A Hotelling model with price-sensitive demand and asymmetric distance costs: the case of strategic transport scheduling*

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Abstract

We analyze the scheduling decisions of competing transport operators, using a horizontal differentiation model with price-sensitive demand and asymmetric distance costs. Two competitors choose fares and departure times in a fixed time interval; consumers' locations indicate their desired departure times. Locations are chosen before prices; we show that the opposite order, like a simultaneous game, does not have a Nash equilibrium. We also discuss Stackelberg games and second-best regulation. Our results show how departure times can be strategic instruments. Services are scheduled closer together than optimal. Optimal regulatory strategies depend on commitment possibilities, and on the value of schedule delay.

Keywords: Horizontal differentiation, scheduling, Hotelling, price-sensitive demand, transport, regulation.

1 Introduction

The Hotelling model is probably the most well-known model for studying product differentiation in markets with multiple competitors. Although originally framed in the context of locational choice along a linear market, it has various possible interpretations. One of these is to consider timing as the relevant measure of product differentiation, in which case Hotelling's 'space' becomes 'clock

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time', and 'transport costs' between consumers and suppliers become 'schedule delay costs'. In this paper, we propose a general formulation of the Hotelling model that has exactly this interpretation.

In particular, we study how competing transport operators, active in the same market, can use the departure times of their services as strategic instruments. This has, so far, received relatively little attention in the literature; usually, only fares and frequencies are considered. However, a better understanding of strategic scheduling is important, not only because it could help explain observed changes in service stability as a result of deregulation (see, for example, Douglas, 1987), but also because departure time choices can have a large impact on transport users, and hence, regulators should know how to deal with them.

There is some existing literature on scheduling, mostly related to the deregulation of the British bus industry in the 1980s. These studies often assume an infinitely repeating schedule (for example, Foster and Golay, 1986; Evans, 1987), such that headways are constant, or focus on head-running and other short-term scheduling decisions (for example, Ellis and Silva, 1998). Most similar to our approach, van Reeven and Janssen (2006) analyze transport scheduling decisions using a circular Salop model. This model builds upon the Hotelling model and avoids its endpoints by making the market circular rather than having two end points that have the maximum distance in the market. In this way, schedules are still infinitely repeated. If there are no transport services during the day, or if there is a distinct off-peak period, this is less appropriate; moreover, van Reeven and Janssen need an additional attribute, such as service quality, for a stable equilibrium to emerge, and travelers have to care more about that attribute than about fares or departure times. This makes the model less attractive. We therefore propose a Hotelling model, in which operators schedule services in a discrete time interval.

Hotelling's (1929) classic paper on horizontal differentiation shows that, when two firms compete on locations only, and a given number of consumers distributed along a linear market buy from the closest firm, the two firms locate as closely together as possible. Later work has generalized Hotelling's model in several ways; most importantly, to also include price-setting behavior, which results in the absence of a pure-strategy equilibrium when combined with linear transport costs, and to maximum differentiation when combined with quadratic transport costs (d'Aspremont et al., 1979).

Most analyses have kept the assumptions that demand at every location is perfectly inelastic, and that the user costs of traveling are independent of direction. There are exceptions; Wauthy (1996*b*) formulates a two-stage model with elastic demand in the context of vertical differentiation. Puu (2002) proposes a Hotelling model with elastic demand, in which locations and prices are determined simultaneously (a mathematical formulation of Smithies, 1941), but his calculations have been shown to be flawed (Sanner, 2005). Colombo (2011) includes elastic demand and asymmetric distance costs, but his model is unidirectional: travel costs in one direction are infinite. Nilssen (1997) and Nilssen and Sørgard (2002) formulate location choice models with asymmetric distance costs, but assume that prices are exogenously fixed, and that there is only one consumer with unit demand at each location. Finally, Gu and Wenzel (2012) formulate a Salop model with elastic demand, but their formulation is unfortunate, in that the distance between consumers and their suppliers negatively affects consumer utility, while their demand is a function of the price only; not of the transport costs. To our knowledge, there are no existing horizontal differentiation models which include price-sensitive demand at every location, allow for asymmetric distance costs, and include price-setting in addition to location choice.

The assumptions mentioned above may yield good approximations in many applications, but for transport scheduling, they are oversimplifications. Transport demand is usually price-sensitive, since people can choose not to travel, or alternative modes of transport may be available. Furthermore, Hotelling's 'distance' or 'travel costs' in this setting represent the costs of schedule delay, and the cost of being late is usually higher than the cost of being early (Small, 1982). Our models generalize Hotelling's horizontal differentiation model to include price-sensitive demand and allow for asymmetric schedule delay costs. Although we do so in the context of transport scheduling, our models can be applied in other instances, such as telecommunications markets (for example, as a generalization of Cancian et al., 1995).

As in a traditional horizontal differentiation model, two competitors choose a location on a fixed interval. In our case, this is an interval in time, such that the two locations are departure times of transport services. The two competitors also set their fares. Consumers are distributed uniformly along the interval; their location indicates their desired departure time, such that they face a schedule delay cost that increases in the deviation from their desired departure time. Hence, they minimize their generalized price, which is the sum of the fare and their schedule delay costs.

Most Hotelling models assume that the two competitors choose their locations or departure times first, after which fares are set. In scheduling, the opposite order is also conceivable, but we show that this game does not have a Nash equilibrium; the same is true for a game in which fares and departure times are chosen simultaneously. We find the equilibrium fares and departure times, examine under which conditions equilibria exist, and also compare them to the social optimum, assuming that schedule delay costs early and late are equal. Contrary to other horizontal differentiation models, notably that of d'Aspremont et al.

(1979), the competitors schedule their services closer together than optimal. We then analyze how these equilibria change if the schedule delay cost late is higher than the schedule delay cost early, and show that the resulting equilibria can still be stable. Finally, we analyze Stackelberg games, which increase the parameter space in which equilibria exist, and comment on optimal second-best regulation.

2 Methodology

Consider a departure time and fare choice game between two duopolistic suppliers of a scheduled transport service. Consumers have different preferences over desirable departure times and, given this desired departure time, there is an elastic demand for trips. Specifically, travel demand $d(t)$, at any time $t \in [-1, 1]$, is a linear function[1] of the generalized price $p(t)$. Since demand cannot be negative

$$d(t) = Max\,[a - bp(t), 0] \tag{1}$$

which holds for all t. The total consumer surplus can then be calculated as

$$CS = \frac{1}{2}\int_{-1}^{1} d(t)\left(\frac{a}{b} - p(t)\right) dt = \frac{1}{2b}\int_{-1}^{1} (d(t))^2 dt \tag{2}$$

Operators choose a fare f_i and departure time t_i; this assumes that each operator schedules only one service in the observed time interval. Operator profits are given by

$$\pi_i = D_i f_i - F \tag{3}$$

where D_i is the total demand for its service and F is the fixed cost associated with the operation of the service; marginal per-passenger costs are assumed to be zero. Social welfare is then simply the sum of consumer surplus and operator profits, $CS + \sum_i \pi_i$.

The generalized price $p_i(t)$ of service i, taken by a consumer with preferred departure time t, is the sum of the fare and the schedule delay cost. Schedule delay costs are assumed to be linear, but not necessarily symmetric, such that the unit cost of schedule delay when a passenger is late (γ) can differ from the unit cost of schedule delay when a passenger is early (β):

$$p_i(t) = f_i + Max\,[\gamma(t_i - t), 0] + Max\,[\beta(t - t_i), 0] \tag{4}$$

Passengers choose the service that minimizes the generalized price they pay, so in equilibrium, their generalized price $p(t) = Min[p_1(t), p_2(t)]$. If the equi-

[1] In some Hotelling models, $t \in [0, 1]$, but the present specification yields more compact expressions, without affecting the conclusions.

librium demand for each operator's service is strictly positive, we can define the inner market boundary point t^*, at which passengers are indifferent between the two operators as

$$t^* = (f_2 - f_1 + \beta t_1 + \gamma t_2)/(\beta + \gamma) \tag{5}$$

where, by construction, operator 1 schedules the first service. The total demand for each operator's service, D_1 and D_2, can then be calculated as

$$D_1 = \int_{-1}^{t_1} Max[a - b(f_1 + \gamma(t_1 - t), 0]dt + \int_{t_1}^{t^*} Max[a - b(f_1 + \beta(t - t_1), 0]dt \tag{6}$$

and

$$D_2 = \int_{t^*}^{t_2} Max[a - b(f_2 + \gamma(t_2 - t), 0]dt + \int_{t_2}^{1} Max[a - b(f_2 + \beta(t - t_2), 0]dt \tag{7}$$

where t^* is given by Eq. 5.

3 Social optimum

Since there are no externalities associated with demand, and assuming that the operator's marginal costs are zero, the social optimum can be found by simply maximizing total demand. This implies that both fares should be equal to zero[2]. Assuming that, in the social optimum, $d(-1), d(t^*), d(1) > 0$, meaning that there is positive demand from all preferred arrival times, maximizing the sum of Eqs. 6 and 7 gives the social welfare-maximizing departure times:

$$\{t_1^{SW}, t_2^{SW}\} = \left\{-\frac{\gamma}{\beta + \gamma}, \frac{\beta}{\beta + \gamma}\right\} \tag{8}$$

This implies that, if $\beta = \gamma$, the socially optimal departure times are at $-1/2$ and $1/2$. If the cost of departing late is higher than the cost of departing early, both departures shift to an earlier time, such that more passengers depart early. Note, however, that this assumes that demand is strictly positive for any $t \in [-1, 1]$. Substituting the zero fares and Eq. 8 into Eq. 1, this implies that

$$\frac{a}{b} - \frac{\beta\gamma}{\beta + \gamma} > 0 \tag{9}$$

If this condition is not met, the social optimum must be a fully separated equilibrium, such that there exists a $t \in [-1, 1]$ for which demand is zero. In

[2]We will disregard the possibility of setting an infinitely negative fare.

that case, the optimal fares are still equal to zero, so any $\{t_1, t_2\}$ that satisfies $d(-1) = d(t^*) = d(1) = 0$ is an equilibrium, as social welfare does not depend on the exact departure times, and therewith the exact desired arrival times served. Shifting to an earlier time will allow more passengers with an earlier desired departure time to travel, but keeps exactly the same number of passengers with later desired departure times from traveling, such that the net effect on welfare is zero.

4 Full market separation

It may be optimal for two operators to set their fares and departure times such that $d(-1) = d(t^*) = d(1) = 0$, so that their markets are fully separated and each operator acts as a monopolist on its own segment. In this case, there exists a desired departure time that is so far away from both services that nobody with this desired departure time travels. To examine when this would happen, consider a single monopolistic operator who can set any departure time and fare, and faces the linear demand function in Eq. 1 for all $t \in \mathbb{R}$, such that it is not constrained by a fixed time period.

Solving Eq. 1 to obtain the passengers with the earliest and latest desired departure times that are traveling in this situation gives

$$\{\underline{t}, \bar{t}\} = \left\{ t_1 - \frac{a - bf_1}{\gamma b}, t_1 + \frac{a - bf_1}{\beta b} \right\} \tag{10}$$

This operator's profits are then

$$\pi_1 = \int_{\underline{t}}^{\bar{t}} d(t)dt - F = f_1 \frac{(\beta + \gamma)(a - bf_1)^2}{2\beta\gamma b} - F \tag{11}$$

Naturally, these profits do not depend on the operator's departure time choice, since there is now no unique fixed time period. The operator's profit in Eq. 11 is maximized when

$$f_1 = a/(3b) \tag{12}$$

Substituting this back in Eq. 10 gives the passengers with the earliest and latest desired departure times, as a function of the parameters:

$$\{\underline{t}, \bar{t}\} = \left\{ t_1 - \frac{2a}{3\gamma b}, t_1 + \frac{2a}{3\beta b} \right\} \tag{13}$$

Only if $\bar{t} - \underline{t} \leq 1$ can two fully separated monopolists with fares as in Eq. 12

operate between $t = -1$ and $t = 1$. This implies that

$$\frac{2(\beta+\gamma)a}{3\beta\gamma b} \leq 1 \tag{14}$$

Hence, a fully separated equilibrium is more likely to occur when the maximum number of passengers for a given t (a) is smaller, when the demand sensitivity (b) is higher[3], for higher costs of schedule delay, and for a larger difference between the cost of schedule delay late and the cost of schedule delay earlier. However, operators can only recover their costs if $\pi_i \geq 0$. Using Eq. 10, this implies that

$$\frac{2(\beta+\gamma)a^3}{27\beta\gamma b^2} \geq F \tag{15}$$

This, conversely, is less likely to occur when the maximum number of passengers is smaller, the demand sensitivity is higher, and for higher costs of schedule delay. In the absence of any subsidies, Eqs. 14 and 15 can only hold simultaneously if $F \leq a^2/(9b)$.

5 Equilibria with covered markets

Having derived when a separated equilibrium occurs, we can now examine the various possibilities for an equilibrium in which the market is entirely covered, such that the two operators compete for the marginal customer. If Eq. 14 does not hold, $D(-1), D(t^*), D(1) > 0$, since it would be suboptimal to stay in a situation where $D_1(-1)$ or $D_2(1)$ equal zero while $D(t^*) > 0$, and vice versa. The resulting equilibrium is considerably more complicated that the fully separated one, which is why we will start by assuming that $\beta = \gamma$, in order to derive tractable results, before we consider asymmetric schedule delay costs.

In this case, the equilibrium fares and departure times depend on the order in which they are chosen if a sequential game structure is allowed. We will examine three possibilities: either fares and departure times are chosen simultaneously, or departure times are chosen first, while fares are chosen only after the departure times have been fixed, or vice versa. In all cases, we initially assume Nash behavior, moving to Stackelberg games in section 6.

5.1 Simultaneous departure time and fare choice

Puu (2002) analyses a Hotelling game in which locations and prices are chosen simultaneously, and derives an equilibrium where both suppliers charge equal prices, and $t_1 = -t_2$. However, as Sanner (2005) shows, this equilibrium only

[3] Note that the effect of b runs via its impact on demand, given a. In particular, for a given price and with equal a, the demand elasticity is independent of b

appears to be stable because of a calculation error. In reality, each operator could obtain a higher profit by choosing the same location as its competitor, while undercutting its competitor's price with an arbitrarily small amount. It would then serve the entire market of its competitor, plus at least part of its own original market. Hence, in this situation, no stable equilibrium exists.

This result continues to hold if there are more than two competitors or if demand functions are nonlinear, for the same reason; it is always possible for one competitor to take its direct neighbor's place and undercut its price by an arbitrarily small amount, and then obtain the full market. As long as the two operators were competing for the marginal customer, the profit of the undercutting competitor will then increase. Only if one of the operators sets its fares and locations before the other can an equilibrium exist; we will briefly examine this game in section 6 below.

5.2 Fares chosen before departure times

We can find the equilibrium by backward induction, by first solving $\frac{\partial \pi_i}{\partial t_i} = 0$ for $i = \{1, 2\}$ to obtain the optimal departure times, substituting these in the operators' profit functions, and then solving for the optimal fares. The optimal timing response functions for both operators are

$$\begin{aligned} t_1 &= -\frac{4}{5} + \frac{1}{5\beta}\left(2\frac{a}{b} - 3f_1 + f_2\right) + \frac{t_2}{5} \\ t_2 &= \frac{4}{5} - \frac{1}{5\beta}\left(2\frac{a}{b} + f_1 - 3f_2\right) + \frac{t_1}{5} \end{aligned} \tag{16}$$

Solving Eq. 16 to obtain the equilibrium departure times $t_i^*(f_1, f_2)$ gives

$$\begin{aligned} t_1^* &= -\frac{2}{3} + \frac{1}{3\beta}\left(\frac{a}{b} - 2f_1 + f_2\right) \\ t_2^* &= \frac{2}{3} - \frac{1}{3\beta}\left(\frac{a}{b} + f_1 - 2f_2\right) \end{aligned} \tag{17}$$

The equilibrium fares can then be obtained by solving $\frac{\partial \pi_i|\{t_1 = t_1^*, t_2 = t_2^*\}}{\partial f_i} = 0$, which gives optimal response functions[4]

$$f_i = \frac{16a}{39b} + \frac{10}{39}(f_j + \beta) - \frac{1}{39}\sqrt{334\left(\frac{a}{b}\right)^2 - 460(f_j + \beta)\frac{a}{b} + 295(f_j + \beta)^2} \tag{18}$$

These derivations are tedious, and the resulting expressions have no intuitive interpretation.

The optimal response functions can be solved to obtain the equilibrium fares. However, this equilibrium is not stable. Substituting the equilibrium fares into

[4] As well as one other root, which corresponds to the minimum profit.

the cross-partial derivatives of Eq. 18, $\partial f_i/\partial f_j$, results in a tedious expression which, however, is smaller than one for any positive β. This means that undercutting strategies are profitable as long as the operators are making positive profits. Instead of setting a fare equal to Eq. 18, any of the two competitors could set its fare an arbitrarily small amount lower. The other would then also adjust its fare, but by a smaller amount, since $\partial f_i/\partial f_j < 1$. In the timing subgame, this competitor could then simply choose the other's departure time; with its lower fare, it would get the entire market. Since both competitors can use this undercutting strategy profitably as long as positive profits are made, the equilibrium is never stable.

This result continues to hold if the value of schedule delay early is higher than the value of schedule delay late. In that case, equilibrium profits are likely to be asymmetric, so undercutting may only be a profitable strategy for one of the operators, but this still results in instability. The same is true if demand or schedule delay function are non-linear; as long as $\partial f_i/\partial f_j < 1$ for at least one of the operators, and as long as the operators compete for the marginal traveler, any equilibrium where positive profits are made is unstable.

5.3 Departure times chosen before fares

5.3.1 Symmetric schedule delay costs ($\beta = \gamma$)

Again, we find the equilibrium by backward induction. Solving $\frac{\partial \pi_i}{\partial f_i} = 0$ for $i = \{1, 2\}$ gives the optimal fare response functions for both operators. Solving these gives the fare equilibrium:

$$f_1^* = \frac{a}{2\beta} + \beta\left(2 + \frac{5}{4}t_1 + \frac{3}{4}t_2\right) - \frac{1}{4b}$$
$$\sqrt{4\left(\frac{a}{b}\right)^2 + \frac{4\beta a}{b}(t_1 - t_2) + \beta^2\left(80 + 112t_1 + 45t_1^2 + 48t_2 + 22t_1t_2 + 13t_2^2\right)} \quad (19)$$

$$f_2^* = \frac{a}{2\beta} + \beta\left(2 - \frac{3}{4}t_1 - \frac{5}{4}t_2\right) - \frac{1}{4b}$$
$$\sqrt{4\left(\frac{a}{b}\right)^2 + \frac{4\beta a}{b}(t_1 - t_2) + \beta^2\left(80 - 112t_2 + 45t_2^2 - 48t_1 + 22t_1t_2 + 13t_1^2\right)} \quad (20)$$

Again, these derivations are tedious, and the resulting equations have no straightforward intuitive interpretation. Substituting them back in the original profit functions and maximizing each operator's profit with respect to its departure times gives the equilibrium departure times. These do not have a closed form, and can only be evaluated numerically, which we will do below. What is important to note here is that in this game undercutting is not a profitable strategy.

Rather than an arbitrarily small deviation from the first-stage subgame equilibrium, a successful undercutting strategy now requires an operator to take its competitor's place; a major deviation. The other operator will respond with an equally large deviation, and take the place previously occupied by the undercutting operator; none of them will gain.

We can also establish the interval in which an equilibrium exists. By construction, $-1 \leq t_1 \leq t_2 \leq 1$. In this game, these conditions are met only when $\beta \geq \frac{6a}{31b}$. For a smaller β, there is no equilibrium. For $\beta \geq \frac{4a}{3b}$, the equilibrium is separated. Using these bounds, it is also possible to calculate the range of $\{t_1^*, t_2^*\}$:

$$\lim_{\beta \to \frac{4a}{3b}} t_1^* = -\frac{1}{2}, \quad \lim_{\beta \to \frac{6a}{31b}} t_1^* = \frac{1}{2} \tag{21}$$

So, the two services are closer together than socially optimal. This is an important difference from many other horizontal differentiation models, notably that of d'Aspremont et al. (1979), in which competitors locate as far apart from each other as possible, such that they can exert local market power. The reason that this does not happen here is that, in our model, demand is price-sensitive. If one operator schedules its service further from the other, this does indeed decrease competition at the inner market boundary, allowing it to increase its fares in the second stage, as Eqs. 19–20 show. However, by doing so, it will also lose costumers with a desired departure time between the two services since, for these travelers, both schedule delay costs and fares have increased. Of course, this will also shift the inner market boundary.

Hence, each operator's departure time must be closer to the inner market boundary than to the closest outer market boundary, precisely because the latter is fixed, while the former moves in the same direction as a change in one operator's departure time. How close it must be exactly depends on the optimal fare, and hence, on the value of schedule delay. When β approaches $6a/(31b)$, the optimal two departure times approach 0, and when β is even smaller, the two operators will continuously swap places; no stable equilibrium emerges. When β is large, however, market areas are small, so the incentive to try and steal a competitor's customers is smaller, and hence, the departure times are set further apart.

Fig. 1 shows the equilibrium fares, profits, departure times and social welfare relative to the optimum, for the range of values of schedule delay where an equilibrium exists. As already indicated by Eq. 21, departure times move further apart if the value of schedule delay increases. Fares and profits, however, are non-monotonic in β. This is because an increase in the value of schedule delay has two effects. Firstly, an increase in β directly decreases the number of travelers, for any set of fares and departure times, as travel costs for all commuters increase. This will reduce optimal fares and profits. However, if travel costs

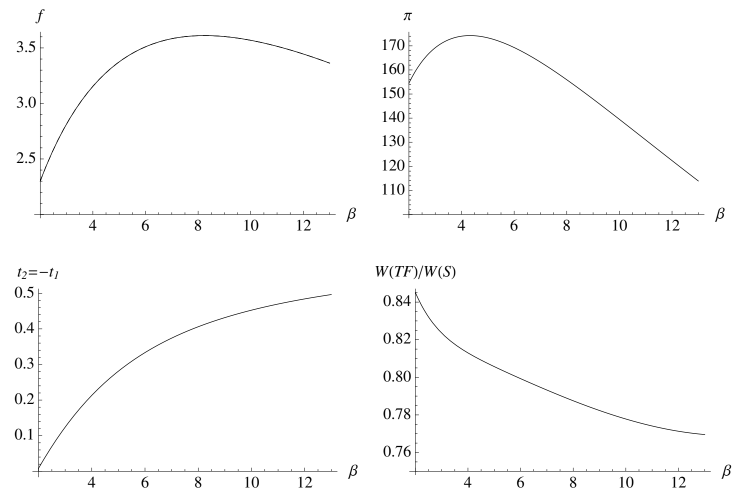

Figure 1: Setting departure times before fares ($a/b = 10$, $\gamma = \beta$)

for all commuters increase, they also increase for the marginal commuters, who have a desired departure time $t = t^*$. Hence, there will also be fewer marginal commuters, which will reduce competition. This allows operators to increase their fares and profits. As Fig. 1 shows, this competitive effect dominates for smaller values of schedule delay. For larger values, the optimal departure times are already so far apart that the demand effect is stronger.

Social welfare, relative to the optimum, always decreases in the value of schedule delay. For low values of schedule delay, this is because fares are increasing in β and thus moving away from the optimum; although the departure times are moving closer to the optimum, this is less important, given that the values of schedule delay are relatively low. For higher values of schedule delay fares start decreasing slightly, but deviations from the optimal departure time are now so costly that although the departure times are moving towards the optimum, they are moving too slowly to offset the negative effect of an increase in β on welfare.

5.3.2 Asymmetric schedule delay functions ($\gamma > \beta$)

If schedule delay functions are not symmetric in each commuter's desired departure time, operators in the resulting equilibrium will charge different fares, and their departure times will not be at equal distances from zero. This complicates the analysis and, hence, this situation can only be evaluated numerically. Fig. 2 shows, for $\beta = 5$ and $a/b = 10$, the fares, departure times, operator profits and social welfare for a range of γ. Although, naturally, the exact functions are specific to these particular parameters, other parameters result in very similar figures. Moreover, all variables only depend on the ratio between β and a/b; not on the individual levels of these parameters.

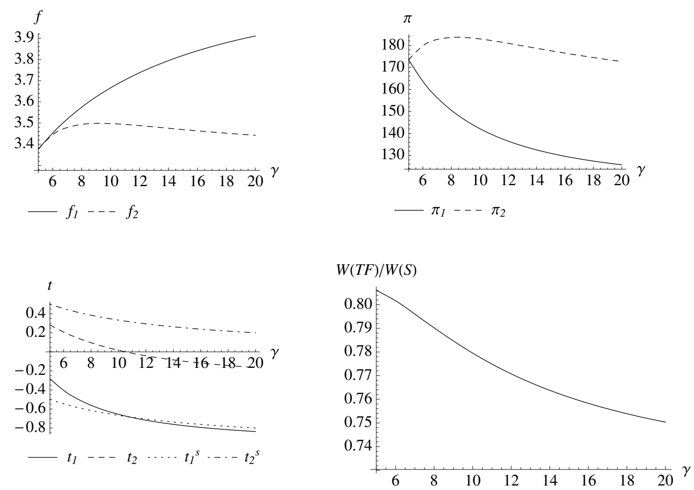

Figure 2: Effects of $\gamma > \beta$ ($a/b = 10$, $\beta = 5$)

Naturally, if the value of schedule delay late increases relative to the value of schedule delay early, both departures will move to an earlier time, such that fewer commuters are late. In an effort to gain the largest market share, both do so at a faster rate than the socially optimal departure times t_i^S. Hence, the first operator's departure time initially moves closer to the optimum, while the second operator's departure time, which is already earlier than optimal, continuously moves away from the optimum.

For moderate deviations of γ from β, this allows both operators to increase their fares. However, the first operator's market size decreases as it is squeezed towards its outer market boundary, and this reduces its profits. For large increases in γ, even the second operator loses, as demand for its service decreases too fast to be offset by its favorable position. Social welfare decreases in γ, relative to the first-best, as a result of higher prices, a less optimal departure time of the second operator and of course, in the same way as an increase in β, simply because suboptimal departure times become more costly.

6 Other games

As we have seen, games with simultaneous fare and timing choices, and games where fares are chosen first, never have pure strategy Nash equilibria. In games where departure times are chosen first equilibria only exist for limited ranges of parameters. It is therefore worth investigating which other game structures could result in equilibria where the above games fail. For the sake of brevity and simplicity, will limit our attention to situations with symmetric schedule delay cost functions, although, like before, it is possible to include asymmetries.

6.1 Stackelberg games

Stackelberg games, in which one operator sets its fare, departure time, or both before the other operator, may be a realistic representation of some real-world transport markets. A large operator, which is active not just in one market but operates many routes may, for example, have to decide on its fares and departure times much earlier than a small, flexible operator that only participates in one market. In this case, the operator that publishes its decisions first can choose them in such a way that it cannot be profitably undercut by the second operator. We will examine the Stackelberg equivalents of the three Nash games above: one situation in which the first operator sets its fare and departure time before the other, one in which the first operator sets its departure time before the other, followed by a separate second stage in which the operators set their fares in the same order, and the reverse, one in which the first operator sets its fare before the other, followed by a sequential departure time choice.

6.1.1 Fares and departure times set simultaneously per firm, and sequentially between firms

If the first operator decides on both its fare and its departure time before the other, and can commit to these decisions, it will choose them such that undercutting is not a profitable strategy for the second operator. This does mean that it has to accept a lower profit than it would get in some of the other games. Starting with the second stage, the second operator's optimal fares and departure times $\{t_2^*(t_1, f_1), f_2^*(t_1, f_1)\}$ can be found by simply setting $\frac{\partial \pi_2}{\partial t_2} = \frac{\partial \pi_2}{\partial f_2} = 0$. The first operator than maximizes its own profits subject to not only $\{t_2^*, f_2^*\}$, but also another constraint, which specifies that the second operator's profit must be greater or equal to the profit it would get if it took the first operator's departure time, and set its fare an arbitrarily small amount lower:

$$\pi_2^* \geq \int_{-1}^{\bar{t}} a - b(f_1 + Max[\gamma(t_1 - t), 0] + Max[\beta(t - t_1), 0]dt - F \tag{22}$$

where π_2^* is the send operator's equilibrium profit, and $\bar{t} = Min\left[t_1 + \frac{a-bf_1}{\beta b}, 1\right]$. Since this constraint will be binding for any set of parameters, Eq. 22 can be solved as a strict equality and used to substitute out one of the first operator's decision variables. Since the constraint is nonlinear, and the resulting equilibrium will have asymmetric departure times and unequal fares, this game can only be solved numerically. It does have a unique pure strategy equilibrium for a large parameter space. Fig. 3 shows the equilibrium fares, profits, departure times and relative welfare for a range of β.

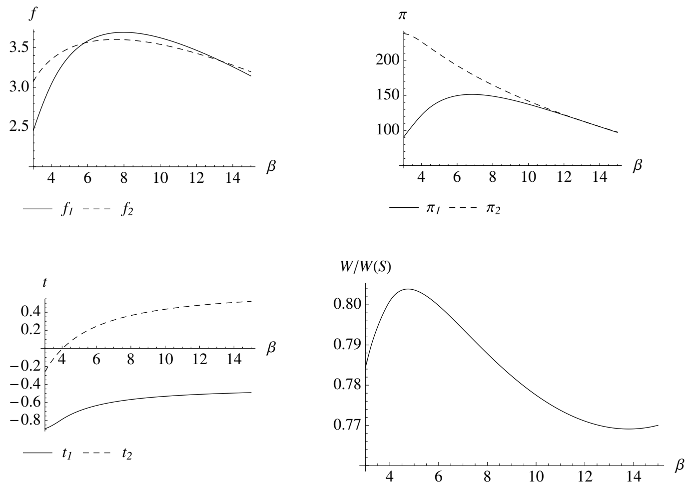

Figure 3: Varying β in a Stackelberg game ($a/b = 10$, $\gamma = \beta$)

As expected, the second mover in this game has an advantage, since the first mover has to choose a position that can not be undercut profitably. Hence, the first operator's departure time is close to the outer market boundary, and its price is lower than in the corresponding Nash game. Naturally, it is therefore optimal for the second operator to set an earlier departure time and higher fare than it would do in a Nash game. As the value of schedule delay increases, resulting in a lower travel demand, the first operator can choose a more favorable position, as undercutting becomes less profitable. For very high values of schedule delay, the second-mover advantage all but disappears, and the equilibrium locations approach those of the Nash game.

6.1.2 Fares chosen before departure times

This game is similar to the previous, but has four separate stages; f_1, f_2, t_1 and t_2 are chosen in that order. Again, the resulting equilibrium is asymmetric; the second operator has a second-mover advantage, so $\pi_1^* < \pi_2^*$. Undercutting is still possible, and can still be profitable, especially for small values of schedule delay, but the first operator can again avoid this. It can be shown numerically that this game has an equilibrium for at least some values of β.

6.1.3 Departure times chosen before fares

The equilibrium of this game, in which t_1, t_2, f_1 and f_2 are chosen in that order, is the most tedious to compute, as only the last subgame has a closed form. However, numerical simulations show that, for all $\beta \geq 0.32a/b$, a pure-strategy equilibrium exists; this is a considerably smaller space than in the Nash game in which both operators set their departure times at the same time, followed by a simultaneous fare decision.

6.2 Regulation

As we have seen, operators can use their departure times as strategic instruments, and hence, regulation may be desired. Even if the first-best solution, in which fares are zero and departure times given by Eq. 8, is not feasible, it may be possible for the regulator to set either fares or departure times. The operators would then be free to set the remaining variables afterwards. This requires that regulators can commit to the choices they announce; operators must be convinced that the announced fares or departure times will not be changed after they have announced their own decisions. If, for any reason, the regulator cannot commit, the game collapses from a two-stage game to a simultaneous game, in which the regulator and the operators effectively set fares and departure times simultaneously.

Hence, at least four different games should be considered; two sequential games, in which regulators set either fares ($F_S T$) or departure times ($T_S F$), and two simultaneous games. Fig. 4 shows the performance of each game, relative to the first-best social optimum, for the range of β in which each game has an equilibrium.

Interestingly, all four games have stable equilibria over a large range of values of schedule delay, while only the first, in which the regulator sets the departure times, followed by a competitive pricing stage, has a stable equivalent in an unregulated market. In reality, completely unregulated transport markets are a rare occurrence, so this may explain why we usually observe stable equilibria. Indeed, there is some evidence that deregulation can lead to service instability (see, for exmaple, Douglas, 1987). However, even the regulatory games do not have equilibria for all possible values of schedule delay. In particular, the games in which the regulator sets fares are unstable for small values of β.

As Fig. 4 shows, regulators have a first-mover advantage, regardless of which variable they are controlling; welfare in each sequential games is always higher than welfare in the corresponding simultaneous game. However, this advantage is relatively small for most parameters. A game in which departure times are chosen by the regulator is preferred, from a social perspective, if values of schedule delay are low. Conversely, when values of schedule delay are high, a game in

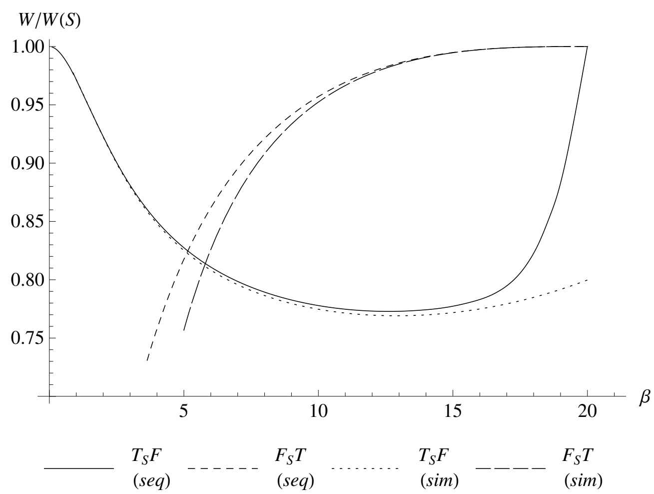

Figure 4: Relative welfare of regulatory games

which the regulator chooses fares performs better. This may sound counterintuitive since, when values of schedule delay are low, suboptimal departure times are relatively unimportant compared to suboptimal fares, so one would expect a more efficient outcome if the regulator set the latter. This does not happen because, for low values of schedule delay, the private operators set low fares anyway; it is better for the regulator to set the departure times, even if they are relatively unimportant. If values of schedule delay are high, on the other hand, the operators would exercise their increased market power and raise fares far above the optimum; it would then be better for the regulator to set the fares instead, even though suboptimal departure times are also relatively costly.

Of the four games in Fig. 4, only the first, in which the regulator sets the departure times, followed by a competitive pricing stage, has a stable equivalent in an unregulated market. A comparison of the relative welfare in this game to the bottom right-hand panel in Fig. 1 shows that the increase in social welfare that results from regulatory intervention is relatively modest, especially for small values of schedule delay. Only when β is very high, such that small deviations from the optimal departure times have a large effect on social welfare, is the regulatory game much more efficient.

7 Discussion

Naturally, our quantitative results depend on the assumptions we have made. One obvious way to generalize the models described above further would be to allow for non-uniform desired departure time distributions (see, for example, Tabuchi and Thisse, 1995; Janssen et al., 2005). This could, for instance, be achieved if Eq. 1 was replaced by

$$d(t) = Max\left[a(t) - bp(t), 0\right] \tag{23}$$

Naturally, the integration in Eqs. 6–7 would be much more complex, though not necessarily impossible, depending on the exact functional form of $a\left(t\right)$.

Depending on the chosen distribution, this generalization would change the relative importance of the inner and outer market boundaries. If, for instance, the distribution was triangular, such that the largest number of passengers preferred a departure time somewhere in the middle of the time interval (for example, $a(t) = a_0 - |x|$), this would increase competition at the inner market boundary, as there would be more potential passengers there. Consequently, we would expect operators to move closer together than they do in the models above. This increased competition at the inner market boundary would also make the parameter space in which a stable equilibrium emerges smaller; in that regard, it could be expected to have a similar effect as a decrease in β in section 5.3.

The addition of more competitors (see, for example, Brenner, 2005), capacity constraints (as in Wauthy, 1996a), nonlinearities, or crowding costs will, of course, also affect the results. However, the qualitative results, and the mechanisms of competition behind them, will remain the same. Circular Salop models, in which the time interval does not have endpoints, are much more difficult to use together with price-sensitive demand (see Gu and Wenzel, 2012, where demand is independent of the distance costs; a more general formulation would be even more complicated), since they have two inner market boundaries, where the operators compete for the marginal customer.

Our models are, themselves, generalizations of earlier Hotelling models. The inclusion of price-sensitive demand and asymmetric distance costs, although adding realism, do create more complexity. One major drawback of our approach is that closed-form solutions can not always be obtained, and if they can, their interpretation is difficult. This is particularly true for the models with asymmetric distance costs. These asymmetries are important in some markets, such as transport, but in others, this generalization will unnecessarily complicate analyses. Price-sensitive demand also makes the results less intuitive, but, contrary to the asymmetric distance costs, it helps establish equilibria in situations where models with fixed demand are unstable. Since the inclusion of price-sensitive demand is less arbitrary than some other measures to prevent instability (such as particular nonlinear formulations for the distance cost function), it might still be an attractive modeling choice.

8 Conclusions

We have proposed a methodology to model the scheduling decisions of competing transport operators, using a generalization of Hotelling's horizontal differentiation model. This model, which includes price-sensitive demand, often has equilibria where other models are unstable. Equilibria in our model can be interior and do not necessarily result in minimum differentiation, and never in maximum differentiation. Indeed, the two competitors normally schedule their services closer together than optimal. This happens because, when demand is price-sensitive, operators have incentives to schedule their services closer to the inner market boundary than to the outer edges of the market, since the inner market boundary can be pushed in the direction of the competitor, while the outer edges of the market are fixed. Games where prices are chosen before or simultaneously with locations, on the other hand, have no stable Nash equilibria.

We have also shown that it is possible to include asymmetric schedule delay functions in our model. Asymmetric schedule delay functions generally lower the relative welfare of the game. Since asymmetric schedule delay functions result in asymmetric equilibria, they do make the calculation of the equilibria much more involved, and for some parameters cause unstability if undercutting is profitable.

A Stackelberg structure, in which one operator sets its decision variables before the other often helps to establish equilibria in games where they are not present with Nash behavior. In these games, the first mover can deliberately choose a position such that its competitor has no incentives to undercut; a consequence is that there is a second-mover advantage. Similarly, regulation can create equilibria in situations where an unregulated market fails to do so. If the socially optimal locations and prices are not attainable, regulating one of these two variables can result in a modest efficiency improvement; the value of schedule delay determines which of the two results in the greatest gain.

Although our quantitative results depend on our assumptions, the qualitative results, and the mechanisms of competition that drive them, are valid for a much larger class of models. Most importantly, the models proposed above show that departure times can be strategic instruments, and should therefore be of interest to regulators

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