# A Model of Mortgage Default

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**Abstract**

This paper solves a dynamic model of a household's decision to default on its mortgage, taking into account labor income, house price, inflation, and interest rate risk. Mortgage default is triggered by negative home equity, which results from declining house prices in a low inflation environment with large mortgage balances outstanding. Not all households with negative home equity default, however. The level of negative home equity that triggers default depends on the extent to which households are borrowing constrained. High loan-to-value ratios at mortgage origination increase the probability of negative home equity. High loan-to-income ratios also increase the probability of default by tightening borrowing constraints. Comparing mortgage types, adjustable-rate mortgage defaults occur when nominal interest rates increase and are substantially affected by idiosyncratic shocks to labor income. Fixed-rate mortgages default when interest rates and inflation are low, and create a higher probability of a default wave with a large number of defaults. Interest-only mortgages trade off an increased probability of negative home equity against a relaxation of borrowing constraints, but overall have the highest probability of a default wave.

# 1 Introduction

Many different factors contributed to the global financial crisis of 2007-09. One such factor seems to have been the growing availability of subprime mortgage credit in the mid-2000s. Households were able to borrow higher multiples of income, with lower required downpayments, often using adjustable-rate mortgages with low initial "teaser" rates. Low initial interest rates made the mortgage payments associated with large loans seem affordable for many households.

The onset of the crisis was characterized by a fall in house prices, an increase in mortgage defaults and home foreclosures, and a decrease in the value of mortgage-backed securities. These events initially affected residential construction and the financial sector, but their negative effects spread quickly to other sectors of the economy. Foreclosures appear also to have had negative feedback effects on the values of neighboring properties, worsening the decline in house prices (Campbell, Giglio, and Pathak 2011). The crisis has emphasized the importance of understanding household incentives to default on mortgages, and the way in which these incentives vary across different types of mortgage contracts. This paper studies the mortgage default decision using a theoretical model of a rational utility-maximizing household.

We solve a dynamic model of a household who finances the purchase of a house with a mortgage, and who must in each period decide how much to consume and whether to default on the loan. Several sources of risk affect household decisions and the value of the option to default on the mortgage, including house prices, labor income, inflation, and real interest rates. We use labor income and house price data from the Panel Study of Income Dynamics (PSID), and interest rate and inflation data published by the Federal Reserve to parameterize these sources of risk.

The existing literature on mortgage default has emphasized the role of house prices and home equity accumulation for the default decision. Deng, Quigley, and Van Order (2000) estimate a model, based on option theory, in which a household's option to default is exercised if it is in the money by some specific amount. Borrowers do not default as soon as home equity becomes negative; they prefer to wait since default is irreversible and house prices may increase. Earlier empirical papers by Vandell (1978) and Campbell and Dietrich (1983) also emphasized the importance of home equity for the default decision.

In our model also, mortgage default is triggered by negative home equity which tends to occur for a particular combination of the several shocks that the household faces: house price declines in a low inflation environment with large nominal mortgage balances outstanding. As in the previous literature, households do not default as soon as home equity becomes negative.

A novel prediction of our model is that the level of negative home equity that triggers default depends on the extent to which households are borrowing constrained; some households with more negative home equity than defaulting households, but who are less borrowing constrained than the defaulters, choose not to default. The degree to which borrowing constraints bind depends on the realizations of income shocks, the endogenously chosen level of savings, the level of interest rates, and the terms of the mortgage contract. For example, adjustable-rate mortgages (ARMs) tend to default when interest rates increase, because high interest rates increase required mortgage payments on ARMs, tightening borrowing constraints and triggering defaults.

We use our model to explore several interesting questions about mortgage defaults. First, we investigate the extent to which the loan-to-value (LTV) and loan-to-income (LTI) ratios at mortgage origination affect default probabilities. The LTV ratio measures the equity stake that households have in the house. Naturally, a lower equity stake at mortgage initiation (i.e. a higher LTV ratio) increases the probability of negative home equity and default. This effect has been documented empirically by Schwartz and Torous (2003) and more recently by Mayer, Pence, and Sherlund (2009). Regulators in many countries, including Austria, Poland, China and Hong Kong, ban high LTV ratios in an effort to control the incidence of mortgage default.

The contribution of the LTI ratio to default is less well understood. LTI and the ratio of mortgage payments to household income (MTI) are measures of mortgage affordability that are often used by mortgage providers to determine the maximum loan amount and the interest rate. These measures have also drawn the attention of regulators, who have imposed LTI and MTI thresholds, either in the form of guidelines or strict limits. Among the countries where that is the case are the Netherlands, Hong Kong, and China. The nature of these thresholds varies. For instance, in Hong Kong, in 1999, the maximum LTV of 70% was increased to 90% provided that borrowers satisfied a set of eligibility criteria based on a maximum debt-to-income ratio, a maximum loan amount, and a maximum loan maturity at mortgage origination.

A clear understanding of the relation between LTV, LTI, and MTI ratios and mortgage defaults is particularly important in light of the recent US experience. Figure 1 plots aggregate ratios for the US over the last couple of decades.[3] This figure shows that there was an increase in the average LTV in the years before the crisis, but to a level that does not seem high by historical standards. What is particularly striking is the large increase in the LTI ratio, from an average of 3.3 during the 1980's and 1990's to a value as high as 4.5 in the mid 2000s. This pattern in the LTI ratio is not confined to the US; in the United Kingdom the average LTI ratio increased from roughly two in the 1970's and 1980's to above 3.5 in the years leading to the credit crunch (Financial Services Authority, 2009). Interestingly, as can be seen from Figure 1, the low interest-rate environment in the 2000s prevented the increase in LTI from driving up MTI to any great extent.

Our model allows us to understand the channels through which LTV and mortgage affordability affect mortgage default. A smaller downpayment increases the probability of negative home equity, and reduces borrowers' incentives to meet mortgage payments. The unconditional default probabilities predicted by the model become particularly large for LTV ratios in excess of ninety percent. The LTI ratio affects default probabilities through a different channel. A higher LTI ratio does not increase the probability of negative equity; however, it reduces mortgage affordability making borrowing constraints more likely to bind. The level of negative home equity that triggers default becomes less negative, and default probabilities accordingly increase. Our model implies that mortgage providers and regulators should think about combinations of LTV and LTI and should not try to control these parameters in isolation.

A second topic we explore is the effect of mortgage contract terms on default rates. We first compare default rates for adjustable-rate mortgages (ARMs) and fixed-rate mortgages (FRMs). We find that even though defaults of a few individuals are a more common occurrence for ARMs, defaults of a large fraction of borrowers have a higher, albeit small, probability for FRMs than for ARMs. This reflects the fact that aggregate shocks are a relatively more important determinant of the decision to default in a FRM contract than in an ARM contract. For the latter, idiosyncratic income shocks are relatively more important, and households are more likely to default for liquidity reasons.

[3]The LTV data are from the monthly interest rate survey of mortgage lenders conducted by the Federal Housing Finance Agency, and the LTI series is calculated as the ratio of average loan amount obtained from the same survey to the median US household income obtained from census data. The survey data is available at www.fhfa.gov.

Unsurprisingly, large default rates on both ARMs and FRMs occur in aggregate states in which there are large declines in house prices. However, for aggregate states characterized by moderate declines in house prices, ARM defaults tend to occur when interest rates are high, whereas the reverse is true for FRMs. Therefore, we find that given moderate house price declines, default rates between ARMs and FRMs are uncorrelated. This creates an opportunity for mortgage investors to diversify default risk at the portfolio level by holding both ARMs and FRMs.

During the recent crisis, interest-only and other alternative mortgage products have been criticized for their higher delinquency and default rates compared to traditional principal-repayment mortgages (Mayer, Pence, and Sherlund, 2009). Interest-only (IO) mortgages defer principal repayments to late in the life of the loan, so the loan amount outstanding at each date is larger, increasing the probability that the household will be faced with negative home equity. This increases the probability of default. On the other hand, IO mortgages have lower cash outlays, or lower mortgage payments relative to income, so that this increases the affordability of these mortgages, relaxes borrowing constraints and reduces default probabilities.

We use our model to study balloon mortgages (IO mortgages with principal repayment at maturity). We find that the relaxation of borrowing constraints dominates early in the life of the mortgage, but default rates become larger than for principal-repayment mortgages late in the life of the mortgage due to the considerably higher probability of negative home equity. Thus default rates for balloon mortgages are less sensitive to drops in house prices in the early years of the loan, but more sensitive to the longer-term evolution of house prices. This also means that mortgage default decisions are more correlated across borrowers for IO mortgages than for other mortgage types, and in this sense, IO mortgages have higher systemic risk.

Households are heterogenous in many respects, for example their human capital characteristics, expected house price appreciation, and risk and time preferences. In a third application of our model, we investigate how such heterogeneity impacts mortgage default rates. For instance, we consider two households who have the same current income, but who differ in terms of the expected growth rate of their labor income. The higher the growth rate, the smaller are the incentives to save, which increases default probabilities. However, we find that this effect is weaker than the direct effect of higher future income on mortgage affordability, as measured for example by the MTI ratio later in the life of the loan. Therefore the mortgage default rate decreases with the expected growth rate of labor income.

Several recent empirical papers study mortgage default. Foote, Gerardi, and Willen (2008) examine homeowners in Massachusetts who had negative home equity during the early 1990s and find that fewer than 10% of these owners eventually lost their home to foreclosure, so that not all households with negative home equity default. Bajari, Chu, and Park (2009) study empirically the relative importance of the various drivers behind subprime borrowers' decision to default. They emphasize the role of the nationwide decrease in home prices as the main driver of default, but also find that the increase in borrowers with high payment to income ratios has contributed to increased default rates in the subprime market. Mian and Sufi (2009) emphasize the importance of an increase in mortgage supply in the mid-2000s, driven by securitization that created moral hazard among mortgage originators.

The contribution of our paper is to propose a dynamic and unified microeconomic model of rational consumption and mortgage default in the presence of house price, labor income, and interest rate risk.[4] Our goal is not to try to derive the optimal mortgage contract (as in Piskorski and Tchistyi, 2010, 2011), but instead to study the determinants of the default decision within an empirically parameterized model, and to compare outcomes across different types of mortgages. In this respect our paper is related to the literature on mortgage choice (see for example Brueckner 1994, Stanton and Wallace 1998, 1999, Campbell and Cocco 2003, and Koijen, Van Hermert, and Van Nieuwerburgh 2010). Our work is also related to the literature on the benefits of homeownership, since default is a decision to abandon homeownership and move to rental housing. For example, we show that the ability of homeownership to hedge fluctuations in housing costs (Sinai and Souleles 2005) plays an important role in deterring default. Similarly, the tax deductibility of mortgage interest not only creates an incentive to buy housing (Glaeser and Shapiro, 2009, Poterba and Sinai, 2011), but also reduces the incentive to default on a mortgage.

Our paper is also related to interesting recent research by Corbae and Quintin (2010). They solve an equilibrium model to try to evaluate the extent to which low downpayments and IO mortgages were responsible for the increase in foreclosures in the late 2000s, and find that mortgages with these features account for 40% of the observed foreclosure increase. Garriga and Schlagenhauf (2009) also solve an equilibrium model of long-term mortgage choice to understand

[4] Ghent (2011) proposes a model of mortgage choice in which borrowers have hyperbolic preferences.

how leverage affects the default decision. Our paper does not attempt to solve for mortgage market equilibrium, and therefore can examine household risks and mortgage terms in more realistic detail, distinguishing the contributions of short- and long-term risks, and idiosyncratic and aggregate shocks, to the default decision. One aspect that we emphasize is the influence of realized and expected inflation on the default decision, an aspect which is absent in real models of mortgage default. In this respect our work complements the research of Piazzesi and Schneider (2010), who show that inflation can have a significant impact on asset prices.

The paper is organized as follows. In section 2 we set up the model, building on Campbell and Cocco (2003) with extensions to study the mortgage default decision. We study unconditional average default rates for standard principal-repayment mortgages, both fixed- and adjustable-rate, and for balloon mortgages in section 3. Section 4 looks at default rates conditional on specific realizations of aggregate state variables, thereby clarifying the relative contributions of aggregate and idiosyncratic shocks to the default decision. Section 5 explores household heterogeneity, and section 6 carries out some robustness exercises. The final section concludes.

# 2 The Model

## 2.1 Setup

### 2.1.1 Time parameters and preferences

We model the consumption and default choices of a household with a $T$-period horizon that uses a mortgage to finance the purchase of a house of fixed size $H$. We assume that household preferences are separable in housing and non-durable consumption, and are given by:

$$\max\ E_1 \sum_{t=1}^{T} \beta^{t-1} \frac{C_t^{1-\gamma}}{1-\gamma} + \beta^T b \frac{W_{T+1}^{1-\gamma}}{1-\gamma}, \tag{1}$$

where $T$ is the terminal age, $\beta$ is the time discount factor, $C_t$ is non-durable consumption, and $\gamma$ is the coefficient of relative risk aversion. The household derives utility from both consumption and terminal real wealth, $W_{T+1}$, which can be interpreted as the remaining lifetime utility from reaching age $T+1$ with wealth $W_{T+1}$. Terminal wealth includes both financial and housing wealth. The parameter $b$ measures the relative importance of the utility derived from terminal wealth.

Since we have assumed that housing and non-durable consumption are separable and that $H$ is fixed, we do not need to include housing explicitly in household preferences. However, the above preferences are consistent with:

$$\max\ E_1 \sum_{t=1}^{T} \beta^{t-1} [\frac{C_t^{1-\gamma}}{1-\gamma} + \theta \frac{H_t^{1-\gamma}}{1-\gamma}] + \beta^T b \frac{W_{T+1}^{1-\gamma}}{1-\gamma}, \qquad (2)$$

for $H_t = H$ fixed and where the parameter $\theta$ measures the importance of housing relative to other non-durable consumption.

Naturally, in reality, $H$ is not fixed and depends on household preferences and income, among other factors. We simplify the analysis here by abstracting from housing choice, but we do study mortgage default for different values of $H$. Later in the paper, in section 6.3, we consider a simple model of housing choice to make sure that our main results are robust to this consideration.

#### 2.1.2 Interest and inflation rates

Nominal interest rates are variable over time. This variability comes from movements in both the expected inflation rate and the ex-ante real interest rate. We use a simple model that captures variability in both these components of the short-term nominal interest rate.

We write the nominal price level at time $t$ as $P_t$, and normalize the initial price level $P_1$=1. We adopt the convention that lower-case letters denote log variables, thus $p_t \equiv \log(P_t)$ and the log inflation rate $\pi_t = p_{t+1} - p_t$. To simplify the model, we abstract from one-period uncertainty in realized inflation; thus expected inflation at time $t$ is the same as inflation realized from $t$ to $t+1$. While clearly counterfactual, this assumption should have little effect on our results since short-term inflation uncertainty is quite modest. We assume that expected inflation follows an AR(1) process. That is,

$$\pi_t = \mu(1-\phi) + \phi\pi_{t-1} + \epsilon_t, \qquad (3)$$

where $\epsilon_t$ is a normally distributed white noise shock with mean zero and variance $\sigma_\epsilon^2$. We assume that the ex-ante real interest rate is time-varying and serially uncorrelated. The expected log real return on a one-period bond, $r_{1t} = \log(1 + R_{1t})$, is given by:

$$r_{1t} = \overline{r} + \varepsilon_t, \tag{4}$$

where $\overline{r}$ is the mean log real interest rate and $\varepsilon_t$ is a normally distributed white noise shock with mean zero and variance $\sigma_\varepsilon^2$.

The log nominal yield on a one-period nominal bond, $y_{1t} = \log(1 + Y_{1t})$, is equal to the log real return on a one-period bond plus expected inflation:

$$y_{1t} = r_{1t} + \pi_t. \tag{5}$$

#### 2.1.3 Labor income

The household is endowed with stochastic gross real labor income in each period, $L_t$, which cannot be traded or used as collateral for a loan. As usual we use a lower case letter to denote the natural log of the variable, so $l_t \equiv \log(L_t)$. The household's log real labor income is exogenous and is given by:

$$l_t = f(t, Z_t) + v_t + \omega_t, \tag{6}$$

where $f(t, Z_t)$ is a deterministic function of age $t$ and other individual characteristics $Z_t$, and $v_t$ and $\omega_t$ are random shocks. In particular, $v_t$ is a permanent shock and assumed to follow a random walk:

$$v_t = v_{t-1} + \eta_t, \tag{7}$$

where $\eta_t$ is an i.i.d. normally distributed random variable with mean zero and variance $\sigma_\eta^2$. The other shock represented by $\omega_t$ is transitory and follows an i.i.d. normal distribution with mean zero and variance $\sigma_\omega^2$. Thus log income is the sum of a deterministic component and two random components, one transitory and one persistent.

We let real transitory labor income shocks, $\omega_t$, be correlated with innovations to the stochastic process for expected inflation, $\epsilon_t$, and denote the corresponding coefficient of correlation $\varphi$. In a world where wages are set in real terms, this correlation is likely to be zero. If wages are set in nominal terms, however, the correlation between real labor income and inflation may be negative.

We model the tax code in the simplest possible way, by considering a linear taxation rule. Gross labor income, $L_t$, and nominal interest earned are taxed at the constant tax rate $\tau$. We allow for deductibility of nominal mortgage interest at the same rate.

### 2.1.4 House prices and other housing parameters

The price of housing fluctuates over time. Let $P_t^H$ denote the date $t$ real price of housing, and let $p_t^H \equiv \log(P_t^H)$. We normalize $P_1^H = 1$ so that $H$ also denotes the value of the house that the household purchases at the initial date. The real price of housing is a random walk with drift, so real house price growth can be written as:

$$\Delta p_t^H = g + \delta_t, \tag{8}$$

where $g$ is a constant and $\delta$ is an i.i.d. normally distributed random shock with mean zero and variance $\sigma_\delta^2$. We assume that the shock $\delta_t$ is uncorrelated with inflation, so in our model housing is a real asset and an inflation hedge. It would be straightforward to relax this assumption.

We assume that innovations to real house prices, $\delta_t$, are correlated with innovations to the permanent component of the household's real labor income, $\eta_t$, and denote by $\rho$ the corresponding coefficient of correlation. When this correlation is positive, states of the world with high house prices are also likely to have high permanent labor income.

We assume that in each period homeowners must pay property taxes, at rate $\tau_p$, proportional to house value, and that property tax costs are income-tax deductible. In addition, homeowners must pay a maintenance cost, $m_p$, proportional to the value of the property. This can be interpreted as the maintenance cost of offsetting property depreciation. The maintenance cost is not income-tax deductible.

### 2.1.5 Mortgage contracts

The household finances the initial purchase of a house of size $H$ with previously accumulated savings and a nominal mortgage loan of $(1-d)H$, where $d$ is the required down-payment. (Recall that we have normalized, without loss of generality, $P_1^H$ and $P_1$ to one.) The LTV and LTI ratios at mortgage origination are therefore given by:

$$LTV = (1-d) \tag{9}$$

$$LTI = \frac{(1-d)H}{L_1}, \tag{10}$$

where $L_1$ denotes the level of household labor income at the initial date.

Required mortgage payments depend on the type of mortgage. We consider several alternative types, including FRM, ARM, and balloon mortgages with loan principal repayment at maturity (we also call these interest-only mortgages).

Let $Y_T^{FRM}$ be the interest rate on a FRM with maturity $T$. It is equal to the expected interest rate over the life of the loan plus an interest rate premium. The date $t$ real mortgage payment, $M_t^{FRM}$, is given by the standard annuity formula:

$$M_t^{FRM} = \frac{(1-d)H\left[\left(Y_T^{FRM}\right)^{-1} - \left(Y_T^{FRM}(1+Y_T^{FRM})^T\right)^{-1}\right]^{-1}}{P_t}. \tag{11}$$

For simplicity we abstract from the refinancing decision. In many countries FRMs do not include an option to refinance. In addition, most households with negative home equity are unable to refinance, so default decisions are little affected by this option.

Let $Y_{1t}^{ARM}$ be the one-period nominal interest rate on an ARM, and let $D_t^{ARM}$ be the nominal principal amount outstanding at date $t$. The date $t$ real mortgage payment, $M_t^{ARM}$, is given by:

$$M_t^{ARM} = \frac{Y_{1t}^{ARM} D_t^{ARM} + \Delta D_{t+1}^{ARM}}{P_t}, \tag{12}$$

where $\Delta D_{t+1}^{ARM}$ is the component of the mortgage payment at date $t$ that goes to pay down principal rather than pay interest. We assume that for the ARM the principal loan repayments, $\Delta D_{t+1}^{ARM}$, equal those that occur for the FRM. This assumption simplifies the solution of the model since the outstanding mortgage balance is not a state variable.

A household with a balloon mortgage pays interest each period but only repays the principal at maturity. Therefore the date $t < T$ real mortgage payment is given by:

$$M_t^{IO} = \frac{Y_{1t}^{IO}(1-d)H}{P_t}, \tag{13}$$

and the principal amount outstanding is constant in nominal terms over the life of the loan. This type of mortgage is available in the UK and some other countries, although in the US the most common type of IO mortgages involve an interest-only period that varies in length, after which the loan resets, and borrowers start paying the principal in addition to the interest.

The date $t$ nominal interest rate for both ARM and IO mortgages is equal to the short rate plus a constant premium:

$$Y_{1t}^i = Y_{1t} + \psi^i. \tag{14}$$

where the mortgage premium $\psi^i$, for $i = ARM, IO$, compensates the lender for default risk. For a FRM the interest rate is fixed over the life of the loan, and equals the average interest rate over the loan maturity plus a premium $\psi^{FRM}$. As previously noted, we assume that mortgage interest payments are tax deductible at the income tax rate $\tau$. IO mortgages maximize the benefits of this income-tax deductibility.

### 2.1.6 Mortgage default and home rental

In each period the household decides whether or not to default on the mortgage loan. The household may be forced to default because it has insufficient cash to meet the mortgage payment. However, the household may also find it optimal to default, even if it has the cash to meet the payment.

We assume that in case of default mortgage providers have no recourse to the household's financial savings or future labor income. The mortgage provider seizes the house, the household is excluded from credit markets, and since it cannot borrow the funds needed to buy another house it is forced into the rental market for the remainder of the time horizon. This is a simplification; in the US households who default are excluded from credit markets for seven years.

We also assume that there is no positive exemption level in the case of bankruptcy. Ghent and Kudlyak (2011) use variation in exemption levels across US states to empirically evaluate their impact on default decisions. Li, White, and Zhu (2010) also study empirically how bankruptcy laws affect mortgage default. It would be straightforward to allow for a positive exemption level in our model. (See also Chatterjee and Eyigungor 2009 and Mitman 2011, who solve equilibrium models of the macroeconomic effects of bankruptcy laws and foreclosure policies.)

The rental cost of housing equals the user cost of housing times the value of the house (Poterba 1994, Diaz and Luengo-Prado 2008). That is, the date $t$ real rental cost $U_t$ for a house of size $H$ is given by:

$$U_t = [Y_{1t} - \mathrm{E}_t[(\exp(\Delta p^H_{t+1} + \pi_t) - 1] + \tau_p + m_p] P^H_t H, \tag{15}$$

where $Y_{1t}$ is the one-period nominal interest rate, $E_t[exp(\Delta p^H_{t+1} + \pi_{1t}) - 1]$ is the expected one-period proportional nominal change in the house price, and $\tau_p$ and $m_p$ are the property tax rate and maintenance costs, respectively. This formula implies that in our model the rent-to-price ratio varies with the level of interest rates.[5]

Relative to owning, renting is costly for two main reasons. First, homeowners benefit from the income-tax deductibility of mortgage interest and property taxes, without having to pay income tax on the implicit rent they receive from their home occupancy. Second, owning provides insurance against future fluctuations in rents and house prices (Sinai and Souleles, 2005). When permanent income shocks are positively correlated with house price shocks, however, households have an economic hedge against rent and house price fluctuations even if they are not homeowners.

We assume that in case of default the household is guaranteed a lower bound of $\underline{X}$ in per-period cash-on-hand, which can be viewed as a subsistence level. This assumption can be motivated by the existence of social welfare programs, such as means-tested income support. In terms of our model it implies that consumption and default decisions are not driven by the probability of extremely high marginal utility, which would be the case for power utility if there was a positive probability of extremely small consumption.

### 2.1.7 Early mortgage termination

We allow households who have accumulated positive home equity to sell their house, repay the outstanding debt, and move into rental accommodation. The house sale is subject to a

[5] Campbell, Davis, Gallin, and Martin (2009) provide an empirical variance decomposition for the rent-to-price ratio.

realtor's commission, a fraction $c$ of the current value of the property. In this way, albeit at a cost, households are able to access their accumulated housing equity, and use it to finance non-durable consumption.

Ideally, we would like to explicitly model households' decisions to refinance their mortgages. Mortgage refinancing can play an important role in consumption smoothing and can have macroeconomic implications (Chen, Michaux, and Roussanov, 2011). Unfortunately this extension would make the model intractable because it would add an additional state variable to the already large number of state variables in our model. However, we have solved the model under alternative assumptions regarding what households are allowed to do when they have accumulated positive home equity (either allowing them to sell and terminate the mortgage contract or not, and with different assumed transactions costs), and such alternative assumptions have little effect on default decisions in states of house price declines which are the focus of our paper.

## 2.2 Solution technique

Our model cannot be solved analytically. The numerical techniques that we use for solving it are standard. We discretize the state-space and the variables over which the choices are made. The state variables of the problem are age ($t$), cash-on-hand ($X_t$), whether the household has previously terminated the mortgage or not ($Def_t^S$, equal to one if previous termination and zero otherwise), real house prices ($P_t^H$), the nominal price level ($P_t$), inflation ($\pi_t$), the real interest rate ($r_{1t}$), and the level of permanent income ($v_t$). The choice variables are consumption ($C_t$), whether to default on the mortgage loan if no default has occurred before ($Def_t^C$, equal to one if the household chooses to default in period $t$ and zero otherwise), and in the case of positive home equity whether to terminate the mortgage contract ($Dterm_t^C$, equal to one if the household chooses to terminate the contract in period $t$ and zero otherwise).

In all periods before the last, if the household has not defaulted on or terminated its mortgage, its cash-on-hand evolves as follows:

$$X_{t+1}^i = (X_t - C_t)\frac{(1 + Y_{1t}(1-\tau))}{(1+\pi_t)} - M_t^i - (m_p + \tau_p)P_t^H H + L_{t+1}(1-\tau) + \frac{Y_{1t}^i D_t \tau}{P_t} + \tau_p P_t^H H\tau, \quad (16)$$

for $i = ARM, IO$. The equation describing the evolution of cash-on-hand for the FRM is similar, except that the mortgage interest tax deduction is calculated using the interest rate on that mortgage. Savings earn interest that is taxed at rate $\tau$. Next period's cash-on-hand is equal to savings plus after-tax interest, minus real mortgage payments (made at the end of the period), minus property taxes and maintenance expenses, plus next period's labor income and the tax deduction on nominal mortgage interest and on property taxes.

If the household has defaulted on or terminated its mortgage and moved to rental housing, the evolution of cash-on-hand is given by:

$$X_{t+1}^{Rent} = (X_t - C_t)\frac{(1 + Y_{1t}(1-\tau))}{(1+\pi_t)} - U_t + L_{t+1}(1-\tau). \tag{17}$$

where $U_t$ denotes the date $t$ real rental payment.

Terminal, i.e. date $T+1$, wealth is given by:

$$W_{T+1}^{i} = \frac{P_{T+1}X_{T+1} + P_{T+1}P_{T+1}^{H}H}{P_{T+1}^{Composite}}, \quad \text{for } i = ARM, FRM \text{ and } Def_{T+1}^{S} = 0 \tag{18}$$

$$W_{T+1}^{IO} = \frac{P_{T+1}X_{T+1} + P_{T+1}P_{T+1}^{H}H - (1-d)H}{P_{T+1}^{Composite}}, \quad \text{for } Def_{T+1}^{S} = 0, \tag{19}$$

$$W_{T+1}^{Rent} = \frac{P_{T+1}X_{T+1}}{P_{T+1}^{Composite}}, \quad \text{for } Def_{T+1}^{S} = 1. \tag{20}$$

For the ARM and FRM contracts, if the household has not previously defaulted or terminated the mortgage contract, terminal wealth is equal to financial wealth plus housing wealth. For the balloon mortgage, with principal repayment at maturity, we need to subtract the balloon payment. In the rental state, households only have financial wealth at the terminal date.

Households derive utility from real terminal wealth, so that in all of the above cases nominal terminal wealth is divided by a composite price index, denoted by $P_{T+1}^{Composite}$. This index is given by:

$$P_{T+1}^{Composite} = [(P_{T+1})^{1-\frac{1}{\gamma}} + \theta^{\frac{1}{\gamma}}(P_{T+1}P_{T+1}^{H})^{1-\frac{1}{\gamma}}]^{\frac{\gamma}{\gamma-1}} \tag{21}$$

where recall that $\gamma$ is the coefficient of relative risk aversion and $\theta$ measures the preference for housing relative to other goods in the preference specification (2). The above composite price index is consistent with our assumptions regarding preferences (Piazzesi, Schneider, and

Tuzel, 2007). The fact that nominal terminal wealth is scaled by a price index that depends on the price of housing implies that even in the penultimate period homeownership serves as an hedge against house price fluctuations. The larger is $\theta$ the stronger is such a hedging motive for homeownership.

We solve this problem by backwards induction starting from period $T + 1$. The shocks are approximated using Gaussian quadrature, assuming two possible outcomes for each of them. This simplifies the numerical solution of the problem since for each period $t$ we only need to keep track of the number of past high/low inflation, high/low permanent income shocks, and high/low house price shocks to determine the date $t$ price level, permanent income, and house prices. For each combination of the state variables, we optimize with respect to the choice variables. We use cubic spline interpolation to evaluate the value function for outcomes that do not lie on the grid for the state variables. In addition, we use a log scale for cash-on-hand. This ensures that there are more grid points at lower levels of cash-on-hand.

## 2.3 Parameterization

### 2.3.1 Time and preference parameters

In order to parameterize the model we assume that each period corresponds to one year. We set the initial age to 30 and the terminal age to 50. Thus mortgage maturity is 20 years. In the baseline parameterization we set the discount factor $\beta$ equal to 0.98 and the coefficient of relative risk aversion $\gamma$ equal to 2. The parameter $\theta$ that measures the preference for housing relative to other consumption is set to 0.3. But we recognize that there is household heterogeneity with respect to preference and other parameters, and later on we study the role that heterogeneity plays in mortgage default. The parameter that measures the relative importance of terminal wealth, $b$, is assumed to be equal to 400. This is large enough to ensure that households have an incentive to save, and that our model generates reasonable values for wealth accumulation. The time and preference parameters that we use in the baseline case are reported in the first panel of Table 1.

### 2.3.2 Labor income

We use data from the Panel Study of Income Dynamics (PSID) for the years 1970 to 2005 to calibrate the labor income process. Our income measure is broadly defined to include total reported labor income, plus unemployment compensation, workers compensation, social security transfers, and other transfers for both the head of the household and his spouse. We use such a broad measure to implicitly allow for the several ways that households insure themselves against risks of labor income that is more narrowly defined. Labor income was deflated using the consumer price index.

It is widely documented that income profile varies across education attainment (see for example Gourinchas and Parker, 2002). To control for this difference, following the existing literature, we partition the sample into three education groups based on the educational attainment of the head of the household. For each education group we regress the log of real labor income on age dummies, controlling for demographic characteristics such as marital status and household size, and allowing for household fixed effects. We use this smoothed income profile to calculate, for each education group, the average household income for an head with age 30 and the average annual growth rate in household income from ages 30 to 50. The estimated real labor income growth rate for households with a high-school degree is 0.8 percent, and we use this value in the benchmark case. The assumption of a constant income growth rate is a simplification of the true income profile that makes it easier to carry out comparative statics and to investigate the role of future income prospects on the default decision.

We use the residuals of the above panel regressions to estimate labor income risk. In order to mitigate the effects of measurement error on estimated income risk, we have winsorized the income residuals at the 5th and 95th percentiles. We follow the procedure of Carroll and Samwick (1997) to decompose the variance of the winsorized residuals into transitory and permanent components. The estimated values are reported in the second panel of Table 1.

### 2.3.3 House prices

We use house price data from the PSID to estimate the parameters of the house price process. In each wave, individuals are asked to assess the current market value of their houses. We obtain real house prices by dividing self-reported house prices by the consumer price index.

House price changes are calculated as the first difference of the logarithm of real house prices, for individuals who are present in consecutive annual interviews, and who report not having moved since the previous year.

In order to address the issue of measurement error, and similarly to labor income, we have winsorized the logarithm of real house price changes at the 5th and 95th percentiles (-36.6 and 40.3 percent, respectively). We use the winsorized data to calculate the expected value and the standard deviation of real house price changes, which are equal to 1.6% and 16.2%, respectively. This fairly large standard deviation probably is due, in part,to measurement error in the data. In the baseline value we use these estimated values, but we consider alternative parameterizations.[6]

#### 2.3.4 Correlation between labor income and house prices

We use household level data to estimate the correlation between labor income shocks and house price shocks. In order to do so we first calculate:

$$\Delta(l_t - \hat{f}_t) = [l_t - \hat{f}(t, Z_t)] - [l_{t-1} - \hat{f}(t-1, Z_{t-1})] = \eta_t + \omega_t - \omega_{t-1}, \tag{22}$$

where the symbol $\hat{f}$ denotes the predicted regression values. We estimate a correlation between (22) and the first differences in log house prices, $\delta_t$, that is positive and statistically significantly, and equal to 0.037. Under the model assumption that temporary labor income shocks, $\omega_t$, are serially uncorrelated and uncorrelated with house price shocks, this value implies a correlation between permanent labor income shocks, $\eta_t$, and house price shocks, $\delta_t$, equal to 0.191. This value reflects the fact that a significant component of the innovations to permanent labor income shocks is of an individual specific nature (and therefore uncorrelated with house prices). We set the remaining model correlations to zero.

[6]The fact that house values in PSID are self assessed and do not correspond to real transactions may raise some concerns. We have obtained data on median US house prices from the Monthly Interest Rate Survey for the years of 1991 to 2007. The average growth rate in real (nominal) house prices over this period was 1.2 (3.9) percent, with a standard deviation of 4.8 percent. This lower standard deviation is due to the fact that it is calculated using an aggregate house price index.

### 2.3.5 Interest and inflation rates

In order to parameterize the stochastic process for the real interest rate we use data on the US Treasury yield with 1-year maturity, published by the Federal Reserve. We calculate the real interest rate by deflating the nominal yield by the inflation rate. The estimated parameters for the process for the real interest rate and the AR(1) process for the inflation rate are reported in the third panel of Table 1.

### 2.3.6 Tax rates and other parameters

We follow Himmelberg, Mayer, and Sinai (2005) in setting the values for the tax rates and other housing parameters. More precisely, we set the income tax rate, $\tau$, equal to 0.25, the property tax rate $\tau_p$ equal to 0.015, and the property maintenance expenses, $m_p$, equal to 0.025. In addition we assume that a house sale is subject to a realtor commission, $c$, equal to 6 percent of the value of the house, which is a fairly standard value. Finally, we set the lower bound on (real) cash-on-hand to one thousand dollars.

### 2.3.7 Loan parameters

Our baseline scenario assumes a mortgage with a downpayment of 10 percent, or a loan-to-value ratio of 0.9, and a loan-to-income ratio equal to 4.5. Naturally we will study how LTV and LTI affect default rates, by solving our model for alternative values for these parameters. In the baseline case we set the credit risk premium on each of the mortgage loans, $\psi^i$, for $i = FRM, ARM, IO$ equal to 1%, which is also the value used by Himmelberg, Mayer, and Sinai (2005) in their calculations. This allows us to compare the determinants of mortgage default across the different types of mortgage loans, for a given premium. Naturally, to the extent that some of these mortgage types have higher default rates than others, the credit risk premium should also be larger. We plan to investigate this issue in future research. The loan parameter values are reported in the last panel of Table 1.

## 2.4 Simulated data

In order to study the determinants of mortgage default we solve, for each mortgage type, for the policy functions, and then use them to generate simulated data. Agents in our model are subject to both aggregate and idiosyncratic shocks. Aggregate shocks are to real house prices, the inflation rate, and real interest rates. Idiosyncratic shocks are innovations to the permanent component of the labor income process (which also have an aggregate component since they are positively correlated with house price shocks) and temporary labor income shocks.

We first generate one realization for the aggregate shocks and then for this realization we generate realizations for the shocks to the labor income process for fifty individuals. We use the model policy functions, the one path for the aggregate variables and the individual income shocks to simulate optimal consumption and default behavior for these fifty individuals. We then repeat the process for a total of eight hundred different paths for the aggregate variables, and for fifty individuals for each of these paths. This yields, for each mortgage type, a total of forty thousand different paths. Naturally we use the same realizations for the shocks to simulate consumption and default behavior for each of the different mortgage types that we study.

In the next section we use the simulated data to predict unconditional default rates, that is average default rates calculated across the different paths for the aggregate and idiosyncratic shocks. Section 4 explores conditional default probabilities, given specific paths for aggregate variables. This analysis allows us to determine the relative contributions of aggregate and idiosyncratic shocks to default.

# 3 Unconditional Default Rates

## 3.1 Mortgage default triggers

To illustrate the determinants of mortgage default, we begin by reporting some results for a standard ARM. Figure 2 plots the age profiles of cross-sectional average real gross income, consumption and cash-on-hand (Panel A) and mortgage default rates (Panel B). Real consumption is on average considerably lower than real gross income. Naturally, the reason is that part of gross income must be paid in taxes, and the individual must also make mortgage payments and other housing related expenditures such as property taxes and maintenance expenses. Part of income is also saved. Although it is not completely visible from Figure 2, there is a slight decline in the average real consumption profile with age. This happens for two main reasons.

First, this is an average profile across many aggregate states, including those with declining house prices (and income). Second, we have estimated an average growth rate of house prices higher than labor income, and house price increases also drive up housing-related expenses. In section 5 we report results for a lower expected growth rate of house prices and a higher expected growth rate of income.

Panel B shows that most defaults occur between the ages of 32 and 38, or between two and eight years into the life of the loan. Schwartz and Torous (2003) have found in regressions aimed at explaining default rates that the age of the mortgage plays an important role. Figure 2 shows that our model is consistent with this empirical finding, with almost no default taking place in the second decade of the mortgage life. The level of default rates in figure 2 may at first seem low, but it is important to remember that these are average default rates calculated across many different aggregate paths, including paths of house price increases.

We are interested in determining what triggers default in our model. A natural candidate is home equity. The empirical literature on mortgage default has emphasized the importance of this variable (see for example Deng, Quigley, and Van Order 2000, or more recently Foote, Gerardi, and Willen 2008 and Bajari, Chu, and Park 2009). We calculate for each household and for each date $t$ the current house value as a fraction of currently outstanding debt:

$$Equity_t = \frac{P_t P_t^H H}{D_t} \tag{23}$$

where $D_t$ denotes the loan principal amount outstanding at date $t$, $P_t$ denotes the price level, and $P_t^H$ the real price of housing. The latter two are a function of the realization of the inflation and house price shocks between periods 1 and $t$, respectively. Taking natural logarithms of the above:

$$\ln(Equity_t) = \ln(P_t) + \ln(P_t^H) + \ln(H) - \ln(D_t). \tag{24}$$

When households are underwater (that is, when they have negative home equity) then $Equity_t$ is less than one and $\ln(Equity_t)$ is less than zero.

Figure 3 plots the date $t$ price level, house prices, and remaining debt against $\ln(Equity_t)$. The top graph shows the data for the households who choose not to default at date $t$, whereas the bottom graph shows the data for the agents who choose to default. The figure illustrates that defaulting households tend to have negative home equity ($\ln(Equity_t) < 0$). We say tend to have since Panel B shows that there is a small number of households who choose to default with small amounts of positive equity. However, these are amounts smaller than the transaction costs of an house sale, so that net of transaction costs these households have negative home equity.

Figure 3 also shows that negative home equity tends to occur for a particular combination of the state variables: declines in house prices (which result in a low $P_t^H$), when the price level is low, and at times when there are large mortgage balances outstanding (early in the life of the loan). Most interestingly, not all households with negative equity choose to default. Panel A of Figure 3 shows that indeed there are households who are underwater but who choose not to default. We focus on these households and construct a variable that measures the ratio of current mortgage payments to household income ($MTI$):

$$MTI_t = \frac{M_t}{L_t} \tag{25}$$

In Figure 4 we plot this variable, for households with negative equity, by default decision. For each level of negative home equity, the columns report the average MTI for households who choose to default and for those who choose not to do so. In addition, the straight line plots default probabilities, conditional on the level of (negative) equity. These probabilities are calculated using one observation per mortgage, so that for those households who choose never to default, in spite of being faced with negative equity, we calculate these probabilities using the lowest level of equity that the household faces during the life of the mortgage. This is similar to the calculations carried out by Bhutta, Dokko, and Shan (2010) who study default rates for non-prime borrowers from Arizona, California, Florida, and Nevada.

Figure 4 shows that many households do not default at low levels of negative home equity. It is only when equity becomes sufficiently negative that default occurs. Thus households only exercise the put option when it is in the money by some amount. This prediction of our model is consistent with the evidence in Bhutta, Dokko, and Shan (2010), who find that the median homeowner does not default until equity falls to -62 percent of their home's value. It is also consistent with the evidence in Foote, Gerardi, and Willen (2008) who study one hundred thousand homeowners in Massachusetts who had negative equity during the early 1990s, and find that fewer than ten percent of these owners lost their home to foreclosure.

The prediction that borrowers do not default as soon as home equity becomes negative is also a prediction of models that take an option theory approach to the mortgage default decision. But as can be seen from Figure 4, in our model the ratio of mortgage payments to household income also plays an important role; for house values between 100 and 80 percent of outstanding loan principal it is those households with a larger value for MTI that tend to default. Large mortgage payments relative to household income, in the presence of borrowing constraints and low savings, force a choice between severe consumption cutbacks and mortgage default. This is an important determinant of default early in the life of the mortgage, and at low levels of negative equity. Elul, Souleles, Chomsisengphet, Glennon and Hut (2010) provide empirical evidence of the importance of liquidity considerations for the mortgage default decisions.

## 3.2 Comparing adjustable-rate and fixed-rate mortgages

Table 2 reports the means of several variables for households who choose to default, for households with negative home equity but who choose not to default, and for households who choose not to default (regardless of the level of home equity). This table shows means for both an ARM and for a FRM. Focusing first on the ARM, we see that households with negative home equity who default tend to have more negative home equity than those with negative home equity but who choose not to default. The current loan to value is 1.37 for the former group, compared to 1.13 for the latter.

In addition, for ARMs, households who choose to default are those with low income and large mortgage payments relative to income. Average real income is forty thousand dollars and the MTI ratio averages 0.49 for households who default, compared to a real income of forty-eight thousand dollars and an average MTI ratio of 0.36 for households with negative equity who choose not to default. Table 2 also reports real rental payments and the difference between mortgage and rent payments scaled by household income. Rental payments are on average much lower than mortgage payments. This is of course due to the fact that mortgage payments cover both interest and principal repayments. For households significantly underwater who choose to default, that decision allows for a reduction in current expenditure of 35 percent of income. For those that choose not to default the reduction would only be 22 percent of income.

Therefore, in our model default occurs as a result of both wealth and cash-flow reasons. House price declines lead to situations of negative home equity. Those households who face larger house price declines, at times when outstanding debt is large, are more likely to default. Since house price shocks are correlated with permanent income shocks, larger house price declines tend to be associated with larger decreases in household income (second panel of Table 2). This forces households to cut back on non-durable consumption. For ARMs such cutbacks are more severe when interest rates (and expected inflation) are high, since they lead to an increase in mortgage payments. The real non-durable consumption of households who choose to default decreases considerably in the periods leading to default. The decision to default allows households to increase their non-durable consumption (third panel of Table 2).

Another metric of the relative importance of wealth and cash-flow motives for mortgage default is the proportion of defaults by households with low cash-on-hand. ARMs have a 2.3 percent unconditional default rate. Of those households who default, 51 percent have cash-on-hand lower than five thousand (real) dollars.

Turning to fixed-rate mortgages, we see that although unconditional default rates are slightly higher for FRMs than ARMs, the proportion of defaults by households with low cash-on-hand is considerably smaller, equal to 22 percent. Furthermore, compared to ARMs, FRMs are characterized by a smaller decrease in consumption prior to default, and by a smaller increase in consumption subsequent to default. Thus cash-flow motives seem to be a relatively less important determinant of the default decision for FRMs than for ARMs. Consistent with this is the fact that default for FRMs tends to occur at higher current loan-to-values, at higher levels of household income, and later in the life of the mortgage, than for ARMs.

Figure 5 plots cumulative default rates, with age, for both types of mortgages. Early in the life of a mortgage, default rates are considerably higher for ARMs than for FRMs. Early ARM defaulters tend to be households who face low levels of negative equity, and large mortgage payments relative to income, who default for cash-flow reasons. Later in the life of the mortgage, at high levels of negative home equity, wealth motives become more important for the default decision. It is at this stage that cumulative default rates become larger for FRMs than for ARMs.

The fact that wealth motives tend to be an important determinant of default decisions at high levels of negative equity is consistent with the empirical findings of Haughwout, Okah, and Tracy (2010). They study mortgage re-default using data on subprime mortgage modifications for borrowers who were seriously delinquent, and whose monthly mortgage payment was reduced as part of the modification. They find that the re-default rate declines relatively more when the payment reduction is achieved through principal forgiveness as compared to lower interest rates. The empirical analysis of Doviak and MacDonald (2011) also emphasizes the role of modifications that reduce loan balances in preventing default.[7]

Finally, the results in Table 2 show another important difference between default decisions for ARMs and FRMs. For ARMs default tends to occur when inflation and nominal interest rates are high, since high interest rates lead to large mortgage payments. On the other hand, for FRMs default tends to occur when inflation and interest rates are low, since low inflation implies a high real debt burden and low real interest rates imply a lower user cost of housing and lower rental payments compared to mortgage payments. Thus even though default tends to occur for both ARMs and FRMs when there are declines in house prices, there is a differential response to interest rates. This is an issue that we will investigate further in the next section, where we study the conditional default rates predicted by our model.[8]

## 3.3 The effects of LTV and LTI on default

We are interested in evaluating how LTV and LTI ratios at mortgage origination relate to subsequent default, so we solve our model for different values for these parameters. We are particularly interested in LTI since Figure 1 shows a significant increase in average LTI during the 2000s. One important advantage of using a model to study the effect of LTI is that we can compare outcomes across LTI for a common set of shocks to the households in the model.

With the previously discussed results on mortgage default triggers in mind, we decompose the probability of default into the probability that the household is faced with negative equity times the probability of default conditional on negative home equity:

$$\Pr(Default) = \Pr(Equity < 0) \times \Pr(Default/Equity < 0). \tag{26}$$

When calculating these probabilities, we also classify as having negative home equity those

[7] Das (2011) and Foote, Gerardi, Goette, and Willen (2009) provide model based analysis of mortgage loan modifications.

[8] Naturally, one way to take advantage of low interest rates in a FRM is to refinance, a feature which is absent from our model. However, it is important to remember that households who default in a FRM have significant levels of negative home equity, which may severely constrain their ability to refinance.

households whose house value net of the transactions costs of a house sale is lower than outstanding debt. Since as previously shown there are a few instances of default when house value is slightly higher than remaining debt, the classification of negative home equity using house value net of transaction costs ensures that the above equation holds exactly. Furthermore, the probability of negative home equity is calculated as the probability that the borrower faces at least one period of negative equity during the life of the mortgage.

The results are reported in Table 3. Panel A shows the results for ARMs: in the top panel we vary the LTV (for a given LTI) and in the bottom panel we vary the LTI (for a given LTV). Unsurprisingly, for higher LTV the probability of negative home equity is also higher. Quantitatively, there is a very large increase in this probability when we move from a LTV ratio of 0.8 to a LTV ratio of 0.9. Krainer, Leroy and Mungpyung (2009) develop an equilibrium valuation model that emphasizes the role of the initial LTV for mortgage default.

On the other hand, the probability of default conditional on negative equity varies non-monotonically with the LTV ratio. When the LTV ratio increases from 0.8 to 0.9 this probability decreases from 0.067 to 0.044. For higher LTV ratios households are more likely to be faced with negative equity early in the life of the loan, and are more likely to wait before defaulting. However, for very high LTV ratios the level of negative equity becomes very large, which reduces the value of the option to wait. This explains why the probability of default conditional on negative equity increases to 0.056 when we increase the LTV further to 0.95. This increase together with the larger probability of negative home equity explains why the unconditional probability of default increases significantly with LTV: from 1.6 percent for 80 percent LTV to 3.2 percent for 95 percent LTV.

In the bottom part of Panel A we vary LTI for a given LTV. Default rates increase with LTI because there is an increase in the probability of default conditional on negative equity. The higher the initial LTI the higher are mortgage payments relative to household income, which makes liquidity constraints more severe, and makes it more likely that households default when faced with negative equity. Quantitatively, it is interesting to see that the effect of LTI on the default probability is non-linear. When the LTI increases from 2.5 to 3.5, there is almost no effect in the probability of default conditional on negative equity. But this probability more than doubles when we increase the LTI further, to 4.5. The probability of negative home equity is fairly insensitive to the LTI.[9]

Panel B of Table 3 shows the results for FRMs. Qualitatively the patterns are similar to ARMs. But there are some interesting quantitative differences. Default rates for FRMs increase much less with LTI than those for ARMs: from 1.6 percent to 2.6 percent for the former compared to from 1 percent to 2.3 percent for the latter (for an increase in LTI from 2.5 to 4.5). Thus default probabilities for ARMs are more sensitive to LTI than default probabilities for FRMs. On the other hand, default probabilities for FRMs are more sensitive to LTV than default probabilities for ARM. For FRMs the default probability increases from a value of 1.5 percent for a LTV ratio of 0.8, to a value of 3.9 percent for a LTV ratio of 0.95. For the same increase in LTV ratios, the probability of default for ARMs increases from the same 1.6 percent to 3.2 percent.

This differential sensitivity of ARMs and FRMs to LTI and LTV ratios can be understood in light of our previous analysis of both types of mortgages. For ARMs a higher proportion of individuals default for cash-flow reasons. A higher LTI implies larger mortgage payments relative to income which makes borrowing constraints more likely to bind. On the other hand, for FRMs, a higher proportion of individuals default for wealth reasons. This makes default rates for these mortgages more sensitive to the LTV ratio. This distinction between cash-flow risk of ARMs and the wealth risk of FRMs has been emphasized by Campbell and Cocco (2003).

The fact that cash-flow risk is higher for ARMs than for FRMs can also be seen by considering the probability of early mortgage termination through a house sale. Recall that we allow those individuals who have accumulated positive home equity to sell their house and cash-out. The last row of each panel of Table 3 shows the proportion of individuals who do so, at some point during the life of the loan. It is important to note that the states under which such house sales take place tend to be orthogonal to the states under which default occurs, which are states of declining house prices. Table 3 shows that the probability of early mortgage termination is higher for ARMs than for FRMs, and that it is increasing in the LTI ratio. Furthermore, the higher the down-payment, the larger is home equity, and the larger are the individual incentives to tap into this equity.

[9] The small decrease shown in Table 3 is due to the fact that for higher values of LTI there is a larger proportion of individuals who terminate the mortgage early on, when they have positive home equity. Therefore, they are never faced with negative equity.

### 3.4 Alternative mortgage products

During the recent financial crisis, mortgage delinquency and default rates have been particularly large for alternative mortgage products. These come in many different forms, but generally share the feature that they postpone principal repayments to later in the life of the loan. We use our model to study these mortgages and to compare them to the more traditional principal-repayment mortgages.[10]

We consider a limiting type of an alternative mortgage, an IO mortgage for which loan principal repayment takes place only at maturity. This type of mortgage, which is also often called a balloon payment mortgage, is available in countries such as the UK, but it is not the most common type employed in the US. In the latter, the most common type of alternative mortgage is characterized by interest-only payments for a given number of years, that then resets to a principal-repayment mortgage. In the US a large number of these mortgages, originated during the mid-2000s, will reset within the next few years, and at the time of the reset there will be a large increase in mortgage payments. The concern is that such an increase will lead households to default on their mortgages.

Panel C of Table 3 decomposes default probabilities into the probability of negative equity and the probability of default conditional on negative equity. Comparing Panels A and C, we see that default rates for IO mortgages are significantly higher than for ARMs. The main reason is that IO mortgages have much higher probabilities of negative home equity. The difference is particularly large for lower levels of the LTV; for LTV of 0.80 the probability of negative home equity is 0.41 for IO mortgages compared to 0.24 for ARMs.

An interesting finding is that the probabilities of default, and of default conditional on negative equity, are less sensitive to LTI for IO mortgages than for ARMs. For an increase in LTI from 2.5 to 4.5, the probability of default conditional on negative equity is almost unchanged for IO mortgages compared to an increase from 0.019 to 0.044 for ARMs. Balloon payment mortgages have much lower mortgage payments relative to income than ARMs do, so that they are subject to lower cash-flow risk, and their default rates conditional on negative home equity are much less sensitive to the initial LTI.

The probabilities reported in Table 3 are calculated over the life of the loan, and hide

[10] Amromin, Huang, Sialm, and Zhong (2011) and Cocco (2011) characterize the households that borrow using these alternative mortgage products.

interesting time variation in default probabilities. In order to investigate this time variation in Figure 6 we plot the per year probability of negative home equity and cumulative default rates for the different types of mortgages. Comparing first ARMs to IO mortgages with principal-repayment at maturity we see that early in the life of the loan default rates are actually higher for the former, in spite of the fact that the probability of negative home equity is higher for the latter. The reason is that mortgage payments are higher for repayment than for IO mortgages, borrowing constraints are more likely to bind, and it is more costly for households to exercise the option to wait. That is, early on households with an ARM default due to cash-flow considerations.

Later in the life of the mortgage, the dominant force becomes the fact that households with an IO mortgage are much more likely to have negative home equity, and their default rates become larger than those for ARMs. In fact for IO mortgages default occurs until maturity, whereas for ARMs default becomes negligible in the second half of the loan. Therefore, default for IO mortgages tends to occur for wealth reasons, and default rates for this type of mortgage are much more sensitive to the longer-term evolution of house prices. However, due to their variable rate nature their wealth value is not as sensitive to inflation as FRMs.

The probability of early mortgage termination through a house sale is lower for IO mortgages than for ARMs (Table 3). There are two reasons for this. First, households have less home equity to tap into. Second, due to the lower level of mortgage payments cash-flow considerations are less important for IOs than for ARMs.

# 4 Conditional Default Rates

In the previous section we have characterized the unconditional default rates predicted by our model, calculated as average rates across the eight hundred different paths for the aggregate variables that we have generated (and across the realizations for the individual labor income shocks). Therefore these are the unconditional default rates that we can expect from an ex-ante point of view. Of course, ex-post only one of the paths for the aggregate variables will be realized.

We now focus on the conditional default probabilities predicted by our model, or on how default probabilities differ across the different paths for the aggregate variables. Naturally, from a policymaker's point of view, the concern is those states with a large incidence of mortgage default. This analysis also allows us to study the relative contribution of aggregate and idiosyncratic shocks to the default decision. Throughout this section we focus on default rates for mortgage loans with a LTV ratio of 0.9 and a LTI ratio equal to 4.5, which are our baseline parameters shown in the bottom panel of Table 1. For each mortgage type, heterogeneity arises solely from the different realizations for the shocks (and the choices that the individuals make given these shocks).

## 4.1 Differences in default across aggregate states

Recall that in our model the aggregate shocks are shocks to real house prices, the inflation rate, and the real interest rate. The past realizations of house price and inflation shocks determine the current level of real house prices and the current price level, respectively. When we refer to an aggregate state, we mean one possible combination of these aggregate shocks, out of the eight hundred that we have generated.

In order to characterize the differences in default rates across aggregate states, we first calculate default frequencies. In Figure 7 we plot the number of aggregate states, with a given number of individual defaults, by mortgage type. The categories that we consider in the horizontal axis are aggregate states with one to five individual defaults (up to 10% of the total), with six to ten individual defaults, and so on up to number of aggregate states in which all individuals default. This figure shows default frequencies for the different types of mortgage, but it is important to note that the aggregate states in which, for example, there are 1 to 5 individual defaults for ARMs are not necessarily the same as those in which there are 1 to 5 individual defaults for FRMs.

Although not shown in Figure 7, there is a large number of aggregate states, over four fifths of the total, for which there is no default. Naturally, in all states characterized by an increase in house prices there will not be any default. For ARMs and FRMs, the second most likely number of defaults is 1 to 5, which happens for roughly ten percent of the aggregate states. These are aggregate states in which up to ten percent of the individuals default. For interest-only mortgages, the second most important category are states in which every individual defaults.

Interestingly, Figure 7 shows that there is more dispersion in defaults across aggregate states for ARMs than for FRMs or interest-only mortgages. This suggests that idiosyncratic shocks are a more important determinant of the default decision for ARMs than for the other mortgage types. In fact for balloon mortgages, in ninety four percent of the aggregate states there is either no default or a one-hundred percent default rate.[11] This finding should be understood in light of our previous discussion that cash-flow risk which varies across individuals due to their different income shocks, is a more important determinant of the decision to default in an ARM than in a FRM or a balloon mortgage, for which wealth motives are relatively more important.

It is also interesting to see that even though for ARMs there are more aggregate states in which some individuals choose to default, these tend to be predominantly states in which a limited number of individuals choose to do so. On the other hand, for FRMs the number of aggregate states in which the majority of the households default, albeit small (fourteen states, or 2.1 percent of the total), is larger than for ARMs. In this sense, FRMs have more systemic risk than ARMs (with interest-only mortgages having the largest systemic risk).

In order to characterize the different aggregate states, in Figure 8 we plot the average evolution of nominal house prices and interest rates for states with 1 to 10 individual defaults (up to a 20 percent default rate, Panel A), and for states with 41 to 50 individual defaults (corresponding to a default rate of over 80 percent, Panel B). We plot such averages for both ARMs and FRMs.

Panel A shows that moderate numbers of ARM defaults tend to occur as result of high interest rates, while the reverse is true for FRMs. High interest rates lead to an increase in the mortgage payments required for ARMs, inducing some individuals to default, especially early on when accumulated financial savings are small. On the other hand, for FRMs, low interest rates imply lower rental payments compared to mortgage payments, which induces default. However, such default tends to occur on average later in the life of the mortgage, and at slightly lower levels of house prices.

Unsurprisingly, for both ARMs and FRMs, aggregate states with high default rates ("default waves") tend to be those which exhibit large falls in house prices, of roughly fifty percent (Panel

[11] This number may sound large, but one should remember that in our model households are ex-ante homogeneous, whereas in reality they are heterogeneus with respect to their preference parameters and the characteristics of their human capital (among others). We study household heterogeneity, which may increase dispersion in default rates, in the next section.

B). Default waves take place until much later in the life of the mortgage, because it takes time for house prices to decline this far after mortgage initiation. In addition, average nominal interest rates are fairly similar for both ARMs and FRMs. This shows that in states of large house price declines, house prices and the level of negative equity become the most important determinant of the default decision, with interest rates being relatively less important.

## 4.2 The role of individual income shocks

The aggregate states in which some individuals choose to default while others choose not to do so are particularly instructive since they highlight the importance of individual specific income shocks for mortgage default. We illustrate this by plotting, for each mortgage type, an example of one aggregate state in which ten percent of the individuals default. More precisely, in Figure 9 we plot the evolution of the aggregate variables, namely of real house prices, the price level, and the nominal interest rate. In addition, we plot average labor income for the individuals who choose to default, and for those who choose not to do so.

Some of the effects of the aggregate shocks previously discussed are clearly visible in this figure. In all of the panels there is a decline in real house prices in the first half of the mortgage, when default takes place. For ARMs and interest-only mortgages (Panels A and C, respectively) default tends to take place when nominal interest rates are high, while the opposite is true for FRMs. For all mortgage types, what explains why some individuals choose to default while others choose not to do so is their income level. Individuals who choose to default have lower average income levels, at least in the periods prior to when they decide to default.

To further evaluate the importance of individual shocks for the default decision, we generate data for an aggregate state with declining house prices, persistently high inflation and high real interest rates. That is, in the simulation we let real house price shocks be negative, while inflation and real interest rates are high in every period. But we generate income shocks that are different across individuals. In Figure 10, Panel A we plot cumulative default rates for the different mortgage types, and for two values for the LTI ratio, equal to 3.5 and 4.5.

Interestingly, we find that for lower values of the LTI ratio, and for a given mortgage type, there is little heterogeneity in default rates. The vast majority (or even all) of the individuals find it optimal to default in the same period, as cumulative default rates jump from zero to one. However, for a higher value for LTI, individual income shocks do matter. For ARMs one in five individuals find it optimal to default earlier than age 41, when the remainder default. The higher is the initial value of the LTI, the higher are mortgage payments relative to income, leading to more cash-flow risk. This cash-flow risk makes individual income shocks matter more, particularly for ARMs which are subject to more cash-flow risk in the first place.

In panel B we carry out a similar experiment, with declining house prices, but with, in each period, low inflation and low real interest rates. In such scenario, individuals who default in an ARM or a balloon mortgage do so primarily for wealth motives. Therefore, there is little or no heterogeneity in default rates, even for the higher LTI value. Individual income shocks matter for FRMs for the higher LTI value, but less than for ARMs. The lower interest rates also means that default tends to occur earlier for FRMs than for ARMs or for interest-only mortgages.

### 4.3 Correlation in defaults

Mortgage default tends to occur, for all mortgage types, in aggregate states with declining house prices. However, default for balloon payment mortgages is more sensitive to the longer term evolution of house prices than for principal-repayment mortgages. Furthermore, as previously documented, interest rate movements have a differential impact on default for FRMs and ARMs. We are interested in studying the extent to which mortgage defaults are correlated across mortgage types. Such correlation is important for evaluating the risk of portfolios of mortgages composed of different types.

We have calculated the correlation between the number of ARM defaults and the number of FRM or IO defaults, where each observation is one aggregate state. In Table 4 we report the estimated values for these correlations. We report results for both a linear correlation (Panel A), and in light of the non-linear nature of our model, a non-parametric correlation (Panel B). Below the estimated correlations we report the corresponding p-values.

The overall correlations are positive, but this positive correlation comes primarily from the aggregate states with the largest house price declines, with $P_T^H < 0.65$. The default correlations between ARMs and FRMs are not significantly different from zero for higher values of $P_T^H$. This lack of correlation creates opportunities for portfolio diversification, but it also creates challenges for monetary authorities conducting a common policy for regions (or countries such as in the

Eurozone) which differ in terms of the importance of ARMs and FRMs for housing finance.

There are some instances of default even for aggregate states with higher real terminal house prices (thus the estimated positive correlations between ARM and IO mortgages for higher house prices). This is due to the fact that there are some aggregate states in which house prices decline in the first years of the mortgages, triggering default, even if terminal house prices turn out to be high.

# 5 Household Heterogeneity

In the previous sections we have studied mortgage default for different initial LTV and LTI ratios, and for different mortgage types, but for given household preference parameters and human capital. In this section we recognize that households are heterogeneous in their preference parameters and in the characteristics of their labor income. Such heterogeneity has effects on portfolio choice (Curcuru, Heaton, Lucas, and Moore, 2010) and it is also likely to affect the type of mortgage that households choose. For example, an individual who faces a steep income profile may be more likely to choose the mortgage that minimizes current payments. With this in mind, we investigate the effects of household characteristics on default rates.

## 5.1 Labor income growth

Households differ in their expected growth rate of labor income. We investigate the impact of this parameter on default probabilities. More precisely, in Table 5 we report results for an average income growth equal to 1.2% (which is lower than the value of 2.7% that we have estimated in the PSID data for households with a college degree, but higher than the baseline value of 0.8%). Compared to the base case we see that the probability of default is now lower, both for the ARM and FRM contracts. Although the probability of negative equity is not affected by household income growth, the probability of default given negative equity is reduced.

When expected income growth is higher, there are two effects. On one hand, households have a lower incentive to save early on, which increases the likelihood of default. On the other hand, the higher income growth leads to a lower future ratio of mortgage payments relative to household income is higher, which improves mortgage affordability. The results in Table 5 show that the latter effect is stronger, and that a crucial parameter when thinking of mortgage affordability is expected income growth.

## 5.2 House price growth

In the fourth column of Table 5 we investigate the effects of a lower expected growth rate of house prices, equal to 1.2% (compared to 1.6% in the baseline case). Lower house price growth increases the probability of negative home equity and the probability of default given negative equity. Both these channels contribute to an increase of the overall default probability, an effect which is larger for FRMs than for ARMs. Housing is now a less attractive investment, so that individuals are more willing to abandon their house, more so in a low inflationary environment where they have negative home equity.

## 5.3 Stigma from mortgage default

In a recent empirical paper Guiso, Sapienza, and Zingales (2009) find that moral and social considerations play an important role in the default decision. Ceteris paribus, people who consider it immoral to default are 77% less likely to declare their intention to do so. We can adapt our model to investigate how such considerations affect default rates for different mortgage types. We assume that in case of default the household incurs a utility loss, *Stigma*. The household will choose to default, setting $Def_t^C = 1$, whenever the continuation utility with default less the stigma cost is higher than the utility without default:

$$V_t(State_t \mid Def_t^c = 1) - Stigma > V_t(State_t \mid Def_t^C = 0). \tag{27}$$

The main difficulty with this extension of our model is determining an appropriate value for *Stigma*. In the fourth column of Table 5 we report the results for $Stigma = 0.05$. In order to give the reader an idea of what this means we have translated this value into an equivalent per-period consumption loss. For the ARM mortgage, $Stigma = 0.05$ is equivalent to a decrease in the constant equivalent consumption stream of 2% per period. The results in table 5 show that this level of *Stigma* has a significant effect on default probabilities, larger for FRMs than for ARMs.

### 5.4 Utility of terminal wealth and discount factor

For tractability, we have truncated our baseline model at age 50, but we have allowed the agent to derive utility from terminal wealth, which can be viewed as the remaining lifetime utility from reaching age 50 with a given wealth level. This also insures that agents in our model have an incentive to save. In the baseline parameterization we have set the parameter $b$ that measures the relative importance of terminal wealth equal to 400. One way to asses how reasonable this value is to study the wealth accumulation generated by the model. For the ARM contract, and at age 50, agents have on average US \$139,900 of accumulated financial wealth. This value should be compared to the financial wealth held by households in checking and saving accounts, mutual funds, and retirement accounts. We have solved our model for an alternative value for $b$, equal to 100, and we report the default probabilities in the fifth column of Table 5. The average financial wealth at age 50 under the ARM contract is \$78,816. The lower are the incentives for individuals to save, the higher are the default probabilities predicted by the model. A similar effect occurs when we decrease the discount factor, as shown in the last column of Table 5. In this case the average financial wealth at age 50 is \$116,848. For both a lower $b$ and for a lower discount factor, the probability that the borrower decides to sell the house so as to access accumulated equity is higher than in the base case.

## 6 Robustness

### 6.1 Hedging

In our model homeownership provides insurance against fluctuations in the price of housing. This happens for two reasons. First, renters must make payments that are proportional to the value of housing (Sinai and Souleles, 2005). Second, households derive utility from terminal real wealth that is calculated using a composite price index that is an average of the price of housing and the price of other goods consumption. In our model labor income acts as a partial hedge against such fluctuations in house prices, since permanent labor income shocks are positively correlated with house price shocks. But the estimated value for this correlation, 0.19, is not very large.

We are interested in evaluating the extent to which hedging motives play an important role in deterring borrowers from defaulting on their mortgages. A simple and clear way to do so in the context of our model is to scale terminal real wealth by the price level, that is, to let $P_{T+1}^{Composite} = P_{T+1}$. This reduces the hedging motives for homeownership, but it does not eliminate them altogether since homeownership still provides insurance against fluctuations in the per period rental cost of housing.

The second column of Table 6 shows unconditional default probabilities with a reduced hedging motive for homeownership. For comparison the first column of Table 6 reports the results for the base case. There is a considerable increase in the unconditional default probability, from 2.3% to 3.7% for the ARM contract, which arises as a result of an increase in the probability of default conditional on negative equity from 4.4% to 7%. There are also horizon effects. As the horizon shortens, the hedging motives for homeownership are reduced, and households have more of an incentive to default. This can clearly be seen in Figure 11 which plots unconditional default rates with age. The difference in default rates between the base case and the no hedging scenarios increases is relatively small early on, but increases from age 36 onwards as the horizon shortens. Overall the default patterns are similar to the base case; the difference is in the levels.

These results are interesting since they illustrate the importance of hedging motives as a default deterrent. The disappearance or reduction of such hedging motives may trigger default. That will be the case for instance for households who are underwater and now expect to have to move to another region for employment reasons.

## 6.2 Unemployment risk

An important source of income risk is unemployment, or the probability of a large drop in income. In our baseline model, unemployment risk is not separately modeled but simply contributes to a higher variance of labor income shocks. If one thinks of unemployment as a temporary decline in labor income, then in terms of our model this would simply be captured by a higher variance of temporary labor income shocks. One concern is that this may not fully capture the risks of unemployment. One important and distinctive feature of unemployment is that it leads to a large drop in labor income.

In order to evaluate the extent to which our results are robust to such a scenario we have solved an alternative version of our model in which temporary labor income shocks $\omega_t$ are in each period equal to $\underline{\omega}$ with probability $p_u$ or to $\overline{\omega}$ with probability $1 - p_u$. The probability $p_u$ refers to the probability that the borrower becomes unemployed. We have used the previously described PSID data to parameterize such probability, which we set equal to 5 percent. To facilitate comparison we choose the values for $\underline{\omega}$ and $\overline{\omega}$ so that the mean and the variance of the temporary labor income shocks are the same as in the base case. This means that in case of unemployment, income is equal to 37.5 percent of its permanent income level, and 105.3 percent of permanent income otherwise.

We report the default probabilities in the third column of Table 6. The main conclusion is that these probabilities are very similar to the base case. Naturally other parameterizations such as a higher probability of unemployment, a larger drop in income in the case of unemployment, or temporary income shocks that are correlated with the other model variables will generate different default probabilities. But it is nonetheless re-assuring to see that for the unemployment parameterization considered the model predicted default probabilities are similar to the base case.

In addition, and to assess how reasonable the drop in income in the case of unemployment is, we have used PSID data and calculated the ratio of the average reported household income for households with an unemployed head to the average reported income for households with an employed head. We find that the former is 57 percent of the latter. This value may seem large, and it is possible that such a large value is the result of sample selection issues (for example, if households whose head becomes unemployed are more likely to disappear from the sample). However, it is also important to remember that we are using a broad measure of labor income, that includes not only wages, but also social security and other transfers that households receive. In addition, we use an annual income measure, and the average duration of unemployment is less than one year. Finally, we measure household income and not individual income. In many households, even if the head becomes unemployed, the spouse may remain employed and receive wage income. Our parameterization takes into account all these different ways that households have to insure themselves against unemployment.

## 6.3 Housing choice

In the baseline model we have assumed for tractability that housing and other goods preferences are separable, and that housing is fixed. This was done for tractability. Existing theoretical models of mortgage default also make this assumption. But there may be interesting interactions between default choice and housing choices. We extend our model to allow for housing choice in the event of default.[12]

More precisely, we allow defaulting households to freely choose between three different levels of housing. The levels that we consider are those that correspond to an initial LTI of 2.5, 3.5 or 4.5. We assume separable preferences between housing and other consumption so as to be consistent with the baseline model. Thus household preferences are given by:

$$U(C_t, H_t) = \frac{C_t^{1-\gamma}}{1-\gamma} + \theta \frac{H_t^{1-\gamma}}{1-\gamma}. \tag{28}$$

In this extended model there is one more choice variable in the default state, but no additional state variables are needed. The results for $\theta$ equal to 0.3 are reported in the penultimate column of Table 6. As we would expect, default rates are higher when households are allowed to choose house size in the default state. Of course this effect comes solely from the probability of default conditional on negative equity. Furthermore, those individuals who default choose the smallest house size in the default state, which helps them to reduce their housing expenditures. Naturally, the degree to which they wish to do so will depend on the degree of substitutability between housing and other goods consumption in household preferences. But overall the results in Table 6 show that the default probability patterns that we have emphasized in this paper are robust to this alternative model in which we allow for house size choice.

## 6.4 Inflation persistence

We have simplified our model by assuming that real interest rate risk is transitory, and that the expected real interest rate is equal to the realized real interest rate. However, this implies that the serial correlation of the one-year nominal yield in our model is lower than in the data.

[12] It would be considerably more complicated to allow for housing choice in the no-default state, since this would require additional state variables, which we cannot handle. In addition it is likely that most households considering default cannot change house size without default, since they are likely to have negative home equity.

Therefore, we consider a robustness exercise in which we increase the persistence of inflation shocks, by setting the value of the $AR(1)$ coefficient equal to 0.95. The results are reported in the last column of Table 6. Comparing them to the base case, we see that higher inflation persistence leads to higher default rates. As expected the effect is stronger for FRMs, with an increase in the default probability from 0.026 to 0.033. When inflation persistence is higher, the capital value of FRMs becomes more sensitive to inflation shocks, so that borrowers are more likely to default, for wealth reasons, in response to a fall in the inflation rate.

# 7 Conclusion

We have proposed a model of mortgage default, in the presence of labor income, house price, inflation and interest rate risk, to show how different shocks contribute to the default decision. We have decomposed our model's predicted default rates into the probability that households face negative home equity and the probability that they choose to default conditional on negative equity. Negative home equity tends to occur for a particular combination of the shocks: house price declines in a low-inflation environment, early in the life of the mortgage when the outstanding principal balance is large. Not all households with negative equity choose to default. For moderate levels of negative home equity, default becomes more likely as borrowing constraints bind more tightly on households.

We have modelled different mortgage types, including adjustable-rate, fixed-rate, and interest-only mortgages. The predicted default rates differ across these types, as do the determinants of the default decision. ARM borrowers tend to default when they face large mortgage payments relative to income, a result of high interest rates and low labor income. FRM borrowers are more likely to default when interest rates are low. IO mortgages are characterized by a higher probability of negative home equity, but not necessarily a higher probability of default in case of negative home equity. Since mortgage payments are lower relative to income, for a given level of negative home equity, households are more likely to exercise the option to wait before defaulting. This also makes default rates for IO mortgages more sensitive to the longer term evolution of house prices.

In the credit boom of the mid-2000s average loan-to-value (LTV) ratios on prime mortgages were relatively stable, but loan-to-income (LTI) ratios increased. We have used our model to calculate default rates, by mortgage type, for different values for these mortgage parameters. Our decomposition of the default rate into the probability of negative home equity and the probability of default given negative home equity shows that a high LTV ratio increases the former whereas a high LTI ratio increases the latter. The model's predicted default rates become particularly large when both LTV and LTI ratios are high.

We have used our model to study the incidence of default waves, in which a large fraction of mortgage borrowers default. We find that default waves are least likely for ARMs, where idiosyncratic income shocks are a more important determinant of the default decision, and most likely for IO mortgages. In this sense FRMs, and particularly IO mortgages, have higher systemic risk than ARMs.

Default waves occur for all mortgage types when house prices experience dramatic declines, but smaller numbers of defaults occur with high interest rates for ARMs and with low interest rates for FRMs. As a result the correlation in defaults across these two mortgage types is large and positive in aggregate states in which there are large declines in house prices, but essentially zero in aggregate states characterized by moderate falls in prices. This implies that mortgage investors can benefit from portfolio diversification across mortgage types in normal conditions, but not in severe housing downturns.

We have also used our model to explore the sensitivity of default rates to household heterogeneity, including variations in expected labor income growth, house price growth, impatience, and inherent reluctance to default.

There are several interesting directions for future research. First, we could use data on mortgage default to structurally estimate our model parameters and to test the predictions of the model across households and mortgage types.

Second, we have investigated the determinants of mortgage default for exogenously given credit risk premia, similar across mortgage contracts. However, such premia should in equilibrium reflect the probability of default and the expected losses given default for mortgage providers. It would be interesting to use our model to determine what credit risk premia should be in a competitive market, in which mortgage providers on average break even. Since default decisions depend on interest rates and mortgage premia, which also affect the expected profits of banks, this would require, for each mortgage contract, solving several iterations of our model to find a fixed point. We could then compare the premia generated from our model to actual data on the premia charged by mortgage providers.

Third, our model could be used to assess the risk, systemic and otherwise, of portfolios of mortgages. Of particular interest is the differential response of FRM and ARM default to interest rate movements. This is particularly relevant for monetary authorities in areas such as the eurozone in which these types of mortgages co-exist.

Finally, our model could be used to study policies of mortgage modification that are intended to reduce the incidence of default in the aftermath of severe declines in house prices.

# 8 References


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Table 1: Baseline parameters.

| Description | Parameter | Value |
|---|---|---|
| Time and preference parameters | | |
| Discount factor | $\beta$ | 0.98 |
| Risk aversion | $\gamma$ | 2 |
| Preference for housing | $\theta$ | 0.3 |
| Initial age | | 30 |
| Terminal age | | 50 |
| Bequest motive | $b$ | 400 |
| Labor income and house prices | | |
| Average income growth | $exp[f(t) - f(t-1)]$ | 0.008 |
| Stdev permanent income shocks | $\sigma_\eta$ | 0.063 |
| Stdev temporary income shocks | $\sigma_\omega$ | 0.225 |
| Expected house price return | $exp(g + \sigma_\delta^2/2) - 1$ | 0.016 |
| Stdev house price return | $\sigma_\delta$ | 0.162 |
| Correl. perm. inc. and house price shocks | $\rho$ | 0.191 |
| Correl. temp. inc. and inflation shocks | $\varphi$ | 0.191 |
| Inflation and real interest rate | | |
| Mean log inflation | $\mu$ | 0.041 |
| Stdev of the inflation rate | $\sigma_\epsilon$ | 0.028 |
| AR(1) coefficient | $\phi$ | 0.723 |
| Mean log real rate | $\bar{r}$ | 0.018 |
| Stdev of the real rate | $\sigma_\varepsilon$ | 0.017 |
| Tax rates and other parameters | | |
| Income tax rate | $\tau$ | 0.25 |
| Property tax rate | $\tau_p$ | 0.015 |
| Property maintenance | $m_p$ | 0.025 |
| Lower bound on cash-on-hand | $\underline{X}$ | $1,000 |
| Transaction costs of house sale | $c$ | 0.06 |
| Loan Parameters | | |
| Loan to income | $LTI$ | 4.5 |
| Loan to value | $LTV$ | 0.90 |
| Down payment | $d$ | 0.10 |
| Credit risk premium | $\psi$ | 0.01 |

Note to Table 1: This table reports the parameter values used in the baseline case.

Table 2: Means for different variables by mortgage type and default decision.

| | ARM | | | FRM | | |
|---|---|---|---|---|---|---|
| Variable | Default | No def/Neg eq | No def | Default | No def/Neg eq | No def |
| Current loan-to-value | 1.37 | 1.13 | 0.49 | 1.48 | 1.13 | 0.48 |
| Price level | 1.22 | 1.16 | 1.50 | 1.26 | 1.16 | 1.55 |
| Real price of housing | 0.49 | 0.63 | 1.09 | 0.39 | 0.63 | 1.09 |
| Real inc at $t$ | 39.59 | 48.03 | 53.97 | 43.08 | 48.00 | 53.89 |
| Real inc at $t-1$ | 40.73 | 48.10 | 53.48 | 44.64 | 48.05 | 53.37 |
| Real inc at $t-2$ | 43.13 | 48.24 | 53.33 | 46.15 | 48.20 | 53.19 |
| Real cons at $t+1$ | 11.74 | 13.91 | 14.50 | 11.41 | 13.51 | 14.45 |
| Real cons at $t$ | 6.71 | 14.07 | 14.62 | 9.44 | 13.70 | 14.60 |
| Real cons at $t-1$ | 9.87 | 14.74 | 14.93 | 10.87 | 14.43 | 14.91 |
| Real cons at $t-2$ | 13.47 | 14.82 | 15.05 | 12.28 | 14.63 | 15.02 |
| Real mortgage payment | 17.64 | 16.08 | 13.98 | 16.12 | 17.37 | 13.86 |
| Mortgage Payment/Inc | 0.48 | 0.36 | 0.29 | 0.41 | 0.39 | 0.28 |
| Real rental payment | 4.78 | 6.15 | 10.53 | 3.73 | 6.20 | 10.53 |
| (Mortgage-Rent)/Income | 0.35 | 0.22 | 0.08 | 0.31 | 0.25 | 0.08 |
| Real interest rate | 0.018 | 0.019 | 0.018 | 0.018 | 0.019 | 0.018 |
| Inflation rate | 0.048 | 0.033 | 0.041 | 0.018 | 0.034 | 0.042 |
| Nominal interest rate | 0.067 | 0.053 | 0.060 | 0.036 | 0.054 | 0.061 |
| Age of default | 35.80 | | | 37.90 | | |
| Probability of default | 0.023 | | | 0.026 | | |
| Prop of def with coh<5 | 0.508 | | | 0.224 | | |

Note to Table 2: This table reports the mean for several variables for ARM and FRM by default decision and whether households have positive home equity. The table reports means across aggregate states and individual shocks. For each mortgage type, the first column reports means for periods in which individuals choose to default, the second column reports means for individuals who have negative home equity but choose not to default, and the third column reports means for individuals who choose not to default.

Table 3: Probability of default predicted by the model.

| Panel A: Adjustable-Rate Mortgage | | | |
|---|---|---|---|
| Loan-to-income = 4.5 | ltv = 0.80 | ltv = 0.9 | ltv = 0.95 |
| Prob(Default) | 0.016 | 0.023 | 0.032 |
| Prob(Home equity<0) | 0.244 | 0.535 | 0.566 |
| Prob(Default/Home equity<0) | 0.067 | 0.044 | 0.056 |
| Prob(Cash-out) | 0.402 | 0.294 | 0.257 |
| Loan-to-value = 0.90 | lti = 2.5 | lti = 3.5 | lti = 4.5 |
| Prob(Default) | 0.010 | 0.010 | 0.023 |
| Prob(Home equity<0) | 0.538 | 0.537 | 0.535 |
| Prob(Default/Home equity<0) | 0.019 | 0.019 | 0.044 |
| Prob(Cash-out) | 0.015 | 0.067 | 0.294 |
| **Panel B: Fixed-Rate Mortgage** | | | |
| Loan-to-income = 4.5 | ltv = 0.80 | ltv = 0.9 | ltv = 0.95 |
| Prob(Default) | 0.015 | 0.026 | 0.039 |
| Prob(Home equity<0) | 0.239 | 0.532 | 0.564 |
| Prob(Default/Home equity<0) | 0.055 | 0.049 | 0.069 |
| Prob(Cash-out) | 0.362 | 0.268 | 0.226 |
| Loan-to-value = 0.90 | lti =2.5 | lti = 3.5 | lti = 4.5 |
| Prob(Default) | 0.016 | 0.019 | 0.026 |
| Prob(Home equity<0) | 0.538 | 0.537 | 0.532 |
| Prob(Default/Home equity<0) | 0.031 | 0.035 | 0.049 |
| Prob(Cash-out) | 0.014 | 0.066 | 0.268 |
| **Panel C: Interest-Only Mortgage** | | | |
| Loan-to-income = 4.5 | ltv = 0.80 | ltv = 0.9 | ltv = 0.95 |
| Prob(Default) | 0.099 | 0.125 | 0.145 |
| Prob(Home equity<0) | 0.412 | 0.651 | 0.675 |
| Prob(Default/Home equity<0) | 0.241 | 0.191 | 0.215 |
| Prob(Cash-out) | 0.179 | 0.114 | 0.092 |
| Loan-to-value = 0.90 | lti =2.5 | lti = 3.5 | lti = 4.5 |
| Prob(Default) | 0.122 | 0.123 | 0.125 |
| Prob(Home equity<0) | 0.654 | 0.654 | 0.651 |
| Prob(Default/Home equity<0) | 0.187 | 0.188 | 0.191 |
| Prob(Cash-out) | 0.006 | 0.028 | 0.114 |

Note to Table 3: This table decomposes the probability of default into probability of negative equity times the probability of default conditional on negative home equity for the FRM, ARM and Interest-only mortgage contracts for different values for LTV and LTI. This table reports probabilities calculated across aggregate states and individual shocks. Negative home equity corresponds to situations when $(1-c)\times$ Nominal house value $<$ Outstanding debt.

Table 4: Correlation in defaults for different mortgage contracts

| Panel A: Linear correlation | | | |
|---|---|---|---|
| | ARM-FRM | ARM-IO | FRM-IO |
| $P_T^H < 0.65$ | 0.67 | 0.24 | 0.32 |
| | [0.00] | [0.00] | [0.00] |
| $0.65 < P_T^H \leq 1.25$ | -0.09 | 0.21 | -0.03 |
| | [0.22] | [0.00] | [0.71] |
| $1.25 < P_T^H \leq 1.73$ | -0.03 | 0.42 | 0.02 |
| | [0.67] | [0.00] | [0.75] |
| $1.73 < P_T^H$ | -0.03 | 0.37 | -0.02 |
| | [0.69] | [0.00] | [0.82] |
| Overall | 0.64 | 0.33 | 0.38 |
| | [0.00] | [0.00] | [0.00] |

| Panel B: Spearman correlation | | | |
|---|---|---|---|
| | ARM-FRM | ARM-IO | FRM-IO |
| $P_T^H < 0.65$ | 0.22 | 0.27 | 0.42 |
| | [0.00] | [0.00] | [0.00] |
| $0.65 < P_T^H \leq 1.25$ | 0.00 | 0.41 | -0.08 |
| | [0.95] | [0.00] | [0.28] |
| $1.25 < P_T^H \leq 1.73$ | -0.09 | 0.32 | 0.05 |
| | [0.18] | [0.00] | [0.45] |
| $1.73 < P_T^H$ | -0.04 | 0.34 | -0.02 |
| | [0.55] | [0.00] | [0.76] |
| Overall | 0.13 | 0.39 | 0.30 |
| | [0.00] | [0.00] | [0.00] |

Note to Table 4: This table reports the correlation in defaults for different mortgage contracts, and aggregate states. The table reports overall correlations, and correlations conditional on the date $T$ real price of housing, corresponding to the quartiles of the date $T$ distribution of real house prices. The table reports p-values below the estimated correlations.

Table 5: Probability of default predicted by the model for different parameters

| Panel A: Adjustable-Rate Mortgage | | | | | | |
|---|---|---|---|---|---|---|
| lti = 4.5, ltv = 0.9 | Base | High inc growth | Low house ret. | Stigma | b = 100 | $\beta = 0.92$ |
| Prob(Default) | 0.023 | 0.021 | 0.033 | 0.019 | 0.032 | 0.031 |
| Prob(Home equity<0) | 0.535 | 0.535 | 0.563 | 0.535 | 0.533 | 0.532 |
| Prob(Default/Home equity<0) | 0.044 | 0.039 | 0.059 | 0.035 | 0.060 | 0.059 |
| Prob(Cash-out) | 0.294 | 0.263 | 0.269 | 0.294 | 0.438 | 0.401 |
| **Panel B: Fixed-Rate Mortgage** | | | | | | |
| lti = 4.5, ltv = 0.9 | Base | High inc growth | Low house ret. | Stigma | b = 100 | $\beta = 0.92$ |
| Prob(Default) | 0.026 | 0.025 | 0.039 | 0.014 | 0.033 | 0.030 |
| Prob(Home equity<0) | 0.532 | 0.532 | 0.561 | 0.532 | 0.531 | 0.530 |
| Prob(Default/Home equity<0) | 0.049 | 0.047 | 0.069 | 0.027 | 0.061 | 0.056 |
| Prob(Cash-out) | 0.268 | 0.235 | 0.237 | 0.268 | 0.382 | 0.339 |

Note to Table 5: This table reports decomposes the probability of default into the probability of negative equity times the probability of default conditional on negative home equity for higher for income growth (equal to 1.2%), lower house price returns (equal to 1.2%), stigma in case of default, lower utility from terminal wealth ($b = 100$), and for a lower discount factor.

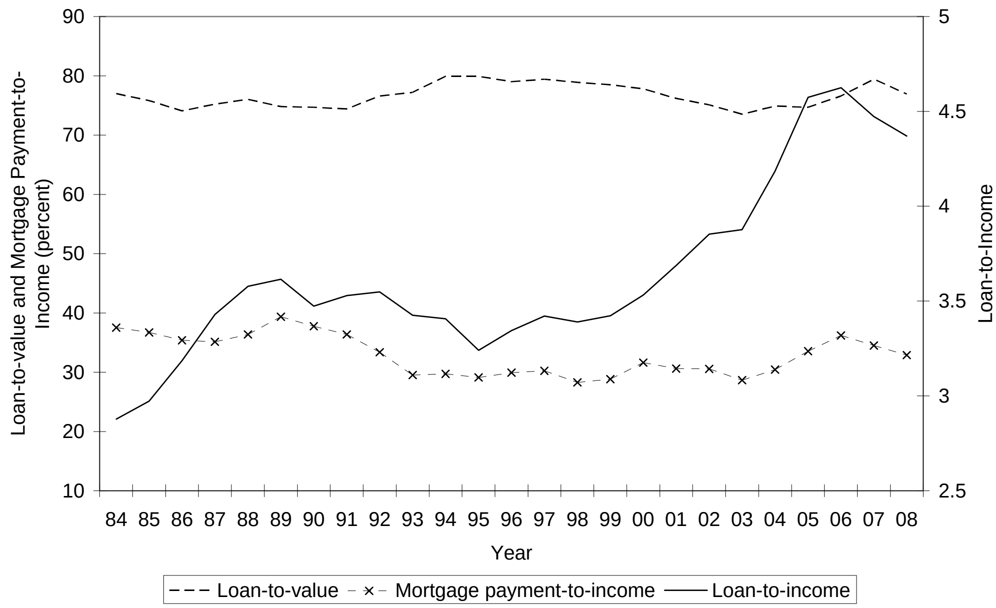
Table 6: Robustness

| Panel A: Adjustable-Rate Mortgage | | | | | |
|---|---|---|---|---|---|
| lti = 4.5, ltv = 0.9 | Base | No hedging | Unemp. | House Choice | Infl. persistence |
| Prob(Default) | 0.023 | 0.037 | 0.022 | 0.051 | 0.025 |
| Prob(Home equity<0) | 0.535 | 0.534 | 0.535 | 0.535 | 0.538 |
| Prob(Default/Home equity<0) | 0.044 | 0.070 | 0.041 | 0.090 | 0.046 |
| Prob(Cash-out) | 0.294 | 0.405 | 0.264 | 0.318 | 0.291 |

| Panel B: Fixed-Rate Mortgage | | | | | |
|---|---|---|---|---|---|
| lti = 4.5, ltv = 0.9 | Base | No hedging | Unemp. | House Choice | Infl. persistence |
| Prob(Default) | 0.026 | 0.038 | 0.026 | 0.053 | 0.033 |
| Prob(Home equity<0) | 0.532 | 0.531 | 0.534 | 0.532 | 0.534 |
| Prob(Default/Home equity<0) | 0.049 | 0.071 | 0.049 | 0.094 | 0.062 |
| Prob(Cash-out) | 0.268 | 0.359 | 0.272 | 0.287 | 0.270 |

Note to Table 6: This table reports the default probabilities for alternative parameterizations. The no hedging column refers to a parameterization in which the price index that we use to scale terminal wealth is the price level, i.e. it does not depend on real house prices. The unemployment column refers to a parameterization in which temporary labor income shocks are such that income is equal to 0.375 of its permanent level with probability 0.05, and equal to 105.3 percent of its permanent level otherwise. The house choice column reports the results for a model in which we allow defaulting individuals to move a smaller house in case of default. The final column reports the results for higher inflation persistence, with $\phi = 0.95$.

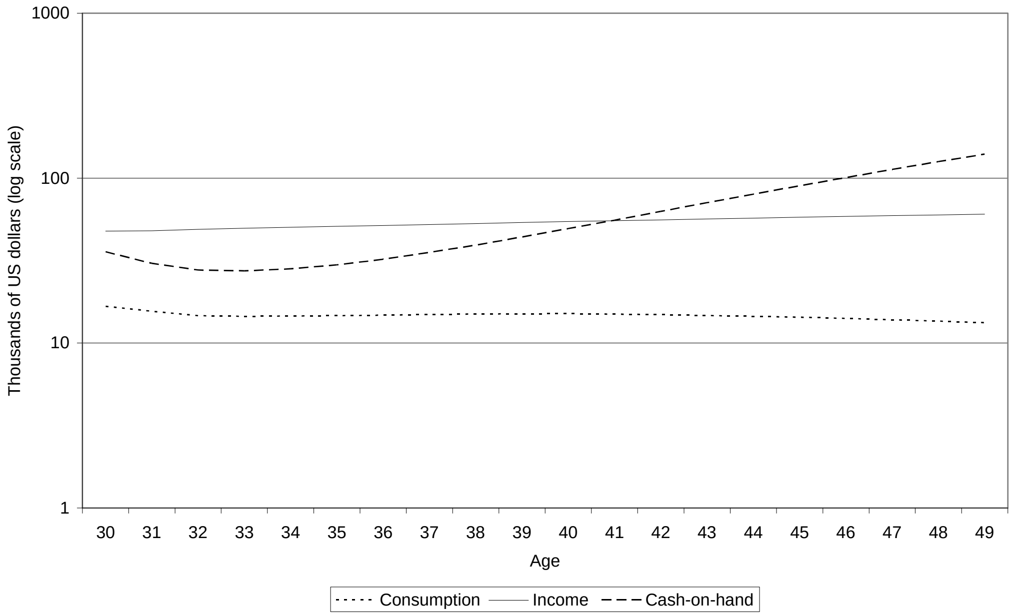
Figure 1: Loan-to-value, mortgage payment-to-income and loan-to-income over time for the US.



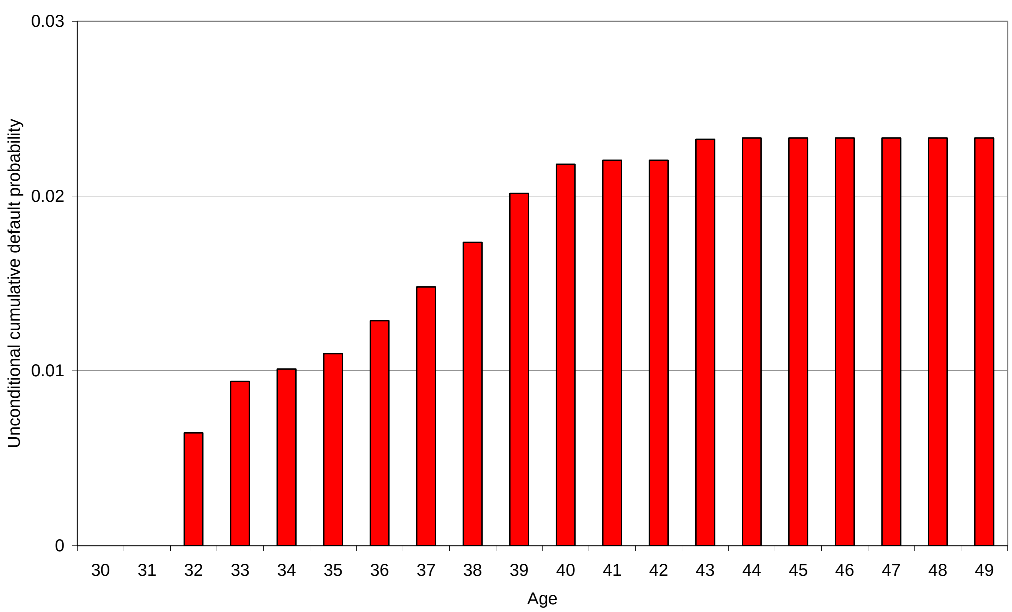

Note to Figure 1: The LTV data are from the Monthly Interest Rate Survey (MIRS), the LTI data are calculated as the ratio of the average loan amount obtained from the same survey to the median US household income obtained from Census data, the mortgage payment to income are calculated using the same income measure and the loan amount, maturity and mortgage interest rate data from the MIRS.

Figure 2: Mean consumption and cumulative default rates predicted by the model

Panel A: Consumption, income, cash-on-hand



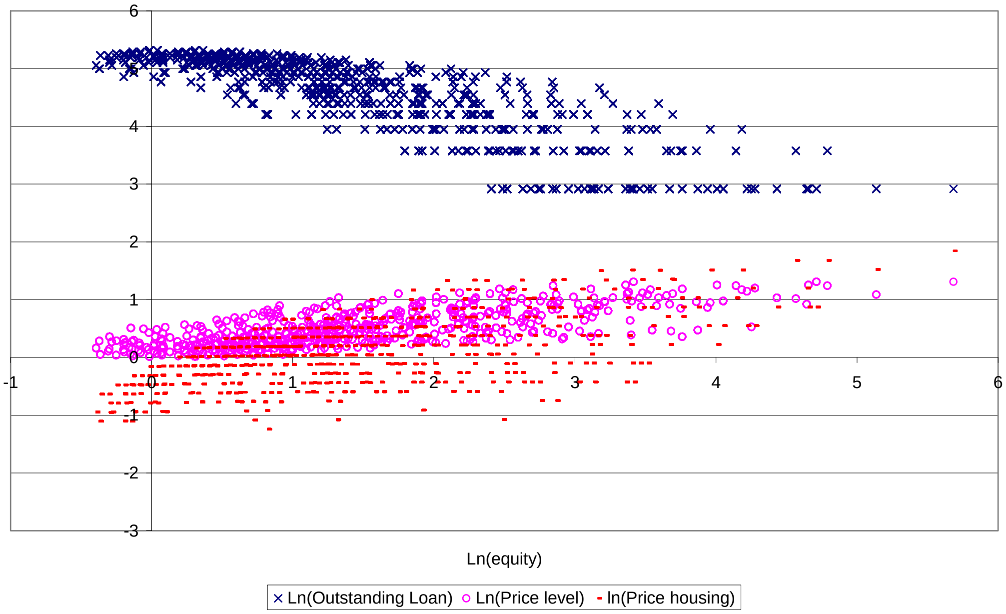

Panel B: Cumulative default rates



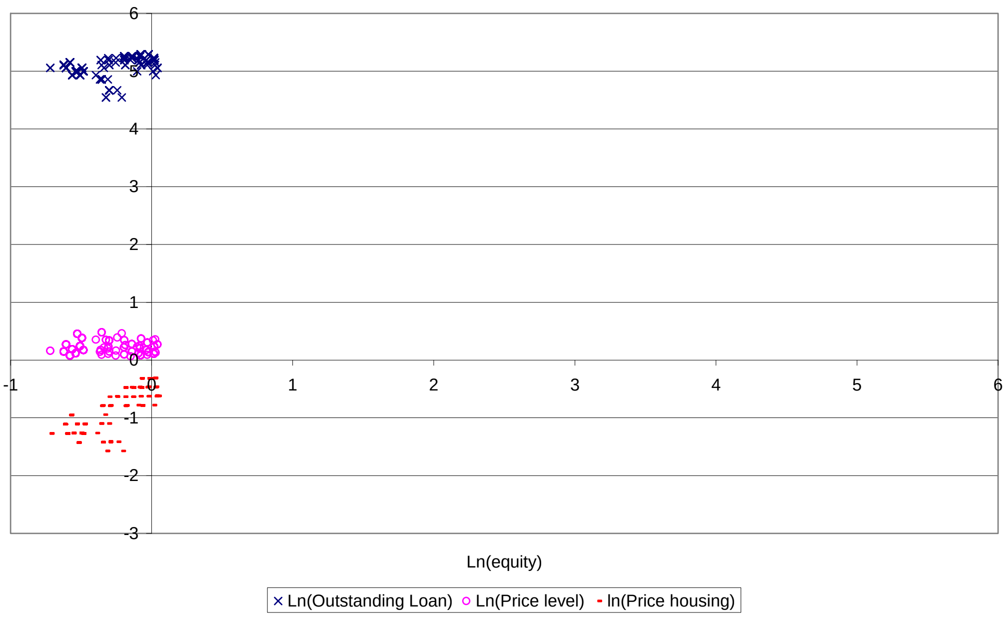

Note to Figure 2: The data is generated from simulating the model for the ARM with the parameters in Table 1.

Figure 3: Logarithm of house prices, price level and outstanding debt as a function of the logarithm of home equity, by default decision.

A: No default

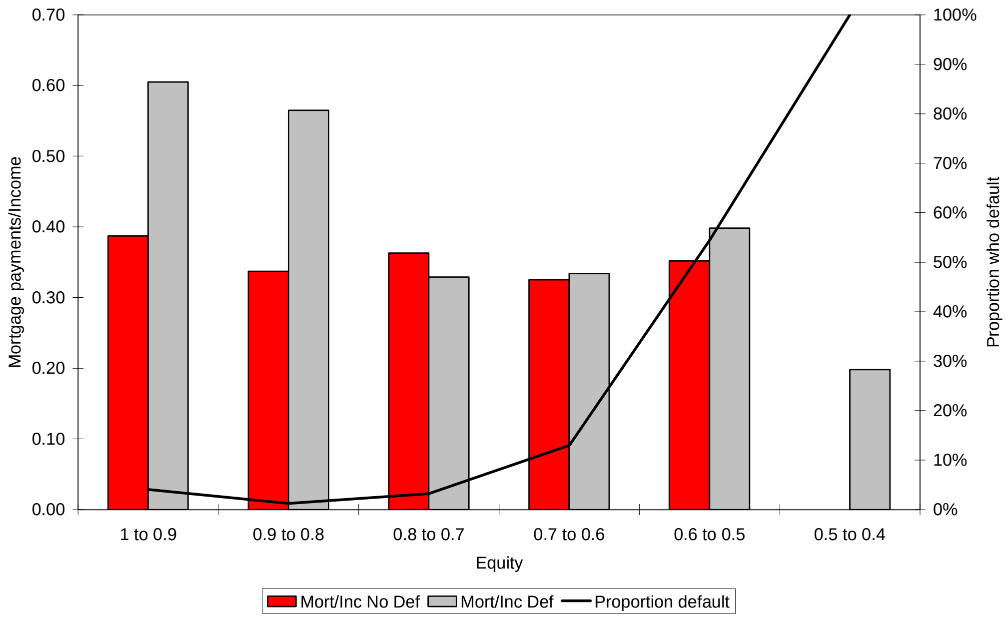



B: Default



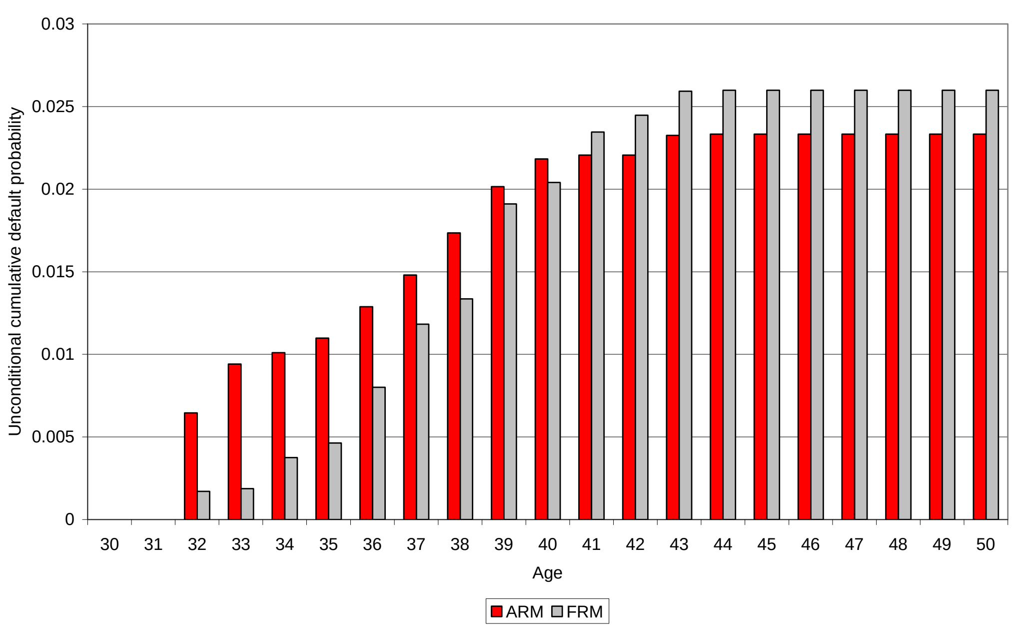

Note to Figure 3: The data is generated from simulating the model for the ARM with the parameters in Table 1.

Figure 4: Mortgage payments to household income by default decision and proportion of defaults as a function of home equity.



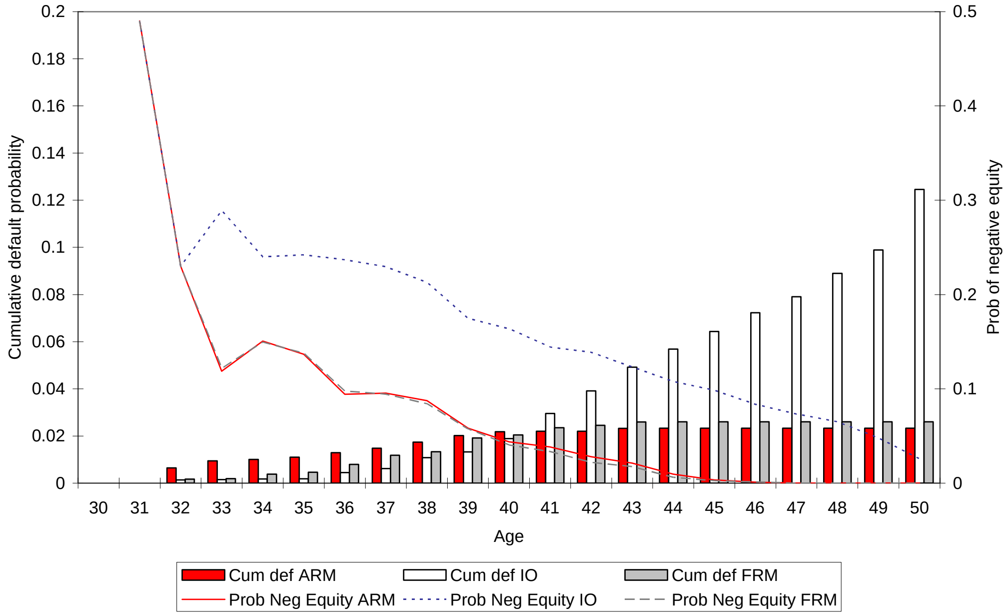

Note to Figure 4: The data is generated from simulating the model for the ARM with the parameters in Table 1, using one observation per household. Equity is calculated as the ratio of the current nominal house value to principal debt outstanding.

Figure 5: Cumulative default rates for different mortgage contracts.



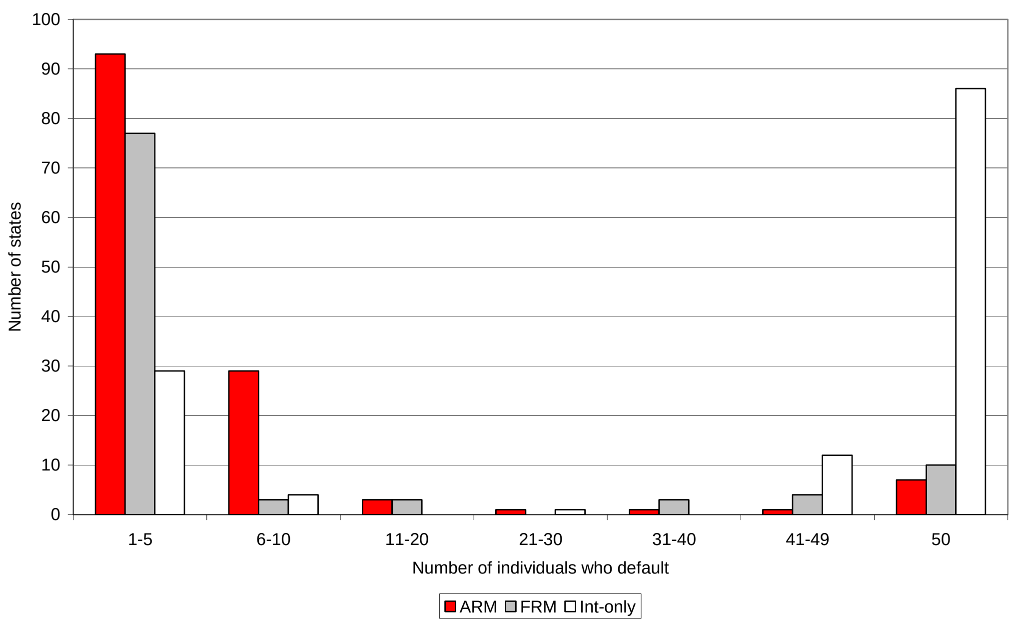

Note to Figure 5: This figure shows cumulative default rates for the FRM contract compared to the ARM contract. The data is generated from simulating the model.

Figure 6: Probability of negative home equity and cumulative default rates with age for different mortgage contracts.




Note to Figure 6: The data is generated from simulating the model. Negative home equity is outstanding loan principal greater than 0.94 x Nominal House value. The probability of negative equity is the probability that the household faces at least one period of negative home equity.

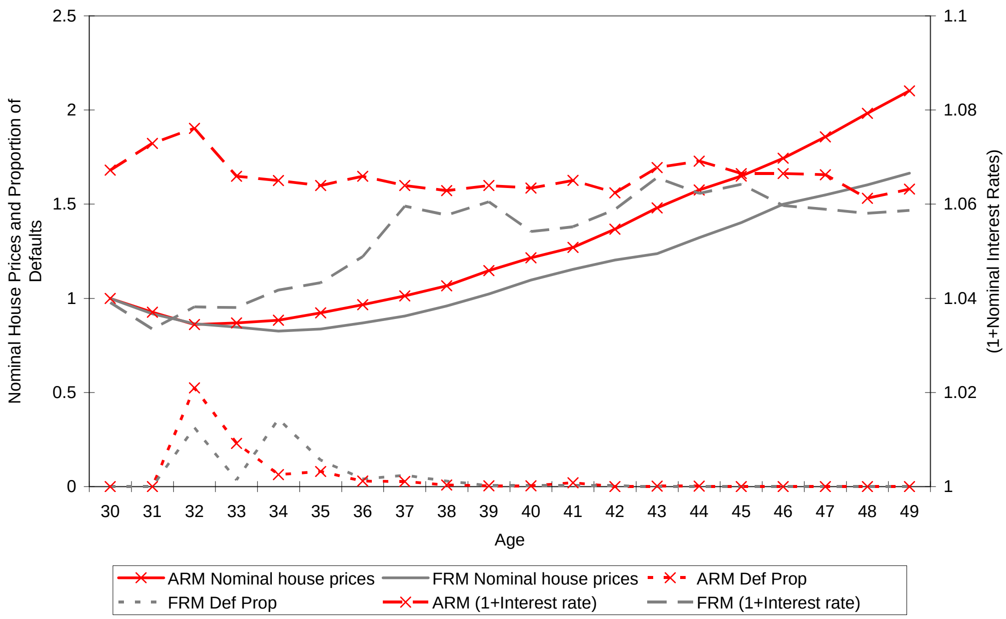
Figure 7: Number of aggregate states with a given number of mortgage defaults, by mortgage type.



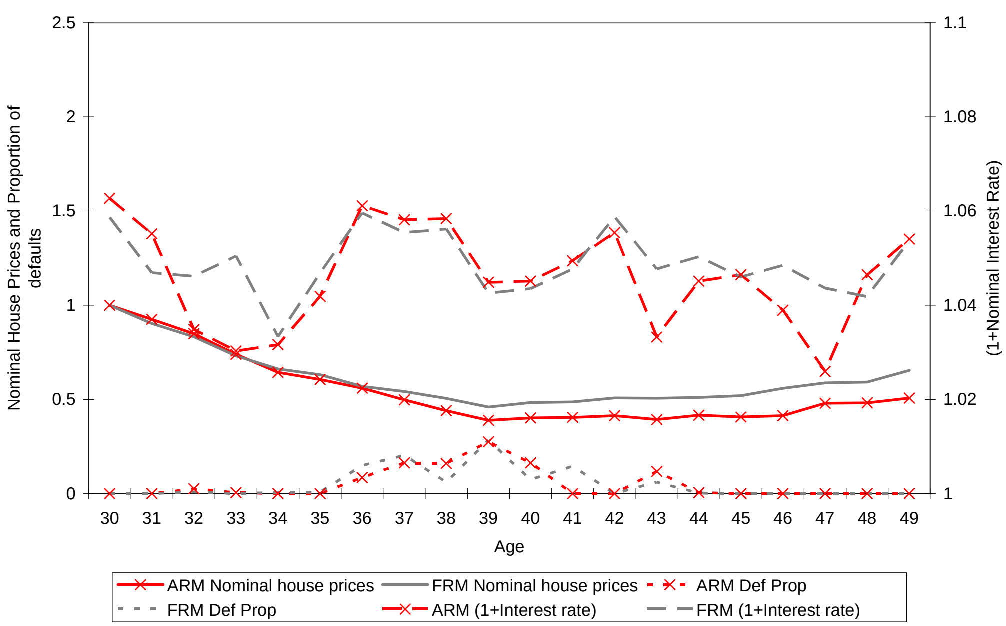

Note to Figure 7: This figure reports the number of aggregate states with a given number of mortgage defaults, by mortgage type. The data is obtained by simulating the model with the parameters shown in Table 1.

Figure 8: Average evolution across aggregate states of nominal house prices and nominal interest rates for states with a given number of individual defaults

A: One to ten individual defaults



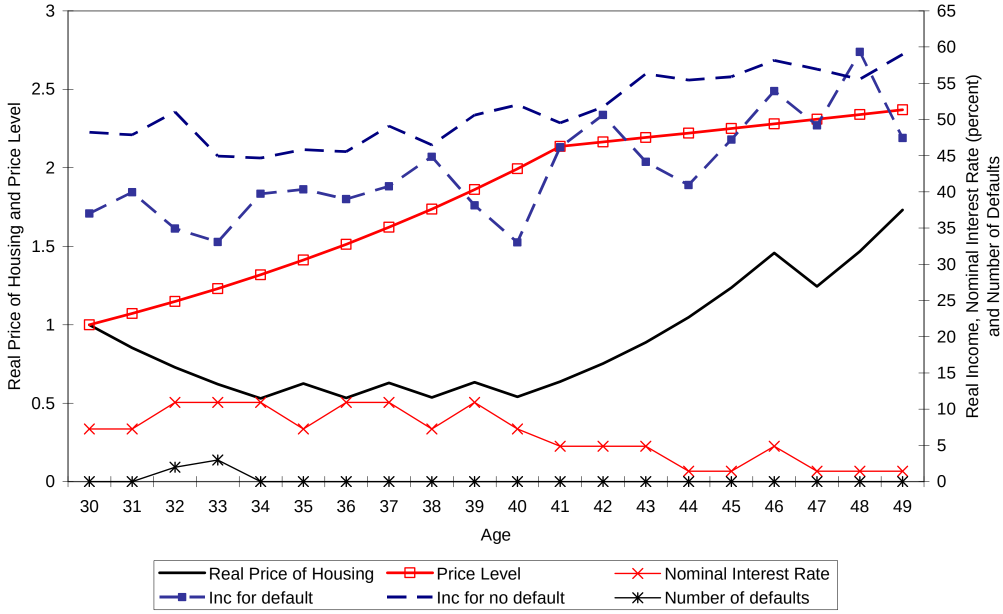

B: Forty-one to fifty individual defaults



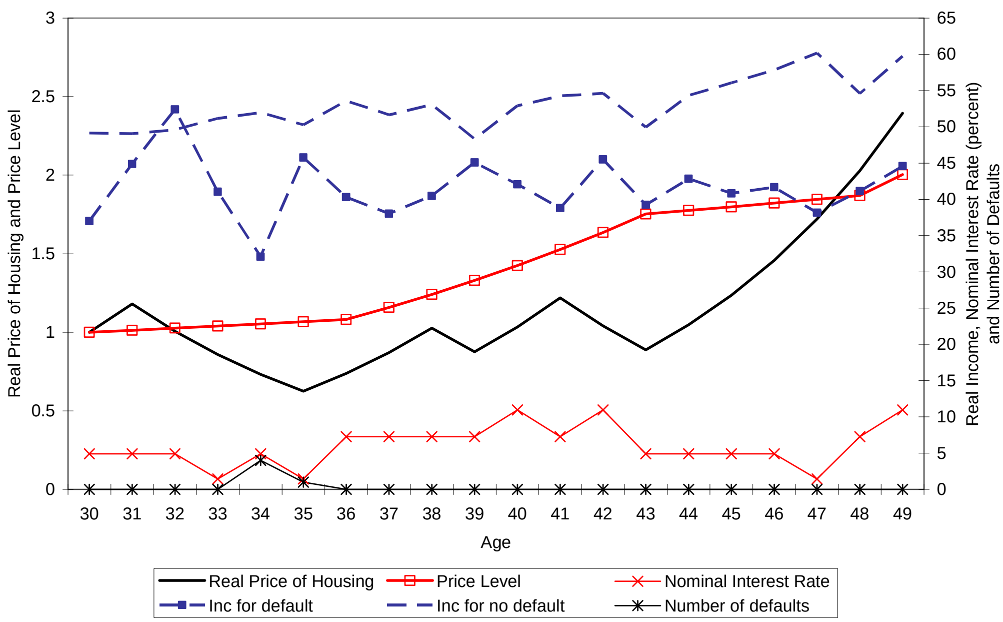

Note to Figure 8: This figure plots average nominal house prices and interest rates for aggregate states with 1 to 10 individual defaults (Panel A) and for aggregate with 41 to 50 individual defaults (Panel B), by mortgage type. The figures also show the proportion of defaults that occur at each age. The aggregate states may differ for the ARM and the FRM contracts.

Figure 9: Evolution of model variables for an aggregate state with a 10% default rate, by mortgage type

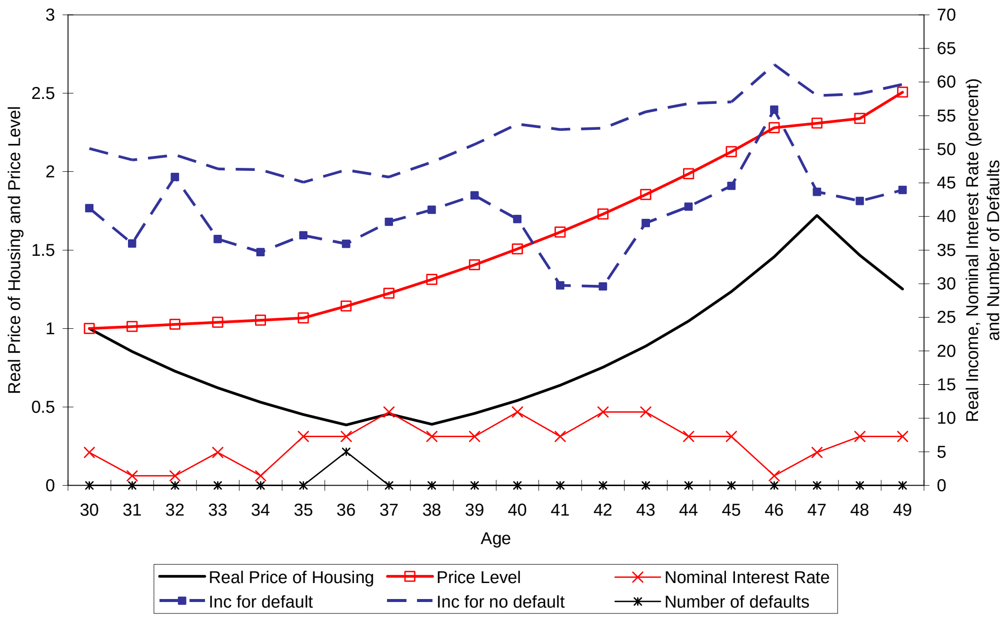
A: ARM




B: FRM



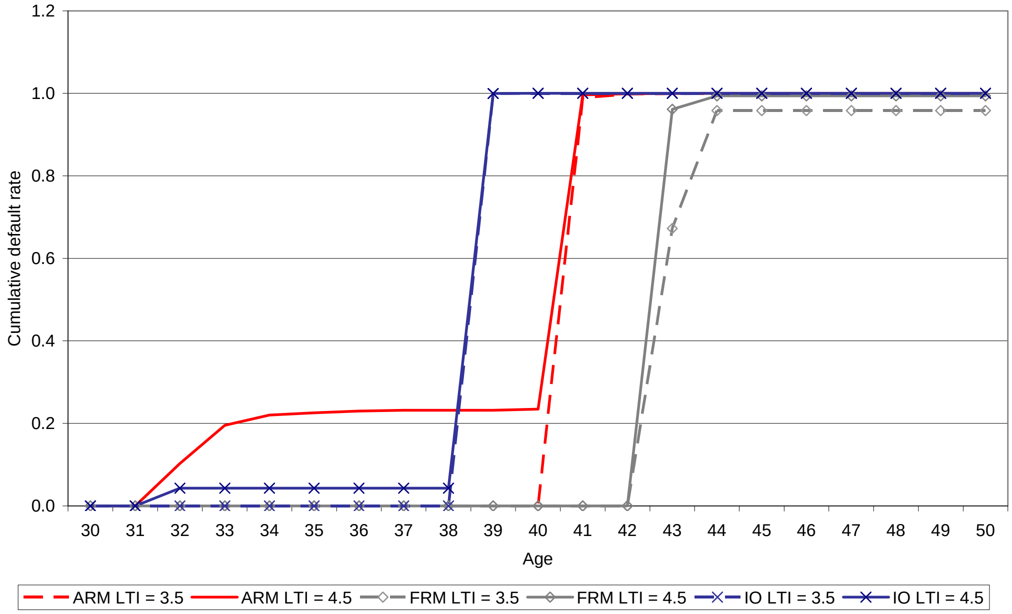
C: Interest-only



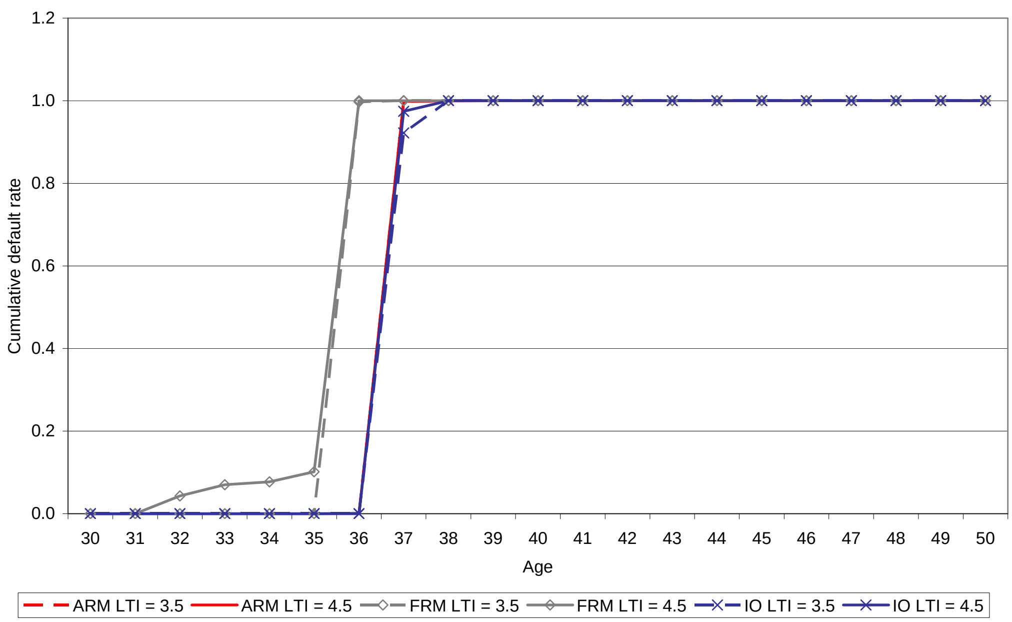

Note to Figure 9: This figure plots real house prices, the price level, and the nominal interest rate for an example of an aggregate state with a 10% default rate. The figure also plots the number of individuals who choose to default at each age, and the average income of individuals who choose to default and not default. The aggregate state with 10% default rate is not the same for the ARM, FRM, and interest-only mortgage.

Figure 10: Cumulative default rates for an aggregate state with declining house prices

A: High inflation rate and high real interest rates

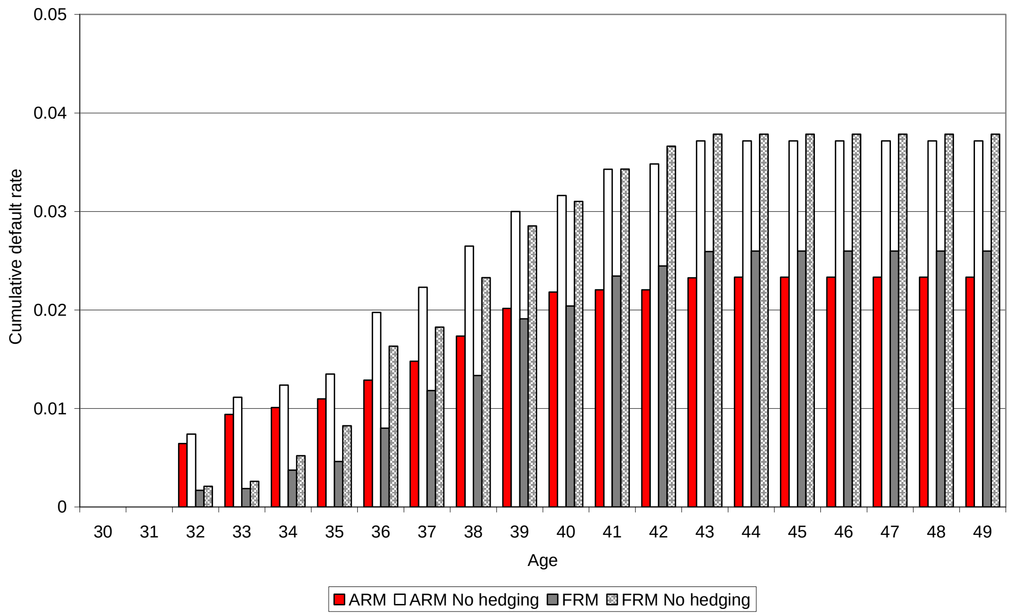



B: Low inflation rate and low real interest rates




Note to Figure 10: This figure plots cumulative default rates for an aggregate state with declining house prices, and high inflation and high real interest rates throughout (Panel A) and low inflation and low real interest rates throughout (Panel B).

Figure 11: Cumulative default rates when there are no hedging motives for terminal house prices




Note to Figure 11: This figure plots cumulative default rates for the base case and for the case when terminal nominal wealth is deflated using the price index $P_{T+1}^{Composite} = P_{T+1}$ , for the ARM and FRM contracts.