Distressed, but not risky: Reconciling the empirical relationship between financial distress, market-based risk indicators, and stock returns (and more)

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First Draft: November, 2010
This Draft: November, 2012

Abstract

Stock prices are a fundamental tool for identifying financially distressed firms. However, contrary to conventional wisdom, distressed firms have lower stock returns, while book-to-market values, frequently associated with distress, are positively related with stock returns. A model that decouples real (observed) from risk-neutral probabilities of default can reconcile these phenomena. This model also fits other empirical regularities, e.g., firms with higher bond yields have higher stock returns, and book-to-market value dominates financial leverage in explaining stock returns. The model predicts that firms with a higher risk-neutral probability of default should have higher stock returns, a hypothesis consistent with recent findings.

JEL Codes: G12, G32, E44

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The recent financial crisis has highlighted the need for economists and policymakers to identify financially distressed firms. Stock prices seem to be the most versatile tool for this purpose because of the frequency and availability of information. Since conventional wisdom suggests that financially distressed firms have higher risk and lower market values than other firms, they should have higher expected stock returns and higher market-based risk indicators, such as book-to-market and earnings-price ratios. Therefore, financial distress may also be the link accounting for the value premium, i.e., the positive relationship between stock returns and market-based indicators, as argued frequently in the literature.[1]

This claim has been scrutinized by several papers that use historical observations of loan defaults to estimate firms' default probabilities as a distress proxy. As Table I illustrates, they reach the puzzling conclusion that book-to-market ratios have a low correlation with default probabilities and that financially distressed firms have lower returns.[2] The solution to this distress premium puzzle is important for academics because it poses a challenge to standard models of rational asset pricing, and for policymakers because they use stock market data to infer the financial health of firms.

This paper argues that the distress premium puzzle can be solved by distinguishing between the default probabilities under real (observed) and risk-neutral probability distributions. While the real distribution describes only the likelihood of different monetary payoffs, the risk-neutral distribution also incorporates information about how investors value these payoffs. Since this additional information is missing from loan default observations, the aforementioned literature concentrates on the estimation of default probabilities under the real distribution. However, this

[1]See Fama and French (1992) for an early example and Gomes and Schmid (2010) for a recent example of this argument.

[2]See, for example, Dichev (1998), Griffin and Lemmon (2002), and Campbell, Hilscher, and Szilagyi (2008). One exception is Vassalou and Xing (2004), who find a positive relationship between stock returns and a distress measure that mimicks KMV's Expected Default Frequency (EDF). Da and Gao (2010) argue that their result is driven by short-term return reversals in a small subset of stocks. Using the actual EDF measure, Garlappi, Shu, and Yan (2008) and Gilchrist, Yankov, and Zakrajsek (2009) find that firms in the highest EDF quintile have the lowest returns.

Table I: Stock returns of portfolios of firms sorted according to earnings/price (E/P), book-to-market (B/M), Ohlson's default likelihood score (O-score), and Campbel-Hilscher-Sziglayi's default likelihood score (CHS).

Portfolio	E/P	B/M	O-Score	CHS
1	9.30	9.20	14.16	16.12
2	9.09	10.29	15.12	14.30
3	10.62	10.85	15.72	13.79
4	9.92	11.02	16.32	13.75
5	10.52	10.68	15.36	13.40
6	12.20	11.64	14.88	12.59
7	13.07	12.34	15.00	8.31
8	13.06	13.09	15.48	4.85
9	14.05	14.00	13.44	6.02
10	15.26	15.87	7.20	-3.32

The growth-value portfolio returns based on earnings-price (E/P) and book-to-market (B/M) ratios are calculated using the data from Ken French's webpage, for the period July 1963 to June 2010. The returns for distress portfolios based on O-score are adapted from Table IV of Dichev (1998) and those based on CHS-score (CHS) are adapted from Table VI of Campbell, Hilscher, Szilagyi (2008). The CHS returns are modified using the monthly T-bill and market return series from Ken French's website and then multiplied by 12 so that all returns are annualized actual returns rather than excess returns. The distress premium implied by O-score and CHS-score are different because the default frequency in the data is low, so the empirical estimates of default probabilities can vary significantly across different methods. All portfolios except CHS are constructed by sorting the firms into deciles, whereas the CHS portfolios include the following percentiles from Campbell, Hilscher, and Szilagyi (2008): 0-5, 5-10, 10-20, 20-40, 40-60, 60-80, 80-90, 90-95, 95-99, 99-100.

real default probability does not necessarily line up with the risk-neutral default probability that governs the market value of equity and the risk indicators based thereon. Therefore, the discrepancy between the real and risk-neutral probability distributions can potentially explain the distress premium puzzle.

The standard asset pricing model tells us that the expected return on a stock can be expressed as the ratio of the stock's expected payoff under the real distribution to the stock's price, the latter of which is proportional to the expected payoff under the risk-neutral distribution.[3] If the real and risk-neutral distributions do not

[3]For a textbook exposition of the standard asset pricing equation, see, for example, the first chapter of Cochrane (2005) which also shows that the proportionality factor is equal to the inverse

comove perfectly, then we anticipate the expected real payoff to be weakly correlated with the risk-neutral default probability, and the expected risk-neutral payoff to be weakly correlated with the real default probability. Therefore, while an increase in the real default probability may decrease the expected real payoff, it may not decrease the expected risk-neutral payoff as much, leading to a decrease in expected returns. Similarly, while an increase in the risk-neutral default probability may decrease the expected risk-neutral payoff, it may not decrease the expected real payoff as much, leading to an increase in expected returns. An additional implication of this argument is that the risk indicators based on the market value of equity should be weakly correlated with the real default probability because market equity is determined by the risk-neutral distribution.[4]

This hypothesis reconciles the studies that find a negative distress premium with the argument that the positive value premium stems from distress risk. On the one hand, as empirical studies suggest, firms with a higher observed (real) likelihood of default should have lower returns, leading to a negative distress premium. On the other hand, firms with a higher default probability under the risk-neutral distribution should have higher market-based risk indicators and higher returns, leading to a positive value premium. This hypothesis is also consistent with Koijen, Lustig, and Van Nieuwerburgh (2012), who connect the value premium to the Cochrane and Piazzesi (2005) factor that explains bond return premia, to the extent that bond premia are related to risk-neutral default probabilities.[5]

This paper shows that this hypothesis can be implemented in a simple dynamic framework that also captures the following empirical regularities in addition to a negative distress premium and a positive value premium:

1- When firms are ranked according to their bond yields, firms with higher bond

of the risk-free rate.

[4]Consistent with this implication, Dichev (1998, Table I) and Griffin and Lemmon (2002, footnote 6) show that the rank and Pearson correlations between book-to-market values and O-scores are both 0.05.

[5]In line with this argument, see also Gilchrist, Yankov, and Zakrajsek (2009) and Gilchrist and Zakrajsek (2012), who show that bond risk premia forecast future economic activity.

yields have higher stock returns, as discussed in Anginer and Yildizhan (2010).

2- Stock returns are positively related with market leverage (Bhandari (1988), Fama and French (1992), Gomes and Schmid (2010)), but are insensitive to book leverage (Gomes and Schmid (2010)).

3- Stock returns are less sensitive to market leverage than to book-to-market ratios.

4- Market leverage is only weakly linked to stock returns after controlling for book-to-market value (Johnson (2004), Gomes and Schmid (2010)).

5- Stock returns remain insensitive to book leverage after controlling for book-to-market value, but they become sensitive to book leverage after controlling for market leverage. (Fama and French (1992)).

Aside from matching these regularities, the paper argues that firms with a higher default probability under the risk-neutral distribution should have higher expected returns, which can be checked using credit default swaps. Given that the CDS instruments are relatively new, are not traded in an exchange, and cover only a subset of the Compustat/CRSP stocks, testing this hypothesis poses some challenges. Nevertheless, Nielsen (2012) finds a positive relationship between CDS spreads and stock returns. Moreover, Friewald, Wagner, and Zechner (2012) find that the stock prices of firms with higher CDS spreads suffered more during the financial crisis, meaning that these firms had lower payoffs during a time period when investors valued monetary payoffs more, consistent with the argument that CDS spreads are closely related with risk-neutral default probabilities. In comparison, the average annual value premium during and in the aftermath of the crisis has been -3.81% versus its historical average of 4.98%, implying that firms with high book-to-market ratios also suffered more from the financial crisis. Together, these findings are consistent with the hypotheses and the intuition in this paper.

I. Literature Review

This paper is closely related to the theoretical literature that focuses on the link between financial distress and the cross-section of stock returns. George and Hwang

(2010) show in a static model that a high cost of distress leads to low leverage, low default probability, and higher returns for the unlevered firm. This mechanism, in turn, generates the negative relationship between the default probability and total firm returns, debt and equity combined.[6] Garlappi, Shu, and Yan (2008) and Garlappi and Yan (2011) model strategic default under violation of the absolute priority rule and potential shareholder recovery in bankruptcies. They show that this mechanism can create a hump-shaped relationship between expected equity returns and the default probability if the shareholders' residual claim upon bankruptcy has low risk.[7] Finally, Avramov, Cederburg, and Hore (2011) present an intuitive link between the negative distress premium and long-run risk: Firms in financial distress are not expected to live long; as a result, they should be less exposed to long-run risk and hence have lower stock returns. The paper contributes to this literature in multiple ways.

First, in the aforementioned literature, real and risk-neutral default probabilities are monotonically related. Therefore, the market-based risk indicators that are governed by risk-neutral default probabilities are highly correlated with the real default probabilities, and the return profile of earnings-price portfolios mimics the return profile of distress portfolios. For example, in Avramov, Cederburg, and Hore (2011), an increase in the long-run dividend share ratios decreases dividend-price ratios and increases expected returns, creating a negative correlation between earnings-price ratios and returns, because dividends are equal to earnings in their model.[8] Similarly, in Garlappi and Yan (2011), the hump-shaped relationship be-

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[6]Johnson et. al. (2011) argue that this mechanism may not generate a negative distress premium in equities only. As a remedy, they propose heterogeneity in other parameters. This remedy works if firms are observed once, right after their capital structure choice, but fails if firms are observed some time after this choice, as seen in their Tables 2 and 3. As discussed in Garlappi and Yan (2011), firms' equity betas explode as the default probability increases, because firms liquidate at debt maturity.

[7]Bharath, Panchapagesan, and Werner (2010) find that the frequency of absolute priority violations declined from 75% before 1990 to 9% for the period 2000-2005. This finding contradicts the explanation of the distress premium via absolute priority violations, since the negative distress premium seems to persist after 1990.

[8]See equation 13, and Figure 2(a) in their paper. The long-run dividend share is defined in their introduction: "Firm dividend growth depends on the long-run share ratio, which is the long-run

tween the default probabilities and stock returns implies a hump-shaped relationship between the earnings-price ratios and stock returns.[9] These results contradict the return profiles presented in Table I, and hence these papers seem to generate the negative distress premium at the expense of the positive value premium.

In contrast, this paper separates real and risk-neutral default probabilities: the former is related to the negative distress premium and the latter is related to the positive value premium. Figure 1 provides a preview of results. These results suggest that while alternative mechanisms in the previous literature play a significant role in explaining the cross-section of returns, it is useful to decouple real and risk-neutral default probabilities if we want to explain the negative distress premium and the positive value premium simultaneously. The paper formalizes the idea of weakening the relationship between real and risk-neutral probabilities across firms, using cross-sectional differences in the exposure of cash flows to systematic risk.

As a second contribution, the model in this paper allows both defaults at the time of debt maturity and strategic defaults before debt maturity. George and Hwang (2010) and Avramov, Cederburg, and Hore (2011) assume that the firm issues a zero coupon bond and that bankruptcy can occur only if the firm cannot meet its payments at the maturity date. Hence, there is no strategic endogenous default in these papers. Garlappi, Shu, and Yan (2008) and Garlappi and Yan (2011) focus on strategic defaults only.

Third, this paper contributes to the growing literature that links firms' capital structure decisions to stock returns. The paper extends the cash-flow model with firms' investment decisions and shows that the extended model successfully captures several empirical patterns that involve book-to-market value, financial lever-

expected dividend share of a firm as a proportion of its current dividend share."

[9]See Figure 4 in the online appendix. Garlappi and Yan (2011) use a model similar to Garlappi, Shu, and Yan (2008) to explain why value premia within different distress portfolios follow a hump-shaped relationship, after sorting portfolios first by EDF and then by book-to-market value. However, their model also implies that the unconditional relationship between earnings-price ratios and stock returns is hump-shaped because real and risk-neutral default probabilities are monotonically related, as discussed in their footnote 11. This unconditional hump-shaped relationship contradicts Table I.

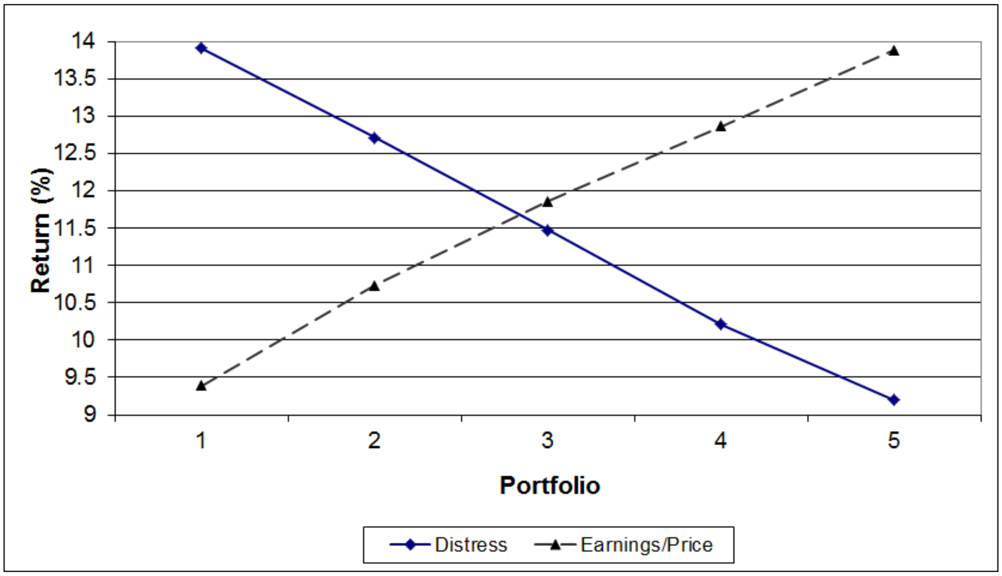

Figure 1: Average returns of simulated portfolios

age, and stock returns. In particular, the effect of book-to-market values on stock returns subsumes the effect of book leverage, defined as the book value of debt divided by the book value of total assets, and the effect of market leverage, defined as the book value of debt divided by the sum of the book value of debt and the market value of equity. Therefore, the paper complements previous literature, such as Whited and Wu (2006), Livdan, Sapriza, and Zhang (2009), and Ozdagli (2012), who look at the effect of risk-free debt capacity on stock returns, and Gomes and Schmid (2010) who link investment growth options and capital structure decisions to study the relationship between financial leverage and stock returns.

Finally, the model predicts that firms with a higher risk-neutral default probability should have higher returns. This prediction seems to be supported by the new literature examining the interaction of stock returns with credit default swaps and bond yields: Anginer and Yildizhan (2010) find a positive relationship between bond yields and stock returns. Nielsen (2012) finds a positive relationship between credit default swap spreads and stock returns, and Friewald, Wagner, and Zechner (2012) find that firms with higher CDS spread suffered more during the financial crisis.[10] These results are also consistent with Kapadia (2011), who connects the value premium to the news about aggregate future firm failures; and Koijen, Lustig, and Van Nieuwerburgh (2012), who connect the value premium to the Cochrane and Piazzesi (2005) factor that explains bond return premia.

II. The Model

As discussed in the introduction, the main intuition in this paper does not make any specific assumption regarding the preferences of investors or payoffs of the assets. We only require a mechanism that decouples the variation of real and risk-neutral probabilities across firms. This section provides a simple, yet realistic, dynamic model of investors' preferences and firms' financing decisions to achieve this requirement.

The investors' preferences for intertemporal substitution and risk are given by a constant risk-free interest rate, r, and price of risk, σ_S:

$$\frac{d\Lambda}{\Lambda} = -rdt - \sigma_S dw_A, \tag{1}$$

where Λ_{t+s}/Λ_t is the stochastic discount factor and dw_A is a Brownian increment that captures macroeconomic shocks. This assumption simplifies the analysis and also has been employed by Carlson, Fisher, and Giammarino (2004), Cooper (2006), and Ozdagli (2012), among others.

The firms differ from one another in the level and riskiness of their cash flows, the latter of which is defined as the exposure of cash-flow growth to systematic risk. When the cash-flow level of a particular firm decreases, both real and risk-neutral default probabilities of this firm increase. However, these probabilities do not comove perfectly across firms because of firms' differences in riskiness of their

[10] While Nielsen (2012, Graph 1) finds that most of the default risk implied by CDS is captured in book-to-market values, Friewald, Wagner, and Zechner (2012) argue that CDS contain some information in addition to market-based risk characteristics. Part of the identification difficulty lies in the fact that the CDS universe includes only about 20% of publicly traded firms and many of them are financial firms or utility companies, which are highly regulated. This identification difficulty is further elevated because reliable CDS data are available only after 2004, a period that overemphasizes the observations from a rare financial crisis.

cash flows, which sets this model apart from the previous literature on the distress premium.

In order to capture this idea in a simple setting, we assume that firm i's cash flow, X_i, follows a geometric Brownian motion

$$\frac{dX_i}{X_i} = \mu_X dt + \sigma \left(\rho_i dw_A + \sqrt{1-\rho_i^2} dw_i \right), \qquad (2)$$

where μ_X and σ are the growth rate and volatility of cash flow, assumed to be the same across firms for the sake of parsimony. The idiosyncratic shocks, dw_i, and the difference in cash flows' riskiness, captured by ρ_i, are the main sources of heterogeneity across firms.[11]

The cross-sectional difference in cash-flow riskiness is a non-standard, but realistic, assumption. For example, fast food and dollar store chains, such as McDonald's and Family Dollar Stores, have less procyclical earnings than their more upscale counterparts, such as Ruby Tuesday and Kohl's. This assumption is similar to that of Berk, Green, and Naik (1999), who study the effects of heterogeneity of cash-flow riskiness on stock returns in the absence of capital structure and loan default decisions. Moreover, this assumption is consistent with the findings of Campbell and Vuolteenaho (2004) and Campbell, Polk, and Vuolteenaho (2009) that value stocks, i.e., stocks with high book-to-market or earnings-price ratios, have higher cash-flow betas.

Figure 2 confirms this point by allowing a comparison of the cash-flow growth rates of the firms above and below median when ranked according to earnings-price ratios. Whereas the firms with high and low earnings-price ratios have similar average cash-flow growth (5.5% versus 4.5%), the cash-flow growth of firms with high earnings-price ratios is more strongly correlated with the aggregate cash-flow growth than the cash-flow growth of firms with low earnings-price ratios, with cor-

[11] While heterogeneity in μ_X and σ is also perceivable, the model focuses on heterogeneity in ρ because changes in μ_X and σ move real and risk-neutral default probabilities in the same direction whereas this paper aims to decouple them. Section VI discusses the implications of heterogeneity in μ_X and σ.

relation coefficients 0.6 versus 0.3. The difference in cyclical properties of cash-flow growth also is present when different cut-offs are used, such as highest and lowest terciles or highest and lowest quintiles of firms. While this pattern and the findings of previous studies provide empirical justification for focusing on the heterogeneity in cash-flow riskiness, Section VI also discusses the implications of heterogeneity in the mean, μ_X, and the standard deviation, σ, of cash-flow growth.

Similar to He and Xiong (2012), the firm's debt takes the form of a coupon bond with a maturity date arriving at an exogenously given rate, λ, and the firm optimally chooses the debt level at date zero. Upon the maturity of existing debt, the firm has two options. Either it refinances by paying off the existing debt and issuing new debt, or it goes bankrupt, leaving the ownership of the firm to the lenders who restructure the capital of the firm after incurring a bankruptcy cost proportional to the after-tax value of the firm, with the proportionality factor η.[12] Similar to Fisher, Heinkel, and Zechner (1989) and Chen (2010), the firm issues the new debt at par value and incurs a cost proportional to the size of the new issue, with the proportionality factor b.

The assumption regarding debt maturity ensures that relatively few firms are close to the endogenous default boundary, so the equity betas of the most distressed firms do not explode, as discussed in Garlappi and Yan (2011). A fixed maturity date would serve the same purpose and would not change the results qualitatively. However, a fixed maturity date would make solution of the model harder because time would enter the model as a state variable. The debt structure in this paper generates the time homogeneity of the problem and allows for closed-form solutions. An alternative interpretation of this assumption is that the firm issues short-term debt that gets rolled over at the same coupon rate in each time period $(t, t + dt)$, with probability $(1 - \lambda dt)$. This interpretation is similar to the one Leland (1994a, p. 1215) proposes for infinite maturity debt when $\lambda = 0$. Other time-homogeneous

[12] An ealier version of this paper follows Chen (2010) and does not allow for lenders to restructure after bankruptcy so that their payoff is $(1 - \eta)(1 - \tau) X / (r - \mu)$. The results do not change qualitatively under this assumption.

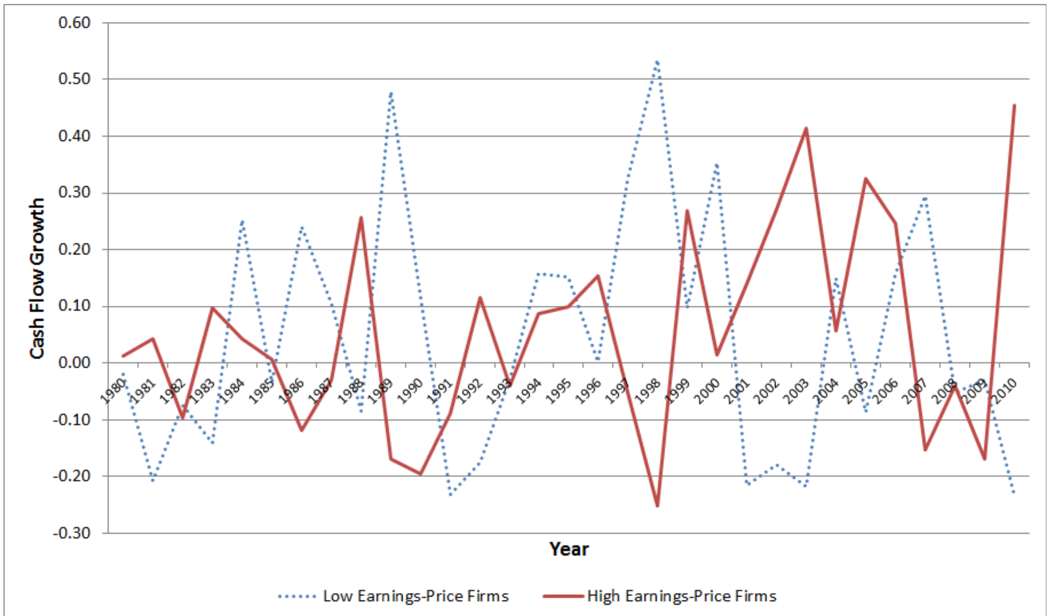

Figure 2: Yearly CPI-adjusted cash-flow growth rates of firms with high and low earnings-price ratios. Cash flows are calculated using annual Compustat data items as income before extraordinary items (IB), plus total income taxes (TXT), minus preferred dividends (DVP), plus interest expense (XINT). Earnings-price ratios are calculated as the ratio of earnings divided by the market value of equity, as in Fama and French (1992). Earnings are measured as income before extraordinary items, plus income-statement deferred taxes (TXDIT), minus preferred dividends. The market value of equity is calculated as shares outstanding times the market price from CRSP. As in Fama and French (1992), the accounting data for all fiscal year-ends in calendar year $t-1$ are matched with market equity at the end of December of year $t-1$. Then, each year, the firms are ranked according to their earnings-price ratios into two portfolios and the within portfolio average of cash flows are used to calculate growth rates in order to correct for the increase in the number of firms over time in the CRSP-Compustat sample. As in Fama and French (1992) and Lettau and Wachter (2007), firms with negative earnings are omitted from the sample. Following Campbell, Hilscher, and Sziglayi (2008), the figure presents the post-1980 period. Firms with less than 5 years of data are excluded. Including all firms produces a similar graph.

settings are presented in Leland (1994b) and Leland and Toft (1996). However, in both models debt is issued continuously, which contradicts Welch's (2004) finding that firms change their debt levels infrequently in response to changes in their stock prices.

A. Equity Valuation

If the firm has coupon payment c, corporate tax rate be $\tau,$ and market value of debt $B(X,c)$, the Hamilton-Jacobi-Bellman (HJB) equation for its market value of equity, $J(X,c)$, becomes

$$\begin{aligned} rJ(X,c) &= (1-\tau)(X-c)+\mu X J_X(X,c)+\frac{1}{2}\sigma^2 X^2 J_{XX}(X,c) \\ &\quad +\lambda\begin{pmatrix} \max\{0,\max_{c'} J(X,c')+(1-b)B(X,c')-B(X_0,c)\} \\ -J(X,c)\end{pmatrix}, \end{aligned} \tag{3}$$

where $\mu=\mu_X-\rho\sigma\sigma_S$ is the risk-adjusted drift of the cash-flow process and X_0 is the value of cash flow at the time of the last debt issue. Since firms issue new debt at par by assumption, $B(X_0,c)$ is equal to the par value of debt. For the sake of parsimony, the model omits personal income taxes, as in Miao (2005), and assumes full loss offset in corporate taxes as in Miao (2005) and Chen (2010). The firm-specific indices are dropped in equation (3) and from here on.

The first line in equation (3) captures the expected continuation value of the firm as the sum of after-tax profits and expected capital gains under the risk-neutral probability measure (distribution) if debt does not mature. The second line of equation (3) captures the effect of debt maturity. When debt matures, the firm can either choose bankruptcy so that shareholders get zero value or it can refinance debt by paying off existing debt, $B(X_0,c)$, choosing new coupon payments, c', and paying restructuring costs proportional to the amount of new debt, $bB(X,c')$.

The model also allows for strategic defaults that can occur before debt matures. In particular, the firm chooses its strategic default boundary, X_B, optimally so that

$J(X,c)$ satisfies the value matching and smooth pasting conditions,

$$J(X_B,c)=J_X(X_B,c)=J_c(X_B,c)=0. \tag{4}$$

B. Debt Valuation

The lenders will receive coupon payments, c, until debt maturity upon which they either receive the face value of debt, $B(X_0,c)$, if the firm remains a going concern, or they receive the ownership of the firm after incurring a cost proportional to the after-tax value of the firm, $\eta(1-\tau)X/(r-\mu)$. Once they receive the ownership of the firm they issue new debt subject to the same cost of issuance, $bB(X,c')$, as previous equityholders.

Accordingly, the HJB equation for the market value of debt becomes

$$\begin{aligned} rB(X,c) &= c+\mu XB_X(X,c)+\frac{1}{2}\sigma^2X^2B_{XX}(X,c) \\ &\quad +\lambda\left(\begin{array}{c} \mathbb{I}_B\left[\max_{c'} J(X,c')+(1-b)B(X,c')-\frac{(1-\tau)\eta X}{r-\mu}\right] \\ +(1-\mathbb{I}_B)B(X_0,c)-B(X,c)\end{array}\right) \end{aligned} \tag{5}$$

where $\mathbb{I}_B$ is the indicator function that is equal to one if the firm prefers bankruptcy at debt maturity and zero if the firm chooses to refinance, that is,

$$\mathbb{I}_B=\begin{cases} 1 \text{ if } \max_{c'} J(X,c')+(1-b)B(X,c')-B(X_0,c)\leq 0 \\ 0 \text{ otherwise}\end{cases}. \tag{6}$$

The first line in equation (5) captures the expected continuation value for the lenders as the sum of coupon payment and expected capital gains under the risk-neutral measure if the debt does not mature. The second line in equation (5) captures the effect of debt maturity. If the firm is not in the bankruptcy zone at debt maturity, i.e. $\mathbb{I}_B=0$, the lenders receive the face value of debt, $B(X_0,c)$. If the firm is in the bankruptcy zone at debt maturity, i.e. $\mathbb{I}_B=1$, the lenders' value will be equal to the value of the restructured firm after incurring restructuring and bankruptcy costs.

Finally, the lenders receive the ownership of the firm and face the same restructuring and bankruptcy costs if the firm defaults strategically. This gives us the final boundary condition at the strategic default boundary, X_B,

$$B(X_B, c) = \max_{c'} J(X_B, c') + (1-b) B(X_B, c') - \frac{(1-\tau)\eta X_B}{r-\mu}. \qquad (7)$$

III. Optimal Policy of the Firm and Stock Returns

The online appendix shows that the market values of debt and equity are homogenous in coupon, c, and cash flow, X, in this model. Therefore, the model can be solved by focusing on a single variable, $y \equiv c/X$, which is also known as the interest coverage ratio. In particular, we can define price-cash flow ratio and market debt-cash flow ratio as $E(y) \equiv J(X,c)/X$ and $D(y) \equiv B(X,c)/X$, respectively. The online appendix shows that the indicator function for the bankruptcy decision at the debt maturity becomes

$$\mathbb{I}_B = \begin{cases} 1 \text{ if } E(y_0) + (1-b) D(y_0) - \frac{y}{y_0} D(y_0) \leq 0 \\ 0 \text{ otherwise} \end{cases}, \qquad (8)$$

where

$$y_0 \equiv \arg\max_{y'} E(y') + (1-b) D(y') \qquad (9)$$

gives the resetting boundary, that is, the interest coverage ratio the firm chooses if it decides to refinance.

This section characterizes basic properties of the firm's optimal policy and leaves the complete characterization to the online appendix. Equation (8) implies that the firm chooses bankruptcy at the time of debt maturity whenever refinancing provides a non-positive value to the shareholders, that is,

$$S(y) \equiv E(y_0) + (1-b) D(y_0) - \frac{y}{y_0} D(y_0) \leq 0. \qquad (10)$$

The following proposition uses this result to characterize the optimal behavior of the firm at the time of debt maturity.

Proposition 1 *There is a threshold level of y, denoted as $\bar{y}$, above (below) which the firm chooses bankruptcy (refinancing) at the time of debt maturity.*

Proof. Note that $y > 0$ and $\lim_{y \to 0^+} S(y) = E(y_0) + (1-b) D(y_0) > 0$, because if $E(y_0) + (1-b) D(y_0) \leq 0$, the firm would choose not to enter the market at its inception. Moreover, $S'(y) < 0$ and $\lim_{y \to \infty} S(y) = -\infty$. Therefore, by the intermediate value theorem, there exists a unique $\bar{y} > 0$ that satisfies $S(\bar{y}) = 0$ and $S(y) \lesseqgtr 0$ if $y \gtreqless \bar{y}$. Since $S(y) \leq 0$ $(S(y) > 0)$ implies bankruptcy (refinancing) this completes the proof. ■

This proposition tells us that the firm chooses bankruptcy at debt maturity if its cash flow falls very short of the scheduled coupon payments so that the shareholders rather pass on the ownership of the firm to the lenders, and that the firm chooses to refinance its debt if its cash flow is high enough so that the shareholders prefer to keep the firm as a going concern. If the firm prefers to refinance, it chooses its debt level so as to maximize its shareholder value, that is, the new coupon payment is set equal to $y_0 X$.

The position of the refinancing boundary, $\bar{y}$, relative to the resetting boundary, y_0, and the strategic default boundary, y_B, allows three possible cases that include $y_0 < \bar{y} < y_B$, $y_B \leq \bar{y}$, and $\bar{y} \leq y_0$. These cases are discussed in online appendix in detail. The following proposition and its corollary refine the properties of optimal policy by showing that the relative positioning of resetting, refinancing, and strategic defaults boundaries satisfies $y_0 < \bar{y} < y_B$ when the cost of debt issuance is small.

Proposition 2 *In the absence of debt issuance costs, that is,* $b = 0$*,* $\bar{y}$ *satisfies* $y_0 < \bar{y} < y_B$*.*

Proof. See the online appendix. ■

To understand the intuition for the relative positioning of $\bar{y}$ and y_0, consider a firm that has chosen coupon c when its cash flow was X_0 at date zero so that $c/X_0 = y_0$. Suppose that the firm's cash flows have increased to a level greater than X_0 by the time its debt matures so that its default probability, and hence expected bankruptcy cost, has decreased for the coupon chosen at date zero. In this case, if there is no cost of debt issuance, the firm can choose a higher debt level and coupon payment to take advantage of the tax deductibility of coupons. Therefore, if cash flow, X, goes above the initial cash flow, X_0, or equivalently if $y < y_0$, the firm finds it optimal to refinance at debt maturity. As a result, $y < y_0$ is a refinancing region and hence the refinancing boundary, $\bar{y}$, cannot be below the resetting boundary, y_0, by definition of $\bar{y}$ in Proposition 1.

To understand the intuition for $\bar{y} < y_B$, suppose that the firm experiences negative cash-flow shocks so that its debt matures at a date when its cash flow is arbitrarily close to the strategic default boundary. If the refinancing boundary, $\bar{y}$, lies above the strategic default boundary, y_B, the firm's optimal choice is to refinance. However, the optimality of refinancing implies that the market value of equity should be strictly positive regardless of how close the firm is to its strategic default boundary, which contradicts the definition of strategic default boundary.

The following corollary follows from the fact that the value functions and boundary conditions are continuous and differentiable in b.

Corollary 1 *For sufficiently small cost of issuing debt,* $y_0 < \bar{y} < y_B$.

The numerical analysis of the calibration in Section IV reveals that the choice of debt issuance cost, b, is small enough so that $y_0 < \bar{y} < y_B$ in the model. Therefore, the analysis in the following sections is based on the case $y_0 < \bar{y} < y_B$, although the intuition derived from the analysis would be similar under different scenarios. Figure 3 illustrates this optimal policy.

Finally, the instantaneous expected stock returns are given by the sum of dividends and expected capital appreciation divided by the current value of the firm,

$$\frac{1}{dt}\mathbb{E}_t\left(dR^e\right) = \frac{1}{dt}\mathbb{E}_t\left(\frac{(1-\tau)\left(X-c\right)dt + dJ\left(X,c\right)}{J\left(X,c\right)}\right) \tag{11}$$

$$= r + \rho\sigma\sigma_S\frac{J_X\left(X,c\right)X}{J\left(X,c\right)} = r + \rho\sigma\sigma_S\left(1 - \frac{E'\left(y\right)y}{E\left(y\right)}\right). \tag{12}$$

The first equality in the second line comes from the HJB equation (3) for the market value of equity and the relationship between the real and the risk-neutral drift of

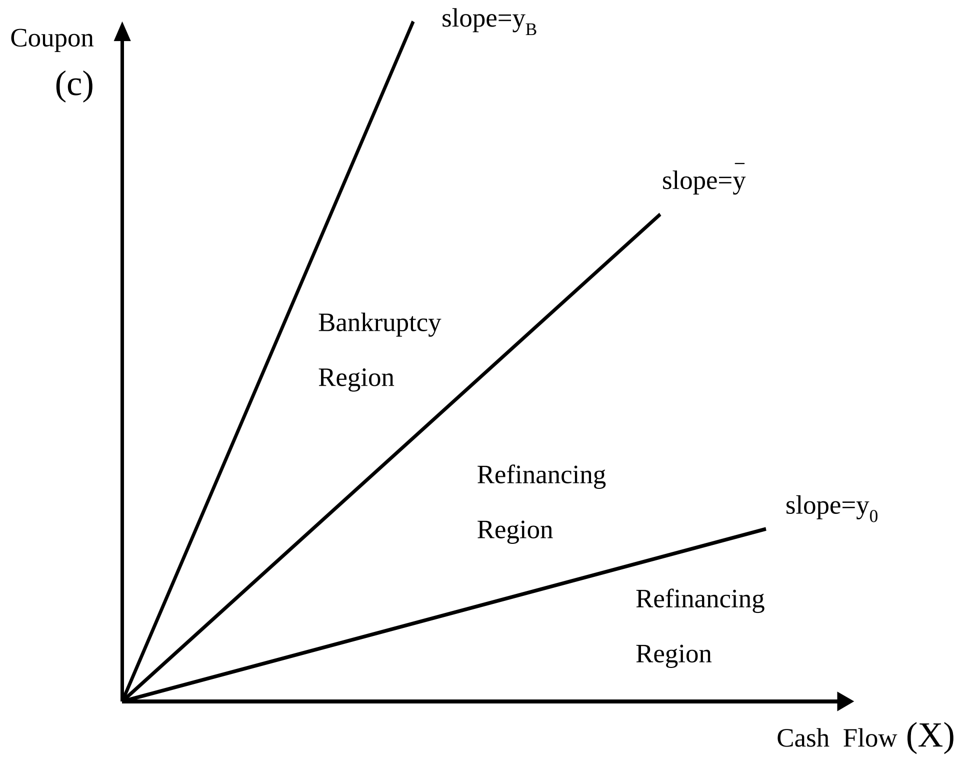

Figure 3: Optimal policy of the firm for different values of coupon, c, **and cash flow,** X**.** The firm goes bankrupt if it hits the endogenous default boundary $c/X = y_B$ before the debt matures, or if the debt matures while the firm is in the bankruptcy region, $\bar{y} < c/X < y_B$. The firm repays the existing debt and issues new debt if the debt maturity arrives in the refinancing region, $c/X < \bar{y}$, upon which the firm returns to the resetting boundary, $c/X = y_0$.

cash flow. The second equality comes from the homogeneity property of the market value of equity.

IV. Calibration and Simulated Portfolios

The annual cash-flow growth is taken as $\mu_X = 0.02$ to match the US post-war real GDP per capita growth rate and cash-flow volatility is taken as $\sigma = 0.35$.[13] The

[13]Miao (2005) and Cooper (2006) use 0.25 for σ, following the standard deviation of aggregated earnings growth of S&P 500 firms. However, this number is likely biased downwards as an estimate of σ because S&P 500 is a diversified portfolio that consists of stocks with a particularly successful history. Nevertheless, the main results are qualitatively unaffected by this choice. The calibration

tax rate, τ, is taken to be 35 percent from Taylor (2003) and Miao (2005). The annual real risk-free rate is taken to be 2 percent, using the time series average of Fama's monthly T-bill returns in the CRSP database from 1963 to 2010. Moreover, the annual risk-price, $\sigma_S = 0.4$, is chosen to match the average monthly Sharpe ratio of the excess market return from 1963 to 2010. The cost of debt issuance is chosen to be the same value as in Fischer, Heinkel, and Zechner (1989) and Chen (2010), that is, $b = 0.01$. Following Huang and Huang (2003) and Glover (2011), the bankruptcy cost is equal to half of the firm's value, that is, $\eta = 0.5$. The support of the distribution for the riskiness of cash flows, ρ, is chosen to match an annual equity premium of 6 percent and it generates an annual 0.3 percent default rate, which is close to 0.5 percent reported by Campbell, Hilscher, and Szilagyi (2008). Accordingly, the cash-flow riskiness is assumed to be uniformly distributed between $\rho_L = 0.2$ and $\rho_H = 0.6$. Compustat balance sheet files suggest that an average debt maturity of three years reasonably approximates the data. Therefore, the expected time to maturity, $1/\lambda$, is set to three years. Nevertheless, various values between two and four years lead to qualitatively similar results. It is also noteworthy that these parameters generate an aggregate cash-flow growth volatility of 0.13 per annum which is similar to 0.11 in the data.[14]

The online appendix shows the derivation of the moment-generating function for the time to default. This function provides us several measures of distress. One way is to use a saddlepoint approximation in order to calculate the probability of default within one year, because the O-score and the CHS-score are based on estimates of the default probability within one year. However, like any other numerical approximation, the saddlepoint approximations are potentially subject to significant numerical errors because the default probabilities within one year are very low. Therefore, this paper uses the moment-generating function to calculate the exact value of the expected time to default and uses its reciprocal as a proxy

here also generates an annualized aggregate CF volatility of 13% which fits the data (11%) better.

[14] In order to control for the increase in the number of firms in CRSP/Compustat sample over years, I first calculate the average earning across firms in each year and then calculate the growth rate of these earnings. This gives a growth rate series with mean 0.02 and standard devation 0.13.

for financial distress.[15] This approach and the calibrated parameters are used to simulate portfolios of interest. The simulation results are presented in Table II and discussed below.

Table II: Simulated portfolio returns.

Portfolio Returns with Different Rankings					
Portfolio	1	2	3	4	5
Distress	13.91	12.71	11.47	10.21	9.20
Risk-Neutral Distress	9.20	10.64	11.89	13.13	14.37
Earnings/Price	9.39	10.73	11.86	12.86	13.88
Bond Yield	10.94	11.48	11.93	12.34	13.97

At the beginning of each year, stocks are ranked according to increasing values of earnings-price ratios, the reciprocal of expected time to default under real and risk-neutral measures, and bond yields. A total of 1200 firms are simulated over 1200 months 100 times. The first 600 months are dropped in each simulation to allow the simulations to converge to the steady state. The table reports the time series means of value weighted annual portfolio returns averaged across simulations, adjusted upwards for inflation. The only exception is that the table reports equally-weighted returns for portfolios ranked according to bond yields to make the results comparable to Anginer and Yildizhan (2010, Table 8) though the value-weighted returns are qualitatively similar.

Distress: The first row of Table II provides portfolio returns when portfolios are formed according to the reciprocal of expected time to default under the real probability measure. We see that firms with greater financial distress earn lower stock returns in the model, implying that the model successfully captures the distress premium puzzle.

The negative distress premium is related to the way firms choose their capital structure in the model. The capital structure is determined by the trade-off between the tax advantage of debt and bankruptcy costs under the risk-neutral measure. The tax advantage of debt results in higher leverage, whereas bankruptcy costs result in lower leverage. The expected bankruptcy costs under the risk-neutral measure increase with firms' cash-flow riskiness because firms with riskier cash flows have lower cash-flow growth and a higher default probability under the risk-neutral mea-

[15] The saddlepoint approximations provide qualitatively similar results when they are used to approximate the probability of default within five years.

sure. Hence, these firms choose lower debt, which increases their distance to default under the real measure and reduces their real default probability. As a result, when we rank the firms according to real default probabilities, the firms with higher rank are those with lower cash-flow risk and hence lower expected equity returns. This leads to a negative distress premium.

Risk-neutral Distress: This part repeats the last exercise using the expected time to default under the risk-neutral probability measure. The second row of Table II provides the returns when portfolios are formed according to decreasing expected time to default under the risk-neutral measure as a distress proxy. We see that the firms with greater financial distress under the risk-neutral measure earn higher stock returns in the model.

Intuitively, firms with a higher risk-neutral default probability are those that have higher coupon payments relative to their cash flow, given cash-flow risk, or those that have higher cash-flow risk given the level of cash flow and coupon payments. Both of these channels increase the riskiness of the firm's equity: The first one levers up the net income of the firm, whereas the second one increases the exposure of the firm to systematic risk.

The predicted relationship between risk-neutral distress and stock returns can be tested using the implied risk-neutral default probabilities from credit default swap (CDS) data. Given that the CDS instruments are relatively new and currently do not cover the whole Compustat/CRSP universe, testing this hypothesis is challenging. Nevertheless, recent work by Nielsen (2012) finds a positive relationship between credit default swap premia and stock returns, a result consistent with this hypothesis. Anginer and Yildizhan (2010) reach a similar conclusion by using bond yields as a proxy for the risk-neutral default probability, which is discussed at the end of this section.

Earnings-price ratio: This part focuses on earnings-price ratios that are used as the basis of the value premium in Lettau and Wachter (2007), whereas the next section focuses on book-to-market values as in Fama and French (1992). The third row of Table II provides the returns for five earnings-price portfolios. The model produces a positive relationship between earnings-price ratios and returns in accordance with the evidence in Lettau and Wachter (2007).

Intuitively, a firm has a high earnings-price ratio because its cash-flow risk is high or because it is close to default under the risk-neutral measure, so that its market value is low relative to its cash flows. Both of these effects make equity riskier and hence increase expected stock returns.

Bond yields: This part ranks the firms according to their bond yields. The reason for this exercise comes from Anginer and Yildizhan (2010) who use bond yields as a proxy for financial distress under the risk-neutral measure and find that firms with higher bond yields have higher stock returns. Since the bond yield is the internal rate of return of the bond under the counterfactual assumption that the firm does not go bankrupt, we have[16]

$$yield = \frac{c + \lambda B\left(X_0, c\right)}{B\left(X, c\right)} - \lambda = \frac{y}{y_0} \frac{y_0 + \lambda D\left(y_0\right)}{D\left(y\right)} - \lambda. \tag{13}$$

At the date of bond issue, that is, when $X = X_0$, the yield is equal to $c/B\left(X_0, c\right)$, which is familiar, since the yield of a bond issued at par is equal to the coupon yield at the time of issue.

The fourth row of Table II provides the returns when portfolios are formed according to bond yields. We see that, in accordance with Anginer and Yildizhan (2010), the firms with greater bond yields earn higher stock returns in the model. In particular, their Table 8 shows that when the firms are ranked in three portfolios according to their yields, these three portfolios earn an equally-weighted average annual return of 11.8, 15.7, and 16.3 percent, respectively.

The firms with higher bond yields have to compensate the lenders more for each

[16]Let $\tilde{B}$ be the discounted value of the payoffs from holding the bond, assuming counterfactually that the bond does not default. Then, since the bond is issued at par, we can write

$$yield * \tilde{B} = c + \lambda \left(B\left(X_0, c\right) - \tilde{B}\right).$$

Solving this for $\tilde{B}$ and setting $\tilde{B} = B\left(X, c\right)$ gives the result above.

dollar they borrow because they tend to have higher cash-flow risk and a higher risk-neutral default probability. This channel creates the relationship between bond yields and stock returns.

In the model, the difference in stock returns across bond yield portfolios is somewhat lower than the difference in stock returns across risk-neutral distress and earnings-price portfolios. This observation suggests that we might need to come up with a clearer measure of risk-neutral distress than raw bond yields, such as the risk-neutral default probabilities implied by credit default swaps, as in Nielsen (2012) and Friewald, Wagner, and Zechner (2012). To see why bond yields are not a perfect measure of risk-neutral distress in the context of this model, note that we can write the bond yields as

$$yield = \frac{c}{B\left(X,c\right)} + \lambda\left(\frac{B\left(X_0,c\right)}{B\left(X,c\right)} - 1\right), \tag{14}$$

where the first term is the coupon yield and the second term captures capital loss by the bondholders as the cash flow changes. The coupon yield is closely related to the risk-neutral default probability, because higher cash-flow risk implies lower bond value for a given coupon value and higher risk-neutral default probability. However, the relation of bondholders' capital loss to the risk-neutral default probability is more ambiguous, because the cash-flow risk affects the par value, $B\left(X_0,c\right)$, and the market value, $B\left(X,c\right)$, of the bond the same way, limiting the effect of risk-neutral distress on the capital loss term.

V. Book-to-Market, Financial Leverage, and Stock Returns

This section discusses the relationship between book-to-market value, financial leverage, and stock returns, and argues that the model can successfully generate the patterns involving these quantities. So far, the paper has focused only on the ability of the model to explain the negative distress premium and the positive value premium simultaneously, in a cash-flow model. In order to be able to talk about book-to-market value and financial leverage, we need to model the amount of physical capital a firm chooses. The next subsection serves this aim, the second subsection discusses the stock return patterns, and the last subsection shows that the distress effect does not disappear after controlling for book-to-market values.

A. Extension with Investment and the Book-to-Market Effect

Although the paper has so far modeled the cash flow of the firm, modeling investment is a straightforward exercise, using arguments similar to those in Miao (2005). If we let δ be the depreciation rate of capital, which is tax-deductible, and r be the rental cost of capital, k, we can write the after-tax profit function of the firm as

$$\pi(k, z, c) = (1-\tau)\left(z^{\alpha} k^{1-\alpha} - \delta k - c\right) - rk, \tag{15}$$

where $z^{\alpha} k^{1-\alpha}$ is the production function and z is the productivity of the firm, which follows geometric Brownian motion

$$\frac{dz}{z} = \mu_z dt + \sigma_z \left(\rho_i dw_A + \sqrt{1-\rho_i^2} dw_i\right). \tag{16}$$

Then, similar to the treatment in Miao (2005), profit maximization implies the neoclassical investment rule that the marginal after-tax product of capital is equal to the user cost of capital,

$$(1-\alpha) z^{\alpha} k^{-\alpha} = \frac{r}{1-\tau} + \delta \tag{17}$$

or equivalently

$$k = \left(\frac{1-\alpha}{r/(1-\tau)+\delta}\right)^{1/\alpha} z. \tag{18}$$

Plugging this back into the profit function $\pi(k, z)$ gives the optimized after-tax profit function $\bar{\pi}(X, c) = (1-\tau)(X-c)$, where

$$X = \left[\left(\frac{1-\alpha}{r/(1-\tau)+\delta}\right)^{(1-\alpha)/\alpha} - \left(\delta + \frac{r}{1-\tau}\right)\left(\frac{1-\alpha}{r/(1-\tau)+\delta}\right)^{1/\alpha}\right] z, \tag{19}$$

which follows the geometric Brownian motion

$$\frac{dX}{X} = \mu_X dt + \sigma \left(\rho_i dw_A + \sqrt{1-\rho_i^2} dw_i\right), \tag{20}$$

where $\mu_X = \mu_z$ and $\sigma = \sigma_z$.

In this extended model, the cash-flow process, X, and the after-tax profit function, $\bar{\pi}(X, c)$, are the same as the ones in the original model without physical investment. Therefore, all the claims regarding returns, financial distress, earnings-price ratios, and bond yields can be carried over to this model with investment.[17]

The main advantage of this extension is that now we can calculate meaningful values for book-to-market ratios. This allows us to check whether the model can successfully generate the book-to-market effect as in Fama and French (1992) and to compare the power of book-to-market value in explaining stock returns with that of financial leverage. This subsection focuses on the book-to-market effect and leaves the comparison of book-to-market value to financial leverage to the next subsection.

The book value of total assets is given by k, and hence $k - B(X_0, c)$ gives the book value of equity, which is measured as the book value of total assets minus the book value of debt, whereas $J(X, c)$ gives the market value of equity. Therefore, if we define

$$\kappa \equiv \frac{1-\alpha}{\alpha\left(\frac{r}{1-\tau}+\delta\right)}, \tag{21}$$

we can write book-to-market value as

$$\begin{aligned} \frac{BE}{ME} &= \frac{k - B(X_0, c)}{J(X, c)} = \frac{\kappa X - B(X_0, c)}{J(X, c)}, &(22)\\ &= \frac{\kappa - y/y_0 D(y_0)}{E(y)}. &(23) \end{aligned}$$

Following Miao (2005), the depreciation rate, δ, is set to 0.1. Moreover, the convexity parameter, α, is set to 0.05. This choice of α can be justified using a

[17]The investment decision omits potential capital adjustment costs for theoretical and empirical reasons. Theoretically, the model focuses on financing decisions which allows a more clear comparion of this model's implications with alternatives, such as Gomes and Schmid (2010). Empirically, Hall (2004) estimates the adjustment cost parameter for capital in a quadratic adjustment cost model without debt and finds that adjustment costs are relatively small and are not an important part of the explanation of the large movements in company values. This is also in line with Ozdagli (2012) who finds that financing decisions play a greater role than investment irreversibility in cross-section of stock returns.

decreasing returns to scale Cobb-Douglas production function with capital and labor inputs, where labor is optimized out.[18] These parameter values generate an almost zero correlation (-0.02) between the reciprocal of expected time to default and book-to-market value. As a comparison, Dichev (1998) and Griffin and Lemmon (2003) find that the correlation of O-score and book-to-market values is 0.05, which is also very close to zero.

The final step calculates the book-to-market values using equation (23) and runs Fama-MacBeth regressions of stock returns on book-to-market values. The first rows of Tables III and IV show that, in accordance with Fama and French (1992), the firms with greater book-to-market values earn higher stock returns both in both the data and the model.

The intuition for the book-to-market effect is similar to the intuition for the earnings-price ratios. Firms with high book-to-market ratios are those with high cash-flow risk or those more likely to default under the risk-neutral measure. The first effect increases the overall business risk of these firms, whereas the second effect implies that the market value of their equity is low relative to the market value of their debt, leading to high market leverage. Both of these effects make the equity of the firms with high book-to-market values riskier, so these firms have higher expected stock returns. This mechanism creates the value premium.

B. Leverage and Stock Return Patterns

Table III shows six regressions of stock returns on book-to-market values and different types of financial leverage, using Compustat and CRSP databases. The following summarizes the regression results and cites examples from previous literature that have similar findings:

1- Stock returns are positively related to market leverage (Bhandari (1988), Fama and French (1992), Gomes and Schmid (2010)), but are insensitive to book leverage (Gomes and Schmid (2010)).

[18] Miao (2005) sets $\alpha = 0.4$. However, his choice generates a significant number of negative book-to-market values in my model, which contradicts the data. Nevertheless, the choice of α does not change the results qualitatively.

Table III: Fama-MacBeth regressions of stock returns on various variables.

log BE/ME	log ML	log BL
0.42 (6.41)		
	0.18 (2.81)	
		-0.04 (0.55)
0.45 (8.11)	-0.06 (1.06)	
0.41 (6.58)		-0.09 (1.37)
	0.63 (6.24)	-0.79 (7.93)

Book-to-market value (BE/ME), market leverage (ML), and book leverage (BL) are values at the beginning of the portfolio formation period. Book equity (BE) is the book value of common equity (ceq/item 60) plus deferred taxes and investment tax credit (txditc/item 35) from annual COMPUSTAT files. Market value of equity (ME) is price of each share times the number of shares outstanding from the CRSP database. Book leverage is total assets (COMPUSTAT item 6/at) minus book equity divided by total assets. Market leverage is total assets minus book equity divided by the quantity of total assets minus book equity plus market equity. The accounting data for all fiscal year-ends in calendar year $t-1$ is matched with market equity at the end of December of year $t-1$ and with the CRSP returns for the period starting July of year t to June of year $t+1$, as in Fama and French (1992). The coefficients are the time series average of regression coefficients for July 1963 to June 2010, and the t-statistics (in parentheses) are the average regression coefficient divided by its time series standard error. Each row corresponds to a separate regression.

2- Stock returns are less sensitive to market leverage than to book-to-market values.

3- Market leverage is only weakly linked to stock returns after controlling for book-to-market value (Johnson (2004), Gomes and Schmid (2010)).

4- Stock returns remain insensitive to book leverage after controlling for book-to-market value, but they become sensitive to book leverage after controlling for market leverage (Fama and French (1992)).

Table IV shows the model-generated regression results using simulations. The model seems to do a good job of capturing the regularities above.

Table IV: Fama-MacBeth Regressions with simulated data.

log BE/ME	log ML	log BL
0.51 (10.91)		
	0.12 (6.69)	
		-0.00 (0.34)
0.50 (10.97)	0.02 (1.96)	
0.51 (10.51)		0.01 (1.38)
	0.64 (19.26)	-0.47 (14.42)

Book-to-market value (BE/ME), market leverage (ML), and book leverage (BL) at the beginning of the portfolio formation period. The coefficients are the time series average of regression coefficients, and the t-statistics (in parentheses) are the average regression coefficient divided by its time series standard error. A total of 1200 firms are simulated over 1200 months 100 times, and the first 600 months are dropped to allow the simulations converge to a steady state. The reported statistics are averages across simulations. Each row corresponds to a separate regression.

How does the model generate the result that book-to-market values are a much stronger predictor of stock returns than financial leverage? In a model without heterogeneity in cash-flow risk, book-to-market value and market leverage are strongly correlated with each other, since firms with higher book-to-market values also have higher real and risk-neutral default probabilities and the default probabilities are strongly correlated with financial leverage. Therefore, book-to-market value has hardly any explanatory power above and beyond that of market leverage in such a model. However, when there is heterogeneity in cash-flow risk across firms, the cash-flow risk affects book-to-market values and market leverage in different ways.

To see this intuition within the context of the model, note that the book-to-market ratio, BE/ME, and market leverage, ML, are given by

$$\frac{BE}{ME} = \frac{AT - D}{ME}, \tag{24}$$

$$ML = \frac{D}{ME + D}, \tag{25}$$

where AT is the book value of firms' total assets, D is the book value of debt, and ME is the market value of equity. Given the levels of cash flow and coupon payments, higher cash-flow risk reduces the market value of equity, ME. This affects book-to-market values and market leverage in the same direction, since equity is in the denominator of both quantities. However, higher cash-flow risk also increases the risk-neutral default probability which, in turn, reduces the amount of debt the firm can borrow, D. This depresses market leverage but increases book-to-market values, as we can see from the equations above. Therefore, book-to-market values are more sensitive to a change in cash-flow risk than is market leverage. Since cash-flow risk is positively related to expected returns, book-to-market values are more strongly correlated with returns than is market leverage and hence book-to-market value subsumes the effect of financial leverage.

This intuition also explains why we have a significantly negative sign on book leverage, after controlling for market leverage. Note that we can write book-to-market value as a combination of book leverage and market leverage, that is

$$\frac{BE}{ME} = \frac{ML}{1 - ML} / \frac{BL}{1 - BL}. \tag{26}$$

where book leverage is given by the ratio of book debt to total book assets, $BL = D/AT$. Because book-to-market values subsume the relationship of financial leverage with stock returns, a regression of returns on market leverage and book leverage should imply a significantly positive sign for market leverage and a significantly negative sign for book leverage.

The cash-flow risk heterogeneity mechanism complements Gomes and Schmid (2010), who argue that growth options provide an additional source of risk for small young firms with lower market leverage, which, in turn, weakens the link between

Table V: Regression of returns on book-to-market values and distress measures.

Dichev (1998)		Model	
BE/ME	O-score	BE/ME	log(1/ETD)
0.32 (3.29)		0.31 (6.78)	
	-0.07 (1.55)		-0.23 (2.27)
0.61 (4.90)	-0.11 (4.59)	0.32 (6.19)	-0.28 (3.79)

The left panel reproduces the regression results in from Dichev (1998, Table IV) and the right panel presents the regressions that result from the simulation. BE/ME is the (normalized) book-to-market ratio, O-score is Ohlson's O-score and ETD is expected time to default under real measure. The details of the simulation are explained in Table IV. The market equity is omitted from the regressions as an explanatory variable since it turns out to be economically and statistically insignificant in Dichev's regressions. The coefficients are the time series average of regression coefficients, and the t-statistics (in parentheses) are the average regression coefficient divided by its time series standard error.

market leverage and stock returns. The heterogeneity mechanism also provides the additional advantage of explaining the negative distress premium and the positive value premium simultaneously, consistent with Fama and French (1992) and Campbell, Hilscher, and Szilagyi (2008).

C. Distress effect survives after controlling for Book-to-Market values

While the correlation between book-to-market values and real distress is very low, -0.02 in simulations here and 0.05 in data according to Dichev (1998) and Griffin and Lemmon (2003), we might be interested in if the distress effect survives after controlling for book-to-market values. Therefore, for the sake of completeness, Table V reproduces the results from Dichev (1998) and presents analogous regressions that come from the simulations where Dichev's distress measure (O-score) is replaced by the log of the reciprocal of expected time to default under the real measure. As it can be seen in this table, the distress effect survives after controlling for book-to-market values due to low correlation of book-to-market values and the distress measure.

VI. Discussion

A. Heterogeneity in other cash-flow parameters

Our analysis so far suggests that the heterogeneity in cash-flow riskiness is a realistic assumption and that this heterogeneity is sufficient in the model to generate many empirically plausible patterns. This section analyzes heterogeneity in average, μ_X, and standard deviation, σ, of cash-flow growth in order to argue that the heterogeneity in cash-flow riskiness, ρ, is a necessary ingredient of this model to match the empirical patterns.

The top panels of Table VI present the simulation results under heterogeneity in μ_X and σ respectively, while the cash-flow riskiness parameter is fixed to the average value in the benchmark model, $\rho = 0.4$. In the first exercise, the distribution of μ_X is chosen so that the risk-adjusted drift, $\mu = \mu_X - \rho\sigma_S\sigma$, has the same distribution as the benchmark model and the results are comparable to the benchmark model. For the same reason, the distribution of σ in the second exercise is chosen so that the risk exposure of cash flows, $\rho\sigma$, has the same distribution as in the benchmark model. As a result, the top left panel in Table VI assumes that μ_X is uniformly distributed between -0.035 and 0.02 and the top right panel assumes that σ is uniformly distributed between 0.18 and 0.53.

The first and third lines in the top panel of Table VI reveal that heterogeneity in average and variance of the cash-flow growth does not help us reconcile the positive value premium and the negative distress premium. The intuition behind this result is consistent with our analysis earlier: In the benchmark model, heterogeneity in cash-flow riskiness decouples real and risk-neutral default probabilities. In contrast, changes in average and variance of cash-flow growth under the real measure moves average and variance of cash-flow growth under the risk-neutral measure in the same direction. Therefore, real and risk-neutral default probabilities remain closely related to each other. As a result, there is a positive relationship between stock returns and both real and risk-neutral distress, as shown by the comparison of first two lines in Table VI.

Table VI: Simulated portfolios under heterogeneity in other cashflow parameters.

	Portfolio Returns with Different Rankings									
	Heterogeneity in									
	Average, μ_X					Standard deviation, σ				
Portfolio	1	2	3	4	5	1	2	3	4	5
Distress	11.46	11.56	11.60	11.67	12.40	9.64	10.61	11.63	12.69	14.06
R.N. Distr.	11.41	11.56	11.67	11.78	12.98	9.63	10.63	11.65	12.79	14.14
Earn./Price	11.29	11.59	11.80	11.95	12.85	9.74	10.68	11.47	12.34	13.56
Bond Yield	11.42	11.57	11.67	11.85	13.06	10.09	11.01	11.98	12.91	14.87
	Systematic component, σ_A					Idiosyncratic component, σ_i				
Distress	12.47	12.40	11.55	10.12	9.25	11.58	11.55	11.51	11.51	11.76
R.N. Distr.	9.21	10.60	11.79	13.05	14.25	11.51	11.50	11.49	11.47	12.21
Earn./Price	9.39	10.71	11.79	12.75	13.78	11.21	11.42	11.59	11.83	12.55
Bond Yield	10.70	11.32	11.93	12.47	13.91	11.21	11.49	11.67	12.01	13.89

At the beginning of each year, stocks are ranked according to increasing values of earnings-price ratios, the reciprocal of expected time to default under real and risk-neutral measures, and bond yields. A total of 1200 firms are simulated over 1200 months 100 times. The first 600 months are dropped in each simulation to allow the simulations to converge to the steady state. The table reports the time series means of value weighted annual portfolio returns averaged across simulations, adjusted upwards for inflation. The only exception is that the table reports equally-weighted returns for portfolios ranked according to bond yields to make the results comparable to Anginer and Yildizhan (2010, Table 8).

Finally, it is important to note that the source of heterogeneity in standard deviation, σ, also matters. The results in the top right panel of Table VI implicitly assume that the idiosyncratic and systematic components of standard deviation moves proportionally. However, the effect of the heterogeneity in systematic, $\sigma_A = \rho\sigma$, and idiosyncratic component, σ_i, of standard deviation can be quite different. A change in systematic component affects the risk neutral drift, $\mu = \mu_X - \sigma_A \sigma_S$, directly and significantly whereas it affects the standard deviation, $\sigma = \sqrt{\sigma_A^2 + \sigma_i^2}$, to a much lesser degree because the idiosyncratic component of the standard deviation dampens the effect. Therefore, a heterogeneity in systematic standard deviation can act like the heterogeneity in cash-flow riskiness and reconcile the negative distress and positive value premia to the extent that it decouples real and risk-neutral default probabilities. In contrast, the idiosyncratic component affects only the standard deviation without altering the risk-neutral drift and hence does not contribute to the reconciliation of the distress and value premia.

The bottom left and right panels of Table VI present the simulation results if the heterogeneity is limited only to the systematic component, σ_A, or only to the idiosyncratic component, σ_i, of the standard deviation, $\sigma^2 = \sqrt{\sigma_A^2 + \sigma_i^2}$. In order to make the degree of heterogeneity in σ_A comparable to the benchmark model, the value of σ_i is fixed to its median value in the benchmark model, i.e. $\sigma_i = 0.35\sqrt{(1 - 0.4^2)}$, and the distribution of σ_A is chosen so that the risk exposure of cash flows, σ_A, has the same distribution as the risk exposure under benchmark model, $\rho\sigma$. Therefore, σ_A, is assumed to be uniformly distributed between 0.07 and 0.21. This generates a very small heterogeneity in σ, with values ranging between 0.34 and 0.39. In order to make the degree of heterogeneity in σ_i comparable with these results, the value of σ_A is fixed to its median value in the benchmark model, i.e. $\sigma_A = 0.35\,(0.4) = 0.14$, and the distribution of σ_i is chosen to be the same as in the benchmark model with support $0.35\sqrt{(1 - 0.6^2)}$ and $0.35\sqrt{(1 - 0.2^2)}$. As we see in Table VI, heterogeneity in only the systematic component of standard deviation can generate similar results as in the original model in reconciling the negative distress and positive value premia to the extent that it decouples real and risk-neutral default probabilities. However, heterogeneity in only the idiosyncratic component generates a slightly positive distress premium. All of these results are consistent with the intuition that comes from the benchmark model.

B. Failure of the Capital Asset Pricing Model

The model has only a single systematic shock, and hence the conditional Capital Asset Pricing Model (CAPM) holds. Although the unconditional version of the CAPM cannot perfectly explain the differences in stock returns, it still explains a significant fraction—more than what is predicted by the data.[19] This is a common property of

[19]For example, the model produces a statistically significant annualized alpha of 1.6% for the difference between returns of highest and lowest real distress quintile portfolios, after controlling for Fama-French factors. However, this number is lower than the alpha reported by Campbell, Hilscher, and Sziglayi (2008) for the sample until 2003 and Kapadia (2011) for the sample until

the cross-sectional asset pricing models that try to explain cross-sectional variation in stock returns with only one shock.[20] However, one reason that the value and distress premia are puzzles is that they cannot be explained by the CAPM. Many investor-based models like the intertemporal capital asset pricing model (ICAPM), as studied by Merton (1973), Campbell and Vuolteenaho (2004), Lettau and Ludvigson (2001), and Lettau and Wachter (2007), suggest that the CAPM fails because it does not price the risk factors correctly when there are multiple aggregate (macroeconomic) shocks. The online appendix shows that the extension of the model with additional macroeconomic shocks can easily generate a similar failure of CAPM without changing any quantitative implications of the original model. The bottom line is that the basic model presented here does a very good job of capturing the empirical regularities in cross-sectional asset pricing literature, at least qualitatively, whereas we can rely on additional macroeconomic factors in order to generate the failure of the CAPM as in the studies above.

VII. Conclusion

This paper captures several empirical regularities in the cross-sectional asset pricing literature in a model where the real and risk-neutral default probabilities do not comove perfectly across firms. In particular, the model reconciles the positive value premium with the negative distress premium, and explains the empirical relationships between stock returns, book-to-market values, and financial leverage. The model also predicts a positive relationship between risk-neutral default probabilities and stock returns, which is consistent with recent empirical evidence.

While the analysis in this paper aims to reconcile several patterns in a simple framework, it leaves out some important questions. For instance, the model does not tell us whether and how the distress premium is related to real-side decisions during crises, such as employment, as recently studied by Duygan-Bump, Levkov,

2010.

[20]Examples include the papers by Garlappi, Shu, and Yan (2008), George and Hwang (2010), Garlappi and Yan (2011), Avramov, Cederburg, and Hore (2012), and also earlier papers by Carlson, Fisher, and Giammarino (2004), Zhang (2005), and Berk, Green, and Naik (1999), among others.

and Montoriol-Garriga (2011). Also, the paper takes no stance on whether we actually have a real or risk-neutral "distress factor," or whether financial distress is just another characteristic that will not survive the tests in Daniel and Titman (1997). Finally, it remains an open question whether a parsimonious behavioral or institutional asset pricing model can also generate the patterns summarized here. The answers to these questions are important not only to financial economists but also to policymakers who want to gauge the viability of the distress premium as an early warning mechanism.

References

Anginer, Deniz, and Çelim Yıldızhan, 2010, Is there a distress risk anomaly? Corporate bond spread as a proxy for default risk, World Bank Policy Research Working Paper 5319.

Avramov, Doron E., Scott Cederburg, and Satadru Hore, 2011, Cross-sectional asset pricing puzzles: An equilibrium perspective, available at http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1314869.

Berk, Jonathan B., Richard C. Green, and Vasant Naik, 1999, Optimal investment, growth options, and security returns, *Journal of Finance* 54, 1153-1607.

Bhandari, Laxmi, 1988, Debt/equity ratio and expected stock returns: Empirical evidence, *Journal of Finance* 43, 507-528.

Bharath, Sreedhar T., Venky Panchapegesan, and Ingrid Werner, 2010, The changing nature of chapter 11, AFA 2010 Atlanta Meetings Paper.

Butler, Ronald, 2007, *Saddlepoint Approximations with Applications* (Cambridge University Press).

Campbell, John Y., Jens Hilscher, and Jan Szilagyi, 2008, In search of distress risk, *Journal of Finance* 63, 2899-2939.

Campbell, John Y. and Tuomo Vuolteenaho, 2004, Bad Beta, Good Beta, *American Economic Review*, 94:5, pp. 1249–75.

Campbell, John Y., Christopher K. Polk, and Tuomo Vuolteenaho, 2009, Growth or Glamour? Fundamentals and Systematic Risk in Stock Returns, *The Review of Financial Studies*. 23:1, pp. 305–44.

Carlson, Murray, Adlai Fisher, and Ron Giammarino, 2004, Corporate investment and asset price dynamics: Implications for the cross-section of returns, *Journal of Finance* 59, 2577–2603.

Chen, Hui, 2010, Macroeconomic conditions and the puzzles of credit spreads and capital structure, *Journal of Finance* 65, 2171–2212.

Cochrane, John, 2005, *Asset Pricing (Revised Edition)* (Princeton University Press).

Cochrane, John, and Monika Piazzesi, 2005, Bond Risk Premia, *American Economic Review* 95, 138–160.

Cooper, Ilan, 2006, Asset pricing implications of nonconvex adjustment costs and irreversibility of investment, *Journal of Finance* 61, 139-170.

Da, Zhi, and Pengjie Gao, 2010, Clientele change, liquidity shock, and the return on financially distressed stocks, *Journal of Financial and Quantitative Analysis* 53, 27-48.

Daniel, Kent D., and Sheridan Titman, 1997, Evidence on the characteristics of cross sectional variation in stock returns, *Journal of Finance* 52, 1-33.

Dichev, Ilia, 1998, Is the risk of bankruptcy a systematic risk?, *Journal of Finance* 53, 1131-1147.

Duygan-Bump, Burcu, Alexey Levkov and Judit Montoriol-Garriga, 2011, Financing constraints and unemployment: evidence from the Great Recession, Federal Reserve Bank of Boston Working Paper No. QAU10-6, available at www.bos.frb.org/bankinfo/qau/wp/2010/qau1006.htm.

Fama, Eugene F., and Kenneth R. French, 1992, The cross-section of expected stock returns, *Journal of Finance* 47, 427-465.

Fischer, Edwin O., Robert Heinkel, and Josef Zechner, 1989, Dynamic capital structure choice: Theory and tests, *Journal of Finance* 44, 19-40.

Friewald, Nils, Christian Wagner, and Josef Zechner, 2012, The cross-section of credit risk premia and equity returns, Vienna University Working paper, available at http://ssrn.com/abstract=1883101.

Garlappi, Lorenzo, Tao Shu, and Hong Yan, 2008, Default risk, shareholder advantage and stock returns, *Review of Financial Studies* 21, 2743-2778.

Garlappi, Lorenzo, and Hong Yan, 2011, Financial distress and the cross section of equity returns, *Journal of Finance* 66, 789-822.

George, Thomas J., and Chuan-Yang Hwang, 2010, A resolution of the distress risk and leverage puzzles in the cross section of stock returns, *Journal of Financial Economics* 96, 56-79.

Gilchrist, Simon, Vladimir Yankov, and Egon Zakrajsek, 2009, Credit market shocks and economic fluctuations: Evidence from corporate bond and stock markets, *Journal of Monetary Economics* 56, 471-493.

Gilchrist, Simon, and Egon Zakrajsek, 2012, Credit Spreads and Business Cycle Fluctuations, *American Economic Review* 102:4, 1692-1720.

Glover, Brent, 2011, Expected cost of default, Western Finance Association 2011 Meeting.

Gomes, Joao F., and Lukas Schmid, 2010, Levered returns, *Journal of Finance* 65, 467-494.

Hall, R. E. 2004. Measuring Factor Adjustment Costs, *Quarterly Journal of Economics* 119, 899–927.

He, Zhiguo, and Wei Xiong, 2012, Dynamic debt runs, *Review of Financial Studies* 25, 1799-1843.

Huang, Jing-Zhi, and Ming Huang, 2003, How much of the Corporate-treasury yield spread is due to credit risk? NYU Working Paper No. FIN-02-040.

Johnson, Tim, 2004, Forecast dispersion and the cross-section of expected returns, *Journal of Finance* 59, 1957-1978.

Johnson, Tim C., C. Chebonenko, I. Cunha, F. D'Almeida, and X. Spencer, 2011, Endogenous leverage and expected stock returns, *Finance Research Letters* 8, 132-145.

Kapadia, Nishad, 2011, Tracking Down Distress Risk, *Journal of Financial Eco-*

nomics 102, 167-182.

Koijen, Ralph, Hanno Lustig, and Stijn Van Nieuwerburgh, 2012, The Cross-Section and Time-Series of Stock and Bond Returns, AFA Atlanta 2010 Meeting paper, available at http://ssrn.com/abstract=1341327.

Leland, Hayne, 1994a, Corporate debt value, bond covenants, and optimal capital structure, *Journal of Finance* 49, 1213-1252.

Leland, Hayne, 1994b, Bond prices, yield spreads, and optimal capital structure with default risk, Working Paper No. 240, IBER, University of California.

Leland, Hayne, and Klaus Toft, 1996, Optimal capital structure, endogenous bankruptcy, and the term structure of credit spreads, *Journal of Finance* 51, 987-1019.

Lettau, Martin, and Sydney Ludvigson, 2001, Resurrecting the (C)CAPM: A cross-sectional test when risk premia are time-varying, *Journal of Political Economy* 109, 1238–1287.

Lettau, Martin, and Jessica A. Wachter, 2007, Why is long-horizon equity less risky? A duration-based explanation of the value premium, *Journal of Finance* 62, 55-92.

Livdan, Dmitry, Horacio Sapriza, and Lu Zhang, 2009, Financially constrained stock returns, *Journal of Finance* 64, 1827-1862.

Miao, Jianjun, 2005, Optimal capital structure and industry dynamics, *Journal of Finance* 60, 2621-2659.

Merton, R. C. 1973. "An Intertemporal Capital Asset Pricing Model." Econometrica 41, 867-87.

Nielsen, Caren, 2012, Is default risk priced in equity returns? available at http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1954677.

Ozdagli, Ali K., 2012, Financial Leverage, Corporate Investment, and Stock Returns, *Review of Financial Studies* 25, 1033-1069.

Taylor, Jack, 2003, Corporation income tax brackets and rates, *Statistics Income Bulletin* Fall, 1909-2002.

Vassalou, Maria, and Yuhang Xing, 2004, Default risk in equity returns, *Journal of*

Finance 59, 831-868.

Welch, Ivo, 2004, Capital structure and stock returns, *Journal of Political Economy* 112, 106-131.

Whited, Toni M., and Guojun Wu, 2006, Financial constraints risk, *Review of Financial Studies* 19, 531-559.

Online Appendix (For Online Publication)

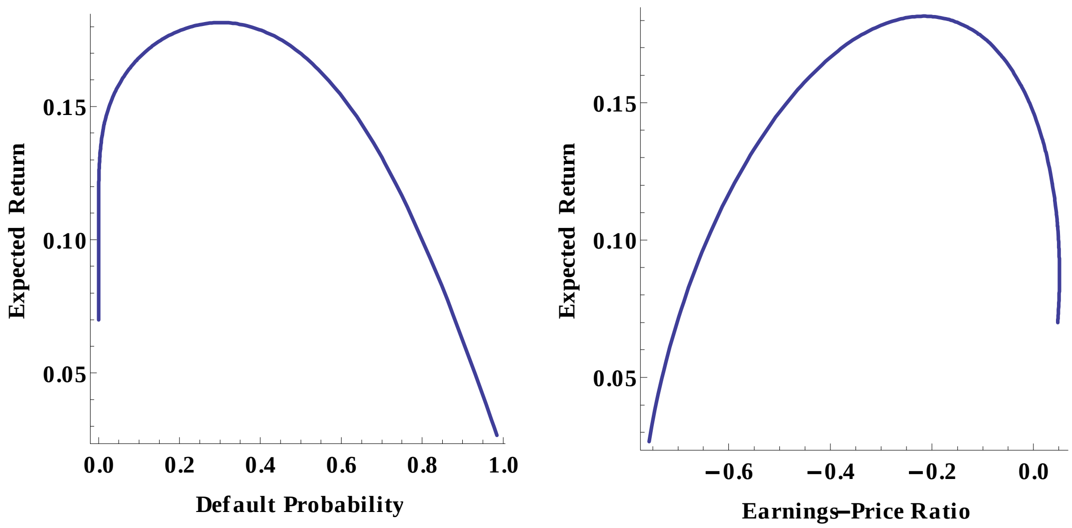

Figure 4: Default probability and earnings-price ratio versus expected returns in Garlappi and Yan (2011). This figure plots the expected returns versus the default probabilities and earnings-price ratios, using the formulas and paramaterization in Figure 1 of Garlappi and Yan (2011) where cash flow follows a geometric Brownian motion.

A. Homogeneity of Market Values

One can solve the joint problem of firms and bondholders directly since the HJB equations for equity and debt can be treated as ordinary differential equations whose solution has constants of integration that depend on coupon c. However, this section employs the guess and verify technique because this technique illuminates the intuition regarding the optimal behavior of the firm, which is studied in section III

Since the payoffs and boundary conditions are homogeneous in X and c, it is natural to start with the guess that both $J(X, c)$ and $B(X, c)$ are linearly homogeneous in X and c. Define interest coverage ratio as $y \equiv c/X$, price-cash flow ratio as $E(y) \equiv J(X, c)/X$, and debt-cash flow ratio as $D(y) \equiv B(X, c)/X$. Then, the HJB equation for market value of equity becomes

$$\begin{aligned}(r-\mu)\,E(y) &= (1-\tau)(1-y)-\mu y E'(y)+\frac{1}{2}\sigma^2 y^2 E''(y) \\ &\quad +\lambda\begin{pmatrix}\max\left\{0,\max_{y'}E(y')+(1-b)\,D(y')-\frac{B(X_0,c)}{X}\right\} \\ -E(y)\end{pmatrix}\end{aligned} \tag{27}$$

with the boundary conditions

$$E(y_B)=E'(y_B)=0, \tag{28}$$

where $y_B \equiv c/X_B$.

Similarly, the HJB equation for debt becomes

$$\begin{aligned}(r-\mu)\,D(y) &= y-\mu y D'(y)+\frac{1}{2}\sigma^2 y^2 D''(y) \\ &\quad +\lambda\begin{pmatrix}\mathbb{I}_B\left[\max_{y'}E(y')+(1-b)\,D(y')-\frac{\eta(1-\tau)}{r-\mu}\right] \\ +(1-\mathbb{I}_B)\,\frac{B(X_0,c)}{X}-D(y)\end{pmatrix}\end{aligned} \tag{29}$$

with boundary condition

$$D(y_B)=\max_{y'}E(y')+(1-b)\,D(y')-\frac{\eta(1-\tau)}{r-\mu} \tag{30}$$

and the indicator function

$$\mathbb{I}_B=\begin{cases}1 \text{ if } \max_{y'}E(y')+(1-b)\,D(y')-B(X_0,c)/X\leq 0 \\ 0 \text{ otherwise}\end{cases}. \tag{31}$$

Finally, one can verify the guess by showing that both of the HJB equations can be represented in terms of y, y_B, $E(y)$, and $D(y)$. In these equations, $B(X_0,c)/X$ is the only term that does not depend exclusively on y. Therefore, it suffices to show that this term can be written as a function of y. For this purpose, define the resetting boundary, y_0, as

$$y_0 \equiv \arg\max_{y'} E(y')+(1-b)\,D(y') \tag{32}$$

and note that y_0 is a constant number because neither $E(y)$ nor $D(y)$ depends explicitly on time. This implies that the firm chooses the same value of $y=y_0$

whenever it issues new debt, including the time of its inception. It follows that $c/X_0 = y_0$, since X_0 is the value of the cash flow at the time of the last debt issue and c is the current value of the coupon payment determined at the time of last issue. This result and the guess of homogeneity of the debt function, $B(X,c)$, leads to

$$\frac{B(X_0,c)}{X} = \frac{X_0 D(y_0)}{X} = \frac{X_0/c}{X/c} D(y_0) = \frac{y}{y_0} D(y_0), \tag{33}$$

and hence the term $B(X_0,c)/X$ can be expressed as a linear function of y.

Substituting $B(X_0,c)/X$ with $(y/y_0) D(y_0)$ in equations (27) and (29) verifies the initial guess of homogeneity, as both of these equations can be represented in terms of functions of y. In particular, the indicator function becomes

$$\mathbb{I}_B = \begin{cases} 1 \text{ if } E(y_0) + (1-b) D(y_0) - \frac{y}{y_0} D(y_0) \leq 0 \\ 0 \text{ otherwise} \end{cases}, \tag{34}$$

which is used section III

B. Determination of Market Values and Optimal Boundaries

Proposition 1 suggests that we can separate the HJB equations for equity and debt into separate regions according to the values of y_0, $\bar{y}$, and y_B. There are three possible cases, depending on the positioning of $\bar{y}$ relative to y_0 and y_B.

Case 1, $y_0 < \bar{y} < y_B$: The optimal policy of the firm suggests that we can separate the HJB equations for debt and equity into two separate equations, one for region $0 < y < \bar{y}$, and one for region $\bar{y} < y < y_B$. If we denote them region 1 and

2, respectively, and note that $y_0 < \bar{y}$, the equations for these regions are

$$\begin{aligned}
(r-\mu+\lambda) E_1(y) &= (1-\tau)(1-y) - \mu y E_1'(y) + \frac{1}{2}\sigma^2 y^2 E_1''(y) \\
&\quad +\lambda\left(E_1(y_0) + (1-b) D_1(y_0) - \frac{y}{y_0} D_1(y_0)\right) && (35)\\
(r-\mu+\lambda) E_2(y) &= (1-\tau)(1-y) - \mu y E_2'(y) + \frac{1}{2}\sigma^2 y^2 E_2''(y) && (36)\\
(r-\mu+\lambda) D_1(y) &= y - \mu y D_1'(y) + \frac{1}{2}\sigma^2 y^2 D_1''(y) + \lambda \frac{y}{y_0} D_1(y_0) && (37)\\
(r-\mu+\lambda) D_2(y) &= y - \mu y D_2'(y) + \frac{1}{2}\sigma^2 y^2 D_2''(y) \\
&\quad +\lambda\left(E_1(y_0) + (1-b) D_1(y_0) - \frac{\eta(1-\tau)}{r-\mu}\right) && (38)
\end{aligned}$$

with boundary conditions

$$\begin{aligned}
E_2(y_B) &= E_2'(y_B) = 0 && (39)\\
D_2(y_B) &= E_1(y_0) + (1-b) D_1(y_0) - \frac{\eta(1-\tau)}{r-\mu} && (40)\\
E_1(\bar{y}) &= E_2(\bar{y}) && (41)\\
E_1'(\bar{y}) &= E_2'(\bar{y}) && (42)\\
D_1(\bar{y}) &= D_2(\bar{y}) && (43)\\
D_1'(\bar{y}) &= D_2'(\bar{y}), && (44)
\end{aligned}$$

where the last four conditions come from the fact that $y = \bar{y}$ is a transitional boundary.[21] Since the HJB equations are second-order ordinary differential equations,

[21] See Dixit (1993, p.30) for the details regarding transitional boundaries.

their solution has the form

$$
\begin{aligned}
E_1(y) &= \frac{1-\tau}{r-\mu+\lambda} + \frac{\lambda}{r-\mu+\lambda}\left(E_1(y_0) + (1-b)D_1(y_0)\right) && (45)\\
&\quad - \left(\frac{1-\tau}{r+\lambda} + \frac{\lambda}{r+\lambda}\frac{D_1(y_0)}{y_0}\right) y + A_1 y^{\beta_1} + A_2 y^{\beta_2} \\
E_2(y) &= \frac{1-\tau}{r-\mu+\lambda} - \frac{1-\tau}{r+\lambda} y + B_1 y^{\beta_1} + B_2 y^{\beta_2} && (46)\\
D_1(y) &= \left(1 + \lambda \frac{D_1(y_0)}{y_0}\right)\frac{y}{r+\lambda} + M_1 y^{\beta_1} + M_2 y^{\beta_2} && (47)\\
D_2(y) &= \frac{y}{r+\lambda} + \frac{\lambda}{r-\mu+\lambda}\left(E_1(y_0) + (1-b)D_1(y_0) - \frac{\eta(1-\tau)}{r-\mu}\right) && (48)\\
&\quad + N_1 y^{\beta_1} + N_2 y^{\beta_2}, && (49)
\end{aligned}
$$

where $\beta_1 < 0$ and $\beta_2 > 1$ are the roots of

$$\frac{1}{2}\sigma^2\beta^2 - \left(\mu + \frac{1}{2}\sigma^2\right)\beta - (r-\mu+\lambda) = 0, \tag{50}$$

because we need $r > \mu$ for convergence of the market values of debt and equity. This implies that we need to find two constants of integration for each of the $E_1(y)$, $E_2(y)$, $D_1(y)$, and $D_2(y)$, and also the values of y_B, y_0, and $\bar{y}$. We need 11 equations to solve for them. Seven of these come from the aforementioned boundary conditions and two more come from the fact that the option value of bankruptcy should not explode as $X \to \infty$ or equivalently $y \to 0^+$, which implies that $A_1 = M_1 = 0$. The last two come from the definitions of y_0 and $\bar{y}$, that is,

$$
\begin{aligned}
E_1'(y_0) + (1-b)D_1'(y_0) &= 0 && (51)\\
E_1(y_0) + (1-b)D_1(y_0) - \frac{\bar{y}}{y_0}D_1(y_0) &= 0. && (52)
\end{aligned}
$$

Case 2, $\bar{y} \geq y_B$: If $\bar{y} \geq y_B$, proposition 1 implies that the firm goes bankrupt only when y hits y_B and never at the time of debt maturity. Therefore, the firms are active only in the region $0 < y < y_B$. This is similar to region 1 of case 1 above, because the firm always refinances at the time of debt maturity. As a result, the HJB equations relevant for this case are

$$
\begin{aligned}
(r-\mu+\lambda) E_1(y) &= (1-\tau)(1-y) - \mu y E_1'(y) + \frac{1}{2}\sigma^2 y^2 E_1''(y) && (53)\\
&\quad +\lambda\left(E_1(y_0) + (1-b) D_1(y_0) - \frac{y}{y_0} D_1(y_0)\right) \\
(r-\mu+\lambda) D_1(y) &= y - \mu y D_1'(y) + \frac{1}{2}\sigma^2 y^2 D_1''(y) + \lambda \frac{y}{y_0} D_1(y_0) && (54)
\end{aligned}
$$

with boundary conditions

$$
\begin{aligned}
E_1(y_B) &= E_1'(y_B) = 0 && (55)\\
D_1(y_B) &= E_1(y_0) + (1-b) D_1(y_0) - \frac{\eta(1-\tau)}{r-\mu}. && (56)
\end{aligned}
$$

Since the HJB equations are second-order ordinary differential equations, their solution has the form

$$
\begin{aligned}
E_1(y) &= \frac{1-\tau}{r-\mu+\lambda} + \frac{\lambda}{r-\mu+\lambda}\left(E(y_0) + (1-b) D(y_0)\right) && (57)\\
&\quad -\left(\frac{1-\tau}{r+\lambda} + \frac{\lambda}{r+\lambda}\frac{D_1(y_0)}{y_0}\right) y + A_1 y^{\beta_1} + A_2 y^{\beta_2} \\
D_1(y) &= \left(1 + \lambda \frac{D_1(y_0)}{y_0}\right)\frac{y}{r+\lambda} + M_1 y^{\beta_1} + M_2 y^{\beta_2}, && (58)
\end{aligned}
$$

where $\beta_1 < 0$ and $\beta_2 > 1$ are the roots of

$$
\frac{1}{2}\sigma^2\beta^2 - \left(\mu + \frac{1}{2}\sigma^2\right)\beta - (r-\mu+\lambda) = 0. \tag{59}
$$

This implies that we need to find two constants of integration for each of the $E_1(y)$ and $D_1(y)$, and also the values of y_B and y_0. We need six equations to solve for them. Three of these come from the aforementioned boundary conditions and two more come from the fact that the option value of bankruptcy should vanish as $X \to \infty$ or equivalently $y \to 0^+$, which implies that $A_1 = M_1 = 0$. The last one comes from the definition of y_0, that is,

$$
E_1'(y_0) + (1-b) D_1'(y_0) = 0. \tag{60}
$$

Case 3, $\bar{y} \leq y_0$: The optimal policy of the firm suggests that we can separate the HJB equations for debt and equity into two separate equations, one for region $0 < y < \bar{y}$, and one for region $\bar{y} < y < y_B$. If we denote them region 1 and 2, respectively, and note that $y_0 \geq \bar{y}$, the equations for these regions are

$$\begin{aligned} (r-\mu+\lambda) E_1(y) &= (1-\tau)(1-y) - \mu y E_1'(y) + \frac{1}{2}\sigma^2 y^2 E_1''(y) \\ &\quad + \lambda \left(E_2(y_0) + (1-b) D_2(y_0) - \frac{y}{y_0} D_2(y_0) \right) && (61) \\ (r-\mu+\lambda) E_2(y) &= (1-\tau)(1-y) - \mu y E_2'(y) + \frac{1}{2}\sigma^2 y^2 E_2''(y) && (62) \\ (r-\mu+\lambda) D_1(y) &= y - \mu y D_1'(y) + \frac{1}{2}\sigma^2 y^2 D_1''(y) + \lambda \frac{y}{y_0} D_2(y_0) && (63) \\ (r-\mu+\lambda) D_2(y) &= y - \mu y D_2'(y) + \frac{1}{2}\sigma^2 y^2 D_2''(y) && (64) \\ &\quad + \lambda \left(E_2(y_0) + (1-b) D_2(y_0) - \frac{\eta(1-\tau)}{r-\mu} \right) && (65) \end{aligned}$$

with boundary conditions

$$\begin{aligned} E_2(y_B) &= E_2'(y_B) = 0 && (66) \\ D_2(y_B) &= E_2(y_0) + (1-b) D_2(y_0) - \frac{\eta(1-\tau)}{r-\mu} && (67) \\ E_1(\bar{y}) &= E_2(\bar{y}) && (68) \\ E_1'(\bar{y}) &= E_2'(\bar{y}) && (69) \\ D_1(\bar{y}) &= D_2(\bar{y}) && (70) \\ D_1'(\bar{y}) &= D_2'(\bar{y}), && (71) \end{aligned}$$

where the last four conditions come from the fact that $y = \bar{y}$ is a transitional boundary. Since the HJB equations are second-order ordinary differential equations, their solution has the form

$$
\begin{aligned}
E_1(y) &= \frac{1-\tau}{r-\mu+\lambda} + \frac{\lambda}{r-\mu+\lambda}\left(E_2(y_0) + (1-b)\,D_2(y_0)\right) && (72)\\
&\quad - \left(\frac{1-\tau}{r+\lambda} + \frac{\lambda}{r+\lambda}\frac{D_2(y_0)}{y_0}\right) y + A_1 y^{\beta_1} + A_2 y^{\beta_2} \\
E_2(y) &= \frac{1-\tau}{r-\mu+\lambda} - \frac{1-\tau}{r+\lambda} y + B_1 y^{\beta_1} + B_2 y^{\beta_2} && (73)\\
D_1(y) &= \left(1+\lambda\frac{D_2(y_0)}{y_0}\right)\frac{y}{r+\lambda} + M_1 y^{\beta_1} + M_2 y^{\beta_2} && (74)\\
D_2(y) &= \frac{y}{r+\lambda} + \frac{\lambda}{r-\mu+\lambda}\left(E_2(y_0) + (1-b)\,D_2(y_0) - \frac{\eta(1-\tau)}{r-\mu}\right) && (75)\\
&\quad + N_1 y^{\beta_1} + N_2 y^{\beta_2}, && (76)
\end{aligned}
$$

where $\beta_1 < 0$ and $\beta_2 > 1$ are the roots of

$$
\frac{1}{2}\sigma^2\beta^2 - \left(\mu + \frac{1}{2}\sigma^2\right)\beta - (r-\mu+\lambda) = 0. \qquad (77)
$$

This implies that we need to find two constants of integration for each of the $E_1(y)$, $E_2(y)$, $D_1(y)$, and $D_2(y)$, and also the values of y_B, y_0 and $\bar{y}$. We need 11 equations to solve them. Seven of these come from the aforementioned boundary conditions and two more come from the fact that the option value of bankruptcy should not explode as $X \to \infty$ or equivalently $y \to 0^+$, which implies that $A_1 = M_1 = 0$. The last two come from the definitions of y_0 and $\bar{y}$, that is,

$$
\begin{aligned}
E_2'(y_0) + (1-b)\,D_2'(y_0) &= 0 && (78)\\
E_2(y_0) + (1-b)\,D_2(y_0) - \frac{\bar{y}}{y_0} D_2(y_0) &= 0. && (79)
\end{aligned}
$$

C. Proof of $y_0 < \bar{y} < y_B$ when $b = 0$ (Proposition 2)

The proof starts by showing that $\bar{y} \geq y_B$, which is Case 2 in the previous section, is not an equilibrium. First, define the normalized net gain of restructuring to the shareholders as

$$
S_{N,b}(y) \equiv E(y_0) + (1-b)\,D(y_0) - \frac{y}{y_0} D(y_0) - E(y), \qquad (80)
$$

which is simply the net gain divided by cash flow after refinancing.

Lemma 1 *If $\bar{y} \geq y_B$, the normalized net gain, $S_{N,b}(y)$, is decreasing in y for $y \geq y_0$.*

Proof. When $\bar{y} \geq y_B$, i.e., Case 2 holds, firms always refinance at debt maturity. Therefore, we can use the definitions of E_1 and D_1 from Case 2 in the previous section to write

$$S_{N,b}(y) = E_1(y_0) + (1-b) D_1(y_0) - \frac{y}{y_0} D_1(y_0) - E_1(y), \tag{81}$$

which has the first and second derivatives

$$S'_{N,b}(y) = -\left(\frac{D_1(y_0)}{y_0} + E'_1(y)\right) \tag{82}$$

$$S''_{N,b}(y) = -E''_1(y). \tag{83}$$

Since the term $A_2 y^{\beta_2}$ in $E_1(y)$ captures the value of the bankruptcy option, which is positive, we have $A_2 > 0$. Combining this with $\beta_2 > 1$ and $A_1 = 0$, we get $E''_1(y) > 0$ and hence $S''_{N,b}(y) < 0$. Moreover, by definition of y_0, we have $E'_1(y_0) + (1-b) D'_1(y_0) = 0$ and $E''_1(y_0) + (1-b) D''_1(y_0) < 0$. Using this with $E''_1(y) > 0$, $\beta_2 > 1$, and $M_1 = 0$ in the formula for $D_1(y)$, it is straightforward to show that $M_2 < 0$. Then, we have

$$S'_{N,b}(y_0) = -\left(\frac{D_1(y_0)}{y_0} - (1-b) D'_1(y_0)\right), \tag{84}$$

$$= -\left[M_2(1-\beta_2) y_0^{\beta_2 - 1} + b D'_1(y_0)\right] < 0. \tag{85}$$

The first term in square brackets is positive because $M_2 < 0$ and $\beta_2 > 1$. The second term in square brackets is also positive because $bD'_1(y_0) = -bE'_1(y_0)/(1-b)$ by definition of y_0, and $E'_1(y_0) < E'_1(y_B) = 0$ using $E''_1(y) > 0$ and the definition of y_B. Combining $S'_{N,b}(y_0) < 0$ and $S''_{N,b}(y) < 0$, we get $S'_{N,b}(y) < 0$ for $y \geq y_0$. ■

This lemma combined with $y_B > y_0$ and $S_{N,b}(y_0) = -bD_1(y_0) < 0$ leads to the following corollary:

Corollary 2 *If $\bar{y} \geq y_B$, $S_{N,b}(y_B) < 0$.*

This corollary helps with the first part of the proof in the following lemma.

Lemma 2 *Case 2, with $\bar{y} \geq y_B$, cannot be an equilibrium.*

Proof. Suppose $\bar{y} \geq y_B$ is an equilibrium. By definition of $\bar{y}$, we have $E_1(y_0) + (1-b) D_1(y_0) - (\bar{y}/y_0) D_1(y_0) = 0$. Moreover, because $y_B \leq \bar{y}$, $E_1(y_0) + (1-b) D_1(y_0) - (y_B/y_0) D_1(y_0) \geq 0$. But this last inequality, combined with $E_1(y_B) = 0$, implies that $S_{N,b}(y_B) \geq 0$, which contradicts Corollary 2. Hence, $\bar{y} \geq y_B$ cannot be an equilibrium. ■

The only remaining task is to show that $\bar{y} > y_0$ if $b = 0$. Let $S_0(y) = E(y_0) + D(y_0) - \frac{y}{y_0} D(y_0)$ be the value of shareholder surplus when $b = 0$. Since $S_0(y_0) = E(y_0) > 0$, $S_0'(y) < 0$ and $S_0(\bar{y}) = 0$ by definition of $\bar{y}$, it immediately follows that $\bar{y} > y_0$.

D. Distribution of Time to Bankruptcy

This section derives the moment-generating function for the distribution of time to bankruptcy.

As we have done with the market value of equity, we divide the the state space into two regions. If we define $a = \ln y/y_B$, and $\bar{a} = \ln(\bar{y}/y_B)$, and $a_0 = \ln(y_0/y_B)$, then we have $a = 0$ as the absorbing barrier. Moreover, a follows the Brownian motion

$$da = mdt + \sigma dw + \left[\mathbb{I}_{a>\bar{a}}(0-a) + (1-\mathbb{I}_{a>\bar{a}})(a_0 - a)\right] dN, \tag{86}$$

where $m = -\left(\mu_X - \frac{1}{2}\sigma^2\right)$, $dw = -\left(\rho dw_A + \sqrt{1-\rho^2} dw_i\right)$, and $\mathbb{I}_{a>\bar{a}}$ is the indicator function that is equal to 1 if $a > \bar{a}$ and 0 otherwise. In addition,

$$dN = \begin{cases} 1 \text{ with probability } \lambda dt \\ 0 \text{ with probability } 1 - \lambda dt \end{cases}, \tag{87}$$

where being hit by the λ-shock when $a > \bar{a}$ implies that the firm goes bankrupt, which we denote as a jump to the absorbing barrier, and being hit by this shock when $a < \bar{a}$ implies resetting to a_0. Note that y_B, y_0, and $\bar{y}$ depend on ρ, and hence a, $\bar{a}$, and a_0 also depend on ρ, which is different across firms.

Using this and denoting the regions $a < \bar{a}$ as region 1 and $a > \bar{a}$ as region 2, we find that the distribution of time to bankruptcy satisfies the following Kolmogorov forward equations in these regions,

$$g_{1,t}(t,a) = mg_{1,a}(t,a) + \frac{1}{2}\sigma^2 g_{1,aa}(t,a) + \lambda\left(g_1(t,a_0) - g_1(t,a)\right) \quad (88)$$

$$g_{2,t}(t,a) = mg_{2,a}(t,a) + \frac{1}{2}\sigma^2 g_{2,aa}(t,a) + \lambda\left(\delta(t) - g_2(t,a)\right) \quad (89)$$

subject to boundary conditions

$$g_2(t,0) = \delta(t) \quad (90)$$

$$g_1(0,a) = g_2(0,a) = 0 \text{ for } a < 0 \quad (91)$$

$$\lim_{a \to -\infty} g_{1,a}(t,a) = 0 \quad (92)$$

$$g_1(t,\bar{a}) = g_2(t,\bar{a}) \quad (93)$$

$$g_{1,a}(t,\bar{a}) = g_{2,a}(t,\bar{a}), \quad (94)$$

where $\delta(t)$ is the Dirac-Delta function and the last two conditions come from the fact that $\bar{a}$ is the transitional boundary.

If we define the Laplace transform as

$$\gamma(s,a) \equiv \int_0^\infty e^{-st} g(t,a)\,dt, \quad (95)$$

we can reduce the Kolmogorov equations to the following second-order ODEs

$$(s+\lambda)\gamma_1(s,a) = m\gamma_{1,a}(s,a) + \frac{1}{2}\sigma^2\gamma_{1,aa}(s,a) + \lambda\gamma_1(s,a_0) \quad (96)$$

$$(s+\lambda)\gamma_2(s,a) = m\gamma_{2,a}(s,a) + \frac{1}{2}\sigma^2\gamma_{2,aa}(s,a) + \lambda, \quad (97)$$

subject to boundary conditions

$$\begin{aligned}
\gamma_2(s,0) &= 1 && (98)\\
\lim_{a\to-\infty}\gamma_{1,a}(s,a) &= 0 && (99)\\
\gamma_1(s,\bar{a}) &= \gamma_2(s,\bar{a}) && (100)\\
\gamma_{1,a}(s,\bar{a}) &= \gamma_{2,a}(s,\bar{a}), && (101)
\end{aligned}$$

which, for $s>-\lambda$, gives us the solution

$$\begin{aligned}
\gamma_1(s,a) &= \tilde{A}\left(\frac{\lambda}{s}e^{\theta_2 a_0}+e^{\theta_2 a}\right) && (102)\\
\gamma_2(s,a) &= \frac{\lambda}{s+\lambda}+\frac{s}{s+\lambda}e^{\theta_2 a}+\tilde{B}\left(e^{\theta_1 a}-e^{\theta_2 a}\right), && (103)
\end{aligned}$$

where $\theta_2>0>\theta_1$ are the roots of

$$\frac{1}{2}\sigma^2\theta^2+m\theta-(s+\lambda)=0 \qquad (104)$$

and $\tilde{A}$ and $\tilde{B}$ satisfy

$$\begin{aligned}
\left(\frac{\lambda}{s}e^{\theta_2 a_0}+e^{\theta_2\bar{a}}\right)\tilde{A} &= \frac{\lambda}{s+\lambda}+\frac{s}{s+\lambda}e^{\theta_2\bar{a}}+\tilde{B}\left(e^{\theta_1\bar{a}}-e^{\theta_2\bar{a}}\right) && (105)\\
\theta_2 e^{\theta_2\bar{a}}\tilde{A} &= \frac{s}{s+\lambda}\theta_2 e^{\theta_2\bar{a}}+\tilde{B}\left(\theta_1 e^{\theta_1\bar{a}}-\theta_2 e^{\theta_2\bar{a}}\right). && (106)
\end{aligned}$$

Having found this, we can derive the moment-generating function and cumulant-generating function.

$$\begin{aligned}
M(s,a) &= \gamma(-s,a) && (107)\\
K(s,a) &= \ln M(s,a), && (108)
\end{aligned}$$

from which we can generate various distress measures, the first of which is simply the mean time to bankruptcy given by $M_s(0,a)=K_s(0,a)=E[t|a]$ and used in this paper. Alternatively, one can use saddlepoint approximation to the probability of default within a given time period.[22]

[22] See sections 1 and 16 in Butler (2007) for an excellent introduction to saddlepoint approxima-

E. Multiple Macroeconomic Shocks and Failure of Conditional CAPM

This section presents a model with multiple macroeconomic shocks that has the same quantitative implications as the benchmark model for the patterns we are interested in, although the conditional, and hence the unconditional, version of the Capital Asset Pricing Model (CAPM) does not hold. Moreover, this section shows that the relationship between CAPM beta and expected excess returns does not even have to be positively correlated in the cross-section at every date when we have mulptiple shocks.

Suppose that the aggregate fluctuations in the economy are caused by two shocks, dw_A and dw_B, but the investors' willingness to be exposed to each shock is different. Then, we can express the investors' stochastic discount factor as

$$\frac{d\Lambda}{\Lambda} = -rdt - \sigma_{S,A}dw_A - \sigma_{S,B}dw_B, \tag{109}$$

where we assume, without loss of generality, $\sigma_{S,A} > \sigma_{S,B}$. Moreover, the cash flows of the individuals firms are allowed to depend on both of these shocks, that is

$$\frac{dX_i}{X_i} = \mu_X dt + \sigma_0\left(\rho_{A,i}dw_A + \sqrt{1-\rho_{A,i}^2}dw_B\right) + \sigma_1 dw_i. \tag{110}$$

Note that this cash-flow process shares the properties of the cash-flow process in the original model because the drift, μ_X, and the variance, $\sigma^2 = \sigma_0^2 + \sigma_1^2$, of the cash-flow process is the same across firms whereas the perceived riskiness of the cash-flow process is increasing in $\rho_{A,i}$. Moreover, the cash-flow process (110) has the drift $\mu = \mu_X - \sigma_0\left(\rho_{A,i}\sigma_{S,A} + \sqrt{1-\rho_{A,i}^2}\sigma_{S,B}\right)$ under the risk-neutral measure. Hence, there is a one-to-one mapping between the risk-neutral drift in this extended model and the original model with a single shock, given by $\rho_{A,i}\sigma_{S,A} + \sqrt{1-\rho_{A,i}^2}\sigma_{S,B} = \rho_i\sigma_S$. As a result, the Hamilton-Jacobi-Bellman equations that govern the market values of equity and debt, and hence the decisions of the firms, will stay the same as in the original model. Therefore, this multi-shock

tions and Wood, Booth, and Butler (1993) for further details about saddlepoint approximation under inverse Gaussian base.

model has the same implications for the cross-section of returns as the original model.

However, unlike the original model, the conditional version of CAPM does not hold anymore. If we let $R_{i,t}$ be the cumulative returns of firm i until date t we can write the instantenous excess returns as

$$\begin{aligned} dR_i - rdt &= \frac{\pi_i dt + dJ_i}{J_i} \\ &= \Delta_i \left[\begin{array}{c} \sigma_0 \left(\rho_{A,i}\sigma_{S,A} + \sqrt{1-\rho_{A,i}^2}\sigma_{S,B} \right) dt \\ +\sigma_0 \left(\rho_{A,i} dw_A + \sqrt{1-\rho_{A,i}^2} dw_B \right) + \sigma_1 dw_i \end{array} \right], \end{aligned} \tag{111}$$

where $\Delta_i \equiv J_{i,X}X_i/J_i$. This leads to the following process for excess market return

$$\begin{aligned} dR_m - rdt &= \frac{\int_i J_i \left(dR_i \right) di}{\int_i J_i di} - rdt \\ &= \sigma_0 \left(\sigma_{S,A}\Delta_{m,A} + \sigma_{S,B}\Delta_{m,B} \right) dt \qquad (112) \\ &\quad +\sigma_0 \left(\Delta_{m,A} dw_A + \Delta_{m,B} dw_B \right) dt, \qquad (113) \end{aligned}$$

where $\Delta_{m,A} \equiv \int_i \rho_{A,i} J_{i,X} X_i di / \int_i J_i di$ and $\Delta_{m,B} \equiv \int_i \sqrt{1-\rho_{A,i}^2} J_{i,X} X_i di / \int_i J_i di$.

In this setting, the conditional beta is given by

$$\beta_{i,t} = \frac{cov_t \left(dR_i, dR_m \right)}{var_t \left(R_m \right)} = \Delta_i \frac{\rho_{A,i}\Delta_{m,A} + \sqrt{1-\rho_{A,i}^2}\Delta_{m,B}}{\Delta_{m,A}^2 + \Delta_{m,B}^2}, \tag{114}$$

and it is straightforward to show that $E_t \left(dR_i - rdt \right) \neq \beta_{i,t} E_t \left(dR_m - rdt \right)$ so that CAPM fails. There does not even have to be a systematically positive relationship between expected returns and betas in this setting because $\Delta_{m,A}$ and $\Delta_{m,B}$ are changing over time and there are instances where $\Delta_{m,A} < \Delta_{m,B}$ and hence higher $\rho_{A,i}$ implies lower beta but higher expected returns because $\sigma_{S,A} > \sigma_{S,B}$.