RISK VERSUS AMBIGUITY AND INTERNATIONAL SECURITY DESIGN*

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Abstract

We study portfolio allocation and characterize contracts issued by firms in the international financial market when investors exhibit ambiguity aversion and perceive ambiguity in assets issued in foreign locations. Increases in the variance of their risky production process cause firms to issue assets with a higher variable payment (equity). Hikes in investors' perceived ambiguity have the opposite effect, and lead to less risk-sharing. Entrepreneurs from capital-scarce countries finance themselves relatively more through debt than equity. They are thus exposed to higher volatility per unit of consumption. Such results do not hold when investors exhibit standard risk-averse preferences.

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1 Introduction

Ambiguity aversion has recently been highlighted as an underlying factor in decision making processes, with concrete consequences for portfolio theory. For example, if investors are ambiguity averse, and have difficulty assessing the probabilities of various events affecting firms in foreign economies that issue risky securities (eg. they may be unsure about what is the proper distribution from which production shocks are drawn), they may not invest as eagerly abroad as in their domestic economy and it is well known that this may give rise to a home bias.[1] In this paper, we study the consequences of such ambiguity aversion for international asset structure, risk sharing, investment and asset holdings. We shall argue that the introduction of ambiguity aversion sheds light on a variety of important, and to a certain extent puzzling, regularities found in the data.

For example, despite the fact that developing countries' GDPs are perceived as being more volatile, they cannot obtain the high degree of risk-sharing – insurance from different shocks provided by well-diversified investors – which would be predicted by a standard model of portfolio choice with risk (see below). In fact, capital flows seem to be rather procyclical to emerging markets (see for example Kaminsky et al. (2004)). Such countries (and firms located there) often issue foreign-denominated fixed-rate bonds (especially of short maturity) even if they have what Reinhart et al. (2003) call "debt intolerance", i.e. run into sovereign debt crises under debt burdens significantly lower than developed countries. Firms with foreign-denominated debt face ballooning liabilities in the case of adverse exchange rate movements (such high exposure has been claimed to be a culprit for the severity of many financial crises). There is a long-lasting concern that too much international financing relies on fixed-rate debt (for example, Lessard and Williamson (1985), Rogoff (1999), Borensztein et al. (2004) or Reinhart and Rogoff (2009)) but proposals to introduce output-linked securities such as Schiller (1993) have not been successfully implemented so far. The available data on the issue, in Figure 1, shows that the share of debt in all public and private external liabilities of capital importers (countries with negative net foreign assets) is markedly higher than that of capital exporters in various subsamples of the Lane and Milesi-Ferretti (2007)

[1]For theoretical studies see for example Uppal and Wang (2003) or Benigno and Nistico (2012). Empirically, for example Gelos and Wei (2005) show that funds invest less in less transparent countries.

data for the most recent years characterized by the freest capital flows.[2] This is not necessarily driven by the "emerging" countries in the sample as the same patterns are observed for OECD countries. Using a different measure for a more homogenous group of Eurozone countries, for the 5 year period 2002-2006 (when the foreign exchange risk between these countries was small), capital importers had a debt/equity liabilities ratio on average of 2.95 (a median of 2.80) whereas among the exporters this average was at 1.56 (a median of 1.58).[3]

We address these issues in a simple setup: firms (entrepreneurs) have access to a risky production technology while investors own productive capital that they would like to invest. Entrepreneurs issue contracts in a monopolistically competitive market that promise a share in the risky outcome and a fixed payout (bond payment). Investors make their portfolio choices after observing the contract terms. Both the firms and the investors are risk averse, and maximize their utility of consumption (of the residual of the payout to investors) and the net proceeds from capital respectively; however, investors also perceive ambiguity in foreign-issued assets, to which they are averse. The problem is static with no default. We analyze the problem under different assumptions on investor ambiguity aversion and the distribution of worldwide capital ownership.

A crucial assumption concerns the difference in the ambiguity perception of investors when considering foreign assets. More precisely, we assume that both home and foreign private investors perceive the same (reduced) distribution of productivity shocks for the asset offered by an entrepreneur, but that foreign investors perceive the asset as ambiguous whereas home investors do not. All investors are assumed to have the same attitude to ambiguity. On the operational side, we use the smooth ambiguity model proposed by Klibanoff et al. (2005), in a specification provided by Gollier (2011). As shown by Maccheroni et al. (2013), this specification is an exact case of the equivalent of the Arrow-Pratt approximation for the case of ambiguity, and can be thought of as a generalization of mean-variance preferences to cover ambiguity. These ingredients allow us to have a tractable model that can be solved and analyzed analytically.

[2]Debt and equity liabilities of all respective countries were summed up as in Lane and Milesi-Ferretti (2007) to obtain these figures.

[3]Capital importers: Austria, Finland, Greece, Italy, Portugal and Spain. Exporters: Belgium, Ireland, France, Germany, Luxembourg and the Netherlands.

1.1 Results

In our model, each entrepreneur has a monopoly on her production process, but faces the following tradeoffs while issuing securities. The higher the risk sharing (the share in the proceeds of the production process offered to outside investors), the lower the risk that the firm carries, but also the lower the potential gains.

Our first main finding is that, whilst the model behaves like the standard "risk-only" model insofar as risk is concerned, the consequences of ambiguity are different, and on certain parameters even diametrically opposite. Just as in standard models of portfolio choice with risk (for example the workhorse CARA-Normal model), increases in the riskiness of the production process cause entrepreneurs to offer contracts with a higher risky share and a lower fixed payout – they effectively seek insurance from investors that are better able to diversify risk (by investing in many non-correlated assets). By contrast, increases in the perceived ambiguity or in the investors' ambiguity aversion cause firms to lower the variable part, therefore insuring (or rather assuring) investors. The effect on the fixed payout can be both positive and negative, depending on other factors.

The second salient property of the model is that the contract an entrepreneur issues depends on the total wealth of his home investors. In countries with low levels of investor wealth (relative to others) firms issue contracts with higher fixed payouts and lower variable parts. Therefore, ceteris paribus, entrepreneurs in capital-scarce countries obtain less risk-sharing than their counterparts in capital-abundant countries. Since, as we show, capital-scarce countries are net capital importers, it follows that firms in capital importing countries obtain less risk-sharing. Again, this is in stark contrast to the standard risk-only model, where contract terms are independent of the country of residence.

Thirdly, the model predicts that, when ambiguity aversion of investors is not too high, firms in countries with relatively lower domestic-based wealth issue assets with lower overall expected returns. Moreover, the firms in these countries attract less capital. Finally, whilst increases in risk alleviate the discrepancy in capital attracted by firms in capital-scarce versus capital-abundant countries, increases in ambiguity exacerbate it.

We argue that these properties of the model suggest that the introduction of ambiguity can provide a unified explanation of some of the phenomena mentioned above. First of all, if we interpret the fixed payouts as bonds and the variable parts as equity, the model can reproduce the patterns observed in Figure 1: the second property mentioned above implies that agents in net capital-importing countries would on average issue relatively more debt.

Secondly, investor ambiguity aversion can also solve the puzzle of why the observed degree of international risk-sharing may be lower than what standard models predict (for a recent study see Bengui et al. (2013)). Firms in countries with a higher volatility of GDP may prefer to issue fixed-rate securities rather than obtain risk-sharing schemes if a) investors are ambiguity averse to foreign-issued assets and b) their domestic markets are relatively capital-scarce.

Thirdly, since an entrepreneur's consumption is more volatile per unit of consumption as the fixed payouts promised to investors is higher, the model offers a story about why developing countries (i.e., associated with low investor wealth) may have more volatile consumption streams.

Finally, given the third prediction of the model, the marginal effective product of capital can be lower in countries with fewer domestic investors, and hence a lower installed capital stock; and this holds even when the marginal return on *physical* capital is high. So, in a world with ambiguity and ambiguity aversion, we can have equalized marginal returns of capital as measured by Caselli and Feyrer (2007) but there would be output gains from reallocating capital. Financial market imperfections may stem from ambiguity aversion and prevent capital flows from "rich" to "poor" countries. Indeed, the model suggests that, unlike increases in risk, sudden increases in ambiguity can generate "capital flight", and increase the home bias (third prediction above).

The model studied involves solely ambiguity perception and aversion, and does not require or assume that foreign investors possess less information than locals. It differs thus from stories based on the assumption that the characteristics of the productivity-generating processes of a country's firms are relatively less known in the outside world (see Appendix A.4 for an extension of our model). More generally, it can be thought of as providing a different, complementary and in a sense unified account of these phenomena, many of which have been tackled on the basis of different underlying mechanisms (eg. lack of proper contract enforcement affecting the marginal product of capital proposed by Matsuyama (2004) or costly state verification as in Boyd and

Smith (1997)). As a side gain, the model can deliver tractable expressions for the home bias or consumption correlations between agents in different countries and match their values obtained in the empirical literature for reasonable parameters.

1.2 Related literature

Our focus is different from the existing research on portfolio selection under ambiguity in that we are interested in the asset structure (which is typically assumed as exogenous), capital allocation among countries with unequal wealth and the risk-sharing and consumption volatility implied by the contracts.[4] Moreover, unlike much of this literature, the decision model used here is the smooth ambiguity model proposed by Klibanoff et al. (2005). It has the advantage that the role of ambiguity – or "model uncertainty" – and attitude to ambiguity – for example, ambiguity aversion – in portfolio decisions can purportedly be neatly decomposed.[5]

The question of the composition of the contracts offered by emerging countries has been a subject of concern for a long time in international economics as witnessed by the policy debates (for example, Lessard and Williamson (1985), Rogoff (1999), Borensztein et al. (2004) or Reinhart and Rogoff (2009)). Empirical investigations of the composition of contracts include for example Claessens et al. (2003), Burger and Warnock (2006) or Kose et al. (2007). There are several papers in the international economics literature such as Alfaro and Kanczuk (2009, 2010) studying the

[4]See Dow and da Costa Werlang (1992), Garlappi et al. (2007) and Boyle et al. (2012) that use the multiple priors approach of Gilboa and Schmeidler (1989) or Uppal and Wang (2003) with intertemporal portfolio choice when investors take into account model misspecification, in the style of Hansen and Sargent (2001, 2007). Benigno and Nistico (2012) use the latter approach to study the home bias puzzle under the assumption that investors have different beliefs about the characteristics of foreign and domestic assets. A general conclusion of this research is that "high" ambiguity leads to portfolio underdiversification. In addition, there is a sizable literature on asset pricing involving ambiguity models. Examples include Epstein and Wang (1994), Cao et al. (2011). Solnik and Zuo (2012) study asset pricing using regret preferences.

[5]For a presentation of the literature on decision under uncertainty and ambiguity with applications to asset markets, see Epstein and Schneider (2010). Two important papers studying portfolio allocation under the smooth ambiguity model are Gollier (2011) and Maccheroni et al. (2013). Both papers identify a solvable specification of the model, which can be thought of as the "equivalent" of the CARA model for risk in the case of ambiguity. Moreover, the latter paper gives a quadratic approximation for the smooth ambiguity model, in the style of the Arrow-Pratt approximation, and uses this to propose a "robust" version of mean-variance preferences, incorporating ambiguity. The aforementioned specification corresponds to the exact case of that approximation (and hence to their robust mean-variance preferences).

macroeconomic tradeoffs of issuing various type of (fixed versus floating, short vs. long term) debt or GDP-linked securities in calibration exercises after imposing a financial structure. Caballero and Krishnamurthy (2003) studied why firms in emerging markets might prefer foreign currency- rather than local currency-denominated debt financing. (For the case of firms, the standard moral hazard argument against governments issuing locally-denominated contracts does not apply.) Broner et al. (2007) and Jeanne (2009) investigated the maturity composition of debt issues. The asset issuance patterns discussed in these papers could be potentially also explained by investor ambiguity aversion either towards foreign-currency denominated bonds or long-term bonds. More generally, in a closed economy, if there is ambiguity aversion, then fixed-rate bonds would seem to be the best solution to insure uncertain investments. Mukerji and Tallon (2004) study why agents would prefer non-indexed bonds even if they are ambiguity averse. They claim that with inflation-indexed bonds, investors are still not covered from the relative price risk in the bundles that they consume. An emerging literature studies the implications of the importance of demand for safe assets based on special models that incorporate ambiguity averse agents ("locally Knightian agents" in Caballero and Farhi (2014)) that warrants different institutional responses as in Gennaioli et al. (2013).

The fact that there is limited international risk sharing between countries has attracted significant attention. Kose et al. (2007) examine empirically the patterns and possible causes. They find that industrial countries do attain a higher degree of risk sharing than developing countries. Emerging countries that had large cross-border capital flows have seen little change in their ability to share risk. The authors claim that this was due to the prevalence of portfolio debt financing. Devereux and Sutherland (2009) note that in recent years more risk sharing between developed and emerging markets has occurred and investigate this in a DSGE model with a fixed asset structure, deriving country portfolios at the steady state. Bengui et al. (2013) investigate how well a standard business cycle model with different frictions can explain both the (low) extent of risk-sharing (in the short run) that is observed in the data and the increase in international cross-holdings of assets since the opening of international markets. Both the canonical model with complete markets and perfect capital mobility and the one with the most severe form of incomplete asset markets (only bonds traded) fail to emulate the low degree of observed risk sharing. A model with portfolio rigidities (modeled as a convex cost of changing the stock of foreign assets) improves greatly the match with the data. This points to the direction taken in this paper – that the extent of risk sharing may be affected by portfolio rigidities. Atkeson (1991) constructs a model with moral hazard and enforcement in lending and shows that countercyclical flows (and hence a lack of risk sharing) may be an outcome of an optimal insurance contract in the presence of the aforementioned frictions. Risk sharing in Mendoza et al. (2009) can be affected also by different levels of financial development (modeled as depending on the extent of an enforcement problem). In particular, a country with a well-developed financial system would have a positive net position in equity while a negative position in bonds. Aguiar and Gopinath (2007) argue that shocks in emerging markets are perceived to be permanent to the trend of growth while they are transitory in developed countries. This helps to explain countercyclical current-account behavior, consumption volatility that exceeds income volatility and sudden stops in emerging markets in an otherwise standard business cycle model of a small open economy with one-period risk-free bonds. In this paper we point to a complementary explanation for some of these phenomena.

A related strand is the traditional home bias literature with informational frictions,[6] concentrated primarily on the equity home bias.[7]. Gehrig (1993) first incorporates an assumption that investors from different countries have different information sets. He assumes that foreign securities are considered as more risky by expected utility maximizers and derives a home bias in equities. This line of modeling is pursued also by Brennan and Cao (1997) and Kang and Stulz (1997). Our framework can also encompass informational differences, but these are not necessary to obtain our results (see Section 2.1). Nieuwerburgh and Veldkamp (2009) discuss a model where potential

[6]The empirical evidence in favor of informational frictions is abundant. Gelos and Wei (2005) show that, in times of crisis, funds invest less in and exit faster from countries that are less transparent. In Andrade and Chhaochharia (2010) past U.S. FDI positions in a country are found to be positively correlated with larger stock portfolio engagements at a later period. Bae et al. (2008) find that local analysts know more about local stocks than foreigners. Mondria et al. (2010) find using web browsing data that investors prefer assets from familiar countries. With an exogenous increase in information about a country, investors increase the asset holdings there. These results can be interpreted as resulting from differing ambiguity perception with respect to foreign or unfamiliar countries, rather than simple differences of information.

[7]Few studies of the home bias of equities include bonds. An exception is Coeurdacier et al. (2010) and Coeurdacier and Gourinchas (2011) where local-currency bonds are included and are reasonable magnitudes of the home bias in a standard business cycle model are obtained. A considerable home bias in bond holdings was documented by Coeurdacier and Rey (2012).

investors can learn additional statistical properties of various assets at home and abroad by incurring some costs. Investors that have better information about locally-issued assets prefer to learn more about their home assets because they profit more from information that the others don't know. Learning amplifies the initial information asymmetry. The asset structure in their model is exogenous. By contrast, as explained in detail in Section 2.1, under the leading interpretation of our model all investors have the same information about all assets; however, foreign investors may perceive more ambiguity than home investors towards a given asset. That is, foreign investors may be less sure about the stochastic properties of an asset and are averse to this "model uncertainty" or ambiguity. The distinction is important when one tries to understand periods of financial crises. One does not need increases in risk, in ambiguity aversion, transaction or information costs to generate "capital flight to safety" – the tendency of capital to move to low-risk capital markets during such events – a general increase in ambiguity (or "model uncertainty") about the stochastic properties of economic fundamentals suffices.

1.3 Organization of the paper

In Section 2 we lay down the assumptions and develop the investors' and firms' problems. In Section 3.1, we provide the general solution to our model. Next, in Section 3.2, we analyze the "standard model with risk" – a benchmark case where investors are not ambiguity averse. We discuss the cases when countries are symmetric in wealth in Section 3.3 and when they aren't in Section 3.4. Section 4 provides a discussion while Section 5 concludes. Proofs are in the Appendix with derivations available online.[8]

2 The model

There are two countries[9], dubbed country 1 and country 2, M entrepreneurs, or firms, of which M_c reside in country c. There is a measure N of investors, each of which resides in a single country (with N_c investors residing in country c), and each with the

[8] https://studies2.hec.fr/jahia/webdav/site/hec/shared/sites/hill/acces_anonyme/Articles/Hill_Michalski_210214_supplementary_material.pdf.

[9] The model developed below can be extended to a multicountry case without gaining significant insight on the qualitative results of interest here.

same productive capital or wealth $w > 0$.

Each entrepreneur residing in a particular country has access to a technology that is governed by an i.i.d. stochastic process; for an entrepreneur n, the risky project which he can run has stochastic return x_n. A typical entrepreneur issues a contract containing two elements. The first, v_n, describes the share of the proceeds (or participation in losses) from his risky project. The second is a riskless return (or payment demanded) of R_n. One interpretation of this capital structure (given the stochastic processes considered) is as a reflection of the standard distinction between debt (the R_n factor) and equity (the v_n part).[10] Like some other international portfolio choice models, we abstract from exchange rate risk, as it has been found that exchange rate volatility is relatively small in comparison to the volatility in equities and it can be potentially hedged away (see, for example, Solnik and Zuo (2012)). Hence all investors, irrespective of their country of residence, perceive the contract terms in the same way.

There is one period within which the timing of events is as follows. First, each entrepreneur communicates to investors the contract terms of hiring capital that they will irrevocably honor. They do this noncooperatively, and they compete monopolistically for funds. Investors observe these contract terms and make their portfolio decisions. Then capital is transferred, productivity draws are realized, output produced, and payouts and consumption take place.

2.1 Investor's problem

We begin by describing the investor's problem. An investor l will allocate a fraction of wealth $0 \leqslant \alpha_{ln} \leqslant 1$ into assets issued by firm n, with $\sum_n \alpha_{ln} \leqslant 1$.[11] Any capital uninvested in one of the firms yields zero return; since the entrepreneurs provide the only investment opportunities in the market, this is capital that the investor lets sit idle.[12] Given that the most interesting situations are where there is scarcity of and

[10] Modeling default or limited liability does not allow us to obtain tractable closed form solutions. Note that there is an alternative interpretation of the setup where the firm offers a contract with a riskless bond and a risky output-linked security.

[11] Therefore, we rule out any short selling. This restriction is, however, nonbinding in the equilibria we consider here.

[12] An alternative interpretation involves the unlimited supply of a risk-free bond to which the investor has access and in which he invests his remaining wealth whenever $\sum_n \alpha_{ln} < 1$; given the use of CARA utility functions, the analysis undertaken immediately extends to this case (using an appropriate normalisation), yielding similar results.

hence competition for capital, we focus on these cases, and assume that investors invest all of their wealth in the firms.[13] The value of an investor's portfolio is $w \sum \alpha_{ln}(v_n x_n + R_n)$.

Investors often feel surer in their judgements about assets from their own country rather than foreign assets. This intuition can be translated by the fact that investors perceive more *ambiguity* with respect to events concerning foreign assets (such as the realisation of the stochastic return x_n) than events concerning domestic firms. If investors have a non-neutral attitude to ambiguity, then the difference in ambiguity may have effects on investment behavior.

To capture this we adopt the smooth ambiguity model proposed by Klibanoff et al. (2005). Rather than assuming that agents have a single ("known") probability distribution P for the returns of an uncertain asset x, the model allows uncertainty about the "true" distribution governing returns, which is represented by a (second-order) probability distribution over the possible distributions. Letting π denote this second-order distribution, decision makers choose assets x to maximise:

$$V(x) = \int_{\Delta} \varphi\left(\mathbb{E}_P(u(x))\right) d\pi = \mathbb{E}_{\pi}\varphi(\mathbb{E}_P(u(x))) \tag{1}$$

where u is a standard von Neumann-Morgenstern utility function, φ is a strictly increasing real-valued function, and Δ is the space of probability distributions over values of x. As standard, u represents the decision maker's risk attitude; by contrast, φ represents the decision maker's *ambiguity attitude* (in the sense of (Klibanoff et al., 2005, §3)). Concave φ corresponds to *ambiguity aversion.*

This decision model has a natural interpretation in terms of model uncertainty (see eg. Klibanoff et al. (2005) and Hansen (2007)). The set Δ can represent the set of possible parameter estimates for a particular model of the stochastic process determining asset returns. The decision maker may be unsure as to which of the parameter values is correct: there may be a set all of which are plausible given the data. This *model uncertainty* is represented by the second-order distribution π. The functional form, with a concave φ, can be thought of as one way of incorporating considerations of robustness of one's choice across the possible parameter values.

[13] Although some of the subsequent discussion relies on this assumption, most of the main properties of the model do not; all Propositions, for example, continue to hold when investors from both countries invest in a zero-yielding risk-free asset.

We assume that an investor considers that there are several possible distributions for the stochastic return x_n ran by a firm n in a foreign country. For simplicity, we assume that he is sure that the returns follow a normal distribution. He is sure about the variance (volatility) of the return, but not about the expected return.[14] So he considers plausible only distributions $\tilde{x_n} \sim N(m, \sigma_n^2)$ for some fixed σ_n^2, and some set of possible means m. His second-order prior over this set of distributions is itself normally distributed, with mean μ_n and variance τ_n^2: $\tilde{m}_n \sim N(\mu_n, \tau^2)$. Following the standard terminology, we shall call σ_n^2 (the variance of the underlying stochastic process) the *risk*. In this specification, the "extent" of the (model) uncertainty about the parameters of the stochastic process is summarized by τ_n^2, which we call the *ambiguity*. When $\tau^2 > 0$, we shall say that there is ambiguity. Note that a Bayesian decision maker, on receiving subjective information corresponding to the two-stage distribution just described would collapse or *reduce* the second-order distribution to a single distribution over returns, obtaining in this case a normal distribution for x_n with mean μ_n and variance $\sigma_n^2 + \tau_n^2$. Such an agent acts as if there is no model uncertainty or ambiguity; all uncertainty has been "bundled together" and is treated as standard stochastic uncertainty, or risk. We assume that investors proceed precisely in this way when considering firms in their home country: they use a single prior for the stochastic return x_n that is normally distributed with mean μ_n and variance $\sigma_n^2 + \tau_n^2$.[15] To the extent that home and foreign investors have the same "reduced" distribution for the stochastic return x_n, this representation is consistent with them having the same data, and hence contrasts with analyses in terms of informational asymmetries between home and foreign investors (as in Gehrig (1993)). The difference between foreign and home investors lies in the fact that the former perceive ambiguity or model uncertainty in x_n (on the basis of the data), whilst the latter do not.[16]

To obtain a tractable model we use the specification of Gollier (2011). Each investor has a constant absolute risk aversion utility function of the form $u(z) = -(1/\theta)e^{-\theta z}$

[14] As Boyle et al. (2012, footnote 8) observe, this is justified by the fact that estimation of expected returns is much more difficult than estimation of second moments; see Merton (1980).

[15] Strictly speaking, given the use of the smooth model, they use a degenerate second-order distribution that puts all probability weight on the distribution $N(\mu_n, \sigma_n^2 + \tau_n^2)$.

[16] Refined versions of the setup in which investors also perceive ambiguity towards domestic assets may be developed; as long as the ambiguity perceived in domestically-issued assets is lower than towards the foreign-issued ones, there is no significant change in the qualitative results. It is also straightforward to extend the general setup to allow for information asymmetries, by allowing different ambiguity and risk parameters for home and foreign investors.

where $\theta > 0$ represents the degree of (absolute) risk aversion. Each investor is ambiguity averse and has constant relative ambiguity aversion; the transformation function is thus of the form $\varphi(U) = -\frac{(-U)^{1+\gamma}}{1+\gamma}$, where $\gamma \geqslant 0$ represents the (degree of) *ambiguity aversion*. When $\gamma > 0$, we shall say that there is ambiguity aversion. We assume no asymmetries in the risk and ambiguity attitudes among investors: risk aversion and ambiguity aversion are the same for all investors.

Hence, for an investor l living in country 1, his optimisation problem is to find:

$$\underset{1 \geqslant \alpha_{ln} \geqslant 0,\ \sum_n \alpha_{ln} = 1}{\arg\max} V_{l1}(\alpha_l) \tag{2}$$

where V_{l1} is defined in equation (1) with u, φ and π as specified above. (Recall that α_{ln} denotes the portfolio allocation of investor l in firm n.) Using a standard technique (see e.g. Gollier (2001)), one obtains the following expression for the expected utility of portfolio α_l for investor l under the distribution P with $x_n \sim N(m_n, \sigma_n^2)$ for all n.

$$\mathbb{E}_P(u(\alpha_l) = -(1/\theta)e^{-\theta w \sum_n [\alpha_{ln}(v_n m_n + R_n) - (\theta w \sigma_n^2/2) v_n^2 \alpha_{ln}^2]} \tag{3}$$

This is the inside term in equation (1); plugging it into that equation under the specification set out above yields:

$$V_{l1}(\alpha_l) = -\frac{1}{\theta^{1+\gamma}(1+\gamma)} e^{-\theta w(1+\gamma)\left(\begin{array}{l} \sum_{i=1}^{M_1} \left(\alpha_{li}\left(\mu_i v_i + R_i\right) - \frac{(\sigma_j^2 + \tau_j^2)\theta w}{2} \left[\alpha_{li} v_i\right]^2 \right) \\ + \sum_{j=M_1+1}^{M} \left(\alpha_{lj}\left(\mu_j v_j + R_j\right) - \frac{(\sigma_j^2 + \tau_j^2(1+\gamma))\theta w}{2} \left[\alpha_{lj} v_j\right]^2 \right) \end{array}\right)} \tag{4}$$

as the evaluation of α_l by an investor l resident in country 1.[17] (There is a similar expression for residents of country 2.) Hence the investor's problem is to maximise this function, under under the constraints that $1 \geqslant \alpha_{ln} \geqslant 0$ for all n and $\sum_n \alpha_{ln} = 1$.

2.2 The entrepreneur's problem

Now consider the entrepreneur's problem. Each entrepreneur must choose the two contract terms v_n and R_n. Once a firm n issues contracts and investors make their

[17] The first M_1 entrepreneurs are in country 1 and the remaining M_2 are in country 2.

portfolio choices, it obtains capital k_n, which it invests in a risky production process. The proceeds can be described by $y_n = x_n f(k_n)$ where $f(k_n) = k_n$ and x_n is stochastic productivity that is unknown prior to investment. We assume that the firm has the same information as investors and, like home investors, perceives no ambiguity with respect to its stochastic process; hence it treats the stochastic return as being distributed according to the normal distribution with $x_n \sim N(\mu_n, \sigma_n^2 + \tau_n^2)$. Finally, we assume that productivity draws across firms in a particular country and across countries are independent, and that this is common knowledge.

The entrepreneur acts as a standard Bayesian decision maker who with a CARA utility function with degree of absolute risk aversion A. He understands that the capital that can be raised in the international capital market will be a function of the contract terms offered by all firms (denote it by a matrix $\mathbf{C} = [\mathbf{v}, \mathbf{R}]$ where $\mathbf{v}$ and $\mathbf{R}$ are vectors of contract terms) and the world distribution of investor wealth wN. The firm chooses contract terms so as to maximize his expected utility from consumption. Hence the problem of the firm n is as follows:

$$\arg\max_{\{v_n, R_n\}} -(1/A) \int e^{-A[(1-v_n)x_n f(k(\mathbf{C},wN)) - R_n k(\mathbf{C},wN)]} \frac{1}{\sqrt{2\pi(\sigma_n^2 + \tau_n^2)}} e^{-(x_n - \mu_n)^2/(2(\sigma_n^2 + \tau_n^2))} dx \tag{5}$$

Plugging in for $f(\cdot)$, we rewrite this problem as

$$\arg\max_{\{v_n, R_n\}} -\frac{1}{A} e^{-A\left([(1-v_n)\mu_n - R_n] k(\mathbf{C},wN) - \frac{(\sigma_n^2 + \tau_n^2) A (1-v_n)^2 (k(\mathbf{C},wN))^2}{2} \right)}. \tag{6}$$

Note that a natural requirement is that $v_n \in [0, 1]$; this does not need to be explicitly imposed as a constraint, because it will turn out to be already satisfied in equilibrium in the cases we consider. Similarly, simple contracts such as $v_n = 0$ and $R_n > 0$ are allowed, but they turn out not to be an equilibrium outcome. The firm can offer $(0, 0)$ or $(1, 0)$ and obtain a utility of at least $-(1/A)$.[18]

[18] To be able to solve the model, we need to assume that both the entrepreneurs and the investors can have negative consumption and that a firm can make payouts to investors even if negative productivity is realized. Given investor and country preferences that admit negative consumption and the widespread usage of the CARA-Normal framework in the finance literature we find the assumptions on the productivity generating process to be awkward but not inadmissible.

2.3 Symmetry assumptions

Throughout the paper we assume symmetry among firms in a given country. That is, we assume that all agents (investors and firms) use the same σ_n^2 and τ_n^2 for all firms in a given country: this corresponds to them perceiving the same values for the risk and ambiguity of all assets in that country (although their treatment of these values differs depending on whether it is a home or foreign country). We also set the means of the stochastic process to be equal across firms and countries $\mu_i = \mu_j = \mu$. For this reason, all subscripts n are dropped in the sequel.

Note finally that the clearing conditions for this market are trivial given that the firms issue only as many contracts as are requested.

3 Analytic results

3.1 The general case

Solving the maximization problem for an investor in country 1 gives the following portfolio allocations to a typical firm in country 1 and 2:

$$\alpha_{11} = S_{11} + \frac{1 - M_1 S_{11} - M_2 S_{12}}{\left(M_1 + M_2 \left(\frac{\sigma_1^2+\tau_1^2}{\sigma_2^2+\tau_2^2(1+\gamma)}\right) \frac{v_{i,1}^2}{v_{j,2}^2}\right)} \tag{7}$$

$$\alpha_{12} = S_{12} + \frac{1 - M_1 S_{11} - M_2 S_{12}}{\left(\frac{v_{j,2}^2}{v_{i,1}^2} \left(\frac{\sigma_2^2+\tau_2^2(1+\gamma)}{\sigma_1^2+\tau_1^2}\right) M_1 + M_2\right)} \tag{8}$$

where σ_1^2 and σ_2^2 are the risk for a firm in countries 1 and 2 respectively and similarly for ambiguity τ_1^2 and τ_2^2; $S_{11} = \frac{R_{i,1}+\mu v_{i,1}}{\left(\sigma_1^2+\tau_1^2\right)\theta w v_{i,1}^2}$ and $S_{12} = \frac{R_{j,2}+\mu v_{j,2}}{\left(\sigma_2^2+\tau_2^2(1+\gamma)\right)\theta w v_{j,2}^2}$; while $v_{i,1}$ and $R_{i,1}$ ($v_{j,2}$ and $R_{j,2}$ respectively) are the v and R set by a typical firm i in country 1 (firm j in country 2).

The first term is reminiscent of the standard solution of the unconstrained optimization problem in the CARA-Normal model: it is the expected return of the "reduced" distribution normalized by a variance. In the case of home assets, the relevant variance is $\sigma_1^2 + \tau_1^2$, the variance of the "reduced" distribution, whereas in the case of the foreign assets, it is a "distorted" variance $\sigma_2^2 + \tau_2^2(1 + \gamma)$. Given that the investor perceives ambiguity towards foreign assets, one could think of these as the "effective" variances (incorporating the ambiguity) he perceives towards home and foreign assets. The second term is a correction term due to the fact that there is a competition for funds between firms. The capital obtained by a firm issuing a contract $\{v_{i,1}, R_{i,1}\}$ is $(N_1\alpha_{11} + N_2\alpha_{21})\, w$.

A representative entrepreneur i located in country 1 maximizes her utility given by (6) anticipating the portfolio choices of investors and offers the following contract terms:

$$v_{i,1} = \frac{\frac{A}{\theta}\left(N_1 + \frac{\sigma_1^2+\tau_1^2}{\sigma_1^2+\tau_1^2(1+\gamma)}N_2\right)}{\left(2 + \frac{A}{\theta}\left(N_1 + \frac{\sigma_1^2+\tau_1^2}{\sigma_1^2+\tau_1^2(1+\gamma)}N_2\right)\right)} \tag{9}$$

$$R_{i,1} = \mu\,(1 - v_{i,1}) - w\theta\Upsilon_{i,1} \tag{10}$$

where $\Upsilon_{i,1}$ is an (involved) function of the parameters of the model (see Appendix A.1). The expected return to investors on a unit of capital is then $R_{i,1} + v_{i,1}\mu = \mu - w\theta\Upsilon_{i,1}$.

Similar expressions are obtained for investors and firms located in country 2. Note that the contract terms are such that $v_{i,1} \in [0,1)$, as one would expect. Moreover, given the symmetry in the risk and ambiguity of firms in the same country, they issue contracts with the same terms. Henceforth we drop the i subscript, using $v_1 = v_{i,1}$ and $R_1 = R_{i,1}$ for any firm i in country 1 and $v_2 = v_{j,2}$ and $R_2 = R_{j,2}$ for any j in country 2.

We can thus obtain closed form solutions for the general case. Already on the basis of these, we can conduct basic comparative statics for the equity part of the contracts offered (the v). To the extent that the equity participation captures the share in the risky process that the entrepreneur offloads to investors, this gives some first results on the effects of the various parameters on risk sharing. First of all, note that the level of an entrepreneur's end exposure to risk in equilibrium depends positively on the ratio of entrepreneur to investor risk aversion A/θ and the total measure of investors (whether home or foreign), as one would expect. Perhaps more interesting are the consequences of risk and ambiguity for risk sharing. In a word, they have the opposite effects. On the one hand, as the risk increases, the v increases as well ($\frac{\partial v}{\partial \sigma^2} > 0$): as the production process becomes more risky, entrepreneurs decide to reduce their equity exposure and issue contracts with higher investor participation. On the other hand, increases in ambiguity (τ^2) lead to a reduction in v ($\frac{\partial v}{\partial \tau^2} < 0$): to attract (foreign) investment, entrepreneurs offload less risk to investors. The same is true for ambiguity aversion (γ): as investors become more ambiguity averse, the amount of risk sharing in the proposed contracts falls ($\frac{\partial v}{\partial \gamma} < 0$).

Proposition 1. *If $N_2 > 0$, $\gamma > 0$ and $\tau^2 > 0$, then risk sharing in country 1 (v_1) falls as either ambiguity (τ_1^2) or ambiguity aversion (γ) increase. By contrast, risk sharing increases with an increase in risk (σ_1^2).*

Note that it is the presence of foreign investors that makes entrepreneurs change the contracts they offer: in the absence of foreign investors, changes in ambiguity (τ^2) have the same effect as changes in risk. In fact, there is a strong dependence on the measure of home investors and foreign investors, as can be seen in the limit risk sharing as risk, ambiguity or ambiguity aversion increase.

Proposition 2. *If $\gamma > 0$, $\tau^2 > 0$, $\lim_{\tau^2 \to \infty} v_1 = \lim_{\gamma \to \infty} v_1 = \frac{\frac{A}{\theta} N_1}{\left(2 + \frac{A}{\theta} N_1\right)}$, whereas $\lim_{\sigma^2 \to \infty} v_1 = \frac{\frac{A}{\theta}(N_1 + N_2)}{\left(2 + \frac{A}{\theta}(N_1 + N_2)\right)}$.*

As ambiguity or ambiguity aversion becomes very large, entrepreneurs rely almost exclusively on home investors, who do not perceive any ambiguity with respect to their production processes. If there are many home investors, entrepreneurs may easily and cheaply obtain insurance from them, and their risk sharing remains high: ambiguity has a mitigated effect. By contrast, if there are few home investors, then it becomes very difficult for entrepreneurs to get risk sharing in contexts with large ambiguity or ambiguity aversion. Once again, risk has the opposite effect: in cases of high risk, the risk sharing is determined by the total measure of investors in the world, and is insensitive to the distribution of wealth between the countries. This could hence provide an explanation for some of the findings mentioned in the Introduction: the model predicts that risk sharing is lower for countries with a small amount of home capital when ambiguity is significant.

As suggested in the Introduction, the case of countries with asymmetric wealth is of particular interest; however, given the complexity of formulas, it is difficult to do any more general comparative statics that would yield insight into that case. In Section 3.4, we consider in detail a simplified, tractable but relevant special case. Before, to provide appropriate comparisons but also to illustrate the power of the model, we shall analyze two other families of special cases: the “standard” model with only risk – a benchmark case, where investors have no ambiguity aversion, corresponding to a standard CARA-Normal model under our assumptions on the economy – and the symmetric case when firms are identical in their stochastic processes and countries are symmetric in the measure of investors and number of firms residing in each country.

3.2 The standard model (risk and no ambiguity aversion)

We begin our analysis with the benchmark case with no ambiguity aversion; ie. $\gamma = 0$. In this case, as noted before, investors act like expected utility maximizers and the portfolio choice problem collapses to a standard CARA-normal problem. We call this case “the standard model with risk”.

We can solve the model for the v's and R's to yield

$$v_1 = v_2 = v = \frac{\frac{A}{\theta}(N_1 + N_2)}{\left(2 + \frac{A}{\theta}(N_1 + N_2)\right)} \tag{11}$$

and

$$R_1 = R_2 = R = \mu \frac{2}{\left(2 + \frac{A}{\theta}(N_1 + N_2)\right)} - wA \frac{\left(4 + \frac{A}{\theta}(N_1 + N_2)\right)(N_1 + N_2)}{\left(\frac{1}{\sigma_1^2+\tau_1^2} M_1 + \frac{1}{\sigma_2^2+\tau_2^2} M_2\right)\left(2 + \frac{A}{\theta}(N_1 + N_2)\right)^2} \tag{12}$$

The expected return is then $R + v\mu = \mu - wA \frac{\left(4+\frac{A}{\theta}(N_1+N_2)\right)(N_1+N_2)}{\left(\frac{1}{\sigma_1^2+\tau_1^2} M_1 + \frac{1}{\sigma_2^2+\tau_2^2} M_2\right)\left(2+\frac{A}{\theta}(N_1+N_2)\right)^2}$ for firms in both countries.

Portfolio shares for investors from country 1 are then: $\alpha_{11} = S_{11} + \frac{1-M_1S_{11}-M_2S_{12}}{M_1+M_2\left(\frac{\sigma_1^2+\tau_1^2}{\sigma_2^2+\tau_2^2}\right)}$ and $\alpha_{12} = S_{12} + \frac{1-M_1S_{11}-M_2S_{12}}{\left(\frac{\sigma_2^2+\tau_2^2}{\sigma_1^2+\tau_1^2}\right)M_1+M_2}$ where $S_{11} = \frac{R+\mu v}{\theta w\left(\sigma_1^2+\tau_1^2\right)v^2}$ and $S_{12} = \frac{R+\mu v}{\theta w\left(\sigma_2^2+\tau_2^2\right)v^2}$. After substitutions we find that $\alpha_{11} = \alpha_{21} = \frac{1}{M_1+\frac{\sigma_1^2+\tau_1^2}{\sigma_2^2+\tau_2^2}M_2}$ while $\alpha_{12} = \alpha_{21} = \frac{1}{\frac{\sigma_2^2+\tau_2^2}{\sigma_1^2+\tau_1^2}M_1+M_2}$ In particular, if $\frac{\sigma_1^2+\tau_1^2}{\sigma_2^2+\tau_2^2} = 1$ we have $\alpha_{11} = \alpha_{12} = \alpha_{21} = \alpha_{22} = \frac{1}{M_1+M_2}$.

Then, a firm in country 1 issuing an asset with characteristics (v, R) will obtain $k_1 = \frac{N_1+N_2}{M_1+\frac{\sigma_1^2+\tau_1^2}{\sigma_2^2+\tau_2^2}M_2} w$ while a firm in country 2 issuing an asset with characteristics (v, R) will obtain $k_2 = \frac{N_1+N_2}{\frac{\sigma_2^2+\tau_2^2}{\sigma_1^2+\tau_1^2}M_1+M_2} w$. We have $\frac{k_2}{k_1} = \frac{\sigma_1^2+\tau_1^2}{\sigma_2^2+\tau_2^2}$.

The result is that all firms issue assets with the same contract terms irrespective of their residence, and investors hold identical portfolios. End capital allocation does not depend at all on where investors or entrepreneurs reside, but only on the risk of the firm's stochastic process: there are no frictions in the capital market. If firms in one country have more risky production processes, then they will choose to obtain less capital. The level of equity offered does not depend on the variance of the firms' stochastic production process. In that sense, in equilibrium the entrepreneurs offload the same share of their idiosyncratic risk. If the stochastic processes are the same in terms of mean and variance, the entrepreneurs obtain the same level of consumption insurance and the same Sharpe ratio of consumption no matter the country in which they reside. Moreover, in this case, expected returns on all assets are the same in both countries.

Finally, as the environment becomes more risky (σ_1^2 or σ_2^2 increase), the interest rates offered by firms in equilibrium on their risk-free bonds fall (as investors value the sure return more). The overall effect of more risk on expected real asset returns is negative.

3.3 Symmetric countries

Now we solve and analyze the case where all countries are identical in terms of the measure of investors ($N_1 = N_2 = \frac{N}{2}$) and the number of firms ($M_1 = M_2 = \frac{M}{2}$), and all firms (irrespective of the country of origin) have the same stochastic properties of their production process. Therefore we assume that $\sigma_1^2 = \sigma_2^2 = \sigma^2$ and $\tau_1^2 = \tau_2^2 = \tau^2$.

Drawing on (9) and (10), imposing symmetry and using the notation above we obtain

$$v = \frac{\frac{A}{\theta}N\left(1 + \frac{\sigma^2+\tau^2}{\sigma^2+\tau^2(1+\gamma)}\right)}{4 + \frac{A}{\theta}N\left(1 + \frac{\sigma^2+\tau^2}{\sigma^2+\tau^2(1+\gamma)}\right)} \tag{13}$$

$$R = \mu\left(1 - v\right) - w\theta\frac{\left(2 - v\right)v}{\frac{M}{2}\left(\frac{1}{\sigma^2+\tau^2} + \frac{1}{\sigma^2+\tau^2(1+\gamma)}\right)} \tag{14}$$

The expected return is then $R + v\mu = \mu - w\theta\frac{(2-v)v}{\frac{M}{2}\left(\frac{1}{\sigma^2+\tau^2}+\frac{1}{\sigma^2+\tau^2(1+\gamma)}\right)}$.

We can now analyze the comparative statics of the interest rate R. The interest rate decreases as the production processes become more risky ($\frac{\partial R}{\partial \sigma^2} < 0$), just as in the standard model with only risk. Investors in a more risky environment are willing to accept a lower risk-free return. By contrast, no such general trend exists for the effect of increases in ambiguity (τ^2) or ambiguity aversion (γ) on the interest rate. There are two opposing forces at play. First, an increase in ambiguity causes investors to value the risk-free return more, and so leads to a drop in interest rates. However, an increase in ambiguity makes home assets also less attractive to foreign investors; this will lead firms to increase the rate offered. (Similar points hold for increases in ambiguity aversion.) Only the first effect is present in the standard (benchmark) model, the latter being specific to the case where investors perceive ambiguity towards foreign assets. So, as one would expect, as one tends to the benchmark case – when ambiguity aversion is low or ambiguity is insignificant – the first force will prevail: $\left[\frac{\partial R}{\partial \tau^2}\right]_{\gamma=0} < 0$ and $\left[\frac{\partial R}{\partial \gamma}\right]_{\tau^2 \to 0} < 0$. The first effect also dominates when one is far from the benchmark case: if the ambiguity or ambiguity aversion is high (so that the share of foreign investors' participation in financing is low) further increases in ambiguity decrease the interest rate: $\left[\frac{\partial R}{\partial \tau^2}\right]_{\tau^2 \to \infty} < 0$ and $\left[\frac{\partial R}{\partial \tau^2}\right]_{\gamma \to \infty} < 0$. In intermediate cases, no tendency can be identified in general: for medium values of ambiguity aversion γ, for instance, the effect of increases of ambiguity, even from small levels, is strongly dependent on the other parameters (the derivative $\left[\frac{\partial R}{\partial \tau^2}\right]_{\tau_2=0}$ cannot be unequivocally signed). For example, if there are many firms (M is very large) relative to the measure of investors N, then increases in ambiguity lead to increases in the interest rate: with lots of competition, entrepreneurs are willing to raise interest rates to avoid losing foreign investors. Therefore, there may be a "hump" shape of the reaction of the interest rate with an increase in ambiguity. As the ambiguity increases from low levels, entrepreneurs try to keep foreign investors allocating their capital in their firms and so increase the interest rate; but with a high "model uncertainty", as the markets become very segmented, firms increasingly turn towards their home investors only. An example of such a reaction of the interest rate is shown in the upper right panel of Figure 2. These effects are quite different from impact of changes of risk on the interest rate in the standard model, which are shown in the upper left panel of Figure 2.

Some other comparative statics on R are immediate. An increase in the mean of the stochastic process (μ) increases the interest rate as equity returns become more attractive; as the number of firms M increases, so the competition for funds gets tougher, equilibrium R grows (as $M \to \infty$, $R \to \mu(1-v)$). As the measure of investors

N or their wealth w increases, with a higher availability of funds, interest rates decrease. Such tendencies are also present in the standard (benchmark) model.

The investors' ambiguity aversion towards foreign assets unequivocally depresses the returns that are offered, even if the fixed payments offered in the contracts might increase. More generally, increases in ambiguity aversion and increases in ambiguity lower expected real asset returns, just like increases in risk in the standard model.

With symmetric countries and entrepreneurs, the level of capital invested will be the same across firms, as in the benchmark risk-only case. However, the shares of domestic and foreign assets in investors' portfolios will differ depending on the different levels of risk (σ^2), ambiguity (τ^2) and ambiguity aversion (γ). For the symmetric case, we can obtain simple results for the portfolio shares of representative firms from country 1 and 2 in a country 1 investor's portfolio:

$$\alpha_{11} = \frac{2}{M\left(1 + \frac{\sigma^2+\tau^2}{\sigma^2+\tau^2(1+\gamma)}\right)} \tag{15}$$

$$\alpha_{12} = \frac{2}{M\left(\frac{\sigma^2+\tau^2(1+\gamma)}{\sigma^2+\tau^2} + 1\right)} \tag{16}$$

As an indication of the difference in investment at home and abroad, we adopt a standard measure of the degree of home bias (see for example Coeurdacier and Rey (2012); Solnik and Zuo (2012)) as one minus the ratio between actual share of foreign holdings and share of foreign equities in the world market portfolio. In this simple symmetric case, where half the world capital is invested in each country, this can be calculated using (15) and (16) as

$$HB = 1 - \frac{2\left(\sigma^2+\tau^2\right)}{\left(\sigma^2+\tau^2\left(1+\gamma\right)\right)+\left(\sigma^2+\tau^2\right)} \tag{17}$$

This quantity takes the value 0 when there is no home bias (in this case, when the investor invests precisely half of his wealth abroad). When it is higher than 0, then there is home bias; when it is 1, the investor holds no foreign assets at all. The observation that this quantity is higher than 0 whenever $\gamma > 0$ and $\tau^2 > 0$ yields the following proposition.

Proposition 3. *Ambiguity and ambiguity aversion towards foreign assets causes home* *bias in asset holdings.*

The fact that in the symmetric case there may be significant home bias without any effect on the total capital invested in each country (with respect to the benchmark risk-only case) is reminiscent of the conclusions of Solnik and Zuo (2012), who find the potential of significant home bias in a symmetric case with no effect on asset prices.

Notice that the portfolio shares depend neither on the risk aversion of investors nor on that of entrepreneurs; what matters is the ratio of "effective" variances of the stochastic production processes of home and foreign firms, as perceived by the investor. This in turn depends on the levels of risk, ambiguity and ambiguity aversion. We have that $\frac{\partial}{\partial\sigma^2}(HB) < 0$, $\frac{\partial}{\partial\tau^2}(HB) > 0$ and $\frac{\partial}{\partial\gamma}(HB) > 0$. As the ambiguity or ambiguity aversion increase, the markets become more segmented, exacerbating the home bias; on the other hand, as the risk increases, the differences between home and foreign assets gets drowned out by the risk factor (which is perceived in the same way by home and foreign investors), thus mitigating the home bias. One can easily obtain high levels of the home bias for reasonable parametrizations. For example, if $\tau^2 = 4\sigma^2$ and $\gamma = 2.5$ then the holdings of foreign assets will only make up 1/4 of the portfolio, and $HB = 0.5$. The same home bias of 0.5 will be obtained if $\tau^2 = \sigma^2$ and $\gamma = 4$ or $\tau^2 = 0.5\sigma^2$ and $\gamma = 6$.[19]

The effect of ambiguity and ambiguity aversion on portfolio holdings affects the correlation of consumption of investors. In our setup it only makes sense to compare the consumption of investors, as entrepreneurs have always an idiosyncratic component stemming from their own production process that they do not diversify away (as we prevent them from investing in any securities). The correlation of consumption among the investors from the two countries in the symmetric case is $Corr(c_1, c_2) = \frac{2\left(\sigma^2+\tau^2\right)\left(\sigma^2+\tau^2(1+\gamma)\right)}{\left(\sigma^2+\tau^2(1+\gamma)\right)^2+\left(\sigma^2+\tau^2\right)^2}$. This yields the following Corollary to Proposition 3.

Corollary 1. *Ambiguity and ambiguity aversion lower the correlation of consumption of investors from the two countries relative to the benchmark case.*

For example, for the sets of parameters displayed above ($\tau^2/\sigma^2 = 4$, $\gamma = 2.5$), ($\tau^2/\sigma^2 = 1$, $\gamma = 4$) or ($\tau^2/\sigma^2 = 0.5$, $\gamma = 6$) the correlation of consumption for investors falls to 0.6.

[19] Solnik and Zuo (2012) report the average HB in developed countries in 2008 at 55%.

3.4 Asymmetric countries

As was made clear in the Introduction, a major motivation for integrating ambiguity into security structure is to analyze the differential effects on firms in countries with differing amounts of capital. In this section, we carry out this analysis. To focus on the question of capital, we assume that the countries are identical as concerns risk, ambiguity and the number of firms ($\sigma_1^2 = \sigma_2^2 = \sigma^2$, $\tau_1^2 = \tau_2^2 = \tau^2$, and $M_1 = M_2 = \frac{M}{2}$), though may differ in the measure of investors. We assume that $N_1 > N_2$, and refer to country 2 as the capital-scarce country.

First of all, it can be shown that the capital invested in the country with a larger measure of investors is higher ($\frac{M}{2}k_1 > \frac{M}{2}k_2$). Moreover, the country with fewer investors is always a net capital importer ($\frac{M}{2}\alpha_{12}N_1 w > \frac{M}{2}\alpha_{21}N_2 w$). This latter fact allows us to identify the capital-importing country with the one with less domestic capital.

We have the following result concerning asset structure and expected returns.

Proposition 4. *Suppose that $M_1 = M_2 = \frac{M}{2}$, $\tau^2 > 0$, $\gamma > 0$ and $N_1 > N_2$. Then a) $v_{i,1} > v_{j,2}$ and b) $R_{i,1} < R_{j,2}$.*

The situation in the presence of ambiguity is markedly different from the benchmark risk-only case considered in Section 3.2. First of all, whereas in a world with no ambiguity aversion, all firms would offer the same contract irrespective of the investor distribution, in the presence of ambiguity, both contract terms are different between the two countries. On the one hand, entrepreneurs from the capital-scarce country issue contracts that "insure" investors more, bearing more risk themselves. On the other hand, they pay a higher interest rate per unit of capital obtained. Both of these factors can be explained by the attempt to have access to the richer capital market in the foreign country.

Proposition 4 has two interesting immediate consequences.

Corollary 2. *Entrepreneurs in the capital-scarce country issue relatively more bonds than equity relative to the entrepreneurs in the capital rich country.*

The value of the bond part of a contract issued by a typical firm is kR while the equity part is $kv\mu$. Hence our model implies that capital importers will have a higher bond/equity ratio in outstanding assets; this is consistent with the patterns founds in

Figure 1 and discussed in the Introduction.[20]

Corollary 3. *The Sharpe ratio of consumption of entrepreneurs in the capital-rich country is higher than in the capital-scarce one.*

The nature of the contracts issued by entrepreneurs in the capital-scarce country also renders entrepreneurial consumption more variable when measured by the Sharpe ratio. This is a consequence of the ambiguity perceived by foreign investors, which renders it more difficult for capital importing countries to obtain insurance from shocks. Accordingly, to attract the desired capital, they must propose contracts that hinder risk sharing (with relatively low equity part v) and that exacerbate the variance of consumption (since more capital is obtained through bond issuance).

Proposition 5. *Suppose that $M_1 = M_2 = \frac{M}{2}$, $\tau^2 > 0$ and $\gamma > 0$ and $N_1 > N_2 > 0$. Then a) As $\lim_{\tau \to 0}$ or $\lim_{\gamma \to 0}$ the expected return on assets is higher in country 1, and b) As $\lim_{\gamma \to \infty}$ the expected return on assets is higher in country 2.*

The intuition behind Proposition **??** is that as ambiguity aversion grows, entrepreneurs turn primarily to domestic investors for funds. In autarky the expected returns are higher in the capital-scarce country as the competition for funds is fiercer. On the other hand, when ambiguity aversion is low, and foreign investors are ready to invest, the firms in the capital-scarce country compete with those from the capital-abundant country. Indeed, when $N_2 = 0$ and country 2 firms compete with country 1 firms for capital then the expected return on assets is always higher in country 1. These patterns are in stark contrast to the case without ambiguity studied in Section 3.2, where the expected returns are independent of the distribution of wealth. As such, it suggests another possible answer to the question of why the rates of return on capital found empirically may be lower than predicted by standard theories in capital-importing (typically also identified with "emerging") countries (see for example the discussion in Caselli and Feyrer (2007)).

Extending the analysis for the symmetric case carried out in Section 3.3, it is possible to consider the effects of asymmetry in country wealth on the home bias. Taking as the world market portfolio the total capital invested in both countries that

[20] Obviously the group of capital importers is very heterogenous. For example, global investors probably have a much better understanding of the stochastic processes governing production in a country like the US than say Greece or Peru.

would be observed under our model, we can calculate the home bias for country 1 and country 2 respectively as $HB_1 = 1 - \frac{\frac{M_2\alpha_{12}}{M_1\alpha_{11}+M_2\alpha_{12}}}{\frac{M_2k_2}{M_1k_1+M_2k_2}}$ and $HB_2 = 1 - \frac{\frac{M_1\alpha_{21}}{M_1\alpha_{21}+M_2\alpha_{22}}}{\frac{M_1k_1}{M_1k_1+M_2k_2}}$. Comparing these two expressions and substituting for the portfolio shares and invested capital, one can show that $HB_2 > HB_1$ if and only if $\alpha_{21}N_2 < \alpha_{12}N_1$. Since, as noted above, the capital-scarce country is a net capital importer, it will thus have a higher home bias in our model. This is because country 2 relies relatively more on foreign capital for investment, and so the relative share of country 2 firms in the portfolios of country 1 investors is going to be closer to the relative global capital share invested in country 2. To the extent that capital importers in the model can be associated with financially less well-developed countries, our model is thus consistent with the empirical analysis by Ahrend and Schwellnus (2012), which concludes that financially less well-developed countries experience a higher home bias. This conclusion is at odds with the view that such countries should have a higher demand for safe assets – and in particular for foreign-issued assets – than financially developed countries (for example, Caballero et al. (2008)).

Further analysis of the model is intractable at this level of generality. To obtain further analytic results, we consider a special yet suggestive case where one country has all the wealth.

3.4.1 One country with no wealth

Beyond the assumptions mentioned at the beginning of Section 3.4, we assume that $N_2 = 0$. We have the following solutions for the contact terms offered by firms in the two countries.

$$v_{i,1} = \frac{\frac{A}{\theta}N_1}{\left(2 + \frac{A}{\theta}N_1\right)} \tag{18}$$

$$R_{i,1} = \mu\left(1 - v_{i,1}\right) - w\theta\frac{\left(2 - v_{i,1}\right)}{\left(\frac{1}{\sigma^2+\tau^2}\frac{M_1}{v_{i,1}} + \frac{1}{\sigma^2+\tau^2(1+\gamma)}\frac{M_2}{v_{j,2}}\right)} \tag{19}$$

$$v_{j,2} = \frac{\frac{A}{\theta}\frac{\sigma^2+\tau^2}{\sigma^2+\tau^2(1+\gamma)}N_1}{\left(2 + \frac{A}{\theta}\frac{\sigma^2+\tau^2}{\sigma^2+\tau^2(1+\gamma)}N_1\right)} \tag{20}$$

$$R_{j,2} = \mu\left(1 - v_{j,2}\right) - w\theta \frac{2 - v_{j,2}}{\left(\frac{1}{\sigma^2+\tau^2}\frac{M_1}{v_{i,1}} + \frac{1}{\sigma^2+\tau^2(1+\gamma)}\frac{M_2}{v_{j,2}}\right)} \quad (21)$$

The portfolio shares then $\alpha_{11} = \frac{1}{M_1 + \frac{\sigma^2+\tau^2}{\sigma^2+\tau^2(1+\gamma)}\frac{v_{i,1}}{v_{j,2}}M_2}$ and $\alpha_{12} = \frac{1}{\frac{\sigma^2+\tau^2(1+\gamma)}{\sigma^2+\tau^2}\frac{v_{j,2}}{v_{i,2}}M_1 + M_2}$ while the ratio of capital obtained by firms is $\frac{k_2}{k_1} = \frac{2+\frac{A}{\theta}\frac{\sigma^2+\tau^2}{\sigma^2+\tau^2(1+\gamma)}N_1}{2+\frac{A}{\theta}N_1}$.

The observations made above carry over to this case. For example, risk sharing is lower for entrepreneurs in country 2: indeed, it follows from Proposition 2 that, with high levels of ambiguity aversion, there will be no risk sharing possible for entrepreneurs in country 2 ($\lim_{\gamma\to\infty} v_2 = 0$). As in the general case discussed above, the Sharpe ratio of consumption of entrepreneurs is higher in country 1.

Moreover, it turns out that the capital invested in a representative entrepreneur in the capital-abundant country is also higher than for the capital-scarce one. Combined with the fact that, when investors are not too ambiguity averse, entrepreneurs in capital-scarce countries offer lower expected returns on each unit of capital invested (see Proposition **??**),[21] this implies that, even with a lower installed capital stock, the marginal effective product of capital can be thus lower in countries with fewer domestic investors, without the marginal return on *physical* capital necessarily being low (as for example in Matsuyama (2004) because of low contract enforcement). In a world with ambiguity and ambiguity aversion, we can thus have equalized marginal returns of capital as measured by Caselli and Feyrer (2007), even in the presence of output gains from reallocating capital. Moreover, the financial market imperfections stemming from ambiguity aversion may play a major role in preventing capital flows from "rich" to "poor" countries. This also points to the following observation. If "rich" countries are more transparent than "poor" ones, in the sense of being less ambiguous, one can have capital flow out of capital-scarce countries if capital flow restrictions are lifted.

Now let us consider in detail the comparative statics in the model as the risk, ambiguity and ambiguity aversion change. Consider first a change in the risk (σ^2) of the production processes. In terms of the issued contracts in the two countries, $\frac{\partial v_1}{\partial \sigma^2} = 0$, $\frac{\partial R_1}{\partial \sigma^2} < 0$ while $\frac{\partial v_2}{\partial \sigma^2} > 0$, $\frac{\partial R_2}{\partial \sigma^2} < 0$. As observed in Proposition 1, increases in risk increase equity participation as long as there are foreign investors, because it

[21] In all simulations that we performed this was true for plausible values of γ (for example, < 50); given the complexity of the expressions involved, a bound of γ could not be found.

mitigates the (relative) importance of ambiguity. Moreover, as in the standard model with risk, an increase in the global riskiness of the underlying production processes increases investors' willingness to hold risk-free assets whether at home or abroad, thus leading to a drop in interest rate. Combining these effects, it is clear that the end capital distribution between the two countries becomes more even with the increase in risk ($\frac{\partial}{\partial \sigma^2}\left(\frac{k_2}{k_1}\right) > 0$). Moreover, the interest differential between the two countries' representatives firms bonds decreases: $\frac{\partial}{\partial \sigma^2}\left(R_2 - R_1\right) < 0$. Indeed, as risk increases and the effect of ambiguity gets "drowned out", the model comes close to the case where there is no ambiguity, and where there is no difference in capital distribution or contract terms between the two countries.

If the investors' ambiguity aversion (γ) changes it is as if they perceive foreign assets as carrying a higher effective variance. Then $\frac{\partial v_1}{\partial \gamma} = 0$ and $\frac{\partial R_1}{\partial \gamma} < 0$, but $\frac{\partial v_2}{\partial \gamma} < 0$, and $\frac{\partial R_2}{\partial \gamma} > 0$ whenever $R_2 > 0$. As ambiguity aversion increases, entrepreneurs from the capital-importing country offer lower equity stakes (as noted in Section 3.1) and "insure" foreign investors by offering higher interest rates on the bond part of the contract. Moreover, the "weakening" of foreign competition allows the firms in the capital-rich countries to offer lower interest rates. In the end, the interest differential offered by firms in the two countries grows: $\frac{\partial}{\partial \gamma}\left(R_2 - R_1\right) > 0$. Despite this, the capital invested in the capital-scarce country decreases. The Sharpe ratio of entrepreneurial consumption in the two countries also diverges: entrepreneurs in country 2 need to accept more risk per unit of expected consumption.

The picture is more complex, however, as concerns changes of ambiguity (τ^2). Qualitatively, an increase in ambiguity is an intermediate scenario between an increase in risk and an increase in ambiguity aversion: the "effective" variance of production processes perceived by investors increases for both home and foreign assets, though more strongly for the latter. Then $\frac{\partial v_1}{\partial \tau^2} = 0$, $\frac{\partial R_1}{\partial \tau^2} < 0$. As in the case of increased ambiguity aversion, firms from the capital-abundant country obtain lower interest rates as investors seek more "safety". Also, as shown in Proposition 1, $\frac{\partial v_2}{\partial \tau^2} < 0$. However, the effect on the interest rate cannot be signed $\frac{\partial R_2}{\partial \tau^2} >< 0$. This is due to the interplay between two forces. On the one hand, an increase in ambiguity leads to an increase in the variance perceived by all investors for all assets, so they are willing to accept a lower remuneration for holding bonds; this is the mechanism driving the interest rate down when the risk increases. On the other hand, the "effective" variance perceived by investors in country 1 towards firms in country 2 increases faster than the variance perceived towards firms in country 1, due to ambiguity aversion; this is the same as the mechanism that drives the interest rates in country 2 up when the ambiguity aversion increases. The interest differential $R_2 - R_1$ on the bond part of the contract may be non-monotonic in ambiguity; for example, it may have a hump shape.

An example of the behavior of the interest rates as τ^2 increases (obtained by simulation) is given by the lower right panel of Figure 2. In comparison, in the standard model with risk the interest rates would be exactly the same in the two countries, as depicted by the left lower panel of Figure 2.

The discrepancy between the capital invested in the capital-abundant and capital-scarce country increases unequivocally. Generally, as long as the proportional increase in ambiguity is greater than that for risk, the capital discrepancy increases.

Proposition 6. *Suppose that σ^2 increases by a factor a and τ^2 increases by a factor b. If $b > a$, then $\frac{k_2}{k_1}$ decreases.*

This observation suggests an interesting interpretation of global crisis events. If investors become suddenly less sure about the stochastic properties of productivity for *all* assets – if their ambiguity or "model uncertainty" increases – the model would predict "capital flight" ("sudden stops") from countries that are capital importers. This could be an explanation for what happens during global crises, like the one in 2008, that does not assume some asymmetric shocks to peripheral capital importers, increases in risk, in ambiguity aversion, transaction or information costs: a general unanticipated shock in ambiguity (or "model uncertainty") about the stochastic properties of economic fundamentals would produce the observed effects. Notice that increases in risk alone do not produce such effects, although simultaneous increases in risk and ambiguity, as long as the proportional increase in ambiguity is larger than that of risk. Indeed, global crises arguably correspond more precisely to situations of this sort: the environments have not got significantly more risky in the technical sense – of having higher variances for the underlying stochastic processes – but there have been large increases in ambiguity – people are less sure about the parameters governing those stochastic processes. A sudden stop may occur even when offered contracts can change.

4 Discussion and further remarks

Inclusion of ambiguity and ambiguity aversion changes the properties of the standard model with risk and makes it more realistic. Increases in risk, ambiguity and ambiguity aversion have radically different, and in many cases opposite effects on the capital invested internationally, the extent of risk sharing given by v and the interest rates R. This allows us to rationalize stylized facts about international capital flows with a simple model; in fact, the model can explain the different patterns of debt/equity issuance at the macroeconomic level. Moreover, as argued above, this allows this simple model to explain a range of puzzling phenomena.

Note that the model supports several alternative interpretations beyond the proposed one. The firms in our model can be interpreted as different idiosyncratic sources of risk in a country. The case where $M_1 = M_2 = 1$ can be interpreted as one source of risk or alternatively in terms of an assumption that all stochastic production processes within that country are perfectly correlated: one could have several separate entities issuing assets with the same characteristics (this would be an equilibrium outcome) that would be treated by investors as if they were the same asset. An alternative interpretation of the case $M_1 = M_2 = 1$ is in terms of holding a market portfolio of equity and bonds from that country. Generalising this interpretation, a version of the model can be developed with several (that is, more than two) individual countries that non-cooperatively compete for funds, yielding qualitatively similar results to those reported above.[22]

Whilst we have deliberately adopted a simple model, in order to better bring out the essential points about the possible consequences of ambiguity and ambiguity aversion for international capital structure, the main conclusions obtained are robust to many refinements of or variations in the model that may come to mind.

For example, whilst it was assumed that the countries are identical as concerns risk and ambiguity $\sigma_1^2 = \sigma_2^2 = \sigma^2$ and $\tau_1^2 = \tau_2^2 = \tau^2$), the literature indicates that many capital-scarce countries experience higher volatility of their production processes. It is thus worth asking what happens when risk or ambiguity are higher in the capital-scarce country – so that either $0 < \sigma_1^2 < \sigma_2^2$ or $0 < \tau_1^2 < \tau_2^2$. In the case when $N_2 = 0$ (as in Section 3.4.1), these cases are relatively easy to analyze, yielding expressions similar

[22] Details are available upon request.

to those in equations (18–21). Generally speaking, the phenomena identified above carry over to this case. Just as in the case studied in the Section 3.4.1, entrepreneurs in the capital-scarce country obtain less risk sharing ($v_1 > v_2$) and they offer high interest rates ($R_2 > R_1$); Moreover, as the production process becomes more risky the entrepreneurs in the capital-scarce country obtain more risk sharing, whereas the opposite is true when ambiguity increases ($\frac{\partial v_2}{\partial \sigma_2^2} > 0$ while $\frac{\partial v_2}{\partial \tau_2^2} < 0$). Also, just as in Section 3.4.1, the interest rate decreases with increases in risk ($\frac{\partial R_2}{\partial \sigma_2^2} < 0$) while the effect of ambiguity ($\frac{\partial R_2}{\partial \tau_2^2}$) cannot be signed in general.

Alternatively, one might consider the assumption that investors perceive no ambiguity with respect to home assets to be strong, and wonder to what extent the results are sensitive to this assumption. In Appendix A.4 we study a version of the model where investors perceive ambiguity in all assets, although the ambiguity of foreign assets is higher than for domestic ones. (Recall that all investors have the same ambiguity aversion, irrespective of the source of the ambiguity, ie. the asset.) This corresponds to a situation where one country would be less transparent than the other one, for example because of a relative lack of sufficient statistical information. In this case, firms with more "ambiguous" statistical processes issue contracts with a higher fixed-payment and a lower equity part, just as in the case analysed in the previous sections. This provides another explanation why risk-sharing in general is limited.

To take yet another example, whilst it is assumed that all investors have the same wealth, one might expect wealth accumulation effects and ask what consequences different wealth levels of investors between countries would have for the results (in particular those in Section 3.4).[23] It turns out that the results concerning the equity and bond elements of proposed contracts hold even when investors have different wealths in the two countries, as long as the investor-wealth differences are compatible with any differences in investor population (eg. the country with poorer investors does not have more investors; this country can thus still be unambiguously referred to as the capital-scarce one). Moreover, the other conclusions, concerning returns, capital attracted all go through when capital-scarcity and capital-abundancy of a country is characterised in terms of the (lesser, respectively greater) wealths of the resident investors rather

[23]Note that the in-depth consideration of cases where there are differing measures of investors covers a certain type of wealth accumulation: namely accumulation resulting in changes in the investor population. Moreover the general solution given in Section 3.1 is wide enough to provide insights into the consequences of firm accumulation, that is unevenness in the number of firms in the two countries.

than their number.

Finally, one might ask whether the results are specific to the decision model used, namely the smooth ambiguity model of Klibanoff et al. (2005)), or whether they continue to hold for other ambiguity models. In Appendix A.3 we study the same questions using the maxmin expected utility model of Gilboa and Schmeidler (1989), of which the Hansen and Sargent (2001) constraint model is a special case. The general qualitative properties identified above continue to hold, with the sole exception that there is no parameter in the maxmin-EU model that, in the context of this problem, yields the same behavior as the ambiguity parameter (τ^2). This is related to a specificity of the smooth ambiguity model that was noted at the outset: it permits a distinction between ambiguity and ambiguity aversion that is absent in most other ambiguity models, and in particular in the maxmin-EU one. The ambiguity-ambiguity aversion parameter in the maxmin-EU model behaves more similarly to the ambiguity aversion parameter in the smooth model (γ) than to the ambiguity one (τ^2).

5 Conclusions

We have studied the implications of ambiguity aversion on the international allocation of capital, risk sharing and security design. Our conclusion is that with ambiguity aversion, risk sharing is impaired. Moreover, firms from capital-scarce countries offer more fixed-income securities relative to the capital-abundant ones. There are marked differences in the contract terms and the composition of capital allocation across the world, which depend on the initial distribution of capital. These differences have consequences for expected returns and capital allocation, and suggest explanations for seemingly puzzling phenomena in the international asset structure.

There are a number of extensions that we would wish to pursue in future research. An interesting avenue to take concerns the idea that investors may be more ambiguity averse towards events further in the future; hence there could be a natural tendency for countries with relatively little domestic capital (owned by “savvy” investors) to issue short-term securities. Firms may issue also a wide array of contracts, and try to discriminate among different investors. Also, moral hazard could be introduced to enrich the analysis. A natural continuation would be to consider a calibration exercise where one would analyse a dynamic general equilibrium system that could match real-world data, clearly beyond the scope of this paper. Another possible extension of the model would be to incorporate exchange rate risk by, for example, allowing the bond part of the contract to be treated as risky by foreign investors. This could explain why economies with few domestic investors issue many foreign-denominated bonds.

The problems created by ambiguity in the model could be addressed by producing more high quality data so that investors feel surer about the environment they invest in (which is the conclusion of empirical studies like Gelos and Wei (2005)). In our context, this calls for more transparency on the side of statistical agencies or central banks, especially in the capital-importing countries. Another solution to the problem of lack of risk sharing and underinvestment caused by ambiguity would be to encourage foreign direct investment and multinational companies: entrepreneurs from different countries could jointly issue contracts and self-insure within such an entity.

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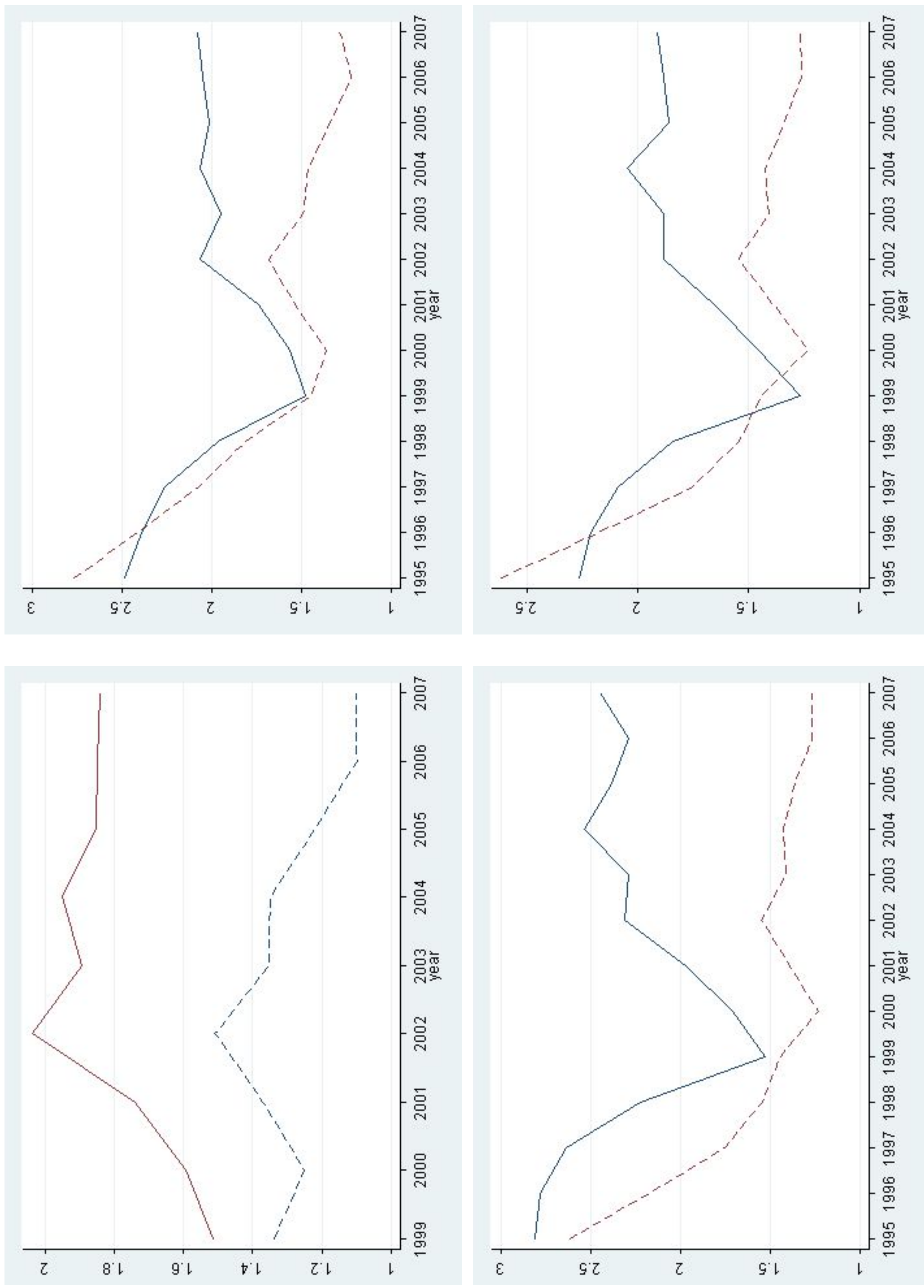

Figure 1: Ratios of debt to equity external liabilities among net capital importers (solid line) and capital exporters (dashed line). Data from the Lane and Milesi-Ferretti (2007) data set. Upper left panel: a sample of all 177 countries available for years 1999-2007. Upper right panel: a sample of 29 OECD members (as of end 1996). Lower left panel: 15 European Union members as of end of 1995. Lower right panel: 11 Euro zone members as of beginning of 1999.

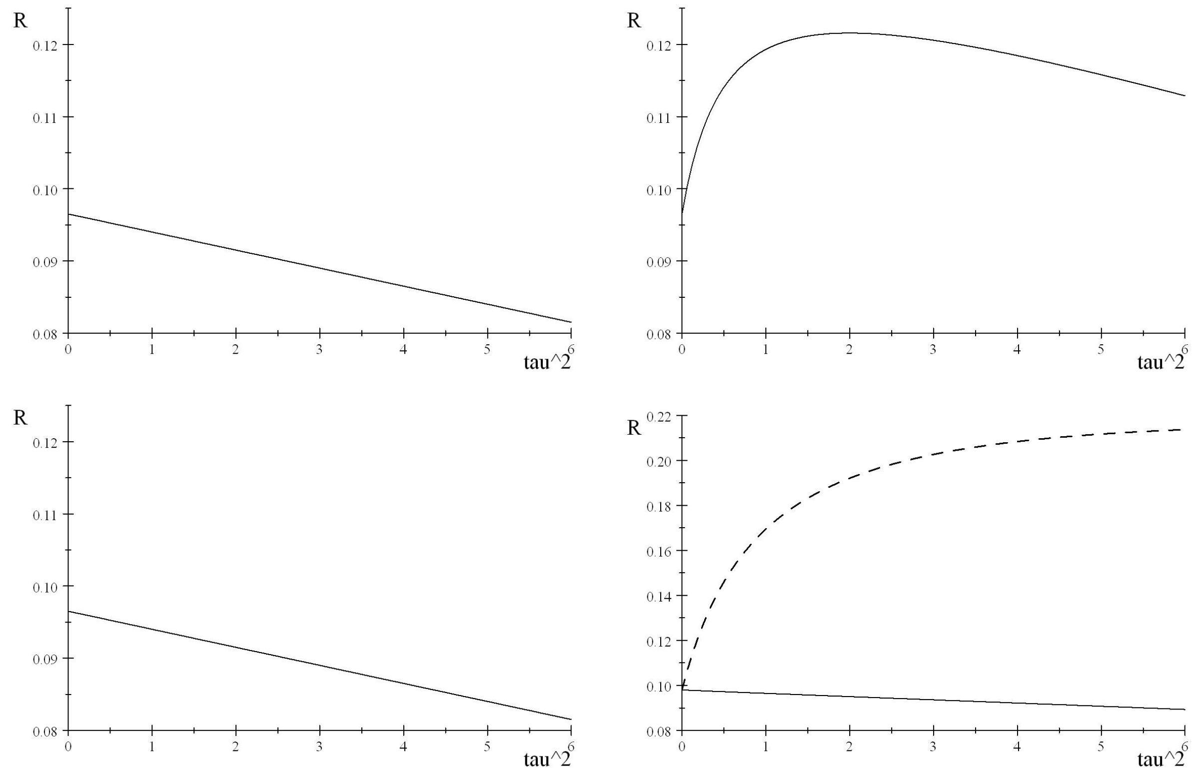

Figure 2: Upper panels: Comparison in the reaction of the interest rate in the symmetric case to increases in ambiguity. Base parameters: $\mu = 10$, $\theta = 1$, $A = 1$, $\sigma = 1$, $N = 100$, $M_1 = 500$, $M_2 = 500$, , $w = 1$. Upper left panel: the standard model, $\gamma = 0$. Upper right panel: the model with ambiguity aversion, $\gamma = 1.5$. Lower panels: Comparison in the reaction of the interest rate in the asymmetric case with $N_2 = 0$ to increases in ambiguity. Base parameters: $\mu = 10$, $\theta = 1$, $A = 1$, $\sigma = 1$, $N_1 = 200$, $N_2 = 0$, $M_1 = 500$, $M_2 = 500$, $w = 1$. Lower left panel: the standard model, $\gamma = 0$. Lower right panel: the model with ambiguity aversion, $\gamma = 1.5$. Solid line: R_1, dashed line: R_2.

A For Online Publication: Technical Appendix

Detailed derivations of all results are available in an online appendix, downloadable at https://studies2.hec.fr/jahia/webdav/site/hec/shared/sites/hill/acces_anonyme/Articles/Hill_Michalski_210214_supplementary_material.pdf.

A.1 General expression of R

The general expression for the interest rate in (10) is

$$R_{i,1} = \mu\left(1 - v_{i,1}\right) - w\theta\Upsilon_{i,1}$$

where

$$\Upsilon_{i,1} = \left(2v_{i,1}^{-1} - 1\right)\frac{\left(\begin{array}{c}\left(\widehat{X}_1N_1 + \widehat{Y}_1N_2\right)\left(\frac{N_1}{\sigma_2^2+\tau_2^2(1+\gamma)} + \frac{N_2}{\sigma_2^2+\tau_2^2}\right) \\ + \left(\widehat{X}_1N_1 + \widehat{Y}_1N_2\right)\left(\frac{\widehat{X}_1N_1}{\sigma_2^2+\tau_2^2(1+\gamma)} + \frac{\widehat{Y}_1N_2}{\sigma_2^2+\tau_2^2}\right)\frac{M_1}{v_{i,1}^2}\left(2v_{j,2}^{-1} - 1\right) \\ + \left(\frac{\sigma_1^2+\tau_1^2}{\sigma_2^2+\tau_2^2(1+\gamma)}\widehat{X}_1N_1 + \frac{\sigma_1^2+\tau_1^2(1+\gamma)}{\sigma_2^2+\tau_2^2}\widehat{Y}_1N_2\right)\left(\frac{\widehat{X}_1N_1}{\sigma_2^2+\tau_2^2(1+\gamma)} + \frac{\widehat{Y}_1N_2}{\sigma_2^2+\tau_2^2}\right)\frac{M_2}{v_2^2}\left(2v_{j,2}^{-1} - 1\right)\end{array}\right)}{\left(\begin{array}{c}\left(\frac{N_1}{\sigma_1^2+\tau_1^2} + \frac{N_2}{\sigma_1^2+\tau_1^2(1+\gamma)}\right)\left(\frac{N_1}{\sigma_2^2+\tau_2^2(1+\gamma)} + \frac{N_2}{\sigma_2^2+\tau_2^2}\right) \\ + \left(\frac{N_1}{\sigma_2^2+\tau_2^2(1+\gamma)} + \frac{N_2}{\sigma_2^2+\tau_2^2}\right)\left(\frac{\widehat{X}_1N_1}{\sigma_2^2+\tau_2^2(1+\gamma)} + \frac{\widehat{Y}_1N_2}{\sigma_2^2+\tau_2^2}\right)\frac{M_2}{v_{j,2}^2}\left(2v_{i,1}^{-1} - 1\right) \\ + \left(\frac{N_1}{\sigma_1^2+\tau_1^2} + \frac{N_2}{\sigma_1^2+\tau_1^2(1+\gamma)}\right)\left(\frac{\widehat{X}_1N_1}{\sigma_2^2+\tau_2^2(1+\gamma)} + \frac{\widehat{Y}_1N_2}{\sigma_2^2+\tau_2^2}\right)\frac{M_1}{v_{i,1}^2}\left(2v_{j,2}^{-1} - 1\right)\end{array}\right)}$$

and $\widehat{X}_1 = \left(\frac{M_1}{v_{i,1}^2} + \frac{\sigma_1^2+\tau_1^2}{\sigma_2^2+\tau_2^2(1+\gamma)}\frac{M_2}{v_{j,2}^2}\right)^{-1}$, $\widehat{Y}_1 = \left(\frac{M_1}{v_{i,1}^2} + \frac{M_2}{v_{j,2}^2}\frac{\sigma_1^2+\tau_1^2(1+\gamma)}{\sigma_2^2+\tau_2^2}\right)^{-1}$.

A.2 Proofs of Propositions

Proof of Proposition 1. Taking respective derivatives from (9) one obtains $\frac{\partial v_{i,1}}{\partial \sigma_1^2} > 0$, $\frac{\partial v_{i,1}}{\partial \tau_1^2} < 0$ and $\frac{\partial v_{i,1}}{\partial \gamma} < 0$.

Proof of Proposition 2. Immediate from taking the appropriate limits in (9).

Proof of Proposition 3. Immediate from (17) where $HB = 0$ if $\gamma = 0$ or $\tau^2 = 0$.

Proof of Proposition 4.

Proof of statement a). Rewrite the expressions for $v_{i,1}$ and $v_{j,2}$ as

$$\begin{aligned} v_{i,1} &= \frac{\eta^2 \frac{A}{\theta}\left[\frac{1}{\eta^2}N_1 + \frac{1}{h^2}N_2\right]}{\left[2+\eta^2\frac{A}{\theta}\left[\frac{1}{\eta^2}N_1+\frac{1}{h^2}N_2\right]\right]} \\ v_{j,2} &= \frac{\zeta^2 \frac{A}{\theta}\left[\frac{1}{z^2}N_1 + \frac{1}{\zeta^2}N_2\right]}{\left[2+\zeta^2\frac{A}{\theta}\left[\frac{1}{z^2}N_1+\frac{1}{\zeta^2}N_2\right]\right]} \end{aligned}$$

where $\eta^2 = \sigma_1^2 + \tau_1^2, h^2 = \sigma_1^2 + \tau_1^2(1+\gamma), \zeta^2 = \sigma_2^2 + \tau_2^2, z^2 = \sigma_2^2 + \tau_2^2(1+\gamma)$.

Proof by contradiction. Suppose that $N_1 > N_2$ but $v_{i,1} < v_{j,2}$. Then by a direct comparison of $v_{i,1}$ and $v_{j,2}$ we obtain

$$N_1\left(1-\frac{\zeta^2}{z^2}\right) < N_2\left(1-\frac{\eta^2}{h^2}\right)$$

and we assumed that $\eta^2 = \zeta^2, h^2 = z^2, \gamma > 0$ so a contradiction.

Proof of part b). We can rewrite

$$\begin{aligned} R_{j,2} - R_{i,1} &= \mu(1-v_{j,2}) - w\theta\Upsilon_j - \mu(1-v_{i,1}) + w\theta\Upsilon_i \\ &= \mu(v_{i,1}-v_{j,2}) - w\theta(\Upsilon_j - \Upsilon_i) \end{aligned} \tag{22}$$

Clearly $(v_{i,1} - v_{j,2}) > 0$ by part a) of the Proposition but it turns out that it might be the case that $(\Upsilon_j - \Upsilon_i) > 0$ so we need to impose conditions on μ.

We rely on the assumption (see Section 2.1) that the constraint concerning how much capital an investor can invest is tight: that is, she would not want to invest some of her assets in a zero-yielding risk-free asset if given the chance. This means that, given the initial parameters, she would like to invest all wealth in risky (firm issued) assets. The assumption that we are in this "constrained" case implies that μ is high enough; we now calculate the precise condition on μ implied.

The portfolio shares are given by (7) and (8), where $S_{11} = \frac{R_{i,1}+\mu v_{i,1}}{\eta^2\theta w v_{i,1}^2}$ and $S_{12} = \frac{R_{j,2}+\mu v_{j,2}}{h^2\theta w v_{j,2}^2}$. Let $W_1 = M_1S_{11} + M_2S_{12}$ and $W_2 = M_1S_{21} + M_2S_{22}$. It is clear from the derivation of (7) and (8) that investors in country 1 are in the constrained case when $W_1 \geqslant 1$, and similarly for investors in country 2 and $W_2 \geqslant 1$. This implies, for W_1:

$$M_1\frac{R_{i,1}+\mu v_{i,1}}{\eta^2\theta w v_{i,1}^2} + M_2\frac{R_{j,2}+\mu v_{j,2}}{h^2\theta w v_{j,2}^2} \geqslant 1$$

or

$$M_1 \frac{\mu - w\theta\Upsilon_i}{\eta^2\theta w v_{i,1}^2} + M_2 \frac{\mu - w\theta\Upsilon_j}{h^2\theta w v_{j,2}^2} \geqslant 1$$

which gives a condition on the minimum $\mu_{1,\min}$ for investors from country 1 to be constrained:

$$\mu_{1,\min} \geqslant \theta w \frac{\left(1 + \frac{M_1}{\eta^2 v_{i,1}^2}\Upsilon_i + \frac{M_2}{h^2 v_{j,2}^2}\Upsilon_j\right)}{\left(\frac{M_1}{\eta^2 v_{i,1}^2} + \frac{M_2}{h^2 v_{j,2}^2}\right)}$$

For W_2, the minimum $\mu_{2,\min}$ for investors of country 2 to be constrained is then $\mu_{2,\min} \geqslant \theta w \frac{\left(1+\frac{M_1}{h^2 v_{i,1}^2}\Upsilon_i+\frac{M_2}{\eta^2 v_{j,2}^2}\Upsilon_j\right)}{\left(\frac{M_1}{h^2 v_{i,1}^2}+\frac{M_2}{\eta^2 v_{j,2}^2}\right)}$. Because both class of investors are assumed to be constrained, it suffices to take any of the minimum μ's to evaluate.

Returning to the evaluation of (22), the equation can be rewritten, assuming that $M_1 = M_2 = \frac{M}{2}$ and $\eta^2 = \zeta^2$, $h^2 = z^2$ (note that, under these assumptions, $\widehat{X}_1 = (\frac{M}{2})^{-1}\left(\frac{1}{v_{i,1}^2} + \frac{\eta^2}{h^2}\frac{1}{v_{j,2}^2}\right)^{-1}$, $\widehat{Y}_1 = (\frac{M}{2})^{-1}\left(\frac{1}{v_{i,1}^2} + \frac{1}{v_{j,2}^2}\frac{h^2}{\eta^2}\right)^{-1}$, $\widehat{X}_2 = (\frac{M}{2})^{-1}\left[\frac{h^2}{\eta^2}\frac{1}{v_{i,1}^2} + \frac{1}{v_{j,2}^2}\right]^{-1}$, $\widehat{Y}_2 = (\frac{M}{2})^{-1}\left(\frac{1}{v_{i,1}^2}\frac{\eta^2}{h^2} + \frac{1}{v_{j,2}^2}\right)^{-1}$):

$$\begin{aligned} & R_{j,2} - R_{i,1} \\ = & \left[\mu\left(v_{i,1} - v_{j,2}\right) - w\theta \frac{\left[\begin{array}{c}\left(\widetilde{X}_2 N_1 + \widetilde{Y}_2 N_2\right)\left[\frac{1}{\eta^2}N_1 + \frac{1}{h^2}N_2\right] \\ -\left(\widetilde{X}_1 N_1 + \widetilde{Y}_1 N_2\right)\left[\frac{1}{z^2}N_1 + \frac{1}{\zeta^2}N_2\right]\end{array}\right]}{\begin{array}{c}\left[\frac{1}{\eta^2}N_1 + \frac{1}{h^2}N_2\right]\left[\frac{1}{z^2}N_1 + \frac{1}{\zeta^2}N_2\right] \\ + \left[\frac{1}{z^2}N_1 + \frac{1}{\zeta^2}N_2\right]\left[\frac{1}{z^2}\widetilde{X}_1 N_1 + \frac{1}{\zeta^2}\widetilde{Y}_1 N_2\right]\frac{M_2}{v_{j,2}^2} \\ + \left[\frac{1}{\eta^2}\widetilde{X}_2 N_1 + \frac{1}{h^2}\widetilde{Y}_2 N_2\right]\left[\frac{1}{\eta^2}N_1 + \frac{1}{h^2}N_2\right]\frac{M_1}{v_{i,1}^2}\end{array}}\right] \end{aligned}$$

We evaluate this expression with the minimum μ for investors from country 1 and, after substitutions we obtain that

$$\frac{\left(1 + \frac{(\frac{M}{2})}{\eta^2 v_{i,1}^2}\Upsilon_i + \frac{(\frac{M}{2})}{h^2 v_{j,2}^2}\Upsilon_j\right)}{\left(\frac{(\frac{M}{2})}{\eta^2 v_{i,1}^2} + \frac{(\frac{M}{2})}{h^2 v_{j,2}^2}\right)}\left(v_{i,1} - v_{j,2}\right) - \left(\Upsilon_j - \Upsilon_i\right) > 0$$

Proof of Proposition 5.

For part a). The difference in expected return on assets between countries 1 and 2 is given by $(\mu v_{i,1} + R_{i,1}) - (\mu v_{i,2} + R_{i,2}) = w\theta\left(\Upsilon_{j,2} - \Upsilon_{i,1}\right)$.

Then

$$
\begin{aligned}
&(\Upsilon_{j,2} - \Upsilon_{i,1}) \\
= &\frac{\begin{pmatrix} \left(2v_{j,2}^{-1} - 1\right)\left(\widehat{X}_2 N_1 + \widehat{Y}_2 N_2\right)\left[\frac{1}{\sigma_1^2+\tau_1^2}N_1 + \frac{1}{\sigma_1^2+\tau_1^2(1+\gamma)}N_2\right] \\ -\left(2v_{i,1}^{-1} - 1\right)\left(\widehat{X}_1 N_1 + \widehat{Y}_1 N_2\right)\left[\frac{1}{\sigma_2^2+\tau_2^2(1+\gamma)}N_1 + \frac{1}{\sigma_2^2+\tau_2^2}N_2\right] \end{pmatrix}}{\begin{pmatrix} \left(\frac{N_1}{\sigma_1^2+\tau_1^2} + \frac{N_2}{\sigma_1^2+\tau_1^2(1+\gamma)}\right)\left(\frac{N_1}{\sigma_2^2+\tau_2^2(1+\gamma)} + \frac{N_2}{\sigma_2^2+\tau_2^2}\right) \\ +\left(\frac{N_1}{\sigma_2^2+\tau_2^2(1+\gamma)} + \frac{N_2}{\sigma_2^2+\tau_2^2}\right)\left(\frac{\widehat{X}_1 N_1}{\sigma_2^2+\tau_2^2(1+\gamma)} + \frac{\widehat{Y}_1 N_2}{\sigma_2^2+\tau_2^2}\right)\frac{M_2}{v_{j,2}^2}\left(2v_{i,1}^{-1} - 1\right) \\ +\left(\frac{N_1}{\sigma_1^2+\tau_1^2} + \frac{N_2}{\sigma_1^2+\tau_1^2(1+\gamma)}\right)\left(\frac{\widehat{X}_1 N_1}{\sigma_2^2+\tau_2^2(1+\gamma)} + \frac{\widehat{Y}_1 N_2}{\sigma_2^2+\tau_2^2}\right)\frac{M_1}{v_{i,1}^2}\left(2v_{j,2}^{-1} - 1\right) \end{pmatrix}}
\end{aligned}
\tag{23}
$$

where $\widehat{X}_2 = \frac{\sigma_1^2+\tau_1^2}{\sigma_2^2+\tau_2^2(1+\gamma)}\widehat{X}_1$ and $\widehat{Y}_2 = \frac{\sigma_1^2+\tau_1^2(1+\gamma)}{\sigma_2^2+\tau_2^2}\widehat{Y}_1$.

The sign of $(\Upsilon_{j,2} - \Upsilon_{i,1})$ depends on the numerator as the denominator is always positive. We can transform the numerator and, assuming $M_1 = M_2$ and simplifying, obtain the following expression:

$$
\begin{pmatrix} \frac{(2-v_{j,2})}{v_{j,2}}\left(N_1 v_{i,1}^2 + N_2 v_{i,1}^2 + N_1\frac{\sigma^2+\tau^2}{\sigma^2+\tau^2(1+\gamma)}v_{j,2}^2 + N_2\frac{\sigma^2+\tau^2(1+\gamma)}{\sigma^2+\tau^2}v_{j,2}^2\right)\left[\frac{1}{\sigma^2+\tau^2}N_1 + \frac{1}{\sigma^2+\tau^2(1+\gamma)}N_2\right] \\ -\frac{(2-v_{i,1})}{v_{i,1}}\left(N_1 v_{j,2}^2 + N_2 v_{j,2}^2 + N_1\frac{\sigma^2+\tau^2(1+\gamma)}{\sigma^2+\tau^2}v_{i,1}^2 + N_2\frac{\sigma^2+\tau^2}{\sigma^2+\tau^2(1+\gamma)}v_{i,1}^2\right)\left[\frac{1}{\sigma^2+\tau^2(1+\gamma)}N_1 + \frac{1}{\sigma^2+\tau^2}N_2\right] \end{pmatrix}
\tag{24}
$$

Note that, when $\gamma = 0$ or $\tau^2 = 0$, $\Upsilon_{j,2} - \Upsilon_{i,1} = 0$, so the sign of $\Upsilon_{j,2} - \Upsilon_{i,1}$ around $\gamma = 0$ (respectively, $\tau^2 = 0$) is indicated by the derivative of $\Upsilon_{j,2} - \Upsilon_{i,1}$ ($\frac{\partial(\Upsilon_{j,2}-\Upsilon_{i,1})}{\partial\gamma}$ and $\frac{\partial(\Upsilon_{j,2}-\Upsilon_{i,1})}{\partial\tau^2}$ respectively). By the form of (23) and (24), these are derivatives of the form $\frac{\partial}{\partial x}\left[\frac{f(x)-g(x)}{h(x)}\right] = \left[\frac{[f'(x)-g'(x)]h(x)-[f(x)-g(x)]h'(x)}{h(x)}\right]$, where, when $\gamma = 0$ or $\tau^2 = 0$, $f(x) = g(x)$. Hence, to sign these derivatives, it suffices to evaluate the derivative of (24).

Evaluating the derivative of the numerator (24) with respect to γ, with $\gamma = 0$, substituting for $\left[\frac{\partial(v_{i,1})}{\partial\gamma}\right]_{\gamma=0} = -\frac{2\frac{A}{\theta}N_2}{\left(2+\frac{A}{\theta}(N_1+N_2)\right)^2}\frac{\tau^2}{[\sigma^2+\tau^2]}$, $\left[\frac{\partial(v_{i,2})}{\partial\gamma}\right]_{\gamma=0} = -\frac{2\frac{A}{\theta}N_1}{\left(2+\frac{A}{\theta}(N_2+N_1)\right)^2}\frac{\tau^2}{[\sigma^2+\tau^2]}$, and noting that when $\gamma = 0$ we have $v_{i,1} = v_{j,2} = v$, we arrive at the condition

$$
\left((N_1 - N_2)\frac{2\frac{A}{\theta}}{\left(2+\frac{A}{\theta}(N_1+N_2)\right)^2}4(N_1+N_2)[N_1+N_2] \right)\frac{\tau^2}{[\sigma^2+\tau^2]^2} > 0
$$

Since $N_1 > N_2$, this is positive, so $\Upsilon_{j,2} - \Upsilon_{i,1}$ is negative close to $\gamma = 0$, as required. For $\tau^2 \to 0$ we do the same; we have $\left[\frac{\partial v_{i,1}}{\partial\tau^2}\right]_{\tau^2=0} = -\frac{2\frac{A}{\theta}\frac{\gamma}{\sigma^2}N_2}{\left(2+\frac{A}{\theta}(N_1+N_2)\right)^2}$, $\left[\frac{\partial v_{j,2}}{\partial\tau^2}\right]_{\tau^2=0} = -\frac{2\frac{A}{\theta}}{\left(2+\frac{A}{\theta}(N_2+N_1)\right)^2}\frac{\gamma}{\sigma^2}N_1$. Taking the derivative of (24), setting $\tau^2 = 0$, substituting for $\frac{\partial v}{\partial\tau^2}$ and noting that when $\tau^2 = 0$

we have $v_{i,1} = v_{j,2} = v$, we arrive at the condition

$$\left([N_1 - N_2] \, 4 \frac{2\frac{A}{\theta}}{\left(2 + \frac{A}{\theta}(N_2 + N_1)\right)^2} \gamma (N_1 + N_2) [N_1 + N_2] \right) \frac{1}{(\sigma^2)^2} > 0$$

Since $N_1 > N_2$, this is positive, so $\Upsilon_{j,2} - \Upsilon_{i,1}$ is negative close to $\tau^2 = 0$, as required.

Proof of part b). At the limit as $\gamma \to \infty$, financial autarky is reached and investors do not investor abroad. Then

$$v_{1,AUT} = \frac{\frac{A}{\theta} N_1}{2 + \frac{A}{\theta} N_1}$$

$$R_{1,AUT} = \mu \frac{2}{2 + \frac{A}{\theta} N_1} - wA \frac{\left(\sigma^2 + \tau^2\right)\left(4 + \frac{A}{\theta} N_1\right) N_1}{M_1 \left(2 + \frac{A}{\theta} N_1\right)^2}$$

$$v_{2,AUT} = \frac{\frac{A}{\theta} N_2}{2 + \frac{A}{\theta} N_2}$$

$$R_{2,AUT} = \mu \frac{2}{2 + \frac{A}{\theta} N_2} - wA \frac{\left(\sigma^2 + \tau^2\right)\left(4 + \frac{A}{\theta} N_2\right) N_2}{M_2 \left(2 + \frac{A}{\theta} N_2\right)^2}$$

If $M_1 = M_2 = \frac{M}{2}$ then

$$(\mu v_{i,1} + R_{i,1}) - (\mu v_{i,2} + R_{i,2}) = \frac{wA\left(\sigma^2 + \tau^2\right)}{\frac{M}{2}} \left[\frac{\left(4 + \frac{A}{\theta} N_2\right) N_2}{\left(2 + \frac{A}{\theta} N_2\right)^2} - \frac{\left(4 + \frac{A}{\theta} N_1\right) N_1}{\left(2 + \frac{A}{\theta} N_1\right)^2} \right]$$

Since $\left[\frac{\left(4 + \frac{A}{\theta} N_2\right) N_2}{\left(2 + \frac{A}{\theta} N_2\right)^2} - \frac{\left(4 + \frac{A}{\theta} N_1\right) N_1}{\left(2 + \frac{A}{\theta} N_1\right)^2} \right] < 0$ the expression is negative.

Proof of Proposition 6. Express $\frac{k_2}{k_1} = \left(2 + \frac{A}{\theta} \frac{1+\chi}{1+\chi(1+\gamma)} N_1\right)\left(2 + \frac{A}{\theta} N_1\right)^{-1}$ where $\chi = \frac{\tau^2}{\sigma^2}$. Then $\frac{\partial}{\partial \chi}\left(\frac{k_2}{k_1}\right) < 0$.

A.3 An Analysis using the Gilboa and Schmeidler (1989) model

A similar analysis could be conducted using the Gilboa and Schmeidler (1989) Maxmin Expected Utility ambiguity model, yielding similar results. In this model, decision makers do not have a single probability distribution P for the relevant issues (in this case, the return of the uncertain asset), but a set of such distributions, $\mathcal{C}$. Decision makers choose an asset x to maximize:

$$V^{GS}(x) = \min_{P \in \mathcal{C}} \mathbb{E}_P(u(x)) \tag{25}$$

Similarly to the interpretation of the smooth model given in Section 2.1, the set $\mathcal{C}$ can be thought of as representing the decision maker's uncertainty about the correct parameter values governing the stochastic process determining asset returns, or his model uncertainty. Unlike the smooth model, however, there is no (clear) separation of ambiguity and ambiguity attitude in this model, there being only one relevant "parameter" in the model, $\mathcal{C}$.

To conduct the analysis, we adopt the following specification. As in the case studied in the main text, we assume that an investor considers that there are several possible distributions for the stochastic return x_n ran by a firm n in a foreign country, all of which follow a normal distribution. However, we assume that he is sure about the expected return μ_n, but not about the variance σ_n^2. We assume that the set of variances he envisages are those which are within a distance δ_n from a particular "reference value" σ_n. So, the set $\mathcal{C}$ is the set of distributions with $\tilde{x_n} \sim N(\mu_n, \sigma_n^2 + \epsilon)$ with $\epsilon \in [-\delta_n, \delta_n]$. σ_n^2 can be thought of as the investor's "best guess" for the variance; δ_n, which parametrizes the "size" of the set $\mathcal{C}$, can be thought of as representing the decision maker's ambiguity aversion (or ambiguity; as noted above, there is no distinction in this model). We assume, as in the case studied above, that investors perceive no ambiguity (or, equivalently here, are not ambiguity averse towards) home assets: for a home asset m, $\delta_m = 0$ and the investor uses a single probability distribution. Moreover, we assume that home and foreign investors have the same σ_n^2 (as well as the same μ_n): this is the equivalent of the assumption in Section 2.1 that they have the same reduced distribution. As was noted, this is consistent with all investors having the same information, but there being a difference in ambiguity (or ambiguity attitude) with respect to foreign assets. As above, we assume that each investor has constant absolute risk aversion, and hence a utility function of the form $u(z) = -(1/\theta)e^{-\theta z}$ where $\theta > 0$ represents the degree of (absolute) risk aversion. All investors are assumed to have the same risk aversion.

Using equation (3), the investor's problem is to maximize:

$$V_{l1}^{GS}(\alpha_1) = \min_{\substack{\epsilon_j \in [-\delta_j, \delta_j], \\ M_1+1 \leqslant j \leqslant M}} -(1/\theta) e^{-\theta w \left(\begin{array}{l} \sum_{i=1}^{M_1} (\alpha_{li}(v_i \mu_i + R_i) + (\theta w \sigma_i^2/2) v_i^2 \alpha_{lj}^2) \\ + \sum_{j=M_1+1}^{M} (\alpha_{lj}(v_j \mu_j + R_j) + (\theta w (\sigma_j^2 + \epsilon_j)/2) v_j^2 \alpha_{lj}^2) \end{array} \right)} \tag{26}$$

Solving for the portfolio allocation for an investor in country 1, under the other assumptions made and notation used in Section 3.1, gives:

$$\alpha_{11}^{GS} = S_{11}^{GS} + \frac{1 - M_1 S_{11}^{GS} - M_2 S_{12}^{GS}}{\left(M_1 + M_2 \left(\frac{\sigma_1^2}{\sigma_2^2 + \delta_{12}} \right) \frac{v_{i,1}^2}{v_{j,2}^2} \right)} \tag{27}$$

$$\alpha_{12}^{GS} = S_{12}^{GS} + \frac{1 - M_1 S_{11}^{GS} - M_2 S_{12}^{GS}}{\left(\frac{v_{j,2}^2}{v_{i,1}^2} \left(\frac{\sigma_2^2 + \delta_{12}}{\sigma_1^2} \right) M_1 + M_2 \right)} \tag{28}$$

where $S_{11}^{GS} = \frac{R_{i,1} + \mu v_{i,1}}{\sigma_1^2 \theta w v_{i,1}^2}$ and $S_{12}^{GS} = \frac{R_{j,2} + \mu v_{j,2}}{(\sigma_2^2 + \delta_{12}) \theta w v_{j,2}^2}$.

Noting that S_{11}^{GS} and S_{12}^{GS} are equal to S_{11} and S_{12} (Section 3.1) once one replaces $\sigma_1^2 + \tau_1^2$ by σ_1^2 and $\tau_1^2 \gamma$ by δ_1, it is clear that the solutions and results in the previous sections translate immediately into results for the maxmin expected utility model. In particular, the behavior of parameters with changes in risk (σ^2) is the same in the smooth and the maxmin expected utility models; and changes in δ have precisely the same effects as changes in γ. There is no natural equivalent of the behavior obtained from changes in τ^2 in the maxmin expected utility model; this is to be expected, given that the distinction between ambiguity and ambiguity aversion is absent in this model. That there are interesting effects of changes in τ^2 (see for example Sections 3.3 and 3.4.1) may indicate the economic utility of the analysis using the smooth ambiguity model, which does incorporate such a distinction.

A.4 Investors ambiguous towards all assets

Suppose that investors now are ambiguity averse towards all assets, also domestic ones with the same degree as towards the foreign ones[24]. The investor's problem can be solved now as in Section 2 with the assumption that the investors have a non-degenerate second-order distribution over the first-order mean of the production process in the case of domestic companies as well. Foreign and domestic assets are now perceived as exactly the same if they share the same σ^2 and τ^2.

One can evaluate the investors utility $V_{l1}(\alpha_1)$ now as:

$$V_{l1}(\alpha_1) = -\frac{1}{\theta^{(1+\gamma)}(1+\gamma)} e^{-\theta w (1+\gamma) \left(\begin{array}{c} \sum_{i=1}^{M_1} \left(\alpha_{li} (\mu_i v_i + R_i) - \frac{(\sigma_i^2 + \tau_i^2 (1+\gamma)) \theta w}{2} [\alpha_{li} v_i]^2 \right) \\ + \sum_{j=M_1+1}^{M} \left(\alpha_{lj} (\mu_j v_j + R_j) - \frac{(\sigma_j^2 + \tau_j^2 (1+\gamma)) \theta w}{2} [\alpha_{lj} v_j]^2 \right) \end{array} \right)}$$

We solve the investor's problem when he invests all his wealth into assets as before in Sections 2 – 3. We assume again that all firms in a given country share the same (σ, τ) characteristics. The firm's problem is the same as before only that now it faces only ambiguity averse investors. We can solve for the contract terms v and R easily and for a representative

[24]Detailed calculations available in the supplementary materials.

firm in country i we have

$$v_i = \frac{\frac{A}{\theta}\frac{\left(\sigma_i^2+\tau_i^2\right)}{\left(\sigma_i^2+\tau_i^2(1+\gamma)\right)}\left(N_1+N_2\right)}{2+\frac{A}{\theta}\frac{\left(\sigma_i^2+\tau_i^2\right)}{\left(\sigma_i^2+\tau_i^2(1+\gamma)\right)}\left(N_1+N_2\right)} \tag{29}$$

$$R_i = \mu\left(1-v_i\right) - \frac{\left(2-v_i\right)\theta w}{\left[\frac{1}{\left(\sigma_i^2+\tau_i^2(1+\gamma)\right)v_i}M_1+\frac{1}{\left(\sigma_j^2+\tau_j^2(1+\gamma)\right)v_j}M_2\right]} \tag{30}$$

Then $\frac{\partial v_i}{\partial \sigma_i^2} > 0$. Firms with more risky processes will try to offload more equity to investors (be they domestic and foreign). This is the same intuition as in the standard portfolio model with risk. However, the impact of more ambiguity is reverse as $\frac{\partial v}{\partial \tau_i^2} < 0$: those companies that are more opaque in the eyes of investors keep more risk to themselves.

We have

$$R_j - R_i = \left(v_i - v_j\right)\left[\mu - \theta w \frac{1}{\left[\frac{1}{\left(\sigma_i^2+\tau_i^2(1+\gamma)\right)v_i}M_1+\frac{1}{\left(\sigma_j^2+\tau_j^2(1+\gamma)\right)v_j}M_2\right]}\right]$$

and $\left[\mu - \theta w \frac{1}{\left[\frac{1}{\left(\sigma_i^2+\tau_i^2(1+\gamma)\right)v_i}M_1+\frac{1}{\left(\sigma_j^2+\tau_j^2(1+\gamma)\right)v_j}M_2\right]}\right] > 0$ (we assume investors are constrained and do not wish to invest in a zero-yielding risk-free asset).

Therefore if $v_i > v_j$ then $R_i < R_j$. We conclude that if a firm is more opaque in the eyes of the investors (has a higher τ) ceteris paribus it will issue contracts with a lower equity participation and a higher interest rate. Consequently, it will offer more fixed debt than equity. This helps to understand why countries that are more opaque would issue more fixed-debt contracts and as more is learnt about the stochastic processes governing their production the share of equity financing should increase. This does not help, however, to explain the pattern in Figure 1 – why in general capital importers would issue more debt than exporters also for OECD, European Union or Eurozone countries (unless they are more opaque, of course). For this it suffices for investors to exhibit *more* ambiguity aversion towards foreign assets as is assumed throughout Sections 2 – 3.