International Liquidity and Exchange Rate Dynamics

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Abstract

We provide a theory of the determination of exchange rates based on capital flows in imperfect financial markets. Capital flows drive exchange rates by altering the balance sheets of financiers that bear the risks resulting from international imbalances in the demand for financial assets. Such alterations to their balance sheets cause financiers to change their required compensation for holding currency risk, thus impacting both the level and volatility of exchange rates. Our theory of exchange rate determination in imperfect financial markets not only rationalizes the empirical disconnect between exchange rates and traditional macroeconomic fundamentals, but also has real consequences for output and risk sharing. Exchange rates are sensitive to imbalances in financial markets and seldom perform the shock absorption role that is central to traditional theoretical macroeconomic analysis. We derive conditions under which heterodox government financial policies, such as currency interventions and taxation of capital flows, can be welfare improving. Our framework is flexible; it accommodates a number of important modeling features within an imperfect financial market model, such as non-tradables, production, money, sticky prices or wages, various forms of international pricing-to-market, and unemployment.

JEL Codes: E42, E44, F31, F32, F41, F42, G11, G15, G20.

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We provide a theory of exchange rate determination based on capital flows in imperfect financial markets. In our model, exchange rates are volatile and are largely disconnected from traditional macroeconomic fundamentals over the medium term; they are instead governed by financial forces. Global shifts in the demand and supply of assets result in large scale capital flows that are intermediated by the global financial system. The demand and supply of assets in different currencies and the willingness of the financial system to absorb the resulting imbalances are first order determinants of exchange rates. A framework to characterize such forces and their implications for welfare and policy, while desirable, has proven elusive.

In our model, financiers absorb part of the currency risk originated by imbalanced global capital flows. Alterations to the size and composition of financiers' balance sheets induce them to differentially price currency risk, thus affecting both the level and the volatility of exchange rates. Our theory of exchange rate determination in imperfect financial markets differs from the traditional open macroeconomic model by introducing financial forces, such as portfolio flows, financiers' balance sheets, and financiers' risk bearing capacity, as first order determinants of exchange rates.

We first present a basic theory of exchange rate financial determination in a two-period two-country model where capital flows are intermediated by global financiers. Each country borrows or lends in its own currency and financiers absorb all currency risk that is generated by the mismatch of global capital flows. Since financiers require compensation for holding currency risk in the form of expected currency appreciation, exchange rates are jointly determined by capital flows and by the financiers' risk bearing capacity. Our theory, therefore, is an elementary one where supply and demand determine a price, the exchange rate, that clears markets.

The exchange rate is disconnected from traditional macroeconomic fundamentals such as imports, exports, output or inflation, because the same fundamentals correspond to a different equilibrium exchange rate depending on the financiers' balance sheets and risk bearing capacity. An extension to a multi-period model strengthens this intuition by solving the exchange rate as a present value relationship. The exchange rate discounts future current account balances, but the rate of discounting is determined in financial markets and therefore hinges on financiers' risk bearing capacity and balance sheet. Changes in such capacity affect both the level and volatility of the exchange rate. Financiers act both as shock absorbers, by using their risk bearing capacity to accommodate flows that result from fundamental shocks, and are themselves the source of financial shocks that distort exchange rates.

Financial determination of exchange rates in imperfect financial markets has real consequences for output and risk sharing. To more fully analyze these consequences, we extend the basic model by introducing nominal exchange rates, monetary policy, and both flexible and sticky prices. In the presence of goods' prices that are sticky in the producers' currencies, a capital inflow or financial shock that produces an overly appreciated exchange rate causes a fall in demand for the inflow-receiving country's exports and a corresponding fall in output.

Our theory yields novel predictions for policy analysis in the presence of flexible exchange rates. In a financial world, exchange rate movements are dominated by financial factors and they seldom perform the benign expenditure switching role that is central to traditional macroeconomic analysis. In fact, the traditional macroeconomic rationale for prescribing pure floating exchange rates is that, in the presence of asymmetric real shocks, the exchange rate acts as a shock absorber by shifting global demand toward the country that has been most negatively affected by the shock via a depreciation of its currency.

By contrast, we show that in our framework the floating exchange rate is itself distorted by imbalances in financial markets and shocks to the financial system's risk bearing capacity; the exchange rate can often be the vehicle of transmission of financial shocks to the real economy. Our policy analysis suggests that novel trade-offs emerge when financial markets are disrupted or are less developed overall, and when output is far below potential so that an exchange rate depreciation increases output via an increase in net exports. Heterodox policies, such as large scale currency interventions and capital controls, are shown to be beneficial in these specific circumstances.

We focus on providing a framework that is not only sufficiently rich to analyze the financial forces at the core of our theory in a full general equilibrium model, but also sufficiently tractable as to provide simple pencil-and-paper solutions that make the analysis as transparent as possible. While tractability requires some assumptions, we also verify that the core forces of our framework remain the leading forces of the determination of exchange rate even in more general setups, where a number of assumptions are relaxed and solutions have to be computed numerically.

Our framework makes sense of a number of fundamental issues in open macroeconomics; these include the exchange rate disconnect from macroeconomics fundamentals, external financial adjustment, the failure of purchasing power parity, the failure of the Backus and Smith risk sharing condition, and the carry trade.

Our framework is flexible in accommodating a number of modeling features that are important in open economy analysis within an imperfect capital market model, such as non-tradables, production, money, sticky prices or wages, and various forms of international pricing-to-market. The framework, therefore, can be employed easily in future research to address a number of open questions in international macroeconomics.

Related Literature Two important papers were published in 1976, the now classic exchange rate overshooting model (Dornbusch (1976)) and the portfolio balance theory model (Kouri

(1976)).[1] While we incorporate important aspects of the Keynesian tradition upon which Dornbusch builds, our model provides modern foundations to the spirit of Kouri's portfolio balance theory of exchange rates. Obstfeld and Rogoff (1995) brought the Keynesian approach into modern international economics by providing micro-foundations to the dynamic version of the Mundell-Fleming-Dornbusch model. Their foundations have been essential not only for the analysis of the determination of exchange rates, but also for that of optimal policy and welfare. However, financial forces play little role in this class of models. In the real version of these models, exchange rates are mostly determined by the demand and supply of domestic and foreign goods. Even in the nominal versions of the models, where the nominal exchange rate is often expressed as the present discounted sum of future monetary policy and other macroeconomic fundamentals, the impact of finance is limited because in most cases the uncovered interest parity holds, the demand for money is tightly linked to consumption expenditures, and/or the model is linearized.

A vast parallel literature has explored the determinants of exchange rates in endowment or real business cycle models following Lucas (1982) and Backus, Kehoe and Kydland (1992). In the context of asset pricing models, Dumas (1992), Lewis (1998), Verdelhan (2010), Colacito and Croce (2011), and Hassan (2013) have explored the relationship between consumption and the real exchange rate following Backus and Smith (1993).[2] Pavlova and Rigobon (2007) analyze a real model with complete markets where countries' representative agents have logarithmic preferences affected by taste shocks.[3] This branch of the literature has mostly maintained the assumption of complete markets.[4] We contribute to this literature not only by providing alternative drivers of exchange rates, financiers' risk bearing capacity and financial imbalances, which break the link between consumption and the exchange rate, but also by focusing on incomplete and imperfect capital markets. These imperfections are crucial to our welfare and policy analysis in open economies.

Recent economic events, such as the global financial and European crises, have rekindled an interest in the analysis of optimal policy and welfare in open economies. Aguiar, Amador and Gopinath (2009), Farhi, Gopinath and Itskhoki (2012), Farhi and Werning (2012*a*,*b*), and Schmitt-Grohé and Uribe (2012) provide innovative analyses of policies such as capital controls, fiscal transfers and fiscal devaluations in the context of the small-open-economy (new-Keynesian) model. We contribute to this literature by analyzing policies in a two-country world where financial flows are direct determinants of exchange rates and where the condition of financial markets is an important policy consideration. We find that public financial policies, which cannot be

[1] Driskill and McCafferty (1980), Kouri (1983), Blanchard, Giavazzi and Sa (2005) build on Kouri (1976).

[2] Amongst others see also: Farhi and Gabaix (2011), Martin (2011), and Stathopoulos (2012).

[3] Similar preferences are also used in Pavlova and Rigobon (2008, 2010).

[4] Notable exceptions are Kehoe and Perri (2002) and Alvarez, Atkeson and Kehoe (2009).

analyzed under the UIP-assumption, complete markets, or simple forms of market incompleteness, can be beneficial in specific circumstances. We characterize the policy instruments (capital controls, currency-swaps, and FX interventions) that can be used to implement these polices.

Relatively little research has been devoted to analyzing the relationship between portfolio flows and exchange rates. Notable exceptions are Evans and Lyons (2002), Hau and Rey (2006), and Bacchetta and Van Wincoop (2010) who consider the price impact of financial flows under different modeling assumptions: microstructure market making, equity portfolio flows, and overlapping generations of investors, respectively. These papers solve for exchange rates in a partial equilibrium framework where the goods market, and therefore exports and imports, are omitted from the modeling. Maggiori (2011), and Bruno and Shin (2013) introduce financial intermediation as an important driver of global risk taking and capital flows.[5]

We summarize our contribution as providing a tractable framework for the determination of exchange rates in financial markets via capital flows and the risk-bearing capacity of financiers. Our general equilibrium framework combines financial forces such as risk taking and financial intermediation in imperfect capital markets with the traditional real economy analysis of production, import and export activities. The resulting equilibrium exchange rate is disconnected from traditional macroeconomic fundamentals and instead connected to financial forces, such as the financiers' balance sheets' exposures and risk-bearing capacity. Based on this framework, we analyze optimal policy and welfare in the presence of both financial and nominal frictions and characterize when and which financial policies can be used to re-equilibrate exchange rates.

1 Basic Gamma Model

Let us start with a minimalistic model of financial determination of exchange rates in imperfect financial markets. This simple real model carries most of the economic intuition and core modeling that we will then extend to more general set-ups.

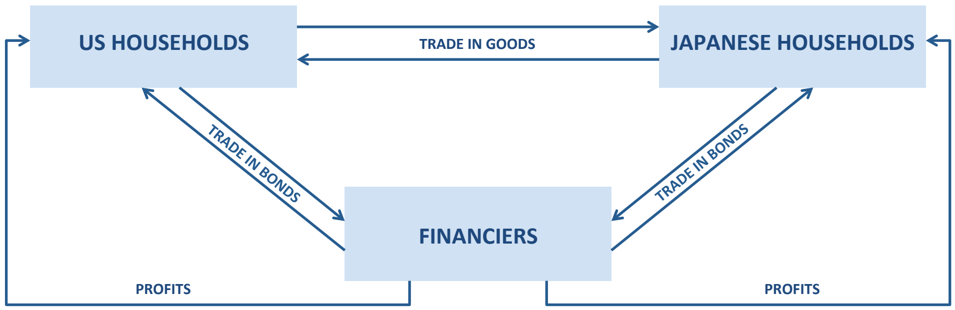
Time is discrete and there are two periods: $t = 0, 1$. There are two countries, the USA and Japan, each populated by a continuum of households. Households produce, trade (internationally) in a market for goods, and invest with financiers in risk-free bonds in their domestic currency.[6] Financiers intermediate the capital flows resulting from the households' investment decisions.

[5] In a closed economy context, an ongoing research effort has introduced financiers and intermediaries as key drivers of both business cycles and asset prices (Kiyotaki and Moore (1997), Garleanu and Pedersen (2011), Brunnermeier and Sannikov (2013), He and Krishnamurthy (2013)).

[6] In the absence of a nominal side to the model, in this section we intentionally abuse the word "currency" to mean a claim to the numéraire of the economy, and "exchange rate" to mean the real exchange rate. Similarly we abuse the words "Dollar or Yen denominated" to mean values expressed in units of non-tradable goods in each economy. As will shortly become clear, even this simple real model is set up so as to generalize immediately to a nominal model.

The basic structure of the model is displayed in Figure 1.

Figure 1: Basic Structure of the Model



The players and structure of the flows in the goods and financial markets in the Basic Gamma Model.

Intermediation is not perfect because of limited commitment on the part of the financiers. The limited-commitment friction induces a downward sloping demand curve for risk taking by financiers. As a result, capital flows from households move financiers up and down their demand curve. Equilibrium is achieved by a relative price, in this case the exchange rate, adjusting so that international financial markets clear given demand and supply of capital denominated in different currencies. In this sense, exchange rates are financially determined in an imperfect capital market.

We now describe each of the model's actors, their optimization problems, and analyze the resulting equilibrium.

1.1 Households

Households in the US derive utility from the consumption of goods according to:

$$\mathbb{E}\left[\theta_0 \ln C_0 + \beta \theta_1 \ln C_1\right], \tag{1}$$

where C is a consumption basket defined as:

$$C_t \equiv \left[(C_{NT,t})^{\chi_t} (C_{H,t})^{a_t} (C_{F,t})^{\iota_t}\right]^{\frac{1}{\theta_t}}, \tag{2}$$

where $C_{NT,t}$ is the US consumption of its non-tradable goods, $C_{H,t}$ is the US consumption of its domestic tradable goods, and $C_{F,t}$ is US consumption of Japanese tradable goods. We use the notation $\{\chi_t, a_t, \iota_t\}$ for non-negative, potentially stochastic, preference parameters and we define $\theta_t \equiv \chi_t + a_t + \iota_t$.

The non-tradable good is the numéraire in each economy and, consequently, its price equals

1 in domestic currency ($p_{NT} = 1$). Non-tradable goods are produced by an endowment process that we assume for simplicity to follow $Y_{NT,t} = \chi_t$, unless otherwise stated.[7]

Households can trade in a frictionless goods market across countries. Financial markets are incomplete and each country trades a risk-free domestic currency bond.[8] *Risk-free* here refers to paying one unit of non-tradable goods in all states of the world and is therefore akin to "nominally risk free".

The households' optimization problem can be divided into two separate problems. The first is a static problem, whereby households decide, given their total consumption expenditure for the period, how to allocate resources to the consumption of the various goods. The second is a dynamic problem, whereby households decide intertemporally how much to save and consume.

The static utility maximization problem takes the form:

$$\max_{C_{NT,t},C_{H,t},C_{F,t}} \quad \chi_t \ln C_{NT,t} + a_t \ln C_{H,t} + \iota_t \ln C_{F,t} + \lambda_t \left(CE_t - C_{NT,t} - p_{H,t}C_{H,t} - p_{F,t}C_{F,t}\right), \tag{3}$$

where CE_t is aggregate consumption expenditure, which is taken as exogenous in this static optimization problem and later endogenized in the dynamic optimization problem, λ_t is the associated Lagrange multiplier, $p_{H,t}$ is the Dollar price in the US of US tradables, and $p_{F,t}$ is the Dollar price in the US of Japanese tradables. Standard optimality conditions imply:

$$C_{NT,t} = \frac{\chi_t}{\lambda_t}; \qquad p_{H,t}C_{H,t} = \frac{a_t}{\lambda_t}; \qquad p_{F,t}C_{F,t} = \frac{\iota_t}{\lambda_t}.$$

Our assumption that $Y_{NT,t} = \chi_t$, combined with the market clearing condition for non-tradables $Y_{NT,t} = C_{NT,t}$, implies that in equilibrium $\lambda_t = 1$. We immediately obtain that:

$$p_{H,t}C_{H,t} = a_t, \qquad p_{F,t}C_{F,t} = \iota_t. \tag{4}$$

These demand functions are very intuitive: the demand for each good depends positively on how much the households like the good and negatively on its price. Notice that the Dollar value of US imports is simply ι_t.

Japanese household derive utility from consumption according to: $\mathbb{E}\left[\theta_0^* \ln C_0^* + \beta^* \theta_1^* \ln C_1^*\right]$, where starred variables denote Japanese quantities and prices. By analogy with the US case, the Japanese consumption basket is: $C_t^* \equiv \left[(C_{NT,t}^*)^{\chi_t^*}(C_{H,t}^*)^{\xi_t}(C_{F,t}^*)^{a_t^*}\right]^{\frac{1}{\theta_t^*}}$, where $\theta_t^* \equiv \chi_t^* + a_t^* + \xi_t$.

[7]The assumption, while stark, makes the analysis of the basic model most tractable. We stress that the assumption is one of convenience, and not necessary for the economics of the paper. The reader might find it useful to think of χ and Y_{NT} as constants and the equality between the two as a normalization that makes the close form solutions of the paper most readable. Section 2 as well as the appendix provide more general results that do not impose this assumption.

[8]The market structure is enriched in the following sections.

The Japanese static utility maximization problem, reported for brevity in the appendix, together with the assumption $Y^*_{NT,t} = \chi^*_t$, lead to demand functions for goods that are entirely analogous to those of US households derived above:

$$p^*_{H,t}C^*_{H,t} = \xi_t, \qquad p^*_{F,t}C^*_{F,t} = a^*_t. \tag{5}$$

Hence, the Yen value of US exports to Japan is: $p^*_{H,t}C^*_{H,t} = \xi_t$.

The exchange rate e_t is defined as the quantity of dollars bought by 1 yen, i.e. the strength of the Yen. Consequently, an increase in e represents a Dollar depreciation.[9] The Dollar value of US exports is: $e_t\xi_t$. US net exports, expressed in dollars, are given by: $NX_t = e_t p^*_{H,t}C^*_{H,t} - p_{F,t}C_{F,t} = \xi_t e_t - \iota_t$.[10] We collect these results in the Lemma below.

Lemma 1 (Net Exports) *Expressed in dollars, US exports to Japan are: $\xi_t e_t$; US imports from Japan are: ι_t; so that US net exports are: $NX_t = \xi_t e_t - \iota_t$.*

Note that this result is independent of the pricing procedure (e.g. price stickiness under either producer or local currency pricing). Under producer currency pricing (PCP) and in the absence of trade costs, the US Dollar price of Japanese tradables is p_H/e, while under local currency pricing (LCP) the price is simply p^*_H.

It follows that under financial autarky, i.e. if trade has to be balanced period by period, the equilibrium exchange rate is: $e_t = \frac{\iota_t}{\xi_t}$. In financial autarky, the Dollar depreciates ($\uparrow e$) whenever US demand for Japanese goods increases ($\uparrow \iota$) or whenever Japanese demand for US goods falls ($\downarrow \xi$). This has to occur because there is no mechanism, in this case, to absorb the excess demand/supply of dollars versus yen that a non-zero trade balance would generate.

The dynamic optimization problem of US households is to maximize the utility in equation (1) by choosing their consumption/savings subject to the state-by-state dynamic budget constraint:

$$\sum_{t=0}^{1} R^{-t}\left(Y_{NT,t} + p_{H,t}Y_{H,t}\right) = \sum_{t=0}^{1} R^{-t}\left(C_{NT,t} + p_{H,t}C_{H,t} + p_{F,t}C_{F,t}\right). \tag{6}$$

The optimization problem leads to a standard optimality condition (Euler equation):

$$1 = \mathbb{E}\left[\beta R \frac{U'_{1,C_{NT}}}{U'_{0,C_{NT}}}\right] = \mathbb{E}\left[\beta R \frac{\chi_1/C_{NT,1}}{\chi_0/C_{NT,0}}\right] = \beta R, \tag{7}$$

[9] In this real model the exchange rate is related to the relative price of non-tradable goods. Section 5.2 provides a full discussion of this exchange rate and its relationship to both the nominal exchange rate, formally introduced in Section 2.1 , and the CPI-based real exchange rate.

[10] Notice that we chose the notation so that *i*mports are denoted by ι_t and e*x*ports by ξ_t.

where $U'_{t,C_{NT}}$ is the marginal utility at time t over the consumption of non-tradables. Given our simplifying assumption that $C_{NT,t} = \chi$ the above Euler equation implies that $R = 1/\beta$. An entirely similar derivation yields: $R^* = 1/\beta^*$.[11]

We stress that the aim of our simplifying assumptions is to create a real structure of the basic economy that captures the main forces (demand and supply of goods), while making the real side of the economy as simple as possible. This will allow us in the next sections to analytically flesh out the crucial forces of the paper in the financial markets without carrying around a burdensome real structure. Should the reader be curious as to the robustness of our model to relaxing some of the assumptions made so far, the quick answer is that it is quite robust. We will make such robustness explicit in Section 2, and in the appendix.

1.2 Financiers

Suppose that global financial markets are imbalanced, such that there is an excess supply of dollars versus yen resulting from, for example, trade or portfolio flows. Who will be willing to absorb such an imbalance by providing Japan those yen, and hold those dollars? We posit that the resulting imbalances are absorbed, at some premium, by global financiers.

We assume that there is a unit mass of global financial firms, each managed by a financier. Agents from the two countries are selected at random to run the financial firms for a single period.[12] Financiers start their jobs with no capital of their own and can trade bonds denominated in both currencies. Therefore, their balance sheet consists of q_0 dollars and $-\frac{q_0}{e_0}$ yen, where q_0 is the Dollar value of Dollar-denominated bonds the financier is long of and $-\frac{q_0}{e_0}$ the corresponding value in yen for Yen-denominated bonds. At the end of (each) the period, financiers pay their profits and losses out to the households.

Our financiers are intended to capture a broad array of financial institutions that intermediate global financial markets. These institutions range from the proprietary desks of global investment banks such as Goldman Sachs and JP Morgan, to macro and currency hedge funds such as Soros Fund Management, to active investment managers such as PIMCO and BlackRock. While there are certainly significant differences across these intermediaries, we stress their common characteristic of being active investors that profit from medium-term imbalances in international financial markets, often by bearing the risks (*taking the other side*) resulting from imbalances in

[11] It might appear surprising that in a model with risk averse agents the equilibrium interest rate equals the rate of time preference. Of course, this occurs here because the marginal utility of non-tradable consumption, in which the bonds are denominated, is constant.

[12] In this set-up being a financier is an occupation for agents in the two countries rather than an entirely separate class of agents. The selection process is governed by a memoryless Poisson distribution. Of course, there are no selection issues in the 1 period basic economy considered here, but we proceed to describe a more general set-up that will also be used in the model extensions.

currency demand due both to trade flows and to financial flows. They also share the characteristic of being subject to financial constraints that limit their ability to take positions, based on their risk bearing capacities and existing balance sheet risks.

It is beyond the scope of this paper to provide the contract-theory foundations underlying which assets and contracts the intermediaries trade in equilibrium. Instead, we take as given the prevalence of short-term debt in different currencies and the presence of frictions and proceed to analyze their equilibrium implications. A similar direct approach to modeling financial imperfections has a long standing tradition with recent contributions by Kiyotaki and Moore (1997), Gromb and Vayanos (2002), Mendoza, Quadrini and Rıos-Rull (2009), Mendoza (2010), Gertler and Kiyotaki (2010), Garleanu and Pedersen (2011).

We assume that each financier maximizes the expected value of her firm:[13]

$$V_0 = \mathbb{E}\left[\beta\left(R - R^* \frac{e_1}{e_0}\right)\right] q_0 = \Omega_0 q_0. \tag{8}$$

In each period, after taking positions but before shocks are realized, the financier can divert part of the funds she intermediates. If the financier diverts the funds, her firm is unwound and the households that had lent to the financier recover a portion $1 - \Gamma\left|\frac{q_0}{e_0}\right|$ of their credit position $\left|\frac{q_0}{e_0}\right|$.[14] Since creditors, when lending to the financier, correctly anticipate the incentives of the financier to divert funds, the financier is subject to a credit constraint of the form:

$$\underbrace{\frac{V_0}{e_0}}_{\substack{\text{Intermediary Value} \\ \text{in yen}}} \geq \underbrace{\left|\frac{q_0}{e_0}\right|}_{\substack{\text{Total} \\ \text{Claims}}} \underbrace{\Gamma\left|\frac{q_0}{e_0}\right|}_{\substack{\text{Diverted} \\ \text{Portion}}} = \underbrace{\Gamma\left(\frac{q_0}{e_0}\right)^2}_{\substack{\text{Total divertable} \\ \text{Funds}}}.$$

Our functional assumption regarding the diversion of funds is not only, as will become clear below, one of convenience, but also stresses the idea that intermediaries are able to divert more and more funds as their balance sheets increase.[15] Limited commitment constraints in a similar spirit have been popular in the literature; for earlier use as well as foundations see amongst others:

[13] We derive this value function explicitly in the appendix. Here we only stress that this function does not require financiers to be risk neutral; in fact, it actually corresponds to the way in which US households would value currency trading, i.e their Euler equation.

[14] Notice that the financier can divert funds from its creditors, but not from its debtors. Given that the balance sheet consists of q_0 dollars and $-\frac{q_0}{e_0}$ yen, the Yen value of the financier's liabilities is always equal to $\left|\frac{q_0}{e_0}\right|$, irrespective of whether q_0 is positive or negative; hence the use of absolute value in the text above. More formally, the financier's creditors can recover a Yen value equal to: $\max\left(1 - \Gamma\left|\frac{q_0}{e_0}\right|, 0\right)\left|\frac{q_0}{e_0}\right|$. See the appendix for further details.

[15] It is outside the scope of this paper to provide deeper foundations for this constraint. However, such foundations could potentially be achieved in models of financial complexity where bigger balance sheets lead to more complex positions. In turn, these more complex positions are more difficult and costly for creditors to unwind when recovering their funds in case of a financier's default.

Caballero and Krishnamurthy (2001), Kiyotaki and Moore (1997), Hart and Moore (1994), and Hart (1995). Here we follow most closely the formulation in Gertler and Kiyotaki (2010) and Maggiori (2011).

The constrained optimization problem of the financier is:

$$\max_{q_0} V_0 = \mathbb{E}\left[\beta\left(R - R^*\frac{e_1}{e_0}\right)\right] q_0, \qquad \text{subject to} \quad V_0 \geq \Gamma\frac{q_0^2}{e_0}. \tag{9}$$

Since the value of the financier's firm is linear in the position q_0, while the right hand side of the constraint is convex in q_0, the constraint always binds.[16] Intuitively, given any non-zero expected excess return in the currency market, the financier will want to either borrow or lend as much as possible in Dollar and Yen bonds. The constraint limits the maximum position and therefore binds. Substituting the intermediary's value into the constraint and re-arranging (using $R = 1/\beta$), we characterize the financier's optimal choice to intermediate global financial flows in different currencies as: $q_0 = \frac{1}{\Gamma}\mathbb{E}\left[e_0 - e_1\frac{R^*}{R}\right]$. Integrating the above demand function over the unit mass of financiers yields the aggregate financiers' demand for assets: $Q_0 = \frac{1}{\Gamma}\mathbb{E}\left[e_0 - e_1\frac{R^*}{R}\right]$. We collect this result in the Lemma below.[17]

Lemma 2 (Financiers' downward sloping demand for dollars) *The financiers' constrained optimization problem implies that the aggregate financial sector optimal demand for Dollar bonds versus Yen bonds follows:*

$$Q_0 = \frac{1}{\Gamma}\mathbb{E}\left[e_0 - e_1\frac{R^*}{R}\right]. \tag{10}$$

The demand for dollars decreases in the strength of the dollar (i.e. increases in e_0), controlling for the future value of the Dollar (i.e. controlling for e_1). Notice that the higher Γ, the lower the financiers' risk bearing capacity, the steeper their demand curve, and the more segmented the asset market. To understand the behavior of this demand, let us consider two polar opposite cases. When $\Gamma = 0$, financiers are able to absorb any imbalances, i.e. they want to take infinite positions, whenever there is a non-zero expected excess return in currency markets. So uncovered interest rate parity (UIP) holds: $\mathbb{E}\left[e_0 - e_1\frac{R^*}{R}\right] = 0$. When $\Gamma \uparrow \infty$, then $Q_0 = 0$; financiers are unwilling to

[16]We make the very mild assumption that the model parameters always imply: $\Omega_0 \geq -1$. That is, we assume that the expected excess returns from currency speculation never exceed -100%. This bound is several order of magnitudes greater than the expected returns in the data (of the order of 0-6%) and has no economic bearing on our model. See appendix for further details.

[17]This demand function could generate deviations not only from perfect risk-taking but also from arbitrage conditions such as covered interest rate parity (CIP). To prevent the existence of these simple arbitrages, we could also assume the existence of arbitrageurs who can freely enter in all risk-less trades. They eliminate trivial arbitrages. As a result, CIP holds. Informally, those arbitrageurs are not constrained because their trades are so simple that their monitoring is very easy.

absorb any imbalances, i.e. they want to take no positions, no matter what the expected returns from risk-taking are.

Since Γ, the financiers' risk bearing capacity, plays a crucial role in our theory, we refer hereafter to the setup described so far as the *basic Gamma model*.

We stress that the above demand function not only captures the spirit of international financial intermediation via the microfoundation of the constrained portfolio problem, but also behaves in aggregate similarly to the demand of a CARA agent with risk aversion Γ.[18] Similar demand functions have been central to the limits of arbitrage theory of De Long et al. (1990*a*,*b*), Shleifer and Vishny (1997), and Gromb and Vayanos (2002).

For simplicity, we assume (for now and for much of this paper) that financiers rebate their profits and losses to the Japanese households, not the US ones. This asymmetry gives much tractability to the model, at fairly little cost to the economics.[19]

Before moving to the equilibrium, let us stress that the structure of the economy described in this Section and the previous one and depicted in Figure 1, while clearly stylized, is meant to capture the fundamental structure of international currency and bond markets. These markets are not only over-the-counter and highly intermediated, but also concentrated in the hands of a few large financial players like Goldman Sachs, Soros Fund Management or PIMCO. Consequently, these players are likely to act as the marginal agent in pricing currencies.

We also emphasize that we are modeling the ability of these players to bear substantial risks over a horizon that goes from a quarter to a few years. Our model is silent on the high frequency market-making activities of currency desks in investment banks. To make this distinction intuitive, let us consider that the typical daily volume of foreign exchange transactions is estimated to be \$5.3 trillion.[20] This trading is highly concentrated among the market making desks of banks and is the subject of attention in the market microstructure literature pioneered by Evans and Lyons (2002). While these microstructure effects are interesting, we completely abstract away from these activities by assuming that there is instantaneous and perfect risk sharing across financiers, so that any trade that matches is executed frictionlessly and nets out. We are only concerned with the ultimate risk, most certainly a small fraction of the total trading volume, that financiers have to bear over quarters and years because households' demand is unbalanced.[21]

[18] Formally, it is equivalent to the first order approximation of CARA demand around a constant volatility of assets returns.

[19] For completeness, note that this assumption had already been implicitly made in deriving the US households' inter-temporal budget constraint in equation (6). This assumption is relaxed in the appendix where we solve for general, as well as symmetric, payoff functions numerically.

[20] Source: Bank of International Settlements (2013).

[21] This is consistent with evidence that market-making desks in large investment banks, for example Goldman Sachs, might intermediate very large volumes daily but are almost always carrying no residual risk, at the end of the business day. In contrast, the proprietary trading desks (before recent changes in legislation) or the investment management

1.3 Supply and Demand of Assets: Equilibrium Exchange Rate

Recall that we are for now, and for simplicity, only considering imbalances resulting from trade flows (imbalances from portfolio flows will come soon). The key equations of the model are the financiers' demand:

$$Q_0 = \frac{1}{\Gamma}\mathbb{E}\left[e_0 - e_1\frac{R^*}{R}\right], \tag{11}$$

and the equilibrium "flow" demand for dollars in the Dollar-Yen market at times $t = 0, 1$:

$$\xi_0 e_0 - \iota_0 + Q_0 = 0, \tag{12}$$

$$\xi_1 e_1 - \iota_1 - RQ_0 = 0. \tag{13}$$

Equation (12) is the market clearing equation for Dollar against Yen market at time zero. It states that the net demand for Dollar against Yen has to be zero for the market to clear. The net demand has two components: $\xi_0 e_0 - \iota_0$, coming from US net exports, and Q_0, coming from financiers. Recall that we assume that US households do not hold any currency exposure: they convert their Japanese sales of ξ_0 yen into dollars, for a demand $\xi_0 e_0$ of dollars. Likewise, Japanese households have ι_0 dollars worth of exports to the US and sell them, as they only keep Yen balances.[22] At time one, equation (13) shows that the same net-export channel generates a demand for dollars of $\xi_1 e_1 - \iota_1$; while the financiers need to sell their dollar position RQ_0, that has accrued interest at rate R.[23] We now explore the equilibrium exchange rate in this simple setup.

Equilibrium exchange rate: a first pass To streamline the algebra and concentrate on the key economic content, we assume for now that $\beta = \beta^* = 1$, which implies $R = R^* = 1$, and that $\xi_t = 1$ for $t = 0, 1$. Adding equations (12) and (13) yields the US external inter-temporal budget constraint:

$$e_1 + e_0 = \iota_0 + \iota_1. \tag{14}$$

Taking expectations on both sides: $\mathbb{E}[e_1] = \iota_0 + \mathbb{E}[\iota_1] - e_0$. From the financiers' demand equation we have:

$$\mathbb{E}[e_1] = e_0 - \Gamma Q_0 = e_0 - \Gamma(\iota_0 - e_0) = (1+\Gamma)e_0 - \Gamma\iota_0,$$

divisions of the same investment banks carry substantial amount of risks over horizons ranging from a quarter to a few years. The latter investment activities are the focus of this paper. Similarly, our financiers capture the risk-taking activities of hedge funds and investment managers that have no market making interests and are therefore not the center of attention in the microstructure literature.

[22]These assumptions are later relaxed in Sections 2.2 and in the appendix where households are allowed to have (limited) foreign currency positions.

[23]At the end of period 0, the financiers own Q_0 dollars and $-Q_0/e_0$ yen. Therefore, at the beginning of period one, they own RQ_0 dollars and $-R^*Q_0/e_0$ yen. At time one, they unwind their position and give the net profits to their principals, which we assumed for simplicity to be the Japanese households. Hence they sell RQ_0 dollars in the Dollar-Yen market at time one.

where the second equality follows from equation (12). Equating the two expressions for the time-one expected exchange rate, we have:

$$\mathbb{E}\left[e_1\right] = \iota_0 + \mathbb{E}[\iota_1] - e_0 = (1+\Gamma)\, e_0 - \Gamma \iota_0.$$

Solving this linear equation for the exchange rate at time zero, we conclude:

$$e_0 = \frac{(1+\Gamma)\, \iota_0 + \mathbb{E}[\iota_1]}{2+\Gamma}.$$

We define $\{X\} \equiv X - \mathbb{E}[X]$ to be the innovation to a random variable X. Then, the exchange rate at time $t = 1$ is:

$$\begin{aligned} e_1 &= \iota_0 + \iota_1 - e_0 = \iota_0 + \mathbb{E}[\iota_1] + \{\iota_1\} - e_0 \\ &= \{\iota_1\} + \iota_0 + \mathbb{E}[\iota_1] - \frac{(1+\Gamma)\, \iota_0 + \iota_1}{2+\Gamma} = \{\iota_1\} + \frac{\iota_0 + (1+\Gamma)\, \mathbb{E}[\iota_1]}{2+\Gamma}. \end{aligned}$$

We collect these results in the Proposition below.

Proposition 1 (Basic Gamma equilibrium exchange rate) *Assume that $\xi_t = 1$ for $t = 0,1$, and that interest rates are zero in both countries. The exchange rate follows:*

$$\begin{aligned} e_0 &= \frac{(1+\Gamma)\, \iota_0 + \mathbb{E}[\iota_1]}{2+\Gamma}, \\ e_1 &= \{\iota_1\} + \frac{\iota_0 + (1+\Gamma)\, \mathbb{E}[\iota_1]}{2+\Gamma}, \end{aligned} \tag{15}$$

where $\{\iota_1\}$ is the time-1 import shock. The expected Dollar appreciation is: $\mathbb{E}\left[\frac{e_0 - e_1}{e_0}\right] = \frac{\Gamma(\mathbb{E}[\iota_1] - \iota_0)}{(1+\Gamma)\iota_0 + \mathbb{E}[\iota_1]}$.

Depending on Γ, the time-zero exchange rate varies between two polar opposites: the UIP-based and the financial-autarky exchange rates, respectively. Both extremes are important benchmarks of open economy analysis, and the choice of Γ allows us to modulate our model between these two useful benchmarks. $\Gamma \uparrow \infty$ results in $e_0 = \frac{\iota_0}{\xi_0}$, which we have shown in Section 1.1 to be the financial autarky value of the exchange rate. Intuitively, financiers have so little risk-bearing capacity that no financial flows can occur between countries and, therefore, trade has to be balanced period by period. When $\Gamma = 0$, UIP holds and we obtain $e_0 = \frac{\iota_o + \mathbb{E}[\iota_1]}{2}$. Intuitively, financiers are so relaxed about risk taking that they are willing to take infinite positions in currencies whenever there is a positive expected excess return from doing so. UIP only imposes a constant exchange rate in expectation $\mathbb{E}[e_1] = e_0$; the level of the exchange rate is then obtained by additionally using the inter-temporal budget constraint in equation (14).

To further understand the effect of Γ, notice that at the end of period 0 (say, time 0^+), the US net foreign asset (NFA) position is $N_{0^+} = \xi_0 e_0 - \iota_0 = \frac{\mathbb{E}[\iota_1]-\iota_0}{2+\Gamma}$. Therefore, the US has positive NFA at $t = 0^+$ *iff* $\iota_0 < \mathbb{E}[\iota_1]$. If the US has a positive NFA position, then financiers are long the Yen and short the Dollar. For financiers to bear this risk, they require a compensation: the Yen needs to appreciate in expectation. The required appreciation is generated by making the Yen weaker at time zero. The magnitude of the effect depends on the extent of the financiers' risk bearing capacity (Γ), as formally shown here by taking partial derivatives: $\frac{\partial e_0}{\partial \Gamma} = \frac{\iota_0 - \mathbb{E}[\iota_1]}{(2+\Gamma)^2} = \frac{-N_{0+}}{2+\Gamma}$. We collect the result in the Proposition below.

Proposition 2 (Effect of financial disruptions on the exchange rate) *In the basic Gamma model, we have:* $\frac{\partial e_0}{\partial \Gamma} = \frac{-N_{0+}}{2+\Gamma}$, *where* $N_{0^+} = \frac{\mathbb{E}[\iota_1]-\iota_0}{2+\Gamma}$. *When there is a financial disruption ($\uparrow \Gamma$), countries that are net external debtors* $(N_{0^+} < 0)$ *experience a currency depreciation ($\uparrow e$), while the opposite is true for net-creditor countries.*

Intuitively, net external-debtor countries have borrowed from the world financial system, thus generating a long exposure for financiers to their currencies. Should the financial system risk bearing capacity be disrupted, these currencies would depreciate to compensate financiers for the increased (perceived) risk. This modeling formalizes a number of external crises where broadly defined global risk aversion shocks, embodied here in Γ, caused large depreciations of the currencies of countries that had recently experienced large capital inflows. Della Corte, Riddiough and Sarno (2013) offer empirical evidence consistent with our theoretical predictions. They show that net-debtor countries' currencies have higher returns that net-creditors' currencies and that net debtor countries' currencies tend to be on the receiving end of carry trade related speculative flows.

To illustrate how the results derived so far readily extend to more general cases, we report below expressions allowing for stochastic US export shocks ξ_t as well as non-zero interest rates. Several more extensions can be found in Section 2.

Proposition 3 *With general trade shocks and interest rates (ι_t, ξ_t, R, R^*), the values of exchange rate at times $t = 0, 1$ are:*

$$e_0 = \frac{\mathbb{E}\left[\frac{\iota_0 + \frac{\iota_1}{R}}{\xi_1}\right] + \frac{\Gamma \iota_0}{R^*}}{\mathbb{E}\left[\frac{\xi_0 + \frac{\xi_1}{R^*}}{\xi_1}\right] + \frac{\Gamma \xi_0}{R^*}}; \qquad e_1 = \mathbb{E}[e_1] + \{e_1\},$$

where we again denote by $\{X\} \equiv X - \mathbb{E}[X]$ *the innovation to a random variable* X*, and*

$$\mathbb{E}[e_1] = \frac{R}{R^*} \frac{\mathbb{E}\left[\frac{R^*}{\xi_1}\left(\iota_0 + \frac{\iota_1}{R}\right)\right] + \Gamma\xi_0\mathbb{E}\left[\frac{R^*}{\xi_1}\frac{\iota_1}{R}\right]}{\mathbb{E}\left[\frac{R^*}{\xi_1}\left(\xi_0 + \frac{\xi_1}{R^*}\right)\right] + \Gamma\xi_0},$$

$$\{e_1\} = \left\{\frac{\iota_1}{\xi_1}\right\} + R\frac{\iota_0 - \mathbb{E}\left[\xi_0 \frac{R^*}{\xi_1}\frac{\iota_1}{R}\right]}{\mathbb{E}\left[\frac{R^*}{\xi_1}\left(\xi_0 + \frac{\xi_1}{R^*}\right)\right] + \Gamma\xi_0}\left\{\frac{1}{\xi_1}\right\}.$$

1.3.1 Supply and Demand Matter in the Gamma Model: The Impact of Portfolio Flows

We now further illustrate how *supply and demand of assets do matter for the financial determination of the exchange rate*. We stress the importance of portfolio flows in addition, and perhaps more importantly than, trade flows for our framework. We introduce here the simplest form of portfolio flows from the households. The rest of the paper, as well as the appendix, extends this minimalistic section to more general flows.

Consider the case where Japanese households have, at time zero, an inelastic demand (e.g. some noise trading) f^* of Dollar bonds funded by an offsetting position $-f^*/e_0$ in Yen bonds. The flow equations are now given by:

$$\xi_0 e_0 - \iota_0 + Q_0 + f^* = 0, \qquad \xi_1 e_1 - \iota_1 - RQ_0 - Rf^* = 0. \tag{16}$$

The financiers' demand is still $Q_0 = \frac{1}{\Gamma}\mathbb{E}\left[e_0 - \frac{R^*}{R}e_1\right]$. The equilibrium exchange rate is derived in the Proposition below.

Proposition 4 (Capital Flows and Exchange Rates) *Assume* $\xi_t = R = R^* = 1$ *for* $t = 0, 1$. *With an inelastic time-zero additional demand* f^* *for Dollar bonds by Japanese households, the exchange rates at time zero and one are:*

$$e_0 = \frac{(1+\Gamma)\,\iota_0 + \mathbb{E}[\iota_1] - \Gamma f^*}{2+\Gamma}; \qquad e_1 = \{\iota_1\} + \frac{\iota_0 + (1+\Gamma)\,\mathbb{E}[\iota_1] + \Gamma f^*}{2+\Gamma}.$$

Hence, an additional demand f^* *for dollars at time zero induces a Dollar appreciation at time zero, and subsequent depreciation at time one. However, the time-average value of the Dollar is unchanged:* $e_0 + e_1 = \iota_0 + \iota_1$*, independently of* f^*.

Proof. Define: $\tilde{\iota}_0 \equiv \iota_0 - f^*$, and $\tilde{\iota}_1 \equiv \iota_1 + f^*$. Given equations (16), our "tilde" economy is isomorphic to the basic economy considered in equations (12) and (13). For instance, import demands are now $\tilde{\iota}_t$ rather than ι_t. Hence, Proposition 1 applies to this "tilde" economy, thus implying that:

$$e_0 = \frac{(1+\Gamma)\,\tilde{\iota}_0 + \mathbb{E}[\tilde{\iota}_1]}{2+\Gamma} = \frac{(1+\Gamma)\,\iota_0 + \mathbb{E}[\iota_1] - \Gamma f^*}{2+\Gamma},$$
$$e_1 = \{\tilde{\iota}_1\} + \frac{\tilde{\iota}_0 + (1+\Gamma)\,\mathbb{E}[\tilde{\iota}_1]}{2+\Gamma} = \{\iota_1\} + \frac{\iota_0 + (1+\Gamma)\,\mathbb{E}[\iota_1] + \Gamma f^*}{2+\Gamma}. \quad \square$$

An increase in Japanese demand for Dollar bonds needs to be absorbed by financiers, who correspondingly need to sell Dollar bonds and buy Yen bonds. To induce financiers to provide the desired bonds, the Dollar needs to appreciate on impact *as a result of the capital flow*, in order to then be expected to depreciate, thus generating an expected gain for the financiers' short Dollar position. This example emphasizes that our model is an elementary one where a relative price, the exchange rate, has to move in order to equate the supply and demand of two assets, Yen and Dollar bonds.

This framework can analyze concrete situations, such as the recent large scale capital flows from developed countries into emerging market local-currency bond markets, say by US investors into Brazilian Real bonds, that put upward pressure on the receiving countries' currencies. While such flows and their impact on currencies have been paramount in the logic of market participants and policy makers, they had thus far proven elusive in a formal theoretical analysis.

Hau, Massa and Peress (2010) provide direct evidence that plausibly exogenous capital flows impact the exchange rate in a manner consistent with the Gamma model. They show that following a restating of the weights of the MSCI World Equity Index, countries that as a result experienced capital inflows (because their weight in the index increased) had their currency appreciate.

To stress the difference between our basic Gamma model of the financial determination of exchange rates in imperfect financial markets and the traditional macroeconomic framework, we next illustrate two polar cases that have been popular in the previous literature: the UIP-based exchange rate, and the complete market exchange rate.

Financial Flows in a UIP Model. Much of the now classic international macroeconomic analysis spurred by Dornbusch (1976) and Obstfeld and Rogoff (1995) either directly assumes that UIP holds or effectively imposes it by solving a first order linearization of the model.[24] The closest analog to this literature in the basic Gamma model is the case where $\Gamma = 0$, such that UIP holds by assumption. In this world, financiers are so relaxed, i.e. their risk bearing capacity is so ample, about supplying liquidity to satisfy shifts in the world demand for assets that such shifts have no impact on expected returns. Consider the example of US investors suddenly wanting to

[24] Intuitively, a first order linearization imposes certainty equivalence on the model and therefore kills any risk premia such as those that could generate a deviation from UIP.

buy Brazilian real bonds; in this case financiers would simply take the other side of the investors' portfolio demand with no effect on the exchange rate between the Dollar and the Real. In fact, equation (17) confirms that if $\Gamma = 0$, then portfolio flow f^* has no impact on the equilibrium exchange rate.

Financial Flows in a Complete Market Model. Another strand of the literature emanating from Lucas (1982) and Backus, Kehoe and Kydland (1992) has analyzed risk premia predominantly under complete markets. We now show that the exchange rate in a setup with complete markets (and no frictions) but otherwise identical to ours is *constant*, and therefore trivially not affected by the flows.

Lemma 3 (Complete Markets) *In an economy identical to the set-up of the basic Gamma model, other than the fact that financial markets are complete and frictionless, the equilibrium exchange rate is constant: $e_t = \nu$, where ν is the relative Negishi weight of Japan.*

Here, we only sketch the logic and the main equations; a full treatment is relegated to the appendix. Under complete markets, the marginal utility of US and Japanese agents must be equal when expressed in a common currency. Intuitively, the full risk sharing that occurs under complete markets calls for Japan and the US to have the same marginal benefit from consuming an extra unit of non-tradables. In our set-up, this risk sharing condition takes the simple form: $\frac{\chi_t/C_{NT,t}}{\chi_t^*/C_{NT,t}^*} e_t = \nu$, where ν is a constant.[25] Simple substitution of the conditions $C_{NT,t} = \chi_t$ and $C_{NT,t}^* = \chi_t^*$ shows that $e_t = \nu$, i.e. the exchange rate is constant.

1.3.2 Flows, not just Stocks, Matter in the Gamma model

In frictionless models only stocks matter, not flows per se. In the Gamma model, instead, flows per se matter. To illustrate this, consider the case where the US has an exogenous Dollar-denominated debt toward Japan, equal to D_0 due at time zero, and D_1 due at time one. For simplicity, assume $\beta = \beta^* = R = R^* = \xi_t = 1$ for $t = 0,1$. Hence, total debt is $D_0 + D_1$. The flow equations now are:

$$e_0 - \iota_0 - D_0 + Q_0 = 0; \qquad e_1 - \iota_1 - D_1 + Q_1 = 0.$$

The exchange rate at time zero is:[26]

$$e_0 = \frac{(1+\Gamma)\,\iota_0 + \mathbb{E}[\iota_1]}{2+\Gamma} + \frac{(1+\Gamma)\,D_0 + D_1}{2+\Gamma}.$$

[25] Formally, the constant is the relative Pareto weight assigned to Japan in the planner's problem that solves for complete-market allocations.

[26] The derivation follows from Proposition 5 by defining the pseudo-imports $\widetilde{\iota}_t \equiv \iota_t + D_t$.

Hence, when finance is imperfect ($\Gamma > 0$) the timing of debt flows matters, as indicated by the the term $(1+\Gamma)D_0 + D_1$, in addition to the total stock of debt ($D_0 + D_1$) in determining exchange rates. The early flow, D_0, receives a higher weight $\left(\frac{1+\Gamma}{2+\Gamma}\right)$ than the late flow, D_1, does $\left(\frac{1}{2+\Gamma}\right)$. In sum, flows, not just stocks, matter for exchange rate determination.

1.3.3 The Exchange Rate Disconnect

The Meese and Rogoff (1983) result on the inability of economic fundamentals such as output, inflation, exports and imports to predict, or even contemporaneously co-move with, exchange rates has had a chilling and long-lasting effect on theoretical research in the field (see Obstfeld and Rogoff (2001)).[27] The disconnect between exchange rates and macro fundamentals is a central feature of the Gamma model. To stress this feature, we here show that two economies with identical macroeconomic fundamentals feature unconnected exchange rates that depend on imbalances in financial markets: i.e. the financiers' risk bearing capacity Γ and their balance sheet exposure Q_0.

Consider a world, which we call *Tranquil Times*, in which financiers' risk-bearing capacity is $\Gamma_T > 0$ and the starting intermediary balance sheet is $Q_{0^-}^T = -f < 0$. Consider an alternate world, which we call *Distressed Times*, in which financial imbalances are higher ($Q_{0^-}^D = -(f+\Delta f) < 0$, where $\Delta f > 0$) and financiers' risk bearing capacity is lower ($\Gamma^D > \Gamma^T$). We stress that the two economies have otherwise identical fundamentals.

Simple algebra, entirely similar to the derivations in the previous sections, reveals that:

$$e_0^T - e_0^D \propto (\Gamma_D - \Gamma_T)\left(\mathbb{E}[\iota_1] - \iota_0 + 2f\right) + \Gamma_D (2+\Gamma_T)\Delta f, \tag{17}$$

The first term on the right hand side has two components: $\mathbb{E}[\iota_1] - \iota_0 + 2f$ summarizes the common fundamentals of the two economies, while $(\Gamma_D - \Gamma_T)$ highlights that the same fundamentals imply different exchange rates based on different financiers' risk bearing capacities.[28] The second term on the right hand side of equation (17) shows that different starting balance sheets for financiers also lead to different equilibrium exchange rates.[29]

[27] Some forecastability of exchange rates using traditional fundamentals appears to occur at very-long horizons (e.g. 10 years) in Mark (1995) or for specific currencies, such as the US Dollar, using transformations of the balance of payments data (Gourinchas and Rey (2007*b*), Gourinchas, Govillot and Rey (2010)).

[28] Recall that whenever $\mathbb{E}[\iota_1] - \iota_0 > 0$, trade flows induce financiers to lend in yen and borrow in dollars. Similarly, a positive f represents a short Dollar and long Yen starting imbalance for financiers. Therefore, the term $\mathbb{E}[\iota_1] - \iota_0 + 2f$ summarizes, based on the common fundamentals of the two economies, the amount of dollars that financiers are short of. Consider the case when $\mathbb{E}[\iota_1] - \iota_0 + 2f > 0$ and financiers are, therefore, short dollars. In this case, the lower the risk bearing capacity (the higher $(\Gamma_D - \Gamma_T) > 0$), the more the Dollar has to appreciate at time zero to induce financiers to hold their positions via expected capital gains ($e_0^T - e_0^D \uparrow$).

[29] Since $\Delta f > 0$, financiers are shorter Dollar and longer Yen in the Distressed than in the Tranquil economy. This additional financial imbalance requires an appreciated Dollar, at time zero, in the Distressed economy compared to

While the Meese and Rogoff (1983) negative result on the empirical relevance of traditional macroeconomic models of exchange rate determination has held up remarkably well, recently new evidence has been building for a strong relationship between capital flows and exchange rates. In addition to the instrumental variable approach in Hau, Massa and Peress (2010) discussed earlier, Hong and Yogo (2012) and Froot and Ramadorai (2005) find that flows and financiers' positions provide information about expected currency returns. Hong and Yogo (2012) show that the speculators' positions in the futures currency market contain information that is useful, beyond the interest rate differential, to forecast future currency returns. Froot and Ramadorai (2005) show that medium-term variation in expected currency returns is mostly associated with capital flows, while long-term variation is more strongly associated with macroeconomic fundamentals.

1.3.4 Endowment Economy

The reader will have noticed that very little has been said so far about output. To build up the intuition for our framework, we consider here a full endowment economy, and consider production economies under both flexible and sticky prices in Section 3.

Let all output stochastic processes $\{Y_{NT,t}, Y_{H,t}, Y^*_{NT,t}, Y_{F,t}\}_{t=0}^{1}$ be exogenous strictly-positive endowments. Assuming that all prices are flexible and that the law of one price (LOP) holds, one has: $p_{H,t} = p^*_{H,t} e_t$, and $p_{F,t} = p^*_{F,t} e_t$.

Summing US and Japanese demand for US tradable goods ($C_{H,t} = \frac{a_t}{p_{H,t}}$ and $C^*_{H,t} = \frac{\xi_t e_t}{p_{H,t}}$, respectively), we obtain the world demand for US tradables: $D_{H,t} \equiv C_{H,t} + C^*_{H,t} = \frac{a_t + \xi_t e_t}{p_{H,t}}$. Clearing the goods market, $Y_{H,t} = D_{H,t}$, yields the equilibrium price in dollars of US tradables: $p_{H,t} = \frac{a_t + \xi_t e_t}{Y_{H,t}}$. An entirely similar argument yields: $p^*_{F,t} = \frac{a^*_t + \frac{\iota_t}{e_t}}{Y_{F,t}}$.

2 Nominal Exchange Rate, Interest Rates, and Capital Flows

We now extend the basic Gamma model from the previous section to account for the nominal side of the economy, direct (but limited) trading of foreign currency bonds by the households, and for a preexisting stock of external debt. Each of the extensions is not only of interest on its own, but also explores the flexibility of our framework by incorporating a number of features that are important in open-economy analysis within an imperfect-market general-equilibrium model.

We introduce each extension separately starting from the basic Gamma model and derive the

the Tranquil one in order for financiers to intermediate the flows. The strength of this effect depends on the level of the risk bearing capacity in the two economies (the term $\Gamma_D (2 + \Gamma_T)$): the lower the risk bearing capacity, the stronger the effect.

extended version of the flow equations in the bond market (extensions of equations (12-13)). In all cases, except in the nominal extension of the model, the financiers' demand equation (equation (10)) is unchanged from the basic Gamma model. Finally, we solve for the equilibrium exchange rate by showing that each extension, as well as all the extensions jointly, can be expressed as a "pseudo" version of the basic Gamma model where the import and export shocks (ι,ξ) are replaced by generalized endogenous quantities $(\tilde{\iota},\tilde{\xi})$. The construction of this "pseudo" economy allows all extensions to be solved for via the simple derivation of the equilibrium of the basic Gamma model in Proposition 3.

2.1 Nominal Exchange Rate

We have thus far considered a real model; we now investigate a nominal version of the Gamma model where the nominal exchange rate is determined, similarly to our baseline model, in an imperfect financial market.[30]

We assume that money is only used domestically and that its demand is captured, in reduced form, in the utility function of consumers in each country.[31] The US consumption basket is now modified to include a real-money-balances term: $C_t \equiv \left[\left(\frac{M_t}{P_t}\right)^{\omega_t}(C_{NT,t})^{\chi_t}(C_{H,t})^{a_t}(C_{F,t})^{\iota_t}\right]^{\frac{1}{\theta_t}}$, where M is the amount of money held by the households and P is the nominal price level so that $\frac{M}{P}$ is real money balances.[32] We maintain the normalization of preference shocks by setting $\theta_t \equiv \omega_t+\chi_t+a_t+\iota_t$. Correspondingly, the Japanese consumption basket is now: $C_t^* \equiv \left[\left(\frac{M_t^*}{P_t^*}\right)^{\omega_t^*}(C_{NT,t}^*)^{\chi_t^*}(C_{H,t}^*)^{\xi_t}(C_{F,t}^*)^{a_t^*}\right]^{\frac{1}{\theta_t^*}}$.

Money is the numéraire in each economy, with local currency price equal to 1. The static utility maximization problem is entirely similar to the one in the basic Gamma model in Section 1.1, and standard optimization arguments lead to demand functions:

$$M_t=\frac{\omega_t}{\lambda_t},\qquad p_{NT,t}C_{NT,t}=\frac{\chi_t}{\lambda_t}, \tag{18}$$
$$p_{H,t}C_{H,t}=\frac{a_t}{\lambda_t},\qquad p_{F,t}C_{F,t}=\frac{\iota_t}{\lambda_t},$$

[30]Notice that we have indeed set up the "real" model in the previous sections in such a way that non-tradables in each country play a role very similar to money and where, therefore, the exchange rate is rather similar to a nominal exchange rate (see Obstfeld and Rogoff (1996)[Ch. 8.3]). In this section we make such analogy more explicit. Section 5.2 provides a full discussion of the CPI-based real exchange rate in our model. Alvarez, Atkeson and Kehoe (2009) provide a model of nominal exchange rates with frictions in the domestic money markets, while our model has frictions in the international capacity to bear exchange-rate risk.

[31]A vast literature has focused on foundations of the demand for money; such foundations are beyond the scope of this paper and consequently we focus on the simplest approach that delivers a plausible demand for money and much tractability.

[32]See section 5.2 for details on the price index.

where, we recall from earlier sections, λ is the Lagrange multiplier on the households' static budget constraint. Money demand, in the top equation, is proportional to total nominal consumption expenditures; the coefficient of proportionality, ω_t, is potentially stochastic.

Let us define $m_t \equiv \frac{M_t^s}{\omega_t}$ and $m_t^* \equiv \frac{M_t^{s*}}{\omega_t^*}$, where M_t^s and M_t^{s*} are the money supplies. Notice that since money is non-tradable across countries or with the financiers (but bonds that pay in units of money are tradable with the financiers as in the previous sections), the money market clearing implies that the central bank can pin down the level of nominal consumption expenditure $(m_t = \lambda_t^{-1}, m_t^* = \lambda_t^{*-1})$.[33] It is convenient to consider the *cashless* limit of our economies by taking the limit case when $\{M_t^s, M_t^{s*}, \omega_t, \omega_t^*\} \downarrow 0$ such that $\{m_t, m_t^*\}$ are finite.[34] The nominal exchange rate e_t is again defined as the strength of the Yen, so that an increase in e_t is a Dollar depreciation.[35]

US nominal imports in dollars are: $p_{F,t}C_{F,t} = \frac{\iota_t}{\lambda_t} = \iota_t m_t$. Similarly, Japanese demand for US tradables is: $p_{H,t}^* C_{H,t}^* = \xi_t m_t^*$. Hence, US nominal exports in dollars are: $p_{H,t}^* C_{H,t}^* \ e_t = \xi_t e_t m_t^*$. We conclude that US nominal net exports in dollars are: $NX_t = p_{H,t}^* C_{H,t}^* e_t - p_{F,t}C_{F,t}$, or more explicitly: $NX_t = \xi_t e_t m_t^* - \iota_t m_t$.

The key equations to solve for the equilibrium nominal exchange rate are the flow equations in the international bond market:

$$\xi_0 e_0 m_0^* - \iota_0 m_0 + Q_0 = 0; \qquad \xi_1 e_1 m_1^* - \iota_1 m_1 - RQ_0 = 0,$$

and the extended financiers' demand curve:[36]

$$Q_t = \frac{m_t^*}{\Gamma} \mathbb{E}_t \left[e_t - e_{t+1} \frac{R_{t+1}^*}{R_{t+1}} \right]. \tag{19}$$

Finally, the nominal interest rates are given by the households' intertemporal optimality conditions (Euler Equations):

$$1 = \mathbb{E}\left[\beta R \frac{U'_{1,C_{NT}}}{U'_{0,C_{NT}}} \frac{p_{NT,0}}{p_{NT,1}}\right] = \mathbb{E}\left[\beta R \frac{\chi_1 / C_{NT,1}}{\chi_0 / C_{NT,0}} \frac{p_{NT,0}}{p_{NT,1}}\right] = \beta R \mathbb{E}\left[\frac{m_0}{m_1}\right],$$

[33] The central bank in each period choses money supply after the preference shocks are realized so that m and m^* are policy variables. We abstract here from issues connected with the zero lower bound (ZLB) on nominal interest rates.

[34] Notice the duality between money in the current setup and non-tradable goods in the basic Gamma model of Section 1. If $M_t = \omega_t$ and $C_{NT,t} = \chi_t$, one recovers the equations in Section 1, because the demand for money implies $\lambda_t = 1$, in which case the demand for non-tradables implies that $p_{NT,t} = 1$.

[35] We intentionally abuse the notation by denoting the nominal exchange rate by e_t the same symbol used for the exchange rate in the basic Gamma model. This allows the notation to be simpler and for the basic concepts of the paper to be more easily compared across a number of different extensions.

[36] Intuitively, scaling by m_t^* makes sure that the demand is invariant to the level of the money supply. See appendix for further details.

so that $R^{-1} = \beta \mathbb{E}\left[\frac{m_0}{m_1}\right]$. Similarly, $R^{*-1} = \beta^* \mathbb{E}\left[\frac{m_0^*}{m_1^*}\right]$. These interest rate determination formulas extend those in equation (7) to the nominal setup.

2.2 Capital Flows

Section 1 focused, for simplicity, on capital flows originated by trade in the goods market. Section 1.3.1 provided a first extension to pure portfolio flows by allowing for a time-zero inelastic, or noise, demand by Japanese households for Dollar bonds in the amount of f^*.

In this section we allow households to directly trade foreign bonds, albeit in limited amounts.[37] We consider here demand functions for foreign bonds that depend on all fundamentals, but that do not directly depend on the exchange rate. These demand functions still allow the model to be solved in closed form. Rather than providing precise foundations for the many possible forms that these demands could take, we focus on a general theory of how they impact the equilibrium exchange rate.[38]

We allow the demand functions for foreign bonds from US and Japanese households, denoted by f and f^* respectively, to depend on all present and expected future fundamentals. We use the shorthand notation f and f^* to denote the generic functions: $f(R, R^*, \iota, \xi, \ldots)$ and $f^*(R, R^*, \iota, \xi, \ldots)$. For example, demand functions that load on a popular trading strategy, the carry trade, that invests in high interest rate currencies while funding the trade in low interest rate currencies can be expressed as $f = b + c(R - R^*)$ and $f^* = d + g(R - R^*)$, for some constants b, c, d, g.[39] The flow equations in the bond market are now given by:

$$e_0 \xi_0 - \iota_0 + Q_0 + f^* - f e_0 = 0; \qquad e_1 \xi_1 - \iota_1 - R Q_0 - R f^* + R^* f e_1 = 0.$$

The above flow equations highlight that the demand for Dollar bonds at time zero is increasing in the Japanese households' demand for these bonds (f^*) and decreasing in the US households' demand for Yen bonds (f) that is funded by a corresponding short position in US bonds ($-f e_0$). The flows are reversed at time 1 after interest has accrued and at the new equilibrium exchange rate e_1.

[37] While it is important that the households are not allowed to *optimally trade unlimited amounts* in foreign currency in order to avoid sidestepping the financial intermediation that is at the core of this paper, limited direct trading or buy-and-hold positions can easily be accommodated in the model.

[38] The appendix extends the present results to demand functions that depend on the exchange rate directly by solving the model numerically.

[39] Possible microfoundations for these strategies range from the "boundedly rational" households who focus on the interest rate when investing without considering future exchange rate changes, to rational models of portfolio delegation where the interest rate is an observable variable that is known, in equilibrium, to load on the sources of risk of the model (see Section 5.1).

2.3 External Debt, Currency Denomination, and Financial Adjustment

We consider here the impact of external debt and of its currency denomination on equilibrium exchange rates. We allow each country to start with a stock of foreign assets and liabilities. The US net foreign liabilities in dollars are D^{US} and Japan net foreign liabilities in yen are D^{J}. The flow equations, therefore, are now extended to be:

$$e_0\xi_0 - \iota_0 + Q_0 - D^{US} + D^J e_0 = 0; \qquad e_1\xi_1 - \iota_1 - RQ_0 = 0.$$

Notice that D^{US} and D^J enter the equations at time $t = 0$ because the stock of debt has to be intermediated and the different signs with which they enter correspond to their respective currency denomination.[40]

2.4 Equilibrium Exchange Rate in the Extended Setup

When we include all the extensions to the basic Gamma model considered in the previous Sections 2.1-2.3, the flow equations for the Dollar-Yen market become:

$$m_0^*\xi_0 e_0 - m_0\iota_0 + Q_0 + f^* - f e_0 - D^{US} + D^J e_0 = 0, \tag{20}$$

$$m_1^*\xi_1 e_1 - m_1\iota_1 - RQ_0 - Rf^* + R^* f e_1 = 0. \tag{21}$$

The above equations in conjunction with the financiers' extended demand curve, equation (19), can be used to solve for the equilibrium exchange rate. We show in the Proposition below that the solution method, even in this more general case, follows the simple derivation of the basic model by representing the current economy as a "pseudo" basic economy.

Proposition 5 *In the richer model above (with money, portfolio flows, external debt, and shocks to imports and exports) the values for the exchange rates e_0 and e_1 are those in Proposition 3, replacing imports (ι_t), exports (ξ_t), and the risk bearing capacity (Γ) by their "pseudo" counterparts $\{\widetilde{\iota}_t, \widetilde{\xi}_t, \widetilde{\Gamma}\}$, defined as: $\widetilde{\iota}_0 \equiv m_0\iota_0 + D^{US} - f^*$; $\widetilde{\xi}_0 \equiv m_0^*\xi_0 + D^J - f$; $\widetilde{\iota}_1 \equiv m_1\iota_1 + Rf^*$; $\widetilde{\xi}_1 \equiv m_1^*\xi_1 + R^* f$; $\widetilde{\Gamma} \equiv \Gamma/m_0^*$.*

Proof: Equations (20)-(21) reduce to the basic flow equations, equations (12)-(13), provided we replace ι_t and ξ_t by $\widetilde{\iota}_t$ and $\widetilde{\xi}_t$. Similarly, equation (19) reduces to equation (10), provided we replace Γ by $\widetilde{\Gamma}$. Then the result follows from the proof of Proposition 3.□

[40] We could have alternatively assumed that only a fraction α of the debt had to be intermediated in which case we would get a flow of αD at time zero and a flow $(1-\alpha)RD$ at time 1.

Intuitively, the pseudo imports $(\widetilde{\iota})$ are composed of factors that lead consumers and firms to sell dollars and hence "force" financiers to be long the Dollar. An entirely symmetric intuition applies to the pseudo exports $\left(\widetilde{\xi}\right)$.

We collect here a number of qualitative results for the generalized economy. While some properties do not strictly depend on $\Gamma > 0$ and therefore can be derived even in UIP models, it is nonetheless convenient to provide a unified treatment in the present model. We assume that $\widetilde{\iota}_t$ and $\widetilde{\xi}_t$ are positive at dates 0 and 1. Otherwise, various pathologies can happen, include the non-existence of an equilibrium (e.g. formally, a negative exchange rate).

Proposition 6 *The Dollar is weaker: 1)* (Imports-Exports) *the higher US import demand for Japanese goods is (higher ι_t); the lower Japanese import demand for US goods is (lower ξ_t). 2)* ("Myopia" from an imperfect financial system) *An increase in Γ affects the effect in point (1) by making current imports matter more than future imports.*[41] *3)* (Debts and their currency denomination) *the higher the US net external liabilities in dollars are (higher D^{US}); the lower the Japanese net external liabilities in Yen are (lower D^{J}). 4)* (Financiers' risk-bearing capacity) *the higher Γ is (the worse the financial conditions are) conditional on Japan being a net creditor at time 0^+, i.e. $N_{0^+} < 0$. 5)* (Demand pressure) *if $\Gamma > 0$, the lower the demand for the Dollar is (lower f^*). 6)* (Interest rates) *the higher the Japanese real interest rate is; the lower the US real interest rate is. 7)* (Money supply) *the higher the US present money supply is (high m_0); the lower the Japanese present money supply is (low m_0^*).*

Point 3 above highlights a valuation channel to the external adjustments of countries. The exchange rate moves in a way that facilitates the re-equilibration of external imbalances. Interestingly, it is not just the net-external position of a country, its net foreign assets, that matters for external adjustment, but actually the (currency) composition of its gross external assets and liabilities (D^{US} and D^{J}). This basic result is consistent with the valuation channel to external adjustment highlighted in Gourinchas and Rey (2007*a*,*b*), Lane and Shambaugh (2010).

3 Production and Price Rigidities

We extend here the endowment economy results from Section 1.3.4 to a production economy with and without price rigidities. Production, particularly in the presence of nominal rigidities, will allow us to illustrate the real effects of the financial determination of exchange rates. These effects will be at the core of the welfare and policy analysis in Section 4.

[41] That is, $\partial e_0/\partial \iota_0$ and $\partial e_0/\partial \iota_1$ are positive and respectively increasing and decreasing in Γ.

Production Without Price Rigidities. Let us introduce a minimal model of production that will allow us to formalize the effects of the exchange rate on output and employment. While we maintain the assumption that non-tradable goods in each country are given by endowment processes, we now assume that tradable goods in each country are produced with a technology linear in labor with unit productivity. In each country, labor L is supplied inelastically and is internationally immobile.

Simple profit maximization at the firm level yields a Dollar wage in the US of $w_{H,t} = p_{H,t}$. Under flexible prices, goods market clearing then implies full employment $Y_{H,t} = L$ and a US tradable price in dollars of: $p^{\circ}_{H,t} = \frac{a_t m_t + \xi_t m^*_t e_t}{L}$, where circle in p° denotes a frictionless quantity. Likewise, for Japanese tradables the equilibrium features both full employment $Y_{F,t} = L$ and a Yen price of: $p^{*\circ}_{F,t} = \frac{a^*_t m^*_t + \iota_t m_t / e_t}{L}$.

Production With Price Rigidities. Let us now assume that wages are "downward rigid" in domestic currency at a preset level of $\{\bar{p}_H, \bar{p}^*_F\}$, where these "prices" are exogenous. Let us further assume that firms do not engage in pricing to market, so that prices are sticky in producer currency (PCP). Firm profit maximization then implies that: $p_{H,t} = \max\left(\overline{p}_H, p^{\circ}_{H,t}\right)$; or more explicitly: $p_{H,t} = \max\left(\overline{p}_H, \frac{a_t m_t + e_t \xi_t m^*_t}{L}\right)$. Hence:

$$Y_{H,t} = \min\left(\frac{a_t m_t + e_t \xi_t m^*_t}{\bar{p}_H}, L\right). \tag{22}$$

If demand is sufficiently low ($a_t m_t + \xi_t m^*_t e_t < \overline{p}_H L$), then output is demand-determined (i.e., it depends directly on: e_t, ξ_t,a_t,m_t, and m^*_t) and there is unemployment: $L - Y_{H,t} > 0$. Notice that in this case the exchange rate has an *expenditure-switching* effect: if the Dollar depreciates ($e_t \uparrow$), unemployment falls and output expands in the US. Intuitively, since US tradables' prices are sticky in dollars, these goods become cheap for Japanese consumers to buy when the Dollar depreciates. In a world that is demand constrained, this expansion in demand for US tradable is met by expanding production, thus raising US output and employment.[42]

The expenditure switching role of exchange rates has been central to the Keynesian analysis of open macroeconomics of Dornbusch (1976), Obstfeld and Rogoff (1995). In the Gamma model, it is enriched by being the central channel for the transmission of financial forces affecting the exchange rate, such as the risk-bearing capacity and balance sheet of the financiers, into output and employment.

The financial determination of exchange rates has real consequences. Let us reconsider our earlier example of a sudden inflow of capital from US investors into Brazilian Real bonds. The

[42] Clearly, a similar expression and mechanism apply to Japanese tradables: $Y_{F,t} = \min\left(\frac{a^*_t m^*_t + \iota_t m_t / e_t}{\overline{p}^*_F}, L\right)$.

exchange rate in this economy with production and sticky prices is still characterized by equation (17). As previously discussed, the capital inflow in Brazil causes the Real to appreciate $\left(\frac{\partial e_0}{\partial f} = -\frac{\Gamma}{2+\Gamma} < 0\right)$, and, if the flow is sufficiently strong (f sufficiently high) or the financiers' risk bearing capacity sufficiently low (Γ sufficiently high), the appreciation (the increase in e_0) can be so strong as to make Brazilian goods uncompetitive on international markets; the corresponding fall in world demand for Brazilian output ($\downarrow C_H^* = \frac{\iota_0}{e_0 \bar{p}_F^*}$) causes an economic slump in Brazil with both falling output and increasing unemployment.[43]

Despite extensive debates on the possibility of this type of adverse effect on the real economy of fluctuations in exchange rates due to capital flows and imbalances in financial markets, little formal modeling has been carried out in the academic literature to provide the necessary theoretical foundations. We focused here on providing such foundations in a positive model; Section 4 provides the corresponding normative analysis.

Let us now briefly consider prices that are sticky in the export destination currency (LCP). Assume that prices for US tradable goods are exogenously set at $\{\bar{p}_H, \bar{p}_H^*\}$ in dollars in the US and in yen in Japan, respectively. US output is now given by: $Y_{H,t} = \min\left(\frac{a_t m_t}{\bar{p}_H} + \frac{\xi_t m_t^*}{\bar{p}_H^*}, L\right)$. Notice that while the exchange rate no longer affects output, US net exports in dollars still have the same value: $NX_t = e_t \xi_t m_t^* - \iota_t m_t$. Consequently the exchange rate is unchanged from the previous formulae.

Devereux and Engel (2003) stressed the absence of exchange rate effects on output under LCP. The empirical evidence shows that, in practice, a combination of PCP, LCP and limited pass-through are present in the data (see Gopinath, Itskhoki and Rigobon (2010), Burstein and Gopinath (2013)). For much of this paper, we focus on PCP as the most illustrative case. Our qualitative analysis can easily accommodate a somewhat more limited pass-through of exchange rate changes to local prices of internationally traded goods.

4 Welfare and Heterodox Policies

The Gamma model of exchange rates considered so far in the positive analysis has made clear that exchange rates are affected by financial forces and that their behavior can be quite different from that implied by the traditional macroeconomic analysis. We have also shown how the financial determination of exchange rates in imperfect financial markets has real consequences for output and risk sharing.

The Mundellian prescription of pure floating exchange rates rests on the idea that a country

[43] The Brazilian Finance Minister Guido Mantega complained, as reported in Forbes Magazine (2011), that: "We have to face the currency war without allowing our productive sector to suffer. If we allow [foreign] liquidity to [freely] enter [the economy], it will bring the Dutch Disease to the economy."

hit by a negative (asymmetric) real shock would be helped by a depreciating currency that in turn, under some form of sticky prices, would boost its exports and therefore alleviate the adverse impact of the shock on output and employment. In our model we focused on an alternative scenario whereby financial shocks and imbalances in financial markets might cause an appreciation of a country's exchange rate and depress its exports and therefore output.

The possibility of such perverse effects of floating exchange rates has been the subject of extensive debates (Rey (2013), Farhi and Werning (2013)), culminating in the threat of *currency wars*, but its theoretical analysis and the development of policies to improve welfare is still in its infancy. The renewed research effort on analyzing the welfare consequences of capital controls has mostly focused on fixed exchange rate regimes in the context of the small-open-economy new-Keynesian model. Farhi, Gopinath and Itskhoki (2012), Farhi and Werning (2012*a*,*b*), Magud, Reinhart and Rogoff (2011), and Schmitt-Grohé and Uribe (2012) provide innovative analyses of policies such as capital controls, fiscal transfers and fiscal devaluations in this context.[44]

We analyze welfare and policy in the Gamma model described in the previous sections. In particular, we focus on economic situations where the financial market imperfections that are at the core of our model, namely having $\Gamma > 0$ in the presence of capital flows, play an important role both in the welfare distortions and in the suggested policy reaction.

4.1 Exchange Rate Manipulation

We first show under which conditions direct government interventions in the currency market can affect the equilibrium exchange rate. Then, we derive optimal currency management policy.

For notational simplicity, we set many parameters at 1: $\iota_t = \xi_t = a_t = a_t^* = \beta = \beta^* = 1$. We allow m_1 to be stochastic (keeping $\mathbb{E}[m_1] = 1$) purely so that currency trading is risky. The reader is encouraged to proceed keeping in mind the intuition coming from the simpler case in which m_1 is not stochastic and set equal to 1.

Positive analysis At time 0, the US government intervenes in the currency market visa à vi the financiers: it buys q yen and sells qe_0 dollars. By Proposition 5 we immediately obtain the result below (as the government creates a flow $f = -q$ in the currency market):

Lemma 4 *If the US government buys q yen and sells qe_0 dollars at time 0, the exchange rates*

[44] See also the literature on macro-prudential regulation, amongst others: Mendoza (2010), Bianchi (2010), Korinek (2011).

satisfy (for small q):

$$e_0 = 1 + \frac{\Gamma}{2+\Gamma} q + O\left(q^2\right), \qquad \mathbb{E}\left[e_1\right] = 1 - \frac{\Gamma}{2+\Gamma} q + O\left(q^2\right).$$

The intervention's impact on the *average* exchange rate is only second order: it cinduces a depreciation at time 0, and an appreciation at time 1. We can call this effect the "*boomerang effect*". A currency intervention can change the level of the exchange rate in a given period, but not the average level of the exchange rate over multiple periods.

Normative analysis We assume that in the short run, i.e. period $t = 0$, US tradables' prices are sticky in domestic currency (PCP) as in Section 3; prices are flexible in the long run, i.e. period $t = 1$. We postulate that at time zero the price is downward rigid at a level $\overline{p}_H$ that is sufficiently high as to cause unemployment in the US tradable sector. Japanese tradable prices are assumed to be flexible. Currency intervention can be welfare improving in this economy.

Proposition 7 (FX intervention) *Assume that $\Gamma > 0$ and that at time zero US tradable goods are downward rigid at a price $\overline{p}_H$ that is sufficiently high to cause unemployment in the US tradable sector. A US government currency intervention, whereby the government buys $q \in [0, q^{opt}]$ worth of Yen bonds and sells qe_0 Dollar bonds at time zero, improves welfare both in the US and in Japan. The welfare improvement is monotonically increasing in the size of the intervention up to size q^{opt}, which is the smallest intervention that restores full employment in the US.*

Note that there are two preconditions for this intervention to be welfare improving. The first one is that prices are sticky (fixed) in the short run at a level that generates a fall in aggregate demand and induces an equilibrium output below the economy's potential. This condition, i.e. being in a *demand driven* state of the world, is central to the Keynesian analysis where a depreciation of the exchange rate leads to an increase in output via an increase in export demand. If this condition is satisfied a first oder welfare loss would occur even in a world of perfect finance.[45]

The second precondition is that financial markets are imperfect, i.e. $\Gamma > 0$. Intuitively, Γ regulates the efficacy of an intervention. In fact, recall from Lemma 4 that the ability of the government to affect the time-zero exchange rate is inversely proportional to Γ. When markets are perfect ($\Gamma = 0$) the government FX policy has no effect on the time-zero exchange rate, even if prices are sticky, because financiers would simply absorb the intervention without requiring a compensation for the resulting risk.[46]

[45] Indeed, in this economy (before the government intervention) financiers optimally choose to not trade at all. The exchange rate is at 1, and is expected to remain at 1 on average, so that there are no gains from trading financial assets. Even if US household where to be allowed to directly trade Yen bonds, they would not change the value of the exchange rate.

[46] We note that after the government intervention the financiers are at their optimum and do not want to change their position. However, US households, that had no incentives to trade Yen bonds in the original equilibrium, would want

Interestingly, the suggested policy is not of the *beggar thy neighbor* type: the US currency intervention, even with its aim to weaken the Dollar, actually increases welfare for both US and Japan. This occurs because the intervention induces first order welfare gains for both US and Japanese consumers by increasing US output, but only induces second order losses due to the ensuing inter-temporal distortions in consumption. We highlight that *currency wars* can only occur when both countries are in a slump and the post-intervention weaker dollar causes a first order output loss in Japan.

More generally, our results highlight that heterodox policies could be welfare improving when the currency appreciation is so strong as to actually cause a slump in domestic employment and when private markets are sufficiently disrupted for the government currency intervention to have a meaningful impact on the exchange rate.

4.2 Taxing International Finance

We now study a second policy instrument, taxation of the financiers, which is a form of capital controls. We consider a proportional (US) government tax on each financier's profits; the tax proceeds are rebated lump sum to financiers as a whole. Recall the imperfect intermediation problem in Section 1.2, we now assume that the after-tax value of the intermediary is $V_t(1-\tau)$, where τ is the tax rate. The financiers' optimality condition, derived in a manner entirely analogous to the optimization problem in equation (9), is now:

$$Q_0 = \frac{\mathbb{E}\left[e_0 - e_1 \frac{R^*}{R}\right](1-\tau)}{\Gamma}. \tag{23}$$

Notice that this is equivalent to changing Γ to an effective $\Gamma^{\text{eff}} \equiv \frac{\Gamma}{1-\tau}$, so that the financiers' demand can be rewritten as $Q_0 = \frac{\mathbb{E}\left[e_0 - e_1 \frac{R^*}{R}\right]}{\Gamma^{\text{eff}}}$. We collect the result in the Lemma below.

Lemma 5 *A tax τ on finance is equivalent to lowering the financiers' risk bearing capacity by increasing Γ to $\Gamma^{\text{eff}} \equiv \frac{\Gamma}{1-\tau}$. A higher tax increases the effective Γ^{eff}, thus reducing the financiers' risk bearing capacity.*

Positive analysis First we note that if the equilibrium before the government intervention features zero risk taking by the financiers ($Q_0 = 0$), as was the case in the economy studied in the previous subsection, then the tax τ is entirely ineffective. Intuitively, this occurs because there

to trade the Yen bonds after the government intervention. Of course, these unlimited trades are not possible due to the frictions in the intermediation process. Therefore the policy success relies on the presence of financial frictions rather than a direct failure of Ricardian equivalence.

are zero expected profits to tax, and therefore the tax has no effect on ex-ante incentives. Equation (23) shows as much: if pre-tax one has $\mathbb{E}\left[e_0 - e_1 \frac{R^*}{R}\right] = 0$, then the tax leaves the financiers' demand unchanged.

More generally we recall from Proposition 2 that an increase in Γ, in this case an increase in Γ^{eff} due to an increase in τ, has the opposite effect on the exchange rate depending on whether the financiers are long or short the Dollar to start with, i.e. depending on the sign of Q_0 before the tax is imposed. For example, the tax would make the Dollar depreciate on impact if the financiers were long dollars to start with ($Q_0 > 0$), but the same tax would make the Dollar depreciate if the financiers' had the opposite position to start with. In practice this means that policy makers who are considering imposing capital controls, or otherwise taxing international finance, should pay close attention to the balance sheets of financial institutions that have exposures to their currency. Basing the policy on reduced form approaches or purely on traditional macroeconomics fundamentals can not only be misguiding, but might actually generate the opposite outcome for the exchange rate from the desired one.

In order to study the impact of our tax policy, we start from an economy that features active risk taking by the financiers. In the interest of tractability, we focus on a special case of the basic Gamma model of Section 1. Namely, we assume that $\mathbb{E}[\iota_1] < \iota_0$ and set all other parameters of the model to 1. We maintain from the previous subsection the assumption that US tradable prices are downward rigid at time 0 and flexible at time 1. In this set-up, the Dollar is so strong compared to the Yen at time zero that US output is below potential and there is unemployment in the US. We study in the Lemma below the impact of the tax policy on the exchange rate.

Lemma 6 *If financiers are taxed at rate τ, the equilibrium exchange rate is:*

$$e_0(\tau) = \frac{\left(1 + \frac{\Gamma}{1-\tau}\right)\iota_0 + \mathbb{E}[\iota_1]}{2 + \frac{\Gamma}{1-\tau}}, \qquad e_1(\tau) = \frac{\iota_0 + \left(1 + \frac{\Gamma}{1-\tau}\right)\mathbb{E}[\iota_1]}{2 + \frac{\Gamma}{1-\tau}} + \{\iota_1\}. \tag{24}$$

Notice that while the tax affects the equilibrium exchange rate in each period, it has no effect on the average rate across the two periods $e_0 + e_1 = \iota_0 + \iota_1$. We, therefore, recover here the same "boomerang effect" that was noted in the previous section for FX interventions. As Γ^{eff} goes from Γ to infinity, i.e. as τ goes from zero to 1, the time zero exchange rate approaches ι_0 monotonically from below. Hence, given our assumption that $\mathbb{E}[\iota_1] < \iota_0$, a tax on capital flows devalues the time zero exchange rate.

Normative analysis To keep the analytics to a minimum, we make the countries symmetric by imposing that both have equal taste for foreign tradable goods, $a_t = \xi_t = 1$, $a_t^* = \iota_t$. We define $\bar{e}_0 \equiv \bar{p}_H L - 1$ as the least-weak value of the Dollar versus the Yen that, in equilibrium, generates full employment in the US.

Proposition 8 (Taxing international finance) *Assume that at time zero: the US Dollar is so strong as to induce a fall in US output below potential ($Y_{H,0} < L$), financial markets are imperfect ($\Gamma > 0$), and the US runs a trade deficit ($\iota_0 > \mathbb{E}[\iota_1]$). A US government tax on the financiers' profits at rate $\tau \in [0, \tau^{opt}]$, improves welfare in both the US and in Japan. The welfare improvement is monotonically increasing in the tax rate up to the tax rate τ^{opt}. This rate is the lowest tax rate that ensures full employment: $e_0(\tau^{opt}) = \overline{e}_0$.*

Note that there are three preconditions for this policy to be welfare improving. As for the case of an FX intervention described in the previous subsection, two preconditions are that output has to be demand driven and that financial markets have to be imperfect. The latter condition is necessary, but in this case no longer sufficient, for the policy to affect the exchange rate. As the reader will recall from our discussion of Lemma 5, a new condition is necessary, and sufficient jointly with $\Gamma > 0$, for the policy to have an effect: financiers need to have a non-zero exposure to currency before the policy is implemented.

4.3 Joint Optimal Monetary and Financial Policy

We have analyzed above optimal financial policy in the form of FX intervention and taxation of international finance in a Gamma world. We analyze here the optimal mix of monetary policy and financial policy.

In order to perform the analysis with minimal algebra, we study a particularly clean case where most model parameters, $\{\beta, \beta^*, \iota_t, \xi_t, \ldots\}$, are set to 1. We initialize $m_0 = m_0^* = m_1^* = 1$ for $t = 0, 1$, while $\mathbb{E}[m_1] = 1$. Therefore, the only uncertainty is monetary and the economy starts at the first best.[47] Before the disruption, the new shock that moves the economy away from first best, the equilibrium exchange rates are $e_0 = 1$ and $e_1 = m_1$. The US economy starts with rigid (in dollars) tradable prices $\overline{p}_H$ at a level that just clears the goods market at full employment (i.e., a slightly higher price would lead to unemployment). We assume that the US government's objective function is: $\mathbb{E}[U_0 + U_1] - g(m_0)$, where U_t is the household utility at time t as in equation (1), and $g(m_0)$ is convex a cost of monetary policy surprises, minimal when $m_0 = 1$.[48]

We consider two types of "shocks": an unexpected increase in the Japanese money supply at time zero, from $m_0^* = 1$ to $m_0^{*'} > 1$, while keeping m_1^* constant so that the interest rate R^* falls, and an increase in the pre-set price for US tradables ($\bar{p}_H$). We first analyze the monetary shock.

[47] Other model parametrizations typically start away from the first best, so that the analysis of the government's policy is less transparent because it achieves two things: the optimal reaction to the new shock, and the correction of the pre-existing distortions.

[48] These costs could be micro-founded in several ways (price dispersion inefficiencies, inflation cost in terms of wealth redistribution, etc...), but here we take the simpler reduced form approach.

Proposition 9 ("Benign neglect" of foreign monetary shocks) *If the Dollar appreciation is simply due to a monetary shock (increase in m_0^*), then there is no real impact, and no unemployment. The best US policy is to do nothing at all: $m_0 = 1$, $q = 0$.*

We next consider the optimal policy reaction to an increase in the pre-set price for US tradables.

Proposition 10 (Trade-off between FX intervention and monetary accommodation) *Suppose that at time zero $\overline{p}_H$ is downwards rigid at a level above the full-employment flexible price, and that at time 1 it is either: 1) flexible ("short-lasting rigidity") or 2) rigid ("long-lasting rigidity"). If the rigidity is short-lasting, then the optimal policy reaction combines both FX intervention and monetary policy, where at the optimum $q > 0$ and $m_0 > 1$. The policy relies more heavily on the FX intervention compared to monetary intervention (i.e. $|m_0 - 1|$ is lower) the higher Γ is. If the rigidity is long-lasting, then the optimal policy reaction employs only monetary policy. In this case, a currency intervention is welfare reducing.*

Intuitively, FX interventions are more desirable when financial markets are more constrained, because an higher Γ makes it is easier for the policy to affect the exchange rate, and when distortions are of shorter duration, because an FX intervention cannot move the exchange rate forever. The reader will recall the "boomerang effect" of a currency intervention: it depreciates the exchange rate at time 0, but then appreciates it at time 1. If the price rigidities are long lasting, in this case last for both periods, then the intervention only generates negative net welfare benefits because of the inter-temporal distortions that it causes.

Note that the government actions would not be taken by the households in the competitive equilibrium. After the shock (in m_0^* or $\overline{p}_H$), there is no incentive for US consumers to intervene and trade the Yen: as the exchange rate is still a martingale ($e_0 = E[e_1]$), there are no excess returns to be gained. The US government chooses to intervene because it internalizes the wage externality (i.e. internalizes the impact on unemployment).

Hence, there are at least three policy tools, (i) monetary policy (ii) FX interventions, (iii) taxation of finance. In general, all might be used, and FX interventions are particularly potent when Γ is high – when financial market are disrupted. While (i) was already well-understood, our analysis allows us to analyze (ii) and (iii) in a way that was difficult in previous treatments that did not have a model of imperfect financial markets.

5 Revisiting Canonical Issues with the Gamma Model

We consider in this section a number of canonical issues of international macroeconomics via the lenses of the Gamma model. This analysis not only provides new insights into these classic issues, but also allows us to illustrate how the framework built in the previous sections rationalizes some of the empirical regularities that are at the center of open-economy analysis.

5.1 The Carry Trade in the Presence of Financial Shocks

In the Gamma model there is a profitable carry trade. Let us give the intuition in terms of the most basic model first and then extend it to a set-up with shocks to the financiers' risk bearing capacity (Γ shocks).

First, imagine a world in which countries are in financial autarky because the financiers have zero risk bearing capacity ($\Gamma = \infty$), suppose that Japan has a 1% interest rate while the US has a 5% interest rate, and that all periods $(t = 0, ..., T)$ are ex-ante identical with $\xi_t = 1$ and ι_t a martingale. Thus, we have $e_t = \iota_t$, and the exchange rate is a random walk $e_0 = \mathbb{E}[e_1] = ... = \mathbb{E}[e_T]$. A small financier with some available risk bearing capacity, e.g. a small hedge fund, could take advantage of this trading opportunity and pocket the 4% interest rate differential. In this case, there is a very profitable carry trade. As the financial sector risk bearing capacity expands (Γ becomes smaller, but still positive), this carry trade becomes less profitable, but does not disappear entirely unless $\Gamma = 0$, in which case the UIP condition holds. Intuitively, the carry trade in the basic Gamma model reflects the risk compensation necessary to induce the financiers to intermediate global financial flows.

In the most basic model, the different interest rates arise from different rates of time preferences, such that $R = \beta^{-1}$ and $R^* = \beta^{*-1}$. Without loss of generality, assume $R < R^*$ so that the Dollar is the "funding" currency, and the Yen the "investment" currency. The return of the carry trade is: $R^c \equiv \frac{R^*}{R}\frac{e_1}{e_0} - 1$. For notational convenience we define the carry trade expected return as $\overline{R}^c \equiv \mathbb{E}[R^c]$. The calculations in Proposition 3 allow us to immediately derive the equilibrium carry trade.

Proposition 11 *Assume $\xi_t = 1$. The expected return to the carry trade in the basic Gamma model is:*

$$\overline{R}^c = \Gamma \frac{\frac{R^*}{R}\mathbb{E}[\iota_1] - \iota_0}{(R^* + \Gamma)\,\iota_0 + \frac{R^*}{R}\mathbb{E}[\iota_1]}. \tag{25}$$

Hence the carry trade return is bigger (i) when the return differential R^*/R is larger (ii) when the funding country is a net foreign creditor.

To gain further intuition on the above result, consider first the case where $\iota_0 = \mathbb{E}[\iota_1]$. The first order approximation to $\overline{R}^c$ in the case of a small interest rate differential $R^* - R$ is: $\overline{R}^c = \frac{\Gamma}{2+\Gamma}(R^* - R)$. Notice that we have both $\frac{\partial \overline{R}^c}{\partial \Gamma} > 0$ and $\frac{\partial \overline{R}^c}{\partial (R^* - R)} > 0$, so that the profitability of the carry trade increases the more limited the risk bearing capacity of the financiers and the larger the interest rate differential.[49]

The effects of broadly defined "global risk aversion", here proxied by Γ, on the profitability of the carry trade have been central to the empirical analysis of for example Brunnermeier, Nagel and Pedersen (2009), Lustig, Roussanov and Verdelhan (2011), and Lettau, Maggiori and Weber (2013). Here we have shown that the carry trade is more profitable the lower the risk bearing capacity of the financiers; we next formally account for shocks to such capacity in the form of a stochastic Γ.

In addition to a pure carry force due to the interest rate differential, our model gives a theoretical foundation to the fact that debtor countries' currencies are riskier (Della Corte, Riddiough and Sarno (2013)). The reader should recall Proposition 2 that showed how net-external-debtor countries' currencies depreciate whenever risk bearing capacity decreases ($\uparrow \Gamma$). We note here that this effect occurs even if both countries have the same interest rate, thus being theoretically separate from the pure carry trade.

The exposure of the carry trade to financial disruptions We now expand on the risks of the carry trade by studying a three period ($t = 0, 1, 2$) model with stochastic shocks to the financiers' risk bearing capacity in the middle period. To keep the analysis streamlined, we take period 2 to be the "long run". Intuitively, period 2 will be a long-run steady state where countries have zero net foreign assets and run a zero trade balance. This allows us to quickly focus on the short-to-medium-run exchange rate dominated by financial forces and the long-run exchange rate completely anchored by fundamentals. We jump into the analysis, and provide many of the background details of this model in the appendix.[50]

We assume that time-1 financial conditions, Γ_1, are stochastic. In the 3-period economy with a long-run last period, the equilibrium exchange rates are:

$$e_0 = \frac{\Gamma_0 \iota_0 + \frac{R^*}{R}\mathbb{E}_0\left[\frac{\Gamma_1 \iota_1 + \iota_2 R^*/R}{\Gamma_1 + 1}\right]}{\Gamma_0 + 1}; \qquad e_1 = \frac{\Gamma_1 \iota_1 + \frac{R^*}{R}\mathbb{E}_1\left[\iota_2\right]}{\Gamma_1 + 1}; \qquad e_2 = \iota_2. \tag{26}$$

Recall that the carry-trade return between period 0 and 1 is: $R^c \equiv \frac{R^*}{R}\frac{e_1}{e_0} - 1$. Interestingly, in this case the carry trade also has "exposure to financial conditions". Notice that $\frac{\partial e_1}{\partial \Gamma_1} < 0$ in the equations above, so that the Dollar (the funding currency) appreciates whenever there is a negative

[49]The first effect occurs because, given an interest rate differential, expected returns to the carry trade have to increase whenever the risk bearing capacity of the financiers goes down to induce them to intermediate financial flows. The second effect occurs because, given a level of risk bearing capacity for the financiers, an increase in the interest rate differential will not be offset one to one by the expected exchange rate change due to the risk premium.

[50]The flow demand equations in the Yen / Dollar market are: $e_t - \iota_t + Q_t = 0$ for $t = 0, 1$, and in the long-run period $e_2 - \iota_2 = 0$, with the financiers' demand for dollars: $Q_t = \frac{e_t - \mathbb{E}[e_{t+1}]\frac{R^*}{R}}{\Gamma_t}$.

shock to the financiers' risk bearing capacity ($\uparrow \Gamma_1$, $\downarrow e_1$). Since in our chosen parametrization the carry trade is short Dollar and long Yen, we correspondingly have: $\frac{\partial R^c}{\partial \Gamma_1} < 0$, the carry trade does badly whenever there is a negative shock to the financiers' risk bearing capacity ($\uparrow \Gamma_1$). This is consistent with the intuition and the empirical findings in Brunnermeier, Nagel and Pedersen (2009); we obtain this effect here in the context of an equilibrium model. We formalize and prove the results obtained so far in the proposition below.

Proposition 12 (Determinants of expected carry trade returns) *Assume that $R^* > R$, $1 = \iota_0 = \mathbb{E}_0[\iota_1]$ and $\iota_1 = \mathbb{E}_1[\iota_2]$. Define the "certainty equivalent" $\overline{\Gamma}_1$ by $\frac{\overline{\Gamma}_1 + R^*/R}{\overline{\Gamma}_1 + 1} \equiv \mathbb{E}_0\left[\frac{\Gamma_1 + R^*/R}{\Gamma_1 + 1}\right]$. Consider the returns to the carry trade, R^c, and the corresponding expected return $\overline{R}^c \equiv \mathbb{E}_0[R^c]$. We have:*

1. *An adverse shock to financiers affects the returns to carry trade negatively : $\frac{\partial R^c}{\partial \overline{\Gamma}_1} < 0$.*
2. *The carry trade has positive expected returns: $\overline{R}^c > 0$.*
3. *The expected return to the carry trade is higher the worse the financial conditions are at time 0 $\left(\frac{\partial \overline{R}^c}{\partial \Gamma_0} > 0\right)$, the better the financial conditions are expected to be at time 1 $\left(\frac{\partial \overline{R}^c}{\partial \overline{\Gamma}_1} < 0\right)$, and the higher the interest rate differential ($\frac{\partial \overline{R}^c}{\partial R^*} > 0$, $\frac{\partial \overline{R}^c}{\partial R} < 0$).*

The Fama Regression The classic UIP regression of Fama (1984) is in levels:[51]

$$\frac{e_1 - e_0}{e_0} = \alpha + \beta\left(R - R^*\right) + \varepsilon_1.$$

Under UIP, we would find $\beta = 1$. However, a long empirical literature finds $\beta < 1$, and sometimes even $\beta < 0$. The proposition below rationalizes these findings in the context of an equilibrium model.

Proposition 13 (Fama regression) *The coefficient of the Fama regression is $\beta = \frac{1 + \overline{\Gamma}_1 - \Gamma_0}{(1+\Gamma_0)\left(1+\overline{\Gamma}_1\right)}$. Therefore, $\beta \leq 1$. In addition, one has $\beta < 0$ iff $\overline{\Gamma}_1 + 1 < \Gamma_0$, i.e. if risk bearing capacity is low in period 0 compared to period 1.*

Intuitively financial market imperfections always lead to $\beta < 1$ and very bad current market imperfections compared to future ones lead to $\beta < 0$. This occurs because any positive Γ leads to a positive risk premium on currencies that the financiers are long of and hence to a deviation from UIP ($\beta < 1$). If, in addition, financial conditions are particularly worse today compared to tomorrow the risk premium is so big as to induce currencies that have temporarily high interest rates to appreciate on average ($\beta < 0$).

[51] The regression is most commonly performed in its logarithmic approximation version, but the levels prove more convenient for our theoretical treatment without loss of economic content.

5.2 Nominal and Real Exchange Rates

We explore here the relationship between the nominal and the real CPI-based exchange rate in our framework. The real exchange rate can be defined as the ratio of two broad price levels, one in each country, expressed in the same numéraire. It is most common to use consumer price indices (CPI) adjusted by the nominal exchange rate, in which case one has:

$$\mathscr{E} \equiv \frac{P^* e}{P}. \tag{27}$$

Notice that a fall in $\mathscr{E}$ is a US Dollar real appreciation. Consider the nominal version of the basic Gamma model in Section 2.1. Standard calculations reported in the appendix imply that the real CPI-based exchange rate is:

$$\mathscr{E} = \tilde{\theta} \frac{(p_H^*)^{\xi'} (p_F^*)^{a^{*'}} (p_{NT}^*)^{\chi^{*'}}}{(p_H)^{a'} (p_F)^{\iota'} (p_{NT})^{\chi'}} e_t, \tag{28}$$

where $\tilde{\theta}$ is a function of exogenous shocks also reported in the appendix, and primed variables are normalized by θ. The above equation is the most general formulation of the relationship between the CPI-RER and the nominal exchange rate in the Gamma model. If we impose further assumptions, we can drive a useful special case.

The Basic Gamma Model Assume that $\omega = \omega^* = 0$ and $p_{NT} = p_{NT}^* = 1$ so that there is no money and the numéraire in each economy is the non-tradable good. Recall that in the basic Gamma model of Section 1 the law of one price holds for tradables, so we have $p_H = p_H^* e$ and $p_F = p_F^* e$. Equation (28) then reduces to: $\mathscr{E} = \tilde{\theta}\, (p_H)^{\xi' - a'} (p_F)^{a^{*'} - \iota'} e^{\chi^{*'}}$. This equation describes the relationship between the RER as defined in the basic Gamma model and the CPI-based RER. Notice that the two are close proxies of each other whenever the baskets' shares of tradables are symmetric across countries (i.e. $\xi' \approx a'$ and $a^{*'} \approx \iota'$) and the non-tradable goods are a large fraction of the Japanese overall basket (i.e. $\chi^{*'} \approx 1$).

5.3 The Backus and Smith Condition

In the spirit of re-deriving some classic results of international macroeconomics with the Gamma model, let us analyze the Backus and Smith condition (Backus and Smith (1993)). Let us first consider the basic Gamma set-up but with the additional assumption of complete markets as in Lemma 3. Then by equating margin utility growth in the two countries and converting, via the exchange rate, in the same units, we have: $\frac{P_0 C_0/\theta_0}{P_1 C_1/\theta_1} = \frac{P_0^* C_0^*/\theta_0^*}{P_1^* C_1^*/\theta_1^*}\frac{e_0}{e_1}$. Re-arranging we conclude:

$$\frac{C_0/\theta_0}{C_1/\theta_1} = \frac{C_0^*/\theta_0^*}{C_1^*/\theta_1^*}\frac{\mathscr{E}_0}{\mathscr{E}_1}, \tag{29}$$

where the reader should recall the definition $\mathscr{E} = \frac{P^* e}{P}$. This is the Backus and Smith condition in our set-up under complete markets: the perfect risk sharing benchmark equation.

Of course, this condition fails in the basic Gamma model because agents not only cannot trade all Arrow-Debreu claims, but also have to trade with financiers in the presence of limited commitment problems. In our framework (Section 1), however, an extended version of this condition holds:

$$\frac{C_0/\theta_0}{C_1/\theta_1} = \frac{C_0^*/\theta_0^*}{C_1^*/\theta_1^*}\frac{\mathscr{E}_0}{\mathscr{E}_1}\frac{e_1}{e_0}. \tag{30}$$

The simple derivation of this result is reported in the appendix. The above equation is the extended Backus-Smith condition that holds in our Gamma model. Notice that our condition in equation (30) differs from the standard Backus-Smith condition in equation (29) by the growth rate of the “nominal" exchange rate $\frac{e_1}{e_0}$. Since exchange rates are much more volatile in the data than consumption, this omitted term creates an ample wedge between the complete market and the Gamma version of the Backus-Smith condition.

6 Conclusion

We presented a theory of exchange rate determination in imperfect capital markets where financiers bear the risks resulting from global imbalances in the demand and supply of international assets. Exchange rates are determined by the balance sheet risks and risk bearing capacity of these financiers. Exchange rates in our model are disconnected from traditional macroeconomic fundamentals, such as output, inflation and the trade balance and are instead more connected to financial forces such as the demand for assets denominated in different currencies. We have shown how seemingly heterodox policies, such as currency interventions and capital controls, can be welfare improving in this context. Our model is tractable, with simple to derive closed form solutions, and can be generalized to address a number of both classic and new issues in international macroeconomic analysis.

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Appendix to "International Liquidity and Exchange Rate Dynamics"

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A.1 Analytical Generalization of the Model

A.1.1 The Exchange Rate with Infinite Horizon

We provide here the infinite-horizon extension of the model. In this model the flow equation is: $NX_t - RQ_{t-1} + Q_t = 0$. We denote Q_0^- to be the initial net foreign liabilities of the US (evaluated in dollars), so that we have: $NX_0 - Q_0^- + Q_0 = 0$.

Proposition A.1 (Exchange rate with infinite horizon) *Assume a non-negative stochastic process for imports ι_t and that $\xi_t = 1$ for all t. Denote Q_0^- the initial US net foreign liabilities to be absorbed by the financiers. The equilibrium exchange rate is:*

$$e_t = \mathbb{E}_t\left[\sum_{s=t}^{\infty} \Lambda^{s-t}(1-\Lambda)\,\iota_s\right] + (1-\Lambda)\,R_t Q_{t-1}, \tag{A.1}$$

where $\Lambda \equiv \frac{(1+R+\Gamma)-\sqrt{(1+R+\Gamma)^2-4R}}{2R} \in (0, \frac{1}{R}]$. When $\Gamma = 0$, then $\Lambda = \frac{1}{R}$, and otherwise Λ is decreasing in Γ.

This Proposition shows that Γ is akin to inducing myopia about future flows. If $\Gamma = 0$ so that UIP holds and $\Lambda = 1/R$, then future flows are discounted at the US interest rate. When $\Gamma > 0$, future flows are discounted at a rate higher than this interest rate.

It is interesting to analyze the model in the presence of portfolio flows, f_t^*, along the lines of Sections 1.3.1 and 2.2. Recall that for analytical tractability we assume that these flows can depend on most fundamental variables, e.g. the interest rate, but cannot depend directly on the exchange rate.[52] We perform a Taylor expansion in the interest rate differential, linearizing around $R_t^* = R_t \approx R$, and assuming that ι_t is close to $\bar{\iota}$ and Q_0^- is close to 0, so that we can expand the exchange rate around $\bar{e} \equiv \bar{\iota}$. Then

$$e_t = \mathbb{E}_t \sum_{s=t}^{\infty} \Lambda^{s-t}\left[(1-\Lambda)(\iota_s - f_s^*) + \Lambda\bar{e}\frac{R_s^* - R_s}{R}\right] + (1-\Lambda)\,R_t Q_{t-1}. \tag{A.2}$$

The above expression is exact in its treatment of the term in $(\iota_s - f_s^*)$, but contains a Taylor expansion in the $R_s^* - R_s$ term.

This generalizes to an infinite horizon the effects of portfolio flows on exchange rate that were the focus of Proposition 6 in the two period model. The Yen is stronger: if Japan is a creditor ($Q_{t-1} > 0$); if there is high import demand for Japanese goods ($\widehat{\iota_s} > 0$); if interest rates are higher in Japan than in the US ($R_s^* - R_s > 0$); and if there is selling pressure on the Dollar ($f_s < 0$). Compared to the UIP case, the first effect is amplified (in general, supply effects are stronger in the Gamma model), the import-demand and interest-rate effects are simply discounted at a higher rate (we have $\Lambda < \frac{1}{R}$ in this Gamma model, rather than $\Lambda = \frac{1}{R}$ in the UIP case). The last effect (the f_s^* term) is entirely specific to the Gamma model.[53]

[52] Section A.2 provides numerical solutions for the case of portfolio flows that depend directly on the exchange rate.

[53] The derivation of equation (A.2) in the Proof section of this appendix provides details for this latter effect.

A.1.1.1 Slow Digestion of Imbalances

To further understand the dynamics of exchange rates, consider a very simple example: there is an initial debt Q_0^- at time 0, but the model is otherwise deterministic, so that $\iota_t = 1$, $r_t = r_t^* = R$ for all t.

Proposition A.2 (Slow digestion of imbalances when $\Gamma > 0$) *Given an initial debt Q_0^-, the dynamics of the exchange rate are:*

$$e_t = 1 + (1-\Lambda) Q_0^- \lambda^t. \tag{A.3}$$

US net foreign liabilities evolve according to: $Q_t = Q_0^- \lambda^t$, where $\lambda \equiv R\Lambda$. If $\Gamma = 0$, then $\lambda = 1$, and otherwise $0 < \lambda < 1$.

In the UIP case ($\Gamma = 0$) the debt and the exchange rate are constant (or more generally a martingale). The Dollar is permanently weak. When $\Gamma > 0$, and therefore $\lambda < 1$, the debt slowly shrinks to 0, and the Dollar slowly mean-reverts to its fundamental value with no debt ($e_\infty = 1$). Intuitively, this "slow digestion"occurs because the US has a debt at time zero and the financiers need to be convinced to hold it. Given that interest rates are the same in both countries, the Dollar needs to be expected to appreciate to give a capital gain to the financiers. Hence, the Dollar is weak initially, and it will appreciate as long at there is debt reminaing. Indeed, it appreciates until $Q_t = 0$, which happens in the limit as $t \to \infty$. Viewed in a different way, the Dollar is depreciated, so US exports are higher, which allows the US to repay its debt slowly.

Hence, the financial imperfections modeled in this Gamma framework lead to qualitatively different dynamics compared to the traditional UIP case.

A.1.2 Japanese Households and The Carry Trade

In most of the main body of the paper consumers do not do the carry trade themselves. In this subsection, we analyze the case where Japanese consumers buy a quantity f of Dollar bonds financing the purchase by shorting an equivalent amount of Yen bonds. We let this demand take the form:

$$f = b(R - R^*).$$

If $b \geq 0$ the Japanese household demand is a form of carry trade because whenever $R \leq R^*$, then we have $f \leq 0$. The flow equations now are:

$$NX_0 + Q + f = 0, \qquad NX_1 - R(Q + f) = 0.$$

We summarize the implications for the equilibrium carry trade in the Proposition below.

Proposition A.3 *When Japanese consumers do the carry trade, the expected return to the carry trade in the basic Gamma model is:*

$$\overline{R}^c = \Gamma \frac{\frac{R^*}{R}\mathbb{E}[\iota_1] - \iota_0 + f(1+R^*)}{(R^* + \Gamma)\iota_0 + \frac{R^*}{R}\mathbb{E}[\iota_1] - \Gamma f}.$$

Hence the carry trade return is bigger (i) when R^*/R is higher (ii) when the funding country is a net foreign creditor (iii) when consumers do the carry trade less (f increases).

If consumers do the carry trade on too large a scale (f too negative), then the carry trade becomes unprofitable, $\overline{R}^c < 0$.

A.2 Numerical Generalization of the Model

Coming soon...

A.3 Further Details For the Main Body of the Paper

A.3.1 Static Utility Maximization Problem of the Japanese Household

We provide here the details, omitted in Section 1, of the static utility maximization problem of the Japanese household:

$$\max_{C^*_{NT,t}, C^*_{H,t}, C^*_{F,t}} \quad \chi^*_t \ln C^*_{NT,t} + \xi_t \ln C^*_{H,t} + a^*_t \ln C^*_{F,t} + \lambda^*_t \left(CE^*_t - C^*_{NT,t} - p^*_{H,t} C^*_{H,t} - p^*_{F,t} C^*_{F,t} \right),$$

where CE^*_t is aggregate consumption expenditure of the Japanese household, λ^*_t is the associated Lagrange multiplier, p^*_H is the Yen price in Japan of US tradables, and p^*_F is the Yen price in Japan of Japanese tradables. Standard optimality conditions imply:

$$C^*_{NT,t} = \frac{\chi^*_t}{\lambda^*_t}; \qquad p^*_{H,t} C^*_{H,t} = \frac{\xi_t}{\lambda^*_t}; \qquad p^*_{F,t} C^*_{F,t} = \frac{a^*_t}{\lambda^*_t}.$$

Our assumption that $Y^*_{NT,t} = \chi^*_t$, combined with the market clearing condition for Japanese non-tradables $Y^*_{NT,t} = C^*_{NT,t}$, implies that in equilibrium $\lambda^*_t = 1$. We obtain:

$$p^*_{H,t} C^*_{H,t} = \xi_t, \qquad p^*_{F,t} C^*_{F,t} = a^*_t.$$

A.3.2 The Euler Equation when there are Several Goods

We state the general Euler equation when there are several goods (this result is well-known, but we re-derive it for completeness).

With utility $u^t(\mathsf{C}_t) + \beta u^{t+1}(\mathsf{C}_{t+1})$, where C_t is the vector of goods consumed (for instance, $\mathsf{C}_t = (C_{NT,t}, C_{H,t}, C_{F,t})$ in our setup), if the consumer is at his optimum, we have:

Lemma A.1 *When there are several goods, the Euler equation is:*

$$\text{Multi-good Euler equation: } 1 = \mathbb{E}_t \left[\beta R \frac{u^{t+1}_{c_{j,t+1}} / p_{j,t+1}}{u^t_{c_{i,t}} / p_{i,t}} \right] \text{ for all } i, j \tag{A.4}$$

This should be understood in "nominal" terms, i.e. the return R is in units of the (potentially arbitrary) numéraire.

Proof. It is a variant on the usual one: the consumer can consume $d\varepsilon$ fewer dollars (assuming that the "dollar" is the local unit of account) worth of good i at time t (hence, consume $dc_{it} = -\frac{d\varepsilon}{p_{it}}$), invest them at rate R, and consume the proceeds, i.e. $R\varepsilon$ more dollars of good j at time $t+1$ (hence, consume $dc_{j,t+1} = \frac{Rd\varepsilon}{p_{j,t+1}}$). The total utility change is:

$$dU = u^t_{c_{i,t}} dc_{i,t} + \beta \mathbb{E}_t u^{t+1}_{c_{j,t+1}} dc_{j,t+1} = \mathbb{E}_t \left(-u^t_{c_{i,t}} / p_{i,t} + \beta R u^{t+1}_{c_{j,t+1}} / p_{j,t+1} \right) d\varepsilon$$

At the margin, the consumer should be indifferent, so $dU = 0$, hence (A.4). □

Applying this to our setup, with $i = j = NT$, with $p_{NT,t} = 1$, $u^t_{c_{NT,t}} = \frac{\chi_t}{C_{NT,t}} = 1$ for $t = 0, 1$, and we obtain: $1 = \mathbb{E}\left[\beta R \frac{1/1}{1/1}\right]$, hence $R = 1/\beta$.

A.3.3 The Financiers' Demand Function

The financiers' optimization problem We clarify here the role of the mild assumption, made in footnote 16, that $\Omega_0 \geq -1$. Formally, the financiers' optimization problem is:

$$\max_{q_0} V_0 = \Omega_0 q_0, \qquad \text{subject to} \quad V_0 \geq \min\left(1, \Gamma\frac{|q_0|}{e_0}\right)|q_0|,$$

where $\Omega_0 \equiv \mathbb{E}_0\left[1 - \frac{R^*}{R}\frac{e_1}{e_0}\right]$. Notice that Ω_0 is unaffected by the individual financier's decisions and can be thought of as exogenous here.

Consider the case in which $\Omega_0 > 0$, then the optimal choice of investment has $q_0 \in (0, \infty)$. Notice that $\Omega_0 \leq 1$ because interest rates and exchange rates are positive, so that $V_0 \leq q_0$. In this case the constraint can be rewritten as: $V_0 \geq \Gamma\frac{q_0^2}{e_0}$, because the constraint will always bind before the portion of assets that the financiers can divert $\Gamma\frac{|q_0|}{e_0}$ reaches 1. This yields the simpler formulation of the constraint adopted in the main text.

Now consider the case in which $\Omega_0 < 0$, then the optimal choice of investment has $q_0 \in (-\infty, 0)$. It is a property of currency excess returns that Ω_0 has no lower bound. In this paper, we assume that the parameters of the model are as such that $\Omega_0 > -1$, i.e. we assume that the worst possible (discounted) expected returns from being long a Dollar bond and being short a Yen bond is -100%. Economically this is an entirely innocuous assumption given that the range of expected excess returns in the data is approximately [-6%,+6%]. With this assumption in hand we have $V_0 \leq |q_0|$, and hence we can once again adopt the simpler formulation of the constraint because the constraint will always bind before the portion of assets that the financiers can divert $\Gamma\frac{|q_0|}{e_0}$ reaches 1.

The financiers' value function and households' valuation of currency trades We analyze next, in the context of the basic Gamma model of Section 1, the connection between the financiers' value function, equation (8), and the households' optimal demand for foreign currency in the absence of frictions. If US households were allowed to trade Yen bonds as well as Dollar bonds we would recover the standard Euler equation:

$$0 = \mathbb{E}\left[\beta\frac{U'_{1,C_{NT}}}{U'_{0,C_{NT}}}\left(R - R^*\frac{e_1}{e_0}\right)\right] = \mathbb{E}\left[\beta\frac{\chi_1/C_{NT,1}}{\chi_0/C_{NT,0}}\left(R - R^*\frac{e_1}{e_0}\right)\right] = \mathbb{E}\left[1 - \frac{R^*}{R}\frac{e_1}{e_0}\right],$$

where the last equality follows from the assumption that $C_{NT,t} = \chi_t$ and the result that $\beta R = 1$ derived in the main text (see equation (7)). US households optimally value the currency trade according to its expected (discounted at R) excess returns. Notice that this mean return criterion occurs despite the households being risk averse. The simplification occurs because variations in marginal utility are exactly offset by variations in the relative price of non-tradable goods, so that marginal utility in terms of the numéraire (the NT good) is constant across states of the world.

The reader can verify from our discussion above that, in the absence of frictions, the first order condition for the investment q_t would be: $0 = \mathbb{E}\left[1 - \frac{R^*}{R}\frac{e_1}{e_0}\right]$, where we have again made use of $\beta R = 1$. We conclude, therefore, that the financiers' value function reflects the risk adjusted value of the currency trade to the US households. In the absence of frictions the financiers are a "veil" and choose exactly the same currency trade as the household would choose. It is the friction $\Gamma > 0$ that makes the financiers' problem interesting in our set-up.

Financiers' demand in the monetary model When we consider set-ups more general than the basic Gamma model of Section 1, we maintain the simpler formulation of the financiers' demand function. We do not directly derive the households' valuation of currency trades in these more general set-ups. Our demand functions are very tractable and carry most of the economic content of more general treatments; we leave it for the extension Section A.2 to characterize numerically financier value functions more complex than those analyzed, in closed form, in the main parts of the paper. We provide here a few details regarding the monetary model. We assume that the financiers solve:

$$\max_{q_0} V_0 = \Omega_0 q_0, \qquad \text{subject to} \quad V_0 \geq \min\left(1, \Gamma \frac{|q_0|}{m_0^* e_0}\right)|q_0|,$$

where $\Omega_0 \equiv \mathbb{E}_0\left[1 - \frac{R^*}{R}\frac{e_1}{e_0}\right]$. Notice that m_0^* is now scaling the portion of nominal assets that the financiers' can divert to ensure that such fraction is scale invariant to the level of the Japanese money supply and hence the nominal value in Yen of the assets.

A.3.4 A "Short-Run" Vs "Long-Run" Analysis

As in undergraduate textbooks, it is handy to have a notion of "long run". Here is a way to introduce it in our model. We have periods of unequal length: we say that period 0 is short, but period "1" lasts for a length T. The equilibrium flow equations in the dollar-yen market become:

$$\begin{aligned} \xi_0 e_0 - \iota_0 + Q_0 &= 0 \\ T\left(\xi_1 e_1 - \iota_1\right) - RQ_0 &= 0 \end{aligned} \tag{A.5}$$

The reason for the "T" is that the imports and exports will occur over T periods. We assume a zero interest rate "within period 1". This already gives a good notion of the "long run".[54]

Some extra clarity is obtained by taking the limit $T \to \infty$. The interpretation is that period 1 is "very long" and period 0 is "very short". The flow equation (A.5) can be written: $\xi_1 e_1 - \iota_1 - \frac{RQ_0}{T} = 0$. So in the large T limit we obtain: $\xi_1 e_1 - \iota_1 = 0$. Economically, it induces the trades absorbed by the financiers to be very small compared to the trades in the goods markets in the long run. We gather the environment, and its solution, in the next Proposition.[55]

Proposition A.4 *Consider a model with a "long-run" last period. Then, the flow equations become:*

$$\xi_0 e_0 - \iota_0 + Q_0 = 0; \qquad e_1 - \iota_1 = 0.$$

while we still have $Q_0 = \frac{1}{\Gamma}\mathbb{E}\left[e_0 - e_1 \frac{R^*}{R}\right]$. *The value of the exchange rates become:*

$$e_0 = \frac{\frac{R}{R^*}\mathbb{E}\left[\frac{\iota_1}{\xi_1}\right] + \Gamma \iota_0}{1 + \Gamma \xi_0}; \qquad e_1 = \frac{\iota_1}{\xi_1}.$$

In that view, the "long run" is determined by fundamentals $e_1 = \frac{\iota_1}{\xi_1}$, while the "short run" is determined both by fundamentals and financial imperfections (Γ) with short-run considerations (ι_0, ξ_0). In the simple case $R = R^* = \xi_t = 1$, we obtain:

$$e_0 = \frac{\Gamma \iota_0 + \mathbb{E}[\iota_1]}{\Gamma + 1}; \qquad e_1 = \iota_1.$$

[54] The solution is simply obtained by Proposition 3, setting $\widetilde{\iota}_1 = T\iota_1, \widetilde{\xi}_1 = T\xi_1$.

[55] One derivation is as follows. Take Proposition 3, set $\widetilde{\iota}_1 = T\iota_1, \widetilde{\xi}_1 = T\xi_1$, and take the limit $T \to \infty$.

Application to carry trade with three periods In the 3-period carry trade model of section 5.1, we take period 2 to be the "long run". That allows us to analyze more clearly the dynamic environment. Without the "long-run" period 2, the expressions are less intelligible, but the economics is the same.

A.3.5 Price Indices, Nominal and Real Exchange Rates

We report here a few details omitted for brevity in Section 5.2.

Let us first derive the price indices $\{P, P^*\}$. The US price index P is defined as the minimum cost, in units of the numéraire (money), of obtaining one unit of the consumption basket:

$$C_t \equiv \left[\left(\frac{M_t}{P_t} \right)^{\omega_t} (C_{NT,t})^{\chi_t} (C_{H,t})^{a_t} (C_{F,t})^{\iota_t} \right]^{\frac{1}{\theta_t}}.$$

Let us define a "primed" variable as being normalized by the sum of the preference coefficients θ_t; so that, for example, $\chi_t^{'} \equiv \frac{\chi_t}{\theta_t}$. Substituting the optimal demand for goods (see equation (18) and similar) in the consumption basket formula we have:

$$1 = \left(\omega^{'} P\right)^{\omega^{'}} \left(a^{'} \frac{P}{p_H}\right)^{a^{'}} \left(\iota^{'} \frac{P}{p_F}\right)^{\iota^{'}} \left(\chi^{'} \frac{P}{p_{NT}}\right)^{\chi^{'}}.$$

Hence:

$$P = (p_H)^{a^{'}} (p_F)^{\iota^{'}} (p_{NT})^{\chi^{'}} \left[(\omega_t^{'})^{-\omega_t^{'}} (\iota_t^{'})^{-\iota_t^{'}} (a_t^{'})^{-a_t^{'}} (\chi_t^{'})^{-\chi_t^{'}} \right].$$

The part in square brackets is a residual and not so interesting. Similarly for Japan, we have:

$$P^* = (p_H^*)^{\xi^{'}} (p_F^*)^{a^{*'}} (p_{NT}^*)^{\chi^{*'}} \left[(\omega_t^{*'})^{-\omega_t^{*'}} (\xi_t^{'})^{-\xi_t^{'}} (a_t^{*'})^{-a_t^{*'}} (\chi_t^{*'})^{-\chi_t^{*'}} \right].$$

The CPI-RER in equation (28) is then obtained by substituting the price indices above in the definition of the real exchange rate in equation (27). For completeness, we report below the full expression for the function $\tilde{\theta}$ that enters in equation (28):

$$\tilde{\theta}_t = \frac{(\omega_t^{*'})^{-\omega_t^{*'}} (\xi_t^{'})^{-\xi_t^{'}} (a_t^{*'})^{-a_t^{*'}} (\chi_t^{*'})^{-\chi_t^{*'}}}{(\omega_t^{'})^{-\omega_t^{'}} (\iota_t^{'})^{-\iota_t^{'}} (a_t^{'})^{-a_t^{'}} (\chi_t^{'})^{-\chi_t^{'}}}.$$

The Basic Complete Market Model The main text of the paper illustrated the relationship between the CPI-RER and the definition of the real exchange rate in the basic Gamma model. For completeness we include here a similar analysis for the case of complete and frictionless markets. We maintain all the assumptions from the paragraph on the Basic Gamma model in Section 5.2, except that we now assume markets to be complete and frictionless. Recall from Lemma 3 that we then obtain $e_t = \nu$. Hence, the CPI-RER now follows: $\mathscr{E} = \tilde{\theta}\ (p_H)^{\xi^{'} - a^{'}} (p_F)^{a^{*'} - \iota^{'}} \nu^{\chi^{*'}}$. Notice that while the real exchange rate (e) is constant in complete markets in the basic Gamma model, the CPI-RER will in general not be constant in as long as the CPI baskets are not symmetric and relative prices of goods move.

A.3.6 Derivation of the Extended Backus and Smith Condition

We report here the simple derivation of the extended version of the Backus and Smith condition that holds in the basic Gamma model. The condition in equation (29) can de verified as follows:

$$\frac{C_0/\theta_0}{C_1/\theta_1} = \frac{C_0^*/\theta_0^*}{C_1^*/\theta_1^*}\frac{\mathscr{E}_0}{\mathscr{E}_1}\frac{e_1}{e_0} \iff \frac{P_0C_0/\theta_0}{P_1C_1/\theta_1} = \frac{P_0^*C_0^*/\theta_0^*}{P_1^*C_1^*/\theta_1^*} \iff \frac{1}{1} = \frac{1}{1},$$

where the first equivalence simply makes use of the definition $\mathscr{E} \equiv \frac{P^*e}{P}$, and the second equivalence follows from $P_tC_t = \theta_t$ and $P_t^*C_t^* = \theta_t^*$ for $t = 0, 1$. These latter equalities (we focus here on the US case) can be recovered by substituting the demand functions for goods in equation (4) in the static household budget constraint:

$$P_tC_t = C_{NT,t} + p_{H,t}C_{H,t} + p_{F,t}C_{F,t} = \chi_t + a_t + \iota_t = \theta_t.$$

A.4 Proofs

A.4.1 Proofs for the Main Body of the Paper

Proof of Proposition 3 The flow equilibrium conditions in the dollar-yen markets are:

$$\xi_0 e_0 - \iota_0 + Q_0 = 0, \tag{A.6}$$
$$\xi_1 e_1 - \iota_1 - RQ_0 = 0. \tag{A.7}$$

Summing R(A.6) and (A.7) gives the intertemporal budget constraint:

$$R(\xi_0 e_0 - \iota_0) + \xi_1 e_1 - \iota_1 = 0 \tag{A.8}$$

From this we obtain:

$$e_1 = \xi_1^{-1}(R\iota_0 + \iota_1 - R\xi_0 e_0) \tag{A.9}$$

The equilibrium in the Dollar / Yen market $\xi_0 e_0 - \iota_0 + \frac{1}{\Gamma}\mathbb{E}\left[e_0 - \frac{R^*}{R}e_1\right] = 0$ gives:

$$\frac{R^*}{R}\mathbb{E}[e_1] = e_0 + \Gamma(\xi_0 e_0 - \iota_0) = (1 + \Gamma\xi_0)e_0 - \Gamma\iota_0 \tag{A.10}$$

Combing (A.9) and (A.10),

$$\mathbb{E}[e_1] = \mathbb{E}\left[\xi_1^{-1}(R\iota_0 + \iota_1)\right] - \mathbb{E}\left[\xi_1^{-1}\right]\xi_0 R e_0 = \frac{R}{R^*}(1 + \Gamma\xi_0)e_0 - \frac{R}{R^*}\Gamma\iota_0$$

i.e.

$$\begin{aligned} e_0 &= \frac{\frac{R}{R^*}\Gamma\iota_0 + \mathbb{E}\left[\xi_1^{-1}(R\iota_0 + \iota_1)\right]}{\frac{R}{R^*}(1 + \Gamma\xi_0) + \mathbb{E}\left[\xi_1^{-1}\right]\xi_0 R} \\ &= \frac{\left(\mathbb{E}\left[R^*\xi_1^{-1}\right] + \Gamma\right)\iota_0 + \mathbb{E}\left[\frac{R^*}{R}\xi_1^{-1}\iota_1\right]}{\left(\mathbb{E}\left[R^*\xi_1^{-1}\right] + \Gamma\right)\xi_0 + 1} \\ &= \frac{\mathbb{E}\left[\frac{R^*}{\xi_1}\left(\iota_0 + \frac{\iota_1}{R}\right)\right] + \Gamma\iota_0}{\mathbb{E}\left[\frac{R^*}{\xi_1}\left(\xi_0 + \frac{\xi_1}{R^*}\right)\right] + \Gamma\xi_0} \end{aligned} \tag{A.11}$$

We can now calculate e_1. We start from its expected value:

$$\begin{aligned}
\frac{R^*}{R}\mathbb{E}\left[e_1\right] &= \left(1+\Gamma\xi_0\right)e_0-\Gamma\iota_0 \\
&= \left(1+\Gamma\xi_0\right)\frac{\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right]+\Gamma\right)\iota_0+\mathbb{E}\left[\frac{R^*}{\xi_1}\frac{\iota_1}{R}\right]}{\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right]+\Gamma\right)\xi_0+1}-\Gamma\iota_0 \\
&= \frac{\left\{\left(1+\Gamma\xi_0\right)\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right]+\Gamma\right)-\Gamma\left[\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right]+\Gamma\right)\xi_0+1\right]\right\}\iota_0+\left(1+\Gamma\xi_0\right)\mathbb{E}\left[\frac{R^*}{\xi_1}\frac{\iota_1}{R}\right]}{\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right]+\Gamma\right)\xi_0+1} \\
&= \frac{\mathbb{E}\left[\frac{R^*}{\xi_1}\right]\iota_0+\left(1+\Gamma\xi_0\right)\mathbb{E}\left[\frac{R^*}{\xi_1}\frac{\iota_1}{R}\right]}{\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right]+\Gamma\right)\xi_0+1} \\
&= \frac{\mathbb{E}\left[\frac{R^*}{\xi_1}\left(\iota_0+\frac{\iota_1}{R}\right)\right]+\Gamma\xi_0\mathbb{E}\left[\frac{R^*}{\xi_1}\frac{\iota_1}{R}\right]}{\mathbb{E}\left[\frac{R^*}{\xi_1}\left(\xi_0+\frac{\xi_1}{R^*}\right)\right]+\Gamma\xi_0}
\end{aligned}$$

To obtain the time-1 innovation, we observe that $e_1=\frac{1}{\xi_1}\left(R\iota_0+\iota_1-R\xi_0e_0\right)$ implies:

$$\{e_1\}=\left\{\frac{\iota_1}{\xi_1}\right\}+R\left(\iota_0-\xi_0e_0\right)\left\{\frac{1}{\xi_1}\right\}$$

As:

$$\iota_0-\xi_0e_0=\iota_0-\xi_0\frac{\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right]+\Gamma\right)\iota_0+\mathbb{E}\left[\frac{R^*}{\xi_1}\frac{\iota_1}{R}\right]}{\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right]+\Gamma\right)\xi_0+1}=\frac{\iota_0-\mathbb{E}\left[\xi_0\frac{R^*}{\xi_1}\frac{\iota_1}{R}\right]}{\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right]+\Gamma\right)\xi_0+1}$$

we obtain:

$$\{e_1\}=\left\{\frac{\iota_1}{\xi_1}\right\}+R\frac{\iota_0-\mathbb{E}\left[\xi_0\frac{R^*}{\xi_1}\frac{\iota_1}{R}\right]}{\left(\mathbb{E}\left[\frac{R^*}{\xi_1}\right]+\Gamma\right)\xi_0+1}\left\{\frac{1}{\xi_1}\right\}\square$$

Proof of Lemma 3 We prove the Lemma, in a slightly more general way. In the decentralized allocation, the consumer's intra-period consumption (3) gives the first order conditions:

$$\begin{aligned}
p_{NT}C_{NT} &= \frac{\chi}{\lambda}, & p^*_{NT}C^*_{NT} &= \frac{\chi^*}{\lambda^*} \\
p_HC_H &= \frac{a}{\lambda}, & \frac{p_H}{e}C^*_H &= \frac{\xi}{\lambda^*} \\
ep^*_FC_F &= \frac{\iota}{\lambda}, & p^*_FC^*_F &= \frac{a^*}{\lambda^*}
\end{aligned} \tag{A.12}$$

so that

$$e_t=\frac{\frac{C^*_H\lambda^*}{\xi}}{\frac{C_H\lambda}{a}}$$

Suppose that the Negishi weight is ν. The planner maximizes $U+\nu U^*$ subject to the resource constraint; hence, in particular $\max_{C_H+C_H^*\leq Y_H} a\ln C_H+\nu\xi\ln C_H^*$, which gives the planner's first order condition

$$\frac{a}{C_H}=\frac{\nu\xi}{C_H^*}$$

Hence, in the first best exchange rate satisfies:

$$e^{FB}=\nu\frac{\lambda^*}{\lambda}=\nu\frac{p_{NT}C_{NT}/\chi}{p_{NT}^*C_{NT}^*/\chi^*}$$

Putting the dates back in for future reference, the first best exchange rate, when the Negishi weights on US and Japan are $(1,\nu)$, is

$$e_t^{FB}=\nu\frac{\lambda_t^*}{\lambda_t}=\nu\frac{p_{NT}C_{NT,t}/\chi_t}{p_{NT}^*C_{NT,t}^*/\chi_t^*}.$$

In the basic case of Lemma 3, we have $\lambda_t=\lambda_t^*=1$, so $e_t^{FB}=\nu$.□

Proof of Proposition 6 We first prove a Lemma.

Lemma A.2 *In the setup of Proposition 3, e_0 is increasing in ι_t and R^* and decreasing in ξ_t and R; $\frac{\partial e_0}{\partial\iota_0}$ increases in Γ. In addition, e_0 increases in Γ if and only the US is a natural net debtor at time 0^+, i.e. $N_{0^+}\equiv\xi_0e_0-\iota_0<0$.*

Proof: The first comparative statics are just by inspection (or direct calculation). The next one is given by:

$$\frac{\partial^2e_0}{\partial\Gamma\partial\iota_0}=\frac{1}{\left(\frac{R^*\xi_0}{\xi_1}+1+\Gamma\xi_0\right)^2}>0$$

The last one comes from:

$$\frac{\partial e_0}{\partial\Gamma}=-N_{0^+}\frac{1}{\frac{R^*\xi_0}{\xi_1}+\xi_0\Gamma}<0.\square$$

This implies all the points of Proposition 6, except for the impact of f^*. By translation invariance in f^* (i.e. redefining $\widetilde{\iota}_t$), it is enough to verify that $\frac{\partial e_0}{\partial f^*}<0$ at $f^*=0$. Taking the derivative at $f^*=0$ reveals that indeed

$$\frac{\partial e_0}{\partial f^*}=-\frac{\Gamma}{\frac{R^*\xi_0}{\xi_1}+1+\Gamma\xi_0}<0$$

We notice that the comparative statics with respect to f are less clear-cut, because f affects the value of $\Gamma\widetilde{\xi}_1$, and hence affects risk-taking. However, we have $de_0/df>0$ for typical values (e.g. $R=R^*=1,\widetilde{\xi}_0=\widetilde{\xi}_1$).
□

Proof of Lemma 4 and Proposition 7 The economy is described by:

$$e_0-1-qe_0+Q_0=0;\qquad e_1-m_1+qe_1-Q_0=0.$$

Using Proposition 3 (with $\widetilde{\xi}_0 = 1-q, \widetilde{\xi}_1 = 1+q$), we have:

$$e_0(q) = 1 + \frac{\Gamma\left(q+q^2\right)}{2+\Gamma\left(1-q^2\right)} \tag{A.13}$$

$$e_1(q) = 1 + \frac{\Gamma\left(-q+q^2\right)}{2+\Gamma\left(1-q^2\right)} + \frac{\{m_1\}}{1+q} \tag{A.14}$$

and $\{e_1\} = \left\{\frac{m_1}{1+q}\right\}$. This implies Lemma 4.

The intervention's impact on the *average* exchange rate is only second order: it creates a depreciation at time 0, and an appreciation at time 1.[56]

US production of tradables at time 0 is (by (22))

$$Y_{H,0} = \min\left(\frac{1+e_0}{\overline{p}_H}, L\right)$$

so $Y_{H,0}$ increases in e for $e \in [0, \overline{e}_0]$, with

$$\overline{e}_0 \equiv \overline{p}_H L - 1.$$

Define $s_t \equiv \frac{C_{H,t}}{C_{H,t}+C^*_{H,t}} = \frac{C_{F,t}}{C_{F,t}+C^*_{F,t}}$ to be the real US share of tradables (US or Japanese) consumption. Given (A.12), $\lambda_t = 1/m_t$, and $\lambda^*_t = 1$, we have:

$$s_t = \frac{1}{1+\frac{e_t}{m_t}}. \tag{A.15}$$

By definition, US consumption of good g is $C_{gt} = s_t Y_{gt}$, where Y_{gt} is the world production of good g. US welfare at time t is (dropping the non-tradables endowment term, and correspondingly setting $\chi = 0$ for algebra convenience):

$$\begin{aligned} U_t &= a_t \ln C_{H,t} + \iota_t \ln C_{F,t} = a_t \ln s_t Y_{Ht} + \iota_t \ln s_t Y_{Ft} \\ &= (a_t + \iota_t) \ln s_t + a_t \ln Y_{Ht} + \iota_t \ln Y^*_{Ft} \end{aligned} \tag{A.16}$$

and intertemporal US welfare is:

$$U(q) = \mathbb{E}_0\left[U_0 + U_1\right] = \mathbb{E}\sum_{t=0}^{1} 2\ln s_t + \ln Y_H + \ln Y^*_F.$$

Hence:

$$U(q) = \ln\min\left(\frac{1+e_0(q)}{\overline{p}_H}, L\right) + h(q) + K, \tag{A.17}$$

where $K = 2\ln L$ is a constant, and $h(q) \equiv 2\mathbb{E}\sum_{t=0}^{1} \ln s_t$ is the sum of the allocational distortions. Hence, US welfare goes up when its share s_t is higher and when world production is higher (Y_H higher).

We shall see that the intervention on a scale q, by inducing a time-0 Yen devaluation (i) improves by a first order effect the production at time 0 (ii) leads only to a second order loss in the total share of consumption $h(q)$ (at least when there are no monetary disturbances). Hence, the intervention is desirable. In attention, it is desirable also for Japan (Japan benefits from the increase in world production, and experiences only a second order change from the inter-temporal distortion). The same argument would show that

[56] This is general. Consider the case $\xi_t = \xi$ for all t. Then, a small q will induce a small change $\delta(e_1 - e_0) = O(q)$, so we'll have $\sum_t \delta e_t = 0 + O\left(q^2\right)$. The exchange rate will go down by de_0, and will compensate next period: $de_1 = -de_0$ $\left(+O\left(q^2\right)\right)$.

the same policy (provided the initially price distortion is not too large) will increase Japanese welfare.

Let us see this analytically. We have:

$$\begin{aligned}
\frac{1}{2}h'(0) &= \mathbb{E}\sum_{t=0}^{1}\left.\frac{d\ln s_t}{dq}\right|_{q=0} \\
&= -\frac{e_0'(0)}{1+e_0(0)} - \mathbb{E}\frac{e_1'(0)}{m_1+e_1(0)} \text{ by (A.15)} \\
&= -\frac{\frac{\Gamma}{2+\Gamma}}{2} - \mathbb{E}\frac{\frac{-\Gamma}{2+\Gamma}-\{m_1\}}{2+2\{m_1\}} \text{ by (A.13)-(A.14) and } m_1 = 1+\{m_1\} \\
&= \mathbb{E}\frac{\left(-\frac{\Gamma}{2+\Gamma}+1\right)\{m_1\}}{2+2\{m_1\}} = \frac{1}{2+\Gamma}\mathbb{E}\frac{\{m_1\}}{1+\{m_1\}}
\end{aligned}$$

We conclude:

$$h'(0) = \frac{2}{2+\Gamma}\mathbb{E}\frac{\{m_1\}}{m_1} \tag{A.18}$$

Hence, we have $h'(0)=0$ when $\{m_1\}=0$. This confirms that intertemporal distortions induced are second order when there are no extra shocks. If there are extra shocks, we get an extra, but small, Jensen's inequality term $\mathbb{E}\frac{\{m_1\}}{1+\{m_1\}}$.

Next, we define, by analogy with (A.17):

$$V(q) \equiv \ln\frac{1+e_0(q)}{\overline{p}_H} + h(q) + K$$

so that $V(q) = U(q)$ when $\frac{1+e_0(q)}{\overline{p}_H} \leq L$, i.e. in the unemployment region. We have: $V'(0) = \frac{e_0'(0)}{1+e_0(0)} + h'(0)$, i.e.

$$V'(0) = \frac{\Gamma}{2(2+\Gamma)} + \frac{2}{2+\Gamma}\mathbb{E}\frac{\{m_1\}}{m_1}$$

by (A.13) and (A.18).

When we assume that $\{m_1\}=0$, then $V'(0)>0$: the welfare improvement is first order. In general, we assume that the disturbance $\{m_1\}$ is sufficiently small, i.e. $\mathbb{E}\frac{\{m_1\}}{m_1} > \frac{-\Gamma}{4}$,[57] that $V'(0)>0$. By continuity, there exists a $q^{\max}$ such that $V'(q)>0$ for $q\in[0,q^{\max})$.

We call $\overline{q}(e_0)$ the inverse of the function $e_0(q)$. As $e_0(q)$ is an increasing function (for $q\geq 0$), so is $\overline{q}(e_0)$. Recall also that the least-devalued exchange rate that ensures full employment is $\overline{e}_0 = \overline{p}_H L - 1$. Define $\overline{p}_H^{\max}$ such that $\overline{q}(\overline{p}_H^{\max}L-1) = q^{\max}$. Finally, recall that $p_{H,0}^{\circ} = \frac{2}{L}$ is the flexible equilibrium price of US tradable. The assumption that there is unemployment initially is equivalent to $\overline{p}_H > p_{H,0}^{\circ}$.

Take an initial price $\overline{p}_H \in (p_{H,0}^{\circ}, \overline{p}_H^{\max})$. To eliminate unemployment cause by $\overline{p}_H$, the government can do the intervention $q^{opt} \equiv \overline{q}(\overline{p}_H L - 1)$. Now, given that $\overline{q}(\cdot)$ is increasing and $\overline{p}_H < \overline{p}_H^{\max}$, and we have[58] $q^{opt} < q^{\max}$. Given that $V'(q)>0$ for $q\in[0,q^{\max})$, and that $U'(q) = V'(q)$ for $q\in[0,q^{opt})$, we have $U'(q)>0$ for $q\in[0,q^{opt})$. This means that welfare is increasing in the size of the intervention, in the range $q\in[0,q^{opt})$.

Japanese welfare. It is intuitive that Japanese welfare will also increase when the US government's FX intervention devalues the Dollar: Japan enjoys a first-order gain from the increase in US production and experiences only a second order change from the inter-temporal distortion. Of course, this hinges on

[57] This condition can be re-expressed $\mathbb{E}\frac{m_1-\mathbb{E}[m_1]}{m_1} > \frac{-\Gamma}{4}$, i.e. $\mathbb{E}[m_1]\,\mathbb{E}\left[\frac{1}{m_1}\right] < 1+\frac{\Gamma}{4}$.

[58] Indeed, $q^{opt} = \overline{q}(\overline{p}_H L-1) < \overline{q}(\overline{p}_H^{\max}L-1) = q^{\max}$.

the assumption that prices are flexible in Japan. Let us record here the more formal arguments. Japanese welfare is:

$$U^*(q) = \mathbb{E}[U_0^* + U_1^*] = \mathbb{E}\sum_{t=0}^{1} 2\ln(1-s_t) + \ln Y_H + \ln Y_F^*$$
$$= \ln\min\left(\frac{1+e_0(q)}{\overline{p}_H}, L\right) + h^*(q) + K^*$$

and $h^*(q) \equiv 2\mathbb{E}\sum_{t=0}^{1}\ln(1-s_t)$. We have

$$h^{*\prime}(0) = 2\mathbb{E}\sum_{t=0}^{1}\left.\frac{d\ln(1-s_t)}{dq}\right|_{q=0} = 2\mathbb{E}\sum_{t=0}^{1} -\left.\frac{ds_t}{dq}\right|_{q=0}\cdot\frac{1}{1-s_t(0)} = 2\mathbb{E}\sum_{t=0}^{1} -\left.\frac{ds_t}{dq}\right|_{q=0}\cdot 2$$
$$= -h'(0)$$
$$= -\frac{2}{2+\Gamma}\mathbb{E}\frac{\{m_1\}}{m_1}.$$

Therefore we write:

$$V^{*\prime}(0) = \frac{\Gamma}{2(2+\Gamma)} - \frac{2}{2+\Gamma}\mathbb{E}\frac{\{m_1\}}{m_1}.$$

Notice that we obtain $V^{*\prime}(0) > 0$ without needing any assumption on $\mathbb{E}\frac{\{m_1\}}{m_1}$, which is nonpositive by Jensen's inequality. By continuity, there is a $q^{\max,*}$ such that $V^{*\prime}(q) > 0$ for $q \in [0, q^{\max,*})$.

Hence, the Proposition also holds for Japanese welfare (i.e., welfare in both US and Japan increases as q increases, for $q \in [0, q^{opt}]$), provided that the initial distortion $\overline{p}_H$ is not too great. More specifically, we should have $\overline{p}_H \in (p_H^{\circ}, \overline{p}_H^{\max})$, where we define $\overline{p}_H^{\max}$ to be the price such that $\overline{q}(\overline{p}_H^{\max}L - 1) = \min(q^{\max}, q^{\max,*})$.□

Proof of Proposition 8 The proof is quite similar to that of Proposition 7. To make the proof more streamlined, we consider the case where ι_1 is deterministic. One can restate the arguments of the proof of Proposition 7 for the more general case with a small variation of ι_1.

Given the countries are symmetric in their taste for foreign tradable goods ($a_t = \xi_t = 1$, $a_t^* = \iota_t$), the share of US consumption for both the US and Japanese tradable good is $s_t = \frac{1}{1+e_t}$.

Intertemporal US welfare is:[59]

$$U = \mathbb{E}[U_0 + U_1] = \mathbb{E}\sum_{t=0}^{1}(\iota_t + \xi_t)\ln s_t + \xi_t \ln Y_H + \iota_t \ln Y_F^*$$

i.e.

$$U(\tau) = \ln\min\left(\frac{1+e_0(\tau)}{\overline{p}_H}, L\right) + h(\tau) + K \tag{A.19}$$

where K is a constant independent of policy, and $h(\tau)$ captures the sum of the allocational distortions:

$$h(\tau) \equiv \mathbb{E}\sum_{t=0}^{1}(1+\iota_t)\ln s_t = \mathbb{E}\sum_{t=0}^{1}(1+\iota_t)\ln\frac{1}{1+e_t(\tau)}$$

[59]The analysis for Japanese welfare is entirely symmetric: the same small tax creates a first-order improvement in Japanese welfare because it increases US tradable output and hence the Japanese equilibrium consumption of US tradable goods, but only creates a smaller distortion in the intertemporal Japanese consumption shares.

We define:

$$V(\tau) = \ln \frac{1+e_0(\tau)}{\overline{p}_H} + h(\tau) + K$$

the counterpart of $U(\tau)$, without the "min" sign ($V(\tau)$ is welfare, assuming that the economy is below full employment).

We have:

$$h'(0) = -\mathbb{E}\sum_{t=0}^{1}(1+\iota_t)\frac{e_t'(0)}{1+e_t(0)}$$

We recall:

$$e_0(\tau) = \frac{\left(1+\frac{\Gamma}{1-\tau}\right)\iota_0 + \mathbb{E}[\iota_1]}{2+\frac{\Gamma}{1-\tau}}$$
$$e_1(\tau) = \frac{\iota_0 + \left(1+\frac{\Gamma}{1-\tau}\right)\mathbb{E}[\iota_1]}{2+\frac{\Gamma}{1-\tau}} + \{\iota_1\}$$

which implies

$$e_0'(0) = \Gamma\frac{\iota_0 - \mathbb{E}\iota_1}{(2+\Gamma)^2} = -e_1'(1)$$

so that

$$h'(0) = -e_t'(0)D$$
$$D = \mathbb{E}\left[\frac{1+\iota_0}{1+e_0(0)} - \frac{1+\mathbb{E}\iota_1}{1+e_1(0)}\right] = \frac{(2+\Gamma)(\iota_0 - \mathbb{E}\iota_1)}{2+\Gamma+\iota_0(1+\Gamma)+\mathbb{E}\iota_1} > 0$$

We are now ready to evaluate the marginal impact of a small tax:

$$V'(0) = \frac{e_0'(0)}{1+e_0} + h'(0)$$
$$= e_0'(0)\left[\frac{1}{1+e_0} - D\right]$$

We assume that $\iota_0 - \mathbb{E}\iota_1$ sufficiently small (remaining positive) so that $D < \frac{1}{1+e_0}$. That implies $V'(0) > 0$. By continuity, there is a $\tau^{\max}$ such that $V'(\tau) > 0$ for $\tau \in [0, \tau^{\max})$.

We call $\overline{\tau}(e_0)$ the inverse of the function $\tau_0(q)$. Define $\overline{p}_H^{\max}$ such that $\overline{\tau}(\overline{p}_H^{\max}L - 1) = \tau^{\max}$. Then, for all $\overline{p}_H \in [p_{H,0}^{\circ}, \overline{p}_H^{\max})$, we have $U'(\tau) > 0$ for $\tau \in [0, \overline{\tau}(\overline{p}_H - 1)]$. This means that welfare is increasing in the size of the intervention. The optimal intervention in that range is $\tau^{opt} = \overline{\tau}(\overline{p}_H - 1)$, the smallest (positive) tax that restores full employment.□.

Proof of Proposition 9 Suppose that $m_0^{*'}$ deviates from its original value, 1, keeping m_1^* constant. Then, $R^{*'} = 1/m_0^{*'}$, hence (by Proposition 3) $e_0' = 1/m_0^{*'}$. Japanese demand for US exports is unchanged because $e_0' m_0^{*'} = 1$. Hence, the US export market is still in equilibrium. We are at the first best, and there is no need for policy intervention. □

Proof of Proposition 10 **Case of the short-lasting rigidity.** We follow the notations and procedure of the proof of Proposition 7. To make the proof readable, we simply consider the case with $m_1 \equiv 1$. One can restate the arguments of the proof of Proposition 7 for the more general case.

We first derive the exchange rate. The pseudo-imports and exports are:

$$\widetilde{\iota}_0 = m_0, \qquad \widetilde{\iota}_1 = 1$$
$$\widetilde{\xi}_0 = 1-q, \qquad \widetilde{\xi}_1 = 1+q,$$

and the gross interest rates are: $R = \frac{1}{m_0}, \qquad R^* = 1$.

The exchange rate is (using Proposition 3 and Proposition 5):

$$e_0 = \frac{\mathbb{E}\left[\frac{\widetilde{\iota}_0 + \frac{\widetilde{\iota}_1}{R}}{\widetilde{\xi}_1}\right] + \frac{\Gamma \widetilde{\iota}_0}{R^*}}{\mathbb{E}\left[\frac{\widetilde{\xi}_0 + \frac{\widetilde{\xi}_1}{R^*}}{\widetilde{\xi}_1}\right] + \frac{\Gamma \widetilde{\xi}_0}{R^*}} = m_0 \frac{2+\Gamma(1+q)}{2+\Gamma(1-q^2)}$$

$$e_1 = \frac{R}{R^*} \frac{\mathbb{E}\left[\frac{R^*}{\widetilde{\xi}_1}\left(\widetilde{\iota}_0 + \frac{\widetilde{\iota}_1}{R}\right)\right] + \Gamma \widetilde{\xi}_0 \mathbb{E}\left[\frac{R^*}{\widetilde{\xi}_1} \frac{\widetilde{\iota}_1}{R}\right]}{\mathbb{E}\left[\frac{R^*}{\widetilde{\xi}_1}\left(\widetilde{\xi}_0 + \frac{\widetilde{\xi}_1}{R^*}\right)\right] + \Gamma \widetilde{\xi}_0} = \frac{1}{m_0} m_0 \frac{2+\Gamma(1-q)}{2+\Gamma(1+q)(1-q)}$$
$$= \frac{2+\Gamma(1-q)}{2+\Gamma(1-q^2)}$$

with $\{e_1\} = 0$. Hence,

$$e_0 = m_0 (1+\eta_0)$$
$$e_1 = 1+\eta_1$$

where

$$\eta_0 = \frac{\Gamma\left(q+q^2\right)}{2+\Gamma(1-q^2)} \tag{A.20}$$
$$\eta_1 = \frac{\Gamma\left(-q+q^2\right)}{2+\Gamma(1-q^2)} \tag{A.21}$$

which are the expressions we found in the proof of Proposition 7.

Output is: $Y_{H,t} = \min\left(\frac{a_t m_t + \xi_t e_t m_t^*}{\overline{p}_H}, L\right) = \min\left(\frac{m_0+e_0}{\overline{p}_H}, L\right)$, i.e.,

$$Y_{H,t} = \min\left(\frac{m_0(2+\eta_0)}{\overline{p}_H}, L\right) \tag{A.22}$$

The US household's utility is given by (A.17), augmented for the cost of monetary distortions:

$$U(q) = \ln \min\left(\frac{1+e_0(q)}{\overline{p}_H}, L\right) + h(q) - g(m_0) + K \tag{A.23}$$

where $K = 2\ln L$ is a constant, and $h(q) \equiv 2\mathbb{E}\sum_{t=0}^{1} \ln s_t$ is the sum of the allocational distortions, with s_t is the share of the home good going to the US: $s_t = \frac{1}{1+\frac{e_t}{m_t}}$, i.e.

$$s_t = \frac{1}{2+\eta_t}$$

We shall use the following Taylor expansions, when q is small, i.e. η_0 and η_1 are small:

$$\eta_0 = \frac{\Gamma}{2+\Gamma} q + O\left(q^2\right)$$

which implies

$$\begin{aligned}
q &= \frac{2+\Gamma}{\Gamma}\eta_0 + O\left(\eta_0^2\right) \\
\eta_1 + \eta_0 &= \frac{2\Gamma q^2}{2+\Gamma\left(1-q^2\right)} = \frac{2\Gamma}{2+\Gamma} q^2 + O\left(q^4\right) \\
&= \frac{2\Gamma}{2+\Gamma}\left(\frac{2+\Gamma}{\Gamma}\eta_0 + O\left(\eta_0^2\right)\right)^2 + O\left(q^4\right) = \frac{2\left(2+\Gamma\right)}{\Gamma}\eta_0^2 + O\left(\eta_0^3\right) \\
&= 2\left(\frac{2}{\Gamma}+1\right)\eta_0^2 + O\left(\eta_0^3\right)
\end{aligned}$$

We are now ready to calculate the distortion term.

$$\begin{aligned}
-\frac{h}{2} &= -\sum_{t=0}^{1} \ln s_t = -\sum_{t=0}^{1} \ln \frac{1}{2+\eta_t} = \ln\left[\left(2+\eta_0\right)\left(2+\eta_1\right)\right] \\
&= \ln\left[4 + 2\left(\eta_0+\eta_1\right) + \eta_0\eta_1\right] \\
&= \ln\left[4 + 2\left(2\left(\frac{2}{\Gamma}+1\right)\eta_0^2 + O\left(\eta_0^3\right)\right) + \eta_0\left(-\eta_0 + O\left(\eta_0^2\right)\right)\right] \\
&= \ln\left[4 + \left(\frac{8}{\Gamma}+3\right)\eta_0^2 + O\left(\eta_0^3\right)\right] \\
&= \ln 4 + \left(\frac{2}{\Gamma}+\frac{3}{4}\right)\eta_0^2 + O\left(\eta_0^3\right)
\end{aligned}$$

hence

$$h = -2\ln 4 - \left(\frac{4}{\Gamma}+\frac{3}{2}\right)\eta_0^2 + O\left(\eta_0^3\right) \tag{A.24}$$

We can now re-consider welfare (A.23). For the purposes of the impact of Γ of the choice of monetary vs FX policy, it is enough to consider the sub-problem of achieving a given employment Y_H, without optimizing for Y_H. The problem is to minimize distortions subject to achieving Y_H:

$$\min_{\eta_0, m_0} h\left(\eta_0\right) - g\left(m_0\right) \text{ s.t. } m_0\left(2+\eta_0\right) = \overline{p}_H Y_H$$

This yields $\eta_0 = \eta_0\left(m_0\right) \equiv \frac{\overline{p}_H Y_H}{m_0} - 2$, so that the optimal monetary intervention is:

$$\max_{m_0} W\left(m_0\right) = h\left(\eta_0\left(m_0\right)\right) - g\left(m_0\right),$$

so that $W_{m_0} = 0$ and $W_{m_0 m_0} < 0$ at the optimum.

We want to ask: "do we have less reliance on monetary policy (rather than FX intervention) when Γ

increases?". The answer is yes if $\frac{\partial m_0}{\partial \Gamma} < 0$. Accordingly, we calculate:

$$\begin{aligned} sign\left(\frac{\partial m_0}{\partial \Gamma}\right) &= sign\left(\frac{-W_{m_0\Gamma}}{W_{m_0 m_0}}\right) \text{ as } W_{m_0} = 0 \text{ implies } \frac{\partial m_0}{\partial \Gamma} = \frac{-W_{m_0\Gamma}}{W_{m_0 m_0}} \\ &= sign\left(W_{m_0\Gamma}\right) \text{ as } W_{m_0 m_0} < 0 \\ &= sign\left(\frac{\partial}{\partial \Gamma} h'\left(\eta_0\left(m_0\right)\right) \eta_0'\left(m_0\right)\right) = -sign\left(\frac{\partial}{\partial \Gamma} h'\left(\eta_0\left(m_0\right)\right)\right) \text{ as } \eta_0'\left(m_0\right) < 0 \\ &< 0 \end{aligned}$$

as (A.24) implies $\frac{\partial}{\partial \Gamma} h'\left(\eta_0\right) > 0$ for a small enough intervention.

Case of the long-lasting rigidity. Here we simply sketch the argument, given that the detailed analytical forces have been already made explicit in the above proof and the proof of Proposition 7. An FX intervention won't improve welfare for the following reason: a high intervention to buy Yen today makes the Yen appreciate today (which helps the US) but makes it (by the "boomerang" effect) depreciate tomorrow, by the same amount. Hence, there's been no net help: the welfare impact of q policies is 0 on the unemployment front, and negative on the intertemporal distortion front. Hence, on net, a q policy has negative impact.

Hence, only monetary policy helps, in the usual way: the government wants to partially inflate, i.e. increase m_0 and m_1 (even though there's a cost g of doing so). □

Proof of Proposition 11

$$\begin{aligned} \overline{R}^c &= \frac{\mathbb{E}\left[\frac{R^*}{R} e_1 - e_0\right]}{e_0} = \frac{-\Gamma Q_0}{e_0} \\ &= -\Gamma\left(\frac{\iota_0 - e_0}{e_0}\right) = \Gamma\left(1 - \frac{\iota_0}{e_0}\right). \end{aligned}$$

Recall that:

$$e_0 = \frac{\left(R^* + \Gamma\right)\iota_0 + \frac{R^*}{R}\mathbb{E}\left[\iota_1\right]}{R^* + \Gamma + 1},$$

so that we conclude:

$$\overline{R}^c = \Gamma\left(1 - \iota_0 \frac{R^* + \Gamma + 1}{\left(R^* + \Gamma\right)\iota_0 + \frac{R^*}{R}\mathbb{E}\left[\iota_1\right]}\right).$$

which rearranged give the announced expression.

Derivation of 3-period economy exchange rates We will use the notation:

$$\mathscr{R}^* \equiv \frac{R^*}{R}. \tag{A.25}$$

From the flow demand equation for $t = 1$, $e_1 - \iota_1 + Q_1 = 0$ and the financiers' demand, $Q_1 = \frac{e_1 - \mathscr{R}^*\mathbb{E}[e_2]}{\Gamma_1}$, we get an expression for e_1:

$$e_1 = \frac{\Gamma_1 \iota_1 + \mathscr{R}^*\mathbb{E}_1\left[e_2\right]}{\Gamma_1 + 1}.$$

The flow demand equation for $t = 2$ gives $e_2 = \iota_2$, so we can rewrite e_1 as:

$$e_1 = \frac{\Gamma_1 \iota_1 + \mathscr{R}^*\mathbb{E}_1\left[\iota_2\right]}{\Gamma_1 + 1}.$$

Similarly for e_0, we have

$$e_0 = \frac{\Gamma_0 \iota_0 + \mathscr{R}^* \mathbb{E}_0 [e_1]}{\Gamma_0 + 1}$$

and we can use our expression for e_1 above to express e_0 as:

$$e_0 = \frac{\Gamma_0 \iota_0 + \mathscr{R}^* \mathbb{E}_0 \left[\frac{\Gamma_1 \iota_1 + \mathscr{R}^* \iota_2}{\Gamma_1 + 1}\right]}{\Gamma_0 + 1} \square$$

Proof of Proposition 12 We have already derived Claim 1. For Claim 2, we can calculate, from the definition of carry trade returns ($R^c \equiv \frac{R^*}{R} \frac{e_1}{e_0} - 1$) and equation (26):

$$\mathbb{E}_0 [R^c] = (\mathscr{R}^* - 1) \Gamma_0 \frac{\overline{\Gamma}_1 + 1 + \mathscr{R}^*}{\overline{\Gamma}_1 (\Gamma_0 + \mathscr{R}^*) + \Gamma_0 + (\mathscr{R}^*)^2} > 0.$$

Hence, the expected carry trade return is positive.

For Claim 3, recall that a function $\frac{ax+b}{cx+d}$ is increasing in x iff $\Delta^x \equiv ad - bc > 0$. For Γ_0,

$$\Delta^{\Gamma_0} = \left(1 + \overline{\Gamma}_1 + \mathscr{R}^*\right) \left(\overline{\Gamma}_1 \mathscr{R}^* + (\mathscr{R}^*)^2\right) > 0,$$

which proves $\frac{\partial \overline{R}^c}{\partial \Gamma_0} > 0$.

For $\overline{\Gamma}_1$, the discriminant is

$$\frac{\Delta^{\overline{\Gamma}_1}}{(\mathscr{R}^* - 1) \Gamma_0} = \Gamma_0 + (\mathscr{R}^*)^2 - (1 + \mathscr{R}^*) (\Gamma_0 + \mathscr{R}^*) = -\mathscr{R}^* (1 + \Gamma_0) < 0,$$

so that $\frac{\partial \overline{R}^c}{\partial \overline{\Gamma}_1} < 0$.

Finally, for $\mathscr{R}^*$, we simply compute:

$$\frac{\partial \overline{R}^c}{\partial \mathscr{R}^*} = \frac{\Gamma_0 (1 + \Gamma_0) (1 + \overline{\Gamma}_1) \left(2\mathscr{R}^* + \overline{\Gamma}_1\right)}{\left(\Gamma_0 \left(1 + \overline{\Gamma}_1\right) + \overline{\Gamma}_1 \mathscr{R}^* + (\mathscr{R}^*)^2\right)^2} > 0. \square$$

Proof of Proposition 13 The regression corresponds to: $\beta = \frac{-\partial \mathbb{E}\left[\frac{e_1}{e_0} - 1\right]}{\partial R^*}$. Calculations show (evaluating at $R^* = R$ for simplicity):

$$\beta = \frac{1 + \overline{\Gamma}_1 - \Gamma_0}{(1 + \Gamma_0) \left(1 + \overline{\Gamma}_1\right)}. \tag{A.26}$$

Hence, $\beta \leq \frac{1 + \overline{\Gamma}_1}{(1 + \Gamma_0)\left(1 + \overline{\Gamma}_1\right)} = \frac{1}{1 + \Gamma_0} < 1$. $\square$

A.4.2 Proofs for Appendix A.1

Proof of Proposition A.1 *Derivation of equation (A.1).* Take $\xi_t = 1$. We start from the flow equation, $NX_t - RQ_{t-1} + Q_t = 0$, and $Q_t = \frac{\mathbb{E}_t[e_t - e_{t+1}]}{\Gamma}$ (dropping the expectations for notational simplicity). Hence:

$$\Gamma (e_t - \iota_t) - R (e_{t-1} - e_t) + e_t - e_{t+1} = 0 \tag{A.27}$$

i.e.

$$e_{t+1} - (1+R+\Gamma)\, e_t + Re_{t-1} + \Gamma\iota_t = 0$$

The characteristic equation is

$$g(X) \equiv X^2 - (1+R+\Gamma)\, X + R = 0 \tag{A.28}$$

and the solutions are:

$$\lambda = \frac{(1+R+\Gamma) - \sqrt{(1+R+\Gamma)^2 - 4R}}{2}$$
$$\lambda' = \frac{R}{\lambda}$$

We define $\Lambda = \frac{1}{\lambda'}$, i.e. $\Lambda = \frac{\lambda}{R}$. The geometry of the solutions of a quadratic equation indicates (via $\lambda < \lambda'$ and $g(1) < 0 < g(0)$) that we have $0 < \lambda < 1 < \lambda'$, hence $\Lambda < \frac{1}{R}$.

Is is well-known that the solution is of the type

$$e_t = AQ_{t-1} + \mathbb{E}_t\left[\sum_{s=t}^{\infty}\left(B\left(\lambda'\right)^{t-s} + C\lambda^{t-s}\right)\iota_s\right]$$

for some constants A, B, C. Because $\lambda < 1$, we need $C = 0$, otherwise the sum would diverge. Hence, we can write:

$$e_t = AQ_{t-1} + B\mathbb{E}_t\left[\sum_{s=t}^{\infty}\Lambda^{s-t}\iota_s\right]$$

Also, when $Q_{t-1} = 0$ and $\iota_s = \iota$ for all $s \geq t$, we must have $e_t = \iota$ (indeed, then e_s is constant, so that $Q_s = 0$ and the dollar flow equation gives $e_s - \iota = 0$). That gives $1 = B\sum_{s=t}^{\infty}\Lambda^{s-t} = \frac{B}{1-\Lambda}$, i.e. $B = 1-\Lambda$. Finally, careful examination of the boundary condition at 0 (or use of the analogy $\widetilde{\iota_0} := \iota_0 + Q_0^-$) gives $AQ_{t-1} = BQ_0^-$, with $Q_0^- \equiv RQ_{-1}$. We conclude:

$$e_t = \mathbb{E}_t\left[\sum_{s=t}^{\infty}\Lambda^{s-t}(1-\Lambda)\,\iota_s\right] + (1-\Lambda)\, Q_t^-.$$

Another proof is possible, relying on the machinery in Blanchard and Kahn (1980). We sketch that alternative proof here. In terms of the Blanchard-Kahn notation, the system is

$$\mathbb{E}_t\begin{pmatrix} Q_t \\ e_{t+1}\end{pmatrix} = \begin{pmatrix} R & -1 \\ -\Gamma R & 1+\Gamma\end{pmatrix}\begin{pmatrix} Q_{t-1} \\ e_t\end{pmatrix} + \begin{pmatrix} 1 \\ -\Gamma\end{pmatrix}\iota_t$$

and the eigenvalues of the matrix are $\lambda_1 = \lambda$, $\lambda_2 = 1/\Lambda$. Then the value of e_t comes from the last equation p.1309 of their paper, with $\mu \equiv (\lambda_1 - a_{11})\,\gamma_1 - a_{12}\gamma_2$ in their notation.

Derivation of (A.2). It is enough to prove (A.2) for $t = 0$. First, the term f_s^* comes simply from using $\widetilde{\iota_s} = \iota_s - f_s^*$ and using (A.1). The more involved term concerns the interest rates. We note $\rho_t \equiv \frac{R_{t+1}^*}{R_{t+1}} - 1$ the interest rate differential. We perform a Taylor expansion in it. The financiers' demand satisfies:

$$
\begin{aligned}
\Gamma Q_t &= \mathbb{E}_t\left[e_t - e_{t+1}\frac{R^*_{t+1}}{R_{t+1}}\right] \\
&= \mathbb{E}_t\left[e_t - e_{t+1}\left(1+\rho_t\right)\right] = \mathbb{E}_t\left[e_t - e_{t+1}\right] - \rho_t \mathbb{E}_t\left[e_{t+1}\right] \\
&= \mathbb{E}_t\left[e_t - e_{t+1}\right] - \rho_t e_* + o\left(\rho_t\right)
\end{aligned}
$$

where we approximate $\mathbb{E}_t\left[e_{t+1}\right]$ by the steady state value, e_*. For instance, $e_* = \frac{\iota_*}{\xi_*}$. Then, the Dollar-Yen balance equation $NX_t - RQ_{t-1} + Q_t = 0$ becomes (dropping for now the $o\left(\rho_t\right)$ terms):

$$
e_t - \iota_t - R\frac{\mathbb{E}_{t-1}\left[e_{t-1} - e_t\right] - \rho_{t-1}e_*}{\Gamma} + \frac{\mathbb{E}_t\left[e_t - e_{t+1}\right] - \rho_t e_*}{\Gamma} = 0
$$

hence we have the same system as before, replacing ι_t by

$$
\widetilde{\iota}_t \equiv \iota_t + \frac{e_*}{\Gamma}\left(\rho_t - R\rho_{t-1}\right) \tag{A.29}
$$

using the convention $\rho_{-1} = 0$ when we calculate e_0 (that cleans up the border conditions). Indeed, we have then:

$$
e_t - \widetilde{\iota}_t - R\frac{\mathbb{E}_{t-1}\left[e_{t-1} - e_t\right]}{\Gamma} + \frac{\mathbb{E}_t\left[e_t - e_{t+1}\right]}{\Gamma} = 0
$$

Hence, (A.1) gives (when $Q_t^- = 0$)

$$
\begin{aligned}
e_0 &= \left(1-\Lambda\right)\mathbb{E}_0\left[\sum_{s=0}^{\infty}\Lambda^s \widetilde{\iota}_s\right] = \left(1-\Lambda\right)\mathbb{E}_0\left[\sum_{s=0}^{\infty}\Lambda^s \iota_s\right] + H \\
H &= \left(1-\Lambda\right)\mathbb{E}_0\left[\sum_{s=0}^{\infty}\Lambda^s \frac{e_*}{\Gamma}\left(\rho_s - R\rho_{s-1}\right)\right] \\
&= \left(1-\Lambda\right)\frac{e_*}{\Gamma}\mathbb{E}_0\left[\sum_{s=0}^{\infty}\Lambda^s\left(1-R\Lambda\right)\rho_s\right] \text{ by rearranging (by “Abel transformation”)} \\
&= \frac{\left(1-\Lambda\right)\left(1-R\Lambda\right)}{\Gamma}e_*\mathbb{E}_0\left[\sum_{s=0}^{\infty}\Lambda^s\rho_s\right]
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\left(1-\Lambda\right)\left(1-R\Lambda\right)}{\Gamma} &= \frac{\left(1-\frac{\lambda}{R}\right)\left(1-\lambda\right)}{\Gamma} \text{ using } \Lambda = \frac{\lambda}{R} \\
&= \frac{\lambda^2 - \left(1+R\right)\lambda + R}{R\Gamma} \\
&= \frac{\Gamma\lambda}{R\Gamma} \text{ as } \lambda \text{ satisfies (A.28)} \\
&= \Lambda
\end{aligned}
$$

so

$$
H = \Lambda e_* \mathbb{E}_0\left[\sum_{s=0}^{\infty}\Lambda^s\rho_s\right]
$$

Hence,

$$
\begin{aligned}
e_0 &= (1-\Lambda)\,\mathbb{E}_0\left[\sum_{s=0}^{\infty}\Lambda^s \iota_s\right] + e_*\mathbb{E}_0\left[\sum_{s=0}^{\infty}\Lambda^{s+1}\rho_s\right] \\
&= (1-\Lambda)\,\mathbb{E}_0\left[\sum_{s=0}^{\infty}\Lambda^s \iota_s\right] + e_*\mathbb{E}_0\left[\sum_{s=1}^{\infty}\Lambda^{s}\frac{R_s^* - R_s}{R_s}\right]
\end{aligned}
$$

up to higher-order terms in $R_s^* - R_s$. $\square$

Finally, we verify that the f_s^* impact is present only when $\Gamma > 0$. The net present value of the noise traders' demand is 0: $\sum_{s=t}^{\infty}\frac{1}{R^{s-t}}f_s = -F_t^{*-}$,where F_t^{*-} is the holding of the f_s^* traders at the beginning of time t. As the US net foreign assets are $N_t = Q_t + F_t^*$, when $\Gamma = 0$ (so that $\Lambda = \frac{1}{R}$) the expression reduces (when $\iota_t = \iota_*$, $r_t^* - r_t = 0$ for simplicity):

$$
\begin{aligned}
e_t &= e_* + \mathbb{E}_t\sum_{s=t}^{\infty}\left(\frac{1}{R}\right)^{s-t}\left[\left(1-\frac{1}{R}\right)(-f_s^*)\right] + \left(1-\frac{1}{R}\right)Q_t^- \\
&= e_* + \left(1-\frac{1}{R}\right)\left(Q_t^- + F_t^{*-}\right) = e_* + \left(1-\frac{1}{R}\right)N_t
\end{aligned}
$$

hence the actions of the noise traders don't have an effect when $\Gamma = 0$.

Appendix References

Blanchard, Olivier, and Charles Kahn. 1980. "The solution of linear difference models under rational expectations." *Econometrica*, 1305–1311.