Informational Frictions and Commodity Markets*

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October 2013

Abstract

This paper develops a model to analyze information aggregation in commodity markets. Through centralized trading, commodity prices aggregate dispersed information about the strength of the global economy among goods producers whose production has complementarity, and serve as price signals to guide producers' production decisions and commodity demand. Our analysis highlights important feedback effects of informational noise originating from supply shocks and futures market trading on commodity demand and spot prices, which are ignored by existing empirical studies and policy discussions.

*This paper supersedes an earlier paper circulated under the title "Feedback Effects of Commodity Futures Prices." We wish to thank Thierry Foucault, Lutz Kilian, Jennifer La'O, Matteo Maggiori, Joel Peress, Ken Singleton, Kathy Yuan, and seminar participants at Asian Meeting of Econometric Society, Bank of Canada, Chicago, Columbia, Emory, HEC-Paris, INSEAD, NBER Meeting on Economics of Commodity Markets, Princeton, the 6th Annual Conference of the Paul Woolley Center of London School of Economics, and Western Finance Association Meetings for helpful discussion and comments. We are especially grateful to Bruno Biais, an Associate Editor, and three referees for many constructive suggestions.

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The boom and bust cycles of commodity markets in 2007-2008 have stimulated intense academic and policy debate regarding the effects of supply and demand shocks and the role of futures market speculation. Despite the attention given to these issues, the academic literature largely ignores a key aspect of commodity markets—informational frictions—by treating different types of shocks as observable to market participants. The markets for key industrial commodities, such as crude oil and copper, have become globalized in recent decades, with supply and demand now stemming from across the world. This globalization exposes market participants, who face significant informational frictions regarding the global supply, demand, and inventory of these commodities, to heightened uncertainty as to the strength of the global economy. In such an environment, commodity prices often serve as important price signals regarding the strength of the global economy for market participants.[1]

This important informational role of commodity prices motivates several conceptual questions not yet addressed by the existing economic literature: How do commodity markets aggregate information about the global economy? How do informational frictions affect commodity prices and demand? In response to significant commodity price volatility in recent years, policy circles have been concerned with speculation in commodity futures markets. How does trading in futures markets affect spot prices and demand in the presence of informational frictions?

This paper develops a theoretical framework to address these questions. Our framework integrates the standard models of asset market trading with asymmetric information, e.g., Grossman and Stiglitz (1980) and Hellwig (1980), with an international macro setting, e.g., Obstfeld and Rogoff (1996) and Angeletos and La'O (2013). In this global economy, a continuum of specialized goods producers whose production has complementarity - which emerges from their need to trade produced goods with each other - demand a key commodity, such as copper, as a common production input. Through trading the commodity, the goods producers aggregate dispersed information regarding unobservable global economic strength, which ultimately determines their demand for the commodity. We start with a baseline model with only a spot market for the commodity to illustrate the key mechanism for informational

[1] For example, in explaining the decision of the European Central Bank (ECB) to raise its key interest rate in March 2008 on the eve of the worst economic recession since the Great Depression, ECB policy reports cite high prices of oil and other commodities as a key factor, suggesting the significant influence of commodity prices on monetary policies. Furthermore, Hu and Xiong (2013) provide evidence that in recent years, stock prices across East Asian economies display significant and positive reactions to overnight futures price changes of a set of commodities traded in the U.S., suggesting that people across the world react to information contained in commodity futures prices.

frictions to affect commodity markets, and then extend the model to incorporate a futures market to further characterize the role of futures market trading.

Our baseline model focuses on a centralized spot market, through which the goods producers acquire the commodity from a group of suppliers, who are subject to an unobservable supply shock. The supply shock prevents the commodity price from perfectly aggregating the goods producers' information as to the global economic strength. Nevertheless, the commodity price provides a useful signal to guide the producers' production decisions and commodity demand. Despite the non-linearity in the producers' production decisions, we derive a unique log-linear, noisy rational expectations equilibrium in closed form. In this equilibrium, each producer's commodity demand is a log-linear function of its private signal and the commodity price, while the commodity price is a log-linear function of the global economic strength and supply shock. This tractability originates from a key feature: the aggregate demand of a continuum of producers remains log-linear as a result of the law of large numbers.

Through its informational role, a higher commodity price motivates each goods producer to produce more goods and thus demand more of the commodity as input, which offsets the usual cost effect of a higher price leading to a lower quantity demanded. The complementarity in production among goods producers magnifies this informational effect through their incentives to coordinate production decisions. Under certain conditions, our model shows that the informational effect can dominate the cost effect and lead to a positive price elasticity of producers' demand for the commodity.

Again through its informational role, the commodity price allows the supply shock to have a subtle feedback effect on commodity demand and price. In a perfect-information benchmark in which global economic strength and supply shocks are both observable, a higher supply shock leads to a lower commodity price and a larger quantity demanded by goods producers. However, in the presence of informational frictions, goods producers partially attribute the lower commodity price to a weaker global economy, which in turn induces them to reduce their commodity demand. This feedback effect thus further amplifies the negative price impact of the supply shock and undermines its impact on commodity demand.

To estimate the effects of supply and demand shocks in commodity markets, it is common for the empirical literature to adopt structural models that ignore informational frictions by letting agents directly observe both demand and supply shocks. As highlighted by our analysis, the price elasticity of demand and the effects of supply shocks are likely to be misspecified when informational frictions are severe.

In practice, trading commodity futures is appealing as it facilitates hedging and speculation without necessarily involving any physical delivery. However, whether and how traders in futures markets might affect commodity prices without taking or making any physical delivery remains illusive. To address this issue, our extended model incorporates a futures market, which allows one round of information aggregation among the goods producers before commodity suppliers observe their supply shock and make physical delivery in the spot market. We also introduce to the futures market a group of financial traders, who always unwind their futures position before delivery and whose aggregate futures position is subject to random noise unrelated to the commodity.

Interestingly, the futures price serves as a useful signal to the goods producers even though they also observe the spot price because the spot price and futures price are traded at different times and are subject to different noise: the spot price contains noise from commodity suppliers' supply shock, while the futures price contains noise from financial traders' futures position. As a result, the futures price is not simply a shadow of the spot price. Instead, it has its own informational effects on commodity demand and the spot price.

This result clarifies a simple yet useful notion that futures market participants, even if not involved in physical delivery, can nevertheless impact commodity markets. It also cautions against a commonly used empirical strategy based on commodity inventory to detect speculative effects, e.g., Kilian and Murphy (2010), Juvenal and Petrella (2012), and Knittel and Pindyck (2013). This strategy is premised on a widely held argument that if speculators distort the spot price of a commodity upward through futures market trading, consumers will find the commodity too expensive and thus reduce consumption, which in turn causes inventories of the commodity to spike. By assuming that consumers are able to recognize the commodity price distortion, this argument ignores realistic informational frictions faced by consumers, which are particularly relevant in times of great economic uncertainty. In contrast, our model shows that informational frictions may cause consumers to react to the distorted price by increasing rather than decreasing their consumption.

Taken together, our analysis systematically illustrates how both spot and futures prices of key industrial commodities can serve as price signals for the strength of the global economy, which in turn allows supply shocks and noise from futures markets to feed back to commodity demand and spot prices. In doing so, our analysis provides a coherent argument for how the large inflow of investment capital to commodity futures markets might have amplified the boom and bust of commodity prices in 2007-2008 by interfering with the price signals.

Our model complements the recent macro literature that analyzes the role of informational frictions on economic growth. Lorenzoni (2009) shows that by influencing agents' expectations, noise in public news can generate sizable aggregate volatility. Angeletos and La'O (2013) focus on endogenous economic fluctuations that result from the lack of centralized communication channels to coordinate the expectations of different households. Our model adopts the setting of Angeletos and La'O (2013) for the goods market equilibrium to derive endogenous complementarity in goods producers' production decisions. We analyze information aggregation through centralized commodity trading, which is absent from their model, and the feedback effects of the equilibrium commodity price.

The literature has long recognized that trading in financial markets aggregates information and the resulting prices can feed back to real world activities, e.g., Bray (1981) and Subrahmanyam and Titman (2001). More recently, the literature points out that such feedback effects can be particularly strong in the presence of strategic complementarity in agents' actions. Morris and Shin (2002) show that in such a setting, noise in public information has an amplified effect on agents' actions and thus on equilibrium outcomes. In our model, the spot and futures prices of the commodity serve such a role in feeding back noise to the goods producers' production decisions. Similar feedback effects are also modeled in several other contexts, such as from stock prices to firm capital investment decisions and from exchange rates to policy choices of central banks (e.g., Ozdenoren, and Yuan (2008), Angeletos, Lorenzoni and Pavan (2010) and Goldstein, Ozdenoren, and Yuan (2011, 2012)). The log-linear equilibrium derived in our model makes the analysis of feedback effects particularly tractable.

This paper also contributes to the emerging literature that analyzes whether the large inflow of financial investment to commodity futures markets in recent years may have affected commodity prices, e.g., Stoll and Whaley (2010), Tang and Xiong (2012), Singleton (2012), Cheng, Kirilenko, and Xiong (2012), Hamilton and Wu (2012), Kilian and Murphy (2012), and Henderson, Pearson, and Wang (2012). Building on realistic informational frictions, our model describes a specific mechanism for trading in futures markets to affect commodity demand and spot prices. This mechanism echoes Singleton (2012), which emphasizes the

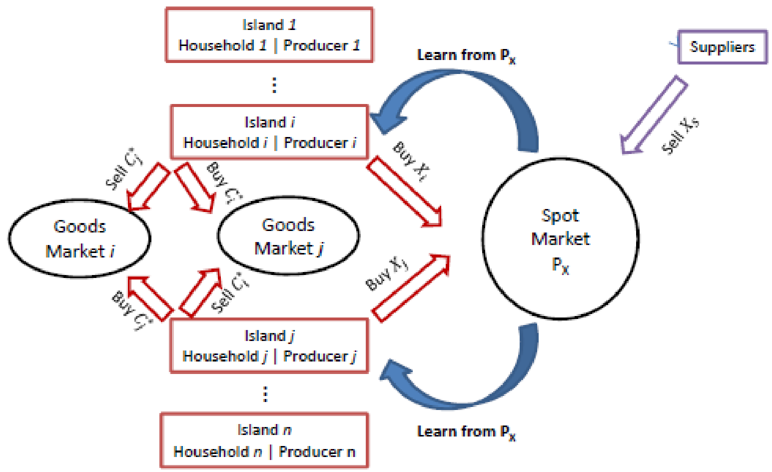

Figure 1: Structure of the Baseline Model

importance of accounting for agents' expectations in order to explain the boom and bust cycles of commodity prices in 2007-2008.

The paper is organized as follows. We first present the baseline model in Section 1, and then the extended model in Section 2. Section 3 concludes the paper. We provide all the technical proofs in the Appendix.

1 The Baseline Model

In this section we develop a baseline model with two dates $t = 1, 2$ to analyze the effects of informational frictions on the market equilibrium related to a commodity. One can think of this commodity as crude oil or copper, which is used across the world as a key production input. We adopt a modified setup of Angeletos and La'O (2013) to model a continuum of islands of total mass 1. Each island produces a single good, which can either be consumed at "home" or traded for another good produced "away" by another island. A key feature of the baseline model is that the commodity market is not only a place for market participants to trade the commodity but also a platform to aggregate private information about the strength of the global economy, which ultimately determines the global demand for the commodity.

1.1 Model setting

Figure 1 illustrates the structure of the model. There are three types of agents: households on the islands, goods producers on the islands, and a group of commodity suppliers. The goods producers trade the commodity with commodity suppliers at $t = 1$ and use the commodity to produce goods at $t = 2$. Their produced goods are distributed to the households on their respective islands at $t = 2$. The households then trade their goods with each other and consume.

1.1.1 Island households

Each island has a representative household. Following Angeletos and La'O (2013), we assume a particular structure for goods trading between households on different islands. Each island is randomly paired with another island at $t = 2$. The households on the two islands trade their goods with each other and consume both goods produced by the islands. For a pair of matched islands, we assume that the preference of the households on these islands over the consumption bundle (C_i, C_i^*), where C_i represents consumption of the "home" good while C_i^* consumption of the "away" good, is determined by a utility function $U(C_i, C_i^*)$. The utility function increases in both C_i and C_i^*. This utility function specifies all "away" goods as perfect substitutes, so that the utility of the household on each island is well-defined regardless of the matched trading partner. The households on the two islands thus trade their goods to maximize the utility of each. We assume that the utility function of the island households takes the Cobb-Douglas form

$$U(C_i, C_i^*) = \left(\frac{C_i}{1-\eta}\right)^{1-\eta} \left(\frac{C_i^*}{\eta}\right)^{\eta} \tag{1}$$

where $\eta \in [0, 1]$ measures the utility weight of the away good. A greater η means that each island values more of the away good and thus relies more on trading its good with other islands. Thus, η eventually determines the degree of complementarity in the islands' goods production.

1.1.2 Goods producers

Each island has a locally-owned representative firm to organize its goods production. We refer to each firm as a producer. The production requires the use of the commodity as an input. To focus on the commodity market equilibrium, we exclude other inputs such as labor from production. Each island has the following decreasing-returns-to-scale production function[2]:

$$Y_i = AX_i^{\phi}, \tag{2}$$

where Y_i is the output produced by island i, and X_i is the commodity input. Parameter $\phi \in (0, 1]$ measures the degree to which the production function exhibits decreasing returns to scale. When $\phi = 1$, the production function has constant returns to scale. A is the common productivity shared by all islands. For simplicity, we ignore the idiosyncratic component of each island's productivity. This simplification is innocuous for our qualitative analysis of how information frictions can affect commodity demand.

For an individual goods producer, A has a dual role—it determines its own output as well as other producers' output. To the extent that demand for the producer's good depends on other producers' output, A represents the strength of the global economy. We assume that A is a random variable, which becomes observable only when the producers complete their production at $t = 2$. This is the key informational friction in our setting. We assume that A has a lognormal distribution:

$$\log A \backsim \mathcal{N}\left(\bar{a}, \tau_A^{-1}\right)$$

where $\bar{a}$ is the mean of $\log A$ and τ_A^{-1} is its variance. At $t = 1$, the goods producer on each island observes a private signal about $\log A$:

$$s_i = \log A + \varepsilon_i$$

where $\varepsilon_i \backsim \mathcal{N}\left(0, \tau_s^{-1}\right)$ is random noise independent of $\log A$ and independent of noise in other producers' signals. τ_s is the precision of the signal. The signal allows the producer to form its expectation of the strength of the global economy, and determine its production decision and commodity demand. The commodity market serves to aggregate the private signals dispersed among the producers. As each producer's private signal is noisy, the publicly observed commodity price also serves as a useful price signal to form its expectation.

At $t = 1$, the producer on island i maximizes its expected profit by choosing its commodity input X_i:

$$\max_{X_i} \; E\left[P_i Y_i \mid \mathcal{I}_i\right] - P_X X_i \tag{3}$$

where P_i is the price of the good produced by the island. The producer's information set $\mathcal{I}_i = \{s_i, P_X\}$ includes its private signal s_i and the commodity price P_X. The goods price P_i,

[2] One can also specify a Cobb-Douglas production function with both commodity and labor as inputs. The model remains tractable although the formulas become more complex and harder to interpret.

which one can interpret as the terms of trade, is determined at $t = 2$ based on the matched trade with another island.

1.1.3 Commodity suppliers

We assume there is a group of commodity suppliers who face a convex labor cost

$$\frac{k}{1+k} e^{-\xi/k} \left(X_S\right)^{\frac{1+k}{k}}$$

in supplying the commodity. X_S is the quantity supplied, $k \in (0,1)$ is a constant parameter, and ξ represents random noise in the supply. As a key source of information frictions in our model, we assume that ξ is observable to the suppliers themselves but not by other market participants. We assume that from the perspective of goods producers, ξ has Gaussian distribution $\mathcal{N}\left(\bar{\xi}, \tau_\xi^{-1}\right)$ with $\bar{\xi}$ as its mean and τ_ξ^{-1} as its variance. The mean captures the part that is predictable to goods producers, while the variance represents uncertainty in supply that is outside goods producers' expectations.

Thus, given a spot price P_X, the suppliers face the following optimization problem:

$$\max_{X_S} \; P_X X_S - \frac{k}{1+k} e^{-\xi/k} \left(X_S\right)^{\frac{1+k}{k}}. \tag{4}$$

It is easy to determine the suppliers' optimal supply curve:

$$X_S = e^{\xi} P_X^k, \tag{5}$$

which shows ξ as uncertainty in the commodity supply and k as the price elasticity.[3]

1.1.4 Joint equilibrium of different markets

Our model features a noisy rational expectations equilibrium of a number of markets: the goods markets between each pair of matched islands and the market for the commodity. The equilibrium requires clearing of each of these markets:

[3] By letting the suppliers sell the commodity according to their marginal cost, our setting ignores any potential feedback effect from the commodity price to the supply side. In a more general setting with multiple rounds of spot market trading, suppliers (or other agents) may have incentives to store the commodity over time based on their expectations of future demand. Then, the commodity price can feed back to these agents' storage decisions. We leave an analysis of such a feedback effect on the supply side to future research and instead focus on the feedback effect on the demand side.

- At $t = 2$, for each pair of randomly matched islands $\{i, j\}$, the households of these islands trade their produced goods and clear the market for each good:

$$\begin{aligned} C_i + C_j^* &= AX_i^{\phi}, \\ C_i^* + C_j &= AX_j^{\phi}. \end{aligned}$$

- At $t = 1$, in the commodity market, the goods producers' aggregate demand equals the supply:

$$\int_{-\infty}^{\infty} X_i\left(s_i, P_X\right) d\Phi\left(\varepsilon_i\right) = X_S\left(P_X\right),$$

  where each producer's commodity demand $X_i\left(s_i, P_X\right)$ depends on its private signal s_i and the commodity price P_X. The demand from producers is integrated over the noise ε_i in their private signals.

1.2 The equilibrium

1.2.1 Goods market equilibrium

We begin our analysis of the equilibrium with the goods markets at $t = 2$. For a pair of randomly matched islands, i and j, the representative household of island i possesses Y_i units of the good produced by the island while the representative household of island j holds Y_j units of the other good.[4] They trade the two goods with each other to maximize the utility function of each given in (1). The following proposition, which resembles a similar proposition in Angeletos and La'O (2013), describes the goods market equilibrium between these two islands.

Proposition 1 *For a pair of randomly matched islands, i and j, their representative households' optimal consumption of the two goods is*

$$C_i = (1-\eta) Y_i, \; C_i^* = \eta Y_j, \; C_j = (1-\eta) Y_j, \; C_j^* = \eta Y_i.$$

The price of the good produced by island i is

$$P_i = \left(\frac{Y_j}{Y_i}\right)^{\eta}. \tag{6}$$

[4] Here we treat a representative household as representing different agents holding stakes in an island's goods production, such as workers, managers, suppliers of inputs, etc. We agnostically group their preferences for the produced goods of their own island and other islands into the preferences of the representative household.

Proposition 1 shows that each household divides its consumption between the home and away goods with fractions $1-\eta$ and η, respectively. When $\eta = 1/2$, the household consumes the two types of goods equally. The price of each good is determined by the relative output of the two matched islands.[5] One island's good is more valuable when the other island produces more. This feature is standard in the international macroeconomics literature (e.g., Obstfeld and Rogoff (1996)) and implies that each goods producer needs to take into account the production decisions of producers of other goods.[6]

1.2.2 Production decision and commodity demand

By substituting the production function in (2) into (3), the expected profit of the goods producer on island i, we obtain the following objective:

$$\max_{X_i} \; E\left[A P_i X_i^{\phi} \middle| s_i, P_X \right] - P_X X_i.$$

In a competitive goods market, the producer will produce to the level that the marginal revenue equals the marginal cost:

$$\phi E\left[A P_i \middle| s_i, P_X \right] X_i^{\phi-1} = P_X.$$

By substituting in P_i from Proposition 1, we obtain

$$X_i = \left\{ \frac{\phi E\left[A X_j^{\phi\eta} \middle| s_i, P_X \right]}{P_X} \right\}^{1/(1-\phi(1-\eta))} \tag{7}$$

which depends on the producer's expectation $E\left[A X_j^{\phi\eta} \middle| s_i, P_X \right]$ regarding the product of global productivity A and the production decision $X_j^{\phi\eta}$ of its randomly matched trading partner, island j. This expression demonstrates the complementarity in the producers' production decisions. A larger η makes the complementarity stronger as the island households engage more in trading the produced goods with each other and the price of each good depends more on the output of other goods.

[5] The goods price P_i given in (6) is the price of good i normalized by the price of good j produced by the other matched island.

[6] Decentralized goods market trading is not essential to our analysis. This feature allows us to conveniently capture endogenous complementarity in goods producers' production decisions with tractability. Alternatively, one can adopt centralized goods markets and let island households consume goods produced by all producers. See Angeletos and La'O (2009) for such a setting. We expect our key insight to carry over to this alternative setting.

The commodity price P_X is a source of information for the producer to form its expectation of $E\left[AX_j^{\phi\eta}\middle| s_i, P_X\right]$, which serves as a channel for the commodity price to feed back into each producer's commodity demand. The presence of complementarity strengthens this feedback effect relative to standard models of asset market trading with asymmetric information.

1.2.3 Commodity market equilibrium

By clearing the aggregate demand of goods producers with the supply of suppliers, we derive the commodity market equilibrium. Despite the nonlinearity in each producer's production decision, we obtain a unique log-linear equilibrium in closed form. The following proposition summarizes the commodity price and each producer's commodity demand in this equilibrium.

Proposition 2 *At $t = 1$, the commodity market has a unique log-linear equilibrium: 1) The commodity price is a log-linear function of $\log A$ and ξ:*

$$\log P_X = h_A \log A + h_\xi \xi + h_0, \tag{8}$$

with the coefficients h_A, and h_ξ given by

$$h_A = -\frac{(1-\phi)\, b + (1-\phi(1-\eta))\, \tau_s^{-1}\tau_\xi b^3}{1 + k(1-\phi)} > 0, \tag{9}$$

$$h_\xi = -\frac{1-\phi + (1-\phi(1-\eta))\, \tau_s^{-1}\tau_\xi b^2}{1 + k(1-\phi)} < 0, \tag{10}$$

where $b < 0$ is given in equation (40), and h_0 given in equation (41).

2) The commodity purchased by goods producer i is a log-linear function of its private signal s_i and $\log P_X$:

$$\log X_i = l_s s_i + l_P \log P_X + l_0, \tag{11}$$

with the coefficients l_s, and l_P given by

$$l_s = -b > 0, \;\; l_P = k + h_\xi^{-1}, \tag{12}$$

and l_0 by equation (42).

Proposition 2 shows that each producer's commodity demand is a log-linear function of its private signal and the commodity price, while the commodity price $\log P_X$ aggregates the producers' dispersed private information to partially reveal the global productivity $\log A$.

The commodity price does not depend on any producer's signal noise as a result of the aggregation across a large number of producers with independent noise. This feature is similar to Hellwig (1980). The commodity price also depends on the supply side noise ξ, which serves the same role as noise trading in the standard models of asset market trading with asymmetric information.

It is well-known that asset market equilibria with asymmetric information are often intractable due to the difficulty in aggregating different participants' positions. The existing literature commonly adopts the setting of Grossman and Stiglitz (1980) and Hellwig (1980), which features CARA utility for agents and normal distributions for asset fundamentals and noise trading. Under this setting, the equilibrium asset price is a linear function of the asset fundamental and the noise from noise trading, while each agent's asset position is a linear function of the price and his own signal. This setting is, however, unsuitable for analyzing real consequences of asset market trading as agents' investment and production decisions tend to make asset fundamentals deviate from normal distributions.

The log-linear equilibrium derived in Proposition 2 resembles the linear equilibrium of Grossman and Stiglitz (1980) and Hellwig (1980) but nevertheless incorporates real consequences of commodity market trading. In fact, each producer's commodity demand has a log-normal distribution (e.g., equation (11)). As shown by equation (33) in the Appendix, the producers' aggregate demand remains log-normal as a result of the law of large numbers. This is the key feature that ensures the tractability of our model.[7]

1.3 Effects of informational frictions

1.3.1 Perfect-information benchmark

To facilitate our analysis of the effects of informational frictions, we first establish a benchmark without any informational friction. Suppose that the global fundamental A and commodity supply shock ξ are both observable by all market participants. Then, the goods producers can choose their optimal production decisions without any noise interference. The following proposition characterizes this benchmark.

[7] It is also worth noting that our setting is different from the setting of Goldstein, Ozdenoren, and Yuan (2012). Their model features stock market trading with asymmetric information and a feedback effect from the equilibrium stock price to firm investment. While the equilibrium stock price is non-linear, they ensure tractability by assuming each trader in the asset market is risk-neutral and faces upper and lower position limits. Our model does not impose any position limit and instead derives each producer's futures position through his interior production choice.

Proposition 3 *When both A and ξ are observed by all market participants, there is a unique equilibrium. In this equilibrium: 1) the goods producers share an identical commodity demand curve: $X_i = X_j = \left(\frac{\phi A}{P_X}\right)^{\frac{1}{1-\phi}}$, $\forall i$ and j; 2) the commodity price is given by*

$$\log P_X = \frac{1}{1+k(1-\phi)} \log A - \frac{1-\phi}{1+k(1-\phi)} \xi + \frac{1}{1+k(1-\phi)} \log \phi,$$

and the aggregate quantity demanded by the goods producers is

$$\log X_S = \frac{k}{1+k(1-\phi)} \log A + \frac{1}{1+k(1-\phi)} \xi + \frac{k}{1+k(1-\phi)} \log \phi.$$

In the absence of any informational frictions, the benchmark features a unique equilibrium despite the complementarity in the goods producers' production decisions because competition between goods producers leads to a downward sloping demand curve for the commodity. This demand curve intersects the suppliers' upward sloping supply curve at the unique commodity price P_X given in the proposition. As a result, the complementarity between goods producers does not lead to multiple equilibria, in which goods producers coordinate on certain high or low demand levels.

Proposition 3 derives the equilibrium commodity price and aggregate quantity demanded. Intuitively, the global fundamental $\log A$ increases both the commodity price and aggregate quantity demanded, while the supply shock ξ reduces the commodity price but increases aggregate quantity demanded.

The following proposition compares the equilibrium derived in Proposition 2 with the perfect-information benchmark.

Proposition 4 *In the presence of informational frictions, coefficients $h_A > 0$ and $h_\xi < 0$ derived in Proposition 2 are both lower than their corresponding values in the perfect-information benchmark, and converge to these values as $\tau_s \to \infty$.*

In the presence of informational frictions, the commodity price deviates from that in the perfect-information benchmark, with the supply shock having a greater price impact (i.e., h_ξ being more negative) and the global fundamental having a smaller impact (i.e., h_A being less positive). Through these price impacts, informational frictions eventually affect goods producers' production decisions and island households' goods consumption, which we analyze step-by-step below.

1.3.2 Price informativeness

In the presence of informational frictions, the equilibrium commodity price $\log P_X = h_A \log A + h_\xi \xi + h_0$ serves as a public signal of the global fundamental $\log A$. This price signal is contaminated by the presence of the supply noise ξ. The informativeness of the price signal is determined by the ratio of the contributions to the price variance of $\log A$ and ξ:

$$\pi = \frac{h_A^2/\tau_A}{h_\xi^2/\tau_\xi}.$$

The following proposition characterizes how the price informativeness measure π depends on several key model parameters: τ_s, τ_ξ, and η.

Proposition 5 *π is monotonically increasing in τ_s and τ_ξ and decreasing in η.*

As τ_s increases, each goods producer's private signal becomes more precise. The commodity price aggregates the goods producers' signals through their demand for the commodity and therefore becomes more informative. τ_ξ measures the amount of noise in the supply shock. As τ_ξ increases, there is less noise from the supply side interfering with the commodity price reflecting $\log A$. Thus the price also becomes more informative.

The effect of η is more subtle. As η increases, there is greater complementarity in each goods producer's production decision. Consistent with the insight of Morris and Shin (2002), such complementarity induces each producer to put a greater weight on the publicly observed price signal and a smaller weight on its own private signal, which in turn makes the equilibrium price less informative.

1.3.3 Price elasticity

The coefficient l_P, derived in (12), measures the price elasticity of each goods producer's commodity demand. The standard cost effect suggests that a higher price leads to a lower quantity demanded. The producer's optimal production decision in equation (7), however, also indicates a second effect through the term in the numerator—a higher price signals a stronger global economy and greater production by other producers. This informational effect motivates each producer to increase its production and thus demand more of the commodity. The price elasticity l_P nets these two offsetting effects. The following proposition shows that under certain necessary and sufficient conditions, the informational effect dominates the cost effect and leads to a positive l_P.

Proposition 6 *Two necessary and sufficient conditions ensure that $l_P > 0$: first*

$$\tau_\xi / \tau_A > 4k^{-1}\left(1 - \phi + k^{-1}\right);$$

and, second, parameter η within a range

$$1 - \frac{1}{\phi} + \frac{k\tau_\xi\tau_s}{4\phi\tau_A^2}\left(1-\rho\right)^2 < \eta \; < 1 - \frac{1}{\phi} + \frac{k\tau_\xi\tau_s}{4\phi\tau_A^2}\left(1+\rho\right)^2,$$

where $\rho = \tau_A^{1/2}\tau_\xi^{-1/2}\sqrt{\tau_\xi/\tau_A - 4k^{-1}\left(1 - \phi + k^{-1}\right)}$.

In order for the informational effect to be sufficiently strong, the commodity price has to be sufficiently informative. The conditions in Proposition 6 reflect this observation. First, the supply noise needs to be sufficiently small (i.e., τ_ξ sufficiently large relative to τ_A) so that the price can be sufficiently informative. Second, η needs to be within an intermediate range, which results from two offsetting forces. On one hand, a larger η implies greater complementarity in producers' production decisions and thus each producer cares more about other producers' production decisions and assigns a greater weight on the public price signal in its own decision making. On the other hand, a larger η also implies a less informative price signal (Proposition 5), which in turn motivates each producer to be less responsive to the price. Netting out these two forces dictates that η needs to be in an intermediate range in order for $l_P > 0$.[8]

This second condition implies that when $\eta = 0$, $l_P < 0$. In other words, in the absence of production complementarity, the price elasticity is always negative, i.e., the cost effect always dominates the informational effect.

1.3.4 Feedback effect on demand

In the perfect-information benchmark (Proposition 3), the supply shock ξ decreases the commodity price and increases the aggregate quantity demanded through the standard cost effect. In the presence of informational frictions, however, the supply shock, by distorting the price signal, has a more subtle effect on commodity demand.

By substituting equation (8) into (11), the commodity demand of producer i is

$$\log X_i = l_s s_i + l_P h_A \log A + l_P h_\xi \xi + l_P h_0 + l_0.$$

[8] Upward sloping demand for an asset may also arise from other mechanisms even in the absence of informational frictions highlighted in our model, such as income effects, complementarity in production, and complementarity in information production (e.g., Hellwig, Kohls and Veldkamp (2012)).

Then, the producers' aggregate commodity demand is

$$\log\left[\int_{-\infty}^{\infty} X_i\left(s_i, P_X\right) d\Phi\left(\varepsilon_i\right)\right] = l_P h_\xi \xi + \left(l_s + l_P h_A\right)\log A + l_0 + l_P h_0 + \frac{1}{2} l_s^2 \tau_s^{-1}.$$

Note that $h_\xi < 0$ (Proposition 2) and the sign of l_P is undetermined (Proposition 6). Thus, the effect of ξ on aggregate demand is also undetermined.

Under the conditions given in Proposition 6, an increase in ξ decreases the aggregate quantity demanded, which is the opposite of the perfect-information benchmark. This effect arises through the informational channel. As ξ rises, the commodity price falls. Since goods producers cannot differentiate a price decrease caused by ξ from one caused by a weaker global economy, they partially attribute the reduced price to a weaker economy. This, in turn, motivates them to cut the quantity of the commodity they demand. Under the conditions given in Proposition 6, this informational effect is sufficiently strong to dominate the effect of a lower cost to acquire the commodity, leading to a lower aggregate quantity of the commodity demanded.

Furthermore, through its informational effect on aggregate demand, ξ can further push down the commodity price, in addition to its price effect in the perfect-information benchmark. This explains why h_ξ is more negative in this economy than in the benchmark (Proposition 4): informational frictions amplify the negative price impact of ξ.[9]

1.3.5 Social welfare

By distorting the commodity price and aggregate demand, informational frictions in turn distort producers' production decisions and households' goods consumption. We now evaluate the unconditional expected social welfare at time 1:

$$W = E\left[\int_0^1 \left(\frac{C_i}{1-\eta}\right)^{1-\eta}\left(\frac{C_i^*}{\eta}\right)^{\eta} di\right] - E\left[\frac{k}{1+k} e^{-\xi/k} X_S^{\frac{1+k}{k}}\right],$$

which contains two parts: the first part comes from aggregating the expected utility from goods consumption of all island households and the second part comes from the commodity suppliers' cost of supplying labor.

[9] One can also evaluate this informational feedback effect of the supply noise by comparing the equilibrium commodity price relative to another benchmark case, in which each goods producer makes his production decision based on only his private signal s_i without conditioning on the commodity price P_X. In this benchmark, the commodity price $\log P_X$ is also a log-linear function of $\log A$ and ξ. Interestingly, despite the presence of informational frictions, the price coefficient on ξ is $-\frac{1-\phi}{1+k(1-\phi)}$, the same as that derived in Proposition 3 for the perfect-information benchmark. This outcome establishes the informational feedback mechanism as the driver for h_ξ to be more negative than that in the perfect-information benchmark.

The next proposition proves that informational frictions reduce the expected social welfare relative to the perfect-information benchmark.

Proposition 7 *In the presence of informational frictions, the expected social welfare is strictly lower than that in the perfect-information benchmark.*

1.3.6 Implications for structural models

The feedback effect of supply shock on commodity demand has important implications for studies of the effects of supply and demand shocks in commodity markets. For example, Hamilton (1983) emphasizes that disruptions to oil supply and the resulting oil price increases can have a significant impact on the real economy, while Kilian (2009) argues that aggregate demand shocks have a bigger impact on the oil market than previously thought. As supply and demand shocks have opposite effects on oil prices, it is important to isolate their respective effects. The existing literature commonly uses structural models to decompose different types of shocks and then estimate their effects, e.g., Kilian (2009). The premise of these structural models is that, while researchers cannot directly observe the shocks that hit commodity markets, agents in the markets are able to perfectly observe the shocks and optimally respond to them. As a result, by imposing certain restrictions on how different types of shocks affect the price of a commodity and its demand, researchers can infer realized shocks from observing the price and quantity of commodity transactions.

As we discussed before, it is unrealistic to assume that agents in commodity markets can perfectly differentiate different types of shocks. Our model shows that in the presence of informational frictions, supply shocks and demand shocks can have effects in sharp contrast to standard intuitions developed from perfect-information settings. For example, the price elasticity of commodity demand can be positive rather than negative, and supply shocks can reduce rather than increase demand for the commodity. These implications challenge identification restrictions commonly used in the existing structural models, such as the price elasticity of demand being negative and supply shocks having a positive impact on demand. Furthermore, our model shows that, by ignoring informational frictions, standard structural models are likely to underestimate the price impact of supply shocks.

Taken together, our model motivates structural models to explicitly build in informational frictions in order to systematically isolate effects of supply and demand shocks.

2 An Extended Model with Futures

In practice, spot markets of commodities are typically decentralized, while centralized trading often occurs in futures markets. As a result, futures markets play an important role in aggregating information regarding supply and demand of many commodities.[10] In this section, we extend our baseline model to incorporate a futures market. This extension allows us to examine how commodity futures prices can serve as price signals even when goods producers also observe spot prices.

2.1 Model setting

We again keep our extension to a minimal setting for analyzing the role of futures market trading in aggregating information. Specifically, we introduce a new date $t = 0$ before the two dates $t = 1$ and 2 in the baseline model, and a centralized futures market at $t = 0$ for delivery of the commodity at $t = 1$. All agents can take positions in the futures market at $t = 0$, and can choose to revise or unwind their positions before delivery at $t = 1$. The flexibility to unwind positions before delivery is an advantage that makes futures market trading appealing in practice.

We keep all of the agents in the baseline model: island households, goods producers, and commodity suppliers and add a group of financial traders. These traders invest in the commodity by taking a long position in the futures market at $t = 0$ and then unwinding this position at $t = 1$ without taking delivery.

To focus on information aggregation through trading in the futures market, we assume that there is no spot market trading at $t = 0$. At $t = 1$, a spot market naturally emerges through commodity delivery for the futures market. Commodity suppliers take a short position in the futures market at $t = 0$ and then make delivery at $t = 1$. The suppliers' marginal cost of supplying the commodity determines the spot price. When a trader chooses to unwind a futures position at $t = 1$, his gain/loss is determined by this spot price.

[10] Roll (1984) systematically analyzes the futures market of orange juice in efficiently aggregating information about weather in Central Florida, which produces more than 98% of the U.S. orange output. Garbade and Silber (1983) provides evidence that futures markets play a more important role in information discovery than cash markets for a set of commodities.

Table 1. Time Line of the Extended Model

	t=0	t=1	t=2
	futures market	spot market	goods market
Households			trade/consume goods
Producers	observe signals long futures	take delivery produce goods	
Com Suppliers	short futures	observe supply shock deliver commodity	
Fin Traders	long/short futures	unwind position	

Table 1 specifies the timeline of the extended model. We keep the same specification for the island households, who trade and consume both home and away goods at $t = 2$ as described in Section 1.1. We modify some of the specifications for goods producers and commodity suppliers and describe our specifications for financial traders below.

2.1.1 Goods producers

As in the baseline model, we allow goods producers to have the same production technology and receive their private signals at $t = 0$. Each producer takes a long position in the futures market at $t = 0$ and then commodity delivery at $t = 1$. The timing of the producer's information flow is key to our analysis. At $t = 0$, producer i's information set $\mathcal{I}_i^0 = \{s_i, F\}$ includes its private signal s_i and the traded futures price F. At $t = 1$, its information set $\mathcal{I}_i^1 = \{s_i, F, P_X\}$ includes the updated spot price P_X.

We allow the producer to use its updated information set at $t = 1$ to revise its futures position for commodity delivery. That is, its production decision is based on not only its private signal and the futures price but also the updated spot price. Thus, it is not obvious that noise in the futures market can affect the producer's production decision and commodity demand. We will examine this key issue with our extended model.

At $t = 1$, the producer optimizes its production decision X_i (i.e., commodity demand) based on its updated information set $\mathcal{I}_i^1$:

$$\max_{X_i} \; E\left[P_i Y_i | \, \mathcal{I}_i^1\right] - P_X X_i + (P_X - F)\tilde{X}_i.$$

The first two terms above represent the producer's expected profit from goods production and the last term is gain/loss from its futures position. Then, the producer's optimal production decision is

$$X_i = \left\{ \phi E \left[A X_j^{\phi\eta} \middle| \mathcal{I}_i^1 \right] \Big/ P_X \right\}^{1/(1-\phi(1-\eta))}. \tag{13}$$

When deciding its futures position at $t = 0$, the producer faces a nuanced issue in that, because it does not need to commit its later production decision to the initial futures position, it may engage in dynamic trading. In other words, it could choose a futures position to maximize its expected trading profit at $t = 0$. This trading motive is not essential for our focus on analyzing aggregation of the producers' information but significantly complicates derivation of the futures market equilibrium. To avoid this complication, we make a simplifying assumption that the producers are myopic at $t = 0$. That is, at $t = 0$, each producer chooses a futures position as if it commits to taking full delivery and using the good for production, even though the producer can revise its production decision based on the updated information at $t = 1$. This simplifying assumption, while it affects each producer's trading profit, is innocuous for our analysis of how the futures price feeds back to the producers' later production decisions because each producer still makes good use of its information and the futures price is informative by aggregating each producer's information.

Specifically, at $t = 0$ the producer chooses a futures position $\tilde{X}_i$ to maximize the following expected production profit based on its information set $\mathcal{I}_i^0$:

$$\max_{\tilde{X}_i} \; E \left[P_i Y_i \middle| \mathcal{I}_i^0 \right] - F \tilde{X}_i,$$

where it treats $\tilde{X}_i$ as its production input at $t = 1$. Throughout the rest of the paper, we use a tilde sign to denote variables and coefficients associated with the futures market at $t = 0$. We maintain the same notations without the tilde sign for variables related to the spot market at $t = 1$. Then, the producer's futures position is

$$\tilde{X}_i = \left\{ \phi E \left[A \tilde{X}_j^{\phi\eta} \middle| \mathcal{I}_i^0 \right] \Big/ F \right\}^{1/(1-\phi(1-\eta))}. \tag{14}$$

2.1.2 Financial traders

Since the mid-2000s, commodity futures markets experienced a large expansion of financial traders as a result of a financialization process through which commodity futures became a new asset class for portfolio investors such as pension funds and endowments (e.g., Tang and Xiong, 2012). These investors regularly allocate a fraction of their portfolios to investing in commodity futures and swap contracts. They take only long positions and typically close out positions without taking any physical delivery. As a result, their trading does not directly affect the supply and demand of commodities. During the same period, hedge funds also expanded their trading operations in commodity futures markets. They are flexible in taking both long and short positions and usually do not hold physical commodities, even though they are allowed to in practice.

To examine whether these financial traders can affect commodity prices, we introduce a group of financial traders, who trade in the futures market at $t = 0$ and unwind their position at $t = 1$ before delivery. For simplicity, we assume that the aggregate position of financial traders and goods producers is given by the aggregate position of producers multiplied by a factor $e^{\kappa \log A + \theta}$:

$$e^{\kappa \log A+\theta} \int_{-\infty}^{\infty} \tilde{X}_i\left(s_i, F\right) d\Phi\left(\varepsilon_i\right),$$

where the factor $e^{\kappa \log A+\theta}$ represents the contribution of financial traders. This multiplicative specification is useful for ensuring the tractable log-linear equilibrium of our model.[11]

We allow the contribution of financial traders $e^{\kappa \log A+\theta}$ to contain a component $\kappa \log A$ with $\kappa > 0$ to capture the possibility that the trading of financial traders is partially driven by their knowledge of the global fundamental $\log A$.

The trading of financial traders also contains a random component θ, which is unobservable by other market participants. This assumption is realistic in two aspects. First, in practice, the trading of financial traders is often driven by portfolio diversification and risk-control purposes unrelated to fundamentals of commodity markets. Second, market participants cannot directly observe others' positions.[12] Specifically, we assume that θ has a normal distribution independent of other sources of uncertainty in the model:

$$\theta \backsim \mathcal{N}\left(\overline{\theta}, \tau_\theta^{-1}\right)$$

with a mean of $\overline{\theta}$ and variance τ_θ^{-1}.

[11] From an economic perspective, this specification implies that the position of financial traders tends to expand and contract with the producers' futures position, which is broadly consistent with the expansion and contraction of the aggregate commodity futures positions of portfolio investors and hedge funds in the recent commodity price boom-and-bust cycle (e.g., Cheng, Kirilenko, and Xiong, 2012). Also note that $e^{\kappa \log A+\theta}$ can be less than one. This implies that financial traders may take a net short position at some point, which is consistent with short positions taken by hedge funds in practice.

[12] Despite the fact that large traders need to report their futures positions to the CFTC on a daily basis, ambiguity in trader classification and netting of positions taken by traders who are involved in different lines of business nevertheless make the aggregate positions provided by the CFTC's weekly Commitment of Traders Report to the public imprecise. See Cheng, Kirilenko, and Xiong (2012) for a more detailed discussion of the trader classification and netting problems in the CFTC's Large Trader Reporting System and a summary of positions taken by commodity index traders and hedge funds.

The presence of financial traders introduces an additional source of uncertainty to the futures market, as both goods producers and commodity suppliers cannot observe θ at $t = 0$. At $t = 1$, financial traders unwind their positions, and commodity suppliers make delivery only to goods producers.

2.1.3 Commodity suppliers

Commodity suppliers take a short position of $\tilde{X}_S$ in the futures market at $t = 0$ and then make delivery of X_S units of the commodity at $t = 1$. We keep the same convex cost function for the suppliers: $\frac{k}{1+k} e^{-\xi/k} (X_S)^{\frac{1+k}{k}}$, where the supply shock ξ has a Gaussian distribution $\mathcal{N}\left(\bar{\xi}, \tau_{\xi}^{-1}\right)$.

We assume that the suppliers observe their supply shock ξ only at $t = 1$, which implies that the supply shock does not affect the futures price at $t = 0$ and instead hits the spot market at $t = 1$. Due to this timing, the supply shock provides a camouflage for the unwinding of financial traders' aggregate futures position at $t = 1$. That is, even after financial traders unwind their position, the commodity spot price does not reveal their position.[13]

In summary, the suppliers' information set at $t = 0$ is $\mathcal{I}_S^0 = \{F\}$, and at $t = 1$ is $\mathcal{I}_S^1 = \{F, P_X, \xi\}$. At $t = 1$, the suppliers face the following optimization problem:

$$\max_{X_S} P_X X_S - \frac{k}{1+k} e^{-\xi/k} X_S^{\frac{1+k}{k}} + (F - P_X)\, \tilde{X}_S,$$

where they choose X_S, the quantity of commodity delivery, to maximize the profit from delivery in the first two terms. The last term is the gain/loss from their initial futures position. It is easy to determine the suppliers' optimal supply curve: $X_S = e^{\xi} P_X^k$, which is identical to their supply curve in the baseline model.

At $t = 0$, like the goods producers, the suppliers also face a nuanced issue related to dynamic trading. As their initial futures position does not necessarily equal their later commodity delivery, they may also choose to maximize the trading profit from $t = 0$ to $t = 1$. To be consistent with our earlier assumption about the myopic behavior of goods producers, we assume that at $t = 0$ the suppliers believe that goods producers will take full delivery of their futures positions and that the suppliers choose their initial short position to myopically

[13] This timing may appear special in our static setting with only one round of futures market trading followed by physical commodity delivery, as there is no particular reason to argue whether letting the suppliers observe the supply shock at $t = 0$ or 1 is more natural. However, if we view this setting as one module of a more realistic setting with many recurrent periods and a supply shock arriving in each period, then there is always a supply shock hitting the market when financial traders unwind their futures position.

maximize the profit from making delivery of $e^{-(\kappa\log A+\theta)}\tilde{X}_S$ units of the commodity to goods producers:

$$\max_{\tilde{X}_S} E\left[Fe^{-(\kappa\log A+\theta)}\tilde{X}_S\middle|\ \mathcal{I}_S^0\right]-E\left[\frac{k}{1+k}e^{-\xi/k}\left(e^{-(\kappa\log A+\theta)}\tilde{X}_S\right)^{\frac{1+k}{k}}\middle|\ \mathcal{I}_S^0\right].$$

Since ξ is independent of θ and $\log A$, it is easy to derive that

$$\tilde{X}_S=e^{\bar{\xi}-\sigma_\xi^2/2k}\left\{E\left[e^{-(\kappa\log A+\theta)}\middle|\ \mathcal{I}_S^0\right]/E\left[e^{-\frac{1+k}{k}(\kappa\log A+\theta)}\middle|\ \mathcal{I}_S^0\right]\right\}^kF^k. \tag{15}$$

which is a function of the futures price F.

2.1.4 Joint equilibrium of different markets

We analyze the joint equilibrium of a number of markets: the goods markets between each pair of matched islands at $t=2$, the spot market for the commodity at $t=1$, and the futures market at $t=0$. The equilibrium requires clearing of each of these markets:

- At $t=2$, for each pair of randomly matched islands $\{i,j\}$, the households of these islands trade their produced goods and clear the market of each good:

$$\begin{aligned} C_i+C_j^* &= AX_i^\phi, \\ C_i^*+C_j &= AX_j^\phi. \end{aligned}$$

- At $t=1$, the commodity supply equals the goods producers' aggregate demand:

$$\int_{-\infty}^{\infty}X\left(s_i,F,P_X\right)d\Phi\left(\varepsilon_i\right)=X_S\left(P_X,\xi\right).$$

- At $t=0$, the futures market clears:

$$e^{\kappa\log A+\theta}\int_{-\infty}^{\infty}\tilde{X}_i\left(s_i,F\right)d\Phi\left(\varepsilon_i\right)=\tilde{X}_S\left(F\right).$$

2.2 The equilibrium

The goods market equilibrium at $t=2$ remains identical to that derived in Proposition 1 for the baseline model. The futures market equilibrium at $t=0$ and the spot market equilibrium at $t=1$ also remain log-linear and can be derived in a similar procedure as the derivation of Proposition 2. The following proposition summarizes the key features of the equilibrium with explicit expressions for all coefficients given in the Appendix.

Proposition 8 *At $t = 0$, the futures market has a unique log-linear equilibrium: The futures price is a log-linear function of $\log A$ and θ:*

$$\log F = \tilde{h}_A \log A + \tilde{h}_\theta \theta + \tilde{h}_0, \tag{16}$$

with the coefficients $\tilde{h}_A > 0$, and $\tilde{h}_\theta > 0$, while the long position taken by goods producer i is a log-linear function of its private signal s_i and $\log F$:

$$\log \tilde{X}_i = \tilde{l}_s s_i + \tilde{l}_F \log F + \tilde{l}_0, \tag{17}$$

with the coefficient $\tilde{l}_s > 0$.

At $t = 1$, the spot market also has a unique log-linear equilibrium: The spot price of the commodity is a log-linear function of $\log A$, $\log F$, and ξ:

$$\log P_X = h_A \log A + h_F \log F + h_\xi \xi + h_0, \tag{18}$$

with the coefficients $h_A > 0$, $h_F > 0$, and $h_\xi < 0$, while the commodity consumed by producer i is a log-linear function of s_i, $\log F$, and $\log P_X$:

$$\log X_i = l_s s_i + l_F \log F + l_P \log P_X + l_0, \tag{19}$$

with the coefficients $l_s > 0$ and $l_F > 0$, and the sign of l_P undetermined.

There are two rounds of information aggregation in the equilibrium. During the first round of trading in the futures market at $t = 0$, goods producers take long positions based on their private signals. The futures price $\log F$ aggregates producers' information, and reflects a linear combination of $\log A$ and θ, as given in (16). The futures price does not fully reveal $\log A$ due to the θ noise originated from the trading of financial traders. The spot price that emerges from the commodity delivery at $t = 1$ represents another round of information aggregation by pooling together the goods producers' demand for delivery. As a result of the arrival of the supply shock ξ, the spot price $\log P_X$ does not fully reveal either $\log A$ or θ, and instead reflects a linear combination of $\log A$ and ξ, as derived in (18).

Despite the updated information from the spot price at $t = 1$, the informational content of $\log F$ is not subsumed by the spot price, and still has an influence on goods producers' expectations of $\log A$. As a result of this informational role, equation (19) confirms that each goods producer's commodity demand at $t = 1$ is increasing with $\log F$, as $l_F > 0$, and equation (18) shows that the spot price is also increasing with $\log F$, as $h_F > 0$. This is the key feedback channel for futures market trading to affect commodity demand and the spot price despite the availability of information from the spot price.

The simplifying assumptions we made regarding the myopic trading of goods producers and commodity suppliers at $t = 0$ are innocuous to the informational role of the futures price at $t = 1$. As long as goods producers trade on their private signals, the futures price would aggregate the information, which in turn establishes the futures price as a useful price signal for the later round at $t = 1$. Our simplifying assumptions have quantitative consequences on goods producers' trading profits and the efficiency of the futures price signal, but should not critically affect the qualitative feedback channel of the futures price, which we characterize in the next subsection.[14]

Interestingly, Proposition 8 also reveals that l_P can be either positive or negative, due to the offsetting cost effect and informational effect of the spot price, similar to our characterization of the baseline model.

2.3 Real effects of futures market trading

2.3.1 Feedback on commodity demand

As financial traders do not take or make any physical delivery, their trading in the futures market does not have any direct effect on either commodity supply or demand. However, their trading affects the futures price, through which it can further impact commodity demand and spot prices. By substituting equation (16) into (18), we express the spot price $\log P_X$ as a linear combination of primitive variables $\log A$, θ, and ξ:

$$\log P_X = \left(h_A + h_F \tilde{h}_A\right) \log A + h_F \tilde{h}_\theta \theta + h_\xi \xi + h_F \tilde{h}_0 + h_0. \tag{20}$$

The θ term arises through the futures price. As $h_F > 0$ and $\tilde{h}_\theta > 0$, θ, the noise from financial traders' trading in the futures market has a positive effect on the spot price.

Furthermore, by substituting the equation above and (16) into (19), we obtain an individual producer's commodity demand as

$$\begin{aligned} \log X_i &= l_s s_i + \left(l_F \tilde{h}_A + l_P \left(h_A + h_F \tilde{h}_A\right)\right) \log A + \left(l_F + l_P h_F\right) \tilde{h}_\theta \theta + l_P h_\xi \xi \\ &\quad + \left(l_F + l_P h_F\right) \tilde{h}_0 + l_P h_0 + l_0, \end{aligned}$$

[14] Note that despite the futures price containing different information content from the spot price, there is no arbitrage between the two prices because the two prices are traded at different points in time and the spot price is exposed to the supply shock realized later.

and the producers' aggregate demand as

$$\log\left[\int_{-\infty}^{\infty} X\left(s_i, F, P_X\right) d\Phi\left(\varepsilon_i\right)\right] = \left[l_s + l_P h_A + l_F \tilde{h}_A + l_P h_F \tilde{h}_A\right] \log A + \left(l_F + l_P h_F\right) \tilde{h}_\theta \theta + l_P h_\xi \xi + \left(l_F + l_P h_F\right) \tilde{h}_0 + l_P h_0 + l_0 + \frac{1}{2} l_s^2 \tau_s^{-1}. \tag{21}$$

By using equation (61) in the proof of Proposition 8, the coefficient of θ in the aggregate commodity demand is

$$l_F + l_P h_F = k h_F > 0.$$

Thus, θ also has a positive effect on aggregate commodity demand.

The effects of θ on commodity demand and the spot price clarify a simple yet important conceptual point that traders in commodity futures markets, who never take or make physical commodity delivery, can nevertheless impact commodity markets through the informational feedback channel of commodity futures prices.

2.3.2 Detecting speculative effects

In the ongoing debate on whether speculation in commodity futures markets affected commodity prices during the commodity market boom and bust of 2007-2008, many studies, e.g., Kilian and Murphy (2010), Juvenal and Petrella (2012), and Knittel and Pindyck (2013), adopt an inventory-based detection strategy. This strategy builds on a widely-held argument that if speculators artificially drive up the futures price of a commodity, say crude oil, then the price spread between the futures price and the spot price of crude oil will motivate the standard textbook cash-and-carry trades by some arbitrageurs. This in turn will cause the spot price to rise with the futures price. Then, consumers will find consuming the commodity too expensive and thus reduce consumption, causing oil inventory to spike.

Under this argument, price increases in the absence of any inventory increases are explained by fundamental demand. Thus, price effects induced by speculation should be limited to price increases accompanied by contemporaneous inventory increases. Motivated by this argument, the literature, as reviewed by Fattouh, Kilian and Mahadeva (2012), tends to use the lack of any pronounced oil inventory spike before the peak of oil prices in July 2008 as evidence to rule out any significant role played by futures market speculation during this commodity price boom.[15]

[15] An exception in this structural VAR literature is Lombardi and Van Robays (2012), who allow non-fundamental shocks to futures prices to cause the futures-spot spread to deviate from its no arbitrage relationship because of frictions to inventory buildup.

Despite the intuitive appeal of this inventory-based detection strategy, it ignores important informational frictions faced by consumers in reality. In particular, it implicitly assumes that oil consumers observe the global economic fundamentals and are thus able to recognize whether current oil prices are too high relative to the fundamentals in making their consumption decisions. This assumption is strong and may prove unrealistic in certain periods with great economic uncertainty and informational frictions.

Our model illustrates a contrasting example in the presence of informational frictions. In this environment, commodity futures prices serve as important price signals. By influencing consumers' beliefs about global economic fundamentals, noise from futures market trading can distort commodity demand and spot prices in the same direction, rather than opposite directions. This insight thus weakens the power of the widely-used, inventory-based detection strategy and cautions against over-interpreting any conclusion building on it.

Our model also conveys a broader message that in the presence of informational frictions, commodity futures prices are not simply the shadow of the spot prices that reflect the spot prices based on the standard no-arbitrage principle. Instead, commodity futures markets may serve as central platforms for aggregating information, and the resulting futures prices can feed back both valuable information and noise to commodity demand and spot prices.

2.3.3 Understanding the commodity price boom in 2007-2008

In the aftermath of the synchronized price boom and bust of major commodities in 2007-2008, a popular view posits that the commodity price boom was the result of a bubble caused by speculation in commodity futures markets (e.g., Masters (2008) and US Senate Permanent Subcommittee on Investigations (2009)). According to this view, the large inflow of investment capital to the long side of commodity futures markets before July 2008 led to a huge price bubble detached from economic fundamentals that collapsed in the second half of 2008. As we discussed earlier, the lack of evidence for reduced oil consumption during the boom makes it difficult for many economists to accept the oil price boom as a bubble.

Another view attributes the price boom to the combination of rapidly growing demand from emerging economies and stagnant supply (e.g., Hamilton (2009)). This argument is compelling for explaining the commodity price increases before 2008. However, oil prices continued to rise over 40% from January to July 2008, to peak at $147 per barrel, at a time when the U.S. had already entered a recession, Bear Stearns had collapsed in March, and most other developed economies were already showing signs of weakness. While China and other emerging economies remained strong at the time, it is difficult to argue, in hindsight, that their growth sped up enough to be able to offset the clear weakness of the developed economies and cause oil prices to rise another 40%.

The informational frictions faced by market participants can help us understand this puzzling price episode. Due to the lack of reliable data on the strength of emerging economies, it was difficult at the time to precisely measure the strength of the emerging economies. As a result, the prices of crude oil and other commodities were regarded as important price signals (see evidence referenced in Footnote 1). This environment makes our model particularly appealing in linking the large commodity price increases in early 2008 to the concurrent large inflow of investment capital, motivated by the intention of many money managers to diversify their portfolio out of declining stock markets into more promising commodity futures markets, e.g., Tang and Xiong (2012). By pushing up commodity futures prices, and sending a wrong price signal, the large investment flow might have confused goods producers across the world into believing that emerging economies were stronger than they actually were. This distorted expectation could have prevented the producers from reducing their demand for the commodity despite the high commodity prices, which in turn made the high prices sustainable. To the extent that more information corrected the producers' expectations over time, the high commodity prices persisted for several months and eventually collapsed in the second half of 2008. Interestingly, after oil prices dropped from its peak of $147 to $40 per barrel at the end of 2008, oil demand largely evaporated and inventory piled up, despite the much lower prices.

Taken together, the commodity price boom in 2007-2008 is not necessarily a price bubble detached from economic fundamentals. Instead, it is plausible to argue that, in the presence of severe informational frictions in early 2008, the large inflow of investment capital might have distorted signals coming from commodity prices and led to confusion among market participants about the strength of emerging economies. This confusion, in turn, could have amplified the boom and bust of commodity prices, which echoes the emphasis of Singleton (2012) to account for agents' expectations in explaining the price cycle. To test this hypothesis would require estimating a structural model that explicitly takes into account these informational frictions.

2.3.4 Implication for market transparency

Information frictions in the futures market, originating from the unobservability of the positions of different participants, are essential in order for the trading of financial traders to impact the demand for the commodity and spot prices. The following proposition confirms that as $\tau_\theta \to \infty$ (i.e., the position of financial traders becomes publicly observable), the spot market equilibrium converges to the perfect-information benchmark.

Proposition 9 *As $\tau_\theta \to \infty$, the spot price and aggregate demand converge to the perfect-information benchmark.*

Proposition 9 shows that by improving transparency of the futures market, one can achieve the perfect-information benchmark because by making the position of financial traders publicly observable, the θ noise no longer interferes with the information aggregation in the futures market. As a result, the futures price fully reveals the global fundamental, which, in turn, allows goods producers to achieve the same efficiency allowed by the perfect-information benchmark. This nice convergence result relies on the assumption that the supply noise ξ does not affect the futures market trading at $t = 0$ and hits the spot market only at $t = 1$. Nevertheless, this result highlights the importance of improving market transparency.[16]

Imposing position limits on speculators in commodity futures markets has occupied much of the post-2008 policy debate, while improving market transparency has received much less attention. By highlighting the feedback effect originating from information frictions as a key channel for noise in futures market trading to affect commodity prices and demand, our model suggests that imposing position limits may not address the central information frictions that confront participants in commodity markets and thus may not be effective in reducing any potential distortion caused by speculative trading. Instead, making trading positions more transparent might be more effective.

[16] While our analysis focuses on the noise effect of their trading, financial traders can also contribute to information aggregation. As κ increases, the futures position of financial traders builds more on the global economic fundamental $\log A$. Then, the futures price $\log F$ becomes more informative of $\log A$. This is because one can prove based on Proposition 8 that $\tilde{h}_A/\tilde{h}_\theta$, the ratio of the loadings of $\log F$ on $\log A$ and θ, increases with κ.

3 Conclusion

This paper develops a theoretical framework to analyze information aggregation in commodity markets. Our baseline model features a commodity spot market to show that, through the informational role of spot prices, the aggregate demand from goods producers can increase with the spot price and the supply shock can have an amplified effect on the price and an undetermined effect on the quantity demanded. Our extended model further incorporates one round of futures market trading to show that futures prices can serve as an important price signal, even though goods producers also observe spot prices. Through the informational role of futures prices, noise in futures market trading can also interfere with goods producers' expectations and distort their production decisions. Taken together, our analysis cautions empirical and policy studies of commodity markets to fully incorporate realistic informational frictions faced by market participants across the world. Our analysis also provides a coherent argument for how the large inflow of investment capital to commodity futures markets, by jamming the commodity price signals and leading to confusion about the strength of emerging economies, might have amplified the boom and bust of commodity prices in 2007-2008.

Appendix Proofs of Propositions

A.1 Proof of Proposition 1

Consider the maximization problem of the household on island i:

$$\max_{C_i, C_i^*} \left(\frac{C_i}{1-\eta}\right)^{1-\eta} \left(\frac{C_i^*}{\eta}\right)^{\eta}$$

subject to the budget constraint

$$P_i C_i + P_j C_i^* = P_i Y_i. \tag{22}$$

The two first order conditions with respect to C_i and C_i^* are

$$\left(\frac{C_i^*}{C_i}\right)^{\eta} \left(\frac{1-\eta}{\eta}\right)^{\eta} = \lambda_i P_i \tag{23}$$

$$\left(\frac{C_i}{C_i^*}\right)^{1-\eta} \left(\frac{\eta}{1-\eta}\right)^{1-\eta} = \lambda_i P_j \tag{24}$$

where λ_i is the Lagrange multiplier for his budget constraint. Dividing equations (23) and (24) leads to $\frac{\eta}{1-\eta}\frac{C_i}{C_i^*} = \frac{P_j}{P_i}$, which is equivalent to $P_j C_i^* = \frac{\eta}{1-\eta} P_i C_i$. By substituting this equation back to the household's budget constraint in (22), we obtain $C_i = (1-\eta) Y_i$.

The market clearing of the island's produced goods requires $C_i + C_j^* = Y_i$, which implies that $C_j^* = \eta Y_i$. The symmetric problem of the household of island j implies that $C_j = (1-\eta) Y_j$, and the market clearing of the goods produced by island j implies $C_i^* = \eta Y_j$.

The first order condition in equation (23) also gives the price of the goods produced by island i. Since the household's budget constraint in (22) is entirely in nominal terms, the price system is only identified up to λ_i, the Lagrange multiplier. Following Angeletos and La'O (2013), we normalize λ_i to 1. Then,

$$P_i = \left(\frac{C_i^*}{C_i}\right)^\eta \left(\frac{1-\eta}{\eta}\right)^\eta = \left(\frac{\eta Y_j}{(1-\eta) Y_i}\right)^\eta \left(\frac{1-\eta}{\eta}\right)^\eta = \left(\frac{Y_j}{Y_i}\right)^\eta .$$

A.2 Proof of Proposition 2

We first conjecture that the commodity price and each goods producer's commodity demand take the following log-linear forms:

$$\log P_X = h_0 + h_A \log A + h_\xi \xi \tag{25}$$

$$\log X_i = l_0 + l_s s_i + l_P \log P_X \tag{26}$$

where the coefficients h_0, h_A, h_ξ, l_0, l_s, and l_P will be determined by equilibrium conditions.

Define

$$z \equiv \frac{\log P_X - h_0 - h_\xi \bar{\xi}}{h_A} = \log A + \frac{h_\xi}{h_A}\left(\xi - \bar{\xi}\right)$$

which is a sufficient statistic of information contained in the commodity price P_X. Then, conditional on observing its private signal s_i and the commodity price P_X, goods producer i's expectation of $\log A$ is

$$E\left[\log A \mid s_i, \log P_X\right] = E\left[\log A \mid s_i, z\right] = \frac{1}{\tau_A + \tau_s + \frac{h_A^2}{h_\xi^2}\tau_\xi}\left(\tau_A \bar{a} + \tau_s s_i + \frac{h_A^2}{h_\xi^2}\tau_\xi z\right),$$

and its conditional variance of $\log A$ is

$$Var\left[\log A \mid s_i, \log P_X\right] = \left(\tau_A + \tau_s + \frac{h_A^2}{h_\xi^2}\tau_\xi\right)^{-1}.$$

According to equation (7),

$$\log X_i = \frac{1}{1-\phi(1-\eta)}\left\{\log\phi + \log\left(E\left[A X_j^{\phi\eta} \mid s_i, \log P_X\right]\right) - \log P_X\right\}. \tag{27}$$

By using equation (26), we obtain

$$
\begin{aligned}
& E\left[AX_j^{\phi\eta} \mid s_i, \log P_X\right] \\
= \; & E\left\{\exp\left[\log A + \phi\eta\left(l_0 + l_s s_j + l_P \log P_X\right) \mid s_i, z\right]\right\} \\
= \; & \exp\left[\phi\eta\left(l_0 + l_P \log P_X\right)\right] \cdot E\left[\exp\left(\left(1+\phi\eta l_s\right)\log A + \phi\eta l_s \varepsilon_j\right) | s_i, \log P_X\right] \\
= \; & \exp\left[\phi\eta\left(l_0 + l_P \log P_X\right)\right] \cdot \exp\left\{\left(1+\phi\eta l_s\right) E\left[\log A \mid s_i, \log P_X\right] + \frac{\left(1+\phi\eta l_s\right)^2}{2} Var\left[\log A \mid s_i, \log P_X\right]\right. \\
& \left. + \frac{\phi^2\eta^2 l_s^2}{2} Var\left[\varepsilon_j \mid s_i, \log P_X\right] + \left(1+\phi\eta l_s\right)\phi\eta l_s Cov\left[\varepsilon_j \log A \mid s_i, \log P_X\right]\right\}.
\end{aligned}
$$

By recognizing that $Cov\left[\varepsilon_j \log A \mid s_i, \log P_X\right] = 0$ and substituting in the expressions of $E\left[\log A \mid s_i, \log P_X\right]$, $Var\left[\log A \mid s_i, \log P_X\right]$, and $Var\left[\varepsilon_j \mid s_i, \log P_X\right]$, we can further simplify the expression of $E\left[AX_j^{\phi\eta} \mid s_i, \log P_X\right]$. Then, equation (27) gives

$$
\begin{aligned}
\log X_i = & \frac{1}{1-\phi\left(1-\eta\right)}\log\phi + \frac{\phi\eta}{1-\phi\left(1-\eta\right)} l_0 + \frac{1}{1-\phi\left(1-\eta\right)}\left(\phi\eta l_P - 1\right)\log P_X \\
& + \left(\frac{1+\phi\eta l_s}{1-\phi\left(1-\eta\right)}\right)\left(\tau_A + \tau_s + \frac{h_A^2}{h_\xi^2}\tau_\xi\right)^{-1}\left(\tau_A \bar{a} + \tau_s s_i + \frac{h_A^2}{h_\xi^2}\tau_\xi \frac{\log P_X - h_0 - h_\xi \bar{\xi}}{h_A}\right) \\
& + \frac{\left(1+\phi\eta l_s\right)^2}{2\left(1-\phi\left(1-\eta\right)\right)}\left(\tau_A + \tau_s + \frac{h_A^2}{h_\xi^2}\tau_\xi\right)^{-1} + \frac{\phi^2\eta^2 l_s^2}{2\left(1-\phi\left(1-\eta\right)\right)}\tau_s^{-1}.
\end{aligned}
$$

For the above equation to match the conjectured equilibrium position in (26), the constant term and the coefficients of s_i and $\log P_X$ have to match. We thus obtain the following equations for determining the coefficients in (26):

$$
\begin{aligned}
l_0 = \; & \frac{\phi\eta}{1-\phi\left(1-\eta\right)} l_0 + \left(\frac{1+\phi\eta l_s}{1-\phi\left(1-\eta\right)}\right)\left(\tau_A + \tau_s + \frac{h_A^2}{h_\xi^2}\tau_\xi\right)^{-1}\left(\tau_A \bar{a} - \frac{h_A}{h_\xi^2}\tau_\xi\left(h_0 + h_\xi \bar{\xi}\right)\right) \quad (28) \\
& + \frac{\left(1+\phi\eta l_s\right)^2}{2\left(1-\phi\left(1-\eta\right)\right)}\left(\tau_A + \tau_s + \frac{h_A^2}{h_\xi^2}\tau_\xi\right)^{-1} + \frac{\phi^2\eta^2 l_s^2}{2\left(1-\phi\left(1-\eta\right)\right)}\tau_s^{-1} + \frac{1}{1-\phi\left(1-\eta\right)}\log\phi, \\
l_s = \; & \left(\frac{1+\phi\eta l_s}{1-\phi\left(1-\eta\right)}\right)\left(\tau_A + \tau_s + \frac{h_A^2}{h_\xi^2}\tau_\xi\right)^{-1}\tau_s, \quad (29) \\
l_P = \; & \frac{\phi\eta}{1-\phi\left(1-\eta\right)} l_P - \frac{1}{1-\phi\left(1-\eta\right)} + \left(\frac{1+\phi\eta l_s}{1-\phi\left(1-\eta\right)}\right)\left(\tau_A + \tau_s + \frac{h_A^2}{h_\xi^2}\tau_\xi\right)^{-1}\frac{h_A}{h_\xi^2}\tau_\xi. \quad (30)
\end{aligned}
$$

By substituting (29) into (30), we have

$$
l_s = \frac{1+\left(1-\phi\right) l_P}{1-\phi\left(1-\eta\right)}\frac{h_\xi^2}{h_A}\tau_s\tau_\xi^{-1}. \quad (31)
$$

By manipulating (29), we also have that

$$l_s = \left(\tau_A + \frac{1-\phi}{1-\phi(1-\eta)}\tau_s + \frac{h_A^2}{h_\xi^2}\tau_\xi\right)^{-1} \frac{\tau_s}{1-\phi(1-\eta)}. \tag{32}$$

We now use the market clearing condition for the commodity market to determine three other equations for the coefficients in the conjectured log-linear commodity price and demand. Aggregating (26) gives the aggregate commodity demand of the goods producers:

$$\begin{aligned}
\int_{-\infty}^{\infty} X(s_i, P_X)\, d\Phi(\varepsilon_i) &= \int_{-\infty}^{\infty} \exp\left[l_0 + l_s s_i + l_P \log P_X\right] d\Phi(\varepsilon_i) \\
&= \int_{-\infty}^{\infty} \exp\left[l_0 + l_s(\log A + \varepsilon_i) + l_P(h_0 + h_A \log A + h_\xi \xi)\right] d\Phi(\varepsilon_i) \\
&= \exp\left[(l_s + l_P h_A)\log A + l_P h_\xi \xi + l_0 + l_P h_0 + \frac{1}{2} l_s^2 \tau_s^{-1}\right]. \qquad (33)
\end{aligned}$$

Equation (5) implies that $\log X_S = k \log P_X + \xi$. Then, the market clearing condition

$$\log\left[\int_{-\infty}^{\infty} X(s_i, P_X)\, d\Phi(\varepsilon_i)\right] = \log X_S(P_X)$$

requires that the coefficients of $\log A$ and ξ and the constant term be identical on both sides:

$$\begin{aligned}
l_s + l_P h_A &= k h_A, \qquad (34)\\
l_P h_\xi &= 1 + k h_\xi, \qquad (35)\\
l_0 + l_P h_0 + \frac{1}{2} l_s^2 \tau_s^{-1} &= k h_0. \qquad (36)
\end{aligned}$$

Equation (35) directly implies that

$$l_P = k + h_\xi^{-1}. \tag{37}$$

Equations (34) and (35) together imply that

$$l_s = -h_\xi^{-1} h_A. \tag{38}$$

By combining this equation with (32), and defining $b = -l_s = h_\xi^{-1} h_A$, we arrive at

$$b^3 + \left(\tau_A + \frac{1-\phi}{1-\phi(1-\eta)}\tau_s\right)\tau_\xi^{-1} b + \frac{\tau_\xi^{-1}\tau_s}{1-\phi(1-\eta)} = 0. \tag{39}$$

b is a real root of a depressed cubic polynomial of the form $x^3+px+q=0$, which has one real and two complex roots. As p and q are both positive, the LHS is monotonically increasing with b while the RHS is fixed. Thus, the real root b is unique and has to be negative: $b < 0$.

Following Cardano's method, the one real root of equation (39) is given by

$$
\begin{aligned}
b &= \left(\frac{\tau_\xi^{-1}\tau_s}{2\left(1-\phi\left(1-\eta\right)\right)}\right)^{1/3}\sqrt[3]{-1+\sqrt{1+\frac{4}{27}\left(\frac{\tau_\xi^{-1}\tau_s}{1-\phi\left(1-\eta\right)}\right)^{-2}\left(\tau_A+\frac{1-\phi}{1-\phi\left(1-\eta\right)}\tau_s\right)^3}} \\
&+\left(\frac{\tau_\xi^{-1}\tau_s}{2\left(1-\phi\left(1-\eta\right)\right)}\right)^{1/3}\sqrt[3]{-1-\sqrt{1+\frac{4}{27}\left(\frac{\tau_\xi^{-1}\tau_s}{1-\phi\left(1-\eta\right)}\right)^{-2}\left(\tau_A+\frac{1-\phi}{1-\phi\left(1-\eta\right)}\tau_s\right)^3}}. \quad (40)
\end{aligned}
$$

Since $b = h_\xi^{-1}h_A$, we have $h_\xi = b^{-1}h_A$, which, together with our expression for l_s and equations (31) and (37), imply that expressions for h_A and h_ξ given in (9) and (10). With h_A and h_ξ determined, l_s is then given by (32), l_P by (37), h_0 by (28) as

$$
\begin{aligned}
h_0 &= \frac{1}{1+k\left(1-\phi\right)}\log\phi - \frac{1-\phi\left(1-\eta\right)}{1+k\left(1-\phi\right)}b\tau_s^{-1}\left(\tau_A\bar{a}-b\tau_\xi\bar{\xi}\right) \\
&+\frac{1}{2}\frac{1-\phi\left(1-\eta\right)}{1+k\left(1-\phi\right)}\left(\left(\frac{1-\phi+\phi^2\eta^2}{1-\phi\left(1-\eta\right)}+\phi\eta\right)b-1\right)\tau_s^{-1}b.
\end{aligned} \quad (41)
$$

and l_0 by equation (36) as

$$
l_0 = \left(k-l_P\right)h_0 - \frac{1}{2}l_s^2\tau_s^{-1}. \quad (42)
$$

A.3 Proof of Proposition 3

We keep the same setting outlined in the main model, except letting A and ξ be observable by all market participants. We first derive the equilibrium. In this setting, each producer's private signal s_i becomes useless as A is directly observable. We can still use equation (7) to derive producer i's optimal commodity demand. As the producers now share the same information about A, they must have the same expectation about their future trading partners' production decisions. As a result, $X_i = X_j$ for any i and j. Then, equation (7) implies that in equilibrium $X_i = \left(\frac{\phi A}{P_X}\right)^{\frac{1}{1-\phi}}$.

Market clearing of the commodity market requires that the producers' aggregate demand equals the commodity supply, i.e., $X_i = X_S$. From equation (5), we must have $\log X_i = k\log P_X + \xi$. Then, we obtain $\log P_X$ and $\log X_i$ stated in the Proposition 3. It is clear that this equilibrium is unique.

A.4 Proof of Proposition 4

As $\tau_s \to \infty$, equation (39) implies that b goes to $-\frac{1}{1-\phi}$. Consequently, as $\tau_s \to \infty$, equation (9) gives that $h_A \to \frac{1}{1+k(1-\phi)}$, and equation (10) gives that $h_\xi \to -\frac{1-\phi}{1+k(1-\phi)}$. Therefore, both h_A and h_ξ converge to their corresponding values in the perfect-information benchmark.

That $|h_\xi|$ is larger than it is in the perfect-information benchmark is apparent since the numerator of $|h_\xi|$ in equation (10) is positive and larger than $1-\phi$. That h_A is lower follows by substituting equation (39) into equation (9) to arrive at

$$h_A = \frac{1+\tau_A\tau_s^{-1}\left(1-\phi\left(1-\eta\right)\right)b}{1+k\left(1-\phi\right)}.$$

Since $b<0$, it follows that $h_A < \frac{1}{1+k(1-\phi)}$, which is the value of h_A in the perfect-information benchmark.

A.5 Proof of Proposition 5

As $l_s = -h_\xi^{-1}h_A$ from (38), $\pi = \frac{h_A^2/\tau_A}{h_\xi^2/\tau_\xi} = l_s^2\frac{\tau_\xi}{\tau_A}$. Since $l_s>0$, it is sufficient to study the behavior of how l_s varies with τ_s and η to understand how π changes with τ_s and η. To see that l_s is monotonically increasing in τ_s, we note that $l_s=-b$ with b as the only real and negative root of equation (39). Then, by the Implicit Function Theorem it is apparent that

$$\frac{\partial b}{\partial \tau_s} = -\frac{\frac{1-\phi}{1-\phi(1-\eta)}\tau_\xi\tau_s b + \frac{\tau_s\tau_\xi}{1-\phi(1-\eta)}}{3b^2+\left(\tau_A+\frac{1-\phi}{1-\phi(1-\eta)}\tau_s\right)\tau_\xi^{-1}}\tau_s^{-1} = \frac{b^3+\tau_A\tau_\xi^{-1}b}{3b^2+\left(\tau_A+\frac{1-\phi}{1-\phi(1-\eta)}\tau_s\right)\tau_\xi^{-1}}\tau_s^{-1}<0.$$

Similarly, we also have

$$\frac{\partial b}{\partial \eta} = \phi\tau_s\tau_\xi^{-1}\frac{\left(1-\left(1-\phi\right)l_s\right)\frac{1}{(1-\phi(1-\eta))^2}}{3l_s^2+\left(\tau_A+\frac{1-\phi}{1-\phi(1-\eta)}\tau_s\right)\tau_\xi^{-1}}>0.$$

Thus, l_s is increasing in τ_s and decreasing in η, which in turn implies that π is increasing in τ_s and decreasing in η.

To analyze the dependence of π on τ_ξ, we have

$$\frac{\partial \pi}{\partial \tau_\xi} = l_s^2\frac{1}{\tau_A}+2l_s\frac{\tau_\xi}{\tau_A}\frac{\partial l_s}{\partial \tau_\xi} = \frac{1}{\tau_A}b\left(b+2\tau_\xi\frac{\partial b}{\partial \tau_\xi}\right).$$

By applying the Implicit Function Theorem again, we obtain

$$\frac{\partial b}{\partial \tau_\xi} = \frac{\left(\tau_A+\frac{1-\phi}{1-\phi(1-\eta)}\tau_s\right)\tau_\xi^{-1}b+\frac{\tau_\xi^{-1}\tau_s}{1-\phi(1-\eta)}}{3b^2+\left(\tau_A+\frac{1-\phi}{1-\phi(1-\eta)}\tau_s\right)\tau_\xi^{-1}}\tau_\xi^{-1} = \frac{-b^3}{3b^2+\left(\tau_A+\frac{1-\phi}{1-\phi(1-\eta)}\tau_s\right)\tau_\xi^{-1}}\tau_\xi^{-1}>0.$$

By substituting this into the above expression for $\frac{\partial\pi}{\partial\tau_\xi}$, we find that

$$\frac{\partial \pi}{\partial \tau_\xi} = \frac{b^2}{\tau_A}\left(\frac{b^2+\left(\tau_A+\frac{1-\phi}{1-\phi(1-\eta)}\tau_s\right)\tau_\xi^{-1}}{3b^2+\left(\tau_A+\frac{1-\phi}{1-\phi(1-\eta)}\tau_s\right)\tau_\xi^{-1}}\right)>0.$$

Therefore, π is monotonically increasing in τ_ξ.

A.6 Proof of Proposition 6

Based on l_P and h_ξ given in equations(37) and (10), $l_P > 0$ is equivalent to $b^2 > \frac{k^{-1}\tau_\xi^{-1}\tau_s}{1-\phi(1-\eta)}$, which, as $l_s = -b > 0$, is in turn equivalent to $l_s > l_s^* = \sqrt{\frac{k^{-1}\tau_\xi^{-1}\tau_s}{1-\phi(1-\eta)}}$. In words, this condition states that the commodity price has to be sufficiently informative. As b is the unique real and negative root of equation (39), this condition is equivalent to the following condition on the left hand side of equation (39): $LHS(-l_s^*) > 0$. By substituting l_s^* into the LHS, we obtain the following condition:

$$-\frac{k^{-3/2}\tau_\xi^{-1}\tau_s}{1-\phi(1-\eta)} - \left(\tau_A + \frac{1-\phi}{1-\phi(1-\eta)}\tau_s\right)\tau_\xi^{-1}k^{-1/2} + \sqrt{\frac{\tau_\xi^{-1}\tau_s}{1-\phi(1-\eta)}} > 0,$$

which, as $1-\phi(1-\eta) > 0$ and by defining $u = \sqrt{1-\phi(1-\eta)}$, can be rewritten as

$$u^2 - u\tau_A^{-1}\sqrt{k\tau_\xi\tau_s} + \left(1-\phi+k^{-1}\right)\tau_A^{-1}\tau_s < 0.$$

Note that the left hand side of this condition $LHS(u)$ is a quadratic form of u, which has its minimum at $u^* = \frac{1}{2\tau_A}\sqrt{k\tau_\xi\tau_s}$. Thus, this condition is satisfied if and only if the following occurs. First, $LHS(u^*) < 0$, which is equivalent to $\tau_\xi/\tau_A > 4k^{-1}\left(1-\phi+k^{-1}\right)$, the first condition given in Proposition 6. Second,

$$LHS(u) = (u-u^*)^2 - \left[(u^*)^2 - \left(1-\phi+k^{-1}\right)\tau_A^{-1}\tau_s\right] < 0$$

which is equivalent to

$$\frac{u - \frac{1}{2\tau_A}\sqrt{k\tau_\xi\tau_s}}{\frac{1}{2}\tau_s^{1/2}\tau_A^{-1}\sqrt{k\tau_\xi - 4\left(1-\phi+k^{-1}\right)\tau_A}} \in (-1,1).$$

This leads to the second condition given in Proposition 6.

A.7 Proof of Proposition 7

We first evaluate the first component of the social welfare from the island households' goods consumption. We denote this component by

$$W^C = E\left[\int_0^1 \left(\frac{C(i)}{1-\eta}\right)^{1-\eta}\left(\frac{C^*(i)}{\eta}\right)^{\eta} di\right].$$

In the perfect-information benchmark, by substituting the symmetric consumption of all island households, the expected social welfare from consumption is

$$\log W^C_{bench} = \log E\left[\int_0^1 AX_i^\phi di\right] = \log E\left[AX_i^\phi\right].$$

Given $\log X_i$ derived in Proposition 3, we have

$$\begin{aligned}\log W^C_{bench} &= \frac{\phi k}{1+k(1-\phi)}\log\phi+\frac{1+k}{1+k(1-\phi)}\bar{a}+\frac{1}{2}\left(\frac{1+k}{1+k(1-\phi)}\right)^2\tau_A^{-1}\\&\quad+\frac{\phi}{1+k(1-\phi)}\bar{\xi}+\frac{1}{2}\left(\frac{\phi}{1+k(1-\phi)}\right)^2\tau_\xi^{-1}.\end{aligned}$$

Note that the total goods output in this economy is given by

$$E\left[Y^{aggr}_{bench}\right]=E\left[\int_0^1 Y_i di\right]=E\left[\int_0^1 AX_i^{\phi}di\right]=W^C_{bench},$$

which indicates that in this symmetric equilibrium with perfect information, the expected social welfare from consumption is equal to the expected aggregate goods output.

In the presence of informational frictions, by using Proposition 1, the expected social welfare from consumption is given by

$$\log W^C=\log E\left[A\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}X\left(s_i,P_X\right)^{\phi(1-\eta)}X\left(s_j,P_X\right)^{\phi\eta}d\Phi\left(\varepsilon_i\right)d\Phi\left(\varepsilon_j\right)\right]$$

where in the second line, an integral over ε_j, i.e., noise in the signal of goods producer of island j, is taken to compute expectation over uncertainty in ε_j. By substituting

$$\log X\left(\varepsilon_i\right)=l_0+l_P\log P_X+l_s s_i=l_0+l_P\log P_X+l_s\left(\log A+\varepsilon_i\right)$$

and $\log P_X=h_0+h_A\log A+h_\xi\xi$, with our expressions for l_s, l_P, h_ξ and h_A and b^3 from equation (39), we obtain

$$\begin{aligned}\log W^C &= \frac{\phi k}{1+k(1-\phi)}\log\phi+\frac{1+k}{1+k(1-\phi)}\bar{a}+\frac{1}{2}\left(1+\phi k h_A\right)^2\tau_A^{-1}+\frac{\phi}{1+k(1-\phi)}\bar{\xi}\\&\quad+\frac{1}{2}\phi^2\left(1+kb^{-1}h_A\right)^2\tau_\xi^{-1}+\frac{1}{2}\left(1-\frac{1}{\phi}-2\eta(1-\eta)\right)\phi^2b^2\tau_s^{-1}\\&\quad-\frac{1}{2}\frac{\phi k\left(1-\phi(1-\eta)\right)}{1+k(1-\phi)}\left(1-\left(\frac{1-\phi+\phi^2\eta^2}{1-\phi(1-\eta)}+\phi\eta\right)b\right)b\tau_s^{-1},\end{aligned}$$

The logarithm of the expected total output in this economy is given by

$$\begin{aligned}\log E\left[Y^{aggr}\right] &= \log E\left[\int_{-\infty}^{\infty}Y_i di\right]=\log E\left[A\int_{-\infty}^{\infty}X\left(\varepsilon_i\right)^{\phi}d\Phi\left(\varepsilon_i\right)\right]\\&= \log E\left[e^{\phi l_0+\phi l_P\log P_X+(1+\phi l_s)\log A}\int_{-\infty}^{\infty}e^{\phi l_s\varepsilon_i}d\Phi\left(\varepsilon_i\right)\right]\\&= \phi l_0+\frac{1}{2}\phi^2 l_s^2\tau_s^{-1}+\phi l_P h_0+\phi l_P h_\xi\bar{\xi}+\frac{1}{2}\phi^2 l_P^2 h_\xi^2\tau_\xi^{-1}+\left(1+\phi l_s+\phi l_P h_A\right)\bar{a}\\&\quad+\frac{1}{2}\left(1+\phi l_s+\phi l_P h_A\right)^2\tau_A^{-1}.\end{aligned}$$

Again by substituting the expressions for l_s, l_P, h_ξ and h_A, we have

$$\begin{aligned}\log E\left[Y^{aggr}\right] &= \frac{\phi k}{1+k(1-\phi)}\log\phi + \left(\frac{1+k}{1+k(1-\phi)}\right)\bar{a} + \frac{1}{2}\left(1+\phi k h_A\right)^2 \tau_A^{-1} \\ &+\phi\left(\frac{1}{1+k(1-\phi)}\right)\bar{\xi} + \frac{1}{2}\phi^2\left(1+kb^{-1}h_A\right)^2\tau_\xi^{-1} - \frac{1}{2}\phi(1-\phi)b^2\tau_s^{-1} \\ &-\frac{1}{2}\frac{\phi k(1-\phi(1-\eta))}{1+k(1-\phi)}\left(1-\left(\frac{1-\phi+\phi^2\eta^2}{1-\phi(1-\eta)}+\phi\eta\right)b\right)b\tau_s^{-1}.\end{aligned}$$

Then, it is easy to compute

$$\log E\left[Y^{aggr}\right] - \log W = 2\phi^2\eta(1-\eta)b^2\tau_s^{-1} > 0.$$

We now compare expected aggregate goods output with and without informational frictions:

$$\begin{aligned}&\log E\left[Y^{aggr}\right] - \log E\left[Y^{aggr}_{bench}\right] \\ =\ & \frac{1}{2}\left(\left(1+\phi k h_A\right)^2 - \left(1+\frac{\phi k}{1+k(1-\phi)}\right)^2\right)\tau_A^{-1} \\ &+\frac{1}{2}\phi^2\left(\left(1+kb^{-1}h_A\right)^2 - \left(1-k\frac{1-\phi}{1+k(1-\phi)}\right)^2\right)\tau_\xi^{-1} - \frac{1}{2}\phi(1-\phi)b^2\tau_s^{-1} \\ &-\frac{1}{2}\frac{\phi k(1-\phi(1-\eta))}{1+k(1-\phi)}\left(1-\left(1-\phi+\phi\eta(1+\phi\eta)\right)b\right)b\tau_s^{-1}.\end{aligned}$$

Substituting with equations (9) and (39), we arrive at

$$\begin{aligned}&\log E\left[Y^{aggr}\right] - \log E\left[Y^{aggr}_{bench}\right] \\ =\ & \frac{1}{2}\frac{\phi k(1-\phi(1-\eta))\tau_s^{-1}b}{(1+k(1-\phi))^2}(1+k-\phi b) - \frac{1}{2}\phi(1-\phi)b^2\tau_s^{-1} \\ &+\frac{1}{2}\frac{\phi k(1-\phi(1-\eta))\tau_s^{-1}b^2}{1+k(1-\phi)}\left(\frac{1-\phi}{1-\phi(1-\eta)}+\phi^2\eta^2+\left(\frac{\phi^2\eta^2}{1-\phi(1-\eta)}-1\right)\phi(1-\eta)\right).\end{aligned}$$

Notice that $\frac{\phi^2\eta^2}{1-\phi(1-\eta)} < 1$ and that the first term is negative since $b < 0$. We further note that

$$\begin{aligned}&\frac{1}{2}\frac{\phi k(1-\phi(1-\eta))\tau_s^{-1}b^2}{1+k(1-\phi)}\frac{1-\phi}{1-\phi(1-\eta)} - \frac{1}{2}\phi(1-\phi)b^2\tau_s^{-1} \\ =\ & -\left(\frac{1-k\phi}{1+k(1-\phi)}\right)\frac{1}{2}\phi(1-\phi)b^2\tau_s^{-1} < 0,\end{aligned}$$

and

$$\frac{1}{2}\frac{\phi k(1-\phi(1-\eta))\tau_s^{-1}b^2}{1+k(1-\phi)}\left(\phi^2\eta^2 - \frac{\phi-(1+k)b^{-1}}{1+k(1-\phi)}\right) < 0,$$

because $\phi^2\eta^2 < \phi$ and, since $\phi < 1$ and $0 > b > -\frac{1}{1-\phi}$, $k(1-\phi)\phi^2\eta^2 + (1+k)b^{-1} < 0$. To see that $b > -\frac{1}{1-\phi}$, we rewrite equation (39) as

$$b^3 + \tau_A\tau_\xi^{-1}b + ((1-\phi)b+1)\frac{\tau_s\tau_\xi^{-1}}{1-\phi(1-\eta)} = 0,$$

from which it follows that $b > -\frac{1}{1-\phi}$. Therefore, we see that

$$\log E\left[Y^{aggr}\right] - \log E\left[Y^{aggr}_{bench}\right] < 0.$$

Given that the expected social welfare from consumption W^C_{bench} is equal to expected aggregate output $E\left[Y^{aggr}_{bench}\right]$ in the perfect-information benchmark and that in the presence of informational frictions the expected social welfare from consumption W^C is strictly less than the expected aggregate goods output $E\left[Y^{aggr}\right]$, the expected social welfare from consumption is lower in the presence of information frictions than in the perfect-information benchmark.

Now we return to the second part of the expected social welfare from commodity suppliers' disutility of labor. We denote this part by

$$W^L = E\left[\frac{k}{1+k} e^{-\xi/k} X_S^{\frac{1+k}{k}}\right].$$

In the perfect-information benchmark, by using $\log X_S$ derived in Proposition 3, we have

$$\begin{aligned}\log W^L_{bench} &= \log\frac{k}{1+k} + \frac{1+k}{1+k\left(1-\phi\right)}\log\phi + \frac{1+k}{1+k\left(1-\phi\right)}\bar{a} + \frac{\phi}{1+k\left(1-\phi\right)}\bar{\xi} \\ &\quad + \frac{1}{2}\left(\frac{1+k}{1+k\left(1-\phi\right)}\right)^2 \tau_A^{-1} + \frac{1}{2}\left(\frac{\phi}{1+k\left(1-\phi\right)}\right)^2 \tau_\xi^{-1}.\end{aligned}$$

In the presence of informational frictions, aggregate demand X_S is given by

$$\log X_S = k\log P_X + \xi = kh_A \log A + \left(kh_\xi + 1\right)\xi + kh_0,$$

and therefore the suppliers' disutility of labor reduces to

$$\begin{aligned}\log W^L &= \log\frac{k}{1+k} + \left(1+k\right)h_0 + \left(1+k\right)h_A\bar{a} + \left(1+\left(1+k\right)h_\xi\right)\bar{\xi} \\ &\quad + \frac{1}{2}\left(1+k\right)^2 h_A^2 \tau_A^{-1} + \frac{1}{2}\left(1+\left(1+k\right)h_\xi\right)^2 \tau_\xi^{-1}.\end{aligned}$$

We now analyze the overall social welfare $W = W^C - W^L$. We can express the relative welfare in the two economies as

$$\frac{W}{W_{bench}} = \frac{W^C - W^L}{W^C_{bench} - W^L_{bench}} = \frac{W^C}{W^C_{bench}}\frac{1 - W^L/W^C}{1 - W^L_{bench}/W^C_{bench}} < \frac{1 - W^L/W^C}{1 - W^L_{bench}/W^C_{bench}},$$

where the last inequality follows from $W^C < W^C_{bench}$, as proved above.

Note that in the perfect-information benchmark,

$$\log W^L_{bench} - \log W^C_{bench} = \log\frac{\phi k}{1+k}.$$

Thus, $1 - W^L_{bench}/W^C_{bench} = 1 - \frac{\phi k}{1+k} > 0$. Therefore, it is sufficient to show that $W^L/W^C \geq W^L_{bench}/W^C_{bench}$ in order to establish that $\frac{W}{W_{bench}} < 1$.

With some manipulation of our expressions for $\log W^L$ and $\log W^C$, by substituting our expressions for h_A and h_ξ and making use of equation (39), we arrive at

$$\begin{aligned}
& \log\left(W^L/W^C\right) \\
= \quad & \log\frac{\phi k}{1+k}+\frac{1}{2}b\tau_s^{-1}\left(\left(1-\phi^2+\phi^2\eta^2+\phi\eta+\phi^2\eta\left(1-\eta\right)\right)b-\left(1-\phi\left(1-\eta\right)\right)\right) \\
& -\left(1-\phi\left(1-\eta\right)\right)\tau_s^{-1}b^2-\frac{1}{2}\left(1-\phi\left(1-\eta\right)\right)^2\tau_s^{-2}b^2\left(\tau_A+\tau_\xi b^2\right).
\end{aligned}$$

Finally, by invoking equation (39) to rewrite the last term, we find that

$$\log\left(W^L/W^C\right)=\log\frac{\phi k}{1+k}.$$

Thus, $\log\left(W^L/W^C\right)=\log\left(W^L_{bench}/W^C_{bench}\right)$, which in turn establishes the proposition.

A.8 Proof of Proposition 8

We follow the same procedure as in the proof of Proposition 2 to derive the futures market equilibrium at $t=0$. We first conjecture the log-linear forms for the futures price and each island producer's long position in (16) and (17) with the coefficients $\tilde{h}_0$, $\tilde{h}_A$, $\tilde{h}_\theta$, $\tilde{l}_0$, $\tilde{l}_s$, and $\tilde{l}_F$ to be determined by equilibrium conditions.

Define z as a sufficient statistic of the information contained in F:

$$z\equiv\frac{\log F-\tilde{h}_0-\tilde{h}_\theta\bar{\theta}}{\tilde{h}_A}=\log A+\frac{\tilde{h}_\theta}{\tilde{h}_A}\left(\theta-\bar{\theta}\right)$$

Then, conditional on observing s_i and F, producer i's expectation of $\log A$ is

$$\begin{aligned}
E\left[\log A\mid s_i,\log F\right] &= E\left[\log A\mid s_i,z\right]=\frac{1}{\tau_A+\tau_s+\frac{\tilde{h}_A^2}{\tilde{h}_\theta^2}\tau_\theta}\left(\tau_A\bar{a}+\tau_s s_i+\frac{\tilde{h}_A^2}{\tilde{h}_\theta^2}\tau_\theta z\right) \\
&= c_0+c_s s_i+c_F\left(\log F-\tilde{h}_0-\tilde{h}_\theta\bar{\theta}\right), \qquad (43)
\end{aligned}$$

where

$$\begin{aligned}
c_0 &= \left(\tau_A+\tau_s+\frac{\tilde{h}_A^2}{\tilde{h}_\theta^2}\tau_\theta\right)^{-1}\left(\tau_A\bar{a}-\frac{\tilde{h}_A^2}{\tilde{h}_\theta^2}\tau_\theta\frac{\tilde{h}_0+\tilde{h}_\theta\bar{\theta}}{\tilde{h}_A}\right), \\
c_s &= \left(\tau_A+\tau_s+\frac{\tilde{h}_A^2}{\tilde{h}_\theta^2}\tau_\theta\right)^{-1}\tau_s, \\
c_F &= \left(\tau_A+\tau_s+\frac{\tilde{h}_A^2}{\tilde{h}_\theta^2}\tau_\theta\right)^{-1}\frac{\tilde{h}_A}{\tilde{h}_\theta^2}\tau_\theta.
\end{aligned}$$

Its conditional variance of $\log A$ is

$$\tilde{\tau}_{A,i} = Var\left[\log A \mid s_i, \log F\right] = \left(\tau_A + \tau_s + \frac{\tilde{h}_A^2}{\tilde{h}_\theta^2}\tau_\theta\right)^{-1}. \tag{44}$$

By substituting equation (17) into producer i's optimal production decision in equation (14), we obtain

$$\begin{aligned}
\log \tilde{X}_i &= \frac{1}{1-\phi(1-\eta)}\log\phi + \frac{\phi\eta}{1-\phi(1-\eta)}\tilde{l}_0 + \frac{1}{1-\phi(1-\eta)}\left(\phi\eta\tilde{l}_F - 1\right)\log F \\
&+ \left(\frac{1+\phi\eta\tilde{l}_s}{1-\phi(1-\eta)}\right)\left(c_0 + c_s s_i + c_F\frac{\log F}{\tilde{h}_A}\right) + \frac{\left(1+\phi\eta\tilde{l}_s\right)^2}{2\left(1-\phi(1-\eta)\right)}\tilde{\tau}_{A,i} + \frac{\phi^2\eta^2\tilde{l}_s^2}{2\left(1-\phi(1-\eta)\right)}\tau_s^{-1}.
\end{aligned}$$

For the above equation to match the conjectured equilibrium position in equation (17), the constant term and the coefficients of s_i and $\log F$ have to be identical:

$$\begin{aligned}
\tilde{l}_0 &= \frac{\phi\eta}{1-\phi(1-\eta)}\tilde{l}_0 + \left(\frac{1+\phi\eta\tilde{l}_s}{1-\phi(1-\eta)}\right)c_0 + \frac{\left(1+\phi\eta\tilde{l}_s\right)^2}{2\left(1-\phi(1-\eta)\right)}\tilde{\tau}_{A,i} \\
&\quad + \frac{\phi^2\eta^2\tilde{l}_s^2}{2\left(1-\phi(1-\eta)\right)}\tau_s^{-1} + \frac{1}{1-\phi(1-\eta)}\log\phi, && (45)\\
\tilde{l}_s &= \left(\frac{1+\phi\eta\tilde{l}_s}{1-\phi(1-\eta)}\right)c_s, && (46)\\
\tilde{l}_F &= \frac{\phi\eta}{1-\phi(1-\eta)}\tilde{l}_F - \frac{1}{1-\phi(1-\eta)} + \left(\frac{1+\phi\eta\tilde{l}_s}{1-\phi(1-\eta)}\right)c_F. && (47)
\end{aligned}$$

By substituting equation (46) into (47), we have

$$\tilde{l}_s = \frac{1+(1-\phi)\tilde{l}_F}{1-\phi(1-\eta)}\frac{\tilde{h}_\theta^2}{\tilde{h}_A}\tau_s\tau_\theta^{-1}. \tag{48}$$

By manipulating equation (46), we also have that

$$\tilde{l}_s = \left(\tau_A + \frac{1-\phi}{1-\phi(1-\eta)}\tau_s + \frac{\tilde{h}_A^2}{\tilde{h}_\theta^2}\tau_\theta\right)^{-1}\frac{\tau_s}{1-\phi(1-\eta)}. \tag{49}$$

We now use the market clearing of the futures market to determine three other equations for the coefficients. Aggregating equation (17) gives the producers' aggregate position:

$$\int_{-\infty}^{\infty}\tilde{X}_i(s_i,F)\,d\Phi(\varepsilon_i) = \exp\left[\left(\tilde{l}_s + \tilde{l}_F\tilde{h}_A\right)\log A + \tilde{l}_F\tilde{h}_\theta\theta + \tilde{l}_0 + \tilde{l}_F\tilde{h}_0 + \frac{1}{2}\tilde{l}_s^2\tau_s^{-1}\right]. \tag{50}$$

Equation (15) gives $\tilde{X}_S$. Define

$$z_\theta \equiv \frac{\log F - \tilde{h}_0 - \tilde{h}_A\bar{a}}{\tilde{h}_\theta} = \frac{\tilde{h}_A}{\tilde{h}_\theta}\left(\log A - \bar{a}\right) + \theta.$$

Then, the suppliers' conditional expection of θ is

$$E[\theta \mid \log F] = E[\theta \mid z_\theta] = \left(\tau_\theta + \frac{\tilde{h}_\theta^2}{\tilde{h}_A^2}\tau_A\right)^{-1}\left[\tau_\theta\bar{\theta} + \frac{\tilde{h}_\theta^2}{\tilde{h}_A^2}\tau_A\left(\frac{\log F - \tilde{h}_0}{\tilde{h}_\theta} - \frac{\tilde{h}_A}{\tilde{h}_\theta}\bar{a}\right)\right],$$

and conditional variance is $Var[\theta \mid \log F] = \left(\tau_\theta + \frac{\tilde{h}_\theta^2}{\tilde{h}_A^2}\tau_A\right)^{-1}$. Their conditional expectation of $\log A$ is

$$E[\log A \mid \log F] = E[\log A \mid z_\theta] = \left(\tau_A + \frac{\tilde{h}_A^2}{\tilde{h}_\theta^2}\tau_\theta\right)^{-1}\left[\tau_A\bar{a} + \frac{\tilde{h}_A^2}{\tilde{h}_\theta^2}\tau_\theta\left(\frac{\log F - \tilde{h}_0 - \tilde{h}_\theta\bar{\theta}}{\tilde{h}_A}\right)\right],$$

and conditional variance is $Var[\log A \mid \log F] = \left(\tau_A + \frac{\tilde{h}_A^2}{\tilde{h}_\theta^2}\tau_\theta\right)^{-1}$. Thus, we obtain an expression for $\log \tilde{X}_S$ in a linear of $\log A$ and θ.

Then, the market clearing condition $\log\left[e^{\kappa \log A + \theta}\int_{-\infty}^{\infty}\tilde{X}_i(s_i, F)\, d\Phi(\varepsilon_i)\right] = \log \tilde{X}_S$ requires that the coefficients of $\log A$ and θ and the constant term be identical on both sides:

$$\kappa + \tilde{l}_s + \tilde{l}_F\tilde{h}_A = k\tilde{h}_A + \left(\tau_\theta + \frac{\tilde{h}_\theta^2}{\tilde{h}_A^2}\tau_A\right)^{-1}\left(\frac{\tilde{h}_\theta}{\tilde{h}_A}\tau_A + \kappa\tau_\theta\right), \tag{51}$$

$$1 + \tilde{l}_F\tilde{h}_\theta = k\tilde{h}_\theta + \left(\tau_\theta + \frac{\tilde{h}_\theta^2}{\tilde{h}_A^2}\tau_A\right)^{-1}\frac{\tilde{h}_\theta}{\tilde{h}_A}\left(\frac{\tilde{h}_\theta}{\tilde{h}_A}\tau_A + \kappa\tau_\theta\right), \tag{52}$$

$$\begin{aligned}\tilde{l}_0 + \tilde{l}_F\tilde{h}_0 + \frac{1}{2}\tilde{l}_s^2\tau_s^{-1} &= k\tilde{h}_0 + \left(\tau_\theta + \frac{\tilde{h}_\theta^2}{\tilde{h}_A^2}\tau_A\right)^{-1}\left(1 + \kappa\frac{\tilde{h}_\theta}{\tilde{h}_A}\right)\tau_\theta\bar{\theta} \\ &\quad - \left(\tau_\theta + \frac{\tilde{h}_\theta^2}{\tilde{h}_A^2}\tau_A\right)^{-1}\frac{\tilde{h}_\theta}{\tilde{h}_A}\left(1 + \kappa\frac{\tilde{h}_\theta}{\tilde{h}_A}\right)\tau_A\bar{a} + \bar{\xi} - \sigma_\xi^2/2k \\ &\quad - \frac{\kappa^2}{2k}(1+2k)\left(\tau_A + \frac{\tilde{h}_A^2}{\tilde{h}_\theta^2}\tau_\theta\right)^{-1} - \frac{1}{2k}(1+2k)\left(\tau_\theta + \frac{\tilde{h}_\theta^2}{\tilde{h}_A^2}\tau_A\right)^{-1}.\end{aligned} \tag{53}$$

Equation (52) directly implies that

$$\tilde{l}_F = k + \left(\tau_\theta + \frac{\tilde{h}_\theta^2}{\tilde{h}_A^2}\tau_A\right)^{-1}\left(\kappa\frac{\tilde{h}_\theta}{\tilde{h}_A} - 1\right)\tau_\theta\tilde{h}_\theta^{-1}. \tag{54}$$

Equations (51) and (52) together imply that

$$\tilde{l}_s = \tilde{h}_\theta^{-1}\tilde{h}_A - \kappa.$$

By combining this equation with (49), we arrive at

$$\tilde{l}_s^3 + 2\kappa\tilde{l}_s^2 + \left(\tau_\theta^{-1}\tau_A + \frac{1-\phi}{1-\phi(1-\eta)}\tau_\theta^{-1}\tau_s + \kappa^2\right)\tilde{l}_s - \frac{\tau_\theta^{-1}\tau_s}{1-\phi(1-\eta)} = 0. \tag{55}$$

By making the convenient substitution $L_s = \tilde{l}_s + \frac{2}{3}\kappa$, called the Tschirnhaus transformation, one can arrive at the depressed cubic polynomial

$$L_s^3 + pL_s + q = 0,$$

where

$$\begin{aligned} p &= \tau_\theta^{-1}\tau_A + \frac{1-\phi}{1-\phi(1-\eta)}\tau_\theta^{-1}\tau_s - \frac{1}{3}\kappa^2, \\ q &= -\frac{2}{3}\kappa\tau_\theta^{-1}\tau_A - \frac{2}{3}\kappa\frac{1-\phi}{1-\phi(1-\eta)}\tau_\theta^{-1}\tau_s - \frac{2}{27}\kappa^3 - \frac{\tau_\theta^{-1}\tau_s}{1-\phi(1-\eta)}. \end{aligned}$$

It is easy to verify that $\frac{q^2}{4} + \frac{p^3}{27} > 0$ and therefore L_s is a real root of this depressed cubic polynomial, which has one real and two complex roots. Following Cardano's method, the one real root of equation (55) is then given by

$$\tilde{l}_s = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} - \frac{2}{3}\kappa.$$

Since the coefficients of equation (55) change sign only once, by Descartes' Rule of Signs the real root must be positive.

Since $\tilde{l}_s = \tilde{h}_\theta^{-1}\tilde{h}_A - \kappa$, we have that

$$\tilde{h}_\theta = \left(\tilde{l}_s + \kappa\right)^{-1}\tilde{h}_A,$$

which, together with our expression for $\tilde{l}_s$ and equations (48) and (54), implies that

$$\tilde{h}_\theta = \left((1-\phi(1-\eta))\,\tau_s^{-1} + \frac{1-\phi}{\tau_\theta\left(\tilde{l}_s+\kappa\right)^2 + \tau_A}\right)\frac{\tau_\theta}{1+k(1-\phi)}\tilde{l}_s\left(\tilde{l}_s+\kappa\right), \tag{56}$$

and therefore

$$\tilde{h}_A = \left((1-\phi(1-\eta))\,\tau_s^{-1} + \frac{1-\phi}{\tau_\theta\left(\tilde{l}_s+\kappa\right)^2 + \tau_A}\right)\frac{\tau_\theta}{1+k(1-\phi)}\tilde{l}_s\left(\tilde{l}_s+\kappa\right)^2. \tag{57}$$

Since by equation (55), $\tilde{l}_s > 0$, one has that $\tilde{h}_A$ and $\tilde{h}_\theta$ must have the same sign. With $\tilde{h}_A$ and $\tilde{h}_\theta$ determined, $\tilde{l}_F$ is then given by equation (54):

$$\tilde{l}_F = k + \left(\tau_\theta + \frac{\tilde{h}_\theta^2}{\tilde{h}_A^2}\tau_A\right)^{-1}\left(\kappa\frac{\tilde{h}_\theta}{\tilde{h}_A} - 1\right)\tau_\theta\tilde{h}_\theta^{-1},$$

$\tilde{h}_0$ by equation (45):

$$
\begin{aligned}
\tilde{h}_0 &= \left(k - \tilde{l}_F + \frac{1-\phi(1-\eta)}{1-\phi}\tilde{l}_s\tau_s^{-1}\frac{\tilde{h}_A}{\tilde{h}_\theta^2}\tau_\theta\right)^{-1} \cdot \\
&\quad \left(\frac{1}{1-\phi}\log\phi - \bar{\xi} + \sigma_\xi^2/2k + \frac{1}{2k}(1+2k)\left(1+\kappa^2\frac{\tilde{h}_\theta^2}{\tilde{h}_A^2}\right)\left(\tau_\theta + \frac{\tilde{h}_\theta^2}{\tilde{h}_A^2}\tau_A\right)^{-1}\right. \\
&\quad + \frac{1}{2}\left(\tilde{l}_s + \frac{1-\phi(1-\eta)}{1-\phi}\left(1+\phi\eta\tilde{l}_s + \frac{\phi^2\eta^2\tilde{l}_s}{1-\phi(1-\eta)}\right)\right)\tilde{l}_s\tau_s^{-1} \\
&\quad \left. + \left(\frac{1-\phi(1-\eta)}{1-\phi}\tilde{l}_s\tau_s^{-1} + \left(\tau_\theta\frac{\tilde{h}_A^2}{\tilde{h}_\theta^2} + \tau_A\right)^{-1}\left(\frac{\tilde{h}_A}{\tilde{h}_\theta} + \kappa\right)\right)\left(\tau_A\bar{a} - \left(\tilde{l}_s + \kappa\right)\tau_\theta\bar{\theta}\right)\right),
\end{aligned}
$$

and $\tilde{l}_0$ by equation (53):

$$
\begin{aligned}
\tilde{l}_0 &= \left(k - \tilde{l}_F\right)\tilde{h}_0 + \bar{\xi} - \sigma_\xi^2/2k + \left(\tau_\theta + \frac{\tilde{h}_\theta^2}{\tilde{h}_A^2}\tau_A\right)^{-1}\left(1+\kappa\frac{\tilde{h}_\theta}{\tilde{h}_A}\right)\tau_\theta\bar{\theta} \\
&\quad - \left(\tau_\theta + \frac{\tilde{h}_\theta^2}{\tilde{h}_A^2}\tau_A\right)^{-1}\frac{\tilde{h}_\theta}{\tilde{h}_A}\left(1+\kappa\frac{\tilde{h}_\theta}{\tilde{h}_A}\right)\tau_A\bar{a} - \frac{1}{2}\tilde{l}_s^2\tau_s^{-1} \\
&\quad - \frac{1}{2k}(1+2k)\left(1+\kappa^2\frac{\tilde{h}_\theta^2}{\tilde{h}_A^2}\right)\left(\tau_\theta + \frac{\tilde{h}_\theta^2}{\tilde{h}_A^2}\tau_A\right)^{-1}.
\end{aligned}
$$

We now derive the spot market equilibrium at $t = 1$. We again first conjecture that the spot price P_X and a goods producer's updated commodity demand take the log-linear forms as given in equations (18) and (19) with the coefficients h_0, h_A, h_F, h_ξ, l_0, l_s, l_F, and l_P to be determined by equilibrium conditions.

The mean and variance of producer i's prior belief over $\log A$ carried from $t = 0$ is derived in (43) and (44). Define

$$
z_p = \frac{\log P_X - h_0 - h_F\log F - h_\xi\bar{\xi}}{h_A} = \log A + \frac{h_\xi}{h_A}\left(\xi - \bar{\xi}\right).
$$

Then, after observing the spot price P_X at $t = 1$, the producer's expectation of $\log A$ is

$$
\begin{aligned}
E\left[\log A \mid s_i, \log F, \log P_X\right] &= E\left[\log A \mid s_i, \log F, z_p\right] \\
&= \left(\tilde{\tau}_{A,i} + \frac{h_A^2}{h_\xi^2}\tau_\xi\right)^{-1}\left(\tilde{\tau}_{A,i}c_s s_i + \frac{h_A^2}{h_\xi^2}\left(\frac{\log P_X - h_0 - h_F\log F - h_\xi\bar{\xi}}{h_A}\right)\right),
\end{aligned}
$$

and its conditional variance is

$$
\tau_{A,i} = Var\left[\log A \mid s_i, \log F, \log P_X\right] = \left(\tilde{\tau}_{A,i} + \frac{h_A^2}{h_\xi^2}\tau_\xi\right)^{-1}.
$$

We use (13) to compute $\log X_i$, and obtain a linear expression of s_i, $\log F$, and P_X. By matching the coefficients of this expression with the conjectured form in (19), we obtain

$$\begin{aligned} l_0 &= \frac{1}{1-\phi}\log\phi + \frac{(1+\phi\eta l_s)^2}{2(1-\phi)}\tau_{A,i} + \frac{1}{2(1-\phi)}\phi^2\eta^2 l_s^2\tau_s^{-1} - \frac{1+\phi\eta l_s}{1-\phi}\tau_{A,i}\frac{h_A}{h_\xi^2}\left(h_0 + h_\xi\bar{\xi}\right) \\ &\quad + \frac{1}{1-\phi}(1+\phi\eta l_s)\tau_{A,i}\tilde{\tau}_{A,i}\left(c_0 - c_F\left(\tilde{h}_0 + \tilde{h}_\theta\bar{\theta}\right)\right), \\ l_s &= \frac{\tilde{\tau}_{A,i}c_s}{(1-\phi(1-\eta))\tau_{A,i}^{-1} - \phi\eta\tilde{\tau}_{A,i}c_s}, \\ l_F &= \frac{1}{1-\phi}(1+\phi\eta l_s)\tau_{A,i}\left(\tilde{\tau}_{A,i}c_F - \frac{h_A}{h_\xi^2}h_F\right), \qquad (58) \\ l_P &= \frac{1}{1-\phi}(1+\phi\eta l_s)\tau_{A,i}\frac{h_A}{h_\xi^2} - \frac{1}{1-\phi}. \qquad (59) \end{aligned}$$

Market clearing of the spot market requires $\int_{-\infty}^{\infty} X_i d\Phi(\varepsilon_i) = X_S$, which implies

$$(k - l_P)\log P_X = l_0 + \frac{1}{2}l_s^2\tau_s^{-1} + l_s\log A + l_F\log F - \xi.$$

By matching coefficients on both sides, we have

$$\begin{aligned} (k-l_P)h_0 &= l_0 + \frac{1}{2}l_s^2\tau_s^{-1}, \\ (k-l_P)h_A &= l_s, \qquad (60) \\ (k-l_P)h_F &= l_F, \qquad (61) \\ (k-l_P)h_\xi &= -1, \qquad (62) \end{aligned}$$

From equations (60) and (62), we have that $l_s = -\frac{h_A}{h_\xi}$, and given our expression for l_0 and l_F above, we also see that

$$\begin{aligned} h_0 &= \left(k - l_P + \frac{1+\phi\eta l_s}{1-\phi}\tau_{A,i}\frac{h_A}{h_\xi^2}\right)^{-1}\cdot\left(\frac{1}{1-\phi}\log\phi + \frac{(1+\phi\eta l_s)^2}{2(1-\phi)}\tau_{A,i} + \frac{1}{2(1-\phi)}\phi^2\eta^2 l_s^2\tau_s^{-1}\right. \\ &\quad \left. + \frac{1}{1-\phi}(1+\phi\eta l_s)\tau_{A,i}\tilde{\tau}_{A,i}\left(c_0 - c_F\left(\tilde{h}_0 + \tilde{h}_\theta\bar{\theta}\right)\right) - \frac{1+\phi\eta l_s}{1-\phi}\tau_{A,i}\frac{h_A}{h_\xi^2}h_\xi\bar{\xi} + \frac{1}{2}l_s^2\tau_s^{-1}\right), \\ h_F &= \left(\frac{1-\phi}{1+\phi\eta l_s}\tau_{A,i}^{-1}(k-l_P) + \frac{h_A}{h_\xi^2}\right)^{-1}\tilde{\tau}_{A,i}c_F. \qquad (63) \end{aligned}$$

From our expression for l_s above and $l_s = -h_A/h_\xi$, we have

$$l_s^3 + \tau_\xi^{-1}\left(\tilde{\tau}_{A,i} - \frac{\phi\eta\tilde{\tau}_{A,i}c_s}{1-\phi(1-\eta)}\right)l_s - \frac{\tau_\xi^{-1}\tilde{\tau}_{A,i}c_s}{1-\phi(1-\eta)} = 0. \qquad (64)$$

This is a depressed cubic polynomial whose unique real and positive root is given by

$$l_s = \sqrt[3]{-\frac{1}{2}\frac{\tau_\xi^{-1}\tilde{\tau}_{A,i}a_s}{1-\phi(1-\eta)} + \sqrt{\frac{1}{4}\left(\frac{\tau_\xi^{-1}\tilde{\tau}_{A,i}a_s}{1-\phi(1-\eta)}\right)^2 + \frac{1}{27}\tau_\xi^{-3}\left(\left(\tilde{\tau}_{A,i} - \frac{\phi\eta\tilde{\tau}_{A,i}a_s}{1-\phi(1-\eta)}\right)\right)^3}}$$
$$+\sqrt[3]{-\frac{1}{2}\left(\frac{\tau_\xi^{-1}\tilde{\tau}_{A,i}a_s}{1-\phi(1-\eta)}\right) - \sqrt{\frac{1}{4}\left(\frac{\tau_\xi^{-1}\tilde{\tau}_{A,i}a_s}{1-\phi(1-\eta)}\right)^2 + \frac{1}{27}\tau_\xi^{-3}\left(\left(\tilde{\tau}_{A,i} - \frac{\phi\eta\tilde{\tau}_{A,i}a_s}{1-\phi(1-\eta)}\right)\right)^3}}.$$

It follows that $l_s > 0$ and from equation (62) that

$$h_A = \frac{(1-\phi)\,l_s + (1+\phi\eta l_s)\left(\tilde{\tau}_{A,i} + l_s^2\tau_\xi\right)^{-1} l_s^2}{1+(1-\phi)\,k} > 0,$$

and, since $l_s = -h_A/h_\xi > 0$, that

$$h_\xi = -\frac{1-\phi + (1+\phi\eta l_s)\left(\tilde{\tau}_{A,i} + l_s^2\tau_\xi\right)^{-1} l_s}{1+(1-\phi)\,k} < 0.$$

We now prove that $l_F > 0$. Given the expression for l_F in (58) and that $l_s > 0$, it is sufficient for $l_F > 0$ if

$$\tilde{\tau}_{A,i}c_F > \frac{h_A}{h_\xi^2}h_F.$$

Given the expression for h_F in (63), and recognizing that $\tilde{\tau}_{A,i} > 0$ and $c_F > 0$, the above condition can be rewritten as

$$1 > \frac{h_A}{h_\xi^2}\left(\frac{1-\phi}{1+\phi\eta l_s}\tau_{A,i}^{-1}(k-l_P) + \frac{h_A}{h_\xi^2}\right)^{-1}.$$

Furthermore, from the expressions for $\tau_{A,i}$ and l_P, this condition can be further expressed as

$$\frac{1}{1+\phi\eta l_s}\left(1+k\,(1-\phi)\right)\left(\tilde{\tau}_{A,i} + \frac{h_A^2}{h_\xi^2}\tau_\xi\right) > \frac{h_A}{h_\xi^2}.$$

Since $l_s = -\frac{h_A}{h_\xi}$, given our expression for $h_\xi < 0$, the condition reduces to

$$\frac{1-\phi}{1+\phi\eta l_s}\left(\tilde{\tau}_{A,i} + l_s^2\tau_\xi\right) > 0,$$

which is always satisfied. Therefore, $l_F > 0$. In addition, since $(k-l_P)\,h_A = l_s$ implies that $k > l_P$, we see from $(k-l_P)\,h_F = l_F$ that $h_F > 0$.

We now examine the sign of l_P. By substituting $l_s = -\frac{h_A}{h_\xi}$ and the expressions of $\tau_{A,i}$ and h_ξ into (59), we have

$$l_P = -\frac{1}{h_\xi\,(1+(1-\phi)\,k)}\left(\tilde{\tau}_{A,i} + l_s^2\tau_\xi\right)^{-1}\left(kl_s - (\tau_\xi - k\phi\eta)\,l_s^2 - \tilde{\tau}_{A,i}\right).$$

Consequently, l_P can be positive or negative depending on the sign of $kl_s - (\tau_\xi - k\phi\eta)\,l_s^2 - \tilde{\tau}_{A,i}$.

A.9 Proof of Proposition 9

In (20), $\log P_X$ is a linear expression of $\log A$, θ, and ξ. We need to show that as $\tau_\theta \to \infty$, the coefficients of $\log A$ and ξ converge to their corresponding values in the perfect-information benchmark (Proposition 3), and the variance of the θ term

$$V_\theta = h_F^2 \tilde{h}_\theta^2 \tau_\theta^{-1} \to 0.$$

We rewrite equation (55) as

$$\left(\tilde{l}_s + \kappa\right)^2 \tilde{l}_s + \tau_\theta^{-1} \left(\tau_A + \frac{1-\phi}{1-\phi(1-\eta)} \tau_s\right) \tilde{l}_s = \frac{\tau_\theta^{-1} \tau_s}{1-\phi(1-\eta)}.$$

As τ_θ becomes sufficiently large, the RHS converges to zero and therefore, since the cubic polynomial has a unique real solution, $\tilde{l}_s \to 0$. By substituting equation (55) into our expression for $\tilde{h}_A$, one can express $\tilde{h}_A$ as

$$\tilde{h}_A = \frac{1-\phi(1-\eta)}{1+k(1-\phi)} \tau_s^{-1} \left(1 + \frac{(1-\phi)\tilde{l}_s}{1-(1-\phi)\tilde{l}_s}\right) \left(\frac{\tau_s}{1-\phi(1-\eta)} - \left(\tau_A + \frac{1-\phi}{1-\phi(1-\eta)}\tau_s\right)\tilde{l}_s\right).$$

As $\tau_\theta \to \infty$, $\tilde{l}_s \to 0$, then $\tilde{h}_A \to \frac{1}{1+k(1-\phi)}$. In addition, we can rewrite (64), by substituting for c_s, as

$$\tau_\xi l_s^3 + \tilde{\tau}_{A,i} l_s = (1+\phi\eta l_s) \frac{\tilde{\tau}_{A,i}^2 \tau_s}{1-\phi(1-\eta)}.$$

Since $\tau_\theta (l_s+\kappa)^2$ grows as τ_θ increases, one also has that $\tilde{\tau}_{A,i} = \left(\tau_s + \tau_A + \tau_\theta (l_s+\kappa)^2\right)^{-1} \to 0$ as $\tau_\theta \to \infty$. It then follows that $l_s \to 0$.

By substituting (64) and our expression for c_s into our expression for h_ξ, we have

$$h_\xi = -\frac{1-\phi}{1+(1-\phi)k} - \frac{1-\phi(1-\eta)}{1+(1-\phi)k} \tau_s^{-1} \left(\tilde{\tau}_{A,i} l_s\right)^2.$$

As $\tau_\theta \to \infty$, $\tilde{\tau}_{A,i} l_s = (1+\phi\eta l_s) \frac{\tilde{\tau}_{A,i}^2 \tau_s}{1-\phi(1-\eta)} - \tau_\xi l_s^3 \to 0$, and therefore $h_\xi \to -\frac{1-\phi}{1+k(1-\phi)}$. Given that $l_s = -\frac{h_A}{h_\xi}$ and our expression for l_P, we have that as $\tau_\theta \to \infty$, the coefficient of ξ in (20)

$$l_P h_\xi = -\frac{1}{1-\phi} (1+\phi\eta l_s) \tau_{A,i} l_s - \frac{1}{1-\phi} h_\xi \to \frac{1}{1+k(1-\phi)}$$

which is its value in the perfect-information benchmark.

Since $l_s = -\frac{h_A}{h_\xi}$, and that as $\tau_\theta \to \infty$, $l_s \to 0$ and $h_\xi \to -\frac{1-\phi}{1+k(1-\phi)}$, we have $h_A \to 0$. By substituting for $\tau_{A,i}$, c_F, l_P, and $\tilde{h}_A / \tilde{h}_\theta$, we can rewrite $h_F \tilde{h}_A$ as

$$\begin{aligned} h_F \tilde{h}_A &= \frac{1-\phi(1-\eta)}{1+k(1-\phi)} \tau_s^{-1} \tau_\theta (l_s+\kappa)^2 l_s \\ &= \frac{1-\phi(1-\eta)}{1+k(1-\phi)} \tau_s^{-1} \left((1+\phi\eta l_s) \frac{\tau_s}{1-\phi(1-\eta)} - \tau_\xi \left(\tilde{\tau}_{A,i}^{-1} l_s^{3/2}\right)^2 - (\tau_s+\tau_A) l_s\right), \end{aligned}$$

where we use substitution with equation (64). As $\tau_\theta \to \infty$, $l_s \to 0$ and $\left(\tilde{\tau}_{A,i}^{-1} l_s^{3/2}\right)^2 \to 0$, the coefficient of $\log A$ in (20) $h_A + h_F \tilde{h}_A \to \frac{1}{1+k(1-\phi)}$, which is its value in the perfect-information benchmark.

By using the expressions of h_F, l_P, l_s, c_F, $\tau_{A,i}$, $\tilde{l}_s$, and $\tilde{h}_\theta$ in Proposition 8 and by manipulating terms, we have

$$h_F \tilde{h}_\theta = \frac{1-\phi(1-\eta)}{1+k(1-\phi)} \tau_s^{-1} l_s \left(\tilde{l}_s + \kappa\right) \tau_\theta.$$

Consequently, we can write V_θ as

$$V_\theta = \left(\frac{1-\phi(1-\eta)}{1+k(1-\phi)} \tau_s^{-1}\right)^2 l_s^2 \left(\tilde{l}_s + \kappa\right)^2 \tau_\theta.$$

We can rewrite equation (55) as

$$\tau_\theta \left(\tilde{l}_s + \kappa\right)^2 = \frac{\tau_s}{1-\phi(1-\eta)} \tilde{l}_s^{-1} - \left(\tau_A + \frac{1-\phi}{1-\phi(1-\eta)} \tau_s\right)$$

By applying the Implicit Function Theorem to equation (55),

$$\frac{\partial \tilde{l}_s}{\partial \tau_\theta} = -\frac{\left(\tilde{l}_s + \kappa\right)^2 \tilde{l}_s^2}{2\tau_\theta \left(\tilde{l}_s + \kappa\right) \tilde{l}_s^2 + \frac{\tau_s}{1-\phi(1-\eta)}} < 0.$$

Consequently, $\tau_\theta \left(\tilde{l}_s + \kappa\right)^2$ is growing in τ_θ. Now we can rewrite equation (64) by substituting for $\tilde{\tau}_{A,i}$ and c_s, as

$$\left(\tau_A + \tau_s + \left(\tilde{l}_s + \kappa\right)^2 \tau_\theta\right) \sqrt{\frac{\tau_\xi l_s^3 + \left(\tau_A + \tau_s + \left(\tilde{l}_s + \kappa\right)^2 \tau_\theta\right)^{-1} l_s}{1 + \phi\eta l_s}} = \sqrt{\frac{\tau_s}{1-\phi(1-\eta)}}.$$

As $\tau_\theta \to \infty$, $l_s \to 0$. Thus, for this equation to hold, we must have $\tau_\theta (l_s + \kappa)^2 \to \infty$. Then, the LHS of the above equation can be expressed as

$$\begin{aligned} &\left(\tau_A + \tau_s + \left(\tilde{l}_s + \kappa\right)^2 \tau_\theta\right) \sqrt{\frac{\tau_\xi l_s^3 + \left(\tau_A + \tau_s + \left(\tilde{l}_s + \kappa\right)^2 \tau_\theta\right)^{-1} l_s}{1 + \phi\eta l_s}} \\ \approx\ & \tau_\theta \left(\tilde{l}_s + \kappa\right)^2 l_s^{3/2} \sqrt{\frac{\tau_\xi}{1+\phi\eta l_s}} + o\left(\tau_\theta^{-1} \left(\tilde{l}_s + \kappa\right)^{-2}\right). \end{aligned}$$

This suggests that $l_s^{3/2}$ must be shrinking at approximately the same rate as $\tau_\theta \left(\tilde{l}_s + \kappa\right)^2$ is growing for the LHS to remain finite. Therefore, l_s^2 must be shrinking at a faster rate and $V_\theta \to 0$ as $\tau_\theta \to \infty$.

Taken together, we have shown that as $\tau_\theta \to \infty$, $\log P_X$ converges with its counterpart in the perfect-information benchmark. We can similarly prove that the producers' aggregate demand also converges.

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