The Efficiency of Real-World Bargaining: Evidence from Wholesale Used-Auto Auctions

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Abstract

This study quantifies the efficiency of a real-world bargaining game with two-sided incomplete information. Myerson and Satterthwaite (1983) and Williams (1987) derived the theoretical efficient frontier for bilateral trade under two-sided uncertainty, but little is known about how well real-world bargaining performs relative to the frontier. The setting is wholesale used-auto auctions, an $80 billion industry where buyers and sellers participate in alternating-offer bargaining when the auction price fails to reach a secret reserve price. Using 270,000 auction/bargaining sequences, this study nonparametrically estimates bounds on the distributions of buyer and seller valuations and then estimates where bargaining outcomes lie relative to the efficient frontier. Findings indicate that the dynamic mechanism attains 80–91% of the surplus which can be achieved on the efficient frontier.

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1 Introduction

From haggling in an open-street market to negotiating a corporate takeover deal, alternating-offer bargaining between a buyer and seller is one of the oldest and most common forms of transaction. When both parties have incomplete information, it is known that equilibrium outcomes are difficult to characterize.[1] Myerson and Satterthwaite (1983) demonstrated that full efficiency is not possible, and theoretical efficiency bounds are derived in Myerson and Satterthwaite (1983) and Williams (1987), but it is unknown how well real-world bargaining performs relative to these bounds. Williams (1987) emphasized that "little is known about whether or not these limits can be achieved with 'realistic' bargaining procedures." This paper is the first attempt to bring data to this question in order to quantify the efficiency of real-world bargaining with two-sided incomplete information. I develop a framework to estimate distributions of private valuations on both sides of the market at wholesale used-auto auctions. I then map these primitives into results from the theoretical mechanism design literature in order to measure the efficiency of bargaining relative to the first-best frontier and the information-constrained, second-best efficient frontier.

The data analyzed in this paper consist of several hundred thousand sequences of back-and-forth bargaining offers between buyers and sellers at wholesale used-auto auctions, a large market where new and used-car dealers buy vehicles from other dealers as well as from rental companies and banks. This industry passes 15 million cars annually through its lanes, selling about 60% of the vehicles, worth a total of $80 billion. Throughout much of the industry, auction houses employ the following mechanism: a secret reserve price set by the seller followed by an ascending price auction, which, when the secret reserve price is not met, is followed by post-auction, alternating-offer bargaining mediated by the auction house. This setting is ideal for studying bargaining under two-sided uncertainty because all players are experienced professionals and are likely to understand well the game being played. Also, because the bargaining takes place after an ascending auction and after the seller's choice of a secret reserve price, the efficiency of bargaining can be studied while imposing only minimal assumptions on the structure or equilibrium of the bargaining game.

After a brief introduction to the industry, Section 2 discusses the data in detail. The data comes from several different auction houses from 2007 to 2010, containing detailed information on each car as well as the actions taken by sellers and buyers in each stage of the game. The data is broken down into two main samples: cars sold by used and new-car dealers (which I refer to as the dealers sample), and cars sold by large institutions, such as rental companies, banks, and fleet companies (which I refer to as the fleet/lease sample). Fleet/lease sellers tend to set reserve prices which are not as high above auction prices as are the reserve prices set by dealers, and hence fleet/lease cars are more likely to sell.

I lay out a simple model in Section 3 which describes the three stages of the game at wholesale auto

[1] Fudenberg and Tirole (1991) stated, "The theory of bargaining under incomplete information is currently more a series of examples than a coherent set of results. This is unfortunate because bargaining derives much of its interest from incomplete information." Fudenberg, Levine, and Tirole (1985) similarly commented "We fear that in this case [of two-sided incomplete information], few generalizations will be possible, and that even for convenient specifications of the functional form of the distribution of valuations, the problem of characterizing the equilibria will be quite difficult."

auctions. The post-auction bargaining is modeled as a general alternating-offer bargaining game. The auction stage is modeled as an ascending auction with private values among bidders, where bidders' values (as well as the seller's value) are correlated through auction-level heterogeneity unobserved by the econometrician. The seller's secret reserve price is chosen optimally before the auction. I prove three preliminary results which motivate an estimation strategy: first, truth-telling is a dominant strategy for bidders in the auction; second, the seller's secret reserve price strategy is monotone; and third, auction-level heterogeneity affects the outcomes of the game in a tractable manner. These three properties allow for nonparametric identification of the distributions of buyer and seller types and the region of the type space in which trade occurs.

Sections 4 and 5 constitute the heart of the paper. Section 4 presents the approach for estimating the distributions of buyer and seller valuations. After controlling for observable covariates and auction house fees, I account for unobserved heterogeneity at the auction level, relying on a deconvolution argument due to Kotlarski (1967) for identification. I then estimate the distribution of buyer valuations using an order statistic inversion. The approach for estimating the distribution of seller types is new. It exploits bounds defined by revealed preferences arguments taken from the seller's response to the auction price. The approach is similar in spirit to Haile and Tamer (2003), using bounds implied by very basic assumptions about players' rationality to learn about model primitives without imposing a complete model of the game or solving for an equilibrium.

Section 5 presents the methods for estimating the efficient frontier and other counterfactual mechanisms from mechanism design theory. Throughout the paper, the terms "efficient" or "second-best" refer to ex-ante incentive efficiency, taking informational asymmetries into account. To refer to full efficiency, I use the terms "ex-post efficient" or "first-best."[2] I also use the terms "surplus" and "gains from trade" interchangeably. The efficient frontier (or Pareto frontier) delineates the best possible outcomes, in terms of buyer and seller surplus, that could be achieved by any bilateral bargaining game in the presence of two-sided incomplete information. Myerson and Satterthwaite (1983) and Williams (1987) demonstrated how this frontier depends on the distributions of buyer and seller valuations. Therefore, the estimated distributions from Section 4 are crucial in solving for these mechanisms. To solve for the Pareto frontier, I adopt an approach described in Williams (1987). I also demonstrate that the direct-revelation mechanism corresponding to the dynamic bargaining mechanism currently used at auto auctions is identified from information on whether trade occurred and from secret reserve and auction prices, relying on the property that the seller's secret reserve price will be strictly monotonic.

I then combine the estimated distributions of buyer and seller valuations and the allocation functions defining the current mechanism and counterfactual mechanisms, putting theoretical bargaining and real-world bargaining on the same footing in order to quantify bargaining efficiency. First, I examine the efficiency loss due to incomplete information. Ideally, a buyer and seller should trade whenever the buyer values a good more than the seller (first-best, ex-post efficient trade). However, incomplete information

[2]For a more detailed taxonomy of ex-ante, interim, and ex-post efficiency under incomplete information, see Holmström and Myerson (1983).

on both sides gives rise to a bilateral monopoly, where each party has some market power. Myerson and Satterthwaite (1983) demonstrated that a deadweight loss occurs as each party trades off the dual incentives of increasing the probability of trade and extracting additional rent from the other party, akin to the deadweight loss in a standard one-sided monopoly pricing model. As a result, some trades fail to occur even when the buyer values the good more than the seller.[3] This deadweight loss is given by the gap between the the second-best mechanism derived in Myerson and Satterthwaite (1983) and first-best trade. I discover that incomplete information need not be a huge problem in this market: The second-best mechanism achieves about 99% of first-best surplus and over 90% of the first-best probability of trade.

Second, I examine the efficiency of post-auction bargaining relative to the information-constrained efficient frontier. Unlike the mechanisms discussed in Myerson and Satterthwaite (1983) and Williams (1987), alternating-offer bargaining with two-sided uncertainty has no clear equilibrium predictions due to signaling by both parties. As a result, it is unknown where alternating-offer bargaining lies within the efficient frontier. Any gap between the efficient frontier and real-world bargaining represents a deadweight loss which could theoretically be eliminated by committing to a static efficient mechanism along the frontier. Therefore, I refer to this as the deadweight loss due to mechanism choice/limited commitment.[4] Findings indicate that the post-auction bargaining achieves 80–84% of the efficient level of surplus in the dealers sample and 87–91% in fleet/lease sample. The deadweight loss due to limited commitment is therefore roughly 9–20% in this market.

In addition to Myerson and Satterthwaite (1983) and Williams (1987), theoretical papers examining the efficient frontier from a mechanism design standpoint include Ausubel and Deneckere (1993), Ausubel, Cramton, and Deneckere (2002), Satterthwaite and Williams (1989), and Chatterjee and Samuelson (1983). Ausubel and Deneckere (1993) and Ausubel, Cramton, and Deneckere (2002) demonstrated theoretically that when buyer and seller distributions have monotone hazard rates and when high weights are placed on the seller or buyer payoff, some equilibria of a dynamic, alternating-offer bargaining game can reach the efficient frontier. Satterthwaite and Williams (1989) studied the k double auction game and found that generically only the $k = 0$ or $k = 1$ double auctions reach the efficient frontier.[5] Chatterjee and Samuelson (1983) demonstrated that in the symmetric uniform case the $k = 1/2$ double auction also reaches the efficient frontier. To my knowledge, the current paper is the first to bring data to the framework of Myerson and Satterthwaite (1983) or Williams (1987).

A large theoretical literature on incomplete-information strategic bargaining has yielded valuable insights through focusing on special cases rather than the full, two-sided incomplete information setting.[6]

[3]Formally, Myerson and Satterthwaite (1983) demonstrated that when the supports of buyer and seller types overlap, there does not exist an incentive-compatible, individually rational mechanism which is ex-post efficient and which also satisfies a balanced budget.

[4]Cramton (1992) and Elyakime, Laffont, Loisel, and Vuong (1997) also referred to this as an issue of commitment.

[5]A k double auction consists of both the buyer and seller simultaneously reporting sealed bids to an intermediary and, if the buyer's bid exceeds the seller's, trading at a price which is a convex combination of the two bids, with weight k.

[6]The incomplete-information bargaining literature focuses on settings of one-sided uncertainty (Gul, Sonnenschein, and Wilson 1986; Gul and Sonnenschein 1988; Fudenberg and Tirole 1983; Sobel and Takahashi 1983; Fudenberg, Levine, and Tirole 1985; Ausubel and Deneckere 1989; Rubinstein 1985a,b; Bikhchandani 1992; Grossman and Perry 1986; Admati and

The set of empirical papers which structurally estimated models of incomplete information bargaining is quite small, including Sieg (2000), Ambrus, Chaney, and Salitsky (2011), and Silveira (2012), who focused on settings of one-sided incomplete-information bargaining, and Keniston (2011), who estimated a model of two-sided uncertainty and compared alternating-offer bargaining to a fixed-price mechanism.[7] One advantage of the current paper over previous structural papers is that, because the bargaining occurs after an auction and after the seller reports a secret reserve price, the model's primitives, namely the distributions of buyer and seller valuations, can be identified from these pre-bargaining actions without relying on much structure or on a complete model of the bargaining game. This is particularly useful given that, unlike auction settings or complete-information bargaining games, there is no canonical model of alternating-offer bargaining under incomplete information with a continuum of types.[8]

2 The Wholesale Auto Auction Industry

The wholesale used-auto auction industry provides liquidity to the supply side of the US used-car market. Each year approximately 40 million used cars are sold in the United States, 15 million of which pass through a wholesale auction house. About 60% of these cars sell, with an average price between $8,000 and $9,000, totaling to over $80 billion in revenue (NAAA 2009). The industry consists of approximately 320 auction houses scattered across the country. The industry leaders, Manheim and Adesa, maintain about a 50% and 25% market share, respectively, and the remaining auction houses are referred to as independents. Each auction house serves as a platform in a two-sided market, competing to attract both sellers and buyers. Throughout the industry, the majority of auction house revenue comes from fees paid by the buyer and seller when trade occurs. Buyers attending wholesale auto auctions are new and

Perry 1987; Cramton 1991), settings of one-sided offers (Cho 1990; Feinberg and Skrzypacz 2005; Cramton 1984; Ausubel and Deneckere 1993), settings with two-types rather than a continuum of types (Chatterjee and Samuelson 1988; Compte and Jehiel 2002), or settings with uncertainty not being about valuations (Abreu and Gul 2000; Watson 1998). Two papers which modeled bargaining as an alternating-offer game and a continuum of types with two-sided incomplete information, where the incomplete information is about players' valuations, are Perry (1986), which predicted immediate agreement or disagreement, and Cramton (1992), which modeled the bargaining game as beginning with a war of attrition and consisting of players signaling their valuations through the length of delay between offers, as in Admati and Perry (1987). Neither of these models fits the type of bargaining observed at wholesale auto auctions. See Binmore, Osborne, and Rubinstein (1992), Ausubel, Cramton, and Deneckere (2002), Roth (1995), and Kennan and Wilson (1993) for additional surveys of the theoretical and experimental bargaining literature.

[7] A separate strand of literature, such as Tang and Merlo (2010), Watanabe (2009), Crawford and Yurukoglu (2012), Grennan (2013), and others, estimated structural models which included elements of bargaining under *complete* information rather than incomplete information.

[8] Several papers analyzed models of auctions followed by complete-information bargaining (Huh and Park 2010; Elyakime, Laffont, Loisel, and Vuong 1997), one-shot bargaining (Bulow and Klemperer 1996; Eklof and Lunander 2003; Menezes and Ryan 2005), or one-sided incomplete information (Wang 2000). Genesove (1991) also discussed post-auction bargaining at wholesale auto auctions. He tested several parametric assumptions for the distributions of buyer and seller valuations, finding that these assumptions performed poorly in explaining when bargaining occurred and when it was successful.

used car dealers.[9] Sellers may be used or new car dealers (whom I will refer to as "dealers") selling off extra inventory, or they may be large institutions, such as banks, manufacturers, or rental companies (whom I will refer to as "fleet/lease") selling repossessed, off-lease, lease-buy-back, or old fleet vehicles. Throughout, I will refer to the seller as "she" and the buyer as "he".

Sellers bring their cars to the auction house and establish a secret reserve price.[10] In the days preceding the auctioning of the car, potential buyers may view car details and pictures online, including a condition report for cars sold by fleet/lease sellers, or may visit the auction house to inspect and test drive cars.[11] The auction sale takes place in a large, warehouse-like room with 8–16 lanes running through it. In each lane there is a separate auctioneer, and lanes run simultaneously. A car is driven to the front of the lane and the auctioneer calls out bids, raising the price until only one bidder remains. The characteristics of the car as well as the current bid are listed on a large monitor near the auctioneer. The entire bidding process usually takes 30–90 seconds.

If the auction price exceeds the secret reserve price, the car is awarded to the high bidder. If the auction price is below the secret reserve, the high bidder is given the option to enter into bargaining with the seller. If the high bidder opts to bargain, the auction house will contact the seller by phone, at which point the seller can accept the auction price, end the negotiations, or propose a counteroffer.[12] If the seller counters, the auction house calls the buyer. Bargaining continues until one party accepts or terminates negotiations. The typical time between calls is 2–3 hours.[13] Auction house employees contacting each party take care not to reveal the other party's identity in order to prevent the buyer and seller from agreeing on a trade outside of the auction house, avoiding auction house fees.[14]

[9]Note that the term "new" means the dealer is authorized to sell new cars from a manufacturer, but can also sell used cars. On the other hand, "used" car dealers can only sell used cars. Genesove (1993) discussed the differences of cars sold by new vs. used-car dealers and found weak evidence of adverse selection among cars sold by used-car dealers. Note also that the general public is not allowed at these auctions; Hammond and Morrill (2014) presented a model explaining this feature of auto auctions.

[10]Most sellers report this secret reserve price to the auction house. Some sellers choose not to report their reserve as they plan to either be present during the auction or to await a phone call from the auction house informing them of the auction price prior to deciding whether to accept.

[11]According to conversations with participants and personal observations, few buyers appear to visit the auction house prior to the day of sale.

[12]If the seller a present during the auctioning of the car, the seller may choose to accept or reject the auction price immediately. If the seller is not present but the auctioneer observes that the auction price and the reserve price are far enough apart that phone bargaining is very unlikely to succeed, the auctioneer may choose to immediately reject the auction price on behalf of the seller.

[13]During the time a car is in the bargaining process, or if that bargaining has ended in no trade, interested buyers other than the high bidder may also contact the auction house and place offers on the car. If the bargaining between the original high bidder and seller ends in disagreement, bargaining commences with the next interested buyer as long as his offer is higher than previous offers the seller has rejected. This occurs for about three percent of the cars in the full dataset. This separate form of dynamics is not accounted for in the model below the observations where this occurs are not included in the analysis.

[14]The identity of a fleet/lease seller is revealed, and these sellers tend to try to build a positive reputation of not setting excessively high reserve prices. Conversations with industry participants reveal that at some auction houses outside of the data sample studied in this paper, the identity of small sellers is also revealed.

A seller accepting the auction price (or bargaining offers) below the reserve price may seem puzzling given that the seller could have accomplished an equivalent outcome by reporting a lower secret reserve price. Industry participants explain this phenomenon as having several potential causes: sellers systemically set high reserve prices due to overly optimistic beliefs about auction prices (Treece 2013) or in attempt to influence auctioneers to achieve higher prices (Lacetera, Larsen, Pope, and Sydnor 2014), or sellers are learning something about demand after observing the auction.[15]

If the auction and/or bargaining does not result in trade the first time the vehicle is up for sale (or first "run"), the vehicle can either be taken back to the seller's business location or, more often, remain at the auction house until the next available sales opportunity, usually several weeks later.[16] The seller can change her reserve price before the next run of the vehicle. If trade takes place but the buyer feels he has been given a lemon, he may, under certain conditions, request arbitration, in which the auction house intervenes to either undo the sale or negotiate a lower sale price. This occurs less than three percent of the time in practice.

The dataset used in this paper is new to the literature. The data come from six auction houses, each maintaining a large market share in the region in which it operates. Between January 2007 and March 2010 these auction houses passed over 600,000 vehicles through their lanes. The data from these auction houses includes detailed information on each car, including make, model, year, trim, and odometer reading; condition report (prepared by the auction house for fleet/lease vehicles); a blue book estimate provided by the auction house; the number of pictures displayed in the online pre-sale profile of the car; disclosure codes for different types of damage reported by the seller; and the identity of the seller.

An observation in the dataset represents a run of the vehicle, that is, a distinct attempt to sell the vehicle through the auction or, if the reserve price is not met, through post-auction bargaining. The total number of runs recorded in the data is approximately 1,000,000, so on average a vehicle passes through the lanes 1.67 times. I treat each run as an independent observation and do not model dynamics between runs. For a given run, the data records the date, time, auction house location, and auction lane, as well as the seller's secret reserve price, the auction price, and, when bargaining occurs over the phone, the full sequence of buyer and seller actions (accept, quit, or counter), and the amounts of any offers/counteroffers.

I drop observations with no recorded auction house blue book estimate; cars less than one year or greater than 16 years old; cars with less than 100 miles or greater than 300,000 miles on the odometer; observations in which the auction sale timestamp is missing; and observations for which the following variables lie outside their respective 0.01 and 0.99 quantiles: auction price, reserve price, blue book price, or the gap between the reserve and auction price. I drop observations for which fewer than ten vehicles were observed at a given make-model-year-trim-age combination or days in which fewer than 100 cars

[15]For the estimation approach I adopt, knowing the precise data-generating process for reserve prices is unnecessary; it is only necessary that secret reserve prices be strictly increasing in the seller's underlying true valuation, a result generated by the model in Section 3.

[16]Genesove (1995) presented a search model to study the seller's decision to reject the auction price and take the car back to her own car lot.

Table 1: Descriptive statistics

	Dealers sample		Fleet/lease sample	
	mean	s.d.	mean	s.d.
Trade	0.710	0.454	0.760	0.427
Reserve price	$7,379	$5,196	$10,307	$5,764
Auction price	$6,256	$4,923	$9,826	$5,846
Blue book	$6,811	$4,853	$10,964	$6,140
Age (years)	6.781	3.380	3.150	2.558
Mileage	97,941	46,518	57,176	40,318
Sample size	135,942		136,453	

Notes: Trade is an indicator for whether trade occurred between the buyer and seller. Blue book is an estimate of the market value of the car, provided by the auction house.

were offered for sale at a given auction house. I drop observations where the auction price or reserve price is missing or is equal to zero and incomplete bargaining sequences.[17] In the end, I am left with 135,942 runs of cars offered for sale by used-car dealers (which I will refer to as the dealers sample), and 136,453 offered for sale by fleet/lease sellers (which I will refer to as the fleet/lease sample).

Summary statistics are displayed in Table 1. The probability of trade is 0.71 in the dealers sample and slightly higher in the fleet/lease sample. In the dealers sample, the average auction price is over $1,000 below the average reserve price and about $600 below the average blue book price. Dealer cars are on average seven years old and have nearly 100,000 miles on the odometer. Fleet/lease cars tend to be newer (three years old and 57,000 miles) and higher priced and have a smaller gap between the reserve and auction prices. Also, unlike dealer cars, in fleet/lease cars the reserve price does not exceed the blue book price on average. All of these descriptive statistics are consistent with conversations with industry participants: Dealer cars tend to be older cars with more aggressive reserve prices and be less likely to sell.

Table 2 displays the characteristics of observations in the dealers sample separated by the period of

[17]Some observations record a secret reserve price or an auction price but not both. These observations, or observations with incorrectly recorded bargaining sequences (such as a seller acceptance followed by a buyer counteroffer) are not suitable for my final analysis but are still useful in controlling for observable heterogeneity as explained in Section 4.2. Missing secret reserve prices typically occur when the seller chooses not to report a reserve price, either planning to be present at the auction sale to accept or reject the auction price in person or planning to have the auction house call her on the phone rather than determining a reserve price ex-ante. Missing auction prices can occur due to the descending/ascending practice of auctioneers: auctioneers do not start the bidding at zero; they start the bidding high and then lower the price until a bidder indicates a willingness to pay, at which point the ascending auction begins. If bidders are slow to participate, the auctioneer will cease to lower bids and postpone the sale of the vehicle until a later date, leaving no auction price recorded. See Lacetera, Larsen, Pope, and Sydnor (2014).

Table 2: Outcome of Game by Period: Dealers Sample

			Full dealers sample				Conditional on trade occurring		
Ending period	Player's turn	# Obs	% of Sample	% Trade	Reserve price	Auction price	Reserve price	Auction price	Final price
1	(Auction)	104,470	76.849%	80.67%	$7,459	$6,447	$6,949	$6,048	$6,048
					($5,243)	($4,976)	($4,933)	($4,702)	($4,702)
2	S	10,993	8.087%	61.91%	$6,609	$5,271	$6,342	$5,282	$5,282
					($4,953)	($4,645)	($4,861)	($4,624)	($4,624)
3	B	14,899	10.960%	13.71%	$7,191	$5,572	$7,760	$6,535	$6,948
					($4,978)	($4,618)	($5,278)	($5,029)	($5,113)
4	S	3,269	2.405%	70.66%	$7,708	$6,299	$7,542	$6,230	$6,485
					($5,151)	($4,846)	($5,019)	($4,730)	($4,807)
5	B	1,879	1.382%	44.01%	$8,175	$6,626	$8,523	$7,005	$7,650
					($5,198)	($4,876)	($5,434)	($5,099)	($5,257)
6	S	242	0.178%	82.23%	$8,305	$6,786	$8,356	$6,867	$7,351
					($5,382)	($5,083)	($5,473)	($5,183)	($5,286)
7	B	157	0.115%	59.87%	$8,306	$6,705	$8,486	$6,929	$7,665
					($5,197)	($4,976)	($5,256)	($5,065)	($5,198)
8	S	24	0.018%	75.00%	$8,863	$7,454	$8,792	$7,506	$8,042
					($5,125)	($5,071)	($5,725)	($5,674)	($5,748)
9	B	6	0.004%	66.67%	$7,583	$6,225	$6,675	$5,188	$5,838
					($4,362)	($4,517)	($2,871)	($2,964)	($2,800)
10	S	3	0.002%	100.00%	$8,333	$6,100	$8,333	$6,100	$7,233
					($5,620)	($4,453)	($5,620)	($4,453)	($5,701)

Notes: Dealers sample. For each period (period 1 = auction and immediately following, period 2 = seller's first turn in post-auction bargaining, period 3 = buyer's first turn, etc.), table reports the number of observations ending in that period, percent of total sample ending in that period, and percent of time which trade occurred. Table also reports reserve price and auction price for observations ending in a given period and, for those observations ending in trade, the reserve price, auction price, and final price conditional on trade.

the game in which the observation ends. Table 3 displays similar results for the fleet/lease sample. For each period of the game, the columns in these tables display the number of observations ending in that period, the percent of the total sample which this number represents, the percent of cases which ended in trade, as well as the unconditional reserve price and auction price (whether or not trade occurred) and the reserve price, auction price, and final price estimated only using cases which did end in trade.

Table 3: Outcome of Game by Period: Fleet/lease Sample

		Full fleet/lease sample					Conditional on trade occurring		
Ending period	Player's turn	# Obs	% of Sample	% Trade	Reserve price	Auction price	Reserve price	Auction price	Final price
1	(Auction)	112,410	82.380%	80.73%	$10,818	$10,559	$10,716	$10,655	$10,655
					($5,883)	($5,886)	($5,993)	($5,990)	($5,990)
2	S	11,012	8.070%	90.98%	$7,412	$6,163	$7,385	$6,231	$6,231
					($4,593)	($4,388)	($4,588)	($4,391)	($4,391)
3	B	10,792	7.909%	13.88%	$8,246	$6,464	$8,459	$7,260	$7,712
					($4,211)	($3,961)	($4,257)	($4,132)	($4,199)
4	S	1,013	0.742%	83.51%	$8,619	$7,053	$8,608	$7,080	$7,404
					($4,476)	($4,236)	($4,473)	($4,242)	($4,316)
5	B	1,116	0.818%	46.68%	$8,932	$7,341	$9,099	$7,650	$8,287
					($4,574)	($4,321)	($4,613)	($4,368)	($4,483)
6	S	47	0.034%	87.23%	$9,083	$7,246	$8,780	$6,967	$7,587
					($4,451)	($4,423)	($3,986)	($3,881)	($4,092)
7	B	56	0.041%	57.14%	$10,876	$8,915	$10,802	$8,680	$9,697
					($6,129)	($5,628)	($6,893)	($6,223)	($6,613)
8	S	4	0.003%	50.00%	$11,250	$9,650	$6,250	$4,800	$5,475
					($7,263)	($6,730)	($1,768)	($1,131)	($1,591)
9	B	3	0.002%	33.33%	$11,633	$10,150	$11,400	$10,600	$10,900
					($5,754)	($5,489)	--	--	--

Notes: Fleet/lease sample. For each period (period 1 = auction and immediately following, period 2 = seller's first turn in post-auction bargaining, period 3 = buyer's first turn, etc.), table reports the number of observations ending in that period, percent of total sample ending in that period, and percent of time which trade occurred. Table also reports reserve price and auction price for observations ending in a given period and, for those observations ending in trade, the reserve price, auction price, and final price conditional on trade.

Period 1 is the auction. Observations ending in period 1 represent cases which ended with the auction or immediately thereafter, and hence cases for which no bargaining actions are recorded.[18]

[18]When the reserve is not met, the game may still end immediately (and very frequently does–61.26% of the time in the dealers sample and 45.65% of the time in the fleet/lease sample (not shown in Tables 2–3)—when one of the following is true: 1) the seller is present at the auction house and can immediately accept or reject the auction price; 2) the auctioneer rejects the auction price on behalf of the seller, knowing that alternating-offer bargaining is unlikely to occur; or 3) the high bidder walks away from bargaining before the seller is contacted. The third case is observable in the data, and occurs 1.32% of the time in the dealers sample and 0.59% of the time in the fleet/lease sample (not shown in Tables 2–3). The first two cases are indistinguishable in the data, and therefore I assume that any time the auctioneer chooses to reject the

The remaining periods are labeled with even numbers for seller turns and odd numbers for buyer turns. Period 2 is the seller's first turn in the post-auction bargaining game, and observations ending in this period represent cases in which the seller accepted the auction price or quit. Period 3 is the buyer's first turn in bargaining, and is reached only if the seller chose to counter in period 2. Play continues back and forth between the buyer and seller until one party accepts or quits.

Table 2 demonstrates that in 77% of the dealers sample the game ends at the auction, and in these cases the final price when trade happens (which occurs 81% of the time) is naturally the auction price. Consider now the fifth period of the game. Only 1.4% of the full sample reaches the this period, but this still consists of nearly 2,000 observations. In the fifth period, when trade does occur, it occurs at an average final price ($7,650) which is over $600 above the average auction price ($7,005), but still does not reach as high as the average reserve price ($8,523). Overall, Table 2 suggests that observations which ended in later periods had somewhat higher reserve prices than those ending in earlier periods, consistent with Coasian dynamics (see Fudenberg and Tirole 1991). Only 3 buyer-seller pairs endured ten periods of the game, all of them coming to agreement in the end, at an average price over $1,000 above the average auction price.

Table 3 displays similar patterns for the fleet/lease sample. 82% of the sample ended the game at the auction. Less than one percent ended in the fifth period, but this still consists of over 1,000 observations. Buyer-seller pairs who traded in this round did so at an average of about $600 above the auction price. Other than observations ending in the first period, observations with phone bargaining ending in later periods had higher average reserve prices than those ending in earlier periods, again consistent with Coasian dynamics. Three pairs remained for nine periods of the game, with only one of the three ending in trade.

3 Model of Post-Auction Bargaining with Secret Reserve Price

This section presents a model of the auction-followed-by-bargaining mechanism used at wholesale auto auctions. I first discuss the timing of the mechanism and set up some general assumptions. I discuss each stage of the game, starting from the end with the post-auction bargaining stage. I then present a model of the ascending auction stage, demonstrating that truth-telling is a weakly dominant strategy, and the stage in which the seller chooses a secret reserve price, demonstrating that the seller's strategy is strictly increasing.

The timing of the game at wholesale auto auctions is as follows:

1. Seller sets a secret reserve price.

2. N bidders bid in an ascending auction.

3. If the auction price exceeds the secret reserve price, the high bidder wins the item.

auction price on behalf of the seller this choice coincides with what the seller would have done given the chance.

4. If the auction price does not exceed the secret reserve price, the high bidder is given the opportunity to walk away, or to enter into bargaining with the seller.[19]

5. If the high bidder chooses to enter bargaining, the auction price becomes the first bargaining offer, and the high bidder and seller enter an alternating-offer bargaining game, mediated by the auction house.

Throughout I maintain the following assumptions:

Assumptions.

(A1) The ascending auction follows a button auction model with $N \geq 2$ risk-neutral bidders participating. For $i = 1, ..., N$, each buyer i has a private valuation $\tilde{B}_i = W + B_i$, with $W \sim F_W$, $B_i \sim F_B$, and with B_i independent of W.

(A2) The risk-neutral seller has a private valuation $\tilde{S} = W + S$, where $S \sim F_S$, with S independent of W and B_i for all i.

(A3) In bargaining players face a per-offer disutility, $(c_B, c_S) > 0$, as well as discount factor, $\delta \in [0, 1)$, where $1 - \delta$ represents the probability that bargaining will break down exogenously.

I further assume F_B, F_S, and F_W have corresponding atomless densities f_B, f_S, and f_W, with supports $[\underline{B}, \overline{B}]$, $[\underline{S}, \overline{S}]$, and $(-\infty, \infty)$. Let realizations of these random variables be denoted b, s, and w.

The random variable W is observed by all buyers and the seller. In the estimation framework in Section 4, I allow W to be unobservable to the econometrician but observed by all players, thus allowing for all players' values to be correlated.[20] Conditional on W, buyers and sellers have independent private values (IPV). A motivation for this framework is that buyers—as well as dealer-type sellers—have valuations arising primarily from their local demand and inventory needs.[21] Also, seller valuations can depend on the value at which the car was assessed as a trade-in, or, for a bank or leasing company, valuations can arise from the size of the defaulted loan.[22] The button auction model is a natural choice given that

[19]At wholesale auto auctions, some large sellers are given the option to elect to eliminate step 4 above, implying that when the auction price does not meet the secret reserve price, the high bidder is not allowed to immediately walk away from bargaining but must wait until the seller responds to the auction price. This situation is referred to as a "binding-if auction." It can be shown that in a binding-if auction, the seller's secret reserve price strategy is only guaranteed to be weakly increasing, rather than strictly as in the non-binding-if case. It can also be shown that bidders will not necessarily drop out of bidding at their valuations but may instead drop out at a price slightly below their valuation to account for the possibility of paying bargaining costs. In the data, there is no way to know if a sale took place in a binding-if setting. I treat all auctions as non-binding-if auctions.

[20]I also introduce characteristics observable to *both* the econometrician and all players and I control for these observables.

[21]Conversations with buyers, sellers, and auction house employees support this assumption: buyers claim to decide upon their willingness to pay before bidding begins, often having a specific retail customer lined up for a particular car. See Lang (2011). Studying Korean auto auctions, Kim and Lee (2014) tested and failed to reject the IPV assumption, while Roberts (2013) found evidence of unobserved auction-level heterogeneity (analogous to W).

[22]These explanations for seller values are due to conversations with industry professionals. Note also that adverse selection from the seller possessing more knowledge about car quality than the buyer is likely small because of auction house information revelation requirements and because sellers are not previous owners/drivers of the vehicles.

jump bidding is rare—as it is the auctioneer who calls out bids—and bid increments are small.[23] The assumption of symmetry is not strong in this setting given that buyer identities are unknown to the seller in bargaining and given the assumption of a private values button auction, implying that bidders' auction strategies will not depend on the identities of other participants.[24] The parameter δ captures the feature that bargaining may end through an auction house employee failing to follow up on a bargaining sequence, occurring in 1–2% of bargaining interactions.[25]

In what follows, I demonstrate three properties which prove useful for estimation: 1) a buyer's auction strategy is to drop out at his value, as in a standard ascending auction, 2) the seller's secret reserve price strategy, $\rho(S)$, is strictly increasing in her type S, and 3) auction-level heterogeneity shifts the outcomes of the game in a tractable manner. I begin by modeling the game conditional on a realization of W and thus omit W for notational simplicity and return to it in Section 3.4. I ignore auction house fees in this theoretical analysis but account for their existence in the empirical analysis in Section 4.1.[26]

3.1 Bargaining Stage

This section describes a simple model of the dynamic, post-auction bargaining game. The game begins with an offer by the buyer in period $t = 1$. At wholesale auto auctions, this offer is the price at the auction. The seller then chooses between accepting (A), quitting (Q)—meaning terminating the negotiations—or making a counteroffer (C). Accepting ends the game, with trade taking place at the accepted price. Quitting also ends the game, with no trade taking place. After a counteroffer by the seller, play returns to the buyer, who then chooses between accepting, quitting, and countering. Thus, at t even it is the seller's turn, and at t odd it is the buyer's turn. Below, I refer to period "t" as being the seller's turn and period "$t+1$" as being the buyer's turn. Where useful for clarification, I also include the superscripts "S" or "B" in notation to denote an action taken by the seller or buyer respectively.

Let $H_t \equiv \{P_\tau\}_{\tau=1}^{t-1}$ represent the set of offers made from period 1 up through period $t-1$. The player whose turn it is at time t has not yet made an offer and so this offer does not enter into H_t. Let $D_t^S \in \{A, Q, C\}$ represent the seller's decision in period t, and let $D_{t+1}^B \in \{A, Q, C\}$ represent the buyer's decision in period $t+1$.

The seller's payoff at time t is given by the following. Conditional on the history of offers H_t, which includes the buyer's most recent offer P_{t-1}^B, a seller of type $S = s$, chooses to accept (A), quit (Q), or

[23] Bid increments lie between \$25 and \$100.

[24] See Coey, Larsen, and Sweeney (2014) for a formal treatment of the implications of asymmetries in ascending auctions.

[25] This number is based on the percent of bargaining sequence records in which trade failed and the sequence of offers was incomplete, ending with an counteroffer. A key reason for modeling discounting (either time discounting or, in this case, a probability of exogenous breakdown) lies in the results of Perry (1986), who showed in a game with two-sided uncertainty that if 1) there is no discounting and 2) bargaining costs take the form of an additive cost common to all buyers and an additive cost common to all sellers then the unique equilibrium is for bargaining to end immediately. Cramton (1991) discusses how allowing for discounting overcomes this feature.

[26] A full theoretical analysis of auction house fees, including the optimal fee schedule, would require knowledge of the competitive structure between auction houses. Appendix B.7.3 discusses the fees which would be optimal for a monopolist auction house.

counter (C), yielding the following payoffs:

$$
\begin{aligned}
&\text{A} : p^B_{t-1} \\
&\text{Q} : s \\
&\text{C} : V^S_t\left(s|H_t\right) \\
&\quad = \max_p \Bigg\{ p\delta \Pr\left(D^B_{t+1} = A|H_{t+1}\right) + s\left(\delta \Pr\left(D^B_{t+1} = Q|H_{t+1}\right) + 1 - \delta\right) \\
&\quad + \delta \Pr\left(D^B_{t+1} = C|H_{t+1}\right)\left(\delta E_{P^B_{t+1}}\left[\max\left\{P^B_{t+1}, s, V^S_{t+2}\left(s|H_{t+2}\right)\right\}\Big| H_{t+1}\right] + s(1-\delta)\right)\Bigg\} - c_S
\end{aligned}
$$

where p is the counteroffer chosen by the seller, which will be included in H_τ for $\tau > t$.[27] The per-period bargaining disutility ($c_S > 0$) is assumed to be common across sellers, and the probability of not terminating exogenously ($\delta < 1$) is assumed to be common across sellers as well as buyers. The seller's counteroffer payoff takes into account that the buyer may either accept, quit, or return a counteroffer. In the latter case, the seller receives her expected payoff from being faced with the decision in period $t+2$ to accept, quit, or counter. Exogenous breakdown may occur in any period, in which case the seller receives s as a payoff.[28]

Similarly, the buyer's payoff at time $t+1$ is given by the following. Conditional on the history of offers H_{t+1}, which includes the seller's most recent offer P^S_t, a buyer of type $B = b$ chooses to accept (A), quit (Q), or counter (C), yielding the following payoffs:

$$
\begin{aligned}
&\text{A} : b - p^S_t \\
&\text{Q} : 0 \\
&\text{C} : V^B_{t+1}\left(b|H_{t+1}\right) \\
&\quad = \delta\Bigg(\max_p \Bigg\{(b-p)\Pr\left(D^S_{t+2} = A|H_{t+2}\right) \\
&\quad + \delta \Pr\left(D^S_{t+2} = C|H_{t+2}\right) E_{P^S_{t+2}}\left[\max\left\{b - P^S_{t+2}, 0, V^B_{t+3}\left(b|H_{t+3}\right)\right\}\Big| H_{t+2}\right]\Bigg\}\Bigg) - c_B
\end{aligned}
$$

where p is the counteroffer chosen by the buyer, which will be included in H_τ for $\tau > t+1$. $c_B > 0$ represents the buyer's per-period bargaining disutility, assumed to be common across buyers. The buyer's outside option is normalized to zero.

[27] H_{t+1} includes all offers in H_t as well as the seller's choice of counteroffer in this period, and H_{t+2} includes the seller's choice in this period, as well as the buyer's choice in the next period, P^B_{t+1}.

[28] In reality, the outside option of both the buyer and seller is a complicated object that cannot be estimated in the scope of this data, as a buyer who exits bargaining has the choice to obtain vehicles from a variety of sources, such as other sales at the same auction house, competing auction houses, online markets, or trade-ins, and a seller who exits bargaining may choose to leave the car at the auction house, return the car to her car lot, or sell it through another source.

3.2 Ascending Auction Stage

This section discusses bidders' strategies in the ascending auction stage of the mechanism. Bidder i's strategy is the price, β_i, at which he stops bidding as a function of his type, $B_i = b_i$, which represents his valuation for the car. Let $R = \rho(S)$ be a random variable representing the secret reserve price of a seller of type S who uses reserve price strategy $\rho(\cdot)$. Let

$$\beta = \max_{k \neq i} \beta_k(B_k)$$

Bidder i will be the highest bidder if and only if $\beta_i > \beta$. The expected payoff of bidder i from following bidding strategy $\beta_i(b_i)$ is given by

$$M(b_i, \beta) = \begin{cases} (b_i - \beta)\Pr(\beta > R) & \\ \qquad + \pi^B(\beta, b_i)\Pr\left(\beta < R, \pi^B(\beta, b_i) > 0\right), & \text{if } \beta_i > \beta. \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

Buyer i would decide to enter bargaining if $\pi_B(\beta, b_i) > 0$, where $\pi^B(\beta, b_i)$ represents the buyer's expected payoff from entering bargaining, equivalent to the counteroffer payoff in the previous section but with the auction price as the buyer's counteroffer, rather than the maximizing counteroffer. In this setup, the following property holds:

Proposition 1. *If in the bargaining game the seller never accepts offers below the auction price, truth-telling is weakly dominant for bidders in the auction stage. That is,* $\beta_i(b_i) = b_i$.

All proofs are found in Appendix A.

This result implies that the winning bid will be the second order statistic from the distribution of buyer valuations, as in standard ascending or second price auctions.[29] Intuitively, the assumption that the seller never accepts bargaining offers below the auction price ensures that buyers will not be tempted to bid beyond their valuations in the ascending auction stage in hopes of bargaining to a lower price in the post-auction bargaining stage. This is supported by the data: the bargained price is not lower than the auction price (see Tables 2–3). Moreover, bidders are not tempted to drop out before the bidding reaches their valuations because if the high bidder learns that the auction price did not meet the secret reserve, he can always opt out of bargaining, and I assume this opting out is costless. And because the auction price is the second-highest valuation of the bidders, the seller cannot infer anything about the valuation of the winner other than learning the point at which the buyer distribution is truncated, eliminating any incentive of buyers to shade bids downward to avoid revealing information to the seller. The argument for identifying the distribution of buyer valuations, discussed in Section 4.4, relies on Proposition 1.

[29] Huh and Park (2010) found the same result in a theoretical model of second price auctions with complete information (rather than incomplete information) post-auction bargaining: bidders' strategies were unaffected by the presence of bargaining.

3.3 Secret Reserve Price Stage

In this section, I discuss the seller's choice of a secret reserve price, chosen before the beginning of the auction to maximize the seller's expected revenue. Applying Proposition 1, the auction price will be $B^{(2)}$, the second order statistic of buyer valuations. In choosing her secret reserve price, $\rho(S)$, a seller of type $S = s$ wishes to maximize her ex-ante payoff, given by

$$E_{B^{(2)},B}\Bigg[B^{(2)} * 1\left\{B^{(2)} > \rho(s)\right\} + s * 1\left\{B^{(2)} < \rho(s), \pi^B(B^{(2)}, B) \leq 0\right\} \tag{2}$$

$$+ \pi^S\left(B^{(2)}, s\right) * 1\left\{B^{(2)} < \rho(s), \pi^B(B^{(2)}, B) > 0\right\}\Bigg] \tag{3}$$

This term consists of three pieces: 1) the auction price, which the seller receives if it exceeds the reserve; 2) the seller's outside option, her type s, which the seller receives if the auction price is below the reserve and the buyer opts out of bargaining; and 3) the seller's bargaining payoff, $\pi^S\left(B^{(2)}, s\right) = \max\left\{B^{(2)}, s, V_2^S\left(s|B^{(2)}\right)\right\}$, which the seller receives when the price is below the reserve and bargaining occurs. I apply a monotone comparative statics result from Edlin and Shannon (1998), a special case of Topkis's Theorem, to obtain the following:

Proposition 2. *The seller's optimal secret reserve price, $\rho^*(s)$, is strictly increasing in s.*

The intuition behind Proposition 2 is that the secret reserve price is never revealed and hence the seller can use a separating strategy without perfectly signaling her type. To prove this result, I first show that bargaining payoffs are weakly increasing in players' types. The strict monotonicity then relies on bargaining being costly to buyers (Assumption A3), such that some buyers will choose to opt out of bargaining when informed that the auction price does not meet the secret reserve. Without costly bargaining, Topkis's Theorem can be used to show that $\rho^*(s)$ will be weakly increasing.

The approach presented in Section 4.5 for identifying the distribution of seller valuations does not formally rely on Proposition 2. However, I do rely on this result for identification of the underlying allocation function (the region of the buyer and seller type space where trade occurs) in the dynamic bargaining game, as discussed in Section 5.2.

3.4 Auction-level Heterogeneity

Sections 3.1–3.3 derived results conditional on a given realization of auction-level heterogeneity. The independence of W, S, and B in the model described above yields the following result:

Proposition 3. *Suppose, when $W = 0$, the secret reserve is r and, for each period t at which the game arrives, the offer is given by $P_t = p_t$ and the decision to accept, quit, or counter is given by $D_t = d_t$. Then when $W = w$ the secret reserve will be $r + w$, the period t offer will be $p_t + w$, and the period t decision will be d_t.*

Proposition 3 is similar to results used elsewhere in the empirical auctions literature (Haile, Hong, and Shum 2003) but is a generalization specific to this setting of a secret reserve price auction followed by bargaining. The result makes it feasible to apply empirical approaches accounting for auction-level heterogeneity, both observed and unobserved, as described in Sections 4.2–4.3. Proposition 3 is also crucial in identifying the region of the buyer and seller type space in which trade occurs, as described in Section 5.2.

4 Estimating Distributions of Buyer and Seller Valuations

In this section, I exploit the model properties derived above in order to estimate the distribution of buyer and seller valuations. The estimation consists of several steps:

1. Accounting for auction house fees
2. Controlling for observed heterogeneity (auction-level characteristics observable to the econometrician)
3. Controlling for unobserved heterogeneity (auction-level heterogeneity observed by the players but not by the econometrician)
4. Estimating the distribution of buyer valuations through an order statistic inversion
5. Estimating bounds on the distribution of seller valuations using revealed preferences arguments

Let $j = 1...J$ represent observations in the data, where each observation contains a complete set of actions for one instance of the game, i.e. a reserve price, auction price, and any bargaining actions. Let R^{raw} and $B^{(2),raw}$ be random variables representing the reserve price and auction price prior to any adjustments for auction house fees or heterogeneity, with realizations for observation j (which I also refer to below as "car j") given by r_j^{raw} and $b_j^{(2),raw}$. Throughout the estimation I treat the dealers and fleet/lease samples separately because the two groups differ in the types of cars they sell and in other behaviors and outcomes, as shown below.

4.1 Auction House Fees

If trade occurs, the buyer and seller pay auction house fees which are approximately linear in the transaction price, p.[30] Let the fee schedules be given by

$$h^S(p) = \alpha_0^S + \alpha_1^S p$$
$$h^B(p) = \alpha_0^B + \alpha_1^B p$$

[30] Knowledge that fees are approximately linear comes from data on realized fees when trade takes place and from auction house documents.

Using estimates of these functions, the auction price for car j can be adjusted upward to account for the fact that, if the auction price were the transaction price, a buyer would be required to pay the auction price as well as the fee. The reserve price can similarly be adjusted downward. Let these adjusted prices be given by

$$R^h \equiv r^{raw} - h^S(R^{raw})$$
$$B^{(2),h} \equiv B^{(2),raw} + h^B(B^{(2),raw})$$

with realizations for car j denoted r_j^h and $b_j^{(2),h}$.

Fees are observed in the data whenever a transaction occurs. Therefore, slope and intercept terms in the fee functions can be estimated with a linear regression using the sample of data in which trade occurs. In practice, the intercept terms, α_0^S and α_0^B, tend to vary by auction house location and year. Therefore, I perform the above steps by estimating separate intercept terms for each auction-location-by-year combination.

4.2 Observed Heterogeneity

To account for auction-level characteristics which are observed to the econometrician as well as the players, I apply Proposition 3. As above, let W be a random variable representing unobserved auction-level heterogeneity. Let X be a random variable representing auction-level heterogeneity observed by the econometrician, with X independent of W, S, and B. Let realizations of X and W be given by x_j and w_j for observation j. I specify the total auction-level heterogeneity for observation j to be $x_j'\gamma + w_j$. Proposition 3 implies that auction prices and reserve prices can be "homogenized" (Haile, Hong, and Shum 2003) by estimating the following joint regression of reserve prices and auction prices on observables:

$$\begin{bmatrix} r_j^h \\ b_j^{(2),h} \end{bmatrix} = \begin{bmatrix} x_j'\gamma + \gamma_r \\ x_j'\gamma \end{bmatrix} + \begin{bmatrix} \tilde{r}_j - \gamma_r \\ \tilde{b}_j^{(2)} \end{bmatrix},$$

where $\tilde{r}_j = r_j + w_j$, $\tilde{b}_j^{(2)} = b_j^{(2)} + w_j$, and the parameter γ_r captures the difference in means between reserve prices and auction prices.

In the vector x_j I include fifth-order polynomial terms (all degrees of the polynomial from one through five) in the auction houses' blue-book estimate, the odometer reading, and the run number of the vehicle.[31] x_j also contains the number of previous attempts to sell the car; the number of pictures displayed online; a dummy for whether or not the odometer reading is considered accurate, and the interaction of this dummy with the odometer reading; the interaction of the odometer reading with a car-make dummies; dummies for each make-model-year-trim-age combination (where age refers to the age of the vehicle in years); dummies for condition report grade (ranging from 1-5, observed only for fleet/lease vehicles);

[31] The run number represents the order in which cars are auctioned. I include fifth-order polynomials for both the run number within an auction-house-by-day combination, and the run number within an auction-house-by-day-by-lane combination.

dummies for auction house location interacted with date of sale and auction house location interacted with hour of sale; dummies for 32 different vehicle damage categories recorded by the auction house; dummies for each seller who appears in at least 500 observations; and dummies for discrete odometer bins.[32] Finally, x_j also includes six measures of the thickness of the market during a given auction sale.[33] The R^2 from this first-stage regression is 0.96 in the fleet/lease sample and 0.95 in the dealers sample, implying that most of the variation in auction prices and reserve prices is explained by observables.[34]

An estimate of $\tilde{r}_j$ is then given by subtracting $x_j'\hat{\gamma}$ from r_j^h, and similarly for $\tilde{b}_j^{(2)}$. Variation in these two quantities is then attributed to unobserved auction-level heterogeneity and to players' private valuations, as detailed below.

4.3 Unobserved Heterogeneity

To account for heterogeneity w_j in the value of car j which is observed by the players but not by the econometrician, I apply a result due to Kotlarski (1967), which implies that observation of $\tilde{r}_j = r_j + w_j$ and $\tilde{b}_j^{(2)} = b_j^{(2)} + w_j$, along with an assumption that $E[b^{(2)}] = 0$, is sufficient to identify the densities f_W, f_R, and $f_{B^{(2)}}$. This identification result has been applied in estimation elsewhere in the auctions literature by directly computing characteristic functions and applying Fourier inversions to perform a deconvolution (e.g. Li, Perrigne, and Vuong 2000; Krasnokutskaya 2011). I adopt a simpler likelihood approach proposed in Freyberger and Larsen (2014), approximating each density with a series of normalized orthogonal Hermite polynomials. The likelihood function be given by

$$\mathcal{L} = \prod_j \left[\int f_{B^{(2)}}(\tilde{b}_j^{(2)} - w) f_R(\tilde{r}_j - w) f_W(w) dw \right] \tag{4}$$

For each random variable Y, the density is approximated by

$$f_Y(y) \approx \frac{1}{\sigma} \left(\sum_{k=0}^{K} \theta_k^Y H_k \left(\frac{y - \mu_Y}{\sigma_Y} \right) \right)^2 \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{y-\mu_Y}{\sigma_Y}\right)^2}$$

where K is a smoothing parameter; θ_Y, μ_Y, and σ_Y are parameters to be estimated; and H_k are Hermite polynomials, defined recursively by $H_1(y) = 1$, $H_2(y) = y$, and $H_k(y) = \frac{1}{\sqrt{k}}[yH_{k-1}(y) - \sqrt{k-1}H_{k-2}(y)]$ for $k > 2$.

I maximize the likelihood in (4) subject to the constraints $\sum_{i=1}^{K} (\theta_i^Y)^2 = 1$ for each random variable Y, which ensures each approximated function is indeed a density function, and also subject to the constraint

[32] Odometer bins are as follows: four equally sized bins for mileage in $[0, 20000)$; eight equally sized bins for mileage in $[20000, 80000)$; four equally sized for mileage in $[100000, 200000)$; one bin for mileage in $[200000, 250000)$; and one bin for mileage greater than 250000.

[33] I compute these market thickness measures as follows: for a given car on a given sale date at a given auction house, I compute the number of remaining vehicles still in queue to be sold at the same auction house on the same day which lie in the same category as the car in consideration. The six categories I consider are make, make-by-model, make-by-age, make-by-model-by-age, age, or seller identity.

[34] In order to improve estimates of γ, these regressions include observations for which the reserve price is recorded but the auction price is missing and vice versa, as well as observations with incorrectly recorded bargaining sequences, as explained in Section 2.

$E[b^{(2)}] = 0$. For the smoothing parameter I choose $K = 5$.[35] The location and scale parameters $\{\mu_Y, \sigma_Y\}_{Y=W,R,B^{(2)}}$ are not required for identification but improve the performance of the estimator and are standard in estimation with Hermite polynomials. I estimate these parameters in an initial step, maximizing (4) with the vectors θ^Y set to zero for each Y, that is, each density f_Y is approximated by a $N(\mu^Y, \sigma^Y)$. I then plug in the estimated values of $\{\hat{\mu}_Y, \hat{\sigma}_Y\}_{Y=W,R,B^{(2)}}$ into (4) and maximize over $\{\theta^Y\}_{Y=W,R,B^{(2)}}$. I perform the integration in (4) by Gauss-Hermite quadrature (See Appendix B.1).

4.4 Distribution of Buyer Valuations

Identification of the underlying distribution of buyer valuations, F_B, follows from Proposition 1, which implies that the auction price will be the second order statistic of buyer valuations. Given knowledge of the distribution of the second order statistic of buyer valuations, $F_{B^{(2)}}$ (which is identified by the argument in the previous section), and assuming a known distribution for N, the number of bidders, it is known that

$$F_{B^{(2)}}(b^{(2)}) = \sum_n \Pr(N = n) \left[nF_B(b^{(2)})^{n-1} - (n-1)F_B(b^{(2)})^n \right] \tag{5}$$

See, for example, Athey and Haile (2007). I assume N follows a Poisson distribution with mean λ, truncated below at $N = 2$, yielding

$$F_{B^{(2)}}(b^{(2)}) = \frac{e^{-\lambda}}{1 - e^{-\lambda}(1+\lambda)} \left(e^{\lambda F_B(b^{(2)})}(1 + \lambda(1 - F_B(b^{(2)}))) - 1 - \lambda \right) \tag{6}$$

See Lemma 5 of Matsuki (2013).[36] Given λ, and given an estimate of $F_{B^{(2)}}$ from Section 4.3, the function $F_B(b^{(2)})$ can be solved for at any point $b^{(2)}$ using standard nonlinear equation approaches.[37] I adopt a bisection method. Based on personal observation of many used-car auctions, I choose $\lambda = 7$ for dealer sales and $\lambda = 10$ for fleet/lease sales.[38] Appendix B.6 demonstrates that results are not sensitive to this choice.

[35] The above framework can be treated as a semi-nonparametric maximum likelihood setting, letting the smoothing parameter K grow appropriately with the sample size and choosing K in practice through cross-validation. I instead fix $K = 5$, treating this as a flexible parametric approximation, as suggested in Kim and Lee (2014), which simplifies inference. I find that choosing K larger than 5 does not affect estimates noticeably.

[36] In (6), I modify Lemma 5 of Matsuki (2013) slightly to apply to the case where N is always at least 2.

[37] Differentiating (5) yields the density:

$$f_B(b^{(2)}) = \frac{f_{B^{(2)}}(b^{(2)})(1 - e^{-\lambda}(1+\lambda))}{\lambda^2 e^{\lambda(F_B(b^{(2)})-1)}(1 - F_B(b^{(2)}))}$$

An estimate of $f_{B^{(2)}}$ comes from the maximum likelihood approach of Section 4.3.

[38] Many more bidders are physically present at auction houses during auction sales; these numbers correspond approximately to the mean number of bidders showing active interest in the auction. Similar numbers are found in Genesove (1991).

4.5 Distribution of Seller Valuations

To identify the distribution of seller valuations, I invoke an argument similar to the Haile and Tamer (2003) bounds in English auction settings. The argument differs from Haile and Tamer (2003), however, in that observation-level bounds are not available. Instead, I obtain bounds on the distribution of seller values relying on probability statements formed from many observations of sellers' initial responses to the auction price.

As in Section 3, let $D_2^S \in \{A, Q, D\}$ represent the seller's choice to accept, quit, or counter in period 2 of the bargaining game. In cases where, in practice, the auction price exceeds the reserve price, or in cases where the auction price falls below the reserve price but there is immediate trade, I consider D_2^S to have a value of accept (A); and in cases where there is immediate disagreement, I consider D_2^S to have a value of quit (Q). Therefore, a realization of $D_2^S = d_2^S$ is available for each observation in the data. As above, let $\tilde{S} \equiv S + W$.

Observe that

$$\Pr(D_2^S = A | B^{raw,(2)} = b^{raw,(2)}) \leq \Pr(\tilde{S} + X'\gamma \leq b^{raw,(2)} - h^S(b^{raw,(2)}))$$
$$\Pr(D_2^S = Q | B^{raw,(2)} = b^{raw,(2)}) \leq \Pr(\tilde{S} + X'\gamma \geq b^{raw,(2)} - h^S(b^{raw,(2)}))$$

Both observations follow from revealed preference arguments. Intuitively, if a seller accepts an auction price of $b^{raw,(2)}$, it must be the case that the seller's total value for the car, $\tilde{S} + X'\gamma$, is *less* than $b^{raw,(2)} - h^S(b^{raw,(2)})$ (the auction price less the auction house fee at that price). Similarly, if the seller quits when the auction price is $b^{raw,(2)}$, it must be the case that the seller values keeping the car herself *more* than $b^{raw,(2)} - h^S(b^{raw,(2)})$.[39] Let $\mathcal{V} = B^{raw,(2)} - X'\gamma - h^S(B^{raw,(2)})$ with realizations v, and define

$$\tilde{L}(v) \equiv \Pr(D_2^S = A | \mathcal{V} = v)$$
$$\tilde{U}(v) \equiv \Pr(D_2^S \neq Q | \mathcal{V} = v)$$

Combining the two revealed preferences arguments yields

$$\tilde{L}(v) \leq F_{\tilde{S}}(v) \leq \tilde{U}(v) \tag{7}$$

Integrating over W then provides upper and lower bounds on F_S, which I denote F_S^U and F_S^L. I perform several steps to improve the estimates of the bounds on $F_{\tilde{S}}$ prior to integrating over W. I first estimate $\tilde{L}(\cdot)$ and $\tilde{U}(\cdot)$ using a Nadayara-Watson kernel regression.[40] I then incorporate information on

[39] The analogous two statements in the Haile and Tamer (2003) English auction setting, for buyer valuations, are that a buyer never bids more than the buyer's value and never lets a competitor win at a price the buyer would have been willing to pay.

[40] The Nadayara-Watson estimator for $\tilde{L}(\cdot)$ is given by

$$\hat{\tilde{L}}(u) = \frac{\sum_j K_h(u - v_j) 1\{d_{2,j}^S = A\}}{\sum_j K_h(u - v_j)}$$

where $1\{d_{2,j}^S = A\}$ is an indicator for whether the seller of car j chose to accept the auction price. K_h is a Gaussian kernel with bandwidth h set to the asymptotically optimal bandwidth for kernel density estimation of $\mathcal{V}$. I replace γ and h^S in the definition of $\mathcal{V}$ with the estimates from Sections 4.1 and 4.2. I follow the same procedure for $\tilde{U}(\cdot)$

$\tilde{r}$ to improve estimates of the upper and lower bounds on $F_{\tilde{S}}$ and to ensure that these bounds correspond to distribution functions. Specifically, note that

$$\tilde{R} > \tilde{S} \Rightarrow F_{\tilde{R}}(v) \leq \tilde{L}(v) \leq \tilde{U}(v) \tag{8}$$

That is, the CDF of $\tilde{R}$ should lie above both $\tilde{L}$ and $\tilde{U}$. Therefore, I enforce that both the upper and lower bounds on $F_{\tilde{S}}$ lie above $F_{\tilde{R}}$. I also impose that the estimates of the lower and upper bounds on $F_{\tilde{S}}$ be weakly increasing, following the rearrangement approach of Chernozhukov, Fernandez-Val, and Galichon (2009).[41] Let $\tilde{L}^*$ and $\tilde{U}^*$ represent the bounds on $F_{\tilde{S}}$ after enforcing (8) and after enforcing monotonicity. Finally, these estimates are not necessarily surjective/onto; that is, there may not exist values of $\mathcal{V}$ at which sellers accept or quit with probability approaching 0 or 1. I fill in these missing parts of the upper and lower bounds by assuming a constant gap between quantiles of $F_{\tilde{R}}$ and those of the upper or lower bounds on $F_{\tilde{S}}$.[42]

After estimating F_S^U and F_S^L, the final upper and lower bounds on F_S, I obtain draws from these CDFs by inverting them at random uniform draws. I estimate the corresponding densities using kernel density estimation of these draws.[43]

4.6 Distribution Estimates

This section presents the results of the estimation procedure described in Sections 4.1–4.5. After controlling for auction house fees and observable auction-level heterogeneity as described in Sections 4.1–4.2, I perform the likelihood approach described in Section 4.3 to account for unobserved auction-level heterogeneity. Figure 1 displays the densities of reserve prices and auction prices before removing unobserved heterogeneity (f_{R+W} and $f_{B^{(2)}+W}$) and after removing unobserved heterogeneity (f_R and $f_{B^{(2)}}$), as well as the estimated density of observed heterogeneity (f_W). In each panel of Figure 1, and in the tables and figures which follow, monetary values are denoted in units of \$1,000 and should be interpreted as

[41] In this setting, rearrangement is performed by simply sorting the vector of lower bounds (estimated on a uniformly spaced grid) and reassigning them to the original grid points, and similarly for the upper bounds. Chernozhukov, Fernandez-Val, and Galichon (2009) demonstrated that, when estimating a monotone function, a rearranged estimate is always an improvement, in terms of estimation error, over an original, non-monotonic estimate. In practice, the approach is sensitive to extreme outliers, and so for this estimation I do not include the top and bottom 0.1% of observations of $\mathcal{V}$, where the kernel regression estimates are noisy. In practice, the rearrangement has only little impact on the estimates, smoothing out small non-monotonic portions of the bounds.

[42] Specifically, to fill in values of $\tilde{L}^*(\cdot)$ and $\tilde{U}^*(\cdot)$ so that these functions are onto, I do the following. Let $\{v_m\}_{m=1,\ldots,M}$ represent the grid of points on which $\tilde{L}^*(\cdot)$ is evaluated, which can lie beyond the range of $\mathcal{V}$ in the data, in which case $\tilde{L}^*(v)$ returns the empty set. Let $\underline{v}^L = \arg\min_{m:\tilde{L}^*(v_m)\neq\emptyset} \tilde{L}^*(v_m)$ and $\overline{v}^L = \arg\max_{m:\tilde{L}^*(v_m)\neq\emptyset} \tilde{L}^*(v_m)$. Then

$$F_{\tilde{S}}^L(v) \equiv \begin{cases} F_{\tilde{R}}(v + \underline{v}^L - v_1) & \text{if } v < \underline{v}^L \\ \tilde{L}^*(v) & \text{if } v \in [\underline{v}^L, \overline{v}^L] \\ F_{\tilde{R}}(v + \overline{v}^L - v_M) & \text{if } v > \overline{v}^L \end{cases}$$

Finally, to remove unobserved heterogeneity, F_S^L is given by $F_S^L(v) \equiv \int F_{\tilde{S}}^L(v-w) f_W(w)dw$. I follow the same steps for $\tilde{U}^*(\cdot)$.

[43] As above, I use a Gaussian kernel with the asymptotically optimal bandwidth.

follows: for a car with vector of observables x, the overall reserve price—less the auction house fee—will have density f_{R+W}, shifted by the quantity $x'\gamma$. Similarly, for a car with observables x and unobserved heterogeneity w, the overall density will be f_R, shifted by the quantity $x'\gamma + w$. A similar interpretation follows for auction prices.

Figure 1 shows that the density after removing unobserved heterogeneity, f_R (in red), is tighter than f_{R+W} (in blue) in both the dealers sample (a) and the fleet/lease sample (b), although more so for panel (a), suggesting that unobserved heterogeneity plays a bigger role in explaining price variation in reserve prices in dealer cars than in fleet/lease cars. Panel (a) suggests that, after accounting for fees and observed and unobserved heterogeneity, the residual of sellers' reserve prices varies principally over a range of about \$4,000, ranging from about \$0 to \$4,000. In this paper, this residual variation is attributed to variation in the seller's private valuation for the car, s.

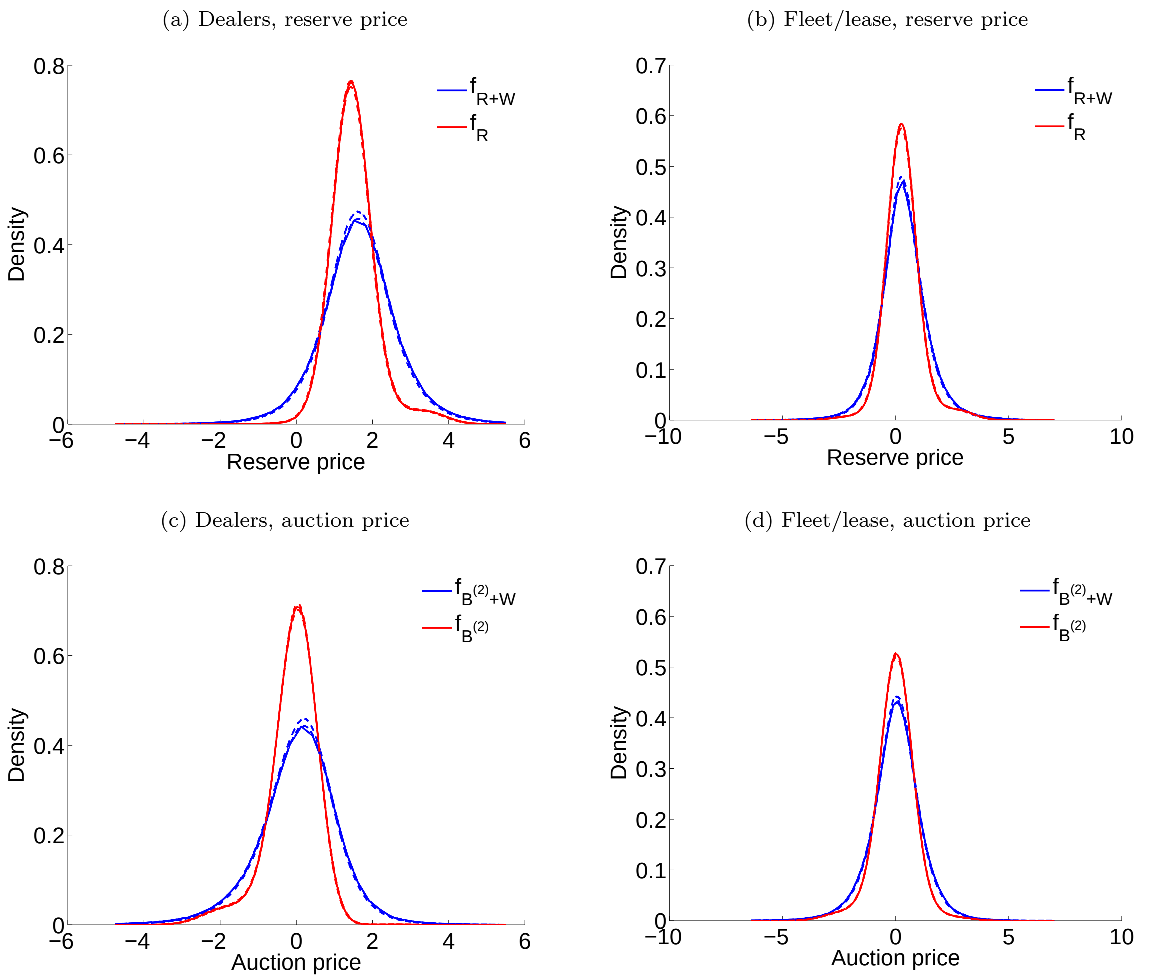
Panels (c) and (d) demonstrate that the results are similar when comparing the density of the auction price with unobserved heterogeneity, $f_{B^{(2)}+W}$, to the density without unobserved heterogeneity, $f_{B^{(2)}}$, in the dealers (c) and fleet/lease samples (d). In each panel the density $f_{B^{(2)}}$ is centered about zero because the estimation procedure of 4.3 constrains $E[B^{(2)}] = 0$ as a normalization required to apply the identification argument due to Kotlarski (1967).

Figure 2 displays the estimated density of unobserved heterogeneity, f_W, in the dealers (a) and fleet/lease samples (b). The majority of the mass of unobserved heterogeneity lies between [-\$2,500, \$2,500].

As a measure of the goodness of fit of this deconvolution procedure, I compare the correlation of the reserve price and auction price (after adjusting for auction house fees) in the raw data to the correlation between simulated draws for $\tilde{R}$ and $\tilde{B}^{(2)}$, generated by 10,000 draws from the estimated distributions for F_R, $F_{B^{(2)}}$, and F_W. The correlations agree quite closely: In the dealers sample, the raw vs. simulated correlation is 0.6146 vs. 0.6149, and in the fleet/lease sample the two quantities are 0.3817 vs. 0.3705. As a further check of fit, note that the model assumes that auction-level heterogeneity—both observed and unobserved—is additively separable. Therefore, the difference between the reserve and auction price in the raw data, after removing auction house fees but with heterogeneity included ($R^h - B^{(2),h}$), should be equal in distribution to the simulated difference between the reserve price and auction price after removing heterogeneity ($R - B^{(2)}$). Figure 3 displays the raw and simulated densities of this difference, which agree well in both samples.

Before discussing the estimated densities of buyer valuations (from the order statistic approach described in Section 4.4, which will be shown below), Figure 4 displays the results from bounding the distribution of seller valuations as described in Section 4.5. Panels (a) and (b) display, for each value of the auction price, the probability a seller accepts ($\Pr(d_2^S = A)$, the lower bound, in red) and the probability the seller does not walk away/quit ($1 - \Pr(d_2^S = Q)$, the upper bound, in green) in the dealers (a) and fleet/lease samples (b). Hollow circles in these panels represent the raw estimated probabilities, and the solid lines represent estimates after monotonizing through the rearrangement approach of Chernozhukov, Fernandez-Val, and Galichon (2009).

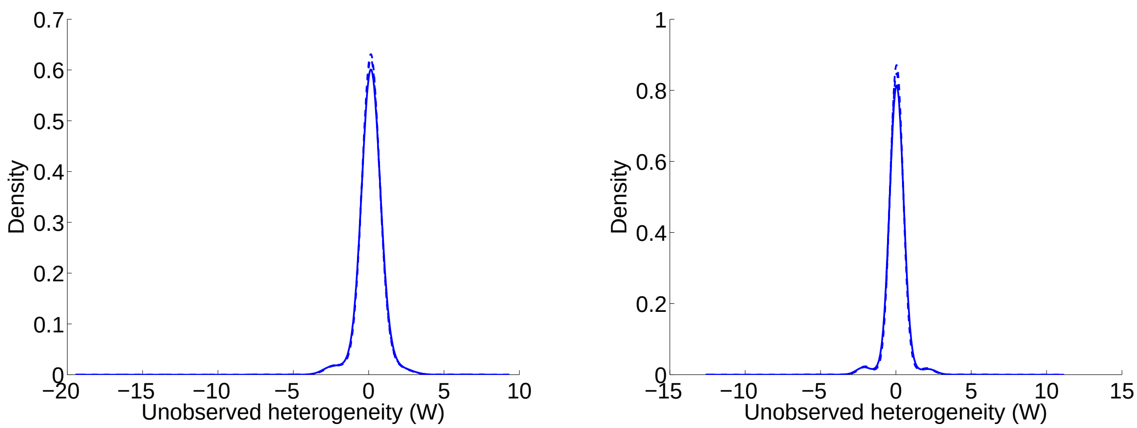
Figure 1: Densities of auction price and reserve price with and without unobserved heterogeneity



Notes: Displays densities prior to removing unobserved heterogeneity (in blue), and after removing unobserved heterogeneity (in red) for reserve price (panels (a) and (b)) and auction price (panels (c) and (d)). Left panels use dealers sample and right panels use fleet/lease sample. Dashed lines mark pointwise 95% confidence bands from 200 bootstrap replications of full estimation procedure. Units = $1,000.

Figure 2: Density of unobserved heterogeneity

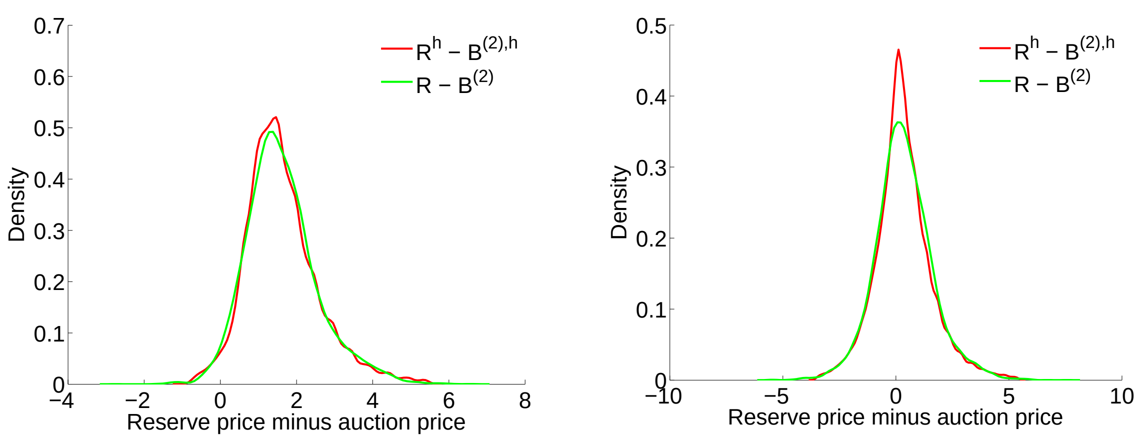
(a) Dealers (b) Fleet/lease



Notes: Displays density of unobserved heterogeneity in dealers sample and fleet/lease sample. Dashed lines mark pointwise 95% confidence bands from 200 bootstrap replications of full estimation procedure. Units = $\$1,000$.

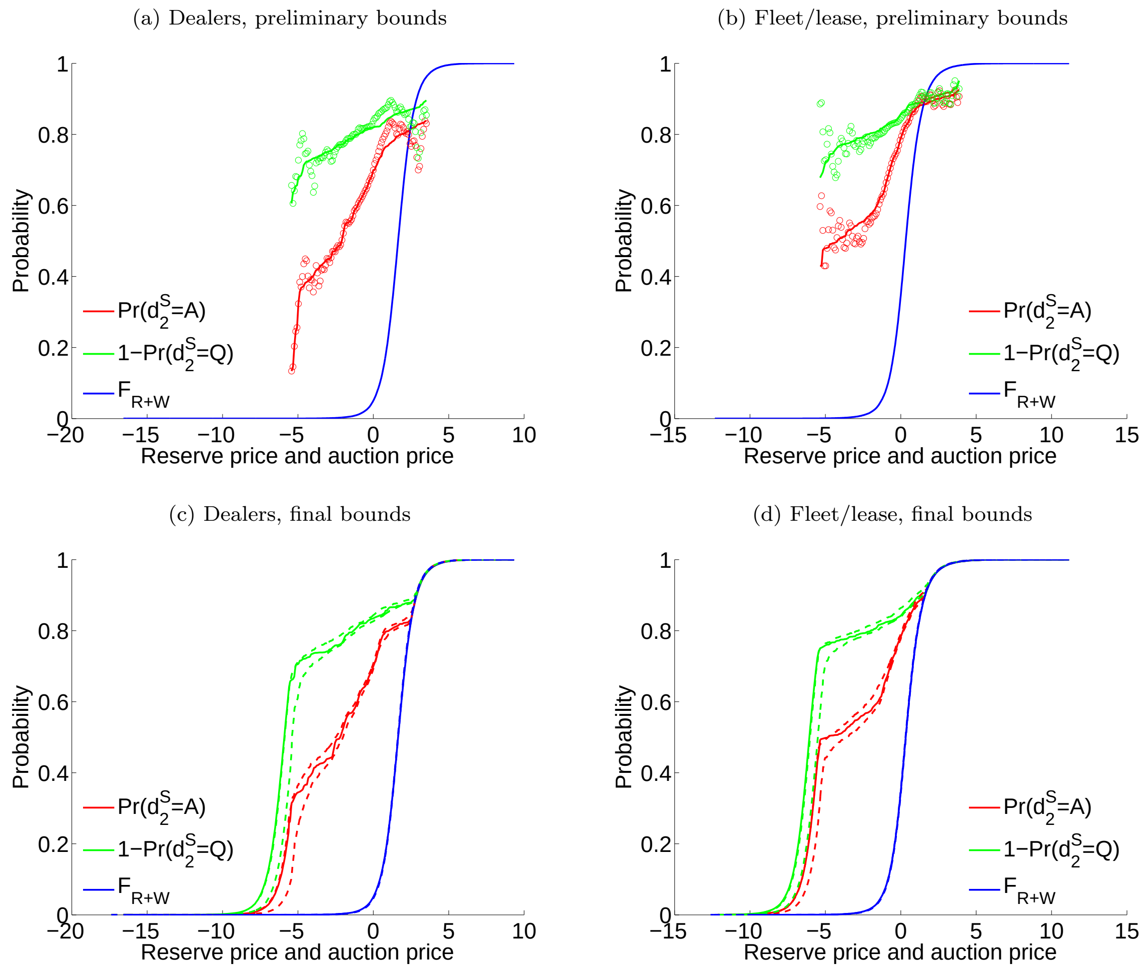
Figure 3: Density of reserve minus auction price, raw and simulated

(a) Dealers (b) Fleet/lease



Notes: Displays density of reserve price minus auction price (after adjusting for auction fees) in the raw data (in red) compared to the density of the reserve price minus auction price with simulated draws from the estimated distributions (in green). Units = $\$1,000$.

Figure 4: Bounds on distribution of $\tilde{S}$



Notes: Bounds on distribution of $\tilde{S}$, with upper bound (green), lower bound (red), and CDF of reserve price with unobserved heterogeneity (F_{R+W}). Panels (a) and (b) display preliminary bounds, prior to additional estimation steps, with hollow circles representing original probability estimates and solid lines representing monotonized (rearranged) estimates. Panels (c) and (d) display final bounds on $\tilde{S}$ after enforcing bounds to lie above F_{R+W} and cover full range in $[0, 1]$. In panels (c) and (d), dashed lines mark pointwise 95% confidence bands from 200 bootstrap replications of full estimation procedure. Units = $1,000.

The solid blue line in each panel of Figure 4 represents the empirical CDF of the seller's reserve price (including observed heterogeneity), which should stochastically dominate the distribution of seller valuations with unobserved heterogeneity included, $F_{\tilde{S}}$. To enforce this, I replace any values of the upper or lower bounds on $F_{\tilde{S}}$ with $F_{\tilde{R}} \equiv F_{R+W}$. The results are displayed in panels (c) and (d). The estimates in panels (c) and (d) also include the additional steps I take to obtain estimated upper and lower bounds which are surjective (covering the range of probabilities in $[0, 1]$) by imposing a fixed distance between quantiles of $F_{\tilde{S}}$ and $F_{\tilde{R}}$ in the range not covered by the raw probabilities, as described in Section 4.5.

Figure 4 suggests that, for dealer cars, panels (a) and (c), when the auction price—less the auction house fee—is about \$2,500 lower than expected (based on observable heterogeneity), sellers choose to accept this auction price with probability about 0.5 and choose to not walk away/quit with probability about 0.75. Therefore, the probability that $\tilde{S}$ is less than -\$2,500 lies in $[0.5, 0.75]$. The bounds are similar for fleet/lease cars, panels (b) and (d), although these bounds appear to be somewhat tighter and the upper and lower bounds individually appear to be less disperse.

Figure 5 displays the estimated densities corresponding to the upper and lower bounds of seller valuations after integrating out the unobserved heterogeneity (W). Panels (a) and (b) display the density corresponding to the upper bound in the dealers and fleet/lease samples, and panels (c) and (d) display similar results for the lower bound. As with the densities for reserve prices and auction prices in Figure 1, the density of seller valuations is tighter after removing unobserved heterogeneity.

The estimates of the density of buyer valuations, f_B, from the order statistic inversion described in Section 4.4 are shown in Figure 6 in green. The density of the auction price, $f_{B^{(2)}}$, in Figure 6 (in blue) is equivalent to the density shown in panels (c) and (d) of Figure 1, after removing unobserved heterogeneity. As should be the case, $f_{B^{(2)}}$ first-order stochastically dominates f_B. Figure 6 also displays (in red) the density corresponding to the upper or lower bound of seller valuations from Figure 5.

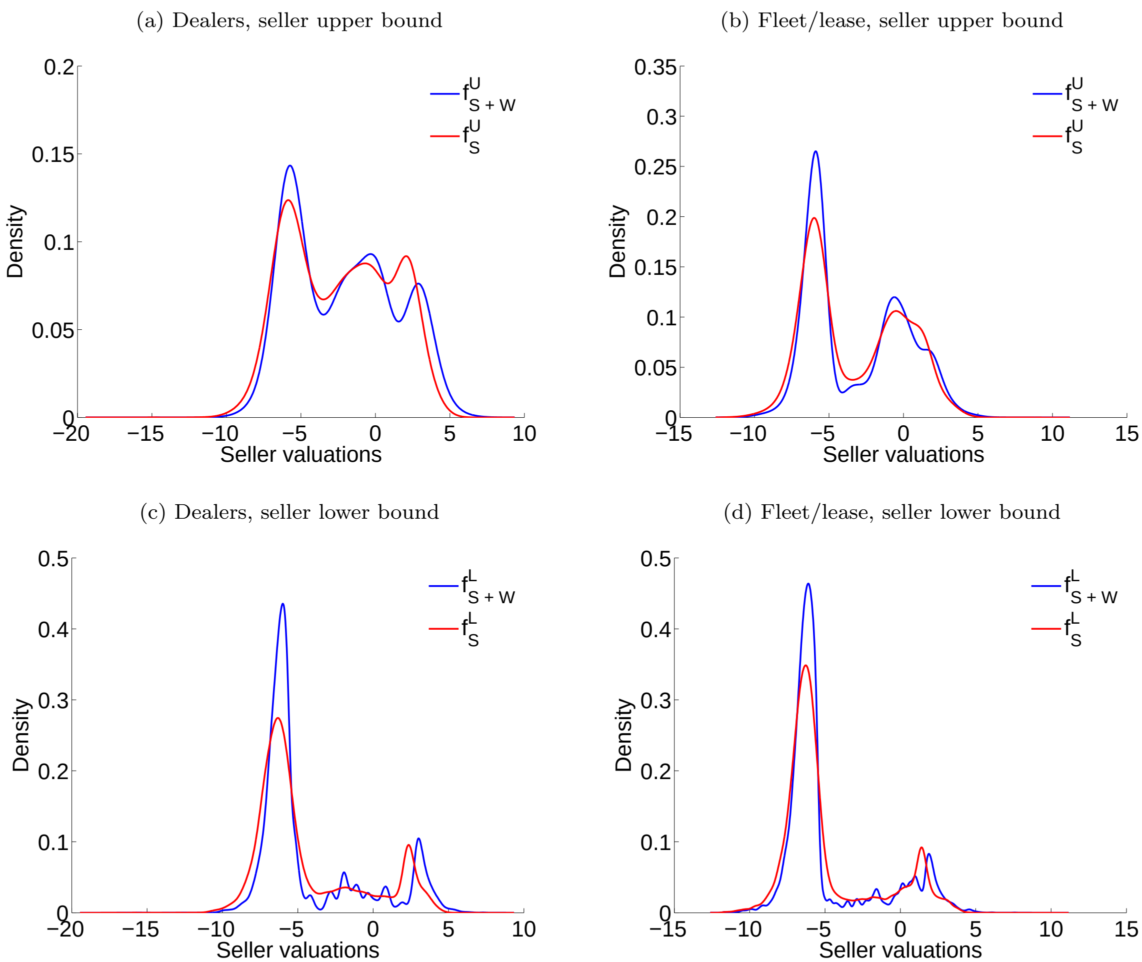
Figure 6 demonstrates that there is significant overlap, both in terms of support and mass, between the density of buyer valuations and seller valuations. This feature of overlapping supports is referred to in the theoretical bargaining literature as the "no gap" case (i.e., there is no gap between the upper bound of the support of seller valuations and the lower bound of the support of buyer valuations, and hence there is uncertainty as to whether gains from trade actually exist), and is the case motivating Myerson and Satterthwaite (1983).[44] Comparing panel (a) to panel (b) shows that fleet/lease sellers tend to have more mass in the lower half of the support of seller types than do dealers, suggesting that trade may be more likely to occur in the fleet/lease sample, as is shown to be the case in Table 1.

5 The Pareto Frontier and Real-World Bargaining

This section describes how the Pareto frontier and other counterfactual mechanisms can be solved for once the distributions of seller and buyer valuations are known. I also describe identification and estimation of the direct mechanism corresponding to the real-world, dynamic bargaining mechanism used at auto

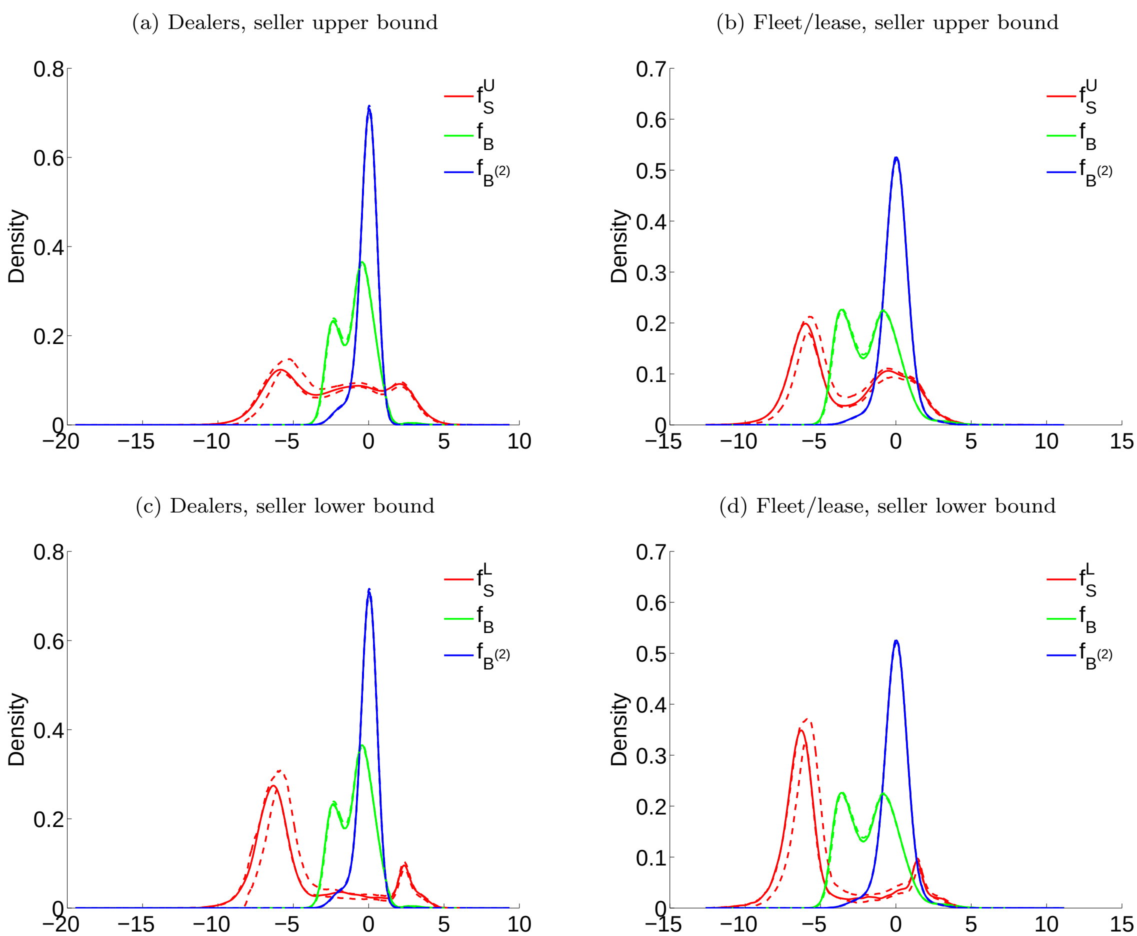
[44] See Fudenberg and Tirole (1991).

Figure 5: Density of seller valuations with and without unobserved heterogeneity



Notes: Densities of seller valuations prior to removing unobserved heterogeneity (in blue), and after removing unobserved heterogeneity (in red). Panels (a) and (b) display seller distribution upper bound. Panels (c) and (d) display lower bound. Units = $1,000.

Figure 6: Densities of seller valuations, buyer valuations, and auction price



Notes: Densities of seller valuations (in red), buyer valuations (in green), and auction price (in blue). Panels (a) and (b) display seller distribution upper bound. Panels (c) and (d) display lower bound. Dashed lines mark pointwise 95% confidence bands from 200 bootstrap replications of full estimation procedure. Units = $\$1,000$.

auctions.[45] I then bring together the estimates from Section 4 to analyze the efficiency of bargaining.

The counterfactuals hold fixed the distribution of buyer and seller types. In reality, changing the mechanism could change the distribution of types participating.[46] Also, in counterfactuals I only change bargaining—I do not change the auction. Each counterfactual mechanism is a mechanism for bilateral trade between the seller and high bidder after a no-reserve ascending auction has occurred.[47] The auction selects the highest-value bidder, and the lower bound for the buyer's support in the post-auction bargaining game becomes $b^{(2)}$, the auction price.[48] Finally, the counterfactual comparisons all consider surplus *after* removing auction-level heterogeneity, both observed and unobserved.

5.1 Solving for the Pareto Frontier

In this section I discuss how I solve for the Pareto frontier and other direct, efficient mechanisms conditional on a realization of $B^{(2)} = b^{(2)}$. By the Revelation Principle (Myerson 1979), any static, incentive-compatible, individually rational, bilateral trade mechanism can be written as a direct-revelation mechanism where players truthfully report their valuations to a broker and then trade occurs with probability $x(s,b)$, with the buyer paying $p(s,b)$ to the seller.[49] Williams (1987) demonstrated that a bilateral bargaining mechanism can alternatively be summarized by the two objects (x,q), rather than (x,p), where q is the expected utility for the type $\overline{S}$. The ex-ante expected utility of the buyer and seller in a mechanism (x,q) is given by

$$\overline{U}_S(x,q) \equiv q + \int_{b^{(2)}}^{\overline{B}} \int_{\underline{S}}^{\overline{S}} x(s,b) F_S(s) \frac{f_B(b)}{1 - F_B(b^{(2)})} ds db \tag{9}$$

$$\overline{U}_B(x,q) \equiv G(x) - q + \int_{b^{(2)}}^{\overline{B}} \int_{\underline{S}}^{\overline{S}} x(s,b) \frac{(1 - F_B(b))}{1 - F_B(b^{(2)})} f_S(s) ds db \tag{10}$$

[45] A "direct-revelation" or "direct" mechanism in a bilateral trade setting is one in which players truthfully report their valuations to a broker and then trade and transfers occur following allocation and payment rules which depend on the distributions of types and the players' reports.

[46] For example, the buyer and seller types choosing to attend the auction house could change if the mechanism were more or less favorable for certain types. Also, the distribution of seller types could change, because embedded in the seller valuations is the option to attempt to sell the car at the auction house at a later date, and the payoff from doing so would change with the mechanism.

[47] Note that I do not work with a direct mechanism in which N buyers and one seller simultaneously report types to a mechanism designer, primarily because this mechanism would be starkly different from mechanisms applied in practice and because I wish to focus on the efficiency of bilateral trade in particular. I also do not consider a secret reserve price in these counterfactual mechanisms.

[48] Setting the lower bound of the buyer's support to be exactly the auction price is an approximation; the actual lower bound of the support of buyer types, from the seller's point of view, may be slightly higher than this in post-auction bargaining given that the buyer's choice to not opt out of bargaining contains some information about the buyer's type. See the proof of Proposition 2 in Appendix A. This approximation error will be small as long as bargaining costs are small.

[49] Note that the notation here is the reverse of Myerson and Satterthwaite (1983) and Williams (1987), in which p represented the probability of trade and x represented the transfer. Also note that the transfer function, p, is not essential for the results here. I report it in Appendix B.2.

where

$$G(x) \equiv \int_{b^{(2)}}^{\overline{B}} \int_{\underline{S}}^{\overline{S}} \left(\phi_B(b) - \phi_S(s)\right) x(s,b) f_S(s) \frac{f_B(b)}{1 - F_B(b^{(2)})} ds db \tag{11}$$

and

$$\phi_S(s) \equiv \phi_S(s,1) \qquad\qquad \phi_B(b) \equiv \phi_B(b,1)$$

$$\phi_S(s,\alpha_1) \equiv s + \alpha_1 \frac{F_S(s)}{f_S(s)} \qquad \text{and} \qquad \phi_B(b,\alpha_2) \equiv b - \alpha_2 \frac{1 - F_B(b)}{f_B(b)}$$

Williams (1987) demonstrated further that the Pareto frontier, that is, the maximized value of

$$\eta \overline{U}_S + (1-\eta)\overline{U}_B \tag{12}$$

for $\eta \in [0,1]$, can be traced out by the class of mechanisms with trading rules, $x(s,b)$, defined by

$$x^{\alpha_1(\eta),\alpha_2(\eta)}(s,b) = 1\left\{\phi_B(b,\alpha_2(\eta)) \geq \phi_S(s,\alpha_1(\eta))\right\}$$

The parameters $(\alpha_1(\eta), \alpha_2(\eta))$ can be solved for at each η using an approach developed in Williams (1987) and described in Appendix B.2. Intuitively, the approach maximizes (12) subject to $G(x^{\alpha_1(\eta),\alpha_2(\eta)}) \geq 0$, where $G(x)$ is defined in (11). This constraint implies that the worst types—the lowest buyer type and highest seller type—must receive a non-negative surplus in order to be willing to participate in the mechanism. A sufficient condition for success of this solution method is that $\phi_S(s)$ and $\phi_B(b)$—traditionally referred to as players' "virtual valuations"—be weakly increasing. I do not impose monotonicity of virtual valuations in my main analysis, but present results doing so in Appendix B.4.[50]

Several mechanisms of interest fit into this framework, such as the following

1. First-best trade (infeasible mechanism where trade occurs whenever buyer values the car more than seller): $\alpha_1 = \alpha_2 = 0$.

2. Second-best trade (the mechanism maximizing the gains from trade): $\eta = 1/2$, $\alpha_1 = \alpha_2 = \alpha^*$, where α^* solves $G(x^{\alpha^*,\alpha^*}) = 0$.

3. Seller-optimal: $\eta = 1$, $\alpha_1 = 0$, $\alpha_2 = 1$.

4. Buyer-optimal: $\eta = 0$, $\alpha_1 = 1$, $\alpha_2 = 0$.

5. Pareto frontier: mechanisms maximizing (12) subject to $G(x^{\alpha_1(\eta),\alpha_2(\eta)}) \geq 0$ for $\eta \in [0,1]$.

Note that an auction followed by the seller-optimal mechanism is equivalent to a public reserve auction.[51] An additional mechanism with $\alpha_1 = \alpha_2 = 1$ is discussed in Myerson and Satterthwaite (1983) and would

[50] An alternative approach to handle non-monotonic virtual valuations would be through "ironing" (Myerson 1981), but this would involve a significantly larger computational burden than the current approach.

[51] See Menezes and Ryan (2005).

maximize the gains to a broker (auction house) with market power. This mechanism is discussed in Appendix B.7.3.

I derive an additional direct-revelation mechanism which maximizes the probability of trade rather than the gains from trade. This result is a corollary to Theorem 2 of Myerson and Satterthwaite (1983) and the proof follows the same line of reasoning as in Myerson and Satterthwaite (1983).[52] The proof of existence relies on strict monotonicity of $\phi_S(s)$ and $\phi_B(b)$.

Corollary 1. *Suppose $\phi_S(s)$ and $\phi_B(b)$ are both strictly increasing. Then the direct mechanism maximizing the probability of trade has allocation rule $x^\kappa(s,b) = 1\{\phi_S(s) - (2\kappa)/(1-\kappa) \leq \phi_B(b)\}$, where $\kappa \in [0,1)$ is the solution to $G(x^\kappa(s,b)) = 0$.*

Once the first-stage auction is taken into account, the ex-ante probability of trade in any of these mechanisms is given by

$$\Pr(trade) = \int_{\underline{B}}^{\overline{B}} \int_{b^{(2)}}^{\overline{B}} \int_{\underline{S}}^{\overline{S}} x(s,b;b^{(2)}) f_S(s) \frac{f_B(b)}{1 - F_B(b^{(2)})} f_{B^{(2)}}(b^{(2)}) ds db db^{(2)} \tag{13}$$

and the expected gain from trade is given by

$$\int_{\underline{B}}^{\overline{B}} \int_{b^{(2)}}^{\overline{B}} \int_{\underline{S}}^{\overline{S}} (b-s) x(s,b;b^{(2)}) f_S(s) \frac{f_B(b)}{1 - F_B(b^{(2)})} f_{B^{(2)}}(b^{(2)}) ds db db^{(2)} \tag{14}$$

To perform this integration, I use Gauss-Chebyshev quadrature, as described in Appendix B.1, with 200 nodes in the s and b dimensions, and 25 nodes in the $b^{(2)}$ dimension.[53] I set the bounds of the support for buyer and seller valuations to the minimum and maximum of 10,000 draws from the estimated densities F_S and F_B. The results are not sensitive to this procedure.

5.2 Estimating the Dynamic Mechanism

This section describes how I solve for surplus in the currently used dynamic mechanism by backing out a direct-revelation mechanism corresponding to the mechanism used at auto auctions. As with the above mechanisms, this direct mechanism can be characterized by functions x and p determining whether or not trade will occur and at what price. x and p will in general depend on the realization of the auction price, $b^{(2)}$, as this is the lower bound of the support of buyer types when bargaining takes place.

Let the allocation function for the dynamic mechanism be written

$$x^D(s,b;b^{(2)}) \equiv 1\left\{b \geq g\left(\rho(s), b^{(2)}\right)\right\} \tag{15}$$

where $g(\cdot)$ is an unknown function.[54] Here I rely on Proposition 2, which demonstrates that ρ is strictly increasing in s and hence the allocation function can be considered to be a function of s or of $r = \rho(s)$.

[52] Note that the expected transfer functions for the mechanism in Corollary 1 are given by (19) and (20) in Appendix B.2

[53] Increasing the number of nodes in any dimension did not change the results. A greater degree of accuracy in the b and s dimensions than in the $b^{(2)}$ dimension is useful, as each mechanism is solved conditional on $b^{(2)}$.

[54] Although the counterfactual mechanisms discussed in Section 5.1 all had binary allocation functions, this is not necessarily the case for the dynamic mechanism, but without assuming an indicator function the allocation function in the real-world mechanism would not be identified.

The secret reserve price being strictly increasing also implies $\rho(s)$ can be written

$$\rho(s) = F_R^{-1}(F_S(s))$$

which can be computed given estimates of F_S and F_R.

As x^D is written in (15), $g(\cdot)$ is not identified given the available data: I observe whether or not trade occurred, but I do not observe realizations of b—or even of r or $b^{(2)}$ because of unobserved heterogeneity. However, for each observation in the data, I do have an estimate of $\tilde{r} = r + w$ and $\tilde{b}^{(2)} = b^{(2)} + w$, which implies an estimate of $\psi \equiv \tilde{r} - \tilde{b}^{(2)} = r - b^{(2)}$. To exploit this result, note that Proposition 3 implies that the allocation function can be simplified to

$$x^D(r, b; b^{(2)}) = 1\left\{b - b^{(2)} \geq g\left(r - b^{(2)}, b^{(2)} - b^{(2)}\right)\right\} \tag{16}$$

$$= 1\left\{b - b^{(2)} \geq g_0\left(r - b^{(2)}\right)\right\} \tag{17}$$

where $g_0(\psi) = g(\psi, 0)$. I refer to g_0 as the trade boundary function. Recall that Proposition 3 demonstrates that the probability of trade in a setting where realizations of the reserve price, auction price, and bargaining buyer's type were given by $(r, b^{(2)}, b)$ would be the same as in a setting where each of these objects were reduced by a common amount (here, by $b^{(2)}$). The probability of trade at any value of ψ, a realization of the random variable $\Psi = R - B^{(2)}$, can then be written

$$\Pr(trade|\Psi = \psi) = \int_{\underline{B}}^{\overline{B}} \frac{1 - F_B\left(g_0(\psi) + b^{(2)}\right)}{1 - F_B\left(b^{(2)}\right)} \frac{f_R\left(\psi + b^{(2)}\right) f_{B^{(2)}}\left(b^{(2)}\right)}{\int_{\underline{B}}^{\overline{B}} f_R(\psi + v) f_{B^{(2)}}(v) dv} db^{(2)} \tag{18}$$

I use (18) to solve for $g_0(\cdot)$ on a grid of ψ using a bisection method. To estimate $\Pr(trade|\Psi = \psi)$, I use a Nadayara-Watson kernel regression.[55] To estimate the right hand side of (18), I use Gauss-Hermite quadrature, plugging in each of the estimated densities. The expected gains from trade under the current mechanism can then be evaluated as in (14), replacing x with the estimate of x^D. Note that this estimate ignores any loss in surplus due to bargaining costs. Appendix B.3 discusses bounds on this loss.

5.3 Putting It All Together: How Efficient Is Bargaining?

This section brings together the density estimates from Section 4 and the mechanisms from Section 5.1–5.2 to present a quantitative comparison of real-world, dynamic bargaining to the efficient frontier. The counterfactual mechanisms I consider include the first-best mechanism, the trade-maximizing mechanism, and the mechanisms defining the second-best Pareto frontier, ranging from the buyer-optimal to the seller-optimal mechanisms and including the mechanism maximizing the total expected gains from trade (which I refer to in the results as "the second-best mechanism").

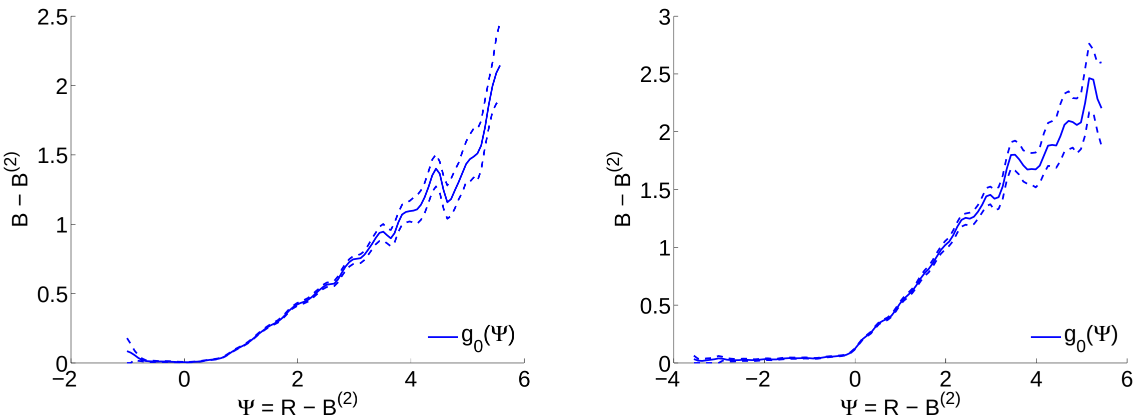
To begin, Figure 7 displays the trade boundary function for the dynamic mechanism, identified by the probability that trade occurs at given value of the gap between the seller's secret reserve price and

[55] I use a Gaussian kernel with bandwidth set to the asymptotically optimal bandwidth for kernel density estimation of Ψ.

Figure 7: Estimated trade boundary function (g_0)

(a) Dealers

(b) Fleet/lease



Notes: Estimate of $g_0(R - B^{(2)})$, defining region where trade occurs. Horizontal axis is gap between reserve price and auction price. Vertical axis is gap between type of buyer who enters bargaining and auction price. Dashed lines mark pointwise 95% confidence bands from 200 bootstrap replications of full estimation procedure. Units = $\$1,000$.

the auction price, as described in Section 5.2. This boundary function, $g_0(R - B^{(2)})$, has the following interpretation: In the dealers sample, panel (a), when the secret reserve price exceeds the auction price by $2,000, the car will sell as long as the valuation of the highest-value buyer exceeds the auction price by about $500. The g_0 function in the fleet/lease sample, panel (b), is slightly steeper; when the reserve price exceeds the auction price by $2,000, the buyer and seller will agree when the buyer values the car at about $1,000 above the reserve price. This reflects the fact that, in fleet/lease cars, secret reserve prices are lower relative to buyer values, and is consistent with personal conversations with industry participants, who explain that fleet/lease cars are more likely to be "priced to sell." Note that 7 does not display separate results for the seller distribution upper and lower bounds because the approach described in Section 5.2 does not rely knowledge of F_S (or bounds of F_S), and instead relies directly on information contained in reserve prices.

The performance of the currently used dynamic mechanism relative to the Pareto frontier is displayed in Figure 8. Each panel of Figure 8 also displays the location of the second-best and trade-maximizing mechanisms with respect to the frontier, and for each mechanism pointwise 95% confidence intervals are displayed. In each panel, the second-best and trade-maximizing mechanisms are nearly indistinguishable from one another on the frontier, and both lie very close to the seller-optimal point of the frontier (the far-right point). Figure 8 suggests that the frontier is approximately linear as it approaches the seller optimal mechanism on the far right point of the frontier. However, magnifying the plot reveals that the frontier is actually quite concave in this region as η, the Pareto weight, moves from 0.5 to 1. In both the dealers and fleet/lease samples, and under both the upper bound and lower bound of the seller valuation distribution, the auction followed by alternating-offer bargaining results in a surplus level which lies relatively close to the Pareto frontier, suggesting that the mechanism may be efficient.

Any gap between the frontier and the dynamic mechanism would represent a deadweight loss—neither bargaining party captures this lost surplus. I refer to this gap as the deadweight loss due to limited commitment or mechanism choice, as this gap could theoretically be eliminated by committing to a static, direct-revelation mechanism which lies on the Pareto frontier. Figure 8 suggests that this deadweight loss may not be large, but is still positive, and some points along the frontier would make both parties better off.[56]

An additional source of deadweight loss is incomplete information. Myerson and Satterthwaite (1983) demonstrated that this deadweight loss is inevitable in bilateral bargaining with two-sided uncertainty. Intuitively, both parties trade off the conflicting incentives of increasing the probability of trade and increasing the rent extracted from the other party, leading buyers to shade their offering price downward and sellers to shade their asking price upward, leading some trades to fail to be consummated even when the buyer values the good more than the seller. This deadweight loss is depicted graphically in Figure 9 as the gap between the Pareto frontier and the first-best (infeasible) surplus line. In both the dealers sample and the fleet/lease sample, the gap is small. In fact, much of the estimated Pareto frontier lies within the pointwise 95% confidence band of the first-best line, indicating that incomplete information per se is likely not the primary reason for inefficiency in this market.

Table 4 contains more detailed information on these mechanisms, displaying the total expected gains from trade, the gains to each player, and the probability of sale. Gains are reported in units of $1,000. The table displays outcomes for the first-best, second-best, buyer-optimal, seller-optimal, and trade-maximizing mechanisms. The current dynamic mechanism is displayed in the final column.[57] Bootstrapped standard errors are reported in parentheses.[58] The percentages in square brackets in the second-best column represent the second-best outcomes as a percentage of the first-best. In the dynamic mechanism column, percentages in square brackets represent the dynamic mechanism outcomes as a percentage of the second-best. Panel A uses the upper bound on the seller distribution and panel B the lower bound.

I focus first on the estimation using the upper bound of seller valuations, panel A, of Table 4. The final column shows that the expected gain from trade in the current mechanism is $4,276, with $548 going to the buyer, $3,509 going to the seller, and the remainder going to the auction house through fees. For buyers, $548 can be interpreted as an expected retail markup above the auction price (net of

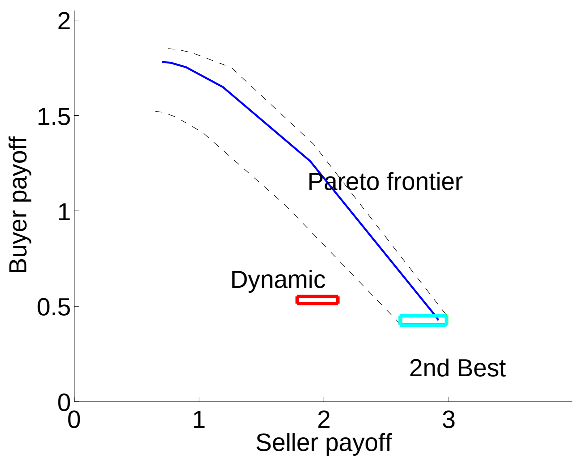
[56] Bazerman, Gibbons, Thompson, and Valley (1998) argued that real-world bargaining can potentially yield *more* efficient outcomes than the theoretical bounds of Myerson and Satterthwaite (1983) and Williams (1987) would predict possible. Bazerman, Gibbons, Thompson, and Valley (1998) reviewed evidence from the behavioral and experimental literature suggesting that more efficient outcomes can occur due to non-traditional utility functions (where one player's utility nests the other's), limits on players' abilities to mimic other types, and other features of bounded rationality.

[57] Because buyer and seller gains take into account fees paid to the auction house, the total expected gains from trade for the dynamic mechanism in the first row is given by the sum of buyer and seller gains and fees paid to the auction house.

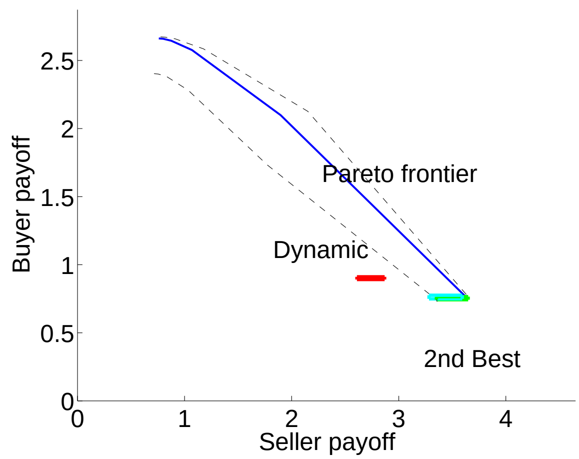
[58] These standard errors come from 200 nonparametric bootstrap replications of the full estimation procedure.

Figure 8: Expected gains from trade in dynamic mechanism and on Pareto frontier

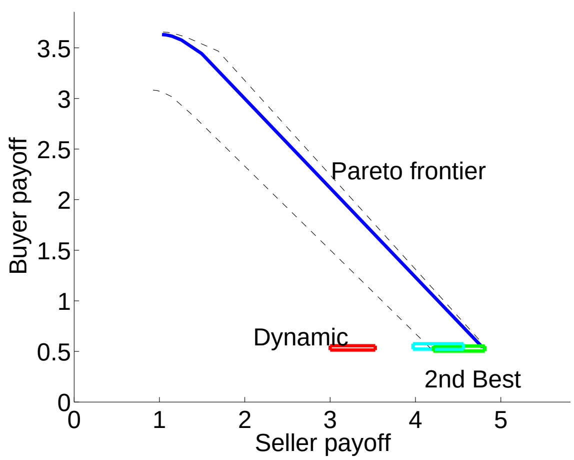
(a) Dealers, seller upper bound



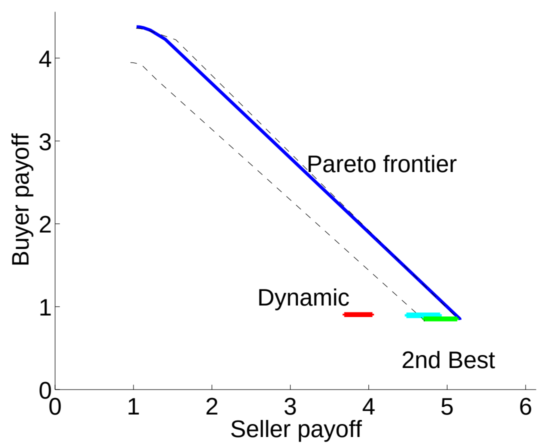
(b) Fleet/lease, seller upper bound



(c) Dealers, seller lower bound

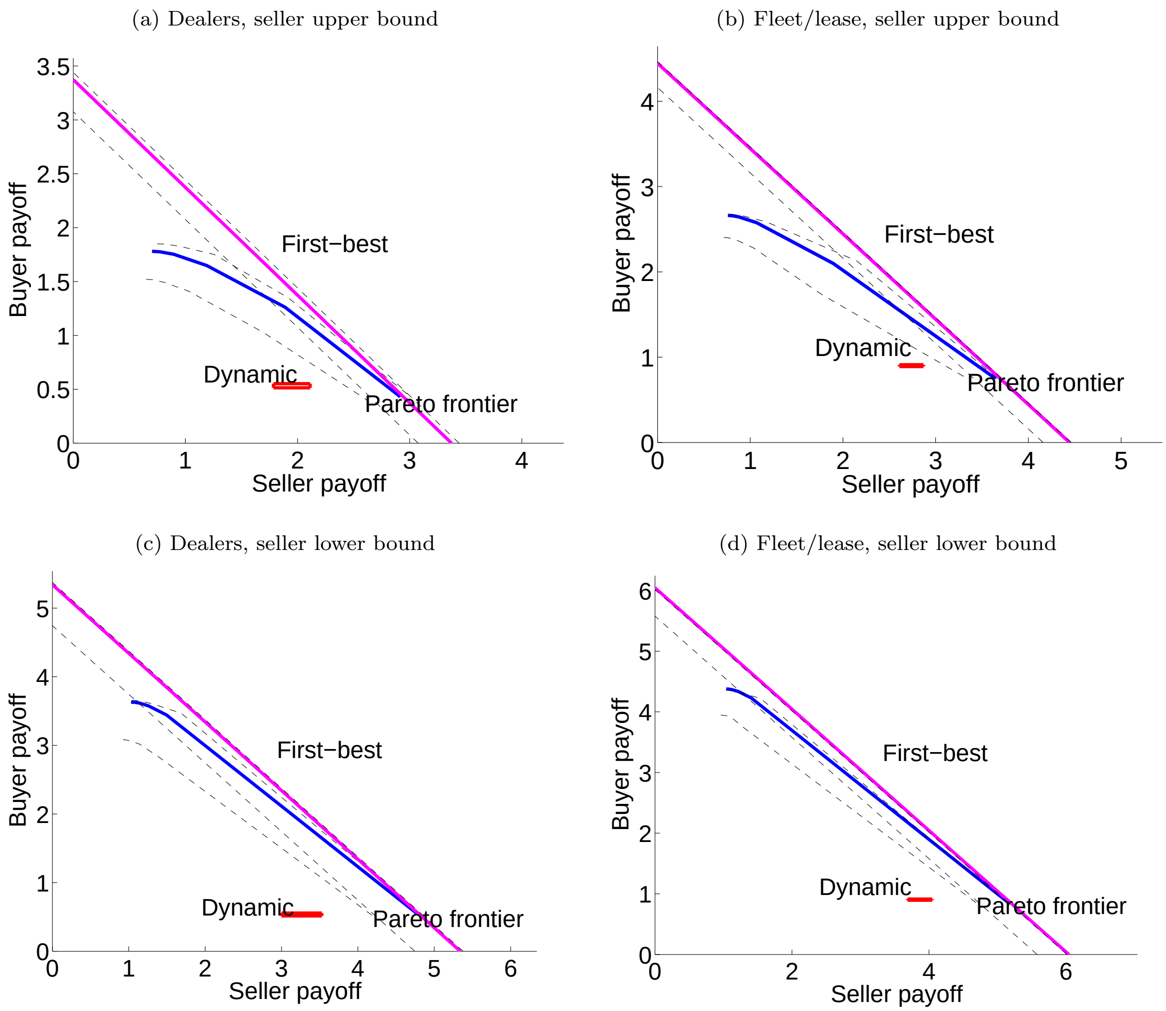


(d) Fleet/lease, seller lower bound



Notes: Expected gains from trade, in buyer and seller payoff space, for Pareto frontier (in blue), current dynamic mechanism (in red), second-best mechanism (in green), and trade-maximizing mechanism (in cyan). Dashed lines surrounding the frontier, and solid lines about other mechanisms, mark pointwise 95% confidence bands from 200 bootstrap replications of full estimation procedure. Units = $1,000.

Figure 9: Expected gains from trade in dynamic mechanism, on Pareto frontier, and on first-best efficient frontier.



Notes: Expected gains from trade, in buyer and seller payoff space, for Pareto frontier (in blue), current dynamic mechanism (in red), and first-best efficient frontier (in magenta). Dashed lines surrounding the frontiers, and solid lines about dynamic mechanism, mark pointwise 95% confidence bands from 200 bootstrap replications of full estimation procedure. Units = $1,000.

Table 4: Dealers sample: Expected gains from trade and prob of trade in counterfactual and current mechanisms

A. Efficiency of bargaining using seller distribution ***upper*** *bound,* ***dealers sample***						
	First-best	Second-best	Buyer-optimal	Seller-optimal	Trade-maximizing	Dynamic mechanism
Expected gains from trade (units=$1,000)	5.342	5.324	4.661	5.319	5.094	4.276
	(0.179)	(0.182)	(0.181)	(0.182)	(0.181)	(0.164)
		[99.7%]				**[80.3%]**
Buyer gains (units=$1,000)		0.541	3.631	0.533	0.564	0.548
		(0.014)	(0.155)	(0.014)	(0.015)	(0.012)
						[101.3%]
Seller gains (units=$1,000)		4.783	1.030	4.786	4.520	3.509
		(0.171)	(0.033)	(0.171)	(0.169)	(0.156)
						[73.4%]
Probability of trade	0.849	0.831	0.644	0.829	0.868	0.705
	(0.016)	(0.020)	(0.019)	(0.020)	(0.021)	(0.015)
		[98.0%]				**[84.8%]**

B. Efficiency of bargaining using seller distribution ***lower*** *bound,* ***dealers sample***						
	First-best	Second-best	Buyer-optimal	Seller-optimal	Trade-maximizing	Dynamic mechanism
Expected gains from trade (units=$1,000)	3.374	3.345	2.483	3.338	3.350	2.811
	(0.103)	(0.108)	(0.097)	(0.108)	(0.109)	(0.097)
		[99.2%]				**[84.0%]**
Buyer gains (units=$1,000)		0.439	1.780	0.426	0.437	0.543
		(0.013)	(0.085)	(0.013)	(0.013)	(0.009)
						[123.9%]
Seller gains (units=$1,000)		2.907	0.702	2.912	2.905	2.049
		(0.099)	(0.024)	(0.099)	(0.099)	(0.093)
						[70.5%]
Probability of trade	0.747	0.693	0.369	0.692	0.694	0.700
	(0.010)	(0.018)	(0.013)	(0.018)	(0.018)	(0.009)
		[92.8%]				**[100.9%]**

Notes: Dealers sample. First-best, second-best, buyer-optimal, seller-optimal, and trade-maximizing mechanisms compared to current dynamic mechanism. Gains are in $1,000 units. Standard errors are reported in parentheses and come from 200 bootstrap replications of the full estimation procedure. Square brackets in second-best column represent second-best outcomes as percentage of first-best. In dynamic mechanism column, square brackets represent dynamic mechanism outcomes as a percentage of second-best. Panel A displays estimates under seller distribution upper bound and Panel B under lower bound.

any other costs of remarketing the vehicle, which would be passed onto the final consumer in the retail price), as a buyer's valuation should be the amount he expects to make on the car. For a seller, \$3,509 is the expected gain from selling the car at the auction today as opposed to selling it through her next-best option. This next-best option includes the possibility of taking the car back to the seller's own lot or leaving it at the auction house to run again in several weeks. Therefore, the relatively high expected gains from trade for the seller suggest that, in a complete-information world, sellers would be willing to sell cars for much less. The characteristic of the data leading to this result is the fact that even when the auction price falls far below expectation (based on auction-level characteristics) sellers still accept (or not walk away from) the auction price with high probability, as discussed in Section 4.6.

The estimated probability of trade in the dynamic mechanism is 0.705. This measure comes from integrating the dynamic allocation function $x^D(\rho(S), B; B^{(2)})$ over all three of its arguments, as described in Section 5.2. Comparing the estimated probability of trade to the raw probability of trade from Table 1 (0.710) serves as one indication of good model fit.

Comparing each of the columns in panel A to each other yields several useful insights. The first-best surplus in panel A is \$5,342, and the second-best surplus captures 99.7% of this surplus (as seen by the percentage in square brackets in the second-best column), as was suggested by Figure 9. Therefore, the deadweight loss due to incomplete information is small. The second-best probability of trade (0.831) is also similarly close to the first-best (0.849), capturing 98% of the first-best volume.[59]

The deadweight loss due to limited commitment can be evaluated by comparing the expected gains from trade in final column of Table 4 to the second-best column. In the used-car dealers sample, the dynamic mechanism achieves 80.3% of the second-best surplus (as seen in square brackets). The probability of trade in the dynamic mechanism achieves 84.8% of the second-best level. The gains to the buyer in the dynamic mechanism are 101.3% of the buyer gains under the second-best mechanism, while the seller gains are only 73.4% of the second-best level. However, the buyer-optimal mechanism would yield much higher gains for the buyer (increasing from \$548 to \$3,631) and the seller-optimal mechanism would yield improvements to the seller (increasing from \$3,509 to \$4,786).

For the most part, the quantities in panel A are maximized by the appropriate mechanism, which serves as another check of the model. Specifically, the expected gains from trade are maximized by the first-best, followed by the second-best. The buyer gains are maximized by the buyer-optimal mechanism and seller gains by the seller-optimal mechanism. One exception to this good fit is the trade-maximizing mechanism, which attains a level of probability of trade (0.868) which exceeds the first-best (0.849). This feature is due to the fact that the virtual valuations (ϕ_B and ϕ_S from Section 5.1) are not necessarily increasing. I address this in Appendix B.4 by imposing monotonicity of virtual valuations and find that this corrects the ranking of first-best vs. the trade-maximizing mechanism; however, this correction also leads to a poorer fit in the probability of trade estimate for the dynamic mechanism.

Panel B of Table 4, which uses the lower bound of seller valuations, demonstrates similar results

[59]Recall that the first-best would have trade occur whenever the buyer's value is higher than the seller's. Therefore, the first-best probability of trade will not be one, but will depend on the distributions of buyer and seller valuations.

to panel A, with several interesting differences. First, under the lower bound of seller valuations, the probability of trade in the dynamic mechanism exceeds slightly the probability of trade in the trade-maximizing mechanism. Second, the trade-maximizing mechanism no longer appears to outperform the first-best. Finally, while the second-best captures 99.2% of the first-best surplus, it captures only 92.8% of the first-best trade volume, highlighting the fact that trades which are captured by the first-best mechanism but missed by the second-best (due to the incomplete information explored in Myerson and Satterthwaite 1983) tend to be relatively low-surplus trades, i.e. cases where the buyer only values the car slightly more than the seller.

The qualitative findings from the dealers sample are similar in the fleet/lease sample (Table 5). The second-best mechanism achieves 99.5% or 98.8% of the first-best gains from trade, depending on whether the upper (panel A) or lower (panel B) bound of seller valuations is used. The second-best probability of trade attains 96.4% or 90.8% of the first-best volume. Comparing Table 5 to Table 4 suggests that the dynamic bargaining mechanism may be slightly more efficient in the fleet/lease sellers sample than in the dealers sample—consistent with conventional wisdom in the industry—capturing 90.9% (panel A) or 86.5% (panel B) of second-best surplus in the fleet/lease sample. The levels of gains as well as the probability of trade are also higher in the fleet/lease sample than in the dealers sample. Under the upper bound of seller valuations, panel A shows that the expected gains from trade in the dynamic mechanism are \$5,201, with buyers capturing \$912, sellers capturing \$4,060, and the remainder going toward auction house fees.

The expected gains from trade in Figures 8-9 and Tables 4–5 do not take into account any surplus lost due to bargaining costs, and thus can be thought of as an upper bound on surplus under dynamic bargaining. Appendix B.3 provides an approach for estimating a lower bound on surplus which takes into account bargaining costs. I find that the surplus lost due to costs in dynamic bargaining is small, less than \$25 for buyers and less than \$50-100 for sellers.

Further checks of model robustness as well as analyses of additional mechanisms are contained in the appendices. Appendix B.5 presents results analogous to those in Tables 4–5 under alternative sample restrictions. Appendix B.6 displays welfare results under alternative assumptions on the mean number of bidders, λ, and demonstrates that alternative assumptions for λ do not alter the paper's conclusions. Appendix B.7.1 analyzes the efficiency of a seller bargaining with a random bidder rather than the high bidder. This mechanism changes not only what dynamic bargaining can achieve, but also shifts the Pareto and first-best frontiers as well. I find that bargaining with a random bidder is less efficient than bargaining with the high bidder, but still achieves 70–88.5% of the second-best gains from trade. Appendix B.7.2 contains results from an auction in which all players commit not to bargain, and in which the seller is only allowed to accept or reject the auction price. I find this mechanism is quite efficient. Finally, Appendix B.7.3 analyzes the mechanism which would maximize revenue for a monopolist auction house.

Table 5: Fleet/lease sample: Expected gains from trade and prob of trade in counterfactual and current mechanisms

*A. Efficiency of bargaining using seller distribution **upper** bound, **fleet/lease sample***						
	First-best	Second-best	Buyer-optimal	Seller-optimal	Trade-maximizing	Dynamic mechanism
Expected gains from trade (units=$1,000)	6.045 (0.117)	6.015 (0.117) **[99.5%]**	5.417 (0.121)	6.004 (0.118)	5.823 (0.116)	5.201 (0.097) **[86.5%]**
Buyer gains (units=$1,000)		0.858 (0.006)	4.378 (0.109)	0.841 (0.006)	0.905 (0.007)	0.912 (0.005) **[106.3%]**
Seller gains (units=$1,000)		5.157 (0.115)	1.040 (0.019)	5.163 (0.114)	4.904 (0.113)	4.060 (0.096) **[78.7%]**
Probability of trade	0.898 (0.003)	0.866 (0.004) **[96.4%]**	0.711 (0.009)	0.859 (0.005)	0.915 (0.006)	0.757 (0.002) **[87.4%]**

*B. Efficiency of bargaining using seller distribution **lower** bound, **fleet/lease sample***						
	First-best	Second-best	Buyer-optimal	Seller-optimal	Trade-maximizing	Dynamic mechanism
Expected gains from trade (units=$1,000)	4.445 (0.081)	4.392 (0.082) **[98.8%]**	3.421 (0.080)	4.382 (0.082)	4.268 (0.082)	3.993 (0.071) **[90.9%]**
Buyer gains (units=$1,000)		0.759 (0.006)	2.662 (0.071)	0.745 (0.006)	0.774 (0.007)	0.908 (0.005) **[119.6%]**
Seller gains (units=$1,000)		3.633 (0.079)	0.759 (0.015)	3.637 (0.079)	3.481 (0.080)	2.856 (0.070) **[78.6%]**
Probability of trade	0.858 (0.003)	0.778 (0.005) **[90.8%]**	0.466 (0.009)	0.771 (0.005)	0.794 (0.005)	0.755 (0.002) **[97.0%]**

Notes: Fleet/lease sample. First-best, second-best, buyer-optimal, seller-optimal, and trade-maximizing mechanisms compared to current dynamic mechanism. Gains are in $1,000 units. Standard errors are reported in parentheses and come from 200 bootstrap replications of the full estimation procedure. Square brackets in second-best column represent second-best outcomes as percentage of first-best. In dynamic mechanism column, square brackets represent dynamic mechanism outcomes as a percentage of second-best. Panel A displays estimates under seller distribution upper bound and Panel B under lower bound.

6 Conclusion

This paper examined the efficiency of bargaining from a real-world setting with two-sided incomplete information. I developed a model and strategy for nonparametrically identifying and estimating the distributions of valuations on both sides of the market without relying on a particular structure or equilibrium solution for the bargaining game. I then mapped these distributions into the static, direct-revelation mechanism framework which traces out the efficient frontier derived in Myerson and Satterthwaite (1983) and Williams (1987). I found that the deadweight loss due to incomplete information—the gap between the first-best trade line and the second-best frontier—is small in the wholesale used-car market. I also found that the deadweight loss due to mechanism choice/limited commitment is roughly 9–20% of the second-best surplus. This result is consistent with the hypothesis of Wilson (1986) and Ausubel and Deneckere (1993) who suggested that it may be that "[dynamic bargaining mechanisms] survive because they employ trading rules that are efficient for a wide class of environments."

The findings of this paper shed some light on the question of why bargaining may be used after an auction, as opposed to sales mechanisms which are more standard in the theoretical literature, such as an auction with no reserve price or an auction with a public reserve price. In this industry, it is the auction house, rather than the seller, who chooses the mechanism. An auction house is a platform in a two-sided market, required to attract both buyers and sellers, each with private information about his or her valuation for the good. A no-reserve auction could drive some high-value sellers out of the market. And while a public reserve auction is optimal for the seller, alternative mechanisms, including post-auction bargaining, may be preferred for the buyer or for the auction house, and may allow the market to achieve a more efficient allocation. Alternating-offer bargaining in particular is a natural mechanism which is easy for players to understand and for the auction house to implement, and which does not require the same level of commitment as static bargaining mechanisms, which, while more efficient, require players to sometimes walk away from negotiations even when it is discovered ex-post that gains from trade exist.

The nature of the wholesale used-car industry as a network of competing platforms also likely affects auction houses' choice of mechanism—both the choice of whether or not to allow post-auction bargaining as well as the choice of fee structure. Studying the role of two-sided uncertainty and competition among auction houses in determining auction houses' choice of mechanism would be a valuable avenue for future research.

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A Proofs

Proof of Proposition 1

Proof. Note that $\pi^B(\beta, b_i)$ is given by

$$\begin{aligned}\pi^B(\beta, b_i) = \delta \Bigg(& (b_i - \beta) \Pr\left(D_2^S = A|\beta\right) \\ & + \delta \Pr\left(D_2^S = C|\beta\right) E_{P_2^S}\left[\max\left\{b_i - P_2^S, 0, V_3^B\left(b_i|\{\beta, P_2^S\}\right)\right\}\middle|\beta\right]\Bigg) - c_B\end{aligned}$$

This expression is the payoff to the buyer from stating the auction auction price as a counteroffer, which is how the post-auction bargaining game begins.

Because the high bidder, after learning that the auction price did not meet the secret reserve price, has the option to immediately walk away without entering bargaining, the payoff $M(\beta, b_i)$ cannot be negative. To see that truth-telling is a dominant strategy, suppose first that bidder i drops out at some $\beta_i < b_i$.

1. If $b_i \leq \beta$, then $\beta_i < b_i \leq \beta$, so bidder i is not the high bidder, and would not have been even if he had bid $\beta_i = b_i$.

2. If $b_i > \beta$, then the following is true:

   (a) If $\beta < \beta_i < b_i$, then bidder i is the high bidder and gets an expected payoff of $M(\beta, b_i)$.

   (b) If $\beta_i < \beta < b_i$, then bidder i loses, but i would have been the high bidder if he had bid b_i, and would have again received $M(\beta, b_i)$.

Thus, dropping out at a price less than b_i will never raise bidder i's payoff, and in some cases may decrease it.

Now, suppose that bidder i drops out at some $\beta_i > b_i$

1. If $\beta \leq b_i$, then $\beta \leq b_i < \beta_i$, then bidder i is the high bidder and gets payoff $M(\beta, b_i)$, but would have received this same payoff dropping out at b_i. Also, as noted above, because it is the auctioneer, rather than the bidders, who calls out bids, a player cannot actually outbid himself in an attempt to win the object while avoiding costly bargaining.

2. If $\beta > b_i$, then the following is true:

   (a) If $b_i < \beta_i < \beta$, then bidder i loses, and would not have been the high bidder even if he had bid $\beta_i = b_i$.

   (b) If $b_i < \beta < \beta_i$, then bidder i is the high bidder, but would not choose to enter bargaining because the condition that the seller never accepts offers below the auction price rules out the possibility that bidder i could receive a positive payoff by bargaining.

□

Proof of Proposition 2

Proof. In order to prove this result, the following lemma is useful

Lemma 1. *For any finite T and history H_t, $V_t^S(s|H_t)$ is weakly increasing in s and $V_{t+1}^B(b|H_{t+1})$ is weakly increasing in b for all $t \leq T$.*

Proof of Lemma 1

The proof proceeds by induction on the number of periods remaining. Suppose there are T total periods in the game and there is currently one period remaining: it is the seller's turn and after her turn the buyer will only be allowed to accept or quit. Let H_{T-1} represent the history at the beginning of period $T-1$ and H_T the history in the final period. The seller's payoff from countering at a price of p is then

$$U_T^S(s, p|H_{T-1}) \equiv \left(p\delta \Pr(D_T^B = A|H_T) + s\left(\delta(1 - \Pr(D_T^B = A|H_T)) + 1 - \delta\right)\right) - c_S$$

Let $p^*(s|H_{T-1}) = \arg\max_p U^S_T(s,p|H_{T-1})$. That is, $V^S_{T-1}(s|H_{T-1}) = U^S_{T-1}(s,p^*(s|H_{T-1})|H_{T-1})$. Now let $V_{T-1}(s,s'|H_{T-1})$ represent the payoff to the seller of type s who mimics type $s' < s$. Clearly $V_{T-1}(s,s|H_{T-1}) \geq V_{T-1}(s,s'|H_{T-1})$ because $V_{T-1}(s,s|H_{T-1})$ is the maximized counteroffer payoff given the seller's true value, s. It remains to be shown that $V_{T-1}(s,s'|H_{T-1}) \geq V_{T-1}(s',s'|H_{T-1})$. Below, let H_T represent the history in period T when the seller of type s has mimicked type s' in period $T-1$. That is, $H_T = \{H_{T-1}, p^*(s'|H_{T-1})\}$. Observe that

$$V_{T-1}(s,s'|H_{T-1}) = \big(p^*(s'|H_{T-1})\delta \Pr(D^B_T = A|H_T) + s(\delta(1-\Pr(D^B_T = A|H_T)) + 1 - \delta)\big) - c_S,$$

and

$$V_{T-1}(s',s'|H_{T-1}) = \big(p^*(s'|H_{T-1})\delta \Pr(D^B_T = A|H_T) + s'(\delta(1-\Pr(D^B_T = A|H_T)) + 1 - \delta)\big) - c_S$$

Thus,

$$\begin{aligned} V_{T-1}(s,s'|H_{T-1}) - V_{T-1}(s',s'|H_{T-1}) &= (s-s')(\delta(1-\Pr(D^B_T = A|H_T)) + 1 - \delta) \\ &\geq 0 \end{aligned}$$

Therefore, $V_{T-1}(s,s|H_{T-1}) \geq V_{T-1}(s',s'|H_{T-1})$, and the seller's counteroffer payoff is weakly increasing in her type when there is one period remaining.

To complete the proof by induction,, let $V^S_{T-(t-1)}(s|H_{T-(t-1)})$ denote the seller's counteroffer payoff with $t-1$ periods remaining, and suppose $V^S_{T-(t-1)}(s|H_{T-(t-1)})$ is weakly increasing s. Note that when there are t periods remaining, $V_{T-t}(s,s|H_{T-t}) \geq V_{T-1}(s,s'|H_{T-t})$ by the same argument as above. It remains to be shown that $V_{T-t}(s,s'|H_{T-t}) \geq V_{T-t}(s',s'|H_{T-t})$. Let

$$\begin{aligned} H_{T-(t-1)} &= \{H_{T-t}, p^*(s'|H_{T-t})\} \\ H_{T-(t-2)} &= \{H_{T-t}, p^*(s'|H_{T-t}), P^B_{T-(t-1)}\} \end{aligned}$$

Note that

$$\begin{aligned} &V_{T-t}(s,s'|H_{T-t}) - V_{T-1}(s',s'|H_{T-t}) \\ &= (s-s')\left(\delta \Pr\left(D^B_{T-(t-1)} = Q|H_{T-(t-1)}\right) + 1 - \delta\right) \\ &+ \ \delta \Pr\left(D^B_{T-(t-1)} = C|H_{T-(t-1)}\right) \\ &\times \left(\delta E_{P^B_{T-(t-1)}}\left[\max\left\{P^B_{T-(t-1)}, s, V^S_{T-(t-1)}\left(s,s'|H_{T-(t-2)}\right)\right\}\right.\right. \\ &- \left.\left.\max\left\{P^B_{T-(t-1)}, s', V^S_{T-(t-1)}\left(s',s'|H_{T-(t-2)}\right)\right\}\right| H_{T-(t-1)}\right] + (s-s')(1+\delta)\Big) \\ &\geq 0 \end{aligned}$$

Therefore, $V_{T-t}(s,s|H_{T-t}) \geq V_{T-t}(s',s'|H_{T-t})$, completing the proof. The proof that the buyer counteroffer payoff, $V^B_{t+1}(b|H_{T+1})$, is increasing follows by the same steps. □

(Continuation of Proof of Proposition 2)

Let $\chi(b)$ be defined by $0 = \pi^B(\chi, b)$, where π^B is defined in the proof of Proposition 1. Intuitively, χ is the auction price which would make a high bidder of type b indifferent between bargaining and not bargaining. Note that, for $b' > b$, $\pi^B(\chi(b), b') > 0$, because $V_3^B(\cdot)$ is increasing in b by Lemma 1. Thus, $\chi(b') > \chi(b)$, and hence χ^{-1}, the inverse, exists and is also strictly increasing. To make notation clear, if $y = \chi(b)$, then this inverse function gives $b = \chi^{-1}(y)$, which defines the lowest buyer type who would enter bargaining when the auction price is y. Also, note that $\chi(b) < b$ because $\pi^B(b, b) < 0$ due to $c_B > 0$.

The seller's payoff can then be re-written as

$$\int_\rho^{\overline{B}} b^{(2)} f_{B^{(2)}}(b^{(2)}) db^{(2)} + \int_{\underline{B}}^\rho \left[\int_{b^{(2)}}^{\chi^{-1}(b^{(2)})} s f_B(b) db + \int_{\chi^{-1}(b^{(2)})}^{\overline{B}} \pi^S \left(b^{(2)}, s\right) f_B(b) db \right] \frac{f_{B^{(2)}}(b^{(2)})}{1 - F_B(b^{(2)})} db^{(2)}$$

$$= \int_\rho^{\overline{B}} b^{(2)} f_{B^{(2)}}(b^{(2)}) db^{(2)} + \int_{\underline{B}}^\rho \left[s \left(F_B(\chi^{-1}(b^{(2)})) - F_B(b^{(2)})\right) + \pi^S \left(b^{(2)}, s\right) \left(1 - F_B(\chi^{-1}(b^{(2)}))\right) \right] \frac{f_{B^{(2)}}(b^{(2)})}{1 - F_B(b^{(2)})} db^{(2)}$$

Differentiating the above expression using Leibniz Rule yields the following first-order condition for ρ:

$$\frac{\partial}{\partial \rho} = -\rho + s \frac{F_B(\chi^{-1}(\rho)) - F_B(\rho)}{1 - F_B(\rho)} + \pi^S(\rho, s) \frac{1 - F_B(\chi^{-1}(\rho))}{1 - F_B(\rho)}$$

Lemma 1 implies that $\pi^S\left(b^{(2)}, s\right)$ is weakly increasing in s, and thus $\frac{\partial}{\partial \rho}$ will be strictly increasing in s because $F_B(\chi^{-1}(\rho)) > F_b(\rho)$ given that $\chi^{-1}(\cdot)$ is strictly increasing and $f_B(\cdot)$ is atomless. Given that $\frac{\partial}{\partial \rho}$ is strictly increasing in s, the Edlin and Shannon (1998) Theorem implies that, as long as the optimal $\rho^*(s)$ lies on the interior of the support of ρ, $\rho^*(s)$ will be strictly increasing in s. The support of ρ is the real line, thus completing the proof. Note that without costly bargaining a weak monotonicity result can be obtained following Topkis Theorem. □

Proof of Proposition 3

Proof. Given the structure of additive separability in the willingness to pay/sell, the goal is to show that the auction price, players' bargaining counteroffers, and the seller's secret reserve price will also be additively separable in the auction-level heterogeneity. Suppose the auction-level heterogeneity is given by a fixed scalar, $W = w$. The buyer's type is given by $\tilde{B} \equiv B + W \sim F_{\tilde{B}}$, with density $f_{\tilde{B}}$. The seller's type is given by $\tilde{S} \equiv S + W$.

That the auction price will be additively separable in W is obvious, given that the bidding function is the identity function by Proposition 1. To demonstrate that bargaining offers are also additively separable, the proof proceeds by induction on the number of periods remaining. Suppose there is currently one period remaining in the bargaining game: it is the seller's turn and after her turn the buyer will only be allowed to accept or quit.

In the final period, a buyer with type $\tilde{B} = \tilde{b}$ will accept a price, $\tilde{p}$, if and only if $\tilde{p} \le \tilde{b}$. In period $T - 1$, the seller of type $\tilde{S} = \tilde{s}$ chooses $\tilde{p}^*$ to solve

$$\tilde{p}^* = \arg\max_{\tilde{p}} \left\{ \delta \tilde{p}(1 - F_{\tilde{B}}(\tilde{p})) + \tilde{s}\left(\delta F_{\tilde{B}}(\tilde{p}) + 1 - \delta\right) - c_S \right\}$$

$$= w + \arg\max_p \{\delta p(1 - F_B(p)) + s(\delta F_B(p) + 1 - \delta) - c_S + w(1 - F_B(p)) + w(\delta F_B(p) + 1 - \delta)\}$$

$$= w + \arg\max_p \{[\delta p(1 - F_B(p)) + s(\delta F_B(p) + 1 - \delta) - c_S] + w\}$$

Therefore, the penultimate bargaining offer in the heterogeneous setting will be w above the bargaining offer from the homogeneous good setting, and similarly for the seller's maximized payoff.

To complete the proof by induction, suppose that offers and payoffs in periods $T-(t-1)$ and $T-(t-2)$ are w higher than their homogeneous good counterparts. It remains to be shown that the same holds true for the offers and payoffs in period $T-t$. Let all $(\tilde{\cdot})$ expressions represent the heterogeneous model expressions. The seller's payoffs from accepting, quitting, or countering in period $T-t$ can be written as follows:

$$\text{A} : \tilde{p}^B_{T-(t+1)} = w + p^B_{T-(t+1)}$$

$$\text{Q} : \tilde{s} = w + s$$

$$\text{C} : \tilde{V}^S_{T-t}\left(\tilde{s}|\tilde{H}_{T-t}\right)$$

$$= \max_{\tilde{p}} \tilde{p}\delta \Pr\left(D^B_{T-(t-1)} = A|\tilde{H}_{T-(t-1)}\right) + \tilde{s}\left(\delta \Pr\left(D^B_{T-(t-1)} = Q|\tilde{H}_{T-(t-1)}\right) + 1 - \delta\right)$$

$$+ \delta \Pr\left(D^B_{T-(t-1)} = C|\tilde{H}_{T-(t-1)}\right)$$

$$\times \left(\delta E_{\tilde{P}^B_{T-(t-1)}}\left[\max\left\{\tilde{P}^B_{T-(t-1)}, \tilde{s}, \tilde{V}^S_{T-(t-2)}\left(\tilde{s}|\tilde{H}_{T-(t-2)}\right)\right\}\middle|\tilde{H}_{T-(t-1)}\right] + \tilde{s}(1-\delta)\right) - c_S$$

$$= w + \max_p p\delta \Pr\left(D^B_{T-(t-1)} = A|H_{T-(t-1)}\right) + s\left(\delta \Pr\left(D^B_{T-(t-1)} = Q|H_{T-(t-1)}\right) + 1 - \delta\right)$$

$$+ \delta \Pr\left(D^B_{T-(t-1)} = C|H_{T-(t-1)}\right)$$

$$\times \left(\delta E_{P^B_{T-(t-1)}}\left[\max\left\{P^B_{T-(t-1)}, s, V^S_{T-(t-2)}\left(s|H_{T-(t-2)}\right)\right\}\middle|H_{T-(t-1)}\right] + s(1-\delta)\right) - c_S$$

The last line follows by removing w from each expression and from the following claim: the probability of the buyer accepting, quitting, or countering in period $T-(t-1)$ will be the same in the heterogeneous good model as in the homogeneous good model. To prove this claim, note that the buyer's payoffs for each action are given by:

$$\text{A} : \tilde{b} - \tilde{p}^S_{T-t} = b - p^S_{T-t}$$

$$\text{Q} : 0$$

$$\text{C} : \tilde{V}^B_{T-(t-1)}\left(\tilde{b}|\tilde{H}_{T-(t-1)}\right)$$

$$= \delta\Bigg(\max_{\tilde{p}}(\tilde{b} - \tilde{p}) \Pr\left(D^S_{T-(t-2)} = A|\tilde{H}_{T-(t-2)}\right)$$

$$+ \delta \Pr\left(D^S_{T-(t-2)} = C|\tilde{H}_{T-(t-2)}\right) E_{\tilde{P}^S_{T-(t-2)}}\left[\max\left\{\tilde{b} - \tilde{P}^S_{T-(t-2)}, 0, \tilde{V}^B_{T-(t-3)}\left(\tilde{b}|\tilde{H}_{T-(t-3)}\right)\right\}\middle|\tilde{H}_{T-(t-2)}\right]\Bigg) - c_B$$

$$= V^B_{T-(t-1)}\left(b|H_{T-(t-1)}\right)$$

Finally, consider the seller's secret reserve price in the setting with auction-level heterogeneity w. Let $\tilde{\chi}$ satisfy $0 = \pi^B(\tilde{\chi}, \tilde{b})$. Note that $\pi^B(\tilde{\chi}, \tilde{b}) = \pi^B(\chi, b)$ by the above arguments for the buyer's bargaining payoff. The first order condition for the seller's secret reserve, $\tilde{r} = \rho(\tilde{s})$, from the proof of Proposition 2, will be given by

$$\begin{aligned} \frac{\partial}{\partial \tilde{r}} &= -\tilde{r} + \tilde{s} \frac{F_{\tilde{B}}(\tilde{\chi}^{-1}(\tilde{r})) - F_{\tilde{B}}(\tilde{r})}{1 - F_{\tilde{B}}(\tilde{r})} + \pi^S(\tilde{r}, \tilde{r}) \frac{1 - F_{\tilde{B}}(\tilde{\chi}^{-1}(\tilde{r}))}{1 - F_{\tilde{B}}(\tilde{r})} \\ &= -\tilde{r} + w + s \frac{F_B(\chi^{-1}(\tilde{r} - w)) - F_B(\tilde{r} - w)}{1 - F_B(\tilde{r} - w)} + \pi^S(\tilde{r} - w, s) \frac{1 - F_B(\chi^{-1}(\tilde{r} - w))}{1 - F_B(\tilde{r} - w)} \end{aligned}$$

Therefore, the optimal secret reserve price in the heterogeneous setting will be w above the optimal reserve in the homogeneous setting, completing the proof.

□

Proof of Corollary 1

Proof. This proof follows similar steps to those in the proof of Theorem 2 of Myerson and Satterthwaite (1983) and relies on results from Theorem 1 of Williams (1987). The problem is to find an allocation rule $x : [b^{(2)}, \overline{B}] \times [\underline{S}, \overline{S}] \rightarrow [0, 1]$ to maximize

$$\int_{b^{(2)}}^{\overline{B}} \int_{\underline{S}}^{\overline{S}} x(s, b) f_S(s) f_B(b) ds db$$

subject to the players' participation constraint, which is

$$0 \leq \int_{b^{(2)}}^{\overline{B}} \int_{\underline{S}}^{\overline{S}} \left(\phi_B(b, 1) - \phi_S(s, 1)\right) x(s, b) f_S(s) f_B(b) ds db$$

See Myerson and Satterthwaite (1983) for more details. Letting λ denote the Lagrange multiplier, the unconstrained problem is to maximize

$$\int_{b^{(2)}}^{\overline{B}} \int_{\underline{S}}^{\overline{S}} \left(1 + \lambda \left(\phi_B(b, 1) - \phi_S(s, 1)\right)\right) x(s, b) f_S(s) f_B(b) ds db$$

For any $\lambda \geq 0$, the Lagrangian is maximized when $x(s, b) = 1$ if and only $(1 + \lambda (\phi_B(b, 1) - \phi_S(s, 1))) \geq 0$. To achieve this result, let

$$\frac{1}{\lambda} = \frac{2\kappa}{1 - \kappa}$$

$\kappa \in [0, 1)$ may then be solved for to equate the participation constraint to zero. That is, let

$$\tilde{G}(\kappa) = \int_{b^{(2)}}^{\overline{B}} \int_{\underline{S}}^{\overline{S}} \left(\phi_B(b, 1) - \phi_S(s, 1)\right) x^{\kappa}(s, b) f_S(s) f_B(b) ds db$$

where

$$x^{\kappa}(s, b) = 1 \left\{ \phi_S(s, 1) - \frac{2\kappa}{1 - \kappa} \leq \phi_B(b, 1) \right\}$$

Observe that $x^\kappa(s,b)$ is decreasing in κ. Therefore, for some $\alpha < \kappa$, $\tilde{G}(\alpha)$ will differ from $\tilde{G}(\kappa)$ only because $0 = x^\alpha(s,b) < x^\kappa(s,b) = 1$ for some (s,b) where $\phi_B(b,1) < \phi_S(s,1) - \frac{2\alpha}{1-\alpha}$, implying that at that same (s,b), it must be the case that $\phi_B(b,1) < \phi_S(s,1)$. Thus, as κ increases, $x^\kappa(s,b)$ yields trade at regions of the type space at which $(\phi_B(b,1) - \phi_S(s,1))$ is negative. Therefore, $\tilde{G}(\kappa)$ is decreasing in κ.

To prove the $\tilde{G}(\kappa)$ is continuous, note that if $\phi_S(s,1)$ and $\phi_B(b,1)$ are both strictly increasing, then given any b and κ, the equation $\phi_B(b,1) = \phi_S(s,1) - \frac{2\alpha}{1-\alpha}$ has at most one solution in s, so $\tilde{G}(\kappa)$ can be written as

$$\tilde{G}(\kappa) = \int_{b^{(2)}}^{\overline{B}} \int_{\underline{S}}^{\tilde{g}(b,\kappa)} \left(\phi_B(b,1) - \phi_S(s,1)\right) f_S(s) f_B(b) ds db$$

where $\tilde{h}(b,\kappa)$ is continuous in b and κ, so $\tilde{G}(\kappa)$ is continuous. Note also that $\tilde{G}(0) >= 0$, and $\lim_{\kappa\to 1}\tilde{G}(\kappa) = -\infty$. Therefore, there exists a unique $\kappa \in [0,1)$ such that $\tilde{G}(\kappa) = 0$. By Theorem 1 of Williams (1987), the transfer function of this mechanism will by given by (19) and (20). □

B Computational Details, Robustness Checks, and Additional Mechanisms

B.1 Gaussian Quadrature

The counterfactual analysis in this paper requires the evaluation of a significant number of integrals, such as (13). In order to achieve accuracy and limit the computational burden, I employ Gauss-Chebyshev integration, as advocated by Judd (1998), with a large number of nodes. Specifically, let $z_k, k = 1, ..., K$ be the Chebyshev nodes, given by $z_k = \cos(\pi(2k-1)/(2K))$. Let $g(v)$ be the function to be integrated. Then

$$\int_{\underline{v}}^{\overline{v}} g(v)dv \approx \frac{\pi(\overline{v} - \underline{v})}{2K} \sum_{k=1}^{K} g(x_k) w_k$$

where $x_k = (1/2)(z_k + 1)(\overline{v} - \underline{v}) + \underline{v}$ and $w_k = (1 - z_k^2)^{1/2}$. For integration in multiple dimensions, I use a tensor product:

$$\int_{\underline{u}}^{\overline{u}} \int_{\underline{v}}^{\overline{v}} g(v,u) dv du \approx \frac{\pi^2(\overline{v} - \underline{v})(\overline{u} - \underline{u})}{(2K)^2} \sum_{j=1}^{K} \sum_{k=1}^{K} g(x_k, x_j) w_k w_j$$

See Kythe and Schäferkotter (2005) or Judd (1998) for additional details. In the estimation of integrals in this paper, I use 200 nodes when integrating in the dimension of the seller's or high bidder's type. Accuracy in these two dimensions is essential, as this is the level at which I solve for counterfactual mechanisms (conditional on the auction price). The integration over the auction price, on the other hand, is not involved in solving for mechanisms, so I use 25 nodes in this dimensions. Increasing the number of nodes beyond 25 does not change results.

In evaluating the Hermite-based likelihood function in Section 4.3, I employ Gauss-Hermite quadrature (recommended by Judd (1998) for integration over the real line), given by

$$\int_{-\infty}^{\infty} g(v)dv \approx \sum_{k=1}^{K} g(x_k)e^{x_k^2} w_k$$

where in this case x_k and w_k are the Gauss-Hermite quadrature nodes and weights described in Kythe and Schäferkotter (2005) or Judd (1998). I choose 10 as the number of nodes and do not find that the results change noticeably as this number increases.

B.2 Solving for the Pareto Frontier and the Transfer Function

The Pareto frontier can be solved for using Theorem 3 of Williams (1987), which states the following. Recall that each mechanism is summarized by two objects, (x, q), defined in Section 5.

Theorem (from Williams 1987).

Suppose $\phi_s(s)$ and $\phi_b(b)$ are weakly increasing. Then

1. *For $0 \leq \eta < 1/2$, if $G(x^{\bar{\alpha}_1,0}) \geq 0$ for $\bar{\alpha}_1 = 1 - \eta/(1-\eta)$, then $(x^{\bar{\alpha}_1,0}, 0)$ is the unique solution maximizing (12) for this η; if $G(x^{\bar{\alpha}_1,0}) < 0$, then there exists a unique (α_1^*, α_2^*) that satisfies the equations $G(x^{\alpha_1,\alpha_2}) = 0$ and $(\alpha_2 - 1) = (\alpha_1 - 1)(1-\eta)/\eta$, and $(x^{\alpha_1^*,\alpha_2^*}, 0)$ is the unique solution maximizing (12) for this η.*

2. *For $1/2 < \eta \leq 1$, if $G(x^{0,\bar{\alpha}_2}) \geq 0$ for $\bar{\alpha}_2 = 1+(\eta-1)/\eta)$, then $(x^{0,\bar{\alpha}_2}, G(x^{0,\bar{\alpha}_2}))$ is the unique solution maximizing (12) for this η; if $G(x^{0,\bar{\alpha}_2}) < 0$, then there exists a unique (α_1^*, α_2^*) that satisfies the equations $G(x^{\alpha_1,\alpha_2}) = 0$ and $(\alpha_2 - 1) = (\alpha_1 - 1)(1-\eta)/\eta$, and $(x^{\alpha_1^*,\alpha_2^*}, 0)$ is the unique solution maximizing (12) for this η.*

For these direct mechanisms defining the Pareto frontier, Theorem 1 of Williams (1987) implies that, given (x, q), the expected transfer for a seller of type s or for a buyer of type b are given by

$$p_S(s) \equiv q + s\int_{b^{(2)}}^{\overline{B}} x(s,b)\frac{f_B(b)}{1-F_B(b^{(2)})}db + \int_{s}^{\overline{S}}\int_{b^{(2)}}^{\overline{B}} x(u,b)\frac{f_B(b)}{1-F_B(b^{(2)})}f_S(u)dbdu \quad (19)$$

$$p_B(b) \equiv G(x) - q + b\int_{\underline{S}}^{\overline{S}} x(s,b)f_S(s)ds + \int_{b^{(2)}}^{b}\int_{\underline{S}}^{\overline{S}} x(s,u)\frac{f_B(u)}{1-F_B(b^{(2)})}f_S(s)dsdu \quad (20)$$

B.3 Bounding Bargaining Costs

This section describes a simple approach to account for bargaining costs incurred during the alternating-offer bargaining game. The parameter δ, the probability that the game does not end exogenously, can be estimated in the data as the empirical probability that a bargaining sequence is not incomplete (where incomplete means the sequence is recorded as having ended with a counteroffer which was not responded to). In practice this probability can vary from period to period. Table 6, panel A, displays these empirical probabilities for periods one through six. In estimation below I use both an upper bound on δ, $\overline{\delta} \equiv 1$, and a lower bound, $\underline{\delta}$, given by the smallest of the period-level δ estimates from Table 6.

Bounds on the parameters c_B and c_S are given by cases in which a player chooses to make a counteroffers. A necessary condition for a party to choose to counter is that the payoff from the player's opponent accepting with probability one must exceed the player's payoff from accepting the current offer on the table. That is,

$$\delta p_2^S - c_S \geq p_1^B$$
$$\delta(b - p_3^B) - c_B \geq b - p_2^S$$

Noting that $\delta < 1$ yields

$$p_2^S - p_1^B \geq c_S \tag{21}$$
$$p_2^S - p_3^B \geq c_B \tag{22}$$

Thus, an upper bound on c_S is given by the minimum gap between period 2 and period 1 offers and an upper bound on c_B is given by the minimum gap between period 2 and period 3 offers (in cases where such offers took place). Rather than use the minimum over all observations, I follow Chernozhukov, Lee, and Rosen (2013) to obtain a bias-corrected, one-sided 95% confidence bound for c_S and c_B and treat these as upper bounds, which I denote $\overline{c}_S$ and $\overline{c}_B$.[60] Panel B of Table 6 displays the estimates.

These estimated bargaining costs can then be used to obtain a lower bound on buyer and seller surplus in the dynamic bargaining mechanism as follows. Let $\mathcal{A}$ be the event that agreement occurs and $\mathcal{T}$ be the period in which the bargaining game ends. Let $P^{B,h}$ and $P^{S,h}$ be random variables representing the final, net payment made by the buyer and received by the seller, respectively, after accounting for auction house fees. Upper bounds on the expected gains from trade for the buyer and seller are given by the following, which ignore bargaining costs,

$$\overline{U}_B^D \equiv E[B - P^{B,h} | \mathcal{A}] \Pr(\mathcal{A})$$
$$\overline{U}_S^D \equiv E[P^{S,h} - S | \mathcal{A}] \Pr(\mathcal{A})$$

These objects are the expected gains from trade shown in Tables 4–5. Incorporating bargaining costs, the expected gains from trade for the buyer and seller in the dynamic bargaining mechanism would be given by

$$U_B^D \equiv \left[\sum_{t=1}^{T} \delta^{t-1} E[B - P^{B,h} | \mathcal{A}, \mathcal{T} = t] \Pr(\mathcal{A} | \mathcal{T} = t) \Pr(\mathcal{T} = t) \right] - \left[c_B \sum_{t=2}^{T} \left(\sum_{i=0}^{\lfloor \frac{t}{2} \rfloor - 1} \delta^{2i} \right) \Pr(\mathcal{T} = t) \right]$$

[60]Chernozhukov, Lee, and Rosen (2013) explained that bounds such as those in those in (21) and (22) will be biased downward given that they are derived from taking a minimum. The authors suggested a bias-corrected $1 - \alpha$ confidence bound (which the authors refer to as being half-median unbiased) given, in this case, by taking the $1 - \alpha$ quantile of bootstrapped estimates of the minimum. I perform this using 200 bootstrap replications.

$$U_S^D \equiv \left[\sum_{t=1}^{T}\delta^{t-1}E[P^{S,h}-S|\mathcal{A},\mathcal{T}=t]\Pr(\mathcal{A}|\mathcal{T}=t)\Pr(\mathcal{T}=t)\right]-\left[c_S\sum_{t=3}^{T}\left(\sum_{i=0}^{\lfloor\frac{t-1}{2}\rfloor-1}\delta^{2i+1}\right)\Pr(\mathcal{T}=t)\right]$$

where $\lfloor\cdot\rfloor$ represents the floor function.

While the approach proposed in Section 5.2 demonstrates that $\overline{U}_B^D$ and $\overline{U}_S^D$ are identified in the data, it does not provide identification of U_B^D and U_S^D. In particular, the sample of observations which continue to later bargaining periods is selected and hence the approach proposed in Section 5.2 does not allows for identification of $E[B-S|\mathcal{A},\mathcal{T}]$ for each realization of $\mathcal{T}$. I circumvent this issue by assuming that observations ending in the first period of bargaining are likely to be situations with higher gains from trade, as in Cramton (1992) and as is common in bargaining settings with Coasian dynamics. Specifically, I assume that

$$\begin{aligned}&\sum_{t=2}^{T}\delta^{t-1}\left(E[B-P^{B,h}|\mathcal{A}]\Pr(\mathcal{A})-E[B-P^{B,h}|\mathcal{A},\mathcal{T}=t]\Pr(\mathcal{A}|\mathcal{T}=t)\right)\Pr(\mathcal{T}=t)\\&\qquad\geq\left(E[B-P^{B,h}|\mathcal{A},\mathcal{T}=1]\Pr(\mathcal{A}|\mathcal{T}=1)-E[B-P^{B,h}|\mathcal{A}]\Pr(\mathcal{A})\right)\Pr(\mathcal{T}=1)\end{aligned}\tag{23}$$

I make an assumption analogous to (23) for seller surplus (replacing $B-P^{B,h}$ with $P^{S,h}-S$). Together, these assumptions imply

$$U_B^D \geq \underline{U}_B^D \equiv \sum_{t=1}^{T}\underline{\delta}^{t-1}E[B-P^{B,h}|\mathcal{A}]\Pr(\mathcal{A})\Pr(\mathcal{T}=t)-\left[\overline{c}_B\sum_{t=2}^{T}\left(\sum_{i=0}^{\lfloor\frac{t}{2}\rfloor-1}\overline{\delta}^{2i}\right)\Pr(\mathcal{T}=t)\right]\tag{24}$$

$$U_S^D \geq \underline{U}_S^D \equiv \sum_{t=1}^{T}\underline{\delta}^{t-1}E[P^{S,h}-S|\mathcal{A}]\Pr(\mathcal{A})\Pr(\mathcal{T}=t)-\left[\overline{c}_S\sum_{t=3}^{T}\left(\sum_{i=0}^{\lfloor\frac{t-1}{2}\rfloor-1}\overline{\delta}^{2i+1}\right)\Pr(\mathcal{T}=t)\right]\tag{25}$$

I estimate $\underline{U}_B^D$ and $\underline{U}_S^D$ plugging in the estimate of $\underline{\delta}$ and the estimated upper bounds $\overline{c}_B$ and $\overline{c}_S$ from above, and setting $\overline{\delta}=1$. The resulting lower bounds on buyer and seller surplus are reported in panel C of Table 6.

B.4 Imposing Monotonicity of Virtual Valuations and Solving for Implied Density/Distribution

I impose that $\phi_S(s)$ and $\phi_B(b)$ be weakly increasing following the rearrangement approach of Chernozhukov, Fernandez-Val, and Galichon (2009). In practice, this operation can be performed as follows. Let a grid of values on $[\underline{S},\overline{S}]$ be given by $z^S=[z_1^S,...,z_K^S]$ and on $[\underline{B},\overline{B}$ be given by $z^B=[z_1^B,...,z_K^B]'$. Let $\hat{\phi}_S(z^S)$ and $\hat{\phi}_B(z^B)$ be the estimates of ϕ_S and ϕ_B obtained by plugging in the estimated distributions and densities from Sections 4.4-4.5 evaluated at the elements of z^B and z^S. Rearrangement is performed by simply sorting the vector $\hat{\phi}_S(z^S)$ and reassigning the sorted values to the original z^S vector, and similarly for $\hat{\phi}_B(z^B)$. Let $\hat{\phi}_S^*(z^S)$ and $\hat{\phi}_B^*(z^B)$ denote the rearranged estimates.

The implied densities and distributions corresponding to the rearranged estimates can then solved for

Table 6: Estimates of bargaining costs and lost surplus

	Dealers sample	Fleet/lease sample
	A. Probability of no exogenous breakdown (δ)	
Period	δ	δ
t=1	0.990	0.992
	(0.000)	(0.000)
t=2	0.964	0.965
	(0.001)	(0.001)
t=3	0.992	0.975
	(0.001)	(0.001)
t=4	0.984	0.957
	(0.002)	(0.004)
t=5	0.998	0.989
	(0.001)	(0.003)
t=6	0.984	0.966
	(0.006)	(0.016)

	Dealers sample	Fleet/lease sample
	B. Per-period bargaining disutility (units=$1,000)	
Buyer	0.025	0.050
($\bar{c}_B$)	(0.010)	(0.000)
Seller	0.104	0.109
($\bar{c}_S$)	(0.002)	(0.036)

C. Gains with and without accounting for bargaining costs (units=$1,000)

	Dealers sample: Seller upper bound		Dealers sample: Seller lower bound		Fleet/lease sample: Seller upper bound		Fleet/lease sample: Seller lower bound	
	w/ costs	no costs	w/ costs	no costs	w/ costs	no costs	w/ costs	no costs
Buyer	0.533	0.548	0.528	0.543	0.890	0.912	0.887	0.908
gains	(0.045)	(0.012)	(0.044)	(0.009)	(0.069)	(0.005)	(0.069)	(0.005)
Seller	3.437	3.509	1.999	2.049	3.997	4.060	2.808	2.856
gains	(0.309)	(0.156)	(0.186)	(0.093)	(0.303)	(0.096)	(0.215)	(0.070)

Notes: Results for dealers sample on the left and fleet/lease sample on the right. Panel A displays, for bargaining periods 1 through 6, the probability that bargaining does not end exogenously. Panel B displays estimated upper bounds on per-period additive costs for buyers and sellers. Panel C displays buyer and seller gains from trade in the dynamic mechanism when costs are not accounted for (from Tables 4–5) and when costs are accounted for. Gains are in $1,000 units. Standard errors are reported in parentheses and come from 200 bootstrap replications of the full estimation procedure.

by noting that $d \ln F_S(s)/ds = f_S(s)/F_S(s)$, which implies

$$\int_{\underline{S}}^{s} \frac{1}{\phi_S(u) - u} du = \ln F_S(s) - \ln F_S(\underline{S}) \qquad \Rightarrow F_S(s) = e^{\left(\int_{\underline{S}}^{s} \frac{1}{\phi_S(u)-u} du + \ln F_S(\underline{S})\right)}$$

and similarly for F_B. Thus,

$$\hat{F}_S^*(z_k^S) = e^{\left(\int_{\underline{S}}^{z_k^S} \frac{1}{\hat{\phi}_S^*(u)-u} du + \ln \hat{F}_S(z_1^S)\right)} \qquad \text{and } \hat{f}_S^*(z_k^S) = \frac{\hat{F}_S^*(z_k^S)}{\hat{\phi}_S^*(z_k^S) - z_k^S}$$

$$\hat{F}_B^*(z_k^B) = 1 - e^{\left(\int_{\underline{B}}^{z_k^B} \frac{1}{\hat{\phi}_B^*(u)-u} du + \ln(1-\hat{F}_B(z_1^S))\right)} \qquad \text{and } \hat{f}_B^*(z_k^B) = \frac{1-\hat{F}_B^*(z_k^B)}{z_k^B - \hat{\phi}_B^*(z_k^B)}$$

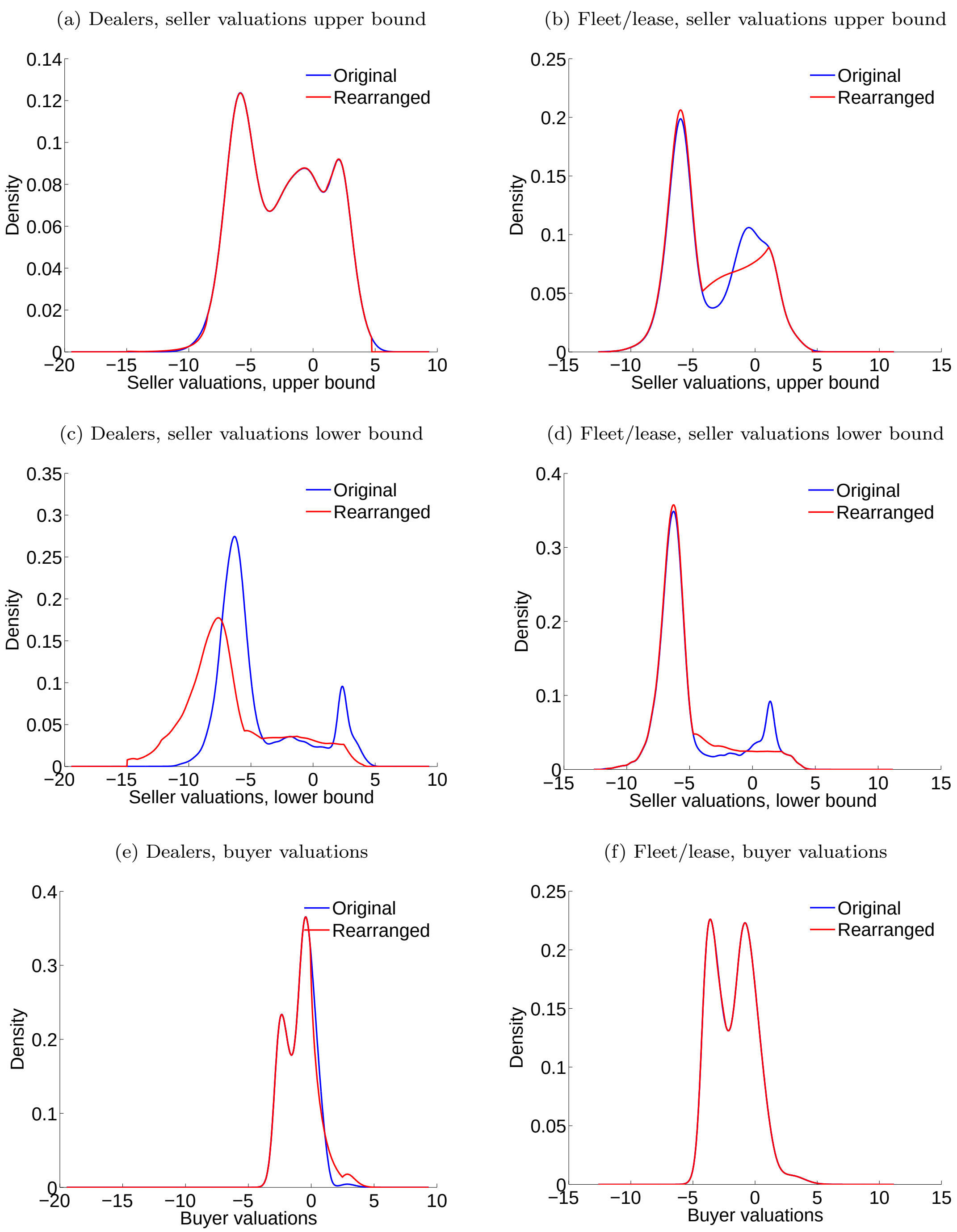
Figure 10 displays the resulting densities from this rearrangement. The first row, panels (a) and (b), displays the density of seller valuations using the seller distribution upper bound in the dealers and fleet/lease samples. Panels (c) and (d) use the seller distribution lower bound. Panels (e) and (f) display the density of buyer valuations. Several densities, including those in panels (a), (e), and (f) are mostly unaffected by the monotonization of virtual valuations. In the remaining panels, the monotonization shifts the mass in particular regions of the seller support.

Table 7 compares the original welfare estimates to those implied under the monotonized virtual valuations. In panel A1, the original estimates suggest that the trade-maximizing mechanism has a higher probability of trade (0.868) than the first-best (0.849). After monotonization, the two mechanism are ranked correctly, with the trade-maximizing mechanism achieving trade with probability 0.897 and the first-best with probability 0.937. However, the monotonized density does not fit the data as well, as can be seen by the predicted probability of trade in the dynamic mechanism, 0.845, which lies much farther from the observed probability of trade in the dealers sample, 0.710 (see Table 1). The results are similar in the fleet/lease sample, where the rearranging corrects the ranking of the trade-maximizing vs. first-best mechanisms, and the loss in model fit is not as drastic in this sample. This is consistent with the evidence in Figure 10, where the densities in the fleet/lease sample, panels (b) and (d), while affected by rearrangement, are not as affected as that in panel (c).

B.5 Model Fit and Robustness Under Alternative Sample Restrictions

As a check of model fit and robustness, Table 8 displays, for alternative sample restrictions in the dealers sample, the expected gains from trade and probability of trade for several mechanisms. Table 9 displays results under the same sample restrictions in the fleet/lease sample. In each table, panel A displays results for the seller distribution upper bound and panel B for the seller distribution lower bound. The first row in each panel displays the full sample results (equivalent to results in Tables 4 and 5). The second row results use only the first time a car is run in order to eliminate any correlation among buyer and seller values for a given car which is run multiple times (the full sample estimation treats these multiple runs as independent). The results are quite similar in each panel to the full sample, although the probability of trade is somewhat higher under each mechanism.

Figure 10: Densities of seller and buyer valuations, imposing monotonic virtual valuations



Notes: Displays original densities of buyer and seller valuations (in blue) and densities after enforcing monotonicity of virtual valuations through rearrangement (in red). Dashed lines mark pointwise 95% confidence bands from 200 bootstrap replications of full estimation procedure. Units = $1,000.

Table 7: Monotonizing virtual valuations through rearrangement

	Expected gains (units=$1,000)			Probability of trade		
	First-best	Second-best	Dynamic mech	First-best	Trade-maximizing	Dynamic mech
A. Monotonizing virtual valuations, ***dealers*** *sample*						
A1. Using seller distribution upper bound						
Original estimates	5.342	5.324	4.276	0.849	0.868	0.705
	(0.179)	(0.182)	(0.164)	(0.016)	(0.021)	(0.015)
		[99.7%]	**[80.3%]**		**[102.3%]**	**[81.2%]**
Monotonized estimates	7.573	7.380	6.971	0.937	0.897	0.845
	(0.902)	(0.856)	(1.024)	(0.024)	(0.028)	(0.045)
		[97.5%]	**[94.5%]**		**[95.7%]**	**[94.2%]**
A2. Using seller distribution lower bound						
Original estimates	3.374	3.345	2.811	0.747	0.694	0.700
	(0.103)	(0.108)	(0.097)	(0.010)	(0.018)	(0.009)
		[99.2%]	**[84.0%]**		**[92.9%]**	**[100.8%]**
Monotonized estimates	3.632	3.573	3.127	0.772	0.693	0.748
	(0.444)	(0.431)	(0.465)	(0.023)	(0.029)	(0.026)
		[98.4%]	**[87.5%]**		**[89.8%]**	**[108.0%]**
B. Monotonizing virtual valuations, ***fleet/lease*** *sample*						
	Expected gains (units=$1,000)			Probability of trade		
	First-best	Second-best	Dynamic mech	First-best	Trade-maximizing	Dynamic mech
B1. Using seller distribution upper bound						
Original estimates	6.045	6.015	5.201	0.898	0.915	0.757
	(0.117)	(0.117)	(0.097)	(0.003)	(0.006)	(0.002)
		[99.5%]	**[86.5%]**		**[101.9%]**	**[82.7%]**
Monotonized estimates	6.355	6.329	5.448	0.940	0.916	0.769
	(0.132)	(0.133)	(0.108)	(0.005)	(0.006)	(0.003)
		[99.6%]	**[86.1%]**		**[97.5%]**	**[83.9%]**
B2. Using seller distribution lower bound						
Original estimates	4.445	4.392	3.993	0.858	0.794	0.755
	(0.081)	(0.082)	(0.071)	(0.003)	(0.005)	(0.002)
		[98.8%]	**[90.9%]**		**[92.6%]**	**[95.1%]**
Monotonized estimates	4.650	4.601	4.172	0.865	0.798	0.763
	(0.105)	(0.106)	(0.092)	(0.004)	(0.007)	(0.004)
		[98.9%]	**[90.7%]**		**[92.3%]**	**[95.6%]**

Notes: Original estimates vs. monotonized (rearranged) estimates of expected gains from trade for first-best, second-best, and dynamic mechanisms, and of probability of trade for first-best, trade-maximizing, and dynamic mechanisms. Panel A displays dealers sample and panel B displays fleet/lease sample. The top half of each panel displays estimates under seller distribution upper bound and the lower half of each panel uses the seller distribution lower bound. Gains are in $1,000 units. Standard errors are reported in parentheses and come from 200 bootstrap replications of the full estimation procedure. Square brackets in second-best column represent second-best outcomes as percentage of first-best. In dynamic mechanism column, square brackets represent dynamic mechanism outcomes as a percentage of second-best expected gains from trade and as a percentage of the trade-maximizing probability of trade.

The third row of each panel uses only the first car offered for sale by a given seller on a given day, addressing the concern that if a seller runs multiple cars on a given day, her bargaining strategy on one car may be affected by the outcomes of her previous sales that day. The sample size is reduced substantially under this restriction, leaving 29,465 runs in the dealers sample and 10,934 in the fleet/lease sample. The estimates using the upper bound of seller valuations suggest that dynamic bargaining is less efficient than implied by the full sample, capturing 76.4% of the second-best surplus in the dealers sample and only 70.0% in the fleet/lease sample. However, using the lower bound of seller surplus, the percentages in rows one and three are more similar.

The fourth row of each panel limits the sample to cars with a blue book value lying below the median. In each panel, row four demonstrates a lower level of expected gains from trade among these lower-priced cars. The efficiency of bargaining appears similar to the full sample. The final row limits the sample to new cars (those with age below the median age). This sample results in higher expected gains from trade but similar efficiency implications to the full sample in most cases. One exception is in panel A of Table 8, which demonstrates that using the upper bound of the seller distribution implies that dynamic bargaining only captures 76% of the second-best surplus.

The second column of Tables 8 and 9 displays the raw probability of trade within the sample. Comparing this column to the final column once again provides a measure of model fit, as the final column comes from integrating the estimated allocation function over buyer values, seller values, and auction prices. In both tables the columns agree quite closely.

B.6 Robustness to Assumption on Mean Number of Bidders

Tables 10–11 display the welfare estimates using the full sample under alternative choices of λ, the mean number of bidders at the auction. The main results of the paper, and all results other than Tables 10–11, use $\lambda = 7$ for the dealers sample and $\lambda = 10$ for the fleet/lease sample. As shown in Tables 10–11, the welfare estimates are not sensitive to this choice.

B.7 Alternative Mechanisms

B.7.1 Bargaining with Random Bidder

Table 12 displays the welfare estimates in a setting where bargaining takes place between the seller and a random bidder rather than the high-value bidder. Note that this setting will shift not only the outcomes achieved in dynamic bargaining, but also the first-best and second-best frontiers.

This setting is simple to simulate for the static mechanisms simply by dropping the integration over the auction price in equations (13) and (14) and setting the lower bound of the support of the buyer type in the bargaining game to the lower bound of buyer valuations, $\underline{B}$. That is, bargaining with a random bidder is equivalent to an auction followed by bargaining where the auction price was the lowest possible price and hence yielded no truncation of the buyer support.

Table 8: **Dealers sample**: Efficiency estimates under alternative sample restrictions

*A. Efficiency of bargaining using seller distribution **upper** bound, **dealers sample***								
			Expected gains (units=$1,000)			Probability of trade		
Sample	Sample size	Sample prob of trade	First-best	Second-best	Dynamic mechanism	First-best	Trade-maximizing	Dynamic mechanism
1) Full sample	135,942	0.710	5.342	5.324	4.276	0.849	0.868	0.705
			(0.179)	(0.182)	(0.164)	(0.016)	(0.021)	(0.015)
				[99.7%]	**[80.3%]**		**[102.3%]**	**[81.2%]**
2) First time car is run	86,666	0.775	5.341	5.327	4.561	0.886	0.908	0.774
			(0.159)	(0.160)	(0.144)	(0.012)	(0.015)	(0.011)
				[99.7%]	**[85.6%]**		**[102.4%]**	**[85.3%]**
3) Seller's first car in day	29,465	0.625	4.733	4.703	3.594	0.839	0.824	0.597
			(0.406)	(0.426)	(0.380)	(0.051)	(0.067)	(0.049)
				[99.4%]	**[76.4%]**		**[98.2%]**	**[72.4%]**
4) Below median blue book value	72,063	0.758	3.544	3.528	2.954	0.899	0.884	0.750
			(0.430)	(0.440)	(0.411)	(0.074)	(0.084)	(0.072)
				[99.6%]	**[83.8%]**		**[98.3%]**	**[84.8%]**
5) Below median age	66,186	0.654	5.714	5.695	4.340	0.824	0.904	0.651
			(0.451)	(0.458)	(0.413)	(0.046)	(0.057)	(0.040)
				[99.7%]	**[76.2%]**		**[109.7%]**	**[72.0%]**
*B. Efficiency of bargaining using seller distribution **lower** bound, **dealers sample***								
1) Full sample	135,942	0.710	3.374	3.345	2.811	0.747	0.694	0.700
			(0.103)	(0.108)	(0.097)	(0.010)	(0.018)	(0.009)
				[99.2%]	**[84.0%]**		**[92.9%]**	**[100.8%]**
2) First time car is run	86,666	0.775	3.666	3.643	3.224	0.809	0.769	0.771
			(0.100)	(0.102)	(0.093)	(0.007)	(0.011)	(0.006)
				[99.4%]	**[88.5%]**		**[95.1%]**	**[100.3%]**
3) Seller's first car in day	29,465	0.625	2.458	2.408	2.087	0.695	0.601	0.606
			(0.173)	(0.202)	(0.163)	(0.023)	(0.054)	(0.022)
				[98.0%]	**[86.7%]**		**[86.4%]**	**[100.8%]**
4) Below median blue book value	72,063	0.758	2.554	2.528	2.186	0.819	0.760	0.753
			(0.281)	(0.307)	(0.273)	(0.052)	(0.084)	(0.051)
				[99.0%]	**[86.5%]**		**[92.9%]**	**[99.0%]**
5) Below median age	66,186	0.654	3.446	3.411	2.789	0.734	0.669	0.644
			(0.239)	(0.261)	(0.232)	(0.031)	(0.048)	(0.023)
				[99.0%]	**[81.8%]**		**[91.2%]**	**[96.3%]**

Notes: Dealers sample. Estimates under various sample restrictions of expected gains from trade for first-best, second-best, and dynamic mechanisms, and of probability of trade for first-best, trade-maximizing, and dynamic mechanisms. Gains are in $1,000 units. Standard errors are reported in parentheses and come from 200 bootstrap replications of the full estimation procedure. Second column displays raw probability of trade within the sample. Square brackets in second-best column represent second-best outcomes as percentage of first-best. In dynamic mechanism column, square brackets represent dynamic mechanism outcomes as a percentage of second-best expected gains from trade and as a percentage of the trade-maximizing probability of trade. Panel A displays estimates under seller distribution upper bound and panel B uses the seller distribution lower bound.

Table 9: **Fleet/lease sample**: Efficiency estimates under alternative sample restrictions

Sample	Sample size	Sample prob of trade	Expected gains (units=$1,000) First-best	Second-best	Dynamic mechanism	Probability of trade First-best	Trade-maximizing	Dynamic mechanism
A. Efficiency of bargaining using seller distribution ***upper*** *bound,* ***fleet/lease sample***								
1) Full sample	136,453	0.760	6.045	6.015	5.201	0.898	0.915	0.757
			(0.117)	(0.117)	(0.097)	(0.003)	(0.006)	(0.002)
				[99.5%]	**[86.5%]**		**[101.9%]**	**[82.7%]**
2) First time car is run	97,935	0.843	6.067	6.040	5.565	0.943	0.927	0.843
			(0.137)	(0.138)	(0.122)	(0.002)	(0.005)	(0.002)
				[99.5%]	**[92.1%]**		**[98.3%]**	**[91.0%]**
3) Seller's first car in day	10,934	0.615	4.066	4.002	2.799	0.759	0.717	0.465
			(0.020)	(0.020)	(0.015)	(0.003)	(0.003)	(0.002)
				[98.4%]	**[70.0%]**		**[94.5%]**	**[64.9%]**
4) Below median blue book value	65,083	0.753	4.479	4.444	3.899	0.882	0.861	0.731
			(0.263)	(0.272)	(0.255)	(0.034)	(0.042)	(0.033)
				[99.2%]	**[87.7%]**		**[97.6%]**	**[85.0%]**
5) Below median age	88,986	0.734	5.567	5.538	4.682	0.873	0.919	0.726
			(0.302)	(0.307)	(0.240)	(0.013)	(0.027)	(0.003)
				[99.5%]	**[84.6%]**		**[105.3%]**	**[78.9%]**
B. Efficiency of bargaining using seller distribution ***lower*** *bound,* ***fleet/lease sample***								
1) Full sample	136,453	0.760	4.445	4.392	3.993	0.858	0.794	0.755
			(0.081)	(0.082)	(0.071)	(0.003)	(0.005)	(0.002)
				[98.8%]	**[90.9%]**		**[92.6%]**	**[95.1%]**
2) First time car is run	97,935	0.843	4.977	4.934	4.656	0.919	0.867	0.842
			(0.110)	(0.111)	(0.100)	(0.002)	(0.004)	(0.002)
				[99.1%]	**[94.4%]**		**[94.4%]**	**[97.1%]**
3) Seller's first car in day	10,934	0.615	2.210	2.069	1.896	0.700	0.528	0.561
			(0.017)	(0.018)	(0.016)	(0.003)	(0.005)	(0.002)
				[93.6%]	**[91.6%]**		**[75.4%]**	**[106.2%]**
4) Below median blue book value	65,083	0.753	3.144	3.075	2.889	0.841	0.740	0.741
			(0.143)	(0.159)	(0.139)	(0.020)	(0.037)	(0.019)
				[97.8%]	**[93.9%]**		**[88.0%]**	**[100.2%]**
5) Below median age	88,986	0.734	4.191	4.138	3.684	0.837	0.767	0.724
			(0.201)	(0.205)	(0.167)	(0.010)	(0.018)	(0.003)
				[98.7%]	**[89.1%]**		**[91.6%]**	**[94.4%]**

Notes: Fleet/lease sample. Estimates under various sample restrictions of expected gains from trade for first-best, second-best, and dynamic mechanisms, and of probability of trade for first-best, trade-maximizing, and dynamic mechanisms. Gains are in $1,000 units. Standard errors are reported in parentheses and come from 200 bootstrap replications of the full estimation procedure. Second column displays raw probability of trade within the sample. Square brackets in second-best column represent second-best outcomes as percentage of first-best. In dynamic mechanism column, square brackets represent dynamic mechanism outcomes as a percentage of second-best expected gains from trade and as a percentage of the trade-maximizing probability of trade. Panel A displays estimates under seller distribution upper bound and panel B uses the seller distribution lower bound.

Table 10: **Dealers sample**: Estimates under various assumptions for mean number of bidders

A. Efficiency of bargaining using seller distribution ***upper*** *bound,* ***dealers sample***						
	Expected gains (units=$1,000)			Probability of trade		
Mean # of bidders (λ)	First-best	Second-best	Dynamic mechanism	First-best	Trade-maximizing	Dynamic mechanism
λ = 3	5.392	5.372	4.379	0.851	0.869	0.704
	(0.180)	(0.183)	(0.167)	(0.016)	(0.021)	(0.016)
		[99.6%]	**[81.5%]**		**[102.1%]**	**[81.0%]**
λ = 7	5.342	5.324	4.276	0.849	0.868	0.705
	(0.179)	(0.182)	(0.164)	(0.016)	(0.021)	(0.015)
		[99.7%]	**[80.3%]**		**[102.3%]**	**[81.2%]**
λ = 10	5.337	5.320	4.264	0.849	0.868	0.705
	(0.179)	(0.182)	(0.164)	(0.016)	(0.021)	(0.015)
		[99.7%]	**[80.2%]**		**[102.3%]**	**[81.2%]**
λ = 20	5.336	5.320	4.259	0.849	0.868	0.705
	(0.179)	(0.182)	(0.164)	(0.016)	(0.021)	(0.015)
		[99.7%]	**[80.1%]**		**[102.3%]**	**[81.2%]**

B. Efficiency of bargaining using seller distribution ***lower*** *bound,* ***dealers sample***						
	Expected gains (units=$1,000)			Probability of trade		
Mean # of bidders (λ)	First-best	Second-best	Dynamic mechanism	First-best	Trade-maximizing	Dynamic mechanism
λ = 3	3.417	3.384	2.908	0.752	0.696	0.698
	(0.104)	(0.109)	(0.098)	(0.010)	(0.018)	(0.009)
		[99.0%]	**[85.9%]**		**[92.5%]**	**[100.3%]**
λ = 7	3.374	3.345	2.811	0.747	0.694	0.700
	(0.103)	(0.108)	(0.097)	(0.010)	(0.018)	(0.009)
		[99.2%]	**[84.0%]**		**[92.9%]**	**[100.8%]**
λ = 10	3.370	3.342	2.799	0.747	0.694	0.700
	(0.103)	(0.108)	(0.097)	(0.010)	(0.018)	(0.009)
		[99.2%]	**[83.7%]**		**[93.0%]**	**[100.9%]**
λ = 20	3.370	3.342	2.798	0.747	0.694	0.700
	(0.103)	(0.108)	(0.097)	(0.010)	(0.018)	(0.009)
		[99.2%]	**[83.7%]**		**[93.0%]**	**[100.9%]**

Notes: Dealers sample. Estimates under various assumptions for the mean number of bidders used in the order statistic inversion estimation step. Table displays expected gains from trade for first-best, second-best, and dynamic mechanisms; and probability of trade for first-best, trade-maximizing, and dynamic mechanisms. Gains are in $1,000 units. Standard errors are reported in parentheses and come from 200 bootstrap replications of the full estimation procedure. Square brackets in second-best column represent second-best outcomes as percentage of first-best. In dynamic mechanism column, square brackets represent dynamic mechanism outcomes as a percentage of second-best expected gains from trade and as a percentage of the trade-maximizing probability of trade. Panel A displays estimates under seller distribution upper bound and panel B uses the seller distribution lower bound.

Table 11: **Fleet/lease sample**: Estimates under various assumptions for mean number of bidders

A. Efficiency of bargaining using seller distribution ***upper*** *bound,* ***fleet/lease sample***						
	Expected gains (units=$1,000)			Probability of trade		
Mean # of bidders (λ)	First-best	Second-best	Dynamic mechanism	First-best	Trade-maximizing	Dynamic mechanism
λ = 3	6.127	6.093	5.327	0.901	0.915	0.758
	(0.117)	(0.117)	(0.098)	(0.003)	(0.006)	(0.003)
		[99.4%]	**[87.4%]**		**[101.6%]**	**[82.8%]**
λ = 7	6.051	6.020	5.211	0.898	0.915	0.756
	(0.117)	(0.117)	(0.097)	(0.003)	(0.006)	(0.002)
		[99.5%]	**[86.6%]**		**[101.9%]**	**[82.7%]**
λ = 10	6.045	6.015	5.201	0.898	0.915	0.757
	(0.117)	(0.117)	(0.097)	(0.003)	(0.006)	(0.002)
		[99.5%]	**[86.5%]**		**[101.9%]**	**[82.7%]**
λ = 20	6.044	6.014	5.204	0.898	0.915	0.757
	(0.117)	(0.117)	(0.097)	(0.003)	(0.006)	(0.002)
		[99.5%]	**[86.5%]**		**[101.9%]**	**[82.8%]**

B. Efficiency of bargaining using seller distribution ***lower*** *bound,* ***fleet/lease sample***						
	Expected gains (units=$1,000)			Probability of trade		
Mean # of bidders (λ)	First-best	Second-best	Dynamic mechanism	First-best	Trade-maximizing	Dynamic mechanism
λ = 3	4.522	4.460	4.099	0.866	0.796	0.753
	(0.082)	(0.082)	(0.072)	(0.003)	(0.005)	(0.003)
		[98.6%]	**[91.9%]**		**[91.9%]**	**[94.7%]**
λ = 7	4.451	4.397	3.998	0.858	0.794	0.754
	(0.081)	(0.082)	(0.071)	(0.003)	(0.005)	(0.002)
		[98.8%]	**[90.9%]**		**[92.5%]**	**[95.0%]**
λ = 10	4.445	4.392	3.993	0.858	0.794	0.755
	(0.081)	(0.082)	(0.071)	(0.003)	(0.005)	(0.002)
		[98.8%]	**[90.9%]**		**[92.6%]**	**[95.1%]**
λ = 20	4.445	4.392	3.990	0.858	0.794	0.755
	(0.081)	(0.082)	(0.071)	(0.003)	(0.005)	(0.002)
		[98.8%]	**[90.9%]**		**[92.6%]**	**[95.1%]**

Notes: Fleet/lease sample. Estimates under various assumptions for the mean number of bidders used in the order statistic inversion estimation step. Table displays expected gains from trade for first-best, second-best, and dynamic mechanisms; and probability of trade for first-best, trade-maximizing, and dynamic mechanisms. Gains are in $1,000 units. Standard errors are reported in parentheses and come from 200 bootstrap replications of the full estimation procedure. Square brackets in second-best column represent second-best outcomes as percentage of first-best. In dynamic mechanism column, square brackets represent dynamic mechanism outcomes as a percentage of second-best expected gains from trade and as a percentage of the trade-maximizing probability of trade. Panel A displays estimates under seller distribution upper bound and panel B uses the seller distribution lower bound.

For the dynamic mechanism, bargaining with a random bidder is more difficult to simulate accurately without estimating a model yielding predictions of equilibrium bargaining behavior, which, as highlighted above, is infeasible. However, following the argument in Section 5.2, the allocation function in the dynamic bargaining is identified for any value of $\Psi = R - B^{(2)}$. Therefore, I evaluate the expected gains from trade by integrating over buyer types and seller types, setting the auction price to $\underline{B}$.

Table 12 presents the results. Panel A1 displays the results for the dealers sample using the seller distribution upper bound, and shows that second-best bargaining with a random bidder captures 96.8% of the first-best surplus. This suggests that, even in the absence of an auction which selects the high-value bidder, the deadweight loss due to incomplete information may not be large. The deadweight loss due to limited commitment does not appear to increase in panel A1 relative to the main results in Table 4, as the dynamic mechanism captures 86.4% of the second-best surplus. In panel A2, using the seller distribution lower bound, the dynamic mechanism only achieves 70.5% of the second-best level. The results for the expected gains from trade in the fleet/lease sample (panel B) are similar.

The results in Table 12 provide some evidence that the model fit is worse when the auction is ignored, as the probability of trade in the dynamic mechanism greatly exceeds the first-best level, suggesting some trades are occurring even the seller values the good more than the buyer. This implies that the fit of the allocation function, g_0, is worse when conditioning on the auction price being fixed at the lower bound of the buyer support, $\underline{B}$, than when integrating over all values of the auction price. Also note that in the random bidder setting, the expected transfer between the buyer and seller is not identified, whereas in the currently used auction-followed-by-bargaining mechanism, the expected transfer is directly observable in the data. For these reasons, I do not focus on the random-bidder-bargaining setting in the main body of the paper.

B.7.2 Auction Followed by Accept/Reject Only

Table 13 displays the comparison of the main results from Tables 4–5 to the results from a setting where all players can commit not to bargain ex-post and where the seller chooses only to accept or reject the auction price. As seen by the percentages in square brackets in the "Accept/reject only" column, this alternative mechanism would capture all or nearly all of the second-best surplus and probability of trade, suggesting that such commitment could eliminate the need for bargaining. Conversations with industry professionals reveal that, while this alternative mechanism is used at some auto auction houses, others have historically found such commitment difficult to maintain, as both the high-value bidder and the seller have incentives ex-post to negotiate when the seller rejects the second-order statistic of buyer valuations.

B.7.3 Broker-Optimal Mechanism

Myerson and Satterthwaite (1983) demonstrated that the mechanism which would maximize revenue for a broker with market power is given by allocation function $x^{1,1}$, with transfers given by $p^B(s,b)$, the amount paid by the buyer to the auction house, and $p^S(s,b)$, the amount which the auction house then passes on to the seller. These transfer functions can be defined in many ways. One such way is given by

Table 12: Efficiency of bargaining with random bidder

A. Efficiency of ***bargaining with random bidder, dealers*** *sample*						
	First-best	Second-best	Buyer-optimal	Seller-optimal	Trade-maximizing	Dynamic mechanism
A1. Using seller distribution ***upper*** *bound*						
Expected gains from trade (units=$1,000)	4.116	3.985	3.585	3.687	3.794	3.444
	(0.159)	(0.166)	(0.158)	(0.165)	(0.149)	(0.161)
		[96.8%]				**[86.4%]**
Probability of trade	0.805	0.727	0.616	0.651	0.742	0.912
	(0.016)	(0.020)	(0.019)	(0.019)	(0.021)	(0.032)
		[90.2%]				**[125.5%]**
A2. Using seller distribution ***lower*** *bound*						
Expected gains from trade	2.383	2.200	1.811	2.046	2.210	1.551
	(0.092)	(0.100)	(0.085)	(0.098)	(0.100)	(0.093)
		[92.3%]				**[70.5%]**
Probability of trade	0.623	0.463	0.332	0.439	0.466	0.911
	(0.010)	(0.016)	(0.012)	(0.015)	(0.016)	0.029
		[74.4%]				**[196.7%]**
B. Efficiency of ***bargaining with random bidder, fleet/lease*** *sample*						
	First-best	Second-best	Buyer-optimal	Seller-optimal	Trade-maximizing	Dynamic mechanism
B1. Using seller distribution ***upper*** *bound*						
Expected gains from trade	3.752	3.592	3.337	2.969	3.573	3.179
	(0.110)	(0.115)	(0.107)	(0.111)	(0.108)	(0.111)
		[95.7%]				**[88.5%]**
Probability of trade	0.818	0.735	0.654	0.518	0.744	0.944
	(0.006)	(0.011)	(0.011)	(0.013)	(0.011)	(0.028)
		[89.9%]				**[128.4%]**
B2. Using seller distribution ***lower*** *bound*						
Expected gains from trade	2.490	2.255	2.026	1.939	2.244	1.633
	(0.072)	(0.074)	(0.066)	(0.067)	(0.073)	(0.086)
		[90.6%]				**[72.4%]**
Probability of trade	0.646	0.476	0.409	0.379	0.484	0.944
	(0.007)	(0.011)	(0.009)	(0.008)	(0.011)	(0.028)
		[73.7%]				**[198.2%]**

Notes: Estimates in setting where seller bargains with random bidder. Table displays expected gains from trade and probability of trade for first-best, second-best, buyer-optimal, seller-optimal, trade-maximizing, and dynamic mechanisms. Panel A displays dealers sample and panel B displays fleet/lease sample. The top half of each panel displays estimates under seller distribution upper bound and the lower half of each panel uses the seller distribution lower bound. Gains are in $1,000 units. Standard errors are reported in parentheses and come from 200 bootstrap replications of the full estimation procedure. Square brackets in second-best column represent second-best outcomes as percentage of first-best. In dynamic mechanism column, square brackets represent dynamic mechanism outcomes as a percentage of second-best expected gains from trade and as a percentage of the trade-maximizing probability of trade.

Table 13: Efficiency of auction followed by accept/reject only

A. Efficiency of ***auction + accept/reject, dealers*** *sample*							
	First-best	Second-best	Buyer-optimal	Seller-optimal	Trade-maximizing	Accept/reject only	Dynamic mechanism
A1. Using seller distribution ***upper*** *bound*							
Expected gains from trade	5.342	5.324	4.661	5.319	5.094	5.328	4.276
	(0.179)	(0.182)	(0.181)	(0.182)	(0.181)	(0.179)	(0.164)
		[99.7%]				**[100.1%]**	**[80.3%]**
Probability of trade	0.849	0.831	0.644	0.829	0.868	0.830	0.705
	(0.016)	(0.020)	(0.019)	(0.020)	(0.021)	(0.016)	(0.015)
		[98.0%]				**[99.9%]**	**[84.8%]**
A2. Using seller distribution lower bound							
Expected gains from trade	3.374	3.345	2.483	3.338	3.350	3.337	2.811
	(0.103)	(0.108)	(0.097)	(0.108)	(0.109)	(0.103)	(0.097)
		[99.2%]				**[99.8%]**	**[84.0%]**
Probability of trade	0.747	0.693	0.369	0.692	0.694	0.694	0.700
	(0.010)	(0.018)	(0.013)	(0.018)	(0.018)	(0.010)	(0.009)
		[92.8%]				**[100.1%]**	**[100.9%]**
B. Efficiency of ***auction + accept/reject, fleet/lease*** *sample*							
	First-best	Second-best	Buyer-optimal	Seller-optimal	Trade-maximizing	Accept/reject only	Dynamic mechanism
B1. Using seller distribution ***upper*** *bound*							
Expected gains from trade	6.045	6.015	5.417	6.004	5.823	6.004	5.201
	(0.117)	(0.117)	(0.121)	(0.118)	(0.116)	(0.118)	(0.097)
		[99.5%]				**[99.8%]**	**[86.5%]**
Probability of trade	0.898	0.866	0.711	0.859	0.915	0.857	0.757
	(0.003)	(0.004)	(0.009)	(0.005)	(0.006)	(0.005)	(0.002)
		[96.4%]				**[99.0%]**	**[87.4%]**
B2. Using seller distribution lower bound							
Expected gains from trade	4.445	4.392	3.421	4.382	4.268	4.365	3.993
	(0.081)	(0.082)	(0.080)	(0.082)	(0.082)	(0.082)	(0.071)
		[98.8%]				**[99.4%]**	**[90.9%]**
Probability of trade	0.858	0.778	0.466	0.771	0.794	0.777	0.755
	(0.003)	(0.005)	(0.009)	(0.005)	(0.005)	(0.005)	(0.002)
		[90.8%]				**[99.8%]**	**[97.0%]**

Notes: Estimates in setting with no bargaining, where seller can only accept or reject auction price. Table displays expected gains from trade and probability of trade for auction followed by first-best, second-best, buyer-optimal, seller-optimal, trade-maximizing, and dynamic mechanisms (from Tables 4–5), as well as the case with only accept/reject. Panel A displays dealers sample and panel B displays fleet/lease sample. The top half of each panel displays estimates under seller distribution upper bound and the lower half of each panel uses the seller distribution lower bound. Gains are in $1,000 units. Standard errors are reported in parentheses and come from 200 bootstrap replications of the full estimation procedure. Square brackets in second-best column represent second-best outcomes as percentage of first-best. In dynamic mechanism column, square brackets represent dynamic mechanism outcomes as a percentage of second-best. In accept/reject only column, square brackets represent outcomes as a percentage of second-best.

Myerson and Satterthwaite (1983) as

$$p^B(s,b) = x^{1,1}(s,b) * \min\{u | u \geq \underline{B}, \phi_B(u) \geq s\}$$
$$p^S(s,b) = x^{1,1}(s,b) * \max\{v | v \leq \overline{S}, \phi_S(v) \leq b\}$$

Revenue is given by the difference between the amount paid by the buyer and the amount received by the seller, or equivalently when virtual valuations are increasing, by $G(x^{1,1})$ (where $G(\cdot)$ is defined in (11)). This expression is the participation constraint which must be satisfied in any individually rational, incentive-compatible mechanism. In the mechanisms which maximize the gains from trade or the probability of trade, this expression is equal to zero. In the mechanism maximizing the auction house revenue, however, the auction house wishes to leave some slack in the participation constraint in order to extract surplus from participants.

Table 14: Performance of broker-optimal mechanism

	Dealers sample				***Fleet/lease sample***			
	Seller upper bound		*Seller lower bound*		*Seller upper bound*		*Seller upper bound*	
	Broker-optimal	Dynamic mech	Broker-optimal	Dynamic mech	Broker-optimal	Dynamic mech	Broker-optimal	Dynamic mech
Auction house revenue	3.712	0.219	1.824	0.219	3.729	0.230	2.241	0.230
	(0.745)	(0.000)	(0.195)	(0.000)	(0.103)	(0.000)	(0.065)	(0.000)
Expected gains from trade	4.601	4.276	2.407	2.811	5.353	5.201	3.351	3.993
	(0.182)	(0.164)	(0.098)	(0.097)	(0.124)	(0.097)	(0.082)	(0.071)
Buyer gains	-0.033	0.548	-0.022	0.543	0.690	0.912	0.445	0.908
	(0.201)	(0.012)	(0.108)	(0.009)	(0.010)	(0.005)	(0.009)	(0.005)
Seller gains	0.922	3.509	0.605	2.049	0.934	4.060	0.665	2.856
	(0.663)	(0.156)	(0.148)	(0.093)	(0.019)	(0.096)	(0.014)	(0.070)
Probability of trade	0.631	0.705	0.352	0.700	0.697	0.757	0.451	0.755
	(0.019)	(0.015)	(0.013)	(0.009)	(0.009)	(0.002)	(0.009)	(0.002)

Notes: Estimates of auction house revenue, expected gains from trade, and probability of trade in broker-optimal vs. dynamic mechanisms. Estimates from dealers sample shown on the left and from fleet/lease sample on the right.

The performance of this mechanism relative to the dynamic mechanism is shown in Table 14. The expected revenue for the auction house in the dynamic mechanism is estimated using data on fees when trade occurs. Table 14 demonstrates that, in the dealers sample, the broker-optimal mechanism would result in auction house revenues of \$3,712 under the seller distribution upper bound, and \$1,824 under the seller distribution lower bound. Under the current dynamic mechanism, these revenues are much lower, at \$219. The comparison is similar in the fleet/lease sample. Both buyer gains and seller gains would be much lower in the broker-optimal mechanism, as the majority of surplus would be taken by the auction house. The probability of trade would also decrease, from about 0.70 to 0.352–0.631 in the dealers sample, and from about 0.76 to 0.451–0.697 in the fleet/lease sample. Comparing the total expected gains from trade in the broker-optimal mechanism to the second-best from Tables 4–5 highlights that this mechanism introduces an additional deadweight loss due to the broker's rent-extraction behavior.

It is difficult to interpret these results given that a shift to this mechanism would likely drive buyers and sellers away from the auction house and toward competing sourcing venues, and this competition is not expressed in the model. Therefore, while auction house revenue is clearly of primary interest to the auction house, competition among auction houses may impede an individual auction house from achieving the payoff of the broker-optimal mechanism. Townsend (1983) demonstrated, in a general equilibrium framework, that competition among auction houses, or even the threat of competition, leads to the Walrasian equilibrium as the number of buyers and sellers grows large. Thus, auction houses may appear to behave as though they were maximizing surplus rather than achieving the optimal revenue for a solo auction house. However, Economides and Siow (1988) showed, in a competition-on-a-line framework, that liquidity provides positive externalities for buyers and sellers which are not fully internalized by the auction house, and this may prevent efficient entry of auction houses and hence prevent the market from achieving the surplus-maximizing allocation. It is theoretically ambiguous how close auction houses would come to achieving the revenue-maximizing outcomes in a setting with two-sided uncertainty. For these reasons, and due to the fact that I have no data on competing auction houses, I do not focus on this mechanism in the main body of results.