Stock Return and Cash Flow Predictability: The Role of Volatility Risk*

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Abstract

We examine the joint predictability of return and cash flow within a present value framework, by imposing the implications from a long-run risk model that allow for both time-varying volatility and volatility uncertainty. We provide new evidence that the expected return variation and the variance risk premium positively forecast both short-horizon returns *and* dividend growth rates. We also confirm that dividend yield positively forecasts long-horizon returns, but that it cannot forecast dividend growth rates. Our equilibrium-based "structural" factor GARCH model permits much more accurate inference than univariate regression procedures traditionally employed in the literature. The model also allows for the direct estimation of the underlying economic mechanisms, including a new volatility leverage effect, the persistence of the latent long-run growth component and the two latent volatility factors, as well as the contemporaneous impacts of the underlying "structural" shocks.

JEL classification: G12, G13, C12, C13.
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Abstract

We examine the joint predictability of return and cash flow within a present value framework, by imposing the implications from a long-run risk model that allow for both time-varying volatility and volatility uncertainty. We provide new evidence that the expected return variation and the variance risk premium positively forecast both short-horizon returns *and* dividend growth rates. We also confirm that dividend yield positively forecasts long-horizon returns, but that it cannot forecast dividend growth rates. Our equilibrium-based "structural" factor GARCH model permits much more accurate inference than univariate regression procedures traditionally employed in the literature. The model also allows for the direct estimation of the underlying economic mechanisms, including a new volatility leverage effect, the persistence of the latent long-run growth component and the two latent volatility factors, as well as the contemporaneous impacts of the underlying "structural" shocks.

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1 Introduction

Counter to the "old" efficient market hypothesis dictum that speculative returns are largely unpredictable over time, it is now generally accepted that equity returns are both time-varying and predictable. It is also widely believed that the predictability of the aggregate stock market as a whole is the strongest over longer multi-year horizons.[1] At the same time, to the extend that a consensus has emerged it suggests that expected dividend growth rates for the aggregate market portfolio, or aggregate cash flows, are much less predictable than the expected returns.[2]

Much of the literature underlying these findings, and the choice of predictor variables in particular, have been guided by the present-value framework pioneered by Campbell and Shiller (1988a,b), and the implication that the dividend-price ratio, or the dividend yield, is identically equal to the expected value of the future returns discounted by the future dividend growth rates. As emphasized by Cochrane (2008, 2011), this intimate link between dividend growth and stock return predictability also implies that the seemingly stronger empirical evidence for long-run return predictability is not surprisingly accompanied by seemingly weaker empirical evidence for long-run dividend growth predictability.

Set against this background, a number of recent studies have argued that the variance risk premium, or the difference between options implied and expected variances, possesses superior forecasting power for stock market returns over shorter within-year horizons; see, e.g., Bollerslev, Tauchen, and Zhou (2009), Drechsler and Yaron (2011), and Kelly (2011). Motivated by these more recent empirical findings, we show how explicitly incorporating priced volatility risk into the present-value framework affords important new insights into the return vis-à-vis dividend growth predictability debate across *all* horizons.

[1] Some of the more popular predetermined variables used in establishing long-run return predictability include: dividend-price, earning-price, and other valuation ratios (Campbell and Shiller, 1988a,b; Fama and French, 1988; Lamont, 1998; Lewellen, 2004); firms' net equity payout (Boudoukh, Richardson, and Roberts, 2007) and equity issuance (Baker and Wurgler, 2000); interest-rate variables such as t-bill and t-bond rates, term spreads, and default spreads (Campbell, 1987; Fama and French, 1989; Hodrick, 1992); and macroeconomic variables like total investment (Cochrane, 1991), the consumption-wealth ratio (Lettau and Ludvigson, 2001), and inflation (Campbell and Vuolteenaho, 2004).

[2] Maio and Santa-Clara (2012) have recently challenged this view, showing that for portfolios comprised of small and value stocks, the dividend-price ratio is primarily related to future changes in cash flows. With a few notable exceptions, however, (e.g., Fama and French, 1988; Lettau and Ludvigson, 2005) cash flow predictability has historically received much less attention in the literature.

The reduced form VAR framework, as exemplified by Hodrick (1992) and Campbell (2001), traditionally used for empirically implementing present value relations does not naturally lend itself to the estimation of models involving priced volatility risk. Instead, we follow Sentana and Fiorentini (2001) and Rigobon (2003) in designing a "structural" factor GARCH model, in which the factors exhibit time-varying volatility. The dynamics of the factors is derived endogenously from an extended long-run risk model explicitly incorporating time-varying consumption volatility and volatility-of-volatility, or economic uncertainty. The resulting econometric model separately identifies the long-run risk, volatility, and economic uncertainty components, as well as the corresponding structural shocks and their contemporaneous impact on both returns and dividend growth.

Estimating the "structural" factor GARCH model by standard GMM techniques on data for the S&P 500 market portfolio, we confirm existing empirical evidence that the dividend-price ratio is useful for predicting long-horizon multi-year returns, but that it has no predictive power for dividend growth.[3] More important, we document a number of new results pertaining to the predictability of the volatility factors. In particular, while the variance risk premium shows significant predictability for returns over short within-year horizons, it also helps predict dividend growth. Similarly, the expected return variation appears to be very informative for predicting dividend growth.

These results are consistent with the findings in Koijen and van Nieuwerburgh (2011) that the high-frequency component of the dividend-price ratio, which in our setup is driven by two separate volatility factors, contains useful information for predicting expected dividend growth. Our results are also related to Binsbergen, Brandt, and Koijen (2012) and their findings that the term structure of equity risk premia is particularly steep in the short end, while standard asset pricing models without priced volatility risk typically imply higher equity premia at the long end.

In addition to the new empirical evidence pertaining to the short-run predictability of returns and dividend growth, by explicitly identifying the systematic risk factors at work, our "struc-

[3]Compared to earlier empirical findings based on univariate regressions (Rozeff, 1984; Fama and French, 1988; Campbell and Shiller, 1988b) and traditional present-value homoskedastic VAR's (Hodrick, 1992; Campbell, 2001; Cochrane, 2008), our "structural" factor GARCH model results in much sharper inference, with the actual point estimates systematically falling within the standard error bands obtained from the more conventional procedures.

tural" factor GARCH approach also helps shed new light on the underlying economic mechanisms. Specifically, we find that the long-run expected growth component is highly persistent with a first-order autocorrelation coefficient close to one ($\rho_x = 0.988$) at the monthly level, consistent with the idea in Bansal and Yaron (2004) that it acts as the most important driver of the risk premium dynamics over long horizons.[4] The model also clearly differentiates and is able to accurately estimate the persistence of the consumption volatility component ($\rho_\sigma = 0.64$) and the volatility-of-volatility, or economic uncertainty, component ($\rho_q = 0.46$), advocated by Bollerslev, Tauchen, and Zhou (2009), both of which are intimately linked to the shorter-run predictability patterns in the data. In terms of the underlying "structural" shocks, we find a negative relationship between the long-run growth and consumption volatility shocks (akin to a "leverage effect"), as well as a negative relationship between the consumption volatility and volatility uncertainty shocks (interpretable as a separate new "leverage effect"). The price-dividend ratio also responds negatively to both consumption volatility and volatility uncertainty shocks.[5]

The basic motivation behind the new "structural" factor GARCH model is in line with a growing recent literature seeking to explicitly incorporate the effect of stochastic volatility in asset pricing models. For example, Bansal, Kiku, Shaliastovich, and Yaron (2012) demonstrate that ignoring the variation in volatility leads to counter-intuitive economic interpretation of risk premium dynamics. Similarly, Campbell, Giglio, Polk, and Turley (2012) examine the cross-sectional return predictability in an ICAPM framework that allows for stochastic volatility.[6] In contrast to these studies, our focus is on the *joint* predictability of returns and cash flows within the context of a "structural" econometric model explicitly designed to accommodate time-varying volatility in an internally consistent fashion. Recent studies by Binsbergen and Koijen (2010) and Piatti and Trojani (2012) have also relied on a latent variable approach with heteroskedastic shocks for incorporating the effect of time-varying volatility within a present-value framework. Importantly,

[4] Nakamura, Sergeyev, and Steinsson (2012) have recently shown how the long-run growth factor may also be identified from cross-country aggregate consumption data under additional simplifying assumptions.

[5] The importance of economic uncertainty for explaining asset prices has also recently been emphasized from different perspectives by Bekaert, Engstrom, and Xing (2009), Nieto and Rubio (2011), and Corradi, Distaso, and Mele (2012), among others.

[6] Our "structural" factor GARCH estimate for the persistence in consumption volatility ρ_σ, and in turn the effect of allowing for time-varying volatility, are much larger than the estimates reported in Campbell, Giglio, Polk, and Turley (2012) based on simple VAR procedures and imprecise variance measures.

however, we differ from both of these studies by specifying an empirically more realistic two-factor volatility structure and by explicitly including both the actual and risk-neutral expected variation in the formulation and estimation of the model.[7]

The rest of the paper is organized as follows. Section 2 describes the data. Section 3 presents the general equilibrium model setup underlying our empirical investigations. Section 4 discusses the formulation of the "structural" factor GARCH model and the GMM-based parameter estimation results. Section 5 details the return and cash flow predictability implied by the model, and contrast the results with those obtained by other less structured reduced form estimation procedures. Section 6 concludes.

2 Data Description

Our empirical investigations are based on end-of-month S&P 500 index returns, as a proxy for the aggregate market portfolio, and the S&P 500 dividend payments, as a proxy for the corresponding aggregate cash flows. All of our S&P 500 data are obtained from DataStream, and cover the period from January 1990 to November 2011, for a total of 262 monthly observations.[8]

Following standard practice in the literature, we use the trailing 12-month dividend-price ratio to account for the strong seasonality inherent in the dividend payouts; see, e.g., the discussion in Bollerslev and Hodrick (1995). Accordingly, the month t log dividend-price ratio dp_t, is formally defined by,

$$dp_t = \log\left(\frac{Div_{t-11} + ... + Div_t}{12P_t}\right), \tag{1}$$

where Div_t denotes the dividend payments from the end-of-month $t-1$ to the end-of-month t, and P_t denotes the end-of-month t price.

Our measures for the month $t+1$ log dividend growth rate Δd_{t+1} and the log returns including

[7]Other recent studies seeking to incorporate more realistic two-factor volatility structures in the standard long-run risk model include Zhou and Zhu (2012), Branger and Vòlkert (2012), and Branger, Rodriguez, and Schlag (2011), among others.

[8]While the S&P 500 data are obviously available over a much longer sample period, some of the key variance measures employed in our analysis are only available starting in 1990.

dividends $r_{t,t+1}$, are similarly defined from this ratio as,

$$\Delta d_{t+1} = \log\left(\frac{Div_{t-10} + ... + Div_{t+1}}{Div_{t-11} + ... + Div_t}\right), \tag{2}$$

$$r_{t,t+1} = \log\left(\frac{P_{t+1} + \frac{Div_{t-10}+...+Div_{t+1}}{12}}{P_t}\right). \tag{3}$$

Longer-run dividend growth rates and returns are defined in an obvious manner by simple summation.

We consider three distinct variation measures: the options implied variation IV_t, the expected return variation ERV_t, and the variance risk premium VRP_t. Our measure for the options implied variation is the square of the Chicago Board of Options Exchange (CBOE) VIX volatility index,

$$IV_t \;=\; VIX_t^2. \tag{4}$$

This model-free measure is (approximately) equal to the market risk-neutral, or $\mathbb{Q}$, expectation of the one-month-ahead return variation under very general assumptions.

To define the corresponding actual, or $\mathbb{P}$, expectation, we first construct the time series of monthly model-free realized variances by summing the daily square returns within each month, $RV_{t,t+1} \equiv \sum_{i=1}^{n} r^2_{t+\frac{i-1}{n},t+\frac{i}{n}}$, where n refers to the number of trading days in month $t+1$.[9] Our measure for the one-month-ahead expected return variation is obtained from the linear projection of these monthly realized variances on their own lagged daily, weekly, and monthly values, along with the lagged implied variance,[10]

$$RV_{t,t+1} \;=\; \alpha_0 + \alpha_1 RV_{t-\frac{1}{n},t} + \alpha_2 RV_{t-\frac{5}{n},t} + \alpha_3 RV_{t-1,t} + \alpha_4 IV_t \;+\; \varsigma_{t+1}. \tag{5}$$

Except for the addition of IV_t as an additional right-hand-side variable, this specification directly mirrors the popular HAR-RV model originally proposed by Corsi (2009).[11] In the sequel, we will

[9]This directly mirrors the use of higher-frequency intraday data in the construction of daily realized volatility measures advocated by Andersen, Bollerslev, Diebold, and Ebens (2001) among many others.

[10]Our estimates of the α's are based on overlapping daily data. The use of daily as opposed to monthly observations in estimating the regression greatly enhances the efficiency, and we consequently ignore the small estimation errors in the α's in our empirical analysis below.

[11]Numerous other more complicated models for forecasting realized volatility have been suggested in the literature, see, e.g., Andersen, Bollerslev, and Diebold (2007) and Corsi, Pirino, and Renò (2010) and the many references therein. However, the relatively simple-to-implement HAR-RV type regression model in (5) is very hard to "beat" for forecasting the monthly volatility.

denote this estimate by ERV_t for short. Finally, our measure for the variance risk premium is simply defined as the difference between our risk-neutral and statistical expectations of the one-month-ahead return variation in (4) and (5), respectively,

$$VRP_t = IV_t - ERV_t. \qquad (6)$$

To illustrate the basic features of the different variables, Figure 1 plots the monthly time series of stock returns, dividend growth rates, dividend-price ratios, and variance risk premia. The large losses in market values and the increased volatility during the recent economic downturn are immediately evident in the plots of the returns and cash flows. The plot for the dividend-yields shows a sharp drop throughout the 1990s, but an increase after the burst of the tech bubble in 2001, reaching a new peak in the fourth quarter of 2008 around the advent of the global financial crisis and the stock market crash.[12] The variance risk premium shown in the last panel is on average positive with occasional negative spikes, the largest of which occur in the fall of 2008 at the onset of the financial crises.

Summary statistics for the same four variables, along with the options implied and expected variation measures underlying the variance risk premium, are reported in Table 1. The annualized mean stock return over the sample equals 8.19 percent with a volatility of 15.33 percent, while the average dividend growth rate was 3.92 percent with a standard deviation of 8.79 percent. The log dividend-price ratio is, of course, highly persistent with a first order autocorrelation coefficient equal to 0.98. Meanwhile, the average implied and expected variances equal 40.30 and 28.54, respectively, on a percentage-squared monthly basis, implying an on average positive variance risk premium of 11.75. Interestingly, while the two individual variance series are strongly positively serially correlated, albeit not as persistent as the dividend-price ratio, the first order autocorrelation of the variance risk premium is only 0.27.

Turning to the sample correlations reported in the bottom panel of the table, the implied and expected variances obviously move closely together. The monthly returns are also highly negatively correlated with both measures, while the returns are only weakly negatively correlated with

[12]The sharp decline observed in the 1990s is often attributed to firms' substitution of dividend payments by share repurchases; see, e.g., Koijen and van Nieuwerburgh (2011), along with the earlier related discussion in Bagwell and Shoven (1989).

dividend yields. The positive contemporaneous sample correlation of 0.15 for the returns and the variance risk premium is largely driven by the negative spike in both series during the financial crisis. Interestingly, the two individual variance measure are both negatively correlated with the dividend growth rate.

We turn next to our new present value framework and "structural" model designed to describe and better understand these dependencies.

3 Asset Pricing Model and Structural Restrictions

Our equilibrium-based approach combines the long-run risk model pioneered by Bansal and Yaron (2004), with the model in Bollerslev, Tauchen, and Zhou (2009) explicitly allowing for stochastic volatility-of-volatility, or time-varying economic uncertainty. This general setup naturally accommodates the magnitude of both the equity and variance risk premia, as well as the long- and short-horizon predictability patterns in the returns and cash flows within a unified framework.

3.1 Model Setup and Assumptions

Following the long-run risk literature, we assume an endowment economy with a representative agent equipped with Epstein and Zin (1991) recursive preferences. The logarithm of the intertemporal marginal substitution for this agent $m_{t+1} \equiv \log(M_{t+1})$, may consequently be expressed as,

$$m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{c,t+1}, \tag{7}$$

where $r_{c,t+1} \equiv \log(R_{c,t+1})$ refers to the logarithmic return on the consumption asset, $\Delta c_{t+1} \equiv \log(C_{t+1}/C_t)$ denotes the growth rate of consumption, $0 < \delta < 1$ is the time discount factor, $\gamma > 0$ denotes the risk aversion parameter, and $\theta \equiv \frac{1-\gamma}{1-\psi^{-1}}$ where $\psi > 0$ refers to the intertemporal elasticity of the substitution. As is standard in the long-run risk literature, we will assume that $\gamma > 1$, implying that the representative agent is more risk averse than log utility, and that $\psi > 1$, and therefore $\theta < 0$, implying a preference for early resolution of uncertainty.

For notational convenience, we collect the consumption growth Δc_t, the log dividend growth

Δd_t, and the latent state variables describing the underlying dynamics in the vector Y_t,

$$Y_t \equiv \begin{pmatrix} \Delta c_t \\ x_t \\ \sigma_t^2 \\ q_t \\ \Delta d_t \end{pmatrix}, \tag{8}$$

where x_t denotes the long-run mean of consumption growth as in Bansal and Yaron (2004), and σ_t^2 and q_t refer to two separate volatility factors along the lines of Bollerslev, Tauchen, and Zhou (2009). The importance of allowing for multiple volatility factors in accurately describing both short- and long-horizon time-varying return and volatility dynamics has also recently been highlighted by Bollerslev, Tauchen, and Zhou (2009), Drechsler and Yaron (2011), Bollerslev, Sizova, and Tauchen (2012b), Zhou and Zhu (2012), Branger and Vòlkert (2012), among others.[13]

We assume that the state vector has affine conditional mean and variance dynamics,

$$Y_{t+1} = \mu + FY_t + HG_t z_{t+1}, \tag{9}$$

where $z_{t+1} \equiv [z_{c,t+1}, z_{x,t+1}, z_{\sigma,t+1}, z_{q,t+1}, z_{d,t+1}]'$ denotes a vector of independent standard normally distributed shocks. The conditional mean of Y_t is in turn determined by the constant vector μ and the loading matrix F. We assume that the loading matrix takes the sparse form,

$$F = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & \rho_x & 0 & 0 & 0 \\ 0 & 0 & \rho_\sigma & 0 & 0 \\ 0 & 0 & 0 & \rho_q & 0 \\ 0 & \phi_{dx} & 0 & 0 & \rho_d \end{pmatrix}, \tag{10}$$

in which the diagonal elements characterize the own lagged dependencies and the off-diagonal elements describe the dynamic first-order cross dependencies. In particular, ϕ_{dx} allows the dividend growth rate Δd_{t+1} to directly load on the lagged long-run consumption growth component x_t. Allowing Δd_{t+1} to also depend on its own lag permits a non-redundant pricing effect of dividend growth risk on the equity premium. Restricting this coefficient ρ_d to be zero reduces the model's

[13]In particular, as discussed in Bollerslev, Tauchen, and Zhou (2009), by allowing for stochastic volatility-of-volatility it is possible to separate the time-varying market price of risk that drives the consumption risk premium from the time-varying volatility risk that drives the volatility risk premium.

growth dynamics to that of a "standard" long-run risk model. However, our estimates of the model discussed below strongly rejects such a specification.

The conditional second-order dynamics of the state vector is determine by the time-varying diagonal volatility matrix G_t and the constant loading matrix H,

$$G_t = \begin{pmatrix} \sigma_t & 0 & 0 & 0 & 0 \\ 0 & \sqrt{q_t} & 0 & 0 & 0 \\ 0 & 0 & \sqrt{q_t} & 0 & 0 \\ 0 & 0 & 0 & \sqrt{q_t} & 0 \\ 0 & 0 & 0 & 0 & \sigma_t \end{pmatrix} \qquad H = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \varphi_x & 0 & 0 & 0 \\ 0 & \varphi_x s_{\sigma,x} & 1 & 0 & 0 \\ 0 & \varphi_x s_{q,x} & s_{q,\sigma} & \varphi_q & 0 \\ 0 & \varphi_x s_{d,x} & s_{d,\sigma} & \varphi_q s_{d,q} & \varphi_d \end{pmatrix}. \tag{11}$$

Our choice of G_t differs from the models in Drechsler and Yaron (2011) and Branger and Vòlkert (2012) by allowing both x_{t+1} and σ^2_{t+1} to have time-varying volatility $\sqrt{q_t}$. As discussed further below, this assumption facilitates our ranking of the "structural" shocks z_{t+1}. Our choice of G_t also nests the model in Bollerslev, Tauchen, and Zhou (2009) by zeroing out the long-run growth component, equating the dividend and consumption growth, and fixing $s_{i,j} = 0$ for $i \neq j$, thereby rendering H diagonal.[14]

Identification of the lower triangular volatility loading matrix H is effectively accomplished through heteroskedasticity, and cross-dependencies between the different state variables implied by the form of the time-varying volatility. We rank the two "structural" consumption shocks $z_{c,t}$ and $z_{x,t}$, before shocks to dividends $z_{d,t}$. Based on the intuition that level shocks are more "fundamental" than shocks to volatility, we also put the two consumption shocks before the volatility shocks $z_{\sigma,t}$ and $z_{q,t}$.

Denoting the columns of $H \equiv [h_1, h_2, h_3, h_4, h_5]$, the "square" of HG_t may be conveniently expressed in affine form as,

$$HG_tG_t'H' = \sum_{j=1,5} h_jh_j'\sigma_t^2 + \sum_{j=2,3,4} h_jh_j'q_t. \tag{12}$$

This two-factor volatility structure is distinctly different from the one-factor setup recently employed in Campbell, Giglio, Polk, and Turley (2012). As discussed in more detail below, it affords

[14]We also experimented with two alternative setups, one closer to Drechsler and Yaron (2011) with $G_t = \text{diag}[\sigma_t, \sqrt{q_t}, \sigma_t, \sqrt{q_t}, \sigma_t]$, and the other one closer to Branger and Vòlkert (2012) with $G_t = \text{diag}[\sigma_t, \sigma_t, \sigma_t, \sqrt{q_t}, \sigma_t]$, resulting in qualitatively similar predictability results to the ones reported below. However, both of these alternative specifications were rejected at conventional significance levels by the corresponding GMM-based J-tests for over-identifying restrictions. Further details concerning these alternative models and empirical results are reported in Appendixes C and D.

an empirically much more realistic description of the return and cash flow dynamics, and in turn the predictability patterns obtained by imposing the equilibrium-based restrictions.

3.2 Model Implications

In order to deduce the "structural" model-implied restrictions that guide our empirical analysis, we begin by solving the consumption-based asset pricing model using similar techniques to the ones in Bansal and Yaron (2004), Bansal, Kiku, and Yaron (2007b), and Drechsler and Yaron (2011). In the spirit of Campbell (1993, 1996), we then substitute out the hard-to-measure consumption and its volatility dynamics with directly observable market return and its variance measures.

Standard solution methods applied in the long-run risk literature readily imply that the stochastic discount factor m_{t+1}, the return on consumption $r_{c,t+1}$, and the market return on dividends r_{t+1}, must satisfy

$$\begin{aligned} m_{t+1} - E_t(m_{t+1}) &= -\Lambda' H G_t z_{t+1}, \\ r_{c,t+1} - E_t(r_{c,t+1}) &= \Lambda_c' H G_t z_{t+1}, \\ r_{t+1} - E_t(r_{t+1}) &= \Lambda_d' H G_t z_{t+1}, \end{aligned} \tag{13}$$

where $\Lambda = \gamma e_1 + \kappa_1(1-\theta)A$ denotes the price of risk for the factor shocks, $\Lambda_c = e_1 + \kappa_1 A$, $\Lambda_d = e_5 + \kappa_{d,1} A_d$, κ_1 and $\kappa_{d,1}$ refer to the Campbell and Shiller (1988b) log-linearization constants based on the "usual" approximations for consumption return $r_{c,t+1} \approx \kappa_0 + \kappa_1 v_{t+1} - v_t + \Delta c_{t+1}$ and dividend return $r_{t+1} \approx \kappa_{d,0} + \kappa_{d,1} w_{t+1} - w_t + \Delta d_{t+1}$, respectively, and the two selection vectors are defined by $e_1 \equiv [1,0,0,0,0]'$ and $e_5 \equiv [0,0,0,0,1]'$.[15] Given these expressions, it is possible to explicitly solve for the market return variance $Var_t(r_{t+1})$, the variance risk premium VRP_t, and the log dividend-price ratio dp_t, as

$$Var_t(r_{t,t+1}) = (1+\kappa_{d,1}A_{d,d})^2 \varphi_d^2 \sigma_t^2 + \sum_{j=2,3,4} \Lambda_d' h_j h_j' \Lambda_d q_t, \tag{14}$$

$$VRP_t = \left(\sum_{j=1,5} \Lambda_d' h_j h_j' \Lambda_d s_{q,1} + \sum_{j=2,3,4} \Lambda_d' h_j h_j' \Lambda_d s_{q,2} \right) q_t, \tag{15}$$

$$dp_t = -A_{0,d} - A_{d,x} x_t - A_{d,\sigma} \sigma_t^2 - A_{d,q} q_t - A_{d,d} \Delta d_t, \tag{16}$$

[15] As further detailed in Appendix A, the market prices of risks also depend implicitly on the coefficients in the wealth-consumption ratio $v_t = A_0 + [0, A_x, A_\sigma, A_q, 0]' Y_t$ and the price-dividend ratio $w_t \equiv -dp_t = A_{d,0} + [0, A_{d,x}, A_{d,\sigma}, A_{d,q}, A_{d,d}]' Y_t$.

where $s_{q,1} = -(\varphi_x s_{\sigma,x} h_2' + h_3')\Lambda$ and $s_{q,2} = -(\varphi_x s_{q,x} h_2' + s_{q,\sigma} h_3' + \varphi_q h_4')\Lambda$. We will impose these "structural" restrictions on the empirical model estimated below.

Even though our empirical strategy of substituting out consumption means that some of the parameters in the autoregressive loading matrix F and the volatility loading matrix H are not identified, the specific structures for the two loading matrices still provide useful guidance on how to restrict the factor dynamics. In particular, denote the sub-vector of Y_t that exclude consumption growth by $f_t \equiv [\sigma_t^2, q_t, \Delta d_t, x_t]'$. The dynamic dependencies in the sub-system defined by f_t may then be expressed as,

$$f_{t+1} = \mu + \rho f_t + S\epsilon_{t+1}, \tag{17}$$

where

$$\rho = \begin{pmatrix} \rho_\sigma & 0 & 0 & 0 \\ 0 & \rho_q & 0 & 0 \\ 0 & 0 & \rho_d & \phi_{dx} \\ 0 & 0 & 0 & \rho_x \end{pmatrix} \qquad S = \begin{pmatrix} 1 & 0 & 0 & s_{\sigma,x} \\ s_{q,\sigma} & 1 & 0 & s_{q,x} \\ s_{d,\sigma} & s_{d,q} & 1 & s_{d,x} \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{18}$$

and the vector of innovations $\epsilon_{t+1} \equiv [\sqrt{q_t} z_{\sigma,t+1}, \varphi_q \sqrt{q_t} z_{q,t+1}, \varphi_d \sigma_t z_{d,t+1}, \varphi_x \sqrt{q_t} z_{x,t+1}]'$ is conditionally heteroskedastic.[16] In our empirical implementation we will use a multivariate GARCH-type model to describe the dynamic dependencies in the ϵ_{t+1} vector.

The state vector f_t is, of course, not directly observable. To circumvent this, we define the "observable" state vector $X_t \equiv [ERV_t, VRP_t, \Delta d_t, dp_t]'$. From the solution of the model discussed in Appendix A, the X_t vector is directly related to the latent f_t vector by the linear equations,

$$X_t = \mu_X + Q f_t \qquad Q = \begin{pmatrix} Q_{1,1} & Q_{1,2} & 0 & 0 \\ 0 & Q_{2,2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -A_{d,\sigma} & -A_{d,q} & -A_{d,d} & -A_{d,x} \end{pmatrix}, \tag{19}$$

where $Q_{1,1} = (1 + \kappa_{d,1} A_{d,d})^2 \varphi_d^2 \rho_\sigma$, $Q_{1,2} = \sum_{j=2,3,4} \Lambda_d h_j h_j' \Lambda_d \rho_q$, and $Q_{2,2} = (1 + \kappa_{d,1} A_{d,d})^2 s_{q,1} + \sum_{j=2,3,4} \Lambda_d' h_j h_j' \Lambda_d s_{q,2}$. Given the standard set of assumptions about the structural parameter values typically employed in the long-run risk literature, all of the Q parameters would be positive. Conversely, $A_{d,\sigma}$, $A_{d,q}$, and $A_{d,d}$ would all be negative, while $A_{d,x}$ is naturally expected to be positive.

[16]The value of μ is immaterial to all of our predictability results. Also, the reordering of the elements in f_t relative to Y_t merely serves to facilitate comparisons with other benchmark models below, and does not affect any of the results.

The relationship between f_t and X_t in equation (19) underlies our estimation of (scaled versions of) the key ρ and S parameter matrices, and the underlying economic mechanisms and different "structural" shocks.

4 Empirical Methodology and Estimation Results

The consumption-based asset pricing model with volatility uncertainty, outlined in the previous section, imposes a number of restrictions pertaining to the dynamic dependencies and possible feedback effects between the expected variance, the variance risk premium, the dividend growth rate, and the dividend-price ratio. Our new "structural" factor GARCH model is designed to honor these restrictions within a tractable econometric framework.

4.1 "Structural" Factor GARCH

Combining the model for f_t in equations (17) and (18) with the expression for X_t in equation (19), it follows that

$$BX_{t+1} = \tilde{\mu} + \tilde{\rho} BX_t + \tilde{S}\tilde{\epsilon}_{t+1}, \qquad \tilde{\epsilon}_{t+1} = \tilde{G}_t z_{t+1}, \tag{20}$$

where $\tilde{G}_t = \text{diag}[Q_{1,1}\sqrt{q_t}, Q_{2,2}\varphi_q\sqrt{q_t}, \varphi_d\sigma_t, -A_{d,x}\varphi_x\sqrt{q_t}]$, and[17]

$$B = \begin{pmatrix} 1 & -\frac{Q_{1,2}}{Q_{2,2}} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{A_{d,\sigma}}{Q_{1,1}} & \frac{Q_{1,1}A_{d,q}-A_{d,\sigma}Q_{1,2}}{Q_{1,1}Q_{2,2}} & \frac{\rho_d}{1-\kappa_{d,1}\rho_d} & 1 \end{pmatrix} \qquad \tilde{\rho} = \begin{pmatrix} \rho_\sigma & 0 & 0 & 0 \\ 0 & \rho_q & 0 & 0 \\ 0 & 0 & \rho_d & \frac{\phi_{dx}}{-A_{d,x}} \\ 0 & 0 & 0 & \rho_x \end{pmatrix} \tag{21}$$

$$\tilde{S} = \begin{pmatrix} 1 & 0 & 0 & \frac{Q_{1,1}}{-A_{d,x}}s_{\sigma,x} \\ \frac{Q_{2,2}}{Q_{1,1}}s_{q,\sigma} & 1 & 0 & \frac{Q_{2,2}}{-A_{d,x}}s_{q,x} \\ \frac{1}{Q_{1,1}}s_{d,\sigma} & \frac{1}{Q_{2,2}}s_{d,q} & 1 & \frac{1}{-A_{d,x}}s_{d,x} \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{22}$$

Multiplying the "structural" VAR in equation (20) by B^{-1}, the corresponding reduced form VAR(1) representation for X_{t+1} becomes,

$$X_{t+1} = B^{-1}\tilde{\mu} + \Phi X_t + u_{t+1}, \tag{23}$$

[17] As explained in more detail in Appendix B, the B matrix is obtained from the Q matrix by normalizing its diagonal elements to unity.

where $\Phi = B^{-1}\tilde{\rho}B$, $u_{t+1} = \Phi_0^{-1}\tilde{\epsilon}_{t+1}$, and $\Phi_0^{-1} = B^{-1}\tilde{S}$. As this representation makes clear, ignoring the heteroskedasticity in the reduced form shocks u_{t+1}, and interpreting the model for X_{t+1} in (20) as a standard homoskedastic VAR(1), the B and $\tilde{S}$ matrices aren't jointly identified. In empirical macroeconomics, this lack of identification is usually "solved" by imposing that Φ_0 is lower triangular. However, as argued by Sentana and Fiorentini (2001), Rigobon (2003) and Rigobon and Sack (2003), among others, under the maintained assumption that the underlying "structural" shocks are independent, it is possible to identify the Φ_0 matrix, and in turn both B and $\tilde{S}$, through the heteroskedasticity in $\tilde{\epsilon}_{t+1}$.

Rather than specifying the time-varying covariance matrix for the "structural" shocks as an explicit function of the latent q_t and σ_t^2 risk factors, in the implementation reported on below we adopt a more flexible and empirically realistic GARCH approach for characterizing the dynamic dependencies in $\tilde{\epsilon}_{t+1}$. Specifically, let Σ_{t+1} denote the conditional covariance matrix of $\tilde{\epsilon}_{t+1}$. We will assume that Σ_{t+1} may be described by the following relatively simple yet flexible diagonal GARCH(1,1) model,

$$\mathrm{diag}(\Sigma_{t+1}) = (I - \Gamma - \Upsilon)\Theta_0^{-1}\varpi_u + \Gamma\mathrm{diag}(\Sigma_t) + \Upsilon\tilde{\epsilon}_t^2, \tag{24}$$

where $\Theta_0 = \Phi_0^{-1} \odot \Phi_0^{-1}$, and ϖ_u denotes the unconditional covariance matrix of the reduced form shocks $u_{t+1} = \Phi_0^{-1}\tilde{\epsilon}_{t+1}$. Consequently, the second order dynamics of u_{t+1} will follow the more complicated non-diagonal GARCH(1,1) structure,[18]

$$\mathrm{vec}(\Omega_{t+1}) = \Theta_1(I - \Gamma - \Upsilon)\Theta_0^{-1}\varpi_u + \Theta_1\Gamma\Theta_0^{-1}\mathrm{diag}(\Omega_t) + \Theta_1\Upsilon\Theta_2\mathrm{vec}(u_t u_t'). \tag{25}$$

By explicitly parameterizing this implied conditional heteroskedasticity in u_{t+1}, it is possible to identify and separately estimate *all* of the "structural" parameters in (20)-(22).

Let ξ denote the vector of stacked parameters comprised of the conditional mean parameters in B, $\tilde{S}$, $\tilde{\mu}$, and $\tilde{\rho}$, along with all of the conditional variance parameters in Γ, Υ, and ϖ_h. Assuming that the reduced form shocks u_{t+1} are jointly normally distributed, the logarithm of the density for X_{t+1} conditional on X_t and Ω_{t+1}, or equivalently the contribution to the log-likelihood function coming

[18]More formally, $\Theta_1 = (\Phi_0^{-1} \otimes \Phi_0^{-1})I_l$, $\Theta_2 = [\mathrm{vec}(\Phi_{0,(1)}^{-1\prime}\Phi_{0,(1)}^{-1}), \mathrm{vec}(\Phi_{0,(2)}^{-1\prime}\Phi_{0,(2)}^{-1}), \mathrm{vec}(\Phi_{0,(3)}^{-1\prime}\Phi_{0,(3)}^{-1}), \mathrm{vec}(\Phi_{0,(4)}^{-1\prime}\Phi_{0,(4)}^{-1})]'$, where $\Phi_{0,(i)}^{-1}$ denotes the i^{th} row of the square matrix Φ_0^{-1}, and the 16×4 matrix I_l helps to transform the vector $\mathrm{vec}(\Omega_t)$ into diagonal matrix form.

from X_{t+1}, may be expressed as,

$$
\begin{aligned}
L_t(X_{t+1}, \xi) &= -2\log 2\pi - \frac{1}{2}\log|\Omega_t| - \frac{1}{2}(X_{t+1} - B^{-1}\tilde{\mu} - \Phi X_t)'\Omega_t^{-1}(X_{t+1} - B^{-1}\tilde{\mu} - \Phi X_t) \\
&= -2\log 2\pi - \frac{1}{2}\log|\Sigma_t| + \log|\tilde{S}^{-1}B| \\
&\quad - \frac{1}{2}(X_{t+1} - B^{-1}\tilde{\mu} - B^{-1}\tilde{\rho}BX_t)'\tilde{S}^{-1}B\Sigma_t^{-1}B'\tilde{S}^{-1\prime}(X_{t+1} - B^{-1}\tilde{\mu} - B^{-1}\tilde{\rho}BX_t).
\end{aligned} \tag{26}
$$

Even if the assumption of conditional normality is violated empirically, the estimate for ξ obtained by maximizing the resulting log-likelihood function, defined by summing (26) over the full sample, remains consist and asymptotically normally distributed under quite general conditions; see, e.g., Bollerslev and Wooldridge (1992).

The diagonal GARCH(1,1) model in (24) freely parametrizes the persistence in the "structural" shocks. Consistent with the implication from the underlying consumption-based asset pricing model, we impose the restriction that the autoregressive dependencies in the GARCH expected variance and the dividend-price ratio are the same, i.e., $\Gamma_{1,1} + \Upsilon_{1,1} = \Gamma_{4,4} + \Upsilon_{4,4} = \rho_q$. Guided by our initial diagnostic tests, we also restrict the dividend growth shock to have only ARCH and no GARCH effect, i.e., $\Gamma_{3,3} = 0$.

The long-run implications from multivariate GARCH models can be very sensitive to estimation errors and small perturbations in a few parameters. To help guard against this, we augment the Gaussian-based score for the "structural" VAR-GARCH model with an additional set of moment conditions designed to ensure that the unconditional variances of the reduced form errors implied by the model match their standard VAR-based analogues.[19] Expressing this additional set of moments in parallel to equation (26) and the contribution to the likelihood function coming from X_{t+1}, we have

$$
W_t(X_{t+1}, \xi) \;=\; \varpi_u - \mathrm{diag}\left((X_{t+1} - \mu^{\mathrm{OLS}} - \Phi^{\mathrm{OLS}}X_t)(X_{t+1} - \mu^{\mathrm{OLS}} - \Phi^{\mathrm{OLS}}X_t)'\right), \tag{27}
$$

where the "OLS" superscript indicates the parameters obtained from equation-by-equation least squares estimation of the reduced form VAR.

[19]This mirrors the variance targeting approach originally advocated by Engle and Mezrich (1996). However, in contrast to that two-step approach, the GMM-based procedure applied here jointly estimates all of the parameters in ξ in a single step.

The estimates for ξ reported below are obtained by applying standard iterated GMM to the conditional set of moments defined by the score for the conditional density in (26), say $\partial_\xi L_t(X_{t+1}, \xi)$, augmented with the moment conditions in (27),

$$g(X_{t+1}, \xi) = \begin{pmatrix} \partial_\xi L_t(X_{t+1}, \xi) \\ W_t(X_{t+1}, \xi) \end{pmatrix}. \tag{28}$$

We turn next to a discussion of the resulting $\hat{\xi}$, and the implications of the estimates in regards to the dynamics of the systematic risk factors and the dependencies among the "structural" shocks.

4.2 Estimation Results and "Structural" Inference

The dynamic dependencies in the observable state vector $X_t = [ERV_t, VRP_t, \Delta d_t, dp_t]'$ underlying our GMM estimation is directly related to the latent state vector $f_t = [\sigma_t^2, q_t, \Delta d_t, x_t]'$ of interest by $X_t = \mu_X + Qf_t$. This allows us to infer both the contemporaneous interaction matrix Q and the autoregressive matrix ρ describing the mean dynamics in $f_{t+1} = \mu + \rho f_t + S\epsilon_{t+1}$ from the estimates for B and $\tilde{\rho}$ based on $BX_{t+1} = \tilde{\mu} + \tilde{\rho}BX_t + \tilde{S}\tilde{\epsilon}_{t+1}$, and the relations in equation (21) above. Similarly, the estimated volatility loading matrix $\tilde{S}$ for the observable state vector X_t allow us to infer the volatility loading matrix S for the latent state vector f_t from equation (22), while the estimated volatility dynamics of the $\tilde{\epsilon}_{t+1}$ shocks effectively determines the implied volatility dynamics of the "structural" ϵ_{t+1} shocks.

We begin with a discussion of the estimates for B and $\tilde{\rho}$,

$$\hat{B} = \begin{pmatrix} 1 & \underset{(0.11)}{-0.02} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \underset{(0.03)}{-0.60} & \underset{(0.10)}{-1.44} & -0.19 & 1 \end{pmatrix} \qquad \hat{\tilde{\rho}} = \begin{pmatrix} \underset{(0.05)}{0.64} & 0 & 0 & 0 \\ 0 & \underset{(0.07)}{0.46} & 0 & 0 \\ 0 & 0 & \underset{(0.03)}{-0.23} & \underset{(0.004)}{-0.002} \\ 0 & 0 & 0 & \underset{(0.009)}{0.988} \end{pmatrix} \tag{29}$$

where the numbers in parentheses represent asymptotic standard errors. With the exception of $B_{1,2}$ and $\tilde{\rho}_{3,4}$, all of the individual parameter estimates are highly statistically significant. All of the estimates also have the "correct" signs vis-a-vis the implications from the equilibrium-based model and the "structural" VAR.

In particular, the negative estimates for the loadings for the dividend price ratio reported in the last row of the B matrix are consistent with the idea that the two volatility components σ_t^2 and

q_t, and cash flow growth Δd_t, are all genuine risk factors with negative market prices of risks.[20] Within the context of the standard Bansal and Yaron (2004) long-run risk model, these negative contemporaneous relationships between the dividend-price ratio and the other state variables, or risk factors, are critically dependent on the risk aversion parameter $\gamma > 1$ and the intertemporal elasticity of substitution $\psi > 1$. As such, our "structural" estimation results indirectly support this commonly invoked set of assumptions.

Our estimate for $\tilde{\rho}_{4,4} = \rho_x = 0.988$ also points to a highly persistent and very accurately estimated long-run risk factor. This contrasts with the typical practice of simply fixing the long-run persistence coefficient at some "large" value, as in, e.g., Bansal, Gallant, and Tauchen (2007a), and clearly highlights the advantages of the more structured GMM estimation approach and richer data sources applied here. Meanwhile, even though our estimate for $\phi_{dx} = \tilde{\rho}_{3,4} = \frac{\phi_{dx}}{-A_{d,x}} = -0.002$ is "correctly" signed, the parameter is not significantly different from zero, and as such offers only limited support to the idea that the long-run risk factor x_t contemporaneously impacts cash flows Δd_t.

Interestingly, our use of more accurate volatility measures results in a much more persistent consumption variance estimate $\tilde{\rho}_{1,1} = \rho_\sigma = 0.64$ compared to the estimates recently reported in Campbell, Giglio, Polk, and Turley (2012). Moreover, our estimates for $\tilde{\rho}_{1,1} = \rho_\sigma = 0.64 > \tilde{\rho}_{2,2} = \rho_q = 0.46$ imply that the consumption variance σ_t^2 is more persistent than the variance-of-variance q_t, or economic uncertainty, which is directly in line with the implicit assumptions invoked in the calibrations reported in Bollerslev, Tauchen, and Zhou (2009).

Turning to our estimates for the volatility dependence matrix $\tilde{S}$,

$$\hat{\tilde{S}} = \begin{pmatrix} 1 & 0 & 0 & \underset{(0.04)}{0.08} \\ \underset{(0.06)}{-0.29} & 1 & 0 & \underset{(0.02)}{-0.09} \\ \underset{(0.05)}{-0.36} & \underset{(0.08)}{-0.09} & 1 & \underset{(0.03)}{0.15} \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{30}$$

all of the individual parameters, except $\tilde{S}_{3,2}$, are again highly statistically significant. This clearly underscores the idea that multiple volatility factors are indeed needed to accurately describe the

[20] Note that the market price of dividend risk $B_{4,3} = -0.19$ is imputed to by the constraint $A_{d,d} = \frac{\rho_d}{1-\kappa_{d,1}\rho_d}$ imposed in equation (21).

dynamic dependencies observed in the data, and that the standard long-run risk model with a single stochastic volatility factor is misspecified. To more fully appreciate this and the other implications of the estimates recall again the relationship between $\tilde{S}$ and the "structural" S matrix for the latent state vector in equation (22).

It follows from this relation that shocks to cash flow growth Δd_t are adversely affected by shocks to the long-run risk component x_t, as $s_{d,x} \propto -\tilde{S}_{3,4} = -0.15$.[21] This is consistent with the idea that companies tend to distribute more in dividends when long-run growth opportunities are poor. The "structural" long-run risk shock affects the two variance processes σ_t^2 and q_t in opposite directions. Good news about long-run consumption growth reduces the consumption variance, as $s_{\sigma,x} \propto -\tilde{S}_{1,4} = -0.08 < 0$, but increases economic uncertainty, as $s_{q,x} \propto -\tilde{S}_{2,4} = 0.09 > 0$. The first effect represents the well known "leverage effect", whereby a negative growth shock is associated with higher volatility, and vice versa. The second effect, however, is more subtle. Since q_t directly affects the time-varying volatility of the long-run risk component, a positive $s_{q,x}$ implies that when a positive $z_{x,t}$ shock occurs, the volatility of next period's $\epsilon_{x,t+1}$ will also be higher, and vice versa. Intuitively, this could happen when good news in consumption growth is accompanied by better investment opportunities, in turn resulting in higher economic uncertainty, possibly due to over-investment. Interestingly, our estimates for $\tilde{S}$ also suggest that $s_{q,\sigma} \propto \tilde{S}_{2,1} = -0.29 < 0$, implying that a positive "structural" shock to consumption volatility σ_t^2 reduces the uncertainty of volatility q_t. This effect is naturally interpreted as a new "leverage effect" between volatility and volatility-of-volatility.[22]

Before turning to our main empirical investigations related to the return and cash flow predictability patterns implied by the "structural" factor GARCH estimates, we will briefly discuss a series of statistical diagnostic tests designed to assess the quality of the fit of the model.

[21] We use the symbol $\propto$ to denote proportional to.

[22] This new equilibrium-based "leverage effect" is also consistent with the asymmetries in daily and high-frequency intraday VIX and S&P 500 returns documented in Aboura and Wagner (2012) and Bollerslev, Osterrieder, Sizova, and Tauchen (2012a), respectively.

4.3 Model Fit and Diagnostics

Our identification and estimation of the "structural" model parameters rely crucially on the presence of time-varying conditional heteroscedasticity in the ϵ_{t+1} shocks. The GMM parameter estimates for the "structural" factor GARCH model describing this heteroscedasticity are reported in Table 2. As the table shows, all of the shocks do indeed exhibit highly significant (G)ARCH effects.[23] The overall good fit of the model is also supported by the conventional J-test statistic for general model misspecification and the minimized value of the GMM objective function equal to 12.76, which has a p-value of 0.12 in the corresponding asymptotic chi-square distribution.[24]

The importance of explicitly allowing for time-varying volatility is further highlighted by the Ljung-Box tests for residual serial correlation reported in Table 3. The tests for the absolute and squared raw residuals ignoring heteroscedasticity reported in the top panel all exceed their relevant quantiles in the chi-square distributions with ten and twenty degrees of freedom, respectively.[25] Meanwhile, the corresponding tests for the standardized raw and absolute residuals reported in the bottom panel are all much smaller and with a few exceptions insignificant when judged by their conventional 95-percent chi-square critical values, thus underscoring the overall satisfactory fit of the "structural" GARCH model.

In order to further gauge the quality of the fit afforded by the model, Figure 2 plots the time-series of "structural" shocks associated with each of the four equations. The top two panels show the volatility shocks $z_{\sigma,t}$ and $z_{q,t}$. Both of these shocks experienced unprecedented large, albeit opposite signed, realizations during the 2007-2009 "Great Recession." Interestingly, neither one of the earlier 1990-1991 and 2001-2002 NBER-dated recessions were accompanied by especially large "structural" volatility shocks. The general time-series pattern of the equilibrium-based cash flow shocks $z_{\Delta d,t}$ appear quite similar to that of the normalized cash flow news in Campbell, Giglio, Polk, and Turley (2012). Although not quite as dramatic as for the two volatility shocks, the

[23]This is, of course, directly in line with the burgeon literature on the estimation of reduced form GARCH and stochastic volatility models for a wide array of different financial and macroeconomic time series.

[24]By contrast, the two alternative specifications discussed in Appendix C and D, one closer to Drechsler and Yaron (2011) with $G_t = \text{diag}[\sigma_t, \sqrt{q_t}, \sigma_t, \sqrt{q_t}, \sigma_t]$, and one closer to Branger and Vòlkert (2012) with $G_t = \text{diag}[\sigma_t, \sigma_t, \sigma_t, \sqrt{q_t}, \sigma_t]$, result in GMM-based J-statistics equal to 26.31 and 37.02, respectively, with corresponding p-values essentially zero.

[25]The 95-percent critical values for the chi-square distributions with ten and twenty degrees of freedom equal 18.3 and 31.4, respectively.

permanent growth shocks $z_{x,t}$ also experienced their most extreme realizations during the "Great Recession." This basic dynamic pattern in the equilibrium-based growth shocks is again quite similar to that of the normalized discount rate news shocks reported in Campbell, Giglio, Polk, and Turley (2012).[26]

In lieu of these findings and generally supportive diagnostic tests for the "structural" factor GARCH model, we turn next to our main empirical investigations, showing how incorporating the additional variance-related state variables in the equilibrium-based model help shed new light on the return and dividend growth predictability patterns inherent in the data.

5 Model Implied Return and Cash Flow Predictability

Our predictability analysis is based on recasting the "structural" factor GARCH model in the form of an expanded VAR system, along with the use of the standard Campbell-Shiller approximation for expressing the return as a function of the observable state variables.

5.1 VAR and Predictability

The first order VAR for the state vector X_t=$[ERV_t, VRP_t, \Delta d_t, dp_t]$ implied by the "structural" factor GARCH model in equation (23) doesn't directly involve the return. However, by the standard Campbell-Shiller approximation, the return may be conveniently expressed as $r_{t,t+1} = \kappa_{d,0} - \kappa_{d,1} dp_{t+1} + dp_t + \Delta d_{t+1}$.[27] Combining this equation for $r_{t,t+1}$ with the VAR for X_{t+1}, it follows that

$$r_{t,t+1} \ = \ \mu_r + (l_1 \Phi \ + \ e_4) X_t + l_1 \Phi_0^{-1} \tilde{\epsilon}_{t+1}, \tag{31}$$

where μ_r collects all of the relevant constant terms, $l_1 \equiv (0, 0, 1, -k_{1,d})$, and the selection vector $e_4 \equiv (0, 0, 0, 1)$. Iterating the VAR for X_t forward, it is therefore possible to derive closed-form expressions for the model-implied multi-period return $r_{t,t+h} = r_{t,t+1} + ... + r_{t+h-1,t+h}$ regressions based any explanatory variable spanned by the X_t state vector.

[26]This is also consistent with the findings in Lettau and Ludvigson (2011), who suggest that large negative permanent growth shocks might have adversely affected housing wealth.

[27]The accuracy of the Campbell-Shiller approximation has recently been corroborated by Engsted, Pedersen, and Tanggaard (2012). By definition $\kappa_{d,1} = \exp(-E(dp_t))[1 + \exp(-E(dp_t))]^{-1}$. In the estimation results reported on below we rely on the sample average of the monthly dividend-price ratio from January 1965 to November 2011 when calculating $E(dp_t)$, implying a value of $\kappa_{d,1} = 0.9976$.

In the analysis reported on below we will focus on the three key predictor variables: the log dividend-price ratio dp_t, the variance risk premium VRP_t, and the expected variation ERV_t. In particular, consider the regression of the h-period returns on the dividend-price ratio,

$$\frac{1}{h}\sum_{i=1}^{h} r_{t,t+i} \;=\; \alpha_{r,dp} \;+\; \beta_{r,dp}(h)\cdot dp_t \;+\; \varsigma_{t,t+h}. \tag{32}$$

By similar arguments to the ones in Hodrick (1992) and Campbell (2001), it is possible to show that

$$\beta_{r,dp}(h) = \frac{(l_1\Phi + e_4)(I-\Phi)^{-1}(I-\Phi^h)C(0)e_4'}{e_4C(0)e_4'} \tag{33}$$

where $C(0) = \sum_{j=0}^{\infty}\Phi^j\Phi_0^{-1}\mathrm{diag}(\Theta_0^{-1}\varpi_u)\Phi_0^{-1\prime}\Phi^{j\prime}$ denotes the model-implied unconditional covariance matrix for X_t, and $e_4 \equiv (0,0,0,1)$.[28] Similarly, the implied coefficients for the return predictability regressions based on VRP_t and ERV_t may be expressed in close form as,

$$\beta_{r,VRP}(h) = \frac{(l_1\Phi + e_4)(I-\Phi)^{-1}(I-\Phi^h)C(0)e_2'}{e_2C(0)e_2'} \tag{34}$$

$$\beta_{r,ERV}(h) = \frac{(l_1\Phi + e_4)(I-\Phi)^{-1}(I-\Phi^h)C(0)e_1'}{e_1C(0)e_1'} \tag{35}$$

where the e_1 and e_2 selection vectors are defined in an obvious manner.[29]

In parallel to equation (31) for the returns, the growth rate dynamics implied by the "structural" factor GARCH may be expressed in linear form as,

$$\Delta d_{t+1} = \mu_d + e_3\Phi X_t + e_3\Phi_0^{-1}\tilde{\epsilon}_{t+1}, \tag{36}$$

where μ_d collects all the relevant constant terms. Thus, replacing $l_1\Phi + e_4$ with $e_3\Phi$ in the formulas for the regression coefficients above, comparable expressions for the cash flow predictability regression coefficients $\beta_{\Delta d,dp}(h)$, $\beta_{\Delta d,VRP}(h)$, and $\beta_{\Delta d,ERV}(h)$ are readily available. When interpreting these coefficients, it is important to keep in mind the relationship $E_t(\Delta d_{t+1}) = \phi_{dx}x_t + \rho_d\Delta d_t$ implied by equations (17) and (18), and the fact that within the "structural" model the expected value of next periods dividend growth rate is linearly related to the lagged dividend growth rate and the long-run risk component.

[28] In the empirical results reported on below, we truncate the infinite sum in the expression for $C(0)$ at 120, or ten years; see Bollerslev and Hodrick (1995) for further discussion along these lines.

[29] Analytical expressions for the R^2s from the regressions may be derived in a similar manner. Specifically, for the dividend-price ratio regression $R^2_{r,dp}(h) = h^2\beta^2_{r,dp}(h)e_4C(0)e_4'/Var(\sum_{j=1}^{h} r_{t,t+j})$, where $Var(\sum_{j=1}^{h} r_{t,t+j}) = h(l_1\Phi + e_3)C(0)(l_1\Phi + e_3)' + hl_1(I-\Phi)C(0)l_1' + \sum_{i=1}^{h-1}2(h-i)((l_1\Phi + e_3)\Phi^iC(0)(l_1\Phi + e_3)' + (l_1\Phi + e_3)\Phi^{i-1}(I-\Phi)C(0)e_3')$.

5.2 Model-Implied Reduced Form VAR Estimates

The reduced form VAR parameter matrix Φ and the unconditional covariance matrix $C(0)$ for X_t entering the expressions for the predictive regression coefficients in equations (33)-(35) could, of course, be estimated directly by OLS equation-by-equation. However, that obviously would ignore any of the equilibrium-based "structural" restrictions. It also would not permit the separate identification of the contemporaneous Φ_0 matrix entering the expressions for the return and dividend growth rate in equations (31) and (36), respectively.

Instead, the Φ_0 and Φ parameter matrices may both be deduced from the "structural" factor GARCH model parameters and the relations $\Phi = B^{-1}\tilde{\rho}B$ and $\Phi_0^{-1} = B^{-1}\tilde{S}$ derived above. Substituting the previously discussed estimates for B, $\tilde{\rho}$ and $\tilde{S}$ into these expressions, yields,

$$\hat{\Phi} = \begin{pmatrix} \underset{(0.05)}{0.64} & \underset{(0.020)}{-0.003} & 0 & 0 \\ 0 & \underset{(0.07)}{0.46} & 0 & 0 \\ \underset{(0.002)}{0.001} & \underset{(0.005)}{0.002} & \underset{(0.03)}{-0.23} & \underset{(0.004)}{-0.002} \\ \underset{(0.03)}{-0.21} & \underset{(0.13)}{-0.76} & \underset{(0.03)}{-0.23} & \underset{(0.087)}{0.988} \end{pmatrix} \qquad \hat{\Phi}_0 = \begin{pmatrix} \underset{(0.033)}{0.995} & \underset{(0.11)}{0.02} & 0 & \underset{(0.04)}{0.08} \\ \underset{(0.06)}{-0.29} & 1 & 0 & \underset{(0.02)}{-0.09} \\ \underset{(0.06)}{-0.34} & \underset{(0.08)}{-0.09} & 1 & \underset{(0.03)}{0.15} \\ \underset{(0.09)}{0.11} & \underset{(0.13)}{1.44} & \underset{(0.02)}{0.19} & \underset{(0.03)}{0.94} \end{pmatrix} \tag{37}$$

where the numbers in parentheses represent standard errors derived by the delta-method.

Based on these estimates for Φ and Φ_0, the return equation in (31) may be expressed numerically as,

$$\begin{aligned} r_{t,t+1} = \; & \underset{(0.03)}{0.05} + \underset{(0.03)}{0.20} ERV_t + \underset{(0.12)}{0.76} VRP_t - \underset{(0.002)}{0.0013} \Delta d_t + \underset{(0.011)}{0.013} dp_t \\ & - \underset{(0.08)}{0.47} \tilde{\epsilon}_{\sigma,t+1} - \underset{(0.11)}{1.52}\, \tilde{\epsilon}_{q,t+1} + \underset{(0.04)}{0.81} \tilde{\epsilon}_{\Delta d,t+1} - \underset{(0.03)}{0.79}\, \tilde{\epsilon}_{x,t+1}. \end{aligned} \tag{38}$$

Of course, this "estimated" return equation does not actually rely on the return data, but instead is deduced from our estimates for the equilibrium-based model and the observable state vector involving the dividend growth rate and the log dividend-price ratio. Again, this mirrors the approach of Cochrane (2008). However, in contrast to the return equation therein, which only involves the dividend-price ratio, we purposely include the two variance variables, both of which enters with highly significant coefficients.

Further underscoring the importance of incorporating the variation measures into the analysis, the model-implied loadings for all of the "structural" shocks are also highly significant. Among the four shocks, the ones for the long-run risk component and the consumption variance uncertainty have the largest impacts, accounting for 43 percent ($z_{x,t}$) and 26 percent ($z_{q,t}$) of the unexpected unconditional return variation, respectively. The "estimated" return equation in (38) also implies that the total one-month explainable return variation equals 9 percent, far exceeding that afforded by traditional univariate return predictability regressions that does not include ERV_t and VRP_t.

Explicitly writing out the second equation for the variance risk premium in the model-implied VAR,

$$VRP_{t+1} = \underset{(0.001)}{0.001} + \underset{(0.07)}{0.46} VRP_t \underset{(0.06)}{-0.29}\, \tilde{\epsilon}_{\sigma,t+1} + \tilde{\epsilon}_{q,t+1} \underset{(0.02)}{-0.09}\, \tilde{\epsilon}_{x,t+1}. \tag{39}$$

shows that the only "structural" shock that enters the return and VRP equations with the opposite sign is $\tilde{\epsilon}_{q,t}$. Indeed, excluding the impact of the economic uncertainty shock from both equations changes the monthly conditional correlation, or "leverage effect," from a negative -0.09 to a positive 0.66, again reinforcing the importance of jointly modeling *all* of the elements in the X_t state vector.

5.3 Model-Implied Predictability Relations

The VAR-based formula for the slope coefficients presented above allow for a direct assessment of the statistical significance of the different predictor variables across different forecast horizons. The formula also allow us to directly assess the enhanced efficiency afforded by the "structural" factor GARCH model compared to the reduced form VAR and simple univariate regression procedures traditionally used in the literature.

To begin, the top panel in Table 4 reports the implied slope coefficients for forecasting returns and cash flows by the dividend-price ratio dp_t over long 1- to 10-year horizons, as previously analyzed in the literature. Although the patterns in the estimated coefficients are generally in line with the estimates reported in the existing literature based on longer calendar time spans of data, taken as a whole there is little evidence for any predictability over these long multi-year horizons in the data analyzed here.[30] The results for the shorter within year "structural" and simply

[30]We also experimented with a traditional two-variable homoskedastic VAR for the dividend-price ratio and the dividend growth rate, as in Cochrane (2008), resulting in similar coefficient estimates, but typically larger standard errors, thus highlighting the more accurate inference afforded by explicitly incorporating the equilibrium-based restrictions and the strong heteroskedasticity inherent in the data in the estimation. Further details concerning these results are

unconstrained univariate regressions reported in the lower panel of the table tell a similar story.

The lack of predictability for the long multi-year horizons, is, of course, not too surprising. With only slightly more than twenty years worth of monthly observations any suggestions about statistically significant long-run predictability should be taken with a grain of salt. For the remainder of this section, we will consequently restrict our attention to within-year horizons only.[31]

Turning to our key empirical findings pertaining to the "new" variance related forecasting variables, Figure 3 shows the regression slope coefficients for the variance risk premium VRP_t implied by the the "structural" factor GARCH model (indicated by dots) along with the corresponding 95 percent confidence intervals (indicated by the shaded area). For comparison purposes, we also include the estimated slope coefficients from simple univariate predictive regressions based on the variance risk premium (indicated by the stars) along with their 95 percent confidence intervals (indicated by the dashed lines). Focusing on the top panel for the returns, both procedures result in significant estimates for up to eight months. It is noteworthy that even though the model-implied point estimates are systematically lower than the unrestricted OLS estimates, they are also less erratic, and the confidence intervals much smaller. Indeed, looking at the numbers in Table 5, the t-statistics for testing the null hypothesis of no return predictability are uniformly larger for the "structural" approach.

This discrepancy in the results across the two approaches is even stronger for the cash flow predictability regressions reported in the bottom panel in Figure 3. Whereas the estimated slope coefficients from the univariate regressions are all insignificant, the t-statistics associated with the VAR-based model-implied coefficients are all negative and exceed conventional significance levels for up to six months. Hence, not only are higher variance risk premia positively related to future returns, as previously documented in the literature, they also predict lower near-term future cash flows.[32] This, of course, contrasts with the view commonly expressed in the literature that dividend growth rates are largely unpredictable over short within-year horizons.

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available upon request.

[31]The univariate return regressions reported in Bollerslev, Tauchen, and Zhou (2009) and Drechsler and Yaron (2011) that in part motivate our analysis also suggest that the return predictability inherent in the variance risk premium is confined to relatively short horizons.

[32]This is also related to the observation by Bloom (2009) that an increase in economic uncertainty causes firms to temporarily reduce their investment and hiring, in turn resulting in a short-term productivity drop.

Of course, the much-studied classical risk-return trade-off is not based on the variance risk premium, but rather the return variation itself. In spite of the intuitively appealing idea behind such a relationship, empirical attempts at establishing a significant risk-return tradeoff have largely proven futile; see, e.g., the discussion in Bollerslev and Zhou (2006) and Guo and Whitelaw (2006), and the many other references therein. The result for the univariate return regressions based on ERV_t reported in the top panel in Figure 4 and Table 6 underscore the elusive nature of a simple linear relationship between the expected returns and the expected variation in the data analyzed here. None of the regression coefficients are significant, and most have the "wrong" sign. By contrast, the VAR-based estimates implied by the "structural" model are all positive and marginally significant for return horizons in excess of 4 months.[33]

The difference in the quality of the inference afforded by standard univariate regression-based procedures traditionally employed in the literature and the "structural" approach advocated here is even more dramatic for the cash flow predictions reported in the bottom panel in Figure 4. While the simple univariate regressions suggest that the 1-6 months dividend growth rate is unpredictable, the regression coefficients implied by the "structural" model are all highly significant. Interestingly, whereas an increase in VRP_t predicts lower future cash flows, and increase in ERV_t is associated with significantly higher future cash flows. Again, this strong empirical evidence for short-run within-year cash flow predictability stands in sharp contrast to the results reported in the existing literature based on other more traditional predictor variables and valuation ratios.

At a more general level, the results for the two different approaches reported in Tables 4-6 and Figures **??**-4 may also be seen as providing indirect support for the equilibrium-based "structural" model, in that the more accurate model-implied predictive relations systematically fall within the wider standard error bands associated with the unrestricted regressions. This, of course, would not necessarily be the case if the assumptions underlying the "structural" model were violated.

[33]The use of $IV_t = VRP_t + ERV_t$ results in qualitatively similar patterns, but slightly more significant coefficient estimates, compared to the ones reported for ERV_t, thus confirming earlier empirical findings in Bollerslev and Zhou (2006) and Guo and Whitelaw (2006) of a stronger risk-return trade-off when using implied as opposed to realized variation. Still, none of the univariate return regressions based on IV_t result in any significant predictability. Further details of these results are available upon request.

5.4 Further Discussion and Interpretation

The contrast between the long-run predictability inherent in the dividend-price ratio, and the variance variables ability to predict both return and cash flow over shorter within-year horizons is intimately related to our equilibrium-based long-run risk model, and the way in which the fundamental risk factors affect the state variables.

In particular, while the dividend-price ratio dp_t loads on the long-run risk factor x_t and both of the volatility factors σ_t^2 and q_t, the expected variation ERV_t depends only on the two volatility factors σ_t^2 and q_t, and the variance risk premium VRP_t is exclusively determined by the volatility-of-volatility factor q_t. Consistent with earlier less formal model calibrations reported in the literature, our GMM-based estimates imply that the long-run risk factor is highly persistent with AR(1) coefficient equal to $\rho_x = 0.988$, while the consumption volatility factor is moderately persistent with AR(1) coefficient equal to $\rho_\sigma = 0.64$, and the consumption volatility-of-volatility factor is quickly mean-revering with AR(1) coefficient equal to $\rho_q = 0.46$.

In light of these estimates for the underlying systematic risk factors, it is therefore not surprising that the "structural" model implied return predictability regressions based on VRP_t, which depends solely on q_t, result in the most significant coefficients over relatively short 1-6 months horizon. Meanwhile, the regressions based on dp_t, which loads heavily on x_t, should show the greatest explanatory power over longer multi-year horizons, which, of course is difficult to detect statistically with the limited time span of data analyzed here. Also, whereas the variance risk premium is most significant over horizons less than 6 months, the expected variation ERV_t displays the most significant predictability over 6-12 months horizons, as the more persistent σ_t^2 process "shifts" the predictable forward.

The documented differences in the degree of cash flow predictability are most easily understood in terms of the correlations among the "structural" shocks. From the model estimates the cash flow shock is more strongly negatively correlated with the contemporaneous variance shock ($s_{d,\sigma} \propto \tilde{S}_{3,1} = -0.36$), than it is with the uncertainty shock ($s_{d,q} \propto \tilde{S}_{3,2} = -0.09$) or the long-run risk shock ($s_{d,x} \propto -\tilde{S}_{3,4} = -0.15$). Since the expected variation loads more heavily on σ_t^2 than q_t, while the dividend-price ratio and the variance risk premium are mostly determined by x_t and q_t, respectively, ERV_t will be more strongly negatively related to Δd_t than either dp_t or VRP_t. Because of the negative autocorrelation in Δd_t ($\rho_d = -0.23 < 0$), this in turn translates into the strongest positive short-run cash flow predictability results for the ERV_t predictor variable implied by the "structural" VAR.

6 Conclusion

We examine the joint predictability of return and dividend growth rates within a present value framework, explicitly imposing the economic equilibrium-based constraints from a long-run risk model with time-varying consumption volatility and volatility-of-volatility risk. The model clearly differentiate the long-run predictability channels associated with the dividend-price ratio from the economic mechanisms responsible for the short-run predictability inherent in the variance risk premium and the expected return variation.

Consistent with Bansal and Yaron (2004), our GMM-based estimates of the "structural" factor GARCH model point to a highly persistent latent long-run risk factor. Our estimates also corroborate the calibrations in Bollerslev, Tauchen, and Zhou (2009), and the notion that consumption volatility is more persistent than consumption volatility-of-volatility. In addition, the "structural" shocks identified within the model reveal that cash flow respond negatively to contemporaneous long-run growth shocks, while consumption volatility decreases with shocks to the long-run growth factor, and volatility uncertainty increases with long-run growth shocks. A new "leverage effect" whereby shocks to consumption volatility is negatively related to volatility-of-volatility also emerges from our "structural" estimation.

By allowing for much sharper and accurate inference than the procedures traditionally employed in the literature, the VAR implied by the "structural" model also provides striking new evidence on the return and cash flow predictability inherent in the data. Specifically, we find that the variance risk premium, and to a lesser extend the expected return variation, significantly predicts short-run within-year returns. On the other hand, the expected return variation, and to a lessor extend the variance risk premium, strongly predicts short-run within-year dividend growth rates. This latter finding stands in sharp contrast to the view expressed by a number of studies in the literature that cash flows are largely unpredictable.

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Table 1 Summary Statistics

The table reports standard summary statistics and correlations for the S&P 500 return $r_{t,t+1}$, dividend growth rate Δd_t, dividend-price ratio dp_t, options implied variance IV_t, expected variance ERV_t, and variance risk premium VRP_t. The returns, dividend growth, and dividend-price ratio are all in annualized percentage form. All of the variance variables are in monthly percentage form. The sample period extends from February 1990 to November 2011, for a total of 262 monthly observations.

	Mean	Std	Skew	Kurt	AC1
$r_{t,t+1}$	8.19	15.33	-0.76	4.48	0.07
Δd_t	3.92	8.79	-0.46	10.02	-0.26
dp_t	-3.91	0.31	0.08	2.32	0.98
IV_t	40.30	36.47	3.23	18.07	0.81
ERV_t	28.54	36.64	4.62	30.08	0.69
VRP_t	11.75	14.93	-3.37	38.42	0.27

	Correlations					
	$r_{t,t+1}$	Δd_t	dp_t	IV_t	ERV_t	VRP_t
$r_{t,t+1}$	1.00	0.34	-0.03	-0.42	-0.48	0.15
Δd_t		1.00	-0.02	-0.25	-0.25	-0.01
dp_t			1.00	-0.05	-0.02	-0.07
IV_t				1.00	0.92	0.19
ERV_t					1.00	-0.21
VRP_t						1.00

Table 2 "Structural" Factor GARCH Estimation

The table reports the GMM estimation result for the conditional variance parameters for the "structural" factor GARCH model discussed in the main text. The column labeled ϖ_u gives the unconditional variance of the reduced form shocks u_t. Υ and Γ denote the ARCH and GARCH parameters, respectively, for the "structural" shocks $\tilde{\epsilon}_t$. The estimates are based on monthly data from February 1990 to November 2011, for a total of 262 observations.

$\tilde{\epsilon}_t$	ϖ_u		Γ		Υ	
ERV_t	0.0011	(0.0002)	0.189		0.273	(0.0751)
VRP_t	0.0003	(0.0000)	0.758	(0.080)	0.239	(0.077)
Δd_t	0.0006	(0.0001)	0		0.524	(0.100)
d_{t+1}/p_t	0.0016	(0.0002)	0.299		0.163	(0.082)

Table 3 Residual Serial Correlation Tests

The table reports the Ljung-Box portmanteau tests for up to tenth and twentieth order serial correlation in the raw $\tilde{\epsilon}_t$ and standardized $\tilde{\epsilon}_t / \sqrt{\Sigma_t}$ "structural" shocks from the estimated factor GARCH model discussed in the main text. The estimates of the model are based on monthly data from February 1990 to November 2011.

	Raw residuals					
	$\tilde{\epsilon}_t$		$\|\tilde{\epsilon}_t\|$		$\tilde{\epsilon}_t^2$	
Lags	10	20	10	20	10	20
ERV_t	20.41	25.35	74.76	75.58	31.30	32.14
VRP_t	36.03	63.04	147.77	181.49	65.95	71.42
Δd_t	16.30	40.63	100.04	108.56	86.42	91.84
d_t/p_t	9.58	17.26	57.55	82.24	38.80	49.20
	Standardized residuals					
	$\tilde{\epsilon}_t / \sqrt{\Sigma_t}$		$\|\tilde{\epsilon}_t / \sqrt{\Sigma_t}\|$		$\tilde{\epsilon}_t^2 / \sqrt{\Sigma_t}$	
Lags	10	20	10	20	10	20
ERV_t	19.04	25.26	13.80	15.53	2.56	2.90
VRP_t	4.82	8.94	6.62	22.75	2.30	7.88
Δd_t	16.27	40.74	11.20	18.74	10.59	14.89
d_t/p_t	11.56	18.25	31.95	45.44	25.72	31.33

Table 4 Predictive Regressions based on the Dividend-Price Ratio

The table reports the slope coefficients in the return and cash flow predictability regressions,

$$\frac{1}{h}\sum_{i=1}^{h} r_{t,t+i} = \alpha_{r,dp} + \beta_{r,dp}(h)\cdot dp_t + \zeta_{t,t+h}$$

$$\frac{1}{h}\sum_{i=1}^{h} \Delta d_{t,t+i} = \alpha_{\Delta d,dp} + \beta_{\Delta d,dp}(h)\cdot dp_t + \zeta_{t,t+h}$$

implied by the parameter estimates for the "structural" factor GARCH model discussed in the main text, with asymptotic standard errors in parentheses. The table also reports the slope coefficients implied by a two-variable reduced form homoskedastic VAR for the dividend growth rate and the dividend-price ratio, as in Cochrane (2008), along with the results from simple univariate predictive regressions. The time horizon h runs from one to ten years in the first two panels, and from one to twelve months in the bottom three panels. All of the results are based on monthly data from February 1990 to November 2011.

Years	1	2	3	4	5	6	7	8	9	10
					Structural Model Implied					
$\beta_{r,dp}(h)$	0.0126	0.0114	0.0106	0.0098	0.0092	0.0086	0.0081	0.0077	0.0072	0.0068
	(0.0099)	(0.0085)	(0.0074)	(0.0065)	(0.0057)	(0.0050)	(0.0045)	(0.0039)	(0.0035)	(0.0031)
$\beta_{\Delta d,dp}(h)$	-0.0013	-0.0012	-0.0011	-0.0011	-0.0010	-0.0009	-0.0009	-0.0008	-0.0008	-0.0007
	(0.0028)	(0.0027)	(0.0026)	(0.0024)	(0.0023)	(0.0022)	(0.0021)	(0.0020)	(0.0019)	(0.0018)

Months	1	2	3	4	5	6	9	12
				Structural Model Implied				
$\beta_{r,dp}(h)$	0.0166	0.0157	0.0150	0.0145	0.0141	0.0138	0.0131	0.0126
	(0.0135)	(0.0127)	(0.0122)	(0.0117)	(0.0114)	(0.0111)	(0.0104)	(0.0099)
$\beta_{\Delta d,dp}(h)$	-0.0022	-0.0017	-0.0016	-0.0015	-0.0015	-0.0014	-0.0014	-0.0013
	(0.0023)	(0.0027)	(0.0028)	(0.0029)	(0.0029)	(0.0029)	(0.0029)	(0.0028)
				Univariate Regression				
$\beta_{r,dp}(h)$	0.0112	0.0119	0.0121	0.0123	0.0129	0.0135	0.0153	0.0161
	(0.0089)	(0.0083)	(0.0078)	(0.0076)	(0.0074)	(0.0072)	(0.0069)	(0.0069)
$\beta_{\Delta d,dp}(h)$	-0.0029	-0.0011	-0.0004	-0.0001	-0.0000	0.0001	0.0002	0.0001
	(0.0030)	(0.0015)	(0.0012)	(0.0008)	(0.0007)	(0.0006)	(0.0005)	(0.0004)

Table 5 Predictive Regressions based on the Variance Risk Premium

The table reports the slope coefficients in the return and cash flow predictability regressions,

$$\frac{1}{h}\sum_{i=1}^{h} r_{t,t+i} = \alpha_{r,VRP} + \beta_{r,VRP}(h)\cdot VRP_t + \zeta_{t,t+h}$$

$$\frac{1}{h}\sum_{i=1}^{h} \Delta d_{t,t+i} = \alpha_{\Delta d,VRP} + \beta_{\Delta d,VRP}(h)\cdot VRP_t + \zeta_{t,t+h}$$

implied by the parameter estimates for the "structural" factor GARCH model discussed in the main text, with asymptotic standard errors in parentheses. The table also reports the slope coefficients from simple univariate predictive regressions. The time horizon h runs from one to twelve months. All of the results are based on monthly data from February 1990 to November 2011.

Months	1	2	3	4	5	6	9	12
				Structural Model Implied				
$\beta_{r,VRP}(h)$	0.5228	0.3571	0.2564	0.1929	0.1514	0.1231	0.0772	0.0557
	(0.1031)	(0.0566)	(0.0396)	(0.0330)	(0.0292)	(0.0263)	(0.0200)	(0.0161)
$\beta_{\Delta d,VRP}(h)$	-0.0393	-0.0147	-0.0103	-0.0074	-0.0058	-0.0047	-0.0029	-0.0020
	(0.0154)	(0.0057)	(0.0041)	(0.0031)	(0.0026)	(0.0023)	(0.0018)	(0.0016)
				Univariate Regression				
$\beta_{r,VRP}(h)$	0.5454	0.4060	0.3620	0.3640	0.3494	0.2683	0.1335	0.0911
	(0.2194)	(0.1450)	(0.1090)	(0.1321)	(0.1400)	(0.1031)	(0.0773)	(0.0540)
$\beta_{\Delta d,VRP}(h)$	-0.1215	0.0035	0.0280	0.0576	0.0183	0.0232	-0.0066	-0.0012
	(0.2420)	(0.1094)	(0.0483)	(0.0640)	(0.0465)	(0.0309)	(0.0237)	(0.0130)

Table 6 Predictive Regressions based on the Expected Variation

The table reports the slope coefficients in the return and cash flow predictability regressions,

$$\frac{1}{h}\sum_{i=1}^{h} r_{t,t+i} = \alpha_{r,ERV} + \beta_{r,ERV}(h)\cdot ERV_t + \zeta_{t,t+h}$$

$$\frac{1}{h}\sum_{i=1}^{h} \Delta d_{t,t+i} = \alpha_{\Delta d,ERV} + \beta_{\Delta d,ERV}(h)\cdot ERV_t + \zeta_{t,t+h}$$

implied by the parameter estimates for the "structural" factor GARCH model discussed in the main text, with asymptotic standard errors in parentheses. The table also reports the slope coefficients from simple univariate predictive regressions. The time horizon h runs from one to twelve months. All of the results are based on monthly data from February 1990 to November 2011.

Months	1	2	3	4	5	6	9	12
				Structural Model Implied				
$\beta_{r,ERV}(h)$	0.0251	0.0387	0.0426	0.0422	0.0398	0.0368	0.0283	0.0223
	(0.0417)	(0.0321)	(0.0264)	(0.0225)	(0.0195)	(0.0171)	(0.0124)	(0.0096)
$\beta_{\Delta d,ERV}(h)$	0.0396	0.0152	0.0107	0.0079	0.0063	0.0052	0.0034	0.0025
	(0.0081)	(0.0029)	(0.0021)	(0.0016)	(0.0013)	(0.0011)	(0.0008)	(0.0006)
				Univariate Regression				
$\beta_{r,ERV}(h)$	-0.1044	-0.0477	-0.0546	-0.0491	-0.0274	-0.0003	0.0185	0.0137
	(0.1273)	(0.1080)	(0.0957)	(0.0911)	(0.0742)	(0.0531)	(0.0307)	(0.0242)
$\beta_{\Delta d,ERV}(h)$	0.1173	0.0079	-0.0004	0.0073	0.0113	0.0152	0.0139	0.0117
	(0.1051)	(0.0404)	(0.0243)	(0.0239)	(0.0132)	(0.0082)	(0.0059)	(0.0035)

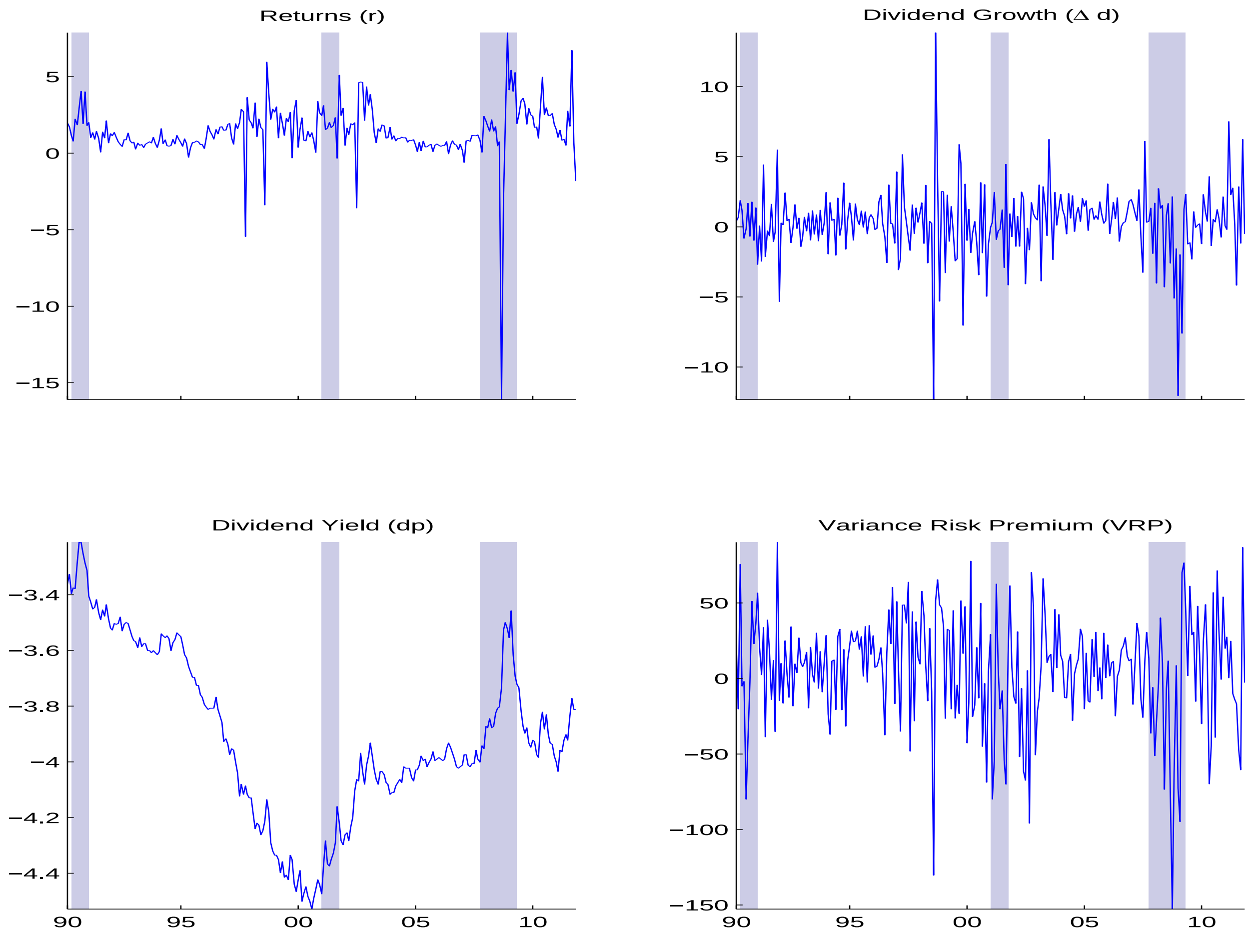

Figure 1 Returns and Dividends

The figure shows the monthly S&P500 returns (upper left panel), the log dividend growth rate (upper right panel), the log dividend-price ratio (lower left panel), and the variance risk premium (lower right panel). The returns, dividend growth, and dividend-price ratio are in annualized percentage form. The variance risk premium is in monthly percentage square form. The sample period extends from February 1990 to November 2011. The shaded areas indicate NBER dated recessions.

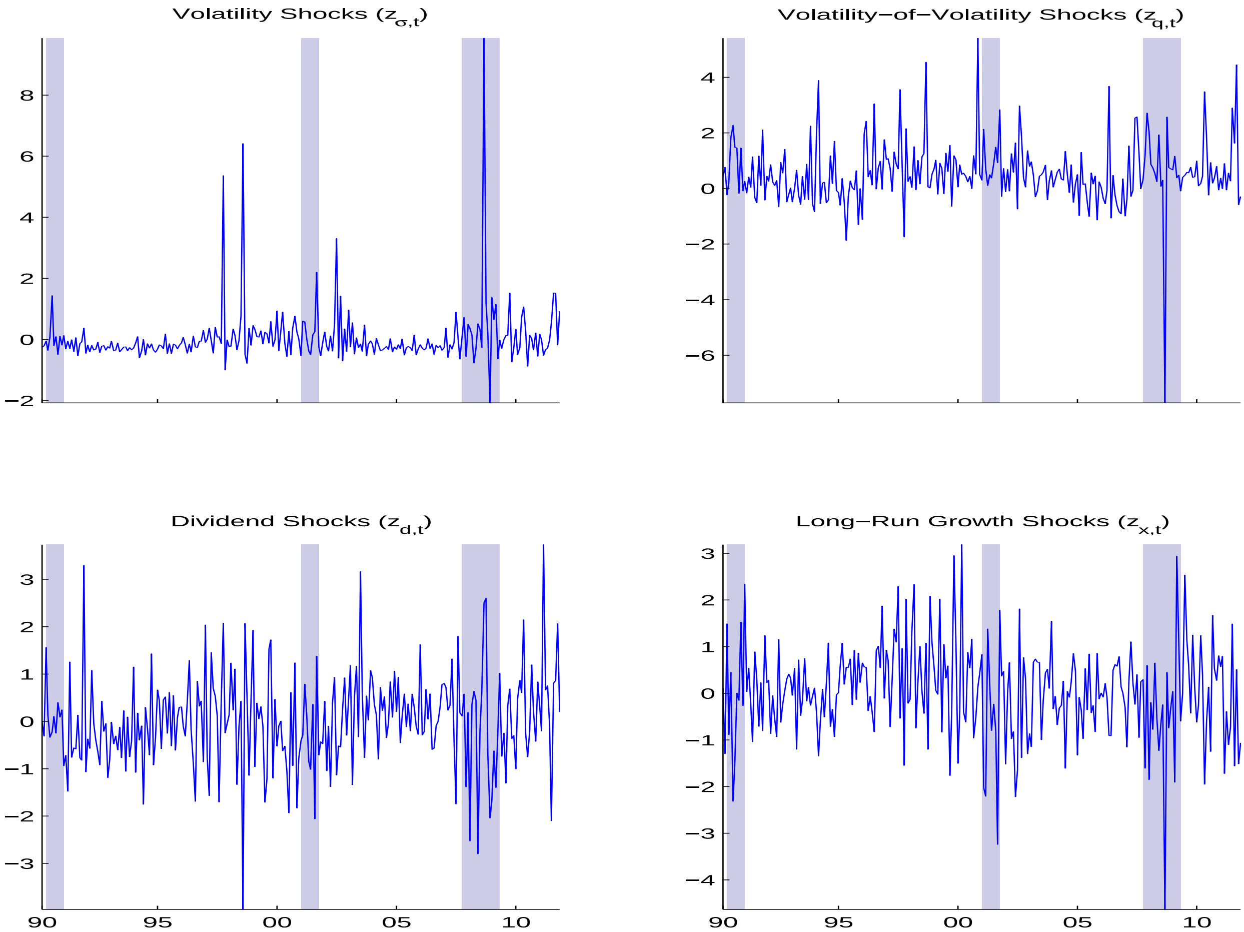

Figure 2 Model Implied Structural Shocks

The figure plots the estimated "structural" shocks z_t from the factor GARCH model discussed in the main text. The sample period extends from February 1990 to November 2011. The shaded areas indicate NBER dated recessions.

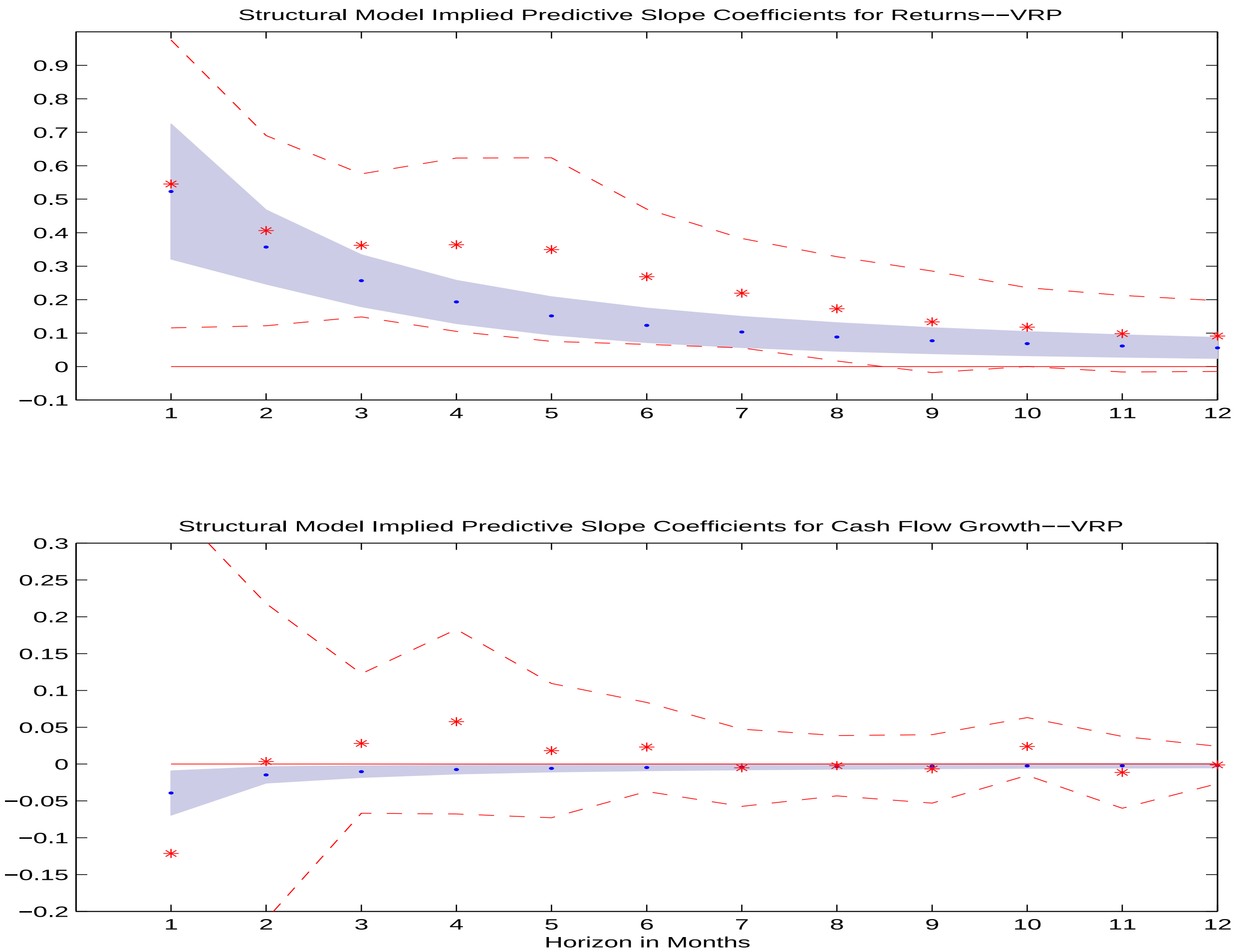

Figure 3 Predictive Regressions based on the Variance Risk Premium

The figure shows the "structural" factor GARCH model implied slope coefficients (dots) for 1-12 months return predictability regressions (upper panel) and cash flow predictability regressions (lower panel) using the variance risk premium as a predictor variable, along with 95% confidence intervals (shaded areas). The figure also shows the estimated slope coefficients from simple univariate predictability regressions using the variance risk premium as a predictor variable (stars), along with their 95% confidence intervals (dashed lines). All of the estimates are based on monthly data from February 1990 to November 2011.

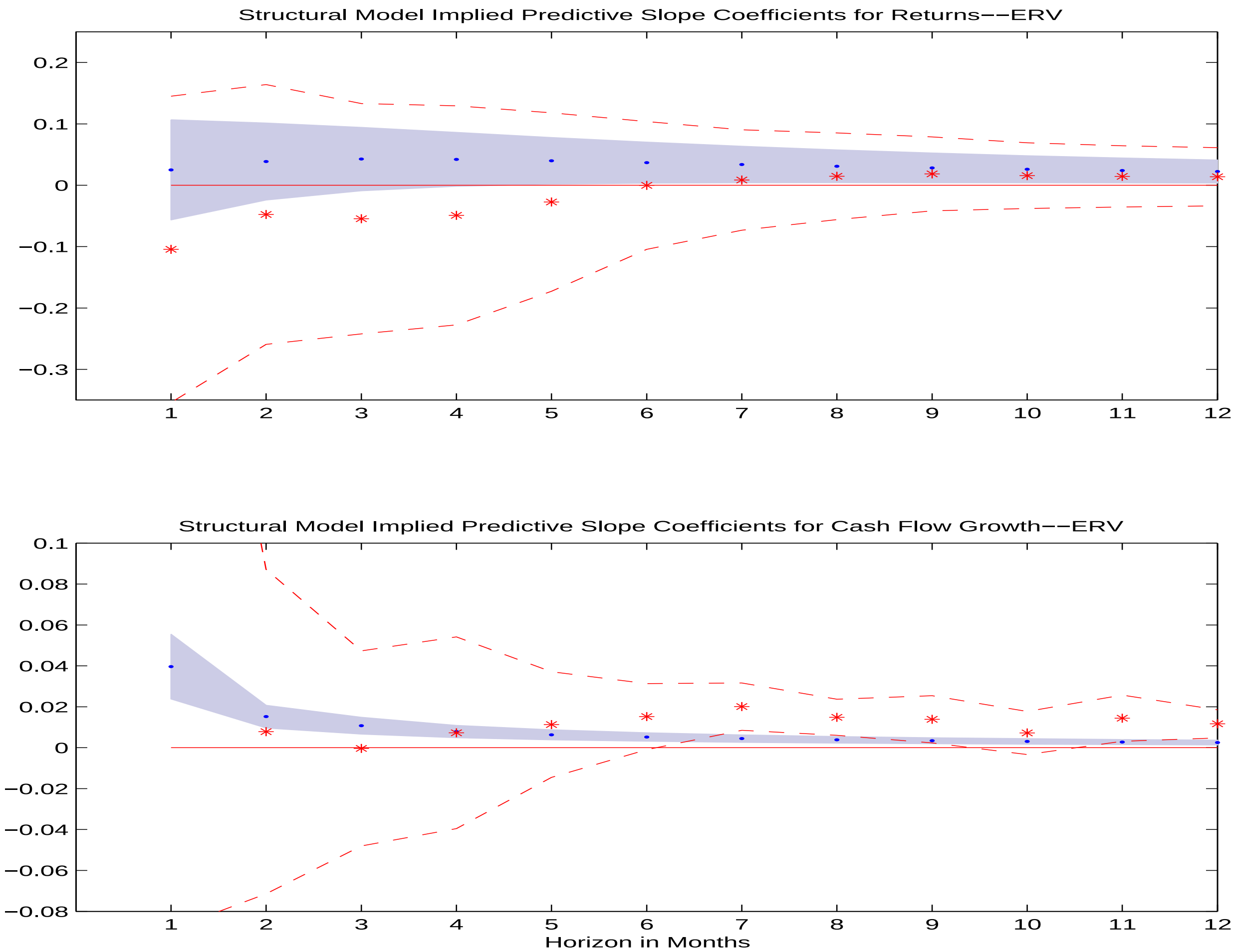

Figure 4 Predictive Regressions based on the Expected Variation

The figure shows the "structural" factor GARCH model implied slope coefficients (dots) for 1-12 months return predictability regressions (upper panel) and cash flow predictability regressions (lower panel) using the expected variation as a predictor variable, along with 95% confidence intervals (shaded areas). The figure also shows the estimated slope coefficients from simple univariate predictability regressions using the expected variation as a predictor variable (stars), along with their 95% confidence intervals (dashed red lines). All of the estimates are based on monthly data from February 1990 to November 2011.

A Model Solution

Our basic solution method for the model is adopted from Bansal and Yaron (2004), Bansal, Kiku, and Yaron (2007b), and Drechsler and Yaron (2011). To begin, we follow Campbell and Shiller (1988b) and solve for the return on consumption by log-linearizing $r_{c,t+1}$ around the unconditional mean of the wealth-consumption ratio v_t,

$$r_{c,t+1} \approx \kappa_0 + \kappa_1 v_{t+1} - v_t + \Delta c_{t+1}, \tag{A.1}$$

where $\kappa_1 = \frac{\exp(E(v))}{1+\exp(E(v))}$, and $\kappa_0 = \log[1 + \exp(E(v))] - \kappa_1 E(v)$. We then conjecture a solution for v_t as a linear function of the state vector Y_t,

$$v_t = A_0 + A' Y_t, \tag{A.2}$$

where A_0 is a scalar, and $A = (0, A_x, A_\sigma, A_q, 0)$ refer to the pricing coefficients. Next, by substituting v_t and v_{t+1} into equation (A.1), both $r_{c,t+1}$ and the stochastic discount factor m_{t+1} in equation (7) may be expressed as linear functions of the state vector,

$$m_{t+1} = \mu_m - (\gamma e_1' + (1-\theta)\kappa_1 A') Y_{t+1} - (\theta - 1) A' Y_t, \tag{A.3}$$

$$r_{c,t+1} = \mu_{r_c} + (e_1' + \kappa_1 A') Y_{t+1} - A' Y_t. \tag{A.4}$$

Going one step further, it follows that the innovations to the pricing kernel and the return on the wealth claim may be expressed as,

$$m_{t+1} - E_t(m_{t+1}) = -\Lambda' H G_t z_{t+1}, \tag{A.5}$$

$$r_{c,t+1} - E_t(r_{c,t+1}) = \Lambda_c' H G_t z_{t+1}, \tag{A.6}$$

where Λ denotes the price of risk for the factor shocks,

$$\Lambda = \gamma e_1 + \kappa_1 (1-\theta) A,$$

for $e_1 \equiv [1, 0, 0, 0, 0]$, and $\Lambda_c = e_1 + \kappa_1 A$. The magnitude and sign of Λ are determined by the preference parameter θ and the pricing coefficient vector A. If investors prefer early resolution of uncertainty, i.e., $\gamma > \phi^{-1}$, A reveals the sensitivity of the market prices for the different shocks to higher order consumption dynamics. When $\gamma = \phi^{-1}$ (CRRA case), Λ collapses to γe_1, and only transient shocks to consumption growth level $z_{g,t+1}$ are priced.

Since the no-arbitrage condition must hold regardless of the realization of the state vector Y_t, it is possible to solve for A by imposing the Euler equation,

$$0 = \mu_m + \mu_{r_c} + [(-\Lambda + \Lambda_c)' F - \theta A'] Y_t + \frac{1}{2}(-\Lambda + \Lambda_c)' H G_t G_t' H' (-\Lambda + \Lambda_c). \tag{A.7}$$

This in turn implies that

$$\theta A_{\{i\}} + (\tilde{\Lambda}_c' F)_{\{i\}} = \frac{1}{2}[1_{i=3} \sum_{j=1,5} (\tilde{\Lambda}_c' h_j)^2 + 1_{i=4} \sum_{j=2,3,4} (\tilde{\Lambda}_c' h_j)^2], \tag{A.8}$$

$$0 = \mu_m + \mu_{r_c}, \tag{A.9}$$

where $\tilde{\Lambda}_c = -\Lambda_c + \Lambda = (\gamma - 1)e_1 - \kappa_1 \theta A$, i refers to the i^{th} element of vector, and $1_{i=n}$ is an indicator function. The solutions are,

$$A_x = -\frac{\gamma - 1}{\theta(1 - \kappa_1 \rho_x)}, \tag{A.10}$$

$$A_\sigma = \frac{(\gamma - 1)^2}{2\theta(1 - \kappa_1 \rho_\sigma)}, \tag{A.11}$$

while A_q solves the equation $\frac{1}{2} a_q \theta^2 A_q^2 + \left(b_q + (1 - \kappa_1 \rho_q)\right)(-\theta A_q) + \frac{1}{2} c_q = 0$, where

$$\begin{aligned} a_q &= \kappa_1^2 (\varphi_x^2 s_{q,x}^2 + s_{q,\sigma}^2 + \varphi_q^2) > 0, \\ b_q &= \kappa_1^2 \left(\varphi_x^2 (-A_x \theta - A_\sigma \theta s_{\sigma,x}) s_{q,x} - A_\sigma \theta s_{q,\sigma}\right), \\ c_q &= \kappa_1^2 \left(\varphi_x^2 (-A_x \theta - A_\sigma \theta s_{\sigma,x})^2 + A_\sigma^2 \theta^2\right) > 0. \end{aligned}$$

Since $a_q > 0$ and $c_q > 0$, the two roots are either negative or positive. We choose the larger root for $-\theta A_q$ if $b_q + (1 - \kappa_1 \rho_q) > 0$, or the smaller root if $b_q + (1 - \kappa_1 \rho_q) < 0$. In both cases A_q reduces to zero when $s_{q,x}$, $s_{q,\sigma}$ and φ_q are zero.

Even though no closed-form expressions for A are available when we consider κ_0 and κ_1 as endogenous, the system of equations are still solvable. As shown in equation (A.8), A depends on κ_1, μ, F, H, as well as the preference parameters. Considering the definitions of κ_1 and κ_0, κ_1 and A are the only unknowns in the constant term in the Euler equation, so that κ_1 may be solved endogenously together with A. Finally, κ_0 and A_0 can be expressed as functions of A and κ_1. For detailed numerical solutions, see Drechsler and Yaron (2011) Appendix A.1 and A.2.

Applying a similar conjecture-evaluation type method, it is possible to solve for the aggregate market return $r_{t,t+1}$. Denote the price-dividend ratio by w_t, and consider the conjecture solution $w_t = A_{d,0} + A_d' Y_t$. Log-linearize $r_{t,t+1}$ around the unconditional mean of the price-dividend ratio the yields,

$$r_{t,t+1} \approx \kappa_{d,0} + \kappa_{d,1} w_{t+1} - w_t + \Delta d_{t+1}. \tag{A.12}$$

Substituting out w_t and w_{t+1} in the above equation, the return on the market may be rewritten as,

$$r_{t,t+1} = \mu_{r_d} + (e_5' + \kappa_{d,1} A_d') Y_{t+1} - A_d' Y_t, \tag{A.13}$$

where $\Lambda_d = e_5 + \kappa_{d,1} A_d$ and $A_d = [0, A_{d,x}, A_{d,\sigma}, A_{d,q}, A_{d,d}]'$ is a vector of pricing coefficients.

Using the same solution method as the one previously used for A, it follows by the no-arbitrage condition,

$$0 = \mu_m + \mu_{r_d} + [(-\Lambda + \Lambda_d)'F - (\theta - 1)A' - A_d']'Y_t + 0.5(-\Lambda + \Lambda_d)'HG_tG_t'H'(-\Lambda + \Lambda_d), \quad (A.14)$$

which implies that

$$(\theta - 1)A_{\{i\}} + A_{d,\{i\}} + (\tilde{\Lambda}_d'F)_{\{i\}} = 0.5[1_{i=3} \sum_{j=1,5} (\tilde{\Lambda}_d'h_j)^2 + 1_{i=4} \sum_{j=2,3,4} (\tilde{\Lambda}_d'h_j)^2], \quad (A.15)$$

$$0 = \mu_m + \mu_{r_d}, \quad (A.16)$$

where $\tilde{\Lambda}_d = -\Lambda_d + \Lambda = \gamma e_1 - e_5 + \kappa_1(1 - \theta)A - \kappa_{d,1}A_d$.

The solution for A_d may therefore be expressed as,

$$A_{d,d} = \frac{\rho_d}{1 - \kappa_{d,1}\rho_d} \quad (A.17)$$

$$\frac{1 - \kappa_1\rho_x}{1 - \kappa_{d,1}\rho_x}(1 - \theta)A_x - A_{d,x} = -\frac{-\gamma + \phi_{dx}(1 + \kappa_{d,1}A_{d,d})}{1 - \kappa_{d,1}\rho_x} \quad (A.18)$$

$$\frac{1 - \kappa_1\rho_\sigma}{1 - \kappa_{d,1}\rho_\sigma}(1 - \theta)A_\sigma - A_{d,\sigma} = -\frac{1}{2}\frac{\gamma^2 + \varphi_d^2(1 + \kappa_{d,1}A_{d,d})^2}{1 - \kappa_{d,1}\rho_\sigma} < 0 \quad (A.19)$$

$$\frac{1 - \kappa_1\rho_q}{1 - \kappa_{d,1}\rho_q}(1 - \theta)A_q - A_{d,q} = -\frac{1}{2}\frac{a_{d,q}\left(\kappa_1(1 - \theta)A_q - \kappa_{d,1}A_{d,q}\right)^2 + 2b_{d,q}\left(\kappa_1(1 - \theta)A_q - \kappa_{d,1}A_{d,q}\right) + c_{d,q}}{1 - \kappa_{d,1}\rho_q} \quad (A.20)$$

where

$$\begin{aligned}
a_{d,q} =& a_q = (\varphi_x^2 s_{q,x}^2 + s_{q,\sigma}^2 + \varphi_q^2) > 0 \\
b_{d,q} =& \left(\varphi_x^2\left(\kappa_1(1 - \theta)A_x - \kappa_{d,1}A_{d,x} + (\kappa_1(1 - \theta)A_\sigma - \kappa_{d,1}A_{d,\sigma})\, s_{\sigma,x} + \frac{-1}{1 - \kappa_{d,1}\rho_d}s_{d,x}\right)s_{q,x}\right) \\
&+ \left(\left(\kappa_1(1 - \theta)A_\sigma - \kappa_{d,1}A_{d,\sigma} + \frac{-1}{1 - \kappa_{d,1}\rho_d}s_{d,\sigma}\right)s_{q,\sigma} + \frac{-1}{1 - \kappa_{d,1}\rho_d}s_{d,q}\varphi_q^2\right) \\
c_{d,q} =& \left(\varphi_x^2\left(\kappa_1(1 - \theta)A_x - \kappa_{d,1}A_{d,x} + (\kappa_1(1 - \theta)A_\sigma - \kappa_{d,1}A_{d,\sigma})\, s_{\sigma,x} + \frac{-1}{1 - \kappa_{d,1}\rho_d}s_{d,x}\right)^2\right) \\
&+ \left(\left(\kappa_1(1 - \theta)A_\sigma - \kappa_{d,1}A_{d,\sigma} + \frac{-1}{1 - \kappa_{d,1}\rho_d}s_{d,\sigma}\right)^2 + \left(\frac{-1}{1 - \kappa_{d,1}\rho_d}s_{d,q}\right)^2\varphi_q^2\right) > 0
\end{aligned}$$

In other words, $A_{d,q}$ solves the equation (A.20) and we choose the root with smaller absolute value. Both A_q and $A_{d,q}$ reduce to zero when $s_{q,x}$, $s_{q,\sigma}$ and φ_q are all zero. We will discuss the sign of $A_{d,q}$ later on in the parameter implication section. We can explicitly express $A_{d,x}$ and $A_{d,\sigma}$ as

$$A_{d,x} = \frac{(1 - \gamma)/\theta - 1 + \phi_{dx}/(1 - \kappa_{d,1}\rho_d)}{1 - \kappa_{d,1}\rho_x} = \frac{-\psi^{-1} + \phi_{dx}/(1 - \kappa_{d,1}\rho_d)}{1 - \kappa_{d,1}\rho_x} \quad (A.21)$$

$$A_{d,\sigma} = \frac{(\gamma - 1)^2 + 2\theta\gamma + \theta(\varphi_d^2/(1 - \kappa_{d,1}\rho_d)^2 - 1)}{2\theta(1 - \kappa_{d,1}\rho_\sigma)}. \quad (A.22)$$

B Variance Risk Premium

In order to determine the factor structure for the variance risk premium, we first need to solve for the second order moment of the return $r_{t,t+1}$. It follows from above that $r_{t,t+1} - E_t(r_{t,t+1}) = \Lambda'_d H G_t z_{t+1}$, so that the conditional variance of the return is affine in σ_t^2 and q_t,

$$
\begin{aligned}
Var_t(r_{t,t+1}) &= \textstyle\sum_{j=1,5} \Lambda'_d h_j h'_j \Lambda_d \sigma_t^2 + \sum_{j=2,3,4} \Lambda'_d h_j h'_j \Lambda_d q_t \\
&= (1+\kappa_{d,1} A_{d,d})^2 \varphi_d^2 \sigma_t^2 + \textstyle\sum_{j=2,3,4} \Lambda'_d h_j h'_j \Lambda_d q_t. \qquad \text{(B.23)}
\end{aligned}
$$

The first term is associated with the volatility of cash flow shocks, and the second term represents the consumption uncertainty. Accordingly, the equity risk premium may be expressed as,

$$
\begin{aligned}
\log(E_t R_{t,t+1}) - r_{f,t} &= E_t(r_{t,t+1}) + \frac{1}{2} Var_t(r_{t,t+1}) - r_{f,t} \qquad \text{(B.24)} \\
&= -Cov_t(m_{t+1}, r_{t,t+1}) = \sum_{j=2,3,4} \Lambda'_d h_j h'_j \Lambda q_t
\end{aligned}
$$

The first equality comes from the normality distribution of $r_{t,t+1} \equiv \log(R_{t,t+1})$, the second equality comes from the no arbitrage condition and $r_{f,t} = \log(E_t m_{t+1} + \frac{1}{2} Var_t(m_{t+1}))$. The expectations of $Var_t(r_{t,t+1})$ under the physical and risk-neutral probability measures are,

$$
E_t(Var_{t+1}(r_{d,t+2})) = \sum_{j=1,5} \Lambda'_d h_j h'_j \Lambda_d (\mu_\sigma + \rho_\sigma \sigma_t^2) + \sum_{j=2,3,4} \Lambda'_d h_j h'_j \Lambda_d (\mu_q + \rho_q q_t), \qquad \text{(B.25)}
$$

$$
E_t^Q(Var_{t+1}(r_{d,t+2})) = \sum_{j=1,5} \Lambda'_d h_j h'_j \Lambda_d (\mu_\sigma + \rho_\sigma \sigma_t^2 + s_{q,1} q_t) + \sum_{j=2,3,4} \Lambda'_d h_j h'_j \Lambda_d (\mu_q + \rho_q q_t + s_{q,2} q_t). \qquad \text{(B.26)}
$$

Under the risk neutral measure, we reweight probabilities according to the pricing kernel $\frac{e^{m_{t,t+1}}}{E_t e^{m_{t,t+1}}}$. If investor prefers early resolution of uncertainty, the shocks z_{t+1}'s conditional mean shifts away from zero. And this shift can be expressed as the conditional covariance between the state vector and SDF $m_{t,t+1}$,

$$
\begin{aligned}
s_{q,1} q_t &= Cov_t(e_3 H G_t z_{t+1}, -\Lambda' H G_t z_{t+1}) \\
&= -(\varphi_x s_{\sigma,x} h'_2 + h'_3) \Lambda q_t, \qquad \text{(B.27)} \\
s_{q,2} q_t &= Cov_t(e_4 H G_t z_{t+1}, -\Lambda' H G_t z_{t+1}) \\
&= -(\varphi_x s_{q,x} h'_2 + s_{q,\sigma} h'_3 + \varphi_q h'_4) \Lambda q_t, \qquad \text{(B.28)}
\end{aligned}
$$

where,

$$
\begin{aligned}
s_{q,1} =& -\kappa_1(1-\theta)\left(A_x\varphi_x^2 s_{\sigma,x} + A_\sigma(\varphi_x^2 s_{\sigma,x}^2 + 1) + A_q(\varphi_x^2 s_{\sigma,x}s_{q,x} + s_{q,\sigma})\right)\\
=& -\kappa_1(1-\theta)(\varphi_x^2 s_{\sigma,x}s_{q,x} + s_{q,\sigma})\left(\frac{A_x\varphi_x^2 s_{\sigma,x} + A_\sigma(\varphi_x^2 s_{\sigma,x}^2 + 1)}{\varphi_x^2 s_{\sigma,x}s_{q,x} + s_{q,\sigma}} + A_q\right)\\
s_{q,2} =& -\kappa_1(1-\theta)\left(A_x\varphi_x^2 s_{q,x} + A_\sigma(\varphi_x^2 s_{\sigma,x}s_{q,x} + s_{q,\sigma}) + A_q(\varphi_x^2 s_{q,x}^2 + s_{q,\sigma}^2 + \varphi_q^2)\right)\\
=& -1/\kappa_1(1-\theta)a_q\left(\frac{b_q}{-\theta a_q} + A_q\right)
\end{aligned}
$$

By definition, $s_{q,1}$ and $s_{q,2}$ represent the market prices of shocks to σ_t^2 and q_t, respectively. Thus, the variance risk premium is naturally defined by,

$$
\begin{aligned}
VRP_t \equiv \quad & E_t^Q(Var_{t+1}(r_{d,t+2})) - E_t(Var_{t+1}(r_{d,t+2}))\\
= \quad & (\textstyle\sum_{j=1,5}\Lambda_d' h_j h_j' \Lambda_d s_{q,1} + \sum_{j=2,3,4}\Lambda_d' h_j h_j' \Lambda_d s_{q,2})q_t.
\end{aligned}
\tag{B.29}
$$

In the main text, we will refer to the expected return variation and the variance risk premium as,

$$
\begin{aligned}
ERV_t =& \frac{Q_{1,1}}{\rho_\sigma}(\mu_\sigma + \rho_\sigma\sigma_t^2) + \frac{Q_{1,2}}{\rho_q}(\mu_q + \rho_q q_t),\\
VRP_t =& Q_{2,2}q_t
\end{aligned}
$$

for short, where

$$Q_{1,1} = \quad \textstyle\sum_{j=1,5}\Lambda_d' h_j h_j' \Lambda_d \rho_\sigma > 0, \tag{B.30}$$

$$Q_{1,2} = \quad \textstyle\sum_{j=2,3,4}\Lambda_d' h_j h_j' \Lambda_d \rho_q > 0, \tag{B.31}$$

$$Q_{2,2} = \quad \tfrac{Q_{1,1}}{\rho_\sigma}s_{q,1} + \tfrac{Q_{1,2}}{\rho_q}s_{q,2}. \tag{B.32}$$

In order to determine the signs of $A_{d,x}$, $A_{d,\sigma}$ and $A_{d,q}$, it is informative to write out the formula in terms of the estimated B and $\tilde{\rho}$ matrices,

$$\frac{\phi_{d,x}}{-A_{d,x}} = \tilde{\rho}_{3,4}, \qquad \frac{A_{d,\sigma}}{Q_{1,1}} = B_{4,1}, \qquad \frac{Q_{1,2}}{Q_{2,2}} = -B_{1,2}, \qquad \frac{A_{d,q}}{Q_{2,2}} = B_{4,2} - B_{1,2}B_{4,1}. \tag{B.33}$$

Since $\tilde{\rho}_{3,4} < 0$, $\phi_{d,x}$ and $A_{d,x}$ must have the same signs. Thus, by definition $Q_{1,1} > 0$ and $Q_{1,2} > 0$, which together with the estimates for $B_{4,1} = -0.60 < 0$ and $B_{1,2} = -0.02 < 0$, imply that $A_{d,\sigma} < 0$ and $Q_{2,2} > 0$. Consequently $A_{d,q} = Q_{2,2}(B_{4,2} - B_{1,2}B_{4,1}) = -1.45Q_{2,2} < 0$.

C Alternative Setups

C.1 Separate Volatility Processes

We will consider the following alternative setup for G_t and H, with F unchanged,

$$G_t = \begin{pmatrix} \sigma_t & 0 & 0 & 0 & 0 \\ 0 & \sqrt{q_t} & 0 & 0 & 0 \\ 0 & 0 & \sigma_t & 0 & 0 \\ 0 & 0 & 0 & \sqrt{q_t} & 0 \\ 0 & 0 & 0 & 0 & \sigma_t \end{pmatrix} \qquad H = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & s_{\sigma,x} & \varphi_\sigma & 0 & 0 \\ 0 & s_{q,x} & 0 & \varphi_q & 0 \\ 0 & s_{d,x} & \varphi_\sigma s_{d,\sigma} & \varphi_q s_{d,q} & \varphi_d \end{pmatrix} \tag{C.34}$$

This setup is related to Bansal and Shaliastovich (2012), where the volatilities of x_t and σ_t^2 are modeled as two separate processes.

For simplicity, we use the same general notation as in the main setup for A, Λ, Λ_c and Λ_d. However, the solutions for the pricing coefficients are obviously different from the main setup, except for $A_{d,d}=\frac{\rho_d}{1-\kappa_{d,1}\rho_d}$,

$$\theta A_{\{i\}} + (\tilde{\Lambda}_c' F)_{\{i\}} = 0.5[1_{i=3} \textstyle\sum_{j=1,3,5} (\tilde{\Lambda}_c' h_j)^2 + 1_{i=4} \sum_{j=2,4} (\tilde{\Lambda}_c' h_j)^2], \tag{C.35}$$

$$(\theta - 1)A_{\{i\}} + A_{d,\{i\}} + (\tilde{\Lambda}_d' F)_{\{i\}} = 0.5[1_{i=3} \textstyle\sum_{j=1,3,5} (\tilde{\Lambda}_d' h_j)^2 + 1_{i=4} \sum_{j=2,4} (\tilde{\Lambda}_d' h_j)^2]. \tag{C.36}$$

Since $r_{t,t+1} - E_t(r_{t,t+1})=\Lambda_d' HG_t z_{t+1}$, the conditional variance of the return is again affine,

$$Var_t(r_{t,t+1}) = \textstyle\sum_{j=1,3,5} \Lambda_d' h_j h_j' \Lambda_d \sigma_t^2 + \sum_{j=2,4} \Lambda_d' h_j h_j' \Lambda_d q_t \tag{C.37}$$

The expectations of $Var_t(r_{t,t+1})$ under the physical and risk-neutral probability measures may further be expressed as,

$$E_t(Var_{t+1}(r_{d,t+2})) = \sum_{j=1,3,5} \Lambda_d' h_j h_j' \Lambda_d (\mu_\sigma + \rho_\sigma \sigma_t^2) + \sum_{j=2,4} \Lambda_d' h_j h_j' \Lambda_d (\mu_q + \rho_q q_t) \tag{C.38}$$

$$E_t^Q(Var_{t+1}(r_{d,t+2})) = \sum_{j=1,3,5} \Lambda_d' h_j h_j' \Lambda_d (\mu_\sigma + \rho_\sigma \sigma_t^2 + s_{q,1} q_t + s_{\sigma,1} \sigma_t^2) + \sum_{j=2,4} \Lambda_d' h_j h_j' \Lambda_d (\mu_q + \rho_q q_t + s_{q,2} q_t) \tag{C.39}$$

If investors prefer early resolution of uncertainty, the conditional means of the z_{t+1} shocks shift away from zero under the risk-neutral measure,

$$\begin{aligned} s_{\sigma,1}\sigma_t^2 + s_{q,1} q_t &= Cov_t(e_3' HG_t z_{t+1}, -\Lambda' HG_t z_{t+1}) \\ &= -\varphi_\sigma h_3' \Lambda \sigma_t^2 - (\varphi_x s_{\sigma,x} h_2') \Lambda q_t, \end{aligned} \tag{C.40}$$

$$\begin{aligned} s_{q,2} q_t &= Cov_t(e_4 HG_t z_{t+1}, -\Lambda' HG_t z_{t+1}) \\ &= -(\varphi_x s_{q,x} h_2' + \varphi_q h_4') \Lambda q_t. \end{aligned} \tag{C.41}$$

Defining the variance risk premium as before,

$$\begin{aligned}VRP_t \equiv & E_t^Q(Var_{t+1}(r_{d,t+2})) - E_t(Var_{t+1}(r_{d,t+2})) \\ = & \sum_{j=1,3,5} \Lambda_d' h_j h_j' \Lambda_d (s_{\sigma,1}\sigma_t^2 + s_{q,1}q_t) + \sum_{j=2,4} \Lambda_d' h_j h_j' \Lambda_d s_{q,2} q_t,\end{aligned} \tag{C.42}$$

we may express the expected return variation and premium in short-hand form as,

$$\begin{aligned}ERV_t = & \frac{Q_{1,1}}{\rho_\sigma}(\mu_\sigma + \rho_\sigma \sigma_t^2) + \frac{Q_{1,2}}{\rho_q}(\mu_q + \rho_q q_t), \\ VRP_t = & Q_{2,1}\sigma_t^2 + Q_{2,2} q_t,\end{aligned}$$

where

$$\begin{aligned}Q_{1,1} = & \sum_{j=1,3,5} \Lambda_d' h_j h_j' \Lambda_d \rho_\sigma > 0 & \qquad Q_{1,2} = & \sum_{j=2,4} \Lambda_d' h_j h_j' \Lambda_d \rho_q > 0 \\ Q_{2,1} = & \frac{Q_{1,1}}{\rho_q} s_{\sigma,1} & \qquad Q_{2,2} = & \frac{Q_{1,1}}{\rho_\sigma} s_{q,1} + \frac{Q_{1,2}}{\rho_q} s_{q,2}.\end{aligned}$$

C.2 Long-Run Stochastic Volatility

We will consider the following alternative setup for G_t, H, and F,

$$F = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & \rho_x & 0 & 0 & 0 \\ 0 & 0 & \rho_\sigma & 1 & 0 \\ 0 & 0 & 0 & \rho_q & 0 \\ 0 & \phi_{dx} & 0 & 0 & \rho_d \end{pmatrix} \quad G_t = \begin{pmatrix} \sigma_t & 0 & 0 & 0 & 0 \\ 0 & \sigma_t & 0 & 0 & 0 \\ 0 & 0 & \sigma_t & 0 & 0 \\ 0 & 0 & 0 & \sqrt{q_t} & 0 \\ 0 & 0 & 0 & 0 & \sigma_t \end{pmatrix} \quad H = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \varphi_x & 0 & 0 & 0 \\ 0 & \varphi_x s_{\sigma,x} & \varphi_\sigma & 0 & 0 \\ 0 & \varphi_x s_{q,x} & 0 & \varphi_q & 0 \\ 0 & \varphi_x s_{d,x} & \varphi_\sigma s_{d,\sigma} & \varphi_q s_{d,q} & \varphi_d \end{pmatrix} \tag{C.43}$$

This setup is motivated by the model analyzed by Branger and Vòlkert (2012), among others, allowing for a time-varying mean of the consumption variance σ_t^2.

Again, for simplicity we will use the same general notation as in the main setup for A, Λ, Λ_c and Λ_d. The solution for $A_{d,d} = \frac{\rho_d}{1-\kappa_{d,1}\rho_d}$ remains the same, but the other the pricing coefficients now take the form,

$$\theta A_{\{i\}} + (\tilde{\Lambda}_c' F)_{\{i\}} = \frac{1}{2}[1_{i=3} \sum_{j=1,2,3,5} (\tilde{\Lambda}_c' h_j)^2 + 1_{i=4} \sum_{j=4} (\tilde{\Lambda}_c' h_j)^2], \tag{C.44}$$

$$(\theta - 1)A_{\{i\}} + A_{d,\{i\}} + (\tilde{\Lambda}_d' F)_{\{i\}} = \frac{1}{2}[1_{i=3} \sum_{j=1,2,3,5} (\tilde{\Lambda}_d' h_j)^2 + 1_{i=4} \sum_{j=4} (\tilde{\Lambda}_d' h_j)^2]. \tag{C.45}$$

As before, $r_{t,t+1} - E_t(r_{t,t+1}) = \Lambda_d' H G_t z_{t+1}$, so that the conditional variance of the return may be expressed as,

$$Var_t(r_{t,t+1}) = \sum_{j=1,2,3,5} \Lambda_d' h_j h_j' \Lambda_d \sigma_t^2 + \sum_{j=4} \Lambda_d' h_j h_j' \Lambda_d q_t. \tag{C.46}$$

The expectation of $Var_t(r_{t,t+1})$ under the physical and risk-neutral probability measures are,

$$E_t(Var_{t+1}(r_{d,t+2})) = \sum_{j=1,2,3,5} \Lambda_d' h_j h_j' \Lambda_d (\mu_\sigma + \rho_\sigma \sigma_t^2 + q_t) + \sum_{j=4} \Lambda_d' h_j h_j' \Lambda_d (\mu_q + \rho_q q_t),$$

$$E_t^Q(Var_{t+1}(r_{d,t+2})) = \sum_{j=1,2,3,5} \Lambda_d' h_j h_j' \Lambda_d (\mu_\sigma + \rho_\sigma \sigma_t^2 + q_t + s_{\sigma,1}\sigma_t^2) + \sum_{j=4} \Lambda_d' h_j h_j' \Lambda_d (\mu_q + \rho_q q_t + s_{\sigma,2}\sigma_t^2 + s_{q,2} q_t).$$

The shifts in the conditional means of the z_{t+1} shocks under the risk-neutral measure become,

$$\begin{aligned} s_{\sigma,1}\sigma_t^2 &= Cov_t(e_3' H G_t z_{t+1}, -\Lambda' H G_t z_{t+1}) \\ &= -\varphi_\sigma h_3' \Lambda \sigma_t^2 - (\varphi_x s_{\sigma,x} h_2') \Lambda \sigma_t^2, \qquad \text{(C.47)} \\ s_{\sigma,2}\sigma_t^2 + s_{q,2} q_t &= Cov_t(e_4 H G_t z_{t+1}, -\Lambda' H G_t z_{t+1}) \\ &= -(\varphi_x s_{q,x} h_2') \Lambda \sigma_t^2 - (\varphi_q h_4') \Lambda q_t. \qquad \text{(C.48)} \end{aligned}$$

As before, the expected return variation and variance risk premium, may be conveniently expressed as,

$$\begin{aligned} ERV_t &= \frac{Q_{1,1}}{\rho_\sigma}(\mu_\sigma + \rho_\sigma \sigma_t^2) + \frac{Q_{1,2}}{\rho_q}(\mu_q + \rho_q q_t), \\ VRP_t &= Q_{2,1}\sigma_t^2 + Q_{2,2} q_t, \end{aligned}$$

where

$$\begin{aligned} Q_{1,1} &= \sum_{j=1,2,3,5} \Lambda_d' h_j h_j' \Lambda_d \rho_\sigma > 0 & \qquad Q_{1,2} &= \sum_{j=4} \Lambda_d h_j h_j' \Lambda_d \rho_q > 0 \\ Q_{2,1} &= \frac{Q_{1,1}}{\rho_\sigma} s_{\sigma,1} + \frac{Q_{1,2}}{\rho_q} s_{\sigma,2} & \qquad Q_{2,2} &= \frac{Q_{1,2}}{\rho_q} s_{q,2}. \end{aligned}$$

D Detailed Derivations for Section 3.2

Substituting f_t by $Q^{-1}(X_t - \mu_X)$ in the basic relation $f_{t+1} = \mu + \rho f_t + S\epsilon_{t+1}$, it follows that

$$Q^{-1}X_{t+1} = \mu + Q^{-1}\mu_X - \rho Q^{-1}\mu_X + \rho Q^{-1} X_t + S\epsilon_{t+1}. \qquad \text{(D.49)}$$

Normalizing each element of $Q^{-1}X_{t+1}$ by the corresponding diagonal element of Q^{-1}, the model may be rewritten as,

$$BX_{t+1} = \tilde{\mu} + \tilde{\rho} B X_t + \tilde{S}\tilde{\epsilon}_{t+1},$$

where

$$B \equiv \left(\frac{1}{\text{diag}(Q^{-1})} \otimes l_{1\times 4}\right) \odot Q^{-1}.$$

To match with equation (D.49),

$$\tilde{\mu} = \left(\frac{1}{\text{diag}(Q^{-1})} \otimes l_{1\times 4}\right) \odot (\mu - \rho Q^{-1}\mu_X),$$

and

$$\begin{aligned}
\tilde{\rho} =& \left[\left(\frac{1}{\text{diag}(Q^{-1})} \otimes l_{1\times 4}\right) \odot (\rho Q^{-1})\right] B^{-1} \\
=& \left[\left(\frac{1}{\text{diag}(Q^{-1})} \otimes l_{1\times 4}\right) \odot (Q^{-1}\text{diag}(\rho) + (\rho - \text{diag}(\rho))Q^{-1})\right] B^{-1} \\
=& \left[\left(\frac{1}{\text{diag}(Q^{-1})} \otimes l_{1\times 4}\right) \odot (Q^{-1} \odot (vec(\text{diag}(\rho)) \otimes l_{1\times 4}) + (\rho - \text{diag}(\rho))Q^{-1})\right] B^{-1} \\
=& \left[B \odot (vec(\text{diag}(\rho)) \otimes l_{1\times 4} + \frac{\rho - \text{diag}(\rho)}{-A_{d,x}} B\right] B^{-1} \\
=& \rho + \frac{\rho - \text{diag}(\rho)}{-A_{d,x}}
\end{aligned} \tag{D.50}$$

or

Defining $\tilde{\epsilon}_{t+1}$ as

$$\tilde{\epsilon}_{t+1} \equiv \frac{1}{\text{diag}(Q^{-1})} \odot \epsilon_{t+1},$$

it follows again from equation (D.49) that

$$\tilde{S} = \left(\frac{1}{\text{diag}(Q^{-1})} \otimes l_{1\times 4}\right) \odot S \odot \frac{1}{\frac{1}{\text{diag}(Q^{-1})'} \otimes l_{1\times 4}}.$$

Based on the formula for Q in the main text, the inverse Q^{-1} and $\frac{1}{\text{diag}(Q^{-1})}$ may be expressed as,

$$Q^{-1} = \begin{pmatrix} \frac{1}{Q_{1,1}} & \frac{-Q_{1,2}}{Q_{1,1}Q_{2,2}} & 0 & 0 \\ 0 & \frac{1}{Q_{2,2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{-A_{d,\sigma}}{Q_{1,1}A_{d,x}} & \frac{-Q_{1,1}A_{d,q}+Q_{1,2}A_{d,\sigma}}{Q_{1,1}Q_{2,2}A_{d,x}} & \frac{-A_{d,d}}{A_{d,x}} & -\frac{1}{A_{d,x}} \end{pmatrix} \qquad \frac{1}{\text{diag}(Q^{-1})} = \begin{pmatrix} Q_{1,1} \\ Q_{2,2} \\ 1 \\ -A_{d,x} \end{pmatrix}$$

Combining the expressions for ρ and S, it therefore follows that

$$B = \begin{pmatrix} 1 & -\frac{Q_{1,2}}{Q_{2,2}} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{A_{d,\sigma}}{Q_{1,1}} & \frac{Q_{1,1}A_{d,q}-A_{d,\sigma}Q_{1,2}}{Q_{1,1}Q_{2,2}} & A_{d,d} & 1 \end{pmatrix} \qquad \tilde{\rho} = \begin{pmatrix} \rho_\sigma & 0 & 0 & 0 \\ 0 & \rho_q & 0 & 0 \\ 0 & 0 & \rho_d & \frac{\phi_{dx}}{-A_{d,x}} \\ 0 & 0 & 0 & \rho_x \end{pmatrix}$$

$$\tilde{S} = \begin{pmatrix} 1 & 0 & 0 & \frac{Q_{1,1}}{-A_{d,x}} s_{\sigma,x} \\ \frac{Q_{2,2}}{Q_{1,1}} s_{q,\sigma} & 1 & 0 & \frac{Q_{2,2}}{-A_{d,x}} s_{q,x} \\ \frac{1}{Q_{1,1}} s_{d,\sigma} & \frac{1}{Q_{2,2}} s_{d,q} & 1 & \frac{1}{-A_{d,x}} s_{d,x} \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \tilde{\epsilon}_{t+1} = \begin{pmatrix} Q_{1,1} \\ Q_{2,2} \\ 1 \\ -A_{d,x} \end{pmatrix} \odot \epsilon_{t+1}.$$

D.1 Separate Volatility Dynamics

In the alternative setup with separate volatility dynamic, ρ, ϵ_{t+1} and S may be expressed as,

$$\rho = \begin{pmatrix} \rho_\sigma & 0 & 0 & 0 \\ 0 & \rho_q & 0 & 0 \\ 0 & 0 & \rho_d & \phi_{dx} \\ 0 & 0 & 0 & \rho_x \end{pmatrix} \qquad \epsilon_{t+1} = \begin{pmatrix} \varphi_\sigma \sigma_t z_{\sigma,t+1} \\ \varphi_q \sqrt{q_t} z_{q,t+1} \\ \varphi_d \sigma_t z_{d,t+1} \\ \sqrt{q_t} z_{x,t+1} \end{pmatrix} \qquad S = \begin{pmatrix} 1 & 0 & 0 & s_{\sigma,x} \\ 0 & 1 & 0 & s_{q,x} \\ s_{d,\sigma} & s_{d,q} & 1 & s_{d,x} \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{D.51}$$

$$X_t = \mu_X + Q f_t \qquad Q = \begin{pmatrix} Q_{1,1} & Q_{1,2} & 0 & 0 \\ Q_{2,1} & Q_{2,2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -A_{d,\sigma} & -A_{d,q} & -A_{d,d} & -A_{d,x} \end{pmatrix}$$

Consequently,,

$$Q^{-1} = \begin{pmatrix} \frac{Q_{2,2}}{Q_{1,1}Q_{2,2}-Q_{1,2}Q_{2,1}} & \frac{-Q_{1,2}}{Q_{1,1}Q_{2,2}-Q_{1,2}Q_{2,1}} & 0 & 0 \\ \frac{-Q_{2,1}}{Q_{1,1}Q_{2,2}-Q_{1,2}Q_{2,1}} & \frac{Q_{1,1}}{Q_{1,1}Q_{2,2}-Q_{1,2}Q_{2,1}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{Q_{2,1}A_{d,q}-Q_{2,2}A_{d,\sigma}}{(Q_{1,1}Q_{2,2}-Q_{1,2}Q_{2,1})A_{d,x}} & \frac{-Q_{1,1}A_{d,q}+Q_{1,2}A_{d,\sigma}}{(Q_{1,1}Q_{2,2}-Q_{1,2}Q_{2,1})A_{d,x}} & \frac{-A_{d,d}}{A_{d,x}} & -\frac{1}{A_{d,x}} \end{pmatrix} \qquad \frac{1}{\mathrm{diag}(Q^{-1})} = \begin{pmatrix} \frac{Q_{1,1}Q_{2,2}-Q_{1,2}Q_{2,1}}{Q_{2,2}} \\ \frac{Q_{1,1}Q_{2,2}-Q_{1,2}Q_{2,1}}{Q_{1,1}} \\ 1 \\ -A_{d,x} \end{pmatrix}$$

Combining these expressions, it follows that

$$B = \begin{pmatrix} 1 & \frac{-Q_{1,2}}{Q_{2,2}} & 0 & 0 \\ \frac{-Q_{2,1}}{Q_{1,1}} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{-Q_{2,1}A_{d,q}+Q_{2,2}A_{d,\sigma}}{Q_{1,1}Q_{2,2}-Q_{1,2}Q_{2,1}} & \frac{+Q_{1,1}A_{d,q}-Q_{1,2}A_{d,\sigma}}{Q_{1,1}Q_{2,2}-Q_{1,2}Q_{2,1}} & A_{d,d} & 1 \end{pmatrix} \qquad \tilde{\rho} = \begin{pmatrix} \rho_\sigma & 0 & 0 & 0 \\ 0 & \rho_q & 0 & 0 \\ 0 & 0 & \rho_d & \frac{\phi_{dx}}{-A_{d,x}} \\ 0 & 0 & 0 & \rho_x \end{pmatrix}$$

$$\tilde{S} = \begin{pmatrix} 1 & 0 & 0 & \frac{Q_{1,1}Q_{2,2}-Q_{1,2}Q_{2,1}}{-Q_{2,2}A_{d,x}} s_{\sigma,x} \\ 0 & 1 & 0 & \frac{Q_{1,1}Q_{2,2}-Q_{1,2}Q_{2,1}}{-Q_{1,1}A_{d,x}} s_{q,x} \\ \frac{Q_{2,2}}{Q_{1,1}Q_{2,2}-Q_{1,2}Q_{2,1}} s_{d,\sigma} & \frac{Q_{1,1}}{Q_{1,1}Q_{2,2}-Q_{1,2}Q_{2,1}} s_{d,q} & 1 & \frac{1}{-A_{d,x}} s_{d,x} \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \tilde{\epsilon}_{t+1} = \begin{pmatrix} \frac{Q_{1,1}Q_{2,2}-Q_{1,2}Q_{2,1}}{Q_{2,2}} \\ \frac{Q_{1,1}Q_{2,2}-Q_{1,2}Q_{2,1}}{Q_{1,1}} \\ 1 \\ -A_{d,x} \end{pmatrix} \odot \epsilon_{t+1}.$$

D.2 Stochastic Volatility in the Long-Run

In the alternative setup with stochastic volatility in the long-run drift, ρ, ϵ_{t+1} and S may be expressed as,

$$\rho = \begin{pmatrix} \rho_\sigma & 1 & 0 & 0 \\ 0 & \rho_q & 0 & 0 \\ 0 & 0 & \rho_d & \phi_{dx} \\ 0 & 0 & 0 & \rho_x \end{pmatrix} \qquad \epsilon_{t+1} = \begin{pmatrix} \varphi_\sigma \sigma_t z_{\sigma,t+1} \\ \varphi_q \sqrt{q_t} z_{q,t+1} \\ \varphi_d \sigma_t z_{d,t+1} \\ \varphi_x \sigma_t z_{x,t+1} \end{pmatrix} \qquad S = \begin{pmatrix} 1 & 0 & 0 & s_{\sigma,x} \\ 0 & 1 & 0 & s_{q,x} \\ s_{d,\sigma} & s_{d,q} & 1 & s_{d,x} \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{D.52}$$

$$X_t = \mu_X + Qf_t \qquad Q = \begin{pmatrix} Q_{1,1} & Q_{1,2} & 0 & 0 \\ Q_{2,1} & Q_{2,2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -A_{d,\sigma} & -A_{d,q} & -A_{d,d} & -A_{d,x} \end{pmatrix}$$

Consequently,

$$Q^{-1} = \begin{pmatrix} \frac{Q_{2,2}}{Q_{1,1}Q_{2,2}-Q_{1,2}Q_{2,1}} & \frac{-Q_{1,2}}{Q_{1,1}Q_{2,2}-Q_{1,2}Q_{2,1}} & 0 & 0 \\ \frac{-Q_{2,1}}{Q_{1,1}Q_{2,2}-Q_{1,2}Q_{2,1}} & \frac{Q_{1,1}}{Q_{1,1}Q_{2,2}-Q_{1,2}Q_{2,1}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{Q_{2,1}A_{d,q}-Q_{2,2}A_{d,\sigma}}{(Q_{1,1}Q_{2,2}-Q_{1,2}Q_{2,1})A_{d,x}} & \frac{-Q_{1,1}A_{d,q}+Q_{1,2}A_{d,\sigma}}{(Q_{1,1}Q_{2,2}-Q_{1,2}Q_{2,1})A_{d,x}} & \frac{-A_{d,d}}{A_{d,x}} & -\frac{1}{A_{d,x}} \end{pmatrix} \qquad \frac{1}{\mathrm{diag}(Q^{-1})} = \begin{pmatrix} \frac{Q_{1,1}Q_{2,2}-Q_{1,2}Q_{2,1}}{Q_{2,2}} \\ \frac{Q_{1,1}Q_{2,2}-Q_{1,2}Q_{2,1}}{Q_{1,1}} \\ 1 \\ -A_{d,x} \end{pmatrix}$$

Combining these expressions, it follows that

$$B = \begin{pmatrix} 1 & \frac{-Q_{1,2}}{Q_{2,2}} & 0 & 0 \\ \frac{-Q_{2,1}}{Q_{1,1}} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{-Q_{2,1}A_{d,q}+Q_{2,2}A_{d,\sigma}}{Q_{1,1}Q_{2,2}-Q_{1,2}Q_{2,1}} & \frac{+Q_{1,1}A_{d,q}-Q_{1,2}A_{d,\sigma}}{Q_{1,1}Q_{2,2}-Q_{1,2}Q_{2,1}} & A_{d,d} & 1 \end{pmatrix} \qquad \tilde{\rho} = \begin{pmatrix} \rho_\sigma & \frac{Q_{1,1}}{Q_{2,2}} & 0 & 0 \\ 0 & \rho_q & 0 & 0 \\ 0 & 0 & \rho_d & \frac{\phi_{dx}}{-A_{d,x}} \\ 0 & 0 & 0 & \rho_x \end{pmatrix}$$

$$\tilde{S} = \begin{pmatrix} 1 & 0 & 0 & \frac{Q_{1,1}Q_{2,2}-Q_{1,2}Q_{2,1}}{-Q_{2,2}A_{d,x}} s_{\sigma,x} \\ 0 & 1 & 0 & \frac{Q_{1,1}Q_{2,2}-Q_{1,2}Q_{2,1}}{-Q_{1,1}A_{d,x}} s_{q,x} \\ \frac{Q_{2,2}}{Q_{1,1}Q_{2,2}-Q_{1,2}Q_{2,1}} s_{d,\sigma} & \frac{Q_{1,1}}{Q_{1,1}Q_{2,2}-Q_{1,2}Q_{2,1}} s_{d,q} & 1 & \frac{1}{-A_{d,x}} s_{d,x} \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \tilde{\epsilon}_{t+1} = \begin{pmatrix} \frac{Q_{1,1}Q_{2,2}-Q_{1,2}Q_{2,1}}{Q_{2,2}} \\ \frac{Q_{1,1}Q_{2,2}-Q_{1,2}Q_{2,1}}{Q_{1,1}} \\ 1 \\ -A_{d,x} \end{pmatrix} \odot \epsilon_{t+1}.$$

Table D.1 Structural Factor GARCH Estimates—Separate Volatility Dynamics

The table reports the "structural" factor GARCH estimates for the alternative setup with separate volatility dynamics described in Sections C.1 and D.1, with the three restrictions: $A_{d,d} = \frac{\rho_d}{1-\kappa_{d,1}\rho_d}$, $\Gamma_{4,4} + \Upsilon_{4,4} = \rho_q$, and $\Gamma_{3,3} = 0$. The resulting J-test with 7 degrees-of-freedom for the GMM-based estimation equals 26.31, corresponding to a p-value 0.0004.

B			ERV_{t+1}		VRP_{t+1}		Δd_{t+1}		d_{t+1}/p_{t+1}	
ERV_{t+1}			1		-0.490	(0.117)	0		0	
VRP_{t+1}			-0.022	(0.030)	1		0		0	
Δd_{t+1}			0		0		1		0	
d_{t+1}/p_{t+1}			-0.110	(0.141)	-1.595	(0.063)	-0.158		1	
$\tilde{\rho}$	constant		ERV_t		VRP_t		Δd_t		d_t/p_t	
ERV_{t+1}	0.009	(0.003)	0.827	(0.089)	0		0		0	
VRP_{t+1}	0.008	(0.002)	0		0.312	(0.071)	0		0	
Δd_{t+1}	-0.002	(0.016)	0		0		-0.187	(0.035)	-0.001	(0.004)
d_{t+1}/p_{t+1}	-0.080	(0.029)	0		0		0		0.980	(0.008)
$\tilde{S}$			$\tilde{\epsilon}_{\sigma^2_{t+1}}$		$\tilde{\epsilon}_{q_{t+1}}$		$\tilde{\epsilon}_{\Delta d_{t+1}}$		$\tilde{\epsilon}_{x,t+1}$	
ERV_{t+1}			1		0		0		0.316	(0.038)
VRP_{t+1}			0		1		0		-0.245	(0.017)
Δd_{t+1}			-0.387	(0.080)	-0.134	(0.160)	1		0.095	(0.034)
d_{t+1}/p_{t+1}			0		0		0		1	
$\tilde{\epsilon}$	ϖ_u		Γ		Υ					
ERV_{t+1}	0.001	(0.000)	0.153	(0.360)	0.388	(0.138)				
VRP_{t+1}	0.000	(0.000)	0.537	(0.335)	0.116	(0.079)				
Δd_{t+1}	0.001	(0.000)	0		0.449	(0.115)				
d_{t+1}/p_{t+1}	0.002	(0.000)	0.167		0.144	(0.106)				

Table D.2 Structural Model Implications—Separate Volatility Dynamics

The table reports the contemporaneous matrix Φ_0, the reduced form matrix Φ, and the return equation, implied by the alternative "structural" factor GARCH model defined in Sections C.1 and D.1).

$\Phi_0^{-1} \equiv B^{-1}\tilde{S}$			ERV_{t+1}		VRP_{t+1}		Δd_{t+1}		d_{t+1}/p_{t+1}	
ERV_{t+1}			1.011	(0.015)	0.496	(0.118)	0		0.198	(0.049)
VRP_{t+1}			0.022	(0.030)	1		0		-0.241	(0.017)
Δd_{t+1}			-0.387	(0.080)	-0.134	(0.160)	1		0.095	(0.034)
d_{t+1}/p_{t+1}			0.085	(0.134)	1.646	(0.079)	0.158	(0.025)	0.652	(0.047)
$\Phi \equiv B^{-1}\tilde{\rho}B$	constant		ERV_t		VRP_t		Δd_t		d_t/p_t	
ERV_{t+1}	0.013	(0.003)	0.833	(0.091)	-0.255	(0.075)	0		0	
VRP_{t+1}	0.008	(0.001)	0.012	(0.016)	0.306	(0.070)	0.000	(0.000)	0.000	(0.000)
Δd_{t+1}	-0.002	(0.016)	0.000	(0.000)	0.002	(0.006)	-0.187	(0.035)	-0.001	(0.004)
d_{t+1}/p_{t+1}	-0.066	(0.030)	0.002	(0.045)	-1.103	(0.130)	-0.185	(0.035)	0.980	(0.008)
	GMM Implied Return Equation									
$r_{t,t+1}$	constant		ERV_t		VRP_t		Δd_t		d_t/p_t	
	0.062	(0.029)	-0.002	(0.044)	1.074	(0.127)	-0.007	(0.002)	0.046	(0.009)
			$\tilde{\epsilon}_{\sigma^2_{t+1}}$		$\tilde{\epsilon}_{q_{t+1}}$		$\tilde{\epsilon}_{\Delta d_{t+1}}$		$\tilde{\epsilon}_{x,t+1}$	
stru-shocks			-0.470	(0.213)	-1.734	(0.178)	0.846	(0.041)	-0.539	(0.036)

Table D.3 Structural Factor GARCH Estimates—Long-Run Stochastic Volatility

The table reports the "structural" factor GARCH estimates for the alternative setup with long-run stochastic volatility described in Sections C.2 and D.2, with the two restrictions: $A_{d,d} = \frac{\rho_d}{1-\kappa_{d,1}\rho_d}$ and $\Gamma_{3,3} = 0$. The resulting J-test with 6 degrees-of-freedom for the GMM-based estimation equals 37.02, corresponding to a p-value 0.0000.

B			ERV_{t+1}		VRP_{t+1}		Δd_{t+1}		d_{t+1}/p_{t+1}	
ERV_{t+1}			1		0.000	(0.189)	0		0	
VRP_{t+1}			0.120	(0.039)	1		0		0	
Δd_{t+1}			0		0		1		0	
d_{t+1}/p_{t+1}			-0.016	(0.086)	-2.007	(0.156)	-0.249		1	
$\tilde{\rho}$	constant		ERV_t		VRP_t		Δd_t		d_t/p_t	
ERV_{t+1}	0.003	(0.002)	1.001	(0.077)	-0.070	(0.255)	0		0	
VRP_{t+1}	0.006	(0.001)	0		0.609	(0.079)	0		0	
Δd_{t+1}	-0.000	(0.014)	0		0		-0.329	(0.040)	-0.001	(0.004)
d_{t+1}/p_{t+1}	-0.075	(0.028)	0		0		0		0.982	(0.007)
$\tilde{S}$			$\tilde{\epsilon}_{\sigma^2_{t+1}}$		$\tilde{\epsilon}_{q_{t+1}}$		$\tilde{\epsilon}_{\Delta d_{t+1}}$		$\tilde{\epsilon}_{x,t+1}$	
ERV_{t+1}			1		0		0		0.332	(0.036)
VRP_{t+1}			0		1		0		-0.186	(0.016)
Δd_{t+1}			-0.524	(0.061)	-0.069	(0.147)	1		0.061	(0.024)
d_{t+1}/p_{t+1}			0		0		0		1	
$\tilde{\epsilon}$	ϖ_u		Γ		Υ					
ERV_{t+1}	0.001	(0.000)	0.001	(0.051)	0.776	(0.062)				
VRP_{t+1}	0.000	(0.000)	0.000	(0.139)	0.322	(0.147)				
Δd_{t+1}	0.001	(0.000)	0		0.454	(0.095)				
d_{t+1}/p_{t+1}	0.002	(0.000)	0.766	(0.061)	0.160	(0.041)				

Table D.4 Structural Model Implications–Long-Run Stochastic Volatility

The table reports the contemporaneous matrix Φ_0, the reduced form matrix Φ, and the return equation, implied by the alternative "structural" factor GARCH model in Sections C.2 and D.2.

$\Phi_0^{-1} \equiv B^{-1}\tilde{S}$			ERV_{t+1}		VRP_{t+1}		Δd_{t+1}		d_{t+1}/p_{t+1}	
ERV_{t+1}			1.000	(0.023)	-0.000	(0.189)	0		0.332	(0.032)
VRP_{t+1}			-0.120	(0.037)	1		0		-0.226	(0.025)
Δd_{t+1}			-0.524	(0.061)	-0.069	(0.147)	1		0.061	(0.024)
d_{t+1}/p_{t+1}			-0.356	(0.052)	1.990	(0.158)	0.249	(0.023)	0.568	(0.051)
$\Phi \equiv B^{-1}\tilde{\rho}B$	constant		ERV_t		VRP_t		Δd_t		d_t/p_t	
ERV_{t+1}	0.003	(0.002)	0.993	(0.089)	-0.070	(0.194)	0		0	
VRP_{t+1}	0.005	(0.001)	-0.046	(0.020)	0.618	(0.069)	0		0	
Δd_{t+1}	-0.000	(0.014)	0.000	(0.000)	0.003	(0.007)	-0.328	(0.040)	-0.001	(0.004)
d_{t+1}/p_{t+1}	-0.065	(0.029)	-0.093	(0.041)	-0.732	(0.174)	-0.326	(0.041)	0.982	(0.007)
	GMM Implied Return Equation									
$r_{t,t+1}$	constant		ERV_t		VRP_t		Δd_t		d_t/p_t	
	0.063	(0.027)	0.090	(0.040)	0.714	(0.169)	-0.011	(0.002)	0.045	(0.008)
			$\tilde{\epsilon}_{\sigma^2_{t+1}}$		$\tilde{\epsilon}_{q_{t+1}}$		$\tilde{\epsilon}_{\Delta d_{t+1}}$		$\tilde{\epsilon}_{x,t+1}$	
stru-shocks			-0.178	(0.124)	-2.004	(0.181)	0.758	(0.050)	-0.491	(0.035)