Characterizing Global and Regional Value Chains

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(Preliminary draft for comments)

This version: December 29, 2015

Abstract

Since the extent of offshoring and production sharing varies by sector and country, we develop measures of GVCs in terms of length, intensity, and location of participation at the levels of country, country-sector, and bilateral sector, and distinguish among pure domestic, directly traded, and indirectly traded production activities. Using these measures, we characterize cross-country production sharing patterns and GVC related trade activities for 35 sectors and 40 countries over 17 years. We find that the production chain for the world as a whole has become longer. While the relative ranking of the length at the sector level is stable across countries, the average length for a given country-sector, of both the domestic and international components, and their participation and position in GVCs in general, do evolve significantly over time. The results contribute to a better understanding of features of global value chains and patterns of participation by individual country-sectors.

Key Words: Production length, Position and Participation in Global Value Chains

JEL Number: F1, F6

The views in the paper are those of the authors and do not necessarily reflect the views and policies of the Asian Development Bank or its Board of Governors or the governments they represent, the USITC and its Commissioners, or any other organization that the authors are affiliated with. Zhi Wang acknowledges the research and financial support by Stanford Center for International Development when he was visiting there in Spring 2015.

1. Introduction

The emergence of global value chains (GVCs) has changed the pattern of international trade in recent decades. Different stages of production now are often conducted by multiple producers located in several countries, with parts and components crossing national borders multiple times. While the deficiency (i.e., due to trade in intermediates) of official trade statistics as a description of true trade patterns has been well recognized, measures of global value chains based on sequential production are still under development.

A "value chain" represents value added at various stages of production, which runs from the initial phase such as R&D and design to the delivery of the final product to consumers. A value chain can be national if all stages of production occur within a country, or global if different stages take place in different countries. In practice, most products or services are produced by a global value chain.

Production length, as a basic measure of GVCs, is defined as the number of stages in a value chain, reflecting the complexity of the production process. Antras et al. (2012) believe that such a measure of relative production-line position is first and foremost the quantitative indicator necessary to assess specialization patterns of countries in relatively upstream versus downstream stages of global production processes. The upstreamness and downstreamness indexes discussed in recent literature (see also Miller and Temurshoev, 2015) are numerical estimates based on production length to measure a sector/country's position in a global production process.

Fally (2012) proposes two measures, "distance to final demand," i.e., the average number of stages between production and final consumption, and "the average number of production stages embodied in each product" to quantify the length of production chains. The first measure, also referred to as "upstreamness" in the literature is further described in Antras et al. (2012); the second measure, also referred to as "downstreamness" in the literature is further explored in Antras and Chor (2013). However, there are two common conceptual caveats for these measures discussed in previous literature: first, they all start from a sector's gross output, which includes not only final goods and services, but also intermediate inputs. As argued by Erik (2005, 2007), a production chain must start from the sector's primary inputs (or value added) such as labor and capital, not its gross output.[1] Second, current "upstreamness" and "downstreamness" measures

[1] It is important to bear in mind that gross outputs are endogenous variables, while primary inputs and final demand are exogenous variables in the standard Leontief model. Converting gross output (gross exports are part of it) into

do not imply each other, and may indicate inconsistent production line positions for the same country/industry pairs.

Therefore, in this paper we define production length as the distance from primary inputs to final products. We show that indexes built on such definition are more consistent and with better economic interpretations. We demonstrate that the average production length of any value chain always equals the ratio of the portion of gross output and the corresponding value-added that induces the output. Most importantly, based on the gross trade accounting framework proposed by Koopman, Wang, and Wei (to be subsequently cited as KWW, 2014) and Wang, Wei, and Zhu (to be subsequently cited as WWZ, 2013), we further split the total production length into a pure domestic segment, a segment related to direct value-added trade, and a segment related to GVCs that reflect deeper cross country production sharing activities. This allows us to define the GVC production length more clearly for the first time in the literature.

We show that there is a conceptual difference between production length measure and production line position measure. Once we define the production length by segments at the bilateral and sector levels, indexes representing a country-sector's position on a GVC can be easily constructed at various levels of disaggregation. With this, we can gauge whether a country or an industry is likely to be located in the upstream or downstream part of a particular global value chain.

We also modify the global value chain participation index defined by Koopman et al. (2010), redefining both the forward and backward industrial linkage based participation indexes by considering not only export production but also production that satisfies domestic final demand through international trade.

We apply these new measures to the recently available Inter Country Input Output (ICIO) database and obtain some interesting results. We show that Fally's result on the lengthening of production chains is not globally representative. More precisely, his main empirical result that the production chain has become shorter, and his main hypothesis that value-added has gradually shifted towards the downstream stage, closer to the final consumers, are both unique to the US input-output tables. We overturn his results with our newly defined GVC production length index and global ICIO databases. First, we show that emerging economies like China have a

final demand is the key technical step to establishing their gross trade accounting framework in both Koopman, Wang, and Wei (2014) and Wang, Wei, and Zhu (2013).

gradual lengthening of the overall production chain and the lengthening of production by these countries dominates shortening of production by others, so that the world as a whole experiences a lengthening of the production process. Second, we decompose changes in total production length into changes in the pure domestic segment, changes in the segment related to direct value-added trade, and changes in the segment related to global value chains. By further separating the production length of GVCs into domestic and international segments, we show that the ratio of international production length versus total production length of GVCs has increased for all countries. Third, we show that all countries in the world increased their GVC participation during 1995–2011. And finally, we use the three types of newly defined GVC indexes as explanatory variables to analyze the role GVCs have played in transmitting economic shocks in the recent global financial crisis and find that a country/sector's GVC position has significant impacts. The further the country/sector pair is located from the final consumption end, the lesser the impact of the global economic shock. In addition, the impact of the financial crisis increases with the length of the international portion of the relevant global value chains.

KWW and WWZ have presented a complete gross trade accounting framework at the country, bilateral, sector, and bilateral-sector levels. While the accounting exercises conducted in the two papers provide useful new measures of production sharing and cross border trade, the determinants and consequences of production sharing and these double counted components are not addressed. To make the decomposition useful for economic analysis, an important first step is to construct various indexes that can measure a country/industry's position and participation in GVCs and systemically ranking all country/industry pairs in available ICIO databases and econometrically studying the determinates of these indexes over time as guided by economic theory. The GVC production length, position and participation indexes defined in this paper are part of our efforts in this direction.

The rest of the paper is organized as follows: Section 2 formally defines the GVC production length, position and participation indexes; Section 3 reports major empirical results based on WIOD; and Section 4 explores the implications of our findings and concludes.

2. Length of Production Chain and GVC Position and Participation Indexes

2.1 The length of production chain in a closed economy

Let us first define the production length measure in an *N*-sector closed economy.

Table 1 Input-Output table in a closed economy

<table>
<tr><td colspan="2" rowspan="2">Outputs
Inputs</td><td>Intermediate Use</td><td rowspan="2">Final use
(Consumption and
Capital Formation)</td><td rowspan="2">Total
Output</td></tr>
<tr><td>1, 2, …, n</td></tr>
<tr><td>Intermediate
Inputs</td><td>1
2
…
N</td><td>Z</td><td>Y</td><td>X</td></tr>
<tr><td colspan="2">Value-added</td><td>Va</td><td></td><td></td></tr>
<tr><td colspan="2">Total input</td><td>X'</td><td></td><td></td></tr>
</table>

where X denotes the gross outputs vector, Y denotes the final goods vector, Z denotes the intermediate goods flow matrix, Va denotes the value added vector, and $'$ denotes matrix transpose operation.

In the Leontief model (Leontief, 1936), the input coefficient matrix can be defined as $A = Z\hat{X}^{-1}$, where $\hat{X}$ denotes a diagonal matrix with the output vector X in its diagonal. The value added coefficient vector can be defined as $V = Va\hat{X}^{-1}$. From the output side, gross outputs can be split into intermediate goods and final goods, $AX + Y = X$. Rearranging terms, we can reach the classical Leontief equation, $X = BY$, where $B = (I - A)^{-1}$ is the well-known Leontief inverse matrix. The value added and final products are linked by the following equation: $Va' = \hat{V}X = \hat{V}BY$.

It is obvious that primary inputs (value added) of sector *i* only can be directly embodied in final products of sector *j* if sector *i* and sector *j* are the same. Therefore, in the first stage of any production process, the value added of sector *i* embodied in final products of sector *j* can be quantified as $\delta_{ij}v_i y_j$, where δ_{ij} is a dummy variable. If *i* and *j* are the same, δ_{ij} equals 1, otherwise it equals 0. At this stage, the length of the production chain is 1.

In the second stage, the value added of sector *i* directly embodied in its gross output that is used as intermediates to produce final products of sector *j* can be measured as $v_i a_{ij} y_j$, which is the value added of sector *i* in the first round indirect value-added embodied in final products of sector *j*. Up to this stage, the length of the production chain is 2.

The indirect value added from sector *i* can be embodied in intermediate goods from any sector. In the third stage, the value added of sector *i* directly embodied in its gross output that is used as intermediates in all sectors to produce their gross outputs which are used as intermediates to produce final goods of sector *j* can be measured as $v_i \sum_k^n a_{ik} a_{kj} y_j$. This is the second round indirect value-added from sector *i* embodied in intermediate goods and absorbed by final goods of sector *j*. At this stage, the length of the production chain is 3.

The same goes for the succeeding stages.

Generalizing the above process to include all rounds of value-added in sector *i* directly and indirectly embodied in final goods of sector *j*, we obtain the following:

$$\delta_{ij} v_i y_j + v_i a_{ij} y_j + v_i \sum_k^n a_{ik} a_{kj} y_j + \ldots \qquad \delta_{ij} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases} \tag{1}$$

Expressing (1) in matrix notation

$$\hat{V}\hat{Y} + \hat{V}A\hat{Y} + \hat{V}AA\hat{Y} + \cdots = \hat{V}(I + A + AA + \cdots)\hat{Y}$$
$$= \hat{V}(I - A)^{-1}\hat{Y} = \hat{V}B\hat{Y} \tag{2}$$

The element of row *i* and column *j* in the matrix at the right side of equation (2), $v_i b_{ij} y_j$, is the total value added of sector *i* embodied in the final goods of sector *j*.

Using the length of each stage as weights and summing across all production stages, we obtain the following equation that gives the length of a particular production chain (sector *i* to sector *j*):

$$\hat{V}\hat{Y} + 2\hat{V}A\hat{Y} + 3\hat{V}AA\hat{Y} + \cdots = \hat{V}(I + 2A + 3AA + \cdots)\hat{Y}$$
$$= \hat{V}(B + AB + AAB + \cdots)\hat{Y} = \hat{V}BB\hat{Y} \tag{3}$$

It captures the footprint of sector value added in each production stage.

The element of row *i* and column *j* in the matrix at the right side of equation (3) is$v_i \sum_k^n b_{ik} b_{kj} y_j$. Dividing by $v_i b_{ij} y_j$, the average length of value added from sector *i* embodied in the final goods of sector *j* can be computed as:

$$vyl_{ij} = \frac{v_i \sum_k^n b_{ik} b_{kj} y_j}{v_i b_{ij} y_j} = \frac{\sum_k^n b_{ik} b_{kj}}{b_{ij}} = (b_{ij})^{-1} \sum_k^n b_{ik} b_{kj} \tag{4}$$

Rearranging equation (4) gives:

$$vyl_{ij} * b_{ij} = \sum_k^n b_{ik} b_{kj} \tag{5}$$

Denoting $VYL = \{vyl_{ij}\}_{\text{nxn}}$ as the matrix of production length from value added to final goods, equation (5) can be expressed in matrix notation as

$$VYL \# B = BB \tag{6}$$

where # is an element-wise matrix multiplication operation,[2] *VYL* is an *n* by *n* matrix of production length. The detailed derivation is given in Appendix A.

Aggregating equation (4) over all products *j*, we obtain the total production length of value added generated in sector *i*, i.e., the production length measure based on forward industrial linkage:

$$vl_i = \sum_j^n \left(\frac{v_i b_{ij} y_j}{\sum_k^n v_i b_{ik} y_k} \times \frac{\sum_k^n b_{ik} b_{kj}}{b_{ij}} \right) = \sum_j^n \left(\frac{\sum_k^n b_{ik} b_{kj} y_j}{\sum_k^n b_{ik} y_k} \right) \tag{7}$$

$$= x_i^{-1} \sum_j^n \sum_k^n b_{ik} b_{kj} y_j = x_i^{-1} \sum_k^n b_{ik} x_k$$

where $\sum_k^n b_{ik} y_k = x_i$ and $\sum_j^n b_{kj} y_j = x_k$. Expressing in matrix notation gives:

$$VL = \hat{X}^{-1} B \hat{X} u' = \hat{X}^{-1} B X \tag{8}$$

We define the output coefficient matrix as $H = \hat{X}^{-1} Z$, and the final products coefficient vector as $F = \hat{X}^{-1} Y$ as in Ghosh (1958). From the input side, gross inputs can be split into intermediate inputs and value added, $X'H + Va = X'$. Rearranging terms, we can reach the classical Ghosh inverse equation, $X' = VaG$, where $G = (I - H)^{-1}$ is the Ghosh inverse matrix.

[2] For example, when a matrix is multiplied by an *n*x1 column vector, each row of the matrix is multiplied by the corresponding row element of the vector.

The linkage between value added and final products can also be expressed as: $Y' = X'\hat{F} = VaG\hat{F}$.

It is easy to derive the linkage between the input and output coefficient matrices as: $\hat{X}^{-1}A\hat{X} = \hat{X}^{-1}Z = H$. Similarly, the linkage between the Leontief inverse and the Ghosh inverse matrices are:

$$\hat{X}^{-1}B\hat{X} = \hat{X}^{-1}(I-A)^{-1}\hat{X} = \left[\hat{X}^{-1}(I-A)\hat{X}\right]^{-1}$$
$$= \left(1 - \hat{X}^{-1}A\hat{X}\right)^{-1} = (1-H)^{-1} = G \tag{9}$$

Based on equation (9), we can further simplify from (8) as

$$VL = \hat{X}^{-1}BX = \hat{X}^{-1}B\hat{X}u' = Gu' \tag{10}$$

where u is a $1\times N$ unit vector with all its elements equal to 1.

It is the sum along the rows of the Ghosh inverse matrix, which equals the total value of gross outputs that are related to one unit of value added created by primary inputs from a particular sector. Therefore, equation (10) measures total gross outputs induced by one unit of value added at the sector level, which are the footprints of each sector's value added in the economy as a whole. The longer the production chain, the greater the number of downstream production stages a sector's value added is counted in the economy. This means that primary inputs of the sector are more to the upstream side of the production chain.

To better understand this point, let us use the diagonal matrix of sectoral value added to multiply with *VL*, obtaining:

$$\widehat{Va}VL = \widehat{Va}\hat{X}^{-1}B\hat{X}u' = \hat{V}BX = \hat{V}X + \hat{V}AX + \hat{V}AAX + \hat{V}AAAX + \cdots \tag{11}$$

Its *i*th element equals

$$Va_i vl_i = Va_i x_i^{-1}\sum_k^n b_{ik}x_k = v_i\sum_k^n b_{ik}x_k = v_i x_i + v_i\sum_k^n a_{ik}x_k + v_i\sum_k^n\sum_j^n a_{ij}a_{jk}x_k + \ldots.$$

On the right side of equation (11), the first term is the value added directly embodied in its own sector's output, and we may name it as the footprint of the sector value added in its own sector gross output; the second term is the value added embodied in its own sector's gross output used by all sectors as intermediates to produce outputs, and we may name it as the footprint of the sector value added directly and indirectly embodied in total gross outputs of this second stage production process. Summing up all terms on the right hand side of (11), we obtain footprints of sector value added in the whole economy, which equals the total value of gross outputs that relates to the sector value added created by primary inputs from a particular sector. Therefore, equation (11) also can be written as

$$\widehat{Va}VL = \widehat{Va}\hat{X}^{-1}B\hat{X}u' = \hat{V}B\hat{X}u' = \hat{V}BX = Xv$$ [3]

where Xv is the gross output induced by sector value added. Therefore, the average production length of sector i based on forward industrial linkages equals the ratio of sector value added induced total gross output in the whole economy and the sector value-added.

Using the shares of sectoral value added in GDP as weights to aggregate equation (11) over all sectors, we obtain:

$$\left(Va\hat{X}^{-1}B\hat{X}u'\right)/(uVa) = (VBX)/GDP = (uX)/GDP \tag{12}$$

where $Va\hat{X}^{-1} = V$, $\hat{X}u' = X$ and $VB = u$.

Equation (12) indicates that the average length of the production chain in a closed economy equals the ratio of total gross outputs to GDP,[4] which can be regarded as a form of complexity of the production process in the economy, i.e., the higher this ratio, the more complex the economy.

Aggregating equation (4) over value-added from all sectors i that have contributed to the final goods and services produced by sector j, we obtain the production length measure based on backward industrial linkages as:

$$yl_j = \sum_i^n \left(\frac{v_i b_{ij} y_j}{\sum_k^n v_k b_{kj} y_j} \times \frac{\sum_k^n b_{ik} b_{kj}}{b_{ij}} \right) = \sum_i^n v_i \sum_k^n b_{ik} b_{kj} = \sum_k^n b_{kj} \tag{13}$$

where $\sum_k^n v_k b_{kj} = \sum_i^n v_i b_{ik} = 1$. Expressing in matrix notation

$$YL = uB \tag{14}$$

This is the sum along the column of the Leontief inverse matrix, which equals the total value of inputs induced by a unit of final product produced in a particular sector. Therefore,

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[3] Please note that $\hat{V}B\hat{X}u' = Xv$ and $u\hat{V}B\hat{X} = X'$. They are the row and column sums of the *GN* by *GN* matrix $\hat{V}B\hat{X}$, respectively. Its row sum is the gross output (across different industries in the whole economy) induced by a particular sector's value-added; its column sum is the gross output with value-added embodied from every sector in the economy. Therefore Xv does not equal X' at the sector level, but equals each other at the aggregate.

[4] This is also recognized by Fally (2012).

equation (13) measures total intermediate inputs induced by a unit value of a particular final product throughout all upstream sectors in the economy, which is called the footprints of final goods and services in the literature. The longer the production chain, the greater the number of upstream production stages a particular final product is counted in the economy, the more to the downstream the products are located.

Using the sectoral ratio of final goods to GDP as weight to aggregate equation (13) over all sectors, we obtain:

$$\left(uB\hat{Y}u'\right)/(uY) = (uBY)/GDP = (uX)/GDP \tag{15}$$

which gives the same gross output to GDP ratio as equation (12) and therefore has the same economic interpretation.

It is worth noting that the length of a production chain based on forward industrial linkages as expressed in equation (10) is mathematically equivalent to the upstreamness index defined by Fally (2012a, 2012b, 2013) and Antras et al. (2012, 2013);[5] On the other hand, the length of a production chain based on backward industrial linkages expressed in equation (13) is mathematically equivalent to the downstreamness index defined by Antras and Chor (2013). However, there are two notable differences. First, similar to Miller and Temurshoev (2013), we define our upstream or downstream indexes by the sum of the rows/columns of the Ghosh/Leontief inverse matrices respectively, which are simpler in mathematics and are part of the classic input-output literature; Second, and most important, we measure a production chain length from primary inputs in sector *i* to final products of sector *j*, starting from primary inputs (value added), not gross outputs (as Fally and Antras did), and provide very clear economic interpretations for both the numerator and denominator in the production line position indexes discussed above.

2.2 The length of production chain within and across national borders

Let's now expand the closed-economy model to an ICIO model. The structure with *M* countries and *N* sectors is described by Table 2:

[5] The proof is provided in Appendix B.

Table 2 General Inter-Country Input-Output table

Outputs / Inputs		Intermediate Use				Final Demand				Total Output
		1	2	…	M	1	2		M	
Intermediate Inputs	1	Z^{11}	Z^{12}	…	Z^{1m}	Y^{11}	Y^{12}	…	Y^{1m}	X^{1}
	2	Z^{21}	Z^{22}	…	Z^{2m}	Y^{21}	Y^{22}	…	Y^{2m}	X^{2}
	…	…	…	…	…	…	…	…	…	
	M	Z^{m1}	Z^{m2}	…	Z^{mm}	Y^{m1}	Y^{m2}	…	Y^{mm}	X^{m}
Value-added		$(VA^1)'$	$(VA^2)'$	…	$(VA^m)'$					
Total input		$(X^1)'$	$(X^2)'$	…	$(X^m)'$					

where Z^{sr} is an $N\times N$ matrix of intermediate input flows that are produced in country s and used in country r; Y^{sr} is an $N\times 1$ vector giving final products produced in country s and consumed in country r; X^s is also an $N\times 1$ vector giving gross outputs in country s; and VA^s denotes an $N\times 1$ vector of direct value added in country s. Both the input coefficient matrix $A = Z\hat{X}^{-1}$ and value added coefficient vector $V = Va\hat{X}^{-1}$ can be defined in a similar way as discussed in the closed economy model.

2.2.1 Production activities with and without cross-country production sharing arrangements

The gross output production and use balance, or the row balance condition of the ICIO table in Table 2 can be written as:

$$X^s = A^{ss}X^s + \sum_{s\neq r}^{M} A^{sr}X^r + Y^{ss} + \sum_{s\neq r}^{M} Y^{sr} = A^{ss}X^s + Y^{ss} + \sum_{s\neq r}^{M} E^{sr} = A^{ss}X^s + Y^{ss} + E^{s*} \qquad (16)$$

where A^{ss} is an $N\times N$ domestic input coefficient matrix of country s (block diagonal), A^{sr} is an $N\times N$ foreign input coefficient matrix of country r (block off diagonal) , and $E^{s*} = \sum_{s\neq r}^{G} E^{sr}$ is the $N\times 1$ vector of total gross exports of country s.

Rearranging the right hand side of (16) yields

$$X^s = (I - A^{ss})^{-1}Y^{ss} + (I - A^{ss})^{-1}E^{s*} \qquad (17)$$

With a further decomposition of gross exports into exports of intermediate/final products and their final destinations of absorption, it can be shown that

$$(I-A^{ss})^{-1}E^{s*}=L^{ss}(\sum_{r\neq s}^{M}Y^{sr}+\sum_{r\neq s}^{M}A^{sr}X^{r})=L^{ss}\sum_{r\neq s}^{M}Y^{sr}+L^{ss}\sum_{r\neq s}^{M}A^{sr}\sum_{u}^{M}B^{ru}\sum_{t}^{M}Y^{ut}$$

$$=L^{ss}\sum_{r\neq s}^{M}Y^{sr}+L^{ss}\sum_{r\neq s}^{M}A^{sr}\sum_{u}^{M}B^{ru}Y^{ur}+L^{ss}\sum_{r\neq s}^{M}A^{sr}\sum_{u}^{M}B^{ru}Y^{us}+L^{ss}\sum_{r\neq s}^{M}A^{sr}\sum_{u}^{M}B^{ru}\sum_{t\neq s,r}^{M}Y^{ut} \quad (18)^{6}$$

where $L^{ss}=(I-A^{ss})^{-1}$ is the local Leontief inverse. B^{ru} s are block matrices in the global Leontief inverse.

Inserting (18) into (17) and pre-multiplying with the direct value-added diagonal matrix $\hat{V}$, we can decompose value-added generated from each industry/country (GDP by industry) into different components:

$$Va^{s}=\hat{V}^{s}X^{s}=\hat{V}^{s}L^{ss}Y^{ss}+\hat{V}^{s}L^{ss}\sum_{r\neq s}^{M}Y^{sr}+\hat{V}^{s}L^{ss}\sum_{r\neq s}^{M}A^{sr}X^{r}$$

$$=\underbrace{\hat{V}^{s}L^{ss}Y^{ss}}_{(1-DVA\_D)}+\underbrace{\hat{V}^{s}L^{ss}\sum_{r\neq s}^{M}(Y^{sr}+A^{sr}L^{rr}Y^{rr})}_{(2-DVA\_RT)}+\underbrace{\hat{V}^{s}L^{ss}\sum_{r\neq s}^{M}A^{sr}\sum_{u}^{M}B^{ru}\sum_{t}^{M}Y^{ut}-\hat{V}^{s}L^{ss}\sum_{r\neq s}^{M}A^{sr}L^{rr}Y^{rr}}_{(3-DVA\_GVC)}$$

$$=\underbrace{\hat{V}^{s}L^{ss}Y^{ss}}_{(1-DVA\_D)}+\underbrace{\hat{V}^{s}L^{ss}\sum_{r\neq s}^{M}(Y^{sr}+A^{sr}L^{rr}Y^{rr})}_{(2-DVA\_RT)}+\underbrace{\hat{V}^{s}L^{ss}\sum_{r\neq s}^{M}A^{sr}(\sum_{u}^{M}B^{ru}Y^{ur}-L^{rr}Y^{rr})}_{(3a-DVA\_GVC\_r)} \quad (19)$$

$$+\underbrace{\hat{V}^{s}L^{ss}\sum_{r\neq s}^{M}A^{sr}\sum_{u}^{M}B^{ru}Y^{us}}_{(3b-DVA\_GVC\_s=RDV\_F)}+\underbrace{\hat{V}^{s}L^{ss}\sum_{r\neq s}^{M}A^{sr}\sum_{u}^{M}B^{ru}\sum_{t\neq s,r}^{M}Y^{ut}}_{(3c-DVA\_GVC\_t)}$$

There are five terms in this decomposition, each representing domestic value-added generated by the industry in its production to satisfy different segments of the global market. These domestic value-added or total GDP in country *s* are generated from the following three types of production activities:

(1) Production of domestically produced and consumed value-added ($\hat{V}^{s}L^{ss}Y^{ss}$). This is domestic value added to satisfy domestic final demand that is not related to international trade, and no cross country production sharing is involved. We label it as DVA_D for short.

(2) Production of "directly" traded value-added, including value-added embodied in both final and intermediate goods and services with domestic factor content embodied in these exports

[6] A detailed mathematical proof of equation (18) is provided in Appendix C.

that are directly absorbed by trading partners. DVA crosses the border only once, with no indirect exports via third countries or re-exports involved. We label it as DVA_RT for short.[7]

(3) Production of "indirectly" traded value-added. It is embodied in intermediate goods and services exports that the source country contributed to global value chains. We label it as DVA_GVC for short. It measures the amount of domestic value added that is generated from the production of such intermediate exports regardless of where these value-added are finally absorbed. It can be further split into three categories according to their different final destinations of absorption:

3a. Indirectly absorbed by partner country *r*. Value-added embodied in intermediate exports to a third country that is used to produce its intermediate or final product exports that are finally consumed in country *r* (i.e., domestic value added to satisfy importing country's final demand indirectly, production sharing between the two partner countries, *s* and *r*, or between the importing country *r* with other third countries, or among *s*, *r*, and third countries, DVA_GVC_r);

3b. Returned (re-imported) to exporting country *s* and finally consumed there. Value-added embodied in intermediate exports that are used by partner country *r* to produce either intermediate or final goods and services and shipped back to the source country (possibly via third countries in the production chain) as imports and consumed there (i.e., domestic value-added to satisfy domestic final demand that is related to international trade, production sharing between home and foreign countries; DVA_GVC_s);

3c. Re-exported to a third country *t* and finally consumed there. Value-added embodied in intermediate exports that is used by partner country *r* to produce intermediate inputs for its own or other countries' production of final goods and services that are eventually re-exported to third countries (i.e., domestic value added to satisfy a third country's final demand, production sharing among at least three countries, DVA_GVC_t).

Such a downstream decomposition based on forward industrial linkages is critical to understanding the measures of international production length or Production Length of the Global Value Chain (PLGVC) that we will define in this paper. It measures the number of production stages the last three parts of the domestic value-added would take to reach the final

[7] Borin and Mancini (2015) have recognized the difference between (2) and (3) and refer to (2) as "Ricardian Trade." However, since we discuss value-added trade and not goods trade here, "Ricardian trade" should only be referred in the sense that this part of value-added crosses borders only once as traditional goods trade. They are not exactly the same, to avoid confusion.

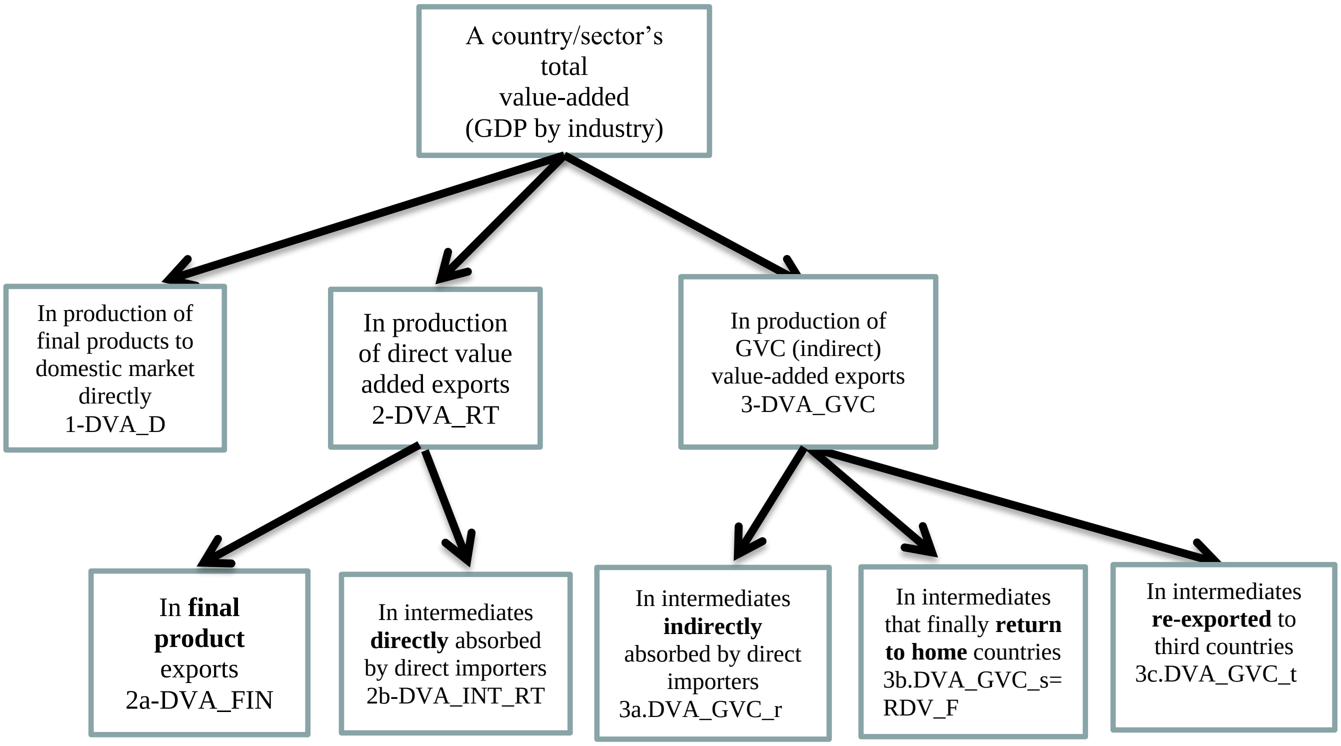
consumer in a particular country/sector pair, including in the home country. However, it excludes domestic value-added measured by the first two terms of equation (19) because those production activities are accomplished either completely within the national boundaries or directly absorbed by trading partners. Therefore, they can be treated as pure domestic production activities (the first term in equation (19)) and production activities related to "direct" value-added trade (the second term in equation (19)), respectively.

Note that we use the term GVC related trade here narrowly to refer to value added in intermediate goods that crosses borders at least twice. A broader definition of "global value chains" trade could also include any value added embedded in intermediate good exports even if they cross borders only once. Indeed, the broadest definition of GVC should also include some of the domestic value added exports that are embedded in the final goods exports absorbed abroad as long as the production of the final products involves foreign value added. For this study, we decide to group value added in intermediate products exports that crosses borders only once as part of the "direct value-added trade" or "Ricardian Trade" in the term used by Borin and Mancini (2015). With this, we reserve the term "GVC related trade" to trade in value added that crosses national borders at least twice.

Note also that the summation in the last four terms indicates that the domestic value-added generated by export production can be further split at the bilateral level into each trading partner's market. The sum of terms 2, 3a, and 3c gives the amount of value-added exports as defined by Johnson and Negara (2012), which is the total (direct and indirect) domestic value added to satisfy foreign final demand, while the sum of 1 and 3b is the total domestic value-added to satisfy domestic final demand. Finally, the sum of (2) and (3) gives the measure domestic value-added (GDP) in gross exports as defined in KWW and WWZ, or DVA related production and trade activities to the most broadly defined "global value chains."

The decomposition is also illustrated in Figure 1.

Figure 1 Decomposition of GDP by industry
— Which types of production and trade activities belong to global production networks?



2.2.2 Length of pure domestic production

Let us first consider the segment of domestic value added that is generated and absorbed by production activities entirely within the country at each stage of production.

We know from equation (19), in an infinite production process, domestic value added of country *s* embodied in its final products that satisfy its domestic final demand equals $\hat{V}^s L^{ss} Y^{ss}$ (DVA_D^{ss}).

Following a similar logic as equation (3) in the closed economy, i.e., using the length of each production stage as weights and summing up all production stages, we obtain an equation that gives the product of the value-added and domestic production length as follows:

$$\begin{aligned} X\_vd^{ss} &= \hat{V}^s Y^{ss} + 2\hat{V}^s A^{ss} Y^{ss} + 3\hat{V}^s A^{ss} A^{ss} Y^{ss} + \ldots \\ &= \hat{V}^s (I - A^{ss})^{-1} (I - A^{ss})^{-1} Y^{ss} = \hat{V}^s L^{ss} L^{ss} Y^{ss} \end{aligned} \qquad (20)^8$$

where $I + A^{ss} + A^{ss}A^{ss} + \ldots = (I - A^{ss})^{-1} = L^{ss}$

[8] A detailed mathematical proof of equation (20) is provided in Appendix D.

Because production activities that generate this part of domestic value-added have no relation with cross border trade, we define its production length as that of pure domestic production. It equals the portion of gross output of country *s* generated by the production of the country's GDP without any cross-border trade activities. Therefore, the average pure domestic production length of country *s* equals the ratio of this portion of gross output to the corresponding domestic value added, and can be expressed as[9]

$$PL\_D^s = \frac{X\_vd^s}{DVA\_d^s} = \frac{\hat{V}^s L^{ss} L^{ss} Y^{ss}}{\hat{V}^s L^{ss} Y^{ss}} = \frac{L^{ss} L^{ss} Y^{ss}}{L^{ss} Y^{ss}} \tag{21}$$

2.2.3 The production length of "direct value-added trade"[10]

Let us now consider the segment of domestic value added that is generated by activities related to trade at each stage of production (terms (2) and (3) in equation (19)).

In a one stage production process, the domestic value added generated from a particular country/sector (for example, sector *i* of country *s*) is directly embodied in its final products that are exported to country *r* and consumed there. It can be measured as $\hat{V}^s Y^{sr}$ and its domestic production length equals 1 and its international production length equals 0.

In a two stage production process, the domestic value added generated from country *s* will be first embodied in its gross output that is used as intermediate input either by country *s* or other countries (through exports) in the production of final product exports. It can be measured as $\hat{V}^s A^{ss} Y^{sr} + \hat{V}^s A^{sr} \sum_t^M Y^{rt}$ and can be decomposed into two parts: $\hat{V}^s A^{ss} Y^{sr}$ and $\hat{V}^s A^{sr} \sum_t^M Y^{rt}$. Their domestic production lengths equal 2 and 1, respectively, and their international production lengths equal 0 and 1, respectively.

In a three stage production process, the domestic value added generated from country *s* will be embodied in the final products produced from the third stage and consumed in all possible destination counties. It can be measured as $\hat{V}^s A^{ss} A^{ss} Y^{sr} + \hat{V}^s A^{ss} A^{sr} \sum_t^M Y^{rt} + \hat{V}^s A^{sr} \sum_u^M A^{ru} \sum_t^M Y^{ut}$ and can be decomposed into three parts: $\hat{V}^s A^{ss} A^{ss} Y^{sr}$, $\hat{V}^s A^{ss} A^{sr} \sum_t^M Y^{rt}$, and $\hat{V}^s A^{sr} \sum_u^M A^{ru} \sum_t^M Y^{ut}$. Their domestic production lengths equal 3, 2, and 1, respectively, and their international production

[9] A division symbol below denotes elements-wide divisions.
[10] A detailed mathematical proof is provided in Appendix E.

lengths equal 0, 1, and 2, respectively. The product of value added in country *s*'s gross intermediate exports and its domestic/international production length can be expressed as $3\hat{V}^s A^{ss} A^{ss} Y^{sr} + 2\hat{V}^s A^{ss} A^{sr} \sum_t^M Y^{rt} + \hat{V}^s A^{sr} \sum_u^M A^{ru} \sum_t^M Y^{ut}$ and $\hat{V}^s A^{ss} A^{sr} \sum_t^M Y^{rt} + 2\hat{V}^s A^{sr} \sum_u^M A^{ru} \sum_t^M Y^{ut}$, respectively.

The same holds for an *n*-stage production process.

Summing over all production stages in an infinite stage production process, we have

$$\begin{aligned} DVA\_F^{sr} &= \hat{V}^s Y^{sr} + (\hat{V}^s A^{ss} Y^{sr} + \hat{V}^s A^{sr} \sum_t^M Y^{rt}) \\ &+ (\hat{V}^s A^{ss} A^{ss} Y^{sr} + \hat{V}^s A^{ss} A^{sr} \sum_t^M Y^{rt} + \hat{V}^s A^{sr} \sum_u^M A^{ru} \sum_t^M Y^{ut}) + \dots \\ &= \hat{V}^s L^{ss} Y^{sr} + \hat{V}^s L^{ss} A^{sr} \sum_u^M B^{ru} \sum_t^M Y^{ut} = \hat{V}^s L^{ss} E^{sr} \end{aligned} \qquad (22)[11]$$

where $\sum_u^M B^{ru}$ is the limit of the series $I + \sum_u^M A^{ru} + \sum_k^M A^{rk} \sum_u^M A^{ku} + \dots$. It measures the amount of domestic value added that can be generated from the production of gross exports E^{sr} in country *s*, regardless of whether these gross exports are finally absorbed in importing country *r* or not. Summing equation (22) over all trading partner countries (i.e., over *r*), we obtain the last 4 terms in equation (19), which are the domestic value-added of country *s* generated from all production activities that are needed in the production of its gross exports to the world.

As equation (19) shows, domestic value added of country *s* embodied in its gross exports can be separated into DVA in direct value-added exports and narrowly defined GVC related exports. "Direct value-added exports" can also refer to "Ricardian trade" (final goods exchange and supply of raw materials) in the following sense: It is the final product exports from country *s* consumed by direct importer *r* or intermediate exports from country *s* used by direct importer *r* in its production of domestically consumed final products. All domestic value added of country *s* in such exports is directly consumed within country *r* and it only crosses national borders once (either for consumption or for production activities). Mathematically, it can be expressed as $DVA\_RT^{sr} = \hat{V}^s L^{ss}(Y^{sr} + A^{sr} L^{rr} Y^{rr})$.

Using a similar logic as equations (3) and (20), we can also obtain an equation that gives the product of the value-added and domestic production length of traditional exports, which equals

[11] A detailed mathematical proof of equation (22) is provided in Appendix E.

the portion of total gross output generated by the corresponding domestic value-added: $X\_RT\_vd^{sr} = \hat{V}^s L^{ss} L^{ss} (Y^{sr} + A^{sr} L^{rr} Y^{rr})$. Therefore, the average domestic production length of country *s*'s direct value-added exports equals the ratio of this portion of gross output to its corresponding domestic value added and can be expressed as

$$PLd\_RT^{sr} = \frac{X\_RT\_vd^{sr}}{DVA\_RT^{sr}} = \frac{L^{ss} L^{ss} (Y^{sr} + A^{sr} L^{rr} Y^{rr})}{L^{ss} (Y^{sr} + A^{sr} L^{rr} Y^{rr})} \tag{23}$$

Because final product exports are consumed by direct importers and do not enter the production process in any foreign country, its international production length equals zero and its total production length is the same as its domestic production length. It can thus be expressed as $\frac{L^{ss} L^{ss} Y^{sr}}{L^{ss} Y^{sr}}$. Intermediate exports used by direct importers in their production of domestically consumed final products are involved in the production process only within the direct importing country; therefore, the international production length of the source countries' domestic value-added embodied in such intermediate exports equals their production length in the direct importing country *r*. Following a similar logic as equations (3) and (20), we obtain the equation that gives the product of this portion of value-added and its production length in country *r* as $X\_RT\_vf^{sr} = \hat{V}^s L^{ss} A^{sr} L^{rr} L^{rr} Y^{rr}$. Therefore, the average international production length of "direct" value-added exports from country *s* to country *r* equals:

$$PLf\_RT^{sr} = \frac{X\_RT\_vf^{sr}}{DVA\_RT^{sr}} = \frac{L^{ss} A^{sr} L^{rr} L^{rr} Y^{rr}}{L^{ss} (Y^{sr} + A^{sr} L^{rr} Y^{rr})} \tag{24}$$

Adding equations (23) and (24), we have the total production length of direct value-added exports as

$$\begin{aligned} PL\_RT^{sr} &= PLd\_RT^{sr} + PLf\_RT^{sr} = \frac{X\_RT\_v^{sr}}{DVA\_RT^{sr}} \\ &= \frac{L^{ss} L^{ss} (Y^{sr} + A^{sr} L^{rr} Y^{rr}) + L^{ss} A^{sr} L^{rr} L^{rr} Y^{rr}}{L^{ss} (Y^{sr} + A^{sr} L^{rr} Y^{rr})} \end{aligned} \tag{25}$$

2.2.4 The production length of narrowly defined Global Value Chain related trade

The production process of GVC related intermediate exports is more complicated than the previous segments. Unlike DVA embodied in direct value-added exports, DVA embodied in GVC related intermediate exports cross national borders at least twice. Subtracting direct value-added exports from equation (22), we obtain the source country's domestic value-added embodied in its GVC related intermediate exports. It can be further decomposed into three parts according to equation (19) as follows:

$$DVA\_GVC^{sr} = \underbrace{\hat{V}^s L^{ss} A^{sr} (\sum_u^M B^{ru} Y^{ur} - L^{rr} Y^{rr})}_{(3a-DVA\_GVC\_r)} + \underbrace{\hat{V}^s L^{ss} A^{sr} \sum_u^M B^{ru} Y^{us}}_{(3b-DVA\_GVC\_s=RDV)} + \underbrace{\hat{V}^s L^{ss} A^{sr} \sum_u^M B^{ru} \sum_{t \neq s,r}^M Y^{ut}}_{(3c-DVA\_GVC\_t=DVA\_INTrex)} \quad (26)^{12}$$

They are the source country's DVA indirectly absorbed in importing country *r* (DVA_GVC_r), returned (re-imported) and absorbed by the source country *s* (DVA_GVC_s) , and re-exported by importing country *r* to third countries *t* and finally consumed there (DVA_GVC_t), respectively. Summing equation (26) over all trading partner countries *r*, we obtain the last 3 terms in equation (19), which are domestic value-added of country *s* generated from all production activities that are needed in the production of its GVC related gross intermediate exports to the world.

Following the same logic to derive equations (3) and (20), i.e., using the domestic or international production length of each stage of gross exports production discussed in the last section as weights and summing across all production stages, we can obtain the average domestic and international production lengths of global value chain related exports as well as its 3 components in a particular bilateral trade route.

For instance, the product of domestic value-added embodied in bilateral GVC related exports and its domestic production length equals the portion of gross output in country *s* (labeled as $X\_GVC\_vd^{sr}$) induced by the production of country *s*'s domestic value-added embodied in its GVC related exports, which can be expressed as:

$$X\_GVC\_vd^{sr} = \underbrace{\hat{V}^s L^{ss} L^{ss} A^{sr} (\sum_u^M B^{ru} Y^{ur} - L^{rr} Y^{rr})}_{(3a) X\_GVC\_r\_vd^{sr}} + \underbrace{\hat{V}^s L^{ss} L^{ss} A^{sr} \sum_{u \neq r}^M B^{ru} Y^{us}}_{(3b) X\_GVC\_s\_vd^{sr}} + \underbrace{\hat{V}^s L^{ss} L^{ss} A^{sr} \sum_u^M B^{ru} \sum_{t \neq s,r}^M Y^{ut}}_{(3c) X\_GVC\_t\_vd^{sr}} \quad (27)^{13}$$

[12] Please note that the first term in equation (26) is part of the second term of equation (21) of WWZ. The second and third terms in equation (26) are exactly the same as the fourth and third terms in equation (21) of WWZ.
[13] The average production length of traditional trade and a detailed mathematical proof of equation (26) is provided in Appendix F.

Term 3a is country s's gross outputs generated by country s's domestic value added in GVC related exports indirectly consumed by trading partners; we label it as $X\_GVC\_r\_vd^{sr}$ for short. Term 3b is country s's gross outputs induced by country s's domestic value added in GVC related exports returned and finally consumed at home; we label it as $X\_GVC\_s\_vd^{sr}$ for short. Term 3c is country r's gross outputs induced by country s's value added in GVC related exports that are re-exported by country r and finally consumed in third countries; we label it as $X\_GVC\_t\_vd^{sr}$ for short. All of these different parts of gross outputs are associated with domestic value-added in GVC related exports before it leaves the country through forward domestic inter-industrial linkage.

Therefore, the average domestic production length of GVC exports can be computed as the weighted sum of the ratio of the portion of gross output to its corresponding domestic value-added of its 3 components in equations (26) and (27) respectively:

$$
\begin{aligned}
PLd\_GVC^{sr} &= \frac{X\_GVC\_vd^{sr}}{DVA\_GVC^{sr}} = \frac{DVA\_GVC\_r^{sr}}{DVA\_GVC^{sr}} \times \underbrace{\frac{L^{ss}L^{ss}A^{sr}(\sum_{u}^{M} B^{ru}Y^{ur} - L^{rr}Y^{rr})}{L^{ss}A^{sr}(\sum_{u}^{M} B^{ru}Y^{ur} - L^{rr}Y^{rr})}}_{(3a)PLd\_GVC\_r^{sr}} \\
&+ \frac{DVA\_GVC\_s^{sr}}{DVA\_GVC^{sr}} \times \underbrace{\frac{L^{ss}L^{ss}A^{sr}\sum_{u}^{M} B^{ru}Y^{us}}{L^{ss}A^{sr}\sum_{u}^{M} B^{ru}Y^{us}}}_{(3b)PLd\_GVC\_s^{sr}} + \frac{DVA\_GVC\_t^{sr}}{DVA\_GVC^{sr}} \times \underbrace{\frac{L^{ss}L^{ss}A^{sr}\sum_{u}^{M} B^{ru}\sum_{t\neq s,r}^{M} Y^{ut}}{L^{ss}A^{sr}\sum_{u}^{M} B^{ru}\sum_{t\neq s,r}^{M} Y^{ut}}}_{(3c)PLd\_GVC\_t^{sr}} \\
&= \frac{L^{ss}L^{ss}A^{sr}(\sum_{u}^{M} B^{ru}\sum_{t}^{M} Y^{ut} - L^{rr}Y^{rr})}{L^{ss}A^{sr}(\sum_{u}^{M} B^{ru}\sum_{t}^{M} Y^{ut} - L^{rr}Y^{rr})}
\end{aligned}
\tag{28}
$$

The average domestic production length of the three components are labeled as $PLd\_GVC\_r^{sr}$, $PLd\_GVC\_s^{sr}$, and $PLd\_GVC\_t^{sr}$ respectively.

Similarly, the product of domestic value-added embodied in bilateral GVC related exports and its international production length (labeled as $X\_GVC\_vf^{sr}$) equals total international (both domestic and foreign) gross outputs induced by domestic value-added of country s embodied in its GVC related intermediate exports, which can be expressed as:

$$X\_GVC\_vf^{sr} = \underbrace{\hat{V}^s L^{ss} A^{sr} (\sum_v^M B^{rv} \sum_u^M B^{vu} Y^{ur} - L^{rr} L^{rr} Y^{rr})}_{(3a) X\_GVC\_r\_vf^{sr}}$$
$$+ \underbrace{\hat{V}^s L^{ss} A^{sr} \sum_v^M B^{rv} \sum_u^M B^{vu} Y^{us}}_{(3b) X\_GVC\_s\_vf^{sr}} + \underbrace{\hat{V}^s L^{ss} A^{sr} \sum_v^M B^{rv} \sum_u^M B^{vu} \sum_{t \neq s,r}^M Y^{ut}}_{(3c) X\_GVC\_t\_vf^{sr}} \quad (29)[14]$$

Term 3a represents international gross outputs generated in the process between domestic value-added of country *s* embodied in its GVC exports arriving at country *r* and the value-added indirectly absorbed by final products consumed in country *r*; we label it as $X\_GVC\_r\_vf^{sr}$ for short. Term 3b represents international gross outputs generated in the process between domestic value-added of country *s* embodied in its GVC exports arriving at country *r* and the value-added shipped back after further processing in country *r* and absorbed by final products that are consumed at home; we label it as $X\_GVC\_s\_vf^{sr}$ for short. Term 3c represents international gross outputs generated in the process between domestic value-added of country *s* embodied in its GVC exports arriving at country *r* and the value-added finally absorbed by final products consumed in third country *t*; we label it as $X\_GVC\_t\_vf^{sr}$ for short. All of these different parts of gross outputs are associated with domestic value-added in GVC exports of country *s* after it leaves the country through forward inter-industrial inter-country linkages. Therefore, the average international production length of country *s*'s GVC exports to country *r* can be computed as the weighted sum of the ratio of the portion of gross output to its corresponding domestic value-added of its 3 components in equations (26) and (29), respectively:

[14] A detailed mathematical proof of equation (29) is provided in Appendix G.

$$PLf\_GVC^{sr}=\frac{X\_GVC\_vf^{sr}}{DVA\_GVC^{sr}}=\frac{DVA\_GVC\_r^{sr}}{DVA\_GVC^{sr}}\times\underbrace{\frac{L^{ss}A^{sr}(\sum_{u}^{M}B^{ru}\sum_{v}^{M}B^{uv}Y^{vr}-L^{rr}L^{rr}Y^{rr})}{L^{ss}A^{sr}(\sum_{u}^{M}B^{ru}Y^{ur}-L^{rr}Y^{rr})}}_{(3a)PLf\_GVC\_r^{sr}}$$

$$+\frac{DVA\_GVC\_s^{sr}}{DVA\_GVC^{sr}}\times\underbrace{\frac{L^{ss}A^{sr}\sum_{u}^{M}B^{ru}\sum_{v}^{M}B^{uv}Y^{us}}{L^{ss}A^{sr}\sum_{u}^{M}B^{ru}Y^{us}}}_{(3b)PLf\_GVC\_s^{sr}}+\frac{DVA\_GVC\_t^{sr}}{DVA\_GVC^{sr}}\times\underbrace{\frac{L^{ss}A^{sr}\sum_{u}^{M}B^{ru}\sum_{v}^{M}B^{uv}\sum_{t\neq s,r}^{M}Y^{vt}}{L^{ss}A^{sr}\sum_{u}^{M}B^{ru}\sum_{t\neq s,r}^{M}Y^{ut}}}_{(3c)PLf\_GVC\_t^{sr}} \quad (30)$$

$$=\frac{L^{ss}A^{sr}(\sum_{u}^{M}B^{ru}\sum_{v}^{M}B^{uv}\sum_{t\neq s,r}^{M}Y^{vt}-L^{rr}L^{rr}Y^{rr})}{L^{ss}A^{sr}(\sum_{u}^{M}B^{ru}\sum_{t}^{M}Y^{ut}-L^{rr}Y^{rr})}$$

The average international production length of the three components are labeled as PLf_GVC_r, PLf_GVC_s, and PLf_GVC_t.

Summing equations (28) and (30), we obtain the total average production length of domestic value-added of country *s* embodied in its bilateral GVC exports as follows:

$$PL\_GVC^{sr}=PLd\_GVC^{sr}+PLf\_GVC^{sr}=\frac{X\_GVC\_v^{sr}}{DVA\_GVC^{sr}}=\frac{X\_GVC\_vd^{sr}+X\_GVC\_vf^{sr}}{DVA\_GVC^{sr}} \quad (31)$$

Obviously, the sum of $X\_GVC\_v^{sr}$ and $X\_RT\_v^{sr}$ measures total world gross outputs generated by domestic value-added of country *s* embodied in its total gross exports. The weighted sum of PL_GVC^{sr} and PL_RT^{sr} defines the average production length of domestic value-added embodied in bilateral gross exports.

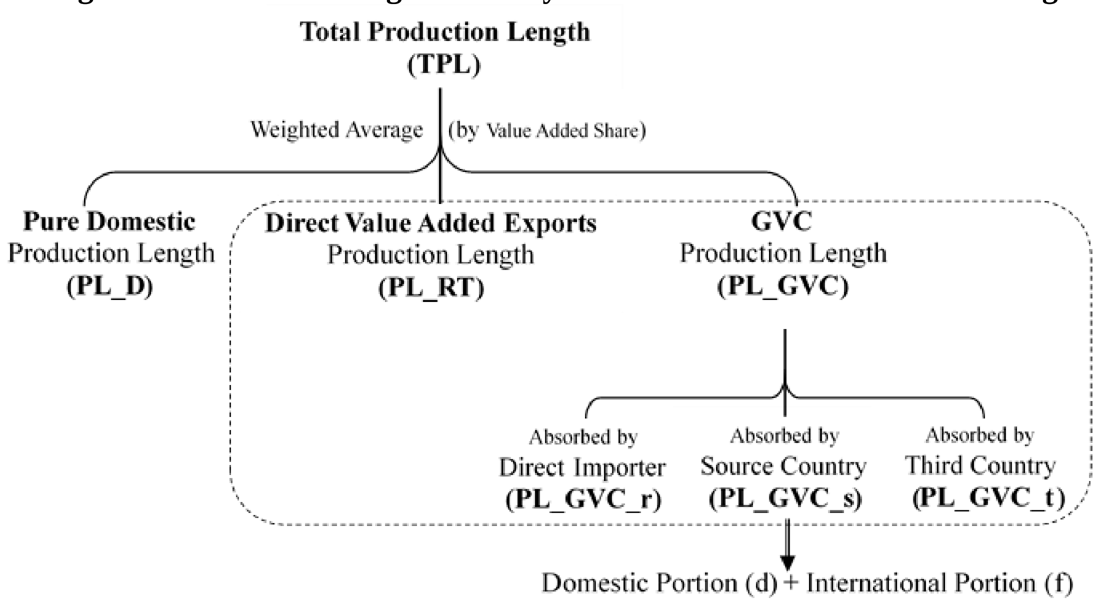
There is a nice symmetry among the terms in equations (26)–(30): all of them are based on the measurement and decomposition of both domestic value-added in global value chain exports and global gross outputs. It is consistent with the gross trade accounting framework proposed in Koopman, Wang, and Wei (2014). Using corresponding components of domestic value-added in GVC related gross exports in equation (26) as the denominators to divide equations (27) and (29) (i.e., the corresponding part of value-added induced gross outputs as numerators), we can obtain the average length of production of each segment and their weighted average in a particular global value chain (equations (28) and (30)). This measures the amount of global gross output that can be generated by one unit of domestic value-added in country *s* and its total subsequent utilization in the global production network.

Summing equations (27) and (29) over all trading partner countries *r*, we obtain

$$\sum_{r \neq s}^{M} X\_GVC\_v^{sr} = \hat{V}^s \sum_{u}^{M} B^{su} \sum_{v}^{M} B^{uv} \sum_{t \neq s,r}^{M} Y^{vt} - \hat{V}^s L^{ss} L^{ss} Y^{ss}$$
$$- (\hat{V}^s L^{ss} L^{ss} \sum_{r \neq s}^{M} Y^{sr} + \hat{V}^s L^{ss} L^{ss} \sum_{r \neq s}^{M} A^{sr} L^{rr} Y^{rr} + \hat{V}^s L^{ss} \sum_{r \neq s}^{M} A^{sr} L^{rr} L^{rr} Y^{rr}) \quad (32)^{15}$$
$$= \hat{V}^s \sum_{u}^{M} B^{su} \sum_{v}^{M} B^{uv} \sum_{t \neq s,r}^{M} Y^{vt} - \sum_{r \neq s}^{M} X\_RT\_v^{sr} - \hat{V}^s L^{ss} L^{ss} Y^{ss}$$

Equation (32) shows clearly that the sum of production length of traditional and GVC exports (equals global total output induced by domestic value-added in gross exports of country *s* to the world) defined in equations (25) and (31) plus the length of pure domestic production defined in equation (21) equals total production length as defined in equation (3), i.e., $\hat{V}BBY$, the product of total value-added and total production length, which, in expression, is the same as what we have defined for a closed economy in Section 2.1. The only difference is that matrix B here represents the global Leontief inverse from the ICIO model of the global economy. The structure and internal linkage of our production length index system can be represented as a tree diagram, as shown in Figure 2.

Figure 2 Production Length Index System: Structure and Internal Linkages



[15] A detailed mathematical proof of equation (32) is provided in Appendix H.

2.2.5 Production length based on backward inter-industry and cross-country linkages (work still in progress)

Similar to the definition based on forward linkages, the specification of production length based on backward linkages starts from a decomposition of final goods and services consumption at each country/sector pair. Following equations (7) and (9) of WWZ (2013), final products consumed by sector *i* in country *s* can be decomposed into its value-added sources as follows:

$$
\begin{aligned}
Y_i^{*s} &= \sum_t^M Y_i^{ts} = \sum_r^M V^r B^{rt} \sum_t^M Y_i^{ts} = V^s \sum_t^M B^{st} Y_i^{ts} + \sum_{r\neq s}^M V^r \sum_t^M B^{rt} Y_i^{ts} \\
&= \underbrace{V^s L^{ss} Y_i^{ss}}_{1-FDY\_D} + \underbrace{\sum_{r\neq s}^M V^r L^{rr} Y_i^{rs} + \sum_{r\neq s}^M V^r L^{rr} A^{rs} L^{ss} Y_i^{ss}}_{2-FDY\_RT} + \underbrace{\sum_r^M V^r L^{rr} \sum_{u\neq r}^M A^{ru} \sum_t^M B^{ut} Y_i^{ts} - \sum_{r\neq s}^M V^r L^{rr} A^{rs} L^{ss} Y_i^{ss}}_{3c-FDY\_GVC} \\
&= \underbrace{V^s L^{ss} Y_i^{ss}}_{1-FDY\_D} + \underbrace{\sum_{r\neq s}^M V^r L^{rr} Y_i^{rs} + \sum_{r\neq s}^M V^r L^{rr} A^{rs} L^{ss} Y_i^{ss}}_{2-FDY\_RT} \\
&\quad + \underbrace{\sum_{r\neq s}^M V^r L^{rr} A^{rs} \sum_u^M B^{su} Y_i^{us} - \sum_{r\neq s}^M V^r L^{rr} A^{rs} L^{ss} Y_i^{ss}}_{3b-FDY\_GVC\_r} + \underbrace{V^s L^{ss} \sum_{r\neq s}^M A^{sr} \sum_u^M B^{ru} Y_i^{us}}_{3a-FDY\_GVC\_s} + \underbrace{\sum_{r\neq s}^M V^r L^{rr} \sum_{u\neq r,s}^M A^{ru} \sum_t^M B^{ut} Y_i^{ts}}_{3c-FDY\_GVC\_t}
\end{aligned}
\tag{33}
$$

where Y_i^{8s} is a scalar, representing final products of sector *i* consumed in country *s*, which is the sum of country *s*'s final consumption sources from all countries, including its own. The first term in equation (33) is value-added in domestically produced final products that satisfy domestic final demand (DFD) without involving any cross border trade and production activities; we label it as pure domestic value-added (FDY_D). The second term has two parts: term 2a is foreign value-added embodied in country *s* final product imports; term 2b is intermediate imports from country *r* used by direct importer *s* in its production of domestically consumed final products. The common feature of both 2a and 2b is that value-added embodied in such imports only cross national borders once, so we label them as direct value-added trade (FDY_RT). Please note that the difference between term 2a in equation (33) and term 2a in equation (19) is that the former includes value-added sourced from all countries in the world, while the later only come from domestic source. Obviously, the third term is value-added from GVC related trade embodied in the final consumed products of country *s*. It also has three parts, corresponding to the three parts in equation (19). However, similar to term 2a and 2b, value-added in these parts are sourced from all countries in the world (foreign value-added), except for the absorption country *s*, so we label them as FDY_GVC_s, FDY_GVC_r, and FDY_GVC_t, respectively.

Following the same logic of Sections 2.2.2 to 2.2.4, we could compute each part's domestic and international total and average production lengths as summarized in the following two equations. Detailed derivations can be found in Appendix I.

$$\begin{aligned}
X_yd_i^s &= \underbrace{V^s L^{ss} L^{ss} Y_i^{ss}}_{1-Xyd_D} + \underbrace{\sum_{r \neq s}^{M} V^r L^{rr} L^{rr} Y_i^{rs} + \sum_{r \neq s}^{M} V^r L^{rr} L^{rr} A^{rs} L^{ss} Y_i^{ss}}_{2-Xyd_RT} \\
&+ \underbrace{\sum_{r \neq s}^{M} V^r L^{rr} L^{rr} A^{rs} \sum_{u}^{M} B^{su} Y_i^{us} - \sum_{r \neq s}^{M} V^r L^{rr} L^{rr} A^{rs} L^{ss} Y_i^{ss}}_{3b-Xyd_GVC_r} \\
&+ \underbrace{V^s L^{ss} L^{ss} \sum_{r \neq s}^{M} A^{sr} \sum_{u}^{M} B^{ru} Y_i^{us}}_{3a-Xyd_GVC_s} + \underbrace{\sum_{r \neq s}^{M} V^r L^{rr} L^{rr} \sum_{u \neq r,s}^{M} A^{ru} \sum_{t}^{M} B^{ut} Y_i^{ts}}_{3c-Xyd_GVC_t}
\end{aligned} \tag{34}$$

$$\begin{aligned}
X_yf_i^s &= \underbrace{0}_{1-Xyf_D} + 0 + \underbrace{\sum_{r \neq s}^{M} V^r L^{rr} A^{rs} L^{ss} L^{ss} Y_i^{ss}}_{2-Xyf_RT} \\
&+ \underbrace{\sum_{r \neq s}^{M} V^r L^{rr} A^{rs} \sum_{v}^{M} B^{sv} \sum_{u}^{M} B^{vu} Y_i^{us} - \sum_{r \neq s}^{M} V^r L^{rr} A^{rs} L^{ss} L^{ss} Y_i^{ss}}_{3b-Xyf_GVC_r} \\
&+ \underbrace{V^s L^{ss} \sum_{r \neq s}^{M} A^{sr} \sum_{v}^{M} B^{rv} \sum_{u}^{M} B^{vu} Y_i^{us}}_{3a-Xyf_GVC_s} + \underbrace{\sum_{r \neq s}^{M} V^r L^{rr} \sum_{u \neq r,s}^{M} A^{ru} \sum_{v}^{M} B^{uv} \sum_{t}^{M} B^{ut} Y_i^{ts}}_{3c-Xyf_GVC_t}
\end{aligned} \tag{35}$$

$X\_yd_i^s$ and $X\_yf_i^s$ are the products of value-added and production length and equal to the domestic and international gross outputs induced by the production of final product Y_i^s in country *s*, respectively. Therefore, the ratio of these gross outputs to Y_i^s is the average domestic and international production length based on backward inter-industry and cross-country linkage.

Sum $X\_yd_i^s$ and $X\_yf_i^s$, we obtain the global gross output driven by the global final demand for final products of sector i at Country s:

$$\begin{aligned}
X_y_i^s &= X_yd_i^s + X_yf_i^s = V^s L^{ss} L^{ss} Y_i^{ss} + \sum_{r \neq s}^{M} V^r L^{rr} L^{rr} Y_i^{rs} \\
&+ \sum_{r}^{M} V^r L^{rr} L^{rr} \sum_{u \neq r}^{M} A^{ru} \sum_{t}^{M} B^{ut} Y_i^{ts} + \sum_{r}^{M} V^r L^{rr} \sum_{u \neq r}^{M} A^{ru} \sum_{v}^{M} B^{uv} \sum_{t}^{M} B^{ut} Y_i^{ts} \\
&= \sum_{r}^{M} V^r \sum_{v}^{M} B^{rv} \sum_{t}^{M} B^{vt} Y_i^{ts} = \sum_{t}^{M} B^{vt} Y_i^{ts}
\end{aligned} \tag{36}$$

Because global final demand always sums to global value-added, the forward and backward based production lengths are equal to each other at the global level. However, they may not be equal at the country or country/sector level due to international trade and cross border production activities. This naturally raises the question: What is the relation between production length measure and production line position? Can production length measure be used directly to infer upstreamness or downstreamness of a country or a country/sector pair? Current literature is not clear on such important questions and often uses production length measures to infer production line position directly. This is the topic we will address in the next section.

2.3 From production length measures to production line position index (work still in progress)

As we have defined GVC related production and trade activities earlier, it is easy to see that a GVC production line not only has a starting and an ending stage, it also has to involve at least one and often many additional middle stages because value-added in global production chains needs to cross national borders at least twice. We thus need to identify and quantify value-added embodied in intermediate goods trade crossing national borders, at which country/sector pair and in what amount, in order to correctly measure the production line position of each specific middle production stage for a particular country/sector pair.

Let us consider a global value chain starting from primary input or value-added at sector *i* of country *s*, embodied in its gross (intermediate) exports used by sector *j* of country *r*, but finally absorbed by final product of sector *k* consumed at country *t*. According to the measure of production length of GVC related trade based on forward linkages described in Section 2.2.4, we can express such a specific GVC production line as follows

$$DVA_{ik}^{st}\_GVC_j^r = V_i^s L^{ss} A_{.j}^{sr} \sum_v^M B_{j.}^{rv} Y_k^{vt} \tag{37}$$

$$\text{Where } V_i^s = \begin{bmatrix} \ddots & 0 & 0 \\ 0 & v_i^s & 0 \\ 0 & 0 & \ddots \end{bmatrix}, \quad A_{.j}^{sr} = \begin{bmatrix} 0 & a_{1j}^{sr} & 0 \\ \vdots & \vdots & \vdots \\ 0 & a_{nj}^{sr} & 0 \end{bmatrix}, \quad B_{j.}^{rv} = \begin{bmatrix} 0 & \cdots & 0 \\ b_{j1}^{ru} & \cdots & b_{jn}^{ru} \\ 0 & \cdots & 0 \end{bmatrix}, , \quad Y_k^{vt} = \begin{bmatrix} 0 \\ y_k^{vt} \\ 0 \end{bmatrix}$$

Following the same logic to derive equations (3) and (20), we can obtain the product of the value-added and production length backward to (*s*,*i*) and forward to (*t*,*k*) from (*r*,*j*) as

$$Xy_{ik}^{st}\_GVC_j^r = V_i^s L^{ss} L^{ss} A_{.j}^{sr} \sum_v^M B_{j.}^{rv} Y_k^{vt} \tag{37.1}$$

$$Xv_{ik}^{st}\_GVC_j^r = V_i^s L^{ss} A_{\cdot j}^{sr} \sum_u^M B_{j\cdot}^{ru} \sum_v^M B^{uv} Y_k^{vt} \tag{37.2}$$

Therefore the total production length of this particular GVC equals

$$X_{ik}^{st}\_GVC_j^r = V_i^s L^{ss} L^{ss} A_{\cdot j}^{sr} \sum_v^M B_{j\cdot}^{rv} Y_k^{vt} + V_i^s L^{ss} A_{\cdot j}^{sr} \sum_u^M B_{j\cdot}^{ru} \sum_v^M B^{uv} Y_k^{vt} \tag{37.3}$$

It can be proved that summing equation (37.3) over *r*, *j*, *t*, and *k* allows us to obtain the same results as equation (32). Therefore, both equations give the same production length for value-added originating from sector *i* of country *s* based on forward inter-industry, cross-country linkages.

Similarly, summing equation (37) over *s*, *i*, *t*, and *k*，we can obtain the value-added and production length backward from (*r*,*j*) to all (*s*,*i*) as：

$$\begin{aligned} DVA\_GVC_j^r &= \sum_{s \neq r}^M V^s L^{ss} A_{\cdot j}^{sr} \sum_v^M B_{j\cdot}^{rv} \sum_t^M Y^{vt} \\ &= \sum_{s \neq r}^M V^s L^{ss} A_{\cdot j}^{sr} \sum_v^M B_{j\cdot}^{rv} Y^{vs} + \sum_{s \neq r}^M V^s L^{ss} A_{\cdot j}^{sr} \sum_v^M B_{j\cdot}^{rv} Y^{vr} + \sum_{s \neq r}^M V^s L^{ss} A_{\cdot j}^{sr} \sum_v^M B_{j\cdot}^{rv} \sum_{t \neq s,r}^M Y^{vt} \end{aligned} \tag{37.4}$$

Summing equation (37.1) over *s*, *i*, *t*, and *k*，we can obtain the product of the value-added and production length backward from (*r*,*j*) to all (*s*,*i*) as：

$$Xg\_f_j^r = \sum_{s \neq r}^M V^s L^{ss} A_{\cdot j}^{sr} \sum_u^M B_{j\cdot}^{ru} \sum_v^M B^{uv} \sum_t^M Y^{vt} \tag{37.5}$$

Summing equation (37.2) over *s*, *i*, *t*, and *k*，we can obtain the product of the value-added and production length forward from (*r*,*j*) to all (*t*,*k*) as：

$$Xg\_d_j^r = \sum_{s \neq r}^M V^s L^{ss} L^{ss} A_{\cdot j}^{sr} \sum_v^M B_{j\cdot}^{rv} \sum_t^M Y^{vt} \tag{37.6}$$

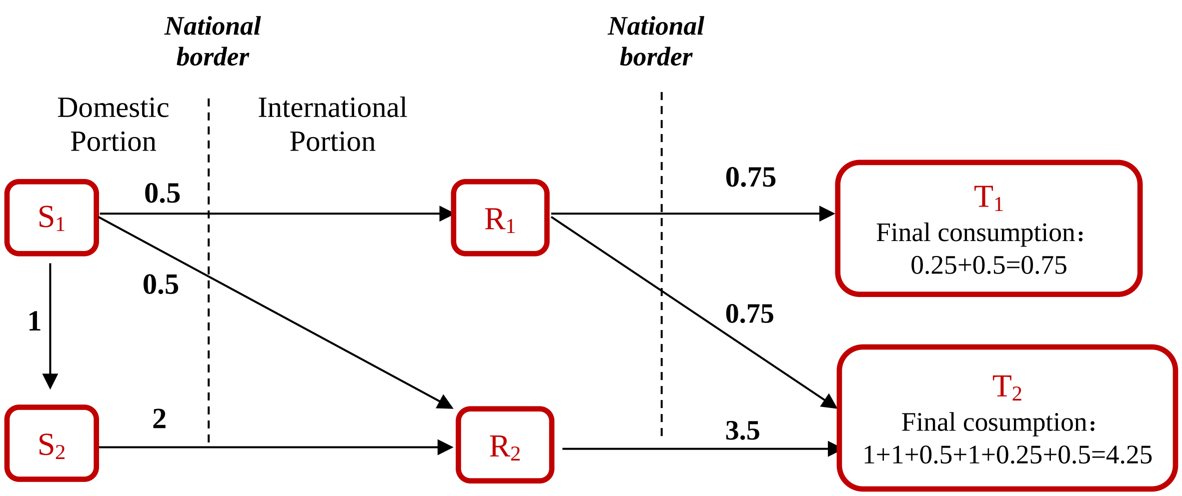
As a special production node in the global production network, the closer sector *j* of country *r* is to these value-added crossing national borders that it used as inputs，the smaller the gross output it can induce (measured by Xy_j^r); the closer sector *j* of country *r* is to these final products that use its value-added as source，the smaller the gross output it is able to push out (measured by Xv_j^r). Therefore, its average production line position in the global value chain can be defined as

$$GVC\_P_j^r = \frac{Xv_j^r}{Xy_j^r + Xv_j^r} \tag{38}$$

This index is bounded by one. The larger the index, the more upstream is the country/pair. Importantly, under our definitions, the upstreamness and downstreamness of a given country sector are really the same thing, thus overcoming the inconsistency of the production position indexes widely used in current literature, such as the N* and D* indexes proposed by Fally (2012) and the Down measure proposed by Atras and Chor (2013).

Let us consider a simple numerical example, illustrated in Figure 3.

Figure 3 GVC position in a 3-country, 2-sector example



There are 3 countries (S, R, and T, respectively) and 2 sectors (1 and 2) in this simple production chain. Countries S and R only produce and do not consume, whereas Country T only consumes and do not produce. The arrows indicate the direction of value-added flows, and the numerical value on each line indicates the gross trade sent from the relevant upstream node to the corresponding downstream node. Thus, the total value added generated in the first node (Country S, sector 1) is assumed to be 2, of which 1 is sent to S2, and 0.5 each is sent to R1 and R2, respectively. The values added in both R1 and R2 are assumed to be 1. The values added in T1 and T2 are zero (because Country T does not produce).

Whenever a node bifurcates into two export routes, it is assumed that the both domestic value added and foreign value added will be evenly split between the two export routes. Thus, from node R1, the gross value of 1.5 is split into an export of 0.75 to T1 and T2, respectively.

There are 4 routes between the value-added originated from S1 and consumed at the final destination T1 or T2：

① S1 —— R1 —— T1

S1 produces intermediate goods and exports to R1，and R1 uses it to produce final exports to T1 and consumed in Country T.

② S1 —— R1—— T2

S1 produces intermediate goods and exports to R1，R1 produces final exports to T2 and consumed by its domestic consumer.

③ S1 —— R2 —— T2

S1 produces intermediate goods and exports to R2，R2 produces final exports to T2 and consumed there.

④ S1 —— S2 —— R2 —— T2

S1 produces intermediate inputs to S2, S2 produces further processed intermediate exports to R2, and R2 produces final exports to T2 and consumed in Country T.

The total value-added of this production network is accounted as:

Total Value-added (TV) = (S1)+ (S2)+ (R1)+ (R2)=2+1+1+1=5

The values of the final products are

① S1 ——— R1 ——— T1

0.25 + 0.5 = **0.75**

② S1 ——— R1——— T2

0.25 + 0.5 = **0.75**

③ S1 ——— R2 ——— T2

0.5 + 0.5 = **1**

④ S1 ——— S2 ——— R2 ——— T2

1 + 1 + 0.5 = **2.5**

Therefore, the total value of final products of the network equals:

FD=0.75+0.75+1+2.5=5, i.e., the value-added and the value of final products are equal to each other at the global level.

There are three ways to compute the average production length:

Firstly, based on forward linkages (sum over the starting node, S1, as example）：

The value added created by S1 in each route is listed below:

① **S1** —— R1 —— T1：0.25

② **S1** —— R1 —— T2：0.25

③ **S1** —— R2 —— T2：0.5

④ **S1** —— S2 —— R2 —— T2：1

Summing them, the total value-added created along this production line equals VA = 0.25+0.25+0.5+1=2

The cost push gross output induced by S1's value added can be measured as

① S1 ——— R1 ——— T1

0.25 + 0.25 = **0. 5**

② S1 ——— R1——— T2

0.25 + 0.25 = **0. 5**

③ S1 ——— R2 ——— T2

0.5 + 0.5 = **1**

④ S1 ——— S2 ——— R2 ——— T2

1 + 1 + 1 = **3**

Therefore, the average production length of value-added created by S1 based on forward linkages can be computed as：

(2×**0.25**+2×**0.25**+2×**0.5**+3×**1**)/2=5/2=2.5

For each route, we can split the gross trade into a "domestic portion" and an "international portion." For S1,

Domestic Portion: (1×**0.25**+1×**0.25**+1×**0.5**+2×**1**)/2 = 3/2=1.5

International Portion: (1×**0.25**+1×**0.25**+1×**0.5**+1×**1**)/2 = 2/2=1

The following identity always holds：

Total production length (2.5) = Domestic Portion (1.5) + International Portion (1)

Similarly, the value-added created by S2 equals：

VA: S2 —— R2 —— T2：**1**

The total output induced by value-added created by S2 equals：

GO: S2 —— R2 —— T2:

1 + 1 = **2**

The average production length of value-added created by S2 based on forward linkages can be computed as: 2/1=2 and its domestic and international portions both equal 1**.**

The above accounting and computation results can be summarized into the following table:

	VA	TO	PL	DPL	FPL
S_1	2	5	2.5	1.5	1
S_2	1	2	2	1	1
S	3	7	7/3	4/3	1
R_1	1	1	1	1	0
R_2	1	1	1	1	0
R	2	2	1	1	0
World	5	9	9/5	6/5	3/5

Note: We assume no value-added at T, so all indexes equal to zero.

Secondly, based on backward linkages (sum over consumption destination, T2, as example)

There are 3 routes contributing to the value-added of the final product consumed at T2. The total value-added absorbed through each route is listed below:

① S1 ——— R1 ——— **T2**:

0.25 + 0.5 = **0.75**

② S1 ——— R2 ——— **T2**:

0.5 + 0.5 = **1**

③ S1 ——— S2 ——— R2 ——— **T2:**

1 + 1 + 0.5 = 2.5

The total value of the final products at T2 equals 0.75+1+2.5=4.25.

To produce such amount of final products, the required gross output produced by each production line equals:

① S1 ——— R1 ——— **T2**:

0.25×2 **+** **0.5**×1 **=** **1**

② S1 ——— R2 ——— **T2**:

0.5×2 + **0.5**×1 = **1.5**

③ S1 ——— S2 ——— R2 ——— **T2:**

1×3 + **1**×2 + **0.5**×1 = **5.5**

Summing the accumulated value-added in each route and dividing by the total value of final products produced at T2, the average production length of value-added absorbed at T2 based on backward linkages can be computed as：

(1+1.5+5.5)/4.25=8/4.25=32/17

It is obvious from such a simple example that the production length computed from forward and backward linkages only equal each other at the global level, not at the country/sector pair; there is no clear implication for upstreamness or downstreamness from production length measures either based on forward or backward linkages because they may give different rankings for each country/sector pair.

The results can be summarized into the following table:

	VA	TO	PL	DPL	FPL
T1	0.75	1	4/3	1	1/3
T2	4.25	8	32/17	21/17	11/17
T	5	9	9/5	6/5	3/5
World	5	9	9/5	6/5	3/5

Note: there are no final goods production for S and R nodes by assumption, so their backward linkage based indexes all equal to zero.

Finally, aggregating for an intermediate production stage (R2 as example to introduce production line position index)

R2 is located in the middle of 2 production and trade routes originating from S1 and ending at T2. Total value-added flow in and out of this production node are：

① S1 —— **R2** —— T2：
0.5 1
② S1 —— S2 ——**R2** —— T2：
1 1 2.5

The total value added embodied in the output of R2 can be measured as 1+2.5=3.5.
The production length of the starting stage (S1) of R2 (total gross output driven by final goods consumption in T2) based on backward linkages equals：

Xy_2^r = (1×**0.5**+2×**1**+1×**1**)/2.5 = 1.4;

The production length to the ending consumption stage (T2) of R2 (total gross output pushed by value-added from R2) equals: Xv_2^r = (1×1+1×2.5)/3.5 = 1. Therefore, the production position of R2 can be computed as

$$GVCP\_P_2^r = \frac{Xv_2^r}{Xv_2^r + Xy_2^r} = 1.4/2.4 = 7/12 = 0.508$$

This implies that all production lines starting from S1 and ending at T2 are located at a relative downstream position, just as shown in Figure 3, closer to final consumption.

The above computation can be summarized into the following table:

	VA1	GO1	PL1	VA2	GO2	PL2	Relative Position
R1	0.5	0.5	1	1.5	1.5	1	½
R2	2.5	3.5	7/5	3.5	3.5	1	7/12
R	3	4	4/3	5	5	1	4/7
S1	0	0	0	2	5	2.5	0
S2	0	0	0	1	2	2	0
S	0	0	0	3	7	7/3	0
T1	0.75	1	4/3	0	0	0	1
T2	4.25	8	32/17	0	0	0	1
T	5	9	9/5	0	0	0	1

This simple numerical example shows clearly that the production line positon index is closely related to the measure of production length, but the production length measure may not directly imply production line position. Only through aggregation, for both forward and backward linkage based production length measures for a particular country/sector pair located in the middle stages of production lines, by first determining its “distance” to both the starting and ending stages of all related production lines, can relative “upstreamness” or “downstreamness” be correctly estimated.

2.3 Global Value-Chain participation index

The amount of Vertical Specialization (measured by both VS and VS1 as proposed by Hummels et al., 2001) as percent of gross exports has been used widely in the literature as the index to quantify the extent of a country’s participation in global value chains (Koopman et al., 2010; OECD, 2013). However, it excludes production to satisfy domestic final demand (which includes both pure domestic and international trade related production activities), and by only considering export activities, may not cover all the possible ways a country could contribute its domestic value-added into the global production network.

Firms in a country/industry may participate in international production chains in three ways:

1. Exporting its domestic value-added in intermediate inputs used by other countries to produce exports directly or indirectly; it is the source country's value-added that shows up as foreign value-added in other countries' production of exports;
2. Using other countries' value-added to produce its exports directly or indirectly; it is the other countries' value-added that shows up as foreign value-added in the source countries' gross exports;
3. Exporting its domestic value-added in intermediate inputs used by other countries to produce other countries' domestic consumed final products indirectly (via the source or a third country).

The global value chain participation indexes used in the literature, such as the VS and VS1 as percent of gross exports, only take the first two channels into consideration, even if the third channel may be quite substantial especially for large economies as both sources and destinations.

Using the decomposition of value-added generated from each industry/country pair (GDP by industry statistics) expressed in equation (19), we can fully identify all the three possible ways a country can realize its domestic value-added in the global production network and construct an index that helps us to measure the full extent to which production factors are employed in a particular country-sector involved in the global production process. Such a GVC participation index based on forward industrial linkage can be defined mathematically as follows:

$$
\begin{aligned}
GVCP_F^s &= \frac{\sum_{r\neq s}^{M}[\hat{V}^s L^{ss} EI^{sr} - \hat{V}^s L^{ss} A^{sr} L^{rr} Y^{rr}]}{\hat{V}^s X^s} = \frac{\sum_{r\neq s}^{M}[\hat{V}^s L^{ss} A^{sr} \sum_{u}^{M}\sum_{v}^{M} B^{ru} Y^{uv} - \hat{V}^s L^{ss} A^{sr} L^{rr} Y^{rr}]}{\hat{V}^s X^s} \\
&= \frac{\hat{V}^s L^{ss} \sum_{r\neq s}^{M} A^{sr} (\sum_{u}^{M} B^{ru} Y^{ur} - L^{rr} Y^{rr})}{\hat{V}^s X^s} + \frac{\hat{V}^s L^{ss} \sum_{r\neq s}^{M} A^{sr} \sum_{u}^{M} B^{ru} Y^{us}}{\hat{V}^s X^s} + \frac{\hat{V}^s L^{ss} \sum_{r\neq s}^{M} A^{sr} \sum_{u}^{M} \sum_{t\neq s,r}^{M} B^{ru} Y^{ut}}{\hat{V}^s X^s} \qquad (39) \\
&= \frac{\sum_{r\neq s}^{M}[DVA\_GVC\_r^s]}{\hat{V}^s X^s} + \frac{\sum_{r\neq s}^{M} DVA\_GVC\_s^s}{\hat{V}^s X^s} + \frac{\sum_{r\neq s}^{M} DVA\_GVC\_t^s}{\hat{V}^s X^s}
\end{aligned}
$$

The denominator of equation (32) is the value-added generated in production from a country/sector pair; the numerator of equation (32) is domestic value added of country *s* embodied in its narrowly defined GVC exports to the world. It excludes domestic value-added embodied in final goods exports (with international production length of zero) and domestic value-added embodied in intermediate exports, but used by the direct importer to produce final products within its border and consumed there without going through a third country. So equation (32) gives domestic value-added generated from GVC related production activities as a share of total sector value added. It differs from the forward industrial linkage based GVC participation index defined in previous literature (VS1 as percent of gross exports) in two ways: (a) it is based on the value-added concept while both VS1 and gross exports are based on the gross concept; (b) it is a production concept, not only trade. It includes domestic value-added embodied in intermediate inputs from the exporting country that is indirectly absorbed by its direct trading partners. Therefore, it completely reflects the degree of participation of production factors employed in a particular country/sector in cross border production sharing activities.

Based on the backward decomposition of final goods production we can define another GVC participation index based on backward industrial linkage as follows:

$$
\begin{aligned}
GVCP\_B_i^s &= \frac{\sum_{r}^{M} V^r L^{rr} \sum_{u\neq r}^{M} A^{ru} B^{us} \sum_{t}^{M} Y_i^{st} - \sum_{r\neq s}^{M} V^r L^{rr} A^{rs} L^{ss} Y_i^{ss}}{Y_i^{s*}} \\
&= \frac{\sum_{r\neq s}^{M} V^r L^{rr} A^{rs} \sum_{u\neq s}^{M} B^{su} Y_i^{us} - \sum_{r\neq s}^{M} V^r L^{rr} A^{rs} L^{ss} Y_i^{ss}}{Y_i^{s*}} + \frac{V^s L^{ss} \sum_{r\neq s}^{M} A^{sr} \sum_{u}^{M} B^{ru} Y_i^{us}}{Y_i^{s*}} + \frac{\sum_{r}^{M} V^r L^{rr} \sum_{u\neq r}^{M} A^{ru} \sum_{t\neq s,r}^{M} B^{ut} Y_i^{ts}}{Y_i^{s*}} \qquad (40) \\
&= \frac{DFY\_GVC\_r_i}{Y_i^{s*}} + \frac{DFY\_GVC\_s_i}{Y_i^{s*}} + \frac{DFY\_GVC\_t_i}{Y_i^{s*}}
\end{aligned}
$$

where $Y_i^{s*} = \sum_{r}^{M} Y_i^{sr}$ the total final goods production of sector i at Country s; the three numerators in (40) give the share of foreign value-added related to GVC trade and cross country production sharing in the total value of final goods produced in country *s*. Its denominator is the value of each country's final goods production (both exports and domestic final use). The global sum of its numerator (and each of its three components) equals the global sum of the numerator in equation (32).[16] Therefore, at the global level, the forward and backward industrial linkage based GVC participation indexes (and each of its three components) equal each other, a similar

[16] The math proof is provided in Appendix J.

property of VS and VS1 based GVC participation indexes. However, it also differs from the backward industrial linkage based GVC participation index defined in previous literature (VS as percent of gross exports) in two ways: (a) it is based on a net concept while both VS and gross exports are based on a gross concept; (b) it is a production concept, not only trade. It includes foreign value-added embodied in intermediate imports that is indirectly absorbed by the importing country (with production sharing activities with the source or third countries). Therefore, it completely reflects the degree of foreign production factors' participation in the home country/sectors' production of final products, and measures international production sharing activities from another perspective: how a country's production relies on other countries' production factors' contribution. Aggregating equations (39) and (40) over all countries, we can show that the forward and backward production linkage based GVC participation indexes are equal to each other at the global level (see Appendix J for details).

3. Estimation Results

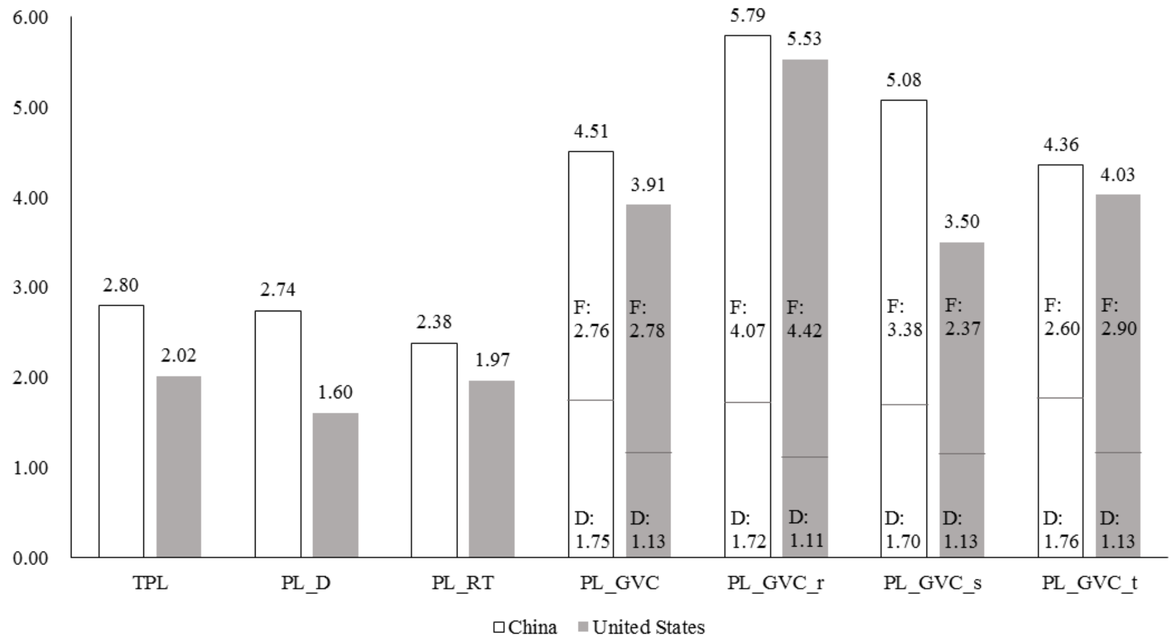
Applying the production length measures as well as both the GVC participation and the position indexes developed in the previous section to WIOD data, a set of indexes can be estimated and used to quantitatively describe the multi-dimensional structures and the evolving trend of various GVCs for 41 countries and 34 industries over 1995–2011. Since all the indexes can be estimated at both the most aggregated "world" and the more disaggregated "country/bilateral-sector" levels, we obtain a large amount of numerical results. To illustrate the estimation outcomes in a manageable manner, we first report a series of examples at various disaggregated levels to highlight the stylized facts based on our new GVC index system and demonstrate their advantages compared to the existing indexes in the literature, we then conduct econometric analysis on the role of GVCs in the economic shocks brought by the recent global financial crisis as a more comprehensive application of our newly developed GVC indexes.

3.1 Production length index

3.1.1 Estimation results

Taking the Electrical Equipment Sector as an example, Figure 4 reports the basic estimation results for China and the US, at the "Country-Sector" Level for 2011.

Figure 4 Production Length of Electrical Equipment Sector, 2011



The estimation results in Figure 4 provide us with the following observations：

(1) The index values are always higher for China than that for the US, which means the value added created by China has to go through more steps before reaching its final uses. In other words, compared with the US, value-added created from China’s Electrical Equipment Sector needs to go through more production stages on average before reaching its final uses.

(2) Compared with the pure domestic and the direct value-added exports production modes, value added created along the GVCs has the longest production length (PL_GVC). This result is intuitively reasonable as more participants and production steps are involved in the GVC production process.

(3) Value added absorbed indirectly by direct importers (PL_GVC_r) have the longest production length. In such case, value added flows back to the GVC network from the direct importing country, further going through several production stages, then returns to the direct importers and is finally absorbed there.

(4) The international portion of GVC production length is always longer than the domestic portion. This finding reflects the global increase in vertical specialization: the more fragmented is the production process, the more participants are involved, and the less is the portion allocated to each participant.

More information can be obtained if we estimate the indexes at the "bilateral-sector" level. Using the US Electrical and Optical Equipment Sector as an example, compared to the value added flows to Canada, Australia, and Russia, the value added imported by some East Asian economies (such as China, Korea, and Taiwan) has to go through more production stages outside the US to reach the final consumers[17]. So the international portion of GVC length is relatively longer for US value added exported to China, Korea, and Taiwan (from 1.9 to 2.5), and shorter for US value added exported to Canada, Australia, and Russia (around 0.81).

Table 3 Length of International Production Portion for Value Added Created by the US Electrical Equipment Sector, 2011

Direct Importers	Length of International Production Portion
TWN	2.403
KOR	2.219
CHN	1.953
CAN	0.815
AUS	0.813
RUS	0.806

TWN=Taiwan; KOR=Korea; CHN=China; CAN=Canada; AUS=Australia; RUS=Russia

3.1.2 Has the length of Global Value Chains become longer or shorter over time?

One important question addressed in the recent GVC related literature is this: Has the global production chain become less or more fragmented?

Most studies conclude that global production has become more fragmented today than decades ago. As shown in Feenstra and Hanson (1996), the imported intermediate inputs in the US have increased from 5.3% to 11.6% between 1972 and 1990. Similarly, Hummels et al. (2001) find that the world VS (Vertical specialization) share of exports has grown almost 30% between 1970 and 1990, which accounts for more than 30% of overall export growth.[18]

Our estimation results clearly show that the Global Value Chain is getting longer, which reflects the increasing fragmentation of GVC related production and trade activities. Moreover,

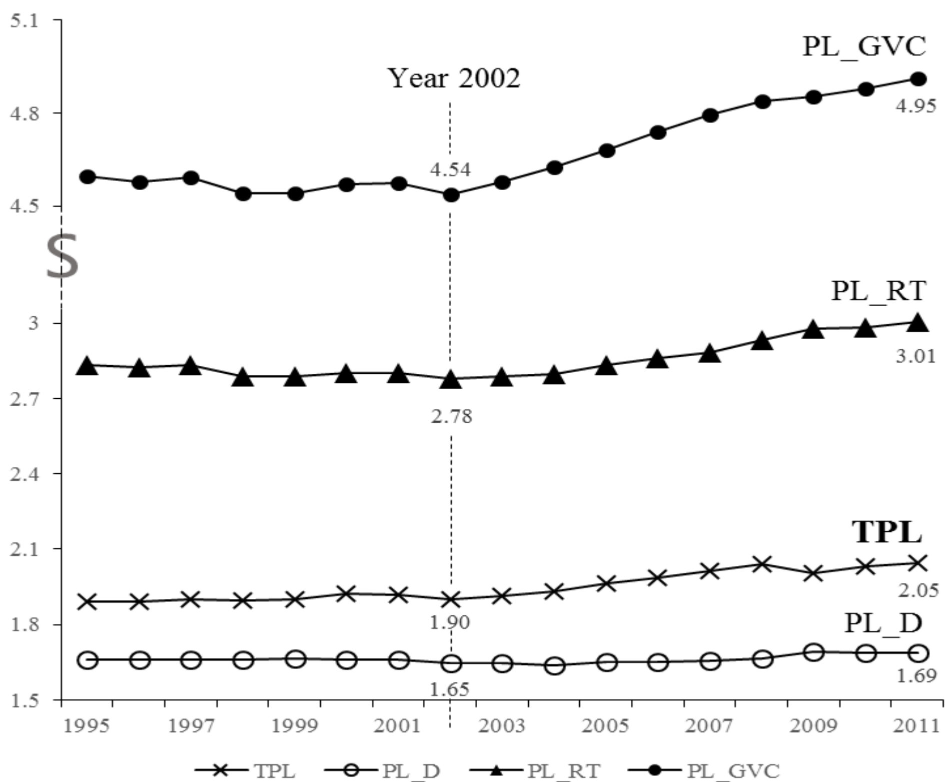
[17] As we will show later in Table 4, the length of global value chains that East Asian countries participate in is significantly longer than in other countries, which means their productions are more globalized relatively than other countries.

[18] Fally (2011) indicates that the production chain (or the distance to final demand) in the US appears to have shortened over time and concludes that such a trend is also a global phenomenon. Consistent with Fally, our calculation also shows that the production length of the US is getting shorter. But this finding is reversed at the global level. In Appendix K, we show that the strong assumption "The same industries have the same production length across countries" is the main factor that leads to the puzzling finding by Fally.

the distinction between different types of production and trade activities enable us to further investigate the major drivers behind the lengthening of GVCs.

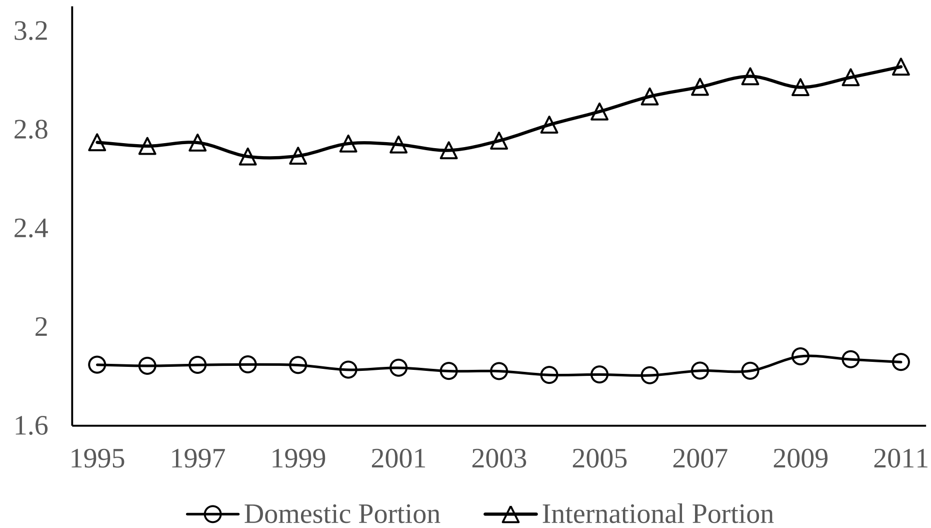
As shown in Figure 5, the world average “Total Production Length” shows a clearly upward trend, especially after year 2002 (this trend was temporarily interrupted by the global financial crisis during 2008 to 2009). Furthermore, the average production length of GVCs has increased by 0.36 from 2002 to 2011, which is much faster than the direct value-added exports and pure domestic production length.

Figure 5 The Upward Trend of Production Length, World Average



In Figure 6, we focus on GVC production activities to investigate the changes of its domestic and international portions. We find that the increasing length of GVCs is primarily driven by the rapid growth of its international portion.

Figure 6 The Production Length of Domestic and International Portion of GVCs, World Average, 1995 to 2011

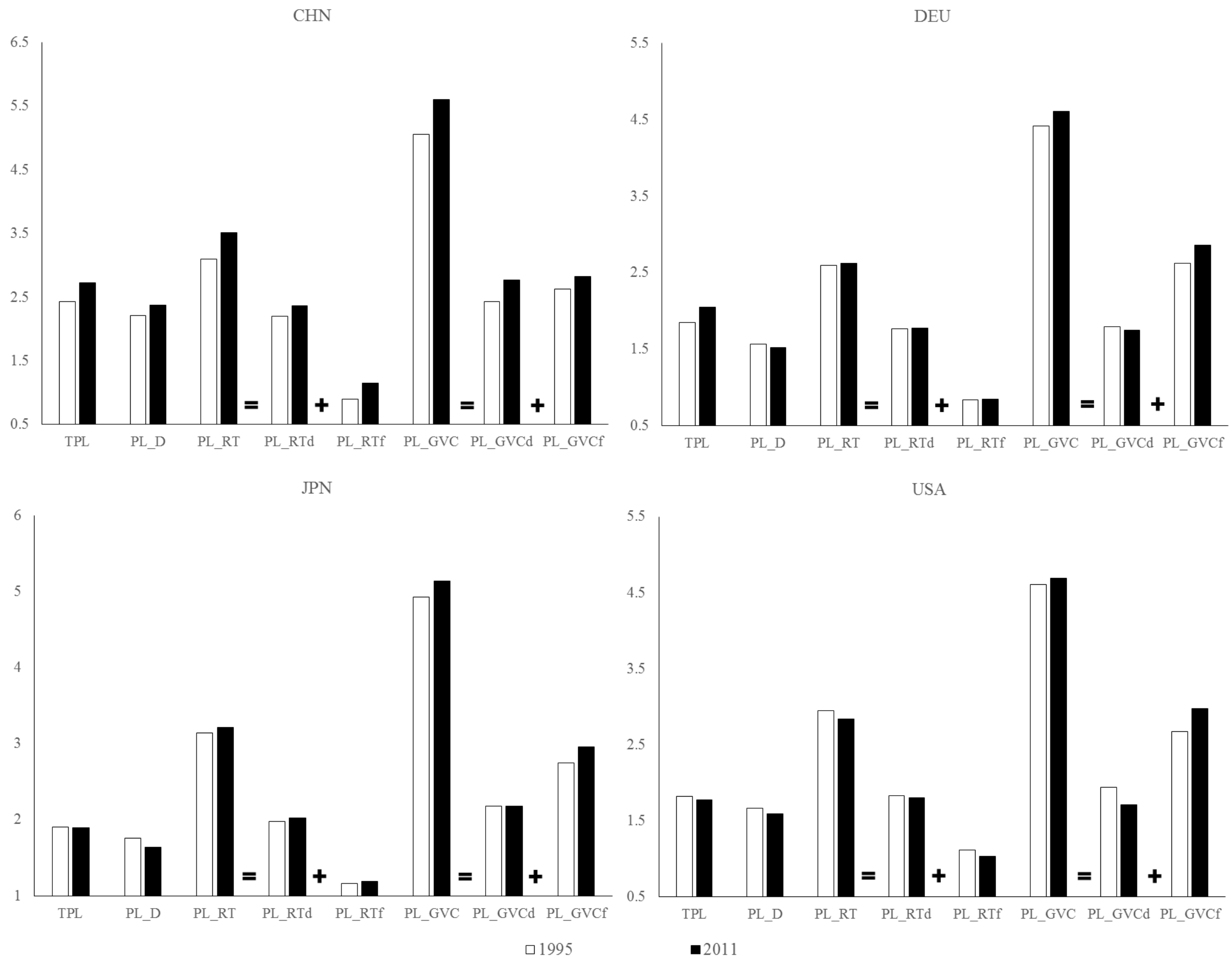


To ensure robustness of results, we further investigate the changes of production length at the country and sectoral level.

In Figure 7, we compare the major portions of production length across countries. For China, the total average production length, as well as all of its portions, is longer in 2011 than in 1995. For Germany, Japan, and the US, their pure domestic and direct value-added exports production lengths have slightly decreased or remained stable during the sample period. But the average GVC production length, especially its international portion, has increased considerably for all countries over this period, even when the total average production length became shorter for Japan and the US.[19]

[19] This may reflect the phenomenon of "offshoring" production activities abroad in these developed economies. When more production activities go abroad, the international portion of GVCs gets longer while its domestic portion becomes shorter.

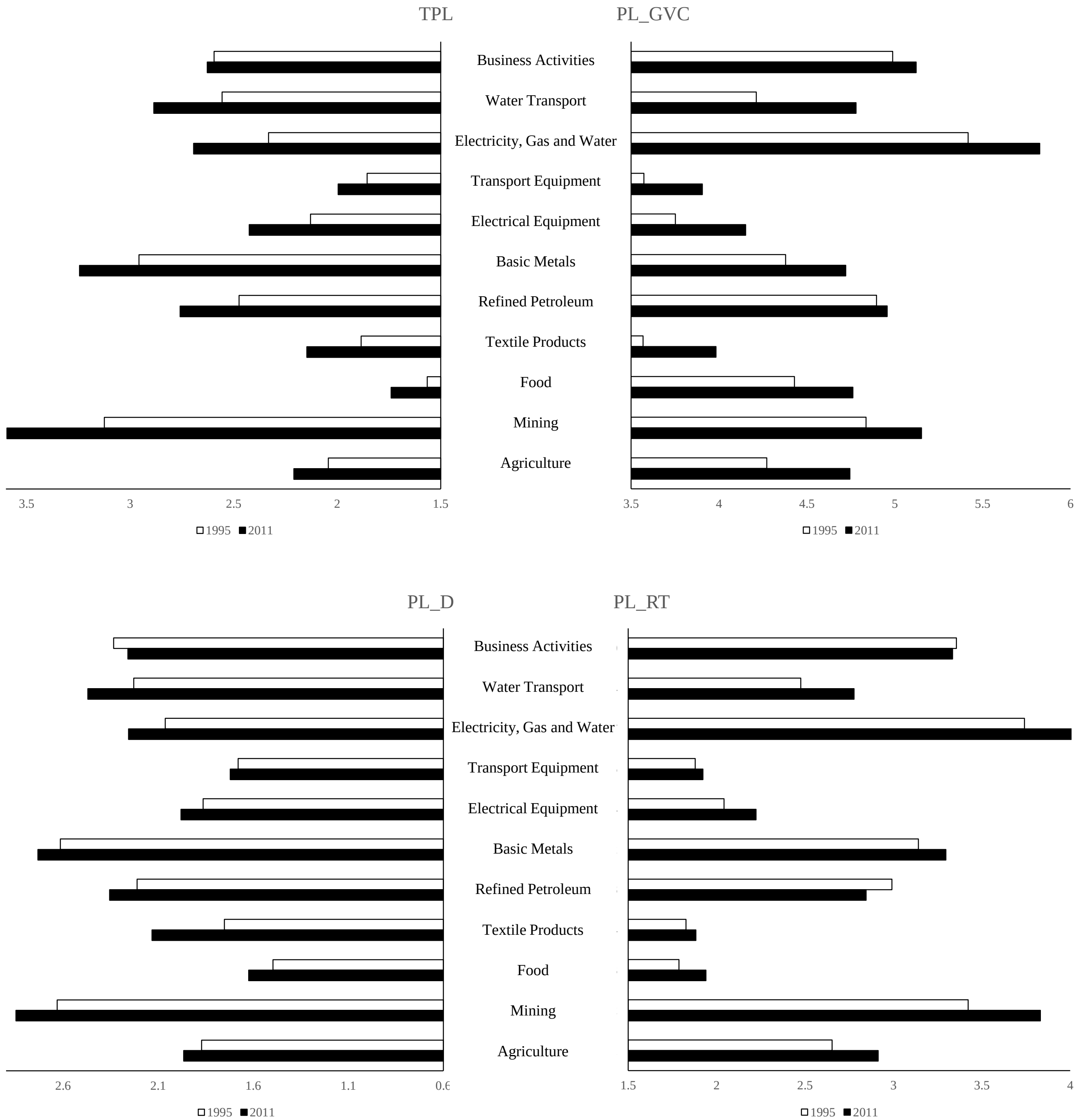
Figure 7 Decomposition of Production Length for Major Economies



CHN=China; DEU=Germany; JPN=Japan; USA=United States

The same results can be found at the sectoral level. Figure 8 shows that the world average production length is longer in year 2011 for all sectors. In addition, compared with pure domestic and direct value-added exports production length, the increasing trend of GVC production length is more significant for almost all sectors.

Figure 8 Decomposition of Production Length for Typical Sectors



In conclusion, using the production length indexes newly defined in this paper, we have observed the increasing trend of fragmentation in production, especially in Global Value Chain related production activities.

3.2 From production length measure to GVC position index [work still in progress]

The GVC position index defined in this paper enables us to focus on a specific value chain (originating from S_i and ending at T_k) and measure the distance from any production stage between the final demand and the initial factor inputs in a production line by a combination of production linkages based on both forward and backward linkages.

More importantly, this measure resolves the puzzling issue in current literature that the "Upstreamness" and "Downstreamness" indexes are incomparable. Our GVC position index measures a middle production stage (any (r,j))'s distance to both ends of the related production line at the most detailed level that starts from (s,i) and ends at (t,k). At the global level, the sum of the forward and backward linkage based production lengths is equal to the total production length of GVC related intermediate exports. Therefore, the forward and backward based production lengths are indeed comparable to our GVC position index. It allows us to accurately quantify the "position" of any particular production node by comparing its forward and backward production length. When the position index's number is larger, it indicates that the forward distance from the production node concerned is relatively longer so the production stage is located away from the final consumption end of the particular production line.

The numerical results at the country level show that during 1995–2011, as covered by WIOD data, China is the country closest to the final consumption end all the times. Another interesting finding is that among countries worldwide, China ranks at the top in terms of average length of value chains it participates in.

Our numerical results are contradictory to Miller et al. (2015). Their results show that, compared with other countries, China is the most upstream country in the world, far away from the final consumption end; but in fact, our results are not actually contradictory with Miller's findings. The reasons for the inconsistency are as follows:

First, the calculation in this paper focuses on "Value-added in Intermediate Exports." The direct value added exports and pure domestic production, which are irrelevant to deep cross border production sharing activities, are excluded from our newly defined measures. Our numerical results thus more accurately measure the positions of different production nodes in Global Value Chains.

Second, when the "Upstreamness" (OU) and "Downstreamness" (ID) indexes of a country/sector pair computed by Miller et al. are high, it means that the distance between the country/sector pair to the factor input/final consumption end is longer. However, as we show in the numerical example, using backward or forward linkage based production lengths alone cannot tell the country/sector pair's relative position in a production line because the ratio of the forward and backward length to each end of the production line could still be relatively shorter or longer. Just as Table 4 shows, the average length of global value chains that China participates in is significantly longer than in other countries, but it can still be located in the most downstream position of GVCs.

Table 4 Country Level GVC Position Index, 2011

Country	Position Index	Average Length of Value Chains that it Engages in
DEU	0.54	4.03
GBR	0.54	3.81
RUS	0.54	4.31
IND	0.53	4.08
USA	0.53	3.94
CAN	0.53	4.02
FRA	0.52	4.02
ITA	0.50	4.01
JPN	0.46	4.35
KOR	0.45	4.71
CHN	0.41	5.00

DEU=Germany; GBR=United Kingdom; RUS=Russia; IND=India; USA=United States; CAN=Canada; FRA=France; ITA=Italy; JPN=Japan; KOR=Korea; CHN=China

The above finding regarding China as the closest to the final end of the value chain is valid at the country-sector level. In Table 5, we rank different countries according to the GVC position index, with the highest country ranked first. It shows that in 2011, most sectors in China are positioned closest to the final consumption end.

Table 5 Country Ranking according to the Position Index

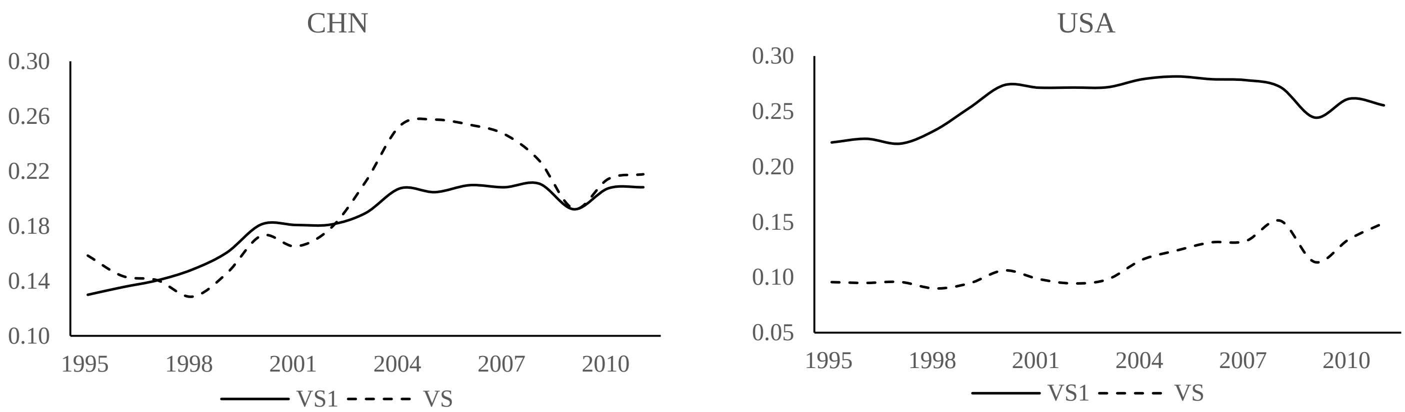
	Sector	China	United States
1	Agriculture	1	3
2	Mining	3	35
3	Food	1	5
4	Textiles Products	1	6
5	Leather and Footwear	1	38
6	Wood Products	1	24
7	Paper and Printing	1	36
8	Refined Petroleum	1	9
9	Chemical Products	1	11
10	Rubber and Plastics	1	24
11	Other Non-Metal	10	29
12	Basic Metals	1	16
13	Machinery	1	35
14	Electrical Equipment	1	35
15	Transport Equipment	1	31
16	Recycling	5	31
17	Electricity, Gas and Water	1	35
18	Construction	38	32
19	Sale of Vehicles and Fuel	-	38
20	Wholesale Trade	1	36
21	Retail Trade	1	40
22	Hotels and Restaurants	1	17
23	Inland Transport	1	23
24	Water Transport	1	39
25	Air Transport	1	37
26	Other Transport	15	23
27	Post and Telecommunications	14	39
28	Financial Intermediation	2	18
29	Real Estate	13	35
30	Business Activities	7	35
31	Public Admin	23	31
32	Education	7	36
33	Health and Social Work	5	28
34	Other Services	1	16

Note: Sector 35 is not included, as the position index for this sector is computable for only 3 countries.

3.3 Participation index

Hummels et al. (2001)'s Vertical Specification Indexes, the VS and VS1 to gross exports ratios, are widely used in the literature to measure the extent of GVC participation since they were first proposed by Koopman et al. (2011). As shown in Figure 9, the VS and VS1 ratios for China and the US can provide us with useful information of GVC participation from at least two aspects: (1) China's participation in Global Vertical Specification has increased dramatically since 1998; (2) The upward trend of Vertical Specification for both China and the US has been temporarily interrupted by the Financial Crisis.

Figure 9 VS and VS1 ratios, 1995 to 2011



CHN=China; USA=United States

However, there are two major shortcomings in those traditional participation indexes:

1) Using gross exports as the denominator. The ratio might be very high just because some sectors may have very little direct exports (e.g., Mining and Service). In such a case, the index value might become very large. In many empirical cases as we will show later, we may not be able to determine whether the index becoming larger is due to the large numerator or the small denominator (in math terms, the index goes to infinity when the denominator goes to zero) and whether the index overestimates GVC participation.

2) Direct value-added exports (only one border crossing) are not excluded from the calculation, which also leads to overestimation. In fact, the ratio of traditional intermediates goods in intermediates exports is declining over time. In the meantime, there is a noticeable rising trend in GVC related trade (two or more border crossings) during the past 30 years.

The GVC participation index developed in this paper has overcome the above-mentioned shortcomings and is able to accurately measure the degree of GVC participation as the share of total value-added production at the bilateral/sector level and can be further decomposed into three parts according to where the value added is absorbed. Such detailed GVC participation measure will provide better indexes that are needed to conduct GVC related empirical analysis.

3.3.1 Estimation results

The forward linkage based participation index proposed in this paper can be understood as "What is the percentage that value added generated by a specific country-sector pair has contributed to the GVC production network?" while the backward linkage based participation index can be understood as "What is the percentage of final products produced by a specific country-sector pair that comes from GVC related production and trade activities?"

(1) Country level

Using China and the US as examples, Figure 10 shows the time series patterns of forward/backward linkage participation indexes. Our results are consistent with the observed upward trend of the traditional VS/VS1 indexes, and the negative impact of the financial crisis has also been clearly reflected. However, the new indexes clearly indicate that China's backward linkage based participation index is consistently higher than its forward linkage based participation index, in contrast with that of the US. This is different from the traditional indexes that provide a mixed picture. More consistent with the fact that compared to the US, China participates in GVCs relatively more from the downstream than upstream. Another point worth noting is that the participation ratio of China, forward or backward notwithstanding, is significantly higher than that of the US.

Figure 10 Forward/Backward Linkage Participation Indexes, 1995 to 2011

CHN=China; USA=United States

(2) Sectoral level

Table 6 lists the forward/backward linkage based participation indexes in year 2011 for 6 sectors and 8 countries, which implies the characteristics of different countries when participating in GVC production.

For example, in the agriculture sector in Finland, the forward linkage based participation ratio is significantly higher than in other countries. This numerical result is in line with the statement that forestry is the dominant industry in Finland. Similarly, since Russia is the giant in energy, its mining sector's forward linkage based participation ratio is as high as 33.8%, in significant contrast to the backward linkage based participation ratio (of only 1.7%).

Table 6 Sectoral Level Participation Index, Forward/Backward Linkage

	Forward Linkage Based Participation Index			
	Agriculture	Mining	Electrical Equipment	Transport Equipment
BRA	6.0%	15.1%	5.0%	2.8%
CHN	2.3%	6.5%	12.1%	4.9%
DEU	7.3%	22.1%	**20.3%**	**14.5%**
FIN	**10.7%**	20.9%	18.6%	11.8%
IDN	2.7%	21.5%	6.6%	2.8%
IND	1.6%	9.9%	9.5%	4.2%
RUS	1.8%	**33.8%**	6.4%	4.3%
USA	3.4%	5.5%	12.9%	7.2%
	Backward Linkage Based Participation Index			
	Agriculture	Mining	Electrical Equipment	Transport Equipment
BRA	2.4%	2.1%	8.1%	8.0%
CHN	1.7%	4.0%	21.3%	8.0%
DEU	7.9%	5.1%	24.7%	28.1%
FIN	4.4%	7.5%	28.6%	21.9%
IDN	1.4%	0.7%	13.0%	6.4%
IND	0.7%	1.2%	10.1%	7.7%
RUS	2.5%	**1.7%**	4.5%	11.3%
USA	4.1%	2.3%	6.7%	14.4%

BRA=Brazil; CHN=China; DEU=Germany; FIN=Finland; IDN=Indonesia; IND=India; RUS=Russia; USA=United States

Regarding the two typical manufacturing industries, "electrical and optical equipment" and "transportation equipment," Germany is the global manufacturing power, so its forward and backward linkage based participation ratios are both higher than that of other countries. With a high forward linkage based participation ratio, a high proportion of value-added generated by Germany has flowed to the network of Global Value Chains. With a high backward linkage based participation ratio, a high proportion of components and parts in the final products produced by Germany are produced by other countries in GVCs.

3.3.2 Why do we need the new "GVC Participation Index"?

(1). To eliminate the sectoral level bias in traditional indexes

As mentioned previously, using gross exports as the denominator may lead to overestimation bias at the bilateral/sectoral level.

For comparison, we use both gross exports and sector GDP as the denominator respectively, to estimate the forward linkage participation index. As shown in Table 7, the overall level of the index value is higher when using gross exports as the denominator. Moreover, the participation ratios for seven sectors (marked with gray background color) are substantially larger than 100%. These six sectors have one thing in common: A great proportion of their value added is exported indirectly, which is embodied in other sectors' exports.

The overestimation problem is more pronounced for energy and service sectors, as a large proportion of their value added is exported indirectly. We choose three typical sectors to illustrate this point. Two of them belong to the energy and service sectors ("Retail Trade", "Electricity, Gas and Water"), while the third one, "Leather and Footwear," is a typical "direct" exporting sector. As we have expected, the overestimation problem is more serious in the energy and service sectors (Table 8).

Table 7 Forward Linkage Participation Index for US sectors, 2011
Comparison between Traditional and New Measures

	Denominator: Exports	Denominator: GDP
Agriculture	10.92%	3.36%
Mining	47.87%	5.46%
Food	2.96%	0.90%
Textiles Products	12.54%	7.64%
Leather and Footwear	4.05%	2.28%
Wood Products	15.90%	3.86%
Paper and Printing	16.98%	4.36%
Refined Petroleum	9.19%	5.19%
Chemical Products	16.06%	10.26%
Rubber and Plastics	18.90%	7.55%
Other Non-Metal	14.41%	3.84%
Basic Metals	23.54%	11.77%
Machinery	9.04%	7.95%
Electrical Equipment	20.74%	12.87%
Transport Equipment	5.08%	7.16%
Recycling	10.32%	5.58%
Electricity, Gas and Water	553.49%	1.61%
Construction	2318.11%	0.37%
Sale of Vehicles and Fuel	743.56%	0.40%
Wholesale Trade	27.46%	4.54%
Retail Trade	2874.46%	0.26%
Hotels and Restaurants	276.53%	0.62%
Inland Transport	24.86%	5.14%
Water Transport	12.88%	6.61%
Air Transport	9.78%	5.48%
Other Transport	51.27%	7.84%
Post and Telecommunications	53.00%	2.62%
Financial Intermediation	29.14%	3.32%
Real Estate	662.26%	0.41%
Business Activities	50.65%	3.72%
Public Admin	27.71%	0.37%
Education	18.84%	0.15%
Health and Social Work	10.70%	0.01%
Other Services	34.68%	1.50%
Private Households	1111.34%	0.40%

Table 8 Comparison between Traditional and New Participation Indexes for Three Typical Sectors

	Electricity, Gas and Water		Retail Trade		Leather and Footwear	
Denominator:	**Exports**	**GDP**	**Exports**	**GDP**	**Exports**	**GDP**
AUS	693.0%	3.6%	62.6%	2.7%	9.5%	5.7%
BRA	112.8%	2.9%	217.2%	2.0%	14.2%	8.3%
CAN	51.5%	5.9%	115.5%	3.6%	2.8%	4.8%
CHN	625.9%	5.5%	27.4%	3.8%	2.6%	3.3%
DEU	50.5%	8.9%	769.2%	6.3%	5.3%	13.5%
ESP	188.5%	5.5%	241.0%	3.6%	3.0%	6.4%
FRA	67.4%	5.2%	2×10^{7}%	4.3%	1.9%	4.2%
GBR	276.0%	4.0%	337.9%	3.6%	5.8%	10.9%
IND	9944.8%	3.0%	893.5%	2.0%	6.8%	5.1%
ITA	300.7%	4.8%	38.4%	4.4%	4.1%	7.6%
JPN	619.9%	3.1%	58.2%	1.0%	20.9%	3.0%
KOR	1729.8%	8.0%	56.8%	3.2%	13.6%	10.5%
MEX	341.7%	2.9%	39.0%	5.1%	9.2%	4.4%
RUS	264.6%	11.8%	35.0%	4.6%	39.5%	5.4%
TWN	8751.4%	15.2%	18700.1%	13.6%	4.6%	17.3%
USA	553.5%	1.6%	2874.5%	0.3%	4.0%	2.3%

AUS=Australia; BRA=Brazil; CAN=Canada; CHN=China; DEU=Germany; ESP=Spain; FRA=France; GBR=United Kingdom; IND=India; ITA=Italy; JPN=Japan; KOR=Korea; MEX=Mexico; RUS=Russia; TWN=Taiwan; USA=United States

(2). To differentiate between deep and shallow cross country production sharing activities

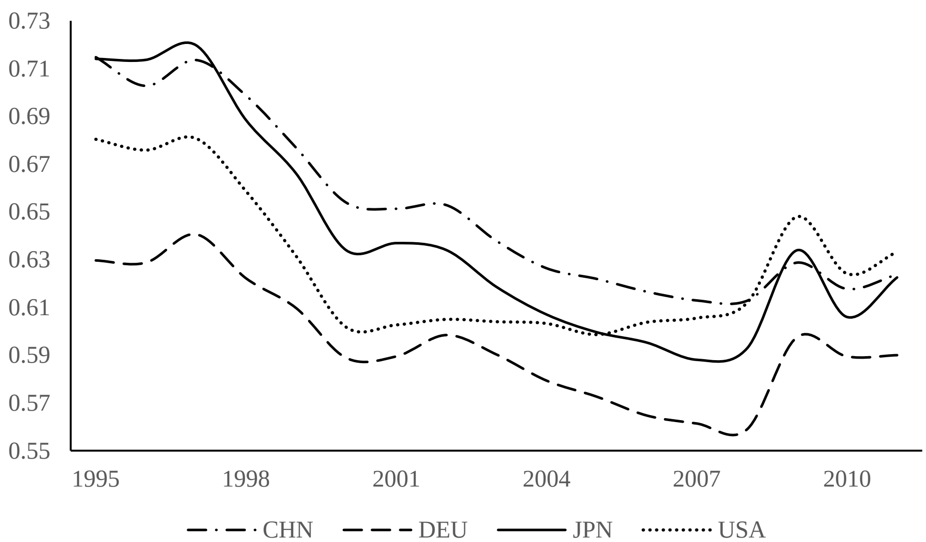
As shown in Equation (19), the value added in gross exports of a certain country can be decomposed into 5 parts from the perspective of forward industrial linkages:

Crossing the national border only once – direct value-added trade, representing the type of cross border specialization that is relatively shallow: (a). Final goods trade (textile from England exchanged for wine made in France); (b). Traditional intermediates trade (absorbed by direct importer without further cross-border production activities; raw material supplies such as coffee beans and crude oil).

Two or more border crossings – GVC related trade, representing the type of cross border specialization that is deeper: (a) Value-added absorbed by direct importers with additional cross-border production activities; (b) Domestic value-added re-imported and absorbed domestically; (c) Value-added re-exported by importing country but absorbed by a third country with additional cross border production activities.

The shallow part of cross country specialization is not included in the numerator of the GVC participation ratio and as shown in Figure 11, the relative importance of "Domestic value added in traditional intermediates exports" is diminishing over time for all sample countries (although the trend was interrupted temporarily by the Global Financial Crisis). Instead, the domestic value added exported via GVC related production activities is increasing dramatically.

Figure 11 DVA in "Traditional Intermediates Exports" as a share of DVA in all Intermediates Exports

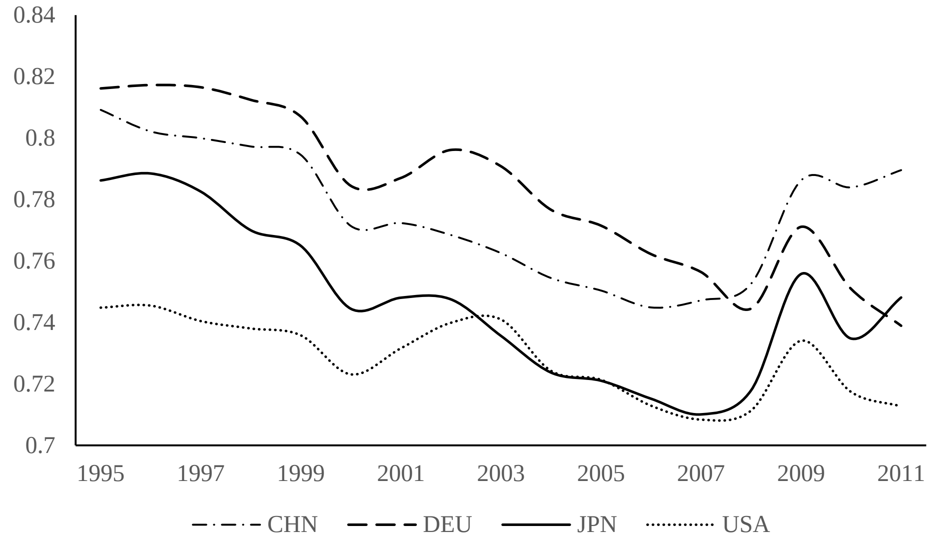


CHN=China; DEU=Germany; JPN=Japan; USA=United States

Similarly, from the perspective of backward linkages, the foreign value added embodied in the final goods produced in a certain country can also be divided into two components: One is created by deep cross border production sharing activities (two or more border crossings along the value chain), and the other is created by shallow cross border specialization (only one time border crossing).

Similar to the forward linkage based participation index, foreign value added embodied in "direct value-added trade" is also excluded, as there is no multinational production activity involved in traditional intermediate goods trade, and the relative importance of "Foreign value added in traditional intermediates imports" is declining over time as shown below in Figure 12.

Figure 12 FVA in "Traditional Intermediates Imports" as a share of FVA in all Intermediate Imports



CHN=China; DEU=Germany; JPN=Japan; USA=United States

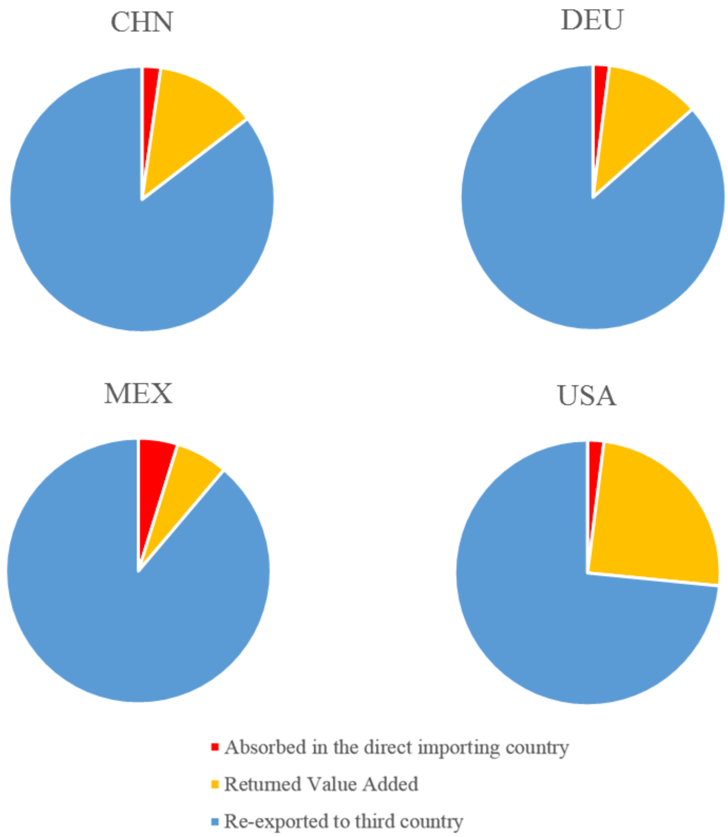
(3) To provide more detailed data for GVC related empirical analysis

As mentioned previously, the domestic value added embodied in GVC related exports (with two or more border crossings before reaching final demand) can be further decomposed into three parts: (A) absorbed by direct importer; (B) re-imported and absorbed domestically; (C) absorbed by a third country.

The pie chart in Figure 13 illustrates that Part C accounts for the largest proportion in all four countries selected. Domestic value added embodied in this part is re-exported by direct importers, and finally absorbed in a third country.

More importantly, the relative sizes of parts A, B, and C may reflect the differences of roles in the GVCs for different countries. For example, part B, i.e., "re-imported and absorbed domestically," accounts for a large proportion in the US, as the US is controlling both ends (design and sales) of the value chain. In contrast, Part B is relatively smaller for Mexico, which is more specialized in processing and assembly activities.

Figure 13 Decomposition of Domestic Value Added in GVC Related Exports, 2011



CHN=China; DEU=Germany; MEX=Mexico; USA=United States

Table 9 lists the forward/backward decomposition results for 16 countries. Part C accounts for the largest proportion in the forward linkage decomposition. This result is very robust for all sample countries.

Table 9 Decomposition of the Value Added in Deep Cross Country Production Sharing Activities

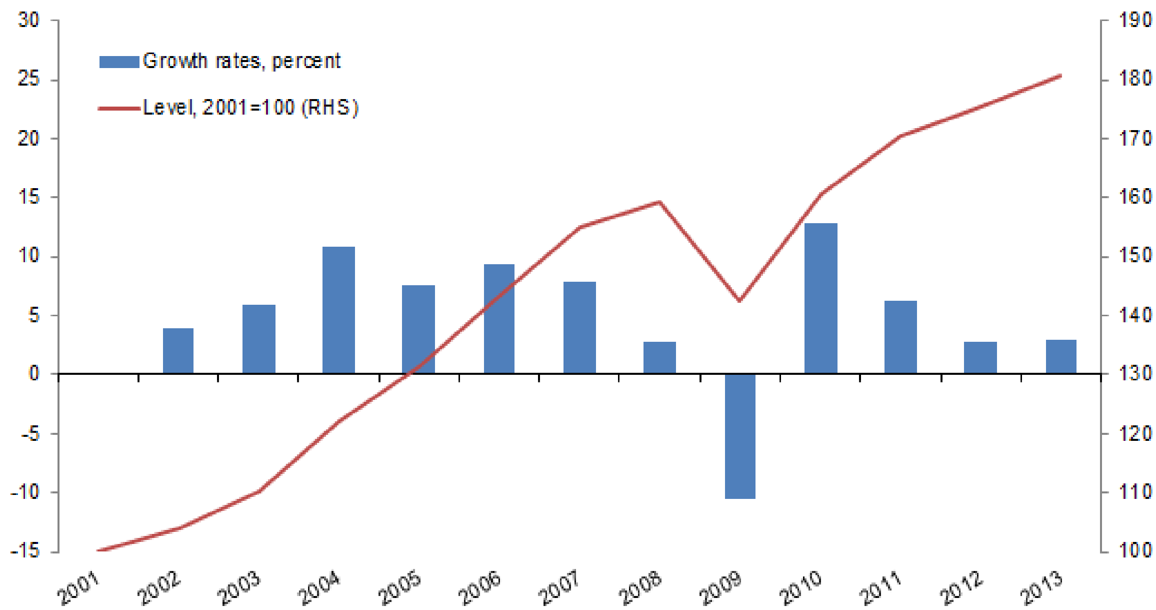
Country	Part A	Part B	Part C
AUS	2.51%	2.64%	94.85%
CAN	4.21%	6.77%	89.02%
CHN	2.27%	12.32%	85.41%
DEU	1.98%	11.46%	86.56%
FIN	2.25%	1.14%	96.61%
FRA	2.24%	6.93%	90.83%
GBR	1.74%	5.74%	92.52%
IDN	2.18%	2.11%	95.71%
IND	2.44%	2.34%	95.22%
ITA	2.34%	4.77%	92.88%
JPN	2.57%	5.91%	91.52%
KOR	2.65%	2.22%	95.13%
MEX	4.81%	6.36%	88.82%
RUS	2.77%	2.25%	94.98%
TWN	2.99%	1.00%	96.01%
USA	2.00%	24.56%	73.44%

AUS=Australia; CAN=Canada; CHN=China; DEU=Germany; FIN=Finland; GBR=United Kingdom; IDN=Indonesia; IND=India; ITA=Italy; JPN=Japan; KOR=Korea; MEX=Mexico; RUS=Russia; TWN=Taiwan; USA=United States

3.4 Index application: GVC length, participation intensity, production line positions and the economic shocks of the recent global financial crisis

In the aftermath of the Global Financial Crisis, as shown in Figure 14, world trade grew by 6.2% in 2011, 2.8% in 2012, and 3.0% in 2013. This growth in trade volumes is substantially lower than the pre-crisis average of 7.1% (1987–2007), and is slightly below the growth rate of world GDP in real terms.

Figure 14 The Growth of World Trade before and after the Financial Crisis



As we analyzed before, value-added created by a country can be decomposed into three parts: pure domestic production and consumption, flow-out through direct value-added trade, and flow-out through GVC trade. Then, in financial crisis, are there differences in the degree of effects on the three types of value added?

Figure 15 The Effects of Financial Crisis to Different Value Added Creating Activities

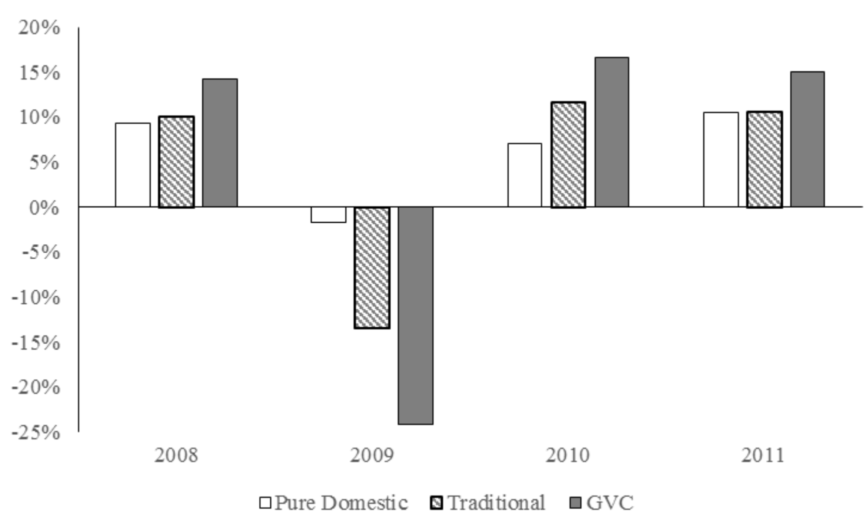


Figure 15 shows the result at the global level: During the financial crisis in 2009, pure domestic production activities were least affected (in comparison with 2008, the fall was only 1.7%), while GVC production and trade activities were mostly affected, as the fall reached 24.2%. However, it is also observed that GVC production and trade activities had the fastest after-crisis-recovery.

Table 10 The Effects of Financial Crisis to Different Value Added Creating Activities (Sectoral Level, 2009)

Sector	China			USA		
	Domestic	Direct	GVC	Domestic	Direct	GVC
Agriculture	8.6%	-4.9%	-15.5%	-14.9%	-29.4%	-36.8%
Mining	16.5%	-16.2%	-33.7%	-26.8%	-28.0%	-47.9%
Food	7.6%	-5.7%	-17.9%	14.8%	5.0%	-12.9%
Textiles Products	21.3%	-6.1%	-12.7%	-22.2%	-12.8%	-25.0%
Leather and Footwear	16.8%	-6.7%	-10.5%	-22.0%	10.4%	-15.4%
Wood Products	14.3%	-17.0%	-27.3%	-17.3%	-23.7%	-36.1%
Paper and Printing	12.7%	-10.7%	-21.8%	-1.7%	-7.3%	-20.1%
Refined Petroleum	15.2%	-18.1%	-26.8%	-24.1%	-28.7%	-47.4%
Chemical Products	16.5%	-10.5%	-25.7%	10.3%	8.4%	-8.6%
Rubber and Plastics	18.5%	-8.4%	-20.2%	-3.1%	-4.8%	-16.0%
Other Non-Metal	9.9%	-19.5%	-33.5%	-2.5%	-2.4%	-20.4%
Basic Metals	20.5%	-17.8%	-40.4%	-16.9%	-15.0%	-33.0%
Machinery	18.4%	-20.4%	-33.7%	-11.3%	-5.8%	-16.4%
Electrical Equipment	25.1%	-7.8%	-17.6%	1.1%	4.9%	-11.8%
Transport Equipment	13.1%	-15.4%	-28.9%	-1.6%	-7.2%	-31.7%

Divided among different countries and sectors, the above phenomenon still holds. Table 10 shows that: pure domestic production is least affected by the financial crisis (China even continued a positive growth). For most sectors, GVC production and trade activities were most affected.

The second issue is this: Are the GVC positions related to the degree of effects of the financial crisis? To test this, we estimate the following regression model:

$$\Delta\, GVCP_{ic} = \beta_0 + \beta_1 \times Position_{ic} + \beta_2 \times PL\_GVC_{ic} + \beta_3 \times PL\_GVC\_f_{ic} + \beta_4 \times W_{ic} + \beta_4 \times Z_c + \gamma_i + u_i$$

where

$\Delta\, GVCP_{ic}$ equals to $GVCP_{ic}$(2009) minus $GVCP_{ic}$(2008), which quantifies the degree of effects on this industry according to the variance of the forward linkage based GVC participation ratio during the financial crisis;

$Position_{ic}$ is the GVC Position Index calculated in this paper. When the value is high, it means that this sector is relatively further from the final consumption end;

PL_GVC_{ic} is the forward linkage based GVC production length, and $PL\_GVC\_f_{ic}$ represents the length of the "International Portion" (GVC Production Length=International Portion + Domestic Portion);

W_{ic} represents the country-sector level control variables, including the logarithm of real capital stock per capita, and hours worked by high-skilled workers (share in total hours);

Z_c represents the country level control variables, including a dummy variable to indicate whether this is an Asian country (=1) and the logarithm of GDP per capita;

We also control for the sector fixed effects by including a sector dummy γ_i in the model.

The benchmark regression results are shown in Table 11. Regression (1) indicates that GVC positions have significant impacts on the degree of effects of the global financial crisis. The further is the position from the final consumption end, the less affected the node would be by the financial crisis. Other than this, capital intensive and high-technology intensive sectors are less affected， and the impact of national economic development (GDP per Capita) is not significant.

Furthermore, Regressions (2)–(4) investigate respectively three possible effects: the forward linkage based GVC production length, the International Portion of the Production Length, and whether there is anything special about Asian countries. The regression results indicate that, given the forward linkage GVC production length, the effects of financial crisis tend to be more severe for countries with a longer international portion of the chain. The average effects of the financial crisis are also significantly higher on Asian countries than on Europe and America.

Table 11 Benchmark Regression Results

	(1)	(2)	(3)	(4)
Position Index	39.97**	47.50***	52.83***	47.11***
	(17.40)	(17.66)	(17.05)	(16.78)
PL_GVC		1.43	3.78***	5.97***
		(1.20)	(1.26)	(1.34)
PL_GVC_f			-18.21***	-19.74***
			(4.41)	(4.46)
Asia				-7.03***
				(1.48)
ln(K/L)	1.20**	1.20**	1.23**	1.68***
	(0.55)	(0.55)	(0.54)	(0.55)
High Skill	12.96***	12.88***	12.45***	14.73***
	(4.39)	(4.42)	(4.27)	(4.26)
ln(GDP per Capita)	-1.37	-1.24	-0.54	-2.10**
	(0.89)	(0.88)	(0.85)	(0.94)
Constant	-9.10	-20.78	5.94	19.37
	(11.64)	(13.88)	(15.04)	(15.08)
Sector Fixed Effects	YES	YES	YES	YES
Observations	742	741	741	741
R-squared	0.23	0.23	0.25	0.27

Note: Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

4. Conclusions

In this paper, we have developed a GVC index system that includes three types of indexes based on both forward and backward inter-industry and cross-country linkages: a production length index for the average number of production stages and complexity of the global value chain; a participation index for the intensity of a country-sector's engagement in global value chains; and a position index for the location of a country sector on a global value chain, or the relative distance of a particular production stage to both ends of a global value chain. While the existing literature has proposed similar measures, our indices contain improvements that we argue are desirable and sensible from the viewpoint of economic intuition.

We thus can provide a comprehensive picture of each country/sector pair's GVC activities from multiple dimensions. All these indexes are built at the decomposition of GDP by industry statistics and can be further divided into different components with clear economic interpretations. By estimating these indexes according to real world data, we produce a large set of indicators.

We hope these indexes could be widely used by both theoretical and empirical economists in advancing studies of global supply chains and become a bridge between economic theories of supply chains and GVC measures based on GDP and gross trade accounting.

These new measures can potentially be linked to productivity growth or changing patterns of comparative advantage as well. We leave such investigation for future research.

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Appendix

Appendix A. Mathematical Proof of Equations in Section 2

Appendix A.1 the detailed mathematical proof of equation (7)

As shown in equation 5 of main text, the average length of value added from sector i embodied in final goods of sector j can be computed as:

$$vyl_{ij} = (b_{ij})^{-1}\sum_{k}^{n} b_{ik}b_{kj} \tag{A1}$$

Denote $VYL=\{vyl_{ij}\}_{n\times n}$ as the matrix of production length from value added to final goods, then equation A1 can be expressed in matrix notation as

$$VYL = \begin{bmatrix} (b_{11})^{-1}\sum_{k}^{n} b_{1k}b_{k1} & (b_{12})^{-1}\sum_{k}^{n} b_{1k}b_{k2} & \cdots & (b_{1n})^{-1}\sum_{k}^{n} b_{1k}b_{kn} \\ (b_{21})^{-1}\sum_{k}^{n} b_{2k}b_{k1} & (b_{22})^{-1}\sum_{k}^{n} b_{2k}b_{k2} & \cdots & (b_{2n})^{-1}\sum_{k}^{n} b_{2k}b_{kn} \\ \vdots & \vdots & \ddots & \vdots \\ (b_{n1})^{-1}\sum_{k}^{n} b_{nk}b_{k1} & (b_{n2})^{-1}\sum_{k}^{n} b_{nk}b_{k2} & \cdots & (b_{nn})^{-1}\sum_{k}^{n} b_{nk}b_{kn} \end{bmatrix} \tag{A2}$$

Appendix B. the detailed mathematical proof of Upstreamness

As defined in Fally (2012a, 2012b, 2013) and Antras et al (2012, 2013), the Upstreamness of an industry's output in the value chain can be measured as

$$\begin{aligned} U_i &= \frac{y_i}{x_i} + 2*\frac{\sum_j^n a_{ij}y_j}{x_i} + 3*\frac{\sum_j^n a_{ij}\sum_k^n a_{jk}y_k}{x_i} + \dots \\ &= \frac{y_i + 2*\sum_j^n a_{ij}y_j + 3*\sum_j^n a_{ij}\sum_k^n a_{jk}y_k + \dots}{x_i} \end{aligned} \tag{B1}$$

The numerator of equation B1 can be expressed in matrix notation as

$$\begin{aligned} &Y + 2AY + 3AAY + \dots \\ &= (Y + AY + AAY + \dots) + A(Y + AY + AY + \dots) + AA(Y + AY + AAY + \dots) + \dots \\ &= BY + ABY + AABY + \dots = BBY = BX \end{aligned}$$

Therefore, Upstreamness of an industry's output can be measured as

$$U_i = \frac{\sum_j^n b_{ij} x_j}{x_i} = x_i^{-1} \sum_k^n b_{ik} x_k \tag{B2}$$

The right side of equation B2 is the same to equation (7) of main text.

Appendix C. the detailed mathematical proof of equation 18

Based on general ICIO model shown in table 2 of main text, classical Leontief inverse equation can be expressed as

$$X = \begin{bmatrix} B^{11} & B^{12} & \cdots & B^{1m} \\ B^{21} & B^{22} & \cdots & B^{2m} \\ \vdots & \vdots & \ddots & \vdots \\ B^{m1} & B^{m2} & \cdots & B^{mm} \end{bmatrix} \begin{bmatrix} Y^{11} + Y^{12} + ... + Y^{1m} \\ Y^{21} + Y^{22} + ... + Y^{2m} \\ ... \\ Y^{m1} + Y^{m2} + ... + Y^{mm} \end{bmatrix} = \begin{bmatrix} \sum_u^M B^{1u} \sum_t^M Y^{ut} \\ \sum_u^M B^{2u} \sum_t^M Y^{ut} \\ ... \\ \sum_u^M B^{mu} \sum_t^M Y^{ut} \end{bmatrix} \tag{C1}$$

Therefore, the gross exports of country s can be expressed as

$$E^{s*} = \sum_{r \neq s}^M Y^{sr} + \sum_{r \neq s}^M A^{sr} X^r = \sum_{r \neq s}^M Y^{sr} + \sum_{r \neq s}^M A^{sr} \sum_u^M B^{ru} \sum_t^M Y^{ut} \tag{C2}$$

Inserting equation (C2) into the second term of equation (17) in main text.

$$\begin{aligned} (I - A^{ss})^{-1} E^{s*} &= L^{ss} (\sum_{r \neq s}^M Y^{sr} + \sum_{r \neq s}^M A^{sr} X^r) = L^{ss} (\sum_{r \neq s}^M Y^{sr} + \sum_{r \neq s}^M A^{sr} \sum_u^M B^{ru} \sum_t^M Y^{ut}) \\ &= L^{ss} \sum_{r \neq s}^M Y^{sr} + L^{ss} \sum_{r \neq s}^M A^{sr} \sum_u^M B^{ru} Y^{ur} + L^{ss} \sum_{r \neq s}^M A^{sr} \sum_u^M B^{ru} Y^{us} + L^{ss} \sum_{r \neq s}^M A^{sr} \sum_u^M B^{ru} \sum_{t \neq s,r}^M Y^{ut} \end{aligned} \tag{C3}$$

Where $L^{ss} = (I - A^{ss})^{-1}$ is the domestic Leontief inverse of country s.

Appendix D. the detailed mathematical proof of equation 21

Multiplying domestic value-added generated from each production stage of section 2.2.2 with production length of that stage and summing all production stages in an infinite stage production process, we can obtain the product of value-added and domestic production length as

$$
\begin{aligned}
& X\_vd^{ss} = \hat{V}^s Y^{ss} + 2\hat{V}^s A^{ss} Y^{ss} + 3\hat{V}^s A^{ss} A^{ss} Y^{ss} + \dots \\
& = (\hat{V}^s Y^{ss} + \hat{V}^s A^{ss} Y^{ss} + \hat{V}^s A^{ss} A^{ss} Y^{ss} + \dots) \\
& + (\hat{V}^s A^{ss} Y^{ss} + \hat{V}^s A^{ss} A^{ss} Y^{ss} + \hat{V}^s A^{ss} A^{ss} A^{ss} Y^{ss} + \dots) \\
& + (\hat{V}^s A^{ss} A^{ss} Y^{ss} + \hat{V}^s A^{ss} A^{ss} A^{ss} Y^{ss} + \hat{V}^s A^{ss} A^{ss} A^{ss} A^{ss} Y^{ss} + \dots) + \dots \\
& = \hat{V}^s (I - A^{ss})^{-1} Y^{ss} + \hat{V}^s A^{ss} (I - A^{ss})^{-1} Y^{ss} + \hat{V}^s A^{ss} A^{ss} (I - A^{ss})^{-1} Y^{ss} + \dots \\
& = \hat{V}^s (I - A^{ss})^{-1} (I - A^{ss})^{-1} Y^{ss} = \hat{V}^s L^{ss} L^{ss} Y^{ss}
\end{aligned} \tag{D1}
$$

Where $I + A^{ss} + A^{ss}A^{ss} + \dots = (I - A^{ss})^{-1} = L^{ss}$

Appendix E. the detailed mathematical proof of equation 22

Based on the definition of Leontief inverse, we can get

$$B^{rr} = I + A^{rr} + \sum_u^M A^{ru} A^{ur} + \dots$$

$$B^{ru} = 0 + A^{ru} + \sum_v^M A^{rv} A^{vu} + \dots \qquad (u \neq r)$$

Summing above equation by country r, we have

$$\sum_u^M B^{ru} = I + \sum_u^M A^{ru} + \sum_v^M A^{rv} \sum_u^M B^{vu} + \dots \tag{E1}$$

Summing all production stages of section 2.2.3 in an infinite stage production process, we have

$$
\begin{aligned}
& DVA\_F^{sr} = \hat{V}^s Y^{sr} + (\hat{V}^s A^{ss} Y^{sr} + \hat{V}^s A^{sr} \sum_t^M Y^{rt}) \\
& + (\hat{V}^s A^{ss} A^{ss} Y^{sr} + \hat{V}^s A^{ss} A^{sr} \sum_t^M Y^{rt} + \hat{V}^s A^{sr} \sum_u^M A^{ru} \sum_t^M Y^{ut}) + \dots \\
& = (\hat{V}^s Y^{sr} + \hat{V}^s A^{ss} Y^{sr} + \hat{V}^s A^{ss} A^{ss} Y^{sr} + \dots) \\
& + (\hat{V}^s A^{sr} \sum_t^M Y^{rt} + \hat{V}^s A^{ss} A^{sr} \sum_t^M Y^{rt} + \hat{V}^s A^{ss} A^{ss} A^{sr} \sum_t^M Y^{rt} + \dots) \\
& + (\hat{V}^s A^{sr} \sum_u^M A^{ru} \sum_t^M Y^{ut} + \hat{V}^s A^{ss} A^{sr} \sum_u^M A^{ru} \sum_t^M Y^{ut} + \dots) + \dots \\
& = \hat{V}^s L^{ss} Y^{sr} + (\hat{V}^s L^{ss} A^{sr} \sum_t^M Y^{rt} + \hat{V}^s L^{ss} A^{sr} \sum_u^M A^{ru} \sum_t^M Y^{ut} + \dots)
\end{aligned} \tag{E2}
$$

Inserting equation E1 into equation E2, we can get

$$DVA\_F^{sr} = \hat{V}^s L^{ss} Y^{sr} + (\hat{V}^s L^{ss} A^{sr} \sum_t^M Y^{rt} + \hat{V}^s L^{ss} A^{sr} \sum_u^M A^{ru} \sum_t^M Y^{ut} + ...)$$
$$= \hat{V}^s L^{ss} Y^{sr} + \hat{V}^s L^{ss} A^{sr} \sum_u^M B^{ru} \sum_t^M Y^{ut} = \hat{V}^s L^{ss} Y^{sr} + \hat{V}^s L^{ss} A^{sr} X^{r} = \hat{V}^s L^{ss} E^{sr}$$

(E3)

Appendix F. the detailed mathematical proof of equation 27

Using the domestic production length of each production stage in section 2.2.3 as weights, summing all production stages, we can obtain a given the domestic production length of a particular global value chain (primary inputs in sector *i* of country s to exports products of sector *j* to country *r*):

$$X\_GVC\_vd^{sr} = GPLd^{sr} \# DVA\_F^{sr}$$
$$= 0 + \hat{V}^s A^{sr} \sum_t^M Y^{rt} + 2\hat{V}^s A^{ss} A^{sr} \sum_t^M Y^{rt} + \hat{V}^s A^{sr} \sum_u^M A^{ru} \sum_t^M Y^{ut} + ...$$
$$= (\hat{V}^s A^{sr} \sum_t^M Y^{rt} + 2\hat{V}^s A^{ss} A^{sr} \sum_t^M Y^{rt} + 3\hat{V}^s A^{ss} A^{ss} A^{sr} \sum_t^M Y^{rt} + ...)$$
$$+ (\hat{V}^s A^{sr} \sum_u^M A^{ru} \sum_t^M Y^{ut} + 2\hat{V}^s A^{ss} A^{sr} \sum_u^M A^{ru} \sum_t^M Y^{ut} + ...) + ...$$
$$- (\hat{V}^s A^{sr} Y^{rr} + 2\hat{V}^s A^{ss} A^{sr} Y^{rr} + 3\hat{V}^s A^{ss} A^{ss} A^{sr} Y^{rr} + ...)$$
$$- (\hat{V}^s A^{sr} A^{rr} Y^{rr} + 2\hat{V}^s A^{ss} A^{sr} A^{rr} Y^{rr} + 3\hat{V}^s A^{ss} A^{ss} A^{sr} A^{rr} Y^{rr} + ...) - ...$$
$$= (\hat{V}^s L^{ss} L^{ss} A^{sr} \sum_t^M Y^{rt} + \hat{V}^s L^{ss} L^{ss} A^{sr} \sum_u^M A^{ru} \sum_t^M Y^{ut} + ...) - \hat{V}^s L^{ss} L^{ss} A^{sr} L^{rr} Y^{rr}$$
$$= \hat{V}^s L^{ss} L^{ss} A^{sr} \sum_u^M B^{ru} \sum_t^M Y^{ut} - \hat{V}^s L^{ss} L^{ss} A^{sr} L^{rr} Y^{rr}$$
$$= \underbrace{\hat{V}^s L^{ss} L^{ss} A^{sr} (\sum_u^M B^{ru} Y^{ur} - L^{rr} Y^{rr})}_{(3a) X\_GVC\_r\_vd^{sr}} + \underbrace{\hat{V}^s L^{ss} L^{ss} A^{sr} \sum_{u \neq r}^M B^{ru} Y^{us}}_{(3b) X\_GVC\_s\_vd^{sr}}$$
$$+ \underbrace{\hat{V}^s L^{ss} L^{ss} A^{sr} \sum_u^M B^{ru} \sum_{t \neq s,r}^M Y^{ut}}_{(3c) X\_GVC\_t\_vd^{sr}} \qquad \text{(F1)}$$

Where $\sum_u^M B^{ru} = I + \sum_u^M A^{ru} + \sum_t^M A^{rt} \sum_u^M A^{tu} + ...$

Appendix G. the detailed mathematical proof of equation 29

Using the international production length of each production stage in section 2.2.3 as weights, summing across all production stages, we can obtain a given the international production length of a particular global value chain (import products of sector *j* in country *r* to final products):

$$
\begin{aligned}
& GPLf^{sr} \# DVA\_F^{sr} = 0*\hat{V}^s Y^{sr} + (0*\hat{V}^s A^{ss} Y^{sr} + \hat{V}^s A^{sr} \sum_t^M Y^{rt}) \\
& + (0*\hat{V}^s A^{ss} A^{ss} Y^{sr} + \hat{V}^s A^{ss} A^{sr} \sum_t^M Y^{rt} + 2\hat{V}^s A^{sr} \sum_u^M A^{ru} \sum_t^M Y^{ut}) + \dots \\
& - (\hat{V}^s A^{sr} Y^{rr} + \hat{V}^s A^{ss} A^{sr} Y^{rr} + \hat{V}^s A^{ss} A^{ss} A^{sr} Y^{rr} + \dots) \\
& - 2(\hat{V}^s A^{sr} A^{rr} Y^{rr} + \hat{V}^s A^{ss} A^{sr} A^{rr} Y^{rr} + \hat{V}^s A^{ss} A^{ss} A^{sr} A^{rr} Y^{rr} + \dots) \\
& - \dots \\
& = (\hat{V}^s A^{sr} \sum_t^M Y^{rt} + \hat{V}^s A^{ss} A^{sr} \sum_t^M Y^{rt} + \hat{V}^s A^{ss} A^{ss} A^{sr} \sum_t^M Y^{rt} + \dots) \\
& + 2(\hat{V}^s A^{sr} \sum_u^M A^{ru} \sum_t^M Y^{ut} + \hat{V}^s A^{ss} A^{sr} \sum_u^M A^{ru} \sum_t^M Y^{ut} + \dots) + \dots \\
& - (\hat{V}^s L^{ss} A^{sr} Y^{rr} + 2\hat{V}^s L^{ss} A^{sr} A^{rr} Y^{rr} + 3\hat{V}^s L^{ss} A^{sr} A^{rr} A^{rr} Y^{rr} + \dots) \\
& = \hat{V}^s L^{ss} A^{sr} \sum_t^M Y^{rt} + 2\hat{V}^s L^{ss} A^{sr} \sum_u^M A^{ru} \sum_t^M Y^{ut} + 3\hat{V}^s L^{ss} A^{sr} \sum_v^M A^{rv} \sum_u^M A^{vu} \sum_t^M Y^{ut} + \dots \\
& - \hat{V}^s L^{ss} A^{sr} L^{rr} L^{rr} Y^{rr} \\
& = \hat{V}^s L^{ss} A^{sr} \sum_v^M B^{rv} \sum_u^M B^{vu} \sum_t^M Y^{ut} - \hat{V}^s L^{ss} A^{sr} L^{rr} L^{rr} Y^{rr} \\
& = \underbrace{\hat{V}^s L^{ss} A^{sr} (\sum_v^M B^{rv} \sum_u^M B^{vu} Y^{ur} - L^{rr} L^{rr} Y^{rr})}_{(3a)\, X\_GVC\_r\_vf^{sr}} + \underbrace{\hat{V}^s L^{ss} A^{sr} \sum_v^M B^{rv} \sum_u^M B^{vu} Y^{us}}_{(3b)\, X\_GVC\_s\_vf^{sr}} \\
& + \underbrace{\hat{V}^s L^{ss} A^{sr} \sum_v^M B^{rv} \sum_u^M B^{vu} \sum_{t \neq s,r}^M Y^{ut}}_{(3c)\, X\_GVC\_t\_vf^{sr}}
\end{aligned}
\tag{G1}
$$

Where $\sum_v^M B^{rv} \sum_u^M B^{vu} = I + 2\sum_u^M A^{ru} + 3\sum_v^M A^{rv} \sum_u^M A^{vu} + \dots$, can be proofed from the expressing of $BB = I + 2A + 3AA + \dots$.

Appendix H. the detailed mathematical proof of equation 32

The equation 32 in main text can be rearranged as

$$\begin{aligned}
&\sum_{r\neq s}^{M} X_GVC_v^{sr} = \sum_{r\neq s}^{M} GPL^{sr} \# DVA_F^{sr} = \hat{V}^{s} L^{ss} L^{ss} \sum_{r\neq s}^{M} Y^{sr} \\
&+\left[\hat{V}^{s} L^{ss} L^{ss} A^{sr} \sum_{u}^{M} B^{ru} Y^{us} + \hat{V}^{s} L^{ss} A^{sr} \sum_{v}^{M} B^{rv} \sum_{u}^{M} B^{vu} Y^{us}\right] \\
&+\left[\hat{V}^{s} L^{ss} L^{ss} A^{sr} \sum_{u}^{M} B^{ru} Y^{ur} + \hat{V}^{s} L^{ss} A^{sr} \sum_{v}^{M} B^{rv} \sum_{u}^{M} B^{vu} Y^{ur}\right] \\
&+\left[\hat{V}^{s} L^{ss} L^{ss} A^{sr} \sum_{u}^{M} B^{ru} \sum_{t\neq s,r}^{M} Y^{ut} + \hat{V}^{s} L^{ss} A^{sr} \sum_{v}^{M} B^{rv} \sum_{u}^{M} B^{vu} \sum_{t\neq s,r}^{M} Y^{ut}\right] \\
&= \hat{V}^{s} L^{ss} L^{ss} \sum_{r\neq s}^{M} Y^{sr} \\
&+ \hat{V}^{s}\left[L^{ss} L^{ss} \sum_{r\neq s}^{M} A^{sr} \sum_{u}^{M} B^{ru} \sum_{t}^{M} Y^{ut} + L^{ss} \sum_{r\neq s}^{M} A^{sr} \sum_{v}^{M} B^{rv} \sum_{u}^{M} B^{vu} \sum_{t}^{M} Y^{ut}\right]
\end{aligned} \tag{H1}$$

The terms in bracket of equation A14 can be further developed as

$$\begin{aligned}
&L^{ss} L^{ss} \sum_{r\neq s}^{M} A^{sr} \sum_{u}^{M} B^{ru} \sum_{t}^{M} Y^{ut} + L^{ss} \sum_{r\neq s}^{M} A^{sr} \sum_{v}^{M} B^{rv} \sum_{u}^{M} B^{vu} \sum_{t}^{M} Y^{ut} \\
&= L^{ss} L^{ss} \sum_{r\neq s}^{M} A^{sr} B^{rs} \sum_{t}^{M} Y^{st} + L^{ss} \sum_{r\neq s}^{M} A^{sr} B^{rs} \sum_{u}^{M} B^{su} \sum_{t}^{M} Y^{ut} \\
&+ L^{ss} L^{ss} \sum_{r\neq s}^{M} A^{sr} \sum_{u\neq s}^{M} B^{ru} \sum_{t}^{M} Y^{ut} + L^{ss} \sum_{r\neq s}^{M} A^{sr} \sum_{v\neq s}^{M} B^{rv} \sum_{u}^{M} B^{vu} \sum_{t}^{M} Y^{ut} \\
&= L^{ss} (B^{ss} - L^{ss}) \sum_{t}^{M} Y^{st} + (B^{ss} - L^{ss}) \sum_{u}^{M} B^{su} \sum_{t}^{M} Y^{ut} \\
&+ L^{ss} \sum_{u\neq s}^{M} B^{su} \sum_{t}^{M} Y^{ut} + \sum_{v\neq s}^{M} B^{sv} \sum_{u}^{M} B^{vu} \sum_{t}^{M} Y^{ut} \\
&= \left[L^{ss} B^{ss} \sum_{t}^{M} Y^{st} + L^{ss} \sum_{u\neq s}^{M} B^{su} \sum_{t}^{M} Y^{ut}\right] - L^{ss} \sum_{u}^{M} B^{su} \sum_{t}^{M} Y^{ut} \\
&+ \left[B^{ss} \sum_{u}^{M} B^{su} \sum_{t}^{M} Y^{ut} + \sum_{v\neq s}^{M} B^{sv} \sum_{u}^{M} B^{vu} \sum_{t}^{M} Y^{ut}\right] - L^{ss} L^{ss} \sum_{t}^{M} Y^{st} \\
&= \sum_{v}^{M} B^{sv} \sum_{u}^{M} B^{vu} \sum_{t}^{M} Y^{ut} - L^{ss} L^{ss} \sum_{t}^{M} Y^{st}
\end{aligned} \tag{H2}$$

Inserting equation H2 into H1， we have

$$\begin{aligned}
&\sum_{r\neq s}^{M} GPL^{sr}\#DVA_F^{sr} = \hat{V}^s L^{ss} L^{ss} \sum_{r\neq s}^{M} Y^{sr} \\
&+\hat{V}^s \left[L^{ss} L^{ss} \sum_{r\neq s}^{M} A^{sr} \sum_{u}^{M} B^{ru} \sum_{t}^{M} Y^{ut} + L^{ss} \sum_{r\neq s}^{M} A^{sr} \sum_{v}^{M} B^{rv} \sum_{u}^{M} B^{vu} \sum_{t}^{M} Y^{ut} \right] \\
&= \hat{V}^s L^{ss} L^{ss} \sum_{r\neq s}^{M} Y^{sr} + \hat{V}^s \sum_{v}^{M} B^{sv} \sum_{u}^{M} B^{vu} \sum_{t}^{M} Y^{ut} - \hat{V}^s L^{ss} L^{ss} \sum_{t}^{M} Y^{st} \\
&= \hat{V}^s \sum_{v}^{M} B^{sv} \sum_{u}^{M} B^{vu} \sum_{t}^{M} Y^{ut} - \hat{V}^s L^{ss} L^{ss} Y^{ss}
\end{aligned} \tag{H3}$$

Appendix I. the detailed mathematical proof of equation 34 and 35

The domestic gross outputs induced by the production of final product Y_i^s can be measured as:

$$\begin{aligned}
&X_yd_i^s = V^s L^{ss} L^{ss} \sum_{t}^{M} Y_i^{st} + \sum_{r}^{M} V^r L^{rr} L^{rr} \sum_{u\neq r}^{M} A^{ru} B^{us} \sum_{t}^{M} Y_i^{st} \\
&= V^s L^{ss} L^{ss} Y_i^{ss} + V^s L^{ss} L^{ss} \sum_{r\neq s}^{M} Y_i^{sr} + \sum_{r}^{M} V^r L^{rr} L^{rr} \sum_{u\neq r}^{M} A^{ru} B^{us} Y_i^{ss} \\
&+ \sum_{r}^{M} V^r L^{rr} L^{rr} \sum_{u\neq r}^{M} A^{ru} B^{us} Y_i^{sr} + \sum_{r}^{M} V^r L^{rr} L^{rr} \sum_{u\neq r}^{M} A^{ru} B^{us} \sum_{t\neq s,r}^{M} Y_i^{st} \\
&= \underbrace{V^s L^{ss} L^{ss} Y_i^{ss}}_{1-Xyd_D} + \underbrace{V^s L^{ss} L^{ss} \sum_{r\neq s}^{M} Y_i^{sr} + \sum_{r\neq s}^{M} V^r L^{rr} L^{rr} A^{rs} L^{ss} Y_i^{ss}}_{2-Xyd_RT} \\
&+ \underbrace{\sum_{r}^{M} V^r L^{rr} L^{rr} \sum_{u\neq r}^{M} A^{ru} B^{us} Y_i^{ss} - \sum_{r\neq s}^{M} V^r L^{rr} L^{rr} A^{rs} L^{ss} Y_i^{ss}}_{3a-Xyd_GVC_s} \\
&+ \underbrace{\sum_{r}^{M} V^r L^{rr} L^{rr} \sum_{u\neq r}^{M} A^{ru} B^{us} Y_i^{sr}}_{3b-Xyd_GVC_r} + \underbrace{\sum_{r}^{M} V^r L^{rr} L^{rr} \sum_{u\neq r}^{M} A^{ru} B^{us} \sum_{t\neq s,r}^{M} Y_i^{st}}_{3c-Xyd_GVC_t}
\end{aligned} \tag{I1}$$

The international gross outputs induced by the production of final product Y_i^s can be measured as:

$$
\begin{aligned}
X\_yf_i^{s} &= \sum_{r}^{M} V^{r} L^{rr} \sum_{u \neq r}^{M} A^{ru} \sum_{v}^{M} B^{uv} B^{vs} \sum_{t}^{M} Y_i^{st} = \sum_{r}^{M} V^{r} L^{rr} \sum_{u \neq s}^{M} A^{ru} \sum_{v}^{M} B^{uv} B^{vs} Y_i^{ss} \\
&+ \sum_{r}^{M} V^{r} L^{rr} \sum_{u \neq s}^{M} A^{ru} \sum_{v}^{M} B^{uv} B^{vs} Y_i^{sr} + \sum_{r}^{M} V^{r} L^{rr} \sum_{u \neq s}^{M} A^{ru} \sum_{v}^{M} B^{uv} B^{vs} \sum_{t \neq s,r}^{M} Y_i^{st} \\
&= \sum_{r \neq s}^{M} V^{r} L^{rr} A^{rs} L^{ss} L^{ss} Y_i^{ss} + \sum_{r}^{M} V^{r} L^{rr} \sum_{u \neq s}^{M} A^{ru} \sum_{v}^{M} B^{uv} B^{vs} Y_i^{ss} - \sum_{r \neq s}^{M} V^{r} L^{rr} A^{rs} L^{ss} L^{ss} Y_i^{ss} \\
&+ \sum_{r}^{M} V^{r} L^{rr} \sum_{u \neq s}^{M} A^{ru} \sum_{v}^{M} B^{uv} B^{vs} Y_i^{sr} + \sum_{r}^{M} V^{r} L^{rr} \sum_{u \neq s}^{M} A^{ru} \sum_{v}^{M} B^{uv} B^{vs} \sum_{t \neq s,r}^{M} Y_i^{st} \\
&= \underbrace{0}_{1-Xyf\_D} + \underbrace{0 + \sum_{r \neq s}^{M} V^{r} L^{rr} A^{rs} L^{ss} L^{ss} Y_i^{ss}}_{2-Xyf\_RT} \\
&+ \underbrace{\sum_{r}^{M} V^{r} L^{rr} \sum_{u \neq r}^{M} A^{ru} \sum_{v}^{M} B^{uv} B^{vs} Y_i^{ss} - \sum_{r \neq s}^{M} V^{r} L^{rr} A^{rs} L^{ss} L^{ss} Y_i^{ss}}_{3a-Xyf\_GVC\_s} \\
&+ \underbrace{\sum_{r}^{M} V^{r} L^{rr} \sum_{u \neq r}^{M} A^{ru} \sum_{v}^{M} B^{uv} B^{vs} Y_i^{sr}}_{3b-Xyf\_GVC\_r} + \underbrace{\sum_{r}^{M} V^{r} L^{rr} \sum_{u \neq r}^{M} A^{ru} \sum_{v}^{M} B^{uv} B^{vs} \sum_{t \neq s,r}^{M} Y_i^{st}}_{3c-Xyf\_GVC\_t}
\end{aligned}
\tag{I2}
$$

Adding up the domestic and international gross outputs induced by the production of final product Y_i^s equals to the total gross outputs induced by the production of final product Y_i^s

$$
\begin{aligned}
& X\_y_i^{s} = X\_yd_i^{s} + X\_yf_i^{s} \\
&= V^{s} L^{ss} L^{ss} \sum_{t}^{M} Y_i^{st} + \sum_{r}^{M} V^{r} L^{rr} L^{rr} \sum_{u \neq r}^{M} A^{ru} B^{us} \sum_{t}^{M} Y_i^{st} \\
&+ \sum_{r}^{M} V^{r} L^{rr} \sum_{u \neq r}^{M} A^{ru} \sum_{v}^{M} B^{uv} B^{vs} \sum_{t}^{M} Y_i^{st} \\
&= V^{s} L^{ss} L^{ss} \sum_{t}^{M} Y_i^{st} + \sum_{r}^{M} V^{r} L^{rr} B^{rs} \sum_{t}^{M} Y_i^{st} - V^{s} L^{ss} L^{ss} \sum_{t}^{M} Y_i^{st} \\
&+ \sum_{r}^{M} V^{r} \sum_{v}^{M} B^{rv} B^{vs} \sum_{t}^{M} Y_i^{st} - \sum_{r}^{M} V^{r} L^{rr} B^{rs} \sum_{t}^{M} Y_i^{st} \\
&= \sum_{r}^{M} V^{r} \sum_{v}^{M} B^{rv} B^{vs} \sum_{t}^{M} Y_i^{st} = u \sum_{v}^{M} B^{vs} \sum_{t}^{M} Y_i^{st}
\end{aligned}
\tag{I3}
$$

Where $\sum_{r}^{M} V^{r} \sum_{v}^{M} B^{rv} = u$, u is a vector which each element of it is 1.

Appendix J. the forward and backward production linkage based GVC participation indexes at global level

As shown in equation 38 and 39, a GVC participation index based on forward and backward industrial linkage can be defined mathematically as follow respectively

$$GVCP\_F^s = \frac{\sum_{r\neq s}^{M}[\hat{V}^s L^{ss} A^{sr} \sum_{u}^{M}\sum_{v}^{M} B^{ru} Y^{uv} - \hat{V}^s L^{ss} A^{sr} L^{rr} Y^{rr}]}{\hat{V}^s X^s} \tag{J1}$$

$$GVCP\_B^s = \frac{\sum_{r}^{M} V^r L^{rr} \sum_{u\neq r}^{M} A^{ru} B^{us} \sum_{t}^{M} Y_i^{st} - \sum_{r\neq s}^{M} V^r L^{rr} A^{rs} L^{ss} Y^{ss}}{\sum_{t}^{M} Y_i^{st}} \tag{J2}$$

Aggregating to the world level

$$GVCP\_F^w = \frac{\sum_{s}^{M} V^s L^{ss} \sum_{r\neq s}^{M} A^{sr} \sum_{u}^{M}\sum_{v}^{M} B^{ru} Y^{uv} - \sum_{s}^{M} V^s L^{ss} \sum_{r\neq s}^{M} A^{sr} L^{rr} Y^{rr}}{\sum_{s}^{M} V^s X^s}$$

$$= \frac{\sum_{r}^{M} V^r L^{rr} \sum_{u\neq r}^{M} A^{ru} \sum_{s}^{M}\sum_{t}^{M} B^{us} Y^{st} - \sum_{r}^{M} V^r L^{rr} \sum_{s}^{M} A^{rs} L^{ss} Y^{ss}]}{\sum_{s}^{M} V^s \sum_{u}^{M}\sum_{v}^{M} B^{su} Y^{uv}} \tag{J3}$$

$$= \frac{\sum_{r}^{M} V^r L^{rr} \sum_{u\neq r}^{M} A^{ru} \sum_{s}^{M}\sum_{t}^{M} B^{us} Y^{st} - \sum_{r}^{M} V^r L^{rr} \sum_{s}^{M} A^{rs} L^{ss} Y^{ss}]}{\sum_{v}^{M} Y^{uv}} = GVCP\_B^w$$

Obviously, GVC participation index of the whole world based on forward and backward industrial linkage are the same.

Appendix K. An Explanation for the finding in Fally (2012)

Fally (2012) showed a finding that the production chain (or the distance to final demand) appears to have shortened over time and he concludes such a trend is also a global phenomenon.

Fally's definition of "production length" (or "Upstreamness") is the average number of production stages from a sector's gross output to the final users. His results rely on the US IO tables, which covers 85 industries from 1947 to 2002, or 540 product categories from 1967 to

1992. To estimate the global production length, Fally (2011) made a strong assumption that "The same industries have the same production length across countries". In this part, we will show that this strong assumption is one of the main factor that leads to the finding "the GVCs are getting shorter at the global level".

First of all, consistent with Fally, our results also show that the production length of the US is getting shorter. Table K1 reports the overall production length for US sectors. The production length has decreased for 26 out of 35 sectors from 1995 to 2011.

Table K1 Production Length (Forward Linkage) of US Sectors, 2011

Sector	Year 1995	Year 2011	Has the Production Length Become Shorter?
Agriculture	2.677	2.583	√
Mining	2.918	2.487	√
Food	1.679	1.688	
Textiles Products	2.227	2.112	√
Leather and Footwear	1.632	1.252	√
Wood Products	2.531	2.597	
Paper and Printing	2.581	2.306	√
Refined Petroleum	2.375	2.305	√
Chemical Products	2.665	2.468	√
Rubber and Plastics	2.659	2.509	√
Other Non-Metal	2.615	2.563	√
Basic Metals	3.025	3.027	
Machinery	1.834	1.784	√
Electrical Equipment	2.187	2.016	√
Transport Equipment	1.802	1.672	√
Recycling	1.570	1.588	
Electricity, Gas and Water	2.061	1.820	√
Construction	1.246	1.295	
Sale of Vehicles and Fuel	1.386	1.324	√
Wholesale Trade	2.154	1.937	√
Retail Trade	1.321	1.204	√
Hotels and Restaurants	1.446	1.435	√
Inland Transport	2.429	2.289	√
Water Transport	2.298	1.740	√
Air Transport	1.806	1.654	√
Other Transport	2.805	2.693	√
Post and Telecommunications	2.266	2.115	√
Financial Intermediation	2.187	2.311	
Real Estate	1.472	1.429	√

Business Activities	2.590	2.453	√
Public Admin	1.103	1.110	
Education	1.254	1.097	√
Health and Social Work	1.036	1.029	
Other Services	1.764	1.785	
Private Households	1.386	1.324	√

Aggregated to the country level, we also find that the average production length for US industries as a whole decreased during the period 1995–2003, but has increased since then until 2008, the global financial crisis, then resumed a declining trend.

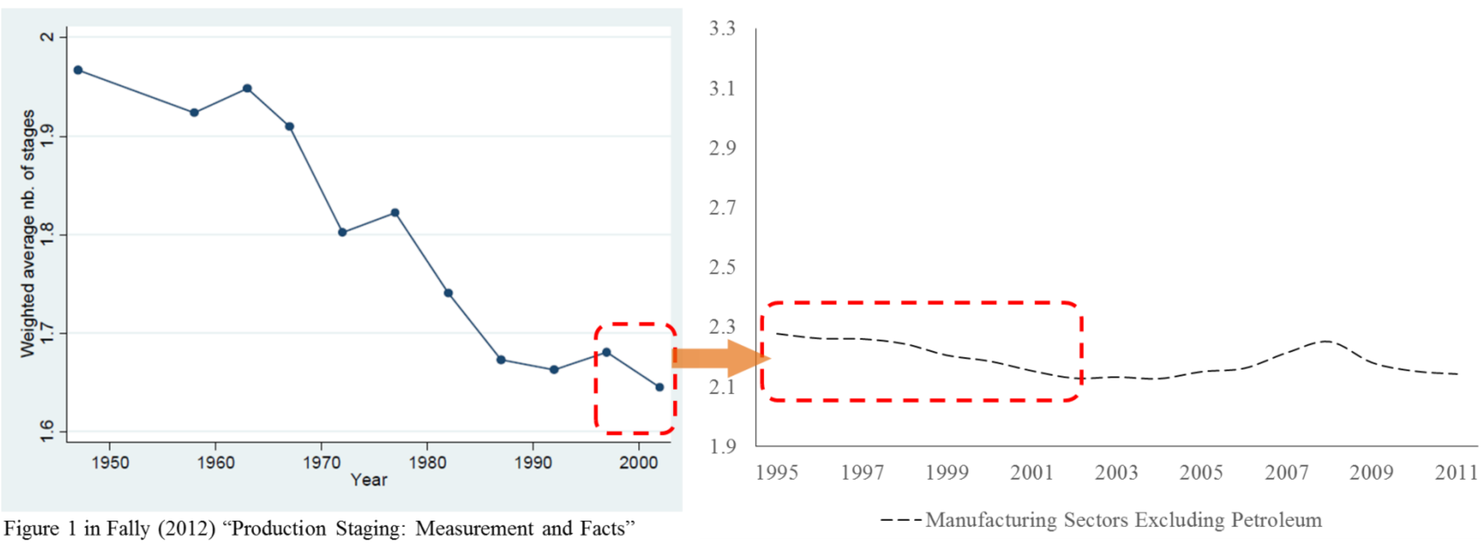

Figure 1 in Fally (2012) "Production Staging: Measurement and Facts"
Aggregate measure of vertical fragmentation (tradable goods excluding petroleum)

Figure K1 Average Production Length for the US

However, this finding is reversed at the global level. As shown in Figure K2, the production length for a certain industry may vary considerably across countries. While the length of production in the United States decreased, it has an opposite pattern in China, which means that the assumption "the same industries have the same production length across countries" does not hold in reality. As a results, for the world as a whole, we have observed that the production chain has become longer.

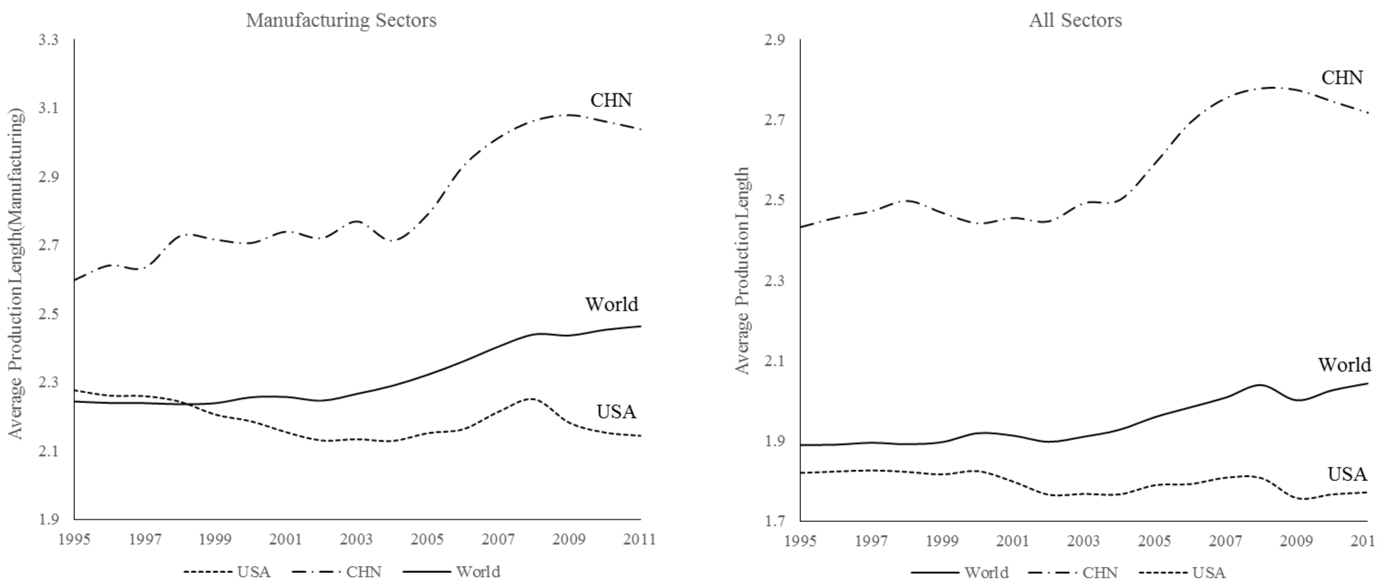

Figure K2 Average Production Length, China, the United States, and the World

To understand why this assumption is crucial to the result, we re-estimate the weighted average global production length with the assumption that the production length of a certain sector is the same across countries and equal to the US. After applying this strong assumption, the upward trend of the global production length in Figure K2 has disappeared, and instead, we see a downward trend in Figure K3.

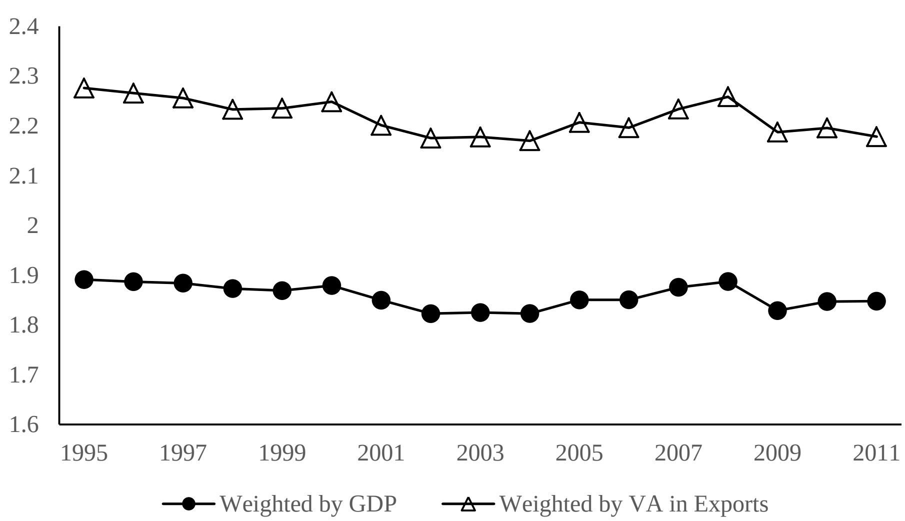

Figure K3 Global Average Production Length under the “Equal Length Assumption”

Appendix L. Sector, Country and Region Code in WIOD

Table L1 WIOD Sectors

Code	NACE	Industry	Description
C01	**AtB**	Agriculture	Agriculture, Hunting, Forestry and Fishing
C02	**C**	Mining	Mining and Quarrying
C03	**15t16**	Food	Food, Beverages and Tobacco
C04	**17t18**	Textiles Products	Textiles and Textile Products
C05	**19**	Leather and Footwear	Leather, Leather and Footwear
C06	**20**	Wood Products	Wood and Products of Wood and Cork
C07	**21t22**	Paper and Printing	Pulp, Paper, Paper, Printing and Publishing
C08	**23**	Refined Petroleum	Coke, Refined Petroleum and Nuclear Fuel
C09	**24**	Chemical Products	Chemicals and Chemical Products
C10	**25**	Rubber and Plastics	Rubber and Plastics
C11	**26**	Other Non-Metal	Other Non-Metallic Mineral
C12	**27t28**	Basic Metals	Basic Metals and Fabricated Metal
C13	**29**	Machinery	Machinery, Nec
C14	**30t33**	Electrical Equipment	Electrical and Optical Equipment
C15	**34t35**	Transport Equipment	Transport Equipment
C16	**36t37**	Recycling	Manufacturing, Nec; Recycling
C17	**E**	Electricity, Gas and Water	Electricity, Gas and Water Supply
C18	**F**	Construction	Construction
C19	**50**	Sale of Vehicles and Fuel	Sale, Maintenance and Repair of Motor Vehicles and Motorcycles; Retail Sale of Fuel
C20	**51**	Wholesale Trade	Wholesale Trade and Commission Trade, Except of Motor Vehicles and Motorcycles
C21	**52**	Retail Trade	Retail Trade, Except of Motor Vehicles and Motorcycles; Repair of Household Goods
C22	**H**	Hotels and Restaurants	Hotels and Restaurants
C23	**60**	Inland Transport	Inland Transport
C24	**61**	Water Transport	Water Transport
C25	**62**	Air Transport	Air Transport
C26	**63**	Other Transport	Other Supporting and Auxiliary Transport Activities; Activities of Travel Agencies
C27	**64**	Post and Telecommunications	Post and Telecommunications
C28	**J**	Financial Intermediation	Financial Intermediation
C29	**70**	Real Estate	Real Estate Activities
C30	**71t74**	Business Activities	Renting of M&Eq and Other Business Activities
C31	**L**	Public Admin	Public Admin and Defense; Compulsory Social Security
C32	**M**	Education	Education
C33	**N**	Health and Social Work	Health and Social Work
C34	**O**	Other Services	Other Community, Social and Personal Services
C35	**P**	Private Households	Private Households with Employed Persons

Table L2 WIOD Country and Region

Label	Country	Region	Label	Country	Region
AUS	Australia	Asia-Pacific	**IRL**	Ireland	Europe
AUT	Austria	Europe	**ITA**	Italy	Europe
BEL	Belgium	Europe	**JPN**	Japan	Asia-Pacific
BGR	Bulgaria	Europe	**KOR**	South Korea	Asia-Pacific
BRA	Brazil	American	**LTU**	Lithuania	Europe
CAN	Canada	American	**LUX**	Luxembourg	Europe
CHN	China	Asia-Pacific	**LVA**	Latvia	Europe
CYP	Cyprus	Europe	**MEX**	Mexico	American
CZE	Czech Republic	Europe	**MLT**	Malta	Europe
DEU	Germany	Europe	**NLD**	Netherlands	Europe
DNK	Denmark	Europe	**POL**	Poland	Europe
ESP	Spain	Europe	**PRT**	Portugal	Europe
EST	Estonia	Europe	**ROM**	Romania	Europe
FIN	Finland	Europe	**RUS**	Russia	Europe
FRA	France	Europe	**SVK**	Slovak Republic	Europe
GBR	United Kingdom	Europe	**SVN**	Slovenia	Europe
GRC	Greece	Europe	**SWE**	Sweden	Europe
HUN	Hungary	Europe	**TUR**	Turkey	Europe
IDN	Indonesia	Asia-Pacific	**TWN**	Taiwan	Asia-Pacific
IND	India	Asia-Pacific	**USA**	United States	American