Banking Competition and Stability: The Role of Leverage*

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Abstract

This paper reexamines the classical issue of the possible trade-offs between banking competition and financial stability by highlighting the key role of leverage. By means of a simple model we show how competition affects portfolio risk, solvency risk, liquidity risk and systemic risk in different ways. The relationships depend crucially on banks' liability structure, and, more precisely on whether banks are financed by insured retail deposits or by uninsured wholesale funding. In addition, we argue that the analysis of the relationship between banking competition and financial stability should carefully distinguish between the different types of risk and should take into account banks' endogenous leverage decisions. This leads us to revisit the existing empirical literature using more precise measures of risk and endogenizing leverage, thus clarifying a number of apparently contradictory empirical results and allowing us to formulate new testable hypotheses.

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1 Introduction

Understanding the link between bank competition and financial stability is essential to the design of an efficient banking industry and its appropriate regulation. Because of the relevance of this topic, there is a large body of literature on the issue with path-breaking contributions from both theoretical and empirical perspectives. Yet, in spite of the critical importance of the subject and notwithstanding today's improved understanding of its complexity there is no clear-cut consensus on the impact of competition on banks' risk taking and the resulting level of stability for the banking industry. Two main contending views rise in the literature: the charter value view and the risk shifting view. The charter value approach, first put forward by [**?**], assumes that banks choose their level of risk and argues that less competition makes banks more cautious in their investment decisions, as in case of bankruptcy they will loose the present value of their future market power rents. Tenants of the risk shifting hypothesis, which originated with [**?**], postulate, instead, that risks result from the borrowing firms' decisions and point out that higher interest rates will lead firms to take more risk and therefore will increase the riskiness of the banks' portfolio of loans, leading to the opposite result.

The theoretical debate on the impact of banking competition cannot be simply solved by resorting to the empirical evidence, which is often ambiguous and contradictory. The ambiguous relationship between competition and financial stability is highlighted by [**?**], who show the relationship displays considerable cross-country variation. Part of the ambiguity stems from the difficulty in the choice of measurements for "financial stability". Indeed, while both [**?**] and [**?**] theoretical approach consider only bank insolvency risk, bank risk has multiple dimensions.

This paper's contribution is first to clarify the different forms of financial instability generated by banks' risk taking decisions and, second and more important, to emphasize the role of leverage as the banks' choice, which allow us to analyze the relationship between bank competition and financial stability from a different perspective.

Regarding the multiple forms of financial instability, the empirical literature reveals a great diversity in the estimated relationship between "competition" and "stability", which varies with stability and competition measures, samples and estimation techniques.[1] This is why a taxonomy of the types of banking risks will help us understand the existing empirical evidence. The need for such a clarification of the different concepts of risk becomes obvious when we consider the other side of the link, the measurement of competition, where the industrial organization literature has established the sensitivity of competition analysis to the specific measures of market power that are used. In comparison, the concept of banks' risk is clearly ambiguous and underdeveloped. We distinguish here four different types of banking risk: portfolio risk, banks' insolvency risk,

[1] Table 2 offers a synthetic survey of the different choices in the measures of competition and risk in the empirical contributions to the analysis of the competition-financial stability link.

illiquidity risk and systemic risk, and show how competition affects these types of risk differently.

More important is the second objective of this paper, to recover the role of leverage in the analysis of the link between competition and financial stability that the literature on banking competition, with the simplifying assumption of exogenous leverage, has largely ignored.[2] Endogenizing banks' leverage is all the more important in that a classical justification of financial intermediation is precisely the role of banks' in security transformation and liquidity insurance. To fulfill their functions, banks have to be able to choose their leverage ratio and to change it rapidly if necessary.[3] On the one hand, via leverage banks actively manage their risk, setting their leverage ratios in response to the riskiness of assets. On the other hand, leverage directly affects banks' solvency and funding liquidity, and plays a key role in financial contagion. The two-way feedback between leverage and risk makes leverage a central hub that connects all types of banking risk. Once we acknowledge the role of leverage as an endogenous variable, the perspective regarding banking competition and its effects on financial stability varies considerably: the riskiness of the banks' portfolio of loans is disentangled from the banks' insolvency, illiquidity and contagion risk. Competition affects the riskiness of banks' portfolios and banks respond by adjusting their leverage, so that leverage ratio, insolvency, illiquidity and systemic risk are all jointly determined.

The introduction of endogenous leverage, of course, implies a higher degree of complexity, but, in exchange, enables us to distinguish the impact of competition on risk at different levels. First, competition changes the banks' loan portfolio characteristics, making the portfolio less profitable but also less risky. Second, competition and the reduced portfolio risk have an impact on banks' leverage, and, ultimately, on its insolvency and illiquidity risk. For example, safer portfolios can lead banks to take on more debt.; and the high leverage erodes the pro-solvency effects of competition. When the debt is short-term, it also increases the bank's exposure to funding liquidity risk. Third, financial contagion from one bank to another is more likely when banks are highly leveraged, resulting in a greater chance of a systemic crisis. Consequently, even if banking competition leads to safer loans, because of the endogeneity of leverage, the insolvency risk of banks is not necessarily reduced, while their funding liquidity risk and systemic risk is increased.

By focusing on the four types of risk involved and endogenizing banks' leverage we point out the complexities of the mechanisms relating competition to banks' leverage and banks' leverage to a specific form of financial stability risk.

[2] For example, [?] considers banks solely financed by debt. [?] assume the cost of equity to be independent of banks' risk. When the constant equity cost is higher than that of debt, banks are again financed purely by debt.

[3] The role of banks in security transformation goes as far back as [?] and their role in liquidity insurance was first rigorously formalized by [?] and [?]

1.1 Related literature

Our approach builds on a large body of literature on banking competition that starts with the seminal paper of [?]. As mentioned before, [?] rightfully point out that the intrinsic countervailing forces of firms' risk-shifting can make the relationship between competition and financial stability ambiguous. [?] further refine Boyd and De Nicolo's argument by showing that the low profit resulting from competition leaves banks little buffer against loan losses and can therefore jeopardize financial stability. [?] considers both banks' and entrepreneurs' incentives to take risk on the portfolio side: once entrepreneurs and banks move sequentially, the overall effect coincides with the charter value hypothesis. The fact that all these contributions focus solely on insolvency risk and take the simplifying assumption of exogenous leverage has been one of the main motivations for our paper.

The study of banks' leverage in a competitive environment is related to [?]. The authors show that as competition decreases charter values, banks' incentives to monitor borrowers are reduced. To provide banks proper incentives to monitor, one way is to have banks hold more capital. Capital and loan rates therefore are alternative ways to improve banks' monitoring incentives. While the paper predicts that banks hold more capital in a competitive market, our paper shows the relationship is richer. Even in the traditional trade-off between bankruptcy cost and tax shield, the relationship between competition and leverage is non-monotonic.

1.2 Overview of the paper

Because we want to explore the impact of competition on the different types of risk, our starting point has to be the microfoundations of borrowing firms' risk taking. Following [?] and [?], in our model firms' investment decisions are subject to moral hazard, such that a higher interest rate leads them to take riskier investment projects. So individual loan defaults diminish with banking competition. To study banks' portfolio risk, we enrich upon the structure by taking into account the rents accruing to market power and allowing for imperfect correlation in loan defaults.[4] The last point is a key realistic assumption that makes leverage relevant in the determination of banks' risk. Otherwise, under perfectly correlated loan defaults the bank's capital level does not affect its solvency risk. At the other extreme, if the correlation is zero, no capital is required because of the law of large numbers. The impact of competition on the bank's portfolio is, on the one hand, a lower portfolio risk and, on the other hand, a reduced profitability that can limit the bank's buffer to cover losses.

Using this framework, we study how banks optimally choose their leverage in response to the changing competitive environment. This is achieved by means of a simplified theoretical framework where banks' leverage choice is explicitly

[4] [?] allows also for imperfectly correlated defaults; but the present model although based on more restrictive assumptions, is more tractable and delivers the analytical solutions that we need to endogenize leverage.

modeled as a response to the cost of funding. When the leverage choice is made endogenous, it presents a countervailing force to portfolio risk, as when the portfolio of assets becomes safer the bank may take higher leverage and increase risks and conversely.[5]

Our analysis allows us to establish that the relationship between insolvency risk and banking competition crucially depends on banks' liability structure, whether the banks are financed by insured retail deposits or by uninsured wholesale funding. The use of a specific model allows us to show that the risk shifting hypothesis is satisfied for low level of insured deposits while the charter value is correct in the opposite case. So the impact of banking competition on financial stability could be opposite for investment and for low leveraged commercial banks.

Going beyond the firms' risk taking and banks' solvency risk, liquidity risk and systemic risk with their own idiosyncrasy should also be analyzed. This implies considering bank runs and the value of assets in the secondary market when a run takes place. The global games approach allow us to show how the probability of a run on illiquid and solvent banks depends upon banks' capital buffer.

As a high leverage increases risks for uninsured short term debt holders, coordination failures are more likely, igniting runs to illiquid but solvent institutions. Our analysis of liquidity risk focuses on funding liquidity, thus emphasize short term wholesale financing as a driving force of runs and disregards banks' holding of liquid securities.[6] [7]

Regarding liquidity risk, defined as the probability that a solvent institution is unable to roll over its debt in the market, the result depends upon whether debt is exogenous given or taken as endogenous. If debt is exogenous, which can be interpreted as capital ratios being binding, competition will always increase liquidity risk. If instead debt is endogenous, the impact of liquidity risk will always move in opposite direction to insolvency risk, and depend on the structure of the bank's liabilities. For low levels of insured deposits pure insolvency risk decreases with competition and funding liquidity increases with competition. Nevertheless, the total credit risk of a bank, defined as the sum of solvency risk plus funding liquidity risk is dominated by the impact of competition on insolvency risk.

We extend the model to incorporate systemic risk and find similar results. For the financial system as a whole, leverage accelerates contagion when bank

[5] This is clearly illustrated in the extreme case where a bank's strategy is to maintain a given insolvency risk, which is in line with the idea of "economic capital". In this case, any changes in portfolio risk are exactly offset by the banks' leverage adjustment.

[6] This point is explored in [?]. The authors study banks' incentive to hold liquid assets in a competitive environment and argue that as competition pushes down returns from risky assets, the opportunity cost of hoarding liquid risk-free assets drops. Banks reshuffle their portfolios, hold more liquid assets and are better protected against bank runs.

[7] In fact, if we allow banks to hold liquid assets of low returns in the model, the hoarding of liquid assets only adds to banks' funding liquidity risk. This is because the low returns of liquid assets makes banks less profitable and less able to build up the capital buffer against fire-sale losses. Coordination failures become more likely as a result.

failures put downward pressure on asset prices. We illustrate this point by a simplified two-bank case. When both banks sell their assets the expected secondary market price becomes lower and the probability of a systemic crisis due to coordination failures increases. As before, and for the same reasons, the risk of two banks being run at the same time increases with leverage.

In sum, banks optimally set their leverage in light of the resulting risks; and at the meanwhile the insolvency, illiquidity and systemic risk are all affected by leverage. Banking risk and leverage interact and are jointly determined by banks' optimization. The two-way feedback generates a rich set of predictions on how competition affects various bank risks. We believe this has strong implications both for regulatory policy and for a possible refinement of the empirical analysis.

The paper proceeds as follows. Section 2 lays out the model. Section 3 establishes the benchmark case, exploring how various risks are affected by banking competition under the assumption of exogenous leverage. Section 4 is the core of the paper: we determine endogenous bank leverage and analyze its impacts on banks' insolvency, illiquidity and systemic risk. The results contrast those under exogenous leverage. We devote section 5 to empirical literature, reinterpreting the empirical findings with the refined definition of "financial stability" and making new testable hypotheses. Relevant policy implications are discussed in section 6. Section 7 concludes.

2 Model Setup

2.1 Portfolio risk and competition

We consider a one good three dates, $t = 0, 1, 2$, economy where all the agents are assumed to be risk neutral. There are three types of active agents: entrepreneurs, banks and banks' wholesale financiers and one type of purely passive agent: retail depositors. Entrepreneurs are penniless but have access to long-term risky projects. A project costs one unit of investment. It yields a gross return of $x > 1$ if succeeds and 0 if fails. The projects are subject to moral hazard: each entrepreneur chooses the probability of success $P \in [0, 1]$ in order to maximize his expected utility,

$$E(U) = P(x - r) - \frac{P^2}{2b}. \tag{1}$$

Here r is the gross loan rate charged by banks. $b \in (0, B]$ represents an entrepreneur's type, with a higher b implying a lower marginal cost of efforts. Entrepreneur types are private information, and in particular, unknown to banks, which hold prior beliefs that b is uniformly distributed in the interval $[0, B]$. Entrepreneurs' reservation utility is normalized to zero.

Because idiosyncratic risk diminishes in a bank's well diversified portfolio of loans, we dispense with the modeling of this type of risk and focus, instead, on a bank level risk that affects the whole bank portfolio in the following way:

whether a project succeeds or not is jointly affected by entrepreneurs' choice P and a risk factor z. The risk is assumed to be identical for all loans in a bank's portfolio, but can change across banks. It is assumed that z follows a standard normal distribution. Following [?] and [?], we assume the failure of a project is represented by a latent random variable y: when $y < 0$, the project fails. y takes the following form

$$y = -\Phi^{-1}(1 - P) + z, \tag{2}$$

where Φ denotes the c.d.f. of standard normal distribution. A project defaults either because of the entrepreneur's moral hazard (a low P) or an unfortunate risk realization that affects the bank's whole portfolio (a low z). For the sake of consistency, note that the probability of success P is given by:

$$Prob(y \geq 0) = 1 - Prob(y < 0) = 1 - Prob(z < \Phi^{-1}(1-P)) = 1 - \Phi(\Phi^{-1}(1-P)) = P.$$

Banks are assumed to invest in a continuum of projects. We further assume the loan market is fully covered and all types of entrepreneurs are financed. The loan portfolio generates a random cash flow denoted by θ. We denote the maximum possible cash flow by $\overline{\theta}$ and the minimum possible level by $\underline{\theta}$.

In order to focus on bank leverage and risks, we dispense with the specific modeling of loan market competition and consider the loan rate r as an exhaustive information on the degree of competition. On the one hand, low prices (loan rates) are predicted by the mainstream competition models[8] and constitute the driving force in reducing risk in the Boyd and De Nicoló setup. On the other hand, low interest margins are also found associated with less concentrated market, [?].

2.2 Funding liquidity risk

Each banks holds one unit portfolio of loans, and finances it with debt and equity. At $t = 0$, a bank finances its total investment of size 1 by raising F from insured retail deposits, V_D from short term wholesale creditors and the rest from equity holders. Because retail depositors are insensitive to the banks' risks and play a purely passive role, we assume its supply is fixed and inelastically set equal to F and that safety net of deposit insurance is offered to banks at no cost.[9] Bank's assets consist exclusively of its portfolio of loans.

The short term debt issued to wholesale financiers is of $t = 2$ face value D, risky and uninsured. It is raised in the market from investors that are risk neutral and require the market interest rate that is here normalized to zero. The debt is jointly financed by a continuum of creditors whose measure is 1. Each creditor holds an equal share of the bank's debt, i.e., D.

[8] The opposite relationship is obtained in models based on Broeker (1990) where an increase in the number of banks raises the probability for a bad borrower to get funded in equilibrium which implies an increases in the equilibrium interest rate.

[9] Assuming a flat deposit insurance premium that is based on the expected equilibrium debt ratio will not change our results.

The short-term nature of debt allows the creditor to withdraw at time $t = 1$, before banks' risky investment matures. In that case she receives qD, where $1 - q \in (0, 1)$ represents an early withdrawal penalty. Equivalently, the debt contract can simply be viewed as promising an interest rate qD at time $t = 1$ and D at time $t = 2$.

The bank's risky loan portfolio takes two periods to mature. When the bank faces early withdraws, it has to sell part (or all) of its portfolio in a secondary market at a discount:[10] for one unit asset with cash flow θ, the bank obtains only[11]

$$\frac{\theta}{1 + \lambda_1}.$$

Here $\lambda_1 > 0$ reflects the illiquid nature of banks' long-term assets. It can be due to moral hazard, e.g., banks' inalienable human capital in monitoring entrepreneurs, or adverse selection due to buyers concern with banks are selling their 'lemon' projects. The maturity mismatch and fire-sale discount together expose banks to the risk of bank runs.

In principle, a bank can fail either at $t = 1$ or $t = 2$. In the former case, the liquidation value of all asset is insufficient to repay early withdrawals. In the latter case, while partial liquidation generates sufficient cash to pay early withdrawals at $t = 1$, the residual portfolio is insufficient to pay creditors who wait until $t = 2$. Once a bank's cash flow is insufficient to repay its debt, either at $t = 1$ or $t = 2$, the bank declares bankruptcy and incurs a bankruptcy cost. For simplicity, we assume the bankruptcy cost is sufficiently high such that once bankruptcy happens, the wholesale financiers get zero payoffs and only the senior deposit insurance company representing retail depositors gets the residual cash flows. When a bank's residual cash flow is insufficient to repay its retail deposits, it is assumed that the deposit insurance company steps in and fills the gap.

At $t = 1$ each wholesale creditor privately observes a noisy signal $x_i = \theta + \epsilon_i$. Based on the information, the creditors play a bank-run game. Each player has two actions: to wait until maturity or to withdraw early. If the bank does not fail at $t = 2$, depositors who wait receive the promised repayment D. For a creditor who chooses to early withdraw, she receives nothing if the bank fails at $t = 1$ and qD if the bank does not fail on the intermediate date. If the bank is only able to pay early withdrawals but goes bankrupt at $t = 2$, the creditors who do not run receive nothing on the final date.

2.3 Contagion

The risk of contagion is illustrated with a two-bank setup: we make a stylized assumption that when both banks need to sell, the fire-sale discount hikes from

[10] The alternative assumption of banks using collateralized borrowing is to generate similar results. See [?].

[11] The proportional form assumes that buyers of the asset can observe better information than banks creditors. Some justification is provided in [?].

λ_1 to λ_2, where the subscripts $1, 2$ denote the number of banks that early liquidate. The assumption captures the observation that the secondary market price tends to fall further when more banks fail and sell, due to either cash-in-the-market pricing or informational contagion. In the former case, market prices are driven down by the limited supply of cash. In the latter, a high number of bank failures lead investors to update their expectations for banks' common risk exposures and lower their willingness to pay for the assets. The exposure to the same asset price provides a channel of financial contagion: when the first bank goes under and sells, the asset price is driven down; this magnifies concerns among debt holders of the other banks', leading to further bank runs.[12]

2.4 Endogenous leverage

Regarding the choice of leverage, we take the simplest standard textbook representation: capital structure is chosen to maximize the equity value of banks' existing shareholders.[13] In particular, each bank chooses to issue debt of face value D. The debt repayment is exempt from the corporate tax, which is assume to be at a constant marginal rate τ. Because of the existence of bankruptcy costs, the optimal leverage ratio will trade off the benefits of debt tax shield with the expected costs of bankruptcy. The existence of a liquidity risk makes the choice of leverage more complex as banks take into account both insolvency risk and illiquidity risk.

2.5 Time line

The timing of the model is summarized in the figure below.

[Timeline (changes have been made according to the handwriting)]

3 Banking risks with exogenous leverage

In this section, we analyze various risks for a fixed level of leverage. We move upward the spectrum of types of risk: from the bottom—individual loan default risk, to the top—the systemic risk of contagion.

3.1 Loan default and risk shifting

In the spirit of [**?**], we first show that bank competition reduces the default risk of individual loans by curbing entrepreneurs' moral hazard.[14] Note the utility

[12] For a full-fledged model that shows asset prices drop with bank runs, see Li and Ma (2012).

[13] It can be shown that this assumption in turn implies the maximization of leveraged firm values.

[14] We discuss the possible different results by taking a alternative priori of charter values in section 4.4.

maximization of an entrepreneur of type b yields the following probability of success.[15]

$$P_b^*(r) = \begin{cases} 1 & \text{if } b \in [1/(x-r), B] \\ b(x-r) & \text{if } b \in (0, 1/(x-r)) \end{cases}$$

An entrepreneur of type $b \geq 1/(x-r)$ will not default for any finite realization of z. This gives us a natural partition between risk-free and risky loans. For the uniform distribution of b, it implies that a fraction α of loans

$$\alpha \equiv 1 - \frac{1}{B(x-r)} \tag{3}$$

are risk free, and the complementary fraction $1-\alpha$ of loans

$$1 - \alpha \equiv \frac{1}{B(x-r)} \tag{4}$$

are risky and have positive probabilities of default, so that α reflects the riskiness of a bank's loan portfolio.

As expected, the risk of the portfolio decreases with bank competition. When banks charge lower loan rates under fierce competition, entrepreneurs have more 'skin in the game' and therefore put in more efforts. As the incentive of risk shifting dampens, entrepreneurs in the risky pool choose safer project and the pool of safe loans grows.

$$\partial\alpha/\partial r = \frac{-1}{B(x-r)^2} < 0 \tag{5}$$

3.2 Portfolio risk: loan loss and cash flow

In order to characterize a bank's portfolio risk we now derive the distribution of loan losses and cash flows that will depend on the distribution of the risk factor z.

Denote the fraction of non-performing loans *in the risky pool* by γ. We show γ follows a uniform distribution on $[0,1]$.

Lemma 1 *The loan loss γ, defined as the fraction of defaults in the risky pool, follows a uniform distribution on* $[0,1]$.

Proof. Take a risky type $\hat{b} < 1/(x-r)$; and define the fraction of entrepreneurs below $\hat{b}$ in the risky pool by $\hat{\gamma}$. We have

$$\hat{\gamma} = \frac{\hat{b} - 0}{1/(x-r) - 0} = \hat{b}(x-r). \tag{6}$$

Consider a critical realization $\hat{z} = \Phi^{-1}(1 - P_{\hat{b}}^*)$ such that an entrepreneur of $\hat{b}$ does not default but all types $b < \hat{b}$ do. So for $z = \hat{z}$, one will have $\gamma = \hat{\gamma}$. To

[15] Note $U_E(P_b^*) \geq 0$ such that the participation constraint of entrepreneurs is always satisfied for optimal P.

derive the distribution of γ, notice that

$$\begin{aligned} F(\hat{\gamma}) &\equiv Prob(\gamma < \hat{\gamma}) = Prob(z > \hat{z}) = 1 - Prob(z < \hat{z}) \\ &= 1 - \Phi(\Phi^{-1}(1 - P^*_{\hat{b}})) = P^*_{\hat{b}} \\ &= \hat{b}(x - r). \end{aligned}$$

By equation (6), we have $\hat{b} = \hat{\gamma}/(x - r)$. Substitution yields

$$F(\hat{\gamma}) = \hat{\gamma},$$

implying $\gamma \sim U(0, 1)$. ■

Lemma 1 entails that the expected loan loss in the risky pool is always 1/2. The riskiness of the portfolio depends only on the proportion of the risky pool, which in turn shrinks with competition and lower loan rates r. That is, banking competition reduces the risk of each individual loans, which in turn reduces the riskiness of a bank's loan portfolio by downsizing the risky pool.

The bank's realized cash flow θ associated with a level of losses γ is given by:

$$\begin{aligned} \theta &\equiv \alpha r + (1 - \alpha)[0 \cdot \gamma + r \cdot (1 - \gamma)] \\ &= r - (1 - \alpha) r \cdot \gamma. \end{aligned}$$

As the loan loss enters the cash flow expression in a linear way, the portfolio cash flow follows a uniform distribution on $[\alpha r, r]$. Denote the length of the support by $w \equiv (1 - \alpha) r$, the cash flow can be represented as

$$\theta = r - w\gamma.$$

The relationship between the portfolio risk and competition is captured by the variance of cash flow distribution. As banking competition intensifies, a bank's cash flow becomes less volatile.

$$Var(\theta) = w^2 \, Var(\gamma) \quad \text{and} \quad \frac{dVar(\theta)}{dr} > 0.$$

To see so, just verify $\partial w/\partial r > 0$. For γ's variance does not depend on the loan rate r, when competition reduces r, w shrinks and the variance of the cash flow decreases. Figure 1 depicts two distribution functions of cash flows, associated with different levels of competition. When competition intensifies, the distribution function becomes steeper, implying a smaller variance.

Lemma 2 *The random cash flow $\theta \sim U(\alpha r, r)$. When banking competition reduces the loan rate r, the variance of the cash flow drops.*

[Figure Here]

3.3 Insolvency risk

In this subsection, we define a bank's insolvency risk for a given level of debt. A bank is solvent if its cash flow meets its liability,

$$\theta = r - w\gamma \geq F + D.$$

The inequality gives a critical level of loan loss

$$g \equiv \frac{r - (F + D)}{w}.$$

A bank with a realized loan loss greater than g is to be insolvent. For $\gamma \sim U(0, 1)$, it is implied that the solvency probability is equal to g. The bank's pure insolvency risk, the risk of failure in the absence of any run, is denoted by SR and takes the following form.

$$SR \equiv 1 - g = \frac{(F + D) - \alpha r}{w} \tag{7}$$

Note that the insolvency risk is not monotonic in r. The reason is as in [**?**]. Banking competition has two countervailing effects on insolvency risk: on the one hand, lower loan rates reduce the risk taking of entrepreneurs so that the portfolio losses decrease (risk-shifting reduction). On the other hand, competition also makes interest margin thinner and banks less profitable, providing less cushion to absorb loan losses (buffer reduction). The overall effect is undetermined.

Proposition 1 *For a given leverage, a bank's insolvency risk is reduced by competition if and only if* $r^2 > x(F + D)$.

Proof. The proposition can be best shown as we examine the complementary probability, $1 - SR = [r - (F + D)]/w$. It has comparative statics

$$\begin{aligned}
\frac{\partial(1 - SR)}{\partial r} &= \frac{1}{w^2}\left[w - \frac{\partial w}{\partial r}\big[r - (F + D)\big]\right] \\
&= \frac{1}{w^2}\left[w - \Big[(1 - \alpha) - \frac{\partial \alpha}{\partial r} r\Big]\big[r - (F + D)\big]\right] \\
&= \frac{1}{w^2}\left[\frac{\partial \alpha}{\partial r} r^2 + \Big[(1 - \alpha) - \frac{\partial \alpha}{\partial r} r\Big](F + D)\right]
\end{aligned}$$

Recall that $\partial\alpha/\partial r = -1/B(x - r)^2$ and $(1 - \alpha) = 1/B(x - r)$. Taking out the common factor, we will have

$$\begin{aligned}
\frac{\partial(1 - SR)}{\partial r} &= \frac{1}{w^2}\frac{\partial \alpha}{\partial r}\left[r^2 + \big[-(x - r) - r\big](F + D)\right] \\
&= \frac{1}{w^2}\frac{\partial \alpha}{\partial r}\big[r^2 - x(F + D)\big]
\end{aligned}$$

Therefore,
$$\frac{\partial SR}{\partial r} = \frac{-1}{w^2}\frac{\partial \alpha}{\partial r}[r^2 - x(F+D)] > 0$$
if and only if
$$r^2 > x(F+D). \tag{8}$$

■

Graphically, this requires two conditions: (1) $\partial\underline{\theta}/\partial r > 0$ so that the distribution function should satisfy a single crossing condition, and (2) the face value of debt should lie to the left of the crossing point. Figure 2 illustrates such a scenario: as banking competition dampens risk-shifting and the loan rate hikes from r to r', the solvency probability drops from g to g'.

039<p type="texpara" tag="subsection" >Funding liquidity risk and bank run038<p type="texpara" tag="Body Text" >In this section we use the global games approach of [**?**] to examine banks' funding liquidity risk and derive a critical level of cash flow for a bank to be solvent but illiquid: the bank is able to repay in full its $t = 2$ liability if no one runs it at $t = 1$, but will default if sufficient many short-term debt holders withdraw early.038<p type="texpara" tag="Body Text" >In principle, a bank can fail either at $t = 1$ or $t = 2$. In the former case, the liquidation value of all assets is insufficient to repay early withdrawals. In the latter case, while partial liquidation generates sufficient cash to pay early withdrawals, the residual portfolio is insufficient to pay creditors who wait until $t = 2$. Once a bank's cash flow is insufficient to repay its debt, either at $t = 1$ or $t = 2$, the bank declares bankruptcy and incurs a bankruptcy cost. For simplicity, we assume the bankruptcy cost is sufficiently high such that wholesale financiers receive zero value. Only insured retail depositors are reimbursed, with any difference between F and residual cash flows being covered by the deposit insurance company.038<p type="texpara" tag="Body Text" >We will focus on the natural case where runs make it more difficult for a bank to meet its debt obligation, which occurs when
$$\frac{1}{1+\lambda_1} < q. \tag{9}$$
The condition is always true as q approaches 1.[16]038<p type="texpara" tag="Body Text" >If condition (9) is not satisfied, the debt repayment can be more easily met in a fire sale, which would be paradoxical.

Denote by L the fraction of wholesale financiers who run the bank. The bank will fail at $t = 1$ if $\theta/(1+\lambda_1) \leq LqD$, or
$$L \geq \frac{\theta}{1+\lambda_1} \cdot \frac{1}{qD} \equiv L'.$$

The bank is to survive $t = 1$ withdraws but fail at $t = 2$ if $(1-f)\theta < F + (1-L)D$, where $f = (1+\lambda_1)LqD/\theta$ denotes the fraction of asset sales to

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meet $t = 1$ withdrawals. In terms of the fraction of wholesale creditors L, the bank survives at $t = 1$ but fails at $t = 2$ if and only if

$$L \geq \frac{\theta - F - D}{[(1+\lambda_1)q - 1)]D} \equiv L''.$$

The lack of common knowledge leads to the so-called Laplacian property of global games: no matter what signal a player i observes, he has no information on the rank of his signal as compared to the signals observed by the others. Denote by M the fraction of players that player i believes to observe a higher signal than his. The Laplacian property implies $M \sim [0, 1]$.[17]

Players take a switching strategy: they run the bank if the observed signal is smaller than s^* and to wait otherwise. As the Laplacian property holds for all players and in particular for the player who observes the critical signal, the player will hold a belief that $M \sim U[0, 1]$ fraction of players will not run the bank and the rest $1 - M$ will run. Consequently, from the perspective of the player who observes the critical signal s^*, the probability for the bank to survive at $t = 1$ is

$$Prob(t = 1 \text{ survival}) = Prob(1 - M \leq L'^*) = \min\{L', 1\}.$$

The probability of $t = 2$ bankruptcy is

$$Prob(t = 2 \text{ survival}) = Prob(1 - M \leq L''^*) = \min\{L'', 1\}.$$

For simplicity, we focus on the case $L' > 1$. The inequality simply states that at $t = 1$ a bank does not have to liquidate all its assets to meet wholesale financiers' withdrawal, even if all of them run.[18] The the stable funding F makes this likely to happen: the presence of retail deposits limits the amount of unstable funding and their potential damage via fire-sale losses.

Depending on the outcome of bank run games, the payoffs for "run" and "wait" are tabulated as follows.

	$t = 2$ failure	$t = 2$ survival
run	qD	qD
wait	0	D

When running the bank, a creditor receives qD with certainty. By waiting, she receives a higher payoff D if the bank survives but risk a chance of having her debt value completely wiped out if $1 - L'' > 0$, in which case . Therefore, in playing the bank run game, the creditors trade off between the risk of receiving zero payoff and the greater remuneration from waiting.

[17] More detailed discussion of the property can be found in [?] and we reproduce the proof in the appendix.

[18] One can verify that the condition can satisfy for the optimal debt level that is solved in section 4.1.

To be consistent with the definition of switching strategy, a creditor who observes the critical signal s^* should be indifferent between running the bank or not.

$$Prob(t = 1 \text{ survival}|s = s^*)\cdot q = Prob(t = 2 \text{ survival}|s = s^*) \quad \text{or} \quad q = \frac{\theta - F - D}{[(1+\lambda_1)q - 1)]D},$$

The indifference condition implies the following critical cash flow $\hat{\theta}$.

$$\theta^* = F + D + [(1+\lambda_1)q - 1]D \tag{10}$$

A run successfully happens when a bank's θ falls below the critical θ^*. Define $\mu_1 \equiv 1 - q[1 - (1+\lambda_1)q]$. A bank is solvent but illiquid if

$$F + D < \theta \leq F + \mu_1 D. \tag{11}$$

We know $\mu_1 > 1$ under our presumption $(1+\lambda_1)q > 1$. Thus in order to survive potential bank runs, a bank has to make more profit than what is in need to be barely solvent. Further notice that the critical cash flow increases in λ_1 and D such that a higher fire sale loss and more exposure to unstable short-term funding lead to a higher chance of illiquidity.

Proposition 2 *There exists a critical level* $\theta^* = F + \mu_1 D$, $\mu_1 = 1 - q[1 - (1+\lambda_1)q] > 1$, *a bank having cash flow* $\theta \in [F + D, \theta^*]$ *becomes solvent but illiquid: being able to sustain in the absence of bank runs, but going bankrupt as a run happens.*

Proposition 2 states that there is pure liquidity risk for banks in the range $[F + D, F + \mu_1 D]$, as those banks are solvent in the absence of bank runs but insolvent if a run occurs. As a bank cash flow $\theta \sim U[\alpha r, r]$, the probability of a pure liquidity crisis, defined as the probability of a bank being solvent but illiquid and denoted by IL, is

$$IL \equiv \frac{(\mu_1 - 1)D}{w}. \tag{12}$$

For a given cash flow distribution, two factors contribute to banks' funding illiquidity: (1) low fire-sale prices of banks' assets, (2) low capital buffer (or equivalently, high debt obligations). For fire-sale price and leverage are held exogenous, competition contributes to illiquidity by reducing the expected cash flows. Contrary to with pure insolvency risk, the amount of stable funds provided by insured deposits, F, is absent from the above measure of risk, as retail depositors do not have the incentives to run the bank.

Once a bank's debt obligation is exogenous, its funding liquidity risk increases with competition. The result follows directly from the first order derivative,

$$\frac{\partial IL}{\partial r} = (\mu_1 - 1)\frac{-D}{w^2}\frac{\partial w}{\partial r} < 0.$$

Intuitively, the lower cash flow due to intensified competition provides a thinner buffer against fire-sale losses. Creditors who withdraw early are more likely to incur a loss to those who wait. As the negative externalities aggravate, the coordination failure intensifies, and bank runs happen more frequently as a results. The following proposition summarizes our result.

Proposition 3 *For a given level of debt obligation, the chance for a bank to be solvent but illiquid increases with competition.*

Proof. To verify the statement, just check the first order derivative

$$\frac{\partial IL}{\partial r} = (\mu_1 - 1)D\frac{-1}{w^2}\frac{\partial w}{\partial r} < 0.$$

As competition reduces the loan rate r, the risk of illiquidity rises. ■

In practice, it is hard to distinguish bank failures due to insolvency and those due to illiquidity. The observational equivalence makes it sensible to examine a bank's total credit risk, the summation of solvency and liquidity risk that measures a bank probability of going bankrupt for either solvency or liquidity reasons. Denoted the total credit risk by TCR, we have

$$TCR = \frac{(F + \mu_1 D) - \alpha r}{w}. \tag{13}$$

Banking competition reduces total credit risk ($\partial TCR/\partial r > 0$) if and only if

$$.r^2 > x(F + \mu_1 D) \tag{14}$$

Note that this condition is more stringent than the condition in Proposition 1. So for a parameter constellation satisfying $x(F + \mu_1 D) > r^2 > x(F + D)$ banking competition would decrease pure insolvency risk but once liquidity risk is taken into account, competition increases total credit risk, that is generates financial instability. In other words, when illiquidity risk is brought in, the set of parameters where the result of [**?**] applies shrinks.

3.4 Systemic risk of contagion

The price for banks' assets tends to fall further when more banks fail and are forced to liquidate, due to either cash-in-the-market pricing or informational contagion. In particular, we make a stylized assumption that when both banks need to sell the fire-sale penalty increases from λ_1 to λ_2, where the subscripts $1, 2$ denote the number of bank failures.

Following the same procedure of the last section, we will be able to derive a critical cash flow level

$$\theta_2^* = F + \mu_2 D > \theta_1^*, \tag{15}$$

where $\mu_2 = 1 - q[1 - (1 + \lambda_2)q] > \mu_1$. A bank whose cash flow falls between $[\theta_1^*, \theta_2^*]$ will be solvent and liquid if the other bank does not face a run, but will become solvent but illiquid if runs happen to the other bank. Namely, a bank whose cash flow falls between $[F + \mu_1 D, F + \mu_2 D]$ is exposed to contagion and will fall, if the other bank's cash flow falls below $F + \mu_2 D$. We therefore define the exposure to contagion (denoted by CTG)

$$CTG \equiv \frac{(\mu_2 - \mu_1)D}{w}. \tag{16}$$

Note that as competition reduces banks' buffer to against fire-sale losses, $\partial w/\partial r > 0$, the exposure to contagion increases with competition.

In this two-bank setup, the systemic risk (denoted by SYS) is captured by the probability that both banks fail at the same time. That is,

$$SYS = Prob(\theta < \theta^{**})^2 = \left(\frac{\theta^{**} - \alpha r}{w}\right) \tag{17}$$

Its first order derivative with respect to r

$$\begin{aligned}
\frac{\partial SYS}{\partial r} &= 2Prob(\theta < \theta^{**})\frac{\partial}{\partial r}\left(\frac{\theta^{**} - \alpha r}{w}\right) \\
&= 2Prob(\theta < \theta^{**})B\big(- \theta^{**}\frac{x}{r^2} + 1\big) \\
&= 2Prob(\theta < \theta^{**})B\big[- (F + \mu_2 D)\frac{x}{r^2} + 1\big]
\end{aligned}$$

implies that competition reduces the systemic risk if and only if

$$r^2 > x(F + \mu_2 D), \tag{18}$$

a counterpart to condition (13).

Therefore, even if banks do not adjust their leverage to the changing competitive environment, we show banking competition can affect different types of risk differently. Focusing on only one dimension of risk can lead to biased judgement for the overall effects. We continue to show, once leverage is endogenous, the situation becomes more complicated: a full range of predictions can rise for the competition-stability relationship. In some instances, the results under exogenous leverage are completely reversed.

4 Endogenous leverage and its impacts

It is important to notice that the portfolio risk is not identical to the default risk of a bank. Leverage plays a crucial role: a low risk portfolio financed with high leverage can still fail with a significant chance. Furthermore, leverage is endogenously chosen based on the portfolio risk: a bank with safer portfolio will be able to use higher leverage and a bank with risky portfolio will find it more costly to issue debts. In this sense, leverage is a countervailing force to the change in asset riskiness. In this section, we endogenize banks' choice of leverage and analyze its impact on different types of risks.

4.1 Endogenous leverage

We assume that the cost of capital is larger than the cost of debt. Denote the premium of capital over debt as k, so that the expected return on capital is $1+k$. One justification for the existence of such a premium could be dilution costs à la Myers-Majluf; Diamond Rajan provide an alternative justification based on the managers renegotiation power; another possible justification would be simply tax benefits of debt.

The optimal level of debt is determined by its marginal cost equaling its marginal benefit: on the one hand, a higher debt level entails a greater chance of bankruptcy. On the other hand, a higher debt level saves on the cost of capital, whether it is dilution cost or tax advantage. Banks rationally set their leverage by taking into account the probability of bankruptcy, caused either by insolvency or illiquidity.

4.2 The general case

Banks choose their capital structure to maximize the leveraged firm value to existing shareholders. If α is the fraction of the bank that is sold to outside shareholders, the value to old shareholders is

$$V_E = (1-\alpha)\int_{F+\mu_1 D}^{\overline{\theta}} [\theta - F - D] h(\theta, r) d\theta,$$

where $h(\theta, r)$ the density function of banks' cash flows. And the market will allow to raise V_S from new shareholders as the present value of an α fraction of the bank, V_D from wholesale short term creditors, and F from insured depositors.

$$\begin{aligned}
V_S &= \frac{1}{1+k}\alpha \int_{F+\mu_1 D}^{\overline{\theta}} [\theta - D - F] h(\theta, r) d\theta \\
V_D &= \int_{F+\mu_1 D} D d\gamma \\
V_F &= F
\end{aligned}$$

The three sources of funding should provide the required amount of investment, that is,

$$V_S + V_D + F = 1.$$

Consequently the optimal leverage structure is the solution to

$$\begin{aligned}
&\max_{\alpha, D} \left\{ (1-\alpha) \int_{F+\mu_1 D}^{\overline{\theta}} [\theta - F - D] h(\theta, r) d\theta \right\} \\
s.t. \qquad & \frac{1}{1+k}\alpha \int_{F+\mu_1 D}^{\overline{\theta}} [\theta - D - F] h(\theta, r) d\theta - V_S = 0 \\
& \int_{F+\mu_1 D}^{\overline{\theta}} D h(\theta, r) d\theta - V_D = 0 \\
& V_S + V_D + F = 1
\end{aligned}$$

Adding the three constraints to the objective function we obtain the unconstrained optimization.

$$\max_D \left\{ \int_{F+\mu_1 D}^{\overline{\theta}} [\theta + k(D+F)]h(\theta,r)d\theta + (1+k)\int_{\underline{\theta}}^{F+\mu_1 D} Fh(\theta,r)d\theta - (1+k) \right\} \tag{19}$$

Notice that if q reflects the cost of a corporate tax τ, $1+k = 1/(1-\tau)$, implying $\tau = \frac{k}{1+k}$, thus providing the familiar expression that combines the the cost of equity, the benefits of tax shield and the benefits of deposit insurance.

The optimization program yields the first order condition

$$-[(F+\mu_1 D)+k(F+\mu_1 D)]h(F+\mu_1 D,r)+\int_{F+\mu_1 D}^{\overline{\theta}} kh(\theta,r)d\theta+(1+k)Fh(F+\mu_1 D,r) = 0,$$

which can be written more compactly as

$$-(1+k)Dh(F+\mu_1 D,r) + \int_{F+\mu_1 D}^{\overline{\theta}} kh(\theta,r)d\theta = 0,$$

or, with H denoting the c.d.f. of θ,

$$D^* = \frac{k[1 - H(F+\mu_1 D^*, r)]}{(1+k)h(F+\mu_1 D^*, r)}.$$

4.2.1 Application to our setup

The derived uniform distributions in the current paper simplify the model. It is especially convenient to work with the stochastic loan losses $\gamma \sim U[0,1]$. To facilitate exposition, we denote

$$g(\mu_1) \equiv 1 - TCR = \frac{r - (F+\mu_1 D)}{w}.$$

$g(\mu_1)$ is a counterpart of g: it denotes the critical loan loss the bank will survive once liquidity risk is taken into account. The optimization program transforms into the following form.

$$\begin{aligned}
& \max_{\alpha, D} \left\{ (1-\alpha)\int_0^{g(\mu_1)} [\theta - F - D]d\gamma \right\} \\
s.t. \qquad & \frac{1}{1+k}\alpha \int_0^{g(\mu_1)} [\theta - D - F]d\gamma - V_S = 0 \\
& \int_0^{g(\mu_1)} Dd\gamma - V_D = 0 \\
& V_S + V_D + F = 1
\end{aligned}$$

With the corresponding unconstrained program:

$$\max_D \left\{ \int_0^{g(\mu_1)} [\theta + k(D+F)]d\gamma + (1+k)\int_{g(\mu_1)}^{1} Fd\gamma - (1+k) \right\} \tag{20}$$

Recall that $\theta = r - w\gamma$. The maximization program has the first order condition

$$\left[r\frac{\partial g(\mu_1)}{\partial D} - \frac{w}{2}2g\frac{\partial g(\mu_1)}{\partial D}\right] + kg(\mu_1) + k(F+D)\frac{\partial g(\mu_1)}{\partial D} - (1+k)F\frac{\partial g(\mu_1)}{\partial D} = 0,$$

which yields the optimal level of risky debt

$$D^* = \frac{r - F}{\mu_1^2/k + 2\mu_1}. \tag{21}$$

And it is straightforward to check that the second order condition satisfies.

$$-\frac{w}{2}2\left(\frac{\partial g(\mu_1)}{\partial D}\right)^2 + 2k\frac{\partial g(\mu_1)}{\partial D} < 0$$

Denote the constant coefficient $c = 1/[\mu_1^2/k + 2\mu_1]$. The optimal debt obligation is written compactly as

$$D^* = c \cdot (r - F). \tag{22}$$

The result is summarized in the following theorem.

Proposition 4 *A bank that maximizes its value by trading off the benefits of debts versus bankruptcy cost sets its debt* $D^* = c \cdot (r - F)$, *where* $c = 1/[\mu_1^2/k + 2\mu_1]$.

The risky debt that a bank issues is proportional to its *maximum* residual cash flow after paying insured deposits F. In particular, the proportion increases in the cost of capital, $\partial c/\partial k > 0$, and decreases in the liquidity risk, $\partial c/\partial \mu_1 < 0$. So will the bank's debt level. Furthermore, note that

$$\lim_{\mu_1 \searrow 1} c = \frac{1}{1/k + 2} < 1.$$

For c monotonically decreases in μ_1, it holds that $c < 1/(1/k+2) < 1$: a bank cannot issue more risky debt claims than its maximum cash flow after paying the risk-free F and is unwilling to issue risky debt more than c fraction of $(r - F)$.

Being junior, risky and demandable, the wholesale funding D is the most relevant debt in the model. Denote a bank's leverage ratio by l^*. It can be measured by the face value of its risky debt over the available expected cash flow:

$$l^* \equiv \frac{D^*}{E(\theta - F)} = \frac{2c(r - F)}{(1+\alpha)r - 2F}. \tag{23}$$

The leverage ratio is not monotonic in banking competition. Its comparative statics with respect to r depends on the relative strength of two countervailing effects: (1) when r increases, a bank generates a higher cash flow and can issue more claims, including risky debts, which we call "margin effects"; and (2) a higher r implies stronger risk-shifting by entrepreneurs, leading to higher portfolio risk and curbs leverage via bankruptcy costs, which we call "risk effects". The overall effect depends on the relative magnitude of the two forces.

Proposition 5 *The leverage ratio $D^*/E(\theta - F)$ decreases with competition (increases with loan rate r) if and only if $xF < r^2$. Otherwise, the result reverses.*

Proof. The result follows directly from the first order condition.

$$\begin{aligned}
\frac{\partial l^*}{\partial r} &= \frac{2c}{[(1+\alpha)r - 2F]^2}\left[\big[(1+\alpha)r - 2F\big] - (r-F)\big[\frac{\partial \alpha}{\partial r}r + (1+\alpha)\big]\right] \\
&= \frac{2c}{[(1+\alpha)r - 2F]^2}\left[-(1-\alpha)F - \frac{\partial \alpha}{\partial r}r(r-F)\right] \\
&= \frac{2c\cdot(1-\alpha)}{[(1+\alpha)r - 2F]^2}\frac{r^2 - xF}{x - r}
\end{aligned}$$

The comparative statics depends on the sign of

$$r^2 - xF. \tag{24}$$

When $r^2 > xF$, $\partial l^*/\partial r > 0$, banking competition leads to a lower leverage ratio. ■

In the current setup, as F decreases, a bank has more cash flow available to its wholesale financiers. In that case, when r increases, the margin effect dominates the risk effect. And overall the leverage ratio rises.

4.3 Risk under endogenous leverage

The banking risk under endogenous leverage D^* is denoted by a superscript star, e.g., SR^* for the pure insolvency risk under endogenous leverage, etc. We show the endogenous leverage has a crucial impact on various risk. In some instances, it reverses the results obtained under exogenous leverage.

Proposition 6 *As competition intensifies, pure insolvency risk and funding liquidity risk move in opposite directions. In particular, if $xF < r^2$, pure insolvency risk decreases with competition and funding liquidity risk increases in competition. Otherwise, the result reverses.*

Proof. The result follows directly the first order derivatives. We start with pure insolvency risk.

$$SR^* \equiv 1 - \frac{r - F - D^*}{w} = 1 - \frac{(1-c)(r-F)}{w}.$$

The comparative statics follows

$$\frac{\partial SR^*}{\partial r} = -(1-c)\frac{\partial}{\partial r}\left(\frac{r-F}{w}\right).$$

For $c < 1$, the expression shares the same sign as

$$-\frac{\partial}{\partial r}\left(\frac{r-F}{w}\right).$$

Now examine liquidity liquidity risk

$$IL^* \equiv (\mu_1 - 1)\frac{D^*}{w} = (\mu_1 - 1)c\frac{r-F}{w}.$$

The comparative statics follow

$$\frac{\partial IL^*}{\partial r} = (\mu_1 - 1)c\frac{\partial}{\partial r}\left(\frac{r-F}{w}\right),$$

whose sign is the same as

$$\frac{\partial}{\partial r}\left(\frac{r-F}{w}\right).$$

Therefore with the change in competition r, the two types of risks move in the opposite direction. Finally, note that

$$\begin{aligned}
\frac{\partial}{\partial r}\left(\frac{r-F}{w}\right) &= \frac{1}{w^2}\left[w - \big[-\frac{\partial \alpha}{\partial r}r + (1-\alpha)\big](r-F)\right] \\
&= \frac{1}{w^2}\left[\frac{\partial \alpha}{\partial r}r^2 + \big[-\frac{\partial \alpha}{\partial r}r + (1-\alpha)\big]F\right] \\
&= \frac{1}{w^2}\frac{\partial \alpha}{\partial r}[r^2 - rF - (x-r)F].
\end{aligned}$$

Simplify the last term yields

$$\frac{1}{w^2}\frac{\partial \alpha}{\partial r}(r^2 - xF). \tag{25}$$

The sign again depends on that of $r^2 - xF$, which also determines the comparative statics of l with respect to r. ■

The results on endogenous total credit risk (TCR^*) and the exposure to contagion (CTG^*) will obtain by the same procedure.

Corollary 1 *The total credit risk of individual bank, i.e., pure insolvency risk plus funding liquidity risk, reduces with competition, if and only if $xF < r^2$. Otherwise, the result reverses.*

Proof. Notice that the total credit risk

$$TCR \equiv 1 - \frac{r - F - \mu D^*}{w} = 1 - (1-\mu c)\frac{r-F}{w}$$

The result follows directly the first order derivative.

$$\frac{\partial TCR}{\partial r} = -(1-\mu c)\frac{\partial}{\partial r}\left(\frac{r-F}{w}\right)$$

Since $\mu c < 1$, the total credit risk is determined by

$$-\frac{\partial}{\partial r}\left(\frac{r-F}{w}\right).$$

Combine this with expression (25). The result is proved. ■

Corollary 2 *The exposure to the risk of contagion, $(\mu_2 - \mu_1)D^*/(1-\alpha)r$, decreases with competition (increases with loan rate r) if and only if $xF < r^2$. Otherwise, the result reverses.*

Proof. The result follows directly the first order derivative:

$$\frac{\partial}{\partial r}\left(\frac{(\mu_2 - \mu_1)D^*}{(1-\alpha)r}\right) = (\mu_2 - \mu_1)\frac{\partial}{\partial r}\left(\frac{r-F}{w}\right).$$

Note that this again gives the expression in equation (25). The comparative statics depends on the sign of $xF < r^2$. ■

As shown by inequality (24), banks' liability structure is central to the relationship between competition and bank risk. Whether a bank has insured retail deposits $F > r^2/x$ makes a critical difference. To link the central condition (24) to the real world financial institutions, consider two types of banks corresponding to the two possible signs of $xF - r^2$. A negative sign, with $xF < r^2$, corresponds to less productive firms, with banks financed through market funding and high interest rates on loans. When this is the case, total credit risk is reduced with competition. This will occur for a low x implying a risky portfolio of loans (P_b^* increases with x for every value of r). Banks have a relatively high average cost of funds because of a low F and a high income as r is high. As a particular case, $F = 0$ in which case x is irrelevant, corresponds to this case and might be interpreted as investment banking. More competition means safer investment banking. A positive sign, with $xF > r^2$, corresponds to highly productive firms, with banks mainly financed through deposits and low interest rates on loans. When this is the case, a high x implies a safer portfolio of loans ($P_B^*(r)$ is higher for every value of r). Banks have a lower average cost of funds because of the higher F and a lower income as r is low. This might be closer to retail banking. The opposite result occurs, and more competition will increase total risk.

4.4 Interpretation

Although our model does not pretend to provide robust results that hold true in every environment, it is worth noticing the key ingredients that determine here the impact of bank competition on the different types of financial stability. As shown by inequality (24), banks' liability structure, and in particular the amount of short term wholesale funding, is central to the relationship between competition and bank risk. Our model's conclusions provide a much richer view of the link between banking competition and financial risk than what is usually considered.

1. To begin with, notice that the result depends upon x. For a given level of deposits and banks' market power, the effect of banking competition on financial stability depends upon how productive the firms are (or how large is the part of self financing in their investment, as mentioned in footnote xxx). In highly productive economies, bank competition constitute a threat to financial stability. The impact of moral hazard is reduced, and the key determinant of the link bank

competition to financial stability is the role of the buffer generated by banks' market power. Comparing Proposition 1 and Proposition 6 we observe that the threshold for x that inverts the relationship from banking competition to financial stability is reached much earlier if we take into account the endogeneity of banks' leverage. This is the case because firms will be more conservative in its choice of leverage, so that the strength of the Boyd and DeNicolo argument is weakened and the charter value dominates.

2. The level of market power is also essential in our framework. For a very high market power competition reduces bank fragility, nevertheless a threshold may exist (provided that $xF > 1$) beyond which the result reserves. This is interesting from a policy perspective as it provides a more nuanced prescription than the usual one: it might be interesting to promote competition up to a certain point.

3. The role of stable funds is critical for our result. In a traditional banking industry funded through deposits and long term bonds, where $xF > r^2$, competition will be detrimental to financial stability. Instead in a banking industry where wholesale short term (possibly interbank) funding is prevalent, the Boyd De Nicolo prediction is more likely to be fulfill.

4 More generally, two types of banks, corresponding to the two possible signs of $xF - r^2$, will coexist and will react in a different way to an increase in competition. For banks that rely less on stable funding $xF < r^2$, in particular for investment banks, an increase in competition will increase financial stability. Instead, for banks with high levels of deposits and lower market power, for which the inequality $xF > r^2$ is fulfilled, the opposite occurs and banking competition main effect is to reduce the banks buffer and to encourage higher leverage.

Figure 3 visualize the channels that banking competition affects risk, either directly through cash flow riskiness, or indirectly through the changing leverage.[19] It emphasizes the role of banking competition in influencing the cash flow characteristics and bank leverage and risk are jointly determined by the optimization behaviors of banks.

It should also be acknowledged that despite of our efforts to build a comprehensive model, the presented still considerably understates the complexity of the issue, for competition also affects banks' portfolio choice, e.g., the correlation of their portfolios, cash hoarding, and so on, which are all abstract from the current setup.

4.5 Comparison to the exogenous leverage case

To emphasize the crucial impact of endogenous leverage, we tabulate in Table 1 the results under exogenous and endogenous bank leverage for a side-by-side comparison.

[Table Here]

[19] For simplicity of the graph, we suppress the feedback from risk to lavage. Yet it should be clear that banks choose their leverage in light of its consequences on risk.

To a large extent, the model presented is a special case where all important comparative statics depend on the sign of $r^2 - xF$. Yet the result conveys the key messages of the paper: (1) banking competition affects different types of risk differently; and (2) the endogenous leverage is a central hub that both responses to the change in the cash flow riskiness and affects all different aspects of banking risk.

4.6 An alternative prior

The current paper takes risk-shifting hypothesis as its starting point. The argument of endogenous leverage however is more general and also applies to models that takes charter value hypothesis as a prior. As competition reduces charter values and induces banks to choose riskier portfolios—portfolios with higher and more volatile cash flows, banks will optimally adjust their leverage to balance the change in the portfolio risk. Again, leverage stands a central hub to which all types of bank risk link. And the leverage as well as risk are all jointly determined by banks' optimization behaviors. The overall results however can be reversed as compared to the current paper.

5 Reinterpreting the empirical literature

The difficulties in analyzing the link between competition and financial stability exponentially increase when we turn to the empirical studies. The empirical analysis has led to a multiplicity of results that are sometimes difficult to reconcile and susceptible of alternative interpretations. As we have emphasized in this paper there are multiple measures of financial stability as there are multiple measures of competition, ranging from franchise or charter value (Tobin's Q), to market structure (e.g., HHI, C-n), to structural measurement (i.e., P-R H-stat., Lerner's index, Boone's indicator), and to institutions (contestability of the market, e.g., activity and entry restrictions).[20] The industrial organization literature does not provide an unambiguous answer to which measurement reflect competition best, even if, in the context of banking the empirical evidence shows that concentration measures are poor proxies for bank competition, [**?**] and [**?**], it still leaves a wide range of possible measures.

The empirical implication of our model is that banks' risks should be measured at four fundamentally different levels: first, at the level of banks' assets; second, at the level of banks' solvency; third, at the level of banks' liquidity risk and fourth at the level of the overall systemic risk and contagion. Competition directly affects the riskiness of banks' cash flows; but because banks react to these changes by altering their leverage, all other types of risks are also affected. Indeed, banks' endogenous leverage constitutes a central hub that connects these three types of risk. The implication is that depending on the magnitude of the direct and indirect forces, a full diversity of predictions can rise.

[20] See [**?**] for a comprehensive review on measuring banking competition.

As a consequence, our reading of the empirical literature results introduces drastic differences depending on whether the evidence concerns the riskiness of banks' assets, the riskiness of banks themselves, either their solvency or their liquidity, or systemic risk.

Our theoretical framework suggests a progressive approach to the understanding of the impact of competition on banks' risk taking by refining the questions that are asked as successive layers.

1. Does competition increase the safety of a banks portfolio of assets? In other words is the Boyd & DeNicole's basic assumption true?

   Next, once we take into account the optimal reallocation of assets and the effect of optimal leverage tuning by the bank, the following issues are to be addressed:

2. Does competition increase the risk of a bank's insolvency?

3. Does competition increase the liquidity risk of banks?

4. Does competition increase banks' systemic risk?

Revisiting the empirical literature through this filter leads us to regroup the empirical results in a more complete and orderly way, refusing to find the different measures either equivalent or complementary in the assessment of the impact of competition on financial stability, without taking into account the changes in leverage it produces. In the end of the section, we summarize in table 2 the empirical literature by highlighting the different key contributions, the measures of risk and competition they utilize and the results regarding the impact of competition on financial instability they obtain.

5.1 Portfolio risk: non-performing loans

The basic postulate of the [**?**] approach is that competition will reduce the riskiness of banks' portfolio, an issue independent of the banks' leverage reaction. The alternative hypothesis put forward by the tenants of the charter value approach is that the banks' overall investment strategy will be more risky as the opportunity cost of bankruptcy is lower. So knowing whether Boyd & De Nicolo's basic assumption is in line with empirical evidence is a crucial step forward. In order to measure the riskiness of assets, measures like stock volatility in [**?**, **?**] are contaminated by leverage. The non-performing loans (NPLs) ratio is the variable that reflects most accurately the riskiness of banks' assets, and a bulk of literature appears to support this view, as it analyzes non-performing loans as one of the key variables in the analysis of the competition-financial stability link.

Restricting the measurement of risk to NPLs implies focusing on a very specific dimension of the broad link between competition and financial stability where we might hope for some consensus on the empirical results. Unfortunately even with this drastic reduction the evidence is mixed. So, in spite of the fact that the charter value and risk shifting theories have completely opposite predictions regarding the impact of banks' competition on non-performing loans, empirical studies give no definitive answer on which one should be the predominant view.

The initial paper on the charter value [?] did not consider NPLs measures but rather estimates of overall bank risk of failure. The prediction on NPLs is backed by more recent works, such as the analysis of [?] and [?]. The authors found an increase in non-performing loans as bank competition increased in Spain and in eight Latin American countries respectively. Support for the risk shifting hypothesis comes from Boyd et al. (2006) and is corroborated by Berger et al.(2008) who find an interesting set of results based upon both loan risk and overall bank risk. Using cross-sectional data on 29 developed countries for the years 1999 through 2005, they find that banks with a higher degree of market power exhibit significantly more loan portfolio risk.

The impact of the US introduction of Nationwide banking also leads to contradictory results: while [?] report that "Loan losses decrease by about 29 basis points in the short run and about 48 basis points in the longer run after statewide branching is permitted", [?] finds out that "charged-off losses over loans (...) appears to increase by 0.4 percentage point following deregulation".[21]

Some caveats are also in order regarding the accuracy of this measurement. First, banks can manipulate NPL by rolling over bad loans. Second, a risky loan granted today will only default in the future (e.g. after a two-year lag if we follow [?] and the rate of default will depend upon the business cycle (Shaffer 98). Although the latter might be corrected by the introduction of macroeconomic risk controls, such as the GDP growth rate, the time lag may be more difficult to correct because of the persistence of the non-performing loans ratios. Third, the riskiness of assets could also be altered by changing the portfolio allocation among the different classes of risks. A bank with higher market power may be willing to take more risks on its assets that will result in higher NPLs in order to obtain a higher expected return while its market power on, say, deposits provides a natural buffer that prevents its financial distress.

5.2 Individual bank risk: insolvency

Because of the endogeneity of banks leverage, changes in the portfolio risk do not translate into equivalent changes on banks' default risk: a poorly capitalized bank can have a high chance to fail, even if its portfolio risk is low. Such

[21] It has been argued that institutional changes can be a more robust measurement than market structure, for its exogeneity facilitates statistical analysis to establish causal relationships. The instrumental approach bears its value for banking deregulation is usually associated with a removal of barriers to entry that will increase competition. Yet, it is not only associated to the removal of barriers to entry, as it might also affect the range of financial products banks are allowed to invest in and the structure of financial institutions. As pointed out by Cubillas and Gonzalez (), bank liberalization has not only an effect on banks' competition, but also an indirect effect on banks' strategies other channels. This implies that the "banking deregulation" measure of market power explores the effect of a package of measures related to market power on financial stability, but market power is only an undistinguishable part of it.

divergence between the impact of competition on the riskiness of banks' assets (as measured by NPL) and its overall risk is perfectly illustrated in [?]: in spite of finding confirmation of the risk shifting hypothesis in the NPL their analysis shows that banks with a higher degree of market power have lower overall risk exposure mainly due to their higher equity capital levels.

Since [?] the literature has been focusing on the risk of individual bank failure. In his classic paper, Kelley considers the market-value capital-to-asset ratio and the interest cost on large, uninsured CD's. Following his approach, [?] use seven different measures of BHCs' risks and in each of them franchise value is statistically significant providing support to the charter value theory.[22] [?] found also corroborating evidence that the standard deviation of stock returns volatility was negatively related to S&L franchise values as measured by the market-to-book asset ratio. Also confirming the charter value perspective, [?] show that capital ratio increases with Tobin's Q, thus providing some evidence on the possible behavior of the (endogenous) leverage ratio.

In our judgement, pure insolvency risk can be best measured by Z-scores,[23] and many empirical works take that as the main risk measurement. Still, there are important nuances in these results. [?] show on average, a positive relationship between banks' market power, as measured by the Lerner's index, and banks' stability, as proxied by the Z-score. Nevertheless, they find large cross-country variation in this relationship. [?] report empirical evidence that supports the franchise value paradigm but only if market power is measured by Lerner's indexes based on bank-specific interest rates and bank risk.

Opposing this view, [?] provided cross-country empirical evidence supporting the risk-shifting model using several proxies to measure bank risk, including using the Z-score.[24] Using a US sample and a cross-country one they consistently find that banks' probability of failure is negatively and significantly related to measures of competition. Confirming this view, [?] analyze the impact of large deposit and loan rents and show that they predict higher probabilities of bank failures and lower bank capitalization.

5.3 Individual bank risk: illiquidity

Funding liquidity risk has largely been overlooked in the empirical study.[25] One might argue that upon observing bank failures, it is difficult, if ever possible, to distinguish pure solvency issue from illiquidity ones, (Goodhart, 1987). However, just as insolvency risk can be measured by Z-scores, illiquidity can be measured with accounting information too. For example, in their study of bond

[22] The risk measurements include annualized standard deviation of weekly stock returns, systematic risk, firm-specific risk, capital-to-assets Ratio, loans-to-assets ratio, commercial and industrial loans-to-assets ratio and loan portfolio concentration.

[23] The measurement is calculated $(RoA - E/A)/\sigma(RoA)$ to capture a bank's distance from insolvency.

[24] The author also use loan losses and dummy for actual bank failures.

[25] On contrast, even though theoretical models made no prediction on how competition affects leverage, which in turn affects insolvency risk, the empirical study has taken into account insolvency risk adjusted by leverage by using measurements like z-scores.

pricing, [?] identify the extra yield due to illiquidity risk: as far as the yield reflects default probability, the liquidity risk can be reflected in bond pricing. While the relationship between funding liquidity risk and competition has not been studied, theoretical models do provide sound guide for estimating the risk: funding liquidity risk is reduced by high returns (e.g., measured by ROA) and high asset market liquidity (e.g., the holding of reserves and cash), and aggravated by the amount of uninsured short-term funding. [?] provide further practical guide. In the context of herding behavior [?] present an attempt to measure the risk by a variety of liquidity ratios.

In addition to the accounting information, funding liquidity risk can also be estimated by market data. [?] suggest constructing bank run index using CDS spreads. In sum, we believe banks' liquidity risk can be measured. Yet how those liquidity measurements link to banking competition invites much future research.

5.4 Systemic risk

The analysis of systemic risk is, obviously, the most difficult one as it often has to deal with cross-country analysis and the main driving force for changes in market power are related to banking deregulation, market entry, deposit insurance and a number of joint measures of which increased competition is only one of the consequences.[26] The precise definition of a banking crisis itself as well as its timing is subject to different interpretations. Thus, while some authors consider the intervention of exceptional measures by the Treasury, or a 10% of the banking industry being affected, others like [?] and De Nicolo et al.(2004) prefer measuring the probability of systemic risk by pairwise distance to default correlation or constructing an indicator of the probability of failure for the five largest banks.

According to [?] on a sample of 69 countries over a 20 year period more concentrated national banking systems are subject to a lower probability of systemic banking crisis. Still, they point out that concentration need not be related to market power, as already mentioned by [?], and that other measures of competition may lead to the opposite result. Contradicting the result of [?], Schaek et al.(2006) show, using the Panzar and Rosse H-Statistic as a measure for competition in 38 countries during 1980-2003, that more competitive banking systems are less prone to systemic crises and that time to crisis is longer in a competitive environment even if concentration and the regulatory environment is controlled for.

Our paper's empirical prediction states here that an increase of competition may have different effects depending upon the amount of insured retail deposits, the profitability of projects and banks' spreads, thus suggesting new lines for future empirical research based on the differentiation of different types of banking systems. It would be interesting to pursue this research by distinguishing

[26] It should be noted that with newly developed measurements on systemic risk such as $CoVaR$ in [?], one can also link a bank's market power to its contribution to the systemic risk, e.g., by regressing an individual bank's $CoVaR$ on its Learner's index.

among different types of banks. If we interpret literally our model, this would be to distinguish banks with low deposit to asset ratios from those with a high deposit to asset ratio. Still, more generally, this could be interpreted as dividing the banks according to their different access to short maturity market funds.

[Empirical Table Here]

6 Empirical predictions and policy implications

Because the aim of our paper is to clarify the multiple concepts of risk and the key role played by leverage, our model has made a number of drastic simplificative assumptions that although leading to simple propositions cannot be easily generalized. So the question arises as to what extent our results might help in formulating new empirical predictions to be testes as well as to improve upon banking competition policy.

Indeed, our framework considerably understates the complexity of the issue, for competition also affects banks' portfolio choice, e.g., the correlation of their portfolios, securitization, cash hoarding, and so on, which are all abstract from the current setup.As a preliminary point, it should be noticed that, as predicted by basic microeconomic theories, competition puts pressure on firms that increases their efficiency through "competitive reallocation effects". The banking industry is no exception and its deregulation has been a fruitful ground where the theoretical predictions have been confirmed. [**?**] concludes "less efficient banks might have been forced to reduce their market shares or even exit the market, allowing surviving banks to enjoy larger market volume." Indeed, this could explain why profit rates are unaffected throughout the period when costs are rising and spreads falling. The point is corroborated by Stiroh and Strahan (2003) and by Schaeck and Cihak ( ) for European banks.

In spite of the obvious limitations of our model, we show that the idea of an identical impact of banking competition on financial stability that would hold accross types of banks and types of firms has no theoretical foundation. As a consequence testing our model prediction that the competition-financial stability link depends upon the type of bank and the state of the economy through firms self financing and productivity may lead to an important step forward in our understanding of the issue.

Regarding policy, our conclusions lead to some caution. The banking sector has long been exempted from the realm of competition law, based on the concern that competition can lead to excessive risk taking. UP TO HERE

Our model states that this view cannot be independ of leverage, i.e. capital regulation. In addition, it is important to discern the different types of risks as the

Consider the removal of deposit rate ceilings and the establishment of credit bureaus for example. While both contribute to a more competitive banking sector, their impacts on stability can be different: the removal of deposit rate ceiling may mainly decrease banks' charter values in deposit markets and therefore induces risk taking. Credit bureaus and information sharing and on the hand can reduce banks' information rent in loan markets, which, as suggested by [**?**] and [**?**], will reduce entrepreneurs' risk taking and promotes financial stability. Thus even if both institutional changes lead to fiercer bank competition, their impacts on stability can be opposite. The argument can also applies to the other driving forces that changes competitive environments, such as foreign bank entries, the deregulation on geographical and functionality restrictions, etc. As financial stability, to study the competition-stability relationship, the concept of competition seems needed to be refined too.

7 Concluding remarks

We develop a model to study banks' risk in competitive environments. We model explicitly the credit risk created by entrepreneurs' moral hazard and examine how banks optimally adjust their leverage in light of various risk. With the theoretical framework, we clarify the concept of financial stability: it has multiple dimensions ranging from portfolio risk to systemic risk of contagion; and competition can affect different types of risk differently. This can help explain the diverse findings in the empirical literature. We further suggest that banks' liability structures affect the relationship between banking competition and financial stability, because leverage responses to portfolio risk and links to risk both on individual bank level, such as solvency and liquidity risk, and risk on the financial system, such as banks' exposure to contagion.

Two caveats are due before we finally conclude. First, the highest financial stability does not necessarily link to the maximum welfare. For example, the financial innovation fueled by competition can be risky but also generates a great social values. The overall welfare consequences needs more close scrutiny. Second, we focus on contagion due to asset price/collateral value, says nothing on common risk exposure. To have a complete picture on systemic risk, banks' incentives to diversify and difference need to be examined. Both invite future research.

[Bib List Here.]