\theta^f - \theta^h) \sigma_N^2.$$

It is efficient to accumulate more capital in the larger country because a larger capital stock in larger country represents a good hedge against global consumption risk. Households around the world are worried about states of the world in which the large country receives a low output from its nontradable sector, because larger countries transmit these shocks to the rest of the world through a higher shadow price of tradable consumption. Although households cannot affect the realization of productivity shocks, they can partially insure against low output in the nontradable sector of large countries by accumulating more capital. This raises expected output in the nontradable sector and dampens the negative effects of a low productivity shock.

4.1.2 Internal effects of a peg

We have described allocations in an economy with freely floating exchange rates. All else equal, larger countries have lower risk-free rates and higher capital investment per capita. Now, we analyze how a country can influence these allocations with an exchange rate peg. We start by analyzing the effect of the exchange rate peg on allocations and prices in the pegging country alone. Afterwards, we analyze the impact of the exchange rate peg on prices and quantities in the rest of the world.

The exchange rate peg makes the price level in the pegging country behave more in line with the price level in the target country.

$$\lambda^p = \lambda^{p*} + (1 - \theta^p)\zeta \frac{\gamma(1 - \tau)}{1 + (\gamma - 1)\tau} (y_{N,2}^p - y_{N,2}^t)$$

Similar to (1), the peg increases the effect of the target country's shock while also decreasing the weight of its own shock. The same is true for the value of the firm's output in the pegging country

$$p_N^p + y_N^p = (p_N^{p*} + y_N^{p*}) + \zeta \frac{(1 - \tau)(\theta^p + (\gamma - 1)\tau)}{\tau(1 + (\gamma - 1)\tau)} (y_N^p - y_N^t).$$

If the target country is sufficiently larger than the pegging country, the exchange rate peg thus increases the covariance of both λ^p and $p_N^p + y_N^p$ with λ_T , lowering the country's interest rate and increasing its capital accumulation.

Proposition 4.2

In the real business cycle model of exchange rate determination with $\gamma > 1$, a country that imposes a hard real exchange rate peg on a target country larger than itself lowers its risk-free rate, increases capital accumulation, and increases the average wage in its country relative to all other countries.

Proof. When the smaller country imposes a hard real exchange rate peg, the interest rate differential becomes

$$r^p + \mathbb{E} s^{p,t} - r^t = cov(\lambda_T, p^t - p^p) = (r^{p*} + \mathbb{E} s^{p,t*} - r^{t*}) - \frac{(1 - \tau)^2 \gamma (\theta^t - \theta^p) (\gamma - 1) \tau}{\tau(1 + (\gamma - 1)\tau)} \sigma_N^2.$$

See Appendix G for the corresponding proof for capital accumulation. ■

Aside from these effects on interest rates and capital accumulation, maintaining the currency peg affects the pegging government's resources (currency reserves). From (21), we already know that the cost of the peg is simply the cost of altering the state contingent purchases of traded goods in world markets. The cost of the peg thus depends on the change in the pegging country's equilibrium consumption of traded goods. We can write this change as

$$c_T^p - c_T^{p*} = \zeta \frac{(1 - \tau)(1 - \theta^p)}{\tau((1 - \tau) + \gamma\tau)} (y_N^t - y_N^p). \quad (23)$$

When the target country receives a relatively bad productivity shock ($y_N^t < y_N^p$), its price of consumption increases. To mirror this increase, the pegging country reduces its consumption of traded goods relative to the freely floating regime, ships additional traded goods to the rest of the world, and thus raises its own marginal utility. Conversely,

when the pegging country receives a relatively bad shock, its price of consumption would ordinarily increase. To off-set this increase it imports even more traded goods than it would have ordinarily to prevent the appreciation.

The peg thus induces the pegging country to sell additional traded goods in response to adverse productivity shocks in the target country and to buy additional traded goods in response to adverse productivity shocks at home. If the target country is larger than the pegging country, traded goods are more expensive in the states in which it sells than in the states in which it buys. In this case, the peg induces the pegging country to provide insurance to the world market, pocketing an insurance premium.

Proposition 4.3

In the real business cycle model of exchange rate determination with $\gamma > 1$, if the pegging country is small, $\theta^p = 0$, then the cost of the peg decreases with the size of the target country and increases with the size of the pegging country.

Additionally, the cost of the peg is negative if and only if

$$\theta^t > \frac{\zeta + (\gamma - 1)\tau}{(\gamma - 1)^2\tau^2}.$$

Proof. The (log) cost of the peg is given by

$$\log(\kappa_{Cost}^p) = \frac{[(\zeta + (\gamma - 1)\tau) - \tau^2(1 - \gamma)^2\theta^t](1 - \tau)^2\zeta\sigma_N^2}{\tau^2(1 + (\gamma - 1)\tau)^2}$$

which is decreasing in the size of the target country. This expression becomes negative if and the target country is large enough. ■

If the target country is sufficiently large relative to the pegging country and risk aversion is sufficiently high, this insurance premium can be so large that the peg generates revenues for the government, building rather than depleting currency reserves.

When the pegging country itself has non-zero mass ($\theta^p > 0$), its purchases and sales of traded goods also affect the equilibrium shadow price of traded goods, λ_T , increasing the cost of the peg. The reason is that pegging effectively increases the volatility of shipments of traded goods to the rest of the world. In states where the pegging country has a bad

productivity shocks, it imports more traded goods than it ordinarily would have. In states where the target country has bad productivity shocks it exports more. The more price impact the pegging country has, the more costly it therefore is to maintain the peg.

4.1.3 External effects of a peg

When the pegging country has non-zero mass, the exchange rate peg affects tradable consumption and prices in the rest of the world. The shadow price of tradable goods is given as

$$\lambda_{T,2} = -(1 - \tau)(\gamma - 1)\bar{y}_N + \frac{\theta^p(1 - \tau)}{\tau}\zeta(y_{N,2}^t - y_{N,2}^p). \quad (24)$$

The second term on the right hand side shows that if the pegging country is large, the peg dampens the effect of the target country's shocks on the shadow price of tradables, reducing the extent to which it spills over to the rest of the world and making it *less* systemic. As a result, the currency peg decreases the covariance between the target country's real exchange rate and $\lambda_{T,2}$, increasing the target country's interest rate and lowering capital accumulation.

Proposition 4.4

If a country becomes the target of a peg imposed by a large country, its risk-free interest rate rises relative to the rest of the world, capital accumulation falls, and average wages fall relative to all other countries.

Proof. The interest rate differential between the target and outside country is

$$r^t + \mathbb{E}s^{t,o} - r^o = (r^{t*} + \mathbb{E}s^{t,o*} - r^{o*}) + \zeta \frac{\theta^p(1 - \tau)^2\gamma}{\tau(1 + (\gamma - 1)\tau)}\sigma_N^2.$$

See Appendix I for the corresponding proof for capital accumulation. ■

In this sense, a currency peg can divert capital from the target country to the pegging country even though it has no effect on the level of the real exchange rate. This finding is particularly interesting because it sheds new light on recent public controversies, for example between Chinese and U.S. officials, that usually focus on the idea that an undervaluation of the Chinese real exchange rate favors Chinese workers at the expense of U.S. workers. By contrast our results suggest, that even a currency peg that manipulates the

variance but not the level of the real exchange rate can have this effect.

4.1.4 Welfare and the Rationale of Pegging

Finally, we may use the real business cycle version of our model to perform a simple welfare calculation. So far we have assumed that a currency peg has two objectives, to reduce the variance of the log real exchange rate (P1) while not distorting its level (P2). For the purposes of this calculation, we now drop the objective (P2) and rebate the cost of the peg to households in the pegging country. Hence, households in the pegging country bear the costs of imposing the exchange rate peg by shifting the level of their tradable consumption in all states of the world.

Changes in the expected utility of households in the pegging country are a result of changes in the expected level of consumption and changes in the variance of consumption. We have already seen that a peg to a larger country can increase the level of consumption by increasing capital accumulation and by generating revenues (a negative cost of the peg). However, the peg also increases the variance of consumption, reducing expected utility. In equilibrium, the latter effect dominates:

Proposition 4.5

In the real business cycle model of exchange rate determination with $\gamma > 1$, a country that imposes an exchange rate peg decreases the welfare of its households.

Proof. See Appendix H. ■

Similarly, the currency peg decreases the volatility of consumption in the target country because it dampens the effect of the target country's shock on the shadow price of traded goods.

Proposition 4.6

In the real business cycle model of exchange rate determination with $\gamma > 1$, a country that becomes the target of an exchange rate peg imposed by a smaller country with positive mass will see the volatility of its households' consumption decrease. Expected utility in the target country increases as a result of the peg.

Proof. See Appendix H. ■

In contrast to the positive results outlined above, these welfare results do not generalize easily to our full model with incomplete markets, inflation, and preference shocks and should therefore be interpreted with some caution.

Moreover, although our model does not deliver a clear welfare-based motive for pegging, it may nevertheless rationalize currency pegs, and in particular the pegs to the US observed in the data, if policymakers have an interest in increasing capital accumulation, increasing wages, or generating revenues from a peg. For example, a peg to the largest economy in the world may be optimal if policymakers in a pegging country maximize a function of the form

$$EU^n + \mu_1 K^n - \mu_2 k^p,$$

where μ_1 and μ_2 are constant that may reflect the political influence of workers, externalities from capital accumulation, or a motive for generating revenues in a way that avoids direct taxation of households or firms.

4.2 Incomplete Markets and Preference Shocks

In the previous section, we established the impact of an exchange rate peg on the equilibrium allocation in a conventional real business cycle model with complete markets. Although the complete-markets model is an important benchmark, it has a number of well-known empirical shortcomings. First, it predicts a perfect negative correlation between appreciations of the real exchange rate and aggregate consumption growth — a currency appreciates when its aggregate consumption falls (Backus and Smith, 1993). Second, the model predicts that consumption should be more correlated across countries than output, whereas the opposite is true in the data (Backus et al., 1994). Third, real exchange rates and terms of trade seem much too volatile to be rationalized exclusively by real (productivity) shocks alone (Chari et al., 2002). As a result, many authors have argued for incomplete market models that allow for an effect of monetary shocks on equilibrium real exchange rates or models with demand shocks.

In this subsection, we analyze the effects of exchange rate manipulation in our full model, featuring inflation shocks, market segmentation, and preference shocks, and show that the intuition and all positive results from the previous section carry over to this

more general model. To simplify the discussion, we derive all results for the case where productivity is deterministic, $\sigma_N = 0$.

The punchline is that both inflation and preference shocks generate a relationship between exchange rates and the shadow price of traded goods identical to the structure in (1) and (2): inflation shocks affect exchange rates by shifting resources within a given country from inactive households who are excluded from financial markets (and thus are irrelevant for prices in international markets) to active households whose marginal utilities price assets in international markets. Inactive households hold nominal bonds denominated in their national currencies and are thus vulnerable to inflation shocks. A positive inflation shock to the price of traded goods in terms of the domestic currency acts as an “inflation tax” on inactive households. The higher the inflation shock, the less their nominal bonds are worth and the less they are able to consume. Since inflation shocks have no bearing on the real resources available for consumption, this reduction of inactive household’s wealth shifts resources towards the country’s active households such that they receive more traded and nontraded goods, depreciating the domestic price of consumption in both real and nominal terms. At the same time, risk-sharing compels the active households to ship some of the additional traded goods to active households in other countries, transmitting part of the inflation shock to active households in other countries via the shadow price of traded goods.

Similarly, preference shocks move exchange rates by shifting the level of utility derived from each unit of consumption. A high preference shock increases the amount of utility derived from each unit of consumption, reduces the marginal utility of households’ consumption, and thus also depreciates the country’s currency in real and nominal terms. Again, risk-sharing with households in other countries then compels domestic households to ship traded goods to the rest of the world, transmitting part of the shock to other countries.

Solving our model with inflation and preference shocks yields

$$\begin{aligned} \lambda^p = & - \frac{(1-\phi)\gamma^2\tau}{\phi(\phi(1-\tau) + \gamma\tau)} \bar{\mu} - \frac{(1-\phi)(1-\tau)\gamma}{\phi(1-\tau) + \gamma\tau} \mu^p - \frac{\gamma\tau(\gamma-1)}{\gamma\tau + (1-\tau)\phi} \bar{\chi} - \frac{(1-\tau)(\gamma-1)\phi}{\gamma\tau + (1-\tau)\phi} \chi^p \\ & + (1-\theta^p)\zeta \frac{\gamma(1-\tau)(1-\phi)}{\phi(1-\tau) + \gamma\tau} (\mu^p - \mu^t) + (1-\theta^p)\zeta \frac{(\gamma-1)(1-\tau)\phi}{\gamma\tau + (1-\tau)\phi} (\chi^p - \chi^t) \end{aligned}$$

and

$$\begin{aligned}\lambda_T &= -\gamma \left(\frac{1-\phi}{\phi} \right) \bar{\mu} - (\gamma-1)\bar{\chi} \\ &\quad + \zeta \frac{\theta^p(1-\tau)}{\gamma\tau} (\gamma(1-\phi)(\mu^t - \mu^p) + \phi(\gamma-1)(\chi^t - \chi^p)),\end{aligned}$$

where $\bar{\mu} = \sum_n \theta^n \mu^n$ and $\bar{\chi} = \sum_n \theta^n \chi^n$ are the weighted sums of inflation and preference shocks in all countries, respectively. The first lines in both expressions show the price of country p 's consumption index and the shadow price of traded goods in the freely floating regime. Note that, as with productivity shocks, a high realization of both inflation and preference shocks depreciates the price of domestic consumption and lowers λ_T in proportion to the size of the country. The second line in both expression shows that, again, a currency peg makes the pegging country's price of consumption behave more like the target country's and that the peg lowers the weight of the target country's shock in λ_T while increasing the weight of the pegging country's.

This change in the size of spill-overs into the shadow price of traded goods results from the fact that active households in the pegging country ship additional traded goods to the rest of the world whenever the target country appreciates and import additional traded goods whenever shocks raise the price of its own consumption relative to that in the target country.

$$\mathcal{C}_{T,2}^p - \mathcal{C}_{T,2}^{p*} = \zeta \Xi_T^p (\gamma(1-\phi)(\mu^t - \mu^p) + \phi(\gamma-1)(\chi^t - \chi^p)), \quad (25)$$

where Ξ_T^p is a positive constant derived in the Appendix J.

It follows directly that all of our positive predictions about the effects of currency pegs carry over to our full model.

Proposition 4.7

In the full model with market segmentation, inflation shocks, preference shocks, and productivity shocks with $\gamma > 1$,

1. *a smaller country that imposes a hard real exchange rate peg on a sufficiently large target country lowers its risk-free rate, increases capital accumulation, and increases the average wage in its country relative to all other countries.*

2. *when the pegging country is small, the cost of the peg decreases with the size of the target country and increases with the size of the pegging country.*
3. *if a country becomes the target of a peg imposed by a large country, its risk-free interest rate rises relative to the rest of the world, capital accumulation falls, and average wages fall relative to all other countries.*

Proof. See Appendix K ■

In addition to re-enforcing the main insights from the complete-markets case, allowing for market incompleteness also improves the quantitative implications of the model along the three dimensions outlined above. The combination of market segmentation, inflation shocks, and preference shocks loosens or even reverses the negative correlation between appreciations of the real exchange rate and aggregate consumption growth, lowers the correlation of aggregate consumption across countries, and increases the volatility of real and nominal exchange rates (Alvarez et al., 2002; Pavlova and Rigobon, 2007; Kollmann, 2012).

4.3 Nominal Pegs

Up until now, we have characterized the internal and external effects of a real exchange rate peg. In practice, nominal exchange rate pegs are often easier to implement. In fact, most exchange rate pegs in the data appear to be nominal.

The log nominal exchange rate between two countries is equal to the ratio of their nominal price indices

$$\tilde{s}^{f,h} = p_2^f - \mu^f - (p_2^h - \mu^h). \quad (26)$$

When markets are complete ($\phi = 1$), inflation shocks impact real allocations only through their impact on the pegging country's tradable consumption. Lemma 4.1 shows that the state contingent tax used to implement a nominal exchange rate peg is identical to that used to impose a real exchange rate peg plus random noise induced by inflation shocks. Naturally, the nominal peg then leads to allocations and risk premia identical to those under the real peg plus an additional noise component. As long as the volatility of the

inflation shock is low relative to productivity and preference shocks (in the data, real and nominal exchange rates are highly correlated), all positive results then continue to hold.

If $\phi < 1$, real and nominal exchange rate pegs both respond to differences in inflation shocks between the pegging and target countries. The only difference is the magnitude of this adjustment. A nominal peg forces households to change tradable consumption to offset the differences in real price levels as well as the differences in inflationary shocks. Hence, we might suspect that a nominal exchange rate peg is just a “stronger” version of the real exchange rate peg, and that there is some equivalence between a nominal exchange rate peg of strength $\tilde{\zeta}$ and a real exchange rate peg of strength ζ . The following proposition shows that this is indeed the case.

Proposition 4.8

Suppose the variance of real shocks and preference shocks are zero ($\sigma_{N,n}, \sigma_{\chi,n} = 0$). A tax on Arrow-Debreu securities purchased by residents of country p of the form

$$z(\omega) = \tilde{\zeta} \frac{\gamma - (\gamma - 1)(1 - \tau)\phi}{\gamma\tau(\tau(1 - \phi) + \phi)}$$

implements a nominal exchange rate peg of strength $\tilde{\zeta}$. This is equivalent to implementing a real exchange rate peg of strength

$$\zeta = \tilde{\zeta} \frac{\gamma - (\gamma - 1)(1 - \tau)\phi}{\gamma(1 - \tau)(1 - \phi)}$$

5 Conclusion

This paper solves an international asset pricing model which endogenizes the stochastic properties of exchange rates, international asset prices, and the level of capital accumulation across countries. It explores the effects of exchange rate pegs on the economies of the target country and of countries outside the peg. We are able to characterize the impact of the peg on the consumption of households in each country, the exchange rates between countries and the spreads on bonds and stocks in the world. Additionally, we solve for the impact of exchange rate pegs on differences in capital investment between countries.

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A Differences in Log Asset Returns

Consider an asset that pays off $X(\omega)$ units of the tradable good. The value of this asset

$$V_X = \mathbb{E} \left[\frac{\Lambda_{T,2}(\omega)}{\Lambda_{T,1}} X(\omega) \right]$$

If we assume asset returns and marginal utilities are log-normally distributed and take logs of both sides of the previous equation

$$v_X = \mathbb{E} [\lambda_{T,2} - \lambda_{T,1} + x] + \frac{1}{2} \text{var} (\lambda_{T,2}) + \frac{1}{2} (x) + \text{cov} (\lambda_{T,2}, x)$$

because $\Lambda_{T,1}$ is deterministic and known in the first period. Hence, the log expected return on an asset with payoff X is

$$\begin{aligned} \log ER[X] &= \log \mathbb{E} \left[\frac{X}{V_X} \right] = \mathbb{E} [x] + \frac{1}{2} \text{var} (x) - v_X \\ &= \lambda_{T,1} - \mathbb{E} [\lambda_{T,2}] - \frac{1}{2} \text{var} (\lambda_{T,2}) - \text{cov} (\lambda_{T,2}, x) \end{aligned}$$

Taking the difference between the log expected returns of two different assets with payoffs X and Z yields

$$\log ER[X] - \log ER[Z] = \text{cov} (\lambda_{T,2}, z - x) \tag{27}$$

The risk free bond in country n pays P^n units of the tradable good in the second period. We plug the bond payments P^h and P^f into equation (27) to derive equation (3).

B Equilibrium Consumption of Inactive Households

Inactive households in country n maximize utility, defined in equation (4), in each state of the world by splitting their wealth $\exp(-\mu_t^n) P_t^n$ optimally between tradable and non-tradable goods. Their optimization problem can be written as maximizing (4) in each

state subject to (8)

$$\begin{aligned} \max_{\hat{C}_{T,2}(i), \hat{C}_{N,2}(i)} & \frac{1}{1-\gamma} \left(e^{\chi^n} \hat{C}_{T,2}(i)^\tau \hat{C}_{N,2}(i)^{1-\tau} \right)^{1-\gamma} \\ \text{s.t.} & \hat{C}_{T,2}(i) + P_{N,2} \hat{C}_{N,2}(i) = \exp(-\mu_t^n) P_t^n \end{aligned} \quad (28)$$

We solve this problem by setting up a Lagrangian and taking first-order conditions with respect to $\hat{C}_{T,2}(i)$ and $\hat{C}_{N,2}(i)$.

Inactive households consume an optimal mix of tradable and non-tradable goods given by

$$\hat{C}_{T,2}^n = \exp(-\mu^n) (\tau P_2^n), \quad \hat{C}_{N,2}^n = \exp(-\mu^n) \left(\frac{(1-\tau) P_2^n}{P_{N,2}^n} \right) \quad (29)$$

where \hat{C}_T^n and \hat{C}_N^n are the consumption of tradable and non-tradable goods by inactive agents in country n , respectively.

C Equilibrium Consumption of Active Households in the Pegging Country

Households in the pegging country maximize utility subject to their budget constraint and the actions of the government in the pegging country.

$$\max_{C_{T,2}^p(\omega)} E \left[\frac{1}{1-\gamma} \left(e^{\chi^n} (C_{T,2}^p(\omega))^\tau (C_{N,2}^p(\omega))^{1-\tau} \right)^{1-\gamma} \right]$$

subject to

$$\begin{aligned} & \int (1+Z(\omega)) Q(\omega) (C_T^p(\omega) + P_N^p(\omega) C_N^p(\omega)) d\omega \leq \\ & (Y_{T,1} + q_1) + \int_{\Omega} Q(\omega) (w_2^p(\omega) + \pi_2^n(\omega)) d\omega + \kappa^p + \kappa_{Tax}^p + \kappa_{Cost}^n \end{aligned} \quad (30)$$

where κ_{Tax}^p represents the lump-sum rebate of tax revenues from the government, κ^p represents the lump sum transfer to decentralize the Social Planner's problem, and κ_{Cost}^n is a lump sum tax or transfer from the government required to impose the peg, which we

define more precisely below.

We set up the Lagrangian take first order conditions with respect to $C_{T,2}^p(\omega)$ and $C_{N,2}^p(\omega)$. Let the Lagrange multiplier on the budget constraint be Λ_W^p . This allows us to derive equations (18) and (19).

We also need to make sure the budget constraint for the pegging household holds. This requires that we formally define the “cost of the peg”, κ_{Cost}^p .

D Proof of Lemma 4.1

We solve for the form of the log-linear tax that implements the exchange rate peg by first assuming a state contingent tax of the form

$$Z(\omega) = \left(\frac{Y_{N,2}^t}{Y_{N,2}^p} \right)^{a_1} \left(\frac{\chi^t}{\chi^p} \right)^{a_2} \left(\frac{\mu^t}{\mu^p} \right)^{a_3}$$

and then searching for the coefficients a_1, a_2, a_3 that decreases the volatility of the exchange rate between the pegging and target country such that (P1) holds.

We re-write the budget constraint of the household in order to more easily identify the components of the lump sum transfer. When markets are complete a household in the pegging country faces the following budget constraint

$$\int (1 + X(\omega)) Q(\omega) (P_2^p(\omega) C_2^p(\omega)) d\omega \leq (Y_{T,1}^p + q_1) + \int Q(\omega) (w_2^p(\omega) + \pi_2^p(\omega)) d\omega + \kappa^p + \kappa_{Tax}^p + \kappa_{Cost}^p + q_1^* (K_N^{p*} - 1) - q_1 (K_N^p - 1)$$

Where we have re-written the exchange rate peg as $X(\omega)$. The exchange rate peg satisfies the two desired properties when the cost of the peg, κ_{Cost}^p , is defined by the previous equation.

The government rebates the tax revenues from the tax back to the households. This lump-sum rebate is

$$\kappa_{Tax}^p = \int X(\omega) Q(\omega) (C_T^p(\omega) + P_N^p(\omega) C_N^p(\omega)) d\omega$$

We use this expression to simplify the pegging country's budget constraint

$$\int Q(\omega) (C_T^p(\omega) + P_N^p(\omega)C_N^p(\omega)) d\omega \leq (Y_{T,1} + q_1) + \int_{\Omega} Q(\omega) (w_2^p(\omega) + \pi_2^n(\omega)) d\omega + \kappa^p + \kappa_{Cost}^n + q_1^* (K_N^{p*} - 1) - q_1 (K_N^p - 1)$$

Substituting in for the firm's profit and using the non-tradable good's market clearing condition ($Y_{N,2}^p = C_{N,2}^p$) yields

$$\int Q(\omega) C_T^p(\omega) d\omega \leq Y_{T,1} + \kappa^p + \kappa_{Cost}^p + q_1^* (K_N^{p*} - 1)$$

Finally, we solve for κ^n from the household's problem in a freely floating regime, and plug that into the equation to recover

$$\int Q(\omega) C_T^p(\omega) d\omega = \int Q^*(\omega) C_{T,2}^{p*} d\omega + \kappa_{Cost}^p$$

Hence, we have derived the expression for the cost of the peg.

E Social Planner's Problem

All active households within a country consume the same bundle $(C_{T,2}^n(\omega), C_{N,2}^n(\omega))$. We can write the economy's resource constraints (11) and (12) as

$$\begin{aligned} \phi C_{N,2}^n + (1 - \phi) \hat{C}_{N,2}^n &= Y_{N,2}^n \\ \sum_{n=p,t,o} \theta^n [\phi C_{T,2}^n + (1 - \phi) \hat{C}_{T,2}^n] &= \sum_{n=p,t,o} \theta^n Y_{T,1}^n \end{aligned}$$

The Lagrangian can be written as

$$\begin{aligned}
L = & \phi \sum_{n=t,o} \frac{\theta^n}{1-\gamma} \left[e^{\chi^n} (C_{T,2}^n)^\tau (C_{T,2}^n)^{1-\tau} \right]^{1-\gamma} \\
& - \Lambda_{T,2} \left[\sum_{n=p,t,o} \theta^n \left(\phi C_{T,2}^n + (1-\phi) \hat{C}_{T,2}^n \right) - \sum_{n=p,t,o} \theta^n Y_{T,1}^n \right] \\
& - \sum_{n=t,o} \Lambda_{N,2}^n \left[\phi C_{N,2}^n + (1-\phi) \hat{C}_{N,2}^n - Y_{N,2}^n \right]
\end{aligned}$$

We take first order conditions with respect to tradable and non-tradable consumption in the target and outside countries. This yields a system of four equations given by (14) and (15)

F Log-linearized System of Equations

We substitute the equilibrium consumption of the inactive households into the resource constraints, and replace prices with ratios of lagrange multipliers. Combining the six first order conditions (18), (19), (14), (15), with three Euler Equations for capital investment (20) and the five resource constraints (11), (12), (13) yields a system of 14 equations. We log-linearize the model around the deterministic solution — the point at which the variances of all shocks are zero ($\sigma_{\chi,n}, \sigma_{N,n}, \tilde{\sigma} = 0$) and all firms have a capital stock that is fixed at the deterministic steady state level.

The log-linear first order conditions for the target and outside countries are

$$\begin{aligned}
(1-\gamma)\chi^n + (1-\gamma)(\tau c_T^n + (1-\tau)c_N^n) - c_T^n + \log \tau &= \lambda_T \\
(1-\gamma)\chi^n + (1-\gamma)(\tau c_T^n + (1-\tau)c_N^n) - c_N^n + \log(1-\tau) &= \lambda_N^n
\end{aligned}$$

The log-linear first order conditions for the pegging country are

$$\begin{aligned}
(1-\gamma)\chi^n + (1-\gamma)(\tau c_T^n + (1-\tau)c_N^n) - c_T^n + \log \tau &= \lambda_T + z \\
(1-\gamma)\chi^n + (1-\gamma)(\tau c_T^n + (1-\tau)c_N^n) - c_N^n + \log(1-\tau) &= \lambda_N^n + z
\end{aligned}$$

where z is the log-linear expression for the tax given by Lemma 4.1.

We log-linearize the euler equation for capital accumulation for each country, given by equation (20)

$$\log(q_1) + \lambda_{T,1} + k_N^n = \mathbb{E}[\lambda_N^n + y_N^n] + \frac{1}{2}var(\lambda_N^n + y_N^n)$$

where we can treat $\log(q_1) + \lambda_{T,1}$ as a single quantity that tells us the cost of a unit of capital in terms of utility.

Finally, the log-linear resource constraints are

$$\begin{aligned} \phi c_N^n + (1 - \phi) \left(-\mu^n - \tau \left(\lambda_N^n - \lambda_T - \log \left(\frac{1 - \tau}{\tau} \right) \right) \right) &= \eta^n + \nu k_N^n = y_N^n \\ \sum_{n=p,t,o} \theta^n \left[\phi c_T^n + (1 - \phi) \left(-\mu^n - (1 - \tau) \left(\lambda_N^n - \lambda_T - \log \left(\frac{1 - \tau}{\tau} \right) \right) \right) \right] &= \sum_{n=p,t,o} \theta^n y_{T,1}^n = 1 \\ \sum_{n=p,t,o} \theta^n k_N^n &= 1 \end{aligned}$$

This set of fourteen equations allows us to solve for the following fourteen unknowns $\{k_N^n, c_N^n, c_T^n, \lambda_N^n\}_{n=p,t,o}$, λ_T and $\log(q_1) + \lambda_{T,1}$. We can write these endogenous variables in terms of the following nine state variables $\{y_N^n, \mu^n, \chi^n\}$ for $n = p, t, o$. Note that agents only care about the total output of non-tradable goods, $y_N^n = \eta^n + \nu k_N^n$, in each country in the second period.

G Proof of Proposition 4.2

In the real business cycle model of exchange rate determination with $\gamma > 1$, a smaller country that imposes a hard real exchange rate peg on a sufficiently large target country lowers its risk-free rate, increases capital accumulation, and increases the average wage in its country relative to all other countries.

The interest rate differential between the pegging and target country is

$$r^p + \mathbb{E} s^{p,t} - r^t = cov(\lambda_T, p^t - p^p) = (r^{p*} + \mathbb{E} s^{p,t*} - r^{t*}) - \zeta \frac{(1 - \tau)^2 \gamma (2\theta^p (1 - \zeta) + (\theta^t - \theta^p) (\gamma - 1) \tau)}{\tau (1 + (\gamma - 1) \tau)} \sigma_N^2$$

When the smaller country imposes a hard real exchange rate peg, this expression simplifies

to

$$r^p + \mathbb{E}s^{p,t} - r^t = cov(\lambda_T, p^t - p^p) = (r^{p*} + \mathbb{E}s^{p,t*} - r^{t*}) - \frac{(1-\tau)^2\gamma(\theta^t - \theta^p)(\gamma-1)\tau}{\tau(1+(\gamma-1)\tau)}\sigma_N^2$$

which implies the exchange rate peg decreases the risk free rate in the pegging country relative to the risk free rate in the target country as long as the target country is larger than the pegging country, $\theta^t > \theta^p$.

The differential incentives to accumulate capital when the pegging country imposes a hard peg in the real business cycle economy are

$$k_N^p - k_N^t = k_N^{p*} - k_N^{t*} + \frac{(\gamma-1)^3(1-\tau)^2\tau(\theta^t - \theta^p)}{(1+(\gamma-1)\tau)^2}\sigma_N^2$$

The last term of the right hand side expression shows that incentives to accumulate capital in the pegging country increase relative to the target country as long as the target country is larger than the pegging country, $\theta^t > \theta^p$.

Because firms are competitive, wages are given by the marginal product of labor. $w^n = (1-\nu)\exp(\eta^n)(K_N^n)^\nu$. Since the marginal product of labor rises with the level of capital accumulation, the exchange rate peg increases wages in the pegging country relative to all other countries.

H Proof of Welfare Results

The volatility of log consumption in the pegging country is

$$var(c^p) = var(c^{p*}) + \zeta \frac{2(1-\theta^p)(1-\tau)^2((1-\theta^p)\zeta - 1 + (\theta^t - \theta^p)(\gamma-1)\tau)}{(1+(\gamma-1)\tau)^2}\sigma_N^2$$

which shows that the volatility of consumption in the pegging country increases with the size of the target country. If

$$\theta^t > \theta^p + \frac{1 - (1 - \theta^p)\zeta}{(\gamma - 1)\tau}$$

then the exchange rate peg increases the volatility of consumption in the pegging country. A corollary of this result is that a small country ($\theta^p = 0$) that imposes a hard exchange rate peg always increases the volatility of consumption of its households, and always decreases the expected utility of its households.

We investigate changes in expected utility by examining $(1 - \gamma)U(i)$. As $(1 - \gamma)U(i)$ increases, utility decreases. For household i in the pegging country, we calculate

$$\frac{d}{d\zeta} \log [(1 - \gamma)U(i)] = \frac{(\gamma - 1)(1 - \tau)^2 ((1 - \theta^p)\zeta + \theta^p(1 + \theta^t - \theta^p)(\gamma - 1)\tau)}{\tau(1 + (\gamma - 1)\tau)} \sigma_N^2 < 0$$

Hence

$$\frac{dU(i)}{d\zeta} \frac{1}{U(i)} > 0$$

If we multiply both sides of the inequality by $U(i)$, we show $\frac{dU(i)}{d\zeta} < 0$ because $U(i) = \frac{1}{1-\gamma} (C_2^p)^{1-\gamma} < 0$. Hence, imposing an exchange rate peg decreases utility in the pegging country.

The volatility of log consumption in the target country is

$$\text{var}(c^t) = \text{var}(c^{t*}) - \zeta \frac{2\theta^p(1 - \tau)^2 (1 - \theta^p\zeta + (\theta^t - \theta^p)(\gamma - 1)\tau)}{(1 + (\gamma - 1)\tau)^2} \sigma_N^2$$

it is clear that $\text{var}(c^t)$ decreases when the pegging country imposes an exchange rate peg if the pegging country is smaller than the target country, $\theta^t > \theta^p$.

Again, we examine the quantity $(1 - \gamma)U(i)$. For household i in the target country, we calculate

$$\frac{d}{d\zeta} \log [(1 - \gamma)U(i)] = -\frac{\theta^p(\gamma - 1)^2(1 - \tau)^2 (2(1 - \theta^p) + \theta^p(1 + \theta^t - \theta^p)(1 + (\gamma - 1)\tau))}{(1 - \theta^p)(1 + (\gamma - 1)\tau)^2} \sigma_N^2 < 0$$

If we multiply both sides of the inequality by $U(i)$, we show $\frac{dU(i)}{d\zeta} > 0$ because $U(i) = \frac{1}{1-\gamma} (C_2^t)^{1-\gamma} < 0$. Hence, expected utility in the target country increases when the pegging country imposes an exchange rate peg.

I Proof of Proposition 4.4

If a country becomes the target of a peg imposed by a large country, its risk-free interest rate rises relative to the rest of the world, capital accumulation falls, and average wages fall relative to all other countries.

The interest rate differential between the target and outside country is

$$r^t + \mathbb{E}s^{t,o} - r^o = cov(\lambda_T, p^o - p^t) = (r^{t*} + \mathbb{E}s^{t,o*} - r^{o*}) + \zeta \frac{\theta^p (1 - \tau)^2 \gamma}{\tau (1 + (\gamma - 1)\tau)} \sigma_N^2$$

which implies the exchange rate peg increases the risk free rate in the target country relative to the risk free rate in the outside country.

The differential incentives to accumulate capital when the pegging country imposes a hard peg in the real business cycle economy are

$$k_N^t - k_N^o = k_N^{t*} - k_N^{o*} - \frac{\theta^p (\gamma - 1)^2 (1 - \tau)^2}{(1 + (\gamma - 1)\tau)^2} \zeta \sigma_N^2$$

The last term of the right hand side expression shows that incentives to accumulate capital in the target country decrease relative to the outside country.

Because firms are competitive, wages are given by the marginal product of labor. Since the marginal product of labor rises with the level of capital accumulation, the exchange rate peg decreases wages in the target country relative to all other countries.

J Expressions for Constants in the Incomplete Markets Model

The following constants are used to define the consumption of tradable goods in the pegging country and the value of non-tradable output in the incomplete markets model, respectively.

$$\Xi_T^p = \frac{(1 - \theta^p) (\tau + (1 - \tau)\phi)^2 + (1 - \tau)(1 - \phi) (\gamma\tau + (1 - \tau)\phi) (1 + (1 - \tau)) (1 - \tau)\phi}{\gamma\tau (\phi + (1 - \tau)\phi) (\gamma\tau + \phi(1 - \tau)) (\tau + (1 - \tau)\phi)}$$

$$\Xi_N^p = \frac{(1-\tau)\phi(\theta^p(\tau(\gamma-\phi)+\phi)+(1-\theta^p)(\gamma-1)\tau)}{\gamma\tau(\tau(1-\phi)+\tau)(\gamma\tau+(1-\tau)\phi)}$$

K Proof of Proposition 4.7

We derive results for the quantities of interest in an economy that is affected by productivity shocks, inflation shocks and preference shocks. We will first prove results for the internal effects of a real exchange rate peg.

The interest rate differential between the pegging and target country when the pegging country imposes a hard exchange rate peg is

$$\begin{aligned} r^p + \mathbb{E}S^{p,t} - r^t = \text{cov}(\lambda_T, p^t - p^p) = \\ (r^{p*} + \mathbb{E}S^{p,t*} - r^{t*}) - \frac{(1-\tau)^2(\gamma-\phi)^2\gamma(\theta^t - \theta^p)}{\phi(\gamma\tau + (1-\tau)\phi)}\sigma_N^2 - \\ \frac{(1-\tau)(1-\phi)^2\gamma^2(\theta^t - \theta^p)}{\phi(\gamma\tau + (1-\tau)\phi)}\tilde{\sigma}^2 - \frac{(\gamma-1)^2(1-\tau)\phi(\theta^t - \theta^p)}{\gamma\tau + (1-\tau)\phi}\sigma_x^2 \end{aligned}$$

which implies the exchange rate peg decreases the risk free rate in the pegging country relative to the risk free rate in the target country as long as the target country is larger than the pegging country, $\theta^t > \theta^p$.

We show that the relative incentives to accumulate capital in the pegging country increase with the size of the target country. Hence, there exists a country size θ_{min} such that a hard exchange rate peg on any country larger than θ_{min} will increase the incentives to accumulate capital in the pegging country.

$$\begin{aligned} \frac{d}{d\theta^t} [k_N^p - k_N^t - (k_N^{p*} - k_N^{t*})] = \frac{\zeta(\gamma-1)(1-\tau)^2\tau(\gamma-\phi)^2}{(\phi + (1-\phi)\tau)(\gamma\tau + (1-\tau)\phi)}\sigma_N^2 \\ + \frac{\zeta(\gamma-1)\gamma(1-\tau)\tau(\gamma-\phi)(1-\phi)^2}{(\phi + (1-\phi)\tau)(\gamma\tau + (1-\tau)\phi)}\tilde{\sigma}^2 + \frac{\zeta(\gamma-1)^3(1-\tau)\tau(\gamma-\phi)\phi^2}{\gamma(\phi + (1-\phi)\tau)(\gamma\tau + (1-\tau)\phi)}\sigma_x^2 \end{aligned}$$

Because firms are competitive, wages are given by the marginal product of labor. Hence, an appropriate exchange rate peg increases wages in the pegging country relative to all other countries.

The interest rate differential between the target country and the outside country is

$$r^t + \mathbb{E}S^{t,o} - r^o = (r^{t*} + \mathbb{E}S^{t,o*} - r^{o*}) + \frac{\zeta\theta^p\gamma(1-\tau)^2}{\tau(\gamma\tau + \phi(1-\tau))}\sigma_N^2 + \frac{\zeta\theta^p\gamma(1-\tau)^2(1-\phi)^2}{\tau(\gamma\tau + \phi(1-\tau))}\tilde{\sigma}^2 + \frac{\theta^p\zeta(\gamma-1)^2(1-\tau)^2\phi^2}{\gamma\tau(\gamma\tau + (1-\tau)\phi)}\sigma_x^2$$

which implies the exchange rate peg increases the risk free rate in the target country relative to the risk free rate in the outside country.

The differential incentives to accumulate capital in the target country relative to the outside country is given by

$$k_N^t - k_N^o = k_N^{t*} - k_N^{o*} - \frac{\theta^p\zeta(1-\tau)^2(\gamma-\phi)^2}{(\gamma\tau + (1-\tau)\phi)^2}\sigma_N^2 - \frac{\theta^p\gamma\zeta(1-\tau)(\gamma-\phi)(1-\phi)^2}{(\gamma\tau + (1-\tau)\phi)^2}\tilde{\sigma}^2 - \frac{\theta^p(\gamma-1)^2\zeta(1-\tau)(\gamma-\phi)\phi^2}{\gamma(\gamma\tau + (1-\tau)\phi)^2}\sigma_x^2$$

Incentives to accumulate capital in the target country decrease relative to the outside country.

Because firms are competitive, wages are given by the marginal product of labor. Since the marginal product of labor rises with the level of capital accumulation, the exchange rate peg decreases wages in the target country relative to all other countries.

Finally, we turn to the cost of the peg. The change in the cost of a hard exchange rate peg as the target country gets larger is

$$\begin{aligned} \frac{d \log(\kappa_{Cost}^p)}{d\theta^t} &= - \frac{\gamma(\gamma-1)(1-\tau)(1-\phi)^2(\tau(1-\phi)(\gamma(1-\tau)+\tau)+\phi)}{\phi(\tau(1-\phi)+\phi)(\gamma\tau-\tau\phi+\phi)^2}\tilde{\sigma}^2 \\ &\quad - \frac{(\gamma-1)(1-\tau)^2(\gamma-\phi)(\tau(1-\phi)(\gamma(1-\tau)+\tau)+\phi)}{\phi(\tau(1-\phi)+\phi)(\gamma\tau-\tau\phi+\phi)^2}\sigma_N^2 \\ &\quad - \frac{(\gamma-1)^3(1-\tau)\phi(\tau(1-\phi)(\gamma(1-\tau)+\tau)+\phi)}{\gamma(\tau(1-\phi)+\phi)(\gamma\tau-\tau\phi+\phi)^2}\sigma_x^2 < 0 \end{aligned}$$

Hence, it is cheaper to peg to a larger country.