The Risky Capital of Emerging Markets*

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Abstract

Emerging markets exhibit high returns to capital, the 'Lucas Paradox,' alongside volatile growth rate regimes. We investigate the role of long-run risks, i.e., risk due to fluctuations in economic growth rates, in leading to return differentials across countries. We take the perspective of a US investor and outline an empirical strategy to identify risky growth shocks and quantify their implications. Long-run risks account for 60-70% of the observed return disparity between the US and a group of the poorest countries. At the individual country level, our model predicts average returns that are highly correlated with those in the data (0.61).

JEL Classification: O4, E22, F21, G12

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1 Introduction

Neoclassical theory predicts that the returns to capital should be equalized across countries. In his seminal paper, Lucas (1990) points out that the data reveal substantial and systematic dispersion in capital returns: poor countries exhibit significantly higher returns than do rich, begging the question of why capital does not flow from rich countries to poor until returns are equalized. In this paper, we revisit the 'Lucas Paradox.' We document significant differences in the average return to capital across 144 countries over the period 1950-2008. These disparities vary systematically with income: poor countries tend to offer higher returns than do rich, suggesting that the former represent untapped investment opportunities. Moreover, dispersion in the returns to capital suggests an inefficient allocation and so a foregone opportunity to increase global output.

The prediction of identical returns to capital, however, holds only in a deterministic setting. In the presence of uncertainty, dispersion in average rates of return should exist so long as there is dispersion in the riskiness of international capital holdings. In short, investors demand higher expected returns from riskier assets. Thus, incorporating risk into an analysis of international capital returns may well generate very different implications than a deterministic setting: there is no reason to expect that rates of return will be identical across space and over time. Indeed, if there are significant differences in the riskiness of international capital holdings, such an outcome would be surprising.

We pursue this line of analysis by asking whether the risk-return tradeoff implied by asset-pricing theory can explain the systematic dispersion in international capital returns, and in particular, the disparities between high return/poor countries and low return/rich ones. Specifically, we investigate the role of 'long-run risks' à la Bansal and Yaron (2004), that is, persistent fluctuations in economic growth prospects, in leading to return differentials across countries. Our approach is motivated in spirit by Aguiar and Gopinath (2007), who show the importance of shocks to trend growth in total factor productivity in accounting for the properties of business cycles in poor/emerging markets, and moreover, in reconciling differences in the behavior of macroeconomic variables between these countries and developed ones.[1] Our analysis focuses on the implications of these shocks for required expected returns to capital for a US investor. We demonstrate that investments in poor countries exhibit high sensitivity to persistent shocks to economic growth rates, which in turn command large risk premia. Quantitatively, we find that these long-run risks can account for 60-70% of the observed return disparity between the US and a group of the poorest countries in the world. Moreover, at the individual country level,

[1] Relatedly, Neumeyer and Perri (2005) demonstrate that country-specific risk premia are intimately linked to the high volatility of macroeconomic variables in emerging markets.

our model of long-run risk predicts average returns that are highly correlated with those in the data (0.61).

We begin our analysis by explicitly laying out our measure of the returns to capital, both in theory and in data. Our focus is on a representative US investor able to invest in both domestic and international capital assets.[2] Our motivation for taking this perspective stems in large part from the fact that many countries import a large share of their capital goods and that this is particularly the case in poor countries.[3] Moreover, the question we want to address is whether there is an arbitrage opportunity on the margin to shift investment from rich countries to poor. We document that average returns across 144 countries over the period 1950-2008 are negatively related to income with a semi-elasticity of -0.016, which is significant both in a statistical sense and an economic one. Next, in order to focus on the component of returns that is systematically related to income, we proceed by constructing 'bundles' or portfolios of countries grouped by income levels, and use these portfolios as the primary unit of analysis (3, 5, and 10 portfolios, with the US always an additional standalone).[4] This procedure creates a spread in mean returns relative to the US that is generally decreasing in income (monotonically so for the case of 3 portfolios) and captures well the systematic component of the relationship at the individual country level. In terms of magnitudes, the poorest one-third of countries offer mean returns about 7 percentage points higher than those in the US, similar to the difference between US stocks and a risk-free bond; the poorest one-tenth of countries offer mean returns 10 percentage points higher than the US.

In order to study risk-return tradeoffs, it is natural to begin with the workhorse power-utility consumption-based capital asset pricing model (CCAPM) in which an asset's risk, and so required mean return, is driven by the covariance of its return with consumption growth. Measuring these covariances, we find that, qualitatively, the portfolio returns rank as predicted by theory, but for reasonable levels of risk aversion, differences in the covariances are far too small to account for the cross-sectional return disparities that we measure.[5] In this light, the

[2] A number of recent papers have taken a similar stance in assessing international return differentials, for example, Lustig and Verdelhan (2007) in the case of high- versus low-inflation currencies and Borri and Verdelhan (2012) in the case of sovereign bonds.

[3] For example, Burstein et al. (2013) document that 80% of the world's capital equipment was produced in just 8 countries in the year 2000; that the median import share of equipment in that year was about 0.75; and that the poorest countries in the world tend to import almost all their equipment. Mutreja et al. (2012) find similarly, and report a correlation between the import to production ratio for capital goods and income of -0.34 (they report, for example, that Malawi imports 39 times as much capital as it produces, and Argentina 19 times as much). Related facts are also in Eaton and Kortum (2001).

[4] The portfolio approach is common practice in empirical asset pricing and macro-finance. It helps to eliminate the diversifiable asset-specific component of returns, thus producing much sharper estimates of the risk-return trade-off of interest. See Fama (1976) for an early exposition, and, e.g., Lustig and Verdelhan (2007) for an application. After reporting our results at the portfolio level, we also report results at the country level.

[5] Parameterized to match the covariance of returns with US consumption growth, the CCAPM requires a

Lucas Paradox resembles the equity premium and other closely related asset pricing puzzles.

We propose a resolution to the Lucas Paradox that builds on long-run risks, i.e., risks due to persistent fluctuations in economic growth rates. Our motivation is twofold: first, persistent shocks to growth rates have been shown to play a key role in emerging market dynamics; second, long-run risks have been able to resolve a number of asset pricing puzzles, including the equity premium puzzle.[6] We outline an international endowment economy featuring shocks to trend growth rates in the spirit of Bansal and Yaron (2004). A representative US investor is endowed with a stream of consumption and payouts from risky capital investments in a number of regions (portfolios or countries) and a risk-free asset. Economic growth rates feature a small but persistent component, which manifests itself in both consumption growth and growth in payments to invested capital ('dividends'). In each region, this component contains both a global piece and a region-specific idiosyncratic one. Regions differ in their exposure to the global shock. With recursive preferences of the Epstein and Zin (1989) form, the value of capital holdings responds sharply to persistent shocks that are global in nature. Regions that are more sensitive to this shock represent riskier investments and so must offer higher risk premia to investors as compensation.

Because of our focus on a US-based investor, the link between long-run risks and returns is not a direct one: in order to command risk premia for this investor, shocks to foreign growth rates must be related to his stochastic discount factor and so, in this sense, must be global in nature. From this perspective, purely idiosyncratic shocks, even those that are persistent in nature, do not lead to higher returns. Ample evidence points to the existence of a global component to fluctuations in expected growth rates, and, moreover, to the high sensitivity of emerging markets to these shocks. For example, Burnside and Tabova (2009) find that about 70% of the cross-sectional variation in the volatility of GDP growth can be explained by countries' differing degrees of sensitivity to global factors and additionally, that low-income countries exhibit greater exposure to these factors.[7] Chen and Crucini (2014) find that global permanent productivity shocks are a key component of the variance of output growth of developing countries. As a last example, a recent IMF report similarly documents the large role of external factors in determining emerging market growth prospects.[8] The role that global

coefficient of relative risk aversion of almost 900 in order to best fit observed return differentials.

[6] Among others, see Bansal and Yaron (2004) and Hansen et al. (2008) for an examination of the equity premium puzzle; Malloy et al. (2009) for the value and size premia and other cross-sectional facts; Chen (2010) for the credit spread puzzle; and Colacito and Croce (2013) and Bansal and Shaliastovich (2013) for the forward premium puzzle in international currency markets.

[7] These include US GDP growth and interest rates, a number of commodity price indices, and the return on the US stock market.

[8] In the 2014 *World Economic Outlook*, the IMF finds that "external factors induce significant fluctuations in emerging market economies' growth, explaining about half the variance in their growth rates...the incidence of external shocks varies across economies...external factors have contributed as much or more than other, mostly

factors play in shifting expected growth rates and cross-sectional differences in exposure to these factors is at the heart of the explanation that we propose.

Quantifying the implications of long-run risks in our model is challenging for two reasons: first, we must disentangle global from purely regional long-run shocks. The former have direct implications for required returns while the latter do not. Second, even having identified common shocks, we must separate those that affect long-run growth prospects from those that are purely transitory in nature. To understand the complication, consider the following: a natural way to identify long-run shocks would be to rely on moments in persistence in growth rates; however, in our context, observed persistence may be due to either common or idiosyncratic shocks, and these moments are not sufficient on their own to disentangle the two. Given this, it would seem that moments in the comovement of growth rates would serve to eliminate purely regional phenomena; in our context, however, comovement may arise due to both common long-run and short-run shocks, and again, these moments are not sufficient to distinguish between the two.

To overcome this hurdle, we use moments in both the persistence and comovement of dividend growth rates and additionally draw on a key prediction of our model: both dividend growth rates and returns depend on both long-run and transitory shocks; however, whereas dividend growth rates and returns respond in an identical manner to transitory shocks, which affect current payments to capital but have no implications for the future, returns respond more sharply to long-run shocks. This is the insight of Lewis and Liu (2012), on which we build closely. Intuitively, because long-lived shocks signify revisions in the long-run value of capital holdings, capital returns exhibit a higher degree of sensitivity to these shocks than do current capital payouts. We exploit this fact and use the comovement of returns *relative* to that in dividend growth rates to infer the degree of comovement that is due to long-run shocks.[9] Together, these moments enable us to infer the sensitivity of each region to the global component of long-run risk (while controlling for the regional component)—the key factor in assessing the quantitative implications of our theory.

We apply our methodology to quantify the implied differences in expected returns between the US and a number of foreign regions. We find that long-run risk can account for a significant portion of the large disparities in returns observed in the data, and more importantly, for the pattern of low income/high return vs high income/low return. In our benchmark specification, which features the US as well as three portfolios of countries sorted according to income, the parameterized model accounts for 66% of the spread in returns between the US and the portfolio of the poorest countries in the world. The figures when further disaggregating countries

internal, factors in explaining emerging markets' growth deviations from the estimated average growth over the past 15 years."

[9]Relatedly, Tabova (2013) points out the importance of the comovement of returns in determining foreign investment in the context of a portfolio allocation problem.

into bundles of five and ten portfolios are 61% and 62%, respectively. At the finest level of granularity, we parameterize the model at the individual country level for a set of 96 countries for which sufficient time-series data are available. The correlation between the model's predicted returns and the actual is 0.61, confirming the key role of long-run risk in driving return differentials. Moreover, at the country level, the model predicts a negative and statistically significant relationship between returns and income, where the slope amounts to 55% of that observed in the data. Our results suggest that the observed allocation of capital is not so distant from that predicted by theory in the presence of long-run risks, despite the large return differentials observed in the data. Thus a potential answer to Lucas: capital does not flow into high return/poor countries until returns are equalized precisely because these countries represent the riskiest investments.

To gain additional insights behind the risk-return relationship, we decompose predicted returns into their short- and long-run risk components. Foreign risk premia stemming solely from short-run risk are generally negative and are actually higher in rich countries than poor. Because period-by-period growth rates in foreign countries exhibit low comovement with US consumption growth, particularly so in poor countries, investments there actually serve as hedges for short-run consumption growth risks. Hence, long-run risks are critical to reconciling the high returns from capital investments in poor countries observed in the data: these risks are systematically higher in poor countries and account for the variation in returns across the income spectrum.

Our paper relates to a growing body of work highlighting the importance of long-run risk in driving a number of asset pricing phenomena. Most closely related are Lewis and Liu (2012), Nakamura et al. (2012), and Colacito and Croce (2011) who add an international dimension to the long-run risk model, albeit in different contexts.[10] Each finds evidence of a global component in long-run risks (generally among developed economies), as do we. We view our findings as building upon theirs: we quantify the implications of common long-run risks in driving differences in the real return to capital across a large group of countries in an explicit international setting.

Our paper also contributes to an extensive literature that explores the 'paradox' of large differences in international capital returns identified by Lucas (1990)—a factor of 58 between the US and India.[11] The spread in returns that we compute is significant, although much smaller than that discussed by Lucas (1990). Important modifications to this original calculation in-

[10] To measure the extent of international risk sharing, assess equity premia across countries, and account for real exchange rate dynamics, respectively.

[11] See Ohanian and Wright (2007) for an evaluation of a number of potential explanations for the Lucas Paradox. The authors find the explanatory power of each to be limited, as none reverse the standard forces pushing for return equalization.

clude accounting for TFP differences, which are well-known to be large, as well as for systematic variation in relative prices, as pointed out by Hsieh and Klenow (2007) and Caselli and Feyrer (2007). Caselli and Feyrer (2007) find that after adjusting capital shares for non-reproducible factors and accounting for differences in the relative price of investment goods, capital returns are approximately equalized across countries in a single year, 1996. A key difference in our analysis is our focus on the behavior of returns over a long period, rather than realizations in any given year, and our use of a stochastic structural environment to highlight the role of risk as predicted by asset pricing theory, an explanation we feel is persuasive and one that goes a great distance in reconciling theory and fact. With respect to the magnitudes of differences in returns across countries, even after applying the important adjustments suggested by these papers, we find economically significant return differentials across countries, ranging, for example, from 7 to 10 percentage points at the portfolio level, which are at least as large as the US equity premium. Using different statistics compiled directly from local national accounts data, Daly (2010) finds average returns in emerging markets exceeded those in developed markets over the period 1981-2008 by a similar amount.

2 Measuring the Returns to Capital

In this section, we lay out our measure of the returns to capital. We first outline a simple theoretical framework to guide our measurement approach and next detail how we map the theory to the data.

2.1 Returns in Theory

A US investor considers pursuing a capital investment, either at home or abroad. He would purchase a unit of the investment good domestically and rent it to a firm either in the US or in some other region. The additional unit of capital is used locally in production and represents a claim on some portion of the local output produced. The payment received by the investor is the rental rate on capital, which represents the period payoff, or 'dividend' from this investment. A portion of the capital depreciates during production and so the investor is left with only a fraction of the unit at the end of the period, which would continue to hold some value. To characterize the returns from this transaction, we describe an explicit accounting framework next.

We consider a world economy consisting of the US and J regions, where regions will correspond to countries, or 'bundles' of countries in our empirical analysis. Production in each region consists of a consumption good sector and an investment good sector, which operate the following technologies, respectively:

$$C_{j,t} = A_{C,j,t} K_{C,j,t}^{\alpha} L_{C,j,t}^{1-\alpha}$$
$$I_{j,t} = A_{I,j,t} K_{I,j,t}^{\alpha} L_{I,j,t}^{1-\alpha}$$

where C, I, K, L denote consumption, investment, capital and labor, respectively. $A_{C,j,t}$ and $A_{I,j,t}$ represent the productivity of each sector in region j and evolve stochastically through time, which is discrete and indexed by t. For simplicity, we assume that the capital elasticity α is common across sectors, regions, and time.[12] The allocation of capital across regions for use in time t production is chosen prior to the realization of the shocks in that period.

Investment goods are freely tradable across regions while consumption goods are not. The law of one price then implies a common price for investment goods across regions $P_{I,t}$, where $P_{I,t}$ denotes the price of investment goods in terms of US consumption goods, which serves as numeraire. Because the price of consumption goods need not equate, the relative price $\frac{P_{I,t}}{P_{C,j,t}}$ may differ across regions. Although the assumption of freely traded capital goods is a clear simplification, it is motivated by the observation that relative price differences that are systematically related to income are largely driven by differences in the price of consumption goods, which tends to be higher in richer countries, whereas the price of investment goods shows no systematic relationship with income.[13]

Firms in each sector act to maximize profits. Using $D_{j,t}$ to denote the period payoff to a unit of capital, which we alternatively refer to as 'dividends,' standard arguments give that dividends in a region are equal to the (price-adjusted) aggregate marginal product of capital:

$$D_{j,t} = \alpha \frac{P_{C,j,t} C_{j,t} + P_{I,t} I_{j,t}}{K_{j,t}} = \alpha \frac{P_{Y,j,t} Y_{j,t}}{K_{j,t}} \tag{1}$$

where $K_{j,t} = K_{C,j,t} + K_{I,j,t}$ is the total stock of capital used in production in region j and $P_{Y,j,t} = \frac{P_{C,j,t} C_{j,t} + P_{I,t} I_{j,t}}{C_{j,t} + I_{j,t}}$ is the price of a unit of output in that region (again relative to the price of US consumption goods) and hence the numerator is simply the value of total local output (in terms of US consumption goods).[14]

In order to make this investment, an investor must purchase a unit of the investment good at price $P_{I,t}$. After production occurs, the investor is left with $(1 - \delta_{j,t})$ units of the good,

[12]In Section 2.2 below, we revisit our assumptions of a common and constant capital share across regions.

[13]See, for example, Hsieh and Klenow (2007). We will also empirically explore a variant on this approach that takes into account different levels of P_I across countries and show that our results do not depend on this assumption.

[14]If capital is perfectly transferable across sectors within a region, (1) is the common (price-adjusted) MPK; if not, it is the capital-weighted average across the sectors.

which is valued at the following period's price $P_{I,t+1}$.[15] Putting the pieces together, the return on capital from region j in period t is given by:

$$
\begin{aligned}
R_{j,t} &= \frac{D_{j,t}}{P_{I,t}} + (1-\delta_{j,t})\frac{P_{I,t+1}}{P_{I,t}} \\
&= \alpha\frac{P_{Y,j,t}Y_{j,t}}{P_{I,t}K_{j,t}} + (1-\delta_{j,t})\frac{P_{I,t+1}}{P_{I,t}} \qquad (2)
\end{aligned}
$$

Equation (2) serves as our guide to measuring the returns to capital. It builds on the insight of Caselli and Feyrer (2007), who show that accounting for differences in relative prices is key when measuring the cross-sectional dispersion in capital returns, and additionally that of Gomme et al. (2011), who point out the importance of changes in the relative price $P_{I,t}$ in driving the time series behavior of capital returns, at least in the US, and in particular, the contribution of this term to the volatility of returns. In one important regard, our measure is closer to that in Gomme et al. (2011) than in Caselli and Feyrer (2007): all prices are expressed in units of US consumption, not of region-specific output. The calculations in Caselli and Feyrer (2007) imply that the investor considers his return in units of output received per unit of output invested; here, as in Gomme et al. (2011), the investor considers units of consumption received per unit of consumption invested, and a corresponding adjustment must be made when mapping (2) to the data. A second departure from Caselli and Feyrer (2007) is in the cost of the original unit of the investment good: there, investors purchase investment goods domestically, that is, in the region where they will be used in production; in our setup, the US investor purchases these goods domestically, no matter the location of production (although given the freely traded nature of investment goods, this distinction is purely semantics in the theory).[16]

2.2 Returns in the Data

In order to measure the quantities in equation (2) we use data from Version 8.0 of the Penn World Tables (PWT)[17], and to measure the relevant prices we rely on data from the US National Income and Product Accounts as reported by the Bureau of Economic Analysis (BEA). We use data spanning the period 1950-2009 (so returns are from 1950-2008). Our final sample consists of 144 countries.[18] For each country, the PWT directly reports real GDP valued at 2005 US dollars, which we will denote $P_{Y,US,2005}Y_{j,t}$, an estimate of the real-valued capital stock $K_{j,t}$

[15] We assume that imported capital is utilized with the same intensity as domestic capital within a region, and so depreciates at the same rate. We will use country-time specific values of δ in our empirical analysis.

[16] As discussed above, the majority of investment goods worldwide are produced in a small number of developed countries.

[17] See Feenstra et al. (2013).

[18] Countries need not be present for the entire period to be included. We describe the sample construction in Appendix A.

and country-time specific depreciation rates $\delta_{j,t}$. Recall that all prices in (2) are relative to US consumption, as that is the relevant tradeoff being made, and that relative prices may vary through time. To map the theory to data, we multiply the reported value of GDP by the relative price of output to consumption in the US, $\frac{P_{US,Y,t}}{P_{US,C,t}} = \frac{\frac{P_{US,Y,t}}{P_{US,Y,2005}}}{P_{US,C,t}}$ in each year t, where $P_{Y,US,2005}$ is normalized to 1. The result gives the value of year t GDP in region j in current units of US consumption, which is the object needed to measure $D_{j,t}$. The price index of US output $P_{US,Y,t}$ is constructed as nominal GDP divided by real GDP, with 2005 serving as the base year as noted. To construct the price index of consumption $P_{US,C,t}$, we divide nominal spending on non-durables and services by the corresponding real values. The ratio of these two series is then the relative price of interest. Data for these latter two computations are obtained from the BEA. It remains to specify a value for α, which we set to 0.3 across all regions following Gollin (2002), although with recent work by Karabarbounis and Neiman (2014) in mind, we relax the assumption of a common/constant α below.

To compute returns, we need the relative price of investment goods in the US. We compute this price as nominal private spending on investment in equipment and structures divided by the corresponding real values, again with data obtained from the BEA. Our approach to measuring the relevant relative prices follows closely that of Gomme et al. (2011). From a strictly empirical point of view, a beneficial by-product of our focus on a US investor is the ability to measure the relevant prices using a widely used data source thought to be highly reliable.

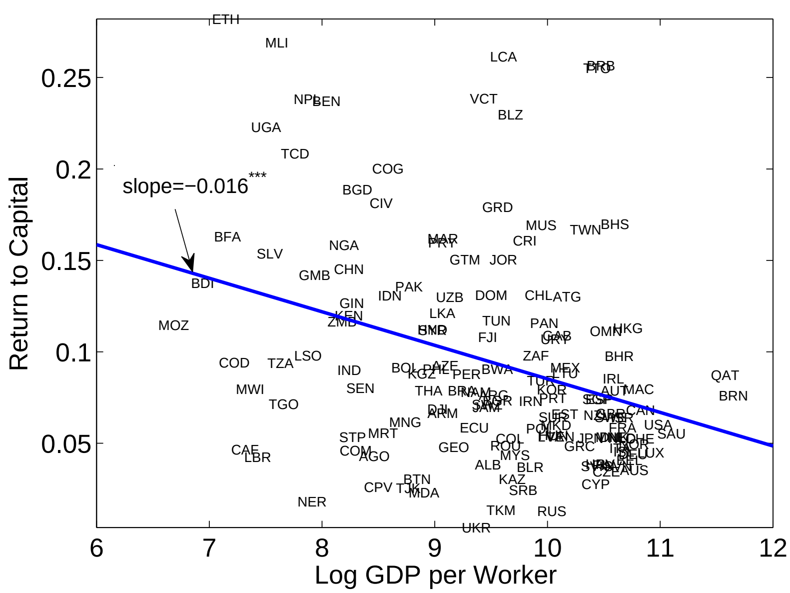

Figure 1: The Cross-Section of Capital Returns

With these pieces in hand, we use (2) to construct returns $R_{j,t}$ for each country in each year in our sample and compute the mean return as the time-series average over the available years for each country. We display the results in Figure 1 across the 144 countries in our sample. The figure shows that capital returns differ significantly around the world and despite a good deal of noise, there is a systematic relationship between returns and income: returns are generally higher in poorer countries. The relationship between returns and income is negative and highly significant, both in a statistical sense and an economic one: each 10% reduction in income is associated on average with a 1.5% increase in expected returns.

Portfolios of countries. The puzzle we are after is why systematic return differences may persist between low return/rich countries and high return/poor ones. To focus on the link between returns and income, we form 'bundles,' or 'portfolios,' of countries, grouped by levels of per-worker income. We use these portfolios as the primary unit of analysis, rather than individual countries, and these will correspond to the J regions to which we have been referring. Our approach here follows widespread practice in empirical asset pricing, which has generally moved from addressing variation in individual asset returns to returns on asset portfolios, sorted by factors that are known to predict returns. This procedure proves useful in eliminating asset-specific diversifiable risk, and so in honing in on the sources of return variation of interest. In our application, it serves to eliminate idiosyncratic factors that drive country-specific returns but are unrelated to countries' levels of economic development.[19] Moreover, we are able to expand the number of countries as data become increasingly available, enabling us to include the largest possible set of countries in our analysis. Lastly, there is an intuitive appeal to analyzing portfolios: by doing so, we are asking whether there are arbitrage opportunities for a US investor to go short in a portfolio of rich country capital assets and long in a portfolio of poor country ones, which is at the heart of the question we are after.

We perform our analysis first on 3 portfolios plus the US and then extend our analysis to 5 and 10 portfolios (with the US always separate). We allocate countries into portfolios based on average income over the sample period. That is, we align average income with average returns with the interpretation being whether average returns in the cross-section are systematically related to average income. To get a sense of our groupings, Figure 2 displays returns at the country-level overlaid with returns in our 3 portfolio grouping.[20] Portfolio 1 contains the poorest set of countries and portfolio 3 the richest, with the US always kept apart, so that higher numbered portfolios are higher income and the US is last, a terminology which will remain consistent throughout the paper. The portfolios eliminate a good deal of the country-level variation in returns even within similar income groups, yet retain the systematic relationship between returns and income. Moreover, the portfolio returns lie quite close to the line of best

[19] The portfolio approach also aids in eliminating measurement error in country-level variables.

[20] Appendix D lists the countries by portfolio and year in which they entered the PWT dataset.

fit, providing some additional reassurance that they capture to a large extent the systematic component of the relation between returns and income.

Figure 2 displays returns in the 3 portfolio case, and visually sums up the key motivation of our paper: capital returns are systematically decreasing in income, both at the country level and when grouped into income-sorted portfolios. The relationship is both statistically and economically significant: for example, as reported in the first column of Table 1, portfolio 1 average returns are 13% compared to 6% in the US, a spread of 7 percentage points.[21]

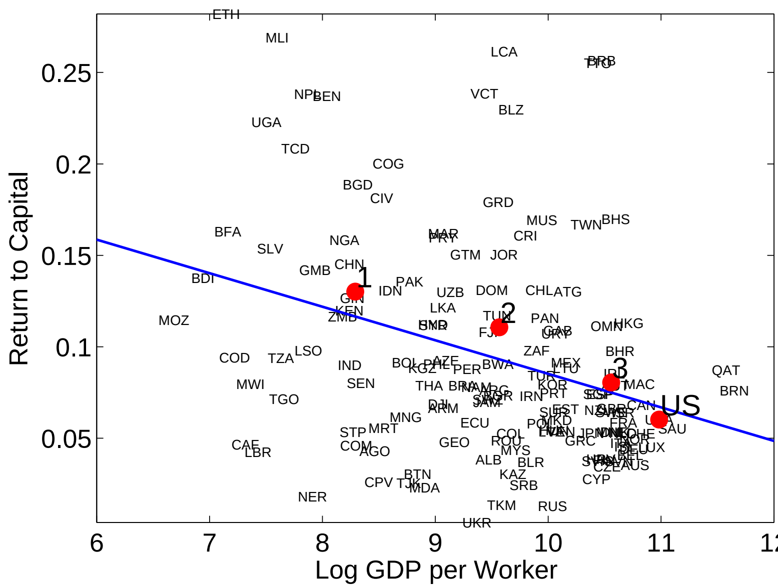

Figure 2: Returns to 3 Portfolios

Alternative measurement approaches. Before moving into our risk-based analysis, it is worth pausing for a moment to consider our measurement approach and explore the sensitivity of our results to several alternatives. Table 1 reports mean returns across the 3 portfolios and the US under a number of variants of approaches. The first column reports our baseline measures, which correspond to the values displayed in Figure 2. In the second column, we relax our assumption of a common price for investment goods. To do so, we use country-specific prices as reported in the PWT for all prices in equation (2). Loosely speaking, this corresponds to the return to an investor who purchases capital goods in the local country and whose payoff is denominated in local consumption goods - in other words, a domestic investor. This is the price adjustment made, for example, in Caselli and Feyrer (2007). Generally, the returns to each portfolio do not change much under this modification; while the dispersion in returns falls slightly, the differences between the returns on different portfolios and the US remain

[21]Similar results obtain for the 5 and 10 portfolio groupings.

significant, both economically and statistically.[22] While this exercise is an informative check, notice that our theory prescribes our baseline measure due to our focus on a US investor, not domestic investors in each country.

Table 1: Capital Returns - A Variety of Approaches

	All Years				1996			
Portfolio	Baseline	Country prices	Country α's	Country prices & α's	Baseline	Country prices	Country α's	Country prices & α's
1	13.02***	12.00***	13.22***	13.63***	5.38**	3.32	8.24**	7.27
	(0.76)	(0.66)	(0.76)	(1.05)	(.78)	(2.81)	(1.25)	(2.00)
2	11.07***	10.53***	13.15***	13.23***	5.21	5.89	8.12*	7.51
	(0.61)	(0.62)	(0.75)	(0.68)	(0.99)	(1.73)	(1.29)	(1.42)
3	8.05***	9.36**	9.17***	11.39***	3.91	10.03*	6.47	14.09***
	(0.45)	(0.33)	(0.49)	(0.44)	(0.85)	(1.48)	(1.29)	(1.22)
US	6.02	8.22	6.20	9.39	3.64	7.23	5.52	9.55
	(0.35)	(0.31)	(0.39)	(0.50)	-	-	-	-

Notes: Table reports the returns to capital across portfolios under a number of measurement approaches. Baseline uses US prices from BEA. Country prices uses country-specific P_Y, P_I, P_C from PWT. Country α's uses country-year α from PWT and subtracts from α the share of payments to non-reproducible capital from WDI, dropping the countries that have negative α for at least one year. Country prices and α's uses country prices and country-year α as described above. Baseline and Country prices cover years from 1950 to 2008. Country α's and Country prices and α's cover years from 1970 to 2008. The portfolios include only countries for which data are available. Standard errors are reported in parentheses. Asterisks denote significance of difference from US values: ***: difference significant at 99%, **: 95%, and *: 90%.

In the third column, we report results obtained using country-year specific capital shares, with an adjustment for the shares of non-reproducible capital, again in the spirit of Caselli and Feyrer (2007). To do so, we obtain data on the shares of payments to natural resources in GDP from the World Bank's World Development Indicators (WDI) database. We compute the reproducible capital share as one minus the labor share minus the natural resource (non-reproducible) share.[23] The results are extremely close to those in our baseline, suggesting that our choice of a single α is not leading to substantial biases in our estimates of capital returns. Particularly important, returns in portfolios 1 to 3 remain significantly higher than in the US, both economically and statistically. It should be noted that payments to natural resources include oil rents, natural gas rents, coal rents, mineral rents, and forest rents, and whether or not these are truly 'non-reproducible' is unclear: consider, for example, an investment by Exxon-Mobil in a new oil well. As a further investigation into this issue, we turn to an outside source of data that does not require assumptions about the form of the production function—namely, stock market returns.

[22]The US changes most, increasing about 2 percentage points simply from using PWT relative prices, rather than those from the BEA.

[23]These data are available for 115 of our original 144 countries only over the period 1970-2008. Because of the differences in countries and time periods, comparisons across columns in Table 1 should not be made.

We obtain MSCI annual country-level returns denominated in US dollars and deflate these to obtain real returns. Sufficient data to form roughly balanced portfolios are only available from 2003-13.[24] Over this admittedly short time period, returns in portfolio 1 averaged about 23%, compared to 8.7% in the US (portfolios 2 and 3 fall in between). Thus, even when examining returns to the capital that we know is fully 'investible,' the data exhibit the systematic patterns we are highlighting.[25]

In the fourth column, we report returns using country-specific prices and capital shares. Similar to our results with only country-specific prices, dispersion falls slightly, and particularly so among portfolios 1 to 3 (although as in column 2, the US shows the largest change). On the other hand, portfolios 1 to 3 continue to exhibit returns that are significantly different from those in the US, and so the main message does not change.

To understand better why we find significant differences in returns where others have not, perhaps most prominently Caselli and Feyrer (2007), we recompute returns for only the year 1996—the year that the authors study—under our baseline approach and each alternative. In other words, we compute the dispersion in returns for a single cross-section rather than over the entire time-period. Under our baseline, the spread in returns in 1996 is much smaller than the average over the period, falling to less than 2% from almost 7%. Although the difference from the US remain statistically significant for portfolio 1, the magnitude is clearly much smaller. Using country-specific prices, statistical significance as well as the systematic pattern across portfolios disappears.[26] Similar patterns hold with country-specific α's and the combination of the two. Thus, under any of these approaches, differences across portfolios are significant—both economically and statistically—when the entire time-period is under examination, but are not under only the single 1996 cross-section.[27] What we conclude is that differences in the time

[24]Over this period, there are roughly 20 countries in each portfolio.

[25]We are able to obtain a substantially longer time series of stock returns when following a rebalancing approach to forming portfolios, i.e., classifying countries based on their income rank in each year, rather than their average rank. Doing so allows us to cover the period 1970-2013. The results are quite similar to the shorter sample: returns in portfolio 1 average about 20%, compared to about 6.5% for the US.

[26]Portfolio 3, which contains the richest countries in the world, enjoys very high returns in 1996 when computed in this fashion.

[27]We should note that one important reason why Caselli and Feyrer (2007) may have chosen to work with year 1996 is because the prices in the PWT 6.1 version that they use correspond to 1996—the benchmark year in PWT 6.1. Prices in PWT are obtained from the International Comparison Program (ICP), which collects prices of narrowly-defined and comparable consumer and capital goods across retail locations in a given year. The prices used outside of the benchmark years are interpolated, so they should be interpreted with caution. As noted earlier, we rely on an entirely different version of the PWT—8.0, where the price data were collected in year 2005. Moreover, in our baseline case, where we compute returns from the point of view of a US investor, we rely on price indices for consumption, investment and output for the US from the BEA, which samples prices annually, thus circumventing the problem of interpolated prices between ICP benchmarks. We do use GDP data (in current 2005 PPP prices) from the PWT, so the price of output of each country relative to the US in all years reflects the 2005 PPP adjustment.

periods under study is the primary reason why we find systematic cross-country differences where some other studies have not.[28] Our proposed risk-based resolution is designed to account for long-run differences in returns, i.e., differences in mean returns over time, not those in any particular year based on some particular realization of the stochastic processes driving returns.

Capital market frictions. One reason that measured returns differ systematically across countries may be the presence of frictions associated with foreign investments in some countries. These capital market distortions may be explicit (ex. trading limits, taxes, etc.) or implicit—for example, Gourinchas and Jeanne (2013) posit that credit market imperfections, expropriation risk, bureaucracy, bribery, and corruption in poor countries may result in a 'wedge' between social and private returns to physical capital there. In our accounting framework, such a wedge may imply that the US investor expects to receive only a fraction of the dividend and/or capital gains yield on investments in poor countries. Hence, in order to invest there, he would demand higher pre-wedge rates of return.

Measuring the types of frictions described above with the intent of adjusting realized returns is very difficult.[29] The existing literature, however, has made attempts to quantify these frictions, commonly referred to as capital controls, and to categorize countries according to their degree of 'capital account openness.' To understand whether systematic differences in openness can account for the observed return differentials in the data, we recompute portfolio returns using only the countries that have open capital accounts. The thought experiment is as follows: if differences in capital controls are the primary source of differences in returns to capital across countries, then returns should be at least approximately equalized among countries with open capital accounts.[30]

Chinn and Ito (2006), Quinn (2003), and Grilli and Milesi-Ferretti (1995) provide measures

[28] We should note that the returns across portfolios over the last decade of the PWT data show some convergence compared to earlier periods. However, insufficient data are yet available to determine whether this is a temporary or more permanent change. For example, as discussed above, stock return data continue to show substantial differences over recent periods (2003-2013).

[29] For example, Gourinchas and Jeanne (2013) impute the capital wedge for each country so as to match the discrepancy between actual investment rates in the data and those predicted by a one-sector deterministic neoclassical growth model with a capital tax and fixed world interest rate. The authors find that the imputed capital wedge is higher in poorer countries—an observation that is consistent with the existence of capital market distortions. As the authors note, however, the wedge is consistent with another mechanism: inefficiencies in producing investment goods in poor countries that distort the relative price of capital to consumption goods as argued by Hsieh and Klenow (2007). It is precisely for this reason that we adopt a two-sector (stochastic) framework in this paper to compute returns to capital. As we demonstrate in Table 1 above, our finding that returns to capital are higher in poorer countries is robust when using country-specific data on prices of investment and consumption.

[30] In an additional exercise, when considering stock returns as discussed above, MSCI reports for a few countries and years returns both before and after withholding taxes. Using these to impute some measure of the effective tax rates, we find no significant relationship between the level of taxes and income.

of capital account openness at the country-year level.[31] The first two indices provide continuous measures of openness, while the last is an indicator function. For each of the first two indices, we compute the median index value over the covered period and we define a country to be open in a given year if its index value exceeds this threshold. In the case of the Grilli/Milesi-Ferretti index, we define a country to be open in every year in the sample the indicator takes on the value of 1.

Having obtained definitions of openness, we turn to the three portfolios analyzed in the baseline case and examine only the countries that are considered open according to one of the three indices described above. The list of open countries according to each measure, classified by portfolio, are reported in Appendix D. Notice that the number of open countries in portfolio 1 is significantly smaller than the number of open countries in portfolios 2 and 3. Thus, there is some evidence that poorer countries are characterized by more strict capital controls. In addition, there is considerable overlap across the different measures of openness, which is reassuring.

Table 2: Capital Returns - Open Countries

	Measure of Openness		
Portfolio	Chinn, Ito	Quinn	Grilli, Milesi-Ferretti
1	10.38***	12.39***	11.48***
	(0.66)	(0.67)	(0.59)
2	8.74***	11.27***	10.28***
	(0.61)	(0.62)	(0.86)
3	5.61	6.66	7.57**
	(0.50)	(0.51)	(0.63)
US	5.22	6.06	5.85
	(0.38)	(0.38)	(0.52)

Notes: Table reports the returns to capital across portfolios for economies that are characterized as open according to three indices: Chinn/Ito, Quinn, and Grilli/Milesi-Ferretti, respectively. Chinn/Ito and Quinn openness cutoff is median value in sample. Grilli/Milesi-Ferretti openness indicator is unity. Standard errors are reported in parentheses. Asterisks denote significance of difference from US values: ***: difference significant at 99%, **: 95%, and *: 90%.

Table 2 reports the portfolio returns in open countries, classified according to each of the three different measures, including the returns on US capital. The returns to capital for the US differ across columns due to the different time periods covered by each openness measure. Overall, portfolios 1 and 2 yield significantly higher rates of return to US investors, regardless

[31]The Grilli/Milesi-Ferretti index covers 61 countries during the 1966-1995 period. Quinn (2003) covers a large number of countries during the 1950-2004 period. Chinn and Ito (2006) build on the work by Quinn (2003) and expand the country coverage to the majority of countries in the world as well as extend the time coverage to 2011.

of the measure of openness employed. Returns are monotonically decreasing across portfolios, as in the baseline. Portfolio 3 remains higher than the US, although the difference is somewhat narrower, and is statistically significant in only one case.[32]

In sum, the negative link between level of income and returns to capital is present among economies that one might classify as "open," suggesting that capital control differences do not account for the majority of the difference in observed returns between rich and poor countries. Consequently, in the next section we propose an alternative explanation for the return differential—namely, one that builds on differences in risk.

3 A Long-Run Risk Resolution

In this section, we explore the potential for a long-run risk based explanation of the cross-sectional patterns in capital returns documented above. In particular, we ask whether differences in the extent of uncertainty in economic growth prospects account for the high return/low income vs. low return/high income pattern observed in the data. To answer this question in a quantitatively precise way, we develop a long-run risk model in the spirit of Bansal and Yaron (2004).[33] Specifically, we place our representative investor in an endowment economy in which both consumption and payments to capital experience shocks to trend growth rates.[34] Each region (e.g., country or portfolio) is exposed to both a global and idiosyncratic component that each impact expected long-run growth rates in the economy. Regions differ in their exposure to the global shock process and in the characteristics of the idiosyncratic one. From the perspective of a US investor, only the former are risky and command a return premium.[35] A key challenge in measuring the quantitative importance of long-run risks is to empirically disentangle these two processes; after outlining our model and its implications for return differentials, we will propose an empirical strategy that does precisely this.

[32] Returns are even closer to the baseline when using a cutoff for openness in the Quinn database that corresponds to the cutoff used by Lustig and Verdelhan (2007).

[33] The two other leading approaches to resolve the asset-pricing puzzles are: external habits (Campbell and Cochrane, 1999), and rare disasters (Barro, 2006; Gabaix, 2008). A model of rare disasters may, potentially, be complementary to our approach.

[34] Our focus on an endowment economy is mostly for tractability and to focus on return implications, given the dynamics of consumption and the total capital stock. Interestingly, recent work by Backus et al. (2014) shows that endogenizing these variables makes little difference for the behavior of asset prices.

[35] In our model, a high degree of long-run risk in some particular region, i.e, volatile or highly persistent growth shocks, does not necessarily mean that the US investor demands a risk premium on his investment there; this is only the case if these shocks are global, in the sense that they affect the investor's SDF. Purely regional long-run shocks do not, and so they do not command significant risk premia.

3.1 The Model

Preferences. We analyze a representative US investor with recursive preferences à la Epstein and Zin (1989). The investor seeks to maximize lifetime utility

$$V_t = \left[(1-\beta) C_t^{\frac{\psi-1}{\psi}} + \beta \nu_t (V_{t+1})^{\frac{\psi-1}{\psi}} \right]^{\frac{\psi}{\psi-1}}, \quad \nu_t (V_{t+1}) = \left(\mathbb{E}_t \left[V_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{1-\gamma}}$$

where ψ denotes the intertemporal elasticity of substitution, γ measures risk aversion, β is the rate of time discount, and $\nu_t (V_{t+1})$ is the certainty equivalent of period $t+1$ utility. Standard arguments give the following Euler equations governing optimal investment in capital in region j and the riskless asset, respectively:

$$\begin{aligned} 1 &= \mathbb{E}_t \left[M_{t+1} R_{j,t+1} \right] \\ 1 &= \mathbb{E}_t \left[M_{t+1} R_{f,t+1} \right] \end{aligned} \tag{3}$$

with the investor's SDF given by

$$M_{t+1} = \beta^\theta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{\theta}{\psi}} R_{c,t+1}^{\theta-1}$$

where $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$ and $R_{c,t+1}$ denotes the return on an asset that pays aggregate consumption as its dividend, or equivalently, the return to aggregate wealth.

Dynamics of consumption and dividends. Growth in each region is exposed to a number of transitory ('short-run') fluctuations, as well as to a small but persistent ('long-run') component that affects expected future growth prospects. The latter is comprised of two pieces: a shock to 'global' growth prospects that impacts all regions and a shock to region-specific idiosyncratic growth. Regions differ in their sensitivity to the global shock, as well in the properties of their idiosyncratic process. An intuitive interpretation of our setup is that there are both global and local long-run risk components that influence growth prospects in a particular region. Although only the former impact both consumption growth of the US-based investor and the menu of assets in which he can invest and so warrant a risk-premium, modeling the latter is key to ensure that we do not incorrectly label local long-run shocks as global and so overstate the implications for return differentials. Additionally, we allow for correlation in short-run shocks so that, first, we do not attribute all comovement to the global long-run shock, and second, so as to include a source of transitory risk and allow for a decomposition of return differentials into long- and short-run components. Disentangling the importance of these two sources of comovement is a second empirical challenge that we need address in our empirical approach.

The following system lays out the joint dynamics of US and foreign consumption and dividends. We denote with an asterisk a representative foreign region, a convention we follow hereafter.

$$\begin{aligned} \Delta c_{t+1} &= \mu_c + x_t + \eta_{t+1} \\ x_{t+1} &= \rho x_t + e_{t+1} \\ \Delta d_{t+1} &= \mu_d + \phi x_t + \pi \eta_{t+1} + \mu_{t+1} \\ \Delta c^*_{t+1} &= \mu^*_c + \xi^* x_t + x^*_t + \pi^*_c \eta_{t+1} + \eta^*_{t+1} \\ x^*_{t+1} &= \rho^* x^*_t + e^*_{t+1} \\ \Delta d^*_{t+1} &= \mu^*_d + \tilde{\phi}^* \left(\xi^* x_t + x^*_t\right) + \pi^* \eta_{t+1} + \pi^*_d \mu_{t+1} + \pi^*_{cd} \eta^*_{t+1} + \mu^*_{t+1} \end{aligned} \tag{4}$$

A detailed description of the environment is as follows: turning first to the US, μ_c is the unconditional mean of consumption growth and x_t a time-varying, small but persistent component of the growth rate, so that the conditional expectation at time t of consumption growth in $t+1$ is $\mu_c + x_t$. The persistent component evolves according to an AR(1) process with persistence ρ and variance in the innovations σ^2_e. Consumption growth is also subject to purely transitory shocks η_{t+1} with variance σ^2_η. Dividend growth in the US has unconditional mean μ_d and a levered exposure to the persistent component of consumption growth, x_t, captured by ϕ. Intuitively, the higher the value of ϕ the more responsive are dividend growth rates to innovations in x. The transitory consumption shock η_{t+1} also influences the dividend process, with the magnitude of this relationship governed by π. This will generate some degree of 'short-run' risk due to period-by-period transitory fluctuations along the lines of that present in a workhorse CCAPM model (or a model with no long-run risk) and enables us to assess the relative contributions of long and short-run risk to return differentials. Dividend growth is also subject to a purely transitory shock μ_{t+1} with variance σ^2_μ. All shocks are i.i.d. and normally distributed.

The dynamics in the foreign regions are similar. Consumption growth has unconditional mean μ^*_c and also contains a small but persistent component, which is now composed of two pieces. First, consumption in each region is exposed to the US long-run shock x, with the degree of exposure governed by ξ^* (the US is normalized to 1) and it is in this sense that x represents a 'global' shock. Additionally, there is a region-specific idiosyncratic long-run component x^*, which again evolves according to an AR(1) process with persistence ρ^* and variance in the innovations $\sigma^2_{e^*}$.[36] Foreign consumption growth exhibits transitory comovement with the US, governed by π^*_c (i.e., there is a global component in transitory shocks to consumption growth) and lastly, is subject to an idiosyncratic transitory shock η^*_{t+1} with variance $\sigma^2_{\eta^*}$. Dividend growth has unconditional mean μ^*_d and a levered exposure to the persistent component of consumption, captured by $\tilde{\phi}^*$. Similar to the US, higher values of $\tilde{\phi}^*$ imply a greater degree of exposure to innovations in x and x^*, so that regions with higher values of $\tilde{\phi}^*$ are more sensitive to changing economic growth conditions. We allow for transitory shocks to foreign dividend growth rates to be correlated with temporary shocks to US dividend growth rates μ_{t+1}, local consumption growth η^*_{t+1}, and US consumption growth η_{t+1}. The strength of these relationships is captured by π^*_d, π^*_{cd}, and π^*, respectively. Lastly, each region is also subject to an idiosyncratic transitory shock μ^*_{t+1} with variance $\tilde{\sigma}^2_{\mu^*}$. As in the US, all shocks are i.i.d. and normally distributed.

In short, our model is quite rich in terms of the dynamics of growth, allowing for both transitory and persistent fluctuations in growth rates, and common and idiosyncratic components of each. When we outline our empirical strategy, we will discuss the role that each element plays in enabling us to accurately pin down the extent of shared long-run risks and to assess the implications for return differentials. Specifically, we will demonstrate that predicted returns from our model depend crucially on properly accounting for regional long-run shocks and distinguishing comovement that arises from long-run and short-run sources; failure to address these issues gives predicted returns that may be substantially biased (generally upward).

Risk premia. We solve the model and derive its asset return implications using standard approximation methods.[37] Risk premia (in excess of the risk-free rate) on the US and foreign capital assets are, respectively:

$$\begin{aligned} \mathbb{E}\left[r^e_t\right] &= \left(\phi - \frac{1}{\psi}\right)\left(\gamma - \frac{1}{\psi}\right)\frac{\kappa_{m,1}}{1-\kappa_{m,1}\rho}\frac{\kappa_1}{1-\kappa_1\rho}\sigma^2_e + \gamma\pi\sigma^2_\eta \\ \mathbb{E}\left[r^{e*}_t\right] &= \left(\phi^* - \frac{1}{\psi}\right)\left(\gamma - \frac{1}{\psi}\right)\frac{\kappa^*_{m,1}}{1-\kappa^*_{m,1}\rho}\frac{\kappa_1}{1-\kappa_1\rho}\sigma^2_e + \gamma\pi^*\sigma^2_\eta, \end{aligned} \tag{5}$$

where $\phi^* \equiv \tilde{\phi}^*\xi^*$. The parameter κ_1 is a constant of linearization that is endogenous and depends in a nonlinear way on the parameters of the US consumption process. Similarly, $\kappa_{m,1}$ and $\kappa^*_{m,1}$ are linearization constants that depend additionally on the parameters of the US and

[36] We model the US x as directly influencing the foreign consumption process. Alternative approaches would be to explicitly include a world x and a US-specific one, or only region-specific x's with some correlation in their innovations. These approaches are all clearly related, and mainly involve a relabeling of the parameter governing the extent of comovement ξ^*.

[37] Because these techniques are widely used, we detail the steps in Appendix B.

foreign dividend processes, respectively. The mean risk free-rate is given by:

$$\mathbb{E}\left[r_{f,t}\right] = -\log\beta + \frac{\mu}{\psi} + \frac{1}{2}\left(\frac{1-\gamma}{\psi} - \gamma\right)\sigma_\eta^2 + \frac{1}{2}(\theta - 1)\left(1 - \frac{1}{\psi}\right)^2\left(\frac{\kappa_1}{1-\kappa_1\rho}\right)^2\sigma_e^2 \tag{6}$$

Intuition. Equation (5) in particular reveals a bit of intuition regarding the behavior of returns in the long-run risk setting. Excess returns are composed of two pieces, the first relating to long-run risks and the second to short-run. First, notice that setting $\gamma = \frac{1}{\psi}$, which is the case of CRRA preferences, eliminates the long-term component so that risk premia are determined only by the transitory comovement in consumption and dividend growth ($\pi\sigma_\eta^2$), which is 'priced' at γ. The same is true if $\sigma_e^2 = 0$, that is, there is no long-run risk in consumption growth, or if $\rho = 0$, simply with an adjustment to reflect the additional variance σ_e^2, which would then be purely transitory in nature. Thus, both recursive preferences, i.e., the disentangling of γ and ψ, as well as the persistent and stochastic nature of growth rate shocks are necessary for risk premia to differ from the case with no long-run risk.

The sensitivity of dividends to changing global growth conditions, ϕ in the US and ϕ^* abroad, although not sufficient statistics, are key in determining risk premia. Intuitively, the higher this sensitivity, the riskier is the asset and the higher the associated risk premium. An important piece of our empirical approach then is to pin down the values of these parameters. In contrast, the idiosyncratic portion of the long-run shock abroad (x_t^*) does not enter the return equations anywhere. Because these shocks are by construction only regional, they do not enter the US investor's SDF and so are not risky from his perspective; in other words, these idiosyncratic shocks are diversifiable and so do not garner risk premia. This does not mean that we can ignore these shocks, however; doing so may bias our estimates of the parameters of the model that do determine returns. We next outline an identification strategy that addresses this challenge, among others.

3.2 Identifying Long-Run Risks

To derive the model's return implications and assess its ability to account for the cross-section of capital returns in the data, we must assign values to the parameters governing the consumption and dividend processes laid out in (4). As we noted above, there are a number of hurdles that make this a challenging task. First, any empirical approach must disentangle common long-run shocks from purely idiosyncratic ones. As we have seen, these two sources of changing growth prospects have very different asset pricing implications; indeed only the first demands a risk premium. Despite this, the second will likely show up in moments that we would otherwise think are informative about common shocks and so must be accounted for. A second empirical challenge is in distinguishing common long-run and short-run shocks. Because both types of shocks generate comovement across regions, it is necessary to account for the latter when using otherwise informative moments regarding comovements to infer the extent of common long-run risk.

In this section, we outline our empirical strategy to address these difficulties. Our approach uses a combination of moments in comovements, persistence, and volatilities to infer the parameters of interest. Specifically, we demonstrate that moments in consumption growth in the US alone, dividend growth in each region, and the comovement of returns in each region with those in the US enable us to pin down all the necessary parameters of our model.

Preferences and US consumption. We begin by assigning values to the preference parameters. We set $\gamma = 10$, $\psi = 1.5$, and $\beta = 0.99$, all standard values in the long-run risk literature. Our choice of β enables the model to approximately match the mean level of the risk-free rate.

Turning to the US investor's consumption process, we first assign a value to the persistence parameter ρ. This parameter is notoriously difficult to identify and rather than attempting to do so, we take guidance from the existing literature and set its value to 0.93.[38] This is reported as the mean estimate from annual US data in Ferson et al. (2013) and is quite close to the annual estimate from Bansal et al. (2012b) of 0.91.[39]

We determine the values of the remaining parameters of the US consumption process in order to match the unconditional mean, variance, and autocovariance of US consumption growth. Specifically, it is straightforward to derive the following expressions relating parameters to observable empirical moments from (4):

$$\begin{aligned} \mathbb{E}\left[\Delta c_t\right] &= \mu_c \\ \operatorname{cov}\left(\Delta c_t, \Delta c_{t+1}\right) &= \rho \frac{\sigma_e^2}{1-\rho^2} \\ \operatorname{var}(\Delta c_t) &= \frac{\sigma_e^2}{1-\rho^2} + \sigma_\eta^2 \end{aligned} \tag{7}$$

The mean growth rate is pinned down by its sample value. The autocovariance in consumption growth depends on the persistence in x, ρ, and the variance of x, $\operatorname{var}(x_t) = \frac{\sigma_e^2}{1-\rho^2}$. The total variance in consumption growth is the sum of the variance in x and the variance in the transitory component, σ_η^2. We use the latter two equations to solve for σ_e^2 and σ_η^2.

[38]Because we are interested in the spread in returns between foreign regions and the US, another approach here would have been to choose ρ to match the mean US return. Our model prediction will be quite close to the actual return for the US, lending an additional degree of confidence in the value of ρ.

[39]Much of the long-run risk literature considers monthly decision frequencies and estimates parameters to match moments aggregated to the annual frequency. We abstract from this issue here and focus only on an annual model.

US Dividends. We pin down the parameters of the US dividend process in a similar fashion. These include: the mean growth rate of dividends μ_d; the exposure to the persistent shock ϕ; the correlation with transitory consumption shocks, governed by π; and the volatility of the transitory shock, σ_μ^2. We set these to match the observed mean growth rate in dividends, the autocovariance of dividend growth relative to that of consumption growth, the covariance of dividend growth with consumption growth, and the variance of dividend growth. To see why these are informative moments, examine the equations in (8), which are straightforward to derive from (4):

$$\begin{aligned} \mathbb{E}\left[\Delta d_t\right] &= \mu_d \\ \sqrt{\frac{\operatorname{cov}\left(\Delta d_{t+1}, \Delta d_t\right)}{\operatorname{cov}\left(\Delta c_{t+1}, \Delta c_t\right)}} &= \phi \\ \operatorname{cov}\left(\Delta d_t, \Delta c_t\right) &= \phi \frac{\sigma_e^2}{1-\rho^2} + \pi \sigma_\eta^2 \\ \operatorname{var}(\Delta d_t) &= \phi^2 \frac{\sigma_e^2}{1-\rho^2} + \pi^2 \sigma_\eta^2 + \sigma_\mu^2 \end{aligned} \tag{8}$$

The mean growth rate μ_d is identified directly from its sample value. The leverage parameter ϕ is pinned down by the ratio of the autocovariance in Δd to that in Δc. This is intuitive: by capturing the exposure to the persistent shock x, ϕ influences the persistence of Δd. At the same time, common to any leverage adjustment, ϕ also affects the volatility of Δd relative to that of Δc in response to the same innovation in x. Thus, it is not only the autocorrelation that provides information on ϕ, but the autocovariance, which encodes both relative persistence and volatility in exactly the right combination.[40] With a value for ϕ in hand, it is straightforward to use the covariance of Δd with Δc to infer π. This covariance is composed of two parts: comovement through mutual dependence on x, and additionally through correlation in the transitory consumption shock. Knowing the value of ϕ, we can distinguish these two components and back out a value for π. Lastly, we use the total variance of Δd to infer a value for σ_μ^2. The variance is composed of three elements: a piece associated with volatility in x, with exposure to the transitory consumption shock, and with the dividend-specific shock. Knowing ϕ and π enables us to compute the first two components; it is then straightforward to back out σ_μ^2.

Foreign Consumption and Dividends. In our model, returns to both US and foreign assets are in large part driven by their exposure to the global persistent shock, governed by ϕ

[40] Note that this identification approach would pin down the correct value of ϕ were we to model the US process as the combination of a world x and US-specific one and so the implications for excess returns would be the same.

in the US and ϕ^* abroad. Therefore, a seemingly natural way to identify ϕ^* would be to follow an approach analogous to the one used above to infer ϕ. Due to the presence of local long-run shocks (i.e., x^*), however, the ratio of the autocovariance of foreign dividend growth to that of US consumption does not deliver the true value of ϕ^*. Indeed, we can derive the following equation from (4), which makes clear the difficulty:

$$\sqrt{\frac{\text{cov}\left(\Delta d^*_{t+1}, \Delta d^*_t\right)}{\text{cov}\left(\Delta c_{t+1}, \Delta c_t\right)}} = \phi^* \sqrt{1 + \frac{1}{\xi^{*2}} \frac{\sigma^2_{e^*}}{\sigma^2_e}} \tag{9}$$

This moment identifies the true ϕ^* only if $\sigma^2_{e^*} = 0$, that is, only if there are no regional long-run shocks. Otherwise, relying on this moment gives a biased estimate of ϕ^*, with the bias corresponding to the term in square root. This term is weakly larger than 1, implying that the estimate of ϕ^* is upwardly biased, which would tend to deliver a higher estimate of the risk premium. We show below that the quantitative impact of this bias can be substantial.

An analogous strategy using the ratio of the autocovariance of foreign dividends to foreign consumption growth is not sufficient. This moment exactly identifies $\tilde{\phi}^*$, but does not hold any information regarding ξ^* and therefore does not pin down ϕ^*, as can be seen from the following expression:

$$\sqrt{\frac{\text{cov}\left(\Delta d^*_{t+1}, \Delta d^*_t\right)}{\text{cov}\left(\Delta c^*_{t+1}, \Delta c^*_t\right)}} = \tilde{\phi}^* \tag{10}$$

In other words, this ratio tells us the translation from persistence in local consumption growth to dividend growth, but does not identify how the local consumption process itself depends on the long-run shock.

Finally, since x^* is orthogonal to x, it may seem that the comovement between domestic and foreign dividend growth would be an informative moment to identify the exposure of foreign dividends to the common component x. Although this moment is indeed unaffected by the foreign long-run shock x^*, because we allow for correlation in the transitory movements of dividend growth rates across countries, this single moment does not provide enough information to disentangle comovement due to long-run and short-run components. To demonstrate this fact, we can derive the following expression:

$$\text{cov}\left(\Delta d^*_t, \Delta d_t\right) = \phi\phi^* \frac{\sigma^2_e}{1-\rho^2} + \pi^*\pi\sigma^2_\eta + \pi^*_d\sigma^2_\mu \tag{11}$$

The US parameters $\left(\phi, \sigma^2_e, \rho, \pi, \sigma^2_\eta, \sigma^2_\mu\right)$ are identified using US data as already described. However, even if we knew the value of π^*, which measures the exposure of foreign dividend growth to the transitory component of US consumption growth, the moment in (11) does not separately identify ϕ^* from π_d^*. Failure to account for the portion of the comovement of dividend growth rates that is due to temporary shocks—π_d^*—may bias the estimate of ϕ^*. We show below that the magnitude of this bias can be substantial, and perhaps more importantly, is systemically related to income levels: it is negative for poorer regions and positive for richer. Thus, the presence of regional long-run shocks in conjunction with correlation in transitory movements in growth rates confounds a number of seemingly available identification approaches.

To overcome this challenge, we propose a strategy that builds on the insight of Lewis and Liu (2012), who face a related problem of having to distinguish correlations in persistent and transitory sources of risk across a few developed economies. Specifically, they show that, in addition to dividend growth comovement, the comovement of returns contains information that can be used to disentangle long-run from short-run correlations. Although our environment is not identical to theirs, the strategy we employ is very much in this spirit.

The six moment conditions we use are given by:

$$
\begin{aligned}
\mathbb{E}\left[\Delta d_t^*\right] &= \mu_d^* & (12)\\
\operatorname{cov}\left(\Delta d_t^*, \Delta c_t\right) &= \phi^* \frac{\sigma_e^2}{1-\rho^2} + \pi^* \sigma_\eta^2 & (13)\\
\operatorname{cov}\left(\Delta d_t^*, \Delta d_t\right) &= \phi \phi^* \frac{\sigma_e^2}{1-\rho^2} + \pi^* \pi \sigma_\eta^2 + \pi_d^* \sigma_\mu^2 & (14)\\
\operatorname{cov}\left(\Delta d_{t+1}^*, \Delta d_t^*\right) &= (\tilde{\phi}^* \sigma_{e^*})^2 \frac{\rho^*}{1-\rho^{*2}} + (\phi^* \sigma_e)^2 \frac{\rho}{1-\rho^2} & (15)\\
\operatorname{var}\left(\Delta d_t^*\right) &= (\phi^*)^2 \frac{\sigma_e^2}{1-\rho^2} + \frac{(\tilde{\phi}^* \sigma_{e^*})^2}{1-\rho^2} + {\pi^*}^2 \sigma_\eta^2 + {\pi_d^*}^2 \sigma_\mu^2 + {\pi_{cd}^*}^2 \sigma_{\eta^*}^2 + \tilde{\sigma}_{\mu^*}^2 & (16)\\
\operatorname{cov}\left(r_{m,t}^*, r_{m,t}\right) &= \frac{1}{\psi^2} \frac{\sigma_e^2}{1-\rho^2} + \pi^* \pi \sigma_\eta^2 + \pi_d^* \sigma_\mu^2 + \\
&\quad \frac{\kappa_{m,1}}{1-\kappa_{m,1}\rho} \frac{\kappa_{m,1}^*}{1-\kappa_{m,1}^*\rho} \left(1 - \frac{1}{\psi}\right) \left(\phi^* - \frac{1}{\psi}\right) \sigma_e^2, & (17)
\end{aligned}
$$

We describe our empirical approach below, and we begin by noting that, because the κ's in (17) are nonlinear functions of the other parameters of the model, we employ a numerical fixed point procedure. We first use (12) to infer the mean dividend growth rate μ_d^* from its sample value. We then guess a candidate value for ϕ^*. Using this guess and the set of US parameters, (13) pins down π^*. The use of this moment is intuitive. π^* measures the exposure of foreign dividend growth to temporary shocks to US consumption growth, while ϕ^* measures the exposure to persistent shocks. Given a value for ϕ^*, the covariance of foreign dividend growth with US consumption growth is informative about π^*. With these values in hand, we next obtain π_d^* from (14). The intuition here is similar. The comovement of dividend growth across countries is governed by their exposure to common consumption shocks, both persistent and temporary, as well as by π_d^*, which measures the correlation in transitory shocks to dividend growth rates. Having already parameterized the first two sources of comovement, (14) pins down the last, π_d^*.

Given ϕ^*, (15) shows that the autocovariance of foreign dividend growth pins down the parameter combination $\tilde{\phi}^*\sigma_{e^*}$ (for a given value of ρ^*). To arrive at this condition we take the following steps. First, it is straightforward to derive the autocovariance of foreign consumption growth as:

$$\text{cov}\left(\Delta c_{t+1}^*, \Delta c_t^*\right) = \frac{\rho}{1-\rho^2}{\xi^*}^2\sigma_e^2 + \frac{\rho^*}{1-{\rho^*}^2}\sigma_{e^*}^2$$

Substituting out $\text{cov}\left(\Delta c_{t+1}^*, \Delta c_t^*\right)$ using expression (10) and rearranging yields expression (15). This expression decomposes the autocovariance of foreign dividend growth into its global and local components. Next, given $\phi^*, \tilde{\phi}^*\sigma_{e^*}^2, \pi^*, \pi_d^*$, (16) shows that the variance of foreign dividend growth pins down the object ${\pi_{cd}^*}^2\sigma_{\eta^*}^2 + \tilde{\sigma}_{\mu^*}^2$, which we will denote by $\sigma_{\mu^*}^2$. Intuitively, after accounting for the variance in foreign dividend growth due to exposure to persistent consumption shocks as well as common transitory shocks in consumption and dividend growth, the remaining variance is due to correlation with temporary shocks to foreign consumption and the idiosyncratic dividend-specific shock. The last two elements need not be disentangled because they affect returns in an equivalent fashion.[41] Finally, we construct the model-implied covariance of returns in expression (17) and iterate on the initial guess of ϕ^* until we match the empirical moment.[42] Notice why (17) is informative about ϕ^*: both the comovement in dividend growth rates (14) and returns (17) depend on both types of common shocks, transitory and persistent. While the former enter both equations in an identical way, return comovement is more sensitive to common long-run shocks than is dividend comovement. Intuitively, a persistent shock leads to large revisions in asset valuations, since expected future growth prospects are changed, which serves to increase the comovement of asset returns *relative* to the comovement in period-by-period dividend growth rates. The higher is ϕ^*, the greater the response of foreign returns to the global shock and the higher the comovement of returns relative to that in dividends.[43]

Our empirical strategy does not require us to separately identify $\sigma_{e^*}^2$ from $\tilde{\phi}^*$, where the first governs the volatility of the persistent shock in the foreign region and the second the sensitivity

[41] We show in Appendix B that $\sigma_{\mu^*}^2 = {\pi_{cd}^*}^2\sigma_{\eta^*}^2 + \tilde{\sigma}_{\mu^*}^2$ and $\tilde{\phi}^*\sigma_{e^*}^2$ are sufficient to compute $\kappa_{m,1}^*$, which is the only place they factor in in determining returns.

[42] Our estimate of return variance in the US is slightly higher than the empirical value (0.035 vs. 0.027). To account for this discrepancy, we normalize both the model-implied and empirical moment by the standard deviation of US returns and match this moment. In other words, we match the 'beta' of foreign returns on US returns, $\frac{\text{cov}\left(r_{m,t}^*, r_{m,t}\right)}{\text{std}(r_{m,t})}$.

[43] This is precisely the intuition pointed out in Lewis and Liu (2012).

of foreign dividend growth to this shock. Hence, our approach does not necessitate the use of any moments in consumption growth in the foreign region. We view this as a significant advantage of the strategy we propose, as we are able to circumvent certain data requirements. For example, alternative approaches that would require separate values of $\sigma^2_{e^*}$ and $\tilde{\phi}^*$ would likely require a consumption series for each country. Ideally, one would use the equivalent data on non-durables and services that we obtain from the BEA for the US. Unfortunately, such data do not exist in the cross-section of countries. At best, we could obtain total consumption series for each country from the PWT. Even if these data are a good proxy for the consumption series we would need, for many countries, the series are quite short, rendering it problematic to compute reliable time series moments.[44] We should note, however, that to implement our (or likely any other) identification strategy, we require a value for ρ^*, a parameter that is highly difficult to estimate for the US, and more so in emerging markets for which data are significantly more limited. We begin by making the simple assumption that $\rho^* = \rho$.[45]

Moments and Parameters. Table 3 reports the target moments for the 3 portfolio case. All measures are consistent with the procedure in Section 2.2. The moments for each portfolio represent the mean values for the countries in that portfolio. This is where the portfolio approach proves useful: our hope is to cleanse country-specific idiosyncratic variation in the moments and isolate the variation that is related to the level of income. We provide further details on our empirical work in Appendix A.

In the top panel of Table 3, we display the moments in US consumption and dividend growth rates. The mean growth rate in consumption is about 2%, with a standard deviation of about 0.02 and an autocovariance of 0.00024, which together imply an autocorrelation of about 0.5. These values are quite close to those found by other authors over similar time periods.[46] Turning to the portfolio moments, portfolios 1 and 2 display a high covariance of returns with those in the US (correlations are actually ordered in the reverse direction; the primary driver of the ordering we find is higher return volatility in poorer countries, scaling up covariances). In addition, dividend growth rates for these portfolios are more volatile. At the same time, these portfolios exhibit the lowest covariance in dividend growth rates: these patterns jointly suggest relatively low contemporaneous correlations in transitory shocks with the US and high exposure to the global long-run shock. Similarly, the covariance of dividend growth rates with US consumption growth is increasing with income across the portfolios, which again suggests

[44] Including durables is problematic since the connection to the marginal flow utility of consumption (i.e., to the agent's SDF), which in this case is likely very different than current consumption expenditures, is much more tenuous. Additionally, measurement error is likely a bigger concern for foreign consumption data.

[45] A similar assumption is made in, for example, Lewis and Liu (2012) and Colacito and Croce (2011). We show below that our results are only negligibly affected by even large changes in this parameter.

[46] See, for example, Bansal and Yaron (2004) and Bansal et al. (2012a).

that developed countries co-move more with the US at high frequencies.

Table 3: Target Moments - 3 Portfolios

US Moments						
Consumption	$\mathbb{E}[\Delta c_t]$	$\text{cov}(\Delta c_{t+1}, \Delta c_t)$	$\text{std}(\Delta c_t)$			
	0.019	0.00024	0.022			
Dividends	$\mathbb{E}[\Delta d_t]$	$\sqrt{\frac{\text{cov}(\Delta d_{t+1}, \Delta d_t)}{\text{cov}(\Delta c_{t+1}, \Delta c_t)}}$	$\text{cov}(\Delta d_t, \Delta c_t)$	$\text{std}(\Delta d_t)$		
	−0.006	2.19	0.00018	0.026		
Foreign Moments						
Portfolio	$\mathbb{E}[\Delta d_t^*]$	$\text{cov}\left(\Delta d_{t+1}^*, \Delta d_t^*\right)$	$\text{cov}(\Delta d_t^*, \Delta c_t)$	$\text{std}(\Delta d_t^*)$	$\text{cov}(\Delta d_t^*, \Delta d_t)$	$\text{cov}(r_t^*, r_t)$
1	−0.017	0.00122	0.00011	0.083	0.00032	0.00109
2	−0.015	0.00156	0.00011	0.074	0.00033	0.00100
3	−0.011	0.00075	0.00015	0.063	0.00040	0.00085

Notes: Table reports target moments for baseline parameterization. Consumption is measured as real per-capita consumption of non-durables and services. Consumption moments are computed over the period 1929-2008, the longest available from the BEA. Dividends are measured in exactly the same manner as described in (1). Portfolio moments are computed over the period 1950-2008 using data from PWT and US prices from BEA. The moments for each portfolio represent the mean values for the countries in that portfolio.

Table 4: Parameter Values - 3 Portfolios

Preferences:	$\gamma = 10$	$\psi = 1.5$	$\beta = 0.99$			
Consumption:	$\rho = 0.93$	$\mu_c = 0.019$	$\sigma_e = 0.006$	$\sigma_\eta = 0.02$		
Portfolio	μ_d	ϕ	π	π_d	σ_μ	$\tilde{\phi}^* \sigma_{e^*}$
1	−0.017	5.14	−1.24	−0.16	0.074	0.005
2	−0.015	4.23	−0.81	−0.00	0.061	0.011
3	−0.011	2.87	0.34	0.27	0.056	0.008
US	−0.006	2.19	0.98	-	0.020	-

Notes: Table reports parameter values that match moments for baseline parameterization reported in Table 3.

We report the resulting parameter estimates (along with those assigned outside the model) in Table 4. The consumption parameters are relatively standard. Turning to the asset-specific parameters, the moments detailed above imply a greater exposure of poor countries to the global long-run shock, captured by a higher value of ϕ. In contrast, the correlation of transitory shocks with the US (captured by π and π_d) is generally increasing in income, and, not surprisingly, is highest in the US. This parameter configuration implies that poor countries are more sensitive to the global long-run shock, even while allowing for richer countries to display the greater period-by-period comovement with the US observed in the data. It is precisely these two forces that our empirical strategy allows us to disentangle. Poor countries also experience more volatile idiosyncratic transitory shocks, captured by σ_μ, a property inherited from the ordering of overall dividend growth volatility seen above.[47] Next, we assess the implication of these cross-region differentials for capital returns, that is, do these patterns across income-sorted portfolios lead to mean return differentials of the order we observe in the data?

3.3 Results

Baseline. We report our baseline results in the first two columns of Table 5. The table reports the average return to capital in each portfolio as measured in the data over the entire time period, r, and the average return predicted by the model, $\hat{r}$, from equations (5) and (6) under the parameter configuration in Table 4. The model predicts returns that are very much in line with those in the data and that generally mimic the decreasing pattern across higher income portfolios. Predicted returns range from about 11% for the poorest portfolio, 1, to about 6% for the US, compared to 13% and 6% in the data. The predicted values for the intermediate portfolios 2 and 3 are 9.3% and 7.2%, respectively, compared to 11% and 8%. Across portfolios, the model predicts a mean level of returns of about 8.2%, slightly below, but in line with, the average of 9.5% observed in the data. Finally, and perhaps most importantly, the model predicts a spread in returns between portfolio 1 and the US of about 4.6%, which represents about 66% of the actual spread of 7%. In other words, given the parameter configuration in Table 4, a long-run risk based explanation can account for a large portion—two thirds—of observed return differentials across income-sorted portfolios. These findings have important implications: despite large observed return differences, the allocation of capital is not so far from that predicted by theory, once uncertainty is taken into account. We generally couch our results in the language of returns, but the closer the predicted returns to the actual, the more the empirical allocation of capital resembles the one that theory would predict.

Disaggregated portfolios. In our baseline analysis, we group countries into 3 income-based portfolios, along with the US. In the remainder of Table 5 we examine the implications of our model for more disaggregated groups of countries, specifically, groups of 5 and 10 portfolios (always along with the US). The middle panel reports the results across a 5-portfolio grouping, and the right-hand panel across 10 portfolios.[48] Not surprisingly, as we move to more disaggregated levels, the spread between the poorest countries in portfolio 1 and the US widens, going from about 7% to about 8% to about 10%. The increase is due to a rise in returns in portfolio 1 (the US, of course, remains the same). The model captures this feature of the data to a large extent, with the predicted spread also increasing, although not quite in-step with the actual,

[47] For completeness, we report the estimated value of $\tilde{\phi}^* \sigma_{e^*}$, although because we cannot separate the two components, this value is difficult to interpret in a meaningful way.

[48] We report moments and parameters for the 5 and 10 portfolio cases in Appendix E.

Table 5: Predicted vs. Actual Returns

3 Portfolios			5 Portfolios			10 Portfolios		
	$\widehat{r}$	r		$\widehat{r}$	r		$\widehat{r}$	r
1	10.55	13.02	1	11.02	14.39	1	12.38	16.40
2	9.30	11.07	2	10.80	10.89	2	9.65	12.38
3	7.18	8.05	3	8.75	11.40	3	9.95	10.28
US	5.94	6.02	4	8.30	10.07	4	11.31	11.38
			5	6.28	6.75	5	7.47	10.65
			US	5.94	6.02	6	10.02	12.00
						7	8.06	9.76
						8	8.50	10.40
						9	6.60	7.63
						10	6.04	6.00
						US	5.94	6.02
Average:	8.24	9.54		8.52	9.92		8.72	10.26
Spread: 1-US	4.61	7.00		5.08	8.36		6.44	10.38
Percent of actual	66			61			62	
corr($\widehat{r}, r$)	1.00			0.92			0.91	

Notes: Table reports the average return to capital in each portfolio as measured in the data, r, and as predicted by the model, $\hat{r}$, from equations (5) and (6) under the parameter configuration in Tables 3, 11, and 12 for 3, 5, and 10 portfolio groupings, respectively.

from 4.6% to about 5% to 6.4%. The result is that the model can account for just over 60% of the observed spread with 5 and 10 portfolios, compared to 66% with 3. As we increase the number of portfolios, the correlation of predicted and actual returns becomes a useful statistic, which remains quite high even in the 10 portfolio case at 0.9. What we glean from Table 5 is that long-run risks play a quantitatively important role in driving return differentials across countries at different stages of development, and that this finding is not an artifact of our baseline choice of 3 portfolios: the model predicts returns that are in line with the data at each of the levels of aggregation we examine (and indeed, as we show below, continues to hold significant explanatory power even at the country level).

The sources of risk. Expressions (5) and (6) show that we can express predicted returns to each asset in our model as the sum of 3 components: the risk-free rate, i.e., the level of returns in the absence of any risk, excess returns due to short-run risk, i.e., risk derived from period-by-period comovement between dividend and consumption growth, and excess returns due to long-run risk, i.e., risk derived from volatility in growth-rate regimes. In Table 6, we report the contribution of each source to the total predicted return.

First, the risk-free rate is only about 1%, as in the data, and by construction, is constant

Table 6: The Composition of Returns

	Actual	Predicted						
Portfolio	r	$\widehat{r}$		$\widehat{r}^f$		$\widehat{r}^e_{sr}$		$\widehat{r}^e_{lr}$
1	13.02	10.55	=	1.29	+	−0.28	+	9.54
2	11.07	9.30	=	1.29	+	−0.18	+	8.20
3	8.05	7.18	=	1.29	+	0.08	+	5.82
US	6.02	5.94	=	1.29	+	0.22	+	4.44

Notes: Table reports the average return to capital in each portfolio as measured in the data, r, and as predicted by the model, $\hat{r}$, from equations (5) and (6) under the parameter configuration in Table 4, decomposed into three components: the risk-free rate, excess returns due to short-run risk, and excess returns due to long-run risk.

across portfolios due to our focus on the US investor. Strikingly, excess returns due to short run risk are negligible, ranging from about -0.3% to 0.2%. Negative values indicate a risk compensation: because transitory fluctuations in portfolios 1 and 2 are negatively correlated with US consumption growth, these assets actually represent good hedges for a US investor and so in the presence of only short-run risk, would provide lower returns than in the US. This is quite intuitive: period-by-period fluctuations in more developed countries are more correlated with those in the US, which implies higher risk premia in those countries, exactly opposite of the patterns actually observed in the data. However, the magnitude of the differences is quite small, with the US demanding 0.5% higher returns than portfolio 1 due to short-run risk. Thus, (more than) the entirety of the systematic return differentials predicted by our model is due to long-run risk. The last column in Table 6 shows that long-run risks command excess returns as high as 9.5% in portfolio 1 compared to about 4.4% in the US, leading to a return differential of about 5%.

These findings show that a model without long-run risks, such as the standard CCAPM with power utility, would likely struggle to account for the capital return differentials across rich and poor countries observed in the data. In this sense, the Lucas Paradox resembles the equity premium and other related asset pricing puzzles. In Appendix C, we formalize this argument using the aforementioned model and the standard covariance formula relating risk premia to the comovement of returns with consumption growth. Such a framework struggles to account for the observed return differentials in the data, with the closest fit requiring a coefficient of relative risk aversion of roughly 900, well above levels deemed reasonable. With a coefficient of relative risk aversion of 10, the one used here, the CCAPM generates negligible return differences.

An alternative bundling approach. As an alternative approach to classifying countries into portfolios, we return to our bundling procedure, and now group countries according to income on an annual basis. Specifically, we 'rebalance' our portfolios on an annual basis, i.e., classify countries in each year based on their rank in the income distribution within that particular year.[49] The main advantage of this approach is that it accounts for the changing place of countries as the income distribution evolves; the drawback is that it introduces a significant degree of turnover in the portfolios, i.e., many countries move in and out of each portfolio, particularly so at finer levels of disaggregation. From our point of view, we are most interested in the robustness of our results to this alternative grouping procedure. We report the results in Table 7.[50] Our results are robust to this alternative grouping procedure. Our model predicts a spread in returns between portfolio 1 and the US of between 4.6% and 5.5%, which accounts for between 62% and 76% of the spread in the data. These results are quite close to those obtained in our baseline approach above. The correlation of predicted and actual returns across portfolios ranges upward from a low of about 0.9.

Table 7: Predicted vs. Actual Returns - Annually Rebalanced Portfolios

3 Portfolios			5 Portfolios			10 Portfolios		
	$\widehat{r}$	r		$\widehat{r}$	r		$\widehat{r}$	r
1	10.56	12.10	1	11.03	13.00	1	11.44	14.91
2	8.74	10.23	2	10.06	10.83	2	10.59	10.92
3	7.04	7.57	3	8.14	10.18	3	9.77	10.49
US	5.94	6.02	4	7.47	8.49	4	10.36	11.17
			5	7.18	7.26	5	8.25	10.51
			US	5.94	6.02	6	8.05	9.88
						7	8.33	8.96
						8	6.62	8.01
						9	7.28	6.87
						10	7.09	7.64
						US	5.94	6.02
Average:	8.07	8.98		8.31	9.30		8.52	9.58
Spread: 1-US	4.61	6.08		5.09	6.98		5.50	8.89
Percent of actual	76			73			62	
corr($\widehat{r}, r$)	1.00			0.96			0.91	

Notes: Table reports the average return to capital in each portfolio as measured in the data, r, and as predicted by the model, $\hat{r}$, from equations (5) and (6) under the parameter configuration in Tables 13, 14, and 15 for 3, 5, and 10 portfolio groupings, respectively.

Identification. In this section, we perform several exercises meant to illustrate the important role played by various features of our model and identification strategy in accurately measuring

[49] This approach has been widely used in examining cross-sectional differences in returns. See, for example, Lustig and Verdelhan (2007).

[50] We report the moments and parameter estimates for this exercise in Appendix E.

the extent of long-run risks around the world. To do so, we return to the 3 portfolio case and parameterize the model under a number of alternative approaches discussed in Section 3.1. First, we parameterize the model ignoring region-specific idiosyncratic long-run shocks, i.e., under the naive assumption that $\sigma_{e^*} = 0$ (or alternatively, that $\rho^* = 0$). In this case, expression (9), i.e., the ratio of autocovariances in foreign dividend and US consumption growth gives an unbiased estimate of ϕ^*. The third column of Table 8 reports the implied returns in the 3 foreign portfolios when following this alternative identification strategy (the US return does not change, since we are only changing assumptions regarding foreign idiosyncratic shocks). The first two columns report the actual returns and those predicted under our baseline approach. A comparison of the second and third columns shows that ignoring regional long-run shocks would lead to substantial bias in the results. As predicted by expression (9), returns are everywhere biased upward. The magnitude of this bias ranges from 0.5% in portfolio 1 to almost 3% in portfolio 3. Clearly, not accounting for idiosyncratic long-run shocks leads to significant bias under an approach that relies solely on autocovariance moments.

Table 8: Predicted Returns - Alternative Empirical Strategies

			$\sigma_{e^*} = 0$		
Portfolio	Actual	Benchmark	Autocovariance	Baseline	$\pi_d^* = 0$
1	13.02	10.55	11.06	10.56	8.76
2	11.07	9.30	12.05	9.33	9.28
3	8.05	7.18	9.59	7.19	10.47

Notes: Table reports the average return to capital in each portfolio as measured in the data, and as predicted by the model from equations (5) and (6) under alternative parameter configurations. Benchmark refers to the parameter configuration in Table 4 obtained following the benchmark approach. Autocovariance sets $\sigma_{e^*} = 0$ and replaces the covariance of return moment (17) in the benchmark approach with the autocovariance moment (9). Baseline sets $\sigma_{e^*} = 0$ and otherwise follows the benchmark approach. $\pi_d^* = 0$ follows the benchmark approach, but imposes that $\pi_d^* = 0$.

In contrast, the fourth column of Table 8 reports the predicted returns under the same assumption that there are no foreign persistent shocks, i.e. $\sigma_{e^*} = 0$, but following our baseline identification strategy. The predicted returns are nearly identical to those obtained from our benchmark approach, without restricting σ_{e^*} to zero. Notice that the last exercise is equivalent to a model where ρ^* is set to zero, and so also serves as a robustness exercise on the value of this parameter. In other words, under our empirical strategy, the results we obtain change only negligibly whether we account for local persistent shocks or not, and are robust to large changes in the persistence of those shocks (recall that the benchmark value of ρ^* is relatively high at 0.93). This robustness to the properties of regional long-run shocks is an attractive feature of our approach and is intuitive when examining our identification equations (12)-(17) and the expressions for predicted returns (5) and (6). Parameters of the foreign shock processes (ρ^* and $\sigma^2_{e^*}$) do not enter anywhere into the latter two, implying that they do not directly impact expected returns; their only effect is indirect by affecting $\kappa^*_{m,1}$, and so additionally our estimate of ϕ^* (through expression (b.5)). However, these indirect effects are quantitatively negligible.

Finally, we compute the predicted returns from our model under the assumption that there is no transitory comovement between dividend growth rates, i.e., $\pi^*_d = 0$. Imposing this restriction implies that correlations in the dividend growth rates across countries are due only to exposure to the common long-run shock, and to their common dependence on the transitory consumption shock governed by π^* and π. In this case, the covariance of dividend growth rates with those in the US and with US consumption growth (expressions (14) and (13)) are clearly sufficient to tease out ϕ^*. We report the results in the last column of Table 8. The implications are very stark: portfolio 3 now yields the highest returns while portfolio 1 yields the lowest. As discussed earlier, since period-by-period comovement between dividend growth in developed economies and the US is much higher than the corresponding comovement between less developed countries and the US, attributing this comovement only to persistent components results in developed countries' portfolios appearing to be riskier. Accounting for these short-run dividend correlations is key to properly measure the various sources of risk faced by the investor.

Country-level analysis. Although the portfolio-based approach lends certain advantages, it is nonetheless worth exploring the implications of our model when the parameters are estimated to match moments of individual countries. To do so, we restrict our analysis to countries with a sufficiently long time-series of available data. Our empirical strategy relies heavily on time-series moments and short samples are thus problematic. Consequently, we examine only countries for which data availability reaches back at least to 1961, so that at least approximately 50 years of data are available.[51] This gives us 96 countries on which we perform our analysis, still a large number, although less than the 144 we include in our portfolio analyses above. We parameterize the model country-by-country and compute predicted returns on this basis.[52]

We analyze the results in a number of ways. First, we simply ask: does the model predict a relationship between returns and income at the country-level that resembles the one we observe in the data? To answer this question, we regress country-level returns on income both in the data and as predicted by the model. We obtain a clear negative relationship between capital returns and income in the data: the line of best fit has a slope of -0.023, which is significant at

[51]Countries tend to be added to the PWT in waves, so we are including here the original 1950 wave, and a second wave that spans 1960 and 1961. The next major wave of additions is not until 1970.

[52]We denote by 1 the countries included in the analysis in Appendix D, column 6, titled Ctry An. A handful of countries feature a configuration of moments that imply a negative σ^2_μ due to its residual nature. In order not to lose these countries, we set $\sigma^2_\mu = 0.05$, which is the average of the other countries. This choice makes very little difference to the results.

the 99% level.[53] The model also predicts a negative relationship: the line of best fit has a slope of -0.013, about 55% the actual, and is significant at the 95% level. In this sense, even when parameterized at the country-level, the model can account for a significant portion (about 55%) of the average relationship between capital returns and income.

To get a sense of how the model predictions line up with actual returns on a country-level basis (similar to the correlations reported at the portfolio level above), we display in Figure 3 the predicted vs. actual values, along with the 45 degree line. If the model were a perfect fit to the data, each point would lie exactly on the line. Although there is a good deal of variation at the country level, as should be expected, the model predicts returns that are generally in line with the data: the correlation between the predicted and actual returns is quite high at about 0.61 and is significant at the 99% level. We view this as a strong confirmation of the explanatory power of our theory: the cross-sectional distribution of returns at the country-level is likely determined by a host of factors specific to each country; that our relatively parsimonious theory predicts returns that are highly correlated with the empirical ones suggests that long-run risks indeed play a key role in leading to the variation in returns observed in the data.

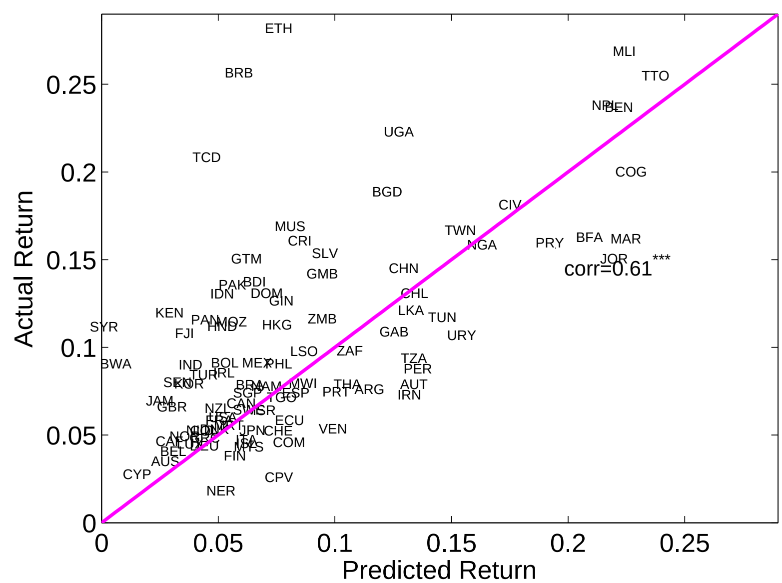

Figure 3: Country-Level Returns - Predicted vs. Actual

[53]This statistic is the same as in Figure 1, but the sample here contains only the subset of countries for which we have sufficient data to pass through the model.

4 Conclusion

Emerging markets exhibit volatile growth rate regimes. Aguiar and Gopinath (2007) show that shocks to TFP trend growth can account for a number of puzzling features of business cycles in these economies, i.e., that "the cycle is the trend." Building on this insight, and the related asset-pricing framework of Bansal and Yaron (2004), we demonstrate that these types of shocks can account for a large portion of observed differences in the returns to capital across the world, and in particular, the systematically higher returns in poor countries—the Lucas Paradox. Parameterized versions of our model can account for 60-70% of the return spread between the US and a group of the poorest countries, and for about 55% of the country-level relationship between income and returns. The implication is that despite large observed return differentials, the allocation of capital is not so distant from that predicted by theory.

Key to our findings is that emerging markets not only feature large fluctuations in growth rates, but also that the shocks are systemically related across countries, i.e., these markets are highly exposed to global growth-rate shocks. We leave for future work a more detailed investigation into the sources of the differences in long-run risk that we measure. The implications of such an analysis would clearly be important in many dimensions; from the point of view of our analysis, in reducing required risk premia associated with investments in poor countries and so potentially attracting additional investment flows. Potential avenues of research include understanding the role that high dependence on the production and export of commodities, whose prices are known to be highly volatile, plays in generating volatility in emerging market macro aggregates. For example, Burnside and Tabova (2009) find a significant relationship between country growth rates and a number of commodity price indices—specifically crude oil, primary metals, and agricultural commodities. Additionally, examining the degree to which institutional differences across countries shape the ability to respond to external shocks may provide further insights into the mechanisms that result in high exposure of emerging markets to global shocks. A common interpretation of the parameter ϕ^* in models of equity markets is that it captures the levered nature of equity claims on total output. In our setup, an interpretation is that foreign investments appear to be in a sense more highly levered from the point of view of the US investor. This could be the case because of institutional differences, for example, poor contract enforcement in poor countries. As the residual claimant, a higher probability of non-payment in poor countries could act similarly to a leverage adjustment for the US investor. A more concrete interpretation of the differences we find across countries would be informative about the nature of the risk we measure.

We have focused on consumption-based risk due to uncertainty regarding the payoffs to capital investments, both in the short- and long-run. By doing so, we have abstracted from a number of other sources of risk that may play a role in leading to return differences, for example, default risk or expropriation risk. Additionally, our model does not shed light on the fundamental source of long-run risk, i.e., changing prospects in technological progress, low frequency movements in relative prices, etc. Further work investigating these issues and their interaction with rates of return on capital around the world could be quite fruitful.

References

Aguiar, M. and G. Gopinath (2007): “Emerging Market Business Cycles: The Cycle Is the Trend,” *Journal of Political Economy*, 115, 69–102.

Backus, D., A. Ferriere, and S. Zin (2014): “Risk and ambiguity in models of business cycles,” *New York University, mimeo.*

Bansal, R., D. Kiku, and A. Yaron (2012a): “An Empirical Evaluation of the Long-Run Risks Model for Asset Prices,” *Critical Finance Review*, 1, 183–221.

——— (2012b): “Risks For the Long Run: Estimation with Time Aggregation,” Working Paper 18305, NBER.

Bansal, R. and I. Shaliastovich (2013): “A Long-Run Risks Explanation of Predictability Puzzles in Bond and Currency Markets,” *Review of Financial Studies*, 26, 1–33.

Bansal, R. and A. Yaron (2004): “Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles,” *Journal of Finance*, 59, 1481–1509.

Barro, R. (2006): “Rare Disasters and Asset Markets in the Twentieth Century,” *Quarterly Journal of Economics*, 121, 823–866.

Borri, N. and A. Verdelhan (2012): “Sovereign Risk Premia,” mimeo, MIT Sloan.

Burnside, C. and A. Tabova (2009): “Risk, Volatility, and the Global Cross-Section of Growth Rates,” Working Paper 15225, NBER.

Burstein, A., J. Cravino, and J. Vogel (2013): “Importing Skill-Biased Technology,” *American Economic Journal: Macroeconomics*, 5, 32–71.

Campbell, J. Y. and J. n. Cochrane (1999): “By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior,” *Journal of Political Economy*, 107, 205–251.

Caselli, F. and J. Feyrer (2007): "The Marginal Product of Capital," *The Quarterly Journal of Economics*, 122, 535–568.

Chen, H. (2010): "Macroeconomic Conditions and the Puzzles of Credit Spreads and Capital Structure," *Journal of Finance*, 65, 2171–2212.

Chen, K. and M. Crucini (2014): "Trends and Cycles in Small Open Economies: Making The Case For A General Equilibrium Approach," mimeo, Vanderbilt University.

Chinn, M. and H. Ito (2006): "What Matters for Financial Development? Capital Controls, Institutions, and Interactions," *Journal of Development Economics*, 81, 163–192.

Colacito, R. and M. M. Croce (2011): "Risks for the Long Run and the Real Exchange Rate," *Journal of Political Economy*, 119, 153 – 181.

——— (2013): "International Asset Pricing with Recursive Preferences," *Journal of Finance*, 68, 2651–2686.

Daly, K. (2010): "Why Doesn't Capital Flow to Where Returns are Highest," Global Economics Paper 205, Goldman Sachs.

Eaton, J. and S. Kortum (2001): "Trade in Capital Goods," *European Economic Review*, 45, 1195–1235.

Epstein, L. G. and S. E. Zin (1989): "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework," *Econometrica*, 57, 937–69.

Fama, E. F. (1976): *Foundations of Finance: Portfolio Decisions and Securities Prices*, New York: Basic Books.

Feenstra, R. C., R. Inklaar, and M. Timmer (2013): "The Next Generation of the Penn World Table," Working Paper 19255, NBER.

Ferson, W., S. Nallareddy, and B. Xie (2013): "The "Out-of-Sample" Performance of Long Run Risk Models," *Journal of Financial Economics*, 107, 537–556.

Gabaix, X. (2008): "Variable Rare Disasters: A Tractable Theory of Ten Puzzles in Macrofinance," *American Economic Review*, 98, 64–67.

Gollin, D. (2002): "Getting Income Shares Right," *Journal of Political Economy*, 110, 458–474.

Gomme, P., B. Ravikumar, and P. Rupert (2011): "The Return to Capital and the Business Cycle," *Review of Economic Dynamics*, 14, 262–278.

Gourinchas, P.-O. and O. Jeanne (2013): "Capital Flows to Developing Countries: The Allocation Puzzle," *Review of Economic Studies*, 80, 1484–1515.

Grilli, V. and G.-M. Milesi-Ferretti (1995): "Economic Effects and Structural Determinants of Capital Controls," *IMF Staff Papers*, 42, 517–551.

Hansen, L. P., J. C. Heaton, and N. Li (2008): "Consumption Strikes Back? Measuring Long-Run Risk," *Journal of Political Economy*, 116, 260–302.

Hsieh, C. and P. Klenow (2007): "Relative Prices and Relative Prosperity," *American Economic Review*, 97, 562–585.

Karabarbounis, L. and B. Neiman (2014): "The Global Decline of the Labor Share," *Quarterly Journal of Economics*, 129, 61–103.

Lewis, K. K. and E. X. Liu (2012): "International Consumption Risk Is Shared After All: An Asset Return View," Working Paper 17872, NBER.

Lucas, Jr., R. (1990): "Why Doesn't Capital Flow from Rich to Poor Countries?" *American Economic Review*, 80, 92–96.

Lustig, H. and A. Verdelhan (2007): "The Cross Section of Foreign Currency Risk Premia and Consumption Growth Risk," *American Economic Review*, 97, 89–117.

Malloy, C. J., T. J. Moskowitz, and A. Vissing-Jørgensen (2009): "Long-Run Stockholder Consumption Risk and Asset Returns," *Journal of Finance*, 64, 2427–2479.

Mehra, R. and E. Prescott (1985): "The Equity Premium: A Puzzle," *Journal of Monetary Economics*, 15, 145–161.

Mutreja, P., B. Ravikumar, and M. Sposi (2012): "Capital Goods Trade and Economic Development," Working Paper 183, Federal Reserve Bank of Dallas.

Nakamura, E., D. Sergeyev, and J. Steinsson (2012): "Growth-Rate and Uncertainty Shocks in Consumption: Cross-Country Evidence," Working Paper 18128, NBER.

Neumeyer, P. A. and F. Perri (2005): "Business Cycles in Emerging Economies: The Role of Interest Rates," *Journal of Monetary Economics*, 52, 345–380.

Ohanian, L. and M. Wright (2007): "Where Did Capital Flow? Fifty Years of International Rate of Return Differentials and Capital Flows," mimeo, UCLA.

Quinn, D. P. (2003): "Capital Account Liberalization and Financial Globalization, 1890-1999: A Synoptic View," *International Journal of Finance & Economics*, 8, 189–204.

Tabova, A. (2013): "Portfolio Diversification and the Cross-Sectional Distribution of Foreign Investment," International Finance Discussion Papers 1091, Board of Governors of the Federal Reserve System.

For Online Publication: Appendix

A Data

As described in the text, we use data on relative prices and US per-capita consumption from the BEA. We obtain data from the Penn World Tables Version 8.0 (PWT) to construct dividends in foreign countries. We exclude countries where insufficient data are available, with clear data errors, or that are large outliers. Altogether, these amount to 23 out of 167 countries, leaving us with the 144 used in the main text.

Due to the long time horizon that we analyze, some notes about the PWT country classification are in order. West Germany proxies Germany prior to the unification of East and West Germany. Data for Ethiopia for the period of 1970-1990 refer to the former territory of Ethiopia, including Eritrea. Czechoslovakia is not contained in PWT. Czech Republic and Slovak Republic are added in 1990. USSR is not contained in PWT. Russia, Azerbaijan, Estonia, Georgia, Kazakhstan, Kyrgyzstan, Lithuania, Latvia and Moldova are added in 1990. Yugoslavia is not contained in PWT. Bosnia and Herzegovina, Croatia, Macedonia, Montenegro, Serbia and Slovenia are added in 1990.

We analyze portfolios of countries during the 1950-2008 period. Countries are added to the PWT in waves. In Appendix D we list the countries we use in our benchmark analysis, classified by the portfolio they fall into. In addition, we list the year the country was added to the PWT database and indicate the set of countries we analyze in the country-level exercises as well as those with open capital accounts. As described in the text, mean returns at the country level are computed as the time-series average for each country. To form portfolio returns, these are averaged across countries within each portfolio and through time. All moments for our empirical work are computed analogously, both at the country and portfolio level.

B Model

Solution. We begin by writing the investor's SDF in logs as

$$m_{t+1} = \theta \log \beta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{c,t+1} \tag{b.1}$$

Covariation with m_{t+1} will determine the risk premia on each asset. To characterize the SDF, we solve for the consumption return $r_{c,t+1}$, which we approximate as

$$r_{c,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + \Delta c_{t+1} \tag{b.2}$$

where $z_t = \log\left(\frac{P_t}{C_t}\right)$ is the log price-consumption ratio and the two κ's are constants of approximation that depend on the unconditional mean of z, $\overline{z}$: $\kappa_1 = \frac{\exp \overline{z}}{1+\exp \overline{z}}$ and $\kappa_0 = \log(1 + \exp \overline{z}) - \overline{z}\kappa_1$. We similarly approximate returns to the US and a representative foreign asset as, respectively,

$$\begin{aligned} r_{m,t+1} &= \kappa_{m,0} + \kappa_{m,1} z_{m,t+1} - z_t + \Delta d_{t+1} \\ r^*_{m,t+1} &= \kappa^*_{m,0} + \kappa^*_{m,1} z^*_{m,t+1} - z^*_t + \Delta d^*_{t+1} \end{aligned}$$

where $z_{m,t} = \log\left(\frac{P_t}{D_t}\right)$ is the US log price-dividend ratio, which has unconditional mean $\overline{z}_m$, and the κ_m's depend on $\overline{z}_m$ in an analogous way to the κ's on $\overline{z}$ above (similar relationships hold for the foreign asset).

Given the endowment nature of the economy, we need find solutions for the price-consumption ratio and the price-dividend ratio for each asset in order to characterize returns. The state variables in the economy are the expected growth rates x_t and x^*_t, and these ratios are approximately linear in the states, i.e.,

$$\begin{aligned} z_t &= A_0 + A_1 x_t + A_2 x^*_t \\ z_{m,t} &= A_{m,0} + A_{m,1} x_t + A_{m,2} x^*_t \\ z^*_{m,t} &= A^*_{m,0} + A^*_{m,1} x_t + A^*_{m,2} x^*_t \end{aligned}$$

Substituting into the Euler equation (3), we can find

$$\begin{aligned} A_1 &= \frac{1-\frac{1}{\psi}}{1-\kappa_1\rho} \\ A_2 &= 0 \\ A_0 &= \frac{\log\beta + \left(1-\frac{1}{\psi}\right)\mu + \kappa_0 + \frac{1}{2}(1-\gamma)\left(1-\frac{1}{\psi}\right)\left(\sigma_\eta^2 + \left(\frac{\kappa_1}{1-\kappa_1\rho}\right)^2\sigma_e^2\right)}{1-\kappa_1} \end{aligned} \tag{b.3}$$

and for the US asset,

$$\begin{aligned} A_{m,1} &= \frac{\phi-\frac{1}{\psi}}{1-\kappa_{m,1}\rho} \\ A_{m,2} &= 0 \\ A_{m,0} &= \frac{\theta\log\beta - \gamma\mu + (\theta-1)(\kappa_0 + A_0(\kappa_1-1)) + \mu_d + \kappa_{m,0}}{1-\kappa_{m,1}} \\ &+ \frac{\frac{1}{2}(\pi-\gamma)^2\sigma_\eta^2 + \frac{1}{2}((\theta-1)\kappa_1 A_1 + \kappa_{m,1}A_{m,1})^2\sigma_e^2 + \frac{1}{2}\sigma_\mu^2}{1-\kappa_{m,1}} \end{aligned} \tag{b.4}$$

and lastly, for the foreign asset,

$$\begin{aligned} A^*_{m,1} &= \frac{\tilde{\phi}^*\xi^* - \frac{1}{\psi}}{1-\kappa^*_{m,1}\rho} \\ A^*_{m,2} &= \frac{\tilde{\phi}^*}{1-\kappa^*_{m,1}\rho^*} \\ A^*_{m,0} &= \frac{\theta\log\beta - \gamma\mu + (\theta-1)(\kappa_0 + A_0(\kappa_1-1)) + \mu_d^* + \kappa^*_{m,0} + \frac{1}{2}\left(\kappa^*_{m,1}A^*_{m,2}\right)^2\sigma^2_{e^*}}{1-\kappa^*_{m,1}} \\ &+ \frac{\frac{1}{2}(\pi^*-\gamma)^2\sigma_\eta^2 + \frac{1}{2}\left((\theta-1)\kappa_1 A_1 + \kappa^*_{m,1}A^*_{m,1}\right)^2\sigma_e^2 + \frac{1}{2}\pi_d^{*2}\sigma_\mu^2 + \frac{1}{2}\pi_{cd}^{*2}\sigma^2_{\eta^*} + \frac{1}{2}\tilde{\sigma}^2_{\mu^*}}{1-\kappa^*_{m,1}} \end{aligned} \tag{b.5}$$

Solving for mean excess returns entails finding the vectors of consumption parameters $\boldsymbol{A}$ and $\boldsymbol{\kappa}$ and the corresponding vectors of return parameters, $\boldsymbol{A_m}$ and $\boldsymbol{\kappa_m}$, both for the US and each foreign asset. This can be done following a simple iterative procedure. As an example, consider the consumption parameters. First, note that $\overline{z} = A_0$. Then, for a candidate value of $\overline{z}$, we can compute values for $\boldsymbol{\kappa}$. We can then compute the vector $\boldsymbol{A}$ using (b.3), which produces an updated value for $\overline{z}$. We then iterate until convergence. We use an analogous procedure to solve for the return parameters, both in the US and each foreign region. The mean risk-free rate depends only on consumption and preference parameters.

Identification. In this section, we prove that $\sigma^2_{\mu^*} = \pi^{*2}_{cd}\sigma^2_{\eta^*} + \tilde{\sigma}^2_{\mu^*}$ and $\tilde{\phi}^*\sigma^2_{e^*}$, along with the parameters identified in equations (12)-(17) are sufficient to compute $\kappa^*_{m,1}$. Combining the expressions for $A^*_{m,1}, A^*_{m,2}$ and $A^*_{m,0}$ using (b.5) and letting $\phi^* \equiv \tilde{\phi}^*\xi^*$ gives the following expression for $A^*_{m,0}$:

$$\begin{aligned} A^*_{m,0} &= \frac{\theta \log \beta - \gamma\mu + (\theta-1)\left(\kappa_0 + A_0\left(\kappa_1 - 1\right)\right) + \mu^*_d + \kappa^*_{m,0} + \frac{1}{2}\left(\frac{\kappa^*_{m,1}}{1-\kappa^*_{m,1}\rho^*}\right)^2 (\tilde{\phi}^*\sigma^*_e)^2}{1-\kappa^*_{m,1}} \\ &+ \frac{\frac{1}{2}\left(\pi^*-\gamma\right)^2\sigma^2_\eta + \frac{1}{2}\left(\left(\theta-1\right)\kappa_1 A_1 + \frac{\kappa^*_{m,1}}{1-\kappa^*_{m,1}\rho}\left(\phi^* - \frac{1}{\psi}\right)\right)^2\sigma^2_e + \frac{1}{2}\pi^{*2}_d\sigma^2_\mu + \frac{1}{2}\sigma^2_{\mu^*}}{1-\kappa^*_{m,1}} \end{aligned}$$

Because only $\sigma^2_{\mu^*}$ and $\tilde{\phi}^*\sigma^2_{e^*}$ appear, these are the two objects that must be identified and we do not need to pin down values for their individual elements separately. Because $\kappa^*_{m,1}$ is simply a nonlinear function of $A^*_{m,0}$, the same argument goes through.

C Lucas Paradox As Another Asset Pricing Puzzle

In this section, we analyze the predictions of the workhorse model of risk, the CCAPM, for the return disparities that we document. In this setting, risk premia are determined by the covariance of returns to each asset with consumption growth of the investor. We show that from this perspective, the dispersion in capital returns takes on the familiar characteristics of a typical asset pricing puzzle.

Consider a representative US investor with CRRA preferences $u(c_t) = \frac{c_t^{1-\gamma}-1}{1-\gamma}$ where γ is the coefficient of relative risk aversion (here also the inverse of the intertemporal elasticity of substitution).

Standard methods give the following Euler equations, one for each risky asset and one for the risk-free asset:

$$\begin{aligned} 1 &= \mathbb{E}_t\left[M_{t+1}R_{j,t+1}\right] \\ 1 &= \mathbb{E}_t\left[M_{t+1}R_{f,t+1}\right] \end{aligned} \tag{b.6}$$

where $M_{t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}$ denotes the US investor's stochastic discount factor (SDF), $R_{j,t}$ the return on capital in region j as defined in equation (2), and $R_{f,t}$ the return on a risk-free bond. To simplify matters, we linearize the SDF around its unconditional mean as $\frac{M_{t+1}}{\mathbb{E}[M_{t+1}]} \approx 1 + m_{t+1} - \mathbb{E}\left[m_{t+1}\right]$ where $m_t = \log M_t$. Using the definition of m_t along with the unconditional expectation of (b.6) gives the standard covariance formula

$$\mathbb{E}\left[R^e_{j,t}\right] = \gamma \text{cov}\left(\Delta c_t, R_{j,t}\right) \tag{b.7}$$

which relates the mean risk premium on an asset to the covariance of its returns with log consumption growth. The degree of risk aversion γ governs the strength of this relationship, that is, it determines how much additional compensation is demanded for each additional unit of covariance risk.

To assess the ability of the CCAPM to account for the patterns in international capital returns, it remains to construct the objects in equation (b.7). Excess returns are computed as returns from equation (2) less the annual real return on a 3 month treasury bond.[54] US consumption is measured as in the text as real per-capita consumption of non-durables and services. The covariance for each portfolio is calculated as the average covariance of the countries within that portfolio (recall that in our baseline approach, countries do not change portfolios). Table 9 reports the mean excess return and the covariance of returns with consumption growth for each of the 3 income-sorted portfolios and for the US. To assess whether the CCAPM can rationalize these patterns, we set a value for γ and evaluate the right hand side of (b.7). This is the excess return predicted by the CCAPM. We first set $\gamma = 10$, which is towards the higher end of the range commonly deemed to be reasonable (and corresponds to the value we use in the main text), and report the implied excess return $\widehat{r}^e$ in the third column of the table.[55] In this case, excess returns range from a low of 0.04% in the US to 0.16% in portfolio 1, the poorest. Quantitatively, these returns are essentially negligible: the model generates excess returns about 2 orders of magnitude below the actual, and only a minimal spread between portfolios. Clearly, the CCAPM cannot rationalize the patterns of capital returns observed in the data, at least not for this level of risk aversion. It is in this sense that we argue that the "Lucas Paradox," i.e., the dispersion in capital returns around the world, takes on the familiar characteristics of an asset pricing puzzle.

Next, we ask what level of risk aversion is needed to best fit the observed levels of returns across portfolios? To answer this, we compute the slope of the line of best fit across portfolios, which is about 890. The predicted excess returns at this extreme level of risk aversion are reported in the last column of the table. The levels are now on par with the data, although the predictions are not quite in line across portfolios, overpredicting portfolio 1 and underpredicting the others, and so generating too large a spread compared to the actual (10.8% vs. 7%). The key point here, however, is that an unreasonably high level of risk aversion is needed to give

[54] We obtain data on nominal returns from the FRED database at the St. Louis Fed. Nominal returns are deflated using the price index for non-durables and services consumption discussed in Section 2.2.

[55] See, for example, Mehra and Prescott (1985).

Table 9: Capital Returns: An Asset Pricing Puzzle

Portfolio	r^e	$\text{cov}(r, \Delta c)$	$\gamma =$	$\widehat{r^e}$ 10	889
1	11.80	0.00016		0.16	14.06
2	9.85	0.00009		0.09	7.93
3	6.83	0.00004		0.04	3.92
US	4.80	0.00004		0.04	3.23
Spread: 1-US	7.00	0.00012		0.12	10.83

Notes: Table reports excess returns across portfolios, r^e, the covariance of returns with US consumption growth, and the predicted excess return from the CCAPM model under two alternative levels of risk aversion.

the CCAPM its best chance at accounting for the patterns in the data.[56]

In sum, the dispersion in international capital returns appears to be an asset pricing puzzle in another guise. The CCAPM cannot generate the levels of returns or the degree of dispersion observed in the data under what are generally deemed reasonable levels of risk aversion.[57]

D List of Countries

Country	3 Letter code	Init. year	Portfolio number 3 portf.	5 portf.	10 portf.	Ctry An.	α	CI-2004	Openness Q-2004	GMF-1995
Albania	ALB	1970	2	3	5	0	0	1	0	0
Angola	AGO	1970	1	2	3	0	0	0	0	0
Antigua/Barbuda	ATG	1970	3	4	8	0	0	1	0	1
Argentina	ARG	1950	2	3	5	1	1	1	0	1
Armenia	ARM	1990	2	2	4	0	1	1	0	0
Australia	AUS	1950	3	5	10	1	1	1	1	1
Austria	AUT	1950	3	5	9	1	1	1	1	1
Azerbaijan	AZE	1990	2	2	4	0	1	0	0	0
Bahamas	BHS	1970	3	5	9	0	1	0	0	0
Bahrain	BHR	1970	3	5	9	0	0	1	0	1
Bangladesh	BGD	1959	1	1	2	1	0	0	0	0
Barbados	BRB	1960	3	4	8	1	1	0	0	0
Belarus	BLR	1990	2	3	6	0	1	0	0	0

[56]Note that the level of risk aversion required here is an order of magnitude above that in Mehra and Prescott (1985). This is because the covariance of real returns with consumption growth is much lower than that of equity market returns. The difference is largely due to the smoothness of the real series.

[57]It is worth pointing out that the above result is not a consequence of our grouping procedure. We repeat the exercise using 5 portfolios, 10 portfolios as well as individual countries. Although γ falls as granularity increases, even at the country level it is about 500, well above the reasonable range.

Country	3 Letter code	Init. year	Portfolio number 3 portf.	5 portf.	10 portf.	Ctry An.	α	Openness CI-2004	Q-2004	GMF-1995
Belgium	BEL	1950	3	5	10	1	1	1	1	1
Belize	BLZ	1970	2	3	6	0	0	0	0	0
Benin	BEN	1959	1	1	2	1	1	0	0	0
Bhutan	BTN	1970	1	2	3	0	0	0	0	0
Bolivia	BOL	1950	1	2	3	1	1	1	1	1
Botswana	BWA	1960	2	3	5	1	1	1	0	0
Brazil	BRA	1950	2	2	4	1	1	1	1	0
Brunei	BRN	1970	3	5	10	0	0	0	0	0
Bulgaria	BGR	1970	2	3	5	0	1	0	1	0
Burkina Faso	BFA	1959	1	1	1	1	1	0	0	0
Burundi	BDI	1960	1	1	1	1	0	0	0	0
Canada	CAN	1950	3	5	10	1	1	1	1	1
Cape Verde	CPV	1960	1	2	3	1	0	0	0	0
Central Afr. Rep.	CAF	1960	1	1	1	1	1	0	0	0
Chad	TCD	1960	1	1	1	1	0	0	0	0
Chile	CHL	1951	2	3	6	1	1	1	1	0
China	CHN	1952	1	1	2	1	1	0	1	0
Colombia	COL	1950	2	3	6	1	1	1	1	0
Comoros	COM	1960	1	1	2	1	0	0	0	0
Congo, Dem. Rep.	COD	1970	1	1	1	0	0	0	0	0
Congo, Rep. of	COG	1960	1	2	3	1	0	0	0	0
Costa Rica	CRI	1950	2	3	6	1	1	1	1	1
Cote d'Ivoire	CIV	1960	1	2	3	1	1	0	0	0
Croatia	HRV	1990	3	4	8	0	1	1	0	0
Cyprus	CYP	1950	3	4	8	1	1	1	0	0
Czech Republic	CZE	1990	3	5	9	0	1	1	1	0
Denmark	DNK	1950	3	5	9	1	1	1	1	1
Djibouti	DJI	1970	1	2	4	0	1	1	0	1
Dominican Rep.	DOM	1951	2	3	5	1	1	1	0	0
Ecuador	ECU	1951	2	2	4	1	1	1	1	1
El Salvador	SLV	1950	1	1	1	1	0	1	1	0
Estonia	EST	1990	3	4	7	0	1	1	0	1
Ethiopia	ETH	1950	1	1	1	1	0	0	0	0
Fiji	FJI	1960	2	3	5	1	1	0	0	0
Finland	FIN	1950	3	4	8	1	1	1	1	1
France	FRA	1950	3	5	10	1	1	1	1	1
Gabon	GAB	1960	3	4	7	1	0	0	0	0
Gambia, The	GMB	1960	1	1	2	1	0	1	0	1
Georgia	GEO	1990	2	2	4	0	1	1	0	0
Germany	DEU	1950	3	5	10	1	1	1	1	1
Greece	GRC	1951	3	4	8	1	1	1	1	0

Country	3 Letter code	Init. year	Portfolio number 3 portf.	5 portf.	10 portf.	Ctry An.	α	Openness CI-2004	Q-2004	GMF-1995
Grenada	GRD	1970	2	3	5	0	0	0	0	0
Guatemala	GTM	1950	2	2	4	1	1	1	1	1
Guinea	GIN	1959	1	1	2	1	0	0	0	0
Honduras	HND	1950	1	2	4	1	1	1	0	1
Hong Kong	HKG	1960	3	5	10	1	1	1	1	1
Hungary	HUN	1970	2	4	7	0	1	1	1	0
Iceland	ISL	1950	3	5	10	1	1	1	0	0
India	IND	1950	1	1	2	1	1	0	1	0
Indonesia	IDN	1960	1	2	3	1	1	1	1	1
Iran	IRN	1955	2	3	6	1	1	1	0	0
Ireland	IRL	1950	3	5	9	1	1	1	0	1
Israel	ISR	1950	3	5	9	1	1	1	1	0
Italy	ITA	1950	3	5	9	1	1	1	1	1
Jamaica	JAM	1953	2	3	5	1	1	1	1	0
Japan	JPN	1950	3	4	8	1	1	1	1	0
Jordan	JOR	1954	2	3	6	1	1	1	0	0
Kazakhstan	KAZ	1990	2	3	6	0	1	0	0	0
Kenya	KEN	1950	1	1	2	1	1	1	1	0
Korea, Rep. of	KOR	1953	2	4	7	1	1	1	1	0
Kyrgyzstan	KGZ	1990	1	2	3	0	1	1	0	0
Latvia	LVA	1990	2	4	7	0	1	1	0	1
Lesotho	LSO	1960	1	1	2	1	1	0	0	0
Liberia	LBR	1964	1	1	1	0	0	1	0	0
Lithuania	LTU	1990	3	4	7	0	1	0	0	1
Luxembourg	LUX	1950	3	5	10	1	1	0	0	0
Macao	MAC	1970	3	5	10	0	1	1	0	0
Macedonia	MKD	1990	3	4	7	0	1	1	0	0
Malawi	MWI	1954	1	1	1	1	0	0	0	0
Malaysia	MYS	1955	2	3	6	1	1	1	0	1
Mali	MLI	1960	1	1	1	1	0	0	0	0
Mauritania	MRT	1960	1	2	3	1	1	0	0	0
Mauritius	MUS	1950	2	4	7	1	1	1	0	0
Mexico	MEX	1950	3	4	8	1	1	1	1	0
Moldova	MDA	1990	1	2	3	0	1	0	0	0
Mongolia	MNG	1970	1	2	3	0	1	1	0	0
Montenegro	MNE	1990	3	5	9	0	0	1	0	0
Morocco	MAR	1950	2	2	4	1	1	0	0	0
Mozambique	MOZ	1960	1	1	1	1	1	0	0	0
Namibia	NAM	1960	2	3	5	1	0	0	0	0
Nepal	NPL	1960	1	1	1	1	0	0	0	0
Netherlands	NLD	1950	3	5	9	1	1	1	1	1

Country	3 Letter code	Init. year	Portfolio number 3 portf.	5 portf.	10 portf.	Ctry An.	α	Openness CI-2004	Q-2004	GMF-1995
New Zealand	NZL	1950	3	4	8	1	1	1	1	1
Niger	NER	1960	1	1	2	1	1	0	0	1
Nigeria	NGA	1950	1	1	2	1	0	0	1	0
Norway	NOR	1950	3	5	10	1	1	1	1	1
Oman	OMN	1970	3	5	9	0	0	1	0	1
Pakistan	PAK	1950	1	2	3	1	0	0	0	0
Panama	PAN	1950	2	4	7	1	1	1	0	1
Paraguay	PRY	1951	1	2	4	1	1	1	0	0
Peru	PER	1950	2	2	4	1	1	1	1	1
Philippines	PHL	1950	1	2	4	1	1	1	1	0
Poland	POL	1970	2	3	6	0	1	1	1	0
Portugal	PRT	1950	2	4	7	1	1	0	1	1
Qatar	QAT	1970	3	5	10	0	0	1	0	1
Romania	ROU	1988	2	3	6	0	1	1	0	0
Russia	RUS	1990	2	4	7	0	1	0	1	0
S.Tome/Principe	STP	1970	1	1	2	0	1	1	0	0
Saudi Arabia	SAU	1970	3	5	10	0	0	1	0	1
Senegal	SEN	1960	1	1	2	1	1	0	1	0
Serbia	SRB	1990	2	3	6	0	1	1	0	0
Singapore	SGP	1960	3	4	8	1	1	1	1	1
Slovak Rep.	SVK	1990	3	4	8	0	1	1	0	0
Slovenia	SVN	1990	3	5	9	0	1	1	0	0
South Africa	ZAF	1950	2	3	6	1	1	0	1	0
Spain	ESP	1950	3	4	8	1	1	1	1	1
Sri Lanka	LKA	1950	2	2	4	1	1	1	1	0
St. Lucia	LCA	1970	2	3	6	0	0	1	0	0
St.Vincent/Gren.	VCT	1970	2	3	5	0	0	0	0	0
Suriname	SUR	1970	2	4	7	0	1	0	0	0
Swaziland	SWZ	1970	2	3	5	0	1	0	0	0
Sweden	SWE	1950	3	5	9	1	1	1	1	1
Switzerland	CHE	1950	3	5	10	1	1	1	0	1
Syria	SYR	1960	1	2	4	1	0	0	0	0
Taiwan	TWN	1951	3	4	8	1	0	1	0	0
Tajikistan	TJK	1990	1	2	3	0	1	0	0	0
Tanzania	TZA	1960	1	1	1	1	1	0	0	0
Thailand	THA	1950	1	2	3	1	1	1	0	0
Togo	TGO	1960	1	1	1	1	0	0	0	0
Trinidad/Tobago	TTO	1950	3	4	8	1	0	0	0	1
Tunisia	TUN	1960	2	3	5	1	1	0	1	0
Turkey	TUR	1950	2	4	7	1	1	0	1	0
Turkmenistan	TKM	1990	2	3	5	0	0	0	0	0

Country	3 Letter code	Init. year	Portfolio number 3 portf.	5 portf.	10 portf.	Ctry An.	α	CI-2004	Openness Q-2004	GMF-1995
Uganda	UGA	1950	1	1	1	1	0	1	1	0
Ukraine	UKR	1990	2	3	5	0	1	0	0	0
United Kingdom	GBR	1950	3	5	9	1	1	1	1	1
United States	USA	1950	4	6	11	1	1	1	1	1
Uruguay	URY	1950	2	4	7	1	1	1	1	0
Uzbekistan	UZB	1990	2	2	4	0	0	0	0	0
Venezuela	VEN	1950	3	4	7	1	1	0	1	0
Zambia	ZMB	1955	1	1	2	1	0	1	0	0

E Additional Tables

Table 11: Target Moments and Parameter Values, 5 Portfolios

Moments						
Portfolio	$\mathbb{E}[\Delta d_t^*]$	$\text{cov}\left(\Delta d_{t+1}^*, \Delta d_t^*\right)$	$\text{cov}\left(\Delta d_t^*, \Delta d_t\right)$	$\text{cov}\left(\Delta d_t^*, \Delta c_t\right)$	$\text{std}\left(\Delta d_t^*\right)$	$\text{cov}\left(r_t^*, r_t\right)$
1	-0.016	0.00148	0.00014	0.08423	0.000	0.00120
2	-0.016	0.00160	0.00005	0.07759	0.000	0.00097
3	-0.016	0.00116	0.00014	0.07716	0.000	0.00099
4	-0.013	0.00102	0.00015	0.06970	0.000	0.00100
5	-0.011	0.00057	0.00015	0.05953	0.000	0.00075
Parameters						
Portfolio	μ_d	ϕ	π	π_d	σ_μ	$\tilde{\phi}^*\sigma_{e^*}$
1	-0.016	5.43402	-1.04919	-0.01392	0.074	0.00661
2	-0.016	5.38104	-2.16723	-0.58453	0.061	0.00804
3	-0.016	3.86334	-0.32746	0.14072	0.069	0.00901
4	-0.013	3.53430	-0.02582	0.34382	0.061	0.00873
5	-0.011	2.40198	0.57627	0.26349	0.054	0.00707

Table 12: Target Moments and Parameter Values, 10 Portfolios

Moments						
Portfolio	$\mathbb{E}\left[\Delta d_t^*\right]$	$\operatorname{cov}\left(\Delta d_{t+1}^*, \Delta d_t^*\right)$	$\operatorname{cov}\left(\Delta d_t^*, \Delta d_t\right)$	$\operatorname{cov}\left(\Delta d_t^*, \Delta c_t\right)$	$\operatorname{std}\left(\Delta d_t^*\right)$	$\operatorname{cov}\left(r_t^*, r_t\right)$
1	-0.019	0.00138	0.00016	0.08918	0.000	0.00131
2	-0.014	0.00161	0.00013	0.07880	0.000	0.00110
3	-0.018	0.00108	0.00005	0.08741	0.000	0.00089
4	-0.015	0.00208	0.00004	0.06885	0.000	0.00101
5	-0.017	0.00114	0.00016	0.08212	0.000	0.00089
6	-0.014	0.00116	0.00012	0.07195	0.000	0.00109
7	-0.017	0.00113	0.00012	0.07425	0.000	0.00095
8	-0.010	0.00091	0.00017	0.06555	0.001	0.00106
9	-0.013	0.00069	0.00014	0.06020	0.000	0.00077
10	-0.009	0.00047	0.00016	0.05885	0.000	0.00073
Parameters						
Portfolio	μ_d	ϕ	π	π_d	σ_μ	$\tilde{\phi}^* \sigma_{e^*}$
1	-0.019	6.52747	-1.43925	-0.20339	0.072	0.01000
2	-0.014	4.39313	-0.67851	0.17838	0.067	0.01103
3	-0.018	4.81915	-1.85258	-0.52808	0.078	0.00461
4	-0.015	5.70850	-2.34323	-0.63151	0.043	0.01064
5	-0.017	3.09174	0.27679	0.28179	0.074	0.01052
6	-0.014	4.69861	-0.98000	0.01693	0.062	0.00634
7	-0.017	3.47803	-0.31028	0.28966	0.065	0.00969
8	-0.010	3.57751	0.22552	0.41122	0.057	0.00764
9	-0.013	2.60449	0.31829	0.24537	0.053	0.00778
10	-0.009	2.24683	0.79181	0.27805	0.054	0.00623

Table 13: Target Moments and Parameter Values - Annually Rebalanced, 3 Portfolios

Moments						
Portfolio	$\mathbb{E}\left[\Delta d_t^*\right]$	$\operatorname{cov}\left(\Delta d_{t+1}^*, \Delta d_t^*\right)$	$\operatorname{cov}\left(\Delta d_t^*, \Delta d_t\right)$	$\operatorname{cov}\left(\Delta d_t^*, \Delta c_t\right)$	$\operatorname{std}\left(\Delta d_t^*\right)$	$\operatorname{cov}\left(r_t^*, r_t\right)$
1	-0.016	0.00150	0.00027	0.00010	0.079	0.00106
2	-0.014	0.00130	0.00035	0.00012	0.071	0.00096
3	-0.011	0.00065	0.00040	0.00014	0.060	0.00084
Parameters						
Portfolio	μ_d	ϕ	π	π_d	σ_μ	$\tilde{\phi}^* \sigma_{e^*}$
1	-0.016	5.14172	-1.47073	-0.23634	0.067	0.00804
2	-0.014	3.85120	-0.50805	0.06715	0.061	0.01011
3	-0.011	2.79865	0.26869	0.28320	0.053	0.00701

Table 14: Target Moments and Parameter Values - Annually Rebalanced, 5 Portfolios

Moments						
Portfolio	$\mathbb{E}[\Delta d_t^*]$	$\text{cov}\left(\Delta d_{t+1}^*, \Delta d_t^*\right)$	$\text{cov}(\Delta d_t^*, \Delta d_t)$	$\text{cov}(\Delta d_t^*, \Delta c_t)$	$\text{std}(\Delta d_t^*)$	$\text{cov}(r_t^*, r_t)$
1	−0.016	0.00128	0.00033	0.00011	0.080	0.00115
2	−0.016	0.00175	0.00018	0.00007	0.076	0.00096
3	−0.014	0.00130	0.00037	0.00014	0.073	0.00092
4	−0.013	0.00081	0.00041	0.00013	0.064	0.00089
5	−0.010	0.00060	0.00038	0.00015	0.058	0.00084
Parameters						
Portfolio	μ_d	ϕ	π	π_d	σ_μ	$\tilde{\phi}^* \sigma_{e^*}$
1	−0.016	5.48566	−1.44265	−0.15540	0.069	0.00338
2	−0.016	4.80609	−1.61850	−0.35426	0.060	0.01097
3	−0.014	3.44629	−0.08783	0.15035	0.063	0.01092
4	−0.013	3.07770	0.01314	0.30387	0.056	0.00792
5	−0.010	2.85864	0.27709	0.23573	0.052	0.00628

Table 15: Target Moments and Parameter Values - Annually Rebalanced, 10 Portfolios

Moments						
Portfolio	$\mathbb{E}[\Delta d_t^*]$	$\text{cov}\left(\Delta d_{t+1}^*, \Delta d_t^*\right)$	$\text{cov}(\Delta d_t^*, \Delta d_t)$	$\text{cov}(\Delta d_t^*, \Delta c_t)$	$\text{std}(\Delta d_t^*)$	$\text{cov}(r_t^*, r_t)$
1	−0.018	0.00109	0.00035	0.00011	0.080	0.00120
2	−0.015	0.00147	0.00031	0.00011	0.079	0.00110
3	−0.016	0.00183	0.00018	0.00007	0.081	0.00094
4	−0.016	0.00167	0.00018	0.00006	0.071	0.00099
5	−0.014	0.00136	0.00037	0.00014	0.073	0.00093
6	−0.014	0.00123	0.00037	0.00014	0.073	0.00091
7	−0.014	0.00093	0.00040	0.00013	0.066	0.00096
8	−0.012	0.00069	0.00042	0.00014	0.061	0.00081
9	−0.009	0.00068	0.00037	0.00013	0.057	0.00085
10	−0.010	0.00051	0.00039	0.00017	0.059	0.00083
Parameters						
Portfolio	μ_d	ϕ	π	π_d	σ_μ	$\tilde{\phi}^* \sigma_{e^*}$
1	−0.018	5.82184	−1.63820	−0.14489	0.063	0.01000
2	−0.015	5.10199	−1.22910	−0.16786	0.067	0.00792
3	−0.016	4.57227	−1.46032	−0.33419	0.066	0.01203
4	−0.016	5.05383	−1.78833	−0.37776	0.055	0.00969
5	−0.014	3.51440	−0.09401	0.13436	0.062	0.01119
6	−0.014	3.39240	−0.09929	0.15923	0.063	0.01060
7	−0.014	3.60101	−0.28566	0.21869	0.058	0.00775
8	−0.012	2.59839	0.29335	0.38395	0.054	0.00780
9	−0.009	2.92365	0.02394	0.25285	0.049	0.00699
10	−0.010	2.79236	0.54137	0.21779	0.054	0.00537