

Understanding the Sources of Risk Underlying the Cross-Section of Commodity Returns*

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Abstract

We show that a model featuring an average commodity factor, a carry factor, and a momentum factor is capable of describing the cross-sectional variation of commodity returns. More parsimonious one- and two-factor models that feature only the average and/or carry factors are rejected. To provide an economic interpretation, we show that innovations in equity volatility can price portfolios formed on carry with a negative risk premium, while innovations in our measure of speculative activity can price portfolios formed on momentum with a positive risk premium. Furthermore, we characterize the relation of the factors with the investment opportunity set.

KEY WORDS: Commodity asset pricing models, futures returns, individual commodities as test assets, average returns, carry, momentum, innovations in equity volatility, and speculative activity, predictive regressions.

JEL CLASSIFICATION CODES: C23, C53, G11, G12, G13, C5, D24, D34.

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1. Introduction

The interaction of storage and convenience yield, and the hedging motives of producers, consumers, and speculators has led to the development of theories about the behavior of commodity futures prices and has spearheaded efforts to understand the evolution of futures prices over time and across maturities.¹ Despite substantial headway, several key questions remain unresolved.

Which commodity asset pricing model is capable of reconciling the stylized patterns of commodity futures returns? Do we garner distinct insights when we apply the models to individual commodities rather than portfolios? How are the commodity pricing factors related to innovations in economic variables? What is the behavior of the commodity factors over the different stages of the business cycle? Are the risk factors that describe the commodity cross-section also able to forecast developments in the real economic activity, bond, equity, and currency markets, as argued by adherents of asset pricing theories?

Our aim is to fill the aforementioned gaps in a market that has grown tremendously with the advent of financialization. In particular, we offer explanations for the patterns in commodity returns with an emphasis on isolating the economic forces behind the positive average returns of the commodity factors.

Using a set of baseline portfolios as well as managed portfolios, we establish that an average commodity factor, a carry factor, and a momentum factor can price the cross-section of commodity returns. Our analysis highlights several findings: (i) the Hansen and Jagannathan (1997) distance test does not reject correct model pricing; (ii) the average pricing errors are not statistically different from zero, when standard errors are computed using the Newey and West (1987) procedure, with and without the Shanken (1992) correction. We also examine the joint pricing ability of the three-factor model using time-series regressions

¹The commodity literature has evolved considerably since Keynes (1930), Hicks (1939), Kaldor (1939), and Samuelson (1965). For a partial list of empirical treatments, we mention Chang (1985), Fama and French (1988a), Bessembinder (1992, 1993), Bessembinder and Chan (1992), Deaton and Laroque (1992), de Roon, Nijman, and Veld (2000), Erb and Harvey (2006), Gorton and Rouwenhorst (2006), Gorton, Hayashi, and Rouwenhorst (2013), Miffre and Rallis (2007), Yang (2013), Szymanowska, de Roon, Nijman, and Goorbergh (2014), and Chiang, Huguen, and Sagi (2014). These studies mainly focus on the economic nature of commodity risk premia and their statistical attributes. Complementing the empirical work, a strand of theoretical research has centered around characterizing the shape of the futures curve. Such studies include Hirshleifer (1988, 1990), Litzenberger and Rabinowitz (1995), Routledge, Seppi, and Spatt (2000), Carlson, Khokher, and Titman (2007), Kogan, Livdan, and Yaron (2009), and Acharya, Lochstoer, and Ramadorai (2013). However, these contributions do not cater to the structure of the cross-sectional relations and are silent about the connections between the slope of the futures curves, average commodity returns, and returns of carry and momentum portfolios. The review articles by Till (2006) and Basu and Miffre (2013) provide a historical perspective.

and find that statistical tests do not reject model adequacy based on the implied alphas. Finally, we show that our results are robust using a randomization exercise.

The model also has broad support when individual commodities are used as test assets, allaying possible concerns regarding the results obtained using portfolios as test assets. In the time-series tests, we adopt the bootstrap procedure of Kosowski, Timmermann, Wermers, and White (2006), which accounts for the unbalanced nature of the panel and the fact that commodities exhibit different return volatilities, and find that none of the individual commodities has a significant alpha. In the cross-sectional tests, we show that it is important to model the time-varying exposure of each commodity to the carry and momentum factors. In this case, the model is not rejected according to χ^2 tests. These findings are crucial because working with individual commodities entails incorporating in the tests the full heterogeneity of commodity returns.

We show that the three-factor model that incorporates the momentum factor appears better aligned with the data compared to a one- or two-factor nested counterpart that contains the carry and/or the average factor, as studied in Szymanowska, de Roon, Nijman, and Goorbergh (2014) and Yang (2013). Our specification tests further show that incorporating an additional commodity value factor (along the lines of Asness, Moskowitz, and Pedersen (2013)) or a commodity volatility factor (along the lines of Menkhoff, Sarno, Schmeling, and Schrimpf (2012)) fails to improve the pricing ability across our set of test assets. Taken all together, our approach shows some promise in reconciling the large average returns to investing in a portfolio of backwardated commodities and high momentum commodities.

Having developed the view that the three factors explain the cross-section of commodity returns, the next core question is: What are the economic forces underlying the carry and momentum factors? First, we show that the commodity carry factor is linked to innovations in equity volatility. During periods of unanticipated high (low) equity volatility, the commodities in backwardation (contango) deliver low (high) returns, which could explain the positive compensation for the carry strategy. We further show that the innovations in equity volatility can price the commodity portfolios sorted on carry. At the same time, we show that innovations in equity volatility cannot price commodity portfolios sorted on momentum. The economic intuition is that the high average returns to carry is compensation for the low payoff of the

strategy in periods of high unexpected volatility.

Second, we demonstrate that innovations in aggregate speculative activity is positively related to the momentum factor, especially the returns of the past winners. We construct this speculative variable using large traders' positions data from CFTC. Our novel result is that innovations in speculative activity are able to price commodity portfolios sorted on momentum, but not the ones sorted on carry. We further show that innovations in speculative activity is negatively correlated with innovations in equity volatility and is mildly procyclical. Overall, our analysis indicates that investors do not like the low returns of momentum strategies during periods of worsening investment opportunity set and accordingly ask for positive remuneration.

Third, we establish that innovations in commodity volatility, log open interest, hedging pressure, scarcity, log of G7 industrial production, US TED spreads, and G7 inflation cannot price portfolios sorted on neither carry nor momentum. Taken altogether, our results isolate the role of equity volatility (speculative activity) in explaining the carry (momentum) factor.

Continuing with our quest to understand the carry and momentum factors we perform two additional exercises. First, both carry and momentum perform particularly well during recessions, which also coincide with negative returns for both the average commodity factor as well as the equity market.

Second, we address whether the commodity factors forecast changes in the investment opportunity set (as suggested in Fama (1991)). Our exercises uncover that the average commodity factor predicts GDP growth and equity returns with a positive sign and Treasury bond returns with a negative sign, and the effect is statistically significant (based on the Hodrick (1992) 1B covariance estimator). Our predictive regressions also show that increases in the carry and momentum factors are associated with future economic slowdowns, higher bond returns, and lower equity returns. Testifying to the global economic nature of the commodity carry and momentum factors, they also forecast the appreciation of the US dollar against the commodity currencies, extending the analysis of Chen, Rogoff, and Rossi (2010).

These findings favor a risk interpretation of the positive returns delivered by the (long-only) average commodity factor. Intriguingly, the carry and momentum factors seem to be valuable hedges against equity

market declines, implying that they should be associated with low average returns. On the other hand, our cross-sectional exercises convey the insight that the carry (momentum) strategy earns negative positive risk premia arising from its exposure to innovations in equity volatility (speculative activity). The takeaway is that the documented high average returns of carry and momentum speak to the multifaceted nature of risk compensation in commodity markets.

Our efforts complement a growing body of literature that strives to comprehend the behavior of commodity returns. Specifically, our work can be differentiated from Erb and Harvey (2006), Gorton and Rouwenhorst (2006), and Gorton, Hayashi, and Rouwenhorst (2013) in that our focus is on commodity asset pricing models and their ability to price the cross-section of commodity returns. Importantly, we evaluate the model using individual commodities in addition to baseline and managed portfolios. Our study also departs from Hong and Yogo (2012), who provide a horse race among alternative predictors of commodity futures returns but do not investigate cross-sectional implications.

We also differ from Asness, Moskowitz, and Pedersen (2013) and Koijen, Pedersen, Moskowitz, and Vrugt (2012), whose focal point is to construct an empirically viable global asset pricing model. Finally, we provide evidence that deviates from the core conclusions in Yang (2013) and Szymanowska, de Roon, Nijman, and Goorbergh (2014). Our empirical work also elaborates on the disparity in the cross-section of commodity futures returns, providing some distinction from the approaches in Deaton and Laroque (1992), Litzenberger and Rabinowitz (1995), Hirshleifer (1988), Routledge, Seppi, and Spatt (2000), Casassus and Collin-Dufresne (2005), and Kogan, Livdan, and Yaron (2009). The compatibility of the three-factor model with the documented commodity return patterns has possible implications for investment theory and practice, which transcends the scope of commodity investments.

2. Data description and commodity futures returns

Our commodity futures returns are constructed from end-of-day data provided by the Chicago Mercantile Exchange (CME). For each commodity and maturity available, the database contains, at the daily fre-

quency, a record of the open, low, high, and closing prices, along with information on open interest and trading volume. Our analysis centers on 29 commodity futures contracts covering four major categories, namely, agriculture, energy, livestock, and metal.

We take the start (end) date for our commodity futures sample to be January 1970 (September 2011). Starting the sample in January 1970 allows us to construct carry and momentum portfolios that contain at least three commodities. The number of commodities available ranges from a minimum of 15 in 1970 to a maximum of 28 in July 1994.

An important element to the calculation of monthly futures returns is the treatment of the first notice day, which varies across commodities (as can be seen from Table Online-I). For each commodity, we take a position in the futures contract with the second shortest maturity at the end of month t , while guaranteeing that its first notice day is *after* the end of month $t + 1$. We follow this treatment because, if the first notice day occurs before a long (short) position is closed, the investor may face a physical delivery (delivery demand) from the counterparty.

Consider our return calculation in the context of crude oil futures between the end of February and March 2011. Let $F_t^{(0)}$ be the price of the front-month futures contract and $F_t^{(1)}$ the price of the next maturity futures contract, both observed at the end of month t . Among the available contracts at the end of February 2011, we take a position in the May 2011 contract (i.e., $F_t^{(1)}$), as its first notice day falls in the middle of April. We do not invest in the April 2011 contract (i.e., $F_t^{(0)}$) because its first notice day falls in the middle of March 2011. The position in the May 2011 contract is closed at the end of March at price $F_{t+1}^{(1)}$. In the same vein, we switch to the June 2011 contract at the end of March 2011.

We calculate the returns of the long and short futures positions as

$$r_{t+1}^{\text{long}} = \frac{1}{F_t^{(1)}} \left(F_{t+1}^{(1)} - F_t^{(1)} \right) + r_t^f \quad \text{and} \quad r_{t+1}^{\text{short}} = -\frac{1}{F_t^{(1)}} \left(F_{t+1}^{(1)} - F_t^{(1)} \right) + r_t^f, \quad (1)$$

where r_t^f reflects the interest earned on the fully collateralized futures position (e.g., Gorton, Hayashi, and

Rouwenhorst (2013, equation (14))). Define

$$\text{er}_{t+1}^{\text{long}} \equiv r_{t+1}^{\text{long}} - r_t^f \quad \text{and} \quad \text{er}_{t+1}^{\text{short}} \equiv r_{t+1}^{\text{short}} - r_t^f, \quad (2)$$

as the excess return of a long and short futures position between the end of month t and $t + 1$, respectively.

Our procedure for constructing futures returns, which accounts for the first notice day, deviates from Shwayder and James (2011) but is broadly consistent with Gorton, Hayashi, and Rouwenhorst (2013) and Hong and Yogo (2012). We note that $F_t^{(0)}$ never enters our return calculation because of the way that the first notice day calendar interacts with our returns, which are based on end of month observations.²

The summary statistics tabulated in Table Online-II show that 20 out of 29 commodities have Sharpe ratios below 0.25, indicating that stand-alone investments in commodities are not attractive. Among other salient features, the commodity returns are serially uncorrelated (the absolute first-order autocorrelations are below 0.1 for 22 commodities) and typically positively skewed. Corn is the most liquid futures contract, as measured by its open interest, and propane is the least liquid.

Our data offers flexibility in two additional ways. First, the availability of daily futures returns allows us to construct monthly realized volatilities for each commodity. Next, futures prices at multiple maturities help to identify whether a commodity is in backwardation or contango.

Inspection of Table Online-II indicates that (i) the fraction of the months in which a commodity is in contango is often greater than when it is in backwardation, and (ii) a predominant portion of the commodities exhibit contango on average. Overall, the magnitudes reported in Table Online-II appear aligned with the corresponding ones in others, for example, Erb and Harvey (2006, Table 4) and Gorton, Hayashi, and Rouwenhorst (2013, Table I).

²Some data limitations are addressed in the following manner. First, when there is a missing observation (for example, palladium on a few occasions), we fill in the corresponding return from Bloomberg to maintain a complete time-series. Second, if there is no recorded futures price for a commodity on the last business day of a given month, we use prices from the second-to-last business day. For example, because there is no trading record for crude oil, natural gas, gasoline, and heating oil on Monday, May 31, 2010, we employ prices from Friday, May 28, 2010.

3. Asset pricing approach and methodology

To outline our approach and empirical tests, we denote the time $t + 1$ excess return of a commodity portfolio i by er_{t+1}^i and collect the returns on all test assets in a vector \mathbf{er}_{t+1} . No-arbitrage implies the existence of a candidate stochastic discount factor (SDF) m_{t+1} such that (Cochrane (2005, chapter 12)):

$$E[m_{t+1} \mathbf{er}_{t+1}] = \mathbf{0}, \quad \text{with} \quad m_{t+1} = 1 - \mathbf{b}'(\mathbf{f}_{t+1} - \boldsymbol{\mu}), \quad (3)$$

where \mathbf{f}_{t+1} is a vector of risk factors and $\boldsymbol{\mu}$ are the factor means.

In our setup, the parameter vector \mathbf{b} is estimated in the system:

$$E \begin{bmatrix} (1 - \mathbf{b}'(\mathbf{f}_{t+1} - \boldsymbol{\mu})) \otimes \mathbf{er}_{t+1} \\ \mathbf{f}_{t+1} - \boldsymbol{\mu} \\ \text{vec}((\mathbf{f}_{t+1} - \boldsymbol{\mu})(\mathbf{f}_{t+1} - \boldsymbol{\mu})') - \text{vec}(\Sigma_{\mathbf{f}}) \end{bmatrix} = \mathbf{0}, \quad (4)$$

using the generalized method of moments of Hansen (1982), where $\Sigma_{\mathbf{f}}$ is the variance-covariance matrix of \mathbf{f}_{t+1} . Our formulation follows that of Burnside, Eichenbaum, Kleshchelski, and Rebelo (2011) and Menkhoff, Sarno, Schmeling, and Schrimpf (2012, equation (A4)) in that our estimates incorporate the uncertainty associated with estimating the means and covariances of \mathbf{f}_{t+1} .

The specification of the SDF in equation (3) implies a beta representation, in which the expected excess returns of each asset depend on the vector of factor risk premia $\boldsymbol{\lambda}$, which is common to all assets, and the vector of risk loadings $\boldsymbol{\beta}_i$, which is asset-specific. More formally,

$$E[er^i] = \boldsymbol{\lambda}'\boldsymbol{\beta}_i, \quad \text{where} \quad \boldsymbol{\lambda} = \Sigma_{\mathbf{f}}\mathbf{b}. \quad (5)$$

As in Cochrane (1996), we first focus on a one-step GMM that uses the identity matrix as a weighting matrix, but we also report results for a two-step GMM that uses the optimal weighting matrix. The standard errors are based on the Newey and West (1987) procedure with lags selected automatically according to

Newey and West (1994).

Additionally, we provide estimates of λ using the cross-sectional regression methodology of Fama and MacBeth (1973). In the first step, we run a time-series regression of returns on the factors. In the second step, we run a cross-sectional regression of average returns on the betas without including a constant. The standard errors of λ are computed using the Newey and West (1987) procedure with automatic lag selection with and without the correction of Shanken (1992).

4. Motivating a model with average, carry, and momentum factors

In this section, we consider an asset pricing model for commodities that incorporates an average factor, a carry factor, and a momentum factor. Our analysis centers on the following specification of the SDF:

$$m_{t+1} = 1 - b_{\text{AVG}} (\text{AVG}_{t+1} - \mu_{\text{AVG}}) - b_{\text{CARRY}} (\text{CARRY}_{t+1} - \mu_{\text{CARRY}}) - b_{\text{CMOM}} (\text{CMOM}_{t+1} - \mu_{\text{CMOM}}), \quad (6)$$

implying that the expected excess returns is a function of its exposure to three factors.

The average factor, denoted by AVG_{t+1} , is the excess return of a long position in all available commodity futures (see equation (2)). AVG_{t+1} is required, because models that do not incorporate this average factor fail to explain the time-series variation in commodity returns (see Section 5.2).

The commodity carry factor, denoted by CARRY_{t+1} , and the momentum factor, denoted by CMOM_{t+1} , deserve further comments, since these factors can be constructed in a variety of ways and depend on the implementation of the underlying carry and momentum strategies. As detailed in Appendix A and the captions of Table Online-III and Table 1, we construct CARRY_{t+1} as the return on a portfolio that is long in the five commodities that are most backwardated (i.e., the lowest $\ln(y_t) < 0$) and short the ones that are most in contango (i.e., the highest $\ln(y_t) > 0$), where $y_t \equiv F_t^{(1)} / F_t^{(0)}$ is the slope of the futures curve.

As noted in Panel A of Table 1, over the past 42 years the carry factor (strategy C5) has been economically profitable, with an average annualized return of 16.34%, several times larger than the returns generated by investing in commodity indexes (see Table Appendix-I). The average return of the carry

factor is statistically different from zero as indicated by the bootstrap confidence intervals. The 95% confidence intervals, denoted as PW, lower CI and upper CI, are based on a stationary bootstrap with 10,000 iterations, where the block size is based on the algorithm of Politis and White (2004).

In our analysis, $CMOM_{t+1}$ is constructed as the return on a portfolio that is long in the five commodities with the highest returns over the previous six months and short the ones with the lowest returns over the previous six months. The average return of the momentum factor is 16.11% (see strategy M5 in Panel C of Table 1), and it is statistically significant.

Several considerations motivate an SDF driven by three factors. First, we conduct a principal component analysis with our test assets, which reveals that AVG_{t+1} is highly correlated to the first principal component, $CMOM_{t+1}$ is highly correlated with the second and third principal components, and $CARRY_{t+1}$ loads only, but substantially, on the third principal component. Importantly, the first three components explain 71% of the variation in the 12 baseline portfolios, which is nontrivial, given that there are 12 potential principal components.

Second, the top and middle panels of Figure 1 reveal the distinct time-series behavior of the carry and momentum factors, with shaded areas representing NBER recessions. The bottom panel plots the return differential between the carry and the momentum factors, that is, $er_t^{cm} \equiv CARRY_t - CMOM_t$, and shows that the standard deviation of er_t^{cm} is 8.5%, the minimum is -40.1% and the maximum is 39.4% . The factors perform differently over different economic conditions. For example, consider March 1980, during which the momentum (carry) factor delivered a return of -28.2% (11.2%). The poor performance of momentum over this month was caused by the sharp decline in silver prices, but such a decline had no impact on the returns to carry. Additionally, $CARRY_t$ and $CMOM_t$ *do not share* the same sign in 41% of the months, which further helps to dichotomize between the two factors.

[Fig. 1 about here.]

The notion that the two strategies are distinct can be further analyzed by computing their conditional correlations. To do so, we estimate the time-varying correlation between the returns generated by the

carry and momentum factors using a dynamic conditional correlation model (Engle (2002)), in which the dynamics of carry and momentum returns are modeled using a bivariate GARCH (1,1) model. The results, reported in Figure 2, indicate that the returns correlation between the two strategies has not increased or decreased over time. Furthermore, their unconditional returns correlation is only 0.27.

[Fig. 2 about here.]

Later we also show that the carry and momentum factors load differently on economic risk variables.

5. The key results on pricing the cross-section of commodity returns

In this section we study the ability of the three-factor model to explain the cross-section of commodity returns. We also compare, statistically and economically, the performance of the three-factor model to nested one-factor and two-factor counterparts, as well as alternative specifications that include a commodity value and a commodity volatility factor.

We report results for (i) 12 baseline portfolios, (ii) managed portfolios constructed using conditioning information (Cochrane (2005, Chapter 8.1)), and (iii) individual commodity returns. As a robustness check, we also present the results from a randomization exercise (Lustig, Roussanov, and Verdelhan (2011)).

5.1. AVG_t , $CARRY_t$, and $CMOM_t$ summarize the cross-section of commodity returns

Can the three-factor model explain the cross-section of commodity returns? Are carry and momentum priced risk factors? Could these factors rationalize the documented average returns across our test assets?

5.1.1. The results from the baseline portfolios are supportive of the model

To answer these questions, we first consider the baseline case of 12 portfolios as test assets: (i) four carry portfolios; (ii) five momentum portfolios; and (iii) three category portfolios, that is, agriculture, livestock,

and metal. We exclude the energy sector as a test asset because of its shorter time-series. Therefore, we estimate 12 parameters using 21 moment conditions.

We report in Panel A of Table 2 the GMM estimates of the factor risk premia λ , the loadings on the SDF \mathbf{b} , and the Hansen and Jagannathan (1997) distance measure. The Newey and West (Shanken) p -values are reported in parentheses (curly brackets).

The estimated risk premia of both carry and momentum are positive, implying that portfolios that covary more with CARRY_{t+1} and CMOM_{t+1} earn extra compensation. In particular, the estimate of 0.018 (0.012) for λ_{CARRY} (λ_{CMOM}) amounts to an annualized risk premium of 21.6% (14.4%) for CARRY_{t+1} (CMOM_{t+1}). The p -values attest to the statistical significance of both factors in pricing the test portfolios.

The average factor helps to describe the cross-section of commodity returns, and such finding contrasts the corresponding one from the equity market (Fama and French (1992)) and the currency market (Lustig, Roussanov, and Verdelhan (2011, Table 4) and Menkhoff, Sarno, Schmeling, and Schrimpf (2012, Table 2)). However, the estimated annualized risk premium for the average factor is about 6%, which is below the risk premia for the other two commodity risk factors.

Displayed in the column “HJ-Dist.” is the Hansen and Jagannathan (1997) distance measure, which quantifies the normalized maximum pricing errors. For the three-factor model, the distance measure is 0.006, with a p -value of 0.21. Consequently, we do not reject correct pricing.

The estimates of λ obtained using the Fama-MacBeth procedure are identical, by construction, to those obtained using the one-step GMM. The p -values based on Newey-West and Shanken are in agreement and establish the statistical significance of the three risk premia, even after accounting for the fact that the β_i 's are estimated. Overall, our evidence strongly supports the presence of priced risk factors.

With a GLS cross-sectional uncentered R^2 of 96.3% (Lewellen, Nagel, and Shanken (2010, Prescription 3)) and an OLS uncentered R^2 of 93.9%, the three factors capture a large fraction of the cross-sectional variation of the commodity portfolios. Furthermore, the χ^2 tests for the null hypothesis that the pricing errors are zero, have p -values equal to 0.22 and 0.25 for Newey and West (1987) and Shanken (1992),

respectively, indicating that the asset pricing model cannot be rejected. The model pricing errors, displayed in Panel A of Figure 3, as measured by the deviation from the 45-degree line, reveal that the unexplained returns are small.

[Fig. 3 about here.]

How can one quantify the contribution of each factor in explaining the returns cross-section? The issue of a possibly redundant factor is addressed from two different perspectives. First, Panels B and C of Table 2 report results for restricted versions of the baseline model that exclude, alternatively, the momentum or the carry factor. In this regard, the χ^2_{SH} tests of the pricing errors show that both restricted models are rejected, with p -values equal to 0.01 for the model that excludes $CMOM_{t+1}$ and 0.00 for the model that excludes $CARRY_{t+1}$. In particular, omitting the carry (momentum) factor worsens the performance, as the GLS R^2 drops to 67.7% (82.9%), and the p -value for the HJ-Dist. drops to 0.00 (0.15). In sum, the three-factor generalization provides a better characterization of commodity returns compared to its nested counterparts.

Building on our analysis, we also perform a two-step GMM estimation based on an optimal weighting matrix (e.g., Cochrane (1996, Table 1) and Lustig, Roussanov, and Verdelhan (2011, row GMM₂ in Table 4)). We first test three exclusion restrictions on the SDF in equation (6): (i) $b_{CARRY} \equiv 0$, (ii) $b_{CMOM} \equiv 0$, and (iii) $b_{CARRY} = b_{CMOM} \equiv 0$, and report the results below:

$b_{CARRY} \equiv 0$	$b_{CMOM} \equiv 0$	$b_{CARRY} = b_{CMOM} \equiv 0$
$\chi^2(1) = 14.18, p\text{-val.}=0.00$	$\chi^2(1) = 5.30, p\text{-val.}=0.02$	$\chi^2(2) = 30.13, p\text{-val.}=0.00$

All the restrictions are rejected, illustrating that both the carry and the momentum factors have loadings on the SDF that are statistically different from zero and, hence, provide additional pricing flexibility. Second, we perform the Hansen (1982) J -test of over-identifying restrictions, which is $\chi^2(9)$ -distributed, and find that it has a p -value of 0.167, reinforcing the conclusion that the three-factor model cannot be rejected.³

³Motivated by Tang and Xiong (2013) and Henderson, Pearson, and Wang (2012), we also study whether the financialization of commodities had an impact on the performance of the model. To address this issue, we compare model ability from 1970:07 to 2003:12 and 2004:01 to 2011:09, and obtain a p -value of 0.15 and 0.24, respectively, for the χ^2_{NW} test of zero pricing errors. Our evidence of correct model pricing on both subsamples shows that the financialization of commodities does not affect model performance.

5.1.2. The evidence from managed portfolios remains supportive of the model

Following Cochrane (2005, Chapter 8.1), this section generates additional test assets by interacting the baseline portfolios with one conditioning variable z_t at a time. Specifically, we define the augmented return vector $\mathbf{er}_{t+1}^* \equiv \mathbf{er}_{t+1} \otimes [1 \ z_t]$ and estimate the parameter vector \mathbf{b} using $E[m_{t+1} \mathbf{er}_{t+1}^*] = \mathbf{0}$.

For each conditioning variable z_t , Table Appendix-II presents the results from the Hansen and Jagannathan (1997) distance test as an overall measure of fit, as well as exclusion tests that analyze in what respects the performance of the model is robust the additional set of managed portfolios. Each set of results are based on 24 portfolios.

We focus on five conditioning variable (defined in Appendix C): (i) open interest growth, (ii) change in commodity volatility ΔVOL_t , (iii) the average slope of the futures curve for the commodities in backwardation (i.e., $\ln(y_t) < 0$), (iv) the currency returns (US dollar index, FX|USD), and (v) industrial production growth (G7 countries). Our choices of z_t are meant to reflect developments in commodity markets and to capture the state of the economy. For example, the aggregate open interest is a slight variation of the one used in Hong and Yogo (2012), while the relation between the commodity risk premia, currency returns, the slope of the futures curve, and macroeconomic variables have been studied by, among others, Bailey and Chan (1993, Table 1), and Chen, Rogoff, and Rossi (2010, Table IV).

In four out of five cases, we are unable to reject the null that the HJ distance measure is equal to zero (Panel A), as the p -values are greater than 0.05. The same holds for the J -test results (Panel B). This implies that when additional test assets are included in the analysis, the performance of our model does not deteriorates substantially. The joint tests of significance for carry and momentum (Panel C) robustly reject the null that the factor loadings are jointly equal to zero. Moreover, the carry factor is significant in all cases and the momentum factor is significant in four out of five cases. Overall, this exercise illustrates that the results established using the baseline portfolios extend to a variety of managed portfolios.⁴

⁴Are our conclusions robust when commodities are double-sorted first by momentum and then by carry, as described in Appendix B. We conduct cross-sectional tests on four portfolios. We find that the χ_{NW}^2 test for the model that excludes the carry (momentum) factor is rejected with a p -value of 0.07 (0.00). On the other hand, the model that includes both is not rejected. For instance, the χ_{NW}^2 has a p -value of 0.38, affirming that the pricing errors are statistically indistinguishable from zero. Furthermore, the magnitudes of the factor risk premia are consistent with those in Table 2. The three-factor model offers considerable flexibility

5.1.3. The results are robust under a randomization procedure

How sensitive are our results to the randomization procedure advocated in Lustig, Roussanov, and Verdelhan (2011)? Following their approach, in the first step we construct the average, carry, and momentum factors based on commodities whose ticker symbol starts with the letter A through the letter M. In the second step we construct two carry, two momentum, and three category portfolios based on commodities whose ticker symbol starts with the letter N through the letter Z. Finally, we use the factors constructed on the first set of commodities to price portfolios formed on the second set of commodities, and we also conduct the reverse exercise.

Such a procedure poses a higher hurdle for the pricing models, given that the commodities included in the pricing factors are different from the ones included in the priced portfolios.

Together, the evidence from Panels A and B of Table Appendix-IV provides justification for the inclusion of both carry and momentum to price the cross-section of commodity returns. In fact, the three-factor model is not rejected on the first set of commodities and is borderline rejected on the second. On the other hand, the two-factor model that features the average and the carry factor is rejected in both cases. The model that features the average and the momentum factors is not rejected in the first subsample, but it is rejected on the second.

5.1.4. Additional tests reject the relevance of commodity volatility and commodity value factors

Could other economically relevant factors drive out the explanatory ability of our commodity factors? To investigate additional models, we augment the three-factor model in equation (6) with either (i) a commodity value factor (in the spirit of Asness, Moskowitz, and Pedersen (2013)) or (ii) a commodity volatility factor (in the spirit of Menkhoff, Sarno, Schmeling, and Schrimpf (2012)).

As reported in Panel A of Table Appendix-III, the Newey-West p -values for the null hypothesis of $\lambda_{\text{VALUE}} = 0$ and $\lambda_{\text{AVOL}} = 0$ are, respectively, 0.21 and 0.94, indicating that these additional risks are not in pricing the various test assets.

priced. Next, to examine the relevance of each additional factor in the SDF specification, we also perform a χ^2 exclusion restriction test in the context of a two-step GMM. The p -values for the $\chi^2(1)$ statistics are 0.51 (0.41) for the value (volatility) factor. Thus, our tests seem to favor a more parsimonious three-factor model specification.⁵

5.2. Time-series regressions: The hypothesis of zero alphas is not rejected

To assess how the three-factor model fares in capturing commodity returns, we perform the following regressions for the baseline test assets indexed by $i = 1, \dots, 12$:

$$er_t^i = \alpha^i + \beta_{AVG}^i AVG_t + \beta_{CARRY}^i CARRY_t + \beta_{CMOM}^i CMOM_t + \varepsilon_t^i \quad \text{for } t = 1, \dots, T. \quad (7)$$

We gauge model adequacy by testing the joint hypothesis that $\boldsymbol{\alpha} = \mathbf{0}$, where $\boldsymbol{\alpha} = [\alpha^1, \dots, \alpha^{12}]'$. This hypothesis of zero pricing errors is tested by constructing the statistic $\hat{\boldsymbol{\alpha}}' \text{var}(\hat{\boldsymbol{\alpha}})^{-1} \hat{\boldsymbol{\alpha}}$ in a GMM setting, which is asymptotically distributed as $\chi^2(df)$, where df is the dimensionality of $\boldsymbol{\alpha}$ (see Cochrane (2005, page 234)). We also test the individual significance of α^i for $i = 1, \dots, 12$.

The results, reported in Table 3, show that the three-factor model can describe the returns variation in the test portfolios. The p -value of 0.11 indicates that we are unable to reject the null hypothesis that $\boldsymbol{\alpha} = \mathbf{0}$. Furthermore, ten out of the 12 α estimates have p -values that exceed 0.05 and the largest (absolute) α has an annualized value of 4.8%, while the majority of α 's are below 2.4%, implying that the departures from the model are economically small.

All the β_{AVG} coefficients are uniformly positive and statistically significant and range from 0.659 to 1.165. The implication of this finding is that the returns of commodity portfolios manifest a strong commodity market component.

As nine (ten) out of 12 p -values on β_{CMOM}^i are below 0.05 (0.1), we also conclude that $CMOM_t$

⁵Additionally, we augment the composition of the test portfolios to include five portfolios that are sorted based on the volatility of commodity returns computed using daily returns over the past month, as in Menkhoff, Sarno, Schmeling, and Schrimpf (2012). The results, reported in Table Online-IV, show that the three-factor model continues to perform well, while the two-factor models are rejected, according to both the χ_{NW}^2 and χ_{SH}^2 tests.

explains the return dynamics of the commodity portfolios. We further note that the β_{CARRY}^i coefficients are statistically significant for carry portfolios. However, CARRY_t does not seem to impact the returns of momentum and commodity category portfolios in a statistically significant manner.

Table 4 reports the performance of the one- and the two-factor models. From a statistical standpoint, the R^2 's associated with the one-factor model, driven by the carry factor alone, are negative for five out of the 12 portfolios and are significantly lower than the ones associated with the three-factor model. This model is fundamentally misspecified with five out of 12 α 's significantly different from zero and a $\chi^2(12)$ statistic, indicating model inadequacy. The two-factor model featuring the average and the carry factor is also rejected according to the $\chi^2(12)$ statistic indicating that momentum has explanatory power beyond the average and carry factor and may be needed to characterize the variation in commodity returns.

Table Appendix-V, which presents the performance of the three-factor model under the randomization approach further reinforces the relevance of the momentum factor. The main message from combining the results from Table 3, Table 4 and Table Appendix-V is that the three-factor generalization can explain the cross-sectional dispersion of commodity returns.

5.3. Model performance using individual commodity returns as test assets

While the tradition is to examine the performance of factor models using portfolios of assets sorted on identifiable characteristics, there appears to be some interest in examining models based on individual assets (e.g., Ang, Liu, and Schwarz (2010), Chordia, Goyal, and Shanken (2013), and Jegadeesh and Noh (2013)). Catering to this line of thinking, we assess model performance using individual commodities as test assets.

Our goal is twofold. First, we seek to evaluate whether the factors we have identified in the context of portfolios are also useful in characterizing the time-series of individual commodity returns. To do so, we implement a bootstrap procedure to account for the nonnormalities in the joint distribution of alphas and the unbalanced nature of the individual commodity data. Second, we aim to study model adequacy and estimate factor risk premia using cross-sectional tests based on individual commodities. We do this under

the setting of both constant and time-varying loadings on the carry and the momentum factors.

5.3.1. Return time-series behavior of individual commodities and bootstrap tests of alphas

For each of the 29 commodities, we perform the time-series regression in equation (7) and report the coefficients in Table 5. There are four commodities with significant α : barley and corn have negative α 's significant at the 5% level, while unleaded gas and heating oil have positive α 's significant at the 10% level. Even if under the truth none of the commodities had a significant unexplained performance, one would expect to observe three commodities with a significant α at the 10% level, by virtue of random variation. Furthermore, it is not feasible to conduct tests of joint α significance because of the unbalanced nature of the panel (trading starts and ends at different dates across commodities).

To partially overcome the above limitations, we follow Kosowski, Timmermann, Wermers, and White (2006, Section III.B.1 and Section IV.D) and resort to a bootstrap procedure. We assess whether individually (and jointly) the commodities in Table 5 with significant α 's maintain their significance once we account for the fact that (i) our cross-section covers a sizable number of commodities, (ii) the volatility levels are different across commodities, and (iii) the commodities series have different time-series lengths. We also estimate whether the average α computed on the whole set of commodities statistically different from zero. As recommended, we focus on the t -statistics associated with the individual commodities α 's to draw our inference.

[Fig. 4 about here.]

Figure 4 conveys that none of the commodities with p -values smaller than 10% in Table 5 is significant once we cast the test in our bootstrap setting. This can be seen by observing the results for the commodity with the most negative alpha t -statistic (i.e., barley, Panel A1), as well as the second-to-most negative alpha t -statistic (i.e., corn, Panel A2). For both commodities, the t -statistics obtained in the sample are not sufficient to reject the null that the α 's are equal to zero and the p -values for the alpha t -statistics associated with the two commodities are 0.40 and 0.13, respectively. The same holds for the commodity with the

highest alpha t -statistic (i.e., unleaded gas, Panel B1), and the commodity with the second-highest alpha t -statistic (i.e., heating oil, Panel B2) whose p -values are 0.53 and 0.34, respectively. Furthermore, we also test whether the worst (best) two commodities have coefficients that are jointly statistically different from zero and obtain a p -value of 0.24 (0.44), indicating that the α 's are not statistically different from zero. Finally, we turn to the null hypothesis that the average α across commodities is equal to zero. The average t -statistic is -0.013 , with a corresponding p -value of 0.46, indicating that the null of zero average α cannot be rejected.

In summary, our results suggest model adequacy for individual commodity returns, consistent with the conclusions we draw using portfolios as test assets.

Turning to the betas, the results in Table 5 indicate that all the β_{AVG} coefficients are positive and statistically significant. In contrast, seven (four) β_{CARRY} (β_{CMOM}) coefficients have $\text{NW}[p]$ values that are less than 0.1. The adjusted R^2 varies from 4.5% for orange juice to 47.2% for soybeans, implying significant variation in the model ability to explain the return time-series of individual commodities.

[Fig. 5 about here.]

How should one interpret the difference in β loadings in Table 5 compared to the portfolio results reported in Table 3? One possibility is that the exposure of individual commodity returns to the carry and momentum factors may be varying due to changes in supply and demand conditions. To formalize this notion, we estimate the time-varying loadings (e.g., Engle (2013, page 23)) of each commodity with respect to the carry and the momentum factors using the Kalman filter (the procedure is outlined in the caption to Figure 5). The likelihood ratio tests indicate that the null of constant loadings on carry and momentum can be rejected for 26 (25) commodities at the 10% (5%) level. We also compute the monthly standard deviation of the time-varying coefficients and find that the 25th, 50th, and 75th percentiles of the distribution equals 0.007, 0.057, and 0.137, respectively. To illustrate the general pattern across commodities, we display in Figure 5 the time-varying coefficients associated with corn and silver. Collectively, these findings contradict the notion of constant betas for individual commodities.

5.3.2. Explaining the cross-sectional returns of individual commodities

The pertinent question now is whether the model also performs well in the cross-section of individual commodity returns. Going beyond the extant literature, we conduct cross-sectional tests using both constant betas as well as time-varying betas estimated with the Kalman filter.

The results for the Fama-MacBeth regressions with constant beta coefficients are reported in Panel A of Table 6. Our estimates support positive risk premia for the average and the carry factors that are quantitatively similar to the portfolio results reported in Table 2. In contrast, the risk premium for the momentum factor is small and insignificant. Furthermore, the χ^2 tests with and without the Shanken correction both reject the model, with p -values equal to 0.04 and 0.07, respectively.

Could one improve model performance by relaxing the assumption of constant betas? Panel B of Table 6 adapts the standard Fama-MacBeth regressions to account for time-varying betas. In the first step, we model the time-varying beta coefficients with respect to the carry and momentum factors using the Kalman filter. In the second step, we follow the more traditional implementation of the Fama-MacBeth procedure described in Cochrane (2005, pages 245–247), as it is no longer appropriate to run a cross-sectional regression of average returns on the betas due to the time-varying nature of the coefficients.⁶

The key finding is that the three-factor model is no longer rejected, as the p -value of the χ^2 test is 0.14. Furthermore, the factor risk premia associated with the average and the carry factors remain economically and statistically significant. While the risk premium of the momentum factor increases to 0.012, it remains statistically insignificant with a p -value of 0.31.

Overall, our findings emphasize the role of time-varying betas in modeling the cross-section of individual commodity returns. Specifically, under the more plausible setting of time-varying exposures of individual commodities on carry and momentum, the three-factor model is not rejected on statistical grounds.

⁶As noted in Cochrane (2005, page 245), this procedure has the limitation of not allowing corrections for the fact that the betas are estimated. We also cannot compute R^2 values.

6. Economic interpretations of the commodity factors

We have established that – taken together – the average, carry and the momentum factors can summarize the cross-sectional variations in commodity returns. Furthermore, our results establish that the exposure of commodities to the carry and momentum factors warrants a positive and sizeable risk premium.

What are the economic interpretations for these findings? Could one identify the sources of risk that underlie the premia of carry and momentum? How do the commodity factors behave over different stages of the business cycle? What is the information content of predictive slope coefficients when the factors are used to forecast the investment opportunity set? Do the long legs of carry and momentum load dynamically on certain economically sensitive commodities and hence explain their high average returns? Answers to these questions could help to get a firmer grasp of the various risks impacting the commodity market.

6.1. The carry factor is linked to innovations in equity volatility

The objective of this section is to show that the carry factor loads negatively on innovations in equity volatility, denoted as $\Delta\text{VOL}_t^{\text{equity}}$. Our motivation for considering $\Delta\text{VOL}_t^{\text{equity}}$ as an economic risk variable is that investors require compensation for holding those assets that pay poorly in periods with positive innovations in equity volatility (e.g., Ang, Hodrick, Xing, and Zhang (2006), Lustig, Roussanov, and Verdelhan (2011, Section 4.5.3), and Menkhoff, Sarno, Schmeling, and Schrimpf (2012)). The idea is that positive changes in equity volatility are perceived to be bad states where the marginal utility is high.

We use a two-pronged approach to isolate the nature of the carry risk premium. First, we establish the statistical relation between the carry factor and $\Delta\text{VOL}_t^{\text{equity}}$ through time-series regressions. Second, and more fundamentally, we use $\Delta\text{VOL}_t^{\text{equity}}$ to directly price the various carry test portfolios.

In the first step, we regress the carry factor on innovations in equity volatility:

$$\text{CARRY}_t = \phi_0 + \phi_1 \Delta\text{VOL}_t^{\text{equity}} + e_t, \quad (8)$$

We take VOL_t^{equity} to be the cross-sectional equity return volatility across the G20 countries (see Appendix C). The results, reported in Panel A of Table 7, show that the carry factor is significantly negatively related to $\Delta Vol_t^{\text{equity}}$, with a p -value of 0.04. The economically sensible inverse relation between the carry factor and $\Delta VOL_t^{\text{equity}}$ is also reflected in the regressions involving the long leg of carry. Specifically, the returns from taking a long position in the backwardated commodities is negatively and significantly exposed to $\Delta VOL_t^{\text{equity}}$, with a p -value on the slope coefficient of 0.01. In contrast, the returns from shorting contangoed commodities bears no relationship with $\Delta VOL_t^{\text{equity}}$, with a p -value on the slope coefficient of 0.61. This result is important, since the bulk of the average carry returns stem from the long leg (as evidenced in Table Appendix-I).

The second and crucial step is to show that $\Delta VOL_t^{\text{equity}}$ is capable of pricing the carry portfolios. Accordingly, we modify our specification in equation (6) to the following:

$$m_{t+1} = 1 - b_{\text{AVG}} (\text{AVG}_{t+1} - \mu_{\text{AVG}}) - b_{\Delta \text{VOL}} \Delta \text{VOL}_{t+1}^{\text{equity}}. \quad (9)$$

The results, reported in Panel B of Table 7, indicate that $\Delta \text{VOL}_{t+1}^{\text{equity}}$ is capable of explaining the cross-section of portfolios formed on carry. First, the risk-premium on the $\Delta \text{VOL}_{t+1}^{\text{equity}}$ factor is negative and statistically significant, consistent with the interpretation that positive changes in equity volatility are associated with a deterioration in the investment opportunity set. Second, the χ^2 tests do not reject the null hypothesis of correct model pricing with a χ_{NW}^2 (χ_{SH}^2) p -value of 0.24 (0.86).

To establish that the effect of equity volatility is specific to the carry portfolios, we also price the five portfolios formed on momentum using the model in equation (9). The key finding is that $\Delta \text{VOL}_{t+1}^{\text{equity}}$ cannot price the five test portfolios as the χ_{NW}^2 equals 12.15, with a Newey-West p -value of 0.01. Our findings therefore identify an economic source underlying carry returns.

Overall, our evidence in Table 7 favor a risk-based explanation for the positive average returns associated with the carry factor.

6.2. The momentum factor is linked to innovations in speculative activity

The thrust of this section is to offer an economic explanation for the ability of the momentum factor to explain commodity returns. To address this, we first propose a proxy for aggregate speculative activity in the commodity market. We then investigate if risks associated with this proxy are priced in the cross-section of momentum sorted portfolios.

We construct a measure of speculation at the individual commodity level using large trader positions from CFTC:

$$\text{Speculation}_{t,j} = \frac{\text{Noncommercial long}_{t,j} + \text{Noncommercial short}_{t,j}}{\text{Total long positions}_{t,j} + \text{Total short positions}_{t,j}}, \quad j = 1, \dots, 29. \quad (10)$$

For example, $\text{Noncommercial long}_{t,j}$ represents the total long positions of noncommercial traders for commodity futures j in month t (see Gorton, Hayashi, and Rouwenhorst (2013, Appendix C) or Haase, Seiler, and Zimmermann (2014, Appendix A) for a description of the CFTC data). We equally weight the commodity-specific speculation variable in equation (10) and denote it by Speculation_t .

We compute the monthly innovations in aggregate speculative activity $\Delta\text{Speculation}_t$ and consider the univariate regression:

$$\text{CMOM}_t = \psi_0 + \psi_1 \Delta\text{Speculation}_t + e_t. \quad (11)$$

The regression results reported in Panel A of Table 8 reveal that the relation between the momentum factor and innovations in speculative activity is positive and statistically significant (the p -value is 0.02). Our analysis further shows that the long leg of the momentum strategy is positively related to $\Delta\text{Speculation}_t$, with a slope coefficient p -value equal to 0.01. On the other hand, we cannot reject that the slope coefficient on the short leg of momentum is equal to zero, as evidenced by a p -value of 0.86.

In the next step, we investigate whether innovations in speculative activity price the five momentum portfolios when the momentum factor is replaced with $\Delta\text{Speculation}_{t+1}$. To this end, we consider an altered

SDF of the type:

$$m_{t+1} = 1 - b_{AVG} (AVG_{t+1} - \mu_{AVG}) - b_{\Delta Speculation} \Delta Speculation_{t+1}. \quad (12)$$

The results, reported in Panel B of Table 8, show that the risk premium on the innovation in speculative activity is positive and significant. The χ^2 tests do not reject the null hypothesis of correct model pricing according to both Newey-West standard errors (p -value of 0.28) and Shanken standard errors (p -value of 0.62).⁷

We also use $\Delta Speculation_{t+1}$ to price the carry portfolios and obtain a χ^2_{NW} statistic of 21.45 with a Newey-West p -value of 0.00, indicating that the role of $\Delta Speculation_{t+1}$ in pricing momentum portfolios is unique.

To build intuition for the new finding that speculative activity explains the cross-sectional variation in momentum sorted portfolios, we conduct additional exercises. First, we find that $\Delta Speculation_t$ is negatively correlated with equity volatility $\Delta VOL_t^{\text{equity}}$, with a correlation coefficient of -0.17. Second, $\Delta Speculation_t$ is mildly procyclical, as it has 0.05 correlation coefficient with G7 industrial production growth. Accordingly, it appears that decreases in speculative activity partly reflect deteriorations in global economic conditions. Therefore, one way to interpret our results is that the positive risk-premium for momentum is compensation for bearing low momentum returns during these periods.

6.3. Innovations in alternative economic variables appear irrelevant for pricing

The analysis reported below lends support to the notion that while innovations equity volatility and speculative activity capture specific risks that matter to commodity markets, innovations in (many) economic variables do not impact the cross-section of commodity returns.

⁷We also considered an aggregated version of the Working's T index, adapting the work of Buyuksahin and Robe (2014, page 50) by equally weighting the Working's T indices across all commodities in each month (see also Haase, Seiler, and Zimmermann (2014, Section 4.1), and Manera, Nicolini, and Vignati (2013, equation (1))). When we use innovations in this measure of speculation to price the cross-section of momentum portfolios, the risk-premium is positive and significant, but the model is rejected with a p -value of 0.048.

To convey this finding in a parsimonious manner, we focus on the cross-sectional Fama-McBeth regressions and use, as criterion, the significance of the χ_{NW}^2 statistic. Throughout, we use alternatively the five momentum portfolios or the four carry portfolios as test assets. The χ_{NW}^2 statistics, along with the associated p -values, are reported in Table 9. Appendix C details the construction of the economic variables.

Innovations in commodity volatility. The first economic variable we consider is innovations in cross-sectional commodity volatility. Unlike the evidence from currency portfolios sorted on the forward discount (Menkhoff, Sarno, Schmeling, and Schrimpf (2012)), innovations in the cross-sectional commodity volatility does not impact returns of commodity carry portfolios, as the model is rejected with a p -value of 0.00. In addition, innovations in commodity volatility is unable to reconcile the return pattern of momentum portfolios, with a p -value of 0.00. The takeaway is that innovations in equity volatility fare better at capturing the return variation in commodity carry portfolios than innovations in commodity volatility;

Innovations in log open interest. Following Hong and Yogo (2012), we construct the change in log open interest and use it as a candidate factor. Our results indicate that this variable is unable to price either the carry or the momentum portfolios, as both p -values are smaller than 0.10;

Innovations in hedging pressure. Following de Roon, Nijman, and Veld (2000, Table II), we construct innovations in hedging pressure and find that this variable fails to price both the carry and the momentum portfolios, as the largest p -value is 0.01;

Innovations in scarcity. Following Gorton, Hayashi, and Rouwenhorst (2013), we examine the role of scarcity. Our results indicate that scarcity does not price the carry nor the momentum portfolios, as the largest p -value is 0.02;

Innovations in log of G7 industrial production. Following Fama (1990), among others, the next variable we consider is innovations in log of industrial production for the G7 countries. With χ_{NW}^2 p -values of 0.00 and 0.00 for the momentum and carry portfolios, respectively, the industrial production growth appears to have limited ability to explain the return cross-section of either set of commodity portfolios;

Innovations in US TED spread. Innovations in US TED spread could help capture changes in liquidity

as argued by Brunnermeier, Nagel, and Pedersen (2009). With a χ^2_{NW} p -value equal to 0.00 for both momentum and momentum portfolios, changes in funding liquidity does not appear to be a source of risk that explains the variation in commodity returns;

Innovations in G7 inflation rate. Guided by the connection between inflation and commodity prices, we lastly consider innovations in G7 inflation (e.g., Ferson and Harvey (1993) and Ang, Bekaert, and Wei (2008)). Our results indicate that a model featuring the average factor and innovations in inflation is not rejected when pricing the momentum portfolios (the p -value is 0.12), and is rejected in pricing the carry portfolios (the p -value is 0.00).

Because the inflation factor is not rejected on the momentum portfolios, we perform additional exercises to distinguish between innovations in inflation and innovations in speculative activity as candidate sources of risk underlying the momentum factor. First, we run a univariate time-series regression using innovations in inflation to explain variations in the momentum factor and obtain a marginally significant p -value of 0.09. The associated R^2 is 0.6%, as opposed to 2.2% with innovations in speculation. Second, we run a horse-race between innovations in inflation and speculation in a bivariate regression. The key finding is that inflation loses its significance, while speculation remains statistically significant.⁸

The big picture is that innovations in equity volatility and speculative activity occupy special roles in pricing the carry and momentum portfolios.

6.4. Characterizing the behavior of commodity factors over the business cycle

In search of a unified economic explanation for our findings, we now turn to analyzing the behavior of the commodity factors in conjunction with equity market returns over the US business cycle. In particular, Table 10 highlights that the commodity carry and momentum factors perform well in recessions, whereas the average commodity factor and equity market returns are negative.

Focusing first on the momentum factor, it is evident that the past losers perform poorly during recessions.

⁸We also assess whether a common set of factors prices both commodity and equity portfolios by replacing the three commodity factors with the four Fama-French equity factors. Our statistical tests reject correct model pricing, implying that the commodity and equity markets may be segmented (Daskalaki, Kostakis, and Skiadopoulos (2013)).

sions (-2.06%). Displaying an opposite pattern, the past winners deliver relatively small but positive returns (0.40%). Our exercise suggests that the performance of commodity momentum during recessions is mainly driven by shorting the past losers.

Another new insight is that the carry strategy also does well during recessions as it delivers a positive monthly return of 3.18%. Unlike the momentum strategy, however, going long in backwarddated commodities is as profitable as shorting contangoed commodities (1.54% versus 1.64%).

In recessions, the average monthly return in equity markets is -0.36%. The return associated with the average commodity factor is also negative and equal to -0.58%. Our results, therefore, suggest that both the momentum and the carry factors can shield investors against equity market declines during recessions.

In periods of economic expansion, the average commodity factor generates a monthly return of 0.73%, while the momentum (carry) factor generates a return of 1.14% (1.03%). Overall, then, our results indicate that the carry and momentum strategies are more profitable in recessions than expansions.

To draw additional insights, we partition the business cycle into (i) early recession, (ii) late recession, (iii) early expansion and (iv) late expansion (i.e., Dangl and Halling (2012, Section 4.2)). This refinement allows us to compare attributes of commodity and equity returns together as the economy transitions from one stage to the next. One salient feature worth highlighting is that the carry and momentum factors provide large monthly returns equal to 4.22% and 5.65% at the beginning of recessions, whereas they only provide 0.29% and -0.03% in late recessions. This is particularly intriguing because equities display the opposite behavior, whereby the equity returns average -2.27% in early recessions and 4.06% in late recessions.

Moreover, our analysis reveals that carry and momentum generate profits from different legs during various stages of the business cycles, while delivering comparable returns. For instance, it is the long position in backwarddated commodities that deliver larger returns in comparison to the short positions in contangoed commodities, with returns equal to 4.40% and 1.26% in early recession, respectively. Behaving differently, the high returns to momentum can be traced to shorting the losers, compared to the returns generated by going long in the winners, with returns equal to 3.17% and 1.05% in early recessions.

Completing the picture, the average commodity factor comoves positively with equity returns in late expansions, but negatively in early and late recession, and in early expansion. Yet at a more broader level, the average commodity returns rise (fall) together with equity returns during expansion (recession), which imparts a risk interpretation to the long-only average commodity factor.

The properties of our spread factors (i.e., carry and momentum) are puzzling at first glance since they could hedge equity declines and yet are characterized by large average returns. However, the returns of carry and momentum may not be puzzling if sources of risk are multifaceted. This is evidenced from the pricing of momentum portfolios with open interest growth and the pricing of carry portfolios with innovations in equity volatility. Since we show that commodities in backwardation (contango) tend to lose (gain) in value when equity volatility rises, backwardated (contangoes) commodities are poor (good) hedges, which is compatible with a positive risk premium for the carry factor.

6.5. The factors help to predict certain dimensions of the investment opportunity set

To understand the cross-sectional pricing ability of the commodity factors, and their associated risk premiums from a different angle, we ask whether they help in forecasting changes in the investment opportunity set.⁹ In what follows, we assume that changes in the investment opportunity set could be summarized by (i) GDP growth, (ii) Treasury bond returns, (iii) equity returns, and (iv) returns of commodity currencies.

6.5.1. The commodity factors have joint predictive content for aggregate economic activity

Motivated by Liew and Vassalou (2000, Table 5), Groen and Pesenti (2009, Section 2), Cespedes and Velasco (2012, Table 1), and Caballero, Farhi, and Gourinchas (2012, Table 9), among others, we consider whether the commodity factors can forecast growth of economic activity. Our framework uses the real

⁹The motivation for our forecasting exercises stems from the approaches in Campbell (1996) and Ferson and Harvey (1999) and is also guided by an analogy in Cochrane (2005, page 445): “*Though Merton’s (1971, 1973) theory says that variables which predict market returns should show up as factors which explain cross-sectional variation in average returns, surprisingly few papers have actually tried to see whether this is true, now that we do have variables that we think forecast the market return.*”

GDP growth of the G7 countries as a measure of economic activity and the following linear model:

$$\ln(\text{GDP}_{t+k}/\text{GDP}_t) = \theta_0 + \boldsymbol{\theta}'\mathbf{f}_t + \varepsilon_{t+k}, \quad k \in \{1, 2, 3, 4\}, \quad (13)$$

where $\mathbf{f}_t \equiv [\text{AVG}_t \text{ CARRY}_t \text{ CMOM}_t]'$. We draw inference about the predictive slope coefficients $\boldsymbol{\theta}$ on the basis of the Newey and West (1987) estimator, with lags automatically selected according to Newey and West (1994), and the Hodrick (1992) 1B covariance estimator under the null of no predictability.

Several features of our results reported in Table 11 deserve discussion. First, a higher AVG_t forecasts stronger global economic growth, whereas a higher CARRY_t and CMOM_t forecast lower economic growth. Second, the statistical significance of the commodity factors is preserved at all horizons, with the exception of AVG_t , which loses its significance at four quarters. Both the Newey and West p -values, denoted by $\text{NW}[p]$, and the Hodrick p -values, denoted by $\text{H}[p]$, are in agreement.

The factors are also relevant from an economic perspective. For example, a one standard deviation increase in the carry factor (i.e., 22%; see Table Appendix-I) is associated with a decline of 88 basis points of annualized GDP in the subsequent quarter. Third, the p -values corresponding to the null hypothesis that none of the predictors is statistically significant, that is, $\boldsymbol{\theta} = \mathbf{0}$, are consistently below 0.02 (reported as $J[p]$). These results affirm the ability of the commodity factors to jointly predict future economic conditions.¹⁰

¹⁰We conduct three additional exercises to demonstrate the reliability of the above findings. First, the commodity factors forecast the G7 and US industrial production growth as well as the US GDP growth (not reported), indicating robustness to alternative measures of economic activity. Next, we build on Liew and Vassalou (2000, Tables 5 and 6) and include the SMB_t and HML_t factors of Fama and French (1996) in our specification (13). Our Wald statistic points to the lack of joint statistical significance of these two equity factors for US GDP growth. Finally, we account for the possible effect of serial correlation in the GDP growth by implementing an ARMAX model. We first select the best model for GDP growth, using the Bayesian information criterion, and find it to be an ARMA(1,1). We then include the commodity factors in an ARMAX specification, and find that they maintain their joint statistical and economic significance. In sum, our results extend beyond the linear forecasting model in equation (13).

6.5.2. The commodity factors predict returns of Treasury bonds and equities

Following Fama and French (1988b) and Hodrick (1992), we now consider the following predictive regressions with overlapping observations for Treasury bonds and equity returns for $K \in \{1, 3, 6, 9, 12\}$:

$$\sum_{k=1}^K er_{t+k}^{\text{bond}} = \xi_0 + \boldsymbol{\xi}' \mathbf{f}_t + \varepsilon_{t+K}, \quad \text{with} \quad er_{t+1}^{\text{bond}} \equiv \ln(1 + r_{t+1}^{\text{bond}}) - \ln(1 + r_t^f), \quad (14)$$

$$\sum_{k=1}^K er_{t+k}^{\text{equity}} = \delta_0 + \boldsymbol{\delta}' \mathbf{f}_t + \varepsilon_{t+K}, \quad \text{with} \quad er_{t+1}^{\text{equity}} \equiv \ln(1 + r_{t+1}^{\text{equity}}) - \ln(1 + r_t^f). \quad (15)$$

We consider both one-year and 30-year US Treasury bond returns (source: CRSP) and US value-weighted equity returns. The results are presented in Tables 12 and 13.

There are three points to glean from the reported slope coefficients. First, the average factor has predictive content for one-year bond returns at all horizons and for 30-year bonds for horizons of one month and three months, but it is generally uncorrelated with future equity returns. When statistically significant, the average factor exerts a negative effect on future bond returns, which intuitively agrees with our findings in Table 11 that the average factor positively predicts output growth.

Our results with respect to the carry factor indicate that it tends to predict positively (negatively) bond (equity) returns. In particular, the estimated coefficients δ_{CARRY} decline with the return horizon, and the negative effect of the carry factor on future equity returns is statistically significant. The forecasting ability of the carry factor also holds in univariate regressions (i.e., when $\delta_{\text{AVG}} = \delta_{\text{CMOM}} = 0$), with δ_{CARRY} estimates that lie between -0.06 and -0.32 . Additionally, the 12-month horizon adjusted R^2 is equal to 2.5%, in line with other equity market predictors (e.g., Cochrane and Piazzesi (2005, Table 3)).¹¹

Finally, the momentum factor is statistically insignificant for equity returns and 30-year bond returns but predictability surfaces for one-year bond returns at horizons ranging from six to 12 months.

¹¹Appreciate, in addition, that the carry factor is not persistent (see column ρ_1 in Table Appendix-I), offering a certain deviation from other predictors of bond and equity returns, such as the term premium and the dividend yield, which are instead highly persistent (see Campbell (2001)).

6.5.3. Relation between the factors and the future behavior of commodity currencies

Chen, Rogoff, and Rossi (2010, Table IV) show that commodity returns have some predictive power for the returns of commodity currencies. Extending their analysis, we consider the predictive regressions:

$$\ln(\text{FX}_{t+k}/\text{FX}_t) = \pi_0 + \boldsymbol{\pi}'\mathbf{f}_t + \varepsilon_{t+k} \quad \text{and} \quad k \in \{1, 2, 3, 4\}, \quad (16)$$

where $\ln(\text{FX}_{t+k}/\text{FX}_t)$ represents currency returns, obtained by equally weighting returns across the commodity currencies (i.e., Australia, Canada, Chile, Norway, New Zealand, and South Africa; see Labuszewski (2012)). Following convention, we maintain the US dollar as the reference currency.

Table 14 reveals three prominent findings. First, an increase in the average factor predicts the depreciation of the US dollar, but the effect weakens after one quarter and becomes statistically insignificant. Next, increases in the carry and momentum factors imply an appreciation of the dollar, with six out of eight $H[p]$ below 0.1. Analogous to our findings in Tables 11 through 13, the signs of π_{AVG} are the opposite of those of π_{CARRY} and π_{CMOM} , sharpening the distinction between the economic nature of the three factors.

6.5.4. Synthesis and interpretation of the predictive regression results

Our Tables 11 through 14 show that a decrease in AVG_t has a profound impact on the macroeconomy: it predicts weaker GDP growth, higher short-term bond returns, and an appreciation of the US dollar against the commodity currencies. Given that low (high) GDP growth corresponds to periods of high (low) marginal utility, our results reinforce a risk interpretation for the positive average returns of AVG_t .

The results with respect to carry are puzzling in a broader economic context. Our evidence shows that the relative spread between the returns of commodities in backwardation versus those in contango contains negative information for aggregate future cash flows, as measured by the GDP growth, and for equity returns, but our evidence also illustrates a positive predictive link to bond returns. Furthermore, negative returns to the carry factor predict a depreciation of the US dollar against the commodity currencies. The sign of the predictive coefficients indicates that the carry factor could hedge adverse changes in the

investment opportunity set. Hence, the positive risk-premia associated with the carry strategy defies a standard textbook interpretation of a factor.

Finally, our results reveal that the momentum factor negatively predicts future GDP growth and positively predicts short-term bond returns. Unlike carry, however, momentum has no predictive ability for equity returns and long-term bond returns. Furthermore, negative momentum returns forecast depreciation of the US dollar. Because high momentum returns are associated with a deterioration in the investment opportunity set, it is hard to justify the high factor premium. Instead, our exercises establish that the momentum factor represents compensation required for bearing open interest growth risk, which could be specific to the commodity markets and represents a missing dimension of the investment opportunity set.

While policymakers have often relied on futures markets to derive price forecasts of commodities (e.g., Ben Bernanke, June 9, 2008), our evidence on the information content of commodity factors for economic activity adds to the extant literature in two new ways. First, the ability of the commodity factors to forecast output growth is not yet recognized. Second, our evidence transcends the predictive role of oil prices on macroeconomic fluctuations (e.g., Hamilton (1983) and the follow-up studies). Specifically, when the growth of oil prices is added as an additional predictor in equation (13), the slope coefficient on oil is insignificant, while the commodity factors remain strongly statistically significant (see Table Online-V).

6.6. Membership in the long legs is not skewed toward economically sensitive commodities

The average returns to a long position in a dynamically rebalanced set of backwardated (or, high momentum) commodities is high, whereas the average returns to a short position in contangoed (or, low momentum) commodities is low (see Table 1, Table Appendix-VI, and the discussion in Appendix B).

To further probe the above features, we analyze the composition of the long and short legs of each strategy. Specifically, we compute the number of months in which each commodity enters the long and short legs of carry and momentum (i.e., C5 and M5, respectively) and report the findings in Table 15.

To elaborate on Table 15, let G_i be the number of months a commodity i enters the long (short) leg of

the M5 portfolio, and let H_i be the number of months the same commodity enters the long (or short) leg of the C5 portfolio. For example, soybean oil appears 126 times in the long leg of M5 and 65 times in the long leg of C5. We perform two OLS estimations: $G_i = \eta_0 + \eta H_i + \varepsilon_i$, where $i = 1, \dots, 29$ and report the η estimates and the p -values (based on White's standard errors) below:

	η	p -val.	R^2 (%)
Aligning long legs of momentum and carry	0.08	0.31	1.7
Aligning short legs of momentum and carry	0.23	0.02	19.3

These regression results allow for new insights into the return variation that underlie the carry and momentum strategies. For one, the slope coefficient η for the long leg is insignificant, implying that the outperformance of the long legs of carry and momentum do not emanate from a similar set of commodities. In contrast, a unit increase in the short leg of the carry strategy membership is associated with a 0.23 increase in the momentum strategy membership, and the effect is statistically significant. The R^2 obtained for the short leg implies that 19.3% of the variation in momentum strategy membership can be explained by variation in the carry strategy membership.

The above finding furnishes additional information content, namely, that the distribution of membership in the long legs is not skewed toward a few economically sensitive commodities. One could ascribe the outperformance of the long legs to their ability to rotate across a set of commodities that reflect the prevailing state of the macroeconomy.

[Fig. 6 about here.]

To additionally assess whether the two strategies are *conditionally* loading on the same set of commodities, we first compute the commodity overlap in the long and short legs of the carry and momentum strategies separately, and then sum the two overlaps. The results, reported in Figure 6, indicate that the two strategies are largely decoupled, with the third quartile (Q3) of the overlap distribution equal to one. Hence, our analysis again reveals that the returns of carry and momentum strategies arise from distinct sources.

7. Concluding remarks

This paper studies the salient attributes of commodity futures returns over a 42-year period, from 1970 to 2011. Using a set of baseline portfolios, managed portfolios, and individual commodities, our empirical estimates indicate that a three-factor model, driven by the average factor, the carry factor, and the momentum factor, outperforms the nested one- and two-factor counterparts in capturing commodity returns.

Throughout, our analysis centers on the economic interpretation of the commodity factors and offers several new results. First, the carry strategy performs poorly when equity volatility increases. Second, the momentum returns are lower when aggregate speculative activity declines. Our results suggest that increases in equity volatility and decreases in speculative activity both appear associated with a deterioration in some aspects of global economic conditions. Therefore, the economic intuition is that investors dislike these adverse changes in the investment opportunity and require positive carry and momentum returns.

Next, expanding on the above analysis, we study whether innovations in other economic variables can explain the cross-section of portfolios sorted on carry and momentum. Our χ^2 tests reject the relevance of innovations in commodity volatility, open interest, hedging pressure, scarcity, G7 industrial production, US TED spread, and G7 inflation as risk factors.

Finally, our exercises establish that the commodity factors predict, in isolation or together, measures of output growth, bond returns, equity returns, and returns of commodity currencies. The signs of the slope coefficients of carry and momentum are not always consistent with their positive risk-premia. For example, we find that both carry and momentum negatively predict future output growth, implying that they should command negative risk-premia. Our predictive results suggest that the risk structure in the commodity markets is multidimensional in nature.

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Appendix A: Carry and momentum strategies in commodity markets

The carry strategy entails a long futures position in a commodity that is in backwardation and a short futures position in a commodity that is in contango. One may express the n -period excess return on such strategy as:

$$\text{er}_{t+n} = \begin{cases} \frac{1}{F_t^{(n)}} (S_{t+n} - F_t^{(n)}) & \text{Long commodity if } \frac{F_t^{(n)}}{S_t} < 1 \\ -\frac{1}{F_t^{(n)}} (S_{t+n} - F_t^{(n)}) & \text{Short commodity if } \frac{F_t^{(n)}}{S_t} > 1, \end{cases} \quad (\text{A1})$$

where the n -period futures price is denoted by $F_t^{(n)}$.

To implement a carry strategy, we cast backwardation and contango in terms of the log price ratio of the two nearest maturity futures. Define $y_t \equiv F_t^{(1)}/F_t^{(0)}$. Then the condition $\ln(y_t) < 0$ ($\ln(y_t) > 0$) maps to a downward (upward) sloping commodity futures curve and, hence, captures backwardation (contango).

We compute

$$\text{er}_{t+1}^{\text{carry}_k} = \frac{\text{er}_{t+1}^{\text{long}_k} + \text{er}_{t+1}^{\text{short}_k}}{k}, \quad k = 1, \dots, 5, \quad (\text{A2})$$

whereby $\text{er}_{t+1}^{\text{carry}_k}$ represents the excess return generated by a carry strategy that consists of k long futures positions in the commodities with the k lowest $\ln(y_t) < 0$ and k short futures positions in the commodities with the k highest $\ln(y_t) > 0$. The strategy is dynamic, as commodities enter and exit a portfolio based on the slope of their futures curve.

Each month t , we also divide the commodity universe into two backwardation portfolios (ranked in ascending order of $\ln(y_t) < 0$) and two contango portfolios (ranked in descending order of $\ln(y_t) > 0$), and compute their equally weighted returns over the following month. Our separation of commodities into two backwardation portfolios and two contango portfolios differentiates our approach from the one adopted in Koijen, Pedersen, Moskowitz, and Vrugt (2012, Table 2) and Yang (2013, Table 2).

The design of momentum strategies often relies on the number of futures contracts that are long and

short, the weight given to each commodity, and the rebalancing frequency. Here we focus on equal weights, monthly rebalancing, and ranking determined by a commodity's prior J month performance:

$$\bar{er}_{t,J} = \left(\prod_{j=1}^J (1 + er_{t-j}) \right)^{\frac{1}{J}} - 1, \quad J = 1, \dots, 12. \quad (\text{A3})$$

Accordingly, the momentum strategy corresponds to k long futures positions in commodities with the k highest $\bar{er}_{t,J}$ (i.e., winners) and k short futures positions in commodities with the k lowest $\bar{er}_{t,J}$ (i.e., losers).

The excess return is

$$er_{t+1}^{\text{momentum}_k} = \frac{er_{t+1}^{\text{long}_k} + er_{t+1}^{\text{short}_k}}{k}, \quad k = 1, \dots, 5. \quad (\text{A4})$$

Each month t , we also rank commodities based on their $\bar{er}_{t,J}$ (in line with Miffre and Rallis (2007, Section 4), Fuertes, Miffre, and Rallis (2010, Table 1), and Gorton, Hayashi, and Rouwenhorst (2013, Table VII)) into quintiles and then compute their equally-weighted return over month $t + 1$.

Appendix B: Return properties of carry and momentum strategies

Our objective is threefold. First, we summarize the return patterns generated by carry and momentum strategies. Second, we establish the statistical significance of the average returns using a stationary bootstrap. Third, we draw the distinction between the long and short legs of each strategy.

Panels A and C of Table 1 present the descriptive statistics of the excess returns generated by carry strategies (denoted by C1 through C5) and momentum strategies (denoted by M1 through M5), respectively. Symmetrically, Panels B and D of Table 1 present the results from carry portfolios (denoted by P1 and P2 for backwardation and P3 and P4 for contango) and momentum portfolios (denoted by Q1 through Q5). While we feature momentum strategies based on a formation period of six months, our conclusions do not appear to be sensitive to this particular choice (as shown in Figure 7).

The carry and momentum strategies are lucrative. For example, the carry strategy C5 that buys the five commodities with the lowest $\ln(y_t) < 0$ and shorts the five commodities with the highest $\ln(y_t) > 0$ delivers

annualized average monthly excess returns of 16.34%. The momentum strategy M5 generates average returns equal to 16.11%. Four out of the five carry strategies (C2 through C5) and four out of the five momentum strategies (M2 through M5) exhibit average returns that are statistically different from zero, as indicated by the bootstrap confidence intervals. The 95% confidence intervals, denoted as PW, lower CI and upper CI, are based on a stationary bootstrap with 10,000 iterations, where the block size is based on the algorithm of Politis and White (2004).¹²

How do these strategies fare relative to futures-based commodity indexes? Table Appendix-I shows that, over the same sample period, the Goldman Sachs Commodity Index (GSCI) and the Commodity Research Bureau (CRB) index deliver annualized average monthly excess returns equal to 5.43% and 3.53%, respectively. The disparity between the average returns of carry and momentum strategies versus the average returns of commodity indexes motivates our search for asset pricing explanations.

The magnitudes of the Sharpe ratios further convey the attractiveness of commodity carry and momentum. Specifically, the Sharpe ratios (denoted by SR) of the C5 and M5 strategies are 0.73 and 0.61 - more than twice the ones associated with the GSCI and CRB index.

Further, note that the commodity carry and momentum returns are generated, at least in part, by a higher percentage of positive return realizations compared to the GSCI, as reflected in the reported $1_{er>0}$. The $1_{er>0}$ for carry strategies increases from 53.13% for C1 to 56.77% for C5. At the same time, the return skewness of the conditional strategies is essentially equal to zero (and similar to that of the GSCI), indicating that none of them generates high returns by loading excessively on possible crash risk.¹³

One unresolved question is whether the profitability of the carry and momentum strategies can be

¹²The carry strategy based on the short-end of the futures curve maximizes the number of observations, the number of commodities included, and is associated with the highest open-interest (as noted in Table Online-III). In contrast, the carry factor of Yang (2013, equation (3)) is obtained by sorting commodities based on the log difference between the 12-month and the one-month futures prices. Our carry factor is also different from that of Szymanowska, de Roon, Nijman, and Goorbergh (2014) in that they construct the factor by going long in an equally weighted portfolio of the 10 commodities that are most backwarddated and short in an equally weighted portfolio of the 10 commodities that are most in contango (Section IV-B.1).

¹³While accommodating the first-notice-day convention, we employ the second-nearest maturity futures contract sampled at the *monthly* frequency. This contrasts Szymanowska, de Roon, Nijman, and Goorbergh (2014), as they construct returns at the *bimonthly* frequency. Furthermore, our dataset features 16 more years of data (it starts in 1970 rather than 1986 and ends in 2011 rather than 2010) and contains 29 commodities rather than 21. The combination of the higher data frequency and the longer sample translates into a larger number of time-series observations (approximately 500 compared to approximately 150) and that could result into a higher precision of our estimates.

traced to their long or short legs. Table Appendix-VI isolates the contribution of each leg and provides three additional insights. First, the profitability emanates from the long legs and not from the short legs. In particular, our bootstrap analysis indicates that the excess returns of the short legs are uniformly not statistically different from zero. Our momentum results therefore deviate from the corresponding strategy in the equity market, where both the long and short legs are equally profitable (e.g., Jegadeesh and Titman (2001, Table 1)). Second, given that the standard deviations of the short and long legs are comparable, the Sharpe ratios of the short legs are overshadowed by those of the long legs. Finally, the long (short) leg of the strategies consistently produce positively (negatively) skewed returns, indicating that the long legs are not generating high returns by loading on possible crash risk.

Reported next in Panels B and D of Table 1 are the descriptive statistics corresponding to the carry and momentum portfolios. Our results show that the returns of the carry portfolios are declining when the commodities are sorted in ascending order of $\ln(y_t)$: the two backwardation portfolios yield positive and statistically significant average returns, whereas the contango portfolios yield negative, but statistically insignificant, average returns. On the other hand, the average returns generated by the momentum portfolios increase when the commodities are sorted on the basis of their past performance. In fact, the monotonicity test of Patton and Timmermann (2010) is supportive of a monotonic pattern in the average returns of carry and momentum portfolios. Specifically, we reject the null of a non-monotonic relation between the carry (momentum) portfolios in favor of the alternative of a monotonically decreasing (increasing) relation with a p -value of 0.000 (0.079).

How profitable is carry, controlling for momentum? To investigate this question, we perform a two-way dependent sort whereby commodities are divided first into two portfolios by momentum, and then each group is divided into two carry portfolios. We then compute the next-month returns on each of the four portfolios. Below we report the average returns, with bootstrap lower and upper CI in square brackets:

	Backwardation	Contango
Low momentum	10.29 [4.80 15.72]	-0.66 [-5.76 4.80]
High momentum	16.29 [8.64 25.08]	5.81 [0.72 10.80]

The main takeaway, in relation to Tables 1 and Appendix-VI, is threefold. First, backwardation is highly profitable across both low and high momentum commodities, highlighting the working of the carry strategy. Second, among commodities that are in backwardation, high and low momentum are both profitable, whereas among commodities that are in contango, the high momentum commodities dominate the low momentum counterparts. Finally, a long position in commodities featuring high momentum and backwardation and a short position in commodities featuring low momentum and contango produces a return of 16.95% (i.e., 16.29% plus 0.66%). Overall, these results reiterate the distinct nature of the two strategies.

[Fig. 7 about here.]

Closing, we ask how sensitive momentum returns are to the formation period J in equation (A3). To address this concern, Figure 7 presents the average annualized returns (top panel) and the Sharpe ratios (bottom panel) for momentum strategy M5, as J varies from one to 12 months. We observe consistently high average annualized returns, ranging from 12.50% to 19.03%, and Sharpe ratios, ranging from 0.42 to 0.62, thereby affirming the profitability of momentum strategies beyond our Tables 1 and Appendix-VI.

Appendix C: Definition of economic variables and alternative commodity risk factors

We adopt the following variables in our empirical work (in alphabetical order):

- **Commodity volatility** (VOL_t): Monthly sum of absolute daily log returns for each commodity. The cross-sectional volatility is the average across all commodities;
- **Currency returns**: The monthly log change of the US dollar index, expressed as $FX|USD$. A rise in the index implies dollar appreciation against major foreign currencies (source: Federal Reserve Board);

- **Equity return:** Returns of the US value-weighted index (source: Kenneth French’s website);
- **Equity volatility** (VOL_t^{equity}): At the end of each month, we compute the monthly sum of absolute daily log equity returns for each of the G20 countries. The (cross-sectional) equity volatility is the average across all countries (Menkhoff, Sarno, Schmeling, and Schrimpf (2012, equation (4)));
- **Hedging pressure:** At the end of each month, we first construct a hedging pressure proxy for each commodity as the ratio of short minus long positions of the commercial traders divided by the total short and long positions of the commercial traders (source: Large trader position data of the CFTC). This variable is the same as that used in de Roon, Nijman, and Veld (2000, equation (5)). The aggregate hedging pressure in Section 6.3 is the average across all commodities in each month;
- **Open interest:** We first calculate the end-of-the-month dollar open interest corresponding to the second nearest maturity futures contract (i.e., $F_t^{(1)}$) for each commodity. We then calculate the cross-sectional average dollar open interest across all commodities each month;
- **Scarcity:** As in Gorton and Rouwenhorst (2006), we proxy the scarcity of a commodity by the slope of the futures curve, defined as $y_t \equiv F_t^{(1)}/F_t^{(0)}$, where $F_t^{(0)}$ is the price of the front-month futures contract and $F_t^{(1)}$ is the price of the next maturity futures contract. The aggregate scarcity variable in Section 6.3 is the average of y_t across all commodities at the end of month t .

We analyze two additional risk factors that are constructed as follows:

- (i) **Commodity value factor** ($VALUE_t$): At the end of each month t , we rank all the commodities by the ratio of $F_{t-60}^{(1)}$ to their time t futures price $F_t^{(1)}$. Then we divide the commodities into five groups and compute the next-month return of each commodity portfolio. $VALUE_t$ is the return spread between the top and bottom quintiles (we deviate from Asness, Moskowitz, and Pedersen (2013, Section I.B), who construct three portfolios);
- (ii) **Commodity volatility factor** (ΔVOL_t): We construct this factor as the monthly change in the commodity cross-sectional volatility, that is, $\Delta VOL_t \equiv VOL_t - VOL_{t-1}$.

As seen from the correlations reported below, the commodity momentum and commodity value fac-

tors display a negative correlation of -0.39 , which mimics its counterpart across asset classes in Asness, Moskowitz, and Pedersen (2013, Panel A of Table II).

	AVG_t	$CARRY_t$	$CMOM_t$	$VALUE_t$
$CARRY_t$	0.09			
$CMOM_t$	0.11	0.27		
$VALUE_t$	-0.23	-0.18	-0.39	
ΔVOL_t	-0.06	0.15	0.05	0.08

Importantly, the correlation between the average factor and the carry (momentum) factor is 0.09 (0.11), while the correlation between carry and momentum is 0.27, implying that the three factors adopted in our SDF specification (6) are mildly correlated. The correlations of ΔVOL_t with other risk factors are small in absolute values.

Table 1

Excess returns of commodity carry and momentum strategies

This table presents the descriptive statistics of the excess returns generated by commodity carry and momentum strategies. Let $y_t \equiv F_t^{(1)}/F_t^{(0)}$, where $F_t^{(0)}$ is the price of the front-month futures contract and $F_t^{(1)}$ is the price of the next maturity futures contract, both observed at the end of month t . A commodity is in backwardation if $\ln(y_t) < 0$ and in contango if $\ln(y_t) > 0$. The carry strategy entails taking a long (short) futures position in a commodity that is in backwardation (contango) at the end of month t , and we compute the returns over the subsequent month. For example, carry strategy C5 (C2) contains an equally weighted portfolio consisting of five (two) commodities with the most negative $\ln(y_t)$ and five (two) commodities with the most positive $\ln(y_t)$. In addition, each month t , we divide the commodity universe into two backwardation portfolios (P1 and P2) and two contango portfolios (P3 and P4), based on their respective rankings of $\ln(y_t)$, and then we compute the next-month returns. For the momentum strategies, the commodities are ranked on the basis of their past six-month performance. Analogously, the momentum strategy M5 (M2) contains an equally weighted portfolio consisting of five (two) commodities with the highest past returns (winners) and five (two) commodities with the lowest past returns (losers). Ranking the commodities by the past six-month performance, the commodities are collected in quintile portfolios Q1 (lowest) through Q5 (highest). For each portfolio, we report the average annualized monthly return and its 95% confidence interval based on a stationary bootstrap (denoted by PW, lower CI and PW, upper CI) with 10,000 bootstrap iterations, in which the block size is based on the algorithm of Politis and White (2004), the annualized monthly standard deviation (SD), the annualized Sharpe ratio (SR), and the monthly skewness. The percentage of months in which the excess return of a strategy is positive is recorded as $1_{er>0}$. There are 501 monthly observations in our sample from January 1970 to September 2011.

	<i>Panel A: Carry Strategies</i>					<i>Panel B: Backwardation/Contango Portfolios</i>				
	Commodities long backwardation and short contango					Commodities sorted based on $\ln(y_t) < 0$ $\ln(y_t) > 0$				
	C1	C2	C3	C4	C5	P1	P2	P3	P4	P1-P4
Mean	9.31	10.08	12.14	14.27	16.34	16.32	13.50	4.63	-0.53	16.85
PW, lower CI	-2.04	0.60	2.04	6.48	9.36	8.28	7.44	-0.96	-6.00	9.48
PW, upper CI	22.32	21.24	22.56	21.84	23.40	26.16	19.56	10.20	5.40	26.16
SD	48.21	34.87	27.46	24.16	22.26	23.23	19.48	17.02	16.94	23.08
SR	0.19	0.29	0.44	0.59	0.73	0.70	0.69	0.27	-0.03	0.73
Skewness	-0.29	0.02	-0.05	-0.03	0.30	0.47	0.25	0.18	0.82	0.45
$1_{er>0}$	53.13	53.54	53.94	55.76	56.77	55.56	58.59	51.92	48.69	57.17

	<i>Panel C: Momentum Strategies</i>					<i>Panel D: Momentum Portfolios</i>					
	Commodities long winners and short losers					Commodities sorted based on past six-month performance					
	M1	M2	M3	M4	M5	Q1	Q2	Q3	Q4	Q5	Q5-Q1
Mean	10.75	17.04	13.88	14.70	16.11	-1.66	1.31	10.07	9.16	14.35	16.00
PW, lower CI	-4.32	5.64	6.00	6.96	9.36	-7.56	-4.08	3.72	3.84	6.60	7.92
PW, upper CI	29.40	30.36	22.92	22.92	22.92	4.44	7.20	17.04	14.76	24.00	24.24
SD	59.75	42.51	34.33	29.19	26.28	20.84	16.76	18.40	18.22	25.58	27.61
SR	0.18	0.40	0.40	0.50	0.61	-0.08	0.08	0.55	0.50	0.56	0.58
Skewness	0.14	0.09	0.11	0.06	0.34	0.53	0.43	1.53	0.45	0.44	0.34
$1_{er>0}$	52.53	54.14	53.54	55.56	57.37	46.67	50.51	55.56	54.55	57.78	55.56

Table 2

Cross-sectional asset pricing results with the average, carry, and momentum factors

Reported are the factor risk premia (λ) and the SDF parameters (\mathbf{b}). The SDF specification in Panel A is of the form: $m_{t+1} = 1 - b_{\text{AVG}} \text{AVG}_{t+1} - b_{\text{CARRY}} \text{CARRY}_{t+1} - b_{\text{CMOM}} \text{CMOM}_{t+1}$, where AVG_{t+1} is the average factor (excess return obtained by holding all commodities available), CARRY_{t+1} is the carry factor (which corresponds to the returns of strategy C5), and CMOM_{t+1} is the momentum factor (which corresponds to the returns of strategy M5). Reported in Panels B and C are results for the restricted versions of the SDF that impose $b_{\text{CMOM}} \equiv 0$ and $b_{\text{CARRY}} \equiv 0$, respectively. In the row marked ‘‘GMM,’’ the parameters are estimated based on the system (4) following a one-step GMM procedure, while those in the row ‘‘Fama-MacBeth’’ are based on a two-step cross-sectional regression. The p -values rely on the Newey and West (1987) procedure with lags selected automatically according to Newey and West (1994) and are reported in parentheses. For the Fama and MacBeth procedure, the p -values are computed using both the Newey and West (1987) procedure without (with) the Shanken (1992) correction in parentheses (curly brackets). We report the GLS cross-sectional uncentered R^2 (Lewellen, Nagel, and Shanken (2010, Prescription 3)) and the OLS uncentered R^2 as $[\cdot]$, and the χ^2 test corresponding to the null hypothesis that the pricing errors are zero, with p -values computed both based on the Newey-West and Shanken standard errors. The Hansen and Jagannathan (1997, equation (29)) distance measure (HJ-Dist.), with the associated p -value, is shown, which tests whether the distance measure is equal to zero.

	Factor risk premia			Loadings on the SDF			Pricing errors			
	λ_{AVG}	λ_{CARRY}	λ_{CMOM}	b_{AVG}	b_{CARRY}	b_{CMOM}	R^2_{GLS} [R^2]	χ^2_{NW} (p -val.)	χ^2_{SH} { p -val.}	HJ-Dist. (p -val.)
<i>Panel A: Three-factor model</i>										
GMM	0.005 (0.01)	0.018 (0.00)	0.012 (0.00)	2.276 (0.07)	3.954 (0.00)	1.067 (0.11)				0.006 (0.22)
Fama-MacBeth	0.005 (0.02) {0.02}	0.018 (0.00) {0.00}	0.012 (0.00) {0.00}				96.3 [93.9]	11.91 (0.22)	11.40 {0.25}	
<i>Panel B: Restricted model omitting a role for CMOM_{t+1}</i>										
GMM	0.005 (0.01)	0.020 (0.00)		2.394 (0.05)	4.661 (0.00)					0.008 (0.15)
Fama-MacBeth	0.005 (0.02) {0.02}	0.020 (0.00) {0.00}					82.9 [91.3]	23.87 (0.01)	22.70 {0.01}	
<i>Panel C: Restricted model omitting a role for CARRY_{t+1}</i>										
GMM	0.006 (0.01)		0.014 (0.00)	2.891 (0.02)		2.300 (0.00)				0.012 (0.00)
Fama-MacBeth	0.006 (0.02) {0.02}		0.014 (0.00) {0.00}				67.7 [77.3]	29.67 (0.00)	28.65 {0.00}	

Table 3

Time-series regressions based on the three-factor model

Results are based on the time-series regression: $er_t^i = \alpha^i + \beta_{AVG}^i AVG_t + \beta_{CARRY}^i CARRY_t + \beta_{CMOM}^i CMOM_t + \varepsilon_t^i$, for $i = 1, \dots, 12$. We report the coefficient estimates and the p -values in parentheses. The p -values are based on the Newey and West (1987) procedure, with lags selected automatically according to Newey and West (1994). To test the null hypothesis that all intercepts are jointly equal to zero, we compute the statistic $\hat{\alpha}' \text{var}(\hat{\alpha})^{-1} \hat{\alpha}$ in a GMM setting (Cochrane (2005, page 234)), which is asymptotically distributed $\chi^2(12)$. The underlying test assets are the four carry portfolios, the five momentum portfolios, and the three category portfolios. We have excluded the energy category as a test asset in the time-series regressions because of its shorter time-series (heating oil started in 1979:04, while crude oil started in 1983:04; see Table Online-I).

Commodity portfolio		α	β_{AVG}	β_{CARRY}	β_{CMOM}	\bar{R}^2 (%)	Joint test on α	
							$\chi^2(12)$	p -val.
P1 (backwardation, lowest y)	Estimate	0.002	0.933	0.566	-0.041	66.4		
	NW[p]	(0.36)	(0.00)	(0.00)	(0.22)			
P2	Estimate	0.004	0.882	0.284	-0.086	53.9		
	NW[p]	(0.01)	(0.00)	(0.00)	(0.00)			
P3	Estimate	-0.001	1.000	-0.063	0.041	70.9		
	NW[p]	(0.43)	(0.00)	(0.01)	(0.05)			
P4 (contango, highest y)	Estimate	-0.001	1.016	-0.303	-0.038	82.9		
	NW[p]	(0.20)	(0.00)	(0.00)	(0.03)			
Q1 (momentum, lowest)	Estimate	-0.002	1.160	0.002	-0.443	84.5		
	NW[p]	(0.15)	(0.00)	(0.96)	(0.00)			
Q2	Estimate	-0.002	0.874	-0.015	-0.091	54.5		
	NW[p]	(0.18)	(0.00)	(0.68)	(0.00)			
Q3	Estimate	0.004	0.950	0.000	-0.054	53.1		
	NW[p]	(0.01)	(0.00)	(0.99)	(0.05)			
Q4	Estimate	0.001	0.892	-0.009	0.146	55.9		
	NW[p]	(0.39)	(0.00)	(0.81)	(0.00)			
Q5 (momentum, highest)	Estimate	-0.002	1.165	0.039	0.552	83.5		
	NW[p]	(0.15)	(0.00)	(0.15)	(0.00)			
Agriculture	Estimate	-0.001	0.983	-0.042	-0.022	69.1		
	NW[p]	(0.38)	(0.00)	(0.31)	(0.54)			
Livestock	Estimate	0.001	0.659	0.004	-0.094	22.3		
	NW[p]	(0.64)	(0.00)	(0.93)	(0.04)			
Metal	Estimate	0.000	0.985	-0.078	0.132	40.3		
	NW[p]	(0.93)	(0.00)	(0.18)	(0.08)			
All portfolios							18.20	0.11

Table 4

Time-series regressions with a two-factor model that excludes the momentum factor

In Panel A, the results are based on the time-series regression: $er_t^i = \alpha^i + \beta_{AVG}^i AVG_t + \beta_{CARRY}^i CARRY_t + \varepsilon_t^i$, for $i = 1, \dots, 12$. In Panel B, the results are based on the time-series regression: $er_t^i = \alpha^i + \beta_{CARRY}^i CARRY_t + \varepsilon_t^i$, for $i = 1, \dots, 12$. We report the coefficient estimates and the p -values in parentheses. The p -values are based on the Newey and West (1987) procedure, with lags selected automatically according to Newey and West (1994). To test the null hypothesis that all intercepts are jointly equal to zero, we compute the statistic $\hat{\alpha}' \text{var}(\hat{\alpha})^{-1} \hat{\alpha}$ in a GMM setting (Cochrane (2005, page 234)), which is asymptotically distributed $\chi^2(12)$. The underlying test assets are the four carry portfolios, the five momentum portfolios, and the three category portfolios. We have excluded the energy category as a test asset in the time-series regressions because of its shorter time-series (the heating oil started in 1979:04, while crude oil started in 1983:04).

Commodity portfolio		Panel A: Two-factor model that excludes the momentum factor				Panel B: One-factor model with the carry factor		
		α	β_{AVG}	β_{CARRY}	\bar{R}^2 (%)	α	β_{CARRY}	\bar{R}^2 (%)
P1 (backwardation, lowest y)	Estimate	0.00	0.927	0.553	66.3	0.005	0.609	33.9
	NW[p]	(0.47)	(0.00)	(0.00)		(0.04)	(0.00)	
P2	Estimate	0.003	0.875	0.255	53.3	0.007	0.309	12.3
	NW[p]	(0.07)	(0.00)	(0.00)		(0.00)	(0.00)	
P3	Estimate	-0.001	1.008	-0.050	71.2	0.004	0.011	-0.2
	NW[p]	(0.56)	(0.00)	(0.03)		(0.14)	(0.79)	
P4 (contango, highest y)	Estimate	-0.001	1.007	-0.315	82.8	0.003	-0.25	10.9
	NW[p]	(0.16)	(0.00)	(0.00)		(0.18)	(0.00)	
Q1 (momentum, lowest)	Estimate	-0.005	1.089	-0.135	55.8	0.000	-0.06	0.3
	NW[p]	(0.00)	(0.00)	(0.00)		(0.87)	(0.24)	
Q2	Estimate	-0.003	0.859	-0.044	53.2	0.001	0.008	-0.2
	NW[p]	(0.09)	(0.00)	(0.22)		(0.68)	(0.87)	
Q3	Estimate	0.004	0.943	-0.017	53.4	0.008	0.040	0.0
	NW[p]	(0.02)	(0.00)	(0.64)		(0.01)	(0.49)	
Q4	Estimate	0.002	0.919	0.036	52.8	0.006	0.092	1.1
	NW[p]	(0.13)	(0.00)	(0.31)		(0.01)	(0.08)	
Q5 (momentum, highest)	Estimate	0.003	1.245	0.212	54.2	0.008	0.287	6.1
	NW[p]	(0.24)	(0.00)	(0.00)		(0.01)	(0.00)	
Agriculture	Estimate	-0.001	0.983	-0.050	69.6	0.003	0.010	-0.2
	NW[p]	(0.30)	(0.00)	(0.16)		(0.22)	(0.87)	
Livestock	Estimate	0.001	0.612	-0.017	19.5	0.003	0.021	-0.1
	NW[p]	(0.79)	(0.00)	(0.76)		(0.18)	(0.71)	
Metal	Estimate	0.001	1.015	-0.038	39.5	0.005	0.024	-0.1
	NW[p]	(0.65)	(0.00)	(0.45)		(0.11)	(0.68)	
$\chi^2(12)$ (p -val.)			28.94 (0.00)				31.28 (0.00)	

Table 5

Time-series regressions with a three-factor model: Individual commodities

Results are based on the time-series regression: $er_t^i = \alpha^i + \beta_{AVG}^i AVG_t + \beta_{CARRY}^i CARRY_t + \beta_{CMOM}^i CMOM_t + \varepsilon_t^i$, for $i = 1, \dots, 29$. We report the coefficient estimates and the p -values in parentheses. The p -values are based on the Newey and West (1987) procedure with lags selected automatically according to Newey and West (1994).

Commodity	α		β_{AVG}		β_{CARRY}		β_{CMOM}		\bar{R}^2 (%)
	Estimate	NW[p]	Estimate	NW[p]	Estimate	NW[p]	Estimate	NW[p]	
Barley	-0.009	(0.02)	0.452	(0.00)	-0.096	(0.08)	0.040	(0.38)	10.8
Cocoa	0.000	(0.95)	0.770	(0.00)	0.108	(0.25)	-0.025	(0.73)	12.0
Coffee	0.006	(0.25)	0.795	(0.00)	-0.003	(0.97)	-0.136	(0.10)	8.5
Corn	-0.006	(0.03)	1.168	(0.00)	-0.050	(0.55)	-0.032	(0.58)	40.3
Cotton	0.001	(0.82)	0.674	(0.00)	0.026	(0.60)	-0.045	(0.34)	13.3
Lumber	-0.006	(0.10)	0.516	(0.00)	-0.090	(0.11)	-0.029	(0.62)	6.8
Oats	-0.004	(0.33)	1.252	(0.00)	-0.208	(0.14)	0.043	(0.65)	31.8
Orange juice	0.003	(0.48)	0.459	(0.00)	-0.015	(0.82)	-0.130	(0.04)	4.5
Rough rice	-0.004	(0.53)	0.643	(0.00)	-0.110	(0.30)	-0.007	(0.93)	6.7
Soybeans	-0.003	(0.21)	1.382	(0.00)	0.002	(0.97)	-0.003	(0.94)	47.2
Soybean meal	-0.001	(0.84)	1.426	(0.00)	-0.025	(0.79)	0.019	(0.82)	37.4
Soybean oil	-0.001	(0.77)	1.394	(0.00)	0.068	(0.38)	-0.039	(0.61)	37.0
Sugar	0.003	(0.60)	1.012	(0.00)	-0.181	(0.19)	0.084	(0.38)	11.9
Wheat	-0.003	(0.26)	1.115	(0.00)	-0.131	(0.01)	-0.019	(0.67)	33.9
Crude oil	0.003	(0.57)	1.415	(0.00)	0.195	(0.08)	-0.027	(0.81)	29.4
Heating oil	0.006	(0.08)	1.287	(0.00)	0.319	(0.01)	0.002	(0.98)	25.8
Natural gas	-0.013	(0.12)	1.595	(0.00)	0.105	(0.58)	0.156	(0.44)	16.7
Propane	0.004	(0.61)	3.128	(0.00)	0.533	(0.08)	0.233	(0.17)	30.8
RBOB gasoline	0.008	(0.40)	1.431	(0.00)	0.300	(0.14)	-0.202	(0.19)	43.8
Unleaded gasoline	0.010	(0.05)	2.076	(0.00)	0.291	(0.02)	-0.056	(0.68)	30.0
Feeder cattle	0.002	(0.29)	0.270	(0.00)	-0.038	(0.34)	-0.027	(0.57)	4.8
Lean hogs	0.003	(0.30)	0.762	(0.00)	-0.028	(0.62)	-0.147	(0.01)	17.8
Live cattle	0.003	(0.14)	0.446	(0.00)	-0.043	(0.28)	-0.024	(0.53)	12.5
Pork belly	-0.003	(0.49)	1.004	(0.00)	0.132	(0.17)	-0.175	(0.01)	15.6
Copper	0.002	(0.58)	0.883	(0.00)	-0.033	(0.58)	0.059	(0.27)	20.5
Gold	-0.001	(0.72)	0.675	(0.00)	-0.068	(0.15)	0.097	(0.14)	22.5
Palladium	0.003	(0.49)	1.401	(0.00)	0.006	(0.95)	0.151	(0.13)	28.2
Platinum	0.000	(0.87)	1.063	(0.00)	-0.071	(0.31)	0.115	(0.16)	31.1
Silver	-0.001	(0.88)	1.173	(0.00)	-0.220	(0.03)	0.235	(0.07)	27.7

Table 6

Cross-sectional asset pricing tests: Individual commodities

Reported are the factor risk premia (λ) estimated using Fama-MacBeth regressions. The underlying SDF is: $m_{t+1} = 1 - b_{\text{AVG}} \text{AVG}_{t+1} - b_{\text{CARRY}} \text{CARRY}_{t+1} - b_{\text{CMOM}} \text{CMOM}_{t+1}$, where AVG_{t+1} is the average factor (excess return obtained by holding all commodities available), CARRY_{t+1} is the carry factor (which corresponds to the returns of strategy C5), and CMOM_{t+1} is the momentum factor (which corresponds to the returns of strategy M5). In Panel A, the parameters are estimated using a Fama-MacBeth specification in which, in the first step, we run a time-series regression of returns on the factors and, in the second step, we run a cross-sectional regression of average returns on the betas. For this specification, the p -values are computed using both the Newey and West (1987) procedure without (with) the Shanken (1992) correction in parentheses (curly brackets), as well as the OLS uncentered R^2 and the χ^2 tests corresponding to the null hypothesis that the pricing errors are zero, with p -values computed both based on the Newey-West and Shanken standard errors. In Panel B, we implement the following modified Fama-MacBeth procedure. In the first step we model the time-variation in beta coefficients on each commodity on carry and momentum using the Kalman Filter. In the second step, we follow the more traditional implementation of the Fama-MacBeth procedure described in Cochrane (2005, pages 245–247). Panel B does not report results using the Shanken correction, as they cannot be computed. The R^2 values are not reported, as they are not comparable to the ones in Panel A. The Newey-West covariance matrix has been adapted for the presence of missing values. In particular, we construct the variance-covariance matrix using pairwise commodity returns series. The procedure requires that each pair of commodities overlaps, and this holds in all cases except for two commodities, i.e., barley and RBOB gasoline. Given that we cannot have missing values in the variance covariance matrix, we omit these two commodities from our analysis and proceed with 27 commodities instead of 29. The variance covariance matrix of the factors uses Newey-West estimates with automatic lags. The one for the variance-covariance matrix of residuals uses Newey-West standard errors with lags equal to the ones of the variance covariance matrix of the factors.

Panel A: Fama-MacBeth results assuming constant betas

Factor risk premia				Pricing errors	
λ_{AVG}	λ_{CARRY}	λ_{CMOM}	R^2	χ_{NW}^2 (p -val.)	χ_{SH}^2 { p -val.}
0.004 (0.06) {0.06}	0.018 (0.07) {0.08}	0.002 (0.88) {0.88}	76.8	37.12 (0.04)	34.84 {0.07}

Panel B: Fama-MacBeth results when time-varying betas are modeled using the Kalman filter

Factor risk premia				Pricing errors
λ_{AVG}	λ_{CARRY}	λ_{CMOM}		χ_{NW}^2 (p -val.)
0.005 (0.04)	0.030 (0.01)	0.012 (0.31)		33.79 (0.14)

Table 7

The commodity carry factor and its relation to innovations in equity volatility

The regression results reported in Panel A are based on the univariate regression: $CARRY_t = \phi_0 + \phi_1 \Delta VOL_t^{\text{equity}} + e_t$. We report the Newey and West (1987) p -values (with lags automatically selected, as in Newey and West (1994)), and denote them by NW[p]. The returns to the long and short legs of carry are constructed as in Appendix A. For the Fama and MacBeth procedure results reported in Panel B, the p -values are computed using both the Newey and West (1987) procedure without (with) the Shanken (1992) correction in parentheses (curly brackets). We report the OLS uncentered R^2 as [.]. The SDF specification is of the form: $m_{t+1} = 1 - b_{\text{AVG}} \text{AVG}_{t+1} - b_{\Delta \text{VOL}} \Delta \text{VOL}_{t+1}^{\text{equity}}$ and the factor risk premia are reported as λ . The χ_{NW}^2 and χ_{SH}^2 tests correspond to the null hypotheses that the pricing errors are zero and their p -values are computed using Newey-West and Shanken standard errors. With four carry test portfolios, the test statistics follow a χ^2 distribution with 2 degrees of freedom.

Panel A: Univariate time-series regressions of carry on innovations in equity volatility

	Constant		Slope		R^2
	ϕ_0	p -val.	ϕ_1	p -val.	
Carry factor	0.015	(0.00)	-0.090	(0.04)	0.9
Long, backwarddated commodities	0.012	(0.00)	-0.108	(0.01)	1.4
Short, contangoed commodities	0.003	(0.29)	0.018	(0.61)	0.1

Panel B: Pricing the carry portfolios with innovations in equity volatility

	λ_{AVG}	$\lambda_{\Delta \text{VOL}}$	R^2	χ_{NW}^2	χ_{SH}^2
Fama-MacBeth	0.006	-0.154	[95.5]	2.89	0.29
	(0.01)	(0.00)		(0.24)	{0.86}
	{0.02}	{0.12}			

Table 8

The commodity momentum factor and its relation to innovations in speculative activity

The regression results reported in Panel A are based on the univariate regression: $\text{CMOM}_t = \psi_0 + \psi_1 \Delta\text{Speculation}_t + e_t$. We report the Newey and West (1987) p -values (with lags automatically selected, as in Newey and West (1994)), and denote them by $\text{NW}[p]$. The returns to the long and short legs of momentum are constructed as in Appendix A. For the Fama and MacBeth procedure results reported in Panel B, the p -values are computed using both the Newey and West (1987) procedure without (with) the Shanken (1992) correction in parentheses (curly brackets). We report the OLS uncentered R^2 as $[\cdot]$. The SDF specification is of the form: $m_{t+1} = 1 - b_{\text{AVG}} \text{AVG}_{t+1} - b_{\Delta\text{Speculation}} \Delta\text{Speculation}_{t+1}$ and the factor risk premia are reported as λ . The χ_{NW}^2 and χ_{SH}^2 tests correspond to the null hypotheses that the pricing errors are zero and their p -values are computed using Newey-West and Shanken standard errors. With five momentum test portfolios, the test statistics follow a χ^2 distribution with 3 degrees of freedom. The sample period ranges from January 1986 through September 2011.

Panel A: Univariate time-series regressions of momentum on innovations in speculative activity

	Constant		Slope		R^2
	ψ_0	$\text{NW}[p]$	ψ_1	$\text{NW}[p]$	
Momentum factor	0.010	(0.01)	0.996	(0.02)	2.2
Long, high momentum (winners)	0.008	(0.02)	0.941	(0.01)	2.5
Short, low momentum (losers)	0.002	(0.55)	0.055	(0.86)	-0.3

Panel B: Pricing the momentum portfolios with innovations in speculative activity

	λ_{AVG}	$\lambda_{\Delta\text{Speculation}}$	R^2	χ_{NW}^2	χ_{SH}^2
Fama-MacBeth	0.005	0.008	[89.8]	3.80	1.78
	(0.05)	(0.02)		(0.28)	{0.62}
	{0.05}	{0.10}			

Table 9

Cross-sectional tests with alternative economic variables

All the results in this table use the SDF specification of the form: $m_{t+1} = 1 - b_{\text{AVG}} \text{AVG}_{t+1} - b_{\Delta X} \Delta X_{t+1}$, where ΔX_{t+1} is the innovation in a economic variable. We use each model specification to alternatively price the five momentum portfolios or the four carry portfolios using Fama and MacBeth (1973) regressions. The reported χ^2_{NW} tests correspond to the null hypotheses that the pricing errors are zero and their p -values are computed using Newey-West standard errors. With five (four) momentum (carry) portfolios, the test statistics follow a χ^2 distribution with 3 (2) degrees of freedom.

ΔX_{t+1}	χ^2_{NW} (p -val.)	
	Momentum portfolios	Carry portfolios
Innovations in commodity volatility	28.37 (0.00)	21.88 (0.00)
Innovations in log open interest	7.30 (0.06)	15.24 (0.00)
Innovations in hedging pressure	12.36 (0.01)	21.08 (0.00)
Innovations in scarcity	10.33 (0.02)	21.14 (0.00)
Innovations in log industrial production	29.14 (0.00)	13.86 (0.00)
Innovations in US TED spread	12.73 (0.01)	19.98 (0.00)
Innovations in G7 Inflation	5.80 (0.12)	21.31 (0.00)

Table 10

Performance of commodity factors and equity market returns over different stages of the business cycle

Reported are the average *monthly* returns of the carry and momentum factors, as well as their long and short positions, over different stages of the business cycle. The returns to the long and short legs of carry and momentum are as constructed in equations (A1)–(A4). The long leg consists of taking long position in five commodities that are most backwardated, or five commodities with the high momentum (winners). The short leg consists of taking short positions in five commodities that are most contangoed, or five commodities with the lowest momentum (losers). We classify the economy in expansions and recessions using the NBER classification. We also characterize early and late expansions as well as early and late recessions following Dangi and Halling (2012, Section 4.2).

	Carry			Momentum			AVG _t	Equity Market	
	CARRY _t	Long	Short	CMOM _t	Long	Short		Returns	Variance
Expansion	1.03	1.11	-0.08	1.14	1.27	-0.13	0.73	0.64	0.206
Recession	3.18	1.54	1.64	2.46	0.40	2.06	-0.58	-0.36	0.265
Late expansion	0.55	-1.53	2.08	1.33	-0.30	1.63	-0.66	-0.33	0.212
Early recession	5.65	4.40	1.26	4.22	1.05	3.17	0.21	-2.27	0.260
Late recession	-0.03	-1.03	1.00	0.29	-0.66	0.95	-0.53	4.06	0.258
Early expansion	-0.23	-1.20	0.97	2.44	1.66	0.78	-0.15	2.36	0.204

Table 11

Relation of the commodity factors to future real GDP growth

We report the coefficient estimates from the following predictive regressions with overlapping observations at the quarterly frequency:

$$\ln(\text{GDP}_{t+k}/\text{GDP}_t) = \theta_0 + \theta_{\text{AVG}} \text{AVG}_t + \theta_{\text{CARRY}} \text{CARRY}_t + \theta_{\text{CMOM}} \text{CMOM}_t + \varepsilon_{t+k} \quad \text{and} \quad k \in \{1, 2, 3, 4\},$$

where GDP refers to the real GDP of the G7 countries (source: Datastream, ticker G7OCMP03D). The p -values based on the Newey and West (1987) covariance estimator, in which lags are automatically selected according to Newey and West (1994), are denoted by $\text{NW}[p]$. Additionally, the p -values based on the Hodrick (1992) 1B covariance estimator under the null of no predictability are denoted by $\text{H}[p]$. Adjusted R^2 is reported as \bar{R}^2 (in %), while the $\text{J}[p]$ column reports the p -values for the null hypothesis that the slope coefficients are jointly equal to zero. The sample period is from 1970:Q1 to 2011:Q3.

Horizon		θ_0	Predictive slope coefficients			\bar{R}^2	$\text{J}[p]$
			θ_{AVG}	θ_{CARRY}	θ_{CMOM}		
1 quarter	Estimate	0.01	0.03	-0.01	-0.01	13.8	
	NW[p]	(0.00)	(0.02)	(0.06)	(0.00)		(0.00)
	H[p]	\langle 0.00 \rangle	\langle 0.04 \rangle	\langle 0.09 \rangle	\langle 0.01 \rangle		\langle 0.02 \rangle
2 quarters	Estimate	0.01	0.04	-0.02	-0.02	11.1	
	NW[p]	(0.00)	(0.03)	(0.06)	(0.00)		(0.02)
	H[p]	\langle 0.00 \rangle	\langle 0.03 \rangle	\langle 0.00 \rangle	\langle 0.00 \rangle		\langle 0.00 \rangle
3 quarters	Estimate	0.02	0.04	-0.03	-0.02	9.3	
	NW[p]	(0.00)	(0.08)	(0.05)	(0.02)		(0.02)
	H[p]	\langle 0.00 \rangle	\langle 0.04 \rangle	\langle 0.00 \rangle	\langle 0.00 \rangle		\langle 0.00 \rangle
4 quarters	Estimate	0.03	0.03	-0.03	-0.03	7.4	
	NW[p]	(0.00)	(0.23)	(0.08)	(0.00)		(0.01)
	H[p]	\langle 0.00 \rangle	\langle 0.16 \rangle	\langle 0.01 \rangle	\langle 0.00 \rangle		\langle 0.00 \rangle

Table 12

Relation of the commodity factors to future excess Treasury bond returns

This table reports the coefficient estimates from the following predictive regressions with overlapping observations at the monthly frequency:

$$\sum_{k=1}^K \text{er}_{t+k}^{\text{bond}} = \xi_0 + \boldsymbol{\xi}' \mathbf{f}_t + \varepsilon_{t+K}, \quad \text{with} \quad \text{er}_{t+1}^{\text{bond}} \equiv \ln(1 + r_{t+1}^{\text{bond}}) - \ln(1 + r_t^f) \quad \text{and} \quad K \in \{1, 3, 6, 9, 12\},$$

where $\mathbf{f}_t \equiv [\text{AVG}_t, \text{CARRY}_t, \text{CMOM}_t]'$. We compute standard errors based on the Hodrick (1992) 1B covariance estimator under the null of no predictability and report the corresponding p -values, denoted by $H[p]$. Adjusted R^2 is reported as \bar{R}^2 (in %). The analysis is conducted for both one-year and 30-year Treasury bonds (source: CRSP), and the sample period is from January 1970 to September 2011. The inference based on the Newey-West p -values agrees with those from $H[p]$, hence, the Newey-West p -values are not reported.

Horizon	Panel A: Bond returns, one-year maturity						Panel B: Bond returns, 30-year maturity					
	Estimate	ξ_0	ξ_{AVG}	ξ_{CARRY}	ξ_{CMOM}	\bar{R}^2	Estimate	ξ_0	ξ_{AVG}	ξ_{CARRY}	ξ_{CMOM}	\bar{R}^2
1 month	$H[p]$	0.00 (0.00)	-0.03 (0.02)	0.00 (0.57)	0.00 (0.50)	4.0	0.02 (0.00)	0.00 (0.12)	-0.20 (0.00)	-0.02 (0.53)	0.02 (0.48)	5.2
3 months	$H[p]$	0.00 (0.00)	-0.04 (0.01)	0.02 (0.01)	-0.01 (0.64)	3.9	0.03 (0.00)	0.01 (0.05)	-0.30 (0.01)	0.03 (0.34)	0.03 (0.61)	3.1
6 months	$H[p]$	0.01 (0.00)	-0.05 (0.01)	0.02 (0.01)	0.00 (0.06)	2.8	0.05 (0.00)	0.01 (0.01)	-0.22 (0.66)	0.05 (0.03)	0.01 (0.95)	0.5
9 months	$H[p]$	0.01 (0.00)	-0.07 (0.00)	0.03 (0.00)	0.01 (0.00)	3.1	0.05 (0.00)	0.02 (0.00)	-0.21 (0.39)	0.15 (0.00)	-0.01 (0.91)	0.9
12 months	$H[p]$	0.01 (0.00)	-0.10 (0.00)	0.02 (0.00)	0.01 (0.00)	3.9	0.05 (0.00)	0.03 (0.00)	-0.24 (0.12)	0.13 (0.00)	0.03 (0.48)	0.5

Table 13

Relation of the commodity factors to future excess equity returns

This table reports the coefficient estimates from the following predictive regressions with overlapping observations at the monthly frequency:

$$\sum_{k=1}^K er_{t+k}^{\text{equity}} = \delta_0 + \boldsymbol{\delta}' \mathbf{f}_t + \varepsilon_{t+K}, \quad \text{with} \quad er_{t+1}^{\text{equity}} \equiv \ln(1+r_{t+1}^{\text{equity}}) - \ln(1+r_t^f) \quad \text{and} \quad K \in \{1, 3, 6, 9, 12\},$$

where er_{t+1}^{equity} is the excess return of the US value-weighted equity index over month t to $t+1$ and $\mathbf{f}_t \equiv [\text{AVG}_t \text{ CARRY}_t \text{ CMOM}_t]'$. We compute standard errors based on the Hodrick (1992) 1B covariance estimator under the null of no predictability and report the corresponding p -values, denoted by $H[p]$. Also reported are the results of the restricted predictive regressions with $\delta_{\text{AVG}} = \delta_{\text{CMOM}} \equiv 0$. Adjusted R^2 is reported as \bar{R}^2 (%), and the sample period is from January 1970 to September 2011. The inference based on the Newey-West p -values agrees with those from $H[p]$, hence, the Newey-West p -values are not reported.

Horizon	Unrestricted predictive regressions					Restricted regressions	
	δ_0	δ_{AVG}	δ_{CARRY}	δ_{CMOM}	\bar{R}^2	$\delta_{\text{AVG}} = \delta_{\text{CMOM}} \equiv 0$ δ_{CARRY}	\bar{R}^2
1 month	0.01 (0.01)	-0.07 (0.35)	-0.06 (0.09)	0.02 (0.45)	0.4	-0.06 (0.10)	0.4
3 months	0.02 (0.01)	0.07 (0.59)	-0.15 (0.01)	0.02 (0.79)	0.7	-0.14 (0.02)	0.9
6 months	0.03 (0.01)	-0.08 (0.67)	-0.16 (0.09)	0.01 (0.89)	0.2	-0.16 (0.08)	0.5
9 months	0.05 (0.01)	-0.32 (0.16)	-0.27 (0.03)	0.07 (0.44)	1.6	-0.27 (0.04)	1.1
12 months	0.06 (0.01)	-0.51 (0.04)	-0.32 (0.04)	0.07 (0.43)	2.5	-0.33 (0.04)	1.4

Table 14

Relation of the commodity factors to the future returns of commodity currencies

This table reports the coefficient estimates from the following predictive regressions with overlapping observations at the quarterly frequency (for comparability with Chen, Rogoff, and Rossi (2010)):

$$\ln(\text{FX}_{t+k}/\text{FX}_t) = \pi_0 + \boldsymbol{\pi}'\mathbf{f}_t + \varepsilon_{t+k} \quad \text{and} \quad k \in \{1, 2, 3, 4\},$$

where $\ln(\text{FX}_{t+k}/\text{FX}_t)$ represents the equally-weighted returns from a set of commodity currencies (i.e., Australia, Canada, Chile, New Zealand, Norway, and South Africa; see Labuszewski (2012)) with the US dollar as the reference currency (i.e., FX|USD) and $\mathbf{f}_t \equiv [\text{AVG}_t \text{ CARRY}_t \text{ CMOM}_t]'$. We compute standard errors based on the Hodrick (1992) 1B covariance estimator under the null of no predictability and report the corresponding p -values, denoted by $\text{H}[p]$. Adjusted R^2 is reported as \bar{R}^2 (in %). The sample period is from 1974:Q1 to 2011:Q3. The inference based on the Newey-West p -values agrees with those from $\text{H}[p]$, hence, the Newey-West p -values are not reported.

Horizon		π_0	Predictive slope coefficients			\bar{R}^2
			π_{AVG}	π_{CARRY}	π_{CMOM}	
1 quarter	Estimate	0.01	-0.12	0.03	0.06	6.0
	$\text{H}[p]$	$\langle 0.02 \rangle$	$\langle 0.06 \rangle$	$\langle 0.25 \rangle$	$\langle 0.03 \rangle$	
2 quarters	Estimate	0.02	-0.09	0.12	0.04	4.0
	$\text{H}[p]$	$\langle 0.02 \rangle$	$\langle 0.27 \rangle$	$\langle 0.01 \rangle$	$\langle 0.25 \rangle$	
3 quarters	Estimate	0.02	-0.06	0.17	0.07	6.0
	$\text{H}[p]$	$\langle 0.02 \rangle$	$\langle 0.64 \rangle$	$\langle 0.00 \rangle$	$\langle 0.07 \rangle$	
4 quarters	Estimate	0.03	-0.03	0.18	0.11	5.0
	$\text{H}[p]$	$\langle 0.02 \rangle$	$\langle 0.81 \rangle$	$\langle 0.00 \rangle$	$\langle 0.01 \rangle$	

Table 15

Membership in the long and short components of the carry and momentum strategies

Entries under the column labeled “Long (Short)” depict how many months the respective commodity has entered the backwardation (contango) component of the carry strategy C5 (Panel A). The next four columns show how many months the commodity has entered the long and short components of the momentum strategy M5 (Panel B). The long leg consists of taking long position in five commodities that are most backwardated, or five commodities with the high momentum (winners). The short leg consists of taking short positions in five commodities that are most contangoed, or five commodities with the lowest momentum (losers). For example, and more concretely, in 171 (126) months the live cattle (soybean oil) has been among the five highest backwardated (momentum) commodities.

Panel A: Carry strategy, C5				Panel B: Momentum strategy, M5			
Long		Short		Long		Short	
Live cattle	171	Oats	225	Soybean oil	126	Sugar	139
Lean hog	168	Lumber	191	Corn	110	Pork belly	123
Pork belly	118	Lean hog	182	Cocoa	99	Lumber	121
Sugar	108	Wheat	165	Crude oil	97	Orange juice	112
Coffee	104	Sugar	158	Cotton	97	Cocoa	111
Oats	100	Corn	156	Feeder cattle	95	Oats	107
Lumber	93	Orange juice	103	Gold	90	Natural gas	102
Orange juice	91	Rough rice	94	Copper	89	Coffee	92
Cocoa	86	Cocoa	92	Heating oil	87	Corn	88
Unleaded gas	86	Cotton	92	Unleaded gas	83	Soybean oil	78
Cotton	85	Coffee	90	Coffee	80	Palladium	75
Feeder cattle	77	Live cattle	89	Lumber	79	Rough rice	72
Soybean meal	74	Natural gas	61	Barley	78	Silver	70
Wheat	69	Platinum	35	Live cattle	75	Lean hog	68
Heating oil	68	Pork belly	34	Lean hog	72	Wheat	68
Soybean oil	65	Soybean meal	32	Natural gas	67	Platinum	67
Copper	57	Feeder cattle	29	Oats	60	Natural gas	65
Crude oil	50	Unleaded gas	22	Orange juice	59	Soybean meal	59
Propane	37	Barley	22	Palladium	59	Cotton	56
Corn	36	Soybean oil	17	Pork belly	57	Heating oil	52
Natural gas	34	Silver	17	Platinum	50	Crude oil	45
Palladium	30	Copper	16	Propane	49	Silver	39
Soybeans	29	Palladium	16	RBOB gasoline	47	Gold	35
RBOB gasoline	28	Silver	13	Rough rice	44	Unleaded gas	30
Platinum	24	Crude oil	9	Soybeans	37	Live cattle	29
Rough rice	20	Heating oil	9	Sugar	32	Feeder cattle	27
Barley	6	RBOB gasoline	6	Silver	32	Propane	26
Silver	1	Propane	5	Soybean meal	25	Barley	15
Gold	0	Gold	0	Wheat	5	RBOB gasoline	9

Table Appendix-I

Excess returns of the commodity factors, the commodity indexes, and the four commodity categories

Panel A first reports the descriptive statistics for each of the factors. The average factor, denoted by AVG, is the excess return of a long position in all available commodity futures. The carry factor, denoted by CARRY, is the return on strategy C5, while the momentum factor, denoted by CMOM, is the return on strategy M5. Panel B corresponds to the excess returns of the Goldman Sachs Commodity Index (GSCI, source: ticker GSCIEXR in Datastream) and the Commodity Research Bureau index (CRB), while Panel C corresponds to the equally weighted commodity returns across four categories. Panel D reports the summary statistics for two additional factors (details of the construction are in Appendix C). Our procedure for constructing the value factor, denoted by VALUE, is similar to that of Asness, Moskowitz, and Pedersen (2013), in that in each month, we rank all the commodities by the ratio of the second nearest maturity futures price five years ago to its current price. We divide the commodities into five groups and compute the next month portfolio returns. The value factor is the return difference between the top and bottom quintiles. The Δ VOL corresponds to the innovation in commodity volatility and is computed following Menkhoff, Sarno, Schmeling, and Schrimpf (2012, equation (4)). For AVG_t , $CARRY_t$, and $CMOM_t$ we investigate seasonality of the form: $f_t = \nu_0 + \sum_{j=2}^{12} \nu_j 1_{j,t} + \varepsilon_t$, where the 1_j 's are dummy variables for the months of February through December. We do not find evidence of seasonality in the factors.

	Mean	PW bootstrap CI		SD	SR	Skewness	ρ_1	$1_{er>0}$
		lower	upper					
<i>Panel A: Commodity factors</i>								
AVG	6.27	1.32	12.60	14.32	0.44	0.22	0.05	57.37
CARRY	16.34	9.36	23.40	22.26	0.73	0.30	0.09	56.77
CMOM	16.11	9.36	22.92	26.28	0.61	0.34	-0.01	57.37
<i>Panel B: Commodity indexes</i>								
GSCI	5.43	-0.84	11.52	20.04	0.27	0.06	0.16	54.75
CRB	3.53	-0.36	8.04	13.56	0.26	-0.06	0.08	52.53
<i>Panel C: Commodity categories</i>								
Agriculture	3.79	-2.88	11.28	16.78	0.23	0.79	0.00	50.71
Livestock	4.29	0.12	14.64	19.65	0.22	0.13	-0.01	52.73
Metal	6.98	-0.84	8.76	22.96	0.30	0.39	0.11	52.53
Energy	14.20	3.84	24.48	35.18	0.40	1.70	0.08	53.59
<i>Panel D: Additional commodity factors</i>								
VALUE	6.15	-2.16	14.28	28.17	0.22	-0.19	-0.01	52.61
Δ VOL	0.02	-0.12	0.12	0.68	-	0.24	-0.25	-

Table Appendix-II

Unconditional tests using conditioning variables to construct additional managed portfolios

Here we expand the exercise reported in Table 2 by including additional test assets constructed using conditioning variables (see Cochrane (2005, Chapter 8.1)). The SDF specification is of the form: $m_{t+1} = 1 - b_{AVG} AVG_{t+1} - b_{CARRY} CARRY_{t+1} - b_{CMOM} CMOM_{t+1}$, where AVG_{t+1} is the average factor (excess return obtained by holding all commodities available), $CARRY_{t+1}$ is the carry factor (which corresponds to the returns of strategy C5), and $CMOM_{t+1}$ is the momentum factor (which corresponds to the returns of strategy M5). Let the vector \mathbf{er}_{t+1} contain the 12 test assets in Table 2, namely the four carry portfolios, the five momentum portfolios and the three sector portfolios. The new set of test assets is defined as $\mathbf{er}_{t+1} \otimes [1 \ z_t]$, generating a total of 24 portfolios. We first report, in Panel A, the Hansen and Jagannathan (1997, equation (29)) distance measure (HJ-Dist.), and the associated p -value, which assesses whether the distance measure is equal to zero. In Panel B we report the p -values from the Hansen (1982) test of over-identifying restrictions (J -test), testing for overall model adequacy. Finally, in Panel C, we test the null hypothesis $b_{CARRY} = b_{CMOM} \equiv 0$ against the alternative that at least one is different from zero, as well as test the individual parameter restrictions $b_{CARRY} \equiv 0$ and $b_{CMOM} \equiv 0$. The p -values rely on the Newey and West (1987) procedure, with lags selected automatically according to Newey and West (1994). Each conditioning variable is as defined in Appendix C.

Conditioning information, z_t	Panel A:		Panel B:	Panel C: Exclusion restrictions					
	HJ Dist. test		J -test	$b_{CARRY} = 0$ $b_{CMOM} = 0$		$b_{CARRY} = 0$		$b_{CMOM} = 0$	
	Dist.	p -val.	p -val.	$\chi^2(2)$	NW[p]	$\chi^2(1)$	NW[p]	$\chi^2(1)$	NW[p]
Open interest growth	0.007	0.30	0.23	28.19	0.00	10.01	0.00	6.60	0.01
ΔVOL_t	0.008	0.09	0.04	37.47	0.00	21.77	0.00	1.65	0.20
$\ln(y_t) 1_{\ln(y_t) < 0}$	0.007	0.17	0.15	32.87	0.00	12.67	0.00	5.10	0.02
Currency returns (FX USD)	0.007	0.32	0.51	39.53	0.00	21.40	0.00	8.23	0.00
Industrial production growth	0.010	0.01	0.09	38.01	0.00	13.45	0.00	5.21	0.02

Table Appendix-III

Exclusion tests that evaluate whether commodity value and volatility are additional priced factors

Reported are the factor risk premia and exclusion tests for the validity of a four-factor asset pricing model, when we incrementally add the value factor and the volatility factor to the SDF specification in equation (6). Specifically, the SDF specification is of the form:

$$m_{t+1} = 1 - b_{\text{AVG}} \text{AVG}_{t+1} - b_{\text{CARRY}} \text{CARRY}_{t+1} - b_{\text{CMOM}} \text{CMOM}_{t+1} - b_{\text{VALUE}} \text{VALUE}_{t+1}$$

or,

$$m_{t+1} = 1 - b_{\text{AVG}} \text{AVG}_{t+1} - b_{\text{CARRY}} \text{CARRY}_{t+1} - b_{\text{CMOM}} \text{CMOM}_{t+1} - b_{\Delta\text{VOL}} \Delta\text{VOL}_{t+1}.$$

Our procedure for constructing the value factor, denoted by VALUE_{t+1} , is similar to that in Asness, Moskowitz, and Pedersen (2013), in that in each month, we rank all the commodities by the ratio of the second-nearest maturity futures price five years ago to its current price. We divide the commodities into five groups and compute the next month portfolio returns. VALUE_{t+1} is the return spread between the top and bottom quintiles (see Appendix C). The ΔVOL_{t+1} corresponds to the innovation in commodity volatility (see Appendix C) and is computed following Menkhoff, Sarno, Schmeling, and Schrimpf (2012, equation (4)). The estimates of the factor risk premia λ are based on a cross-sectional regression. The p -values are computed using both the Newey and West (1987) procedure and the Shanken (1992) correction (in curly brackets). The test of individual parameter restriction $b_{\text{VALUE}} \equiv 0$ and $b_{\Delta\text{VOL}} \equiv 0$ is based on the two-step GMM χ^2 test, in which the p -values rely on the Newey and West (1987) procedure with lags selected automatically according to Newey and West (1994).

		Fama-MacBeth		GMM
		Value factor	Volatility factor	(two-step)
Panel A: Factor risk premia				
λ_{AVG}	Estimate	0.003	0.005	
	NW[p]	(0.11)	(0.02)	
	Shanken[p]	{0.11}	{0.02}	
λ_{CARRY}	Estimate	0.017	0.018	
	NW[p]	(0.00)	(0.00)	
	Shanken[p]	{0.00}	{0.00}	
λ_{CMOM}	Estimate	0.012	0.012	
	NW[p]	(0.00)	(0.00)	
	Shanken[p]	{0.00}	{0.00}	
λ_{VALUE}	Estimate	-0.014		
	NW[p]	(0.21)		
	Shanken[p]	{0.22}		
$\lambda_{\Delta\text{VOL}}$	Estimate		0.000	
	NW[p]		(0.94)	
	Shanken[p]		{0.94}	
Panel B: Exclusion tests for the loadings on the SDF				
$H_0: b_{\text{VALUE}} \equiv 0$	$\chi^2(1)$ (p -val.)			0.43 (0.51)
$H_0: b_{\Delta\text{VOL}} \equiv 0$	$\chi^2(1)$ (p -val.)			0.68 (0.41)

Table Appendix-IV

Cross-sectional asset pricing results with the average, carry, and momentum factors - alphabetical sorts A through M or N to Z, based on the ticker symbol

In Panel A we conduct a randomization exercise in the style of Lustig, Roussanov, and Verdellhan (2011), in which we construct the average, carry, and momentum factors based on commodities whose ticker symbol starts with the letter A through the letter M. Next we construct two carry, two momentum, and three category portfolios based on commodities whose ticker symbol starts with the letter N through the letter Z. Reported are the factor risk premia (λ) and the SDF parameters (\mathbf{b}). The baseline SDF specification is of the form: $m_{t+1} = 1 - b_{AVG} AVG_{t+1} - b_{CARRY} CARRY_{t+1} - b_{CMOM} CMOM_{t+1}$. Also reported are results for the restricted versions of the SDF that impose $b_{CMOM} \equiv 0$ and $b_{CARRY} \equiv 0$, respectively. In the row marked "GMM," the parameters are estimated based on the system (8) following a one-step GMM procedure, while those in the row "Fama-MacBeth" are based on a two-step cross-sectional regression. The p -values rely on the Newey and West (1987) procedure with lags selected automatically according to Newey and West (1994), and reported in parentheses. For the Fama and MacBeth procedure, the p -values are computed using both the Newey and West (1987) procedure without (with) the Shanken (1992) correction in parentheses (curly brackets). We report the GLS cross-sectional uncentered R^2 (Lewellen, Nagel, and Shanken (2010, Prescription 3)) and the OLS uncentered R^2 as $[\cdot]$, and the χ^2 test corresponding to the null hypothesis that the pricing errors are zero, with p -values computed both based on the Newey-West and Shanken standard errors. The Hansen and Jagannathan (1997, equation (29)) distance measure (HJ-Dist.), with the associated p -value, is shown, which tests whether the distance measure is equal to zero. In Panel B we repeat the exercise by constructing factors based on commodities whose ticker symbol starts with the letter N through the letter Z and test portfolios based on commodities whose ticker symbol starts with the letter A through the letter M.

	Panel A. Alphabetical Sorting A through M						Panel B. Alphabetical Sorting N through Z											
	Factor risk premia			Loadings on the SDF			Pricing errors			Loadings on the SDF			Pricing errors					
	λ_{AVG}	λ_{CARRY}	λ_{CMOM}	b_{AVG}	b_{CARRY}	b_{CMOM}	R_{GLS}^2 [R^2]	χ_{SH}^2 { p -val.}	HJ-Dist. { p -val.}	λ_{AVG}	λ_{CARRY}	λ_{CMOM}	b_{AVG}	b_{CARRY}	b_{CMOM}	R_{GLS}^2 [R^2]	χ_{SH}^2 { p -val.}	HJ-Dist. { p -val.}
<i>Three-factor model</i>																		
GMM	0.003 {0.54}	0.024 {0.19}	0.048 {0.03}	0.736 {0.79}	4.377 {0.42}	16.976 {0.02}			0.004 {0.61}	0.016 {0.01}	0.077 {0.04}	0.035 {0.22}	6.026 {0.01}	15.937 {0.03}	5.355 {0.35}			0.007 {0.04}
Fama-MacBeth	0.003 {0.43} {0.49}	0.024 {0.06} {0.14}	0.048 {0.00} {0.02}				50.8 [92.6]	4.32 {0.37}	2.45 {0.65}	0.016 {0.00}	0.077 {0.00}	0.035 {0.03} {0.13}				63.2 [81.8]	19.44 {0.00}	9.43 {0.05}
<i>Restricted model omitting a role for CMOM_{t+1}</i>																		
GMM	0.005 {0.08}	-0.004 {0.75}		3.601 {0.06}	-1.614 {0.70}				0.008 {0.04}	0.014 {0.00}	0.049 {0.01}	5.303 {0.00}	10.780 {0.01}					0.008 {0.02}
Fama-MacBeth	0.005 {0.00} {0.10}	-0.004 {0.75}					35.7 [68.4]	12.36 {0.03}	12.16 {0.03}	0.014 {0.00}	0.049 {0.01}					60.4 [78.7]	21.51 {0.00}	15.01 {0.01}
<i>Restricted model omitting a role for CARRY_{t+1}</i>																		
GMM	0.003 {0.45}	0.034 {0.04}	1.498 {0.52}			12.763 {0.04}			0.005 {0.48}	0.010 {0.02}	-0.014 {0.26}	3.953 {0.02}		-3.388 {0.24}				0.010 {0.00}
Fama-MacBeth	0.003 {0.42} {0.45}	0.034 {0.04}	1.498 {0.08}				54.2 [88.2]	5.41 {0.37}	3.98 {0.55}	0.010 {0.01}	-0.014 {0.22}					41.3 [66.0]	27.84 {0.00}	25.78 {0.00}

Table Appendix-V

Time-series regressions with a three-factor model - alphabetical sorts A through M or N through Z, based on the ticker symbol

In Panel A we conduct a randomization exercise in the style of Lustig, Roussanov, and Verdelhan (2011), in which we construct the average, carry, and momentum factors based on commodities whose ticker symbol starts with the letter A through the letter M. Next, we construct two carry, two momentum, and three category portfolios based on commodities whose ticker symbol starts with the letter N through the letter Z. Results are based on the time-series regression: $er_t^i = \alpha^i + \beta_{AVG}^i AVG_t + \beta_{CARRY}^i CARRY_t + \beta_{CMOM}^i CMOM_t + \varepsilon_t^i$, for $i = 1, \dots, 7$. We report the coefficient estimates and the p -values in parentheses. The p -values are based on the Newey and West (1987) procedure, with lags selected automatically according to Newey and West (1994). To test the null hypothesis that all intercepts are jointly equal to zero, we compute the statistic $\hat{\alpha}' \text{var}(\hat{\alpha})^{-1} \hat{\alpha}$ in a GMM setting (Cochrane (2005, page 234)), which is asymptotically distributed $\chi^2(7)$. We have excluded the energy category as a test asset in the time-series regressions because of its shorter time-series. In Panel B we repeat the exercise by constructing factors based on commodities whose ticker symbol starts with the letter N through the letter Z and test portfolios based on commodities whose ticker symbol starts with the letter A through the letter M.

Panel A. Alphabetical Sorting A through M

Commodity portfolio		α	β_{AVG}	β_{CARRY}	β_{CMOM}	\bar{R}^2 (%)	Joint test on α	
							$\chi^2(7)$	p -val.
P1 (backwardation, lowest y)	Estimate	0.003	0.784	0.089	0.069	19.7		
	NW[p]	(0.358)	(0.000)	(0.059)	(0.389)			
P2 (contango, highest y)	Estimate	-0.001	0.939	-0.161	0.132	46.9		
	NW[p]	(0.553)	(0.000)	(0.000)	(0.001)			
Q1 (momentum, lowest)	Estimate	-0.004	0.960	-0.005	-0.004	39.1		
	NW[p]	(0.043)	(0.000)	(0.926)	(0.937)			
Q2 (momentum, highest)	Estimate	0.005	0.912	-0.197	0.256	34.0		
	NW[p]	(0.029)	(0.000)	(0.002)	(0.000)			
Agriculture	Estimate	-0.004	0.839	0.118	-0.080	9.5		
	NW[p]	(0.415)	(0.000)	(0.224)	(0.389)			
Livestock	Estimate	0.004	0.950	-0.292	0.178	22.2		
	NW[p]	(0.236)	(0.000)	(0.001)	(0.020)			
Metal	Estimate	-0.001	0.836	-0.094	0.087	31.6		
	NW[p]	(0.731)	(0.000)	(0.078)	(0.103)			
All portfolios							13.41	0.063

Panel B. Alphabetical Sorting N through Z

Commodity portfolio		α	β_{AVG}	β_{CARRY}	β_{CMOM}	\bar{R}^2 (%)	Joint test on α	
							$\chi^2(7)$	p -val.
P1 (backwardation, lowest y)	Estimate	0.012	0.503	0.081	-0.122	23.2		
	NW[p]	(0.000)	(0.000)	(0.014)	(0.009)			
P2 (contango, highest y)	Estimate	-0.001	0.493	-0.075	-0.006	39.9		
	NW[p]	(0.360)	(0.000)	(0.001)	(0.877)			
Q1 (momentum, lowest)	Estimate	0.002	0.482	-0.008	-0.103	30.9		
	NW[p]	(0.425)	(0.000)	(0.823)	(0.007)			
Q2 (momentum, highest)	Estimate	0.006	0.572	-0.028	0.026	38.6		
	NW[p]	(0.001)	(0.000)	(0.321)	(0.419)			
Agriculture	Estimate	0.004	0.301	0.002	-0.096	10.3		
	NW[p]	(0.098)	(0.000)	(0.958)	(0.024)			
Livestock	Estimate	0.002	0.507	-0.138	0.133	23.6		
	NW[p]	(0.518)	(0.000)	(0.002)	(0.025)			
Metal	Estimate	0.001	0.562	0.018	-0.054	34.5		
	NW[p]	(0.747)	(0.000)	(0.525)	(0.250)			
All portfolios							34.11	0.000

Table Appendix-VI

Excess returns of long and short legs of the commodity carry and momentum strategies

This table presents the descriptive statistics of the excess returns generated by the long and short legs of the commodity carry and momentum strategies. Let $y_t \equiv F_t^{(1)}/F_t^{(0)}$, where $F_t^{(0)}$ is the price of the front-month futures contract and $F_t^{(1)}$ is the price of the next maturity futures contract, both observed at the end of month t . A commodity is in backwardation if $\ln(y_t) < 0$ and in contango if $\ln(y_t) > 0$. The carry strategy entails taking a long (short) futures position in a commodity that is in backwardation (contango) at the end of month t , and we compute the returns over the subsequent month. For example, the long (short) leg of the carry strategy C5 contains an equally weighted portfolio consisting of five commodities with the most negative (positive) $\ln(y_t)$. For the momentum strategies, the commodities are ranked on the basis of their past six-month performance. Analogously, the long (short) leg of the momentum strategy M5 contains an equally weighted portfolio consisting of five commodities with the highest (lowest) past returns. For each of the long and short legs of the strategies, we report the average annualized monthly return and its 95% confidence interval based on a stationary bootstrap (denoted by PW, lower CI and PW, upper CI) with 10,000 bootstrap iterations, in which the block size is based on the algorithm of Politis and White (2004), the annualized monthly standard deviation (SD), the annualized Sharpe ratio (SR), and the monthly skewness. The percentage of months in which the excess return of a strategy is positive is recorded as $1_{er>0}$. There are 501 monthly observations in our sample from January 1970 to September 2011.

	<i>Panel A: Long leg of carry strategy</i>					<i>Panel B: Short leg of carry strategy</i>				
	Commodities long backwardation					Commodities short contango				
	C1	C2	C3	C4	C5	C1	C2	C3	C4	C5
Mean	15.34	10.47	11.38	12.93	14.14	-6.03	-0.40	0.76	1.34	2.20
PW, lower CI	2.76	-0.12	2.88	4.92	6.84	-14.16	-6.36	-6.72	-5.64	-4.20
PW, upper CI	30.72	22.44	22.68	21.84	23.16	1.68	6.24	8.04	7.44	8.28
SD	37.02	28.68	24.01	21.57	20.89	35.16	25.33	21.88	19.55	18.08
SR	0.41	0.37	0.47	0.60	0.68	-0.17	-0.02	0.03	0.07	0.12
Skewness	0.98	0.83	0.67	0.48	0.59	-1.56	-1.23	-1.44	-0.80	-0.48
$1_{er>0}$	50.91	52.12	54.95	57.58	55.35	49.29	51.31	50.10	51.31	52.93

	<i>Panel C: Long leg of momentum strategy</i>					<i>Panel D: Short leg of momentum strategy</i>				
	Commodities long winners based on past six-month performance					Commodities short losers based on past six-month performance				
	M1	M2	M3	M4	M5	M1	M2	M3	M4	M5
Mean	13.00	15.62	11.99	12.39	13.63	-2.25	1.42	1.89	2.31	2.48
PW, lower CI	-0.12	4.80	3.00	3.84	6.00	-11.04	-6.48	-6.00	-4.20	-3.48
PW, upper CI	27.60	28.80	22.44	23.04	22.08	6.48	9.00	9.12	8.64	8.64
SD	44.37	34.13	28.40	25.86	24.12	40.50	30.11	25.97	22.98	20.63
SR	0.29	0.46	0.42	0.48	0.56	-0.06	0.05	0.07	0.10	0.12
Skewness	0.84	0.82	0.45	0.16	0.19	-1.36	-1.39	-0.86	-0.53	-0.34
$1_{er>0}$	54.75	54.55	53.94	56.77	56.97	52.73	51.72	53.94	53.94	53.94

Table Online-I

First notice day conventions adopted by the CME and start and end dates for the commodity futures sample

This table describes the first notice day (denoted by FN) convention for each of the 29 commodities. FN is either specified in the contract specification or can be inferred from the product calendar (see, for instance, Shwayder and James (2011), Gorton, Hayashi, and Rouwenhorst (2013), Hong and Yogo (2012), and Yang (2013)). Our futures data is constructed using end-of-day data provided by the Chicago Mercantile Exchange (CME).

Category	Commodity futures	Start	End	First notice day convention
Agriculture	Barley	1994:07	2003:05	FN is usually the last business day of the month preceding the contract month
	Cocoa	1970:01	2011:09	FN is usually ten business days prior to first business day of contract month
	Coffee	1973:09	2011:09	FN is usually seven business days prior to first business day of contract month
	Corn	1970:01	2011:09	FN is usually the last business day of the month preceding the contract month
	Cotton	1970:01	2011:09	FN is usually around the 15th of the contract month
	Lumber	1972:12	2011:09	FN is usually around the 15th of the contract month
	Oats	1970:02	2011:09	FN is usually the last business day of the month preceding the contract month
	Orange juice	1970:01	2011:09	FN is usually the first business day of contract month
	Rough rice	1986:09	2011:09	FN is usually the last business day of the month preceding the contract month
	Soybeans	1970:01	2011:09	FN is usually the last business day of the month preceding the contract month
	Soybean meal	1970:01	2011:09	FN is usually the last business day of the month preceding the contract month
	Soybean oil	1970:01	2011:09	FN is usually the last business day of the month preceding the contract month
	Sugar	1970:01	2011:09	FN is usually the first business day of the contract month
	Wheat	1970:01	2011:09	FN is usually the last business day of the month preceding the contract month
	Energy	Crude oil	1983:04	2011:09
Heating oil		1979:04	2011:09	FN is usually within the first two business days of the contract month
Natural gas		1990:05	2011:09	FN is usually the last business day of the month preceding the contract month
Propane		1987:09	2006:10	FN is usually within the first two business days of the contract month
RBOB gasoline		2005:11	2011:09	FN is usually within the first two business days of the contract month
Unleaded gasoline		1985:01	2006:12	FN is usually within the first two business days of the contract month
Livestock		Feeder cattle	1973:10	2011:05
	Lean hogs	1970:01	2011:09	No first notice day, last trade day is usually around the 10th of the contract month
	Live cattle	1970:01	2011:09	FN is usually within the first four to ten days of the contract month
	Pork belly	1970:01	2011:07	FN is usually within the first four to ten days of the contract month
Metal	Copper	1970:01	2011:09	FN is usually the last business day of the month preceding the contract month
	Gold	1975:01	2011:09	FN is usually the last business day of the month preceding the contract month
	Palladium	1977:02	2011:09	FN is usually the last business day of the month preceding the contract month
	Platinum	1970:01	2011:09	FN is usually the last business day of the month preceding the contract month
	Silver	1970:01	2011:09	FN is usually the last business day of the month preceding the contract month

Table Online-II

Descriptive statistics of commodity futures excess returns, frequency of contango, and open interest

The monthly excess returns are computed using equation (2), which takes into account the first notice day conventions and also incorporates the interest earned on a fully collateralized futures position. Displayed are the number of observations (N), the annualized mean, standard deviation (SD), Sharpe ratio (SR), monthly skewness, and the first-order autocorrelation (ρ_1). Also reported are (i) the fraction of the months in which a commodity is in contango, denoted by $1_{\ln(y_t) > 0}$, where $y_t \equiv F_t^{(1)}/F_t^{(0)}$, $F_t^{(0)}$ is the price of the front-month futures contract and $F_t^{(1)}$ is the price of the next maturity futures contract; (ii) the mean of $\ln(y_t)$; and (iii) the end of the month open interest, as measured by the number of contracts. Our sample starts in January 1970 and ends in September 2011, and the futures data is constructed using end-of-day data provided by the CME.

Commodity futures	N	Mean	SD	SR	Skewness	ρ_1	$1_{\ln(y_t) > 0}$	$\ln(y_t)$ (Mean)	Open interest (number)
Barley	107	-9.83	11.72	-0.84	0.34	-0.01	0.90	0.015	413
Cocoa	501	5.51	32.21	0.17	0.72	0.00	0.72	0.003	19,587
Coffee	457	7.81	37.46	0.21	1.18	-0.02	0.68	0.004	21,076
Corn	501	-0.76	25.91	-0.03	1.12	0.01	0.83	0.018	146,178
Cotton	501	4.67	25.81	0.18	0.58	0.10	0.69	0.005	21,925
Lumber	466	-6.42	27.87	-0.23	0.09	0.06	0.66	0.020	2,792
Oats	500	0.51	31.88	0.02	2.31	-0.04	0.72	0.018	4,967
Orange juice	501	3.55	31.53	0.11	1.83	-0.04	0.65	0.007	7,886
Rough rice	301	-2.41	28.70	-0.08	1.28	0.12	0.87	0.021	3,740
Soybeans	501	4.93	28.55	0.17	1.34	0.03	0.77	0.006	60,111
Soybean meal	501	8.22	33.12	0.25	2.18	0.06	0.62	0.001	23,806
Soybean oil	501	8.63	32.74	0.26	1.40	-0.04	0.75	0.001	30,674
Sugar	501	9.00	42.02	0.21	1.17	0.17	0.62	0.013	58,129
Wheat	501	0.42	27.10	0.02	0.73	0.07	0.76	0.014	52,989
Crude oil	342	11.62	33.47	0.35	0.42	0.19	0.46	-0.002	140,095
Heating oil	390	16.26	35.88	0.45	1.14	0.05	0.65	-0.005	5,452
Natural gas	257	-4.68	51.02	-0.09	0.60	0.10	0.77	0.018	69,613
Propane	230	29.31	64.88	0.45	7.01	-0.07	0.65	-0.008	142
RBOB gasoline	71	18.58	40.77	0.46	-0.59	0.22	0.46	-0.001	6,649
Unleaded gasoline	264	25.84	40.70	0.63	1.03	0.02	0.42	-0.012	2,044
Feeder cattle	452	2.68	16.28	0.16	-0.53	-0.01	0.45	0.000	4,102
Lean hogs	501	5.41	26.04	0.21	0.04	-0.03	0.53	0.013	17,334
Live cattle	501	5.34	17.51	0.31	-0.26	-0.01	0.49	-0.002	32,505
Pork belly	499	2.01	36.77	0.05	0.55	-0.07	0.45	0.003	3,309
Copper	501	7.91	27.94	0.28	0.33	0.17	0.69	0.000	11,504
Gold	441	2.11	19.57	0.11	0.49	0.02	1.00	0.008	44,645
Palladium	416	11.47	35.62	0.32	0.41	0.04	0.74	0.005	4,403
Platinum	501	6.46	27.84	0.23	0.47	0.00	0.76	0.008	7,703
Silver	501	6.41	34.35	0.19	1.47	0.09	0.98	0.009	19,380

Table Online-III

Open interest and number of commodities across the futures curve

For a given commodity, let $F_t^{(0)}$ be the price of the front-month futures contract, $F_t^{(1)}$ be the price of the next maturity futures contract, and likewise $F_t^{(n)}$ be the price of the final futures contract available for trading, where n can vary across commodities. The futures prices $F_t^{(0)}$ through $F_t^{(n)}$ describe the futures curve for a given commodity. Tabulated below are the number of commodities, the open interest, and the number of observations across different points on the futures curve. For example, we have 29 commodities to construct the carry strategy if we use the first contract, while we have only 17 commodities available to construct the carry strategy if we use the sixth contract.

Slope of the futures curve is based on:

	$\ln\left(\frac{F_t^{(1)}}{F_t^{(0)}}\right)$	$\ln\left(\frac{F_t^{(2)}}{F_t^{(0)}}\right)$	$\ln\left(\frac{F_t^{(3)}}{F_t^{(0)}}\right)$	$\ln\left(\frac{F_t^{(4)}}{F_t^{(0)}}\right)$	$\ln\left(\frac{F_t^{(5)}}{F_t^{(0)}}\right)$	$\ln\left(\frac{F_t^{(6)}}{F_t^{(0)}}\right)$
Number of commodities	29	29	29	27	23	17
Open interest (end of month)	27,904	15,529	9,761	7,045	5,631	5,161
Number of observations	12,207	12,071	11,831	10,659	6,958	4,532

Table Online-IV

Cross-sectional asset pricing results with the average, carry, and momentum factors and expanded set of test portfolios

This table assesses the performance of the three-factor model and its two-factor nested counterparts, when the number of test portfolios is expanded to additionally include five variance portfolios. The variance portfolios are constructed as follows. At the end of each month we compute the monthly sum of absolute daily log returns for each commodity following Menkhoff, Sarno, Schmeling, and Schrimpf (2012, equation (4)). We then sort commodities in five equally weighted portfolios according to their variance and compute the next month returns for each portfolio. The SDF of the three-factor model is of the form: $m_{t+1} = 1 - b_{AVG} AVG_{t+1} - b_{CARRY} CARRY_{t+1} - b_{CMOM} CMOM_{t+1}$, where AVG_{t+1} is the average factor (excess return obtained by holding all commodities available), $CARRY_{t+1}$ is the carry factor, and $CMOM_{t+1}$ is the momentum factor. Reported are also results for the restricted versions of the SDF that impose $b_{CMOM} \equiv 0$ and $b_{CARRY} \equiv 0$, respectively. We report the χ^2 test corresponding to the null hypothesis that the pricing errors are zero, with p -values computed both based on the Newey-West and Shanken standard errors. The Hansen and Jagannathan (1997, equation (29)) distance measure (HJ-Dist.), with the associated p -value, is shown, which tests whether the distance measure is equal to zero.

Portfolios	Three factor model			Omitting momentum			Omitting carry		
	χ^2_{NW}	χ^2_{SH}	HJ-Dist.	χ^2_{NW}	χ^2_{SH}	HJ-Dist.	χ^2_{NW}	χ^2_{SH}	HJ-Dist.
Carry, Momentum, Industry, Variance (17 portfolios)	16.40 (0.29)	15.71 {0.33}	0.007 (0.281)	25.64 (0.04)	24.39 {0.06}	0.008 (0.21)	35.74 (0.00)	34.53 {0.00}	0.013 (0.00)
Carry, Momentum, Variance (14 portfolios)	12.89 (0.30)	12.35 {0.34}	0.007 (0.135)	22.81 (0.03)	21.68 {0.04}	0.008 (0.10)	28.93 (0.00)	27.91 {0.01}	0.012 (0.00)

Table Online-V

Relation of the commodity factors to future real GDP growth, accounting for oil price growth

We report the coefficient estimates from the following predictive regressions with overlapping observations at the quarterly frequency:

$$\ln(\text{GDP}_{t+k}/\text{GDP}_t) = \theta_0 + \theta_{\text{AVG}} \text{AVG}_t + \theta_{\text{CARRY}} \text{CARRY}_t + \theta_{\text{CMOM}} \text{CMOM}_t + \psi_{\text{OIL}} \text{OIL_growth}_t + \varepsilon_{t+k},$$

where $k \in \{1, 2, 3, 4\}$, GDP refers to the real GDP of the G7 countries (source: Datastream, ticker G7OCMP03D), and OIL_growth_{*t*} is the growth rate of West Texas intermediate oil price (Source: Federal Reserve Bank–St. Louis). The *p*-values based on the Newey and West (1987) covariance estimator, with lags automatically selected according to Newey and West (1994), are denoted by NW[*p*]. Additionally, the *p*-values based on the Hodrick (1992) 1B covariance estimator under the null of no predictability are denoted by H[*p*]. Adjusted R^2 is reported as \bar{R}^2 (in %). The sample period is from 1970:Q1 to 2011:Q3.

Horizon		θ_0	Predictive slope coefficients			ψ_{OIL}	\bar{R}^2
			θ_{AVG}	θ_{CARRY}	θ_{CMOM}		
1 quarter	Estimate	0.01	0.03	-0.01	-0.01	0.00	13.3
	NW[<i>p</i>]	(0.00)	(0.01)	(0.07)	(0.01)	(0.86)	
	H[<i>p</i>]	<0.00>	<0.02>	<0.12>	<0.01>	<0.90>	
2 quarters	Estimate	0.01	0.05	-0.01	-0.02	0.00	10.7
	NW[<i>p</i>]	(0.00)	(0.02)	(0.05)	(0.01)	(0.72)	
	H[<i>p</i>]	<0.00>	<0.01>	<0.01>	<0.00>	<0.78>	
3 quarters	Estimate	0.02	0.04	-0.03	-0.02	0.00	8.9
	NW[<i>p</i>]	(0.00)	(0.06)	(0.06)	(0.02)	(0.61)	
	H[<i>p</i>]	<0.00>	<0.02>	<0.00>	<0.00>	<0.64>	
4 quarters	Estimate	0.03	0.04	-0.03	-0.03	-0.01	7.2
	NW[<i>p</i>]	(0.00)	(0.16)	(0.10)	(0.01)	(0.32)	
	H[<i>p</i>]	<0.00>	<0.08>	<0.02>	<0.00>	<0.35>	

Table Online-VI

Correlations between the long and short legs of the carry and momentum strategies

We report the correlation of the commodity factors and the long and short components of carry and momentum strategies. The long (short) leg of the carry trade returns is denoted by $CARRY_t^{\text{long}}$ ($CARRY_t^{\text{short}}$). Similarly, the long (short) leg of the momentum returns is denoted by $CMOM_t^{\text{long}}$ ($CMOM_t^{\text{short}}$). The calculation of the returns of the long and short legs of the strategies follows the procedure described in Appendix A and implemented in Appendix B.

	AVG_t	$CARRY_t$	$CMOM_t$	$CARRY_t^{\text{long}}$	$CARRY_t^{\text{short}}$	$CMOM_t^{\text{long}}$
$CARRY_t^{\text{long}}$	0.71	0.65	0.21			
$CARRY_t^{\text{short}}$	-0.71	0.48	0.10	-0.35		
$CMOM_t^{\text{long}}$	0.74	0.25	0.67	0.62	-0.41	
$CMOM_t^{\text{short}}$	-0.73	0.06	0.49	-0.46	0.61	-0.32

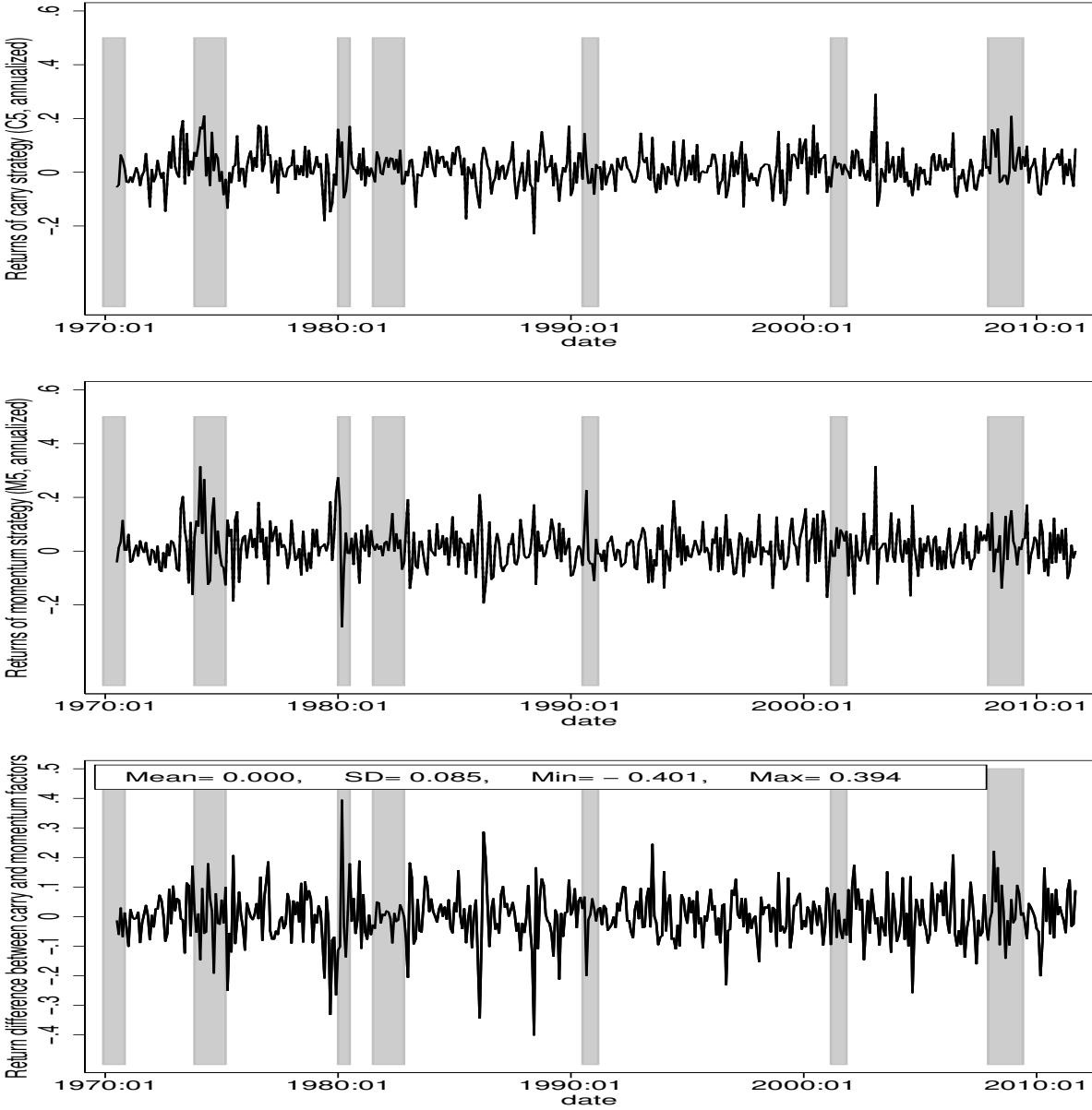


Fig. 1. Returns of commodity carry and momentum strategies

Plotted in the top (middle) panel is the time-series of the excess returns generated by the carry factor $CARRY_t$ (the momentum factor $CMOM_t$). The bottom panel plots the time-series of $er_t^{cm} \equiv CARRY_t - CMOM_t$, for $t = 1, \dots, T$ and reports the mean, standard deviation, minimum and maximum of the er_t^{cm} series. The shaded areas indicate NBER recessions. Let $y_t \equiv F_t^{(1)}/F_t^{(0)}$, whereby, at the end of month t , a commodity is in backwardation if $\ln(y_t) < 0$ and in contango if $\ln(y_t) > 0$. The carry strategy entails taking a long position in the five commodities with the lowest $\ln(y_t)$ and a short position in the five commodities with the highest $\ln(y_t)$. The momentum strategy entails taking a long position in the five commodities with the highest returns over the previous six months and a short position in the five commodities with the lowest returns over the previous six months. Our sample period is January 1970 to September 2011. The monthly returns of the carry and momentum strategies co-move with a correlation of 0.27.

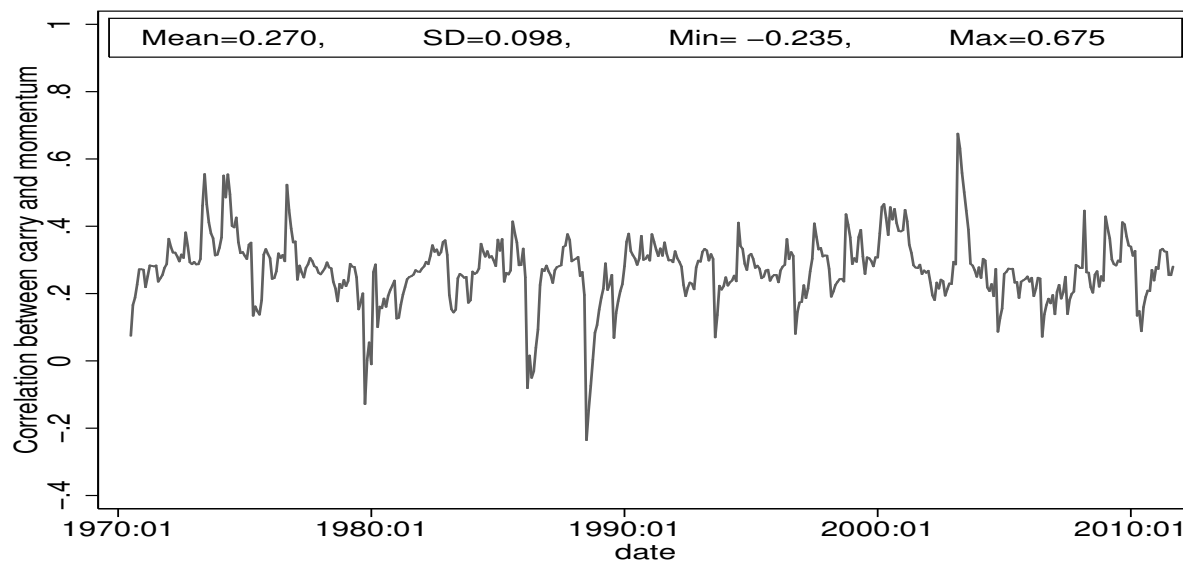


Fig. 2. Dynamic correlation between the carry and momentum factors

The plotted correlation between the carry and momentum factors is based on the dynamic conditional correlation model of Engle (2002). The estimated correlation relies on a bivariate GARCH (1,1) model for carry and momentum factors. We report the mean, standard deviation, minimum, and maximum of the correlation series. The carry factor entails taking a long position in the five commodities with the lowest $\ln(y_t)$ and a short position in the five commodities with the highest $\ln(y_t)$. The momentum factor entails taking a long position in the five commodities with the highest returns over the previous six months and a short position in the five commodities with the lowest returns over the previous six months. Our sample period is January 1970 to September 2011.

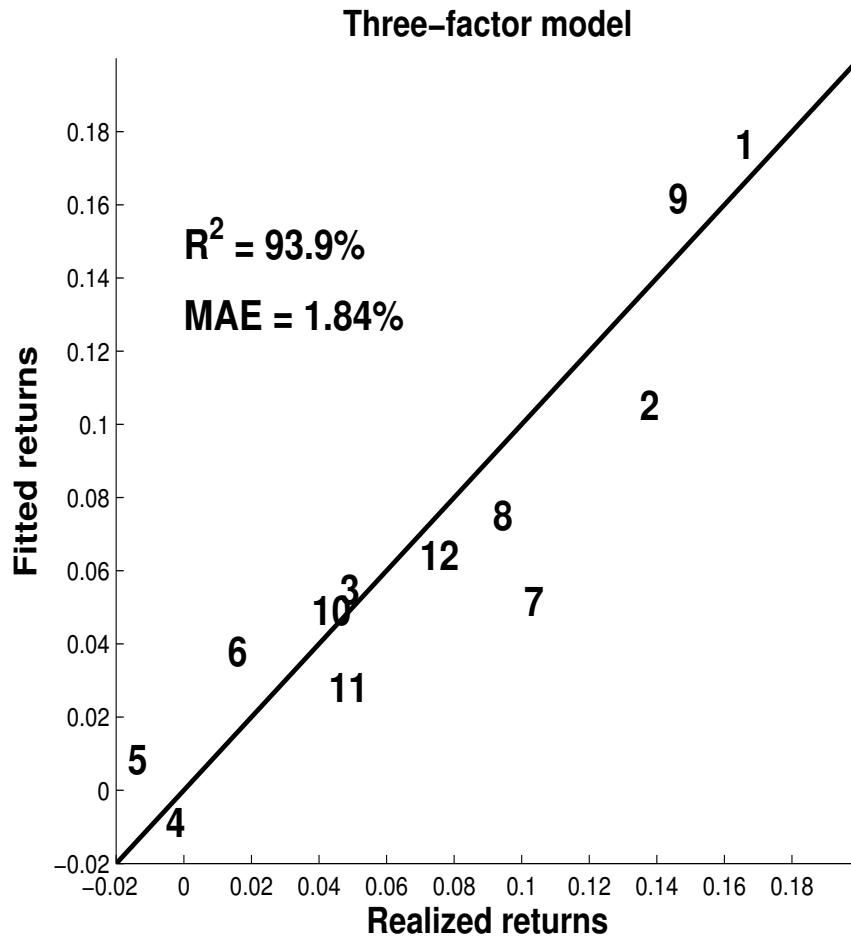


Fig. 3. Realized versus fitted returns across the commodity portfolios

Plotted are the realized returns (x -axis) and the fitted returns (y -axis) corresponding to the commodity portfolios indexed from 1 to 12 (see Table 3). The fitted average returns are based on equation (5). We also display the uncentered R^2 's and the mean absolute errors (denoted by MAE), as goodness-of-fit yardsticks. The MAE is computed as $(1/12) \sum_{i=1}^{12} |\text{Fitted}_i - \text{Realized}_i|$, in monthly percentage units.

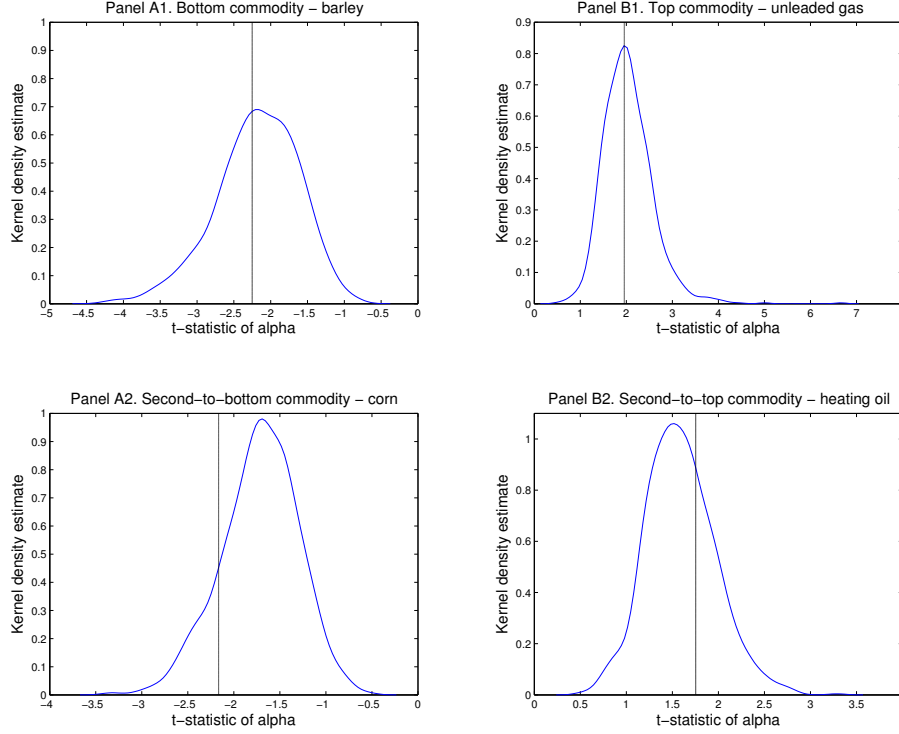


Fig. 4. Estimated alpha t -statistics vs. kernel density estimates of the bootstrapped distributions

Plotted are the estimated alpha t -statistics versus the kernel density estimates of the bootstrapped distribution of the alpha t -statistic, computed following the procedure described in Kosowski, Timmermann, Wermers, and White (2006, Section III.B.1). The bootstrap procedure is as follows:

1. For each commodity $i = 1, \dots, 29$, we estimate the model:

$$er_t^i = \alpha^i + \beta_{AVG}^i AVG_t + \beta_{CARRY}^i CARRY_t + \beta_{CMOM}^i CMOM_t + \varepsilon_t^i,$$

and save $\{\hat{\alpha}^i, \hat{t}_{\hat{\alpha}^i}, \hat{\beta}_{AVG}^i, \hat{\beta}_{CARRY}^i, \hat{\beta}_{CMOM}^i\}$ and $\{\hat{\varepsilon}_t^i, t = t_{i,\min}, \dots, t_{i,\max}\}$.

2. For each commodity i , we generate $b = 1, \dots, 1000$ samples by imposing the null $\alpha^i = 0$:

$$er_t^{i,b} = \hat{\beta}_{AVG}^i AVG_t + \hat{\beta}_{CARRY}^i CARRY_t + \hat{\beta}_{CMOM}^i CMOM_t + \hat{\varepsilon}_t^{i,b}.$$

3. We estimate:

$$er_t^{i,b} = \alpha^{i,b} + \beta_{AVG}^{i,b} AVG_t + \beta_{CARRY}^{i,b} CARRY_t + \beta_{CMOM}^{i,b} CMOM_t + \varepsilon_t^{i,b},$$

and save $\{\hat{t}_{\hat{\alpha}^{i,b}}, i = 1, \dots, 29; b = 1, \dots, 1000\}$.

4. The distribution of alpha t -statistic for the bottom commodity (barley, reported in Panel A1) is constructed as the distribution of the minimum alpha t -statistic generated across all bootstraps. We follow a similar procedure for the other three individual commodities that have t -statistics significant at the 10% level in Table 5, that is, corn (Panel A2), unleaded gas (Panel B1), and heating oil (Panel B2).

The x -axis shows the t -statistic, and the y -axis shows the kernel density. The vertical line represents the t -statistic of alpha estimated using the return time-series.

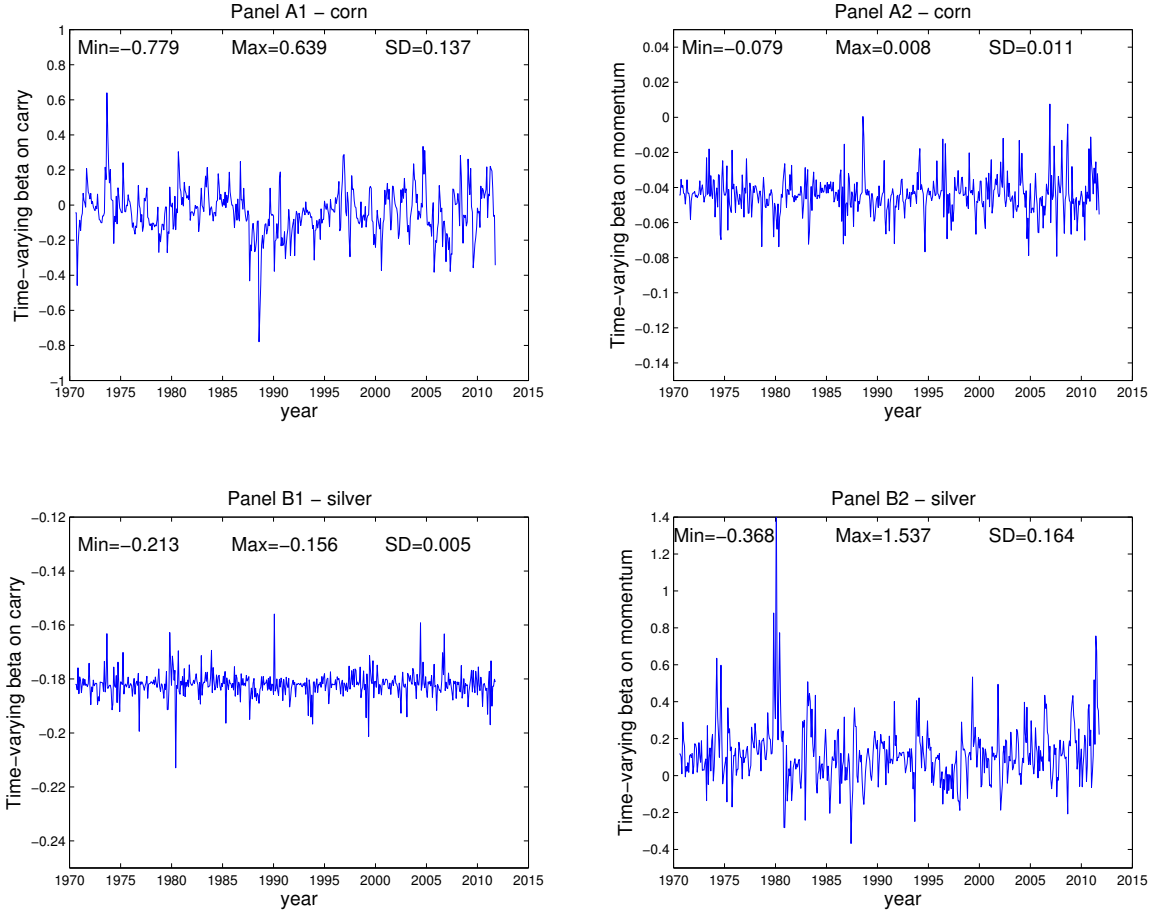


Fig. 5. Time-varying coefficients estimated using the Kalman filter

Plotted are the coefficients $\hat{\beta}_{\text{CARRY}, t|t-1}^i$ and $\hat{\beta}_{\text{CMOM}, t|t-1}^i$ for corn and silver, estimated using the Kalman filter. For each commodity $i = 1, \dots, 29$, the model is specified as follows:

$$er_t^i = \alpha^i + \beta_{\text{AVG}}^i \text{AVG}_t + \beta_{\text{CARRY}, t}^i \text{CARRY}_t + \beta_{\text{CMOM}, t}^i \text{CMOM}_t + \varepsilon_t^i \quad \varepsilon_t^i \sim \mathcal{N}(0, R^i),$$

where

$$\begin{aligned} \beta_{\text{CARRY}, t}^i &= \mu_1^i + F_1^i \beta_{\text{CARRY}, t-1}^i + v_{1,t}^i & v_{1,t}^i &\sim \mathcal{N}(0, Q_1^i), \\ \beta_{\text{CMOM}, t}^i &= \mu_2^i + F_2^i \beta_{\text{CMOM}, t-1}^i + v_{2,t}^i & v_{2,t}^i &\sim \mathcal{N}(0, Q_2^i). \end{aligned}$$

The model is cast in state-space form and estimated via maximum likelihood, following Kim and Nelson (1999, chapter 3). The following percentiles of the standard deviation distributions of $\hat{\beta}_{\text{CARRY}, t|t-1}^i$ and $\hat{\beta}_{\text{CMOM}, t|t-1}^i$ across the 29 commodities are illustrative of the time-varying nature of the coefficients.

Percentile	10th	25th	50th	75th	90th
$\hat{\beta}_{\text{CARRY}, t t-1}^i$	0.002	0.004	0.057	0.137	0.253
$\hat{\beta}_{\text{CMOM}, t t-1}^i$	0.002	0.010	0.072	0.130	0.277
All	0.002	0.007	0.057	0.137	0.278

The standard deviations are expressed in monthly units.

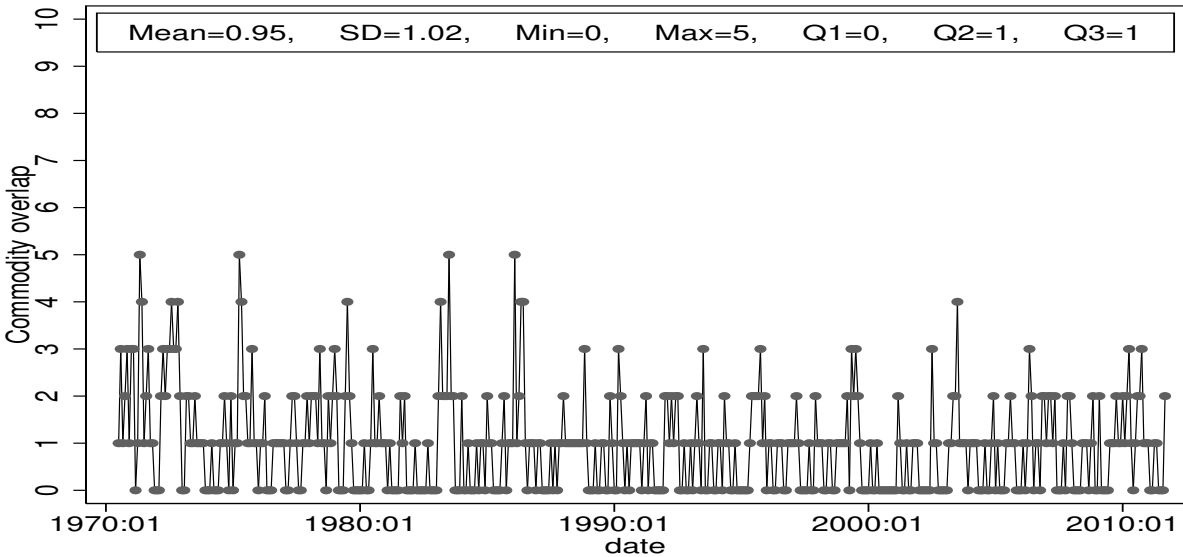


Fig. 6. Overlap in the commodities selected by the carry and momentum factors

Plotted is the time-series of the number of commodities selected by both the carry and the momentum factors. We compute this overlap in two steps. First, we identify the commodities in the long legs of the carry and momentum strategies each month and compute the overlap (the maximum overlap is five). Second, we compute the same quantity for the short legs of the strategies (the maximum overlap is five). The plotted overlap is the sum of the overlaps of the long and short legs of the strategies. We report the mean, standard deviation, minimum, maximum, as well as the three quartiles (Q1, Q2, Q3) of the overlap distribution. The carry factor entails taking a long position in the five commodities with the lowest $\ln(y_t)$ and a short position in the five commodities with the highest $\ln(y_t)$. The momentum factor entails taking a long position in the five commodities with the highest returns over the previous six months and a short position in the five commodities with the lowest returns over the previous six months. Our sample period is January 1970 to September 2011.

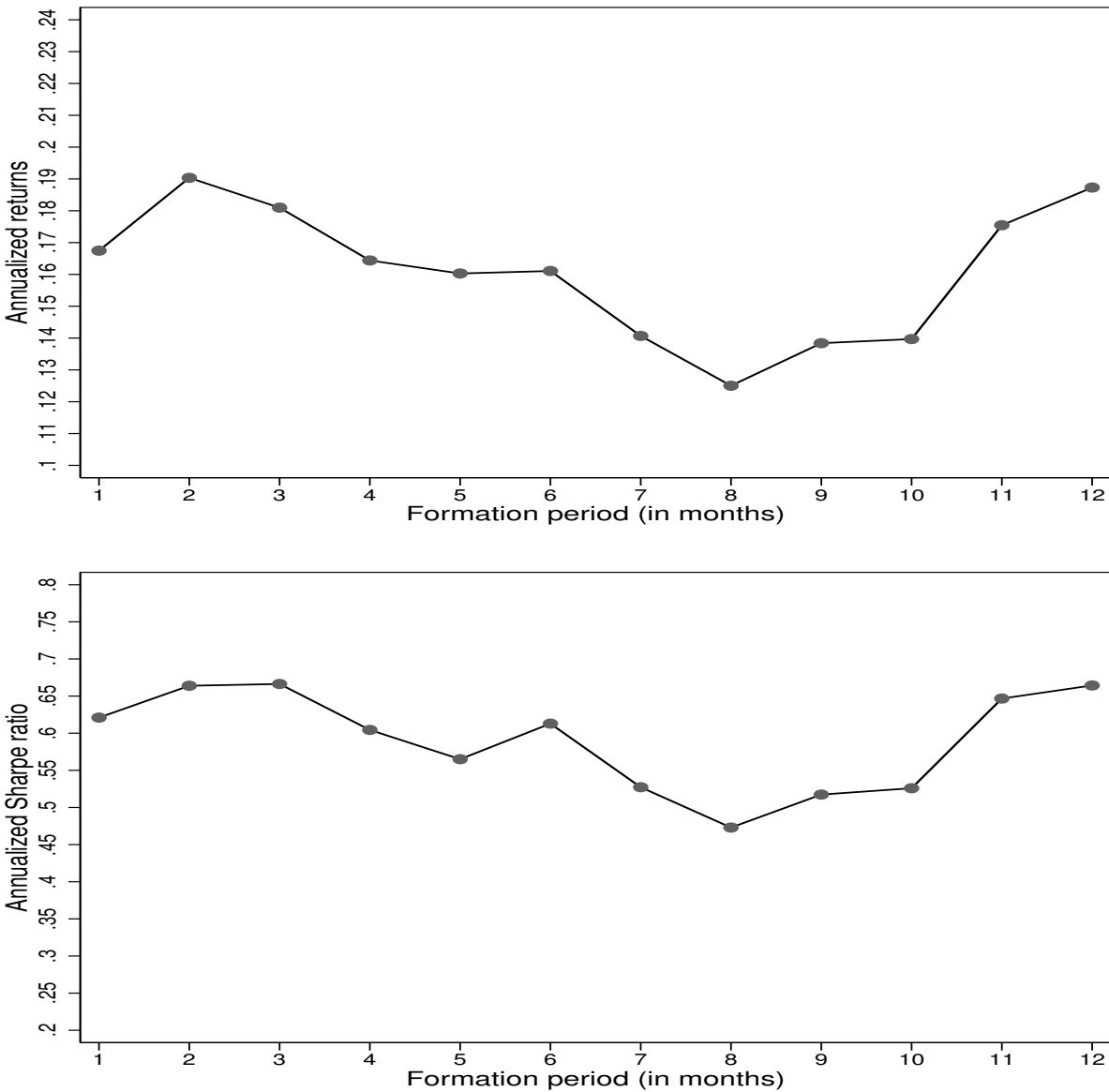


Fig. 7. Average returns and Sharpe ratios of momentum strategies across formation periods

Plotted are the annualized average monthly returns (top panel) and Sharpe ratios (bottom panel) of momentum strategies with formation periods J that range from 1 to 12 months. Specifically, we report the results for strategy M5 (see also the caption to Table 1), which entails taking a long position in the five commodities with the highest returns over the previous J months and a short position in the five commodities with the lowest returns over the previous J months. We measure past returns using the geometric average (equation (A3)). Our sample period is January 1970 to September 2011.