

# Dynamic bidding

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## Abstract

Consider a second-price auction with costly bidding in which bidders with i.i.d. private values have multiple opportunities to bid. If bids are publicly observable, the resulting dynamic-bidding game generates greater expected total welfare than when bids are sealed or, if the seller commits to an optimal reserve, greater expected revenue. If the seller cannot commit to a bid-revelation policy, however, equilibrium outcomes are the same as if bids cannot be revealed.

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# 1 Introduction

Bids in sealed-bid auctions may arrive at different times but, since they are sealed, equilibrium play is the same as if bids were simultaneous. This paper considers the welfare and revenue implications of an alternative policy of publicly revealing all bids as they arrive, prior to an otherwise standard second-price auction with reserve price  $r$ , in which bidders have i.i.d. private values  $v_i$  and submitting a bid costs  $c > 0$ . In particular, I consider a dynamic-bidding game in which there are  $K$  “bidding rounds” at which bids can be simultaneously submitted, with bids made in each round automatically revealed prior to the next round.

Bidders with higher values submit earlier bids in equilibrium, allowing them to deter lower-value bidders from competing. Such bid deterrence benefits higher-value bidders, by allowing them to obtain the object at a lower expected price, while also benefitting lower-value bidders as they are able to avoid costly losing auction contests (Proposition []). For any given reserve price, the effect of dynamic bidding on seller expected revenue is ambiguous – the revenue from new sales to lower-value bidders may or may not dominate the lost revenue from selling to higher-value bidders at lower prices – but expected total welfare is always higher under dynamic bidding than under sealed bidding (Proposition []).

If the seller is able to commit to an optimal reserve price,<sup>1</sup> expected revenue is higher under dynamic bidding than under sealed bidding (Proposition []). Intuitively, the reason is that since dynamic bidding makes the auction more attractive to bidders at any

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<sup>1</sup>The seller’s reserve price is set before the game begins and, in particular, does not depend on the realized timing of entry into the auction. The seller can obviously do even better if able to commit to a reserve price that changes over time until the first bidder entry (see Li and Conitzer (2013)) or to more general dynamic mechanisms, the analysis of which is beyond the scope of this paper.

given reserve price, the seller can raise the reserve without losing sales. Indeed, raising the reserve price allows the seller to extract all the welfare gains associated with better bidder coordination in the form of greater expected revenue.

Dynamic bidding increases expected welfare by reducing inefficient bidder congestion in costly bidding contests, but there are limits to the entry coordination made possible by dynamic bidding. In particular, even in the limit as the number of bidding rounds goes to infinity, costly bidding contests arise in equilibrium with positive probability (Corollary to Theorem [ ]). To build intuition for this result, suppose that only a small mass of bidders (with values in the range  $[\hat{v}, \infty)$ ) were to bid (and bid truthfully) in the first round. Since first-round bids are unlikely, any bidder with value  $v_i$  such that  $r + c \ll v_i \ll \hat{v}$  will find it profitable to bid  $\hat{v}$  in the first round, since doing so (i) deters all those with values below  $\hat{v}$  from participating, allowing bidder  $i$  to win the object at the reserve price with probability close to one, and (ii) ensures that bidder  $i$  does not win at a price above his true value in the low-probability event that someone with value greater than  $\hat{v}$  also enters in the first round.

The paper focuses on a setting in which bidding is costly and bids are publicly observable, but all of the analysis carries over to an alternative setting in which bidding is costless and unobservable but there are costs associated with participating in the auction and other bidders can observe when these costs are incurred. The notion that participation can be costly is well-accepted in the auctions literature,<sup>2</sup> but the analysis here also depends on the notion that (i) these costs can be incurred prior to the auction itself and (ii) the act of incurring these costs is observable to other bidders. For instance, the

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<sup>2</sup>For instance, in a study of eBay coin auctions, Bajari and Hortacsu (2003) estimate that bidders faced participation cost of \$3.20, a significant amount given that expected revenue in these auctions ranged from \$40 to \$50.

cost of travelling to an auction site would *not* fit within the framework studied here, since bidders cannot observe who has travelled to the auction site until they have already incurred the travel cost.

That said, there are several sorts of potentially substantial participation costs that must be incurred before an auction and which therefore have the potential, if observed or revealed, to influence other bidders' decisions whether to participate.

*Example: Building personal trust.* In a corporate acquisition, top managers of each potential acquirer may need to meet at length with the target firm's management to become acquainted and build personal trust.

*Example: Establishing one's qualifications.* Bidders in auctions of high-value assets are often required to post bond and/or to receive third-party certification of their ability to pay. Similarly, bidders in complex procurement auctions must often first establish that they are capable of delivering the desired products or services.

*Example: Securing necessary expertise.* When bidding to provide expert services, a bidder first needs to secure the services of a qualified expert. This may entail substantial cost even if the bid is unsuccessful, if the experts' services are needed to prepare the bid.

Some of these "pre-participation costs" could potentially be kept secret. For instance, when a corporate acquisition target and potential suitor's management teams meet, both sides could choose to keep quiet about it. However, this paper's results show that there are situations in which the acquisition target and/or the suitor would prefer to reveal that such a meeting has occurred, undermining its secrecy. Why? From the suitor's point of view, revealing that a meeting has taken place serves to signal its interest in the target, deterring others from competing for the deal. Moreover, from the target's point of view, committing ahead of time to a transparent policy can increase expected revenue

if the target can also commit to a reserve price.

Other times, the seller may have little influence over whether bidders can observe each other's pre-participation activities and/or be unable to observe when such costs have been incurred. For instance, consider again the example in which bidders need to secure an expert's services in order to prepare a bid. In a "small world" with only a few qualified experts, each of whom is in regular contact with all of the bidders, each bidder would naturally be able to observe whenever anyone else secured an expert's services, as that expert would then be unavailable. On the other hand, the seller might not be able to observe anything about which bidders have secured an expert until the auction itself.

The rest of the paper is organized as follows. The introduction continues with some discussion of related literature. Section 2 presents the model of dynamic bidding. Section 3 provides the main analysis, while Section ?? considers an extension in which the seller decides which bids to reveal. Section 4 offers concluding remarks.

**Related literature.** This paper fits into the literature following Samuelson (1985) on auctions with costly bidding, the novel feature here being that bidders have multiple opportunities to enter the auction. A key finding is that bidders with higher values enter the auction earlier in order to signal their strength and deter others from entering the auction later. As such, the paper is similar in spirit to the jump-bidding literature (e.g. Avery (1998) and Horner and Sahuguet (2007)) and to the diverse literature that explores other mechanisms for bidder-to-bidder signaling in auctions.<sup>3</sup>

Another large related strand of literature features sequential bidder arrivals, where early entry also can serve to deter later entry. See e.g. McAfee and McMillan (1987)

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<sup>3</sup> See e.g. Eso and Schummer (2004), where higher bribes signal higher values; McAdams and Schwarz (2007), where waiting until the deadline to bid signals a high value; and Daley, Schwarz, and Sonin (2012), where bidders can burn money and/or make costly investments.

and Bulow and Klemperer (2009). A crucial difference is that, in such sequential-entry models, bidders who arrive earlier to the auction are typically assumed to be ex ante identical to those who arrive later. By contrast, because bidders here *choose* when to enter the auction, earlier entrants' values are drawn from a higher distribution.

Several other reasons have been explored in the literature for why bidders can have an incentive to bid early or wait until the last minute to bid, including: *Common values*: When the good has unknown quality, revealing one's interest may convey a positive signal about quality, prompting others to bid more aggressively. This gives bidders an incentive to shroud their interest, including by waiting until the very last moment to bid (Bajari and Hortacsu (2003), McAdams (2013)). *Endogenous information acquisition*: Fishman (1988) and Hirschliefer and Png (1989) provide an alternative explanation of jump bidding, that such early bids can deter others from acquiring information during the auction. On the other hand, Rasmusen (2006) shows that early bidding may also provoke others to invest in information acquisition, providing a possible explanation of the commonly-observed flurry of last-minute bids on online-auction sites such as eBay. Still more potential reasons for late bidding on sites like eBay include *overlapping auctions* of substitute goods with different deadlines (Wang (2003), Zeithammer (2006)), *unsophisticated bidders* (Ockenfels and Roth (2006), Ariely and Simonsohn (2008)), and *random delays* in bid transmission (Ockenfels and Roth (2006)).

My focus here is on comparing equilibrium auction outcomes under sealed vs dynamic bidding, not on characterizing the optimal sales mechanism. Several papers in the dynamic mechanism-design literature have characterized optimal dynamic mechanisms in somewhat related settings. See e.g. Board and Skrzypacz (2010), Ely, Garrett, and Hinnosaar (2012), Gershkov and Moldavanu (2009), Hinnosaur (2011), and Pai and Vohra (2013). A common theme of this literature is that bidders arrive to the auction accord-

ing to an exogenous process. However, when participation is costly and bidders observe private information about their values before deciding whether/when to participate, the bidder-arrival process is inherently endogenous. This makes characterizing the optimal mechanism potentially a very challenging problem.

The most closely related paper is Levin and Peck (2003) (hereafter “LP”). LP considers a game in which two firms<sup>4</sup> with i.i.d. entry costs have multiple opportunities to enter a new market. Each firm enjoys monopoly revenue  $R_m$  if it is the only one to enter or duopoly revenue  $R_d$  if both enter. When  $R_d = 0$ , LP’s game can be interpreted as a second-price auction with zero reserve price, where bidders have known common value  $v = R_m$  and i.i.d. entry costs. The basic structure of equilibrium entry is similar here and in LP, but the papers take different (and complementary) analytical approaches. For instance, whereas LP use a contraction argument to establish equilibrium uniqueness, I provide a direct proof and a simple algorithmic method to compute the equilibrium.

[[ADD COST DISCUSSION: See slides from Stanford talk. Plus Alessandro Pavan idea about “qualification cost” e.g. bank pre-approves your ability to pay up to a certain level; this is costly but not related to bid submitted.]]

## 2 Model

Potential bidders  $i = 1, \dots, N$  observe i.i.d. private values  $v_i$  with continuous c.d.f.  $F(\cdot)$ , p.d.f.  $f(\cdot)$ , and full support on  $[0, \bar{V}]$ . The seller then holds a second-price auction with reserve price  $r$ , with the novel feature that there are  $K \geq 1$  “bidding rounds” at which bidders simultaneously decide whether to incur cost  $c > 0$  to submit a publicly observable bid. (For simplicity, I assume that each bidder submits at most one bid.) The analysis

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<sup>4</sup>LP provide an extension to  $n > 2$  firms, under the assumption that duopoly revenue  $R_d = 0$ .

will focus on symmetric threshold equilibria.

**Definition 1** (Symmetric threshold equilibrium). A “*symmetric threshold equilibrium (STE)*” is a perfect Bayesian equilibrium in which each bidder  $i$  bids (and bids truthfully) in round  $k = 1, \dots, K$  if and only if  $v_i \geq e^{k|K}$  and no one has bid previously, where  $e^{1|K} \geq \dots \geq e^{K|K}$  are entry thresholds.

Let  $v_{-i} = \max_{j \neq i} v_j$  and let  $G(\cdot)$  and  $g(\cdot)$  denote the c.d.f. and p.d.f. of  $v_{-i}$ .

**Benchmark case: Sealed bids.** Samuelson (1985) characterized the unique STE when there is just one round of bids or, equivalently, when there are multiple rounds but bids are sealed. In this equilibrium, each bidder enters iff  $v_i \geq e^{1|1}$ , where the “simultaneous-entry threshold”  $e^{1|1}$  is defined by the entry-indifference condition:

$$(e^{1|1} - r)G(e^{1|1}) = c. \tag{1}$$

### 3 Dynamic bidding

Section 3.1 characterizes the unique symmetric threshold equilibrium (STE) when there are  $K$  bidding rounds, as well as the limit as  $K \rightarrow \infty$ . Section 3.2 then explores the welfare and revenue effects of dynamic bidding.

#### 3.1 Symmetric threshold equilibrium

**Theorem 1.** *The  $K$ -round bidding game has a unique symmetric threshold equilibrium, with entry thresholds  $(e^{1|K}, \dots, e^{K|K})$  satisfying  $r + c < e^{K|K} < \dots < e^{1|K} < e^{0|K} = \bar{V}$  and*



characterized by the following system of equations:

$$(e^{K|K} - r)G(e^{K|K}) = cG(e^{K-1|K}) \quad (2)$$

$$\int_{e^{k+1|K}}^{e^{k|K}} (v_{-i} - r)dG(x) = c(G(e^{k-1|K}) - G(e^{k|K})) \text{ for } k = 1, \dots, K - 1 \quad (3)$$

where by convention  $e^{0|K} = \bar{V}$ .

*Proof.* The proof is in the Appendix. □

*Discussion:* (2) is the equilibrium zero-profit condition for bidders having value  $v_i = e^{K|K}$ , akin to Samuelson (1985)'s condition (1). Similarly, (3) is the equilibrium indifference condition for bidders having value  $v_i = e^{k|K}$  between bidding in round  $k$  and waiting until round  $k + 1$ , for any  $k = 1, \dots, K - 1$ . (Entering in round  $k$  forces bidder  $i$  to incur a loss of  $c$  whenever others also enter in round  $k$ , i.e. when  $v_{-i} \in (e^{k|K}, e^{k-1|K})$ , while deterring others from entering leads bidder  $i$  to pay  $v_{-i} - r$  less whenever others would have entered in round  $k + 1$ , i.e. when  $v_{-i} \in (e^{k+1|K}, e^k)$ ).

**Corollary 1 to Theorem 1.** *In the unique STE, the ex ante probability that the first bid arrives in round  $k$  is decreasing in  $k$ .*

*Proof.* The first bid arrives in round  $k$  when  $\max_i v_i \in [e^{k|K}, e^{k-1|K})$ , i.e. with probability  $F(e^{k-1|K})^N - F(e^{k|K})^N$ . Since  $e^{0|K} > e^{1|K} > \dots > e^{K|K}$ ,  $F(e^{k-1|K})^N - F(e^{k|K})^N$  is decreasing in  $k$  if and only if  $G(e^{k-1|K}) - G(e^{k|K}) = F(e^{k-1|K})^{N-1} - F(e^{k|K})^{N-1}$  is decreasing in  $k$ . (Details for this step are straightforward and omitted.) Note that conditions (2,3) can be rewritten as

$$\frac{e^{K|K} - r}{c} = \frac{G(e^{K-1|K})}{G(e^{K|K})} \quad (4)$$

$$\frac{E[v_{-i} - r | v_{-i} \in (e^{k+1|K}, e^{k|K})]}{c} = \frac{G(e^{k-1|K}) - G(e^{k|K})}{G(e^{k|K}) - G(e^{k+1|K})} \quad (5)$$

for all  $k = 1, \dots, K - 1$ . Since  $e^{k+1|K} > r + c$ , the right-hand side of (5) is greater than one, implying  $G(e^{k-1}) - G(e^k) > G(e^k) - G(e^{k+1})$  for all  $k = 1, \dots, K - 1$ , as desired.  $\square$

**Corollary 2 to Theorem 1.**  $e^{K|K}$  is decreasing in  $K$ .

*Proof.* Suppose for the sake of contradiction that  $e^{K'|K'} \geq e^{K|K}$  for some  $K' > K$ . This implies  $G(e^{K'|K'}) \geq G(e^{K|K})$  and, by (4),  $\frac{G(e^{K'-1|K'})}{G(e^{K'|K'})} \geq \frac{G(e^{K-1|K})}{G(e^{K|K})}$ . So,  $e^{K'-1|K'} \geq e^{K-1|K}$  and  $G(e^{K'-1|K'}) - G(e^{K'|K'}) \geq G(e^{K-1|K}) - G(e^{K|K})$ . By (5), this then recursively implies both  $e^{K'-t|K'} \geq e^{K-t|K}$  and  $G(e^{K'-t|K'}) - G(e^{K'-t+1|K'}) \geq G(e^{K-t|K}) - G(e^{K-t+1|K})$  for all  $t = 2, \dots, K$ . Why? Consider  $t = 2$ . Since  $e^{K'|K'} \geq e^{K|K}$  and  $e^{K'-1|K'} \geq e^{K-1|K}$ ,  $E[v_{-i} - r | v_{-i} \in (e^{K'|K'}, e^{K'-1|K'})] \geq E[v_{-i} - r | v_{-i} \in (e^{K|K}, e^{K-1|K})]$ . Since  $G(e^{K'-1|K'}) - G(e^{K'|K'}) \geq G(e^{K-1|K}) - G(e^{K|K})$ , condition (5) for  $k = K - 1$  then implies that  $G(e^{K'-2|K'}) - G(e^{K'-1|K'}) \geq G(e^{K-2|K}) - G(e^{K-1|K})$ .  $e^{K'-2} \geq e^{K-2}$  then follows immediately from  $e^{K'-1} \geq e^{K-1}$ . Repeating this argument recursively, using condition (5) for  $k = K - 2, \dots, 1$ , establishes that  $e^{K'-K|K'} \geq e^{0|K} = \bar{V}$ . But this is a contradiction since, by Theorem 1,  $e^{K'-K|K'} < \bar{V}$ .  $\square$

**Theorem 2.**  $e^{k|K}$  is increasing in  $K$  for all  $k$ , with  $\{e^{k|\infty} = \lim_{K \rightarrow \infty} e^{k|K} : k = 1, 2, \dots\}$  determined recursively by

$$(e^{1|\infty} - r)G(e^{1|\infty}) - \int_{r+c}^{e^{1|\infty}} G(x)dx = c \quad (6)$$

$$(e^{k|\infty} - r)G(e^{k|\infty}) - \int_{r+c}^{e^{k|\infty}} G(x)dx = cG(e^{k-1|\infty}) \text{ for } k = 2, 3, \dots \quad (7)$$

*Proof. Step One: Derive analogues to (6,7) for all finite  $K$ .* By a standard Envelope Theorem argument, each bidder's interim expected equilibrium payoff in the unique STE takes the integral form

$$\Pi^K(v_i) = \int_{e^{K|K}}^{v_i} G(x)dx \text{ for all } v_i \geq e^{K|K}, \quad (8)$$

where  $G(x)$  is the probability that each bidder wins given value  $v_i = x$ . (A bidder with value  $v_i < e^{K|K}$  never enters and earns zero expected payoff.) At the same time, at each threshold  $e^{k|K}$ , we can also express  $\Pi^K(e^{k|K}) = (e^{k|K} - r)G(e^{k|K}) - cG(e^{k-1|K})$  since the threshold type  $e^{k|K}$  only wins if no one else enters in round  $k$ . In particular, for every  $K$  and  $k \leq K$ ,  $e^{k|K}$  must be the (unique) solution in  $v_i$  to

$$(v_i - r)G(v_i) - \int_{e^{K|K}}^{v_i} G(x)dx = cG(e^{k-1|K}), \quad (9)$$

where  $(e^{k-1|K}, e^{K|K})$  are viewed in parameters that vary with  $K$ .

*Step Two:*  $e^{k|K}$  is increasing in  $K$  for all  $k$ . The fact that  $e^{k|K}$  is increasing in  $K$  emerges as a simple comparative static of this solution. Consider first  $k = 1$ . Since  $e^{0|K} = \bar{V}$  for all  $K$ , the right-hand side of (9) does not depend on  $K$ . On the other hand, since  $e^{K|K}$  is decreasing in  $K$  (Corollary 2 to Theorem 1), the left-hand side of (9) is decreasing in  $K$ . Since  $\frac{d[(v_i - r)G(v_i)]}{dv_i} - \frac{d[\int_{e^{K|K}}^{v_i} G(x)dx]}{dv_i} = (v_i - r)g(v_i) > 0$ , the solution  $e^{1|K}$  of (9) must therefore be increasing in  $K$ . The rest of the proof is by induction on  $k$ . As long as  $e^{k-1|K}$  is increasing in  $K$ , the right-hand side of (9) is increasing in  $K$  while, as in the  $k = 1$  case, the left-hand side of (9) is decreasing in  $K$ . So, the solution  $e^{k|K}$  of (9) must be increasing in  $K$ , and the limit  $e^{k|\infty}$  exists for all  $k$ .

*Step Three:*  $\lim_{K \rightarrow \infty} e^{K|K} = r + c$ . Let  $\underline{v} = \lim_{K \rightarrow \infty} e^{K|K}$ . Suppose for the sake of contradiction that  $\underline{v} > r + c$ , and fix  $\hat{v} \in (r + c, \underline{v})$ . Since  $e^{K|K}$  is decreasing in  $K$ , a bidder with value  $\hat{v}$  must find it unprofitable to enter in every round  $k$  of every  $K$ -round bidding game. Entering in round  $k$  would allow a bidder with value  $\hat{v}$  to win at the reserve price with (ex ante) probability  $G(e^{k|K})$  while incurring cost  $c$  with probability  $G(e^{k-1|K})$ . For this to be unprofitable, the conditional probability that someone else enters in round  $k$  must be sufficiently high, namely,  $\frac{G(e^{k-1|K}) - G(e^{k|K})}{G(e^{k-1|K})} \geq \frac{\hat{v} - r - c}{\hat{v} - r}$ . Now, consider any  $K > \frac{\hat{v} - r}{G(r+c)(\hat{v} - r - c)}$  and define  $\hat{k} \in \arg \min_k (G(e^{k-1|K}) - G(e^{k|K}))$ .

Since  $e^{1|K} > \dots > e^{K|K}$ ,  $\min_k (G(e^{k-1|K}) - G(e^{k|K})) \leq \frac{1}{K} < \frac{G(r+c)(\hat{v}-r-c)}{\hat{v}-r}$ . So,  $\frac{\hat{v}-r-c}{\hat{v}-r} > \frac{G(e^{\hat{k}-1|K})-G(e^{\hat{k}|K})}{G(r+c)} > \frac{G(e^{\hat{k}-1|K})-G(e^{\hat{k}|K})}{G(e^{\hat{k}-1|K})}$  and entering in round  $\hat{k}$  is profitable for a bidder with value  $\hat{v}$ , a contradiction.

To complete the proof, note that (4) and (5) follow from (9) for  $k = 1$  and  $k > 1$ , respectively, by continuity in the limit as  $e^{K|K} \rightarrow r + c$ .  $\square$

*Discussion:* The limit-thresholds  $(e^{1|\infty}, e^{2|\infty}, \dots)$  provide a lower bound of sorts on bidding activity in each round. Since  $e^{k|\infty} > e^{k|K}$  for all  $(k, K)$ , any bidder with value  $v_i > e^{1|\infty}$  always bid in the first round, no matter how many rounds there may be, anyone with value  $v_i \in (e^{2|\infty}, e^{1|\infty})$  bids no later than the second round, and so on. Because the system (6,7) characterizes the limit-thresholds recursively, they are easy to compute. For instance, suppose that there are two bidders with i.i.d. values uniformly distributed on  $[0, 1]$ , the reserve price is zero, and the bidding cost  $c = \frac{1}{6}$ . The simultaneous-entry threshold  $e^S$  solves (1):  $(e^S)^2 - c = 0$ , or  $e^S = \sqrt{c} \approx .408$ . By contrast,  $e^{1|\infty}$  solves (6):  $(e^{1|\infty})^2 - c = \int_c^{e^{1|\infty}} x dx$ , or  $e^{1|\infty} = \sqrt{2c - c^2} \approx .552$ . Another interesting fact here is that  $e^{2|\infty} = \sqrt{2e^{1|\infty}c - c^2} \approx .396 < e^S$ . So, any bidder who waits until after the second round to bid in any  $K$ -round game would not bid at all if bids were sealed.

### 3.2 Welfare and revenue effects of dynamic bidding

This section investigates the welfare and revenue effects of having multiple bidding rounds ( $K > 1$ ) compared to having sealed bids ( $K = 1$ ). The main findings are that dynamic bidding (i) increases expected revenue under an optimal reserve price (Proposition 1) and (ii) increases both interim expected surplus and expected total welfare under any fixed reserve price (Proposition 2).

**Proposition 1.** *If the seller can commit to a reserve price, expected revenue is strictly higher when  $K > 1$  than when  $K = 1$ .*

*Proof.* Let  $e^S(r)$ ,  $e^{K|K}(r)$  denote the simultaneous-entry threshold in the 1-round game and the  $K$ -th round entry threshold for some  $K > 1$ , respectively, viewed as functions of the reserve price  $r$ . By inspection and comparison of (1) and (2),  $e^{K|K}(r) < e^S(r)$  for all  $r < \bar{V}$ . Moreover, it is straightforward to show that  $e^{K|K}(r)$  is continuous and strictly increasing in  $r$ .

Let  $r^S$  denote the optimal reserve price when bids are sealed. Define  $\hat{r} > r^S$  so that  $e^{K|K}(\hat{r}) = e^S(r^S)$ , and note that the  $K$ -round game with reserve  $\hat{r}$  has strictly less equilibrium entry than the 1-round game with reserve  $r^S$ . (Bidders with values in  $(e^{K|K}(\hat{r}), e^{1|K}(\hat{r}))$  always enter when bids are sealed, but stay out when  $\max_i v_i > e^{1|K}(\hat{r})$  in the  $K$ -round game.) On the other hand, the object's allocation is identical in both cases – the object is sold to the highest-value bidder when  $\max_i v_i > e^S(r^S)$  and otherwise not sold – as is interim expected surplus by (8) since  $e^S(r^S) = e^{K|K}(\hat{r})$ . Expected revenue must therefore be strictly higher in the  $K$ -round game with reserve  $\hat{r}$ , by an amount equal to the expected cost savings of having less equilibrium entry. This completes the proof since seller expected revenue in the  $K$ -round game is even higher under an *optimal* reserve.  $\square$

**Proposition 2.** *For any given reserve price, bidders' interim expected surplus and expected total surplus are each strictly higher in the unique STE when  $K > 1$  than when  $K = 1$ .*

*Proof.* The fact that bidders' interim expected surplus  $\Pi^K(v_i)$  is increasing in  $K$  follows immediately from (8), since  $e^{K|K}$  is decreasing in  $K$ . The rest of the proof focuses on expected total surplus.

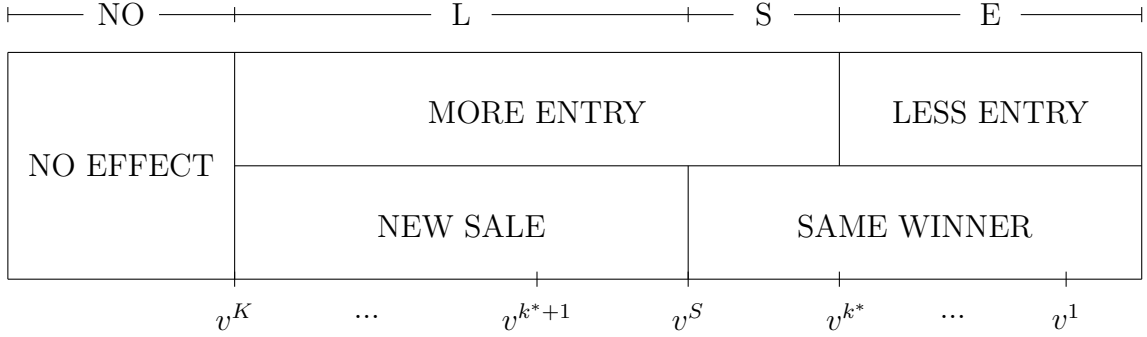


Figure 1: How having multiple bidding rounds changes equilibrium entry and sales.

Let  $k^*$  be the last bidding round in the  $K$ -round game in which all entrants would have entered in the simultaneous-move game, i.e.,  $v^{k^*} \geq v^S > v^{k^*+1}$ . Figure 1 illustrates the effect of moving from one to  $K$  rounds on equilibrium outcomes, in terms of entry and sales. For convenience, the value-space is divided into four regions  $\{E, S, L, NO\}$  with mnemonic labels: when  $\max_i v_i = v^{(1)} \in E = (v^{k^*}, \bar{V})$ , some bidder enters Early (in rounds  $1, \dots, k^*$ ) in the  $K$ -round game and would enter simultaneously; when  $v^{(1)} \in S = (v^S, v^{k^*})$ , no one enters Early but at least one bidder would enter Simultaneously; when  $v^{(1)} \in L = (v^{k^*+1}, v^S)$ , no one enters Simultaneously but someone would enter Late (in rounds  $k^* + 1, \dots, K$ ); finally, when  $v^{(1)} \in NO = [0, v^K)$ , no one ever enters.

*Step 1: when  $v^{(1)} \in E$ , total welfare rises.* When  $v^{(1)} \in E$ , the winner enters in round  $\hat{k} \leq k^*$  and is the same as under simultaneous entry. Ex post total welfare is (weakly) higher since bidders with values in  $(v^S, v^{\hat{k}})$  do not enter the auction.

*Step 2: when  $v^{(1)} \in S$ , expected total welfare falls by the same amount as  $L$ -type bidders' surplus falls.* When  $v^{(1)} \in S$ , the winner enters in round  $k^* + 1$  and is the same as under simultaneous entry. Ex post total welfare is (weakly) lower, however, since bidders with values in  $(v^{k^*+1}, v^S) \subset L$  enter and lose, when they would not have under simultaneous bidding. Since these  $L$ -type bidders do not win, their surplus falls and, importantly, falls by exactly the same amount in aggregate as total welfare falls.

*Step 3: when  $v^{(1)} \in L$ , expected total welfare rises by more than L-type bidders' surplus rises.* When  $v^{(1)} \in L$ , the seller's revenue is higher in the  $K$ -round game, obviously, since no entry would have occurred under simultaneous entry. The overall change in expected total welfare in this event is therefore strictly greater than the change in type-L bidders' surplus.

*Step 4: when  $v^{(1)} \in L \cup S$ , expected total welfare rises.* Combining Steps 2-3, the net welfare effect of having multiple bidding rounds when  $v^{(1)} \in L \cup S$  is greater than the net effect on type-L bidders in this event. Note that type-L bidders earn zero surplus when  $v^{(1)} \in E$ , whether or not preemptive bidding is allowed. So, the net effect of having multiple bidding rounds on type-L bidders when  $v^{(1)} \in L \cup S$  is positive iff its effect on type-L bidders' interim expected surplus is non-negative. As shown in Part One, however, bidders' interim expected surplus is greater when there are multiple bidding rounds.

All together, we conclude that expected total welfare is strictly higher when there are multiple bidding rounds, conditional on the event  $v^{(1)} \in L \cup S \cup E$ . This completes the proof, since bidder surplus and revenue are zero in the remaining event  $v^{(1)} \in NO$  in which no sales ever occur. □

## 4 Concluding Remarks

A standard argument in favor of conducting a sealed-bid auction is that sealed bids can make it more difficult for bidder-cartels to monitor and enforce a collusive agreement (Marshall and Marx (2012)). This paper emphasizes a countervailing upside associated with making bidding activity observable during the auction, that such “dynamic bidding” facilitates bidder coordination and hence reduces excess entry. This improved coordina-

tion increases bidder interim expected surplus and total expected welfare for any given reserve price (Propositions ??-2) and increases expected revenue when the seller sets an optimal reserve (Proposition 1). That said, equilibrium entry remains excessive, even in the limit as the number of bidding rounds goes to infinity, with a positive measure of high-value bidders entering immediately in the first round (Theorem 2).<sup>5</sup>

If bidders can costlessly communicate with the seller, this remaining entry inefficiency can be easily addressed.<sup>6</sup> Consider a class of mechanisms that specify who should pay the entry cost as well as who wins and what price they pay. The optimal mechanism in this context can be implemented by conducting a “virtual auction” based on bidders’ costless reports, inducing only the winner of this virtual auction to pay the entry cost, and then charging this winner the final price in the virtual auction. Working to identify optimal mechanisms when communication is costly is a worthwhile goal for future research. For recent progress on this problem, see e.g. Mookherjee and Tsumagari (2012).

## Appendix

### Proof of Theorem 1

*Part One: (2,3) are sufficient for STE.* Consider bidder  $i$ ’s best response, if all bidders  $j \neq i$  adopt  $(e^{1|K}, \dots, e^{K|K})$ -threshold strategies where  $(e^{1|K}, \dots, e^{K|K})$  satisfy (2, 3). Suppose first that  $v_{-i} < e^{K-1}$ , so that round  $K$  can be reached with no prior bids (if bidder  $i$  does

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<sup>5</sup>This pooling feature of the equilibrium differentiates this paper’s signaling mechanism from those in other auction models, such as Daley, Schwarz, and Sonin (2012), in which bidders can perfectly signal their value by burning money.

<sup>6</sup>Others who have explored how costless communication can help coordinate entry when participation is costly include Campbell (1998) and Miralles (2010), who allow for cheap talk among the bidders before the auction, and Quint and Hendricks (2013), who allow for indicative bidding.



not bid prior to round  $K$ ). By entering in round  $K$ , bidder  $i$  wins at the reserve price when  $v_{-i} < e^K$  and wins at price  $v_{-i}$  when  $v_{-i} \in (e^K, \min\{v_i, e^{K-1}\})$ , yielding expected payoff (expressed for convenience in ex ante terms)

$$\begin{aligned} X^K(v_i) &= (v_i - r)G(e^K) - cG(e^{K-1}) \text{ if } v_i \leq e^K \\ &= (v_i - r)G(e^K) + \int_{e^K}^{\min\{v_i, e^{K-1}\}} (v_i - v_{-i})dG(v_{-i}) - cG(e^{K-1}) \text{ if } v_i \geq e^K \end{aligned} \quad (10)$$

Note that  $X^K(v_i)$  is strictly increasing in  $v_i$  and, by (2),  $X^K(e^K) = 0$ . So, entering in round  $K$  is bidder  $i$ 's best response if and only  $v_i \geq e^K$ .

Next, suppose that  $v_{-i} < e^{k-1}$ , so that round  $k = 1, \dots, K - 1$  can be reached with no prior bids. Relative to waiting and entering in round  $k + 1$ , entering in round  $k$  has three effects, depending on others' values.

*Case #1:*  $v_{-i} < e^{k+1}$ . No one else enters in round  $k$  or would enter in round  $k + 1$ , so bidder  $i$  wins at the reserve price (for ex post payoff  $v_i - r - c$ ) whether he enters in round  $k$  or waits to enter in round  $k + 1$ .

*Case #2:*  $v_{-i} \in (e^{k+1}, e^k)$ . Bidder  $i$  is better off entering in round  $k$ , since doing so deters others from entering in round  $k + 1$ . Such entry deterrence allows bidder  $i$  to win at the reserve price rather than at price  $v_{-i}$  when  $v_i \geq v_{-i}$  (for ex post gain  $v_{-i} - r$ ), or to avoid losing the auction when  $v_i < v_{-i}$  (for ex post gain  $v_i - r$ ). Overall, then, bidder  $i$ 's (ex ante) expected gain due to entering in round  $k$  when  $v_{-i} \in (e^{k+1}, e^k)$  is

$$\begin{aligned} Y^k(v_i) &= \int_{e^{k+1}}^{e^k} (v_{-i} - r)dG(v_{-i}) \text{ if } v_i \geq e^k \\ &= (v_i - r)(G(e^k) - G(\max\{v_i, e^{k+1}\})) + \int_{e^{k+1}}^{\max\{v_i, e^{k+1}\}} (v_{-i} - r)dG(v_{-i}) \text{ if } v_i \leq e^k \end{aligned} \quad (11)$$

*Case #3:*  $v_{-i} \in (e^k, e^{k-1})$ . Bidder  $i$  is at least weakly worse off entering in round  $k$ , since there is an option value to waiting and observing what others' round- $K$  bids before

deciding whether to enter the auction. In particular, waiting until round  $k + 1$  allows bidder  $i$  to avoid incurring a loss of  $c - \max\{0, v_i - v_{-i}\}$  when  $v_{-i} > e^k$  and  $v_{-i} > v_i - c$ .<sup>7</sup> Overall, bidder  $i$ 's (ex ante) expected loss due entering in round  $k$  when  $v_{-i} \in (e^k, e^{k-1})$  is

$$\begin{aligned} Z^k(v_i) &= c(G(e^{k-1}) - G(e^k)) \text{ if } v_i \leq e^k & (12) \\ &= \int_{\max\{e^k, v_i - c\}}^{\min\{e^{k-1}, v_i\}} (c - v_i + v_{-i}) dG(v_{-i}) + c(G(e^{k-1}) - G(\min\{e^{k-1}, v_i\})) \text{ if } e^k \leq v_i \leq e^{k-1} + c \\ &= 0 \text{ if } v_i \geq e^{k-1} + c \end{aligned}$$

(To parse (12) in the most complex case when  $v_i \in (e^k, e^{k-1} + c)$ , note that (i) round- $k$  entry leads to a loss of  $c$  when others also enter and bidder  $i$  loses, i.e. when  $v_{-i} \in (e^k, e^{k-1})$  and  $v_{-i} > v_i$ , and (ii) round- $k$  entry leads to a loss of  $c - v_i + v_{-i}$  when others also enter and bidder  $i$  wins at a price greater than  $v_i - c$ , i.e. when  $v_{-i} \in (e^k, e^{k-1})$ ,  $v_{-i} < v_i$ , and  $v_{-i} > v_i - c$ .)

By (3),  $X^k(e^k) = Y^k(e^k)$ . Note further by inspection of (11,12) that, for any  $v'' > e^k > v'$ ,  $Y^k(v'') = Y^k(e^k) > Y^k(v')$  and  $Z^k(v'') < Z^k(e^k) = Z^k(v')$ . So,  $Y^k(v_i) \geq Z^k(v_i)$  for all  $v_i \geq e^k$  while  $Y^k(v_i) < Z^k(v_i)$  for all  $v_i < e^k$ . So, bidder  $i$ 's best response is to enter in round  $k$  when  $v_i \geq e^k$  but not enter in round  $k$  when  $v_i < e^k$ .

So far, I have shown that bidder  $i$ 's best response if any round  $k$  is reached with no prior bids is to enter (and bid truthfully<sup>8</sup>) if and only if  $v_i \geq e^k$ . To complete the proof,

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<sup>7</sup>If bidder  $i$  waits until round  $k + 1$  and  $v_{-i} > e^k$  but  $v_{-i} < v_i - c$ , bidder  $i$  will jump in and outbid the highest round- $k$  bidder once round  $k + 1$  is reached. So, in this case, he wins at price  $v_{-i}$  whether he bids in round  $k$  or waits until round  $k + 1$ .

<sup>8</sup>Bidding more than one's true value could be optimal, if such overbidding deters others from bidding later. However, since any round- $k$  bid greater than  $e^k$  is sufficient to deter all future entry, and bidder  $i$ 's value  $v_i \geq e^k$  whenever he enters in round  $k$ , truthful bidding is sufficient to deter all future entry. Moreover, as usual, truthful bidding ensures that bidder  $i$  only wins in round  $k$  when his value exceeds

observe that if bidder  $i$  follows this rule in all subgames with no prior bids, bidder  $i$ 's best response is never to bid in subgames with prior bids. Why? Suppose that the first bids received were in round  $k' < k$  and that bidder  $i$  has not bid prior to round  $k$ . All bids submitted in round  $k'$  are at least  $e^{k'}$  but, since bidder  $i$  did not bid in rounds  $1, \dots, k'$ , bidder  $i$ 's own value must be less than  $e^{k'}$ . Clearly, then bidder  $i$  prefers not to bid. All together, then, bidder  $i$ 's best response when all others adopt  $(e^1, \dots, e^K)$ -threshold strategies is to do so as well. This completes the proof that (2,3) are necessary and sufficient for existence of a STE with thresholds  $(e^1, \dots, e^K)$ .

*Part Two: Uniqueness of STE.* To establish uniqueness, I need to show that (2,3) has a unique solution. Note that (2,3) can be re-written as

$$G(e^{K-1}) = G(e^K) \left( \frac{e^K - r}{c} \right) \text{ and} \quad (13)$$

$$G(e^{k-1}) - G(e^k) = (G(e^k) - G(e^{k+1})) \frac{E[v_{-i} - r | v_{-i} \in [e^{k+1}, e^k]]}{c} \quad (14)$$

for all  $k = 1, \dots, K - 1$ . (Recall that  $e^0 = \bar{V}$ , so this is a system of  $K$  equations with  $K$  unknowns.)

Suppose for a moment that  $e^K = r + c$ . If so, (2) implies that  $e^{K-1} = r + c$  while (3) implies that  $e^{K-2} = \dots = e^1 = e^0 = r + c$ . This is a contradiction, clearly, since  $e^0 = \bar{V} > r + c$ . Similarly, for  $e^K > r + c$ , (2) determines  $e^{K-1}$  as a function of  $e^K$  (call it  $e^{K-1}(e^K)$ ) while (3) inductively determines the other thresholds  $(e^{K-2}, \dots, e^1, e^0)$  as functions  $(e^{K-2}(e^K), \dots, e^1(e^K), e^0(e^K))$  of  $e^K$ . ((14) determines  $e^{K-2}$  as a function of  $(e^{K-1}, e^K)$ . Since  $e^{K-1}$  is determined by  $e^K$ , so is  $e^{K-2}$ . Repeating this logic inductively determines  $e^{K-3}, \dots, e^1, e^0$  as functions of  $e^K$ .)

$(e^0 = \bar{V}, e^{1*}, \dots, e^{K*})$  solves (2,3) if and only if (i)  $e^{1*} = e^1(e^{K*}), \dots, e^{K-1*} = e^{K-1}(e^{K*})$  and (ii)  $e^0(e^{K*}) = \bar{V}$  or, equivalently,  $G(e^0(e^{K*})) = 1$ . Next, note that (13) implies that

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the price that he will pay to win.

$G(e^{K-1}(e^K)) - G(e^K) > 0$  is continuous and strictly increasing in  $e^K$ , while (14) implies by induction that  $G(e^{k-1}(e^K)) - G(e^k(e^K))$  is continuous and strictly increasing in  $e^K$ , for all  $k = 1, \dots, K - 1$ . In particular,  $G(e^0(e^K))$  is continuous and strictly increasing in  $e^K$ . So, there is a unique solution  $e^{K*}$  to  $G(e^0(e^{K*})) = 1$  and hence a unique solution  $(e^{1*}, \dots, e^{K*})$  to (2,3).  $\square$

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