Agent-Based Computational Models - A Formal Heuristic for Institutionalist Pattern Modelling?

Claudius Gräbner*

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Abstract

This paper investigates the consistency of agent-based computational models with the institutionalist research program as outlined by Myrdal, Wilber and Harrison, Hodgson and other institutionalists and discusses whether such models can be a useful heuristic for "pattern modelling".

I study the ability of agent-based computational models to provide a holistic, systemic and evolutionary picture of the economy, the conception of agents in ACE models, and discuss potentials and challenges of their application in institutionalist research.

Keywords: Original Institutional Economics, Methodology, Epistemology, Ontology, Agent-Based Computational Economics

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1. Introduction

Institutionalists have always been criticizing the neoclassical way of modelling the economy, especially because of its obsession to a strict formalism. On the other hand, there have been a number of attempts to introduce more formal modelling tools to institutionalist economics, including the social fabric matrix (Hayden, 1982), system dynamics (Radzicki, 1988) and evolutionary game theory (Elsner, 2012). In this article I

*Claudius Gräbner, Institute for Insitutional and Innovation Economics (iino), University of Bremen, Hochschulring 4, D-28359 Bremen, Germany. Email: graebnerc@uni-bremen.de

discuss whether the framework of agent based computational economic (ACE) models can provide a useful formal extension to the research program of original institutional economics (OIE). ACE models are commonly seen as a typical tool of the rising research program of complexity economics.[1] While some of consider complexity economics to be an interesting extension to neoclassical economics (e.g. Durlauf (2005)), others consider them to belong to a completely new way of thinking about economics (Arthur, 2013) and critisize the "analytical straitjacket" of neoclassical economics from a complexity perspective (Farmer, 2012). The relation between original institutional economics (OIE) and complexity economics is largely unexplored. Many concepts of complexity economics, though, have been anticipated by institutionalists: Although often using a different vocabulary, complexity economists speak about cumulative causation, realistic agents, dynamic relations among individuals and the necessity to see the economy as an organic whole rather than from an atomistic and reductionist perspective. Consequently, ACE models should not be left unconsidered by institutionalist economists and the study of whether ACE models can be a useful tool for institutionalist research may give hints about potential convergences of institutionalist and complexity economics.

This paper investigates whether the use of ACE models is consistent with the research program of original institutional economics (OIE) as it was outlined by Myrdal (1978), Wilber and Harrison (1978), Hodgson (1988) and other institutionalists. After giving a short introduction to the ACE framework in Section 2, I will study the compatibility of ACE with key aspects of institutionalist modelling in Section 3. Then main potentials and dangers of ACE modelling are discussed in Section 4. Section 5 concludes and points to directions for future research.

2. What Agent-Based-Computational Models are about

ACE models are expressed via a programming language and help to understand how individual actions lead to patterns, how these patterns in turn shape individual behaviour and what dynamics result from this interplay on the level of the societal system as a whole. Here I focus on the general idea behind ACE models. The interested reader might consult appendix A for a more technical description of the formal structure of ACE models.

ACE models differ from the strict analytical framework of conventional economics as the modeller is not forced to make assumptions in such a way that an equilibrium path results in the model. They allow a realistic representation of the system under investigation in the sense of an evolutionary science according to Veblen (1898).

The basic idea is to specify the fundamental entities (esp. the economic agents and their relations) in an adequate manner and to study the systemic and dynamic consequences of the configuration. Because the resulting system is usually very complicated, one relies on simulation to solve it. Logically one proceeds from the assumptions about the system to the conclusions regarding the overall dynamics. This contrasts the practice in general equilibrium modelling (esp. Computable and Dynamic Stochastic General Equilibrium

[1] Although not all complexity economists agree to the use of ACE models, see e.g. Durlauf (2006).

modelling): While these models are said to be microfounded, one has to specify the assumptions on the micro level not solely based on their adequateness, but in a way such that they stay mathematically tractable and are suitable to yield a stable equilibrium for the overall dynamic. So while the equilibrium is formally a conclusion of the model, it is better seen as an implicit macro assumption that dominates the micro assumptions of these models.

ACE models on the other hand can be evaluated on all levels: The model agents for example are suited for *microcalibration*. This involves a direct test of the adequateness of the agent design, e.g. through the consultation of field experts (see Section 3.3). Because the behavioural specification of the agents is done via computer code, there is no upper limit for the complexity of the rules other than accountability considerations ((Chen, 2012), see also Section 3.3). More generally, ACE models allow heterogeneous and boundedly rational agents in the sense of Herbert Simon (rather than in the sense of modern behavioural economics) that are not atomistic, but directly interdependent and socially embedded, as proposed especially by sociologists since Granovetter (1985). The embeddedness is modeled via an underlying, possibly changing, graph. Such graph could represent a simple grid or an (potentially empirical) interaction structure among the agents. Thus the interdependence of the economic agents is modelled explicitly via the underlying spatial structure so that group formation and dynamic power relations among the agents can explicitly be taken into account. Agents can also be capable of communicating with each other and to hide or share information with each other, with important consequences for the overall dynamic of the system (Moss, 2002).

The agents do not have to be represented via convenient equations but are more intuitively specified via a computer language: Agents are instantiated as a digital object that has attributes and different rules (called "methods") according to which these attributes change. Such a specification of the agents allows the natural implementation of heuristics, learning behaviour and habits into the methods of the agent objects.

On the meso level, ACE models allow the natural inclusion of institutions, rules and networks (Elsner and Heinrich, 2009). This is the case as the methods of the agents use not only the current state of the agent itself as an input, but may also consider the states of her neighbours, a group of agents or the state of the system as a whole.

It is therefore straightforward to study phenomena such as reconstitutive downward effects in these models: Consider the model of Hodgson and Knudsen (2004) as an example. The authors study the emergence and evolution of a simple traffic convention.

In their model, agents drive cars on a ring structure, half of them clockwise the other half anti-clockwise. Every round, each driver has to decide whether she wants to drive on the right or the left. The authors clarify that the experimentation with different decision rules in their model helped them to identify a surprisingly easy, but very effective decision procedure (Hodgson and Knudsen, 2004, p. 23): Drivers develop a habituation of driving either on the left or right side and the model shows how the presence of habituation fosters a convergence to a drive-left or drive-right convention. The model also shows that habit formation is not the only relevant mechanism, but in the end a combination of mechanisms leads to the emergence of the convention. Due to the modular structure of their ACE model, the authors were also able to study what happens if habit is substituted by pure inertia, and that the functioning of institutions is best interpreted as influencing habits rather than behaviour or preferences. This shows how ACE models can be used to study different mechanism and their mutual influences on each other in one coherent model.

In general, the macro dynamics of the ACE model are the result of the (non-trivial) aggregated behaviour of direct interdependent agents. It is an interesting consequence of the generality of the agent-based approach that it contains the formal models of neoclassical economics as one particular special case.

3. ACE Models and the Methodology of OIE

The tradition of OIE has its direct origins in the work of Thorstein Veblen, John R. Commons and Gunnar Myrdal among others.[2] Because the critique of the neoclassical approach to economics has always been an important aspect of institutionalism, it is not straightforward to identify the methodological core of this vital and pluralistic research program.

A very good starting point is the classical paper of Myrdal (1978) under the heading "What is institutionalism?". In the same year, Wilber and Harrison (1978) published a paper about the methodology of institutionalist economics in which they characterized the institutionalist way of modeling as *pattern modelling* in which they come to very similar answers as Myrdal (1978). The criteria identified by the authors are still representative for the way most economists identifying themselves as original institutionalists work and I will use them as a starting point for the question of whether ACE models can play a role in institutionalist economics today.

Five main criteria can be identified: the models are necessarily *holistic*, *systemic*, and *evolutionary* and they pay particular attention to *conflict and power relations* within a society while being based on a *realistic conception of economic agency* and therefore reject the models based on the neoclassical conception of rational individuals. I will now scrutinize these points one by one.

3.1. Holism: The Relevance of Downward Effects

Wilber and Harrison (1978) explicitly distinguished between *holism* and *systemism.* For them holism, which is considered to be the opposite of *atomism*, entails a focus on the pattern of relations among the agents and the economy as a whole (Wilber and Harrison, 1978, p. 71). This expresses the belief that the whole is not only greater than the sum of its parts, "but that the parts are so related that their functioning is conditioned by their interrelations" (Wilber and Harrison, 1978, p. 74). Their distinction to systemism, according to which they expect the economy to show patterns that emerged from the joint behaviour of the agents but that cannot be derived from the behaviour of a single agent in isolation, is not entirely clear.

[2] The indirect roots go back in history, of course. Veblen received important influence from pragmatist philosopher Charles Peirce (Hall and Whybrow, 2009).

When discussing holism in the social sciences, epistemological and ontological arguments are too often mixed up: If holism has an ontological meaning, the study of agency, individual incentives and the relation among the parts making up the whole becomes unnecessary. Such a view must not be compatible with institutionalist theory. Institutionalists have always stressed the learning capacities of individuals, the variety of reasons guiding their decision making and Veblen himself stressed the individual's instinct of workmenship and their idle curiosity. Such concepts are worthless to the ontological holist as they would be mere derivatives of the social structure in which the individuals exist. More adequately, holism is understood in the epistemological sense: In order to understand the behavior of individuals, a deep understanding of the social structure into which they are embedded is required. This is what Wilber and Harrison (1978, p. 71) mean when the argue that the parts of the system under investigation must make up a coherent whole and must be understood in their relation to the whole. Their use of the the concept of holism suggests that both the relations among individuals and the relation between different ontological levels of the economy are important. This idea is most precisely developed in the institutionalist concept of *reconstitutive downward effects* (Hodgson, 2002, 2006, 2011) according to which individuals, groups and the entire population are strongly interconnected and none of the three can be successfully understood without considering the other two. Or, in the words of Wilber and Harrison (1978, p. 71): "The process of social change is not purely mechanical; it is the product of human action, but action which is definitely shaped and limited by the society in which it has its roots."

Institutionalists use the concept of reconstitutive downward effects to study patterns that emerge because of the fact that different ontological levels of the economy are strongly interdependent. These *emergent* patterns then shape the consciousness and behaviour of the agents on the individual level again. They are independent from the support of the single individual agent but can only sustain if they are supported by a critical mass of agents. Because the existence of these effects has its reasons on the lower micro level, but this micro level is influenced by the effects themselves and still necessary for the persistence of these patterns, they are called *reconstitutive* downward effects (see the model of Hodgson and Knudsen (2004) outlined in Section 2). Following the current conventions, a theory considering *reconstitutive* downward effects would not be termed holistic, but *systemic*. Wilber and Harrison (1978) made use of the term holism probably to reject neoclassical individualism, but individualism is also rejected by systemism alone (Bunge, 2000). Can ACE models be consistent with a view of the economy that stresses the mutual interdependence of its different layers? Brian Arthur, one of the leading figures of the complexity movement in economics and an advocate of ACE models,[3] described the process in the models like this: "Behaviour creates pattern; and pattern in turn influences behaviour" (Arthur, 2006, 1552). This is the same as to say that "parts are at once conditioning and conditioned by the whole" (Wilber and Harrison, 1978, p. 80). In an ACE model one specifies the agents and how they behave in

[3]He advocates ACE models not for their own sake, but because frequently no analytical alternative exits.

certain situations. The trigger for their behaviour can be, as explained above, their own state, the state of their direct environment, the state of a certain group or the state of the global system. As other agents, groups and the system as a whole are also influenced by the agent herself, it is straightforward to see how the concept of interdependent levels can be accounted for in ACE models.

3.2. Systemism: Organized Complexity as a Central Property of Social Systems

While neoclassical economics requires all models to be microfounded in the sense that the whole must be understood by reducing it entirely to the level of its individual parts (and is therefore to be classified as an individualistic framework), institutionalist models try to explain the whole by its parts, but not by the parts in isolation but by their interaction with and by their inter-relation to the whole itself.

They reject the atomistic perception of neoclassical economics not only because they claim the importance of the interrelation among the different levels of the economy, but also because of the importance of the relation among the different entities on the individual level, the economic agents.

Some might respond that state-of-the-art neoclassical models include heterogeneous agents and deal with questions of network structure. This objection is misplaced in many ways, but most importantly, although there are many general equilibrium models including either heterogeneous agents or an explicit network structure, it has been shown that the simultaneous presence of heterogeneity and an explicit network structure poses significant difficulties to general equilibrium models (Page, 2012). Furthermore, the systemic conception is not only about introducing new facts and mechanisms into the models, but also to understand how different mechanisms influence each other in their consequences.

The idea underlying the institutionalist conception of the economy and their focus on the relation among the individual components seems to be strongly related to the reasoning of Warren Weaver about simple and complex scientific problems: According to Weaver (1948) *simple* problems include only very few variables and were studied by pre-1900 physics and engineering. All problems involving living organisms can never fall into this category as they involve many different aspects and cannot be studied under the *ceteris paribus* assumption because of the interrelatedness of the variables (Weaver, 1948, p. 537-538). What Weaver has taken for granted is obviously not accepted by many economists who make extensive use of *ceteris paribus* assumptions when studying social systems. Weaver then further distinguished between *organized* and *disorganized* complexity:

A system consisting of many components shows disorganized complexity, if some emergent pattern exists because the linear interactions between the different elements smooth each other out. The Law of Large Numbers can be interpreted as such an emergent pattern. Econometric theory generally assumes this kind of complexity when it assumes error terms to be identically and independently distributed.

In contrast, a system showing organized complexity shows patterns, which emerge because the interactions of the different elements do *not* smooth each other out (i.e. are non-linear). This is the case if there exists a kind of self-organization of the system such that the factors are interrelated into an organic whole (Weaver, 1948, p. 539).

When arguing for the need of systemic models, institutionalists implicitly say that the economy exhibits organised complexity.

The analytical models of neoclassical economics in contrast presume the economy to show disorganized complexity. Their unambiguous results can only be obtained by assuming mechanical agents that interact in a linear fashion.

Many ACE models are motivated with the argument that the economy exhibits organised complexity (Miller and Page, 2007). The implementation is straightforward: Heterogeneous agents interact with each other and their environment. As there is no requirement for the system to exhibit a particular dynamic (esp. an equilibrium path), assumptions can be made on entirely proper considerations. One can then conduct artificial experiments by changing an aspect of the model and check whether an emergent pattern is the result of the change or not. One can model the system with an adequate specification without a compulsive formalism, but with the obligation to state any assumed process explicitly.

Examples of where ACE models were successfully applied include the paper of Hodgson and Knudsen (2004), but also Elsner and Heinrich (2009), where the authors study the meso level of the economy: The authors study the emergence of a cooperative institution in a group smaller than the whole population and assess the importance of different circumstances and mechanisms such as monitoring, reputation, partner selection and different agencies. The ACE framework does not only allow the joint consideration of these mechanisms and to understand their mutual influence, but also to overcome the dichotomy between micro and macro level.

If one studies the economy from a systemic viewpoint, it becomes inevitable to consider to relax or drop assumptions about fictitious central planning mechanisms such as the Walrasian auctioneer, but to study the economy as a *self-organizing* system *without* central control. In this case, one has to deduce the overall dynamics from the interaction of its constituent parts and has to account for the interplay of the different levels, which is exactly what ACE models were invented for (see Section 3.4.

In particular, such a perspective allows the study of *self-organizing criticality*, which is a property of open (i.e. dissipative) systems without a central control that are characterized by many interacting agents. Only from a systemic perspective can one understand this concept that is in the tradition of Granovetter (1985) who argues that most economic behaviour is closely embedded in networks of interpersonal relations and uses this argument to criticize neoclassical economics from a sociological perspective.An open system shows self-organized criticality if the interactions of the agents yield a state of the system that is robust to small changes in the agents behaviour, but frequently experiences "avalanches" of change, after the individual interactions cumulated in a specific way. The concept is most famously related to the example of a sandpile to which one can add single grains without anything happening, but if a certain threshold of grains is passed, many grains fall down the pile (Bak, 1996). An economic example is the study of fluctuations of markets as a consequence of factor-demand linkages among different producers, i.e. if agents produce and demand goods simultaneously (Scheinkman and Woodford, 1994). Self-organized criticality cannot be explained with neoclassical theories as neoclassical agents are not socially embedded, are not able to communicate with each other, are rational utility maximizing individuals, and because the equilibrium concept is incompatible with that of self-organized criticality. Furthermore, self-organized criticality only emerges in open systems, while neoclassical economic focuses on closed systems. Therefore, models studying self-organized criticality were mainly of an agent-based nature and while the concept is heavily used in the natural science (and was frequently proposed by physicists and biologists to understand financial crises and financial markets), its application in the social sciences seems to be negligible. It is, however, an interesting concept for institutionalists and an example of how ACE models might help to include a concept fruitfully into more general institutionalist theories.

3.3. Evolution and Agency

Institutionalists have been criticizing the static nature of neoclassical economics ever since.

As an alternative to neoclassical equilibrium analysis they developed concepts such as *circular cumulative causation*, *path dependence* and *reconstitutive downward effects* to explain the overall dynamics of the economy.

ACE models have already proofed their ability to include everlasting change of behaviour (Lindgren, 1992; Edmonds, 1999; Arthur, 2006). One reason is their ability to constitute non-linear dynamical systems that exhibit non-ergodic properties, something that is not possible in current neoclassical models. In this way, they are perfectly suitable to resemble the principle of (circular) cumulative causation and the path dependence of real world dynamics.

But the evolutionary flavour of ACE models is present not only on the aggregated level: The agents themselves are not static and rational, but can be boundedly rational and adaptive. Their reasoning is not necessarily deductive, but, following psychological evidence, can be inductive and based on heuristics. They are not isolated representative entities, but active and communicating, socially embedded agents (Edmonds, 1999). An adequate representation of the economic agents is not only an important tool to make models evolutionary in the institutionalist sense and to allow for an explicit and dynamic representation of direct interaction (and power relations) in the model economy. It is also important to contrast the instrumentalist use of rational (and often representative) agents of neoclassical economics with a representation of the economic actor that involves an adequate level of descriptive accuracy.

Agents in institutionalist models usually have a thirst for power and adventure, a sense of independence, their motivations include altruism, their instinct of workmenship, idle curiosity, custom, and habit depending on the situation they face. Consumption decisions are explained not by the empty concept of utility maximization, but with concrete motives that frequently contrast the rationality assumptions of neoclassical economics, e.g. via the concept of conspicuous consumption and waste (see Rengs and Wäckerle (2014b) for a concrete example).

Chen (2012) gives an introduction to the various historical origins of ACE modelling and the different approaches to the representation of economic agents in such models. One practical example is the CRISIS project that tries to develop an ACE model of the European Union in order to get a deeper understanding of the financial crisis. As in many ACE projects, the modellers make extensive use of *microcalibration* (see Section 2): This includes the consultation of field experts or the substitution of the artificial agent with a human being acting in a laboratory to assess the adequateness of behavioural assumptions.

Neoclassical economics have never been interested in the realism of their agents which is why they generally reject microcalibration. The utility-maximising homo oeconomicus was built in a way such that mathematical analysis gets as easy and intuitive as possible. Departing from the standard utility-maximizer, the preference relations become more and more complicated and dynamics were introduced by allowing agents to maximize their utility inter-temporarily. This is a sad example of what Fontana (2010) recently termed the "oil-spot dynamic". When the unrealistic conception of the economic agent by neoclassical economics received more and more critique, most prominently by Herbert Simon and his conception of bounded rationality, neoclassical economists reacted by building more and more *complex* utility functions including social preferences or by making the optimization problem more complicated by adding certain "decision defects".

What really was the essence of Simon's ideas of bounded rationality is that agents do not have the computational capacity to maximize their utility and therefore will never do so. They will employ heuristics to cope with the complexity of their environment, rely on institutions and make their decisions more inductively than deductively.[4]

Especially the work of Herbert Simon highlighted how adequately the reasoning of humans can be represented via computational machines. His results have been important for psychologists and logicicists since the "logical theorist"[5] and show why ACE models are well suited to study the effects of realistic decision procedures. Consider the classical model of Arthur (1994) as an example: There is a population with 100 agents who all consider to visit a bar. Every time step the agents have to decide whether they go or stay at home. If there are more than 60 agents in the bar, it will be overcrowded and the evening is spoiled for everybody. If there are fewer agents, everyone attending enjoys the evening. Arthur shows that rational agents are not able to solve this problem, but inductive agents are. In the model, agents can choose between different (inductive) forecasting rules, and the system self-organizes to a configuration where constantly approximately 60 agents visit the bar.

One could easily include more sophisticated decision procedures into the models, e.g.

[4]Velupillai and Zambelli (2011) used the term "modern behavioural economics" to contrast the neoclassical approach to bounded rationality in contrast to the "classical" approach from Herbert Simon and Alan Turing, among others.

[5]The logical theorist was one of the machines built by Herbert Simon and Allen Newell that were able to proof mathematical theorems and solve logical puzzles *on their own* and in the same manner than humans do.

genetic algorithms. A genetic algorithm is a heuristic that helps to solve optimization problems in a *satifsactory* way. They are applied if a straightforward solution to an optimization problem is not feasible. A genetic algorithm starts with a set of possible solutions to a problem, evaluates them according to a criterion, combines them randomly based on their performance and evaluates the resulting combinations again. By proceeding this way, the results usually become better and better. Our organic world is full of examples of genetic algorithms. They can explain very well how certain instincts and behavioural habits have come into existence (Mitchell, 1999).

When employed to agents, they can help to simulate the learning behaviour of agents and their way of adapting to their environmental requirements. This is one source for the ability of ACE models to describe non-reversible dynamics and thus to resemble the principle of (circular) cumulative causation and the path dependence of real world dynamics. The application of genetic algorithms always involves the danger to "get stuck" with bad solutions, i.e. to get *locked in*, but rather than seeing this critically it should be seen as a realistic feature of the model, exactly in the spirit of institutionalist pattern modelling.

3.4. Social Structure and Power

The distribution of power in society and its economic, social, and political stratification has always been an important institutionalist research area (Myrdal, 1978, 774). The explicit consideration of network and power structures is already a prerequisite for a model to be holistic and systemic, but as this aspect has been highlighted by institutionalists again and again it will receive particular attention in this article as well.

The study of networks has been a lively area of research and developed a plausible taxonomy for empirical real-world networks and provided several proposals for theories of how these networks could have come into existence. Institutionalists should build on these insights and embed them into a broader theory about the society, especially because networks are difficult to discuss in a purely verbal analysis as they are difficult to describe verbally and the relation between network structure and the economic outcomes is usually not intuitive, especially when the network structures are non static, but changing. In this case simulations are a very strong ally in visualizing the processes underlying real-world dynamics.

Much use of ACE models was motivated by the wish to take the underlying social structure of an economy explicitly into account and not to follow the implicit practice of neoclassical economics to assume complete (or trivial) networks. By the assumption of the representative agent or at least many identical agents, one assumes implicitly a complete network because all agents have to be the same. And the complete network is the only network structure where each agent has exactly the same neighbourhood, namely all other agents. But recent studies of networks have shown that small-world or scale-free networks are much more likely to exist in reality.[6] Small world networks are

[6]The issue is much more complex than presented here and empirical networks can be described using many different statistics, but for the sake of the argument, the following coarse grained description

characterized by small average path lengths between the nodes and comparatively high degree of clustering. The constituent feature of scale free networks is that there are very few nodes with a lot of connections to other nodes, and very many nodes with very few connections to other nodes. More precisely, the distribution of the number of neighbours (i.e. the degree of the nodes) follows a power law. How these networks influence the distributional properties of an economic system can be studied via simulations, and even if they cannot provide a complete explanation, they are necessary to deal with such abstract structure as networks. Furthermore, the existence of certain network structures (e.g. particular scale-free networks) were proposed to give rise to self-organised criticality, a concept that should be of interest for institutionalists (see Section 3.2). Consider the following examples illustrating the usefulness of ACE models in this context: Albin and Foley (1992) and Gintis (2007) simulated the distributional effects of changing network structures in the general equilibrium framework and showed how a shift from central to de-central organization has severe distributional effects. Their models remained very abstract and one would not classify them as institutionalist models. They exemplify, however, how big the consequences of a small change in the underlying network structure can be. This insight is important for institutionalists when describing the stratification of real-world economies, because networks probably play an important role for the observed stratification (and unequal distribution of wealth). In order to figure out how this role looks like, one must build on simulations of these network structures, particularly because network structures are a catalysator to other aspects (Page (2012), see above). To have these results in mind is important for the construction of purely verbal models as well. We live in a world where economic networks of directly interdependent agents acting without any central control become more and more important. The increasing fragmentation of valued added chains, the growing importance of network-based information and telecommunication technologies, and the ever more centralized industrial structure with few huge corporations and many smaller, globally dispersed, sub-contractors (and the resulting hub and spoke networks) make it essential to pay attention to the underlying network structure in the economic system under consideration. To *explain* rather than just to describe the role of network structures, simulations can be a powerful heuristic. Institutionalists do not really have an alternative to ACE models in this respect, as problems including network strucutures very quickly become intractable in an analytical sense. Purely verbal models, on the other hand, are not accurate enough to capture important differences of various network structures, even in a qualitative way.

4. Discussion

4.1. Major Chances...

While the proceeding section focused on how ACE models fit into the general criteria of institutionalist pattern modeling, I now make some more general statements about how ACE models can help to increase the quality of institutionalist models.

suffices for our purposes.

Clarification of the relevant factors ACE models can help to identify whether a factor or mechanism is sufficient or necessary to produce a certain pattern. In sharp contrast to the *ceteris paribus* analysis in neoclassical economics, it is well suited to study the dynamic interaction effects of several factors or mechanisms such as the role of networks as a catalysator for other factors as discussed in Section 3.4. Such interaction effects are also very difficult to identify in purely verbal models. Furthermore, ACE models can help to study how in an open system different initial states and trajectories can lead to the same long term behaviour. For open systems, this property is known as *equifinality*. Equifinality is very important to identify because in order to explain an observed phenomenon in an open system, it might be insufficient to provide one universal explanation. This is what neoclassical economists do when they require everything to be explained in terms of the utility maximizing behaviour of some representative agents. But in open systems, the same phenomenon can be reached via very different ways and from very different initial conditions. It is therefore very important to provide a *constructive* explanation of a phenomenon, i.e. to show the exact mechanism that leads to the presence of the phenomenon of interest, and what other factors can yield the same result. Such a constructive explanation is naturally implemented in simulation studies.

Generalization of case study results Case studies are much more common in the institutionalist literature than in conventional economics. This may partly be because facing the trade-off between accuracy and generality, mainstream economists tend to favour the latter in order to allow a wide area of applicability (Gilboa et al., 2014), while institutionalists favour accuracy above generality.

There are, of course, some exceptional, more general theories developed by great instutitional minds such as Gunnar Myrdal (circular cumulative causation, backwash and spread effects), Clarence Ayres (the nature of technology and skills), and Thorstein Veblen (e.g. conspicious consumption, institutional life cycle), among others.

Although Diesing (1971, p. 198) considers these concepts to be a mere grouping of real cases (using the term *real type*), I think they are better seen as mechanisms taking place in many real world situations. To identify such mechanisms is of particular scientific value and ACE models may facilitate their exploration by helping to construct and to test models with a medium range of abstraction.

Improving contextual validation Institutionalists should formulate hypothesis about the system under investigation and evaluate the hypothesis via *contextual validation* (Wilber and Harrison, 1978, p. 76). This involves " comparing it with other kinds of evidence on the same point" or by "evaluating the source of the evidence by locating other kinds of evidence about that source."

Unfortunately, contextual validation lacks the rigour of more formal models and can only indicate varying degrees of plausibility (Wilber and Harrison, 1978, p. 76). This plausibility can be increased via the application of ACE models that can explicitly show constructively how the proposed mechanisms yield a pattern observed in reality. By doing so, they help to consider the openness and equifinality of real world economic systems (see above). They do so not only by validating the resulting pattern, but also the whole adjustment process leading to the pattern and therefore allow a much more detailed examination of the hypothesis than mere statistical hypothesis testing.

The question of how to implement empirical studies has always been lively discussed by institutionalists. Wesley Mitchell at the latest has put the issue on the institutionalist agenda (Hirsch, 1994). His (constructive) critique that the veblen-ian kind of investigation is too little data-driven and too speculative has always been controversial, especially because of the (legitimate) critique of the praxis in econometrics. ACE models can be a valuable tool to confront institutionalist theories with the data. But unlike most other statistical methods, ACE models are accessible to a wide range of verification techniques. Especially the fact that ACE models can be evaluated on different scales (microcalibration, macrocalibration,...) and the possibility to consider concrete mechanisms allows to test ACE models much more transparently than most econometric models. They can help to compare institutionalist models much more concisely with observed data than a verbal analysis can, but more transparently and appropriately than a purely econometric analysis.

Multidimensional models Another advantage of ACE models is that the state of each agent is not necessarily expressed in only one dimension. Because every agent can have several attributes, ACE models are able to provide a *multidimensional* perspective on economic phenomena. While neoclassical economics tend to express everything in "utility" or "income", much less effort was put into the development of multidimensional models. As argued above and shown in the appendix, the attributes of agents can include inconvertible properties such as "literacy" and "income". This makes ACE models particularly useful for the development and testing of multidimensional theories, which have been advocated by institutionalist ever since (Myrdal, 1958) and are considered to be more and more important to tackle the most important challenges of our time (Nussbaum and Sen, 1993; Sen, 1999; Stiglitz et al., 2010).

Consideration of scaling effects Not all properties of a system are emergent. Some are only an aggregation of the individual components and or are present on a lower level of the entire system as well. An interesting question is therefore to what size a bigger system, e.g. the society, can be reduced (or "scaled") without losing its emergent properties. Emergent properties that are the result of interactions among the components might require a certain minimum group size of components that interact directly and indirectly with each other. To understand this minimum group size means to study the degree of scale invariance of the properties and it is something that can be conveniently carried out via ACE models, as it is typically easy to alter the number of agents in the model and to study the change of the overall dynamics. It is clear that different properties might have different degrees of scale invariance. The degree of scale invariance or event the probability whether a certain property will be present on a higher level, can be dependent on the properties and actions of the individuals, which, as elaborated above, can themselves be partly dependent of the state of the whole. Such a study is relevant for many applied institutionalist concepts such as the varieties of capitalism research program, the investigation of meso levels, and the evolution of cooperation in communities.

Increased transparency Although recent incidents such as the misleading study of Reinhart and Rogoff (2010)[7] make it difficult to argue for the superior transparency of formal models over verbal description, ACE models allow in principle an extremely rigorous test of their validity: Because the researcher has to write every single assumption into the computer code, a reviewer can reconstruct any train of thought. The ideal case would be to publish the code after the publication of the model such that everybody can check how the results depend on the assumptions and a replicability of the study gets simplified enormously. Many authors already distribute their code on request which makes it easy to use their models for educational means, e.g. in graduate training.

Better policy advice Increased transparency is strongly related to the ability to simulate potential effects of policy. Institutionalists have always seen the necessity for an informed and active role of public policy. Simulations can guide the decision making process for policy makers, as they may not only provide analogies which have then to be interpreted in the public discourse and can therefore be instrumentalized easily by opinion leaders, but provide concrete simulation results which are subject to replicability and critical scrutiny by all parties involved. In this respect, they can be discussed in an easier way than mere verbal descriptions and can enhance the ability of institutionalists to provide reasonable policy advice on a transparent base.

Another important advantage in this context is the constructive character of ACE models: The simulation not only provides results, it also shows how the results emerge. In a dynamic economy, it is important to take adjustment paths into account. And while many purely formal models are not capable to describe the exact adjustment paths (e.g. game theoretic models hinting to an existing Nash equilibrium), ACE models can show the effects on all involved agents on all points of time. ACE models truly *generate* their results step by step in a transparent manner (Epstein and Axtell, 1996).

4.2. ...and Major Challenges

But there are of course also possible pitfalls of applying ACE models in an institutionalist framework. These are in particular the following:

[7] Based on econometric "evidence", Reinhart and Rogoff (2010) argued that a country with a gross external debt rate above 90 percent of its GDP will experience a decline of annual growth by two per cent, with even worse consequences for higher debt rates. Herndon et al. (2014) showed that the conclusions was a consequence of bad data management and does not hold if the data is used correctly. The original paper played a major role in discussions about austerity policies, especially in the European Union.

Instrumental tendencies ACE models tempt researchers to take a constructionist-instrumental standpoint that seems to be incompatible with institutionalist epistemology and ontology. Instrumentalists do not try to describe the reality accurately but consider their theories to be mere instruments replicating what is going on in reality.

As institutionalists have been sceptical to the idea that economic outcomes can be predictable at all, their focus has always been on building explanatory models. ACE models are of course simplified abstractions of real world economies. But strictly speaking they share this property with any model, including the mainly verbal models elaborated by institutionalist economists. But as ACE models do not necessarily include the dominant equilibrium condition, they can include different mechanisms, others than the aggregation of the behavior of utility-maximising agents. This allows a focus on the study of concrete mechanisms and not on providing a prediction of the overall system.[8]

Implicit focus on predictive power Related to the preceding difficulty, ACE models are frequently used to predict economic outcomes. In this sense, they are perfectly compatible with Friedmans methodological instrumentalism. If one tries to explain an economic phenomenon of a certain time period via the use of an ACE model one might be tempted to let the model run a few time periods more in order to predict the further development of the system. One might then tune the model in a way that the predictions fit one's theoretical convictions and in turn accept a lower level of explanatory accuracy. Such an approach is difficult to be identified later on and requires an intensive review of the ACE model.

Overerparametrization and decreased transparency The possibility of an implicit focus on predictive power relates to another problem, namely that ACE models tend to be overparametrized. This means on adds variables, processes and methods until one gets a very good fit to data or is able to create the patterns one wishes to explain. Overparametrization yields extremely complicated models that are very hard to review and very hard to discuss. While good ACE models can help to identify important factors and to increase the transparency of a study, bad ACE models do the reverse. And the distinction among the two classes is not always straightforward. The problem of overparametrization of ACE models is well known in the community and there has been an enormous progress in developing methods to test for overparametrization. Such tests are difficult and cumbersome, however. They require excellent knowledge of the relevant literature. Especially institutionalists not yet very proficient with ACE modelling must rely on the judgements of others. But in defense of ACE models one must mention that other quantitative models, but verbal models as well, are also vulnerable to overparametrization. It is therefore important to have this problem in mind, but not

[8]This is particularly the case since the dominance of the programming paradigm called *object oriented programming* (OOP). The idea behind OOP is to build programs by defining objects, which should correspond to some entity in the real world, and methods on these objects, which should correspond to processes in the real world. Because objects can have different attributes that can be accessed by different methods, the attributes to not have to be expressed in the same unit and one can study multi-dimensional phenomena.

to throw the baby out with the bath water. One must never forget that the contrary, "underparametrization", can be dangerous as well.

4.3. ACE models are compatible with the Institutionalist Approach

Based on the above said one can conclude that ACE models may be affine to institutionalist pattern modelling. There are some qualifications to this conclusion, however. Firstly, ACE models are abstract mathematical models and must be embedded into a more general process story to get explanatory significance. This process story should be consistent with the criteria for institutionalist storytelling and should provide strong theoretical underpinnings for the ACE model. Especially, the assumptions must be justified and the range of applicability of the models must be clarified. The overall insights of a study can never be reduced to the simulation results - only the interpretation of the results in the broader and therefore necessarily verbal discussion yield scientific progress. ACE models are not self-contained and should be considered to be a heuristic in the overall attempt to address a real-world problem. The formal model has no value in itself and must be judged on its ability to support the researcher to understand aspects of the overall problem that she would otherwise not be able to explain. The assumptions made in order to build the formal model should be interpreted as heuristic assumptions in the sense of Musgrave (1981): They get dropped later in the conclusion of the study and the fundamental relationships one has identified can be expressed in a purely verbal manner.

5. Conclusion

I have argued that ACE models can be a valuable heuristic and analytical tool for institutionalist research. In particular they are useful for building holistic, systemic and evolutionary models in the sense of Wilber and Harrison (1978) and Myrdal (1978). Here, this article also contributed to institutionlist methodology in a more general way by showing that it is modern systemism that best describes the institutionalist perspective on the economy. Wilber and Harrison (1978) would have chosen this term to describe institutionalist pattern models if they had written their article today.

All considered, this article suggests institutionalists to be open-minded to the application of ACE models as they entail a large potential to clarify important and thus far unresolved questions. This is particular true for phenomena such as networks, aggregation (and scaling) behaviour and the consequences of policy measures. Here they entail the desirable rigor of a more formal analysis, but avoid the compulsory formalism of neoclassical economics: They could be seen as the golden middle between a purely verbal and a formalistic approach to modelling: Because they are difficult to build and one has to state one's assumptions and considerations in an explicit way during the modelling process, even the process of building a model may already help to get new insights into the subject. As such, they can also be considered to be a *topoi*, i.e. a concept making the researcher ask important questions about the subject of investigation.

Further research may use ACE models in practical applications of institutionalist analysis to investigate further its applicability in the institutionalist research program. There are already institutionalist studies based on ACE models, e.g. Hodgson and Knudsen (2004), Elsner and Heinrich (2009), Wäckerle et al. (2014) and Rengs and Wäckerle (2014a), but as institutionalist researchers become less sceptical against the method, there will emerge a huge number of questions to be addressed via ACE models embedded in an institutionalist process story. The attempt to elaborate on more general concepts from case studies and to further investigate the role of certain network structures for the social stratification are only two examples for starting points.

Further research should also address whether the application of ACE models is consistent with critical realism, which has been becoming more popular as a philosophical basis for institutionalist modelling and tends to be even more sceptical against any kind of formalization. Such an investigation might further clarify the relation between critical realism and institutionalism and may further strengthen the methodological base of institutionalist modelling.

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A. Formal Foundations of ACE Modelling

In this short appendix I quickly review the technical underpinnings of ACE modelling.

In a model with N agents, the state of agent i is represented by the variable $x_i \in \{x_1, ..., x_n\} = \boldsymbol{x}$.

The set of possible states for every $x_j \in \boldsymbol{x}$ is given by X_j. Generally, agents in ACE models consist of the same class, therefore $X_j = X_k \forall j, k \in N$.[9] That agents are modeled as a class means that they have several attributes. The attributes can include many different, non-convertible properties: One attribute can be the level of literacy of the agent, another attribute can be her wealth and a third attribute can describe her relationship status. In the case of more than one attribute, the x_i are tuples, not integers. The overall state of an agent can be represented with a state vector and is given by

$$X \equiv X_1 \times X_2 \times X_3 \times ... \times X_n \text{ and } X \subseteq \mathbb{R}^n.$$

How the states of the individual agents change is specified by the local update functions $\mathcal{F} = \{f_1, ..., f_n\}$ with

$$f_i : X_i \to X_i, \quad \forall i \in N.$$

The inputs of the function f_i are the state of the agent i and all the other agents in the neighbourhood of i, $Z_0(i)$.

The neighborhood results from the topological structure of the model which is naturally expressed via a *dependency graph* $Y(V, E)$ in which the number of vertices in Y corresponds to the number of agents. For two arbitrary vertices $i, j \in V : \exists \langle i, j \rangle \in E$ iff f_i depends *inter alia* on x_j.[10] For simplicity define the neighbourhood of agent i as $Z_0(i)$ including the vertices that are adjacent to vertex i. The input of f_i is then $Z_0(i) \cup \{i\}$. In the simplest case, the dependency relation is fixed, but there are also natural extensions to cases in which the dependence relations are subject to change.

The global dynamics of the model result from the composition of local update functions:

$$\Phi = (f_1, ..., f_n) : X^n \to X^n.$$

It is specified by $\Phi(\mathcal{F}, Y, \pi)$ as the composition of the local update functions according to a given update schedule π and a given dependency graph Y.

For the case of synchronous updating (and thus a *parallel dynamical system*), the resulting system is:

$$\Phi(x_1, ..., x_n) = (f_1(x_1, ..., x_n), ..., f_n(x_1, ..., x_n)),$$

while the sequential case (*sequential dynamical system* with update order $\pi = (\pi_1, ..., \pi_t)$ is given by:[11]

$$\Phi_\pi = f_{\pi_t} \circ f_{\pi_{n-1}} \circ ... \circ f_{\pi_1}.$$

[9]Heterogeneity is taken into account by allowing $x_j \neq x_k \forall j, k \in \boldsymbol{x}$.

[10]If the dependency of agents is defined symmetrically, i.e. x_i depends on x_j if and only if x_j depends on x_i, then $Y(V, E)$ is an undirected graph and $\langle i, j \rangle = \langle j, i \rangle$.

[11]In the economics literature the mathematical study of ACE models is less well known but there is a huge literature studying the conditions necessary for the existence of fixed points, reachability

problems (Barrett et al., 2003) and predecessor existence problems (Barrett et al., 2007). Although it was shown that these problems are very complex and not solvable for the more interesting systems (Barrett et al., 2006), there are valuable results about how to express the *equivalence* between different models (Barrett et al. (2001), Laubenbacher and Pareigis (2001), Laubenbacher et al. (2009), Reidys (2005)) and about the mathematical *category* of such systems (Laubenbacher et al., 2009).