Agent-Based Computational Economics: Simulation Tools for Heterodox Research

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Abstract

This chapter introduces Agent-Based Modeling (ABM) as a research tool that possesses advantages for heterodox research programs. We introduce the approach in four steps. First, we discuss the uniqueness of ABMs, which lies primarily in the flexibility to incorporate vastly heterogeneous agents and to address models with high degrees of freedom. Second, we argue that the flexibility of ABMs makes them an appropriate tool for the questions raised by Classical and (Post-)Keynesian economists. To demonstrate this point we briefly sketch two ABMs, one which constructs an environment that captures the Classical-Marxian processes of gravitation, thereby opening new pathways in value theory, and the other is a nuanced analysis of Keynesian effective demand problems and the existence of chaotic cycles in a capitalist economy. The last section revisits the flexibility of ABMs in order to discuss their capability of incorporating dimensions from across the broad variety of heterodox research programs.

Keywords: Agent-Based Model; Computational Simulation; Heterodox Economics; Gravitation; Chaotic Cycles.

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1 Introduction

This chapter introduces Agent-Based Modeling (ABM) as a research tool that possesses advantages for current heterodox research programs. An ABM consists of a computer simulation of many interacting heterogeneous agents that produces an economic phenomena of interest. The purpose of producing, or generating, economic phenomena with an ABM is to develop an account of how the phenomena in question are generated and to study the processes through which these phenomena evolve. The dynamics produced by ABMs are often complex and the ABM approach in general overlaps with complexity approaches to economics.[1] ABMs are extremely flexible in their construction and can incorporate many aspects of economic behavior and socioeconomic situations that are of paramount interest to heterodox economists.

The general approach of ABM and how it might be applied to existing heterodox research programs is introduced in four steps. First, the uniqueness of ABM, which lies primarily in the flexibility to incorporate vastly heterogeneous agents and to address models with high degrees of freedom, is presented and discussed. Second, it is argued that the flexibility of ABMs makes them an appropriate tool for the questions raised by Classical and (Post-)Keynesian economists. This argument is demonstrated by briefly sketching two existing ABMs, one which constructs an environment that captures the Classical-Marxian processes of gravitation, thereby opening new pathways in value theory, and another which presents a nuanced analysis of Keynesian effective demand problems and the existence of chaotic cycles in a capitalist economy. The last section revisits the flexibility of ABMs in order to discuss the open possibilities of applying the ABM toolset to different visions from across the variety of heterodox bodies of thought.

2 Why ABM?

Agent-based Modeling (ABM) presents a framework and set of tools that hold promise for advancing research within many heterodox bodies of thought. To address the pertinent question of "why should heterodox economists be interested in agent-based models?" a brief, and abstract, sketch of the basic features of an agent-based model (ABM) is presented below and discussed in relation to current research programs. Detailed introductions to the construction of ABMs in general can be found in Railsback and Grimm (2012)

[1] Discussions of complexity theory and economics that may be of interest to heterodox economists can be found in the volumes by Colander (2000) and Rosser (2009), as well as in the work of Kirman (2004) and Rosser (1999, 2008). In a similar thread, Holt, Rosser, and Colander (2011) describe the recent emergence of what they deem to be the "complexity era" of economics.

and Salamon (2011). The exact choice of programming language or the software in which an ABM is developed depends on the researcher. *NetLogo* (Wilensky 1999) is a popular package that is easy to become acquainted with and has free introductions available online. Other languages/software packages that are used in developing ABM are *Mathematica*, *Swarm*, and *Repast*. These are more difficult to pick up without any prior background in computer programming, but they can be more computationally powerful than *NetLogo*. For instance, if an ABM is built in *Mathematica* it is possible to use the software to analytically study the properties of the model.

2.1 The Basics of ABM

The first thing to consider when constructing an ABM is the design of a set of agents. The agents are heterogeneous to varying degrees, depending on what is desirable for the model being constructed, and a single model can feature multiple sets of agents: e.g. consumers, households, firms, government, financial institutions, and/or other institutions. Once the "taxonomy of agents" (LeBaron and Tesfatsion 2008, 246) is decided upon, the agents are then given a set of characteristics appropriate for the model at hand. These characteristics could be endowments of commodities, preferences, a production technology, and/or abilities to process information.

Next, the agents must also be given rules through which they interact. The interactions can range from simple to complex or from one-off to repeated interactions, and anything in between. Examples of possible interactions could be exchange, wage bargaining, competition between firms, management-employee interactions, and/or interactions between individual members of a household. The benefit of an ABM is that it can capture many of these interactions between heterogeneous agent sets within one model.

Lastly, the "scale of the model must be suitable for the particular purpose at hand" (LeBaron and Tesfatsion 2008, 246). Stated another way, the agents must be situated within a world that makes sense for the problem being investigated and the number of agents should be appropriately large enough. Similarly, when dealing with heterogeneous sets of agents, the proportions of different agent types should fit the problem/phenomena being investigated.

Broadly speaking, ABMs fit into what can be described as a "generativist" methodology (Epstein 2006a). The purpose of a generativist methodology is to develop a micro-specification (the agents and their rules of interaction) that *grows* some type of more macro-level phenomenon through the interaction(s) of the many agents. Thus, some type of macro-behavior, or regularity, is generated from the micro-specification of the model and the model itself provides an account of how the macro-behavior is attained. As an ABM simulation unfolds the micro-specification generates macro-structures, which feed back into how the micro-specification updates and produces future macro-structures, thus the micro and macro "co-evolve" (Epstein 2006a, 6). The focus on the formation of macro-behavior or a "macrostructure" (Epstein 2006a, 8) should not be taken to mean that all ABMs are macroeconomic models; it simply means that ABMs are focused on economic phenomena that cannot be explained at the level of an individual agent. According to Epstein (2006a), ABMs "provide computational demonstrations that a given microspecification is in fact *sufficient to generate* a macrostructure of interest", and the account of this generation is how ABMs explain (Epstein 2006a, 8).

In the context of discussing how ABMs are built and explain economic phenomena it should be noted that, while ABMs are built around individual agents, these agents are not representative agents. One can examine a particular agent in an ABM over the course of a simulation, but the state of this agent over the simulation may or may not provide any information regarding the aggregate behavior of the model. In many ways, ABMs inherently accept Kirman (1992)'s point that the representative agent is unjustified and attempt to move beyond this modeling convention. Furthermore, the micro-specification of an ABM is not the same the microfoundations found in many economic models. The micro-specification of an ABM sounds suspiciously similar to the microfoundations found in many neoclassical macroeconomic models, but the interaction of the agents in an ABM provides a degree of freedom between the micro-specification and the macrostructure that emerges from the simulation.

2.2 The Advantages

The advantages of using ABM comes in several forms. As Tesfatsion (2006) notes, ABMs are particularly well-adapted to incorporate asymmetric information, strategic interactions/choices, learning behavior, and the existence of multiple equilibria. The ability to incorporate the aforementioned features stems from the heterogeneity of the agents and the flexibility in their construction. ABMs are flexible in construction to the point that is it theoretically possible to include all of the characteristics mentioned by Tesfatsion (2006) in a model that demonstrates highly complex behavior. Similarly, this flexibility can allow ABMs to be 'tuned' (or 'calibrated' if one prefers) to replicate multiple empirically observable behaviors in the same model—something that can be difficult with more traditional modeling techniques. Arguing the point further, Dosi, Fagiolo, and Roventini (2010) remark on the fact that an ABMs construction allows it to be "empirically quite robust" because they can account "for a large number of empirical regularities" rather than just a few moments or stylized facts observable in time series data (Dosi et al. 2010, 1759).

It is not uncommon for proponents of ABM to stress that the purpose of an ABM is to grow or have it produce some empirically observable pattern(s) in economic data. There is language to this effect in Dosi et al. (2010), Epstein (2006a), LeBaron and Tesfatsion (2008), and Tesfatsion (2006). The empirical robustness aspect of ABM is of great importance for further establishing ABM as an acceptable modeling choice, but the benefits of ABMs do not lie exclusively in their ability to mimic empirical data. ABMs can contribute to the development of economic theory by: (1) allowing exploration of complex model setups that cannot be solved with traditional techniques; and (2) by extending existing theory through adding to our understanding of how relevant phenomena are produced within the theory (Arthur 2006). Point (2) is particularly so in the case of Classical and Marxian Political Economy, as shown in Section 3, and in the case of the macro-dynamics explored in Section 4.

Agent-based modeling holds promise for heterodox economics because of the inherent flexibility in construction. As discussed above, the flexibility can be how agents are designed and granted capabilities to process (or not process) information, how the interaction of agents is setup, the possible array of agents present in the model, and the complexity of agent interactions. This flexibility can allow the further development of both theory and modeling in some of the major heterodox traditions. This particular point is discussed further in the remaining sections.

2.3 ABMs of Interest

Agent-based modeling already has some history in economic modeling, with examples dating back roughly twenty years. However, many aspects of ABM are still underdeveloped, particularly its possible applications to existing research programs in the heterodox traditions. One of the earliest instances of an economic model built around interacting agents can be found in Albin and Foley (1992). Other early examples of agent-based modeling, or thinking of the economy as a complex system, can be found in Anderson, Arrow, and Pines (1988), as well as in Arthur, Durlauf, and Lane (1997), with the contributions by Kirman (1997) and Tesfatsion (1997) speaking directly to the modeling of economies as systems of interacting agents. There is also a growing number of collected volumes containing a variety of examples of agent-based models of economic phenomena. Notable among these are Epstein (2006b) and Tesfatsion and Judd (2006). ABMs, including the plea for more attention to the benefits of ABM, have also found their way into some academic journals. LeBaron and Tesfatsion (2008), for instance, is in *The American Economic Review* and presentations of ABMs have been published in the *Journal of Economic Behavior and Organization*, the *Journal of Economic Dynamics and Control*, *Advances in Complex Systems*, and the

Journal of Economic Interaction and Coordination was founded with the express purpose of furthering the development of ABMs.

Recent ABMs that may prove interesting for heterodox researchers to consider, some of which could be considered heterodox in their own right, can be found in the following works: Axtell (2010); Delli Gatti, Giulmi, Gaffeo, Gianfranco Giulioni, and Palestrini (2004); Di Guilmi and Chiarella (2011, 2013); Di Guilmi, Gallegati, Landini, and Stiglitz (2012); Dosi, Fagiolo, and Roventini (2006); Dosi et al. (2010); Foley (2010); Gintis (2007, 2012); Kinsella, Greiff, and Nell (2011); LeBaron (2006); Ussher (2008); and Wright (2008, 2011a).

The models presented in Axtell (2010) and Delli Gatti et al. (2004) study the distribution of firm size and the associated dynamics over business cycles. In Axtell (2010)'s case an ABM is developed to replicate the distribution of firm size that is observable in economic data because the neoclassical theory of the firm is lacking in its ability to speak to empirically observable aspects of firms. Continuing with the more micro-oriented of the aforementioned models, Foley (2010) and Gintis (2007, 2012) present models of exchange processes in the general equilibrium tradition, but with a distinctly non-Walrasian flavor. Instead of employing a Walrasian auctioneer or having agents always trade at equilibrium prices Foley (2010) and Gintis (2007, 2012) have the agents discover the equilibrium through their exchanges. Ongoing work by LeBaron (2006) and Ussher (2008) furthers understanding the behavior of financial markets in terms of the interaction of economic agents. Wright (2008, 2011a)'s contributions situate Classical-Marxian Political Economy in an agent-based environment in order to construct models that demonstrate the emergence of the labor theory of value of the Classical-Marxian tradition. The model presented in Section 3 is heavily influenced by Wright's pioneering contributions.

The remaining models listed above are more macro-oriented, although Delli Gatti et al. (2004) span both groups. Dosi et al. (2006, 2010) develop and ABM built around a Schumpeterian engine of technical change and economic growth that eventually incorporates Keynesian demand management (see Dosi et al. (2010)) in order to study the potential positive effects of complementary creative destruction and demand management policies. Di Guilmi et al. (2012) further explore and develop analytical techniques for ABMs and Di Guilmi and Chiarella (2011, 2013) explore financial instability, with Di Guilmi and Chiarella (2013) focusing on Minskyan dynamics, and transmission mechanisms of financial stress/instability to the real economy. Kinsella et al. (2011) develop an agent-based macro-model based on the stock-flow consistent approach to macroeconomics found in Godley and Lavoie (2006), which replicates a number of distribution patterns found macroeconomic data (e.g. distributions of firm size and income). Kinsella et al. (2011)'s contribution may be of particular interest to those working in heterodox macroeconomics.

This is just a sample of the burgeoning literature on ABMs that may be of interest to heterodox researchers in further developing their own research programs. In the remaining sections of this chapter the applications of ABM to specific heterodox bodies of thought is explicitly discussed.

3 ABM for Classical-Marxian Value Theory

One thread of heterodox research that benefits from an agent-based setting is Clasical-Marxian value theory. The complementarity between Classical-Marxian value theory and ABM becomes clear when reading Marx and the Classicals (Smith and Ricardo) as employing a long-period method[2] in their development of the labor theory of value. This particular reading also treats the Classicals (Smith and Ricardo in particular) and Marx as viewing capitalist economies to be complex systems (Foley 2003). The long-period method is briefly reconstructed below, with specific focus on Marx, in order to present the characteristics that make it a natural choice for an agent-based framework.

Marx's long-period method, as developed in Foley and Duménil (2008a,b) and Foley (2011), begins with an abstraction in which there are a large number of commodity producers, with access to, or very low cost, means of production, laboring across many lines of production to create commodities. These producers are also taken to be mobile across lines of production. The producers then engage in direct exchange with one another and the prices of commodities that emerge are proportional to the labor-time required for their production. If prices are not proportional to labor-time requirements then the producers will migrate across lines of production until prices are once again proportional to labor-time requirements. This movement of producers across industries is conceived to be ongoing, thus the equilibrium at which prices are exactly proportional to labor-time requirements—or prices are proportional to values—emerges as a center of gravity for the constant oscillations of prices and the allocation of producers across lines of production.

The abstraction outlined above is referred to as the "commodity law of exchange" (Foley and Duménil 2008a; Foley 2011) and can be expanded to include ownership of means of production and costly capital goods. As a result of including costly capital goods, the prices that form become prices of production (Marx 1981, 297-298). The expanded abstraction is referred to as the "capitalist law of exchange" (Foley and Duménil 2008a; Foley 2011) and "transcends" (Foley 2011, 22) the commodity law of exchange so that

[2]See Garegnani (1970, 1976, 1984)

the mobility of producers becomes the mobility of labor and capital across lines production. The mobility of labor and capital then entail that the relevant central tendencies of the capitalist law of exchange are the independent equalization of rates of surplus value across sectors (Cogliano 2013) and the equalization of profit rates.

The focus of Classical-Marxian value theory on the emergence of centers of gravity (or central tendencies) lends itself to an agent-based approach, which has the flexibility to capture the type of open-ended oscillations described above. The ABM approach carries the additional benefit of allowing for detailed study of the exchange process taking place in the commodity or capitalist laws of exchange—a story that is largely absent from presentations of the Classical-Marxian theory of value. Examples of recent work that situates the labor theory of value in an ABM can be found in Cogliano (2013) and Wright (2008, 2011a,b). The approach and main results of Cogliano (2013) are presented below to demonstrate the advantages of an ABM approach to the study of Classical-Marxian value theory.

3.1 The Beaver-Deer World

The flexibility of ABM to capture the open-ended processes of gravitation described by Marx can be demonstrated with the simple two-commodity model of the commodity law of exchange initially presented by Cogliano (2013). This particular model can be viewed as a version of Smith's beaver-deer thought experiment through which he develops the basic insights of the labor theory of value (Smith 2000). This model lacks certain features of Marx's theory of value yet still holds insight for Marxian political economy given Marx's acceptance and approval of Smith's abstract approach to developing the labor theory of value.[3]

The model consists of a set of N agents similar to the producers in the commodity law of exchange within a two-commodity world and occurs in discrete time steps t for some total length of time T. The commodities held by the agents are denoted by x_1 and x_2. The agents produce commodities, trade with one another, consume commodities, and decide where to allocate their productive capacity. Commodities are produced with labor as the only input in order to capture the open access to means of production in the commodity law of exchange. The output of each agent during one time step of the model is given by $x_i = 1/l_i$ with l_i denoting the labor value or labor-time requirement of producing commodity i. With $l_i \in (0, 1]$ for all i, the speed of production can be fairly rapid, with each agent producing at least

[3]The closeness of Smith and Marx on the development of the labor theory of value is discussed in detail by Cogliano (2013) and the passages in Marx's own work can be found in volume three of *Capital* (Marx 1981, 241-242), the introduction to the *Grundrisse* (Marx 1973, 104), and *Theories of Surplus Value* (Marx 1988, 376-411).

one commodity during each time step of the simulation. Each agent produces one of the two commodities at a time but consumes a fixed proportion of their holdings of both during each time step of the model.

Once the agents have produced their commodities, they enter the market to exchange for the commodity they do not produce. There is no money in the model since the presence of money in a two-commodity world does not add to the story, thus all exchanges take place via barter. The agents enter the market and determine their initial offer prices from a Cobb-Douglas utility function $u[x_1, x_2] = x_1^{\alpha} x_2^{\beta}$ with $\alpha = \beta$. With the utility function, the agents' willingness to trade is given by their marginal rate of substitution between the two commodities.[4] Hence, with x_2 as the numéraire, the initial offer prices are given by

$$p = \frac{\partial u[x_1, x_2]/\partial x_1}{\partial u[x_1, x_2]/\partial x_2} = \frac{\alpha x_2}{\beta x_1} = \frac{x_2}{x_1} \tag{1}$$

The agents in each sector are then randomly matched with agents in the other sector and they determine a final exchange price as the geometric mean of their offer prices, e.g. for some pair of agents j and k in sectors 1 and 2 respectively, their exchange price will be $\rho = (p_{1,j} \cdot p_{2,k})^{1/2}$. Once an exchange price ρ is struck between the agents, they exchange a given quantity of the numéraire good and an appropriate amount of x_1 based on the exchange price: $x_1 = \bar{x}_2/\rho$.

Agents can engage in multiple exchanges during each time step of the model, thus multiple exchange prices manifest during each time step t, and exchanges continue to take place until the average offer prices across the two sectors are close. The exact exchange prices and the allocation of commodities across agents depends on the path taken to reach the equilibrium and varies over each iteration of the model. This type of exchange procedure and the corresponding equilibrium it reaches are referred to as "catallactic" (Foley 2010; Julius 2013), which entails that "agents who have identical preferences and endowments may have different commodity bundles and utility levels" (Foley 2010, 119) even in equilibrium.[5]

After the exchange procedure finishes, agents consume a fixed proportion of their holdings of both commodities and decide whether or not to reallocate their productive capacity across sectors in response to how they fared in exchange. Agents make this decision by comparing a moving average of the prices at which they exchanged to the average exchange price in the other

[4] This type of trading procedure can first be found in one of the early agent-based exchange procedures developed by Albin and Foley (1992).

[5] Axtell (2005), Fisher (1983), and Smale (1976) study exchange processes with a similar non-Walrasian flavor, while Gintis (2007, 2012) develops an exchange procedure in an agent-based setting that is also non-Walrasian.

sector. E.g. an agent j in sector 1 producing x_1 will compare a moving average of their ρ_j to the average price in sector 2 given by $\bar{\rho}_2$. The comparison is made with a logistic function that takes the following form for an agent currently engaged in sector 1:

$$\Theta = \frac{1}{1 + e^{\gamma(\varepsilon(\bar{\rho}_2 - \rho_j) + (1-\varepsilon)(x_2/x_1)^{-1})}} \tag{2}$$

The ε is a binary term that denotes whether or not the agent successfully completed an exchange in the market. If the agent did not exchange then $\varepsilon = 0$ and the agent could switch sectors if their offer price becomes small enough (or large enough in the case of agents in sector 2). The γ term in Equation (2) is a damping parameter. Equation (2) can be interpreted as yielding the inverse probability of an agent switching sectors. This probability Θ then updates the following equation:

$$s_t = s_{t-1} + \theta(\Theta - s_{t-1}) \tag{3}$$

The agent then compares s_t to a number that is drawn randomly from a normal distribution with $\mu = 0$ and $\sigma = 0.75$. If the number drawn is greater than s_t then the agent will switch sectors and produce the other commodity in the next time step of the model. The result of Equation (2) is fed into Equation (3) in order to make each agent's decision to switch a gradual one based on some history of their experience in production and exchange.

The movement of producers across sectors in response to price signals occurs during each time step of the model. Thus the prices that emerge from the market during one time step t will determine the allocation of producers across sectors in the next time step $t+1$, which effectively determines the available supply of commodities. The available supply of commodities, in large part, determines the prices that emerge from the market in $t+1$, which then determines the allocation of producers in $t+2$.[6] Hence, the relative price and the allocation of producers across sectors co-evolve and the labor theory of value equilibrium emerges as a center of gravity as the simulation unfolds.

3.2 Results & Considerations

Figure (1) below demonstrates the co-evolution of relative price and producer allocation for a typical run of the model that begins out of equilibrium with $l_1 = l_2$. Figure (2) demonstrates how the deviations of relative price and the

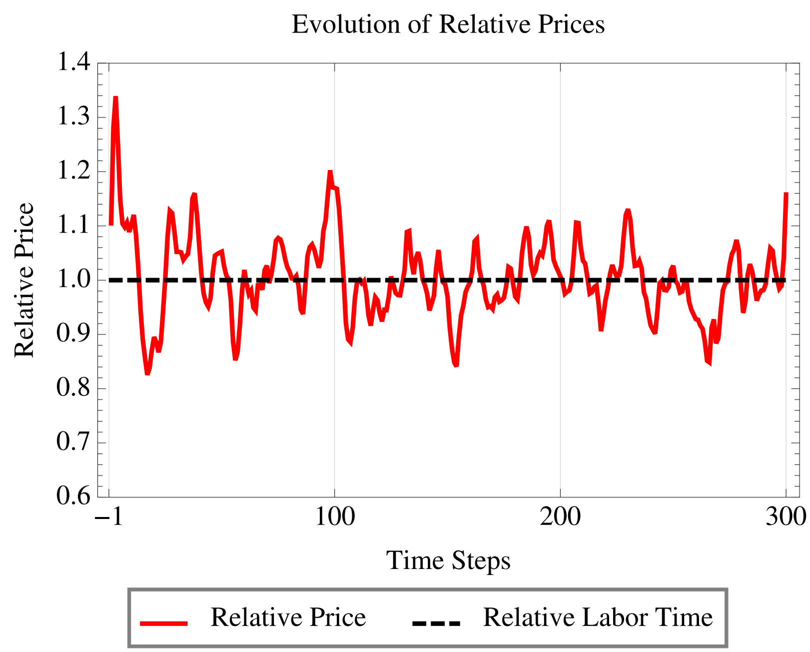
[6] The other factor in determining the price(s) that manifest in the market is the particular path that producer trades take from the starting point to the equilibrium.

allocation of producers from the equilibrium fit a tight pattern about the equilibrium for a longer period of time.[7]

The gravitation of price and producer allocation about the equilibrium continues for any length of time T without settling down or converging to the equilibrium. This model helps frame the labor theory of value not only as a theory of price formation, but also as a theory of the allocation of productive labor across sectors of an economy—an important nuance of the labor theory of value emphasized by Cogliano (2013). Thus this simple ABM captures the open-ended processes of gravitation that are a central aspect of the Classical-Marxian vision.

[7]Figure (2) features the deviation in the number of producers in sector 1 from the equilibrium because the number of producers in one sector provides information regarding the overall allocation across sectors since the model is built with two sectors.

Figure 1: Evolution of Relative Price and Allocation of Producers for: $N = 300$ and $l_1 = l_2 = 0.20$ for 300 time steps.



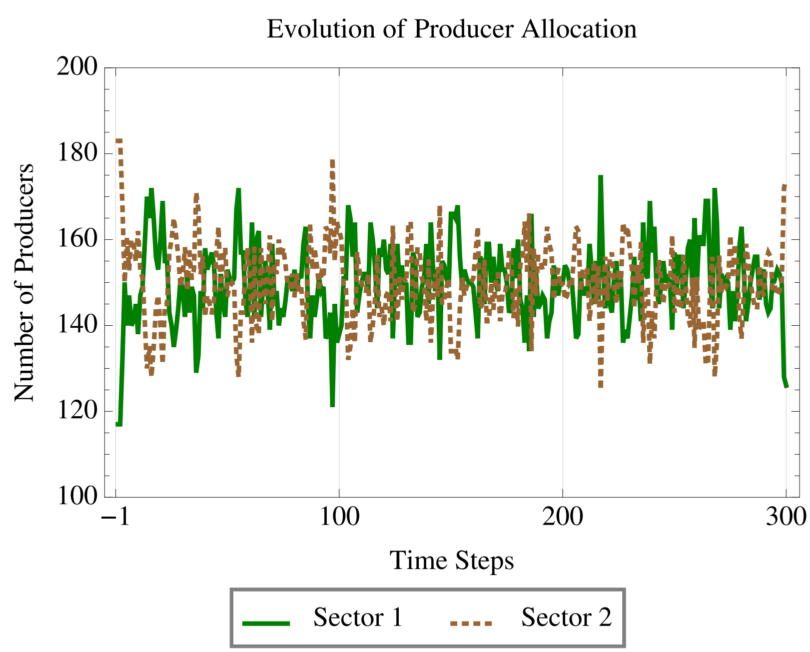
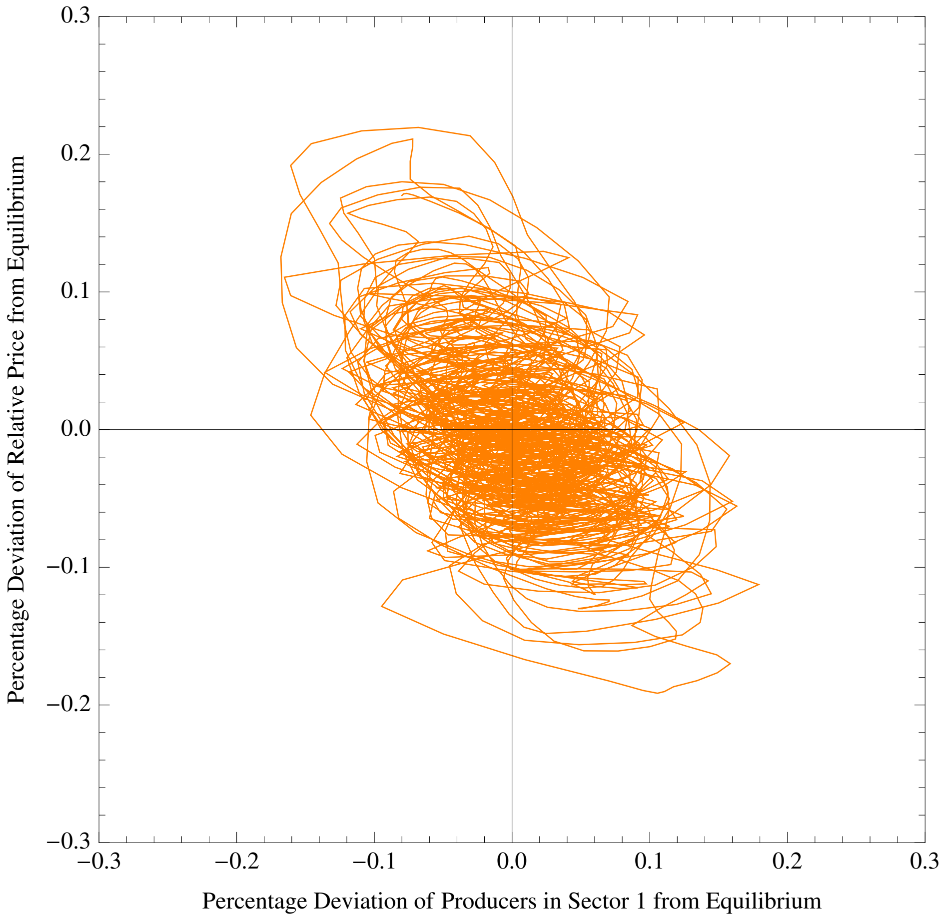


Figure 2: Percentage Deviation of Relative Price and Allocation of Producers from Equilibrium for: $N = 300$ and $l_1 = l_2 = 0.20$ for 900 time steps (excludes initialization period).



4 ABM for (Post-)Keynesian Macro-Dynamics

This section introduces a firm-level agent-based simulation model that is capable of generating chaotic cycles endogenously from an artificial economy. This model is particularly heterodox in the sense that it is a circuit of capital model in the Marxian tradition, and, at the same time, it imports various insights from Post-/Keynesian macroeconomics. Perhaps more importantly, this section also intends to show that the method of ABM is particularly useful in addressing some research questions that are quite heterodox in nature, such as: what are the dynamics implied by complex interactions of profit-seeking firms; and what are the structural causes of growth and instability in a capitalist economy?

4.1 An Overview of the Structure of Circuit of Capital

Marx's circuit of capital model[8] was an attempt to conceptualize the economy as a whole by studying the macroeconomy in a way that is stock and flow consistent and compatible with the labor theory of value. The circuit of capital was originally written down by Marx (1978) as $M - C ... P ... C' - M'$. Money (or financial capital) M is first transformed into commodities C through advancing capital outlays, with C taking on the form of constant and variable capital.[9] C is then thrown into the production process P and is transformed into a new commodity C', which contains surplus value. Finally, this commodity is sold in the market and transformed back into money M', with M' being greater than the money value M that began the circuit. The increase of $M' > M$ presupposes that M' contains realized surplus value.

4.2 An Agent-Based Circuit of Capital Model

Rooted in Marx's circuit of capital and the formal model developed by Foley (1982, 1986), this model of an artificial economy consists of n profit-seeking firms, each owning two stocks: financial capital (M) and productive capital (X); and two flows: sales (S) and capital outlays (C). In the beginning, each firm is endowed with some amount of initial financial and productive capital and they have to first make investment, lending, and borrowing decisions. These decisions are modeled with an investment function in the Post-Keynesian tradition:

$$C_{i,t+1} = C_{i,t} + a[m_{i,t}, i_t - r_{i,t}]C_{i,t}, C_m > 0, C_{i-r} < 0 \tag{4}$$

In Equation (4), m is the liquidity ratio, which is the ratio between financial capital and productive capital (M/X). There is a positive relation between the growth rate of capital outlays ($a[m, i-r]$) and m because higher liquidity will encourage greater investment on the part of firms. The growth rate of capital outlays also depends on the difference between the interest and profit rates ($i - r$). The higher the difference, the more a firm is inclined to lend its money to banks to earn interest rather than investing in real production to earn a profit, and vice versa.

The next step is the introduction of the demand closure. Assuming a portion of capital outlays of each firm goes to wages, and wages are spent instantaneously, then firms' capital outlays in the form of means of production and means of consumption must be shared across all other firms in

[8] First appearing in its developed form in volume two of *Capital* (Marx 1978).

[9] It is important to note here that one of the C commodities is labor-power.

the form of sales. This model further assumes that there is no capitalist consumption, hence there is no leakage from the sales. These constructs therefore yield (5): the sum capital outlays equals to the sum of sales; in other words, total effective demand always meets total supply.

$$\sum_{n=1}^{N} C_{n,t} = \sum_{n=1}^{N} S_{n,t} \tag{5}$$

The specific way in which capital outlays and sales match on the firm level in this system depends on the construction of the matrix A. Let A be an $n \times n$ transition matrix of coefficients with its columns summing to one. Right-multiplying it by the vector of capital outlays $\boldsymbol{C}$ will result in the vector of sales $\boldsymbol{S}$.

$$A \cdot \boldsymbol{C} = \boldsymbol{S} \tag{6}$$

The A matrix essentially distributes capital outlays across firms as their sales. In fact, the shape of the A matrix is what makes this model "agent-based" because it determines the heterogeneity of the firms. The A matrix creates a network of supply and demand (capital-outlays and sales) amongst the n firms in the same way as a typical input-output matrix. Although Equation (5) must hold in the aggregate, at the individual level, each firm might have its capital outlays (supply) above or below the sales (demand from the rest of the firms). In other words, the Keynesian problem of effective demand arises at individual level. The effective demand problem is modeled by the A matrix with each column summing to one, but each row sums to a number that is either above or below one (and yet the sum of column sums and the sum of row sums are kept equal so Equation (5) still holds).

For each firm, its sales minus capital outlays is its profit, and the share of profit out of sales is represented by the profit margin q.

$$q_i = \frac{S_i - C_i}{S_i} \tag{7}$$

The accounting framework of this model deducts sales after profit, therefore, while updating the productive capital, the profit margin is discounted.

$$X_{i,t+1} = X_{i,t} + C_{i,t+1} - S_{i,t+1} \cdot (1 - q_i) \tag{8}$$

Equation (8) states that a firm's capital outlays add to its productive capital, and its sales (discounted by its profit margin) reduce its productive capital. Next, a firm's financial capital is updated in a similar fashion given by Equation (9) below.

$$M_{i,t+1} = (1 + i_t)M_{i,t} - C_{i,t+1} + S_{i,t+1} \tag{9}$$

A firm receives (or pays out) interest on its financial capital first. Capital outlays reduce a firm's financial capital and sales increases the firm's financial capital. Finally, the firm's profit rate is determined as follows:

$$r_{i,t+1} = (q_i \cdot S_{i,t})/X_{i,t} \tag{10}$$

There is a central bank that determines a single new interest rate for all firms by looking at the average liquidity ratio ($\overline{m}$) of the entire economy. The central bank issues a lower interest rate when the average liquidity is high, and vice versa. The determination of the interest rate is expressed by Equation (11) below.

$$i_{t+1} = \phi[\overline{m_t}], i_{\overline{m}} < 0 \tag{11}$$

Once the interest rate is determined, the round of interaction ends and the new round starts with updated variables.

4.3 Simulation Results

For the purposes of simulation, the equations in the system are parameterized, the A matrix is generated with desired properties as discussed in the last subsection, the number of firms and the uniform profit margin are again set to be 200 and 0.4 respectively, and finally, the initial values for each firm's stock variables are randomly assigned from a uniform distribution bounded between 0 and 200. The simulation is carried using *Mathematica*.

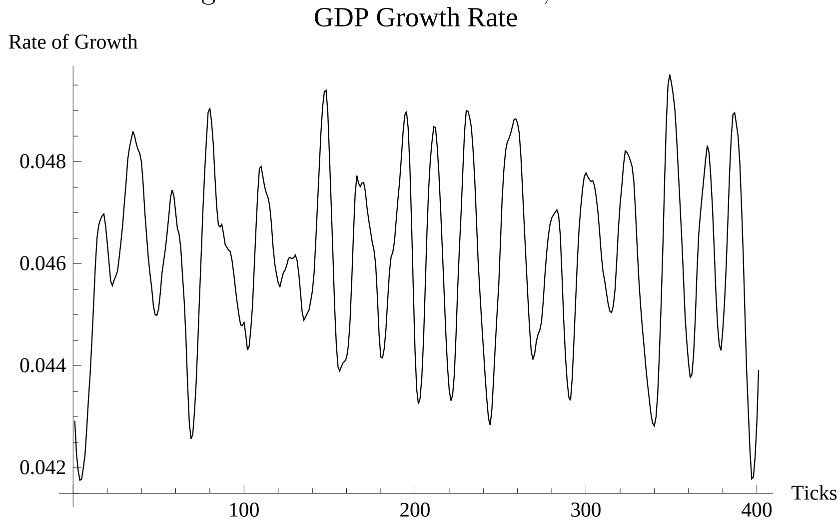
Since the model itself is an artificial economy, some measure of GDP can be constructed similar to how GDP is computed in actual economies. For this particular economy GDP can be measured using the income approach. In this model, there are only two sources of income: wages and profit. Wages in this model are specified as a portion of capital outlays and all wages are spent instantaneously. Profit is equal to sales multiplied by the profit margin, as stated in part of Equation (10), but profit is claimed by firms after goods are sold, which is one period after wages are paid and consumed. Therefore, the GDP in this model is calculated by following equation.

$$\mathrm{GDP}_t = kC_{t-1} + qS_t \tag{12}$$

With the proportion of capital outlays that goes to wages set at 30%, the growth rate of GDP in the simulation is illustrated in Figure 3 below. The growth rate of GDP in this model fluctuates within a range of 1 percent and displays a great deal of irregularity. In fact, disregarding the scale of the model,[10] the GDP growth rate trajectory of this system is strikingly similar to the actual U.S. GDP growth rate path. It is worth noting that

[10] A model as such can be calibrated and scaled while its dynamics are still preserved, but the discussion of scaling and calibrating is beyond the aims of the current paper.

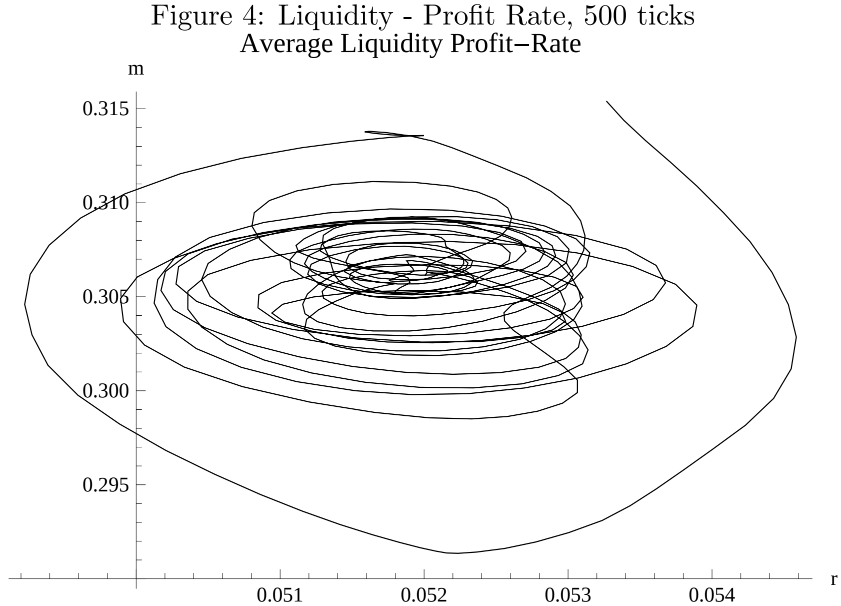
Figure 3: GDP Growth Rate, 400 ticks



the cycles and turbulence appearing in the simulated GDP growth rate path are results of the nonlinear deterministic interactions of agents rather than random exogenous shocks. This in turn raises the important methodological question of how macroeconomic outcomes should be assessed and studied.

Shifting focus to the average liquidity (m) and profit rate (r), this model is able to generate chaotic average liquidity-profit rate cycles, which can be seen in Figure 4 below. Similar to Foley (1987), the basic cycles in this

Figure 4: Liquidity - Profit Rate, 500 ticks



model are generated via accelerator-multiplier effect in the tradition of Hicks (1950), Kalecki (1969), and Goodwin (1982) with the major distinction being that the this model generates accumulation cycles endogenously rather than exogenously. Although, differing from Foley (1987), cycles in this model exhibit chaotic patterns that are qualitatively similar to the Goodwin cycles that have been empirically estimated by several authors[11]. In fact, what is shown in Figure (4) is a projection of a multi-dimensional orbit into two dimensions. Hence, the qualitative similarity between the output in Figure (4) and previous empirical studies interestingly suggests that the observed patterns of macroeconomic fluctuations can very well be generated by a high dimensional nonlinear deterministic system rather than a stochastic process.

This model can also be solved analytically for its steady-states and it can be shown that the steady-states are centers of gravitation for the whole dynamic system. Mathematically, this set of steady-states forms a trapping region in the dynamic system which ensures the chaotic trajectories of the system always remain in the neighborhood of the steady-state (global stability). In economic terms, they are essentially a set of long-period positions, as can be found in Classical and Marxian Political Economy. Since the relationship between the long-period method and ABM has been one of the emphases of Section 3, it will not be further discussed here.

4.4 Discussions

A key feature of this model is that, as a deterministic model, it is able to produce chaotic trajectories that qualitatively resemble what one often observes in actual macroeconomic data. In fact, such a capability comes from the construction of the capital outlays-sales matrix (A) that models the Keynesian problem of effective demand on individual level. It has been shown elsewhere[12] that the same model, but without the Keynesian problem of effective demand, will produce cycles and fluctuations that are regular rather than chaotic. Hence, this model provides a heterodox interpretation of the chaotic characteristic of capitalist accumulation. As soon as firms fail to coordinate their sales and capital outlays in a way that the effective demand is rightly met for each firm, the size of the firms (measured by their stock variables) becomes very different, hence this model becomes one with multiple heterogenous agents, and the trajectories of the system become chaotic due to the fact that the system goes through numerous bifurcations as the agents interact.

The immediate political economy implication is that cycles and chaos in capital accumulation are neither due to "mistakes" that firms make nor

[11] See, Barbosa and Taylor (2006), Tarassow (2010), and Rezai (2012).
[12] See Jiang (2013).

mysterious shocks from outside the system. Instead, they are embedded in the logic and structure of the circuit of capital—a decentralized system of production and distribution conceptualized from the perspective of profit-seeking enterprises. It is also worth emphasizing that, although the model is highly abstract at the present stage, what is *not* being abstracted from in building this model is the coexistence of individualistic profit-seeking actions (the heterogenous micro behavior of the firms) and the interdependence of the firms within a network of supply and demand (the A matrix). Such coexistence is in fact an important feature of capitalism and it is precisely due to the dialectical tensions generated from such coexistence that the system under investigation is dynamic and complex in nature, which in turn demands methodologies such as nonlinear dynamics and ABM.

5 Heterodox Visions, ABM Methodology, and the Road Ahead

This section discusses the possibilities of ABM being applied to different dimensions of the broad variety of heterodox research programs. Heterodox economics is an umbrella term that covers various approaches, schools, or traditions (e.g. Classical-Marxian (Smith, Ricardo, Marx), Post-/Keynesian, Feminist, Institutionalist, Austrian, etc.). Despite its much celebrated diversity, heterodox economics tends to share one theoretical commonality, that is, economic outcomes are, to a large extent, determined by the relation(s) between socioeconomic structure and the agents who reside in it. This section will argue that this theoretical commonality makes ABM a particularly useful research method for heterodox economics.

Taking capital-labor relations in Marxian political economy as an example. The socioeconomic structure of capitalism produces two distinct classes of agents—wage-laborers and capitalists. Agents from each class have distinct (and often conflicting) intensions and patterns of behavior, and their interactions generate various economic outcomes, such as increasing inequality, the falling rate of profit, global expansion of capital, etc. A parallel to the capital-labor relations is the gender relations emphasized by Feminist economics. Socially constructed gender differences endow economic agents with gender-biased roles in economic life, and agents endowed with gender-biased roles interacting in a market economy results in a series of economic phenomena of central concern to Feminist economists. Among these concerns are gender segregation, feminization of global production, the widening and persistence of gender-wage gaps, etc. "Institutions" are either products of some socioeconomic structures (e.g. labor unions, international organizations, healthcare systems etc.) or subsets of some socioeconomic structures (e.g. the market system, marriage, property rights, the legal system).[13] Institutions shape the behaviors of economic agents, and economic agents' behavior in turn influences the evolution of institutions. Moreover, the importance of institutions has also been stressed by Austrian economists in their attempts to understand market order. To them, the study of market order is fundamentally about exchange behavior and the institutions within which exchanges take place. Finally, the relation between socioeconomic structure and individual agents plays a central role in the Post-/Keynesian line of thought. The capitalist structure of production and distribution determines the behaviors of economic agents in the system, and a set of macroeconomic phenomena (e.g. inflation, unemployment, cycles and fluctuations, etc.) emerge out of the interactions amongst those agents as unintended consequences.

The previous paragraph is intended to show that heterodox schools of thought tend to share a theoretical commonality, that is many economic processes happen to be conceptualized as the interplay amongst different economic agents and the socioeconomic structure in which they reside. Although this theoretical commonality cannot and should not be generalized to all heterodox economics, it is not a stretch to see it as an important characteristic brands of heterodox economics happen to share. ABM holds some unique advantages to modeling this characteristic as it appears in the different bodies of heterodox thought. Chief among these advantages are ABM's flexibility in construction and the heterogeneity of agents (as mentioned in Section 2.2). Socioeconomic structures can be modeled as either a general environment that agents have to respond to, or characteristics agents take on. For example, in the model introduced in Section 3, the structure of production and distribution that characterizes the commodity law of exchange was constructed by building the Beaver-Deer World which endows a set of behavioral rules on the agents. In the model in Section 4, the productive and distributive outcomes of the system depend on the structure of the circuit of capital and the network of supply and effective demand.

Furthermore, given the flexibility of ABM, gender and social class differentiated behavioral traits can be easily built into agents' behavior. With well-specified behavioral rules that are consistent with the relevant socioeconomic structures, interactions amongst those agents will be capable of producing fruitful results that help the understanding of human (gender and/or capital-labor) relations. Institutions can be modeled as individual agents, such as the State, the central bank, the labor union, etc., that di-

[13] Judging whether an institution is a product or a part of a particular socioeconomic structure is an extremely controversial task, and it is beyond of the scope of this chapter. The key is to show that institutions are inseparable from the notion of socioeconomic structure.

rectly interact with economic agents, or as behavioral rules economic agents might follow (e.g. to get married, to emulate others, to sell labor power, to file lawsuits, etc.). More importantly, with the "generativist" nature of ABM, the modeled institutions are capable of self-evolving via the feedback mechanism(s) between the macro-structure and the micro-specification as mentioned in Section 2.1. Finally, the heterogeneity of agents and high flexibility in agent construction make ABM an extremely useful methodology for some important Post-/Keynesian research programs involving the explorations of emergent macroeconomic properties, unintended consequences, and the fallacy of composition.

Finally, it is important to point out here that this chapter argues that ABM is a useful methodology for heterodox economics given its unique advantages. This chapter should not be taken as arguing the superiority of ABM over other research methods for heterodox economics. ABM cannot replace formal mathematical modeling, empirical research, and nuanced qualitative analysis. While ABM is particularly well-suited for exploring ways to model human interactions and the relationship(s) between micro-behaviors and macro-outcomes, at the same time, given its shortcomings,[14] it might very well be impotent in many other areas of research taken up by heterodox economists. After all, it is the research question itself that demands the right methodology, not the other way around. Thus, the toolset provided by ABM should be viewed as a valuable complement to the already vibrant research programs in the heterodox traditions.

[14] Some of the limitations of ABM are discussed by Arthur (2006), Epstein (2006a), LeBaron and Tesfatsion (2008), and Tesfatsion (2006).

References

Albin, P. S. and D. K. Foley (1992). "Decentralized, dispersed exchange without an auctioneer". *Journal of Economic Behavior and Organization 18*(1), pp. 27–51.

Anderson, P. W., K. J. Arrow, and D. Pines (Eds.) (1988). *The Economy as an Evolving Complex System*, Redwood City, CA. Addison-Wesley. A proceedings volume in the Santa Fe Institute Studies in the Sciences of Complexity.

Arthur, W. B. (2006). "Out-of-Equilibrium Economics and Agent-Based Modeling". In K. L. Judd and L. Tesfatsion (Eds.), *Handbook of Computational Economics, Volume 2*. Amsterdam: Elsevier B.V.

Arthur, W. B., S. N. Durlauf, and D. A. Lane (Eds.) (1997). *The Economy as an Evolving Complex System II*, Reading, MA. Perseus Books. A proceedings volume in the Santa Fe Institute Studies in the Sciences of Complexity.

Axtell, R. (2005). "The Complexity of Exchange". *The Economic Journal 115*(504), pp. F193–F210.

Axtell, R. (2010). "Many Agent Firms". Working Paper, Center for Social Complexity, Krasnow Institute for Advanced Study, George Mason University. Version: July 2010.

Barbosa, N. H. and L. Taylor (2006). "Distributive and Demand Cycles in the US Economy—A Structuralist Goodwin Model". *Metroeconomica 57*(3), pp. 389–411.

Cogliano, J. F. (2013). *New Directions in Political Economy: Value Theory, Agent-Based Computational Modeling, and the Dynamics of Labor Mobility*. Ph. D. thesis, New School for Social Research.

Colander, D. (Ed.) (2000). *Complexity and the History of Economic Thought*, London. Routledge.

Delli Gatti, D., C. D. Giulmi, E. Gaffeo, M. G. Gianfranco Giulioni, and A. Palestrini (2004). "Business Cycle Fluctuations and Firm' Size Distribution Dynamics". *Advances in Complex Systems 7*(2), pp. 223–240.

Di Guilmi, C. and C. Chiarella (2011). "The financial instability hypothesis: A stochastic microfoundation framework". *Journal of Economic Dynamics and Control 35*(8), pp. 1151–1171.

Di Guilmi, C. and C. Chiarella (2013). “A Reconsideration of the Formal Minskyan Analysis: Microfoundations, Endogenous Money and the Public Sector”. In G. I. Bischi, C. Chiarella, and I. Sushiko (Eds.), *Global Analysis of Dynamic Models in Economics and Finance*. Heidelberg: Springer.

Di Guilmi, C., M. Gallegati, S. Landini, and J. E. Stiglitz (2012). “Towards an Analytical Solution for Agent Based Models: an Application to a Credit Network Economy”. In M. Aoki, K. Binmore, S. Deakin, and H. Gintis (Eds.), *Complexity and Institutions: Markets, Norms and Corporations*. London: Palgrave Macmillan.

Dosi, G., G. Fagiolo, and A. Roventini (2006). “An Evolutionary Model of Endogenous Business Cycles”. *Computational Economics 27*(1), pp. 3–34.

Dosi, G., G. Fagiolo, and A. Roventini (2010). “Schumpeter meeting keynes: A policy-friendly model of endogenous growth and business cycles”. *Journal of Economic Dynamics & Control 34*(9), pp. 1748–1767.

Epstein, J. M. (2006a). “Agent-Based Computational Models and Generative Social Science”. In J. M. Epstein (Ed.), *Generative Social Science: Studies in Agent-Based Computational Modeling*. Princeton, NJ: Princeton University Press.

Epstein, J. M. (Ed.) (2006b). *Generative Social Science: Studies in Agent-Based Computational Modeling*, Princeton, NJ. Princeton University Press.

Fisher, F. M. (1983). *Disequilibrium Foundations of Equilibrium Economics*. Cambridge, UK: Cambridge University Press.

Foley, D. and G. Duménil (2008a). “Marxian transformation problem”. In S. N. Durlauf and L. E. Blume (Eds.), *The New Palgrave Dictionary of Economics*. Basingstoke: Palgrave Macmillan. *The New Palgrave Dictionary of Economics Online*. Palgrave Macmillan. 28 January 2010 http://www.dictionaryofeconomics.com/article?id=pde2008_M000400.

Foley, D. and G. Duménil (2008b). “Marx’s analysis of capitalist production”. In S. N. Durlauf and L. E. Blume (Eds.), *The New Palgrave Dictionary of Economics*. Basingstoke: Palgrave Macmillan. *The New Palgrave Dictionary of Economics Online*. Palgrave Macmillan. 28 January 2010 http://www.dictionaryofeconomics.com/article?id=pde2008_M000399.

Foley, D. K. (1982). “Realization and Accumulation in a Marxian Model of the Circuit of Capital”. *Journal of Economic Theory 28*(2), pp. 300–319.

Foley, D. K. (1986). *Money, Accumulation and Crisis*. Chur, Switzerland: Harwood.

Foley, D. K. (1987). "Liquidity-profit rate cycles in a capitalist economy". *Journal of Economic Behavior & Organization 8*(3), pp. 363–376.

Foley, D. K. (2003). *Unholy Trinity: Labor, capital, and land in the new economy*. New York, NY: Routledge.

Foley, D. K. (2010). "What's wrong with the fundamental existence and welfare theorems?". *Journal of Economic Behavior & Organization 75*(2), pp. 115–131.

Foley, D. K. (2011). "The Long-Period Method and Marx's Theory of Value". In V. Caspari (Ed.), *The Evolution of Economic Theory: Essays in Honour of Bertram Schefold*. Abingdon, Oxon: Routledge.

Garegnani, P. (1970). "Heterogeneous Capital, the Production Function and the Theory of Distribution". *Review of Economic Studies 37*(3), pp. 407–436.

Garegnani, P. (1976). "On a change in the notion of equilibrium in recent work on value and distribution: a comment on Samuelson". In M. Brown, K. Sato, and P. Zarembka (Eds.), *Essays in Modern Capital Theory*. Amsterdam: North-Holland.

Garegnani, P. (1984). "Value and Distribution in the Classical Economists and Marx". *Oxford Economic Papers 36*(2), pp. 291–325.

Gintis, H. (2007). "The Dynamics of General Equilibrium". *The Economic Journal 117*(523), pp. 1280–1309.

Gintis, H. (2012). "The Dynamics of Pure Market Exchange". In M. Aoki, K. Binmore, S. Deakin, and H. Gintis (Eds.), *Complexity and Institutions: Markets, Norms, and Corporations*. Basingstoke, UK: Palgrave Macmillan.

Godley, W. and M. Lavoie (2006). *Monetary Economics: An Integrated Approach to Credit, Money, Income, Production and Wealth*. Hampshire: Palgrave Macmillan.

Goodwin, R. M. (1982). *Essays in Economic Dynamics*. London: Macmillan.

Hicks, J. R. (1950). *A Contribution to the Theory of the Trade Cycle*. Oxford: Clarendon Press.

Holt, R. P., J. B. Rosser, and D. Colander (2011). "The Complexity Era in Economics". *Review of Political Economy 23*(3), pp. 357–369.

Jiang, X. (2013). "From Limit-Cycle to Chaos: An Inquiry into the Macrodynamics of a Profit-Seeking Economy Through the Lens of Circuit of Capital". Unpibished manuscript.

Julius, A. J. (2013). "Class in catallaxy". In L. Taylor, A. Rezai, and T. Michl (Eds.), *Social Justice and Economics: Economics Essays in the Spirit of Duncan Foley*. Abingdon, Oxon: Routledge.

Kalecki, M. (1969). *Studies in the Theory of Business Cycles: 1933-1939*. Homewood: Irwin.

Kinsella, S., M. Greiff, and E. J. Nell (2011). "Income Distribution in a Stock-Flow Consistent Model with Education and Technological Change". *Eastern Economic Journal 37*(1), pp. 134–149.

Kirman, A. P. (1992). "Whom or What Does the Representative Individual Represent?". *Journal of Economic Perspectives 6*(2), pp. 117–136.

Kirman, A. P. (1997). "The Economy as an Interactive System". In W. B. Arthur, S. N. Durlauf, and D. A. Lane (Eds.), *The Economy as an Evolving Complex System II*. Reading, MA: Perseus Books. A proceedings volume in the Santa Fe Institute Studies in the Sciences of Complexity.

Kirman, A. P. (2004). "Economics and Complexity". *Advances in Complex Systems 7*(2), pp. 139–155.

LeBaron, B. (2006). "Agent-Based Financial Markets: Matching Stylized Facts with Style". In D. Colander (Ed.), *Post Walrasian Macroeconomics*. Cambridge: Cambridge University Press.

LeBaron, B. and L. Tesfatsion (2008). "Modeling Macroeconomies as Open-Ended Dynamic Systems of Interacting Agents". *The American Economic Review 98*(2), pp. 246–250. Papers and Proceedings of the One Hundred Twentieth Annual Meeting of the American Economic Association.

Marx, K. (1973). *Grundrisse*. New York, NY: Penguin Group.

Marx, K. (1978). *Capital: Volume II*. New York, NY: Penguin Group.

Marx, K. (1981). *Capital: Volume III*. New York, NY: Penguin Group.

Marx, K. (1988). *Collected Works*, Volume 30. New York, NY: International Publishers. Theories of Surplus Value in Collected Works edition.

Railsback, S. F. and V. Grimm (2012). *Agent-Based and Individual-Based Modeling: A Practical Introduction*. Princeton, NJ: Princeton University Press.

Rezai, A. (2012). "Goodwin Cycles, Distributional Conflict and Productivity Growth". *Metroeconomica 63*(1), pp. 29–39.

Rosser, J. B. (1999). "On the Complexities of Complex Economic Dynamics". *Journal of Economic Perspectives 13*(4), pp. 169–192.

Rosser, J. B. (2008). "Econophysics and Economic Complexity". *Advances in Complex Systems 11*(5), pp. 745–760.

Rosser, J. B. (Ed.) (2009). *Handbook of Research on Complexity*, Cheltenham. Edward Elgar.

Salamon, T. (2011). *Design of Agent-Based Models.* Czech Republic: Tomas Bruckner.

Smale, S. (1976). "Exchange Processes with Price Adjustment". *Journal of Mathematical Economics 3*(3), pp. 211–226.

Smith, A. (2000). *The Wealth of Nations.* New York, NY: Random House, Inc. Modern Library Edition. Originally published in 1776 as: An Inquiry into the Nature and Causes of the Wealth of Nations.

Tarassow, A. (2010). "The empirical relevance of Goodwin's business cycle model for the US economy". MPRA Paper No. 22271.

Tesfatsion, L. (1997). "How Economists Can Get ALife". In W. B. Arthur, S. N. Durlauf, and D. A. Lane (Eds.), *The Economy as an Evolving Complex System II.* Reading, MA: Perseus Books. A proceedings volume in the Santa Fe Institute Studies in the Sciences of Complexity.

Tesfatsion, L. (2006). "Agent-Based Computational Economics: A Constructive Approach to Economic Theory". In L. Tesfatsion and K. L. Judd (Eds.), *Handbook of Computational Economics, Volume 2.* Amsterdam: Elsevier.

Tesfatsion, L. and K. L. Judd (Eds.) (2006). *Handbook of Computational Economics, Volume 2*, Amsterdam. Elsevier.

Ussher, L. J. (2008). "A Speculative Futures Market with Zero-Intelligence". *Eastern Economic Journal 34*(4), pp. 518–549.

Wilensky, U. (1999). Netlogo. http://ccl.northwestern.edu/netlogo/. Center for Connected Learning and Computer-Based Modeling, Northwestern University. Evanston, IL.

Wright, I. (2008). "The Emergence of the Law of Value in a Dynamic Simple Commodity Economy". *Review of Political Economy 20*(3), pp. 367–391.

Wright, I. (2011a). “Classical macrodynamics and the labor theory of value”. Open Discussion Papers in Economics, no. 76. Milton Keynes: The Open University.

Wright, I. (2011b). “Convergence to natural prices in simple production”. Open Discussion Papers in Economics, no. 75. Milton Keynes: The Open University.